

Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.3-General/1.1.3.8-P-x-c-x^m-
a+b-xⁿ-^p

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3.422	$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3} dx$.2375
3.423	$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3} dx$.2388
3.424	$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3} dx$.2397
3.425	$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^3} dx$.2406
3.426	$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)^3} dx$.2415
3.427	$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)^3} dx$.2422
3.428	$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)^3} dx$.2429
3.429	$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)^3} dx$.2443
3.430	$\int \frac{x^3(c+dx+ex^2)}{\sqrt{a+bx^3}} dx$.2450
3.431	$\int \frac{x^2(c+dx+ex^2)}{\sqrt{a+bx^3}} dx$.2457
3.432	$\int \frac{x(c+dx+ex^2)}{\sqrt{a+bx^3}} dx$.2463
3.433	$\int \frac{c+dx+ex^2}{\sqrt{a+bx^3}} dx$.2469
3.434	$\int \frac{c+dx+ex^2}{x\sqrt{a+bx^3}} dx$.2474
3.435	$\int \frac{c+dx+ex^2}{x^2\sqrt{a+bx^3}} dx$.2480
3.436	$\int \frac{c+dx+ex^2}{x^3\sqrt{a+bx^3}} dx$.2486
3.437	$\int \frac{x^5(c+dx+ex^2)}{(a+bx^3)^{3/2}} dx$.2492

3.438	$\int \frac{x^4(c+dx+ex^2)}{(a+bx^3)^{3/2}} dx$.2499
3.439	$\int \frac{x^3(c+dx+ex^2)}{(a+bx^3)^{3/2}} dx$.2505
3.440	$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^{3/2}} dx$.2511
3.441	$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^{3/2}} dx$.2516
3.442	$\int \frac{c+dx+ex^2}{(a+bx^3)^{3/2}} dx$.2522
3.443	$\int \frac{c+dx+ex^2}{x(a+bx^3)^{3/2}} dx$.2527
3.444	$\int \frac{c+dx+ex^2}{x^2(a+bx^3)^{3/2}} dx$.2534
3.445	$\int x^3\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4) dx$.2541
3.446	$\int x^2\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4) dx$.2549
3.447	$\int x\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4) dx$.2556
3.448	$\int \sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4) dx$.2563
3.449	$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x} dx$.2570
3.450	$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^2} dx$.2577
3.451	$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^3} dx$.2584
3.452	$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^4} dx$.2591
3.453	$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^5} dx$.2598
3.454	$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^6} dx$.2605
3.455	$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^7} dx$.2612
3.456	$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^8} dx$.2619
3.457	$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^9} dx$.2627
3.458	$\int x^3(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4) dx$.2636
3.459	$\int x^2(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4) dx$.2645
3.460	$\int x(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4) dx$.2652
3.461	$\int (a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4) dx$.2659
3.462	$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x} dx$.2666
3.463	$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^2} dx$.2673

3.464	$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^3} dx$.2680
3.465	$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^4} dx$.2687
3.466	$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^5} dx$.2694
3.467	$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^6} dx$.2701
3.468	$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^7} dx$.2709
3.469	$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^8} dx$.2717
3.470	$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^9} dx$.2726
3.471	$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{10}} dx$.2734
3.472	$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{11}} dx$.2741
3.473	$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{12}} dx$.2749
3.474	$\int (c+dx+ex^2)(a+bx^3)^p dx$.2758
3.475	$\int x(c+dx+ex^2)(a+bx^3)^p dx$.2763
3.476	$\int x^2(c+dx+ex^2)(a+bx^3)^p dx$.2767
3.477	$\int (c+dx+ex^2+fx^3)(a+bx^4) dx$.2771
3.478	$\int x^3(c+dx+ex^2+fx^3)(a+bx^4) dx$.2774
3.479	$\int (c+dx+ex^2+fx^3)(a+bx^4)^2 dx$.2777
3.480	$\int x^3(c+dx+ex^2+fx^3)(a+bx^4)^2 dx$.2781
3.481	$\int (c+dx+ex^2+fx^3)(a+bx^4)^3 dx$.2785
3.482	$\int x^3(c+dx+ex^2+fx^3)(a+bx^4)^3 dx$.2789
3.483	$\int (c+dx+ex^2+fx^3)(a+bx^4)^4 dx$.2793
3.484	$\int x^3(c+dx+ex^2+fx^3)(a+bx^4)^4 dx$.2797
3.485	$\int \frac{c+dx+ex^2+fx^3}{a-bx^4} dx$.2801
3.486	$\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4} dx$.2806
3.487	$\int \frac{c+dx+ex^2+fx^3}{a+bx^4} dx$.2812
3.488	$\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4} dx$.2818
3.489	$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^2} dx$.2825
3.490	$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2} dx$.2832
3.491	$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3} dx$.2839

3.492	$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3} dx$.2846
3.493	$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4} dx$.2853
3.494	$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4} dx$.2860
3.495	$\int x^4(c+dx+ex^2+fx^3)\sqrt{a+bx^4} dx$.2867
3.496	$\int x^3(c+dx+ex^2+fx^3)\sqrt{a+bx^4} dx$.2874
3.497	$\int x^2(c+dx+ex^2+fx^3)\sqrt{a+bx^4} dx$.2880
3.498	$\int x(c+dx+ex^2+fx^3)\sqrt{a+bx^4} dx$.2886
3.499	$\int (c+dx+ex^2+fx^3)\sqrt{a+bx^4} dx$.2892
3.500	$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x} dx$.2898
3.501	$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2} dx$.2904
3.502	$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3} dx$.2911
3.503	$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4} dx$.2917
3.504	$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5} dx$.2924
3.505	$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6} dx$.2930
3.506	$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7} dx$.2937
3.507	$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8} dx$.2943
3.508	$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9} dx$.2949
3.509	$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^{10}} dx$.2956
3.510	$\int x^4(c+dx+ex^2+fx^3)(a+bx^4)^{3/2} dx$.2963
3.511	$\int x^3(c+dx+ex^2+fx^3)(a+bx^4)^{3/2} dx$.2970
3.512	$\int x^2(c+dx+ex^2+fx^3)(a+bx^4)^{3/2} dx$.2977
3.513	$\int x(c+dx+ex^2+fx^3)(a+bx^4)^{3/2} dx$.2984
3.514	$\int (c+dx+ex^2+fx^3)(a+bx^4)^{3/2} dx$.2990
3.515	$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x} dx$.2996
3.516	$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^2} dx$.3003
3.517	$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^3} dx$.3011
3.518	$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^4} dx$.3019

3.519	$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^5} dx$	3027
3.520	$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^6} dx$	3035
3.521	$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^7} dx$	3043
3.522	$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^8} dx$	3051
3.523	$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^9} dx$	3059
3.524	$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{10}} dx$	3066
3.525	$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{11}} dx$	3073
3.526	$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{12}} dx$	3080
3.527	$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{13}} dx$	3087
3.528	$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{14}} dx$	3095
3.529	$\int \frac{x^4(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}} dx$	3103
3.530	$\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}} dx$	3109
3.531	$\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}} dx$	3115
3.532	$\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}} dx$	3120
3.533	$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}} dx$	3125
3.534	$\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}} dx$	3130
3.535	$\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}} dx$	3136
3.536	$\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}} dx$	3142
3.537	$\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}} dx$	3148
3.538	$\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}} dx$	3154
3.539	$\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}} dx$	3160
3.540	$\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}} dx$	3166
3.541	$\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}} dx$	3173
3.542	$\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}} dx$	3179

3.543	$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}} dx$	3185
3.544	$\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}} dx$	3191
3.545	$\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}} dx$	3197
3.546	$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}} dx$	3202
3.547	$\int \frac{c+dx+ex^2+fx^3}{x(a+bx^4)^{3/2}} dx$	3207
3.548	$\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx^4)^{3/2}} dx$	3213
3.549	$\int \frac{c+dx+ex^2+fx^3}{x^3(a+bx^4)^{3/2}} dx$	3220
3.550	$\int \frac{c+dx+ex^2+fx^3}{x^4(a+bx^4)^{3/2}} dx$	3227
3.551	$\int (gx)^m (c+dx+ex^2+fx^3)(a+bx^4)^p dx$	3235
3.552	$\int (c+dx+ex^2+fx^3)(a+bx^4)^p dx$	3240
3.553	$\int x^3(c+dx+ex^2+fx^3)(a+bx^4)^p dx$	3245
3.554	$\int \frac{1+x+x^2+x^3+x^4}{1-x^5} dx$	3250
3.555	$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6} dx$	3253
3.556	$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6} dx$	3256
3.557	$\int \frac{81+36x^2+16x^4}{729-64x^6} dx$	3259
3.558	$\int \frac{81+54x-24x^3-16x^4}{729-64x^6} dx$	3262
3.559	$\int \frac{3-2x}{729-64x^6} dx$	3266
3.560	$\int \frac{3+2x}{729-64x^6} dx$	3270
3.561	$\int \frac{9-6x+4x^2}{729-64x^6} dx$	3274
3.562	$\int \frac{9+6x+4x^2}{729-64x^6} dx$	3278
3.563	$\int \frac{27-8x^3}{729-64x^6} dx$	3282
3.564	$\int \frac{27+36x+24x^2+8x^3}{729-64x^6} dx$	3286
3.565	$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{(729-64x^6)^2} dx$	3290
3.566	$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{(729-64x^6)^2} dx$	3295
3.567	$\int \frac{81+36x^2+16x^4}{(729-64x^6)^2} dx$	3300
3.568	$\int \frac{81+54x-24x^3-16x^4}{(729-64x^6)^2} dx$	3304

3.569	$\int \frac{3-2x}{(729-64x^6)^2} dx$	3309
3.570	$\int \frac{3+2x}{(729-64x^6)^2} dx$	3314
3.571	$\int \frac{9-6x+4x^2}{(729-64x^6)^2} dx$	3319
3.572	$\int \frac{9+6x+4x^2}{(729-64x^6)^2} dx$	3324
3.573	$\int \frac{27-8x^3}{(729-64x^6)^2} dx$	3329
3.574	$\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2} dx$	3334
3.575	$\int \frac{x(27-2x^3)}{729-64x^6} dx$	3339
3.576	$\int \frac{(cx)^m(d+ex^n+fx^{2n}+gx^{3n})}{a+bx^n} dx$	3344
3.577	$\int (c+dx^{-1+n})(a+bx^n)^3 dx$	3348
3.578	$\int (c+dx^{-1+n})(a+bx^n)^2 dx$	3353
3.579	$\int (c+dx^{-1+n})(a+bx^n) dx$	3357
3.580	$\int (c+dx^{-1+n}) dx$	3361
3.581	$\int \frac{c+dx^{-1+n}}{a+bx^n} dx$	3364
3.582	$\int \frac{c+dx^{-1+n}}{(a+bx^n)^2} dx$	3368
3.583	$\int \frac{c+dx^{-1+n}}{(a+bx^n)^3} dx$	3372
3.584	$\int \frac{(cx)^m(d+ex^n+fx^{2n}+gx^{3n})}{\sqrt{a+bx^n}} dx$	3376
3.585	$\int \frac{-ahx^{-1+\frac{n}{4}}+bfx^{-1+\frac{n}{2}}+bgx^{-1+n}+bhx^{-1+\frac{5n}{4}}}{(a+bx^n)^{3/2}} dx$	3380
3.586	$\int (cx)^m(d+ex+fx^2+gx^3)(a+bx^n)^p dx$	3384
3.587	$\int (cx)^m(a+bx^n)^p(d+ex^n+fx^{2n}+gx^{3n}) dx$	3388
3.588	$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2} dx$	3392
3.589	$\int \frac{ac+2(bc+ad)x^2+3bdx^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}} dx$	3396
3.590	$\int \frac{1+x^3}{(1-x^4)\sqrt[4]{1+x^4}} dx$	3399
3.591	$\int (a+bx^n)^{\frac{-1-n}{n}}(c+dx^n)^{\frac{-1-n}{n}}(ac-bdx^{2n}) dx$	3404
3.592	$\int (hx)^{-1-n-np}(a+bx^n)^p(c+dx^n)^p(ac-bdx^{2n}) dx$	3407
3.593	$\int (a+bx^n)^p(c+dx^n)^p\left(e+\frac{(bc+ad)e(1+n+np)x^n}{ac}+\frac{bde(1+2n+2np)x^{2n}}{ac}\right) dx$	3411
3.594	$\int (hx)^m(a+bx^n)^p(c+dx^n)^p\left(e+\frac{(bc+ad)e(1+m+n+np)x^n}{ac(1+m)}+\frac{bde(1+m+2n+2np)x^{2n}}{ac(1+m)}\right) dx$	3414

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [594]. This is test number [29].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (594)	% 0. (0)
Mathematica	% 100. (594)	% 0. (0)
Maple	% 97.14 (577)	% 2.86 (17)
Maxima	% 34.68 (206)	% 65.32 (388)
Fricas	% 56.9 (338)	% 43.1 (256)
Sympy	% 76.43 (454)	% 23.57 (140)
Giac	% 70.37 (418)	% 29.63 (176)

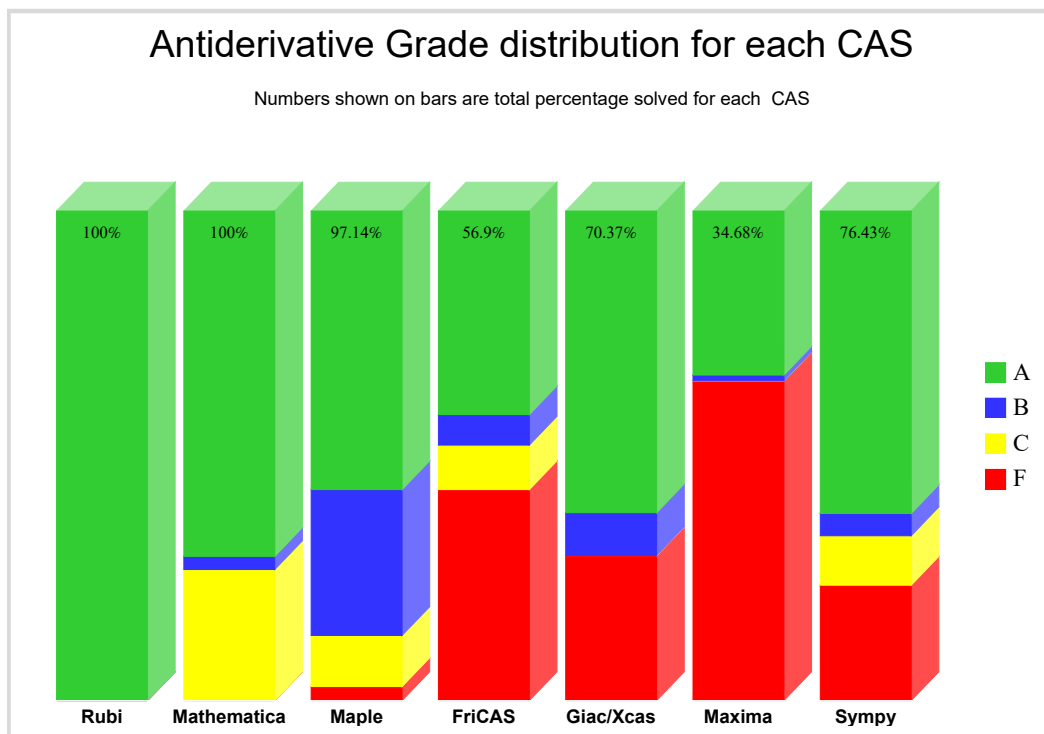
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

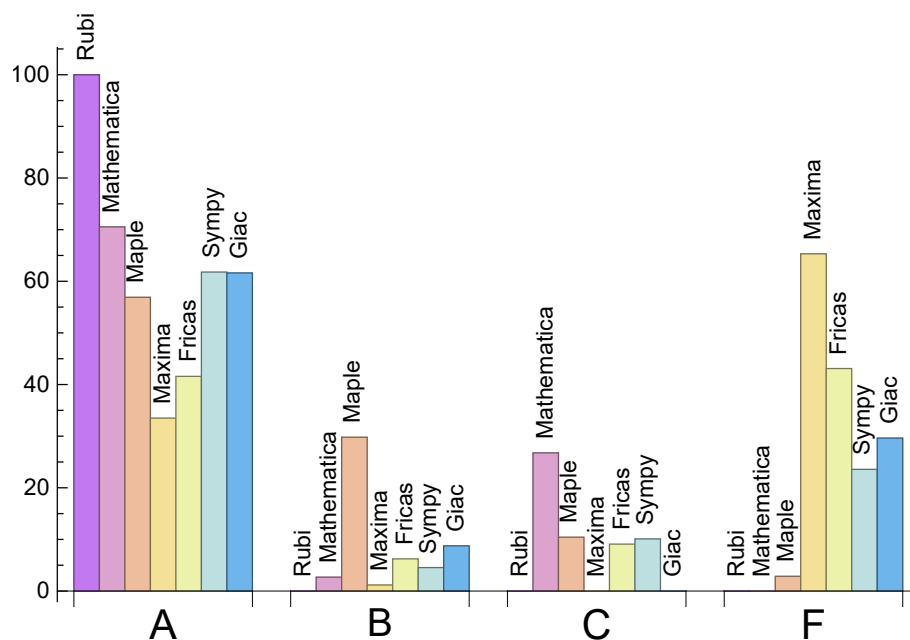
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	70.54	2.69	26.77	0.
Maple	56.9	29.8	10.44	2.86
Maxima	33.5	1.18	0.	65.32
Fricas	41.58	6.23	9.09	43.1
Sympy	61.78	4.55	10.1	23.57
Giac	61.62	8.75	0.	29.63

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.27	245.1	1.	221.5	1.
Mathematica	0.16	170.81	0.91	154.	0.95
Maple	0.01	388.7	1.44	289.	1.27
Maxima	1.12	139.43	1.33	105.5	1.28
Fricas	4.23	2531.22	11.34	402.5	2.93
Sympy	9.88	221.46	1.25	129.	0.94
Giac	1.09	304.82	1.7	266.5	1.44

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {41, 567, 590}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via `sagemath`) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in>

```
def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

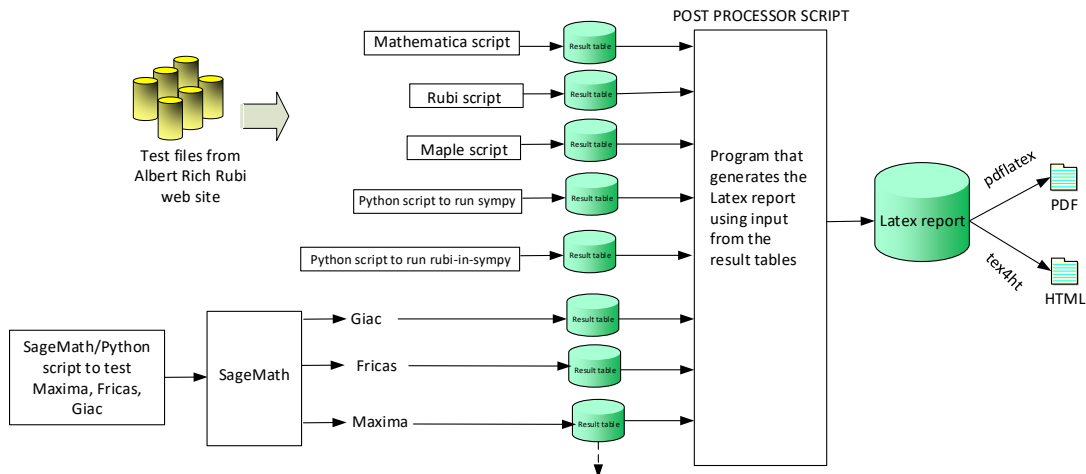
```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507,

508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 37, 38, 39, 40, 42, 43, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 70, 71, 72, 73, 74, 75, 76, 77, 78, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 551, 552, 553, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 591, 592, 593, 594 }

B grade: { 21, 32, 33, 34, 35, 36, 41, 44, 45, 46, 47, 369, 370, 371, 372, 557 }

C grade: { 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 124, 161, 210, 211, 212, 213, 214, 220, 221, 222, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 567, 590 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 22, 23, 24, 25, 26, 27, 28, 33, 36, 39, 42, 44, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 70, 71, 72, 76, 77, 78, 80, 93, 94, 97, 98, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 126, 128, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 151, 152, 153, 154, 155, 157, 159, 160, 161, 162, 163, 165, 166, 167, 170, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 189, 195, 198, 201, 202, 203, 204, 205, 206, 207, 208, 209, 211, 212, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 288, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 422, 423, 424, 425, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 441, 442, 443, 444, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 577, 578, 579, 580, 589, 593 }

B grade: { 6, 20, 21, 29, 30, 31, 32, 34, 35, 37, 38, 40, 41, 43, 45, 46, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 73, 74, 75, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 95, 96, 99, 100, 101, 102, 103, 104, 105, 106, 123, 125, 127, 129, 149, 150, 156, 158, 164, 168, 169, 171, 172, 186, 187, 188, 190, 191, 192, 193, 194, 196, 197, 199, 200, 221, 222, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 287, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 370, 371, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 426, 427, 428, 429, 440, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 557 }

C grade: { 210, 213, 214, 220, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 592, 594 }

F grade: { 474, 475, 476, 551, 552, 553, 576, 581, 582, 583, 584, 585, 586, 587, 588, 590, 591 }

2.1.4 Maxima

A grade: { 1, 2, 4, 5, 13, 14, 15, 16, 17, 18, 19, 27, 28, 39, 42, 48, 49, 50, 51, 52, 53, 54, 55, 56, 76, 77, 78, 123, 124, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 180, 181, 182, 183, 184, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 251, 252, 253, 254, 255, 256, 257, 258, 259, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 365, 366, 367, 368, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386,

387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 477, 478, 479, 480, 481, 482, 483, 484, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 589, 592, 593

B grade: { 3, 6, 161, 179, 185, 557, 594 }

C grade: { }

F grade: { 7, 8, 9, 10, 11, 12, 20, 21, 22, 23, 24, 25, 26, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 43, 44, 45, 46, 47, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 125, 126, 127, 128, 129, 130, 131, 132, 149, 150, 151, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 220, 221, 222, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 369, 370, 371, 372, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 590, 591 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 48, 49, 50, 51, 52, 53, 54, 55, 56, 76, 77, 78, 123, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 153, 159, 161, 167, 180, 181, 182, 183, 184, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 477, 478, 479, 480, 481, 482, 483, 484, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 571, 572, 573, 574, 575, 579, 580, 585, 589, 593, 594 }

B grade: { 40, 41, 44, 45, 46, 47, 152, 155, 156, 160, 163, 164, 168, 179, 185, 221, 222, 283, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 557, 569, 570, 577, 578, 591, 592 }

C grade: { 7, 8, 9, 10, 11, 12, 24, 25, 26, 57, 58, 70, 71, 72, 73, 337, 338, 339, 340, 341, 342, 343,

344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 414, 415, 416, 417, 419, 421, 422, 423, 424, 425, 428 }

F grade: { 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 74, 75, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 149, 150, 151, 154, 157, 158, 162, 165, 166, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 220, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 418, 420, 426, 427, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 576, 581, 582, 583, 584, 586, 587, 588, 590 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 22, 23, 24, 25, 26, 29, 31, 37, 38, 39, 42, 43, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 70, 71, 72, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 126, 128, 130, 132, 133, 134, 135, 136, 137, 138, 139, 141, 142, 143, 144, 145, 146, 147, 148, 152, 153, 154, 155, 156, 157, 159, 160, 162, 163, 164, 165, 167, 168, 169, 170, 180, 181, 182, 183, 184, 211, 212, 217, 218, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 251, 252, 253, 254, 255, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 294, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 344, 345, 346, 351, 352, 353, 358, 359, 360, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 477, 478, 479, 480, 481, 482, 483, 484, 489, 490, 491, 495, 496, 497, 498, 499, 510, 511, 512, 513, 514, 515, 516, 517, 518, 529, 530, 531, 532, 533, 540, 541, 542, 543, 544, 545, 546, 552, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 577, 578, 579, 580, 581 }

B grade: { 73, 74, 125, 127, 129, 131, 140, 149, 150, 158, 166, 171, 172, 175, 176, 179, 185, 403, 404, 405, 406, 407, 485, 486, 487, 488, 557 }

C grade: { 18, 19, 20, 21, 27, 28, 30, 32, 33, 34, 35, 36, 49, 123, 161, 210, 213, 214, 215, 216, 219, 365, 366, 367, 368, 369, 370, 371, 372, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 534, 535, 536, 537, 538, 539, 547, 548, 549, 550, 582 }

F grade: { 6, 40, 41, 44, 45, 46, 47, 65, 68, 69, 75, 151, 173, 174, 177, 178, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 230, 231, 232, 247, 248, 249, 250, 256, 257, 258, 259, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281,

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2.1.7 Giac

A grade: { 1, 2, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 35, 36, 39, 42, 43, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 70, 71, 72, 73, 74, 75, 76, 77, 78, 116, 118, 120, 122, 124, 126, 128, 130, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 170, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 189, 190, 191, 195, 196, 197, 201, 202, 203, 207, 208, 209, 215, 216, 217, 218, 219, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 477, 478, 479, 480, 481, 482, 483, 484, 487, 488, 489, 490, 491, 492, 493, 494, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 579, 580 }

B grade: { 3, 6, 30, 32, 33, 34, 44, 45, 115, 117, 119, 121, 123, 125, 127, 129, 131, 149, 150, 151, 161, 169, 171, 172, 173, 174, 179, 186, 187, 188, 192, 193, 194, 198, 199, 200, 204, 205, 206, 254, 369, 370, 371, 485, 486, 557, 577, 578, 591, 592, 593, 594 }

C grade: { }

F grade: { 29, 31, 37, 38, 40, 41, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 210, 211, 212, 213, 214, 220, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 576, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	53	53	104	127	223	105
normalized size	1	1.	0.74	0.74	1.44	1.76	3.1	1.46
time (sec)	N/A	0.033	0.071	0.023	0.954	1.262	7.773	1.108

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	155	194	320	433	644	320
normalized size	1	1.	0.96	1.2	1.99	2.69	4.	1.99
time (sec)	N/A	0.105	0.152	0.005	0.931	1.283	50.669	1.107

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	294	495	709	1067	1406	710
normalized size	1	1.	1.07	1.81	2.59	3.89	5.13	2.59
time (sec)	N/A	0.194	0.538	0.006	0.958	1.281	116.817	1.111

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	82	91	173	216	354	174
normalized size	1	1.	0.72	0.8	1.52	1.89	3.11	1.53
time (sec)	N/A	0.071	0.126	0.045	0.962	1.191	15.201	1.08

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	303	447	675	1000	1365	697
normalized size	1	1.	0.95	1.4	2.11	3.12	4.27	2.18
time (sec)	N/A	0.244	0.457	0.007	0.991	1.269	117.588	1.115

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	708	708	678	1417	1836	2993	0	1909
normalized size	1	1.	0.96	2.	2.59	4.23	0.	2.7
time (sec)	N/A	0.625	2.263	0.007	1.053	1.385	0.	1.181

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	124	186	0	4590	76	216
normalized size	1	1.	0.77	1.16	0.	28.51	0.47	1.34
time (sec)	N/A	0.111	0.055	0.044	0.	7.861	0.634	1.115

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	180	238	0	4961	105	252
normalized size	1	1.	0.95	1.26	0.	26.25	0.56	1.33
time (sec)	N/A	0.139	0.151	0.003	0.	7.973	1.103	1.109

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	205	272	0	5554	146	279
normalized size	1	1.	0.95	1.27	0.	25.83	0.68	1.3
time (sec)	N/A	0.187	0.154	0.004	0.	8.437	1.904	1.124

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	229	306	0	5960	185	312
normalized size	1	1.	0.95	1.27	0.	24.83	0.77	1.3
time (sec)	N/A	0.223	0.192	0.004	0.	7.257	3.36	1.103

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	125	186	0	4590	76	197
normalized size	1	1.	0.78	1.16	0.	28.51	0.47	1.22
time (sec)	N/A	0.121	0.058	0.048	0.	6.319	1.243	1.113

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	125	188	0	4563	78	155
normalized size	1	1.	0.78	1.17	0.	28.34	0.48	0.96
time (sec)	N/A	0.1	0.049	0.004	0.	6.228	1.211	1.079

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	22	58	26	22
normalized size	1	1.	1.	0.89	1.16	3.05	1.37	1.16
time (sec)	N/A	0.015	0.005	0.022	1.446	0.954	0.104	1.079

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	22	58	26	22
normalized size	1	1.	1.	0.89	1.16	3.05	1.37	1.16
time (sec)	N/A	0.013	0.004	0.001	1.432	0.892	0.097	1.159

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	53	17	23
normalized size	1	1.	1.	0.77	1.	2.41	0.77	1.05
time (sec)	N/A	0.012	0.004	0.004	1.439	0.917	0.096	1.192

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	24	54	17	26
normalized size	1	1.	1.	0.86	1.09	2.45	0.77	1.18
time (sec)	N/A	0.013	0.004	0.004	1.472	1.009	0.092	1.119

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	33	43	112	44	45
normalized size	1	1.	1.	0.8	1.05	2.73	1.07	1.1
time (sec)	N/A	0.027	0.008	0.004	1.442	0.948	0.135	1.088

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	31	35	46	72	54	38
normalized size	1	1.	1.07	1.21	1.59	2.48	1.86	1.31
time (sec)	N/A	0.022	0.01	0.067	1.459	0.903	0.157	1.119

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	34	45	72	53	35
normalized size	1	1.	1.	1.17	1.55	2.48	1.83	1.21
time (sec)	N/A	0.02	0.007	0.002	1.437	1.027	0.213	1.059

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	195	0	323	88	65
normalized size	1	1.	0.9	5.	0.	8.28	2.26	1.67
time (sec)	N/A	0.027	0.017	0.043	0.	1.091	0.431	1.117

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	129	228	0	339	105	78
normalized size	1	1.	3.15	5.56	0.	8.27	2.56	1.9
time (sec)	N/A	0.043	0.049	0.01	0.	1.083	0.477	1.098

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	90	94	0	760	26	155
normalized size	1	1.	0.76	0.8	0.	6.44	0.22	1.31
time (sec)	N/A	0.126	0.011	0.041	0.	0.99	0.135	1.079

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	90	94	0	765	22	155
normalized size	1	1.	0.76	0.8	0.	6.48	0.19	1.31
time (sec)	N/A	0.106	0.012	0.043	0.	1.009	0.156	1.116

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	124	186	0	4590	76	224
normalized size	1	1.	0.77	1.16	0.	28.51	0.47	1.39
time (sec)	N/A	0.166	0.039	0.043	0.	6.199	0.658	1.085

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	122	108	0	2425	75	162
normalized size	1	1.	0.91	0.81	0.	18.1	0.56	1.21
time (sec)	N/A	0.113	0.023	0.043	0.	6.385	0.299	1.102

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	123	111	0	2820	70	128
normalized size	1	1.	0.92	0.83	0.	21.04	0.52	0.96
time (sec)	N/A	0.089	0.03	0.041	0.	6.3	0.414	1.095

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	72	43	57	95	60	50
normalized size	1	1.	1.95	1.16	1.54	2.57	1.62	1.35
time (sec)	N/A	0.058	0.02	0.048	1.468	0.878	0.513	1.082

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	71	45	59	95	60	51
normalized size	1	1.	1.82	1.15	1.51	2.44	1.54	1.31
time (sec)	N/A	0.042	0.018	0.007	1.43	0.999	0.434	1.074

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	76	117	0	387	58	0
normalized size	1	1.	1.58	2.44	0.	8.06	1.21	0.
time (sec)	N/A	0.037	0.019	0.044	0.	1.081	0.491	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	72	84	0	122	85	150
normalized size	1	1.	1.53	1.79	0.	2.6	1.81	3.19
time (sec)	N/A	0.034	0.024	0.047	0.	1.02	0.443	1.098

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	99	122	0	466	58	0
normalized size	1	1.	1.74	2.14	0.	8.18	1.02	0.
time (sec)	N/A	0.069	0.029	0.006	0.	1.053	0.583	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	106	110	0	130	95	132
normalized size	1	1.	2.26	2.34	0.	2.77	2.02	2.81
time (sec)	N/A	0.061	0.039	0.007	0.	1.025	0.468	1.138

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	146	87	0	138	100	224
normalized size	1	1.	2.92	1.74	0.	2.76	2.	4.48
time (sec)	N/A	0.077	0.052	0.044	0.	1.05	0.536	1.126

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	150	135	0	140	110	219
normalized size	1	1.	2.83	2.55	0.	2.64	2.08	4.13
time (sec)	N/A	0.081	0.059	0.007	0.	1.033	0.554	1.122

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	149	132	0	140	109	123
normalized size	1	1.	2.76	2.44	0.	2.59	2.02	2.28
time (sec)	N/A	0.06	0.047	0.003	0.	0.983	0.523	1.093

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	147	90	0	138	102	115
normalized size	1	1.	2.77	1.7	0.	2.6	1.92	2.17
time (sec)	N/A	0.058	0.043	0.005	0.	1.044	0.623	1.083

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	95	117	0	463	70	0
normalized size	1	1.	1.56	1.92	0.	7.59	1.15	0.
time (sec)	N/A	0.041	0.017	0.005	0.	1.051	0.669	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	116	122	0	539	73	0
normalized size	1	1.	1.66	1.74	0.	7.7	1.04	0.
time (sec)	N/A	0.072	0.028	0.007	0.	1.087	0.766	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	50	32	42	107	42	43
normalized size	1	1.	1.25	0.8	1.05	2.68	1.05	1.08
time (sec)	N/A	0.03	0.01	0.005	1.457	1.001	0.123	1.204

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	122	310	0	1058	0	0
normalized size	1	1.	1.74	4.43	0.	15.11	0.	0.
time (sec)	N/A	0.067	0.046	0.004	0.	11.304	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	238	345	0	1129	0	0
normalized size	1	1.	2.7	3.92	0.	12.83	0.	0.
time (sec)	N/A	0.112	0.526	0.006	0.	9.575	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	12	12	16	22	7	18
normalized size	1	1.	1.09	1.09	1.45	2.	0.64	1.64
time (sec)	N/A	0.011	0.002	0.039	0.995	0.918	0.069	1.502

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	218	0	51	20	22
normalized size	1	1.	1.	10.38	0.	2.43	0.95	1.05
time (sec)	N/A	0.015	0.003	0.005	0.	1.018	0.312	1.472

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	247	121	0	948	0	362
normalized size	1	1.	3.48	1.7	0.	13.35	0.	5.1
time (sec)	N/A	0.094	0.267	0.005	0.	5.691	0.	1.249

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	288	345	0	972	0	351
normalized size	1	1.	3.79	4.54	0.	12.79	0.	4.62
time (sec)	N/A	0.102	0.177	0.006	0.	5.776	0.	1.102

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	253	340	0	968	0	180
normalized size	1	1.	3.24	4.36	0.	12.41	0.	2.31
time (sec)	N/A	0.108	0.251	0.005	0.	5.751	0.	1.13

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	244	124	0	952	0	169
normalized size	1	1.	3.25	1.65	0.	12.69	0.	2.25
time (sec)	N/A	0.105	0.261	0.007	0.	5.992	0.	1.089

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	31	36	35	77	24	36
normalized size	1	1.	0.97	1.12	1.09	2.41	0.75	1.12
time (sec)	N/A	0.034	0.012	0.045	1.443	1.28	0.31	1.064

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	62	87	63	158	323	70
normalized size	1	1.	1.13	1.58	1.15	2.87	5.87	1.27
time (sec)	N/A	0.057	0.033	0.043	1.454	1.344	0.891	1.071

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	18	5	9
normalized size	1	1.	1.	0.88	1.	2.25	0.62	1.12
time (sec)	N/A	0.006	0.001	0.001	0.978	1.196	0.072	1.079

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	33	43	101	5	45
normalized size	1	1.	1.	1.1	1.43	3.37	0.17	1.5
time (sec)	N/A	0.03	0.009	0.005	1.446	1.238	0.121	1.067

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	22	46	15	23
normalized size	1	1.	1.	0.94	1.22	2.56	0.83	1.28
time (sec)	N/A	0.019	0.005	0.004	1.443	1.401	0.112	1.073

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	113	98	131	232	117	131
normalized size	1	1.	1.	0.87	1.16	2.05	1.04	1.16
time (sec)	N/A	0.099	0.004	0.001	0.985	1.118	0.087	1.085

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	75	100	180	90	100
normalized size	1	1.	1.	0.85	1.14	2.05	1.02	1.14
time (sec)	N/A	0.06	0.003	0.001	0.96	1.108	0.079	1.087

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	68	120	58	68
normalized size	1	1.	1.	0.85	1.13	2.	0.97	1.13
time (sec)	N/A	0.038	0.002	0.001	0.99	1.08	0.076	1.063

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	23	8	14
normalized size	1	1.	1.	0.92	1.17	1.92	0.67	1.17
time (sec)	N/A	0.014	0.001	0.002	0.949	1.253	0.069	1.063

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	124	186	0	4590	76	216
normalized size	1	1.	0.77	1.16	0.	28.51	0.47	1.34
time (sec)	N/A	0.098	0.045	0.002	0.	8.105	0.688	1.08

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	180	238	0	4961	105	252
normalized size	1	1.	0.95	1.26	0.	26.25	0.56	1.33
time (sec)	N/A	0.127	0.148	0.004	0.	7.999	1.111	1.084

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	585	585	78	1618	0	0	265	0
normalized size	1	1.	0.13	2.77	0.	0.	0.45	0.
time (sec)	N/A	0.464	0.04	0.009	0.	0.	5.448	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	556	556	76	1546	0	0	170	0
normalized size	1	1.	0.14	2.78	0.	0.	0.31	0.
time (sec)	N/A	0.327	0.032	0.008	0.	0.	3.5	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	525	525	75	1480	0	0	163	0
normalized size	1	1.	0.14	2.82	0.	0.	0.31	0.
time (sec)	N/A	0.239	0.029	0.009	0.	0.	2.861	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	490	490	75	1536	0	0	78	0
normalized size	1	1.	0.15	3.13	0.	0.	0.16	0.
time (sec)	N/A	0.158	0.032	0.011	0.	0.	2.872	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	522	522	96	1662	0	0	163	0
normalized size	1	1.	0.18	3.18	0.	0.	0.31	0.
time (sec)	N/A	0.249	0.051	0.009	0.	0.	12.033	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	554	554	123	1782	0	0	163	0
normalized size	1	1.	0.22	3.22	0.	0.	0.29	0.
time (sec)	N/A	0.317	0.089	0.088	0.	0.	102.473	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	581	581	138	1902	0	0	0	0
normalized size	1	1.	0.24	3.27	0.	0.	0.	0.
time (sec)	N/A	0.41	0.117	0.057	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	590	590	135	1491	0	0	187	0
normalized size	1	1.	0.23	2.53	0.	0.	0.32	0.
time (sec)	N/A	0.556	0.121	0.046	0.	0.	3.088	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	594	594	130	1547	0	0	189	0
normalized size	1	1.	0.22	2.6	0.	0.	0.32	0.
time (sec)	N/A	0.433	0.121	0.005	0.	0.	18.522	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	628	628	170	1673	0	0	0	0
normalized size	1	1.	0.27	2.66	0.	0.	0.	0.
time (sec)	N/A	0.5	0.185	0.006	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	676	676	196	1793	0	0	0	0
normalized size	1	1.	0.29	2.65	0.	0.	0.	0.
time (sec)	N/A	0.675	0.253	0.006	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	200	211	0	10904	156	262
normalized size	1	1.	1.08	1.13	0.	58.62	0.84	1.41
time (sec)	N/A	0.178	0.084	0.002	0.	8.11	1.034	1.094

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	214	325	0	15101	245	327
normalized size	1	1.	0.96	1.46	0.	68.02	1.1	1.47
time (sec)	N/A	0.319	0.17	0.005	0.	14.948	2.644	1.092

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	280	277	446	0	18692	325	427
normalized size	1	0.99	0.98	1.58	0.	66.28	1.15	1.51
time (sec)	N/A	0.443	0.244	0.003	0.	57.848	5.48	1.085

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	270	269	444	0	26617	546	383
normalized size	1	0.99	0.99	1.63	0.	97.86	2.01	1.41
time (sec)	N/A	0.49	0.284	0.004	0.	15.671	8.799	1.07

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	439	837	0	0	1314	648
normalized size	1	1.	1.06	2.01	0.	0.	3.16	1.56
time (sec)	N/A	0.7	0.451	0.005	0.	0.	86.1	1.068

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	645	643	678	1339	0	0	0	1060
normalized size	1	1.	1.05	2.08	0.	0.	0.	1.64
time (sec)	N/A	1.097	0.349	0.007	0.	0.	0.	1.088

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	54	38	50	120	44	51
normalized size	1	1.	1.26	0.88	1.16	2.79	1.02	1.19
time (sec)	N/A	0.078	0.012	0.005	1.401	1.556	0.124	1.087

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	54	38	50	122	46	51
normalized size	1	1.	1.17	0.83	1.09	2.65	1.	1.11
time (sec)	N/A	0.084	0.012	0.005	1.416	1.516	0.121	1.08

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	38	50	122	48	51
normalized size	1	1.	1.	0.86	1.14	2.77	1.09	1.16
time (sec)	N/A	0.043	0.008	0.004	1.426	1.549	0.12	1.071

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	47	407	0	0	92	0
normalized size	1	1.	0.2	1.77	0.	0.	0.4	0.
time (sec)	N/A	0.06	0.02	0.013	0.	0.	1.639	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	43	368	0	0	97	0
normalized size	1	1.	0.17	1.43	0.	0.	0.38	0.
time (sec)	N/A	0.067	0.016	0.016	0.	0.	2.356	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	63	407	0	0	82	0
normalized size	1	1.	0.44	2.83	0.	0.	0.57	0.
time (sec)	N/A	0.027	0.024	0.013	0.	0.	2.362	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	67	370	0	0	99	0
normalized size	1	1.	0.5	2.74	0.	0.	0.73	0.
time (sec)	N/A	0.033	0.026	0.011	0.	0.	1.65	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	468	468	90	1003	0	0	122	0
normalized size	1	1.	0.19	2.14	0.	0.	0.26	0.
time (sec)	N/A	0.121	0.056	0.068	0.	0.	3.347	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	481	481	91	949	0	0	128	0
normalized size	1	1.	0.19	1.97	0.	0.	0.27	0.
time (sec)	N/A	0.139	0.057	0.087	0.	0.	3.507	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	92	952	0	0	112	0
normalized size	1	1.	0.34	3.51	0.	0.	0.41	0.
time (sec)	N/A	0.066	0.04	0.035	0.	0.	3.547	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	93	1012	0	0	129	0
normalized size	1	1.	0.35	3.8	0.	0.	0.48	0.
time (sec)	N/A	0.062	0.043	0.028	0.	0.	3.535	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	520	520	89	1004	0	0	124	0
normalized size	1	1.	0.17	1.93	0.	0.	0.24	0.
time (sec)	N/A	0.213	0.048	0.034	0.	0.	2.167	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	533	533	89	950	0	0	129	0
normalized size	1	1.	0.17	1.78	0.	0.	0.24	0.
time (sec)	N/A	0.172	0.044	0.036	0.	0.	2.77	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	90	953	0	0	114	0
normalized size	1	1.	0.35	3.72	0.	0.	0.45	0.
time (sec)	N/A	0.083	0.032	0.019	0.	0.	2.753	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	92	1013	0	0	131	0
normalized size	1	1.	0.37	4.04	0.	0.	0.52	0.
time (sec)	N/A	0.064	0.034	0.012	0.	0.	2.279	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	49	407	0	0	92	0
normalized size	1	1.	0.39	3.2	0.	0.	0.72	0.
time (sec)	N/A	0.019	0.021	0.013	0.	0.	1.597	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	45	368	0	0	97	0
normalized size	1	1.	0.32	2.59	0.	0.	0.68	0.
time (sec)	N/A	0.025	0.016	0.011	0.	0.	2.419	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	63	407	0	0	82	0
normalized size	1	1.	0.24	1.54	0.	0.	0.31	0.
time (sec)	N/A	0.054	0.026	0.008	0.	0.	2.439	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	67	370	0	0	97	0
normalized size	1	1.	0.27	1.5	0.	0.	0.39	0.
time (sec)	N/A	0.054	0.024	0.008	0.	0.	1.669	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	47	407	0	0	92	0
normalized size	1	1.	0.37	3.23	0.	0.	0.73	0.
time (sec)	N/A	0.025	0.015	0.01	0.	0.	2.302	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	43	368	0	0	97	0
normalized size	1	1.	0.3	2.57	0.	0.	0.68	0.
time (sec)	N/A	0.025	0.011	0.009	0.	0.	1.719	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	63	407	0	0	82	0
normalized size	1	1.	0.24	1.55	0.	0.	0.31	0.
time (sec)	N/A	0.05	0.02	0.009	0.	0.	1.689	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	67	370	0	0	97	0
normalized size	1	1.	0.27	1.49	0.	0.	0.39	0.
time (sec)	N/A	0.052	0.019	0.011	0.	0.	2.395	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	90	1003	0	0	122	0
normalized size	1	1.	0.35	3.92	0.	0.	0.48	0.
time (sec)	N/A	0.045	0.062	0.038	0.	0.	3.327	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	90	949	0	0	128	0
normalized size	1	1.	0.34	3.61	0.	0.	0.49	0.
time (sec)	N/A	0.042	0.062	0.04	0.	0.	3.556	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	497	497	91	952	0	0	112	0
normalized size	1	1.	0.18	1.92	0.	0.	0.23	0.
time (sec)	N/A	0.133	0.039	0.014	0.	0.	3.545	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	488	488	93	1012	0	0	128	0
normalized size	1	1.	0.19	2.07	0.	0.	0.26	0.
time (sec)	N/A	0.115	0.05	0.017	0.	0.	3.563	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	89	1004	0	0	124	0
normalized size	1	1.	0.37	4.17	0.	0.	0.51	0.
time (sec)	N/A	0.069	0.051	0.03	0.	0.	2.175	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	89	950	0	0	129	0
normalized size	1	1.	0.36	3.83	0.	0.	0.52	0.
time (sec)	N/A	0.061	0.044	0.033	0.	0.	2.77	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	549	549	90	953	0	0	114	0
normalized size	1	1.	0.16	1.74	0.	0.	0.21	0.
time (sec)	N/A	0.218	0.032	0.014	0.	0.	2.731	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	540	540	92	1013	0	0	129	0
normalized size	1	1.	0.17	1.88	0.	0.	0.24	0.
time (sec)	N/A	0.163	0.039	0.016	0.	0.	2.293	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	490	490	75	720	0	0	78	0
normalized size	1	1.	0.15	1.47	0.	0.	0.16	0.
time (sec)	N/A	0.148	0.029	0.003	0.	0.	1.626	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	503	503	75	681	0	0	82	0
normalized size	1	1.	0.15	1.35	0.	0.	0.16	0.
time (sec)	N/A	0.149	0.035	0.004	0.	0.	1.722	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	515	515	76	683	0	0	73	0
normalized size	1	1.	0.15	1.33	0.	0.	0.14	0.
time (sec)	N/A	0.163	0.025	0.004	0.	0.	1.701	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	508	508	78	726	0	0	83	0
normalized size	1	1.	0.15	1.43	0.	0.	0.16	0.
time (sec)	N/A	0.154	0.025	0.005	0.	0.	1.722	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	42	291	0	0	61	0
normalized size	1	1.	0.17	1.18	0.	0.	0.25	0.
time (sec)	N/A	0.081	0.01	0.004	0.	0.	1.385	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	38	267	0	0	65	0
normalized size	1	1.	0.14	0.99	0.	0.	0.24	0.
time (sec)	N/A	0.087	0.009	0.005	0.	0.	1.461	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	58	291	0	0	56	0
normalized size	1	1.	0.21	1.06	0.	0.	0.2	0.
time (sec)	N/A	0.087	0.022	0.004	0.	0.	1.43	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	62	269	0	0	66	0
normalized size	1	1.	0.24	1.03	0.	0.	0.25	0.
time (sec)	N/A	0.078	0.022	0.004	0.	0.	1.448	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	134	101	0	0	126	304
normalized size	1	1.	1.54	1.16	0.	0.	1.45	3.49
time (sec)	N/A	0.065	0.035	0.004	0.	0.	0.77	1.092

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	184	151	0	0	124	288
normalized size	1	1.	0.84	0.69	0.	0.	0.57	1.32
time (sec)	N/A	0.171	0.082	0.003	0.	0.	0.759	1.081

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	168	142	0	0	155	343
normalized size	1	1.	1.53	1.29	0.	0.	1.41	3.12
time (sec)	N/A	0.082	0.16	0.004	0.	0.	1.276	1.091

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	224	188	0	0	155	321
normalized size	1	1.	0.93	0.78	0.	0.	0.64	1.33
time (sec)	N/A	0.202	0.204	0.004	0.	0.	1.258	1.078

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	193	180	0	0	194	367
normalized size	1	1.	1.42	1.32	0.	0.	1.43	2.7
time (sec)	N/A	0.11	0.146	0.006	0.	0.	2.194	1.089

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	249	222	0	0	192	346
normalized size	1	1.	0.94	0.83	0.	0.	0.72	1.3
time (sec)	N/A	0.23	0.207	0.006	0.	0.	2.155	1.083

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	217	177	0	0	231	400
normalized size	1	1.	1.34	1.09	0.	0.	1.43	2.47
time (sec)	N/A	0.13	0.178	0.013	0.	0.	6.646	1.073

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	274	225	0	0	231	378
normalized size	1	1.	0.94	0.77	0.	0.	0.79	1.3
time (sec)	N/A	0.268	0.284	0.014	0.	0.	6.626	1.08

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	42	44	47	119	313	50
normalized size	1	1.	1.75	1.83	1.96	4.96	13.04	2.08
time (sec)	N/A	0.018	0.015	0.004	1.442	1.293	0.459	1.053

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	99	68	116	0	83	116
normalized size	1	1.	1.01	0.69	1.18	0.	0.85	1.18
time (sec)	N/A	0.067	0.067	0.004	1.536	0.	0.411	1.064

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	187	161	0	0	471	396
normalized size	1	1.	1.61	1.39	0.	0.	4.06	3.41
time (sec)	N/A	0.095	0.051	0.002	0.	0.	5.494	1.083

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	229	280	0	0	466	371
normalized size	1	1.	0.83	1.01	0.	0.	1.68	1.34
time (sec)	N/A	0.197	0.1	0.003	0.	0.	5.469	1.087

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	211	228	0	0	507	440
normalized size	1	1.	1.45	1.56	0.	0.	3.47	3.01
time (sec)	N/A	0.128	0.201	0.003	0.	0.	7.278	1.086

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	305	344	0	0	505	413
normalized size	1	1.	0.99	1.12	0.	0.	1.64	1.34
time (sec)	N/A	0.255	0.33	0.004	0.	0.	7.284	1.095

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	244	286	0	0	563	481
normalized size	1	1.	1.36	1.6	0.	0.	3.15	2.69
time (sec)	N/A	0.167	0.213	0.005	0.	0.	10.206	1.101

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	337	396	0	0	558	454
normalized size	1	1.	0.99	1.16	0.	0.	1.64	1.33
time (sec)	N/A	0.311	0.307	0.006	0.	0.	10.127	1.098

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	276	274	0	0	612	531
normalized size	1	1.	1.31	1.3	0.	0.	2.9	2.52
time (sec)	N/A	0.211	0.258	0.013	0.	0.	16.479	1.098

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	372	369	394	0	0	610	504
normalized size	1	1.	0.99	1.06	0.	0.	1.64	1.35
time (sec)	N/A	0.379	0.394	0.013	0.	0.	16.342	1.089

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	24	34	55	27	34
normalized size	1	1.	0.96	0.86	1.21	1.96	0.96	1.21
time (sec)	N/A	0.011	0.002	0.04	0.956	1.056	0.06	1.069

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	27	36	66	29	36
normalized size	1	1.	0.97	0.82	1.09	2.	0.88	1.09
time (sec)	N/A	0.014	0.001	0.039	1.055	1.065	0.061	1.044

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	68	123	58	68
normalized size	1	1.	1.	0.85	1.13	2.05	0.97	1.13
time (sec)	N/A	0.062	0.002	0.039	0.982	1.	0.066	1.055

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	36	66	31	36
normalized size	1	1.	1.	0.82	1.09	2.	0.94	1.09
time (sec)	N/A	0.013	0.001	0.041	0.986	1.053	0.059	1.082

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	68	123	60	68
normalized size	1	1.	1.	0.85	1.13	2.05	1.	1.13
time (sec)	N/A	0.027	0.002	0.041	0.975	1.056	0.069	1.047

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	54	72	134	61	72
normalized size	1	1.	1.	0.83	1.11	2.06	0.94	1.11
time (sec)	N/A	0.098	0.003	0.04	1.01	1.087	0.068	1.057

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	92	77	103	190	90	103
normalized size	1	1.	1.	0.84	1.12	2.07	0.98	1.12
time (sec)	N/A	0.053	0.003	0.041	0.981	1.123	0.07	1.059

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	33	27	20	66	29	22
normalized size	1	1.	1.94	1.59	1.18	3.88	1.71	1.29
time (sec)	N/A	0.005	0.001	0.039	1.014	1.055	0.059	1.061

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	60	51	68	123	58	68
normalized size	1	1.	1.33	1.13	1.51	2.73	1.29	1.51
time (sec)	N/A	0.019	0.002	0.038	0.993	1.044	0.065	1.074

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	65	54	72	134	60	72
normalized size	1	1.	1.3	1.08	1.44	2.68	1.2	1.44
time (sec)	N/A	0.02	0.003	0.042	1.002	1.025	0.066	1.054

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	92	77	103	190	88	103
normalized size	1	1.	1.19	1.	1.34	2.47	1.14	1.34
time (sec)	N/A	0.051	0.003	0.04	1.004	1.096	0.068	1.058

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	65	54	72	134	61	72
normalized size	1	1.	1.3	1.08	1.44	2.68	1.22	1.44
time (sec)	N/A	0.023	0.003	0.039	1.044	1.089	0.066	1.056

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	92	77	103	190	90	103
normalized size	1	1.	1.19	1.	1.34	2.47	1.17	1.34
time (sec)	N/A	0.042	0.004	0.039	0.942	0.986	0.068	1.051

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	97	80	107	201	92	107
normalized size	1	1.	1.18	0.98	1.3	2.45	1.12	1.3
time (sec)	N/A	0.084	0.004	0.04	0.942	1.071	0.071	1.06

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	124	103	138	258	121	142
normalized size	1	1.	1.14	0.94	1.27	2.37	1.11	1.3
time (sec)	N/A	0.08	0.006	0.	0.933	1.154	0.073	1.059

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	180	151	203	375	180	208
normalized size	1	1.	1.19	1.	1.34	2.48	1.19	1.38
time (sec)	N/A	0.106	0.004	0.001	0.95	0.924	0.085	1.054

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	220	248	0	0	518	452
normalized size	1	1.	1.42	1.6	0.	0.	3.34	2.92
time (sec)	N/A	0.118	0.139	0.003	0.	0.	15.127	1.084

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	253	326	0	0	583	505
normalized size	1	1.	1.35	1.73	0.	0.	3.1	2.69
time (sec)	N/A	0.155	0.219	0.006	0.	0.	66.184	1.084

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	286	280	0	0	0	555
normalized size	1	1.	1.3	1.27	0.	0.	0.	2.52
time (sec)	N/A	0.189	0.308	0.012	0.	0.	0.	1.087

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	78	114	166	882	88	131
normalized size	1	1.	0.77	1.13	1.64	8.73	0.87	1.3
time (sec)	N/A	0.096	0.029	0.043	1.477	1.666	0.359	1.111

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	16	20	54	19	20
normalized size	1	1.	1.	0.73	0.91	2.45	0.86	0.91
time (sec)	N/A	0.013	0.009	0.04	1.424	1.498	0.09	1.082

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	107	129	198	0	88	155
normalized size	1	1.	0.87	1.05	1.61	0.	0.72	1.26
time (sec)	N/A	0.103	0.049	0.003	1.486	0.	0.454	1.098

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	78	114	166	869	88	131
normalized size	1	1.	0.77	1.13	1.64	8.6	0.87	1.3
time (sec)	N/A	0.077	0.015	0.041	1.455	1.444	0.362	1.084

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	113	226	225	5063	68	177
normalized size	1	1.	0.8	1.6	1.6	35.91	0.48	1.26
time (sec)	N/A	0.098	0.051	0.002	1.456	1.62	0.365	1.108

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	99	129	198	0	85	154
normalized size	1	1.	0.8	1.05	1.61	0.	0.69	1.25
time (sec)	N/A	0.118	0.044	0.022	1.46	0.	0.459	1.11

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	129	241	252	0	292	193
normalized size	1	1.	0.79	1.48	1.55	0.	1.79	1.18
time (sec)	N/A	0.124	0.071	0.003	1.447	0.	3.075	1.113

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	31	10	15
normalized size	1	1.	1.	0.92	1.15	2.38	0.77	1.15
time (sec)	N/A	0.004	0.003	0.039	0.943	1.392	0.076	1.075

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	108	125	201	1041	51	147
normalized size	1	1.	0.95	1.1	1.76	9.13	0.45	1.29
time (sec)	N/A	0.099	0.031	0.043	1.452	1.61	0.3	1.131

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	65	28	153	86	53	126
normalized size	1	1.	1.81	0.78	4.25	2.39	1.47	3.5
time (sec)	N/A	0.031	0.03	0.042	1.47	1.476	0.306	1.088

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	128	140	231	0	199	169
normalized size	1	1.	0.94	1.03	1.7	0.	1.46	1.24
time (sec)	N/A	0.118	0.058	0.041	1.455	0.	1.189	1.109

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	108	125	205	743	70	147
normalized size	1	1.	0.95	1.1	1.8	6.52	0.61	1.29
time (sec)	N/A	0.121	0.028	0.001	1.45	1.647	0.273	1.109

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	148	237	263	5162	148	185
normalized size	1	1.	0.96	1.54	1.71	33.52	0.96	1.2
time (sec)	N/A	0.117	0.092	0.041	1.458	1.951	1.078	1.105

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	125	140	235	0	189	167
normalized size	1	1.	0.92	1.03	1.73	0.	1.39	1.23
time (sec)	N/A	0.14	0.063	0.046	1.454	0.	1.171	1.103

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	164	252	279	0	580	201
normalized size	1	1.	0.93	1.43	1.59	0.	3.3	1.14
time (sec)	N/A	0.144	0.105	0.003	1.481	0.	6.537	1.122

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	18	5	9
normalized size	1	1.	1.	0.88	1.	2.25	0.62	1.12
time (sec)	N/A	0.008	0.001	0.	0.954	1.487	0.056	1.05

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	50	102	103	440	73	95
normalized size	1	1.	0.94	1.92	1.94	8.3	1.38	1.79
time (sec)	N/A	0.042	0.035	0.003	1.437	1.561	0.324	1.062

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	203	171	0	0	187	392
normalized size	1	1.	1.64	1.38	0.	0.	1.51	3.16
time (sec)	N/A	0.087	0.05	0.004	0.	0.	1.029	1.09

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	283	286	0	0	187	365
normalized size	1	1.	1.02	1.03	0.	0.	0.68	1.32
time (sec)	N/A	0.196	0.188	0.003	0.	0.	0.997	1.063

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	249	244	0	0	2394	539
normalized size	1	1.	1.68	1.65	0.	0.	16.18	3.64
time (sec)	N/A	0.203	0.081	0.005	0.	0.	156.777	1.088

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	221	289	0	0	1406	528
normalized size	1	1.	1.28	1.68	0.	0.	8.17	3.07
time (sec)	N/A	0.165	0.314	0.008	0.	0.	169.346	1.085

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	263	328	0	0	0	595
normalized size	1	1.	1.19	1.48	0.	0.	0.	2.69
time (sec)	N/A	0.263	0.492	0.011	0.	0.	0.	1.082

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	313	368	0	0	0	662
normalized size	1	1.	1.18	1.38	0.	0.	0.	2.49
time (sec)	N/A	0.32	0.324	0.012	0.	0.	0.	1.099

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	311	429	0	0	2392	459
normalized size	1	1.	0.97	1.34	0.	0.	7.5	1.44
time (sec)	N/A	0.351	0.318	0.005	0.	0.	158.354	1.104

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	319	482	0	0	1406	493
normalized size	1	1.	0.94	1.41	0.	0.	4.12	1.45
time (sec)	N/A	0.305	0.196	0.009	0.	0.	170.756	1.079

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	394	366	519	0	0	0	562
normalized size	1	1.	0.93	1.32	0.	0.	0.	1.43
time (sec)	N/A	0.439	0.278	0.01	0.	0.	0.	1.098

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	437	437	411	560	0	0	0	629
normalized size	1	1.	0.94	1.28	0.	0.	0.	1.44
time (sec)	N/A	0.531	0.366	0.014	0.	0.	0.	1.093

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	9	8	20	41	15	20
normalized size	1	1.	0.82	0.73	1.82	3.73	1.36	1.82
time (sec)	N/A	0.013	0.001	0.002	0.964	1.728	0.076	1.103

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	14	8	16	26	8	16
normalized size	1	1.	1.27	0.73	1.45	2.36	0.73	1.45
time (sec)	N/A	0.012	0.001	0.001	0.94	1.682	0.066	1.065

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	9	19	5	9
normalized size	1	1.	1.	0.89	1.	2.11	0.56	1.
time (sec)	N/A	0.009	0.	0.001	0.936	1.671	0.057	1.05

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	18	5	9
normalized size	1	1.	1.	0.88	1.	2.25	0.62	1.12
time (sec)	N/A	0.007	0.001	0.	0.943	1.702	0.057	1.053

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	9	16	5	9
normalized size	1	1.	1.	1.14	1.29	2.29	0.71	1.29
time (sec)	N/A	0.016	0.001	0.001	0.96	1.664	0.08	1.059

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	9	8	16	28	10	9
normalized size	1	1.	0.82	0.73	1.45	2.55	0.91	0.82
time (sec)	N/A	0.018	0.001	0.002	0.937	1.661	0.097	1.068

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	9	8	23	41	17	9
normalized size	1	1.	0.82	0.73	2.09	3.73	1.55	0.82
time (sec)	N/A	0.019	0.001	0.	0.942	1.695	0.103	1.053

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	256	296	0	0	0	539
normalized size	1	1.	1.55	1.79	0.	0.	0.	3.27
time (sec)	N/A	0.261	0.257	0.045	0.	0.	0.	1.084

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	301	367	0	0	0	807
normalized size	1	1.	1.6	1.95	0.	0.	0.	4.29
time (sec)	N/A	0.327	0.376	0.045	0.	0.	0.	1.09

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	318	393	0	0	0	828
normalized size	1	1.	1.55	1.92	0.	0.	0.	4.04
time (sec)	N/A	0.313	0.351	0.045	0.	0.	0.	1.102

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	342	462	0	0	0	506
normalized size	1	1.	1.01	1.37	0.	0.	0.	1.5
time (sec)	N/A	0.399	0.325	0.003	0.	0.	0.	1.083

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	384	384	427	603	0	0	0	759
normalized size	1	1.	1.11	1.57	0.	0.	0.	1.98
time (sec)	N/A	0.565	0.307	0.004	0.	0.	0.	1.097

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	402	402	445	627	0	0	0	780
normalized size	1	1.	1.11	1.56	0.	0.	0.	1.94
time (sec)	N/A	0.567	0.309	0.006	0.	0.	0.	1.097

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	257	340	0	0	0	574
normalized size	1	1.	1.4	1.85	0.	0.	0.	3.12
time (sec)	N/A	0.204	0.163	0.008	0.	0.	0.	1.076

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	302	409	0	0	0	848
normalized size	1	1.	1.49	2.01	0.	0.	0.	4.18
time (sec)	N/A	0.274	0.222	0.009	0.	0.	0.	1.093

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	338	431	0	0	0	884
normalized size	1	1.	1.5	1.92	0.	0.	0.	3.93
time (sec)	N/A	0.31	0.24	0.011	0.	0.	0.	1.12

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	359	515	0	0	0	537
normalized size	1	1.	1.02	1.46	0.	0.	0.	1.52
time (sec)	N/A	0.34	0.242	0.009	0.	0.	0.	1.107

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	395	415	654	0	0	0	795
normalized size	1	1.	1.05	1.66	0.	0.	0.	2.01
time (sec)	N/A	0.492	0.344	0.01	0.	0.	0.	1.096

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	417	417	460	675	0	0	0	833
normalized size	1	1.	1.1	1.62	0.	0.	0.	2.
time (sec)	N/A	0.536	0.328	0.011	0.	0.	0.	1.112

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	309	389	0	0	0	656
normalized size	1	1.	1.28	1.61	0.	0.	0.	2.72
time (sec)	N/A	0.339	0.287	0.011	0.	0.	0.	1.129

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	359	472	0	0	0	942
normalized size	1	1.	1.34	1.76	0.	0.	0.	3.51
time (sec)	N/A	0.435	0.352	0.011	0.	0.	0.	1.12

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	380	488	0	0	0	986
normalized size	1	1.	1.33	1.71	0.	0.	0.	3.46
time (sec)	N/A	0.391	0.277	0.012	0.	0.	0.	1.124

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	413	413	411	561	0	0	0	620
normalized size	1	1.	1.	1.36	0.	0.	0.	1.5
time (sec)	N/A	0.486	0.364	0.012	0.	0.	0.	1.108

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	463	473	716	0	0	0	892
normalized size	1	1.	1.02	1.55	0.	0.	0.	1.93
time (sec)	N/A	0.686	0.531	0.011	0.	0.	0.	1.124

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	480	480	500	731	0	0	0	936
normalized size	1	1.	1.04	1.52	0.	0.	0.	1.95
time (sec)	N/A	0.666	0.429	0.013	0.	0.	0.	1.117

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	360	434	0	0	0	738
normalized size	1	1.	1.23	1.48	0.	0.	0.	2.52
time (sec)	N/A	0.431	0.395	0.013	0.	0.	0.	1.084

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	422	522	0	0	0	1044
normalized size	1	1.	1.27	1.58	0.	0.	0.	3.15
time (sec)	N/A	0.567	0.368	0.015	0.	0.	0.	1.103

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	439	538	0	0	0	1087
normalized size	1	1.	1.26	1.54	0.	0.	0.	3.11
time (sec)	N/A	0.524	0.368	0.015	0.	0.	0.	1.112

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	462	462	461	607	0	0	0	703
normalized size	1	1.	1.	1.31	0.	0.	0.	1.52
time (sec)	N/A	0.619	0.462	0.014	0.	0.	0.	1.093

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	516	516	530	767	0	0	0	992
normalized size	1	1.	1.03	1.49	0.	0.	0.	1.92
time (sec)	N/A	0.85	0.703	0.014	0.	0.	0.	1.107

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	534	534	555	783	0	0	0	1035
normalized size	1	1.	1.04	1.47	0.	0.	0.	1.94
time (sec)	N/A	0.824	0.553	0.014	0.	0.	0.	1.109

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	79	96	0	0	61	0
normalized size	1	1.	0.65	0.79	0.	0.	0.5	0.
time (sec)	N/A	0.068	0.047	0.073	0.	0.	1.84	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	81	90	0	0	97	0
normalized size	1	1.	0.93	1.03	0.	0.	1.11	0.
time (sec)	N/A	0.062	0.047	0.015	0.	0.	1.962	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	83	95	0	0	92	0
normalized size	1	1.	0.93	1.07	0.	0.	1.03	0.
time (sec)	N/A	0.06	0.042	0.019	0.	0.	1.965	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	85	101	0	0	66	0
normalized size	1	1.	0.67	0.8	0.	0.	0.52	0.
time (sec)	N/A	0.072	0.045	0.018	0.	0.	1.965	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	131	193	0	0	102	0
normalized size	1	1.	0.51	0.75	0.	0.	0.4	0.
time (sec)	N/A	0.124	0.103	0.005	0.	0.	2.157	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	28	80	16
normalized size	1	1.	1.	0.93	1.14	2.	5.71	1.14
time (sec)	N/A	0.006	0.009	0.043	1.059	1.387	7.557	1.078

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	27	24	34	74	104	43
normalized size	1	1.	0.93	0.83	1.17	2.55	3.59	1.48
time (sec)	N/A	0.023	0.037	0.042	1.073	1.35	11.095	1.074

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	27	24	31	69	109	30
normalized size	1	1.	1.08	0.96	1.24	2.76	4.36	1.2
time (sec)	N/A	0.028	0.037	0.043	1.082	1.505	10.461	1.085

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	35	59	93	133	42
normalized size	1	1.	1.	0.92	1.55	2.45	3.5	1.11
time (sec)	N/A	0.03	0.042	0.042	1.078	1.302	13.776	1.097

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	24	58	14
normalized size	1	1.	1.	0.92	1.17	2.	4.83	1.17
time (sec)	N/A	0.003	0.005	0.004	1.544	1.309	2.615	1.067

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	385	281	516	0	0	260	0
normalized size	1	1.	0.73	1.34	0.	0.	0.68	0.
time (sec)	N/A	0.424	0.224	0.017	0.	0.	5.734	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	51	173	0	2955	36	136
normalized size	1	1.	0.47	1.59	0.	27.11	0.33	1.25
time (sec)	N/A	0.064	0.01	0.019	0.	8.737	0.372	1.107

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	47	169	0	2493	36	136
normalized size	1	1.	0.43	1.55	0.	22.87	0.33	1.25
time (sec)	N/A	0.041	0.009	0.017	0.	8.558	0.373	1.093

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	187	266	282	429	192	332
normalized size	1	1.	0.9	1.28	1.36	2.06	0.92	1.6
time (sec)	N/A	0.316	0.09	0.005	0.956	1.273	1.167	1.057

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	154	218	228	351	155	266
normalized size	1	1.	0.91	1.28	1.34	2.06	0.91	1.56
time (sec)	N/A	0.242	0.066	0.004	0.955	1.234	1.137	1.064

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	119	170	174	267	117	200
normalized size	1	1.	0.9	1.29	1.32	2.02	0.89	1.52
time (sec)	N/A	0.183	0.053	0.003	0.953	1.273	1.082	1.069

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	88	124	123	190	83	136
normalized size	1	1.	0.92	1.29	1.28	1.98	0.86	1.42
time (sec)	N/A	0.14	0.043	0.003	0.949	1.195	1.007	1.069

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	75	97	104	171	68	107
normalized size	1	1.	0.94	1.21	1.3	2.14	0.85	1.34
time (sec)	N/A	0.12	0.031	0.005	0.958	1.588	4.913	1.058

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	77	94	104	180	70	128
normalized size	1	1.	0.95	1.16	1.28	2.22	0.86	1.58
time (sec)	N/A	0.116	0.043	0.007	0.966	1.503	11.439	1.059

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	88	116	126	213	85	170
normalized size	1	1.	0.93	1.22	1.33	2.24	0.89	1.79
time (sec)	N/A	0.129	0.069	0.007	0.948	1.527	51.496	1.055

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	128	161	169	269	0	248
normalized size	1	1.	1.	1.26	1.32	2.1	0.	1.94
time (sec)	N/A	0.162	0.064	0.007	0.953	1.471	0.	1.067

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	164	210	224	355	0	317
normalized size	1	1.	1.	1.28	1.37	2.16	0.	1.93
time (sec)	N/A	0.181	0.062	0.009	0.955	1.513	0.	1.068

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	194	260	281	441	0	387
normalized size	1	1.	0.95	1.27	1.37	2.15	0.	1.89
time (sec)	N/A	0.209	0.144	0.01	0.959	1.667	0.	1.066

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	351	592	0	767	450	613
normalized size	1	1.	1.01	1.7	0.	2.2	1.29	1.76
time (sec)	N/A	0.333	0.079	0.003	0.	1.398	1.874	1.062

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	311	554	0	726	496	595
normalized size	1	1.	0.98	1.75	0.	2.3	1.57	1.88
time (sec)	N/A	0.306	0.098	0.003	0.	1.444	1.451	1.082

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	306	544	0	679	411	541
normalized size	1	1.	0.98	1.74	0.	2.18	1.32	1.73
time (sec)	N/A	0.298	0.093	0.004	0.	1.376	1.822	1.072

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	266	502	0	635	459	521
normalized size	1	1.	0.95	1.8	0.	2.28	1.65	1.87
time (sec)	N/A	0.273	0.092	0.004	0.	1.407	1.383	1.075

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	264	492	0	566	371	467
normalized size	1	1.	0.96	1.8	0.	2.07	1.35	1.7
time (sec)	N/A	0.267	0.091	0.003	0.	1.297	1.719	1.089

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	231	450	0	1289	422	466
normalized size	1	1.	0.94	1.84	0.	5.26	1.72	1.9
time (sec)	N/A	0.216	0.16	0.003	0.	1.519	1.473	1.084

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	229	442	0	1350	340	414
normalized size	1	1.	0.95	1.84	0.	5.62	1.42	1.72
time (sec)	N/A	0.153	0.147	0.004	0.	1.361	1.89	1.075

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	224	419	0	1260	406	428
normalized size	1	1.	0.99	1.85	0.	5.55	1.79	1.89
time (sec)	N/A	0.193	0.128	0.004	0.	1.444	2.515	1.095

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	218	414	0	1280	326	378
normalized size	1	1.	0.97	1.85	0.	5.71	1.46	1.69
time (sec)	N/A	0.17	0.117	0.005	0.	1.537	3.058	1.076

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	220	412	0	1251	411	417
normalized size	1	1.	0.97	1.81	0.	5.51	1.81	1.84
time (sec)	N/A	0.186	0.111	0.007	0.	1.386	8.047	1.081

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	220	410	0	1312	328	370
normalized size	1	1.	0.98	1.82	0.	5.83	1.46	1.64
time (sec)	N/A	0.17	0.082	0.005	0.	1.308	9.841	1.08

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	231	440	0	1359	432	444
normalized size	1	1.	0.95	1.82	0.	5.62	1.79	1.83
time (sec)	N/A	0.188	0.111	0.007	0.	1.435	30.419	1.075

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	231	441	0	1353	348	401
normalized size	1	1.	0.95	1.81	0.	5.55	1.43	1.64
time (sec)	N/A	0.174	0.112	0.008	0.	1.378	52.712	1.074

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	266	491	0	597	473	508
normalized size	1	1.	0.96	1.77	0.	2.16	1.71	1.83
time (sec)	N/A	0.222	0.112	0.009	0.	1.41	142.622	1.085

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	266	493	0	667	0	456
normalized size	1	1.	0.95	1.76	0.	2.38	0.	1.63
time (sec)	N/A	0.199	0.115	0.008	0.	1.396	0.	1.072

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	308	546	0	711	0	566
normalized size	1	1.	0.98	1.74	0.	2.27	0.	1.81
time (sec)	N/A	0.238	0.099	0.009	0.	1.343	0.	1.078

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	311	548	0	759	0	531
normalized size	1	1.	0.99	1.74	0.	2.41	0.	1.69
time (sec)	N/A	0.229	0.097	0.009	0.	1.384	0.	1.077

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	346	600	0	798	0	640
normalized size	1	1.	0.99	1.71	0.	2.27	0.	1.82
time (sec)	N/A	0.258	0.11	0.01	0.	1.272	0.	1.104

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	205	288	300	656	224	405
normalized size	1	1.	0.93	1.31	1.36	2.98	1.02	1.84
time (sec)	N/A	0.341	0.139	0.012	0.958	1.244	11.509	1.066

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	167	240	243	555	180	335
normalized size	1	1.	0.93	1.33	1.35	3.08	1.	1.86
time (sec)	N/A	0.265	0.111	0.013	0.965	1.259	11.093	1.058

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	129	192	186	432	138	293
normalized size	1	1.	0.92	1.37	1.33	3.09	0.99	2.09
time (sec)	N/A	0.199	0.098	0.01	0.959	1.305	10.644	1.064

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	93	142	132	305	97	278
normalized size	1	1.	0.9	1.38	1.28	2.96	0.94	2.7
time (sec)	N/A	0.145	0.058	0.01	0.959	1.135	9.22	1.058

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	95	125	135	293	95	169
normalized size	1	1.	0.95	1.25	1.35	2.93	0.95	1.69
time (sec)	N/A	0.125	0.122	0.011	0.956	1.435	33.825	1.073

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	97	132	157	340	0	177
normalized size	1	1.	0.89	1.21	1.44	3.12	0.	1.62
time (sec)	N/A	0.142	0.077	0.014	0.955	1.494	0.	1.059

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	118	167	186	431	0	271
normalized size	1	1.	0.91	1.28	1.43	3.32	0.	2.08
time (sec)	N/A	0.154	0.115	0.015	0.979	1.428	0.	1.07

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	160	229	244	549	0	371
normalized size	1	1.	0.91	1.31	1.39	3.14	0.	2.12
time (sec)	N/A	0.203	0.105	0.018	1.049	1.62	0.	1.067

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	198	282	305	670	0	447
normalized size	1	1.	0.93	1.32	1.43	3.13	0.	2.09
time (sec)	N/A	0.234	0.203	0.02	1.017	1.608	0.	1.074

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	364	622	0	1145	490	609
normalized size	1	1.	0.99	1.69	0.	3.1	1.33	1.65
time (sec)	N/A	0.467	0.353	0.01	0.	1.413	10.508	1.078

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	335	319	584	0	1057	530	597
normalized size	1	1.	0.95	1.74	0.	3.16	1.58	1.78
time (sec)	N/A	0.705	0.166	0.01	0.	1.298	37.454	1.087

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	315	567	0	992	440	532
normalized size	1	1.	0.96	1.73	0.	3.02	1.34	1.62
time (sec)	N/A	0.369	0.248	0.01	0.	1.366	10.915	1.08

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	282	529	0	2094	484	537
normalized size	1	1.	0.95	1.78	0.	7.03	1.62	1.8
time (sec)	N/A	0.463	0.157	0.009	0.	1.705	35.782	1.084

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	277	514	0	2125	398	471
normalized size	1	1.	0.96	1.78	0.	7.38	1.38	1.64
time (sec)	N/A	0.326	0.157	0.01	0.	1.447	10.133	1.1

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	255	495	0	1935	461	494
normalized size	1	1.	0.94	1.83	0.	7.14	1.7	1.82
time (sec)	N/A	0.289	0.153	0.009	0.	1.482	16.851	1.076

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	251	482	0	1917	376	433
normalized size	1	1.	0.95	1.83	0.	7.26	1.42	1.64
time (sec)	N/A	0.264	0.145	0.009	0.	1.524	6.359	1.072

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	255	474	0	1877	457	477
normalized size	1	1.	0.96	1.79	0.	7.08	1.72	1.8
time (sec)	N/A	0.253	0.153	0.012	0.	1.48	26.608	1.076

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	250	463	0	1952	381	417
normalized size	1	1.	0.96	1.78	0.	7.51	1.47	1.6
time (sec)	N/A	0.246	0.152	0.011	0.	1.463	58.686	1.081

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	255	486	0	1968	473	483
normalized size	1	1.	0.95	1.81	0.	7.32	1.76	1.8
time (sec)	N/A	0.288	0.168	0.013	0.	1.525	138.391	1.082

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	253	477	0	1991	0	429
normalized size	1	1.	0.94	1.77	0.	7.37	0.	1.59
time (sec)	N/A	0.272	0.162	0.011	0.	1.559	0.	1.081

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	281	529	0	2159	0	522
normalized size	1	1.	0.95	1.78	0.	7.27	0.	1.76
time (sec)	N/A	0.384	0.179	0.015	0.	1.465	0.	1.117

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	280	520	0	2182	0	468
normalized size	1	1.	0.94	1.75	0.	7.35	0.	1.58
time (sec)	N/A	0.37	0.178	0.014	0.	1.555	0.	1.087

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	319	575	0	1034	0	590
normalized size	1	1.	0.96	1.72	0.	3.1	0.	1.77
time (sec)	N/A	0.457	0.181	0.016	0.	1.372	0.	1.101

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	335	317	566	0	1099	0	528
normalized size	1	1.	0.95	1.69	0.	3.28	0.	1.58
time (sec)	N/A	0.434	0.187	0.017	0.	1.384	0.	1.086

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	370	631	0	1188	0	651
normalized size	1	1.	0.99	1.68	0.	3.17	0.	1.74
time (sec)	N/A	0.534	0.36	0.016	0.	1.361	0.	1.081

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	246	361	371	883	0	471
normalized size	1	1.	0.92	1.36	1.39	3.32	0.	1.77
time (sec)	N/A	0.436	0.143	0.015	0.954	1.234	0.	1.078

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	208	313	315	765	0	402
normalized size	1	1.	0.92	1.38	1.39	3.38	0.	1.78
time (sec)	N/A	0.331	0.12	0.013	0.963	1.346	0.	1.097

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	170	266	258	635	0	319
normalized size	1	1.	0.91	1.43	1.39	3.41	0.	1.72
time (sec)	N/A	0.269	0.099	0.012	0.95	1.294	0.	1.079

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	145	213	198	470	0	197
normalized size	1	1.	0.99	1.46	1.36	3.22	0.	1.35
time (sec)	N/A	0.201	0.074	0.013	0.953	1.26	0.	1.088

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	105	156	147	329	0	135
normalized size	1	1.	0.96	1.43	1.35	3.02	0.	1.24
time (sec)	N/A	0.152	0.054	0.01	0.951	1.165	0.	1.078

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	104	147	174	374	0	173
normalized size	1	1.	0.91	1.29	1.53	3.28	0.	1.52
time (sec)	N/A	0.155	0.106	0.015	0.953	1.368	0.	1.07

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	121	163	194	497	0	234
normalized size	1	1.	0.9	1.22	1.45	3.71	0.	1.75
time (sec)	N/A	0.171	0.091	0.014	0.962	1.4	0.	1.097

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	149	213	246	652	0	255
normalized size	1	1.	0.91	1.31	1.51	4.	0.	1.56
time (sec)	N/A	0.2	0.106	0.017	0.98	1.31	0.	1.073

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	200	293	313	837	0	437
normalized size	1	1.	0.92	1.34	1.44	3.84	0.	2.
time (sec)	N/A	0.264	0.137	0.02	0.981	1.443	0.	1.101

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	238	349	378	984	0	513
normalized size	1	1.	0.92	1.35	1.47	3.81	0.	1.99
time (sec)	N/A	0.304	0.18	0.019	0.988	1.64	0.	1.077

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	411	706	0	1589	0	675
normalized size	1	1.	0.99	1.7	0.	3.82	0.	1.62
time (sec)	N/A	0.742	0.482	0.013	0.	1.326	0.	1.091

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	384	384	380	668	0	1496	0	663
normalized size	1	1.	0.99	1.74	0.	3.9	0.	1.73
time (sec)	N/A	1.05	0.402	0.013	0.	1.352	0.	1.092

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	362	651	0	1424	0	598
normalized size	1	1.	0.97	1.74	0.	3.8	0.	1.59
time (sec)	N/A	0.606	0.374	0.012	0.	1.443	0.	1.088

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	329	611	0	2915	0	601
normalized size	1	1.	0.95	1.77	0.	8.45	0.	1.74
time (sec)	N/A	0.763	0.224	0.013	0.	1.481	0.	1.092

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	323	596	0	2986	0	539
normalized size	1	1.	0.96	1.77	0.	8.89	0.	1.6
time (sec)	N/A	0.509	0.313	0.012	0.	1.461	0.	1.108

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	300	574	0	2723	0	558
normalized size	1	1.	0.95	1.82	0.	8.62	0.	1.77
time (sec)	N/A	0.505	0.206	0.011	0.	1.535	0.	1.093

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	294	561	0	2716	0	495
normalized size	1	1.	0.96	1.83	0.	8.85	0.	1.61
time (sec)	N/A	0.412	0.202	0.011	0.	1.608	0.	1.092

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	284	550	0	2538	0	522
normalized size	1	1.	0.94	1.83	0.	8.43	0.	1.73
time (sec)	N/A	0.368	0.2	0.011	0.	1.493	0.	1.094

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	279	539	0	2587	418	463
normalized size	1	1.	0.96	1.85	0.	8.86	1.43	1.59
time (sec)	N/A	0.307	0.19	0.011	0.	1.499	170.751	1.097

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	286	547	0	2591	0	525
normalized size	1	1.	0.94	1.81	0.	8.55	0.	1.73
time (sec)	N/A	0.339	0.223	0.014	0.	1.505	0.	1.112

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	283	539	0	2618	0	486
normalized size	1	1.	0.94	1.79	0.	8.7	0.	1.61
time (sec)	N/A	0.329	0.213	0.013	0.	1.844	0.	1.084

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	303	574	0	2773	0	547
normalized size	1	1.	0.96	1.81	0.	8.75	0.	1.73
time (sec)	N/A	0.37	0.232	0.013	0.	2.072	0.	1.101

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	299	566	0	2785	0	491
normalized size	1	1.	0.95	1.79	0.	8.81	0.	1.55
time (sec)	N/A	0.368	0.233	0.017	0.	1.83	0.	1.1

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	328	611	0	3031	0	586
normalized size	1	1.	0.96	1.78	0.	8.84	0.	1.71
time (sec)	N/A	0.569	0.238	0.02	0.	1.761	0.	1.099

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	324	603	0	3013	0	532
normalized size	1	1.	0.95	1.77	0.	8.84	0.	1.56
time (sec)	N/A	0.547	0.236	0.017	0.	1.777	0.	1.098

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	366	659	0	1472	0	656
normalized size	1	1.	0.96	1.73	0.	3.86	0.	1.72
time (sec)	N/A	0.714	0.392	0.019	0.	1.619	0.	1.098

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	380	380	376	651	0	1543	0	594
normalized size	1	1.	0.99	1.71	0.	4.06	0.	1.56
time (sec)	N/A	0.669	0.429	0.018	0.	1.625	0.	1.08

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	424	424	419	716	0	1638	0	717
normalized size	1	1.	0.99	1.69	0.	3.86	0.	1.69
time (sec)	N/A	0.853	0.466	0.022	0.	1.686	0.	1.105

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	59	45	59	140	53	61
normalized size	1	1.	1.09	0.83	1.09	2.59	0.98	1.13
time (sec)	N/A	0.072	0.014	0.005	1.415	1.555	0.129	1.05

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	73	24	34
normalized size	1	1.	1.	0.83	1.07	2.43	0.8	1.13
time (sec)	N/A	0.04	0.004	0.004	1.423	1.515	0.09	1.075

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	53	38	50	117	44	51
normalized size	1	1.	1.2	0.86	1.14	2.66	1.	1.16
time (sec)	N/A	0.06	0.009	0.003	1.412	1.499	0.123	1.061

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	50	35	46	112	42	47
normalized size	1	1.	1.22	0.85	1.12	2.73	1.02	1.15
time (sec)	N/A	0.042	0.007	0.004	1.412	1.546	0.131	1.058

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	53	37	49	126	46	51
normalized size	1	1.	1.26	0.88	1.17	3.	1.1	1.21
time (sec)	N/A	0.049	0.008	0.006	1.423	1.493	0.175	1.088

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	60	44	58	144	49	61
normalized size	1	1.	1.22	0.9	1.18	2.94	1.	1.24
time (sec)	N/A	0.05	0.015	0.008	1.472	1.534	0.195	1.065

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	38	85	27	39
normalized size	1	1.	1.	0.84	1.19	2.66	0.84	1.22
time (sec)	N/A	0.033	0.004	0.007	1.401	1.423	0.109	1.047

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	47	35	46	112	42	47
normalized size	1	1.	1.15	0.85	1.12	2.73	1.02	1.15
time (sec)	N/A	0.041	0.007	0.004	1.407	1.548	0.138	1.068

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	53	33	43	108	41	45
normalized size	1	1.	1.36	0.85	1.1	2.77	1.05	1.15
time (sec)	N/A	0.04	0.01	0.005	1.408	1.501	0.128	1.056

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	44	58	112	49	61
normalized size	1	1.	1.	0.8	1.05	2.04	0.89	1.11
time (sec)	N/A	0.055	0.003	0.002	0.928	1.295	0.064	1.064

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	44	58	112	49	61
normalized size	1	1.	1.	0.8	1.05	2.04	0.89	1.11
time (sec)	N/A	0.038	0.002	0.001	0.946	1.284	0.064	1.067

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	41	54	104	46	57
normalized size	1	1.	1.	0.82	1.08	2.08	0.92	1.14
time (sec)	N/A	0.025	0.002	0.002	0.95	1.283	0.065	1.049

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	39	51	103	44	55
normalized size	1	1.	1.	0.85	1.11	2.24	0.96	1.2
time (sec)	N/A	0.025	0.004	0.003	0.949	1.491	0.272	1.083

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	39	51	113	41	55
normalized size	1	1.	1.	0.89	1.16	2.57	0.93	1.25
time (sec)	N/A	0.035	0.005	0.004	0.943	1.439	0.296	1.067

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	39	51	111	42	55
normalized size	1	1.	1.	0.89	1.16	2.52	0.95	1.25
time (sec)	N/A	0.034	0.004	0.005	0.937	1.504	0.366	1.062

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	97	80	107	198	92	111
normalized size	1	1.	1.18	0.98	1.3	2.41	1.12	1.35
time (sec)	N/A	0.059	0.004	0.002	0.939	1.221	0.074	1.08

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	97	80	107	196	94	111
normalized size	1	1.	1.18	0.98	1.3	2.39	1.15	1.35
time (sec)	N/A	0.051	0.003	0.	0.933	1.316	0.071	1.06

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	92	77	103	185	88	107
normalized size	1	1.	1.19	1.	1.34	2.4	1.14	1.39
time (sec)	N/A	0.061	0.003	0.001	0.939	1.362	0.098	1.063

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	75	100	184	88	105
normalized size	1	1.	1.	0.85	1.14	2.09	1.	1.19
time (sec)	N/A	0.052	0.008	0.001	0.949	1.495	0.353	1.051

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	74	99	204	82	104
normalized size	1	1.	1.	0.89	1.19	2.46	0.99	1.25
time (sec)	N/A	0.063	0.008	0.005	0.948	1.534	0.34	1.052

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	84	75	100	198	85	105
normalized size	1	1.	1.	0.89	1.19	2.36	1.01	1.25
time (sec)	N/A	0.064	0.008	0.006	0.943	1.466	0.395	1.053

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	139	116	155	288	138	161
normalized size	1	1.	1.26	1.05	1.41	2.62	1.25	1.46
time (sec)	N/A	0.079	0.004	0.001	0.941	1.307	0.075	1.059

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	139	116	155	285	138	161
normalized size	1	1.	1.26	1.05	1.41	2.59	1.25	1.46
time (sec)	N/A	0.069	0.004	0.001	0.919	1.268	0.076	1.082

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	134	113	151	274	134	157
normalized size	1	1.	1.28	1.08	1.44	2.61	1.28	1.5
time (sec)	N/A	0.097	0.004	0.001	0.934	1.284	0.075	1.083

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	127	110	147	265	131	154
normalized size	1	1.	1.	0.87	1.16	2.09	1.03	1.21
time (sec)	N/A	0.074	0.009	0.001	0.953	1.492	0.376	1.073

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	125	110	147	305	128	154
normalized size	1	1.	1.	0.88	1.18	2.44	1.02	1.23
time (sec)	N/A	0.092	0.008	0.006	0.94	1.453	0.399	1.058

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	126	111	149	308	129	155
normalized size	1	1.	1.	0.88	1.18	2.44	1.02	1.23
time (sec)	N/A	0.086	0.009	0.006	0.936	1.467	0.461	1.055

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	181	152	204	373	184	211
normalized size	1	1.	1.31	1.1	1.48	2.7	1.33	1.53
time (sec)	N/A	0.099	0.005	0.001	0.949	1.304	0.087	1.062

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	181	152	204	371	185	211
normalized size	1	1.	1.31	1.1	1.48	2.69	1.34	1.53
time (sec)	N/A	0.093	0.004	0.002	0.94	1.301	0.087	1.051

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	173	148	198	355	178	205
normalized size	1	1.	1.33	1.14	1.52	2.73	1.37	1.58
time (sec)	N/A	0.145	0.004	0.001	0.935	1.283	0.084	1.064

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	166	145	194	347	175	203
normalized size	1	1.	1.	0.87	1.17	2.09	1.05	1.22
time (sec)	N/A	0.109	0.009	0.003	0.936	1.468	0.456	1.061

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	162	145	194	433	168	203
normalized size	1	1.	1.	0.9	1.2	2.67	1.04	1.25
time (sec)	N/A	0.133	0.009	0.006	0.942	1.49	0.473	1.046

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	166	147	197	413	173	205
normalized size	1	1.	1.	0.89	1.19	2.49	1.04	1.23
time (sec)	N/A	0.125	0.009	0.006	0.949	1.527	0.558	1.055

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	191	231	0	10954	178	302
normalized size	1	1.	0.93	1.13	0.	53.43	0.87	1.47
time (sec)	N/A	0.261	0.105	0.003	0.	9.928	1.597	1.081

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	184	221	0	9513	150	285
normalized size	1	1.	0.95	1.15	0.	49.29	0.78	1.48
time (sec)	N/A	0.248	0.089	0.003	0.	9.578	1.501	1.078

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	200	209	0	9754	160	257
normalized size	1	1.	1.09	1.14	0.	53.3	0.87	1.4
time (sec)	N/A	0.226	0.056	0.003	0.	9.561	1.653	1.075

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	176	200	0	10437	160	246
normalized size	1	1.	0.99	1.13	0.	58.97	0.9	1.39
time (sec)	N/A	0.132	0.092	0.003	0.	9.515	1.272	1.073

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	176	207	0	10657	0	267
normalized size	1	1.	0.96	1.12	0.	57.92	0.	1.45
time (sec)	N/A	0.206	0.089	0.005	0.	10.685	0.	1.09

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	184	216	0	10066	0	279
normalized size	1	1.	0.96	1.12	0.	52.43	0.	1.45
time (sec)	N/A	0.214	0.239	0.007	0.	9.231	0.	1.083

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	192	225	0	9917	0	285
normalized size	1	1.	0.95	1.11	0.	48.85	0.	1.4
time (sec)	N/A	0.194	0.178	0.005	0.	8.22	0.	1.078

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	174	219	0	4988	109	258
normalized size	1	1.	0.92	1.15	0.	26.25	0.57	1.36
time (sec)	N/A	0.166	0.129	0.009	0.	6.236	2.422	1.074

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	186	228	0	5293	124	273
normalized size	1	1.	0.93	1.14	0.	26.46	0.62	1.36
time (sec)	N/A	0.153	0.174	0.007	0.	6.774	1.667	1.103

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	189	253	0	5011	116	266
normalized size	1	1.	0.95	1.27	0.	25.18	0.58	1.34
time (sec)	N/A	0.132	0.179	0.002	0.	6.246	1.284	1.079

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	199	274	0	12189	0	311
normalized size	1	1.	0.9	1.23	0.	54.91	0.	1.4
time (sec)	N/A	0.313	0.157	0.01	0.	7.164	0.	1.095

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	213	275	0	11634	0	328
normalized size	1	1.	0.92	1.19	0.	50.36	0.	1.42
time (sec)	N/A	0.343	0.262	0.012	0.	7.342	0.	1.1

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	221	276	0	12122	0	344
normalized size	1	1.	0.91	1.14	0.	50.09	0.	1.42
time (sec)	N/A	0.345	0.169	0.013	0.	9.802	0.	1.094

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	225	289	0	12670	0	373
normalized size	1	1.	0.86	1.1	0.	48.36	0.	1.42
time (sec)	N/A	0.404	0.175	0.013	0.	10.179	0.	1.078

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	198	255	0	5122	148	288
normalized size	1	1.	0.92	1.19	0.	23.82	0.69	1.34
time (sec)	N/A	0.197	0.179	0.01	0.	9.744	5.124	1.113

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	214	256	0	5632	170	306
normalized size	1	1.	0.9	1.07	0.	23.56	0.71	1.28
time (sec)	N/A	0.204	0.282	0.01	0.	9.66	3.385	1.103

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	213	308	0	5612	163	301
normalized size	1	1.	0.95	1.37	0.	24.94	0.72	1.34
time (sec)	N/A	0.188	0.286	0.003	0.	8.387	2.117	1.127

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	229	331	0	13407	0	359
normalized size	1	1.	0.89	1.29	0.	52.17	0.	1.4
time (sec)	N/A	0.414	0.178	0.013	0.	10.692	0.	1.116

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	248	334	0	13300	0	377
normalized size	1	1.	0.93	1.25	0.	49.81	0.	1.41
time (sec)	N/A	0.462	0.221	0.017	0.	10.085	0.	1.117

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	253	337	0	13346	0	390
normalized size	1	1.	0.92	1.22	0.	48.36	0.	1.41
time (sec)	N/A	0.5	0.21	0.016	0.	9.171	0.	1.093

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	255	351	0	14256	0	421
normalized size	1	1.	0.86	1.18	0.	47.84	0.	1.41
time (sec)	N/A	0.589	0.37	0.017	0.	12.219	0.	1.085

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	230	275	0	5952	201	333
normalized size	1	1.	0.93	1.11	0.	24.	0.81	1.34
time (sec)	N/A	0.242	0.233	0.01	0.	8.785	13.595	1.075

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	241	278	0	6597	214	346
normalized size	1	1.	0.89	1.03	0.	24.43	0.79	1.28
time (sec)	N/A	0.254	0.346	0.012	0.	8.837	7.049	1.088

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	239	360	0	6020	202	333
normalized size	1	1.	0.96	1.44	0.	24.08	0.81	1.33
time (sec)	N/A	0.222	0.232	0.004	0.	6.224	4.049	1.084

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	259	394	0	15555	0	409
normalized size	1	1.	0.89	1.35	0.	53.45	0.	1.41
time (sec)	N/A	0.517	0.228	0.016	0.	7.407	0.	1.094

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	279	397	0	15244	0	427
normalized size	1	1.	0.93	1.32	0.	50.64	0.	1.42
time (sec)	N/A	0.601	0.292	0.018	0.	7.531	0.	1.073

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	284	400	0	15485	0	441
normalized size	1	1.	0.92	1.29	0.	49.95	0.	1.42
time (sec)	N/A	0.656	0.286	0.019	0.	7.404	0.	1.071

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	284	415	0	15115	0	459
normalized size	1	1.	0.84	1.22	0.	44.46	0.	1.35
time (sec)	N/A	0.773	0.478	0.019	0.	12.297	0.	1.069

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	57	29	35	80	54	36
normalized size	1	1.	1.97	1.	1.21	2.76	1.86	1.24
time (sec)	N/A	0.058	0.014	0.007	1.419	0.992	0.287	1.062

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	57	29	35	80	54	36
normalized size	1	1.	1.97	1.	1.21	2.76	1.86	1.24
time (sec)	N/A	0.035	0.005	0.006	1.423	1.005	0.286	1.077

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	58	29	38	81	54	39
normalized size	1	1.	1.87	0.94	1.23	2.61	1.74	1.26
time (sec)	N/A	0.056	0.013	0.006	1.432	1.024	0.287	1.057

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	58	29	38	81	54	39
normalized size	1	1.	1.87	0.94	1.23	2.61	1.74	1.26
time (sec)	N/A	0.037	0.006	0.005	1.417	0.941	0.289	1.057

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	146	87	0	139	100	235
normalized size	1	1.	2.92	1.74	0.	2.78	2.	4.7
time (sec)	N/A	0.087	0.041	0.004	0.	1.082	0.414	1.111

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	149	135	0	142	110	223
normalized size	1	1.	2.81	2.55	0.	2.68	2.08	4.21
time (sec)	N/A	0.093	0.069	0.006	0.	1.101	0.436	1.094

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	148	132	0	142	109	131
normalized size	1	1.	2.74	2.44	0.	2.63	2.02	2.43
time (sec)	N/A	0.077	0.042	0.003	0.	1.122	0.421	1.068

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	147	90	0	139	102	122
normalized size	1	1.	2.77	1.7	0.	2.62	1.92	2.3
time (sec)	N/A	0.076	0.053	0.005	0.	1.032	0.431	1.092

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	80	107	239	90	117
normalized size	1	1.	1.	0.82	1.1	2.46	0.93	1.21
time (sec)	N/A	0.121	0.028	0.001	0.954	0.844	0.071	1.095

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	80	107	234	90	117
normalized size	1	1.	1.	0.82	1.1	2.41	0.93	1.21
time (sec)	N/A	0.101	0.028	0.001	0.936	0.875	0.072	1.066

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	80	107	231	90	117
normalized size	1	1.	1.	0.82	1.1	2.38	0.93	1.21
time (sec)	N/A	0.091	0.024	0.001	0.941	0.839	0.071	1.321

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	80	107	228	90	117
normalized size	1	1.	1.	0.82	1.1	2.35	0.93	1.21
time (sec)	N/A	0.08	0.017	0.002	0.951	0.893	0.071	1.054

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	92	77	103	217	87	113
normalized size	1	1.	1.	0.84	1.12	2.36	0.95	1.23
time (sec)	N/A	0.073	0.013	0.	0.933	0.847	0.07	1.067

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	81	100	192	85	112
normalized size	1	1.	1.	0.92	1.14	2.18	0.97	1.27
time (sec)	N/A	0.058	0.029	0.003	0.944	0.993	0.34	1.057

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	86	81	100	212	82	112
normalized size	1	1.	1.	0.94	1.16	2.47	0.95	1.3
time (sec)	N/A	0.066	0.041	0.004	0.948	1.02	0.348	1.071

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	78	78	100	205	82	108
normalized size	1	1.	0.91	0.91	1.16	2.38	0.95	1.26
time (sec)	N/A	0.072	0.06	0.005	0.94	1.113	0.456	1.066

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	76	76	101	205	82	107
normalized size	1	1.	0.88	0.88	1.17	2.38	0.95	1.24
time (sec)	N/A	0.071	0.061	0.006	0.943	1.246	0.732	1.052

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	77	76	101	197	82	104
normalized size	1	1.	0.9	0.88	1.17	2.29	0.95	1.21
time (sec)	N/A	0.073	0.061	0.005	1.205	1.235	2.885	1.076

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	163	152	204	414	167	216
normalized size	1	1.	1.	0.93	1.25	2.54	1.02	1.33
time (sec)	N/A	0.21	0.04	0.001	0.956	1.117	0.095	1.074

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	163	152	204	409	167	216
normalized size	1	1.	1.	0.93	1.25	2.51	1.02	1.33
time (sec)	N/A	0.159	0.028	0.001	0.936	1.084	0.14	1.066

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	150	152	204	405	167	216
normalized size	1	1.	0.95	0.96	1.29	2.56	1.06	1.37
time (sec)	N/A	0.126	0.076	0.001	0.959	0.862	0.1	1.073

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	163	152	204	400	167	216
normalized size	1	1.	1.03	0.96	1.29	2.53	1.06	1.37
time (sec)	N/A	0.129	0.024	0.001	0.947	0.862	0.102	1.052

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	125	149	200	387	163	212
normalized size	1	1.	0.82	0.97	1.31	2.53	1.07	1.39
time (sec)	N/A	0.127	0.074	0.002	0.947	0.9	0.087	1.057

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	154	153	197	351	162	211
normalized size	1	1.	1.03	1.03	1.32	2.36	1.09	1.42
time (sec)	N/A	0.106	0.044	0.003	0.934	0.938	0.473	1.068

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	152	152	197	379	156	209
normalized size	1	1.	1.03	1.03	1.34	2.58	1.06	1.42
time (sec)	N/A	0.128	0.061	0.006	0.958	0.992	0.5	1.055

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	127	150	197	383	156	207
normalized size	1	1.	0.86	1.02	1.34	2.61	1.06	1.41
time (sec)	N/A	0.128	0.071	0.006	0.939	1.	0.551	1.074

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	123	149	198	375	156	207
normalized size	1	1.	0.81	0.98	1.3	2.47	1.03	1.36
time (sec)	N/A	0.118	0.081	0.005	0.926	0.991	0.873	1.07

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	125	149	198	371	155	205
normalized size	1	1.	0.82	0.98	1.3	2.44	1.02	1.35
time (sec)	N/A	0.117	0.073	0.006	0.948	0.991	2.775	1.064

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	223	224	293	591	246	315
normalized size	1	1.	1.	1.	1.31	2.65	1.1	1.41
time (sec)	N/A	0.293	0.051	0.001	0.957	0.873	0.1	1.068

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	223	224	293	586	246	315
normalized size	1	1.	1.	1.	1.31	2.63	1.1	1.41
time (sec)	N/A	0.228	0.047	0.002	0.932	0.87	0.101	1.077

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	223	224	293	583	246	315
normalized size	1	1.	1.05	1.06	1.38	2.75	1.16	1.49
time (sec)	N/A	0.179	0.048	0.001	0.943	0.828	0.099	1.079

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	223	224	293	578	246	315
normalized size	1	1.	1.05	1.06	1.38	2.73	1.16	1.49
time (sec)	N/A	0.177	0.033	0.	0.955	0.877	0.1	1.078

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	170	221	289	564	243	311
normalized size	1	1.	0.82	1.07	1.4	2.72	1.17	1.5
time (sec)	N/A	0.177	0.101	0.001	0.948	0.833	0.095	1.065

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	214	224	286	497	240	308
normalized size	1	1.	1.07	1.12	1.43	2.48	1.2	1.54
time (sec)	N/A	0.146	0.078	0.004	0.961	0.997	0.605	1.069

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	172	224	286	570	236	308
normalized size	1	1.	0.87	1.13	1.44	2.88	1.19	1.56
time (sec)	N/A	0.182	0.137	0.007	0.946	1.021	0.633	1.061

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	174	222	286	551	236	305
normalized size	1	1.	0.88	1.12	1.44	2.78	1.19	1.54
time (sec)	N/A	0.197	0.117	0.008	0.951	1.011	0.716	1.063

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	172	220	286	551	235	304
normalized size	1	1.	0.82	1.05	1.37	2.64	1.12	1.45
time (sec)	N/A	0.18	0.126	0.006	0.947	1.006	1.061	1.062

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	170	220	286	528	233	302
normalized size	1	1.	0.81	1.05	1.37	2.53	1.11	1.44
time (sec)	N/A	0.177	0.121	0.007	0.948	0.977	2.979	1.074

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	334	533	0	0	874	513
normalized size	1	1.	1.01	1.61	0.	0.	2.64	1.55
time (sec)	N/A	1.07	0.438	0.006	0.	0.	29.166	1.078

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	299	505	0	0	842	477
normalized size	1	1.	0.96	1.61	0.	0.	2.69	1.52
time (sec)	N/A	0.988	0.238	0.005	0.	0.	29.526	1.095

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	290	483	0	0	789	450
normalized size	1	1.	0.99	1.64	0.	0.	2.68	1.53
time (sec)	N/A	0.976	0.262	0.003	0.	0.	63.182	1.083

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	272	455	0	0	811	428
normalized size	1	1.	0.99	1.65	0.	0.	2.95	1.56
time (sec)	N/A	0.921	0.373	0.003	0.	0.	33.016	1.083

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	257	254	429	0	0	804	397
normalized size	1	0.99	0.98	1.66	0.	0.	3.1	1.53
time (sec)	N/A	0.373	0.294	0.003	0.	0.	28.682	1.08

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	256	258	426	0	0	0	409
normalized size	1	0.99	1.	1.65	0.	0.	0.	1.59
time (sec)	N/A	0.471	0.212	0.006	0.	0.	0.	1.063

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	257	423	0	0	0	404
normalized size	1	1.	1.02	1.67	0.	0.	0.	1.6
time (sec)	N/A	0.454	0.28	0.007	0.	0.	0.	1.092

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	258	257	423	0	0	0	393
normalized size	1	0.99	0.99	1.63	0.	0.	0.	1.51
time (sec)	N/A	0.38	0.321	0.007	0.	0.	0.	1.082

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	274	264	442	0	0	0	423
normalized size	1	0.99	0.96	1.6	0.	0.	0.	1.53
time (sec)	N/A	0.436	0.44	0.006	0.	0.	0.	1.096

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	334	562	0	0	0	512
normalized size	1	1.	0.99	1.67	0.	0.	0.	1.52
time (sec)	N/A	0.717	0.398	0.011	0.	0.	0.	1.091

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	294	533	0	0	0	475
normalized size	1	1.	0.95	1.71	0.	0.	0.	1.53
time (sec)	N/A	0.64	0.212	0.011	0.	0.	0.	1.086

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	288	280	506	0	27702	0	444
normalized size	1	0.99	0.97	1.74	0.	95.52	0.	1.53
time (sec)	N/A	0.498	0.198	0.01	0.	11.473	0.	1.092

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	285	502	0	27753	0	451
normalized size	1	1.	0.99	1.74	0.	96.03	0.	1.56
time (sec)	N/A	0.505	0.196	0.01	0.	13.028	0.	1.088

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	268	462	0	27752	0	429
normalized size	1	1.	0.97	1.67	0.	100.55	0.	1.55
time (sec)	N/A	0.37	0.176	0.008	0.	12.37	0.	1.084

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	287	269	507	0	28260	0	452
normalized size	1	0.99	0.93	1.75	0.	97.79	0.	1.56
time (sec)	N/A	0.559	0.198	0.012	0.	156.896	0.	1.086

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	285	517	0	0	0	473
normalized size	1	1.	0.95	1.72	0.	0.	0.	1.57
time (sec)	N/A	0.593	0.299	0.014	0.	0.	0.	1.102

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	304	292	527	0	28467	0	483
normalized size	1	0.99	0.95	1.72	0.	93.03	0.	1.58
time (sec)	N/A	0.577	0.488	0.013	0.	121.35	0.	1.078

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	338	336	303	561	0	0	0	520
normalized size	1	0.99	0.9	1.66	0.	0.	0.	1.54
time (sec)	N/A	0.727	0.454	0.014	0.	0.	0.	1.096

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	342	619	0	30286	0	541
normalized size	1	1.	0.99	1.79	0.	87.79	0.	1.57
time (sec)	N/A	0.891	0.305	0.011	0.	34.187	0.	1.084

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	315	515	0	29083	0	512
normalized size	1	1.	0.97	1.58	0.	89.49	0.	1.58
time (sec)	N/A	0.642	0.253	0.012	0.	28.726	0.	1.09

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	287	490	0	15354	0	454
normalized size	1	1.	0.97	1.65	0.	51.7	0.	1.53
time (sec)	N/A	0.43	0.234	0.011	0.	19.229	0.	1.122

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	297	498	0	15553	0	481
normalized size	1	1.	0.92	1.54	0.	48.15	0.	1.49
time (sec)	N/A	0.482	0.3	0.011	0.	22.23	0.	1.091

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	295	506	0	15406	0	467
normalized size	1	1.	0.94	1.62	0.	49.22	0.	1.49
time (sec)	N/A	0.429	0.238	0.008	0.	20.815	0.	1.09

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	345	311	618	0	0	0	529
normalized size	1	0.99	0.9	1.78	0.	0.	0.	1.52
time (sec)	N/A	0.723	0.274	0.017	0.	0.	0.	1.098

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	362	362	336	622	0	0	0	556
normalized size	1	1.	0.93	1.72	0.	0.	0.	1.54
time (sec)	N/A	0.83	0.57	0.016	0.	0.	0.	1.092

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	357	337	626	0	31293	0	568
normalized size	1	0.99	0.94	1.74	0.	86.92	0.	1.58
time (sec)	N/A	0.814	0.606	0.017	0.	139.296	0.	1.115

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	392	352	680	0	0	0	612
normalized size	1	0.99	0.89	1.72	0.	0.	0.	1.55
time (sec)	N/A	1.006	0.591	0.017	0.	0.	0.	1.096

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	583	583	132	793	0	0	129	0
normalized size	1	1.	0.23	1.36	0.	0.	0.22	0.
time (sec)	N/A	0.733	0.129	0.016	0.	0.	3.132	0.

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	560	560	121	773	0	0	107	0
normalized size	1	1.	0.22	1.38	0.	0.	0.19	0.
time (sec)	N/A	0.484	0.079	0.007	0.	0.	2.87	0.

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	537	537	114	753	0	0	107	0
normalized size	1	1.	0.21	1.4	0.	0.	0.2	0.
time (sec)	N/A	0.323	0.062	0.006	0.	0.	2.689	0.

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	509	509	107	735	0	0	105	0
normalized size	1	1.	0.21	1.44	0.	0.	0.21	0.
time (sec)	N/A	0.173	0.046	0.004	0.	0.	1.981	0.

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	518	518	128	740	0	0	105	0
normalized size	1	1.	0.25	1.43	0.	0.	0.2	0.
time (sec)	N/A	0.205	0.141	0.007	0.	0.	3.335	0.

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	547	547	126	759	0	0	107	0
normalized size	1	1.	0.23	1.39	0.	0.	0.2	0.
time (sec)	N/A	0.343	0.124	0.007	0.	0.	2.948	0.

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	569	569	131	778	0	0	112	0
normalized size	1	1.	0.23	1.37	0.	0.	0.2	0.
time (sec)	N/A	0.464	0.194	0.009	0.	0.	2.966	0.

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	594	594	134	836	0	0	129	0
normalized size	1	1.	0.23	1.41	0.	0.	0.22	0.
time (sec)	N/A	0.641	0.111	0.02	0.	0.	27.031	0.

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	574	574	127	817	0	0	129	0
normalized size	1	1.	0.22	1.42	0.	0.	0.22	0.
time (sec)	N/A	0.469	0.103	0.009	0.	0.	18.138	0.

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	542	542	118	800	0	0	129	0
normalized size	1	1.	0.22	1.48	0.	0.	0.24	0.
time (sec)	N/A	0.327	0.095	0.007	0.	0.	13.504	0.

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	522	522	107	779	0	0	109	0
normalized size	1	1.	0.2	1.49	0.	0.	0.21	0.
time (sec)	N/A	0.262	0.088	0.007	0.	0.	11.561	0.

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	561	561	108	782	0	0	109	0
normalized size	1	1.	0.19	1.39	0.	0.	0.19	0.
time (sec)	N/A	0.321	0.064	0.005	0.	0.	11.124	0.

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	532	532	109	785	0	0	107	0
normalized size	1	1.	0.2	1.48	0.	0.	0.2	0.
time (sec)	N/A	0.246	0.047	0.004	0.	0.	11.264	0.

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	579	579	119	810	0	0	265	0
normalized size	1	1.	0.21	1.4	0.	0.	0.46	0.
time (sec)	N/A	0.4	0.103	0.007	0.	0.	22.444	0.

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	607	607	121	825	0	0	267	0
normalized size	1	1.	0.2	1.36	0.	0.	0.44	0.
time (sec)	N/A	0.56	0.098	0.008	0.	0.	34.63	0.

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	733	733	172	1674	0	0	238	0
normalized size	1	1.	0.23	2.28	0.	0.	0.32	0.
time (sec)	N/A	1.91	0.299	0.019	0.	0.	5.106	0.

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	681	681	158	1197	0	0	223	0
normalized size	1	1.	0.23	1.76	0.	0.	0.33	0.
time (sec)	N/A	1.422	0.254	0.008	0.	0.	4.524	0.

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	667	667	143	1311	0	0	223	0
normalized size	1	1.	0.21	1.97	0.	0.	0.33	0.
time (sec)	N/A	1.047	0.194	0.006	0.	0.	4.307	0.

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	639	639	135	1557	0	0	194	0
normalized size	1	1.	0.21	2.44	0.	0.	0.3	0.
time (sec)	N/A	0.719	0.148	0.005	0.	0.	3.902	0.

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	620	620	185	1118	0	0	235	0
normalized size	1	1.	0.3	1.8	0.	0.	0.38	0.
time (sec)	N/A	0.547	0.317	0.008	0.	0.	9.255	0.

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	638	638	211	1248	0	0	236	0
normalized size	1	1.	0.33	1.96	0.	0.	0.37	0.
time (sec)	N/A	0.65	0.244	0.009	0.	0.	5.507	0.

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	640	640	218	1529	0	0	255	0
normalized size	1	1.	0.34	2.39	0.	0.	0.4	0.
time (sec)	N/A	0.765	0.411	0.009	0.	0.	6.011	0.

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	637	637	254	1114	0	0	265	0
normalized size	1	1.	0.4	1.75	0.	0.	0.42	0.
time (sec)	N/A	0.844	0.387	0.01	0.	0.	6.899	0.

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	694	694	253	1286	0	0	274	0
normalized size	1	1.	0.36	1.85	0.	0.	0.39	0.
time (sec)	N/A	1.084	0.387	0.01	0.	0.	7.315	0.

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	652	652	180	1571	0	0	240	0
normalized size	1	1.	0.28	2.41	0.	0.	0.37	0.
time (sec)	N/A	0.787	0.26	0.009	0.	0.	6.551	0.

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	659	659	211	1180	0	0	304	0
normalized size	1	1.	0.32	1.79	0.	0.	0.46	0.
time (sec)	N/A	0.98	0.425	0.008	0.	0.	9.384	0.

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	711	711	213	1376	0	0	308	0
normalized size	1	1.	0.3	1.94	0.	0.	0.43	0.
time (sec)	N/A	1.123	0.452	0.01	0.	0.	10.029	0.

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	743	743	192	1679	0	0	304	0
normalized size	1	1.	0.26	2.26	0.	0.	0.41	0.
time (sec)	N/A	1.331	0.191	0.009	0.	0.	9.386	0.

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	791	791	179	1764	0	0	512	0
normalized size	1	1.	0.23	2.23	0.	0.	0.65	0.
time (sec)	N/A	2.111	0.527	0.017	0.	0.	10.768	0.

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	742	742	162	1269	0	0	525	0
normalized size	1	1.	0.22	1.71	0.	0.	0.71	0.
time (sec)	N/A	1.552	0.35	0.008	0.	0.	9.442	0.

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	723	723	148	1383	0	0	525	0
normalized size	1	1.	0.2	1.91	0.	0.	0.73	0.
time (sec)	N/A	1.239	0.272	0.006	0.	0.	8.804	0.

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	694	694	139	1629	0	0	444	0
normalized size	1	1.	0.2	2.35	0.	0.	0.64	0.
time (sec)	N/A	0.899	0.177	0.006	0.	0.	7.325	0.

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	676	676	215	1188	0	0	473	0
normalized size	1	1.	0.32	1.76	0.	0.	0.7	0.
time (sec)	N/A	0.707	0.439	0.009	0.	0.	21.367	0.

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	692	692	224	1317	0	0	474	0
normalized size	1	1.	0.32	1.9	0.	0.	0.68	0.
time (sec)	N/A	0.791	0.34	0.009	0.	0.	10.297	0.

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	694	694	232	1613	0	0	462	0
normalized size	1	1.	0.33	2.32	0.	0.	0.67	0.
time (sec)	N/A	0.888	0.356	0.009	0.	0.	10.501	0.

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	692	692	243	1193	0	0	484	0
normalized size	1	1.	0.35	1.72	0.	0.	0.7	0.
time (sec)	N/A	0.957	0.612	0.01	0.	0.	11.966	0.

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	741	741	246	1342	0	0	495	0
normalized size	1	1.	0.33	1.81	0.	0.	0.67	0.
time (sec)	N/A	1.24	0.603	0.01	0.	0.	12.167	0.

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	689	689	191	1606	0	0	476	0
normalized size	1	1.	0.28	2.33	0.	0.	0.69	0.
time (sec)	N/A	0.921	0.244	0.009	0.	0.	11.759	0.

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	692	692	240	1196	0	0	524	0
normalized size	1	1.	0.35	1.73	0.	0.	0.76	0.
time (sec)	N/A	1.004	0.507	0.01	0.	0.	15.539	0.

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	746	746	240	1375	0	0	536	0
normalized size	1	1.	0.32	1.84	0.	0.	0.72	0.
time (sec)	N/A	1.284	0.781	0.009	0.	0.	16.457	0.

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	705	705	202	1663	0	0	527	0
normalized size	1	1.	0.29	2.36	0.	0.	0.75	0.
time (sec)	N/A	1.008	0.461	0.01	0.	0.	15.513	0.

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	714	714	226	1273	0	0	573	0
normalized size	1	1.	0.32	1.78	0.	0.	0.8	0.
time (sec)	N/A	1.123	0.645	0.025	0.	0.	23.72	0.

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	764	764	227	1470	0	0	576	0
normalized size	1	1.	0.3	1.92	0.	0.	0.75	0.
time (sec)	N/A	1.33	0.507	0.008	0.	0.	24.308	0.

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	796	796	194	1773	0	0	541	0
normalized size	1	1.	0.24	2.23	0.	0.	0.68	0.
time (sec)	N/A	1.534	0.4	0.025	0.	0.	22.843	0.

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	120	114	0	0	0	112	0
normalized size	1	1.18	1.12	0.	0.	0.	1.1	0.
time (sec)	N/A	0.076	0.062	0.264	0.	0.	91.678	0.

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	125	116	0	0	0	114	0
normalized size	1	1.17	1.08	0.	0.	0.	1.07	0.
time (sec)	N/A	0.091	0.062	0.227	0.	0.	145.318	0.

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	125	116	0	0	0	0	0
normalized size	1	1.17	1.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	0.076	0.234	0.	0.	0.	0.

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	55	73	142	63	76
normalized size	1	1.	1.	0.81	1.07	2.09	0.93	1.12
time (sec)	N/A	0.044	0.005	0.001	0.917	1.381	0.063	1.088

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	58	77	155	66	80
normalized size	1	1.	1.	0.79	1.05	2.12	0.9	1.1
time (sec)	N/A	0.064	0.003	0.	0.919	1.433	0.066	1.058

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	124	103	138	258	121	142
normalized size	1	1.	1.14	0.94	1.27	2.37	1.11	1.3
time (sec)	N/A	0.074	0.004	0.	0.915	1.489	0.075	1.085

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	129	106	142	273	124	146
normalized size	1	1.	1.13	0.93	1.25	2.39	1.09	1.28
time (sec)	N/A	0.084	0.004	0.001	0.917	1.438	0.079	1.083

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	180	151	203	375	180	208
normalized size	1	1.	1.19	1.	1.34	2.48	1.19	1.38
time (sec)	N/A	0.108	0.005	0.	0.911	1.382	0.088	1.063

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	185	154	207	390	184	212
normalized size	1	1.	1.19	0.99	1.33	2.5	1.18	1.36
time (sec)	N/A	0.113	0.005	0.001	0.916	1.441	0.084	1.078

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	236	199	267	490	241	274
normalized size	1	1.	1.22	1.03	1.38	2.54	1.25	1.42
time (sec)	N/A	0.156	0.006	0.001	0.931	1.481	0.09	1.106

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	241	202	271	504	245	278
normalized size	1	1.	1.22	1.02	1.37	2.55	1.24	1.4
time (sec)	N/A	0.15	0.006	0.001	0.92	1.498	0.095	1.088

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	214	177	0	0	952	419
normalized size	1	1.	1.61	1.33	0.	0.	7.16	3.15
time (sec)	N/A	0.124	0.066	0.004	0.	0.	13.139	1.075

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	221	208	0	0	887	443
normalized size	1	1.	1.36	1.28	0.	0.	5.48	2.73
time (sec)	N/A	0.203	0.084	0.004	0.	0.	12.548	1.076

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	296	294	0	0	950	392
normalized size	1	1.	1.01	1.	0.	0.	3.24	1.34
time (sec)	N/A	0.222	0.215	0.004	0.	0.	13.21	1.081

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	311	325	0	0	886	416
normalized size	1	1.	0.97	1.01	0.	0.	2.76	1.3
time (sec)	N/A	0.334	0.173	0.004	0.	0.	12.174	1.101

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	315	362	0	0	517	427
normalized size	1	1.	0.99	1.14	0.	0.	1.63	1.34
time (sec)	N/A	0.27	0.34	0.005	0.	0.	16.141	1.088

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	294	334	0	0	508	409
normalized size	1	1.	0.95	1.08	0.	0.	1.64	1.32
time (sec)	N/A	0.274	0.269	0.009	0.	0.	31.183	1.093

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	347	432	0	0	578	478
normalized size	1	1.	0.99	1.23	0.	0.	1.65	1.36
time (sec)	N/A	0.318	0.328	0.005	0.	0.	68.514	1.101

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	329	373	0	0	0	456
normalized size	1	1.	0.97	1.1	0.	0.	0.	1.34
time (sec)	N/A	0.328	0.35	0.012	0.	0.	0.	1.096

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	382	379	400	0	0	0	528
normalized size	1	1.	0.99	1.05	0.	0.	0.	1.38
time (sec)	N/A	0.406	0.409	0.014	0.	0.	0.	1.081

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	380	380	366	403	0	0	0	513
normalized size	1	1.	0.96	1.06	0.	0.	0.	1.35
time (sec)	N/A	0.402	0.421	0.012	0.	0.	0.	1.092

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	418	202	390	0	0	252	0
normalized size	1	1.	0.48	0.93	0.	0.	0.6	0.
time (sec)	N/A	0.381	0.636	0.032	0.	0.	7.096	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	394	215	380	0	0	212	0
normalized size	1	1.	0.55	0.96	0.	0.	0.54	0.
time (sec)	N/A	0.334	0.63	0.008	0.	0.	6.297	0.

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	182	361	0	0	212	0
normalized size	1	1.	0.49	0.98	0.	0.	0.57	0.
time (sec)	N/A	0.298	0.721	0.01	0.	0.	6.005	0.

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	354	211	337	0	0	158	0
normalized size	1	1.	0.6	0.95	0.	0.	0.45	0.
time (sec)	N/A	0.273	0.184	0.013	0.	0.	4.281	0.

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	171	313	0	0	156	0
normalized size	1	1.	0.52	0.95	0.	0.	0.47	0.
time (sec)	N/A	0.194	0.128	0.007	0.	0.	4.143	0.

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	208	339	0	0	204	0
normalized size	1	1.	0.6	0.98	0.	0.	0.59	0.
time (sec)	N/A	0.255	0.357	0.015	0.	0.	8.392	0.

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	208	339	0	0	206	0
normalized size	1	1.	0.61	0.99	0.	0.	0.6	0.
time (sec)	N/A	0.263	0.391	0.012	0.	0.	5.704	0.

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	342	204	360	0	0	230	0
normalized size	1	1.	0.6	1.05	0.	0.	0.67	0.
time (sec)	N/A	0.262	0.233	0.016	0.	0.	5.271	0.

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	205	362	0	0	235	0
normalized size	1	1.	0.57	1.01	0.	0.	0.66	0.
time (sec)	N/A	0.295	0.301	0.013	0.	0.	5.414	0.

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	175	385	0	0	211	0
normalized size	1	1.	0.53	1.17	0.	0.	0.64	0.
time (sec)	N/A	0.281	0.234	0.016	0.	0.	5.735	0.

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	179	404	0	0	216	0
normalized size	1	1.	0.5	1.12	0.	0.	0.6	0.
time (sec)	N/A	0.324	0.234	0.015	0.	0.	6.044	0.

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	352	145	361	0	0	189	0
normalized size	1	1.	0.41	1.03	0.	0.	0.54	0.
time (sec)	N/A	0.316	0.243	0.017	0.	0.	5.702	0.

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	145	385	0	0	192	0
normalized size	1	1.	0.39	1.03	0.	0.	0.51	0.
time (sec)	N/A	0.362	0.24	0.018	0.	0.	6.313	0.

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	400	400	146	408	0	0	246	0
normalized size	1	1.	0.36	1.02	0.	0.	0.62	0.
time (sec)	N/A	0.386	0.165	0.019	0.	0.	7.904	0.

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	425	425	148	429	0	0	246	0
normalized size	1	1.	0.35	1.01	0.	0.	0.58	0.
time (sec)	N/A	0.433	0.167	0.018	0.	0.	8.464	0.

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	476	476	225	462	0	0	462	0
normalized size	1	1.	0.47	0.97	0.	0.	0.97	0.
time (sec)	N/A	0.44	0.865	0.039	0.	0.	18.042	0.

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	452	452	238	434	0	0	398	0
normalized size	1	1.	0.53	0.96	0.	0.	0.88	0.
time (sec)	N/A	0.408	0.747	0.007	0.	0.	15.031	0.

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	427	427	205	413	0	0	398	0
normalized size	1	1.	0.48	0.97	0.	0.	0.93	0.
time (sec)	N/A	0.353	0.863	0.01	0.	0.	14.209	0.

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	409	409	196	392	0	0	396	0
normalized size	1	1.	0.48	0.96	0.	0.	0.97	0.
time (sec)	N/A	0.324	0.701	0.015	0.	0.	10.174	0.

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	382	175	368	0	0	394	0
normalized size	1	1.	0.46	0.96	0.	0.	1.03	0.
time (sec)	N/A	0.255	0.553	0.006	0.	0.	9.642	0.

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	403	224	411	0	0	405	0
normalized size	1	1.	0.56	1.02	0.	0.	1.	0.
time (sec)	N/A	0.353	0.512	0.019	0.	0.	26.863	0.

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	404	222	411	0	0	406	0
normalized size	1	1.	0.55	1.02	0.	0.	1.	0.
time (sec)	N/A	0.348	0.523	0.013	0.	0.	11.26	0.

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	406	406	194	409	0	0	377	0
normalized size	1	1.	0.48	1.01	0.	0.	0.93	0.
time (sec)	N/A	0.343	0.362	0.02	0.	0.	9.062	0.

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	408	408	194	408	0	0	381	0
normalized size	1	1.	0.48	1.	0.	0.	0.93	0.
time (sec)	N/A	0.336	0.358	0.014	0.	0.	9.128	0.

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	386	163	409	0	0	379	0
normalized size	1	1.	0.42	1.06	0.	0.	0.98	0.
time (sec)	N/A	0.347	0.219	0.021	0.	0.	10.707	0.

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	387	165	409	0	0	386	0
normalized size	1	1.	0.43	1.06	0.	0.	1.	0.
time (sec)	N/A	0.349	0.223	0.015	0.	0.	11.021	0.

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	392	392	163	408	0	0	406	0
normalized size	1	1.	0.42	1.04	0.	0.	1.04	0.
time (sec)	N/A	0.342	0.202	0.02	0.	0.	9.893	0.

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	412	164	411	0	0	415	0
normalized size	1	1.	0.4	1.	0.	0.	1.01	0.
time (sec)	N/A	0.394	0.221	0.015	0.	0.	11.456	0.

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	174	416	0	0	444	0
normalized size	1	1.	0.46	1.1	0.	0.	1.18	0.
time (sec)	N/A	0.371	0.294	0.023	0.	0.	12.967	0.

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	405	405	174	437	0	0	449	0
normalized size	1	1.	0.43	1.08	0.	0.	1.11	0.
time (sec)	N/A	0.425	0.314	0.018	0.	0.	13.571	0.

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	399	171	417	0	0	398	0
normalized size	1	1.	0.43	1.05	0.	0.	1.	0.
time (sec)	N/A	0.42	0.326	0.019	0.	0.	15.602	0.

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	424	424	172	441	0	0	401	0
normalized size	1	1.	0.41	1.04	0.	0.	0.95	0.
time (sec)	N/A	0.462	0.381	0.02	0.	0.	15.493	0.

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	149	462	0	0	403	0
normalized size	1	1.	0.33	1.03	0.	0.	0.9	0.
time (sec)	N/A	0.49	0.191	0.023	0.	0.	20.177	0.

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	474	474	151	483	0	0	403	0
normalized size	1	1.	0.32	1.02	0.	0.	0.85	0.
time (sec)	N/A	0.549	0.184	0.02	0.	0.	21.79	0.

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	361	212	335	0	0	177	0
normalized size	1	1.	0.59	0.93	0.	0.	0.49	0.
time (sec)	N/A	0.3	0.131	0.018	0.	0.	6.138	0.

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	212	325	0	0	156	0
normalized size	1	1.	0.63	0.97	0.	0.	0.46	0.
time (sec)	N/A	0.265	0.145	0.007	0.	0.	5.023	0.

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	193	248	0	0	156	0
normalized size	1	1.	0.63	0.81	0.	0.	0.51	0.
time (sec)	N/A	0.222	0.169	0.006	0.	0.	4.724	0.

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	160	229	0	0	129	0
normalized size	1	1.	0.54	0.77	0.	0.	0.43	0.
time (sec)	N/A	0.198	0.095	0.005	0.	0.	3.56	0.

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	150	208	0	0	128	0
normalized size	1	1.	0.54	0.75	0.	0.	0.46	0.
time (sec)	N/A	0.142	0.094	0.004	0.	0.	2.627	0.

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	159	222	0	0	126	0
normalized size	1	1.	0.56	0.78	0.	0.	0.44	0.
time (sec)	N/A	0.18	0.227	0.013	0.	0.	4.07	0.

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	157	299	0	0	128	0
normalized size	1	1.	0.51	0.97	0.	0.	0.41	0.
time (sec)	N/A	0.224	0.227	0.009	0.	0.	3.31	0.

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	148	293	0	0	126	0
normalized size	1	1.	0.49	0.98	0.	0.	0.42	0.
time (sec)	N/A	0.216	0.14	0.015	0.	0.	3.096	0.

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	149	316	0	0	131	0
normalized size	1	1.	0.46	0.98	0.	0.	0.41	0.
time (sec)	N/A	0.258	0.158	0.01	0.	0.	3.441	0.

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	147	335	0	0	158	0
normalized size	1	1.	0.42	0.97	0.	0.	0.46	0.
time (sec)	N/A	0.281	0.147	0.017	0.	0.	4.537	0.

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	134	354	0	0	163	0
normalized size	1	1.	0.36	0.94	0.	0.	0.43	0.
time (sec)	N/A	0.327	0.208	0.01	0.	0.	4.883	0.

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	365	220	378	0	0	202	0
normalized size	1	1.	0.6	1.04	0.	0.	0.55	0.
time (sec)	N/A	0.523	0.173	0.027	0.	0.	34.669	0.

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	176	358	0	0	172	0
normalized size	1	1.	0.51	1.04	0.	0.	0.5	0.
time (sec)	N/A	0.367	0.152	0.016	0.	0.	25.754	0.

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	166	340	0	0	172	0
normalized size	1	1.	0.53	1.08	0.	0.	0.55	0.
time (sec)	N/A	0.268	0.146	0.008	0.	0.	19.986	0.

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	181	331	0	0	156	0
normalized size	1	1.	0.61	1.11	0.	0.	0.53	0.
time (sec)	N/A	0.2	0.129	0.007	0.	0.	17.232	0.

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	165	331	0	0	156	0
normalized size	1	1.	0.5	0.99	0.	0.	0.47	0.
time (sec)	N/A	0.251	0.196	0.009	0.	0.	15.565	0.

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	116	250	0	0	133	0
normalized size	1	1.	0.38	0.83	0.	0.	0.44	0.
time (sec)	N/A	0.193	0.077	0.011	0.	0.	14.639	0.

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	116	250	0	0	131	0
normalized size	1	1.	0.42	0.91	0.	0.	0.48	0.
time (sec)	N/A	0.117	0.054	0.006	0.	0.	13.983	0.

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	125	336	0	0	289	0
normalized size	1	1.	0.39	1.04	0.	0.	0.89	0.
time (sec)	N/A	0.292	0.127	0.017	0.	0.	24.447	0.

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	123	355	0	0	291	0
normalized size	1	1.	0.36	1.03	0.	0.	0.85	0.
time (sec)	N/A	0.383	0.114	0.01	0.	0.	34.144	0.

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	367	367	140	363	0	0	316	0
normalized size	1	1.	0.38	0.99	0.	0.	0.86	0.
time (sec)	N/A	0.478	0.106	0.016	0.	0.	42.437	0.

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	387	136	383	0	0	321	0
normalized size	1	1.	0.35	0.99	0.	0.	0.83	0.
time (sec)	N/A	0.606	0.119	0.011	0.	0.	41.397	0.

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	174	0	0	0	0	0
normalized size	1	1.	0.65	0.	0.	0.	0.	0.
time (sec)	N/A	0.256	0.214	0.314	0.	0.	0.	0.

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	170	147	0	0	0	141	0
normalized size	1	1.19	1.03	0.	0.	0.	0.99	0.
time (sec)	N/A	0.131	0.111	0.25	0.	0.	70.572	0.

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	145	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.182	0.115	0.222	0.	0.	0.	0.

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	18	5	9
normalized size	1	1.	1.	0.88	1.	2.25	0.62	1.12
time (sec)	N/A	0.01	0.001	0.	0.916	1.346	0.06	1.054

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	24	7	12
normalized size	1	1.	1.	0.9	1.1	2.4	0.7	1.2
time (sec)	N/A	0.015	0.001	0.001	0.928	1.346	0.07	1.059

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	26	8	12
normalized size	1	1.	1.	0.9	1.1	2.6	0.8	1.2
time (sec)	N/A	0.013	0.001	0.001	0.921	1.359	0.068	1.052

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	21	18	23	53	15	20
normalized size	1	1.	2.1	1.8	2.3	5.3	1.5	2.
time (sec)	N/A	0.01	0.003	0.006	0.895	1.353	0.093	1.048

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	17	22	58	24	22
normalized size	1	1.	1.	0.71	0.92	2.42	1.	0.92
time (sec)	N/A	0.024	0.006	0.003	1.372	1.355	0.114	1.052

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	39	51	128	46	53
normalized size	1	1.	1.	0.78	1.02	2.56	0.92	1.06
time (sec)	N/A	0.049	0.017	0.007	1.388	1.433	0.162	1.083

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	46	39	51	128	46	53
normalized size	1	1.	0.92	0.78	1.02	2.56	0.92	1.06
time (sec)	N/A	0.048	0.013	0.007	1.387	1.405	0.165	1.049

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	56	47	62	157	56	65
normalized size	1	1.	0.93	0.78	1.03	2.62	0.93	1.08
time (sec)	N/A	0.051	0.013	0.007	1.374	1.435	0.181	1.046

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	52	47	62	157	56	65
normalized size	1	1.	0.87	0.78	1.03	2.62	0.93	1.08
time (sec)	N/A	0.049	0.011	0.006	1.384	1.434	0.18	1.072

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	39	51	126	48	47
normalized size	1	1.	1.	0.78	1.02	2.52	0.96	0.94
time (sec)	N/A	0.026	0.006	0.005	1.373	1.375	0.138	1.044

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	39	51	124	48	53
normalized size	1	1.	1.	0.78	1.02	2.48	0.96	1.06
time (sec)	N/A	0.043	0.01	0.004	1.367	1.415	0.146	1.062

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	100	85	113	342	105	116
normalized size	1	1.	0.91	0.77	1.03	3.11	0.95	1.05
time (sec)	N/A	0.118	0.087	0.01	1.405	1.497	0.359	1.072

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	97	85	113	342	105	116
normalized size	1	1.	0.88	0.77	1.03	3.11	0.95	1.05
time (sec)	N/A	0.117	0.077	0.009	1.381	1.438	0.351	1.074

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	122	68	82	259	70	85
normalized size	1	1.	1.51	0.84	1.01	3.2	0.86	1.05
time (sec)	N/A	0.069	0.431	0.01	1.375	1.527	0.201	1.061

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	84	73	100	338	82	103
normalized size	1	1.	0.91	0.79	1.09	3.67	0.89	1.12
time (sec)	N/A	0.116	0.03	0.01	1.386	1.654	0.285	1.056

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	119	115	142	736	124	150
normalized size	1	1.	0.8	0.78	0.96	4.97	0.84	1.01
time (sec)	N/A	0.172	0.057	0.016	1.374	1.683	0.503	1.07

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	121	115	142	736	124	150
normalized size	1	1.	0.83	0.79	0.97	5.04	0.85	1.03
time (sec)	N/A	0.173	0.082	0.013	1.381	1.506	0.501	1.062

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	111	111	128	539	116	143
normalized size	1	1.	0.78	0.78	0.9	3.8	0.82	1.01
time (sec)	N/A	0.148	0.063	0.013	1.376	1.462	0.475	1.065

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	111	111	128	539	116	143
normalized size	1	1.	0.78	0.78	0.9	3.8	0.82	1.01
time (sec)	N/A	0.145	0.062	0.013	1.377	1.461	0.469	1.06

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	103	102	117	377	110	120
normalized size	1	1.	0.91	0.9	1.04	3.34	0.97	1.06
time (sec)	N/A	0.082	0.05	0.013	1.381	1.427	0.384	1.059

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	111	102	128	531	119	134
normalized size	1	1.	0.85	0.78	0.98	4.05	0.91	1.02
time (sec)	N/A	0.148	0.064	0.013	1.395	1.443	0.468	1.05

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	91	76	101	257	102	93
normalized size	1	1.	0.92	0.77	1.02	2.6	1.03	0.94
time (sec)	N/A	0.063	0.014	0.007	1.375	1.444	0.328	1.063

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	130	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.167	0.341	0.384	0.	0.	0.	0.

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	108	130	0	660	1251	529
normalized size	1	1.	1.29	1.55	0.	7.86	14.89	6.3
time (sec)	N/A	0.056	0.163	0.015	0.	1.545	6.769	1.069

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	120	87	0	347	552	265
normalized size	1	1.	1.97	1.43	0.	5.69	9.05	4.34
time (sec)	N/A	0.04	0.112	0.012	0.	1.537	3.252	1.074

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	42	45	0	128	163	88
normalized size	1	1.	1.02	1.1	0.	3.12	3.98	2.15
time (sec)	N/A	0.023	0.096	0.011	0.	1.485	1.509	1.056

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	36	15	16
normalized size	1	1.	1.	1.08	0.	3.	1.25	1.33
time (sec)	N/A	0.003	0.001	0.041	0.	1.501	0.058	1.041

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	0	0	65	0
normalized size	1	1.	1.	0.	0.	0.	1.55	0.
time (sec)	N/A	0.03	0.056	0.339	0.	0.	9.521	0.

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	313	0
normalized size	1	1.	1.	0.	0.	0.	7.11	0.
time (sec)	N/A	0.029	0.075	0.36	0.	0.	22.753	0.

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	63	0	0	0	0	0
normalized size	1	1.	1.37	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	0.074	0.355	0.	0.	0.	0.

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	206	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.232	0.374	0.464	0.	0.	0.	0.

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	153	0	0
normalized size	1	1.	1.	0.	0.	3.4	0.	0.
time (sec)	N/A	0.395	0.215	0.487	0.	1.895	0.	0.

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	178	0	0	0	0	0
normalized size	1	1.	0.65	0.	0.	0.	0.	0.
time (sec)	N/A	0.185	0.243	0.437	0.	0.	0.	0.

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	204	0	0	0	0	0
normalized size	1	1.	0.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.208	0.273	0.385	0.	0.	0.	0.

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	147	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	0.338	0.432	0.	0.	0.	0.

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	27	47	0	0
normalized size	1	1.	1.	0.88	1.12	1.96	0.	0.
time (sec)	N/A	0.053	0.148	0.009	1.105	1.618	0.	0.

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	93	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.103	0.168	0.387	0.	0.	0.	0.

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	0	0	128	0	308
normalized size	1	1.	1.	0.	0.	4.57	0.	11.
time (sec)	N/A	0.101	0.307	0.817	0.	2.001	0.	1.119

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	46	138	104	312	0	320
normalized size	1	1.	1.02	3.07	2.31	6.93	0.	7.11
time (sec)	N/A	0.157	0.355	0.51	1.365	2.007	0.	1.146

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	52	80	115	0	155
normalized size	1	1.	1.	1.68	2.58	3.71	0.	5.
time (sec)	N/A	0.205	0.524	0.153	1.257	1.798	0.	1.171

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	41	136	124	223	0	209
normalized size	1	1.	0.91	3.02	2.76	4.96	0.	4.64
time (sec)	N/A	0.553	0.767	0.429	1.367	1.734	0.	1.189

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [124] had the largest ratio of [0.6923]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.	20	0.05
2	A	2	1	1.	22	0.045
3	A	2	1	1.	22	0.045
4	A	2	1	1.	25	0.04
5	A	2	1	1.	27	0.037
6	A	2	1	1.	27	0.037
7	A	6	6	1.	15	0.4
8	A	7	7	1.	15	0.467
9	A	8	7	1.	15	0.467
10	A	9	7	1.	15	0.467
11	A	6	6	1.	15	0.4
12	A	6	6	1.	16	0.375
13	A	3	3	1.	11	0.273
14	A	3	3	1.	15	0.2
15	A	3	3	1.	13	0.231
16	A	3	3	1.	13	0.231
17	A	6	6	1.	15	0.4
18	A	3	3	1.	19	0.158
19	A	3	3	1.	21	0.143
20	A	3	3	1.	31	0.097
21	A	3	3	1.	36	0.083
22	A	12	10	1.	35	0.286
23	A	11	9	1.	33	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
24	A	10	8	1.	36	0.222
25	A	10	10	1.	19	0.526
26	A	9	9	1.	18	0.5
27	A	4	4	1.	27	0.148
28	A	4	4	1.	28	0.143
29	A	4	4	1.	24	0.167
30	A	4	4	1.	24	0.167
31	A	4	4	1.	26	0.154
32	A	4	4	1.	26	0.154
33	A	4	4	1.	28	0.143
34	A	4	4	1.	30	0.133
35	A	4	4	1.	29	0.138
36	A	4	4	1.	29	0.138
37	A	4	4	1.	29	0.138
38	A	4	4	1.	32	0.125
39	A	6	6	1.	13	0.462
40	A	4	4	1.	49	0.082
41	A	4	4	1.	57	0.07
42	A	2	2	1.	31	0.065
43	A	2	2	1.	42	0.048
44	A	4	4	1.	42	0.095
45	A	4	4	1.	45	0.089
46	A	4	4	1.	45	0.089
47	A	4	4	1.	44	0.091
48	A	3	3	1.	20	0.15
49	A	6	6	1.	20	0.3
50	A	2	2	1.	16	0.125
51	A	5	5	1.	20	0.25
52	A	3	3	1.	18	0.167
53	A	2	1	1.	30	0.033
54	A	2	1	1.	30	0.033
55	A	2	1	1.	28	0.036

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	2	1	1.	30	0.033
57	A	7	7	1.	30	0.233
58	A	8	8	1.	30	0.267
59	A	7	5	1.	32	0.156
60	A	6	5	1.	32	0.156
61	A	5	5	1.	32	0.156
62	A	4	4	1.	32	0.125
63	A	5	5	1.	32	0.156
64	A	6	5	1.	32	0.156
65	A	7	5	1.	32	0.156
66	A	7	6	1.	32	0.188
67	A	6	6	1.	32	0.188
68	A	5	5	1.	32	0.156
69	A	6	6	1.	32	0.188
70	A	8	8	1.	17	0.471
71	A	10	9	1.	17	0.529
72	A	10	9	0.99	17	0.529
73	A	10	9	0.99	22	0.409
74	A	10	9	1.	22	0.409
75	A	10	9	1.	22	0.409
76	A	9	8	1.	17	0.471
77	A	9	8	1.	19	0.421
78	A	8	7	1.	18	0.389
79	A	3	3	1.	18	0.167
80	A	3	3	1.	22	0.136
81	A	1	1	1.	20	0.05
82	A	1	1	1.	20	0.05
83	A	3	3	1.	33	0.091
84	A	3	3	1.	35	0.086
85	A	1	1	1.	36	0.028
86	A	1	1	1.	36	0.028
87	A	3	3	1.	30	0.1

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	3	3	1.	32	0.094
89	A	1	1	1.	33	0.03
90	A	1	1	1.	33	0.03
91	A	1	1	1.	20	0.05
92	A	1	1	1.	24	0.042
93	A	3	3	1.	22	0.136
94	A	3	3	1.	22	0.136
95	A	1	1	1.	20	0.05
96	A	1	1	1.	20	0.05
97	A	3	3	1.	18	0.167
98	A	3	3	1.	22	0.136
99	A	1	1	1.	35	0.029
100	A	1	1	1.	37	0.027
101	A	3	3	1.	38	0.079
102	A	3	3	1.	38	0.079
103	A	1	1	1.	32	0.031
104	A	1	1	1.	34	0.029
105	A	3	3	1.	35	0.086
106	A	3	3	1.	35	0.086
107	A	3	3	1.	17	0.176
108	A	3	3	1.	18	0.167
109	A	3	3	1.	19	0.158
110	A	3	3	1.	20	0.15
111	A	3	3	1.	15	0.2
112	A	3	3	1.	17	0.176
113	A	3	3	1.	15	0.2
114	A	3	3	1.	17	0.176
115	A	7	5	1.	16	0.312
116	A	13	9	1.	15	0.6
117	A	8	6	1.	16	0.375
118	A	14	10	1.	15	0.667
119	A	9	6	1.	16	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
120	A	15	10	1.	15	0.667
121	A	10	6	1.	16	0.375
122	A	16	10	1.	15	0.667
123	A	7	5	1.	15	0.333
124	A	13	9	1.	13	0.692
125	A	7	5	1.	21	0.238
126	A	13	9	1.	20	0.45
127	A	8	6	1.	21	0.286
128	A	14	10	1.	20	0.5
129	A	9	6	1.	21	0.286
130	A	15	10	1.	20	0.5
131	A	10	6	1.	21	0.286
132	A	16	10	1.	20	0.5
133	A	3	2	1.	11	0.182
134	A	3	2	1.	12	0.167
135	A	2	1	1.	15	0.067
136	A	3	2	1.	14	0.143
137	A	2	1	1.	17	0.059
138	A	3	2	1.	19	0.105
139	A	2	1	1.	20	0.05
140	A	2	2	1.	14	0.143
141	A	4	3	1.	17	0.176
142	A	4	3	1.	19	0.158
143	A	3	2	1.	20	0.1
144	A	4	3	1.	21	0.143
145	A	3	2	1.	22	0.091
146	A	4	3	1.	24	0.125
147	A	3	2	1.	25	0.08
148	A	3	2	1.	25	0.08
149	A	8	6	1.	26	0.231
150	A	9	7	1.	26	0.269
151	A	10	7	1.	26	0.269

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
152	A	10	7	1.	11	0.636
153	A	3	3	1.	12	0.25
154	A	13	9	1.	15	0.6
155	A	10	7	1.	14	0.5
156	A	9	6	1.	17	0.353
157	A	14	10	1.	19	0.526
158	A	13	9	1.	20	0.45
159	A	2	2	1.	14	0.143
160	A	12	8	1.	17	0.471
161	A	5	5	1.	19	0.263
162	A	15	11	1.	20	0.55
163	A	13	9	1.	21	0.429
164	A	12	8	1.	22	0.364
165	A	16	12	1.	24	0.5
166	A	15	11	1.	25	0.44
167	A	2	2	1.	19	0.105
168	A	11	8	1.	17	0.471
169	A	9	7	1.	20	0.35
170	A	15	11	1.	19	0.579
171	A	11	8	1.	31	0.258
172	A	8	6	1.	31	0.194
173	A	9	7	1.	31	0.226
174	A	10	8	1.	31	0.258
175	A	17	12	1.	30	0.4
176	A	14	10	1.	30	0.333
177	A	15	11	1.	30	0.367
178	A	16	12	1.	30	0.4
179	A	2	2	1.	21	0.095
180	A	2	2	1.	21	0.095
181	A	2	1	1.	19	0.053
182	A	2	2	1.	19	0.105
183	A	2	2	1.	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
184	A	2	2	1.	21	0.095
185	A	2	2	1.	21	0.095
186	A	13	9	1.	36	0.25
187	A	13	9	1.	41	0.22
188	A	13	9	1.	46	0.196
189	A	19	13	1.	35	0.371
190	A	19	13	1.	40	0.325
191	A	19	13	1.	45	0.289
192	A	8	6	1.	36	0.167
193	A	8	6	1.	41	0.146
194	A	10	8	1.	46	0.174
195	A	14	10	1.	35	0.286
196	A	14	10	1.	40	0.25
197	A	16	12	1.	45	0.267
198	A	9	7	1.	36	0.194
199	A	9	7	1.	41	0.171
200	A	9	7	1.	46	0.152
201	A	15	11	1.	35	0.314
202	A	15	11	1.	40	0.275
203	A	15	11	1.	45	0.244
204	A	10	8	1.	36	0.222
205	A	10	8	1.	41	0.195
206	A	10	8	1.	46	0.174
207	A	16	12	1.	35	0.343
208	A	16	12	1.	40	0.3
209	A	16	12	1.	45	0.267
210	A	6	5	1.	17	0.294
211	A	7	6	1.	18	0.333
212	A	7	6	1.	19	0.316
213	A	6	5	1.	20	0.25
214	A	8	7	1.	22	0.318
215	A	1	1	1.	23	0.043

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
216	A	1	1	1.	26	0.038
217	A	1	1	1.	28	0.036
218	A	1	1	1.	31	0.032
219	A	1	1	1.	15	0.067
220	A	12	10	1.	42	0.238
221	A	3	2	1.	11	0.182
222	A	3	2	1.	15	0.133
223	A	3	2	1.	30	0.067
224	A	3	2	1.	30	0.067
225	A	3	2	1.	30	0.067
226	A	3	2	1.	30	0.067
227	A	3	2	1.	30	0.067
228	A	3	2	1.	30	0.067
229	A	3	2	1.	30	0.067
230	A	3	2	1.	30	0.067
231	A	3	2	1.	30	0.067
232	A	3	2	1.	30	0.067
233	A	9	8	1.	30	0.267
234	A	9	8	1.	30	0.267
235	A	9	8	1.	30	0.267
236	A	9	8	1.	30	0.267
237	A	9	8	1.	30	0.267
238	A	9	8	1.	28	0.286
239	A	8	7	1.	27	0.259
240	A	8	7	1.	30	0.233
241	A	8	7	1.	30	0.233
242	A	8	7	1.	30	0.233
243	A	8	7	1.	30	0.233
244	A	8	7	1.	30	0.233
245	A	8	7	1.	30	0.233
246	A	8	7	1.	30	0.233
247	A	8	7	1.	30	0.233

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
248	A	8	7	1.	30	0.233
249	A	8	7	1.	30	0.233
250	A	8	7	1.	30	0.233
251	A	3	2	1.	30	0.067
252	A	3	2	1.	30	0.067
253	A	3	2	1.	30	0.067
254	A	3	2	1.	30	0.067
255	A	3	2	1.	30	0.067
256	A	3	2	1.	30	0.067
257	A	3	2	1.	30	0.067
258	A	3	2	1.	30	0.067
259	A	3	2	1.	30	0.067
260	A	9	8	1.	30	0.267
261	A	12	10	1.	30	0.333
262	A	9	8	1.	30	0.267
263	A	11	10	1.	30	0.333
264	A	9	8	1.	30	0.267
265	A	10	9	1.	28	0.321
266	A	9	9	1.	27	0.333
267	A	9	8	1.	30	0.267
268	A	9	8	1.	30	0.267
269	A	9	8	1.	30	0.267
270	A	9	8	1.	30	0.267
271	A	9	8	1.	30	0.267
272	A	9	8	1.	30	0.267
273	A	9	8	1.	30	0.267
274	A	9	8	1.	30	0.267
275	A	9	8	1.	30	0.267
276	A	3	2	1.	30	0.067
277	A	3	2	1.	30	0.067
278	A	3	2	1.	30	0.067
279	A	3	2	1.	30	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
280	A	3	2	1.	30	0.067
281	A	3	2	1.	30	0.067
282	A	3	2	1.	30	0.067
283	A	3	2	1.	30	0.067
284	A	3	2	1.	30	0.067
285	A	3	2	1.	30	0.067
286	A	10	9	1.	30	0.3
287	A	14	10	1.	30	0.333
288	A	10	9	1.	30	0.3
289	A	13	10	1.	30	0.333
290	A	10	9	1.	30	0.3
291	A	12	10	1.	30	0.333
292	A	10	10	1.	30	0.333
293	A	10	10	1.	28	0.357
294	A	9	9	1.	27	0.333
295	A	9	9	1.	30	0.3
296	A	9	9	1.	30	0.3
297	A	10	9	1.	30	0.3
298	A	10	9	1.	30	0.3
299	A	10	8	1.	30	0.267
300	A	10	8	1.	30	0.267
301	A	10	8	1.	30	0.267
302	A	10	8	1.	30	0.267
303	A	10	8	1.	30	0.267
304	A	8	7	1.	16	0.438
305	A	5	4	1.	16	0.25
306	A	8	7	1.	16	0.438
307	A	6	6	1.	14	0.429
308	A	6	5	1.	16	0.312
309	A	6	5	1.	16	0.312
310	A	3	2	1.	16	0.125
311	A	6	6	1.	14	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
312	A	6	6	1.	16	0.375
313	A	2	1	1.	21	0.048
314	A	2	1	1.	19	0.053
315	A	2	1	1.	18	0.056
316	A	2	1	1.	21	0.048
317	A	2	1	1.	21	0.048
318	A	2	1	1.	21	0.048
319	A	3	2	1.	23	0.087
320	A	3	2	1.	21	0.095
321	A	3	2	1.	20	0.1
322	A	2	1	1.	23	0.043
323	A	2	1	1.	23	0.043
324	A	2	1	1.	23	0.043
325	A	3	2	1.	23	0.087
326	A	3	2	1.	21	0.095
327	A	3	2	1.	20	0.1
328	A	2	1	1.	23	0.043
329	A	2	1	1.	23	0.043
330	A	2	1	1.	23	0.043
331	A	3	2	1.	23	0.087
332	A	3	2	1.	21	0.095
333	A	3	2	1.	20	0.1
334	A	2	1	1.	23	0.043
335	A	2	1	1.	23	0.043
336	A	2	1	1.	23	0.043
337	A	10	9	1.	23	0.391
338	A	10	9	1.	23	0.391
339	A	10	9	1.	21	0.429
340	A	8	8	1.	20	0.4
341	A	10	9	1.	23	0.391
342	A	10	9	1.	23	0.391
343	A	10	9	1.	23	0.391

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
344	A	7	7	1.	23	0.304
345	A	7	7	1.	21	0.333
346	A	7	7	1.	20	0.35
347	A	11	10	1.	23	0.435
348	A	11	10	1.	23	0.435
349	A	11	10	1.	23	0.435
350	A	11	10	1.	23	0.435
351	A	8	8	1.	23	0.348
352	A	8	8	1.	21	0.381
353	A	8	8	1.	20	0.4
354	A	12	10	1.	23	0.435
355	A	12	10	1.	23	0.435
356	A	12	10	1.	23	0.435
357	A	12	10	1.	23	0.435
358	A	9	8	1.	23	0.348
359	A	9	9	1.	21	0.429
360	A	9	8	1.	20	0.4
361	A	13	10	1.	23	0.435
362	A	13	10	1.	23	0.435
363	A	13	10	1.	23	0.435
364	A	13	10	1.	23	0.435
365	A	5	5	1.	20	0.25
366	A	4	4	1.	18	0.222
367	A	5	5	1.	20	0.25
368	A	4	4	1.	18	0.222
369	A	4	4	1.	27	0.148
370	A	4	4	1.	29	0.138
371	A	4	4	1.	28	0.143
372	A	4	4	1.	28	0.143
373	A	2	1	1.	36	0.028
374	A	2	1	1.	36	0.028
375	A	2	1	1.	36	0.028

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	2	1	1.	34	0.029
377	A	2	1	1.	33	0.03
378	A	2	1	1.	36	0.028
379	A	2	1	1.	36	0.028
380	A	2	1	1.	36	0.028
381	A	2	1	1.	36	0.028
382	A	2	1	1.	36	0.028
383	A	2	1	1.	38	0.026
384	A	2	1	1.	38	0.026
385	A	3	2	1.	38	0.053
386	A	3	2	1.	36	0.056
387	A	3	2	1.	35	0.057
388	A	3	2	1.	38	0.053
389	A	3	2	1.	38	0.053
390	A	3	2	1.	38	0.053
391	A	2	1	1.	38	0.026
392	A	2	1	1.	38	0.026
393	A	2	1	1.	38	0.026
394	A	2	1	1.	38	0.026
395	A	3	2	1.	38	0.053
396	A	3	2	1.	36	0.056
397	A	3	2	1.	35	0.057
398	A	3	2	1.	38	0.053
399	A	3	2	1.	38	0.053
400	A	3	2	1.	38	0.053
401	A	2	1	1.	38	0.026
402	A	2	1	1.	38	0.026
403	A	13	10	1.	38	0.263
404	A	13	10	1.	38	0.263
405	A	13	10	1.	38	0.263
406	A	13	10	1.	36	0.278
407	A	10	9	0.99	35	0.257

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
408	A	10	9	0.99	38	0.237
409	A	10	9	1.	38	0.237
410	A	10	9	0.99	38	0.237
411	A	10	9	0.99	38	0.237
412	A	11	10	1.	38	0.263
413	A	11	10	1.	38	0.263
414	A	11	10	0.99	38	0.263
415	A	11	10	1.	36	0.278
416	A	9	9	1.	35	0.257
417	A	11	10	0.99	38	0.263
418	A	11	10	1.	38	0.263
419	A	11	10	0.99	38	0.263
420	A	11	10	0.99	38	0.263
421	A	12	11	1.	38	0.29
422	A	10	10	1.	38	0.263
423	A	8	8	1.	38	0.21
424	A	8	8	1.	36	0.222
425	A	8	8	1.	35	0.229
426	A	12	10	0.99	38	0.263
427	A	12	10	1.	38	0.263
428	A	12	10	0.99	38	0.263
429	A	12	10	0.99	38	0.263
430	A	10	7	1.	25	0.28
431	A	8	7	1.	25	0.28
432	A	6	6	1.	23	0.261
433	A	5	5	1.	22	0.227
434	A	7	7	1.	25	0.28
435	A	8	8	1.	25	0.32
436	A	9	8	1.	25	0.32
437	A	8	7	1.	25	0.28
438	A	7	7	1.	25	0.28
439	A	6	6	1.	25	0.24

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
440	A	4	4	1.	25	0.16
441	A	6	6	1.	23	0.261
442	A	4	4	1.	22	0.182
443	A	10	10	1.	25	0.4
444	A	11	11	1.	25	0.44
445	A	13	9	1.	35	0.257
446	A	11	9	1.	35	0.257
447	A	9	8	1.	33	0.242
448	A	8	7	1.	32	0.219
449	A	11	11	1.	35	0.314
450	A	11	11	1.	35	0.314
451	A	10	9	1.	35	0.257
452	A	11	9	1.	35	0.257
453	A	12	9	1.	35	0.257
454	A	10	10	1.	35	0.286
455	A	11	10	1.	35	0.286
456	A	12	10	1.	35	0.286
457	A	13	10	1.	35	0.286
458	A	14	9	1.	35	0.257
459	A	12	9	1.	35	0.257
460	A	10	8	1.	33	0.242
461	A	9	7	1.	32	0.219
462	A	12	11	1.	35	0.314
463	A	12	11	1.	35	0.314
464	A	11	9	1.	35	0.257
465	A	12	9	1.	35	0.257
466	A	13	9	1.	35	0.257
467	A	11	11	1.	35	0.314
468	A	12	11	1.	35	0.314
469	A	13	11	1.	35	0.314
470	A	11	10	1.	35	0.286
471	A	12	10	1.	35	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
472	A	13	10	1.	35	0.286
473	A	14	10	1.	35	0.286
474	A	8	7	1.18	20	0.35
475	A	7	4	1.17	21	0.19
476	A	7	4	1.17	23	0.174
477	A	2	1	1.	23	0.043
478	A	2	1	1.	26	0.038
479	A	3	2	1.	25	0.08
480	A	3	2	1.	28	0.071
481	A	3	2	1.	25	0.08
482	A	3	2	1.	28	0.071
483	A	3	2	1.	25	0.08
484	A	3	2	1.	28	0.071
485	A	9	7	1.	26	0.269
486	A	12	9	1.	29	0.31
487	A	15	11	1.	25	0.44
488	A	18	13	1.	28	0.464
489	A	14	10	1.	25	0.4
490	A	14	10	1.	28	0.357
491	A	15	11	1.	25	0.44
492	A	15	11	1.	28	0.393
493	A	16	11	1.	25	0.44
494	A	16	11	1.	28	0.393
495	A	14	12	1.	30	0.4
496	A	13	11	1.	30	0.367
497	A	12	11	1.	30	0.367
498	A	12	11	1.	28	0.393
499	A	11	10	1.	27	0.37
500	A	14	13	1.	30	0.433
501	A	14	13	1.	30	0.433
502	A	14	13	1.	30	0.433
503	A	15	14	1.	30	0.467

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
504	A	13	13	1.	30	0.433
505	A	14	14	1.	30	0.467
506	A	12	12	1.	30	0.4
507	A	13	12	1.	30	0.4
508	A	14	13	1.	30	0.433
509	A	15	13	1.	30	0.433
510	A	16	12	1.	30	0.4
511	A	15	11	1.	30	0.367
512	A	14	11	1.	30	0.367
513	A	14	11	1.	28	0.393
514	A	13	10	1.	27	0.37
515	A	16	13	1.	30	0.433
516	A	16	14	1.	30	0.467
517	A	16	15	1.	30	0.5
518	A	16	14	1.	30	0.467
519	A	15	15	1.	30	0.5
520	A	15	15	1.	30	0.5
521	A	15	15	1.	30	0.5
522	A	16	16	1.	30	0.533
523	A	14	13	1.	30	0.433
524	A	15	14	1.	30	0.467
525	A	13	12	1.	30	0.4
526	A	14	12	1.	30	0.4
527	A	15	13	1.	30	0.433
528	A	16	13	1.	30	0.433
529	A	12	10	1.	30	0.333
530	A	11	9	1.	30	0.3
531	A	10	9	1.	30	0.3
532	A	10	9	1.	28	0.321
533	A	9	8	1.	27	0.296
534	A	12	11	1.	30	0.367
535	A	13	12	1.	30	0.4

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
536	A	11	10	1.	30	0.333
537	A	12	10	1.	30	0.333
538	A	13	11	1.	30	0.367
539	A	14	11	1.	30	0.367
540	A	12	11	1.	30	0.367
541	A	11	10	1.	30	0.333
542	A	10	9	1.	30	0.3
543	A	9	8	1.	30	0.267
544	A	10	9	1.	30	0.3
545	A	7	6	1.	28	0.214
546	A	4	4	1.	27	0.148
547	A	11	10	1.	30	0.333
548	A	13	12	1.	30	0.4
549	A	15	12	1.	30	0.4
550	A	17	13	1.	30	0.433
551	A	14	4	1.	30	0.133
552	A	12	8	1.19	25	0.32
553	A	13	7	1.	28	0.25
554	A	2	2	1.	22	0.091
555	A	2	2	1.	35	0.057
556	A	2	2	1.	35	0.057
557	A	2	2	1.	22	0.091
558	A	3	3	1.	25	0.12
559	A	6	5	1.	15	0.333
560	A	6	5	1.	15	0.333
561	A	7	6	1.	20	0.3
562	A	7	6	1.	20	0.3
563	A	7	7	1.	17	0.412
564	A	7	6	1.	25	0.24
565	A	11	6	1.	35	0.171
566	A	11	6	1.	35	0.171
567	A	8	5	1.	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
568	A	11	7	1.	25	0.28
569	A	17	7	1.	15	0.467
570	A	17	7	1.	15	0.467
571	A	14	7	1.	20	0.35
572	A	14	7	1.	20	0.35
573	A	15	9	1.	17	0.529
574	A	14	7	1.	25	0.28
575	A	13	7	1.	18	0.389
576	A	7	4	1.	36	0.111
577	A	4	3	1.	19	0.158
578	A	4	3	1.	19	0.158
579	A	4	2	1.	17	0.118
580	A	1	0	1.	9	0.
581	A	3	3	1.	19	0.158
582	A	3	3	1.	19	0.158
583	A	3	3	1.	19	0.158
584	A	13	4	1.	38	0.105
585	A	2	2	1.	58	0.034
586	A	10	3	1.	30	0.1
587	A	13	4	1.	36	0.111
588	A	4	4	1.	35	0.114
589	A	1	1	1.	46	0.022
590	A	10	8	1.	24	0.333
591	A	1	1	1.	48	0.021
592	A	1	1	1.	45	0.022
593	A	1	1	1.	69	0.014
594	A	1	1	1.	86	0.012

Chapter 3

Listing of integrals

3.1

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx}} dx$$

Optimal. Leaf size=72

$$\frac{2\sqrt{a+bx}(a^2e - abd + b^2c)}{b^3} + \frac{2(a+bx)^{3/2}(bd - 2ae)}{3b^3} + \frac{2e(a+bx)^{5/2}}{5b^3}$$

[Out] $(2*(b^2*c - a*b*d + a^2*e)*\text{Sqrt}[a + b*x])/b^3 + (2*(b*d - 2*a*e)*(a + b*x)^{(3/2)})/(3*b^3) + (2*e*(a + b*x)^{(5/2)})/(5*b^3)$

Rubi [A] time = 0.0330131, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$, Rules used = {698}

$$\frac{2\sqrt{a+bx}(a^2e - abd + b^2c)}{b^3} + \frac{2(a+bx)^{3/2}(bd - 2ae)}{3b^3} + \frac{2e(a+bx)^{5/2}}{5b^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/Sqrt[a + b*x], x]

[Out] $(2*(b^2*c - a*b*d + a^2*e)*\text{Sqrt}[a + b*x])/b^3 + (2*(b*d - 2*a*e)*(a + b*x)^{(3/2)})/(3*b^3) + (2*e*(a + b*x)^{(5/2)})/(5*b^3)$

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{c + dx + ex^2}{\sqrt{a + bx}} dx = \int \left(\frac{b^2c - abd + a^2e}{b^2\sqrt{a + bx}} + \frac{(bd - 2ae)\sqrt{a + bx}}{b^2} + \frac{e(a + bx)^{3/2}}{b^2} \right) dx$$

$$= \frac{2(b^2c - abd + a^2e)\sqrt{a + bx}}{b^3} + \frac{2(bd - 2ae)(a + bx)^{3/2}}{3b^3} + \frac{2e(a + bx)^{5/2}}{5b^3}$$

Mathematica [A] time = 0.0710323, size = 53, normalized size = 0.74

$$\frac{2\sqrt{a + bx} (8a^2e - 2ab(5d + 2ex) + b^2(15c + x(5d + 3ex)))}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/Sqrt[a + b*x], x]
```

```
[Out] (2*Sqrt[a + b*x]*(8*a^2*e - 2*a*b*(5*d + 2*e*x) + b^2*(15*c + x*(5*d + 3*e*
x))))/(15*b^3)
```

Maple [A] time = 0.023, size = 53, normalized size = 0.7

$$\frac{6ex^2b^2 - 8abex + 10b^2dx + 16a^2e - 20abd + 30b^2c}{15b^3} \sqrt{bx + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x+a)^(1/2), x)
```

```
[Out] 2/15*(b*x+a)^(1/2)*(3*b^2*e*x^2-4*a*b*e*x+5*b^2*d*x+8*a^2*e-10*a*b*d+15*b^2
*c)/b^3
```


Maxima [A] time = 0.954447, size = 104, normalized size = 1.44

$$\frac{2 \left(15 \sqrt{bx+ac} + \frac{5 \left((bx+a)^{\frac{3}{2}} - 3 \sqrt{bx+aa} \right) d}{b} + \frac{\left(3 (bx+a)^{\frac{5}{2}} - 10 (bx+a)^{\frac{3}{2}} a + 15 \sqrt{bx+aa^2} \right) e}{b^2} \right)}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="maxima")

[Out] 2/15*(15*sqrt(b*x + a)*c + 5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*d/b + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*e/b^2)/b

Fricas [A] time = 1.26161, size = 127, normalized size = 1.76

$$\frac{2 \left(3b^2ex^2 + 15b^2c - 10abd + 8a^2e + (5b^2d - 4abe)x \right) \sqrt{bx+a}}{15b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="fricas")

[Out] 2/15*(3*b^2*e*x^2 + 15*b^2*c - 10*a*b*d + 8*a^2*e + (5*b^2*d - 4*a*b*e)*x)*sqrt(b*x + a)/b^3

Sympy [A] time = 7.77256, size = 223, normalized size = 3.1

$$\frac{\left(\frac{2ac}{\sqrt{a+bx}} + \frac{2ad \left(-\frac{a}{\sqrt{a+bx}} - \sqrt{a+bx} \right)}{b} + \frac{2ae \left(\frac{a^2}{\sqrt{a+bx}} + 2a\sqrt{a+bx} - \frac{(a+bx)^{\frac{3}{2}}}{3} \right)}{b^2} + 2c \left(-\frac{a}{\sqrt{a+bx}} - \sqrt{a+bx} \right) + \frac{2d \left(\frac{a^2}{\sqrt{a+bx}} + 2a\sqrt{a+bx} - \frac{(a+bx)^{\frac{3}{2}}}{3} \right)}{b} + \frac{2e \left(-\frac{a^3}{\sqrt{a+bx}} - 3a^2\sqrt{a+bx} + a(a+bx)^{\frac{3}{2}} - \frac{(a+bx)^{\frac{5}{2}}}{5} \right)}{b^2} \right)}{\frac{cx + \frac{dx^2}{2} + \frac{ex^3}{3}}{\sqrt{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/(b*x+a)**(1/2),x)

```
[Out] Piecewise((-2*a*c/sqrt(a + b*x) + 2*a*d*(-a/sqrt(a + b*x) - sqrt(a + b*x))
/b + 2*a*e*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b*
*2 + 2*c*(-a/sqrt(a + b*x) - sqrt(a + b*x)) + 2*d*(a**2/sqrt(a + b*x) + 2*a
*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b + 2*e*(-a**3/sqrt(a + b*x) - 3*a**2*
sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**2)/b, Ne(b, 0))
, ((c*x + d*x**2/2 + e*x**3/3)/sqrt(a), True))
```

Giac [A] time = 1.1084, size = 105, normalized size = 1.46

$$\frac{2 \left(15 \sqrt{bx + ac} + \frac{5 \left((bx+a)^{\frac{3}{2}} - 3 \sqrt{bx+aa} \right) d}{b} + \frac{\left(3 (bx+a)^{\frac{5}{2}} - 10 (bx+a)^{\frac{3}{2}} a + 15 \sqrt{bx+aa^2} \right) e}{b^2} \right)}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(b*x + a)*c + 5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*d/b + (3
*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*e/b^2)/b
```

$$3.2 \quad \int \frac{(c+dx+ex^2)^2}{\sqrt{a+bx}} dx$$

Optimal. Leaf size=161

$$\frac{2(a+bx)^{5/2}(-6a^2e^2+6abde+b^2(-2ce+d^2))}{5b^5} + \frac{4(a+bx)^{3/2}(bd-2ae)(a^2e-abd+b^2c)}{3b^5} + \frac{2\sqrt{a+bx}(a^2e-abd+b^2c)}{b^5}$$

[Out] (2*(b^2*c - a*b*d + a^2*e)^2*Sqrt[a + b*x])/b^5 + (4*(b*d - 2*a*e)*(b^2*c - a*b*d + a^2*e)*(a + b*x)^(3/2))/(3*b^5) - (2*(6*a*b*d*e - 6*a^2*e^2 - b^2*(d^2 + 2*c*e))*(a + b*x)^(5/2))/(5*b^5) + (4*e*(b*d - 2*a*e)*(a + b*x)^(7/2))/(7*b^5) + (2*e^2*(a + b*x)^(9/2))/(9*b^5)

Rubi [A] time = 0.105497, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {698}

$$\frac{2(a+bx)^{5/2}(-6a^2e^2+6abde+b^2(-2ce+d^2))}{5b^5} + \frac{4(a+bx)^{3/2}(bd-2ae)(a^2e-abd+b^2c)}{3b^5} + \frac{2\sqrt{a+bx}(a^2e-abd+b^2c)}{b^5}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)^2/Sqrt[a + b*x], x]

[Out] (2*(b^2*c - a*b*d + a^2*e)^2*Sqrt[a + b*x])/b^5 + (4*(b*d - 2*a*e)*(b^2*c - a*b*d + a^2*e)*(a + b*x)^(3/2))/(3*b^5) - (2*(6*a*b*d*e - 6*a^2*e^2 - b^2*(d^2 + 2*c*e))*(a + b*x)^(5/2))/(5*b^5) + (4*e*(b*d - 2*a*e)*(a + b*x)^(7/2))/(7*b^5) + (2*e^2*(a + b*x)^(9/2))/(9*b^5)

Rule 698

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_ Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))

Rubi steps

$$\int \frac{(c + dx + ex^2)^2}{\sqrt{a + bx}} dx = \int \left(\frac{(b^2c - abd + a^2e)^2}{b^4\sqrt{a + bx}} + \frac{2(bd - 2ae)(b^2c - abd + a^2e)\sqrt{a + bx}}{b^4} + \frac{(-6abde + 6a^2e^2 + b^2(d^2 + 2e^2))}{b^4} \right) dx$$

$$= \frac{2(b^2c - abd + a^2e)^2\sqrt{a + bx}}{b^5} + \frac{4(bd - 2ae)(b^2c - abd + a^2e)(a + bx)^{3/2}}{3b^5} - \frac{2(6abde - 6a^2e^2 - b^2(d^2 + 2e^2))}{3b^5}$$

Mathematica [A] time = 0.15182, size = 155, normalized size = 0.96

$$\frac{2\sqrt{a + bx}(24a^2b^2(2e(7c + ex^2) + 7d^2 + 6dex) - 32a^3be(9d + 2ex) + 128a^4e^2 - 4ab^3(21c(5d + 2ex) + x(21d^2 + 27dex + 2e^2))) + b^4(315c^2 + 42cx(5d + 3ex) + x^2(63d^2 + 90dex + 35e^2))}{315b^5}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)^2/Sqrt[a + b*x], x]

[Out] (2*Sqrt[a + b*x]*(128*a^4*e^2 - 32*a^3*b*e*(9*d + 2*e*x) + 24*a^2*b^2*(7*d^2 + 6*d*e*x + 2*e*(7*c + e*x^2)) - 4*a*b^3*(21*c*(5*d + 2*e*x) + x*(21*d^2 + 27*d*e*x + 10*e^2*x^2)) + b^4*(315*c^2 + 42*c*x*(5*d + 3*e*x) + x^2*(63*d^2 + 90*d*e*x + 35*e^2*x^2)))/(315*b^5)

Maple [A] time = 0.005, size = 194, normalized size = 1.2

$$\frac{70e^2x^4b^4 - 80ab^3e^2x^3 + 180b^4dex^3 + 96a^2b^2e^2x^2 - 216ab^3dex^2 + 252b^4cex^2 + 126b^4d^2x^2 - 128a^3be^2x + 288a^2b^2dex - 288a^3bde + 336a^2b^2c^2 + 168a^2b^2d^2 - 420ab^3cd + 315b^4c^2}{315b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)^2/(b*x+a)^(1/2), x)

[Out] 2/315*(b*x+a)^(1/2)*(35*b^4*e^2*x^4-40*a*b^3*e^2*x^3+90*b^4*d*e*x^3+48*a^2*b^2*e^2*x^2-108*a*b^3*d*e*x^2+126*b^4*c*e*x^2+63*b^4*d^2*x^2-64*a^3*b*e^2*x+144*a^2*b^2*d*e*x-168*a*b^3*c*e*x-84*a*b^3*d^2*x+210*b^4*c*d*x+128*a^4*e^2-288*a^3*b*d*e+336*a^2*b^2*c^2+168*a^2*b^2*d^2-420*a*b^3*c*d+315*b^4*c^2)/b^5

[In] integrate((e*x**2+d*x+c)**2/(b*x+a)**(1/2),x)

[Out] Piecewise((-2*a*c**2/sqrt(a + b*x) + 4*a*c*d*(-a/sqrt(a + b*x) - sqrt(a + b*x))/b + 4*a*c*e*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b**2 + 2*a*d**2*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b**2 + 4*a*d*e*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**3 + 2*a*e**2*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b**4 + 2*c**2*(-a/sqrt(a + b*x) - sqrt(a + b*x)) + 4*c*d*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b + 4*c*e*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**2 + 2*d**2*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**2 + 4*d*e*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b**3 + 2*e**2*(-a**5/sqrt(a + b*x) - 5*a**4*sqrt(a + b*x) + 10*a**3*(a + b*x)**(3/2)/3 - 2*a**2*(a + b*x)**(5/2) + 5*a*(a + b*x)**(7/2)/7 - (a + b*x)**(9/2)/9)/b**4)/b, Ne(b, 0)), ((c**2*x + c*d*x**2 + d*e*x**4/2 + e**2*x**5/5 + x**3*(2*c*e + d**2)/3)/sqrt(a), True))

Giac [A] time = 1.1067, size = 320, normalized size = 1.99

$$2 \left(315 \sqrt{bx + ac^2} + \frac{210 \left((bx+a)^{\frac{3}{2}} - 3 \sqrt{bx+aa} \right) cd}{b} + \frac{21 \left(3 (bx+a)^{\frac{5}{2}} - 10 (bx+a)^{\frac{3}{2}} a + 15 \sqrt{bx+aa^2} \right) d^2}{b^2} + \frac{42 \left(3 (bx+a)^{\frac{5}{2}} - 10 (bx+a)^{\frac{3}{2}} a + 15 \sqrt{bx+aa^2} \right) ce}{b^2} + \dots \right)$$

315 b

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="giac")

[Out] 2/315*(315*sqrt(b*x + a)*c^2 + 210*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*c*d/b + 21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*d^2/b^2 + 42*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*c*e/b^2 + 18*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d*e/b^3 + (35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*e^2/b^4)/b

$$3.3 \quad \int \frac{(c+dx+ex^2)^3}{\sqrt{a+bx}} dx$$

Optimal. Leaf size=274

$$\frac{2e(a+bx)^{9/2}(-5a^2e^2+5abde+b^2(-ce+d^2))}{3b^7} - \frac{2(a+bx)^{7/2}(bd-2ae)(-10a^2e^2+10abde+b^2(-6ce+d^2))}{7b^7} - 6$$

[Out] $(2*(b^2*c - a*b*d + a^2*e)^3*\text{Sqrt}[a + b*x])/b^7 + (2*(b*d - 2*a*e)*(b^2*c - a*b*d + a^2*e)^2*(a + b*x)^{(3/2)})/b^7 - (6*(b^2*c - a*b*d + a^2*e)*(5*a*b*d*e - 5*a^2*e^2 - b^2*(d^2 + c*e))*(a + b*x)^{(5/2)})/(5*b^7) - (2*(b*d - 2*a*e)*(10*a*b*d*e - 10*a^2*e^2 - b^2*(d^2 + 6*c*e))*(a + b*x)^{(7/2)})/(7*b^7) - (2*e*(5*a*b*d*e - 5*a^2*e^2 - b^2*(d^2 + c*e))*(a + b*x)^{(9/2)})/(3*b^7) + (6*e^2*(b*d - 2*a*e)*(a + b*x)^{(11/2)})/(11*b^7) + (2*e^3*(a + b*x)^{(13/2)})/(13*b^7)$

Rubi [A] time = 0.193835, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {698}

$$\frac{2e(a+bx)^{9/2}(-5a^2e^2+5abde+b^2(-ce+d^2))}{3b^7} - \frac{2(a+bx)^{7/2}(bd-2ae)(-10a^2e^2+10abde+b^2(-6ce+d^2))}{7b^7} - 6$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)^3/Sqrt[a + b*x], x]

[Out] $(2*(b^2*c - a*b*d + a^2*e)^3*\text{Sqrt}[a + b*x])/b^7 + (2*(b*d - 2*a*e)*(b^2*c - a*b*d + a^2*e)^2*(a + b*x)^{(3/2)})/b^7 - (6*(b^2*c - a*b*d + a^2*e)*(5*a*b*d*e - 5*a^2*e^2 - b^2*(d^2 + c*e))*(a + b*x)^{(5/2)})/(5*b^7) - (2*(b*d - 2*a*e)*(10*a*b*d*e - 10*a^2*e^2 - b^2*(d^2 + 6*c*e))*(a + b*x)^{(7/2)})/(7*b^7) - (2*e*(5*a*b*d*e - 5*a^2*e^2 - b^2*(d^2 + c*e))*(a + b*x)^{(9/2)})/(3*b^7) + (6*e^2*(b*d - 2*a*e)*(a + b*x)^{(11/2)})/(11*b^7) + (2*e^3*(a + b*x)^{(13/2)})/(13*b^7)$

Rule 698

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_ Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]

$$2e^x x^4 - 1600a^3 b^3 e^3 x^3 + 5200a^2 b^4 d e^2 x^3 - 5720a b^5 c e^2 x^3 - 5720a^2 b^5 d^2 e^2 x^3 + 12870b^6 c d e^2 x^3 + 2145b^6 d^3 x^3 + 1920a^4 b^2 e^3 x^2 - 6240a^3 b^3 d e^2 x^2 + 6864a^2 b^4 c e^2 x^2 + 6864a^2 b^4 d^2 e^2 x^2 - 15444a b^5 c d e^2 x^2 - 2574a b^5 d^3 x^2 + 9009b^6 c^2 e^2 x^2 + 9009b^6 c d^2 x^2 - 2560a^5 b e^3 x + 8320a^4 b^2 d e^2 x - 9152a^3 b^3 c e^2 x - 9152a^3 b^3 d^2 e^2 x + 20592a^2 b^4 c d e^2 x + 3432a^2 b^4 d^3 x - 12012a b^5 c^2 e^2 x - 12012a b^5 c d^2 x + 15015b^6 c^2 d x + 5120a^6 e^3 - 16640a^5 b d e^2 + 18304a^4 b^2 c e^2 + 18304a^4 b^2 d^2 e - 41184a^3 b^3 c d e - 6864a^3 b^3 d^3 + 24024a^2 b^4 c^2 e + 24024a^2 b^4 c d^2 - 30030a b^5 c^2 d + 15015b^6 c^3) / b^7$$

Maxima [B] time = 0.957792, size = 709, normalized size = 2.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="maxima")

[Out]
$$\frac{2}{15015} \left(15015 \sqrt{bx+a} c^3 + 3003 c^2 (5((bx+a)^{3/2} - 3\sqrt{bx+a}) a) d/b + (3(bx+a)^{5/2} - 10(bx+a)^{3/2} a + 15\sqrt{bx+a}) a^2 e/b^2 + 143 c (21(3(bx+a)^{5/2} - 10(bx+a)^{3/2} a + 15\sqrt{bx+a}) a^2) d^2/b^2 + 18(5(bx+a)^{7/2} - 21(bx+a)^{5/2} a + 35(bx+a)^{3/2} a^2 - 35\sqrt{bx+a}) a^3) d e/b^3 + (35(bx+a)^{9/2} - 180(bx+a)^{7/2} a + 378(bx+a)^{5/2} a^2 - 420(bx+a)^{3/2} a^3 + 315\sqrt{bx+a}) a^4) e^2/b^4 + 429(5(bx+a)^{7/2} - 21(bx+a)^{5/2} a + 35(bx+a)^{3/2} a^2 - 35\sqrt{bx+a}) a^3) d^3/b^3 + 143(35(bx+a)^{9/2} - 180(bx+a)^{7/2} a + 378(bx+a)^{5/2} a^2 - 420(bx+a)^{3/2} a^3 + 315\sqrt{bx+a}) a^4) d^2 e/b^4 + 65(63(bx+a)^{11/2} - 385(bx+a)^{9/2} a + 990(bx+a)^{7/2} a^2 - 1386(bx+a)^{5/2} a^3 + 1155(bx+a)^{3/2} a^4 - 693\sqrt{bx+a}) a^5) d e^2/b^5 + 5(231(bx+a)^{13/2} - 1638(bx+a)^{11/2} a + 5005(bx+a)^{9/2} a^2 - 8580(bx+a)^{7/2} a^3 + 9009(bx+a)^{5/2} a^4 - 6006(bx+a)^{3/2} a^5 + 3003\sqrt{bx+a}) a^6) e^3/b^6) / b$$

Fricas [A] time = 1.2812, size = 1067, normalized size = 3.89

$$2 \left(1155 b^6 e^3 x^6 + 15015 b^6 c^3 - 30030 a b^5 c^2 d + 24024 a^2 b^4 c d^2 - 6864 a^3 b^3 d^3 + 5120 a^6 e^3 + 315 (13 b^6 d e^2 - 4 a b^5 e^3) x^5 + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.


```

2)/5 - (a + b*x)**(7/2)/7)/b**3 + 2*d**3*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(
a + b*x) - 2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7
/2)/7)/b**3 + 6*c*e**2*(-a**5/sqrt(a + b*x) - 5*a**4*sqrt(a + b*x) + 10*a**
3*(a + b*x)**(3/2)/3 - 2*a**2*(a + b*x)**(5/2) + 5*a*(a + b*x)**(7/2)/7 - (
a + b*x)**(9/2)/9)/b**4 + 6*d**2*e*(-a**5/sqrt(a + b*x) - 5*a**4*sqrt(a + b
*x) + 10*a**3*(a + b*x)**(3/2)/3 - 2*a**2*(a + b*x)**(5/2) + 5*a*(a + b*x)*
*(7/2)/7 - (a + b*x)**(9/2)/9)/b**4 + 6*d*e**2*(a**6/sqrt(a + b*x) + 6*a**5
*sqrt(a + b*x) - 5*a**4*(a + b*x)**(3/2) + 4*a**3*(a + b*x)**(5/2) - 15*a**
2*(a + b*x)**(7/2)/7 + 2*a*(a + b*x)**(9/2)/3 - (a + b*x)**(11/2)/11)/b**5
+ 2*e**3*(-a**7/sqrt(a + b*x) - 7*a**6*sqrt(a + b*x) + 7*a**5*(a + b*x)**(3
/2) - 7*a**4*(a + b*x)**(5/2) + 5*a**3*(a + b*x)**(7/2) - 7*a**2*(a + b*x)*
*(9/2)/3 + 7*a*(a + b*x)**(11/2)/11 - (a + b*x)**(13/2)/13)/b**6)/b, Ne(b,
0)), ((c**3*x + 3*c**2*d*x**2/2 + d*e**2*x**6/2 + e**3*x**7/7 + x**5*(3*c*e
**2 + 3*d**2*e)/5 + x**4*(6*c*d*e + d**3)/4 + x**3*(3*c**2*e + 3*c*d**2)/3)
/sqrt(a), True))

```

Giac [B] time = 1.11144, size = 710, normalized size = 2.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="giac")
```

```

[Out] 2/15015*(15015*sqrt(b*x + a)*c^3 + 15015*((b*x + a)^(3/2) - 3*sqrt(b*x + a)
*a)*c^2*d/b + 3003*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x
+ a)*a^2)*c*d^2/b^2 + 3003*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*s
qrt(b*x + a)*a^2)*c^2*e/b^2 + 429*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a
+ 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d^3/b^3 + 2574*(5*(b*x +
a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)
*a^3)*c*d*e/b^3 + 143*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*
x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*d^2*e/b
^4 + 143*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*
a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*c*e^2/b^4 + 65*(63*(
b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b
*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*d*e^2
/b^5 + 5*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(
9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x
+ a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*e^3/b^6)/b

```

$$3.4 \quad \int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}} dx$$

Optimal. Leaf size=114

$$\frac{2\sqrt{a+bx}(a^2be+a^3(-f)-ab^2d+b^3c)}{b^4} + \frac{2(a+bx)^{3/2}(3a^2f-2abe+b^2d)}{3b^4} + \frac{2(a+bx)^{5/2}(be-3af)}{5b^4} + \frac{2f(a+bx)^{7/2}}{7b^4}$$

[Out] (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x])/b^4 + (2*(b^2*d - 2*a*b*e + 3*a^2*f)*(a + b*x)^(3/2))/(3*b^4) + (2*(b*e - 3*a*f)*(a + b*x)^(5/2))/(5*b^4) + (2*f*(a + b*x)^(7/2))/(7*b^4)

Rubi [A] time = 0.0707695, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$, Rules used = {1850}

$$\frac{2\sqrt{a+bx}(a^2be+a^3(-f)-ab^2d+b^3c)}{b^4} + \frac{2(a+bx)^{3/2}(3a^2f-2abe+b^2d)}{3b^4} + \frac{2(a+bx)^{5/2}(be-3af)}{5b^4} + \frac{2f(a+bx)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3)/Sqrt[a + b*x], x]

[Out] (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x])/b^4 + (2*(b^2*d - 2*a*b*e + 3*a^2*f)*(a + b*x)^(3/2))/(3*b^4) + (2*(b*e - 3*a*f)*(a + b*x)^(5/2))/(5*b^4) + (2*f*(a + b*x)^(7/2))/(7*b^4)

Rule 1850

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])]

Rubi steps

$$\begin{aligned} \int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}} dx &= \int \left(\frac{b^3c-ab^2d+a^2be-a^3f}{b^3\sqrt{a+bx}} + \frac{(b^2d-2abe+3a^2f)\sqrt{a+bx}}{b^3} + \frac{(be-3af)(a+bx)^{3/2}}{b^3} + \frac{f(a+bx)^{5/2}}{5b^3} \right) dx \\ &= \frac{2(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx}}{b^4} + \frac{2(b^2d-2abe+3a^2f)(a+bx)^{3/2}}{3b^4} + \frac{2(be-3af)(a+bx)^{5/2}}{5b^4} + \frac{2f(a+bx)^{7/2}}{7b^4} \end{aligned}$$

Mathematica [A] time = 0.125911, size = 82, normalized size = 0.72

$$\frac{2\sqrt{a+bx}\left(8a^2b(7e+3fx)-48a^3f-2ab^2(35d+x(14e+9fx))+b^3(105c+x(35d+3x(7e+5fx)))\right)}{105b^4}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3)/Sqrt[a + b*x], x]

[Out] (2*Sqrt[a + b*x]*(-48*a^3*f + 8*a^2*b*(7*e + 3*f*x) - 2*a*b^2*(35*d + x*(14*e + 9*f*x)) + b^3*(105*c + x*(35*d + 3*x*(7*e + 5*f*x))))/(105*b^4)

Maple [A] time = 0.045, size = 91, normalized size = 0.8

$$\frac{-30fx^3b^3 + 36ab^2fx^2 - 42b^3ex^2 - 48a^2bfx + 56ab^2ex - 70b^3dx + 96a^3f - 112a^2be + 140ab^2d - 210b^3c}{105b^4} \sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)/(b*x+a)^(1/2), x)

[Out] -2/105*(b*x+a)^(1/2)*(-15*b^3*f*x^3+18*a*b^2*f*x^2-21*b^3*e*x^2-24*a^2*b*f*x+28*a*b^2*e*x-35*b^3*d*x+48*a^3*f-56*a^2*b*e+70*a*b^2*d-105*b^3*c)/b^4

Maxima [A] time = 0.961995, size = 173, normalized size = 1.52

$$\frac{2\left(105\sqrt{bx+ac} + \frac{35\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+aa}\right)d}{b} + \frac{7\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+aa^2}\right)e}{b^2} + \frac{3\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+aa^3}\right)}{b^3}\right)}{105b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x+a)^(1/2), x, algorithm="maxima")

[Out] 2/105*(105*sqrt(b*x + a)*c + 35*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*d/b + 7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*e/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*f/b^3)/b

Fricas [A] time = 1.19074, size = 216, normalized size = 1.89

$$\frac{2(15b^3fx^3 + 105b^3c - 70ab^2d + 56a^2be - 48a^3f + 3(7b^3e - 6ab^2f)x^2 + (35b^3d - 28ab^2e + 24a^2bf)x)\sqrt{bx+a}}{105b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="fricas")

[Out] 2/105*(15*b^3*f*x^3 + 105*b^3*c - 70*a*b^2*d + 56*a^2*b*e - 48*a^3*f + 3*(7*b^3*e - 6*a*b^2*f)*x^2 + (35*b^3*d - 28*a*b^2*e + 24*a^2*b*f)*x)*sqrt(b*x + a)/b^4

Sympy [A] time = 15.2014, size = 354, normalized size = 3.11

$$\left\{ \begin{array}{l} \frac{\frac{2ac}{\sqrt{a+bx}} + \frac{2ad\left(-\frac{a}{\sqrt{a+bx}} - \sqrt{a+bx}\right)}{b} + \frac{2ae\left(\frac{a^2}{\sqrt{a+bx}} + 2a\sqrt{a+bx} - \frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b^2} + \frac{2af\left(-\frac{a^3}{\sqrt{a+bx}} - 3a^2\sqrt{a+bx} + a(a+bx)^{\frac{3}{2}} - \frac{(a+bx)^{\frac{5}{2}}}{5}\right)}{b^3} + 2c\left(-\frac{a}{\sqrt{a+bx}} - \sqrt{a+bx}\right) + \frac{2d\left(\frac{a^2}{\sqrt{a+bx}} + 2a\sqrt{a+bx} - \frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b} \right. \\ \left. cx + \frac{dx^2}{2} + \frac{ex^3}{3} + \frac{fx^4}{4} \right\} / \sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x+a)**(1/2),x)

[Out] Piecewise((-2*a*c/sqrt(a + b*x) + 2*a*d*(-a/sqrt(a + b*x) - sqrt(a + b*x)) /b + 2*a*e*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b* *2 + 2*a*f*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**3 + 2*c*(-a/sqrt(a + b*x) - sqrt(a + b*x)) + 2*d*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b + 2*e*(-a** 3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5 /2)/5)/b**2 + 2*f*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b**3)/b, Ne(b, 0)), ((c*x + d*x**2/2 + e*x**3/3 + f*x**4/4)/sqrt(a), True))

Giac [A] time = 1.08042, size = 174, normalized size = 1.53

$$2 \left(105 \sqrt{bx + ac} + \frac{35 \left((bx+a)^{\frac{3}{2}} - 3 \sqrt{bx+aa} \right) d}{b} + \frac{7 \left(3 (bx+a)^{\frac{5}{2}} - 10 (bx+a)^{\frac{3}{2}} a + 15 \sqrt{bx+aa^2} \right) e}{b^2} + \frac{3 \left(5 (bx+a)^{\frac{7}{2}} - 21 (bx+a)^{\frac{5}{2}} a + 35 (bx+a)^{\frac{3}{2}} a^2 - 35 \sqrt{bx+aa^3} \right) f}{b^3} \right) / 105 b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="giac")

[Out] 2/105*(105*sqrt(b*x + a)*c + 35*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*d/b + 7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*e/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*f/b^3)/b

, 0] || EqQ[n, 1])

Rubi steps

$$\int \frac{(c + dx + ex^2 + fx^3)^2}{\sqrt{a + bx}} dx = \int \left(\frac{(b^3c - ab^2d + a^2be - a^3f)^2}{b^6\sqrt{a + bx}} + \frac{2(b^2d - 2abe + 3a^2f)(b^3c - ab^2d + a^2be - a^3f)\sqrt{a + bx}}{b^6} \right) dx$$

$$= \frac{2(b^3c - ab^2d + a^2be - a^3f)^2\sqrt{a + bx}}{b^7} + \frac{4(b^2d - 2abe + 3a^2f)(b^3c - ab^2d + a^2be - a^3f)}{3b^7}$$

Mathematica [A] time = 0.457357, size = 303, normalized size = 0.95

$$2 \left(\frac{1}{5} (a + bx)^{5/2} (6a^2b^2(2df + e^2) - 20a^3bef + 15a^4f^2 - 6ab^3(cf + de) + b^4(2ce + d^2)) + \frac{2}{7} (a + bx)^{7/2} (10a^2bef - 10a^3f^2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3)^2/Sqrt[a + b*x], x]

[Out] (2*((b^3*c - a*b^2*d + a^2*b*e - a^3*f)^2*Sqrt[a + b*x] + (2*(b^2*d - 2*a*b*e + 3*a^2*f)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a + b*x)^(3/2))/3 + ((b^4*(d^2 + 2*c*e) - 20*a^3*b*e*f + 15*a^4*f^2 - 6*a*b^3*(d*e + c*f) + 6*a^2*b^2*(e^2 + 2*d*f))*(a + b*x)^(5/2))/5 + (2*(10*a^2*b*e*f - 10*a^3*f^2 + b^3*(d*e + c*f) - 2*a*b^2*(e^2 + 2*d*f))*(a + b*x)^(7/2))/7 - ((10*a*b*e*f - 15*a^2*f^2 - b^2*(e^2 + 2*d*f))*(a + b*x)^(9/2))/9 + (2*f*(b*e - 3*a*f)*(a + b*x)^(11/2))/11 + (f^2*(a + b*x)^(13/2))/13))/b^7

Maple [A] time = 0.007, size = 447, normalized size = 1.4

$$6930 f^2 x^6 b^6 - 7560 a b^5 f^2 x^5 + 16380 b^6 e f x^5 + 8400 a^2 b^4 f^2 x^4 - 18200 a b^5 e f x^4 + 20020 b^6 d f x^4 + 10010 b^6 e^2 x^4 - 9600$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)^2/(b*x+a)^(1/2), x)

[Out] 2/45045*(b*x+a)^(1/2)*(3465*b^6*f^2*x^6-3780*a*b^5*f^2*x^5+8190*b^6*e*f*x^5+4200*a^2*b^4*f^2*x^4-9100*a*b^5*e*f*x^4+10010*b^6*d*f*x^4+5005*b^6*e^2*x^4)

$$\begin{aligned}
& -4800a^3b^3f^2x^3 + 10400a^2b^4e*fx^3 - 11440a*b^5d*fx^3 - 5720a*b^5e^2x^3 + 12870b^6c*fx^3 + 12870b^6d*e*fx^3 + 5760a^4b^2f^2x^2 - 12480a^3b^3e*fx^2 + 13728a^2b^4d*fx^2 + 6864a^2b^4e^2x^2 - 15444a*b^5c*fx^2 - 15444a*b^5d*e*fx^2 + 18018b^6c*e*fx^2 + 9009b^6d^2x^2 - 7680a^5b*fx + 16640a^4b^2e*fx - 18304a^3b^3d*fx - 9152a^3b^3e^2x + 20592a^2b^4c*fx + 20592a^2b^4d*e*fx - 24024a*b^5c*e*fx - 12012a*b^5d^2x + 30030b^6c*d*x + 15360a^6f^2 - 33280a^5b*e*fx + 36608a^4b^2d*fx + 18304a^4b^2e^2 - 41184a^3b^3c*fx - 41184a^3b^3d*e + 48048a^2b^4c*e + 24024a^2b^4d^2 - 60060a*b^5c*d + 45045b^6c^2)/b^7
\end{aligned}$$

Maxima [A] time = 0.991374, size = 675, normalized size = 2.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="maxima")

[Out] $2/45045*(45045*\sqrt{b*x + a}*c^2 + 858*c*(35*((b*x + a)^{(3/2)} - 3*\sqrt{b*x + a})*a)*d/b + 7*(3*(b*x + a)^{(5/2)} - 10*(b*x + a)^{(3/2)}*a + 15*\sqrt{b*x + a})*a^2)*e/b^2 + 3*(5*(b*x + a)^{(7/2)} - 21*(b*x + a)^{(5/2)}*a + 35*(b*x + a)^{(3/2)}*a^2 - 35*\sqrt{b*x + a})*a^3)*f/b^3 + 3003*(3*(b*x + a)^{(5/2)} - 10*(b*x + a)^{(3/2)}*a + 15*\sqrt{b*x + a})*a^2)*d^2/b^2 + 143*(35*(b*x + a)^{(9/2)} - 180*(b*x + a)^{(7/2)}*a + 378*(b*x + a)^{(5/2)}*a^2 - 420*(b*x + a)^{(3/2)}*a^3 + 315*\sqrt{b*x + a})*a^4)*e^2/b^4 + 286*(35*(b*x + a)^{(9/2)}*f + 45*(b*e - 4*a*f)*(b*x + a)^{(7/2)} - 189*(a*b*e - 2*a^2*f)*(b*x + a)^{(5/2)} + 105*(3*a^2*b*e - 4*a^3*f)*(b*x + a)^{(3/2)} - 315*(a^3*b*e - a^4*f)*\sqrt{b*x + a})*d/b^4 + 130*(63*(b*x + a)^{(11/2)} - 385*(b*x + a)^{(9/2)}*a + 990*(b*x + a)^{(7/2)}*a^2 - 1386*(b*x + a)^{(5/2)}*a^3 + 1155*(b*x + a)^{(3/2)}*a^4 - 693*\sqrt{b*x + a})*a^5)*e*f/b^5 + 15*(231*(b*x + a)^{(13/2)} - 1638*(b*x + a)^{(11/2)}*a + 5005*(b*x + a)^{(9/2)}*a^2 - 8580*(b*x + a)^{(7/2)}*a^3 + 9009*(b*x + a)^{(5/2)}*a^4 - 6006*(b*x + a)^{(3/2)}*a^5 + 3003*\sqrt{b*x + a})*a^6)*f^2/b^6)/b$

Fricas [A] time = 1.26904, size = 1000, normalized size = 3.12

$$2(3465b^6f^2x^6 + 45045b^6c^2 - 60060ab^5cd + 24024a^2b^4d^2 + 18304a^4b^2e^2 + 15360a^6f^2 + 630(13b^6ef - 6ab^5f^2)x^5 + \dots)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3465*b^6*f^2*x^6 + 45045*b^6*c^2 - 60060*a*b^5*c*d + 24024*a^2*b^4*d^2 + 18304*a^4*b^2*e^2 + 15360*a^6*f^2 + 630*(13*b^6*e*f - 6*a*b^5*f^2)*x^5 + 35*(143*b^6*e^2 + 120*a^2*b^4*f^2 + 26*(11*b^6*d - 10*a*b^5*e)*f)*x^4 + 10*(1287*b^6*d*e - 572*a*b^5*e^2 - 480*a^3*b^3*f^2 + 13*(99*b^6*c - 88*a*b^5*d + 80*a^2*b^4*e)*f)*x^3 + 3*(3003*b^6*d^2 + 2288*a^2*b^4*e^2 + 1920*a^4*b^2*f^2 + 858*(7*b^6*c - 6*a*b^5*d)*e - 52*(99*a*b^5*c - 88*a^2*b^4*d + 80*a^3*b^3*e)*f)*x^2 + 6864*(7*a^2*b^4*c - 6*a^3*b^3*d)*e - 416*(99*a^3*b^3*c - 88*a^4*b^2*d + 80*a^5*b*e)*f + 2*(15015*b^6*c*d - 6006*a*b^5*d^2 - 4576*a^3*b^3*e^2 - 3840*a^5*b*f^2 - 1716*(7*a*b^5*c - 6*a^2*b^4*d)*e + 104*(99*a^2*b^4*c - 88*a^3*b^3*d + 80*a^4*b^2*e)*f)*x)*sqrt(b*x + a)/b^7
```

Sympy [A] time = 117.588, size = 1365, normalized size = 4.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)**2/(b*x+a)**(1/2),x)
```

```
[Out] Piecewise((-2*a*c**2/sqrt(a + b*x) + 4*a*c*d*(-a/sqrt(a + b*x) - sqrt(a + b*x))/b + 4*a*c*e*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b**2 + 2*a*d**2*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b**2 + 4*a*c*f*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**3 + 4*a*d*e*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**3 + 4*a*d*f*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b**4 + 2*a*e**2*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b**4 + 4*a*e*f*(-a**5/sqrt(a + b*x) - 5*a**4*sqrt(a + b*x) + 10*a**3*(a + b*x)**(3/2)/3 - 2*a**2*(a + b*x)**(5/2) + 5*a*(a + b*x)**(7/2)/7 - (a + b*x)**(9/2)/9)/b**5 + 2*a*f**2*(a**6/sqrt(a + b*x) + 6*a**5*sqrt(a + b*x) - 5*a**4*(a + b*x)**(3/2) + 4*a**3*(a + b*x)**(5/2) - 15*a**2*(a + b*x)**(7/2)/7 + 2*a*(a + b*x)**(9/2)/3 - (a + b*x)**(11/2)/11)/b**6 + 2*c**2*(-a/sqrt(a + b*x) - sqrt(a + b*x)) + 4*c*d*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b + 4*c*e*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**2 + 2*d**2*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**2 + 4*c*f*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b**3 + 4*d*e*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b**3
```

```

2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b**3 + 4*d*f*(-a**5/sqrt(
a + b*x) - 5*a**4*sqrt(a + b*x) + 10*a**3*(a + b*x)**(3/2)/3 - 2*a**2*(a +
b*x)**(5/2) + 5*a*(a + b*x)**(7/2)/7 - (a + b*x)**(9/2)/9)/b**4 + 2*e**2*(-
a**5/sqrt(a + b*x) - 5*a**4*sqrt(a + b*x) + 10*a**3*(a + b*x)**(3/2)/3 - 2*
a**2*(a + b*x)**(5/2) + 5*a*(a + b*x)**(7/2)/7 - (a + b*x)**(9/2)/9)/b**4 +
4*e*f*(a**6/sqrt(a + b*x) + 6*a**5*sqrt(a + b*x) - 5*a**4*(a + b*x)**(3/2)
+ 4*a**3*(a + b*x)**(5/2) - 15*a**2*(a + b*x)**(7/2)/7 + 2*a*(a + b*x)**(9
/2)/3 - (a + b*x)**(11/2)/11)/b**5 + 2*f**2*(-a**7/sqrt(a + b*x) - 7*a**6*s
qrt(a + b*x) + 7*a**5*(a + b*x)**(3/2) - 7*a**4*(a + b*x)**(5/2) + 5*a**3*(
a + b*x)**(7/2) - 7*a**2*(a + b*x)**(9/2)/3 + 7*a*(a + b*x)**(11/2)/11 - (a
+ b*x)**(13/2)/13)/b**6)/b, Ne(b, 0)), ((c**2*x + c*d*x**2 + e*f*x**6/3 +
f**2*x**7/7 + x**5*(2*d*f + e**2)/5 + x**4*(2*c*f + 2*d*e)/4 + x**3*(2*c*e
+ d**2)/3)/sqrt(a), True))

```

Giac [A] time = 1.11509, size = 697, normalized size = 2.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="giac")
```

```

[Out] 2/45045*(45045*sqrt(b*x + a)*c^2 + 30030*((b*x + a)^(3/2) - 3*sqrt(b*x + a)
*a)*c*d/b + 3003*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x +
a)*a^2)*d^2/b^2 + 6006*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(
b*x + a)*a^2)*c*e/b^2 + 2574*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35
*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*c*f/b^3 + 2574*(5*(b*x + a)^(7
/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)
*d*e/b^3 + 286*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(
5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*d*f/b^4 + 143*
(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420
*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*e^2/b^4 + 130*(63*(b*x + a)^(
11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5
/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*f*e/b^5 + 15*(2
31*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 -
8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*
a^5 + 3003*sqrt(b*x + a)*a^6)*f^2/b^6)/b

```

$$3.6 \quad \int \frac{(c+dx+ex^2+fx^3)^3}{\sqrt{a+bx}} dx$$

Optimal. Leaf size=708

$$\frac{2(a+bx)^{7/2}(-30a^2b^4(2cef+d^2f+de^2)+20a^3b^3(3cf^2+6def+e^3)-105a^4b^2f(df+e^2)+168a^5bef^2-84a^6f^3+7b^{10}}{7b^{10}}$$

[Out] $(2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^3*\text{Sqrt}[a + b*x])/b^{10} + (2*(b^2*d - 2*a*b*e + 3*a^2*f)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^2*(a + b*x)^{(3/2)})/b^{10} + (6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(b^4*(d^2 + c*e) - 16*a^3*b*e*f + 12*a^4*f^2 - a*b^3*(5*d*e + 3*c*f) + a^2*b^2*(5*e^2 + 9*d*f))*(a + b*x)^{(5/2)})/(5*b^{10}) - (2*(168*a^5*b*e*f^2 - 84*a^6*f^3 - b^6*(d^3 + 6*c*d*e + 3*c^2*f) - 105*a^4*b^2*f*(e^2 + d*f) + 12*a*b^5*(d^2*e + c*e^2 + 2*c*d*f) - 30*a^2*b^4*(d*e^2 + d^2*f + 2*c*e*f) + 20*a^3*b^3*(e^3 + 6*d*e*f + 3*c*f^2))*(a + b*x)^{(7/2)})/(7*b^{10}) + (2*(70*a^4*b*e*f^2 - 42*a^5*f^3 - 35*a^3*b^2*f*(e^2 + d*f) + b^5*(d^2*e + c*e^2 + 2*c*d*f) - 5*a*b^4*(d*e^2 + d^2*f + 2*c*e*f) + 5*a^2*b^3*(e^3 + 6*d*e*f + 3*c*f^2))*(a + b*x)^{(9/2)})/(3*b^{10}) - (6*(56*a^3*b*e*f^2 - 42*a^4*f^3 - 21*a^2*b^2*f*(e^2 + d*f) - b^4*(d*e^2 + d^2*f + 2*c*e*f) + 2*a*b^3*(e^3 + 6*d*e*f + 3*c*f^2))*(a + b*x)^{(11/2)})/(11*b^{10}) + (2*(84*a^2*b*e*f^2 - 84*a^3*f^3 - 21*a*b^2*f*(e^2 + d*f) + b^3*(e^3 + 6*d*e*f + 3*c*f^2))*(a + b*x)^{(13/2)})/(13*b^{10}) - (2*f*(8*a*b*e*f - 12*a^2*f^2 - b^2*(e^2 + d*f))*(a + b*x)^{(15/2)})/(5*b^{10}) + (6*f^2*(b*e - 3*a*f)*(a + b*x)^{(17/2)})/(17*b^{10}) + (2*f^3*(a + b*x)^{(19/2)})/(19*b^{10})$

Rubi [A] time = 0.625186, antiderivative size = 708, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {1850}

$$\frac{2(a+bx)^{7/2}(-30a^2b^4(2cef+d^2f+de^2)+20a^3b^3(3cf^2+6def+e^3)-105a^4b^2f(df+e^2)+168a^5bef^2-84a^6f^3+7b^{10}}{7b^{10}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3)^3/Sqrt[a + b*x], x]

[Out] $(2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^3*\text{Sqrt}[a + b*x])/b^{10} + (2*(b^2*d - 2*a*b*e + 3*a^2*f)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^2*(a + b*x)^{(3/2)})/b^{10} + (6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(b^4*(d^2 + c*e) - 16*a^3*b*e*f + 12*a^4*f^2 - a*b^3*(5*d*e + 3*c*f) + a^2*b^2*(5*e^2 + 9*d*f))*(a + b*x)^{(5/2)})/(5*b^{10}) - (2*(168*a^5*b*e*f^2 - 84*a^6*f^3 - b^6*(d^3 + 6*c*d*e +$

$$\begin{aligned}
& 3c^2f) - 105a^4b^2f(e^2 + df) + 12ab^5(d^2e + ce^2 + 2c*df) - \\
& 30a^2b^4(d^2e + d^2f + 2c*ef) + 20a^3b^3(e^3 + 6d*ef + 3c*f^2) \\
&)*(a + bx)^{(7/2)}/(7b^{10}) + (2*(70a^4b*ef^2 - 42a^5f^3 - 35a^3b^2 \\
& *f*(e^2 + df) + b^5(d^2e + ce^2 + 2c*df) - 5ab^4(d^2e + d^2f + 2 \\
& *c*ef) + 5a^2b^3(e^3 + 6d*ef + 3c*f^2))*(a + bx)^{(9/2)}/(3b^{10}) - \\
& (6*(56a^3b*ef^2 - 42a^4f^3 - 21a^2b^2f*(e^2 + df) - b^4(d^2e + d \\
& ^2f + 2c*ef) + 2ab^3(e^3 + 6d*ef + 3c*f^2))*(a + bx)^{(11/2)}/(11b \\
& ^{10}) + (2*(84a^2b*ef^2 - 84a^3f^3 - 21ab^2f*(e^2 + df) + b^3(e^3 \\
& + 6d*ef + 3c*f^2))*(a + bx)^{(13/2)}/(13b^{10}) - (2f*(8ab*ef - 12a \\
& ^2f^2 - b^2(e^2 + df))*(a + bx)^{(15/2)}/(5b^{10}) + (6f^2*(b*e - 3a*f) \\
& *(a + bx)^{(17/2)}/(17b^{10}) + (2f^3*(a + bx)^{(19/2)}/(19b^{10})
\end{aligned}$$

Rule 1850

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand [Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned}
\int \frac{(c + dx + ex^2 + fx^3)^3}{\sqrt{a + bx}} dx &= \int \left(\frac{(b^3c - ab^2d + a^2be - a^3f)^3}{b^9\sqrt{a + bx}} + \frac{3(b^2d - 2abe + 3a^2f)(b^3c - ab^2d + a^2be - a^3f)^2\sqrt{a + bx}}{b^9} \right. \\
&= \frac{2(b^3c - ab^2d + a^2be - a^3f)^3\sqrt{a + bx}}{b^{10}} + \frac{2(b^2d - 2abe + 3a^2f)(b^3c - ab^2d + a^2be - a^3f)^2}{b^{10}}
\end{aligned}$$

Mathematica [A] time = 2.26284, size = 678, normalized size = 0.96

$$\frac{2\left(\frac{1}{7}(a + bx)^{7/2}\left(30a^2b^4(2cef + d^2f + de^2) - 20a^3b^3(3cf^2 + 6def + e^3) + 105a^4b^2f(df + e^2) - 168a^5bef^2 + 84a^6f^3 - 12\right)\right)}{b^{10}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3)^3/Sqrt[a + b*x], x]

[Out] (2*((b^3*c - a*b^2*d + a^2*b*e - a^3*f)^3*Sqrt[a + b*x] + (b^2*d - 2*a*b*e + 3*a^2*f)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^2*(a + b*x)^(3/2) + (3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(b^4*(d^2 + c*e) - 16*a^3*b*ef + 12*a^4*f^2 - a*b^3*(5*d*e + 3*c*f) + a^2*b^2*(5*e^2 + 9*d*f))*(a + b*x)^(5/2))/5 + ((-168*a^5*b*ef^2 + 84*a^6*f^3 + b^6*(d^3 + 6*c*d*e + 3*c^2*f) + 105*a^4*b^2

$$\begin{aligned} & *f*(e^2 + d*f) - 12*a*b^5*(d^2*e + c*e^2 + 2*c*d*f) + 30*a^2*b^4*(d*e^2 + d \\ & ^2*f + 2*c*e*f) - 20*a^3*b^3*(e^3 + 6*d*e*f + 3*c*f^2))*(a + b*x)^{(7/2)}/7 \\ & + ((70*a^4*b*e*f^2 - 42*a^5*f^3 - 35*a^3*b^2*f*(e^2 + d*f) + b^5*(d^2*e + c \\ & *e^2 + 2*c*d*f) - 5*a*b^4*(d*e^2 + d^2*f + 2*c*e*f) + 5*a^2*b^3*(e^3 + 6*d* \\ & e*f + 3*c*f^2))*(a + b*x)^{(9/2)}/3 + (3*(-56*a^3*b*e*f^2 + 42*a^4*f^3 + 21* \\ & a^2*b^2*f*(e^2 + d*f) + b^4*(d*e^2 + d^2*f + 2*c*e*f) - 2*a*b^3*(e^3 + 6*d* \\ & e*f + 3*c*f^2))*(a + b*x)^{(11/2)}/11 + ((84*a^2*b*e*f^2 - 84*a^3*f^3 - 21*a \\ & *b^2*f*(e^2 + d*f) + b^3*(e^3 + 6*d*e*f + 3*c*f^2))*(a + b*x)^{(13/2)}/13 + \\ & (f*(-8*a*b*e*f + 12*a^2*f^2 + b^2*(e^2 + d*f))*(a + b*x)^{(15/2)}/5 + (3*f^2 \\ & *(b*e - 3*a*f)*(a + b*x)^{(17/2)}/17 + (f^3*(a + b*x)^{(19/2)}/19))/b^10 \end{aligned}$$

Maple [B] time = 0.007, size = 1417, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((f*x^3+e*x^2+d*x+c)^3/(b*x+a)^{(1/2)}, x)$

[Out]
$$\begin{aligned} & -2/4849845*(b*x+a)^{(1/2)}*(-255255*b^9*f^3*x^9+270270*a*b^8*f^3*x^8-855855*b \\ & ^9*e*f^2*x^8-288288*a^2*b^7*f^3*x^7+912912*a*b^8*e*f^2*x^7-969969*b^9*d*f^2 \\ & *x^7-969969*b^9*e^2*f*x^7+310464*a^3*b^6*f^3*x^6-983136*a^2*b^7*e*f^2*x^6+1 \\ & 044582*a*b^8*d*f^2*x^6+1044582*a*b^8*e^2*f*x^6-1119195*b^9*c*f^2*x^6-223839 \\ & 0*b^9*d*e*f*x^6-373065*b^9*e^3*x^6-338688*a^4*b^5*f^3*x^5+1072512*a^3*b^6*e \\ & *f^2*x^5-1139544*a^2*b^7*d*f^2*x^5-1139544*a^2*b^7*e^2*f*x^5+1220940*a*b^8* \\ & c*f^2*x^5+2441880*a*b^8*d*e*f*x^5+406980*a*b^8*e^3*x^5-2645370*b^9*c*e*f*x^ \\ & 5-1322685*b^9*d^2*f*x^5-1322685*b^9*d*e^2*x^5+376320*a^5*b^4*f^3*x^4-119168 \\ & 0*a^4*b^5*e*f^2*x^4+1266160*a^3*b^6*d*f^2*x^4+1266160*a^3*b^6*e^2*f*x^4-135 \\ & 6600*a^2*b^7*c*f^2*x^4-2713200*a^2*b^7*d*e*f*x^4-452200*a^2*b^7*e^3*x^4+293 \\ & 9300*a*b^8*c*e*f*x^4+1469650*a*b^8*d^2*f*x^4+1469650*a*b^8*d*e^2*x^4-323323 \\ & 0*b^9*c*d*f*x^4-1616615*b^9*c*e^2*x^4-1616615*b^9*d^2*e*x^4-430080*a^6*b^3* \\ & f^3*x^3+1361920*a^5*b^4*e*f^2*x^3-1447040*a^4*b^5*d*f^2*x^3-1447040*a^4*b^5 \\ & *e^2*f*x^3+1550400*a^3*b^6*c*f^2*x^3+3100800*a^3*b^6*d*e*f*x^3+516800*a^3*b \\ & ^6*e^3*x^3-3359200*a^2*b^7*c*e*f*x^3-1679600*a^2*b^7*d^2*f*x^3-1679600*a^2* \\ & b^7*d*e^2*x^3+3695120*a*b^8*c*d*f*x^3+1847560*a*b^8*c*e^2*x^3+1847560*a*b^8 \\ & *d^2*e*x^3-2078505*b^9*c^2*f*x^3-4157010*b^9*c*d*e*x^3-692835*b^9*d^3*x^3+5 \\ & 16096*a^7*b^2*f^3*x^2-1634304*a^6*b^3*e*f^2*x^2+1736448*a^5*b^4*d*f^2*x^2+1 \\ & 736448*a^5*b^4*e^2*f*x^2-1860480*a^4*b^5*c*f^2*x^2-3720960*a^4*b^5*d*e*f*x^ \\ & 2-620160*a^4*b^5*e^3*x^2+4031040*a^3*b^6*c*e*f*x^2+2015520*a^3*b^6*d^2*f*x^ \\ & 2+2015520*a^3*b^6*d*e^2*x^2-4434144*a^2*b^7*c*d*f*x^2-2217072*a^2*b^7*c*e^2 \\ & *x^2-2217072*a^2*b^7*d^2*e*x^2+2494206*a*b^8*c^2*f*x^2+4988412*a*b^8*c*d*e* \\ & x^2+831402*a*b^8*d^3*x^2-2909907*b^9*c^2*e*x^2-2909907*b^9*c*d^2*x^2-688128 \end{aligned}$$

```

*a^8*b*f^3*x+2179072*a^7*b^2*e*f^2*x-2315264*a^6*b^3*d*f^2*x-2315264*a^6*b^
3*e^2*f*x+2480640*a^5*b^4*c*f^2*x+4961280*a^5*b^4*d*e*f*x+826880*a^5*b^4*e^
3*x-5374720*a^4*b^5*c*e*f*x-2687360*a^4*b^5*d^2*f*x-2687360*a^4*b^5*d*e^2*x
+5912192*a^3*b^6*c*d*f*x+2956096*a^3*b^6*c*e^2*x+2956096*a^3*b^6*d^2*e*x-33
25608*a^2*b^7*c^2*f*x-6651216*a^2*b^7*c*d*e*x-1108536*a^2*b^7*d^3*x+3879876
*a*b^8*c^2*e*x+3879876*a*b^8*c*d^2*x-4849845*b^9*c^2*d*x+1376256*a^9*f^3-43
58144*a^8*b*e*f^2+4630528*a^7*b^2*d*f^2+4630528*a^7*b^2*e^2*f-4961280*a^6*b
^3*c*f^2-9922560*a^6*b^3*d*e*f-1653760*a^6*b^3*e^3+10749440*a^5*b^4*c*e*f+5
374720*a^5*b^4*d^2*f+5374720*a^5*b^4*d*e^2-11824384*a^4*b^5*c*d*f-5912192*a
^4*b^5*c*e^2-5912192*a^4*b^5*d^2*e+6651216*a^3*b^6*c^2*f+13302432*a^3*b^6*c
*d*e+2217072*a^3*b^6*d^3-7759752*a^2*b^7*c^2*e-7759752*a^2*b^7*c*d^2+969969
0*a*b^8*c^2*d-4849845*b^9*c^3)/b^10

```

Maxima [B] time = 1.05296, size = 1836, normalized size = 2.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/4849845*(4849845*sqrt(b*x + a)*c^3 + 138567*c^2*(35*((b*x + a)^(3/2) - 3*
sqrt(b*x + a)*a)*d/b + 7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sq
rt(b*x + a)*a^2)*e/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b
*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*f/b^3) + 323*c*(3003*(3*(b*x + a)
^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*d^2/b^2 + 143*(35*(b*
x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x +
a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*e^2/b^4 + 286*(35*(b*x + a)^(9/2)*f
+ 45*(b*e - 4*a*f)*(b*x + a)^(7/2) - 189*(a*b*e - 2*a^2*f)*(b*x + a)^(5/2)
+ 105*(3*a^2*b*e - 4*a^3*f)*(b*x + a)^(3/2) - 315*(a^3*b*e - a^4*f)*sqrt(b*
x + a))*d/b^4 + 130*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x
+ a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693
*sqrt(b*x + a)*a^5)*e*f/b^5 + 15*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11
/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a
)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*f^2/b^6) +
138567*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2
- 35*sqrt(b*x + a)*a^3)*d^3/b^3 + 4199*(315*(b*x + a)^(11/2)*f + 385*(b*e -
5*a*f)*(b*x + a)^(9/2) - 990*(2*a*b*e - 5*a^2*f)*(b*x + a)^(7/2) + 1386*(3
*a^2*b*e - 5*a^3*f)*(b*x + a)^(5/2) - 1155*(4*a^3*b*e - 5*a^4*f)*(b*x + a)
^(3/2) + 3465*(a^4*b*e - a^5*f)*sqrt(b*x + a))*d^2/b^5 + 1615*(231*(b*x + a)
^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x +
a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*s

```


$$\begin{aligned} & ^7*c - 12*a^3*b^6*d)*e)*f)*x^3 + 537472*(11*a^4*b^5*c - 10*a^5*b^4*d)*e^2 + \\ & 19456*(255*a^6*b^3*c - 238*a^7*b^2*d + 224*a^8*b*e)*f^2 + 3*(969969*b^9*c* \\ & d^2 - 277134*a*b^8*d^3 + 206720*a^4*b^5*e^3 - 172032*a^7*b^2*f^3 + 67184*(1 \\ & 1*a^2*b^7*c - 10*a^3*b^6*d)*e^2 + 2432*(255*a^4*b^5*c - 238*a^5*b^4*d + 224 \\ & *a^6*b^3*e)*f^2 + 46189*(21*b^9*c^2 - 36*a*b^8*c*d + 16*a^2*b^7*d^2)*e - 64 \\ & 6*(1287*a*b^8*c^2 - 2288*a^2*b^7*c*d + 1040*a^3*b^6*d^2 + 896*a^5*b^4*e^2 + \\ & 160*(13*a^3*b^6*c - 12*a^4*b^5*d)*e)*f)*x^2 + 369512*(21*a^2*b^7*c^2 - 36* \\ & a^3*b^6*c*d + 16*a^4*b^5*d^2)*e - 5168*(1287*a^3*b^6*c^2 - 2288*a^4*b^5*c*d \\ & + 1040*a^5*b^4*d^2 + 896*a^7*b^2*e^2 + 160*(13*a^5*b^4*c - 12*a^6*b^3*d)*e \\ &)*f + (4849845*b^9*c^2*d - 3879876*a*b^8*c*d^2 + 1108536*a^2*b^7*d^3 - 8268 \\ & 80*a^5*b^4*e^3 + 688128*a^8*b*f^3 - 268736*(11*a^3*b^6*c - 10*a^4*b^5*d)*e^ \\ & 2 - 9728*(255*a^5*b^4*c - 238*a^6*b^3*d + 224*a^7*b^2*e)*f^2 - 184756*(21*a \\ & *b^8*c^2 - 36*a^2*b^7*c*d + 16*a^3*b^6*d^2)*e + 2584*(1287*a^2*b^7*c^2 - 22 \\ & 88*a^3*b^6*c*d + 1040*a^4*b^5*d^2 + 896*a^6*b^3*e^2 + 160*(13*a^4*b^5*c - 1 \\ & 2*a^5*b^4*d)*e)*f)*x)*sqrt(b*x + a)/b^10 \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)**3/(b*x+a)**(1/2),x)

[Out] Timed out

Giac [B] time = 1.18098, size = 1909, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="giac")

[Out]
$$\begin{aligned} & 2/4849845*(4849845*sqrt(b*x + a)*c^3 + 4849845*((b*x + a)^(3/2) - 3*sqrt(b* \\ & x + a)*a)*c^2*d/b + 969969*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*s \\ & qrt(b*x + a)*a^2)*c*d^2/b^2 + 969969*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2 \\ &)*a + 15*sqrt(b*x + a)*a^2)*c^2*e/b^2 + 138567*(5*(b*x + a)^(7/2) - 21*(b*x \\ & + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d^3/b^3 + 41 \end{aligned}$$

$$\begin{aligned}
& 5701*(5*(b*x + a)^{(7/2)} - 21*(b*x + a)^{(5/2)}*a + 35*(b*x + a)^{(3/2)}*a^2 - 3 \\
& 5*\sqrt{b*x + a}*a^3)*c^2*f/b^3 + 831402*(5*(b*x + a)^{(7/2)} - 21*(b*x + a)^{(5/2)}*a + 35*(b*x + a)^{(3/2)}*a^2 - 35*\sqrt{b*x + a}*a^3)*c*d*e/b^3 + 92378*(\\
& 35*(b*x + a)^{(9/2)} - 180*(b*x + a)^{(7/2)}*a + 378*(b*x + a)^{(5/2)}*a^2 - 420*(b*x + a)^{(3/2)}*a^3 + 315*\sqrt{b*x + a}*a^4)*c*d*f/b^4 + 46189*(35*(b*x + a) \\
&)^{(9/2)} - 180*(b*x + a)^{(7/2)}*a + 378*(b*x + a)^{(5/2)}*a^2 - 420*(b*x + a)^{(3/2)}*a^3 + 315*\sqrt{b*x + a}*a^4)*d^2*e/b^4 + 20995*(63*(b*x + a)^{(11/2)} - \\
& 385*(b*x + a)^{(9/2)}*a + 990*(b*x + a)^{(7/2)}*a^2 - 1386*(b*x + a)^{(5/2)}*a^3 + 1155*(b*x + a)^{(3/2)}*a^4 - 693*\sqrt{b*x + a}*a^5)*d^2*f/b^5 + 46189*(35*(\\
& b*x + a)^{(9/2)} - 180*(b*x + a)^{(7/2)}*a + 378*(b*x + a)^{(5/2)}*a^2 - 420*(b*x + a)^{(3/2)}*a^3 + 315*\sqrt{b*x + a}*a^4)*c*e^2/b^4 + 41990*(63*(b*x + a)^{(11/2)} - \\
& 385*(b*x + a)^{(9/2)}*a + 990*(b*x + a)^{(7/2)}*a^2 - 1386*(b*x + a)^{(5/2)}*a^3 + 1155*(b*x + a)^{(3/2)}*a^4 - 693*\sqrt{b*x + a}*a^5)*c*f*e/b^5 + 4845 \\
& *(231*(b*x + a)^{(13/2)} - 1638*(b*x + a)^{(11/2)}*a + 5005*(b*x + a)^{(9/2)}*a^2 - 8580*(b*x + a)^{(7/2)}*a^3 + 9009*(b*x + a)^{(5/2)}*a^4 - 6006*(b*x + a)^{(3/2)}*a^5 + 3003*\sqrt{b*x + a}*a^6)*c*f^2/b^6 + 20995*(63*(b*x + a)^{(11/2)} - 3 \\
& 85*(b*x + a)^{(9/2)}*a + 990*(b*x + a)^{(7/2)}*a^2 - 1386*(b*x + a)^{(5/2)}*a^3 + 1155*(b*x + a)^{(3/2)}*a^4 - 693*\sqrt{b*x + a}*a^5)*d*e^2/b^5 + 9690*(231*(b \\
& x + a)^{(13/2)} - 1638*(b*x + a)^{(11/2)}*a + 5005*(b*x + a)^{(9/2)}*a^2 - 8580*(b*x + a)^{(7/2)}*a^3 + 9009*(b*x + a)^{(5/2)}*a^4 - 6006*(b*x + a)^{(3/2)}*a^5 + \\
& 3003*\sqrt{b*x + a}*a^6)*d*f*e/b^6 + 2261*(429*(b*x + a)^{(15/2)} - 3465*(b*x + a)^{(13/2)}*a + 12285*(b*x + a)^{(11/2)}*a^2 - 25025*(b*x + a)^{(9/2)}*a^3 + 3 \\
& 2175*(b*x + a)^{(7/2)}*a^4 - 27027*(b*x + a)^{(5/2)}*a^5 + 15015*(b*x + a)^{(3/2)}*a^6 - 6435*\sqrt{b*x + a}*a^7)*d*f^2/b^7 + 1615*(231*(b*x + a)^{(13/2)} - 16 \\
& 38*(b*x + a)^{(11/2)}*a + 5005*(b*x + a)^{(9/2)}*a^2 - 8580*(b*x + a)^{(7/2)}*a^3 + 9009*(b*x + a)^{(5/2)}*a^4 - 6006*(b*x + a)^{(3/2)}*a^5 + 3003*\sqrt{b*x + a} \\
& *a^6)*e^3/b^6 + 2261*(429*(b*x + a)^{(15/2)} - 3465*(b*x + a)^{(13/2)}*a + 1228 \\
& 5*(b*x + a)^{(11/2)}*a^2 - 25025*(b*x + a)^{(9/2)}*a^3 + 32175*(b*x + a)^{(7/2)}* \\
& a^4 - 27027*(b*x + a)^{(5/2)}*a^5 + 15015*(b*x + a)^{(3/2)}*a^6 - 6435*\sqrt{b*x \\
& + a}*a^7)*f*e^2/b^7 + 133*(6435*(b*x + a)^{(17/2)} - 58344*(b*x + a)^{(15/2)}* \\
& a + 235620*(b*x + a)^{(13/2)}*a^2 - 556920*(b*x + a)^{(11/2)}*a^3 + 850850*(b*x \\
& + a)^{(9/2)}*a^4 - 875160*(b*x + a)^{(7/2)}*a^5 + 612612*(b*x + a)^{(5/2)}*a^6 - \\
& 291720*(b*x + a)^{(3/2)}*a^7 + 109395*\sqrt{b*x + a}*a^8)*f^2*e/b^8 + 21*(121 \\
& 55*(b*x + a)^{(19/2)} - 122265*(b*x + a)^{(17/2)}*a + 554268*(b*x + a)^{(15/2)}*a \\
& ^2 - 1492260*(b*x + a)^{(13/2)}*a^3 + 2645370*(b*x + a)^{(11/2)}*a^4 - 3233230* \\
& (b*x + a)^{(9/2)}*a^5 + 2771340*(b*x + a)^{(7/2)}*a^6 - 1662804*(b*x + a)^{(5/2)} \\
& *a^7 + 692835*(b*x + a)^{(3/2)}*a^8 - 230945*\sqrt{b*x + a}*a^9)*f^3/b^9)/b
\end{aligned}$$

3.7 $\int \frac{c+dx}{a+bx^3} dx$

Optimal. Leaf size=161

$$-\frac{\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}} + \frac{\left(\sqrt[3]{bc} - \sqrt[3]{ad}\right) \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{3a^{2/3}b^{2/3}} - \frac{\left(\sqrt[3]{ad} + \sqrt[3]{bc}\right) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}$$

[Out] -(((b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(2/3)) - ((c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))

Rubi [A] time = 0.111138, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {1860, 31, 634, 617, 204, 628}

$$-\frac{\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}} + \frac{\left(\sqrt[3]{bc} - \sqrt[3]{ad}\right) \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{3a^{2/3}b^{2/3}} - \frac{\left(\sqrt[3]{ad} + \sqrt[3]{bc}\right) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)/(a + b*x^3), x]

[Out] -(((b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(2/3)) - ((c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))

Rule 1860

Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]

Rule 31

```
Int[((a_) + (b_.)*(x_))^-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx}{a + bx^3} dx &= \frac{\int \frac{\sqrt[3]{a}(2\sqrt[3]{bc} + \sqrt[3]{ad}) + \sqrt[3]{b}(-\sqrt[3]{bc} + \sqrt[3]{ad})x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} dx}{3a^{2/3}\sqrt[3]{b}} + \frac{\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{bx}} dx}{3a^{2/3}} \\
&= \frac{\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{2/3}\sqrt[3]{b}} - \frac{(\sqrt[3]{bc} - \sqrt[3]{ad}) \int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} dx}{6a^{2/3}b^{2/3}} + \frac{1}{2} \left(\frac{c}{\sqrt[3]{a}} + \frac{d}{\sqrt[3]{b}}\right) \int \frac{1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} \\
&= \frac{\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{2/3}\sqrt[3]{b}} - \frac{(\sqrt[3]{bc} - \sqrt[3]{ad}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{6a^{2/3}b^{2/3}} + \frac{(\sqrt[3]{bc} + \sqrt[3]{ad}) \operatorname{Subst}\left(\int \frac{1}{-3-x^2}\right)}{a^{2/3}b^{2/3}} \\
&= -\frac{(\sqrt[3]{bc} + \sqrt[3]{ad}) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}} + \frac{\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{2/3}\sqrt[3]{b}} - \frac{(\sqrt[3]{bc} - \sqrt[3]{ad}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{6a^{2/3}b^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.0553796, size = 124, normalized size = 0.77

$$\frac{(\sqrt[3]{bc} - \sqrt[3]{ad}) \left(2 \log(\sqrt[3]{a} + \sqrt[3]{bx}) - \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)\right) - 2\sqrt{3}(\sqrt[3]{ad} + \sqrt[3]{bc}) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{6a^{2/3}b^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/(a + b*x^3), x]

[Out] (-2*Sqrt[3]*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + (b^(1/3)*c - a^(1/3)*d)*(2*Log[a^(1/3) + b^(1/3)*x] - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(2/3))

Maple [A] time = 0.044, size = 186, normalized size = 1.2

$$\frac{c}{3b} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{c}{6b} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{c\sqrt{3}}{3b} \arctan\left(\frac{\sqrt{3}}{3} \left(2x \frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{d}{3b} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)/(b*x^3+a), x)

```
[Out] 1/3*c/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/6*c/b/(1/b*a)^(2/3)*ln(x^2-(1/b
*a)^(1/3)*x+(1/b*a)^(2/3))+1/3*c/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)
*(2/(1/b*a)^(1/3)*x-1))-1/3*d/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/6*d/b/(
1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3*d*3^(1/2)/b/(1/b*a)^(
1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 7.86126, size = 4590, normalized size = 28.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/6*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d
^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*
d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I
*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)
- 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*
c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d - 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)
*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(
2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)
/(a^2*b^2))^(1/3)))*a*b*c^2 + 2*a*c*d^2 + (b*c^3 + a*d^3)*x) + 1/12*((1/2)^(
1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2)
)^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^
2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)) + 3*sqrt(1/3)*sqrt(-(((1/2)^(1/3)*(I
*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)
- 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*
c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a*b + 16*c*d)/(a*b))*log(-1/4*((1/2)^(1/
3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(
```

$$\begin{aligned}
& \left(\frac{1}{3} - 2 \left(\frac{1}{2} \right)^{2/3} c d (-I \sqrt{3} + 1) / (a b ((b c^3 + a d^3) / (a^2 b^2) + (b c^3 - a d^3) / (a^2 b^2))^{1/3}) \right)^2 a^2 b d + \frac{1}{2} \left(\left(\frac{1}{2} \right)^{1/3} (I \sqrt{3} + 1) \left((b c^3 + a d^3) / (a^2 b^2) + (b c^3 - a d^3) / (a^2 b^2) \right)^{1/3} - 2 \left(\frac{1}{2} \right)^{2/3} c d (-I \sqrt{3} + 1) / (a b ((b c^3 + a d^3) / (a^2 b^2) + (b c^3 - a d^3) / (a^2 b^2))^{1/3}) \right) a b c^2 - 2 a^* c d^2 + 2 (b c^3 + a d^3) x + \frac{3}{4} \sqrt[3]{1/3} \left(\left(\frac{1}{2} \right)^{1/3} (I \sqrt{3} + 1) \left((b c^3 + a d^3) / (a^2 b^2) + (b c^3 - a d^3) / (a^2 b^2) \right)^{1/3} - 2 \left(\frac{1}{2} \right)^{2/3} c d (-I \sqrt{3} + 1) / (a b ((b c^3 + a d^3) / (a^2 b^2) + (b c^3 - a d^3) / (a^2 b^2))^{1/3}) \right) a^2 b d + 2 a b c^2 \sqrt{-\left(\left(\frac{1}{2} \right)^{1/3} (I \sqrt{3} + 1) \left((b c^3 + a d^3) / (a^2 b^2) + (b c^3 - a d^3) / (a^2 b^2) \right)^{1/3} - 2 \left(\frac{1}{2} \right)^{2/3} c d (-I \sqrt{3} + 1) / (a b ((b c^3 + a d^3) / (a^2 b^2) + (b c^3 - a d^3) / (a^2 b^2))^{1/3}) \right)^2 a b + 16 c d} / (a b)) + \frac{1}{12} \left(\left(\frac{1}{2} \right)^{1/3} (I \sqrt{3} + 1) \left((b c^3 + a d^3) / (a^2 b^2) + (b c^3 - a d^3) / (a^2 b^2) \right)^{1/3} - 2 \left(\frac{1}{2} \right)^{2/3} c d (-I \sqrt{3} + 1) / (a b ((b c^3 + a d^3) / (a^2 b^2) + (b c^3 - a d^3) / (a^2 b^2))^{1/3}) \right) - 3 \sqrt[3]{1/3} \sqrt{-\left(\left(\frac{1}{2} \right)^{1/3} (I \sqrt{3} + 1) \left((b c^3 + a d^3) / (a^2 b^2) + (b c^3 - a d^3) / (a^2 b^2) \right)^{1/3} - 2 \left(\frac{1}{2} \right)^{2/3} c d (-I \sqrt{3} + 1) / (a b ((b c^3 + a d^3) / (a^2 b^2) + (b c^3 - a d^3) / (a^2 b^2))^{1/3}) \right)^2 a b + 16 c d} / (a b)) \log(-\frac{1}{4} \left(\left(\frac{1}{2} \right)^{1/3} (I \sqrt{3} + 1) \left((b c^3 + a d^3) / (a^2 b^2) + (b c^3 - a d^3) / (a^2 b^2) \right)^{1/3} - 2 \left(\frac{1}{2} \right)^{2/3} c d (-I \sqrt{3} + 1) / (a b ((b c^3 + a d^3) / (a^2 b^2) + (b c^3 - a d^3) / (a^2 b^2))^{1/3}) \right)^2 a^2 b d + \frac{1}{2} \left(\left(\frac{1}{2} \right)^{1/3} (I \sqrt{3} + 1) \left((b c^3 + a d^3) / (a^2 b^2) + (b c^3 - a d^3) / (a^2 b^2) \right)^{1/3} - 2 \left(\frac{1}{2} \right)^{2/3} c d (-I \sqrt{3} + 1) / (a b ((b c^3 + a d^3) / (a^2 b^2) + (b c^3 - a d^3) / (a^2 b^2))^{1/3}) \right) a b c^2 - 2 a^* c d^2 + 2 (b c^3 + a d^3) x - \frac{3}{4} \sqrt[3]{1/3} \left(\left(\frac{1}{2} \right)^{1/3} (I \sqrt{3} + 1) \left((b c^3 + a d^3) / (a^2 b^2) + (b c^3 - a d^3) / (a^2 b^2) \right)^{1/3} - 2 \left(\frac{1}{2} \right)^{2/3} c d (-I \sqrt{3} + 1) / (a b ((b c^3 + a d^3) / (a^2 b^2) + (b c^3 - a d^3) / (a^2 b^2))^{1/3}) \right) a^2 b d + 2 a b c^2 \sqrt{-\left(\left(\frac{1}{2} \right)^{1/3} (I \sqrt{3} + 1) \left((b c^3 + a d^3) / (a^2 b^2) + (b c^3 - a d^3) / (a^2 b^2) \right)^{1/3} - 2 \left(\frac{1}{2} \right)^{2/3} c d (-I \sqrt{3} + 1) / (a b ((b c^3 + a d^3) / (a^2 b^2) + (b c^3 - a d^3) / (a^2 b^2))^{1/3}) \right)^2 a b + 16 c d} / (a b))
\end{aligned}$$

Sympy [A] time = 0.633847, size = 76, normalized size = 0.47

$$\text{RootSum} \left(27 t^3 a^2 b^2 + 9 t a b c d + a d^3 - b c^3, \left(t \mapsto t \log \left(x + \frac{9 t^2 a^2 b d + 3 t a b c^2 + 2 a c d^2}{a d^3 + b c^3} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x**3+a),x)

[Out] RootSum(27*_t**3*a**2*b**2 + 9*_t*a*b*c*d + a*d**3 - b*c**3, Lambda(_t, _t*log(x + (9*_t**2*a**2*b*d + 3*_t*a*b*c**2 + 2*a*c*d**2)/(a*d**3 + b*c**3)))

)

Giac [A] time = 1.11479, size = 216, normalized size = 1.34

$$\frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}bc - \left(-ab^2\right)^{\frac{2}{3}}d\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^2} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}ab^3c + \left(-ab^2\right)^{\frac{2}{3}}d\right)}{a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x^3+a),x, algorithm="giac")

[Out] -1/3*(d*(-a/b)^(1/3) + c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a + 1/3*sqrt(3)*((-a*b^2)^(1/3)*b*c - (-a*b^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) + 1/6*((-a*b^2)^(1/3)*a*b^3*c + (-a*b^2)^(2/3)*a*b^2*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^4)

$$3.8 \quad \int \frac{c+dx}{(a+bx^3)^2} dx$$

Optimal. Leaf size=189

$$-\frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{18a^{5/3}b^{2/3}} + \frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{5/3}b^{2/3}} - \frac{(\sqrt[3]{ad} + 2\sqrt[3]{bc}) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} + \frac{x}{3a}$$

[Out] (x*(c + d*x))/(3*a*(a + b*x^3)) - ((2*b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(5/3)*b^(2/3)) + ((2*b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x]/(9*a^(5/3)*b^(2/3)) - ((2*b^(1/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(18*a^(5/3)*b^(2/3))

Rubi [A] time = 0.139398, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1855, 1860, 31, 634, 617, 204, 628}

$$-\frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{18a^{5/3}b^{2/3}} + \frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{5/3}b^{2/3}} - \frac{(\sqrt[3]{ad} + 2\sqrt[3]{bc}) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} + \frac{x}{3a}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)/(a + b*x^3)^2, x]

[Out] (x*(c + d*x))/(3*a*(a + b*x^3)) - ((2*b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(5/3)*b^(2/3)) + ((2*b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x]/(9*a^(5/3)*b^(2/3)) - ((2*b^(1/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(18*a^(5/3)*b^(2/3))

Rule 1855

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] & & PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^( -1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^( -1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx}{(a + bx^3)^2} dx &= \frac{x(c + dx)}{3a(a + bx^3)} - \frac{\int \frac{-2c - dx}{a + bx^3} dx}{3a} \\
&= \frac{x(c + dx)}{3a(a + bx^3)} - \frac{\int \frac{\sqrt[3]{a}(-4\sqrt[3]{bc} - \sqrt[3]{ad}) + \sqrt[3]{b}(2\sqrt[3]{bc} - \sqrt[3]{ad})x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} dx}{9a^{5/3}\sqrt[3]{b}} + \frac{\left(2c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{bx}} dx}{9a^{5/3}} \\
&= \frac{x(c + dx)}{3a(a + bx^3)} + \frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{5/3}b^{2/3}} - \frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} dx}{18a^{5/3}b^{2/3}} + \frac{\left(2c + \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{9a^{5/3}} \\
&= \frac{x(c + dx)}{3a(a + bx^3)} + \frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{5/3}b^{2/3}} - \frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{18a^{5/3}b^{2/3}} + \frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{18a^{5/3}b^{2/3}} \\
&= \frac{x(c + dx)}{3a(a + bx^3)} - \frac{(2\sqrt[3]{bc} + \sqrt[3]{ad}) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} + \frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{5/3}b^{2/3}} - \frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{18a^{5/3}b^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.150732, size = 180, normalized size = 0.95

$$\frac{\frac{(a^{2/3}d - 2\sqrt[3]{a}\sqrt[3]{bc}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{b^{2/3}} + \frac{2(2\sqrt[3]{a}\sqrt[3]{bc} - a^{2/3}d) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{b^{2/3}} - \frac{2\sqrt{3}\sqrt[3]{a}(\sqrt[3]{ad} + 2\sqrt[3]{bc}) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{2/3}} + \frac{6ax(c+dx)}{a+bx^3}}{18a^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/(a + b*x^3)^2, x]

[Out] ((6*a*x*(c + d*x))/(a + b*x^3) - (2*sqrt[3]*a^(1/3)*(2*b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]])/b^(2/3) + (2*(2*a^(1/3)*b^(1/3)*c - a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((-2*a^(1/3)*b^(1/3)*c + a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(18*a^2)

Maple [A] time = 0.003, size = 238, normalized size = 1.3

$$\frac{cx}{3a(bx^3 + a)} + \frac{2c}{9ab} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{c}{9ab} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{2c\sqrt{3}}{9ab} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)/(b*x^3+a)^2,x)`

[Out] $\frac{1}{3}c*x/a/(b*x^3+a)+2/9*c/a/b/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})-1/9*c/a/b/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})+2/9*c/a/b/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))+1/3*d*x^2/a/(b*x^3+a)-1/9*d/a/b/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})+1/18*d/a/b/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})+1/9*d/a*3^{(1/2)}/b/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [C] time = 7.97279, size = 4961, normalized size = 26.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")`

[Out] $\frac{1}{36}*(12*d*x^2 - 2*(a*b*x^3 + a^2)*((1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^{(1/3)} + 4*(1/2)^{(2/3)}*c*d*(I*\sqrt{3} - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^{(1/3)}))*\log(1/4*((1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^{(1/3)} + 4*(1/2)^{(2/3)}*c*d*(I*\sqrt{3} - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^{(1/3)}))^{(1/3)} + 4*(1/2)^{(2/3)}*c*d*(I*\sqrt{3} - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^{(1/3)}))*a^2*b*c^2 + 4*a*c*d^2 + (8*b*c^3 + a*d^3)*x) + 12*c*x + ((a*b*x^3 + a^2)*(($

Sympy [A] time = 1.10274, size = 105, normalized size = 0.56

$$\text{RootSum}\left(729t^3a^5b^2 + 54ta^2bcd + ad^3 - 8bc^3, \left(t \mapsto t \log\left(x + \frac{81t^2a^4bd + 36ta^2bc^2 + 4acd^2}{ad^3 + 8bc^3}\right)\right)\right) + \frac{cx + dx^2}{3a^2 + 3abx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x**3+a)**2,x)

[Out] RootSum(729*_t**3*a**5*b**2 + 54*_t*a**2*b*c*d + a*d**3 - 8*b*c**3, Lambda(_t, _t*log(x + (81*_t**2*a**4*b*d + 36*_t*a**2*b*c**2 + 4*a*c*d**2)/(a*d**3 + 8*b*c**3)))) + (c*x + d*x**2)/(3*a**2 + 3*a*b*x**3)

Giac [A] time = 1.10932, size = 252, normalized size = 1.33

$$\frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 2c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^2} + \frac{dx^2 + cx}{3(bx^3 + a)a} + \frac{\sqrt{3}\left(2(-ab^2)^{\frac{1}{3}}bc - (-ab^2)^{\frac{2}{3}}d\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^2b^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x^3+a)^2,x, algorithm="giac")

[Out] -1/9*(d*(-a/b)^(1/3) + 2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1/3*(d*x^2 + c*x)/((b*x^3 + a)*a) + 1/9*sqrt(3)*(2*(-a*b^2)^(1/3)*b*c - (-a*b^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^2) + 1/18*(2*(-a*b^2)^(1/3)*a*b^3*c + (-a*b^2)^(2/3)*a*b^2*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^4)

$$3.9 \quad \int \frac{c+dx}{(a+bx^3)^3} dx$$

Optimal. Leaf size=215

$$-\frac{(5\sqrt[3]{bc}-2\sqrt[3]{ad})\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2)}{54a^{8/3}b^{2/3}}+\frac{(5\sqrt[3]{bc}-2\sqrt[3]{ad})\log(\sqrt[3]{a}+\sqrt[3]{bx})}{27a^{8/3}b^{2/3}}-\frac{(2\sqrt[3]{ad}+5\sqrt[3]{bc})\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}}+$$

[Out] (x*(c + d*x))/(6*a*(a + b*x^3)^2) + (x*(5*c + 4*d*x))/(18*a^2*(a + b*x^3)) - ((5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(8/3)*b^(2/3)) + ((5*b^(1/3)*c - 2*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x]/(27*a^(8/3)*b^(2/3)) - ((5*b^(1/3)*c - 2*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(54*a^(8/3)*b^(2/3))

Rubi [A] time = 0.187046, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1855, 1860, 31, 634, 617, 204, 628}

$$-\frac{(5\sqrt[3]{bc}-2\sqrt[3]{ad})\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2)}{54a^{8/3}b^{2/3}}+\frac{(5\sqrt[3]{bc}-2\sqrt[3]{ad})\log(\sqrt[3]{a}+\sqrt[3]{bx})}{27a^{8/3}b^{2/3}}-\frac{(2\sqrt[3]{ad}+5\sqrt[3]{bc})\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}}+$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)/(a + b*x^3)^3, x]

[Out] (x*(c + d*x))/(6*a*(a + b*x^3)^2) + (x*(5*c + 4*d*x))/(18*a^2*(a + b*x^3)) - ((5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(8/3)*b^(2/3)) + ((5*b^(1/3)*c - 2*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x]/(27*a^(8/3)*b^(2/3)) - ((5*b^(1/3)*c - 2*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(54*a^(8/3)*b^(2/3))

Rule 1855

Int[(Pq_)*((a_) + (b_)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] & & PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]

Rule 1860

Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]

Rule 31

Int[((a_) + (b_)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
\int \frac{c+dx}{(a+bx^3)^3} dx &= \frac{x(c+dx)}{6a(a+bx^3)^2} - \frac{\int \frac{-5c-4dx}{(a+bx^3)^2} dx}{6a} \\
&= \frac{x(c+dx)}{6a(a+bx^3)^2} + \frac{x(5c+4dx)}{18a^2(a+bx^3)} + \frac{\int \frac{10c+4dx}{a+bx^3} dx}{18a^2} \\
&= \frac{x(c+dx)}{6a(a+bx^3)^2} + \frac{x(5c+4dx)}{18a^2(a+bx^3)} + \frac{\int \frac{\sqrt[3]{a}(20\sqrt[3]{bc}+4\sqrt[3]{ad})+\sqrt[3]{b}(-10\sqrt[3]{bc}+4\sqrt[3]{ad})x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2} dx}{54a^{8/3}\sqrt[3]{b}} + \frac{(5c-\frac{2\sqrt[3]{ad}}{\sqrt[3]{b}}) \int \frac{1}{\sqrt[3]{a}+\sqrt[3]{bx}} dx}{27a^{8/3}} \\
&= \frac{x(c+dx)}{6a(a+bx^3)^2} + \frac{x(5c+4dx)}{18a^2(a+bx^3)} + \frac{(5\sqrt[3]{bc}-2\sqrt[3]{ad}) \log(\sqrt[3]{a}+\sqrt[3]{bx})}{27a^{8/3}b^{2/3}} - \frac{(5\sqrt[3]{bc}-2\sqrt[3]{ad}) \int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2} dx}{54a^{8/3}b^{2/3}} \\
&= \frac{x(c+dx)}{6a(a+bx^3)^2} + \frac{x(5c+4dx)}{18a^2(a+bx^3)} + \frac{(5\sqrt[3]{bc}-2\sqrt[3]{ad}) \log(\sqrt[3]{a}+\sqrt[3]{bx})}{27a^{8/3}b^{2/3}} - \frac{(5\sqrt[3]{bc}-2\sqrt[3]{ad}) \log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx})}{54a^{8/3}b^{2/3}} \\
&= \frac{x(c+dx)}{6a(a+bx^3)^2} + \frac{x(5c+4dx)}{18a^2(a+bx^3)} - \frac{(5\sqrt[3]{bc}+2\sqrt[3]{ad}) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}} + \frac{(5\sqrt[3]{bc}-2\sqrt[3]{ad}) \log(\sqrt[3]{a}+\sqrt[3]{bx})}{27a^{8/3}b^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.153778, size = 205, normalized size = 0.95

$$\frac{\frac{(2a^{2/3}d-5\sqrt[3]{a}\sqrt[3]{bc}) \log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2)}{b^{2/3}} + \frac{2(5\sqrt[3]{a}\sqrt[3]{bc}-2a^{2/3}d) \log(\sqrt[3]{a}+\sqrt[3]{bx})}{b^{2/3}} + \frac{9a^2x(c+dx)}{(a+bx^3)^2} - \frac{2\sqrt{3}\sqrt[3]{a}(2\sqrt[3]{ad}+5\sqrt[3]{bc}) \tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{3ax(5c+4dx)}{a+bx^3}}{54a^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/(a + b*x^3)^3, x]

[Out] ((9*a^2*x*(c + d*x))/(a + b*x^3)^2 + (3*a*x*(5*c + 4*d*x))/(a + b*x^3) - (2*sqrt[3]*a^(1/3)*(5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]])/b^(2/3) + (2*(5*a^(1/3)*b^(1/3)*c - 2*a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((-5*a^(1/3)*b^(1/3)*c + 2*a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(54*a^3)

Maple [A] time = 0.004, size = 272, normalized size = 1.3

$$\frac{cx}{6a(bx^3+a)^2} + \frac{5cx}{18a^2(bx^3+a)} + \frac{5c}{27ba^2} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{5c}{54ba^2} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{5c\sqrt{3}}{27ba^2} \arctan$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)/(b*x^3+a)^3,x)`

[Out] `1/6*c/a*x/(b*x^3+a)^2+5/18*c/a^2*x/(b*x^3+a)+5/27*c/a^2/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-5/54*c/a^2/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+5/27*c/a^2/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/6*d/a*x^2/(b*x^3+a)^2+2/9*d/a^2*x^2/(b*x^3+a)-2/27*d/a^2/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/27*d/a^2/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2/27*d/a^2*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))`

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [C] time = 8.43691, size = 5554, normalized size = 25.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")`

[Out] `1/108*(24*b*d*x^5 + 30*b*c*x^4 + 42*a*d*x^2 + 48*a*c*x - 2*(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4)*((1/2)^(1/3)*(I*sqrt(3) + 1))*((125*b*c^3 + 8*a*d^3)/(a^8*`

$$\begin{aligned}
& b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^{(1/3)} - 20*(1/2)^{(2/3)}*c*d*(-I*\sqrt{3} \\
& (3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a \\
& ^8*b^2))^{(1/3)})) * \log(1/2*((1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((125*b*c^3 + 8*a*d^3 \\
&)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^{(1/3)} - 20*(1/2)^{(2/3)}*c*d*(\\
& -I*\sqrt{3} + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a* \\
& d^3)/(a^8*b^2))^{(1/3)}))^{2*a^6*b*d} - 25/2*((1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((125 \\
& *b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^{(1/3)} - 20*(\\
& 1/2)^{(2/3)}*c*d*(-I*\sqrt{3} + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (\\
& 125*b*c^3 - 8*a*d^3)/(a^8*b^2))^{(1/3)})) * a^3*b*c^2 + 40*a*c*d^2 + (125*b*c^3 \\
& + 8*a*d^3)*x) + ((a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4)*((1/2)^{(1/3)}*(I*\sqrt{3} \\
& + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^{(\\
& 1/3)} - 20*(1/2)^{(2/3)}*c*d*(-I*\sqrt{3} + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a \\
& ^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^{(1/3)})) + 3*\sqrt{1/3}*(a^2*b^2*x \\
& ^6 + 2*a^3*b*x^3 + a^4)*\sqrt{-(((1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((125*b*c^3 + 8 \\
& *a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^{(1/3)} - 20*(1/2)^{(2/3)} \\
& *c*d*(-I*\sqrt{3} + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 \\
& - 8*a*d^3)/(a^8*b^2))^{(1/3)}))^{2*a^5*b} + 160*c*d)/(a^5*b))} * \log(-1/2*((1/2)^ \\
& (1/3)* (I*\sqrt{3} + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d \\
& ^3)/(a^8*b^2))^{(1/3)} - 20*(1/2)^{(2/3)}*c*d*(-I*\sqrt{3} + 1)/(a^5*b*((125*b*c \\
& ^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^{(1/3)}))^{2*a^6*b* \\
& d} + 25/2*((1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (1 \\
& 25*b*c^3 - 8*a*d^3)/(a^8*b^2))^{(1/3)} - 20*(1/2)^{(2/3)}*c*d*(-I*\sqrt{3} + 1)/ \\
& (a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^{(\\
& 1/3)})) * a^3*b*c^2 - 40*a*c*d^2 + 2*(125*b*c^3 + 8*a*d^3)*x + 3/2*\sqrt{1/3} * \\
& (((1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 \\
& - 8*a*d^3)/(a^8*b^2))^{(1/3)} - 20*(1/2)^{(2/3)}*c*d*(-I*\sqrt{3} + 1)/(a^5*b*(\\
& (125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^{(1/3)})) * \\
& a^6*b*d + 25*a^3*b*c^2)*\sqrt{-(((1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((125*b*c^3 + 8 \\
& *a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^{(1/3)} - 20*(1/2)^{(2/3)} \\
& *c*d*(-I*\sqrt{3} + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 \\
& - 8*a*d^3)/(a^8*b^2))^{(1/3)}))^{2*a^5*b} + 160*c*d)/(a^5*b))} + ((a^2*b^2*x^6 \\
& + 2*a^3*b*x^3 + a^4)*((1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((125*b*c^3 + 8*a*d^3)/(a \\
& ^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^{(1/3)} - 20*(1/2)^{(2/3)}*c*d*(-I*s \\
& qrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3) \\
& / (a^8*b^2))^{(1/3)})) - 3*\sqrt{1/3}*(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4)*\sqrt{- (\\
& ((1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 \\
& - 8*a*d^3)/(a^8*b^2))^{(1/3)} - 20*(1/2)^{(2/3)}*c*d*(-I*\sqrt{3} + 1)/(a^5*b*((\\
& 125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^{(1/3)}))^{2 \\
& *a^5*b} + 160*c*d)/(a^5*b))} * \log(-1/2*((1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((125*b*c \\
& ^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^{(1/3)} - 20*(1/2) \\
& ^{(2/3)}*c*d*(-I*\sqrt{3} + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125* \\
& b*c^3 - 8*a*d^3)/(a^8*b^2))^{(1/3)}))^{2*a^6*b*d} + 25/2*((1/2)^{(1/3)}*(I*\sqrt{3} \\
&) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^{(\\
& 1/3)} - 20*(1/2)^{(2/3)}*c*d*(-I*\sqrt{3} + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(\\
& a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^{(1/3)})) * a^3*b*c^2 - 40*a*c*d^2
\end{aligned}$$

$$+ 2*(125*b*c^3 + 8*a*d^3)*x - 3/2*\sqrt{1/3}*(((1/2)^(1/3)*(I*\sqrt{3}) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*\sqrt{3}) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*a^6*b*d + 25*a^3*b*c^2)*\sqrt{-((1/2)^(1/3)*(I*\sqrt{3}) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*\sqrt{3}) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))-2*a^5*b + 160*c*d)/(a^5*b)))/(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4)$$

Sympy [A] time = 1.90385, size = 146, normalized size = 0.68

$$\text{RootSum}\left(19683t^3a^8b^2 + 810ta^3bcd + 8ad^3 - 125bc^3, \left(t \mapsto t \log\left(x + \frac{1458t^2a^6bd + 675ta^3bc^2 + 40acd^2}{8ad^3 + 125bc^3}\right)\right)\right) + \frac{8acx +}{18a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x**3+a)**3,x)

[Out] RootSum(19683*_t**3*a**8*b**2 + 810*_t*a**3*b*c*d + 8*a*d**3 - 125*b*c**3, Lambda(_t, _t*log(x + (1458*_t**2*a**6*b*d + 675*_t*a**3*b*c**2 + 40*a*c*d**2)/(8*a*d**3 + 125*b*c**3)))) + (8*a*c*x + 7*a*d*x**2 + 5*b*c*x**4 + 4*b*d*x**5)/(18*a**4 + 36*a**3*b*x**3 + 18*a**2*b**2*x**6)

Giac [A] time = 1.12425, size = 279, normalized size = 1.3

$$\frac{\left(2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 5c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^3} + \frac{\sqrt{3}\left(5(-ab^2)^{\frac{1}{3}}bc - 2(-ab^2)^{\frac{2}{3}}d\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^3b^2} + \frac{4bdx^5 + 5bc}{18(b^2x^3 + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x^3+a)^3,x, algorithm="giac")

[Out] -1/27*(2*d*(-a/b)^(1/3) + 5*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^3 + 1/27*sqrt(3)*(5*(-a*b^2)^(1/3)*b*c - 2*(-a*b^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^2) + 1/18*(4*b*d*x^5 + 5*b*c*x^4 + 7*a*d*x^2 + 8*a*c*x)/((b*x^3 + a)^2*a^2) + 1/54*(5*(-a*b^2)^(1/3)*a*b^

$$\frac{3c + 2(-ab^2)^{2/3}ab^{2d}\log(x^2 + x(-a/b)^{1/3} + (-a/b)^{2/3})}{a^4b^4}$$

$$3.10 \quad \int \frac{c+dx}{(a+bx^3)^4} dx$$

Optimal. Leaf size=240

$$-\frac{(20\sqrt[3]{bc} - 7\sqrt[3]{ad}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{243a^{11/3}b^{2/3}} + \frac{2(20\sqrt[3]{bc} - 7\sqrt[3]{ad}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{243a^{11/3}b^{2/3}} - \frac{2(7\sqrt[3]{ad} + 20\sqrt[3]{bc}) \tan^{-1}\left(\frac{\sqrt[3]{a}-2}{\sqrt{3}}\right)}{81\sqrt{3}a^{11/3}b^{2/3}}$$

[Out] (x*(c + d*x))/(9*a*(a + b*x^3)^3) + (x*(8*c + 7*d*x))/(54*a^2*(a + b*x^3)^2) + (2*x*(10*c + 7*d*x))/(81*a^3*(a + b*x^3)) - (2*(20*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(81*Sqrt[3]*a^(11/3)*b^(2/3)) + (2*(20*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x]/(243*a^(11/3)*b^(2/3)) - ((20*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(243*a^(11/3)*b^(2/3))

Rubi [A] time = 0.223105, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1855, 1860, 31, 634, 617, 204, 628}

$$-\frac{(20\sqrt[3]{bc} - 7\sqrt[3]{ad}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{243a^{11/3}b^{2/3}} + \frac{2(20\sqrt[3]{bc} - 7\sqrt[3]{ad}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{243a^{11/3}b^{2/3}} - \frac{2(7\sqrt[3]{ad} + 20\sqrt[3]{bc}) \tan^{-1}\left(\frac{\sqrt[3]{a}-2}{\sqrt{3}}\right)}{81\sqrt{3}a^{11/3}b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)/(a + b*x^3)^4, x]

[Out] (x*(c + d*x))/(9*a*(a + b*x^3)^3) + (x*(8*c + 7*d*x))/(54*a^2*(a + b*x^3)^2) + (2*x*(10*c + 7*d*x))/(81*a^3*(a + b*x^3)) - (2*(20*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(81*Sqrt[3]*a^(11/3)*b^(2/3)) + (2*(20*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x]/(243*a^(11/3)*b^(2/3)) - ((20*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(243*a^(11/3)*b^(2/3))

Rule 1855

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &

& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]

Rule 1860

Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
\int \frac{c + dx}{(a + bx^3)^4} dx &= \frac{x(c + dx)}{9a(a + bx^3)^3} - \frac{\int \frac{-8c-7dx}{(a+bx^3)^3} dx}{9a} \\
&= \frac{x(c + dx)}{9a(a + bx^3)^3} + \frac{x(8c + 7dx)}{54a^2(a + bx^3)^2} + \frac{\int \frac{40c+28dx}{(a+bx^3)^2} dx}{54a^2} \\
&= \frac{x(c + dx)}{9a(a + bx^3)^3} + \frac{x(8c + 7dx)}{54a^2(a + bx^3)^2} + \frac{2x(10c + 7dx)}{81a^3(a + bx^3)} - \frac{\int \frac{-80c-28dx}{a+bx^3} dx}{162a^3} \\
&= \frac{x(c + dx)}{9a(a + bx^3)^3} + \frac{x(8c + 7dx)}{54a^2(a + bx^3)^2} + \frac{2x(10c + 7dx)}{81a^3(a + bx^3)} - \frac{\int \frac{\sqrt[3]{a}(-160\sqrt[3]{bc}-28\sqrt[3]{ad})+\sqrt[3]{b}(80\sqrt[3]{bc}-28\sqrt[3]{ad})x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2} dx}{486a^{11/3}\sqrt[3]{b}} + \frac{2(20\sqrt[3]{bc}-7\sqrt[3]{ad})\log(\sqrt[3]{a}+\sqrt[3]{bx})}{243a^{11/3}b^{2/3}} - \frac{(20\sqrt[3]{bc})}{243a^{11/3}b^{2/3}} \\
&= \frac{x(c + dx)}{9a(a + bx^3)^3} + \frac{x(8c + 7dx)}{54a^2(a + bx^3)^2} + \frac{2x(10c + 7dx)}{81a^3(a + bx^3)} + \frac{2(20\sqrt[3]{bc}-7\sqrt[3]{ad})\log(\sqrt[3]{a}+\sqrt[3]{bx})}{243a^{11/3}b^{2/3}} - \frac{(20\sqrt[3]{bc})}{243a^{11/3}b^{2/3}} \\
&= \frac{x(c + dx)}{9a(a + bx^3)^3} + \frac{x(8c + 7dx)}{54a^2(a + bx^3)^2} + \frac{2x(10c + 7dx)}{81a^3(a + bx^3)} + \frac{2(20\sqrt[3]{bc}-7\sqrt[3]{ad})\log(\sqrt[3]{a}+\sqrt[3]{bx})}{243a^{11/3}b^{2/3}} - \frac{(20\sqrt[3]{bc})}{243a^{11/3}b^{2/3}} \\
&= \frac{x(c + dx)}{9a(a + bx^3)^3} + \frac{x(8c + 7dx)}{54a^2(a + bx^3)^2} + \frac{2x(10c + 7dx)}{81a^3(a + bx^3)} - \frac{2(20\sqrt[3]{bc}+7\sqrt[3]{ad})\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{11/3}b^{2/3}} + \frac{2(20\sqrt[3]{bc})}{243a^{11/3}b^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.192324, size = 229, normalized size = 0.95

$$\frac{2(7a^{2/3}d-20\sqrt[3]{a}\sqrt[3]{bc})\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2)}{b^{2/3}} + \frac{4(20\sqrt[3]{a}\sqrt[3]{bc}-7a^{2/3}d)\log(\sqrt[3]{a}+\sqrt[3]{bx})}{b^{2/3}} + \frac{54a^3x(c+dx)}{(a+bx^3)^3} + \frac{9a^2x(8c+7dx)}{(a+bx^3)^2} - \frac{4\sqrt{3}\sqrt[3]{a}(7\sqrt[3]{ad}+20\sqrt[3]{bc})\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{2/3}}$$

$$486a^4$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/(a + b*x^3)^4, x]

[Out] ((54*a^3*x*(c + d*x))/(a + b*x^3)^3 + (9*a^2*x*(8*c + 7*d*x))/(a + b*x^3)^2 + (12*a*x*(10*c + 7*d*x))/(a + b*x^3) - (4*sqrt[3]*a^(1/3)*(20*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]])/b^(2/3) + (4*(20

$$\frac{a^{1/3}b^{1/3}c - 7a^{2/3}d \operatorname{Log}[a^{1/3} + b^{1/3}x]/b^{2/3} + (2(-20a^{1/3}b^{1/3}c + 7a^{2/3}d) \operatorname{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2])/b^{2/3}}{(486a^4)}$$

Maple [A] time = 0.004, size = 306, normalized size = 1.3

$$\frac{cx}{9a(bx^3+a)^3} + \frac{4cx}{27a^2(bx^3+a)^2} + \frac{20cx}{81a^3(bx^3+a)} + \frac{40c}{243a^3b} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{20c}{243a^3b} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)/(b*x^3+a)^4,x)`

[Out] $\frac{1}{9}c/a*x/(b*x^3+a)^3 + \frac{4}{27}c/a^2*x/(b*x^3+a)^2 + \frac{20}{81}c/a^3*x/(b*x^3+a) + \frac{40}{243}c/a^3/b/(1/b*a)^{2/3}*\ln(x+(1/b*a)^{1/3}) - \frac{20}{243}c/a^3/b/(1/b*a)^{2/3}*\ln(x^2-(1/b*a)^{1/3}*x+(1/b*a)^{2/3}) + \frac{40}{243}c/a^3/b/(1/b*a)^{2/3}*3^{1/2}*arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1)) + \frac{1}{9}d/a*x^2/(b*x^3+a)^3 + \frac{7}{54}d/a^2*x^2/(b*x^3+a)^2 + \frac{14}{81}d/a^3*x^2/(b*x^3+a) - \frac{14}{243}d/a^3/b/(1/b*a)^{1/3}*\ln(x+(1/b*a)^{1/3}) + \frac{7}{243}d/a^3/b/(1/b*a)^{1/3}*\ln(x^2-(1/b*a)^{1/3}*x+(1/b*a)^{2/3}) + \frac{14}{243}d/a^3*3^{1/2}/b/(1/b*a)^{1/3}*arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1))$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)/(b*x^3+a)^4,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [C] time = 7.25727, size = 5960, normalized size = 24.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x^3+a)^4,x, algorithm="fricas")

[Out] $\frac{1}{972} \cdot (168b^2dx^8 + 240b^2c^2x^7 + 462ab^2dx^5 + 624abc^2x^4 + 402a^2d^2x^2 + 492a^2c^2x - 2(a^3b^3x^9 + 3a^4b^2x^6 + 3a^5b^2x^3 + a^6) \cdot (4^{1/3} \cdot (I\sqrt{3} + 1) \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2)))^{1/3} - 140 \cdot 4^{2/3} \cdot c \cdot d \cdot (-I\sqrt{3} + 1)/(a^7 \cdot b \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2)))^{1/3}) \cdot \log(7/4 \cdot (4^{1/3} \cdot (I\sqrt{3} + 1) \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2)))^{1/3} - 140 \cdot 4^{2/3} \cdot c \cdot d \cdot (-I\sqrt{3} + 1)/(a^7 \cdot b \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2)))^{1/3}))^2 \cdot a^8 \cdot b \cdot d - 400 \cdot (4^{1/3} \cdot (I\sqrt{3} + 1) \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2)))^{1/3} - 140 \cdot 4^{2/3} \cdot c \cdot d \cdot (-I\sqrt{3} + 1)/(a^7 \cdot b \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2)))^{1/3})) \cdot a^4 \cdot b \cdot c^2 + 7840 \cdot a \cdot c \cdot d^2 + 4 \cdot (8000b^3c^3 + 343ad^3) \cdot x + ((a^3b^3x^9 + 3a^4b^2x^6 + 3a^5b^2x^3 + a^6) \cdot (4^{1/3} \cdot (I\sqrt{3} + 1) \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2)))^{1/3} - 140 \cdot 4^{2/3} \cdot c \cdot d \cdot (-I\sqrt{3} + 1)/(a^7 \cdot b \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2)))^{1/3})) + 3 \cdot \sqrt{1/3} \cdot (a^3b^3x^9 + 3a^4b^2x^6 + 3a^5b^2x^3 + a^6) \cdot \sqrt{-((4^{1/3} \cdot (I\sqrt{3} + 1) \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2)))^{1/3} - 140 \cdot 4^{2/3} \cdot c \cdot d \cdot (-I\sqrt{3} + 1)/(a^7 \cdot b \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2)))^{1/3}))^2 \cdot a^7 \cdot b + 8960 \cdot c \cdot d)/(a^7 \cdot b)) \cdot \log(-7/4 \cdot (4^{1/3} \cdot (I\sqrt{3} + 1) \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2)))^{1/3} - 140 \cdot 4^{2/3} \cdot c \cdot d \cdot (-I\sqrt{3} + 1)/(a^7 \cdot b \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2)))^{1/3}))^2 \cdot a^8 \cdot b \cdot d + 400 \cdot (4^{1/3} \cdot (I\sqrt{3} + 1) \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2)))^{1/3} - 140 \cdot 4^{2/3} \cdot c \cdot d \cdot (-I\sqrt{3} + 1)/(a^7 \cdot b \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2)))^{1/3})) \cdot a^4 \cdot b \cdot c^2 - 7840 \cdot a \cdot c \cdot d^2 + 8 \cdot (8000b^3c^3 + 343ad^3) \cdot x + 3/4 \cdot \sqrt{1/3} \cdot (7 \cdot (4^{1/3} \cdot (I\sqrt{3} + 1) \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2)))^{1/3} - 140 \cdot 4^{2/3} \cdot c \cdot d \cdot (-I\sqrt{3} + 1)/(a^7 \cdot b \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2)))^{1/3})) \cdot a^8 \cdot b \cdot d + 1600 \cdot a^4 \cdot b \cdot c^2) \cdot \sqrt{-((4^{1/3} \cdot (I\sqrt{3} + 1) \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2)))^{1/3} - 140 \cdot 4^{2/3} \cdot c \cdot d \cdot (-I\sqrt{3} + 1)/(a^7 \cdot b \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2)))^{1/3}))^2 \cdot a^7 \cdot b + 8960 \cdot c \cdot d)/(a^7 \cdot b)) + ((a^3b^3x^9 + 3a^4b^2x^6 + 3a^5b^2x^3 + a^6) \cdot (4^{1/3} \cdot (I\sqrt{3} + 1) \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2)))^{1/3} - 140 \cdot 4^{2/3} \cdot c \cdot d \cdot (-I\sqrt{3} + 1)/(a^7 \cdot b \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2)))^{1/3})) - 3 \cdot \sqrt{1/3} \cdot (a^3b^3x^9 + 3a^4b^2x^6 + 3a^5b^2x^3 + a^6) \cdot \sqrt{-((4^{1/3} \cdot (I\sqrt{3} + 1) \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2)))^{1/3} - 140 \cdot 4^{2/3} \cdot c \cdot d \cdot (-I\sqrt{3} + 1)/(a^7 \cdot b \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2)))^{1/3}))^2 \cdot a^7 \cdot b + 8960 \cdot c \cdot d)/(a^7 \cdot b))$

$$7*b*((8000*b*c^3 + 343*a*d^3)/(a^{11}*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^{11}*b^2))^{(1/3)})^2*a^7*b + 8960*c*d)/(a^7*b)))*\log(-7/4*(4^{(1/3)}*(I*\sqrt{3} + 1))*((8000*b*c^3 + 343*a*d^3)/(a^{11}*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^{11}*b^2))^{(1/3)} - 140*4^{(2/3)}*c*d*(-I*\sqrt{3} + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^{11}*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^{11}*b^2))^{(1/3)}))^2*a^8*b*d + 400*(4^{(1/3)}*(I*\sqrt{3} + 1))*((8000*b*c^3 + 343*a*d^3)/(a^{11}*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^{11}*b^2))^{(1/3)} - 140*4^{(2/3)}*c*d*(-I*\sqrt{3} + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^{11}*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^{11}*b^2))^{(1/3)})))*a^4*b*c^2 - 7840*a*c*d^2 + 8*(8000*b*c^3 + 343*a*d^3)*x - 3/4*\sqrt{1/3}*(7*(4^{(1/3)}*(I*\sqrt{3} + 1))*((8000*b*c^3 + 343*a*d^3)/(a^{11}*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^{11}*b^2))^{(1/3)} - 140*4^{(2/3)}*c*d*(-I*\sqrt{3} + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^{11}*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^{11}*b^2))^{(1/3)})))*a^8*b*d + 1600*a^4*b*c^2)*\sqrt{-((4^{(1/3)}*(I*\sqrt{3} + 1))*((8000*b*c^3 + 343*a*d^3)/(a^{11}*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^{11}*b^2))^{(1/3)} - 140*4^{(2/3)}*c*d*(-I*\sqrt{3} + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^{11}*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^{11}*b^2))^{(1/3)}))^2*a^7*b + 8960*c*d)/(a^7*b)))/((a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^6)$$

Sympy [A] time = 3.36028, size = 185, normalized size = 0.77

$$\text{RootSum}\left(14348907t^3a^{11}b^2 + 408240ta^4bcd + 2744ad^3 - 64000bc^3, \left(t \mapsto t \log\left(x + \frac{413343t^2a^8bd + 194400ta^4bc^2 + 7840a^4c^2d}{1372ad^3 + 32000bc^3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x**3+a)**4,x)

[Out] RootSum(14348907*_t**3*a**11*b**2 + 408240*_t*a**4*b*c*d + 2744*a*d**3 - 64000*b*c**3, Lambda(_t, _t*log(x + (413343*_t**2*a**8*b*d + 194400*_t*a**4*b*c**2 + 7840*a*c*d**2)/(1372*a*d**3 + 32000*b*c**3)))) + (82*a**2*c*x + 67*a**2*d*x**2 + 104*a*b*c*x**4 + 77*a*b*d*x**5 + 40*b**2*c*x**7 + 28*b**2*d*x**8)/(162*a**6 + 486*a**5*b*x**3 + 486*a**4*b**2*x**6 + 162*a**3*b**3*x**9)

Giac [A] time = 1.10275, size = 312, normalized size = 1.3

$$\frac{2\left(7d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 20c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243a^4} + \frac{2\sqrt{3}\left(20(-ab^2)^{\frac{1}{3}}bc - 7(-ab^2)^{\frac{2}{3}}d\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243a^4b^2} + \frac{28b^2dx^8}{243a^4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] -2/243*(7*d*(-a/b)^(1/3) + 20*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^
4 + 2/243*sqrt(3)*(20*(-a*b^2)^(1/3)*b*c - 7*(-a*b^2)^(2/3)*d)*arctan(1/3*s
qrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b^2) + 1/162*(28*b^2*d*x^8 +
40*b^2*c*x^7 + 77*a*b*d*x^5 + 104*a*b*c*x^4 + 67*a^2*d*x^2 + 82*a^2*c*x)/(
(b*x^3 + a)^3*a^3) + 1/243*(20*(-a*b^2)^(1/3)*a*b^3*c + 7*(-a*b^2)^(2/3)*a*
b^2*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5*b^4)
```

$$3.11 \quad \int \frac{a+bx}{d+ex^3} dx$$

Optimal. Leaf size=161

$$\frac{\left(a - \frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right) \log\left(d^{2/3} - \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}} - \frac{(b\sqrt[3]{d} - a\sqrt[3]{e}) \log\left(\sqrt[3]{d} + \sqrt[3]{ex}\right)}{3d^{2/3}e^{2/3}} - \frac{(a\sqrt[3]{e} + b\sqrt[3]{d}) \tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{ex}}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}}$$

[Out] -(((b*d^(1/3) + a*e^(1/3))*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(2/3))) - ((b*d^(1/3) - a*e^(1/3))*Log[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(2/3)) - ((a - (b*d^(1/3)))/e^(1/3))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3)*e^(1/3))

Rubi [A] time = 0.121466, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {1860, 31, 634, 617, 204, 628}

$$\frac{\left(a - \frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right) \log\left(d^{2/3} - \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}} - \frac{(b\sqrt[3]{d} - a\sqrt[3]{e}) \log\left(\sqrt[3]{d} + \sqrt[3]{ex}\right)}{3d^{2/3}e^{2/3}} - \frac{(a\sqrt[3]{e} + b\sqrt[3]{d}) \tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{ex}}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/(d + e*x^3), x]

[Out] -(((b*d^(1/3) + a*e^(1/3))*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(2/3))) - ((b*d^(1/3) - a*e^(1/3))*Log[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(2/3)) - ((a - (b*d^(1/3)))/e^(1/3))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3)*e^(1/3))

Rule 1860

Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]

Rule 31

```
Int[((a_) + (b_)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^-1, x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx}{d+ex^3} dx &= \frac{\left(a - \frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right) \int \frac{1}{\sqrt[3]{d} + \sqrt[3]{ex}} dx}{3d^{2/3}} + \frac{\int \frac{\sqrt[3]{d}(b\sqrt[3]{d} + 2a\sqrt[3]{e}) + (b\sqrt[3]{d} - a\sqrt[3]{e})\sqrt[3]{ex}}{d^{2/3} - \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2} dx}{3d^{2/3}\sqrt[3]{e}} \\
&= \frac{\left(a - \frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right) \log(\sqrt[3]{d} + \sqrt[3]{ex})}{3d^{2/3}\sqrt[3]{e}} + \frac{1}{2} \left(\frac{a}{\sqrt[3]{d}} + \frac{b}{\sqrt[3]{e}}\right) \int \frac{1}{d^{2/3} - \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2} dx + \frac{(b\sqrt[3]{d} - a\sqrt[3]{e}) \int \frac{-\sqrt[3]{d}\sqrt[3]{e} + 2e^{2/3}}{d^{2/3} - \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2}}{6d^{2/3}e^{2/3}} \\
&= \frac{\left(a - \frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right) \log(\sqrt[3]{d} + \sqrt[3]{ex})}{3d^{2/3}\sqrt[3]{e}} + \frac{(b\sqrt[3]{d} - a\sqrt[3]{e}) \log(d^{2/3} - \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2)}{6d^{2/3}e^{2/3}} + \frac{(b\sqrt[3]{d} + a\sqrt[3]{e}) \text{Subst}\left(\int \frac{1}{-3-x^2}\right)}{d^{2/3}e^{2/3}} \\
&= -\frac{(b\sqrt[3]{d} + a\sqrt[3]{e}) \tan^{-1}\left(\frac{\sqrt[3]{d} - 2\sqrt[3]{ex}}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}} + \frac{\left(a - \frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right) \log(\sqrt[3]{d} + \sqrt[3]{ex})}{3d^{2/3}\sqrt[3]{e}} + \frac{(b\sqrt[3]{d} - a\sqrt[3]{e}) \log(d^{2/3} - \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2)}{6d^{2/3}e^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.0579368, size = 125, normalized size = 0.78

$$\frac{-(b\sqrt[3]{d} - a\sqrt[3]{e}) \left(2 \log(\sqrt[3]{d} + \sqrt[3]{ex}) - \log(d^{2/3} - \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2)\right) - 2\sqrt{3} (a\sqrt[3]{e} + b\sqrt[3]{d}) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{ex}}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{6d^{2/3}e^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)/(d + e*x^3), x]

[Out] (-2*Sqrt[3]*(b*d^(1/3) + a*e^(1/3))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]] - (b*d^(1/3) - a*e^(1/3))*(2*Log[d^(1/3) + e^(1/3)*x] - Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2]))/(6*d^(2/3)*e^(2/3))

Maple [A] time = 0.048, size = 186, normalized size = 1.2

$$\frac{a}{3e} \ln\left(x + \sqrt[3]{\frac{d}{e}}\right) \left(\frac{d}{e}\right)^{-\frac{2}{3}} - \frac{a}{6e} \ln\left(x^2 - \sqrt[3]{\frac{d}{e}}x + \left(\frac{d}{e}\right)^{\frac{2}{3}}\right) \left(\frac{d}{e}\right)^{-\frac{2}{3}} + \frac{a\sqrt{3}}{3e} \arctan\left(\frac{\sqrt{3}}{3} \left(2x \frac{1}{\sqrt[3]{\frac{d}{e}}} - 1\right)\right) \left(\frac{d}{e}\right)^{-\frac{2}{3}} - \frac{b}{3e} \ln\left(x + \sqrt[3]{\frac{d}{e}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)/(e*x^3+d), x)


```
[Out] 1/3*a/e/(d/e)^(2/3)*ln(x+(d/e)^(1/3))-1/6*a/e/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)
)*x+(d/e)^(2/3))+1/3*a/e/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1
/3)*x-1))-1/3*b/e/(d/e)^(1/3)*ln(x+(d/e)^(1/3))+1/6*b/e/(d/e)^(1/3)*ln(x^2-
(d/e)^(1/3)*x+(d/e)^(2/3))+1/3*b*3^(1/2)/e/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(
2/(d/e)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(e*x^3+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 6.31883, size = 4590, normalized size = 28.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(e*x^3+d),x, algorithm="fricas")
```

```
[Out] -1/6*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3
*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^
3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I
*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)
- 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2) - (b^
3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e - 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1
))*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(
2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)
/(d^2*e^2))^(1/3)))*a^2*d*e + 2*a*b^2*d + (b^3*d + a^3*e)*x) + 1/12*((1/2)^(
1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2
))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^
2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)) + 3*sqrt(1/3)*sqrt(-(((1/2)^(1/3)*(I
*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)
- 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2) - (b^
3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*d*e + 16*a*b)/(d*e))*log(-1/4*((1/2)^(1/
3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(
```

$$\begin{aligned}
& \left(\frac{1}{3} - 2 \left(\frac{1}{2} \right)^{\frac{2}{3}} a b (-I \sqrt{3} + 1) / (d e ((b^3 d + a^3 e) / (d^2 e^2) - (b^3 d - a^3 e) / (d^2 e^2))^{\frac{1}{3}}) \right)^2 b d^2 e + \frac{1}{2} \left(\left(\frac{1}{2} \right)^{\frac{1}{3}} (I \sqrt{3} + 1) ((b^3 d + a^3 e) / (d^2 e^2) - (b^3 d - a^3 e) / (d^2 e^2))^{\frac{1}{3}} - 2 \left(\frac{1}{2} \right)^{\frac{2}{3}} a b (-I \sqrt{3} + 1) / (d e ((b^3 d + a^3 e) / (d^2 e^2) - (b^3 d - a^3 e) / (d^2 e^2))^{\frac{1}{3}}) \right) a^2 d e - 2 a b^2 d + 2 (b^3 d + a^3 e) x + \frac{3}{4} \sqrt[3]{1} \left(\left(\frac{1}{2} \right)^{\frac{1}{3}} (I \sqrt{3} + 1) ((b^3 d + a^3 e) / (d^2 e^2) - (b^3 d - a^3 e) / (d^2 e^2))^{\frac{1}{3}} - 2 \left(\frac{1}{2} \right)^{\frac{2}{3}} a b (-I \sqrt{3} + 1) / (d e ((b^3 d + a^3 e) / (d^2 e^2) - (b^3 d - a^3 e) / (d^2 e^2))^{\frac{1}{3}}) \right) b d^2 e + 2 a^2 d e \sqrt{-\left(\left(\frac{1}{2} \right)^{\frac{1}{3}} (I \sqrt{3} + 1) ((b^3 d + a^3 e) / (d^2 e^2) - (b^3 d - a^3 e) / (d^2 e^2))^{\frac{1}{3}} - 2 \left(\frac{1}{2} \right)^{\frac{2}{3}} a b (-I \sqrt{3} + 1) / (d e ((b^3 d + a^3 e) / (d^2 e^2) - (b^3 d - a^3 e) / (d^2 e^2))^{\frac{1}{3}}) \right)^2 d e + 16 a b} / (d e)) + \frac{1}{12} \left(\left(\frac{1}{2} \right)^{\frac{1}{3}} (I \sqrt{3} + 1) ((b^3 d + a^3 e) / (d^2 e^2) - (b^3 d - a^3 e) / (d^2 e^2))^{\frac{1}{3}} - 2 \left(\frac{1}{2} \right)^{\frac{2}{3}} a b (-I \sqrt{3} + 1) / (d e ((b^3 d + a^3 e) / (d^2 e^2) - (b^3 d - a^3 e) / (d^2 e^2))^{\frac{1}{3}}) \right) - 3 \sqrt[3]{1} \sqrt{-\left(\left(\frac{1}{2} \right)^{\frac{1}{3}} (I \sqrt{3} + 1) ((b^3 d + a^3 e) / (d^2 e^2) - (b^3 d - a^3 e) / (d^2 e^2))^{\frac{1}{3}} - 2 \left(\frac{1}{2} \right)^{\frac{2}{3}} a b (-I \sqrt{3} + 1) / (d e ((b^3 d + a^3 e) / (d^2 e^2) - (b^3 d - a^3 e) / (d^2 e^2))^{\frac{1}{3}}) \right)^2 d e + 16 a b} / (d e)) \log(-\frac{1}{4} \left(\left(\frac{1}{2} \right)^{\frac{1}{3}} (I \sqrt{3} + 1) ((b^3 d + a^3 e) / (d^2 e^2) - (b^3 d - a^3 e) / (d^2 e^2))^{\frac{1}{3}} - 2 \left(\frac{1}{2} \right)^{\frac{2}{3}} a b (-I \sqrt{3} + 1) / (d e ((b^3 d + a^3 e) / (d^2 e^2) - (b^3 d - a^3 e) / (d^2 e^2))^{\frac{1}{3}}) \right)^2 b d^2 e + \frac{1}{2} \left(\left(\frac{1}{2} \right)^{\frac{1}{3}} (I \sqrt{3} + 1) ((b^3 d + a^3 e) / (d^2 e^2) - (b^3 d - a^3 e) / (d^2 e^2))^{\frac{1}{3}} - 2 \left(\frac{1}{2} \right)^{\frac{2}{3}} a b (-I \sqrt{3} + 1) / (d e ((b^3 d + a^3 e) / (d^2 e^2) - (b^3 d - a^3 e) / (d^2 e^2))^{\frac{1}{3}}) \right) a^2 d e - 2 a b^2 d + 2 (b^3 d + a^3 e) x - \frac{3}{4} \sqrt[3]{1} \left(\left(\frac{1}{2} \right)^{\frac{1}{3}} (I \sqrt{3} + 1) ((b^3 d + a^3 e) / (d^2 e^2) - (b^3 d - a^3 e) / (d^2 e^2))^{\frac{1}{3}} - 2 \left(\frac{1}{2} \right)^{\frac{2}{3}} a b (-I \sqrt{3} + 1) / (d e ((b^3 d + a^3 e) / (d^2 e^2) - (b^3 d - a^3 e) / (d^2 e^2))^{\frac{1}{3}}) \right) b d^2 e + 2 a^2 d e \sqrt{-\left(\left(\frac{1}{2} \right)^{\frac{1}{3}} (I \sqrt{3} + 1) ((b^3 d + a^3 e) / (d^2 e^2) - (b^3 d - a^3 e) / (d^2 e^2))^{\frac{1}{3}} - 2 \left(\frac{1}{2} \right)^{\frac{2}{3}} a b (-I \sqrt{3} + 1) / (d e ((b^3 d + a^3 e) / (d^2 e^2) - (b^3 d - a^3 e) / (d^2 e^2))^{\frac{1}{3}}) \right)^2 d e + 16 a b} / (d e))
\end{aligned}$$

Sympy [A] time = 1.24309, size = 76, normalized size = 0.47

$$\text{RootSum} \left(27 t^3 d^2 e^2 + 9 t a b d e - a^3 e + b^3 d, \left(t \mapsto t \log \left(x + \frac{9 t^2 b d^2 e + 3 t a^2 d e + 2 a b^2 d}{a^3 e + b^3 d} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)/(e*x**3+d),x)

[Out] RootSum(27*_t**3*d**2*e**2 + 9*_t*a*b*d*e - a**3*e + b**3*d, Lambda(_t, _t*log(x + (9*_t**2*b*d**2*e + 3*_t*a**2*d*e + 2*a*b**2*d)/(a**3*e + b**3*d)))

)

Giac [A] time = 1.11335, size = 197, normalized size = 1.22

$$\frac{\sqrt{3} \left((-de^2)^{\frac{1}{3}} ae - (-de^2)^{\frac{2}{3}} b \right) \arctan \left(\frac{\sqrt{3} \left(2x + (-de^{-1})^{\frac{1}{3}} \right)}{3(-de^{-1})^{\frac{1}{3}}} \right) e^{(-2)}}{3d} - \frac{(-de^{-1})^{\frac{1}{3}} \left((-de^{-1})^{\frac{1}{3}} b + a \right) \log \left(\left| x - (-de^{-1})^{\frac{1}{3}} \right| \right)}{3d} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)/(e*x^3+d),x, algorithm="giac")

[Out] 1/3*sqrt(3)*((-d*e^2)^(1/3)*a*e - (-d*e^2)^(2/3)*b)*arctan(1/3*sqrt(3)*(2*x + (-d*e^(-1))^(1/3))/(-d*e^(-1))^(1/3))*e^(-2)/d - 1/3*(-d*e^(-1))^(1/3)*((-d*e^(-1))^(1/3)*b + a)*log(abs(x - (-d*e^(-1))^(1/3)))/d + 1/6*((-d*e^2)^(2/3)*b*d*e^2 + (-d*e^2)^(1/3)*a*d*e^3)*e^(-4)*log(x^2 + (-d*e^(-1))^(1/3)*x + (-d*e^(-1))^(2/3))/d^2

3.12 $\int \frac{a+bx}{d-ex^3} dx$

Optimal. Leaf size=161

$$\frac{(a\sqrt[3]{e} + b\sqrt[3]{d}) \log(d^{2/3} + \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2)}{6d^{2/3}e^{2/3}} - \frac{(a\sqrt[3]{e} + b\sqrt[3]{d}) \log(\sqrt[3]{d} - \sqrt[3]{ex})}{3d^{2/3}e^{2/3}} - \frac{(b\sqrt[3]{d} - a\sqrt[3]{e}) \tan^{-1}\left(\frac{\sqrt[3]{d} + 2\sqrt[3]{ex}}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}}$$

[Out] -(((b*d^(1/3) - a*e^(1/3))*ArcTan[(d^(1/3) + 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))]/(Sqrt[3]*d^(2/3)*e^(2/3))) - ((b*d^(1/3) + a*e^(1/3))*Log[d^(1/3) - e^(1/3)*x]/(3*d^(2/3)*e^(2/3))) + ((b*d^(1/3) + a*e^(1/3))*Log[d^(2/3) + d^(1/3)*e^(1/3)*x + e^(2/3)*x^2]/(6*d^(2/3)*e^(2/3)))

Rubi [A] time = 0.100357, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1861, 31, 634, 617, 204, 628}

$$\frac{(a\sqrt[3]{e} + b\sqrt[3]{d}) \log(d^{2/3} + \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2)}{6d^{2/3}e^{2/3}} - \frac{(a\sqrt[3]{e} + b\sqrt[3]{d}) \log(\sqrt[3]{d} - \sqrt[3]{ex})}{3d^{2/3}e^{2/3}} - \frac{(b\sqrt[3]{d} - a\sqrt[3]{e}) \tan^{-1}\left(\frac{\sqrt[3]{d} + 2\sqrt[3]{ex}}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/(d - e*x^3), x]

[Out] -(((b*d^(1/3) - a*e^(1/3))*ArcTan[(d^(1/3) + 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))]/(Sqrt[3]*d^(2/3)*e^(2/3))) - ((b*d^(1/3) + a*e^(1/3))*Log[d^(1/3) - e^(1/3)*x]/(3*d^(2/3)*e^(2/3))) + ((b*d^(1/3) + a*e^(1/3))*Log[d^(2/3) + d^(1/3)*e^(1/3)*x + e^(2/3)*x^2]/(6*d^(2/3)*e^(2/3)))

Rule 1861

Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 3]], s = Denominator[Rt[-(a/b), 3]]}, Dist[(r*(B*r + A*s))/(3*a*s), Int[1/(r - s*x), x], x] - Dist[r/(3*a*s), Int[(r*(B*r - 2*A*s) - s*(B*r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && NegQ[a/b]

Rule 31

```
Int[((a_) + (b_)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^-1, x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx}{d-ex^3} dx &= \frac{\left(a + \frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right) \int \frac{1}{\sqrt[3]{d}-\sqrt[3]{ex}} dx}{3d^{2/3}} - \frac{\int \frac{\sqrt[3]{d}(b\sqrt[3]{d}-2a\sqrt[3]{e})-(b\sqrt[3]{d}+a\sqrt[3]{e})\sqrt[3]{ex}}{d^{2/3}+\sqrt[3]{d}\sqrt[3]{ex}+e^{2/3}x^2} dx}{3d^{2/3}\sqrt[3]{e}} \\
&= -\frac{(b\sqrt[3]{d}+a\sqrt[3]{e})\log(\sqrt[3]{d}-\sqrt[3]{ex})}{3d^{2/3}e^{2/3}} - \frac{1}{2}\left(-\frac{a}{\sqrt[3]{d}}+\frac{b}{\sqrt[3]{e}}\right) \int \frac{1}{d^{2/3}+\sqrt[3]{d}\sqrt[3]{ex}+e^{2/3}x^2} dx + \frac{(b\sqrt[3]{d}+a\sqrt[3]{e}) \int \frac{\sqrt[3]{d}\sqrt[3]{ex}}{d^{2/3}+\sqrt[3]{d}\sqrt[3]{ex}+e^{2/3}x^2}}{6d^{2/3}e^{2/3}} \\
&= -\frac{(b\sqrt[3]{d}+a\sqrt[3]{e})\log(\sqrt[3]{d}-\sqrt[3]{ex})}{3d^{2/3}e^{2/3}} + \frac{(b\sqrt[3]{d}+a\sqrt[3]{e})\log(d^{2/3}+\sqrt[3]{d}\sqrt[3]{ex}+e^{2/3}x^2)}{6d^{2/3}e^{2/3}} + \frac{(b\sqrt[3]{d}-a\sqrt[3]{e})\text{Subst}\left(\int \frac{1}{u^2+1} du\right)}{d^{2/3}e^{2/3}} \\
&= -\frac{(b\sqrt[3]{d}-a\sqrt[3]{e})\tan^{-1}\left(\frac{\sqrt[3]{d}+2\sqrt[3]{ex}}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}} - \frac{(b\sqrt[3]{d}+a\sqrt[3]{e})\log(\sqrt[3]{d}-\sqrt[3]{ex})}{3d^{2/3}e^{2/3}} + \frac{(b\sqrt[3]{d}+a\sqrt[3]{e})\log(d^{2/3}+\sqrt[3]{d}\sqrt[3]{ex}+e^{2/3}x^2)}{6d^{2/3}e^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.0489133, size = 125, normalized size = 0.78

$$\frac{-(a\sqrt[3]{e}+b\sqrt[3]{d})\left(2\log(\sqrt[3]{d}-\sqrt[3]{ex})-\log(d^{2/3}+\sqrt[3]{d}\sqrt[3]{ex}+e^{2/3}x^2)\right)-2\sqrt{3}(b\sqrt[3]{d}-a\sqrt[3]{e})\tan^{-1}\left(\frac{\frac{2\sqrt[3]{ex}+1}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{6d^{2/3}e^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)/(d - e*x^3), x]

[Out] (-2*Sqrt[3]*(b*d^(1/3) - a*e^(1/3))*ArcTan[(1 + (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]] - (b*d^(1/3) + a*e^(1/3))*(2*Log[d^(1/3) - e^(1/3)*x] - Log[d^(2/3) + d^(1/3)*e^(1/3)*x + e^(2/3)*x^2]))/(6*d^(2/3)*e^(2/3))

Maple [A] time = 0.004, size = 188, normalized size = 1.2

$$-\frac{a}{3e} \ln\left(x - \sqrt[3]{\frac{d}{e}}\right) \left(\frac{d}{e}\right)^{-\frac{2}{3}} + \frac{a}{6e} \ln\left(x^2 + \sqrt[3]{\frac{d}{e}}x + \left(\frac{d}{e}\right)^{\frac{2}{3}}\right) \left(\frac{d}{e}\right)^{-\frac{2}{3}} + \frac{a\sqrt{3}}{3e} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{d}{e}}} + 1\right)\right) \left(\frac{d}{e}\right)^{-\frac{2}{3}} - \frac{b}{3e} \ln\left(x - \sqrt[3]{\frac{d}{e}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)/(-e*x^3+d), x)


```

+ a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d
- a^3*e)/(d^2*e^2))^(1/3))^2*b*d^2*e + 1/6*(9*(I*sqrt(3) + 1)*(-1/54*(b^3
*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sq
r t(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^
2*e^2))^(1/3)))*a^2*d*e + 2*a*b^2*d - 2*(b^3*d - a^3*e)*x + 1/12*sqrt(1/3)*
(9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/
(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^
2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*b*d^2*e + 6*a^2*d*e)*sqrt(-((9
*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d
^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2)
- 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*d*e - 144*a*b)/(d*e))) + 1/36*
(9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/
(d^2*e^2))^(1/3) - 3*sqrt(1/3)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^
3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) +
1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(
1/3)))^2*d*e - 144*a*b)/(d*e)) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d +
a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*log(-1/36*(9*(I
*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*
e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) -
1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e + 1/6*(9*(I*sqrt(3) + 1)*
(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) +
a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d -
a^3*e)/(d^2*e^2))^(1/3)))*a^2*d*e + 2*a*b^2*d - 2*(b^3*d - a^3*e)*x - 1/12*
sqrt(1/3)*((9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*
d - a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3
*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*b*d^2*e + 6*a^2*d*e
)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d
- a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e
)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*d*e - 144*a*b)/(d*e
)))

```

Sympy [A] time = 1.21121, size = 78, normalized size = 0.48

$$-\text{RootSum}\left(27t^3d^2e^2 - 9tabde - a^3e - b^3d, \left(t \mapsto t \log\left(x + \frac{9t^2bd^2e - 3ta^2de - 2ab^2d}{a^3e - b^3d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)/(-e*x**3+d), x)

[Out] -RootSum(27*_t**3*d**2*e**2 - 9*_t*a*b*d*e - a**3*e - b**3*d, Lambda(_t, _t
*log(x + (9*_t**2*b*d**2*e - 3*_t*a**2*d*e - 2*a*b**2*d)/(a**3*e - b**3*d))

))

Giac [A] time = 1.07933, size = 155, normalized size = 0.96

$$\frac{\sqrt{3}\left(bd^{\frac{2}{3}}e^{\frac{4}{3}} - ad^{\frac{1}{3}}e^{\frac{5}{3}}\right) \arctan\left(\frac{\sqrt{3}\left(d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)} + 2x\right)e^{\frac{1}{3}}}{3d^{\frac{1}{3}}}\right) e^{(-2)}}{3d} - \frac{\left(bd^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)} + a\right) e^{\left(-\frac{1}{3}\right)} \log\left(\left|-d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)} + x\right|\right)}{3d^{\frac{2}{3}}} + \frac{\left(bd^{\frac{2}{3}}e^{\frac{4}{3}} + ad^{\frac{1}{3}}e^{\frac{5}{3}}\right) e^{(-2)}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)/(-e*x^3+d),x, algorithm="giac")

[Out] $-1/3*\sqrt{3}*(b*d^{(2/3)}*e^{(4/3)} - a*d^{(1/3)}*e^{(5/3)})*\arctan(1/3*\sqrt{3}*(d^{(1/3)}*e^{(-1/3)} + 2*x)*e^{(1/3)}/d^{(1/3)})*e^{(-2)}/d - 1/3*(b*d^{(1/3)}*e^{(-1/3)} + a)*e^{(-1/3)}*\log(\text{abs}(-d^{(1/3)}*e^{(-1/3)} + x))/d^{(2/3)} + 1/6*(b*d^{(2/3)}*e^{(4/3)} + a*d^{(1/3)}*e^{(5/3)})*e^{(-2)}*\log(d^{(1/3)}*x*e^{(-1/3)} + x^2 + d^{(2/3)}*e^{(-2/3)})/d$

3.13 $\int \frac{1+x}{1+x^3} dx$

Optimal. Leaf size=19

$$\frac{2 \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] (-2*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3]

Rubi [A] time = 0.0152534, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1586, 618, 204}

$$\frac{2 \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x)/(1 + x^3), x]

[Out] (-2*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3]

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1+x}{1+x^3} dx &= \int \frac{1}{1-x+x^2} dx \\ &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{-3-x^2} dx, x, -1+2x\right)\right) \\ &= -\frac{2 \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.0048812, size = 19, normalized size = 1.

$$\frac{2 \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x)/(1 + x^3), x]

[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3]

Maple [A] time = 0.022, size = 17, normalized size = 0.9

$$\frac{2\sqrt{3}}{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x)/(x^3+1), x)

[Out] 2/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))

Maxima [A] time = 1.44645, size = 22, normalized size = 1.16

$$\frac{2}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x^3+1),x, algorithm="maxima")

[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1))

Fricas [A] time = 0.954347, size = 58, normalized size = 3.05

$$\frac{2}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x - 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x^3+1),x, algorithm="fricas")

[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1))

Sympy [A] time = 0.103765, size = 26, normalized size = 1.37

$$\frac{2\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x**3+1),x)

[Out] 2*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3

Giac [A] time = 1.07871, size = 22, normalized size = 1.16

$$\frac{2}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x - 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x^3+1),x, algorithm="giac")

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1))
```

$$3.14 \quad \int \frac{1-x}{1-x^3} dx$$

Optimal. Leaf size=19

$$\frac{2 \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right)}{\sqrt{3}}$$

[Out] (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3]

Rubi [A] time = 0.0133185, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1586, 618, 204}

$$\frac{2 \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x)/(1 - x^3), x]

[Out] (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3]

Rule 1586

Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1-x}{1-x^3} dx &= \int \frac{1}{1+x+x^2} dx \\ &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1+2x\right)\right) \\ &= \frac{2 \tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.0043036, size = 19, normalized size = 1.

$$\frac{2 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x)/(1 - x^3), x]

[Out] (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3]

Maple [A] time = 0.001, size = 17, normalized size = 0.9

$$\frac{2\sqrt{3}}{3} \arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-x)/(-x^3+1), x)

[Out] 2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)

Maxima [A] time = 1.43222, size = 22, normalized size = 1.16

$$\frac{2}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)/(-x^3+1),x, algorithm="maxima")

[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1))

Fricas [A] time = 0.892496, size = 58, normalized size = 3.05

$$\frac{2}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)/(-x^3+1),x, algorithm="fricas")

[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1))

Sympy [A] time = 0.097308, size = 26, normalized size = 1.37

$$\frac{2\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)/(-x**3+1),x)

[Out] 2*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3

Giac [A] time = 1.15932, size = 22, normalized size = 1.16

$$\frac{2}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)/(-x^3+1),x, algorithm="giac")


```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1))
```

3.15 $\int \frac{1+x}{1-x^3} dx$

Optimal. Leaf size=22

$$\frac{1}{3} \log(x^2 + x + 1) - \frac{2}{3} \log(1 - x)$$

[Out] $(-2*\text{Log}[1 - x])/3 + \text{Log}[1 + x + x^2]/3$

Rubi [A] time = 0.0121661, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1861, 31, 628}

$$\frac{1}{3} \log(x^2 + x + 1) - \frac{2}{3} \log(1 - x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 + x)/(1 - x^3), x]$

[Out] $(-2*\text{Log}[1 - x])/3 + \text{Log}[1 + x + x^2]/3$

Rule 1861

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 3]], s = Denominator[Rt[-(a/b), 3]]}, Dist[(r*(B*r + A*s))/(3*a*s), Int[1/(r - s*x), x], x] - Dist[r/(3*a*s), Int[(r*(B*r - 2*A*s) - s*(B*r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && NegQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1+x}{1-x^3} dx = -\left(\frac{1}{3} \int \frac{-1-2x}{1+x+x^2} dx\right) + \frac{2}{3} \int \frac{1}{1-x} dx$$

$$= -\frac{2}{3} \log(1-x) + \frac{1}{3} \log(1+x+x^2)$$

Mathematica [A] time = 0.003773, size = 22, normalized size = 1.

$$\frac{1}{3} \log(x^2 + x + 1) - \frac{2}{3} \log(1 - x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x)/(1 - x^3), x]

[Out] (-2*Log[1 - x])/3 + Log[1 + x + x^2]/3

Maple [A] time = 0.004, size = 17, normalized size = 0.8

$$-\frac{2 \ln(-1 + x)}{3} + \frac{\ln(x^2 + x + 1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x)/(-x^3+1), x)

[Out] -2/3*ln(-1+x)+1/3*ln(x^2+x+1)

Maxima [A] time = 1.43893, size = 22, normalized size = 1.

$$\frac{1}{3} \log(x^2 + x + 1) - \frac{2}{3} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(-x^3+1), x, algorithm="maxima")

[Out] $\frac{1}{3}\log(x^2 + x + 1) - \frac{2}{3}\log(x - 1)$

Fricas [A] time = 0.917206, size = 53, normalized size = 2.41

$$\frac{1}{3} \log(x^2 + x + 1) - \frac{2}{3} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x)/(-x^3+1),x, algorithm="fricas")`

[Out] $\frac{1}{3}\log(x^2 + x + 1) - \frac{2}{3}\log(x - 1)$

Sympy [A] time = 0.09598, size = 17, normalized size = 0.77

$$-\frac{2 \log(x - 1)}{3} + \frac{\log(x^2 + x + 1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x)/(-x**3+1),x)`

[Out] $-2*\log(x - 1)/3 + \log(x**2 + x + 1)/3$

Giac [A] time = 1.19189, size = 23, normalized size = 1.05

$$\frac{1}{3} \log(x^2 + x + 1) - \frac{2}{3} \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x)/(-x^3+1),x, algorithm="giac")`

[Out] $\frac{1}{3}\log(x^2 + x + 1) - \frac{2}{3}\log(\text{abs}(x - 1))$

$$3.16 \quad \int \frac{1-x}{1+x^3} dx$$

Optimal. Leaf size=22

$$\frac{2}{3} \log(x+1) - \frac{1}{3} \log(x^2 - x + 1)$$

[Out] (2*Log[1 + x])/3 - Log[1 - x + x^2]/3

Rubi [A] time = 0.0126704, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1860, 31, 628}

$$\frac{2}{3} \log(x+1) - \frac{1}{3} \log(x^2 - x + 1)$$

Antiderivative was successfully verified.

[In] Int[(1 - x)/(1 + x^3), x]

[Out] (2*Log[1 + x])/3 - Log[1 - x + x^2]/3

Rule 1860

Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]

Rule 31

Int[((a_) + (b_)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}\int \frac{1-x}{1+x^3} dx &= \frac{1}{3} \int \frac{1-2x}{1-x+x^2} dx + \frac{2}{3} \int \frac{1}{1+x} dx \\ &= \frac{2}{3} \log(1+x) - \frac{1}{3} \log(1-x+x^2)\end{aligned}$$

Mathematica [A] time = 0.0036034, size = 22, normalized size = 1.

$$\frac{2}{3} \log(x+1) - \frac{1}{3} \log(x^2-x+1)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x)/(1 + x^3), x]

[Out] (2*Log[1 + x])/3 - Log[1 - x + x^2]/3

Maple [A] time = 0.004, size = 19, normalized size = 0.9

$$\frac{2 \ln(1+x)}{3} - \frac{\ln(x^2-x+1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-x)/(x^3+1), x)

[Out] 2/3*ln(1+x)-1/3*ln(x^2-x+1)

Maxima [A] time = 1.47228, size = 24, normalized size = 1.09

$$-\frac{1}{3} \log(x^2-x+1) + \frac{2}{3} \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)/(x^3+1), x, algorithm="maxima")

[Out] -1/3*log(x^2 - x + 1) + 2/3*log(x + 1)

Fricas [A] time = 1.00901, size = 54, normalized size = 2.45

$$-\frac{1}{3} \log(x^2 - x + 1) + \frac{2}{3} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)/(x^3+1),x, algorithm="fricas")

[Out] -1/3*log(x^2 - x + 1) + 2/3*log(x + 1)

Sympy [A] time = 0.091979, size = 17, normalized size = 0.77

$$\frac{2 \log(x + 1)}{3} - \frac{\log(x^2 - x + 1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)/(x**3+1),x)

[Out] 2*log(x + 1)/3 - log(x**2 - x + 1)/3

Giac [A] time = 1.11873, size = 26, normalized size = 1.18

$$-\frac{1}{3} \log(x^2 - x + 1) + \frac{2}{3} \log(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)/(x^3+1),x, algorithm="giac")

[Out] -1/3*log(x^2 - x + 1) + 2/3*log(abs(x + 1))

3.17 $\int \frac{3-x}{1-x^3} dx$

Optimal. Leaf size=41

$$\frac{1}{3} \log(x^2 + x + 1) - \frac{2}{3} \log(1 - x) + \frac{4 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] (4*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 - x])/3 + Log[1 + x + x^2]/3

Rubi [A] time = 0.0272263, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {1861, 31, 634, 618, 204, 628}

$$\frac{1}{3} \log(x^2 + x + 1) - \frac{2}{3} \log(1 - x) + \frac{4 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x)/(1 - x^3), x]

[Out] (4*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 - x])/3 + Log[1 + x + x^2]/3

Rule 1861

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 3]], s = Denominator[Rt[-(a/b), 3]]}, Dist[(r*(B*r + A*s))/(3*a*s), Int[1/(r - s*x), x], x] - Dist[r/(3*a*s), Int[(r*(B*r - 2*A*s) - s*(B*r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && NegQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```



```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{3-x}{1-x^3} dx &= -\left(\frac{1}{3} \int \frac{-7-2x}{1+x+x^2} dx\right) + \frac{2}{3} \int \frac{1}{1-x} dx \\ &= -\frac{2}{3} \log(1-x) + \frac{1}{3} \int \frac{1+2x}{1+x+x^2} dx + 2 \int \frac{1}{1+x+x^2} dx \\ &= -\frac{2}{3} \log(1-x) + \frac{1}{3} \log(1+x+x^2) - 4 \operatorname{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1+2x\right) \\ &= \frac{4 \tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3} \log(1-x) + \frac{1}{3} \log(1+x+x^2) \end{aligned}$$

Mathematica [A] time = 0.0075016, size = 41, normalized size = 1.

$$\frac{1}{3} \log(x^2 + x + 1) - \frac{2}{3} \log(1-x) + \frac{4 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x)/(1 - x^3), x]
```

[Out] $(4*\text{ArcTan}[(1 + 2*x)/\text{Sqrt}[3]])/\text{Sqrt}[3] - (2*\text{Log}[1 - x])/3 + \text{Log}[1 + x + x^2]/3$

Maple [A] time = 0.004, size = 33, normalized size = 0.8

$$-\frac{2 \ln(-1+x)}{3} + \frac{\ln(x^2+x+1)}{3} + \frac{4\sqrt{3}}{3} \arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3-x)/(-x^3+1),x)`

[Out] $-2/3*\ln(-1+x)+1/3*\ln(x^2+x+1)+4/3*\arctan(1/3*(1+2*x)*3^{(1/2)})*3^{(1/2)}$

Maxima [A] time = 1.4424, size = 43, normalized size = 1.05

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{3}\log(x^2+x+1) - \frac{2}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3-x)/(-x^3+1),x, algorithm="maxima")`

[Out] $4/3*\text{sqrt}(3)*\arctan(1/3*\text{sqrt}(3)*(2*x + 1)) + 1/3*\log(x^2 + x + 1) - 2/3*\log(x - 1)$

Fricas [A] time = 0.948494, size = 112, normalized size = 2.73

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{3}\log(x^2+x+1) - \frac{2}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3-x)/(-x^3+1),x, algorithm="fricas")`

[Out] $4/3*\text{sqrt}(3)*\arctan(1/3*\text{sqrt}(3)*(2*x + 1)) + 1/3*\log(x^2 + x + 1) - 2/3*\log(x - 1)$

Sympy [A] time = 0.135038, size = 44, normalized size = 1.07

$$-\frac{2 \log(x-1)}{3} + \frac{\log(x^2+x+1)}{3} + \frac{4\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3-x)/(-x**3+1),x)

[Out] -2*log(x - 1)/3 + log(x**2 + x + 1)/3 + 4*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3

Giac [A] time = 1.0882, size = 45, normalized size = 1.1

$$\frac{4}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x+1)\right) + \frac{1}{3} \log(x^2+x+1) - \frac{2}{3} \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3-x)/(-x^3+1),x, algorithm="giac")

[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/3*log(x^2 + x + 1) - 2/3*log(abs(x - 1))

$$3.18 \quad \int \frac{c+dx}{c^3+d^3x^3} dx$$

Optimal. Leaf size=29

$$-\frac{2 \tan^{-1}\left(\frac{c-2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

[Out] $(-2*\text{ArcTan}[(c - 2*d*x)/(\text{Sqrt}[3]*c)])/(\text{Sqrt}[3]*c*d)$

Rubi [A] time = 0.0216336, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1586, 617, 204}

$$-\frac{2 \tan^{-1}\left(\frac{c-2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)/(c^3 + d^3*x^3), x]$

[Out] $(-2*\text{ArcTan}[(c - 2*d*x)/(\text{Sqrt}[3]*c)])/(\text{Sqrt}[3]*c*d)$

Rule 1586

$\text{Int}[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] \rightarrow \text{Int}[u*\text{PolynomialQuotient}[Px, Qx, x]^{p*Qx^{(p+q)}, x} /; \text{FreeQ}[q, x] \ \&\& \ \text{PolyQ}[Px, x] \ \&\& \ \text{PolyQ}[Qx, x] \ \&\& \ \text{EqQ}[\text{PolynomialRemainder}[Px, Qx, x], 0] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ \text{LtQ}[p*q, 0]$

Rule 617

$\text{Int}[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]^{-1}, x_Symbol] \rightarrow \text{With}[\{q = 1 - 4*Simplify[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4*a*c]) /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 204

$\text{Int}[(a_) + (b_.)*(x_)^2]^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[\text{Rt}[-a, 2], \text{Rt}[-b, 2]])$

a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{c+dx}{c^3+d^3x^3} dx &= \int \frac{1}{c^2-cdx+d^2x^2} dx \\ &= \frac{2 \operatorname{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1-\frac{2dx}{c}\right)}{cd} \\ &= -\frac{2 \tan^{-1}\left(\frac{c-2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd} \end{aligned}$$

Mathematica [A] time = 0.0101002, size = 31, normalized size = 1.07

$$\frac{2 \tan^{-1}\left(\frac{2dx-c}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/(c^3 + d^3*x^3), x]

[Out] (2*ArcTan[(-c + 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)

Maple [A] time = 0.067, size = 35, normalized size = 1.2

$$\frac{2\sqrt{3}}{3cd} \arctan\left(\frac{(2d^2x - cd)\sqrt{3}}{3cd}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)/(d^3*x^3+c^3), x)

[Out] 2/3*3^(1/2)/c/d*arctan(1/3*(2*d^2*x-c*d)*3^(1/2)/c/d)

Maxima [A] time = 1.45929, size = 46, normalized size = 1.59

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2d^2x-cd)}{3cd}\right)}{3cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(d^3*x^3+c^3),x, algorithm="maxima")

[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d^2*x - c*d)/(c*d))/(c*d)

Fricas [A] time = 0.903478, size = 72, normalized size = 2.48

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2dx-c)}{3c}\right)}{3cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(d^3*x^3+c^3),x, algorithm="fricas")

[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x - c)/c)/(c*d)

Sympy [C] time = 0.156581, size = 54, normalized size = 1.86

$$\frac{-\frac{\sqrt{3}i\log\left(x+\frac{-c-\sqrt{3}ic}{2d}\right)}{3} + \frac{\sqrt{3}i\log\left(x+\frac{-c+\sqrt{3}ic}{2d}\right)}{3}}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(d**3*x**3+c**3),x)

[Out] (-sqrt(3)*I*log(x + (-c - sqrt(3)*I*c)/(2*d))/3 + sqrt(3)*I*log(x + (-c + sqrt(3)*I*c)/(2*d))/3)/(c*d)

Giac [A] time = 1.11921, size = 38, normalized size = 1.31

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2dx-c)}{3c}\right)}{3cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(d^3*x^3+c^3),x, algorithm="giac")

[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x - c)/c)/(c*d)

$$3.19 \quad \int \frac{c-dx}{c^3-d^3x^3} dx$$

Optimal. Leaf size=29

$$\frac{2 \tan^{-1}\left(\frac{c+2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

[Out] (2*ArcTan[(c + 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)

Rubi [A] time = 0.0203174, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1586, 617, 204}

$$\frac{2 \tan^{-1}\left(\frac{c+2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

Antiderivative was successfully verified.

[In] Int[(c - d*x)/(c^3 - d^3*x^3), x]

[Out] (2*ArcTan[(c + 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p+q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```


a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{c - dx}{c^3 - d^3 x^3} dx &= \int \frac{1}{c^2 + cdx + d^2 x^2} dx \\ &= \frac{2 \operatorname{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 + \frac{2dx}{c}\right)}{cd} \\ &= \frac{2 \tan^{-1}\left(\frac{c+2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd} \end{aligned}$$

Mathematica [A] time = 0.0074607, size = 29, normalized size = 1.

$$\frac{2 \tan^{-1}\left(\frac{c+2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

Antiderivative was successfully verified.

[In] Integrate[(c - d*x)/(c^3 - d^3*x^3),x]

[Out] (2*ArcTan[(c + 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)

Maple [A] time = 0.002, size = 34, normalized size = 1.2

$$\frac{2\sqrt{3}}{3cd} \arctan\left(\frac{(2d^2x + cd)\sqrt{3}}{3cd}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-d*x+c)/(-d^3*x^3+c^3),x)

[Out] 2/3*3^(1/2)/c/d*arctan(1/3*(2*d^2*x+c*d)*3^(1/2)/c/d)

Maxima [A] time = 1.43697, size = 45, normalized size = 1.55

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2d^2x+cd)}{3cd}\right)}{3cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-d*x+c)/(-d^3*x^3+c^3),x, algorithm="maxima")

[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d^2*x + c*d)/(c*d))/(c*d)

Fricas [A] time = 1.02726, size = 72, normalized size = 2.48

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2dx+c)}{3c}\right)}{3cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-d*x+c)/(-d^3*x^3+c^3),x, algorithm="fricas")

[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x + c)/c)/(c*d)

Sympy [C] time = 0.213179, size = 53, normalized size = 1.83

$$\frac{\frac{\sqrt{3}i\log\left(x+\frac{c-\sqrt{3}ic}{2d}\right)}{3} + \frac{\sqrt{3}i\log\left(x+\frac{c+\sqrt{3}ic}{2d}\right)}{3}}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-d*x+c)/(-d**3*x**3+c**3),x)

[Out] (-sqrt(3)*I*log(x + (c - sqrt(3)*I*c)/(2*d))/3 + sqrt(3)*I*log(x + (c + sqrt(3)*I*c)/(2*d))/3)/(c*d)

Giac [A] time = 1.05908, size = 35, normalized size = 1.21

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2dx+c)}{3c}\right)}{3cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-d*x+c)/(-d^3*x^3+c^3),x, algorithm="giac")

[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x + c)/c)/(c*d)

$$3.20 \quad \int \frac{\sqrt[3]{a} \sqrt[3]{b} B + b^{2/3} B x}{a + b x^3} dx$$

Optimal. Leaf size=39

$$\frac{2B \tan^{-1} \left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3} \sqrt[3]{a}} \right)}{\sqrt{3} \sqrt[3]{a}}$$

[Out] $(-2*B*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*a^{(1/3)})$

Rubi [A] time = 0.0266326, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1586, 617, 204}

$$\frac{2B \tan^{-1} \left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3} \sqrt[3]{a}} \right)}{\sqrt{3} \sqrt[3]{a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a^{(1/3)}*b^{(1/3)}*B + b^{(2/3)}*B*x)/(a + b*x^3), x]$

[Out] $(-2*B*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*a^{(1/3)})$

Rule 1586

$\text{Int}[(u_*)*(P x_*)^{(p_*)}*(Q x_*)^{(q_*)}, x_Symbol] \rightarrow \text{Int}[u*PolynomialQuotient[Px, Qx, x]^p*Qx^{(p+q)}, x] /; \text{FreeQ}[q, x] \ \&\& \ \text{PolyQ}[Px, x] \ \&\& \ \text{PolyQ}[Qx, x] \ \&\& \ \text{EqQ}[\text{PolynomialRemainder}[Px, Qx, x], 0] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ \text{LtQ}[p*q, 0]$

Rule 617

$\text{Int}[(a_*) + (b_*)*(x_*) + (c_*)*(x_*)^2]^{(-1)}, x_Symbol] \rightarrow \text{With}[\{q = 1 - 4*Simplify[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4*a*c]) /; \text{FreeQ}\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 204

$\text{Int}[(a_*) + (b_*)*(x_*)^2]^{(-1)}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[\text{Rt}[-a, 2], \text{Rt}[-b, 2]])$

a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{\sqrt[3]{a}\sqrt[3]{b}B + b^{2/3}Bx}{a + bx^3} dx &= \int \frac{1}{\frac{a^{2/3}}{\sqrt[3]{b}B} - \frac{\sqrt[3]{ax}}{B} + \frac{\sqrt[3]{bx^2}}{B}} dx \\ &= \frac{(2B) \operatorname{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 - \frac{2\sqrt[3]{bx}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}} \\ &= -\frac{2B \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}} \end{aligned}$$

Mathematica [A] time = 0.0165907, size = 35, normalized size = 0.9

$$-\frac{2B \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

Antiderivative was successfully verified.

[In] Integrate[(a^(1/3)*b^(1/3)*B + b^(2/3)*B*x)/(a + b*x^3), x]

[Out] (-2*B*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(1/3))

Maple [B] time = 0.043, size = 195, normalized size = 5.

$$\frac{B}{3}\sqrt[3]{a} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) b^{-\frac{2}{3}} \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{B}{6}\sqrt[3]{a} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) b^{-\frac{2}{3}} \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{B\sqrt{3}}{3}\sqrt[3]{a} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) b^{-\frac{2}{3}} \left(\frac{a}{b}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^(1/3)*b^(1/3)*B+b^(2/3)*B*x)/(b*x^3+a), x)

[Out] $\frac{1}{3}B/b^{2/3}a^{1/3}/(1/b*a)^{2/3}*\ln(x+(1/b*a)^{1/3})-1/6*B/b^{2/3}a^{1/3}/(1/b*a)^{2/3}*\ln(x^2-(1/b*a)^{1/3}*x+(1/b*a)^{2/3})+1/3*B/b^{2/3}a^{1/3}/(1/b*a)^{2/3}*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1))-1/3*B/b^{1/3}/(1/b*a)^{1/3}*\ln(x+(1/b*a)^{1/3})+1/6*B/b^{1/3}/(1/b*a)^{1/3}*\ln(x^2-(1/b*a)^{1/3}*x+(1/b*a)^{2/3})+1/3*B/b^{1/3}*3^{1/2}/(1/b*a)^{1/3}*\arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1))$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a^(1/3)*b^(1/3)*B+b^(2/3)*B*x)/(b*x^3+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 1.09054, size = 323, normalized size = 8.28

$$\left[\sqrt{\frac{1}{3}}B\sqrt{\frac{1}{a^{\frac{1}{3}}}} \log\left(\frac{2bx^3 - 3a^{\frac{2}{3}}b^{\frac{1}{3}}x + 3\sqrt{\frac{1}{3}}\left(2a^{\frac{2}{3}}b^{\frac{2}{3}}x^2 + ab^{\frac{1}{3}}x - a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{2} - a}}{bx^3 + a}} \right), \frac{2\sqrt{\frac{1}{3}}B \arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2b^{\frac{1}{3}}x - a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a^(1/3)*b^(1/3)*B+b^(2/3)*B*x)/(b*x^3+a),x, algorithm="fricas")`

[Out] `[sqrt(1/3)*B*sqrt(-1/a^(2/3))*log((2*b*x^3 - 3*a^(2/3)*b^(1/3)*x + 3*sqrt(1/3)*(2*a^(2/3)*b^(2/3)*x^2 + a*b^(1/3)*x - a^(4/3))*sqrt(-1/a^(2/3)) - a)/(b*x^3 + a), 2*sqrt(1/3)*B*arctan(sqrt(1/3)*(2*b^(1/3)*x - a^(1/3))/a^(1/3))/a^(1/3)]`

Sympy [C] time = 0.430803, size = 88, normalized size = 2.26

$$\frac{B \left(-\frac{\sqrt{3}i \log \left(x + \frac{-B \sqrt[3]{a} - \sqrt{3}iB \sqrt[3]{a}}{2B \sqrt[3]{b}} \right)}{3} + \frac{\sqrt{3}i \log \left(x + \frac{-B \sqrt[3]{a} + \sqrt{3}iB \sqrt[3]{a}}{2B \sqrt[3]{b}} \right)}{3} \right)}{\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**(1/3)*b**(1/3)*B+b**(2/3)*B*x)/(b*x**3+a),x)

[Out] B*(-sqrt(3)*I*log(x + (-B*a**(1/3) - sqrt(3)*I*B*a**(1/3))/(2*B*b**(1/3)))/3 + sqrt(3)*I*log(x + (-B*a**(1/3) + sqrt(3)*I*B*a**(1/3))/(2*B*b**(1/3)))/3)/a**(1/3)

Giac [A] time = 1.11651, size = 65, normalized size = 1.67

$$\frac{2 \sqrt{3} B b^{\frac{1}{3}} \arctan \left(\frac{\sqrt{3} \left(2 b^{\frac{2}{3}} x - a^{\frac{1}{3}} b^{\frac{1}{3}} \right)}{3 \sqrt{a^{\frac{2}{3}} b^{\frac{2}{3}}}} \right)}{3 \sqrt{a^{\frac{2}{3}} b^{\frac{2}{3}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(1/3)*b^(1/3)*B+b^(2/3)*B*x)/(b*x^3+a),x, algorithm="giac")

[Out] 2/3*sqrt(3)*B*b^(1/3)*arctan(1/3*sqrt(3)*(2*b^(2/3)*x - a^(1/3)*b^(1/3))/sqrt(a^(2/3)*b^(2/3)))/sqrt(a^(2/3)*b^(2/3))

$$3.21 \quad \int \frac{\sqrt[3]{a} \sqrt[3]{-b} B - (-b)^{2/3} B x}{a + b x^3} dx$$

Optimal. Leaf size=41

$$\frac{2B \tan^{-1} \left(\frac{\sqrt[3]{a} + 2\sqrt[3]{-bx}}{\sqrt{3}\sqrt[3]{a}} \right)}{\sqrt{3}\sqrt[3]{a}}$$

[Out] (2*B*ArcTan[(a^(1/3) + 2*(-b)^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3))

Rubi [A] time = 0.0431953, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1586, 617, 204}

$$\frac{2B \tan^{-1} \left(\frac{\sqrt[3]{a} + 2\sqrt[3]{-bx}}{\sqrt{3}\sqrt[3]{a}} \right)}{\sqrt{3}\sqrt[3]{a}}$$

Antiderivative was successfully verified.

[In] Int[(a^(1/3)*(-b)^(1/3)*B - (-b)^(2/3)*B*x)/(a + b*x^3), x]

[Out] (2*B*ArcTan[(a^(1/3) + 2*(-b)^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3))

Rule 1586

Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204


```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B - (-b)^{2/3}Bx}{a + bx^3} dx = \int \frac{1}{-\frac{a^{2/3}(-b)^{2/3}}{bB} + \frac{\sqrt[3]{ax}}{B} + \frac{\sqrt[3]{-bx^2}}{B}} dx$$

$$= \frac{(2B) \operatorname{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 + \frac{2\sqrt[3]{-bx}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}}$$

$$= \frac{2B \tan^{-1}\left(\frac{\sqrt[3]{a} + 2\sqrt[3]{-bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

Mathematica [B] time = 0.0490526, size = 129, normalized size = 3.15

$$\frac{\sqrt[3]{-b}B \left((\sqrt[3]{-b} + \sqrt[3]{b}) (2 \log(\sqrt[3]{a} + \sqrt[3]{bx}) - \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)) + 2\sqrt{3}(\sqrt[3]{-b} - \sqrt[3]{b}) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) \right)}{6\sqrt[3]{ab^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(1/3)*(-b)^(1/3)*B - (-b)^(2/3)*B*x)/(a + b*x^3), x]
```

```
[Out] ((-b)^(1/3)*B*(2*Sqrt[3]*((-b)^(1/3) - b^(1/3))*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + ((-b)^(1/3) + b^(1/3))*(2*Log[a^(1/3) + b^(1/3)*x] - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(1/3)*b^(2/3))
```

Maple [B] time = 0.01, size = 228, normalized size = 5.6

$$\frac{B\sqrt[3]{-1}}{3}\sqrt[3]{a}\ln\left(x + \sqrt[3]{\frac{a}{b}}\right)b^{-\frac{2}{3}}\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{B\sqrt[3]{-1}}{6}\sqrt[3]{a}\ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^{-\frac{2}{3}}\left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{B\sqrt[3]{-1}\sqrt{3}}{3}\sqrt[3]{a}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^(1/3)*(-b)^(1/3)*B-(-b)^(2/3)*B*x)/(b*x^3+a),x)`

[Out] $\frac{1}{3} \frac{B}{b^{2/3}} (-1)^{1/3} a^{1/3} / (1/b*a)^{2/3} \ln(x + (1/b*a)^{1/3}) - \frac{1}{6} \frac{B}{b^{2/3}} (-1)^{1/3} a^{1/3} / (1/b*a)^{2/3} \ln(x^2 - (1/b*a)^{1/3} * x + (1/b*a)^{2/3}) + \frac{1}{3} \frac{B}{b^{2/3}} (-1)^{1/3} a^{1/3} / (1/b*a)^{2/3} * 3^{1/2} * \arctan(1/3 * 3^{1/2} * (2/(1/b*a)^{1/3} * x - 1)) + \frac{1}{3} \frac{B}{b^{2/3}} (-1)^{1/3} * (-b)^{1/3} / (1/b*a)^{1/3} * \ln(x + (1/b*a)^{1/3}) - \frac{1}{6} \frac{B}{b^{2/3}} (-1)^{1/3} * (-b)^{1/3} / (1/b*a)^{1/3} * \ln(x^2 - (1/b*a)^{1/3} * x + (1/b*a)^{2/3}) - \frac{1}{3} \frac{B}{b^{2/3}} (-1)^{1/3} * (-b)^{1/3} * 3^{1/2} / (1/b*a)^{1/3} * \arctan(1/3 * 3^{1/2} * (2/(1/b*a)^{1/3} * x - 1))$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a^(1/3)*(-b)^(1/3)*B-(-b)^(2/3)*B*x)/(b*x^3+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 1.08293, size = 339, normalized size = 8.27

$$\left[\sqrt{\frac{1}{3}} B \sqrt{\frac{1}{a^{\frac{2}{3}}}} \log \left(\frac{2bx^3 + 3a^{\frac{2}{3}}(-b)^{\frac{1}{3}}x - 3\sqrt{\frac{1}{3}} \left(2a^{\frac{2}{3}}(-b)^{\frac{2}{3}}x^2 - a(-b)^{\frac{1}{3}}x - a^{\frac{4}{3}} \right) \sqrt{\frac{1}{a^{\frac{3}}}} - a}{bx^3 + a} \right), \frac{2\sqrt{\frac{1}{3}} B \arctan \left(\frac{\sqrt{\frac{1}{3}} \left(2(-b)^{\frac{1}{3}}x + a^{\frac{1}{3}} \right)}{a^{\frac{1}{3}}} \right)}{a^{\frac{1}{3}}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a^(1/3)*(-b)^(1/3)*B-(-b)^(2/3)*B*x)/(b*x^3+a),x, algorithm="fricas")`

[Out] $[\sqrt{1/3} * B * \sqrt{-1/a^{2/3}} * \log((2*b*x^3 + 3*a^{2/3}*(-b)^{1/3}*x - 3*\sqrt{1/3}*(2*a^{2/3}*(-b)^{2/3}*x^2 - a*(-b)^{1/3}*x - a^{4/3})*\sqrt{-1/a^{2/3}}) - a)/(b*x^3 + a), 2*\sqrt{1/3} * B * \arctan(\sqrt{1/3} * (2*(-b)^{1/3}*x + a^{1/3})) / a^{1/3}]$

/3))/a^(1/3))/a^(1/3)]

Sympy [C] time = 0.47654, size = 105, normalized size = 2.56

$$B \left[\frac{\sqrt{3}i \log \left(-\frac{\sqrt[3]{a}(-b)^{\frac{2}{3}}}{2b} - \frac{\sqrt{3}i \sqrt[3]{a}(-b)^{\frac{2}{3}}}{2b} + x \right)}{3} + \frac{\sqrt{3}i \log \left(-\frac{\sqrt[3]{a}(-b)^{\frac{2}{3}}}{2b} + \frac{\sqrt{3}i \sqrt[3]{a}(-b)^{\frac{2}{3}}}{2b} + x \right)}{3} \right]$$

$$\frac{\quad}{\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**(1/3)*(-b)**(1/3)*B-(-b)**(2/3)*B*x)/(b*x**3+a), x)

[Out] -B*(-sqrt(3)*I*log(-a**(1/3)*(-b)**(2/3)/(2*b) - sqrt(3)*I*a**(1/3)*(-b)**(2/3)/(2*b) + x)/3 + sqrt(3)*I*log(-a**(1/3)*(-b)**(2/3)/(2*b) + sqrt(3)*I*a**(1/3)*(-b)**(2/3)/(2*b) + x)/3)/a**(1/3)

Giac [A] time = 1.09758, size = 78, normalized size = 1.9

$$\frac{2\sqrt{3}Bb \arctan \left(\frac{\sqrt{3} \left(2(-b)^{\frac{2}{3}}x + a^{\frac{1}{3}}(-b)^{\frac{1}{3}} \right)}{3\sqrt{a^{\frac{2}{3}}(-b)^{\frac{2}{3}}}} \right)}{3\sqrt{a^{\frac{2}{3}}(-b)^{\frac{2}{3}}(-b)^{\frac{2}{3}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(1/3)*(-b)^(1/3)*B-(-b)^(2/3)*B*x)/(b*x^3+a), x, algorithm="giac")

[Out] 2/3*sqrt(3)*B*b*arctan(-1/3*sqrt(3)*(2*(-b)^(2/3)*x + a^(1/3)*(-b)^(1/3))/sqrt(a^(2/3)*(-b)^(2/3)))/(sqrt(a^(2/3)*(-b)^(2/3))*(-b)^(2/3))

$$3.22 \quad \int \left(-\frac{Cx^2}{a+bx^3} + \frac{Bx+Cx^2}{a+bx^3} \right) dx$$

Optimal. Leaf size=118

$$\frac{B \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{6\sqrt[3]{ab^{2/3}}} - \frac{B \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3\sqrt[3]{ab^{2/3}}} - \frac{B \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{ab^{2/3}}}$$

[Out] -((B*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(2/3))) - (B*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*b^(2/3)) + (B*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(2/3))

Rubi [A] time = 0.126368, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {260, 1593, 1871, 12, 292, 31, 634, 617, 204, 628}

$$\frac{B \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{6\sqrt[3]{ab^{2/3}}} - \frac{B \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3\sqrt[3]{ab^{2/3}}} - \frac{B \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{ab^{2/3}}}$$

Antiderivative was successfully verified.

[In] Int[-((C*x^2)/(a + b*x^3)) + (B*x + C*x^2)/(a + b*x^3), x]

[Out] -((B*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(2/3))) - (B*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*b^(2/3)) + (B*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(2/3))

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 1593

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Dist[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(-\frac{Cx^2}{a+bx^3} + \frac{Bx+Cx^2}{a+bx^3} \right) dx &= -\left(C \int \frac{x^2}{a+bx^3} dx \right) + \int \frac{Bx+Cx^2}{a+bx^3} dx \\
&= -\frac{C \log(a+bx^3)}{3b} + \int \frac{x(B+Cx)}{a+bx^3} dx \\
&= -\frac{C \log(a+bx^3)}{3b} + C \int \frac{x^2}{a+bx^3} dx + \int \frac{Bx}{a+bx^3} dx \\
&= B \int \frac{x}{a+bx^3} dx \\
&= -\frac{B \int \frac{1}{\sqrt[3]{a}+\sqrt[3]{bx}} dx}{3\sqrt[3]{a}\sqrt[3]{b}} + \frac{B \int \frac{\sqrt[3]{a}+\sqrt[3]{bx}}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2} dx}{3\sqrt[3]{a}\sqrt[3]{b}} \\
&= -\frac{B \log(\sqrt[3]{a}+\sqrt[3]{bx})}{3\sqrt[3]{ab^{2/3}}} + \frac{B \int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2} dx}{6\sqrt[3]{ab^{2/3}}} + \frac{B \int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2} dx}{2\sqrt[3]{b}} \\
&= -\frac{B \log(\sqrt[3]{a}+\sqrt[3]{bx})}{3\sqrt[3]{ab^{2/3}}} + \frac{B \log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2)}{6\sqrt[3]{ab^{2/3}}} + \frac{B \operatorname{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1-\frac{2\sqrt[3]{b}}{\sqrt[3]{a}}\right)}{\sqrt[3]{ab^{2/3}}} \\
&= -\frac{B \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{ab^{2/3}}} - \frac{B \log(\sqrt[3]{a}+\sqrt[3]{bx})}{3\sqrt[3]{ab^{2/3}}} + \frac{B \log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2)}{6\sqrt[3]{ab^{2/3}}}
\end{aligned}$$

Mathematica [A] time = 0.011431, size = 90, normalized size = 0.76

$$\frac{B \left(\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2) - 2 \log(\sqrt[3]{a} + \sqrt[3]{bx}) - 2\sqrt{3} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) \right)}{6\sqrt[3]{ab^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[-((C*x^2)/(a + b*x^3)) + (B*x + C*x^2)/(a + b*x^3), x]
```

```
[Out] (B*(-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3)
+ b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(1/3)*
```

$b^{(2/3)}$

Maple [A] time = 0.041, size = 94, normalized size = 0.8

$$-\frac{B}{3b} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \frac{1}{\sqrt[3]{\frac{a}{b}}} + \frac{B}{6b} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \frac{1}{\sqrt[3]{\frac{a}{b}}} + \frac{B\sqrt{3}}{3b} \arctan\left(\frac{\sqrt{3}}{3} \left(2x \frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) \frac{1}{\sqrt[3]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-C*x^2/(b*x^3+a)+(C*x^2+B*x)/(b*x^3+a),x)`

[Out] `-1/3*B/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/6*B/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3*B*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))`

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x)/(b*x^3+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 0.99017, size = 760, normalized size = 6.44

$$\frac{3\sqrt{\frac{1}{3}}Bab\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}} \log\left(\frac{2b^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2(-ab^2)^{\frac{2}{3}}x^2+(-ab^2)^{\frac{1}{3}}a\right)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}-3(-ab^2)^{\frac{2}{3}}x}{bx^3+a}}{6ab^2}\right) + (-ab^2)^{\frac{2}{3}}B \log\left(b^2x^2 + (-ab^2)^{\frac{1}{3}}bx + \dots\right)}{6ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x)/(b*x^3+a),x, algorithm="fricas")

[Out] [1/6*(3*sqrt(1/3)*B*a*b*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + (-a*b^2)^(2/3)*B*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*B*log(b*x - (-a*b^2)^(1/3)))/(a*b^2), 1/6*(6*sqrt(1/3)*B*a*b*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (-a*b^2)^(2/3)*B*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*B*log(b*x - (-a*b^2)^(1/3)))/(a*b^2)]

Sympy [A] time = 0.134989, size = 26, normalized size = 0.22

$$B \text{RootSum}\left(27t^3ab^2 + 1, \left(t \mapsto t \log(9t^2ab + x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-C*x**2/(b*x**3+a)+(C*x**2+B*x)/(b*x**3+a),x)

[Out] B*RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(9*_t**2*a*b + x)))

Giac [A] time = 1.07905, size = 155, normalized size = 1.31

$$\frac{B\left(-\frac{a}{b}\right)^{\frac{2}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a} - \frac{\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}} B \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^2} + \frac{\left(-ab^2\right)^{\frac{2}{3}} B \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x)/(b*x^3+a),x, algorithm="giac")

[Out] -1/3*B*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a - 1/3*sqrt(3)*(-a*b^2)^(2/3)*B*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) + 1/6*(-a*b^2)^(2/3)*B*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2)

$$3.23 \quad \int \left(-\frac{Cx^2}{a+bx^3} + \frac{A+Cx^2}{a+bx^3} \right) dx$$

Optimal. Leaf size=118

$$-\frac{A \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{6a^{2/3}\sqrt[3]{b}} + \frac{A \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{2/3}\sqrt[3]{b}} - \frac{A \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}}$$

[Out] -((A*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(1/3))) + (A*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)) - (A*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))

Rubi [A] time = 0.106268, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {260, 1871, 12, 200, 31, 634, 617, 204, 628}

$$-\frac{A \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{6a^{2/3}\sqrt[3]{b}} + \frac{A \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{2/3}\sqrt[3]{b}} - \frac{A \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[-((C*x^2)/(a + b*x^3)) + (A + C*x^2)/(a + b*x^3), x]

[Out] -((A*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(1/3))) + (A*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)) - (A*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))

Rule 260

Int[(x_)^m_./((a_) + (b_.)*(x_)^n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 1871

Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Dist[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(-\frac{Cx^2}{a+bx^3} + \frac{A+Cx^2}{a+bx^3} \right) dx &= -\left(C \int \frac{x^2}{a+bx^3} dx \right) + \int \frac{A+Cx^2}{a+bx^3} dx \\
&= -\frac{C \log(a+bx^3)}{3b} + C \int \frac{x^2}{a+bx^3} dx + \int \frac{A}{a+bx^3} dx \\
&= A \int \frac{1}{a+bx^3} dx \\
&= \frac{A \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{bx}} dx}{3a^{2/3}} + \frac{A \int \frac{2\sqrt[3]{a} - \sqrt[3]{bx}}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} dx}{3a^{2/3}} \\
&= \frac{A \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{2/3}\sqrt[3]{b}} + \frac{A \int \frac{1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} dx}{2\sqrt[3]{a}} - \frac{A \int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} dx}{6a^{2/3}\sqrt[3]{b}} \\
&= \frac{A \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{2/3}\sqrt[3]{b}} - \frac{A \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{6a^{2/3}\sqrt[3]{b}} + \frac{A \operatorname{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 - \frac{2\sqrt[3]{b}}{\sqrt[3]{a}}x\right)}{a^{2/3}\sqrt[3]{b}} \\
&= -\frac{A \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}} + \frac{A \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{2/3}\sqrt[3]{b}} - \frac{A \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{6a^{2/3}\sqrt[3]{b}}
\end{aligned}$$

Mathematica [A] time = 0.0124255, size = 90, normalized size = 0.76

$$-\frac{A \left(\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2) - 2 \log(\sqrt[3]{a} + \sqrt[3]{bx}) + 2\sqrt{3} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) \right)}{6a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Integrate[-((C*x^2)/(a + b*x^3)) + (A + C*x^2)/(a + b*x^3), x]

[Out] -(A*(2*sqrt(3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt(3)] - 2*Log[a^(1/3) + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))

Maple [A] time = 0.043, size = 94, normalized size = 0.8

$$\frac{A}{3b} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{A}{6b} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{A\sqrt{3}}{3b} \arctan\left(\frac{\sqrt{3}}{3} \left(2x \frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-C*x^2/(b*x^3+a)+(C*x^2+A)/(b*x^3+a),x)`

[Out] $\frac{1}{3} \frac{A}{b} \frac{1}{(1/b*a)^{2/3}} \ln(x + (1/b*a)^{1/3}) - \frac{1}{6} \frac{A}{b} \frac{1}{(1/b*a)^{2/3}} \ln(x^2 - (1/b*a)^{1/3} * x + (1/b*a)^{2/3}) + \frac{1}{3} \frac{A}{b} \frac{1}{(1/b*a)^{2/3}} * 3^{1/2} * \arctan(1/3 * 3^{1/2} * (2/(1/b*a)^{1/3} * x - 1))$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-C*x^2/(b*x^3+a)+(C*x^2+A)/(b*x^3+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 1.00941, size = 765, normalized size = 6.48

$$\frac{3 \sqrt{\frac{1}{3}} A a b \sqrt{-\frac{(a^2 b)^{\frac{1}{3}}}{b}} \log \left(\frac{2 a b x^3 - 3 (a^2 b)^{\frac{1}{3}} a x - a^2 + 3 \sqrt{\frac{1}{3}} \left(2 a b x^2 + (a^2 b)^{\frac{2}{3}} x - (a^2 b)^{\frac{1}{3}} a \right) \sqrt{-\frac{(a^2 b)^{\frac{1}{3}}}{b}}}{b x^3 + a} \right) - (a^2 b)^{\frac{2}{3}} A \log \left(a b x^2 - (a^2 b)^{\frac{2}{3}} x + (a^2 b)^{\frac{1}{3}} \right)}{6 a^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-C*x^2/(b*x^3+a)+(C*x^2+A)/(b*x^3+a),x, algorithm="fricas")`

[Out] $\frac{1}{6} * (3 * \sqrt{1/3} * A * a * b * \sqrt{-(a^2 * b)^{1/3} / b} * \log((2 * a * b * x^3 - 3 * (a^2 * b)^{1/3} * a * x - a^2 + 3 * \sqrt{1/3} * (2 * a * b * x^2 + (a^2 * b)^{2/3} * x - (a^2 * b)^{1/3} * a) * \sqrt{-(a^2 * b)^{1/3} / b}) / (b * x^3 + a)) - (a^2 * b)^{2/3} * A * \log(a * b * x^2 - (a^2 * b)^{2/3} * x + (a^2 * b)^{1/3} * a) + 2 * (a^2 * b)^{2/3} * A * \log(a * b * x + (a^2 * b)^{2/3})) / (a^2 * b), \frac{1}{6} * (6 * \sqrt{1/3} * A * a * b * \sqrt{(a^2 * b)^{1/3} / b} * \arctan(\sqrt{1/3} * (2 * (a^2 * b)^{1/3} * x - 1) / (a^2 * b)^{1/3}))$

$$(2*(a^2*b)^{(2/3)}*x - (a^2*b)^{(1/3)}*a)*\sqrt{(a^2*b)^{(1/3)}/b}/a^2 - (a^2*b)^{(2/3)}*A*\log(a*b*x^2 - (a^2*b)^{(2/3)}*x + (a^2*b)^{(1/3)}*a) + 2*(a^2*b)^{(2/3)}*A*\log(a*b*x + (a^2*b)^{(2/3)})/(a^2*b)]$$

Sympy [A] time = 0.155791, size = 22, normalized size = 0.19

$$A \operatorname{RootSum}\left(27t^3a^2b - 1, (t \mapsto t \log(3ta + x))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-C*x**2/(b*x**3+a)+(C*x**2+A)/(b*x**3+a), x)

[Out] A*RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a + x)))

Giac [A] time = 1.11605, size = 155, normalized size = 1.31

$$\frac{A\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a} + \frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}} A \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab} + \frac{\left(-ab^2\right)^{\frac{1}{3}} A \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+A)/(b*x^3+a), x, algorithm="giac")

[Out] -1/3*A*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a + 1/3*sqrt(3)*(-a*b^2)^(1/3)*A*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) + 1/6*(-a*b^2)^(1/3)*A*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b)

$$3.24 \quad \int \left(-\frac{Cx^2}{a+bx^3} + \frac{A+Bx+Cx^2}{a+bx^3} \right) dx$$

Optimal. Leaf size=161

$$\frac{\left(A - \frac{\sqrt[3]{a}B}{\sqrt[3]{b}}\right) \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}} + \frac{\left(A\sqrt[3]{b} - \sqrt[3]{a}B\right) \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{3a^{2/3}b^{2/3}} - \frac{\left(\sqrt[3]{a}B + A\sqrt[3]{b}\right) \tan^{-1}\left(\frac{\sqrt[3]{a-2\sqrt[3]{bx}}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}$$

[Out] -(((A*b^(1/3) + a^(1/3)*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((A*b^(1/3) - a^(1/3)*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(2/3)) - ((A - (a^(1/3)*B)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))

Rubi [A] time = 0.166297, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {260, 1871, 1860, 31, 634, 617, 204, 628}

$$\frac{\left(A - \frac{\sqrt[3]{a}B}{\sqrt[3]{b}}\right) \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}} + \frac{\left(A\sqrt[3]{b} - \sqrt[3]{a}B\right) \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{3a^{2/3}b^{2/3}} - \frac{\left(\sqrt[3]{a}B + A\sqrt[3]{b}\right) \tan^{-1}\left(\frac{\sqrt[3]{a-2\sqrt[3]{bx}}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[-((C*x^2)/(a + b*x^3)) + (A + B*x + C*x^2)/(a + b*x^3), x]

[Out] -(((A*b^(1/3) + a^(1/3)*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((A*b^(1/3) - a^(1/3)*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(2/3)) - ((A - (a^(1/3)*B)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 1871

Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Dist[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a

/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

Rule 1860

Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]

Rule 31

Int[((a_) + (b_)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
\int \left(-\frac{Cx^2}{a+bx^3} + \frac{A+Bx+Cx^2}{a+bx^3} \right) dx &= -\left(C \int \frac{x^2}{a+bx^3} dx \right) + \int \frac{A+Bx+Cx^2}{a+bx^3} dx \\
&= -\frac{C \log(a+bx^3)}{3b} + C \int \frac{x^2}{a+bx^3} dx + \int \frac{A+Bx}{a+bx^3} dx \\
&= \frac{\sqrt[3]{a}(2A\sqrt[3]{b} + \sqrt[3]{a}B) + \sqrt[3]{b}(-A\sqrt[3]{b} + \sqrt[3]{a}B)x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx + \frac{\left(A - \frac{\sqrt[3]{a}B}{\sqrt[3]{b}}\right) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3a^{2/3}} \\
&= \frac{\left(A - \frac{\sqrt[3]{a}B}{\sqrt[3]{b}}\right) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{2/3}\sqrt[3]{b}} - \frac{(A\sqrt[3]{b} - \sqrt[3]{a}B) \int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{6a^{2/3}b^{2/3}} + \frac{1}{2} \left(\frac{A}{\sqrt[3]{a}} + \frac{B}{\sqrt[3]{b}} \right) \\
&= \frac{\left(A - \frac{\sqrt[3]{a}B}{\sqrt[3]{b}}\right) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{2/3}\sqrt[3]{b}} - \frac{(A\sqrt[3]{b} - \sqrt[3]{a}B) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6a^{2/3}b^{2/3}} + \frac{(A\sqrt[3]{b} - \sqrt[3]{a}B) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}} + \frac{\left(A - \frac{\sqrt[3]{a}B}{\sqrt[3]{b}}\right) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{2/3}\sqrt[3]{b}} - \frac{(A\sqrt[3]{b} - \sqrt[3]{a}B) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6a^{2/3}b^{2/3}} + \frac{(A\sqrt[3]{b} - \sqrt[3]{a}B) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.0388091, size = 124, normalized size = 0.77

$$\frac{(A\sqrt[3]{b} - \sqrt[3]{a}B) \left(2 \log(\sqrt[3]{a} + \sqrt[3]{b}x) - \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2) \right) - 2\sqrt{3}(\sqrt[3]{a}B + A\sqrt[3]{b}) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6a^{2/3}b^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[-((C*x^2)/(a + b*x^3)) + (A + B*x + C*x^2)/(a + b*x^3), x]

[Out] (-2*Sqrt[3]*(A*b^(1/3) + a^(1/3)*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + (A*b^(1/3) - a^(1/3)*B)*(2*Log[a^(1/3) + b^(1/3)*x] - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(2/3)*b^(2/3))

Maple [A] time = 0.043, size = 186, normalized size = 1.2

$$\frac{A}{3b} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{A}{6b} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{A\sqrt{3}}{3b} \arctan\left(\frac{\sqrt{3}}{3} \left(2x \frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{B}{3b} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(-C*x^2/(b*x^3+a)+(C*x^2+B*x+A)/(b*x^3+a), x)$

[Out] $\frac{1}{3}A/b/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})-\frac{1}{6}A/b/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})+\frac{1}{3}A/b/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))-\frac{1}{3}B/b/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})+\frac{1}{6}B/b/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})+\frac{1}{3}B*3^{(1/2)}/b/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(-C*x^2/(b*x^3+a)+(C*x^2+B*x+A)/(b*x^3+a), x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

Fricas [C] time = 6.19941, size = 4590, normalized size = 28.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(-C*x^2/(b*x^3+a)+(C*x^2+B*x+A)/(b*x^3+a), x, \text{algorithm}="fricas")$

[Out]
$$-1/6*((1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^{(1/3)} - 2*(1/2)^{(2/3)}*A*B*(-I*\sqrt{3} + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^{(1/3)}))*\log(1/4*((1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^{(1/3)} - 2*(1/2)^{(2/3)}*A*B*(-I*\sqrt{3} + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^{(1/3)}))^{2*B*a^2*b} - 1/2*((1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^{(1/3)} - 2*(1/2)^{(2/3)}*A*B*(-I*\sqrt{3} + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^{(1/3)}))*A^2*a*b + 2*A*B^2*a + (B^3*a + A^3*b)*x) + 1/12*((1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^{(1/3)} - 2*(1/2)^{(2/3)}*A*B*(-I*\sqrt{3} + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^{(1/3)}))$$

$$\begin{aligned}
& 2) - (B^3a - A^3b)/(a^2b^2)^{(1/3)} + 3\sqrt{1/3}\sqrt{-(((1/2)^{(1/3)}(I \\
& \sqrt{3} + 1)*((B^3a + A^3b)/(a^2b^2) - (B^3a - A^3b)/(a^2b^2))^{(1/3)} \\
& - 2*(1/2)^{(2/3)}*A*B*(-I\sqrt{3} + 1)/(a*b*((B^3a + A^3b)/(a^2b^2) - (B^ \\
& 3a - A^3b)/(a^2b^2))^{(1/3)})^2*a*b + 16*A*B)/(a*b)))*\log(-1/4*((1/2)^{(1/ \\
& 3)}*(I\sqrt{3} + 1)*((B^3a + A^3b)/(a^2b^2) - (B^3a - A^3b)/(a^2b^2))^{(\\
& 1/3)} - 2*(1/2)^{(2/3)}*A*B*(-I\sqrt{3} + 1)/(a*b*((B^3a + A^3b)/(a^2b^2) \\
& - (B^3a - A^3b)/(a^2b^2))^{(1/3)})^2*B*a^2*b + 1/2*((1/2)^{(1/3)}*(I\sqrt{3} \\
&) + 1)*((B^3a + A^3b)/(a^2b^2) - (B^3a - A^3b)/(a^2b^2))^{(1/3)} - 2*(1 \\
& /2)^{(2/3)}*A*B*(-I\sqrt{3} + 1)/(a*b*((B^3a + A^3b)/(a^2b^2) - (B^3a - A \\
& ^3b)/(a^2b^2))^{(1/3)})^2*a*b - 2*A*B^2*a + 2*(B^3a + A^3b)*x + 3/4*sq \\
& rt(1/3)*(((1/2)^{(1/3)}*(I\sqrt{3} + 1)*((B^3a + A^3b)/(a^2b^2) - (B^3a - \\
& A^3b)/(a^2b^2))^{(1/3)} - 2*(1/2)^{(2/3)}*A*B*(-I\sqrt{3} + 1)/(a*b*((B^3a \\
& + A^3b)/(a^2b^2) - (B^3a - A^3b)/(a^2b^2))^{(1/3)})^2*B*a^2*b + 2*A^2*a*b \\
&)*\sqrt{-(((1/2)^{(1/3)}*(I\sqrt{3} + 1)*((B^3a + A^3b)/(a^2b^2) - (B^3a - \\
& A^3b)/(a^2b^2))^{(1/3)} - 2*(1/2)^{(2/3)}*A*B*(-I\sqrt{3} + 1)/(a*b*((B^3a \\
& + A^3b)/(a^2b^2) - (B^3a - A^3b)/(a^2b^2))^{(1/3)})^2*a*b + 16*A*B)/(a* \\
& b))) + 1/12*((1/2)^{(1/3)}*(I\sqrt{3} + 1)*((B^3a + A^3b)/(a^2b^2) - (B^3* \\
& a - A^3b)/(a^2b^2))^{(1/3)} - 2*(1/2)^{(2/3)}*A*B*(-I\sqrt{3} + 1)/(a*b*((B^3 \\
& *a + A^3b)/(a^2b^2) - (B^3a - A^3b)/(a^2b^2))^{(1/3)}) - 3*\sqrt{1/3}*sq \\
& rt(-(((1/2)^{(1/3)}*(I\sqrt{3} + 1)*((B^3a + A^3b)/(a^2b^2) - (B^3a - A^3* \\
& b)/(a^2b^2))^{(1/3)} - 2*(1/2)^{(2/3)}*A*B*(-I\sqrt{3} + 1)/(a*b*((B^3*a + A^3 \\
& *b)/(a^2b^2) - (B^3a - A^3b)/(a^2b^2))^{(1/3)})^2*a*b + 16*A*B)/(a*b))) * \\
& \log(-1/4*((1/2)^{(1/3)}*(I\sqrt{3} + 1)*((B^3a + A^3b)/(a^2b^2) - (B^3a - \\
& A^3b)/(a^2b^2))^{(1/3)} - 2*(1/2)^{(2/3)}*A*B*(-I\sqrt{3} + 1)/(a*b*((B^3*a \\
& + A^3b)/(a^2b^2) - (B^3a - A^3b)/(a^2b^2))^{(1/3)})^2*B*a^2*b + 1/2*((1 \\
& /2)^{(1/3)}*(I\sqrt{3} + 1)*((B^3a + A^3b)/(a^2b^2) - (B^3a - A^3b)/(a^2 \\
& *b^2))^{(1/3)} - 2*(1/2)^{(2/3)}*A*B*(-I\sqrt{3} + 1)/(a*b*((B^3a + A^3b)/(a^ \\
& 2*b^2) - (B^3a - A^3b)/(a^2b^2))^{(1/3)})^2*a*b - 2*A*B^2*a + 2*(B^3*a \\
& + A^3b)*x - 3/4*\sqrt{1/3}*(((1/2)^{(1/3)}*(I\sqrt{3} + 1)*((B^3a + A^3b)/(\\
& a^2b^2) - (B^3a - A^3b)/(a^2b^2))^{(1/3)} - 2*(1/2)^{(2/3)}*A*B*(-I\sqrt{3} \\
& + 1)/(a*b*((B^3a + A^3b)/(a^2b^2) - (B^3a - A^3b)/(a^2b^2))^{(1/3)})^2 \\
& *B*a^2*b + 2*A^2*a*b)*\sqrt{-(((1/2)^{(1/3)}*(I\sqrt{3} + 1)*((B^3a + A^3b)/(\\
& a^2b^2) - (B^3a - A^3b)/(a^2b^2))^{(1/3)} - 2*(1/2)^{(2/3)}*A*B*(-I\sqrt{3} \\
& + 1)/(a*b*((B^3a + A^3b)/(a^2b^2) - (B^3a - A^3b)/(a^2b^2))^{(1/3)})^2 \\
& *a*b + 16*A*B)/(a*b))}
\end{aligned}$$

Sympy [A] time = 0.657534, size = 76, normalized size = 0.47

$$\text{RootSum}\left(27t^3a^2b^2 + 9tABab - A^3b + B^3a, \left(t \mapsto t \log\left(x + \frac{9t^2Ba^2b + 3tA^2ab + 2AB^2a}{A^3b + B^3a}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-C*x**2/(b*x**3+a)+(C*x**2+B*x+A)/(b*x**3+a),x)

[Out] RootSum(27*_t**3*a**2*b**2 + 9*_t*A*B*a*b - A**3*b + B**3*a, Lambda(_t, _t*log(x + (9*_t**2*B*a**2*b + 3*_t*A**2*a*b + 2*A*B**2*a)/(A**3*b + B**3*a))))

Giac [A] time = 1.08512, size = 224, normalized size = 1.39

$$-\frac{\left(Bb\left(-\frac{a}{b}\right)^{\frac{1}{3}} + Ab\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}} Ab - \left(-ab^2\right)^{\frac{2}{3}} B\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^2} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}} Aab^3\right)}{3ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x+A)/(b*x^3+a),x, algorithm="giac")

[Out] -1/3*(B*b*(-a/b)^(1/3) + A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b) + 1/3*sqrt(3)*((-a*b^2)^(1/3)*A*b - (-a*b^2)^(2/3)*B)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) + 1/6*((-a*b^2)^(1/3)*A*a*b^3 + (-a*b^2)^(2/3)*B*a*b^2)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^4)

3.25 $\int \frac{bx+cx^2}{d+ex^3} dx$

Optimal. Leaf size=134

$$\frac{b \log(d^{2/3} - \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2)}{6\sqrt[3]{de^{2/3}}} - \frac{b \log(\sqrt[3]{d} + \sqrt[3]{ex})}{3\sqrt[3]{de^{2/3}}} - \frac{b \tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{ex}}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}\sqrt[3]{de^{2/3}}} + \frac{c \log(d + ex^3)}{3e}$$

[Out] $-\left(\frac{b \operatorname{ArcTan}\left[\frac{d^{1/3} - 2e^{1/3}x}{\sqrt{3}d^{1/3}}\right]}{\sqrt{3}d^{1/3}e^{2/3}}\right) - \frac{b \operatorname{Log}\left[d^{1/3} + e^{1/3}x\right]}{3d^{1/3}e^{2/3}} + \frac{b \operatorname{Log}\left[d^{2/3} - d^{1/3}e^{1/3}x + e^{2/3}x^2\right]}{6d^{1/3}e^{2/3}} + \frac{c \operatorname{Log}\left[d + ex^3\right]}{3e}$

Rubi [A] time = 0.113145, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1593, 1871, 12, 292, 31, 634, 617, 204, 628, 260}

$$\frac{b \log(d^{2/3} - \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2)}{6\sqrt[3]{de^{2/3}}} - \frac{b \log(\sqrt[3]{d} + \sqrt[3]{ex})}{3\sqrt[3]{de^{2/3}}} - \frac{b \tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{ex}}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}\sqrt[3]{de^{2/3}}} + \frac{c \log(d + ex^3)}{3e}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}\left[\frac{b*x + c*x^2}{d + e*x^3}, x\right]$

[Out] $-\left(\frac{b \operatorname{ArcTan}\left[\frac{d^{1/3} - 2e^{1/3}x}{\sqrt{3}d^{1/3}}\right]}{\sqrt{3}d^{1/3}e^{2/3}}\right) - \frac{b \operatorname{Log}\left[d^{1/3} + e^{1/3}x\right]}{3d^{1/3}e^{2/3}} + \frac{b \operatorname{Log}\left[d^{2/3} - d^{1/3}e^{1/3}x + e^{2/3}x^2\right]}{6d^{1/3}e^{2/3}} + \frac{c \operatorname{Log}\left[d + ex^3\right]}{3e}$

Rule 1593

$\operatorname{Int}\left[(u_.) * ((a_.) * (x_.)^{(p_.)} + (b_.) * (x_.)^{(q_.)})^{(n_.)}, x_Symbol\right] \rightarrow \operatorname{Int}\left[u * x^{(n*p)} * (a + b*x^{(q-p)})^n, x\right] /;$ FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1871

$\operatorname{Int}\left[\frac{P2_}{(a_.) + (b_.) * (x_.)^3}, x_Symbol\right] \rightarrow \operatorname{With}\left[\{A = \operatorname{Coeff}[P2, x, 0], B = \operatorname{Coeff}[P2, x, 1], C = \operatorname{Coeff}[P2, x, 2]\}, \operatorname{Int}\left[\frac{A + B*x}{a + b*x^3}, x\right] + \operatorname{Dist}\left[C, \operatorname{Int}\left[\frac{x^2}{a + b*x^3}, x\right], x\right] /;$ EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a

/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 292

Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 31

Int[((a_) + (b_)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^-1, x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{bx + cx^2}{d + ex^3} dx &= \int \frac{x(b + cx)}{d + ex^3} dx \\
&= c \int \frac{x^2}{d + ex^3} dx + \int \frac{bx}{d + ex^3} dx \\
&= \frac{c \log(d + ex^3)}{3e} + b \int \frac{x}{d + ex^3} dx \\
&= \frac{c \log(d + ex^3)}{3e} - \frac{b \int \frac{1}{\sqrt[3]{d} + \sqrt[3]{ex}} dx}{3\sqrt[3]{d}\sqrt[3]{e}} + \frac{b \int \frac{\sqrt[3]{d} + \sqrt[3]{ex}}{d^{2/3} - \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2} dx}{3\sqrt[3]{d}\sqrt[3]{e}} \\
&= -\frac{b \log(\sqrt[3]{d} + \sqrt[3]{ex})}{3\sqrt[3]{de}^{2/3}} + \frac{c \log(d + ex^3)}{3e} + \frac{b \int \frac{-\sqrt[3]{d}\sqrt[3]{e} + 2e^{2/3}x}{d^{2/3} - \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2} dx}{6\sqrt[3]{de}^{2/3}} + \frac{b \int \frac{1}{d^{2/3} - \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2} dx}{2\sqrt[3]{e}} \\
&= -\frac{b \log(\sqrt[3]{d} + \sqrt[3]{ex})}{3\sqrt[3]{de}^{2/3}} + \frac{b \log(d^{2/3} - \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2)}{6\sqrt[3]{de}^{2/3}} + \frac{c \log(d + ex^3)}{3e} + \frac{b \operatorname{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 - \frac{2\sqrt[3]{ex}}{\sqrt[3]{d}}\right)}{\sqrt[3]{de}^{2/3}} \\
&= -\frac{b \tan^{-1}\left(\frac{\sqrt[3]{d} - 2\sqrt[3]{ex}}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}\sqrt[3]{de}^{2/3}} - \frac{b \log(\sqrt[3]{d} + \sqrt[3]{ex})}{3\sqrt[3]{de}^{2/3}} + \frac{b \log(d^{2/3} - \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2)}{6\sqrt[3]{de}^{2/3}} + \frac{c \log(d + ex^3)}{3e}
\end{aligned}$$

Mathematica [A] time = 0.0230979, size = 122, normalized size = 0.91

$$\frac{b\sqrt[3]{e} \log(d^{2/3} - \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2) - 2b\sqrt[3]{e} \log(\sqrt[3]{d} + \sqrt[3]{ex}) - 2\sqrt{3}b\sqrt[3]{e} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{ex}}{\sqrt[3]{d}}}{\sqrt{3}}\right) + 2c\sqrt[3]{d} \log(d + ex^3)}{6\sqrt[3]{de}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2)/(d + e*x^3),x]

[Out] (-2*Sqrt[3]*b*e^(1/3)*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]] - 2*b*e^(1/3)*Log[d^(1/3) + e^(1/3)*x] + b*e^(1/3)*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2] + 2*c*d^(1/3)*Log[d + e*x^3])/(6*d^(1/3)*e)

Maple [A] time = 0.043, size = 108, normalized size = 0.8

$$-\frac{b}{3e} \ln\left(x + \sqrt[3]{\frac{d}{e}}\right) \frac{1}{\sqrt[3]{\frac{d}{e}}} + \frac{b}{6e} \ln\left(x^2 - \sqrt[3]{\frac{d}{e}}x + \left(\frac{d}{e}\right)^{\frac{2}{3}}\right) \frac{1}{\sqrt[3]{\frac{d}{e}}} + \frac{b\sqrt{3}}{3e} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{d}{e}}} - 1\right)\right) \frac{1}{\sqrt[3]{\frac{d}{e}}} + \frac{c \ln(ex^3 + d)}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x)/(e*x^3+d), x)

[Out] $-1/3*b/e/(d/e)^{(1/3)}*\ln(x+(d/e)^{(1/3)})+1/6*b/e/(d/e)^{(1/3)}*\ln(x^2-(d/e)^{(1/3)}*x+(d/e)^{(2/3)})+1/3*b*3^{(1/2)}/e/(d/e)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(d/e)^{(1/3)}*x-1))+1/3*c*\ln(e*x^3+d)/e$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)/(e*x^3+d), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [C] time = 6.38466, size = 2425, normalized size = 18.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)/(e*x^3+d), x, algorithm="fricas")

[Out] $-1/12*(12*\sqrt{1/3}*e*\sqrt{((3*(I*\sqrt{3}) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^{(1/3)} - 2*c/e)^2*e^2 + 4*(3*(I*\sqrt{3}) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^{(1/3)} - 2*c/e)*c*e + 4*c^2/e^2)*\arctan(1/8*\sqrt{1/3}*((3*(I*\sqrt{3}) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^{(1/3)} - 2*c/$

$$e)^2 d e^2 + 4(3(\sqrt{3} + 1)(-1/54 c^3/e^3 + 1/54 b^3/(d e^2) + 1/54 (c^3 d - b^3 e)/(d e^3))^{1/3} - 2c/e) c d e - 8 b^2 e x + 4 b^2 e \sqrt{-(3(\sqrt{3} + 1)(-1/54 c^3/e^3 + 1/54 b^3/(d e^2) + 1/54 (c^3 d - b^3 e)/(d e^3))^{1/3} - 2c/e)^2 d e^2 x - 4 b^2 e x^2 + 4 c^2 d x - 4 b c d + 2(2 c d e x - b d e)(3(\sqrt{3} + 1)(-1/54 c^3/e^3 + 1/54 b^3/(d e^2) + 1/54 (c^3 d - b^3 e)/(d e^3))^{1/3} - 2c/e)/(b^2 e)) + 4 c^2 d) \sqrt{((3(\sqrt{3} + 1)(-1/54 c^3/e^3 + 1/54 b^3/(d e^2) + 1/54 (c^3 d - b^3 e)/(d e^3))^{1/3} - 2c/e)^2 e^2 + 4(3(\sqrt{3} + 1)(-1/54 c^3/e^3 + 1/54 b^3/(d e^2) + 1/54 (c^3 d - b^3 e)/(d e^3))^{1/3} - 2c/e) c e + 4 c^2)/e^2)/b^3} + 2(3(\sqrt{3} + 1)(-1/54 c^3/e^3 + 1/54 b^3/(d e^2) + 1/54 (c^3 d - b^3 e)/(d e^3))^{1/3} - 2c/e) e \log(1/4(3(\sqrt{3} + 1)(-1/54 c^3/e^3 + 1/54 b^3/(d e^2) + 1/54 (c^3 d - b^3 e)/(d e^3))^{1/3} - 2c/e)^2 d e^2 + (3(\sqrt{3} + 1)(-1/54 c^3/e^3 + 1/54 b^3/(d e^2) + 1/54 (c^3 d - b^3 e)/(d e^3))^{1/3} - 2c/e) c d e + b^2 e x + c^2 d) - ((3(\sqrt{3} + 1)(-1/54 c^3/e^3 + 1/54 b^3/(d e^2) + 1/54 (c^3 d - b^3 e)/(d e^3))^{1/3} - 2c/e) e + 6 c) \log(-1/4(3(\sqrt{3} + 1)(-1/54 c^3/e^3 + 1/54 b^3/(d e^2) + 1/54 (c^3 d - b^3 e)/(d e^3))^{1/3} - 2c/e)^2 d e^2 x + b^2 e x^2 - c^2 d x + b c d - 1/2(2 c d e x - b d e)(3(\sqrt{3} + 1)(-1/54 c^3/e^3 + 1/54 b^3/(d e^2) + 1/54 (c^3 d - b^3 e)/(d e^3))^{1/3} - 2c/e)))/e$$

Sympy [A] time = 0.299123, size = 75, normalized size = 0.56

$$\text{RootSum}\left(27t^3de^3 - 27t^2cde^2 + 9t^2de + b^3e - c^3d, \left(t \mapsto t \log\left(x + \frac{9t^2de^2 - 6tcde + c^2d}{b^2e}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x)/(e*x**3+d),x)

[Out] RootSum(27*_t**3*d*e**3 - 27*_t**2*c*d*e**2 + 9*_t*c**2*d*e + b**3*e - c**3*d, Lambda(_t, _t*log(x + (9*_t**2*d*e**2 - 6*_t*c*d*e + c**2*d)/(b**2*e)))

Giac [A] time = 1.10152, size = 162, normalized size = 1.21

$$\frac{\sqrt{3}(-de^2)^{\frac{2}{3}} b \arctan\left(\frac{\sqrt{3}\left(2x + (-de^{(-1)})^{\frac{1}{3}}\right)}{3(-de^{(-1)})^{\frac{1}{3}}}\right) e^{(-2)}}{3d} + \frac{1}{3} ce^{(-1)} \log(|x^3e + d|) + \frac{(-de^2)^{\frac{2}{3}} be^{(-2)} \log\left(x^2 + (-de^{(-1)})^{\frac{1}{3}}x + (-de^{(-1)})^{\frac{2}{3}}\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x)/(e*x^3+d),x, algorithm="giac")`

[Out]
$$\begin{aligned} & -\frac{1}{3}\sqrt{3}(-d e^2)^{2/3} b \arctan\left(\frac{1}{3}\sqrt{3}(2x + (-d e^{-1})^{1/3})\right) / \\ & (-d e^{-1})^{1/3} e^{-2} / d + \frac{1}{3} c e^{-1} \log(\operatorname{abs}(x^3 e + d)) + \frac{1}{6} (-d e^2)^{2/3} b e^{-2} \log(x^2 + (-d e^{-1})^{1/3} x + (-d e^{-1})^{2/3}) / d - \\ & \frac{1}{3} (-d e^{-1})^{2/3} b \log(\operatorname{abs}(x - (-d e^{-1})^{1/3})) / d \end{aligned}$$

3.26 $\int \frac{a+cx^2}{d-ex^3} dx$

Optimal. Leaf size=134

$$\frac{a \log(d^{2/3} + \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2)}{6d^{2/3}\sqrt[3]{e}} - \frac{a \log(\sqrt[3]{d} - \sqrt[3]{ex})}{3d^{2/3}\sqrt[3]{e}} + \frac{a \tan^{-1}\left(\frac{\sqrt[3]{d+2}\sqrt[3]{ex}}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}\sqrt[3]{e}} - \frac{c \log(d - ex^3)}{3e}$$

[Out] (a*ArcTan[(d^(1/3) + 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))]/(Sqrt[3]*d^(2/3)*e^(1/3)) - (a*Log[d^(1/3) - e^(1/3)*x])/(3*d^(2/3)*e^(1/3)) + (a*Log[d^(2/3) + d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3)*e^(1/3)) - (c*Log[d - e*x^3])/(3*e))

Rubi [A] time = 0.0891273, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {1871, 12, 200, 31, 634, 617, 204, 628, 260}

$$\frac{a \log(d^{2/3} + \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2)}{6d^{2/3}\sqrt[3]{e}} - \frac{a \log(\sqrt[3]{d} - \sqrt[3]{ex})}{3d^{2/3}\sqrt[3]{e}} + \frac{a \tan^{-1}\left(\frac{\sqrt[3]{d+2}\sqrt[3]{ex}}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}\sqrt[3]{e}} - \frac{c \log(d - ex^3)}{3e}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^2)/(d - e*x^3), x]

[Out] (a*ArcTan[(d^(1/3) + 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))]/(Sqrt[3]*d^(2/3)*e^(1/3)) - (a*Log[d^(1/3) - e^(1/3)*x])/(3*d^(2/3)*e^(1/3)) + (a*Log[d^(2/3) + d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3)*e^(1/3)) - (c*Log[d - e*x^3])/(3*e))

Rule 1871

Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Dist[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + cx^2}{d - ex^3} dx &= c \int \frac{x^2}{d - ex^3} dx + \int \frac{a}{d - ex^3} dx \\
&= -\frac{c \log(d - ex^3)}{3e} + a \int \frac{1}{d - ex^3} dx \\
&= -\frac{c \log(d - ex^3)}{3e} + \frac{a \int \frac{1}{\sqrt[3]{d} - \sqrt[3]{ex}} dx}{3d^{2/3}} + \frac{a \int \frac{2\sqrt[3]{d} + \sqrt[3]{ex}}{d^{2/3} + \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2} dx}{3d^{2/3}} \\
&= -\frac{a \log(\sqrt[3]{d} - \sqrt[3]{ex})}{3d^{2/3}\sqrt[3]{e}} - \frac{c \log(d - ex^3)}{3e} + \frac{a \int \frac{1}{d^{2/3} + \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2} dx}{2\sqrt[3]{d}} + \frac{a \int \frac{\sqrt[3]{d}\sqrt[3]{e} + 2e^{2/3}x}{d^{2/3} + \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2} dx}{6d^{2/3}\sqrt[3]{e}} \\
&= -\frac{a \log(\sqrt[3]{d} - \sqrt[3]{ex})}{3d^{2/3}\sqrt[3]{e}} + \frac{a \log(d^{2/3} + \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2)}{6d^{2/3}\sqrt[3]{e}} - \frac{c \log(d - ex^3)}{3e} - \frac{a \operatorname{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 + \frac{2\sqrt[3]{ex}}{\sqrt[3]{d}}\right)}{d^{2/3}\sqrt[3]{e}} \\
&= \frac{a \tan^{-1}\left(\frac{\sqrt[3]{d} + 2\sqrt[3]{ex}}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}\sqrt[3]{e}} - \frac{a \log(\sqrt[3]{d} - \sqrt[3]{ex})}{3d^{2/3}\sqrt[3]{e}} + \frac{a \log(d^{2/3} + \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2)}{6d^{2/3}\sqrt[3]{e}} - \frac{c \log(d - ex^3)}{3e}
\end{aligned}$$

Mathematica [A] time = 0.0299887, size = 123, normalized size = 0.92

$$\frac{ae^{2/3} \log(d^{2/3} + \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2) - 2ae^{2/3} \log(\sqrt[3]{d} - \sqrt[3]{ex}) + 2\sqrt{3}ae^{2/3} \tan^{-1}\left(\frac{2\sqrt[3]{ex} + 1}{\sqrt{3}\sqrt[3]{d}}\right) - 2cd^{2/3} \log(d - ex^3)}{6d^{2/3}e}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^2)/(d - e*x^3),x]

[Out] (2*Sqrt[3]*a*e^(2/3)*ArcTan[(1 + (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]] - 2*a*e^(2/3)*Log[d^(1/3) - e^(1/3)*x] + a*e^(2/3)*Log[d^(2/3) + d^(1/3)*e^(1/3)*x + e^(2/3)*x^2] - 2*c*d^(2/3)*Log[d - e*x^3])/(6*d^(2/3)*e)

Maple [A] time = 0.041, size = 111, normalized size = 0.8

$$-\frac{a}{3e} \ln\left(x - \sqrt[3]{\frac{d}{e}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}} + \frac{a}{6e} \ln\left(x^2 + \sqrt[3]{\frac{d}{e}}x + \left(\frac{d}{e}\right)^{\frac{2}{3}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}} + \frac{a\sqrt{3}}{3e} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{d}{e}}} + 1\right)\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}} - \frac{c \ln(ex^3 - d)}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+a)/(-e*x^3+d),x)`

[Out]
$$-1/3*a/e/(d/e)^{(2/3)}*\ln(x-(d/e)^{(1/3)})+1/6*a/e/(d/e)^{(2/3)}*\ln(x^2+(d/e)^{(1/3)}*x+(d/e)^{(2/3)})+1/3*a/e/(d/e)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(d/e)^{(1/3)}*x+1))-1/3*c/e*\ln(e*x^3-d)$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+a)/(-e*x^3+d),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [C] time = 6.29991, size = 2820, normalized size = 21.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+a)/(-e*x^3+d),x, algorithm="fricas")`

[Out]
$$\begin{aligned} & 1/12*(12*\sqrt{1/3}*e*\sqrt{(((1/2)^{(1/3)}*(I*\sqrt{3}) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^{(1/3)} + 2*c/e)^2*e^2 - 4*((1/2)^{(1/3)}*(I*\sqrt{3}) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^{(1/3)} + 2*c/e)*c*e + 4*c^2)/e^2)*\arctan(-1/8*(2*\sqrt{1/3}*(((1/2)^{(1/3)}*(I*\sqrt{3}) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^{(1/3)} + 2*c/e)*a*d*e^2 - 2*a*c*d*e)*\sqrt{(((1/2)^{(1/3)}*(I*\sqrt{3}) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^{(1/3)} + 2*c/e)^2*e^2 - 4*((1/2)^{(1/3)}*(I*\sqrt{3}) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^{(1/3)} + 2*c/e)*c*e + 4*c^2)/e^2)*\sqrt{(((1/2)^{(1/3)}*(I*\sqrt{3}) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^{(1/3)} + 2*c/e)^2*d^2*e^2 + 4*a^2*e^2*x^2 - 4*a*c*d*e*x + 4*c^2*d^2 + 2*(a*d*e^2*x - 2*c*d^2*e)*((1/2)^{(1/3)}*(I*\sqrt{3}) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^{(1/3)} + 2*c/e))/(a^2*e^2)) - \sqrt{1/3}*(((1/2)^{(1/3)}*(I*\sqrt{3}) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^{(1/3)} + 2*c/e)^2*d^2*e^2 - 8*a*c*d*e*x + 4*c^2*d^2 + 4*(a*d*e^2*x - c*d^2*e)*((1/2)^{(1/3)}*(I* \end{aligned}$$

$$\begin{aligned} & \sqrt{3} + 1) * (c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^{(1/3)} \\ & + 2*c/e) * \sqrt{((1/2)^{(1/3)} * (I*\sqrt{3} + 1) * (c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^{(1/3)} \\ & + 2*c/e)^2 * e^2 - 4 * ((1/2)^{(1/3)} * (I*\sqrt{3} + 1) * (c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^{(1/3)} \\ & + 2*c/e) * c * e + 4 * c^2)/e^2)} / (a^3 * e) - 2 * ((1/2)^{(1/3)} * (I*\sqrt{3} + 1) * (c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^{(1/3)} \\ & + 2*c/e) * e * \log(-1/2 * ((1/2)^{(1/3)} * (I*\sqrt{3} + 1) * (c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^{(1/3)} \\ & + 2*c/e) * d * e + a * e * x + c * d) + (((1/2)^{(1/3)} * (I*\sqrt{3} + 1) * (c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^{(1/3)} \\ & + 2*c/e) * e - 6 * c) * \log(1/4 * ((1/2)^{(1/3)} * (I*\sqrt{3} + 1) * (c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^{(1/3)} \\ & + 2*c/e)^2 * d^2 * e^2 + a^2 * e^2 * x^2 - a * c * d * e * x + c^2 * d^2 + 1/2 * (a * d * e^2 * x - 2 * c * d^2 * e) * ((1/2)^{(1/3)} * (I*\sqrt{3} + 1) * (c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^{(1/3)} \\ & + 2*c/e))) / e \end{aligned}$$

Sympy [A] time = 0.413526, size = 70, normalized size = 0.52

$$-\text{RootSum}\left(27t^3d^2e^3 - 27t^2cd^2e^2 + 9tc^2d^2e - a^3e^2 - c^3d^2, \left(t \mapsto t \log\left(x + \frac{-3tde + cd}{ae}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+a)/(-e*x**3+d),x)

[Out] -RootSum(27*_t**3*d**2*e**3 - 27*_t**2*c*d**2*e**2 + 9*_t*c**2*d**2*e - a**3*e**2 - c**3*d**2, Lambda(_t, _t*log(x + (-3*_t*d*e + c*d)/(a*e))))

Giac [A] time = 1.09519, size = 128, normalized size = 0.96

$$-\frac{1}{3}ce^{(-1)}\log(|x^3e-d|) + \frac{\sqrt{3}a \arctan\left(\frac{\sqrt{3}\left(d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}+2x\right)e^{\frac{1}{3}}}{3d^{\frac{1}{3}}}\right)}{3d^{\frac{2}{3}}} + \frac{ae^{\left(-\frac{1}{3}\right)}\log\left(d^{\frac{1}{3}}xe^{\left(-\frac{1}{3}\right)}+x^2+d^{\frac{2}{3}}e^{\left(-\frac{2}{3}\right)}\right)}{6d^{\frac{2}{3}}} - \frac{ae^{\left(-\frac{1}{3}\right)}\log\left(-d\right)}{3d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)/(-e*x^3+d),x, algorithm="giac")

[Out] -1/3*c*e^(-1)*log(abs(x^3*e - d)) + 1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(d^(1/3)*e^(-1/3) + 2*x)*e^(1/3)/d^(1/3))*e^(-1/3)/d^(2/3) + 1/6*a*e^(-1/3)*log(d

$$\frac{d^{1/3} * x * e^{-1/3} + x^2 + d^{2/3} * e^{-2/3}}{d^{2/3}} - \frac{1}{3} * a * e^{-1/3} * \log(\text{abs}(-d^{1/3} * e^{-1/3} + x)) / d^{2/3}$$

$$3.27 \quad \int \frac{2a^2 + b^2 x^2}{a^3 + b^3 x^3} dx$$

Optimal. Leaf size=37

$$\frac{\log(a + bx)}{b} - \frac{2 \tan^{-1}\left(\frac{a-2bx}{\sqrt{3a}}\right)}{\sqrt{3}b}$$

[Out] $(-2*\text{ArcTan}[(a - 2*b*x)/(\text{Sqrt}[3]*a)])/(\text{Sqrt}[3]*b) + \text{Log}[a + b*x]/b$

Rubi [A] time = 0.0576785, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1868, 31, 617, 204}

$$\frac{\log(a + bx)}{b} - \frac{2 \tan^{-1}\left(\frac{a-2bx}{\sqrt{3a}}\right)}{\sqrt{3}b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2*a^2 + b^2*x^2)/(a^3 + b^3*x^3), x]$

[Out] $(-2*\text{ArcTan}[(a - 2*b*x)/(\text{Sqrt}[3]*a)])/(\text{Sqrt}[3]*b) + \text{Log}[a + b*x]/b$

Rule 1868

```
Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Dist[C/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(−1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(−1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
```


$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 204

$\text{Int}[\frac{(a_.) + (b_.)(x_)^2}{(a^2 - 4b^2x^2)^{-1}}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[\frac{\text{Rt}[-b, 2]*x}{\text{Rt}[-a, 2]}] / \frac{\text{Rt}[-a, 2]*\text{Rt}[-b, 2]}{\text{Rt}[-a, 2]*\text{Rt}[-b, 2]}, x] / ; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \frac{2a^2 + b^2x^2}{a^3 + b^3x^3} dx &= \frac{a \int \frac{1}{\frac{a^2}{b^2} - \frac{ax}{b} + x^2} dx}{b^2} + \frac{\int \frac{1}{\frac{a}{b} + x} dx}{b} \\ &= \frac{\log(a + bx)}{b} + \frac{2 \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 - \frac{2bx}{a}\right)}{b} \\ &= -\frac{2 \tan^{-1}\left(\frac{a-2bx}{\sqrt{3a}}\right)}{\sqrt{3b}} + \frac{\log(a + bx)}{b} \end{aligned}$$

Mathematica [A] time = 0.0202841, size = 72, normalized size = 1.95

$$\frac{-\log(a^2 - abx + b^2x^2) + \log(a^3 + b^3x^3) + 2\log(a + bx) + 2\sqrt{3} \tan^{-1}\left(\frac{2bx-a}{\sqrt{3a}}\right)}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[(2*a^2 + b^2*x^2)/(a^3 + b^3*x^3), x]

[Out] (2*sqrt(3)*ArcTan[(-a + 2*b*x)/(sqrt(3)*a)] + 2*Log[a + b*x] - Log[a^2 - a*b*x + b^2*x^2] + Log[a^3 + b^3*x^3])/(3*b)

Maple [A] time = 0.048, size = 43, normalized size = 1.2

$$\frac{2\sqrt{3}}{3b} \arctan\left(\frac{(2b^2x - ab)\sqrt{3}}{3ab}\right) + \frac{\ln(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b^2*x^2+2*a^2)/(b^3*x^3+a^3),x)`

[Out] $\frac{2}{3} \cdot 3^{1/2} / b \cdot \arctan(1/3 \cdot (2 \cdot b^2 \cdot x - a \cdot b) \cdot 3^{1/2} / a / b) + \ln(b \cdot x + a) / b$

Maxima [A] time = 1.46757, size = 57, normalized size = 1.54

$$\frac{2\sqrt{3} \arctan\left(\frac{\sqrt{3}(2b^2x-ab)}{3ab}\right)}{3b} + \frac{\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b^2*x^2+2*a^2)/(b^3*x^3+a^3),x, algorithm="maxima")`

[Out] $\frac{2}{3} \cdot \sqrt{3} \cdot \arctan(1/3 \cdot \sqrt{3} \cdot (2 \cdot b^2 \cdot x - a \cdot b) / (a \cdot b)) / b + \log(b \cdot x + a) / b$

Fricas [A] time = 0.878056, size = 95, normalized size = 2.57

$$\frac{2\sqrt{3} \arctan\left(\frac{\sqrt{3}(2bx-a)}{3a}\right) + 3 \log(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b^2*x^2+2*a^2)/(b^3*x^3+a^3),x, algorithm="fricas")`

[Out] $\frac{1}{3} \cdot (2 \cdot \sqrt{3} \cdot \arctan(1/3 \cdot \sqrt{3} \cdot (2 \cdot b \cdot x - a) / a) + 3 \cdot \log(b \cdot x + a)) / b$

Sympy [C] time = 0.513175, size = 60, normalized size = 1.62

$$\frac{\frac{\sqrt{3}i \log\left(x + \frac{-a - \sqrt{3}ia}{2b}\right)}{3} + \frac{\sqrt{3}i \log\left(x + \frac{-a + \sqrt{3}ia}{2b}\right)}{3}}{b} + \log\left(\frac{a}{b} + x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*x**2+2*a**2)/(b**3*x**3+a**3),x)`

[Out] $(-\sqrt{3}I\log(x + (-a - \sqrt{3}Ia)/(2b))/3 + \sqrt{3}I\log(x + (-a + \sqrt{3}Ia)/(2b))/3 + \log(a/b + x))/b$

Giac [A] time = 1.08222, size = 50, normalized size = 1.35

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2bx-a)}{3a}\right)}{3b} + \frac{\log(|bx+a|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b^2*x^2+2*a^2)/(b^3*x^3+a^3),x, algorithm="giac")`

[Out] $2/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*b*x - a)/a)/b + \log(\text{abs}(b*x + a))/b$

$$3.28 \quad \int \frac{2a^2 + b^2x^2}{a^3 - b^3x^3} dx$$

Optimal. Leaf size=39

$$\frac{2 \tan^{-1}\left(\frac{a+2bx}{\sqrt{3a}}\right)}{\sqrt{3b}} - \frac{\log(a-bx)}{b}$$

[Out] (2*ArcTan[(a + 2*b*x)/(Sqrt[3]*a)])/(Sqrt[3]*b) - Log[a - b*x]/b

Rubi [A] time = 0.0421376, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1868, 31, 617, 204}

$$\frac{2 \tan^{-1}\left(\frac{a+2bx}{\sqrt{3a}}\right)}{\sqrt{3b}} - \frac{\log(a-bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[(2*a^2 + b^2*x^2)/(a^3 - b^3*x^3), x]

[Out] (2*ArcTan[(a + 2*b*x)/(Sqrt[3]*a)])/(Sqrt[3]*b) - Log[a - b*x]/b

Rule 1868

```
Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Dist[C/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(−1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(−1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
```

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 204

$\text{Int}[\{(a_) + (b_) * (x_)^2\}^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2] * x] / \text{Rt}[-a, 2]] / (\text{Rt}[-a, 2] * \text{Rt}[-b, 2]), x] \ /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \frac{2a^2 + b^2x^2}{a^3 - b^3x^3} dx &= \frac{a \int \frac{1}{\frac{a^2}{b^2} + \frac{ax}{b} + x^2} dx}{b^2} - \int \frac{1}{-\frac{a}{b} + x} dx \\ &= -\frac{\log(a - bx)}{b} - \frac{2 \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 + \frac{2bx}{a}\right)}{b} \\ &= \frac{2 \tan^{-1}\left(\frac{a+2bx}{\sqrt{3}a}\right)}{\sqrt{3}b} - \frac{\log(a - bx)}{b} \end{aligned}$$

Mathematica [A] time = 0.0178963, size = 71, normalized size = 1.82

$$\frac{\log(a^2 + abx + b^2x^2) - \log(a^3 - b^3x^3) - 2\log(a - bx) + 2\sqrt{3} \tan^{-1}\left(\frac{a+2bx}{\sqrt{3}a}\right)}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[(2*a^2 + b^2*x^2)/(a^3 - b^3*x^3), x]

[Out] (2*Sqrt[3]*ArcTan[(a + 2*b*x)/(Sqrt[3]*a)] - 2*Log[a - b*x] + Log[a^2 + a*b*x + b^2*x^2] - Log[a^3 - b^3*x^3])/(3*b)

Maple [A] time = 0.007, size = 45, normalized size = 1.2

$$\frac{2\sqrt{3}}{3b} \arctan\left(\frac{(2b^2x + ab)\sqrt{3}}{3ab}\right) - \frac{\ln(bx - a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b^2*x^2+2*a^2)/(-b^3*x^3+a^3),x)`

[Out] $2/3*3^{(1/2)}/b*\arctan(1/3*(2*b^2*x+a*b)*3^{(1/2)}/a/b)-1/b*\ln(b*x-a)$

Maxima [A] time = 1.42968, size = 59, normalized size = 1.51

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2b^2x+ab)}{3ab}\right)}{3b} - \frac{\log(bx-a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b^2*x^2+2*a^2)/(-b^3*x^3+a^3),x, algorithm="maxima")`

[Out] $2/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*b^2*x + a*b)/(a*b))/b - \log(b*x - a)/b$

Fricas [A] time = 0.998721, size = 95, normalized size = 2.44

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2bx+a)}{3a}\right) - 3\log(bx-a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b^2*x^2+2*a^2)/(-b^3*x^3+a^3),x, algorithm="fricas")`

[Out] $1/3*(2*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*b*x + a)/a) - 3*\log(b*x - a))/b$

Sympy [C] time = 0.433734, size = 60, normalized size = 1.54

$$\frac{\frac{\sqrt{3}i\log\left(x+\frac{a-\sqrt{3}ia}{2b}\right)}{3} - \frac{\sqrt{3}i\log\left(x+\frac{a+\sqrt{3}ia}{2b}\right)}{3} + \log\left(-\frac{a}{b} + x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*x**2+2*a**2)/(-b**3*x**3+a**3),x)`

[Out] $-\frac{\sqrt{3}i \log(x + (a - \sqrt{3}i)a)/(2b)}{3} - \frac{\sqrt{3}i \log(x + (a + \sqrt{3}i)a)/(2b)}{3} + \frac{\log(-a/b + x)}{b}$

Giac [A] time = 1.07399, size = 51, normalized size = 1.31

$$\frac{2\sqrt{3} \arctan\left(\frac{\sqrt{3}(2bx+a)}{3a}\right)}{3b} - \frac{\log(|bx-a|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b^2*x^2+2*a^2)/(-b^3*x^3+a^3),x, algorithm="giac")`

[Out] $\frac{2}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2bx+a)/a\right) / b - \log(\text{abs}(bx-a)) / b$

$$3.29 \quad \int \frac{8C + b^{2/3}Cx^2}{8 + bx^3} dx$$

Optimal. Leaf size=48

$$\frac{C \log(\sqrt[3]{bx} + 2)}{\sqrt[3]{b}} - \frac{2C \tan^{-1}\left(\frac{1 - \sqrt[3]{bx}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

[Out] $(-2*C*ArcTan[(1 - b^{(1/3)}*x)/Sqrt[3]])/(Sqrt[3]*b^{(1/3)}) + (C*Log[2 + b^{(1/3)}*x])/b^{(1/3)}$

Rubi [A] time = 0.0369016, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1863, 31, 617, 204}

$$\frac{C \log(\sqrt[3]{bx} + 2)}{\sqrt[3]{b}} - \frac{2C \tan^{-1}\left(\frac{1 - \sqrt[3]{bx}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(8*C + b^{(2/3)}*C*x^2)/(8 + b*x^3), x]$

[Out] $(-2*C*ArcTan[(1 - b^{(1/3)}*x)/Sqrt[3]])/(Sqrt[3]*b^{(1/3)}) + (C*Log[2 + b^{(1/3)}*x])/b^{(1/3)}$

Rule 1863

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Dist[C/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617


```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{8C + b^{2/3}Cx^2}{8 + bx^3} dx &= \frac{(2C) \int \frac{1}{\frac{4}{b^{2/3}} - \frac{2x}{\sqrt[3]{b}} + x^2} dx}{b^{2/3}} + \frac{C \int \frac{1}{\frac{2}{\sqrt[3]{b}} + x} dx}{\sqrt[3]{b}} \\ &= \frac{C \log(2 + \sqrt[3]{b}x)}{\sqrt[3]{b}} + \frac{(2C) \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 - \sqrt[3]{b}x\right)}{\sqrt[3]{b}} \\ &= -\frac{2C \tan^{-1}\left(\frac{1 - \sqrt[3]{b}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} + \frac{C \log(2 + \sqrt[3]{b}x)}{\sqrt[3]{b}} \end{aligned}$$

Mathematica [A] time = 0.018627, size = 76, normalized size = 1.58

$$\frac{C \left(-\log(b^{2/3}x^2 - 2\sqrt[3]{b}x + 4) + \log(bx^3 + 8) + 2 \log(\sqrt[3]{b}x + 2) + 2\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{b}x - 1}{\sqrt{3}}\right) \right)}{3\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8*C + b^(2/3)*C*x^2)/(8 + b*x^3), x]
```

```
[Out] (C*(2*Sqrt[3]*ArcTan[(-1 + b^(1/3)*x)/Sqrt[3]] + 2*Log[2 + b^(1/3)*x] - Log
[4 - 2*b^(1/3)*x + b^(2/3)*x^2] + Log[8 + b*x^3]))/(3*b^(1/3))
```

Maple [B] time = 0.044, size = 117, normalized size = 2.4

$$\frac{C\sqrt[3]{8}}{3b} \ln\left(x + \sqrt[3]{8}\sqrt[3]{b^{-1}}\right) (b^{-1})^{-\frac{2}{3}} - \frac{C\sqrt[3]{8}}{6b} \ln\left(x^2 - \sqrt[3]{8}\sqrt[3]{b^{-1}}x + 8^{\frac{2}{3}}(b^{-1})^{\frac{2}{3}}\right) (b^{-1})^{-\frac{2}{3}} + \frac{C\sqrt[3]{8}\sqrt{3}}{3b} \arctan\left(\frac{\sqrt{3}}{3}\left(\frac{8^{\frac{2}{3}}x}{4} - \frac{1}{\sqrt[3]{b^{-1}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*C+b^(2/3)*C*x^2)/(b*x^3+8),x)

[Out] 1/3*C/b*8^(1/3)/(1/b)^(2/3)*ln(x+8^(1/3)*(1/b)^(1/3))-1/6*C/b*8^(1/3)/(1/b)^(2/3)*ln(x^2-8^(1/3)*(1/b)^(1/3)*x+8^(2/3)*(1/b)^(2/3))+1/3*C/b*8^(1/3)/(1/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(1/4*8^(2/3)/(1/b)^(1/3)*x-1))+1/3*C/b^(1/3)*ln(b*x^3+8)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*C+b^(2/3)*C*x^2)/(b*x^3+8),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.08095, size = 387, normalized size = 8.06

$$\left[\frac{\sqrt{\frac{1}{3}}Cb\sqrt{-\frac{1}{2}}\log\left(\frac{bx^3+6\sqrt{\frac{1}{3}}\left(bx^2+b^{\frac{2}{3}}x-2b^{\frac{1}{3}}\right)\sqrt{-\frac{1}{2}}-6b^{\frac{1}{3}}x-4}}{bx^3+8}\right)+Cb^{\frac{2}{3}}\log\left(bx+2b^{\frac{2}{3}}\right)}{b}, \frac{2\sqrt{\frac{1}{3}}Cb^{\frac{2}{3}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(b^{\frac{2}{3}}x-b^{\frac{1}{3}}\right)}{b^{\frac{1}{3}}}\right)+Cb^{\frac{2}{3}}\log\left(\frac{\sqrt{\frac{1}{3}}\left(b^{\frac{2}{3}}x-b^{\frac{1}{3}}\right)}{b^{\frac{1}{3}}}\right)}{b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*C+b^(2/3)*C*x^2)/(b*x^3+8),x, algorithm="fricas")

```
[Out] [(sqrt(1/3)*C*b*sqrt(-1/b^(2/3))*log((b*x^3 + 6*sqrt(1/3)*(b*x^2 + b^(2/3))*
x - 2*b^(1/3))*sqrt(-1/b^(2/3)) - 6*b^(1/3)*x - 4)/(b*x^3 + 8)) + C*b^(2/3)
*log(b*x + 2*b^(2/3)))/b, (2*sqrt(1/3)*C*b^(2/3)*arctan(sqrt(1/3)*(b^(2/3)*
x - b^(1/3))/b^(1/3)) + C*b^(2/3)*log(b*x + 2*b^(2/3)))/b]
```

Sympy [A] time = 0.49057, size = 58, normalized size = 1.21

$$\text{RootSum}\left(3t^3b^{\frac{5}{3}} - 3t^2Cb^{\frac{4}{3}} + tC^2b - C^3b^{\frac{2}{3}}, \left(t \mapsto t \log\left(x + \frac{3t\sqrt[3]{b} - C}{C\sqrt[3]{b}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*C+b**(2/3)*C*x**2)/(b*x**3+8), x)
```

```
[Out] RootSum(3*_t**3*b**(5/3) - 3*_t**2*C*b**(4/3) + _t*C**2*b - C**3*b**(2/3),
Lambda(_t, _t*log(x + (3*_t*b**(1/3) - C)/(C*b**(1/3)))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*C+b^(2/3)*C*x^2)/(b*x^3+8), x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.30 \quad \int \frac{a^{2/3}C + 2Cx^2}{a + 8x^3} dx$$

Optimal. Leaf size=47

$$\frac{1}{4}C \log(\sqrt[3]{a} + 2x) - \frac{C \tan^{-1}\left(\frac{\sqrt[3]{a}-4x}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}}$$

[Out] $-(C \operatorname{ArcTan}[(a^{1/3} - 4x)/(\operatorname{Sqrt}[3] * a^{1/3})]) / (2 * \operatorname{Sqrt}[3]) + (C * \operatorname{Log}[a^{1/3} + 2x]) / 4$

Rubi [A] time = 0.0343279, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1863, 31, 617, 204}

$$\frac{1}{4}C \log(\sqrt[3]{a} + 2x) - \frac{C \tan^{-1}\left(\frac{\sqrt[3]{a}-4x}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a^{2/3} * C + 2 * C * x^2) / (a + 8 * x^3), x]$

[Out] $-(C * \operatorname{ArcTan}[(a^{1/3} - 4x) / (\operatorname{Sqrt}[3] * a^{1/3})]) / (2 * \operatorname{Sqrt}[3]) + (C * \operatorname{Log}[a^{1/3} + 2x]) / 4$

Rule 1863

$\operatorname{Int}[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] \rightarrow \operatorname{With}[\{A = \operatorname{Coeff}[P2, x, 0], B = \operatorname{Coeff}[P2, x, 1], C = \operatorname{Coeff}[P2, x, 2]\}, \operatorname{With}[\{q = a^{1/3}/b^{1/3}\}, \operatorname{Dist}[C/b, \operatorname{Int}[1/(q + x), x], x] + \operatorname{Dist}[(B + C*q)/b, \operatorname{Int}[1/(q^2 - q*x + x^2), x], x]] \;/; \operatorname{EqQ}[A*b^{2/3} - a^{1/3}*b^{1/3}*B - 2*a^{2/3}*C, 0]] \;/; \operatorname{FreeQ}[\{a, b\}, x] \ \&\& \operatorname{PolyQ}[P2, x, 2]$

Rule 31

$\operatorname{Int}[(a_) + (b_)*(x_)^{(-1)}, x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/b, x] \;/; \operatorname{FreeQ}[\{a, b\}, x]$

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{a^{2/3}C + 2Cx^2}{a + 8x^3} dx &= \frac{1}{4}C \int \frac{1}{\frac{\sqrt[3]{a}}{2} + x} dx + \frac{1}{8}(\sqrt[3]{a}C) \int \frac{1}{\frac{a^{2/3}}{4} - \frac{\sqrt[3]{ax}}{2} + x^2} dx \\ &= \frac{1}{4}C \log(\sqrt[3]{a} + 2x) + \frac{1}{2}C \text{Subst} \left(\int \frac{1}{-3 - x^2} dx, x, 1 - \frac{4x}{\sqrt[3]{a}} \right) \\ &= -\frac{C \tan^{-1} \left(\frac{\sqrt[3]{a} - 4x}{\sqrt{3}\sqrt[3]{a}} \right)}{2\sqrt{3}} + \frac{1}{4}C \log(\sqrt[3]{a} + 2x) \end{aligned}$$

Mathematica [A] time = 0.023975, size = 72, normalized size = 1.53

$$\frac{1}{12}C \left(-\log(a^{2/3} - 2\sqrt[3]{ax} + 4x^2) + \log(a + 8x^3) + 2\log(\sqrt[3]{a} + 2x) - 2\sqrt{3} \tan^{-1} \left(\frac{1 - \frac{4x}{\sqrt[3]{a}}}{\sqrt{3}} \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(2/3)*C + 2*C*x^2)/(a + 8*x^3), x]
```

```
[Out] (C*(-2*Sqrt[3]*ArcTan[(1 - (4*x)/a^(1/3))/Sqrt[3]] + 2*Log[a^(1/3) + 2*x] -
Log[a^(2/3) - 2*a^(1/3)*x + 4*x^2] + Log[a + 8*x^3]))/12
```

Maple [B] time = 0.047, size = 84, normalized size = 1.8

$$\frac{C8^{\frac{2}{3}}}{24} \ln \left(x + \frac{8^{\frac{2}{3}}}{8} \sqrt[3]{a} \right) - \frac{C8^{\frac{2}{3}}}{48} \ln \left(x^2 - \frac{8^{\frac{2}{3}}x}{8} \sqrt[3]{a} + \frac{\sqrt[3]{8}}{8} a^{\frac{2}{3}} \right) + \frac{C8^{\frac{2}{3}}\sqrt{3}}{24} \arctan \left(\frac{\sqrt{3}}{3} \left(2 \frac{\sqrt[3]{8x}}{\sqrt[3]{a}} - 1 \right) \right) + \frac{C \ln(8x^3 + a)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((a^{(2/3)}*C+2*C*x^2)/(8*x^3+a),x)$

[Out] $\frac{1}{24}*C*8^{(2/3)}*\ln(x+1/8*8^{(2/3)}*a^{(1/3)})-1/48*C*8^{(2/3)}*\ln(x^2-1/8*8^{(2/3)}*a^{(1/3)}*x+1/8*8^{(1/3)}*a^{(2/3)})+1/24*C*8^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2*8^{(1/3)}/a^{(1/3)}*x-1))+1/12*C*\ln(8*x^3+a)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((a^{(2/3)}*C+2*C*x^2)/(8*x^3+a),x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

Fricas [A] time = 1.01958, size = 122, normalized size = 2.6

$$\frac{1}{6} \sqrt{3} C \arctan\left(\frac{4 \sqrt{3} a^{\frac{2}{3}} x - \sqrt{3} a}{3 a}\right) + \frac{1}{4} C \log\left(2 x + a^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((a^{(2/3)}*C+2*C*x^2)/(8*x^3+a),x, \text{algorithm}="fricas")$

[Out] $\frac{1}{6}*\sqrt{3}*C*\arctan(1/3*(4*\sqrt{3}*a^{(2/3)}*x - \sqrt{3}*a)/a) + 1/4*C*\log(2*x + a^{(1/3)})$

Sympy [C] time = 0.442604, size = 85, normalized size = 1.81

$$C \left(\frac{\log\left(\frac{\sqrt[3]{a}}{2} + x\right)}{4} - \frac{\sqrt{3}i \log\left(x + \frac{-C\sqrt[3]{a} - \sqrt{3}iC\sqrt[3]{a}}{4C}\right)}{12} + \frac{\sqrt{3}i \log\left(x + \frac{-C\sqrt[3]{a} + \sqrt{3}iC\sqrt[3]{a}}{4C}\right)}{12} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**(2/3)*C+2*C*x**2)/(8*x**3+a),x)

[Out] C*(log(a**(1/3)/2 + x)/4 - sqrt(3)*I*log(x + (-C*a**(1/3) - sqrt(3)*I*C*a**(1/3))/(4*C))/12 + sqrt(3)*I*log(x + (-C*a**(1/3) + sqrt(3)*I*C*a**(1/3))/(4*C))/12)

Giac [B] time = 1.09818, size = 150, normalized size = 3.19

$$\frac{\sqrt{3}(\sqrt{3}i|a| + a)C \arctan\left(\frac{\sqrt{3}\left(4x+(-a)^{\frac{1}{3}}\right)}{3(-a)^{\frac{1}{3}}}\right)}{12a} + \frac{(\sqrt{3}i|a| + 3a)C \log\left(x^2 + \frac{1}{2}(-a)^{\frac{1}{3}}x + \frac{1}{4}(-a)^{\frac{2}{3}}\right)}{24a} - \frac{(C(-a)^{\frac{2}{3}} + 2Ca^{\frac{2}{3}})(-a)^{\frac{1}{3}}}{12a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(2/3)*C+2*C*x^2)/(8*x^3+a),x, algorithm="giac")

[Out] 1/12*sqrt(3)*(sqrt(3)*i*abs(a) + a)*C*arctan(1/3*sqrt(3)*(4*x + (-a)^(1/3)) / (-a)^(1/3))/a + 1/24*(sqrt(3)*i*abs(a) + 3*a)*C*log(x^2 + 1/2*(-a)^(1/3)*x + 1/4*(-a)^(2/3))/a - 1/12*(C*(-a)^(2/3) + 2*C*a^(2/3))*(-a)^(1/3)*log(abs(x - 1/2*(-a)^(1/3)))/a

$$3.31 \quad \int \frac{8C+(-b)^{2/3}Cx^2}{-8+bx^3} dx$$

Optimal. Leaf size=57

$$\frac{2C \tan^{-1}\left(\frac{1-\sqrt[3]{-bx}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{-b}} - \frac{C \log(\sqrt[3]{-bx} + 2)}{\sqrt[3]{-b}}$$

[Out] (2*C*ArcTan[(1 - (-b)^(1/3)*x)/Sqrt[3]])/(Sqrt[3]*(-b)^(1/3)) - (C*Log[2 + (-b)^(1/3)*x])/(-b)^(1/3)

Rubi [A] time = 0.0689679, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1864, 31, 617, 204}

$$\frac{2C \tan^{-1}\left(\frac{1-\sqrt[3]{-bx}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{-b}} - \frac{C \log(\sqrt[3]{-bx} + 2)}{\sqrt[3]{-b}}$$

Antiderivative was successfully verified.

[In] Int[(8*C + (-b)^(2/3)*C*x^2)/(-8 + b*x^3), x]

[Out] (2*C*ArcTan[(1 - (-b)^(1/3)*x)/Sqrt[3]])/(Sqrt[3]*(-b)^(1/3)) - (C*Log[2 + (-b)^(1/3)*x])/(-b)^(1/3)

Rule 1864

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a)^(1/3)/(-b)^(1/3)}, Dist[C/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /; EqQ[A*(-b)^(2/3) - (-a)^(1/3)*(-b)^(1/3)*B - 2*(-a)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617


```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{8C + (-b)^{2/3}Cx^2}{-8 + bx^3} dx &= \frac{(2C) \int \frac{1}{\frac{4}{(-b)^{2/3}} - \frac{2x}{\sqrt[3]{-b}} + x^2} dx}{(-b)^{2/3}} - \frac{C \int \frac{1}{\frac{2}{\sqrt[3]{-b}} + x} dx}{\sqrt[3]{-b}} \\ &= -\frac{C \log(2 + \sqrt[3]{-bx})}{\sqrt[3]{-b}} - \frac{(2C) \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 - \sqrt[3]{-bx}\right)}{\sqrt[3]{-b}} \\ &= \frac{2C \tan^{-1}\left(\frac{1 - \sqrt[3]{-bx}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{-b}} - \frac{C \log(2 + \sqrt[3]{-bx})}{\sqrt[3]{-b}} \end{aligned}$$

Mathematica [A] time = 0.0291527, size = 99, normalized size = 1.74

$$\frac{C \left(-b^{2/3} \log(b^{2/3}x^2 + 2\sqrt[3]{bx} + 4) + 2b^{2/3} \log(2 - \sqrt[3]{bx}) - 2\sqrt{3}b^{2/3} \tan^{-1}\left(\frac{\sqrt[3]{bx} + 1}{\sqrt{3}}\right) + (-b)^{2/3} \log(8 - bx^3) \right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8*C + (-b)^(2/3)*C*x^2)/(-8 + b*x^3), x]
```

```
[Out] (C*(-2*Sqrt[3]*b^(2/3)*ArcTan[(1 + b^(1/3)*x)/Sqrt[3]] + 2*b^(2/3)*Log[2 -
b^(1/3)*x] - b^(2/3)*Log[4 + 2*b^(1/3)*x + b^(2/3)*x^2] + (-b)^(2/3)*Log[8
- b*x^3]))/(3*b)
```

Maple [B] time = 0.006, size = 122, normalized size = 2.1

$$\frac{C\sqrt[3]{8}}{3b} \ln\left(x - \sqrt[3]{8}\sqrt[3]{b^{-1}}\right) (b^{-1})^{-\frac{2}{3}} - \frac{C\sqrt[3]{8}}{6b} \ln\left(x^2 + \sqrt[3]{8}\sqrt[3]{b^{-1}}x + 8^{\frac{2}{3}}(b^{-1})^{\frac{2}{3}}\right) (b^{-1})^{-\frac{2}{3}} - \frac{C\sqrt[3]{8}\sqrt{3}}{3b} \arctan\left(\frac{\sqrt{3}}{3}\left(\frac{8^{\frac{2}{3}}x}{4} + \frac{1}{\sqrt[3]{b^{-1}}}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*C+(-b)^(2/3)*C*x^2)/(b*x^3-8),x)

[Out] 1/3*C/b*8^(1/3)/(1/b)^(2/3)*ln(x-8^(1/3)*(1/b)^(1/3))-1/6*C/b*8^(1/3)/(1/b)^(2/3)*ln(x^2+8^(1/3)*(1/b)^(1/3)*x+8^(2/3)*(1/b)^(2/3))-1/3*C/b*8^(1/3)/(1/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(1/4*8^(2/3)/(1/b)^(1/3)*x+1))+1/3*C*(-b)^(2/3)/b*ln(b*x^3-8)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*C+(-b)^(2/3)*C*x^2)/(b*x^3-8),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.05274, size = 466, normalized size = 8.18

$$\frac{\sqrt{\frac{1}{3}}Cb\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}} \log\left(\frac{bx^3-6\sqrt{\frac{1}{3}}\left(bx^2-(-b)^{\frac{2}{3}}x+2(-b)^{\frac{1}{3}}\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}+6(-b)^{\frac{1}{3}}x+4}}{bx^3-8}}\right) + C(-b)^{\frac{2}{3}} \log\left(bx-2(-b)^{\frac{2}{3}}\right)}{b}, -\frac{2\sqrt{\frac{1}{3}}Cb\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}} \arctan\left(\frac{\sqrt{3}}{3}\left(\frac{8^{\frac{2}{3}}x}{4} + \frac{1}{\sqrt[3]{b^{-1}}}\right) + 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*C+(-b)^(2/3)*C*x^2)/(b*x^3-8),x, algorithm="fricas")

```
[Out] [(sqrt(1/3)*C*b*sqrt((-b)^(1/3)/b)*log((b*x^3 - 6*sqrt(1/3)*(b*x^2 - (-b)^(2/3)*x + 2*(-b)^(1/3))*sqrt((-b)^(1/3)/b) + 6*(-b)^(1/3)*x + 4)/(b*x^3 - 8)
) + C*(-b)^(2/3)*log(b*x - 2*(-b)^(2/3)))/b, -(2*sqrt(1/3)*C*b*sqrt(-(-b)^(1/3)/b)*arctan(sqrt(1/3)*((-b)^(2/3)*x - (-b)^(1/3))*sqrt(-(-b)^(1/3)/b)) -
C*(-b)^(2/3)*log(b*x - 2*(-b)^(2/3)))/b]
```

Sympy [A] time = 0.582586, size = 58, normalized size = 1.02

$$\text{RootSum}\left(3t^3b^2 - 3t^2Cb(-b)^{\frac{2}{3}} + tC^2(-b)^{\frac{4}{3}} - C^3b, \left(t \mapsto t \log\left(-\frac{3t}{C} + x + \frac{(-b)^{\frac{2}{3}}}{b}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*C+(-b)**(2/3)*C*x**2)/(b*x**3-8),x)
```

```
[Out] RootSum(3*_t**3*b**2 - 3*_t**2*C*b*(-b)**(2/3) + *_t*C**2*(-b)**(4/3) - C**3
*b, Lambda(_t, *_t*log(-3*_t/C + x + (-b)**(2/3)/b)))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*C+(-b)^(2/3)*C*x^2)/(b*x^3-8),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.32 \quad \int \frac{(-a)^{2/3}C+2Cx^2}{a-8x^3} dx$$

Optimal. Leaf size=47

$$\frac{C \tan^{-1}\left(\frac{1-\frac{4x}{\sqrt[3]{-a}}}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{4}C \log(\sqrt[3]{-a} + 2x)$$

[Out] (C*ArcTan[(1 - (4*x)/(-a)^(1/3))/Sqrt[3]])/(2*Sqrt[3]) - (C*Log[(-a)^(1/3) + 2*x])/4

Rubi [A] time = 0.061084, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1864, 31, 617, 204}

$$\frac{C \tan^{-1}\left(\frac{1-\frac{4x}{\sqrt[3]{-a}}}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{4}C \log(\sqrt[3]{-a} + 2x)$$

Antiderivative was successfully verified.

[In] Int[((-a)^(2/3)*C + 2*C*x^2)/(a - 8*x^3),x]

[Out] (C*ArcTan[(1 - (4*x)/(-a)^(1/3))/Sqrt[3]])/(2*Sqrt[3]) - (C*Log[(-a)^(1/3) + 2*x])/4

Rule 1864

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a)^(1/3)/(-b)^(1/3)}, Dist[C/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /; EqQ[A*(-b)^(2/3) - (-a)^(1/3)*(-b)^(1/3)*B - 2*(-a)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(1-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
 implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(-a)^{2/3}C + 2Cx^2}{a - 8x^3} dx &= -\left(\frac{1}{4}C \int \frac{1}{\frac{\sqrt[3]{-a}}{2} + x} dx\right) - \frac{1}{8}(\sqrt[3]{-a}C) \int \frac{1}{\frac{1}{4}(-a)^{2/3} - \frac{1}{2}\sqrt[3]{-a}x + x^2} dx \\ &= -\frac{1}{4}C \log(\sqrt[3]{-a} + 2x) - \frac{1}{2}C \text{Subst}\left(\int \frac{1}{-3 - x^2} dx, x, 1 - \frac{4x}{\sqrt[3]{-a}}\right) \\ &= \frac{C \tan^{-1}\left(\frac{1 - \frac{4x}{\sqrt[3]{-a}}}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{4}C \log(\sqrt[3]{-a} + 2x) \end{aligned}$$

Mathematica [B] time = 0.0390344, size = 106, normalized size = 2.26

$$\frac{C \left((-a)^{2/3} \log(a^{2/3} + 2\sqrt[3]{ax} + 4x^2) - a^{2/3} \log(8x^3 - a) - 2(-a)^{2/3} \log(\sqrt[3]{a} - 2x) + 2\sqrt{3}(-a)^{2/3} \tan^{-1}\left(\frac{\frac{4x}{\sqrt[3]{a}} + 1}{\sqrt{3}}\right) \right)}{12a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-a)^(2/3)*C + 2*C*x^2)/(a - 8*x^3), x]
```

```
[Out] (C*(2*Sqrt[3]*(-a)^(2/3)*ArcTan[(1 + (4*x)/a^(1/3))/Sqrt[3]] - 2*(-a)^(2/3)
*Log[a^(1/3) - 2*x] + (-a)^(2/3)*Log[a^(2/3) + 2*a^(1/3)*x + 4*x^2] - a^(2/
3)*Log[-a + 8*x^3]))/(12*a^(2/3))
```

Maple [B] time = 0.007, size = 110, normalized size = 2.3

$$-\frac{C8^{\frac{2}{3}}}{24}(-a)^{\frac{2}{3}}\ln\left(x-\frac{8^{\frac{2}{3}}}{8}\sqrt[3]{a}\right)a^{-\frac{2}{3}}+\frac{C8^{\frac{2}{3}}}{48}(-a)^{\frac{2}{3}}\ln\left(x^2+\frac{8^{\frac{2}{3}}x}{8}\sqrt[3]{a}+\frac{\sqrt[3]{8}}{8}a^{\frac{2}{3}}\right)a^{-\frac{2}{3}}+\frac{C8^{\frac{2}{3}}\sqrt{3}}{24}(-a)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}}{3}\left(2\frac{\sqrt[3]{8}x}{\sqrt[3]{a}}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((−a)^(2/3)*C+2*C*x^2)/(−8*x^3+a),x)

[Out] −1/24*C*(−a)^(2/3)*8^(2/3)/a^(2/3)*ln(x−1/8*8^(2/3)*a^(1/3))+1/48*C*(−a)^(2/3)*8^(2/3)/a^(2/3)*ln(x^2+1/8*8^(2/3)*a^(1/3)*x+1/8*8^(1/3)*a^(2/3))+1/24*C*(−a)^(2/3)*8^(2/3)/a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*8^(1/3)/a^(1/3)*x+1))−1/12*C*ln(8*x^3−a)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((−a)^(2/3)*C+2*C*x^2)/(−8*x^3+a),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.02475, size = 130, normalized size = 2.77

$$\frac{1}{6}\sqrt{3}C\arctan\left(\frac{4\sqrt{3}(-a)^{\frac{2}{3}}x+\sqrt{3}a}{3a}\right)-\frac{1}{4}C\log\left(2x+(-a)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((−a)^(2/3)*C+2*C*x^2)/(−8*x^3+a),x, algorithm="fricas")

[Out] 1/6*sqrt(3)*C*arctan(1/3*(4*sqrt(3)*(-a)^(2/3)*x + sqrt(3)*a)/a) − 1/4*C*log(2*x + (-a)^(1/3))

Sympy [C] time = 0.467872, size = 95, normalized size = 2.02

$$-C \left(\frac{\log\left(-\frac{a}{2(-a)^{\frac{2}{3}}} + x\right)}{4} + \frac{\sqrt{3}i \log\left(\frac{a}{4(-a)^{\frac{2}{3}}} - \frac{\sqrt{3}ia}{4(-a)^{\frac{2}{3}}} + x\right)}{12} - \frac{\sqrt{3}i \log\left(\frac{a}{4(-a)^{\frac{2}{3}}} + \frac{\sqrt{3}ia}{4(-a)^{\frac{2}{3}}} + x\right)}{12} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a)**(2/3)*C+2*C*x**2)/(-8*x**3+a), x)

[Out] -C*(log(-a/(2*(-a)**(2/3)) + x)/4 + sqrt(3)*I*log(a/(4*(-a)**(2/3)) - sqrt(3)*I*a/(4*(-a)**(2/3)) + x)/12 - sqrt(3)*I*log(a/(4*(-a)**(2/3)) + sqrt(3)*I*a/(4*(-a)**(2/3)) + x)/12)

Giac [B] time = 1.13767, size = 132, normalized size = 2.81

$$\frac{\sqrt{3}(\sqrt{3}i|a| - a)C \arctan\left(\frac{\sqrt{3}\left(4x+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{12a} + \frac{(\sqrt{3}i|a| - 3a)C \log\left(x^2 + \frac{1}{2}a^{\frac{1}{3}}x + \frac{1}{4}a^{\frac{2}{3}}\right)}{24a} - \frac{(2C(-a)^{\frac{2}{3}} + Ca^{\frac{2}{3}}) \log\left(\left|x - \frac{1}{2}a^{\frac{1}{3}}\right|\right)}{12a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a)^(2/3)*C+2*C*x^2)/(-8*x^3+a), x, algorithm="giac")

[Out] 1/12*sqrt(3)*(sqrt(3)*i*abs(a) - a)*C*arctan(1/3*sqrt(3)*(4*x + a^(1/3))/a^(1/3))/a + 1/24*(sqrt(3)*i*abs(a) - 3*a)*C*log(x^2 + 1/2*a^(1/3)*x + 1/4*a^(2/3))/a - 1/12*(2*C*(-a)^(2/3) + C*a^(2/3))*log(abs(x - 1/2*a^(1/3)))/a^(2/3)

$$3.33 \quad \int \frac{2\left(\frac{a}{b}\right)^{2/3} C + Cx^2}{a + bx^3} dx$$

Optimal. Leaf size=50

$$\frac{C \log\left(\sqrt[3]{\frac{a}{b}} + x\right)}{b} - \frac{2C \tan^{-1}\left(\frac{1 - \frac{2x}{\sqrt[3]{a}}}{\sqrt[3]{b}}\right)}{\sqrt{3}b}$$

[Out] $(-2*C*ArcTan[(1 - (2*x)/(a/b)^{(1/3)})/Sqrt[3]])/(Sqrt[3]*b) + (C*Log[(a/b)^{(1/3)} + x])/b$

Rubi [A] time = 0.0765061, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1867, 31, 617, 204}

$$\frac{C \log\left(\sqrt[3]{\frac{a}{b}} + x\right)}{b} - \frac{2C \tan^{-1}\left(\frac{1 - \frac{2x}{\sqrt[3]{a}}}{\sqrt[3]{b}}\right)}{\sqrt{3}b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2*(a/b)^{(2/3)}*C + C*x^2)/(a + b*x^3), x]$

[Out] $(-2*C*ArcTan[(1 - (2*x)/(a/b)^{(1/3)})/Sqrt[3]])/(Sqrt[3]*b) + (C*Log[(a/b)^{(1/3)} + x])/b$

Rule 1867

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[C/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```


Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{2\left(\frac{a}{b}\right)^{2/3} C + Cx^2}{a + bx^3} dx &= \frac{C \int \frac{1}{\sqrt[3]{\frac{a}{b} + x}} dx}{b} + \frac{\left(\sqrt[3]{\frac{a}{b}} C\right) \int \frac{1}{\left(\frac{a}{b}\right)^{2/3} - \sqrt[3]{\frac{a}{b}} x + x^2} dx}{b} \\ &= \frac{C \log\left(\sqrt[3]{\frac{a}{b}} + x\right)}{b} + \frac{(2C) \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 - \frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)}{b} \\ &= -\frac{2C \tan^{-1}\left(\frac{1 - \frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b} + \frac{C \log\left(\sqrt[3]{\frac{a}{b}} + x\right)}{b} \end{aligned}$$

Mathematica [B] time = 0.0519632, size = 146, normalized size = 2.92

$$\frac{C \left(-b^{2/3} \left(\frac{a}{b}\right)^{2/3} \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right) + a^{2/3} \log(a + bx^3) + 2b^{2/3} \left(\frac{a}{b}\right)^{2/3} \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right) - 2\sqrt{3}b^{2/3} \left(\frac{a}{b}\right)^{2/3} \tan^{-1}\left(\frac{1 - \frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right) \right)}{3a^{2/3}b}$$

Antiderivative was successfully verified.

[In] Integrate[(2*(a/b)^(2/3)*C + C*x^2)/(a + b*x^3), x]

[Out] (C*(-2*Sqrt[3]*(a/b)^(2/3)*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3)]/Sqrt[3]] + 2*(a/b)^(2/3)*b^(2/3)*Log[a^(1/3) + b^(1/3)*x] - (a/b)^(2/3)*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(2/3)*Log[a + b*x^3]))/(

$3*a^{(2/3)*b}$

Maple [A] time = 0.044, size = 87, normalized size = 1.7

$$\frac{2C}{3b} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) - \frac{C}{3b} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) + \frac{2C\sqrt{3}}{3b} \arctan\left(\frac{\sqrt{3}}{3} \left(2x \frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) + \frac{C \ln(bx^3 + a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*(1/b*a)^(2/3)*C+C*x^2)/(b*x^3+a),x)

[Out] 2/3*C*ln(x+(1/b*a)^(1/3))/b-1/3*C/b*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2/3*C/b*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/3*C/b*ln(b*x^3+a)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*(a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.04965, size = 138, normalized size = 2.76

$$\frac{2\sqrt{3}C \arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}} - \sqrt{3}a}{3a}\right) + 3C \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*(a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="fricas")

[Out] $\frac{1}{3} * (2 * \sqrt{3} * C * \arctan(1/3 * (2 * \sqrt{3}) * b * x * (a/b)^{2/3} - \sqrt{3} * a) / a) + 3 * C * \log(x + (a/b)^{1/3}) / b$

Sympy [C] time = 0.536165, size = 100, normalized size = 2.

$$C \left(\log \left(\frac{a}{b \left(\frac{a}{b} \right)^{2/3}} + x \right) - \frac{\sqrt{3}i \log \left(-\frac{a}{2b \left(\frac{a}{b} \right)^{2/3}} - \frac{\sqrt{3}ia}{2b \left(\frac{a}{b} \right)^{2/3}} + x \right)}{3} + \frac{\sqrt{3}i \log \left(-\frac{a}{2b \left(\frac{a}{b} \right)^{2/3}} + \frac{\sqrt{3}ia}{2b \left(\frac{a}{b} \right)^{2/3}} + x \right)}{3} \right) / b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*(a/b)**(2/3)*C+C*x**2)/(b*x**3+a), x)`

[Out] $C * (\log(a / (b * (a/b)^{2/3}) + x) - \sqrt{3} * I * \log(-a / (2 * b * (a/b)^{2/3})) - \sqrt{3} * I * a / (2 * b * (a/b)^{2/3} + x) / 3 + \sqrt{3} * I * \log(-a / (2 * b * (a/b)^{2/3})) + \sqrt{3} * I * a / (2 * b * (a/b)^{2/3} + x) / 3) / b$

Giac [B] time = 1.12556, size = 224, normalized size = 4.48

$$\frac{\sqrt{3} \left(ab^2 + \sqrt{3} \sqrt{a^2 b^4} i \right) C \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b} \right)^{1/3} \right)}{3 \left(-\frac{a}{b} \right)^{1/3}} \right)}{3 ab^3} - \frac{\left(C b^2 \left(-\frac{a}{b} \right)^{2/3} + 2 \left(ab^2 \right)^{2/3} C \right) \left(-\frac{a}{b} \right)^{1/3} \log \left(\left| x - \left(-\frac{a}{b} \right)^{1/3} \right| \right)}{3 ab^2} + \frac{\left(3 ab^2 + \sqrt{3} \sqrt{a^2 b^4} i \right) C \log \left(x + \left(-\frac{a}{b} \right)^{1/3} \right)}{3 ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*(a/b)^(2/3)*C+C*x^2)/(b*x^3+a), x, algorithm="giac")`

[Out] $\frac{1}{3} * \sqrt{3} * (a * b^2 + \sqrt{3} * \sqrt{a^2 * b^4} * i) * C * \arctan(1/3 * \sqrt{3} * (2 * x + (-a/b)^{1/3}) / (-a/b)^{1/3}) / (a * b^3) - 1/3 * (C * b^2 * (-a/b)^{2/3} + 2 * (a * b^2)^{2/3} * C) * (-a/b)^{1/3} * \log(\text{abs}(x - (-a/b)^{1/3})) / (a * b^2) + 1/6 * (3 * a * b^2 + \sqrt{3} * \sqrt{a^2 * b^4} * i) * C * \log(x^2 + x * (-a/b)^{1/3} + (-a/b)^{2/3}) / (a * b^3)$

$$3.34 \quad \int \frac{2\left(-\frac{a}{b}\right)^{2/3} C + Cx^2}{a - bx^3} dx$$

Optimal. Leaf size=53

$$\frac{2C \tan^{-1}\left(\frac{1 - \frac{2x}{\sqrt[3]{-a/b}}}{\sqrt{3}}\right)}{\sqrt{3}b} - \frac{C \log\left(\sqrt[3]{-\frac{a}{b}} + x\right)}{b}$$

[Out] (2*C*ArcTan[(1 - (2*x)/(-a/b))^(1/3))/Sqrt[3]]/(Sqrt[3]*b) - (C*Log[(-a/b))^(1/3) + x])/b

Rubi [A] time = 0.0808304, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1867, 31, 617, 204}

$$\frac{2C \tan^{-1}\left(\frac{1 - \frac{2x}{\sqrt[3]{-a/b}}}{\sqrt{3}}\right)}{\sqrt{3}b} - \frac{C \log\left(\sqrt[3]{-\frac{a}{b}} + x\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[(2*(-a/b))^(2/3)*C + C*x^2)/(a - b*x^3),x]

[Out] (2*C*ArcTan[(1 - (2*x)/(-a/b))^(1/3))/Sqrt[3]]/(Sqrt[3]*b) - (C*Log[(-a/b))^(1/3) + x])/b

Rule 1867

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[C/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{2\left(-\frac{a}{b}\right)^{2/3} C + Cx^2}{a - bx^3} dx &= -\frac{C \int \frac{1}{\sqrt[3]{-\frac{a}{b}+x}} dx}{b} - \frac{\left(\sqrt[3]{-\frac{a}{b}}C\right) \int \frac{1}{\left(-\frac{a}{b}\right)^{2/3} - \sqrt[3]{-\frac{a}{b}}x+x^2} dx}{b} \\ &= -\frac{C \log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b} - \frac{(2C) \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 - \frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)}{b} \\ &= \frac{2C \tan^{-1}\left(\frac{1 - \frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b} - \frac{C \log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b} \end{aligned}$$

Mathematica [B] time = 0.0587242, size = 150, normalized size = 2.83

$$\frac{C \left(b^{2/3} \left(-\frac{a}{b}\right)^{2/3} \log\left(a^{2/3} + \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right) - a^{2/3} \log\left(a - bx^3\right) - 2b^{2/3} \left(-\frac{a}{b}\right)^{2/3} \log\left(\sqrt[3]{a} - \sqrt[3]{bx}\right) + 2\sqrt{3}b^{2/3} \left(-\frac{a}{b}\right)^{2/3} \tan^{-1}\left(\frac{1 - \frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right) \right)}{3a^{2/3}b}$$

Antiderivative was successfully verified.

[In] Integrate[(2*(-(a/b))^(2/3)*C + C*x^2)/(a - b*x^3), x]

[Out] (C*(2*Sqrt[3]*(-(a/b))^(2/3)*b^(2/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*(-(a/b))^(2/3)*b^(2/3)*Log[a^(1/3) - b^(1/3)*x] + (-(a/b))^(2/3)*b^(2/3)*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - a^(2/3)*Log[a - b*

$$x^3]))/ (3*a^{(2/3)*b})$$

Maple [B] time = 0.007, size = 135, normalized size = 2.6

$$-\frac{2C}{3b} \left(-\frac{a}{b}\right)^{\frac{2}{3}} \ln\left(x - \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{C}{3b} \left(-\frac{a}{b}\right)^{\frac{2}{3}} \ln\left(x^2 + \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{2C\sqrt{3}}{3b} \left(-\frac{a}{b}\right)^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}}{3} \left(1 + 2x \frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*(-1/b*a)^(2/3)*C+C*x^2)/(-b*x^3+a),x)

[Out] -2/3*C*(-1/b*a)^(2/3)/b/(1/b*a)^(2/3)*ln(x-(1/b*a)^(1/3))+1/3*C*(-1/b*a)^(2/3)/b/(1/b*a)^(2/3)*ln(x^2+(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2/3*C*(-1/b*a)^(2/3)/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*(1+2/(1/b*a)^(1/3)*x)*3^(1/2))-1/3*C/b*ln(b*x^3-a)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*(-a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.03322, size = 140, normalized size = 2.64

$$\frac{2\sqrt{3}C \arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}} + \sqrt{3}a}{3a}\right) - 3C \log\left(x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*(-a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="fricas")

[Out] $1/3*(2*\sqrt{3}*C*\arctan(1/3*(2*\sqrt{3})*b*x*(-a/b)^{(2/3)} + \sqrt{3}*a)/a) - 3*C*\log(x + (-a/b)^{(1/3)))/b$

Sympy [C] time = 0.553843, size = 110, normalized size = 2.08

$$\frac{C \left(\log \left(-\frac{a}{b \left(-\frac{a}{b} \right)^{\frac{2}{3}}} + x \right) + \frac{\sqrt{3}i \log \left(\frac{a}{2b \left(-\frac{a}{b} \right)^{\frac{2}{3}}} - \frac{\sqrt{3}ia}{2b \left(-\frac{a}{b} \right)^{\frac{2}{3}}} + x \right)}{3} - \frac{\sqrt{3}i \log \left(\frac{a}{2b \left(-\frac{a}{b} \right)^{\frac{2}{3}}} + \frac{\sqrt{3}ia}{2b \left(-\frac{a}{b} \right)^{\frac{2}{3}}} + x \right)}{3} \right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*(-a/b)**(2/3)*C+C*x**2)/(-b*x**3+a), x)`

[Out] $-C*(\log(-a/(b*(-a/b)**(2/3))) + x) + \sqrt{3}*I*\log(a/(2*b*(-a/b)**(2/3))) - \sqrt{3}*I*a/(2*b*(-a/b)**(2/3)) + x)/3 - \sqrt{3}*I*\log(a/(2*b*(-a/b)**(2/3))) + \sqrt{3}*I*a/(2*b*(-a/b)**(2/3)) + x)/b$

Giac [B] time = 1.12249, size = 219, normalized size = 4.13

$$\frac{\sqrt{3}(ab^2 - \sqrt{3}\sqrt{a^2b^4}i)C \arctan \left(\frac{\sqrt{3} \left(2x + \left(\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{3ab^3} - \frac{\left(Cb^2 \left(\frac{a}{b} \right)^{\frac{2}{3}} + 2(-ab^2)^{\frac{2}{3}} C \right) \left(\frac{a}{b} \right)^{\frac{1}{3}} \log \left(\left| x - \left(\frac{a}{b} \right)^{\frac{1}{3}} \right| \right)}{3ab^2} - \frac{(3ab^2 - \sqrt{3}\sqrt{a^2b^4}i)C}{3ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*(-a/b)^(2/3)*C+C*x^2)/(-b*x^3+a), x, algorithm="giac")`

[Out] $-1/3*\sqrt{3}*(a*b^2 - \sqrt{3}*\sqrt{a^2*b^4}*i)*C*\arctan(1/3*\sqrt{3}*(2*x + (a/b)^{(1/3)})/(a/b)^{(1/3)})/(a*b^3) - 1/3*(C*b^2*(a/b)^{(2/3)} + 2*(-a*b^2)^{(2/3)}*C)*(a/b)^{(1/3)}*\log(\text{abs}(x - (a/b)^{(1/3)}))/(a*b^2) - 1/6*(3*a*b^2 - \sqrt{3}*\sqrt{a^2*b^4}*i)*C*\log(x^2 + x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(a*b^3)$

$$3.35 \quad \int \frac{2\left(-\frac{a}{b}\right)^{2/3} C + Cx^2}{a + bx^3} dx$$

Optimal. Leaf size=54

$$\frac{C \log\left(\sqrt[3]{-\frac{a}{b}} - x\right)}{b} - \frac{2C \tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}} + 1}{\sqrt{3}}\right)}{\sqrt{3}b}$$

[Out] $(-2*C*ArcTan[(1 + (2*x)/(-a/b))^{(1/3)})/Sqrt[3]]/(Sqrt[3]*b) + (C*Log[(-(a/b))^{(1/3)} - x])/b$

Rubi [A] time = 0.0598418, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1869, 31, 617, 204}

$$\frac{C \log\left(\sqrt[3]{-\frac{a}{b}} - x\right)}{b} - \frac{2C \tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}} + 1}{\sqrt{3}}\right)}{\sqrt{3}b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2*(-a/b))^{(2/3)}*C + C*x^2]/(a + b*x^3), x]$

[Out] $(-2*C*ArcTan[(1 + (2*x)/(-a/b))^{(1/3)})/Sqrt[3]]/(Sqrt[3]*b) + (C*Log[(-(a/b))^{(1/3)} - x])/b$

Rule 1869

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a/b)^(1/3)}, -Dist[C/b, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2), x], x]] /; EqQ[A + (-a/b)^(1/3)*B - 2*(-a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(1/3), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```


Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{2\left(-\frac{a}{b}\right)^{2/3} C + Cx^2}{a + bx^3} dx &= -\frac{C \int \frac{1}{\sqrt[3]{-\frac{a}{b}-x}} dx}{b} - \frac{\left(\sqrt[3]{-\frac{a}{b}} C\right) \int \frac{1}{\left(-\frac{a}{b}\right)^{2/3} + \sqrt[3]{-\frac{a}{b}} x + x^2} dx}{b} \\ &= \frac{C \log\left(\sqrt[3]{-\frac{a}{b}} - x\right)}{b} + \frac{(2C) \operatorname{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 + \frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)}{b} \\ &= -\frac{2C \tan^{-1}\left(\frac{1 + \frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b} + \frac{C \log\left(\sqrt[3]{-\frac{a}{b}} - x\right)}{b} \end{aligned}$$

Mathematica [B] time = 0.0465669, size = 149, normalized size = 2.76

$$\frac{C \left(-b^{2/3} \left(-\frac{a}{b}\right)^{2/3} \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right) + a^{2/3} \log\left(a + bx^3\right) + 2b^{2/3} \left(-\frac{a}{b}\right)^{2/3} \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right) - 2\sqrt{3}b^{2/3} \left(-\frac{a}{b}\right)^{2/3} \tan^{-1}\left(\frac{1 + \frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right) \right)}{3a^{2/3}b}$$

Antiderivative was successfully verified.

[In] Integrate[(2*(-(a/b))^(2/3)*C + C*x^2)/(a + b*x^3), x]

[Out] (C*(-2*Sqrt[3]*(-(a/b))^(2/3)*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(-(a/b))^(2/3)*b^(2/3)*Log[a^(1/3) + b^(1/3)*x] - (-(a/b))^(2/3)*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(2/3)*Log[a + b

$*x^3]))/(3*a^{(2/3)*b}$

Maple [B] time = 0.003, size = 132, normalized size = 2.4

$$\frac{2C}{3b} \left(-\frac{a}{b}\right)^{\frac{2}{3}} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{C}{3b} \left(-\frac{a}{b}\right)^{\frac{2}{3}} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{2C\sqrt{3}}{3b} \left(-\frac{a}{b}\right)^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}}{3} \left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*(-1/b*a)^(2/3)*C+C*x^2)/(b*x^3+a),x)

[Out] $\frac{2}{3}C(-1/b*a)^{(2/3)}/b/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})-1/3*C*(-1/b*a)^{(2/3)}/b/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})+2/3*C*(-1/b*a)^{(2/3)}/b/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))+1/3*C/b*\ln(b*x^3+a)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*(-a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 0.982751, size = 140, normalized size = 2.59

$$\frac{2\sqrt{3}C \arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}} - \sqrt{3}a}{3a}\right) + 3C \log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*(-a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="fricas")

[Out] $1/3*(2*\sqrt{3}*C*\arctan(1/3*(2*\sqrt{3})*b*x*(-a/b)^{(2/3)} - \sqrt{3}*a)/a) + 3*C*\log(x - (-a/b)^{(1/3)))/b$

Sympy [C] time = 0.523065, size = 109, normalized size = 2.02

$$C \left(\log \left(\frac{a}{b \left(-\frac{a}{b}\right)^{\frac{2}{3}}} + x \right) - \frac{\sqrt{3}i \log \left(-\frac{a}{2b \left(-\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\sqrt{3}ia}{2b \left(-\frac{a}{b}\right)^{\frac{2}{3}}} + x \right)}{3} + \frac{\sqrt{3}i \log \left(-\frac{a}{2b \left(-\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\sqrt{3}ia}{2b \left(-\frac{a}{b}\right)^{\frac{2}{3}}} + x \right)}{3} \right)$$

b

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*(-a/b)**(2/3)*C+C*x**2)/(b*x**3+a), x)`

[Out] $C*(\log(a/(b*(-a/b)**(2/3)) + x) - \sqrt{3}*I*\log(-a/(2*b*(-a/b)**(2/3)) - \sqrt{3}*I*a/(2*b*(-a/b)**(2/3)) + x)/3 + \sqrt{3}*I*\log(-a/(2*b*(-a/b)**(2/3)) + \sqrt{3}*I*a/(2*b*(-a/b)**(2/3)) + x)/3)/b$

Giac [A] time = 1.09283, size = 123, normalized size = 2.28

$$\frac{2\sqrt{3}C \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}} \right)}{3b} - \frac{\left(Cb^2 \left(-\frac{a}{b}\right)^{\frac{2}{3}} + 2 \left(-ab^2\right)^{\frac{2}{3}} C \right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log \left(\left| x - \left(-\frac{a}{b}\right)^{\frac{1}{3}} \right| \right)}{3ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*(-a/b)^(2/3)*C+C*x^2)/(b*x^3+a), x, algorithm="giac")`

[Out] $-2/3*\sqrt{3}*C*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)))/(-a/b)^{(1/3)))/b - 1/3*(C*b^2*(-a/b)^{(2/3)} + 2*(-a*b^2)^{(2/3)}*C)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)))/(a*b^2)$

$$3.36 \quad \int \frac{2\left(\frac{a}{b}\right)^{2/3} C + Cx^2}{a - bx^3} dx$$

Optimal. Leaf size=53

$$\frac{2C \tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{a}} + 1}{\sqrt[3]{b}}\right)}{\sqrt{3}b} - \frac{C \log\left(\sqrt[3]{\frac{a}{b}} - x\right)}{b}$$

[Out] (2*C*ArcTan[(1 + (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C*Log[(a/b)^(1/3) - x])/b

Rubi [A] time = 0.0582328, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1869, 31, 617, 204}

$$\frac{2C \tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{a}} + 1}{\sqrt[3]{b}}\right)}{\sqrt{3}b} - \frac{C \log\left(\sqrt[3]{\frac{a}{b}} - x\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[(2*(a/b)^(2/3)*C + C*x^2)/(a - b*x^3),x]

[Out] (2*C*ArcTan[(1 + (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C*Log[(a/b)^(1/3) - x])/b

Rule 1869

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a/b)^(1/3)}, -Dist[C/b, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2), x], x]] /; EqQ[A + (-a/b)^(1/3)*B - 2*(-a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3} C + Cx^2}{a - bx^3} dx = \frac{C \int \frac{1}{\sqrt[3]{\frac{a}{b}-x}} dx}{b} + \frac{\left(\sqrt[3]{\frac{a}{b}}C\right) \int \frac{1}{\left(\frac{a}{b}\right)^{2/3} + \sqrt[3]{\frac{a}{b}}x + x^2} dx}{b}$$

$$= -\frac{C \log\left(\sqrt[3]{\frac{a}{b}} - x\right)}{b} - \frac{(2C) \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 + \frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)}{b}$$

$$= \frac{2C \tan^{-1}\left(\frac{1 + \frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b} - \frac{C \log\left(\sqrt[3]{\frac{a}{b}} - x\right)}{b}$$

Mathematica [B] time = 0.0430835, size = 147, normalized size = 2.77

$$\frac{C \left(b^{2/3} \left(\frac{a}{b}\right)^{2/3} \log\left(a^{2/3} + \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right) - a^{2/3} \log(a - bx^3) - 2b^{2/3} \left(\frac{a}{b}\right)^{2/3} \log\left(\sqrt[3]{a} - \sqrt[3]{bx}\right) + 2\sqrt{3}b^{2/3} \left(\frac{a}{b}\right)^{2/3} \tan^{-1}\left(\frac{2\sqrt[3]{\frac{a}{b}}x + 1}{\sqrt{3}}\right) \right)}{3a^{2/3}b}$$

Antiderivative was successfully verified.

[In] Integrate[(2*(a/b)^(2/3)*C + C*x^2)/(a - b*x^3), x]

[Out] (C*(2*Sqrt[3]*(a/b)^(2/3)*b^(2/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3)]/Sqrt[3]] - 2*(a/b)^(2/3)*b^(2/3)*Log[a^(1/3) - b^(1/3)*x] + (a/b)^(2/3)*b^(2/3)*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - a^(2/3)*Log[a - b*x^3]))/(3

$*a^{(2/3)*b}$

Maple [A] time = 0.005, size = 90, normalized size = 1.7

$$-\frac{2C}{3b} \ln\left(x - \sqrt[3]{\frac{a}{b}}\right) + \frac{C}{3b} \ln\left(x^2 + \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) + \frac{2C\sqrt{3}}{3b} \arctan\left(\frac{\sqrt{3}}{3} \left(1 + 2x \frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\right) - \frac{C \ln(bx^3 - a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*(1/b*a)^(2/3)*C+C*x^2)/(-b*x^3+a),x)`

[Out] $-2/3*C/b*\ln(x-(1/b*a)^{(1/3)})+1/3*C/b*\ln(x^2+(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})+2/3*C*\arctan(1/3*(1+2/(1/b*a)^{(1/3)}*x)*3^{(1/2)})/b*3^{(1/2)}-1/3*C/b*\ln(b*x^3-a)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*(a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 1.0441, size = 138, normalized size = 2.6

$$\frac{2\sqrt{3}C \arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}} + \sqrt{3}a}{3a}\right) - 3C \log\left(x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*(a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="fricas")`

[Out] $\frac{1}{3} \cdot (2 \cdot \sqrt{3} \cdot C \cdot \arctan(\frac{1}{3} \cdot (2 \cdot \sqrt{3}) \cdot b \cdot x \cdot (a/b)^{2/3} + \sqrt{3} \cdot a) / a) - 3 \cdot C \cdot \log(x - (a/b)^{1/3}) / b$

Sympy [C] time = 0.622831, size = 102, normalized size = 1.92

$$\frac{C \left(\log \left(-\frac{a}{b \left(\frac{a}{b} \right)^{2/3}} + x \right) + \frac{\sqrt{3} i \log \left(\frac{\frac{a}{2b \left(\frac{a}{b} \right)^{2/3}} - \frac{\sqrt{3} i a}{2b \left(\frac{a}{b} \right)^{2/3}} + x}{3} \right) - \frac{\sqrt{3} i \log \left(\frac{\frac{a}{2b \left(\frac{a}{b} \right)^{2/3}} + \frac{\sqrt{3} i a}{2b \left(\frac{a}{b} \right)^{2/3}} + x}{3} \right)}{3} \right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*(a/b)**(2/3)*C+C*x**2)/(-b*x**3+a), x)`

[Out] $-C \cdot (\log(-a / (b \cdot (a/b)^{2/3})) + x) + \sqrt{3} \cdot I \cdot \log(a / (2 \cdot b \cdot (a/b)^{2/3})) - \sqrt{3} \cdot I \cdot a / (2 \cdot b \cdot (a/b)^{2/3} + x) / 3 - \sqrt{3} \cdot I \cdot \log(a / (2 \cdot b \cdot (a/b)^{2/3})) + \sqrt{3} \cdot I \cdot a / (2 \cdot b \cdot (a/b)^{2/3} + x) / 3) / b$

Giac [A] time = 1.08276, size = 115, normalized size = 2.17

$$\frac{2 \sqrt{3} C \arctan \left(\frac{\sqrt{3} \left(2x + \left(\frac{a}{b} \right)^{1/3} \right)}{3 \left(\frac{a}{b} \right)^{1/3}} \right)}{3b} - \frac{\left(C b^2 \left(\frac{a}{b} \right)^{2/3} + 2 \left(a b^2 \right)^{2/3} C \right) \left(\frac{a}{b} \right)^{1/3} \log \left(\left| x - \left(\frac{a}{b} \right)^{1/3} \right| \right)}{3 a b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*(a/b)^(2/3)*C+C*x^2)/(-b*x^3+a), x, algorithm="giac")`

[Out] $\frac{2}{3} \cdot \sqrt{3} \cdot C \cdot \arctan(\frac{1}{3} \cdot \sqrt{3} \cdot (2 \cdot x + (a/b)^{1/3}) / (a/b)^{1/3}) / b - \frac{1}{3} \cdot (C \cdot b^2 \cdot (a/b)^{2/3} + 2 \cdot (a \cdot b^2)^{2/3} \cdot C) \cdot (a/b)^{1/3} \cdot \log(\text{abs}(x - (a/b)^{1/3})) / (a \cdot b^2)$

$$3.37 \quad \int \frac{2a^{2/3}C + b^{2/3}Cx^2}{a + bx^3} dx$$

Optimal. Leaf size=61

$$\frac{C \log(\sqrt[3]{a} + \sqrt[3]{bx})}{\sqrt[3]{b}} - \frac{2C \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

[Out] $(-2*C*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*b^{(1/3)})$
 $+ (C*Log[a^{(1/3)} + b^{(1/3)}*x])/b^{(1/3)}$

Rubi [A] time = 0.0412275, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1863, 31, 617, 204}

$$\frac{C \log(\sqrt[3]{a} + \sqrt[3]{bx})}{\sqrt[3]{b}} - \frac{2C \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2*a^{(2/3)}*C + b^{(2/3)}*C*x^2)/(a + b*x^3), x]$

[Out] $(-2*C*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*b^{(1/3)})$
 $+ (C*Log[a^{(1/3)} + b^{(1/3)}*x])/b^{(1/3)}$

Rule 1863

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Dist[C/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617


```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{2a^{2/3}C + b^{2/3}Cx^2}{a + bx^3} dx &= \frac{(\sqrt[3]{a}C) \int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{ax}}{\sqrt[3]{b}} + x^2} dx}{b^{2/3}} + \frac{C \int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}} + x} dx}{\sqrt[3]{b}} \\ &= \frac{C \log(\sqrt[3]{a} + \sqrt[3]{bx})}{\sqrt[3]{b}} + \frac{(2C) \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 - \frac{2\sqrt[3]{bx}}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}} \\ &= -\frac{2C \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{b}} + \frac{C \log(\sqrt[3]{a} + \sqrt[3]{bx})}{\sqrt[3]{b}} \end{aligned}$$

Mathematica [A] time = 0.0173002, size = 95, normalized size = 1.56

$$\frac{C \left(-\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2) + \log(a + bx^3) + 2 \log(\sqrt[3]{a} + \sqrt[3]{bx}) - 2\sqrt{3} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) \right)}{3\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a^(2/3)*C + b^(2/3)*C*x^2)/(a + b*x^3), x]
```

```
[Out] (C*(-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*Log[a^(1/3)
+ b^(1/3)*x] - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + Log[a + b*x
^3]))/(3*b^(1/3))
```

Maple [B] time = 0.005, size = 117, normalized size = 1.9

$$\frac{2C}{3b}a^{\frac{2}{3}}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{C}{3b}a^{\frac{2}{3}}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{2C\sqrt{3}}{3b}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{C\ln\left(\frac{a}{b}\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*a^(2/3)*C+b^(2/3)*C*x^2)/(b*x^3+a),x)

[Out] 2/3*C*a^(2/3)/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/3*C*a^(2/3)/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2/3*C*a^(2/3)/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/3*C/b^(1/3)*ln(b*x^3+a)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a^(2/3)*C+b^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.05073, size = 463, normalized size = 7.59

$$\left[\frac{\sqrt{\frac{1}{3}}Cb\sqrt{-\frac{1}{2}\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{2bx^3-3a^{\frac{2}{3}}b^{\frac{1}{3}}x+3\sqrt{\frac{1}{3}}\left(2a^{\frac{1}{3}}bx^2+a^{\frac{2}{3}}b^{\frac{2}{3}}x-ab^{\frac{1}{3}}\right)\sqrt{-\frac{1}{2}\frac{1}{b^{\frac{2}{3}}}-a}}{bx^3+a}}\right)+Cb^{\frac{2}{3}}\log\left(bx+a^{\frac{1}{3}}b^{\frac{2}{3}}\right)}{b}, \frac{2\sqrt{\frac{1}{3}}Cb^{\frac{2}{3}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2a^{\frac{2}{3}}b^{\frac{2}{3}}x-ab^{\frac{1}{3}}\right)}{ab^{\frac{1}{3}}}\right)}{b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a^(2/3)*C+b^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="fricas")

```
[Out] [(sqrt(1/3)*C*b*sqrt(-1/b^(2/3))*log((2*b*x^3 - 3*a^(2/3)*b^(1/3)*x + 3*sqrt(1/3)*(2*a^(1/3)*b*x^2 + a^(2/3)*b^(2/3)*x - a*b^(1/3))*sqrt(-1/b^(2/3)) - a)/(b*x^3 + a) + C*b^(2/3)*log(b*x + a^(1/3)*b^(2/3)))/b, (2*sqrt(1/3)*C*b^(2/3)*arctan(sqrt(1/3)*(2*a^(2/3)*b^(2/3)*x - a*b^(1/3))/(a*b^(1/3))) + C*b^(2/3)*log(b*x + a^(1/3)*b^(2/3)))/b]
```

Sympy [A] time = 0.668899, size = 70, normalized size = 1.15

$$\text{RootSum}\left(3t^3b^{\frac{5}{3}} - 3t^2Cb^{\frac{4}{3}} + tC^2b - C^3b^{\frac{2}{3}}, \left(t \mapsto t \log\left(x + \frac{3t\sqrt[3]{a}\sqrt[3]{b} - C\sqrt[3]{a}}{2C\sqrt[3]{b}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a**(2/3)*C+b**(2/3)*C*x**2)/(b*x**3+a), x)
```

```
[Out] RootSum(3*_t**3*b**(5/3) - 3*_t**2*C*b**(4/3) + _t*C**2*b - C**3*b**(2/3),
Lambda(_t, _t*log(x + (3*_t*a**(1/3)*b**(1/3) - C*a**(1/3))/(2*C*b**(1/3))))
))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a^(2/3)*C+b^(2/3)*C*x^2)/(b*x^3+a), x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.38 \quad \int \frac{-2a^{2/3}C - (-b)^{2/3}Cx^2}{a+bx^3} dx$$

Optimal. Leaf size=70

$$\frac{C \log(\sqrt[3]{a} - \sqrt[3]{-bx})}{\sqrt[3]{-b}} - \frac{2C \tan^{-1}\left(\frac{\sqrt[3]{a+2\sqrt[3]{-bx}}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{-b}}$$

[Out] $(-2*C*ArcTan[(a^{(1/3)} + 2*(-b)^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*(-b)^{(1/3)}) + (C*Log[a^{(1/3)} - (-b)^{(1/3)}*x])/(-b)^{(1/3)}$

Rubi [A] time = 0.0718664, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1866, 31, 617, 204}

$$\frac{C \log(\sqrt[3]{a} - \sqrt[3]{-bx})}{\sqrt[3]{-b}} - \frac{2C \tan^{-1}\left(\frac{\sqrt[3]{a+2\sqrt[3]{-bx}}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{-b}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-2*a^{(2/3)}*C - (-b)^{(2/3)}*C*x^2)/(a + b*x^3), x]$

[Out] $(-2*C*ArcTan[(a^{(1/3)} + 2*(-b)^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*(-b)^{(1/3)}) + (C*Log[a^{(1/3)} - (-b)^{(1/3)}*x])/(-b)^{(1/3)}$

Rule 1866

$\text{Int}[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] \rightarrow \text{With}[\{A = \text{Coeff}[P2, x, 0], B = \text{Coeff}[P2, x, 1], C = \text{Coeff}[P2, x, 2]\}, \text{With}[\{q = a^{(1/3)}/(-b)^{(1/3)}\}, -\text{Dist}[C/b, \text{Int}[1/(q - x), x], x] + \text{Dist}[(B - C*q)/b, \text{Int}[1/(q^2 + q*x + x^2), x], x]] \;/; \text{EqQ}[A*(-b)^{(2/3)} + a^{(1/3)}*(-b)^{(1/3)}*B - 2*a^{(2/3)}*C, 0]] \;/; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[P2, x, 2]$

Rule 31

$\text{Int}(((a_) + (b_)*(x_))^{(-1)}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] \;/; \text{FreeQ}[\{a, b\}, x]$

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{-2a^{2/3}C - (-b)^{2/3}Cx^2}{a + bx^3} dx &= \frac{(\sqrt[3]{a}C) \int \frac{1}{\frac{a^{2/3}}{(-b)^{2/3} + \sqrt[3]{-b}} + x^2} dx}{(-b)^{2/3}} - C \int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{-b}} - x} dx \\ &= \frac{C \log(\sqrt[3]{a} - \sqrt[3]{-bx})}{\sqrt[3]{-b}} + \frac{(2C) \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 + \frac{2\sqrt[3]{-bx}}{\sqrt[3]{a}}\right)}{\sqrt[3]{-b}} \\ &= -\frac{2C \tan^{-1}\left(\frac{\sqrt[3]{a+2\sqrt[3]{-bx}}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{-b}} + \frac{C \log(\sqrt[3]{a} - \sqrt[3]{-bx})}{\sqrt[3]{-b}} \end{aligned}$$

Mathematica [A] time = 0.0283125, size = 116, normalized size = 1.66

$$\frac{C \left(-b^{2/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2) + 2b^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{bx}) - 2\sqrt{3}b^{2/3} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + (-b)^{2/3} \log(a + bx^3) \right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*a^(2/3)*C - (-b)^(2/3)*C*x^2)/(a + b*x^3), x]
```

```
[Out] -(C*(-2*Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*b^(
2/3)*Log[a^(1/3) + b^(1/3)*x] - b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2] + (-b)^(2/3)*Log[a + b*x^3])/(3*b)
```

Maple [B] time = 0.007, size = 122, normalized size = 1.7

$$-\frac{2C}{3b}a^{\frac{2}{3}}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{C}{3b}a^{\frac{2}{3}}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{2C\sqrt{3}}{3b}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{C\ln}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2*a^(2/3)*C-(-b)^(2/3)*C*x^2)/(b*x^3+a),x)

[Out] -2/3*C*a^(2/3)/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+1/3*C*a^(2/3)/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-2/3*C*a^(2/3)/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/3*C*(-b)^(2/3)/b*ln(b*x^3+a)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*a^(2/3)*C-(-b)^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.08685, size = 539, normalized size = 7.7

$$\left[\frac{\sqrt{\frac{1}{3}}Cb\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(\frac{2bx^3+3a^{\frac{2}{3}}(-b)^{\frac{1}{3}}x-3\sqrt{\frac{1}{3}}\left(2a^{\frac{1}{3}}bx^2+a^{\frac{2}{3}}(-b)^{\frac{2}{3}}x+a(-b)^{\frac{1}{3}}\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}-a}{bx^3+a}}\right)}{b}, -C(-b)^{\frac{2}{3}}\log\left(bx+a^{\frac{1}{3}}(-b)^{\frac{2}{3}}\right), 2\sqrt{\frac{1}{3}}Cb\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*a^(2/3)*C-(-b)^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="fricas")

```
[Out] [(sqrt(1/3)*C*b*sqrt((-b)^(1/3)/b)*log((2*b*x^3 + 3*a^(2/3)*(-b)^(1/3)*x -
3*sqrt(1/3)*(2*a^(1/3)*b*x^2 + a^(2/3)*(-b)^(2/3)*x + a*(-b)^(1/3))*sqrt((-
b)^(1/3)/b) - a)/(b*x^3 + a)) - C*(-b)^(2/3)*log(b*x + a^(1/3)*(-b)^(2/3))
/b, -(2*sqrt(1/3)*C*b*sqrt(-(-b)^(1/3)/b)*arctan(sqrt(1/3)*(2*a^(2/3)*(-b)^(
2/3)*x + a*(-b)^(1/3))*sqrt(-(-b)^(1/3)/b)/a) + C*(-b)^(2/3)*log(b*x + a^(
1/3)*(-b)^(2/3)))/b]
```

Sympy [A] time = 0.765978, size = 73, normalized size = 1.04

$$-\text{RootSum}\left(3t^3b^2 - 3t^2Cb(-b)^{\frac{2}{3}} + tC^2(-b)^{\frac{4}{3}} - C^3b, \left(t \mapsto t \log\left(\frac{3t\sqrt[3]{a}}{2C} - \frac{\sqrt[3]{a}(-b)^{\frac{2}{3}}}{2b} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a**(2/3)*C-(-b)**(2/3)*C*x**2)/(b*x**3+a), x)
```

```
[Out] -RootSum(3*_t**3*b**2 - 3*_t**2*C*b*(-b)**(2/3) + _t*C**2*(-b)**(4/3) - C**
3*b, Lambda(_t, _t*log(3*_t*a**(1/3)/(2*C) - a**(1/3)*(-b)**(2/3)/(2*b) + x
)))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a^(2/3)*C-(-b)^(2/3)*C*x^2)/(b*x^3+a), x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.39 \quad \int \frac{-3+x^2}{-1+x^3} dx$$

Optimal. Leaf size=40

$$\frac{5}{6} \log(x^2 + x + 1) - \frac{2}{3} \log(1 - x) + \sqrt{3} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

[Out] Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - (2*Log[1 - x])/3 + (5*Log[1 + x + x^2])/6

Rubi [A] time = 0.0301783, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {1875, 31, 634, 618, 204, 628}

$$\frac{5}{6} \log(x^2 + x + 1) - \frac{2}{3} \log(1 - x) + \sqrt{3} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] Int[(-3 + x^2)/(-1 + x^3), x]

[Out] Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - (2*Log[1 - x])/3 + (5*Log[1 + x + x^2])/6

Rule 1875

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = -(a/b)^(1/3)}, Dist[(q*(A + B*q + C*q^2))/(3*a), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q - C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 - b*A^3, 0] && NeQ[A + B*q + C*q^2, 0] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2] && LtQ[a/b, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(1-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```


`t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rule 618

`Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

Rule 204

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

Rule 628

`Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`

Rubi steps

$$\begin{aligned}
 \int \frac{-3 + x^2}{-1 + x^3} dx &= -\left(\frac{1}{3} \int \frac{-7 - 5x}{1 + x + x^2} dx\right) + \frac{2}{3} \int \frac{1}{1 - x} dx \\
 &= -\frac{2}{3} \log(1 - x) + \frac{5}{6} \int \frac{1 + 2x}{1 + x + x^2} dx + \frac{3}{2} \int \frac{1}{1 + x + x^2} dx \\
 &= -\frac{2}{3} \log(1 - x) + \frac{5}{6} \log(1 + x + x^2) - 3 \operatorname{Subst}\left(\int \frac{1}{-3 - x^2} dx, x, 1 + 2x\right) \\
 &= \sqrt{3} \tan^{-1}\left(\frac{1 + 2x}{\sqrt{3}}\right) - \frac{2}{3} \log(1 - x) + \frac{5}{6} \log(1 + x + x^2)
 \end{aligned}$$

Mathematica [A] time = 0.0098046, size = 50, normalized size = 1.25

$$\frac{1}{2} \log(x^2 + x + 1) + \frac{1}{3} \log(1 - x^3) - \log(1 - x) + \sqrt{3} \tan^{-1}\left(\frac{2x + 1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(-3 + x^2)/(-1 + x^3), x]

[Out] $\text{Sqrt}[3] \cdot \text{ArcTan}[(1 + 2x)/\text{Sqrt}[3]] - \text{Log}[1 - x] + \text{Log}[1 + x + x^2]/2 + \text{Log}[1 - x^3]/3$

Maple [A] time = 0.005, size = 32, normalized size = 0.8

$$-\frac{2 \ln(-1+x)}{3} + \frac{5 \ln(x^2+x+1)}{6} + \arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^2-3)/(x^3-1), x)$

[Out] $-2/3 \cdot \ln(-1+x) + 5/6 \cdot \ln(x^2+x+1) + \arctan(1/3 \cdot (1+2x) \cdot 3^{(1/2)}) \cdot 3^{(1/2)}$

Maxima [A] time = 1.45667, size = 42, normalized size = 1.05

$$\sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x+1)\right) + \frac{5}{6} \log(x^2+x+1) - \frac{2}{3} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((x^2-3)/(x^3-1), x, \text{algorithm}="maxima")$

[Out] $\text{sqrt}(3) \cdot \arctan(1/3 \cdot \text{sqrt}(3) \cdot (2x+1)) + 5/6 \cdot \log(x^2+x+1) - 2/3 \cdot \log(x-1)$

Fricas [A] time = 1.00098, size = 107, normalized size = 2.68

$$\sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x+1)\right) + \frac{5}{6} \log(x^2+x+1) - \frac{2}{3} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((x^2-3)/(x^3-1), x, \text{algorithm}="fricas")$

[Out] $\text{sqrt}(3) \cdot \arctan(1/3 \cdot \text{sqrt}(3) \cdot (2x+1)) + 5/6 \cdot \log(x^2+x+1) - 2/3 \cdot \log(x-1)$

Sympy [A] time = 0.123212, size = 42, normalized size = 1.05

$$-\frac{2 \log(x-1)}{3} + \frac{5 \log(x^2+x+1)}{6} + \sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2-3)/(x**3-1),x)

[Out] -2*log(x - 1)/3 + 5*log(x**2 + x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)

Giac [A] time = 1.20438, size = 43, normalized size = 1.08

$$\sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x+1)\right) + \frac{5}{6} \log(x^2+x+1) - \frac{2}{3} \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-3)/(x^3-1),x, algorithm="giac")

[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/6*log(x^2 + x + 1) - 2/3*log(abs(x - 1))

$$3.40 \quad \int \frac{\sqrt[3]{a} \sqrt[3]{b} B + 2a^{2/3} C + b^{2/3} Bx + b^{2/3} Cx^2}{a + bx^3} dx$$

Optimal. Leaf size=70

$$\frac{C \log(\sqrt[3]{a} + \sqrt[3]{bx})}{\sqrt[3]{b}} - \frac{2 \left(\frac{B}{\sqrt[3]{a}} + \frac{C}{\sqrt[3]{b}} \right) \tan^{-1} \left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3} \sqrt[3]{a}} \right)}{\sqrt{3}}$$

[Out] $(-2*(B/a^{(1/3)} + C/b^{(1/3)})*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/Sqrt[3] + (C*Log[a^{(1/3)} + b^{(1/3)}*x])/b^{(1/3)}$

Rubi [A] time = 0.0665512, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.082$, Rules used = {1863, 31, 617, 204}

$$\frac{C \log(\sqrt[3]{a} + \sqrt[3]{bx})}{\sqrt[3]{b}} - \frac{2 \left(\frac{B}{\sqrt[3]{a}} + \frac{C}{\sqrt[3]{b}} \right) \tan^{-1} \left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3} \sqrt[3]{a}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a^{(1/3)}*b^{(1/3)}*B + 2*a^{(2/3)}*C + b^{(2/3)}*B*x + b^{(2/3)}*C*x^2)/(a + b*x^3), x]$

[Out] $(-2*(B/a^{(1/3)} + C/b^{(1/3)})*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/Sqrt[3] + (C*Log[a^{(1/3)} + b^{(1/3)}*x])/b^{(1/3)}$

Rule 1863

$\text{Int}[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] \rightarrow \text{With}[\{A = \text{Coeff}[P2, x, 0], B = \text{Coeff}[P2, x, 1], C = \text{Coeff}[P2, x, 2]\}, \text{With}[\{q = a^{(1/3)}/b^{(1/3)}\}, \text{Dist}[C/b, \text{Int}[1/(q + x), x], x] + \text{Dist}[(B + C*q)/b, \text{Int}[1/(q^2 - q*x + x^2), x], x]] /; \text{EqQ}[A*b^{(2/3)} - a^{(1/3)}*b^{(1/3)}*B - 2*a^{(2/3)}*C, 0]] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[P2, x, 2]$

Rule 31

$\text{Int}[(a_) + (b_)*(x_)^{(-1)}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}[\{a, b\}, x]$

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
 implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B + 2a^{2/3}C + b^{2/3}Bx + b^{2/3}Cx^2}{a + bx^3} dx = \frac{C \int \frac{1}{\sqrt[3]{a} + x} dx}{\sqrt[3]{b}} + \frac{(\sqrt[3]{b}B + \sqrt[3]{a}C) \int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{ax}}{\sqrt[3]{b}} + x^2} dx}{b^{2/3}}$$

$$= \frac{C \log(\sqrt[3]{a} + \sqrt[3]{bx})}{\sqrt[3]{b}} + \left(2 \left(\frac{B}{\sqrt[3]{a}} + \frac{C}{\sqrt[3]{b}}\right)\right) \text{Subst} \left(\int \frac{1}{-3 - x^2} dx, x, 1 - \frac{2\sqrt[3]{bx}}{\sqrt[3]{a}} \right)$$

$$= -\frac{2 \left(\frac{B}{\sqrt[3]{a}} + \frac{C}{\sqrt[3]{b}}\right) \tan^{-1} \left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}} \right)}{\sqrt{3}} + \frac{C \log(\sqrt[3]{a} + \sqrt[3]{bx})}{\sqrt[3]{b}}$$

Mathematica [A] time = 0.0464287, size = 122, normalized size = 1.74

$$\frac{\sqrt[3]{a}C \left(-\log \left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2 \right) + \log \left(a + bx^3 \right) + 2 \log \left(\sqrt[3]{a} + \sqrt[3]{bx} \right) - 2\sqrt{3} \left(\sqrt[3]{a}C + \sqrt[3]{b}B \right) \tan^{-1} \left(\frac{1 - \frac{2\sqrt[3]{bx}}{\sqrt[3]{a}}}{\sqrt{3}} \right) \right)}{3\sqrt[3]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(1/3)*b^(1/3)*B + 2*a^(2/3)*C + b^(2/3)*B*x + b^(2/3)*C*x^2)/(
a + b*x^3), x]
```

```
[Out] (-2*Sqrt[3]*(b^(1/3)*B + a^(1/3)*C)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]] + a^(1/3)*C*(2*Log[a^(1/3) + b^(1/3)*x] - Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2] + Log[a + b*x^3]))/(3*a^(1/3)*b^(1/3))
```

Maple [B] time = 0.004, size = 310, normalized size = 4.4

$$\frac{B}{3} \sqrt[3]{a} \ln \left(x + \sqrt[3]{\frac{a}{b}} \right) b^{-\frac{2}{3}} \left(\frac{a}{b} \right)^{-\frac{2}{3}} + \frac{2C}{3b} a^{\frac{2}{3}} \ln \left(x + \sqrt[3]{\frac{a}{b}} \right) \left(\frac{a}{b} \right)^{-\frac{2}{3}} - \frac{B}{6} \sqrt[3]{a} \ln \left(x^2 - \sqrt[3]{\frac{a}{b}} x + \left(\frac{a}{b} \right)^{\frac{2}{3}} \right) b^{-\frac{2}{3}} \left(\frac{a}{b} \right)^{-\frac{2}{3}} - \frac{C}{3b} a^{\frac{2}{3}} \ln \left(x^2 - \sqrt[3]{\frac{a}{b}} x + \left(\frac{a}{b} \right)^{\frac{2}{3}} \right) b^{-\frac{2}{3}} \left(\frac{a}{b} \right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^(1/3)*b^(1/3)*B+2*a^(2/3)*C+b^(2/3)*B*x+b^(2/3)*C*x^2)/(b*x^3+a), x)

[Out] 1/3*B/b^(2/3)*a^(1/3)/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+2/3*C*a^(2/3)/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/6*B/b^(2/3)*a^(1/3)/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-1/3*C*a^(2/3)/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3*B/b^(2/3)*a^(1/3)/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+2/3*C*a^(2/3)/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/3*B/b^(1/3)/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/6*B/b^(1/3)/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3*B/b^(1/3)*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/3*C/b^(1/3)*ln(b*x^3+a)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(1/3)*b^(1/3)*B+2*a^(2/3)*C+b^(2/3)*B*x+b^(2/3)*C*x^2)/(b*x^3+a), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 11.3042, size = 1058, normalized size = 15.11

$$\left[\sqrt{\frac{1}{3}} b \sqrt{\frac{C^2 a b^{\frac{1}{3}} + 2 B C a^{\frac{2}{3}} b^{\frac{2}{3}} + B^2 a^{\frac{1}{3}} b}{ab}} \log \left(\frac{C^3 a^2 + B^3 ab - 2(C^3 ab + B^3 b^2)x^3 + 3(C^3 a + B^3 b)a^{\frac{2}{3}} b^{\frac{1}{3}} x - 3 \sqrt{\frac{1}{3}} \left((2 B^2 b x^2 + C^2 a x + B C a) a^{\frac{2}{3}} b^{\frac{2}{3}} + (2 C^2 a b x^2 - B C a b x) \right)}{bx^3 + a}} \right) \right] b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(1/3)*b^(1/3)*B+2*a^(2/3)*C+b^(2/3)*B*x+b^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="fricas")

[Out] [(sqrt(1/3)*b*sqrt(-(C^2*a*b^(1/3) + 2*B*C*a^(2/3)*b^(2/3) + B^2*a^(1/3)*b)/(a*b))*log(-(C^3*a^2 + B^3*a*b - 2*(C^3*a*b + B^3*b^2)*x^3 + 3*(C^3*a + B^3*b)*a^(2/3)*b^(1/3)*x - 3*sqrt(1/3)*((2*B^2*b*x^2 + C^2*a*x + B*C*a)*a^(2/3)*b^(2/3) + (2*C^2*a*b*x^2 - B*C*a*b*x - B^2*a*b)*a^(1/3) - (2*B*C*a*b*x^2 - B^2*a*b*x + C^2*a^2)*b^(1/3)))*sqrt(-(C^2*a*b^(1/3) + 2*B*C*a^(2/3)*b^(2/3) + B^2*a^(1/3)*b)/(a*b)))/(b*x^3 + a)) + C*b^(2/3)*log(b*x + a^(1/3)*b^(2/3)))/b, (2*sqrt(1/3)*b*sqrt((C^2*a*b^(1/3) + 2*B*C*a^(2/3)*b^(2/3) + B^2*a^(1/3)*b)/(a*b))*arctan(sqrt(1/3)*((2*C^2*x + B*C)*a^(2/3)*b^(2/3) - (2*B*C*b*x + B^2*b)*a^(1/3) + (2*B^2*b*x - C^2*a)*b^(1/3)))*sqrt((C^2*a*b^(1/3) + 2*B*C*a^(2/3)*b^(2/3) + B^2*a^(1/3)*b)/(a*b)))/(C^3*a + B^3*b)) + C*b^(2/3)*log(b*x + a^(1/3)*b^(2/3)))/b]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**(1/3)*b**(1/3)*B+2*a**(2/3)*C+b**(2/3)*B*x+b**(2/3)*C*x**2)/(b*x**3+a),x)

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*b^(1/3)*B+2*a^(2/3)*C+b^(2/3)*B*x+b^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] Timed out
```


$$3.41 \quad \int \frac{\sqrt[3]{a} \sqrt[3]{-b} B - 2a^{2/3} C - (-b)^{2/3} Bx - (-b)^{2/3} Cx^2}{a + bx^3} dx$$

Optimal. Leaf size=88

$$\frac{2(\sqrt[3]{a}(-b)^{2/3}C + bB) \tan^{-1}\left(\frac{\sqrt[3]{a+2\sqrt[3]{-b}x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{ab}} + \frac{C \log(\sqrt[3]{a} - \sqrt[3]{-bx})}{\sqrt[3]{-b}}$$

[Out] (2*(b*B + a^(1/3)*(-b)^(2/3)*C)*ArcTan[(a^(1/3) + 2*(-b)^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3)*b) + (C*Log[a^(1/3) - (-b)^(1/3)*x])/(-b)^(1/3)

Rubi [A] time = 0.112343, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 57, $\frac{\text{number of rules}}{\text{integrand size}} = 0.07$, Rules used = {1866, 31, 617, 204}

$$\frac{2(\sqrt[3]{a}(-b)^{2/3}C + bB) \tan^{-1}\left(\frac{\sqrt[3]{a+2\sqrt[3]{-b}x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{ab}} + \frac{C \log(\sqrt[3]{a} - \sqrt[3]{-bx})}{\sqrt[3]{-b}}$$

Antiderivative was successfully verified.

[In] Int[(a^(1/3)*(-b)^(1/3)*B - 2*a^(2/3)*C - (-b)^(2/3)*B*x - (-b)^(2/3)*C*x^2)/(a + b*x^3), x]

[Out] (2*(b*B + a^(1/3)*(-b)^(2/3)*C)*ArcTan[(a^(1/3) + 2*(-b)^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3)*b) + (C*Log[a^(1/3) - (-b)^(1/3)*x])/(-b)^(1/3)

Rule 1866

Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/(-b)^(1/3)}, -Dist[C/b, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2), x], x]] /; EqQ[A*(-b)^(2/3) + a^(1/3)*(-b)^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B - 2a^{2/3}C - (-b)^{2/3}Bx - (-b)^{2/3}Cx^2}{a + bx^3} dx = -\frac{C \int \frac{1}{\sqrt[3]{a}-x} dx}{\sqrt[3]{-b}} + \frac{(\sqrt[3]{-b}B - \sqrt[3]{a}C) \int \frac{1}{\frac{a^{2/3}}{(-b)^{2/3}} + \frac{\sqrt[3]{ax}}{\sqrt[3]{-b}} + x^2} dx}{(-b)^{2/3}}$$

$$= \frac{C \log(\sqrt[3]{a} - \sqrt[3]{-b}x)}{\sqrt[3]{-b}} - \left(2 \left(\frac{B}{\sqrt[3]{a}} + \frac{bC}{(-b)^{4/3}}\right)\right) \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x\right)$$

$$= \frac{2 \left(\frac{B}{\sqrt[3]{a}} + \frac{bC}{(-b)^{4/3}}\right) \tan^{-1}\left(\frac{\sqrt[3]{a} + 2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}} + \frac{C \log(\sqrt[3]{a} - \sqrt[3]{-b}x)}{\sqrt[3]{-b}}$$

Mathematica [B] time = 0.526485, size = 238, normalized size = 2.7

$$\frac{(2\sqrt[3]{ab}\sqrt[3]{-b}C + b^{5/3}B + (-b)^{5/3}B) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2) - 2b(2\sqrt[3]{a}\sqrt[3]{-b}C + (b^{2/3} - (-b)^{2/3})B) \log(\sqrt[3]{a} + \sqrt[3]{bx}) - 2\sqrt[3]{a}(-b)^{2/3}\sqrt[3]{-b}^2C \log(a + bx^3)}{\sqrt[3]{-b^2}} + 2\sqrt{3}\sqrt[3]{b} \left(2\sqrt[3]{a}\sqrt[3]{-b}C + b^{5/3}B + (-b)^{5/3}B\right)$$

$$6\sqrt[3]{ab}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a^(1/3)*(-b)^(1/3)*B - 2*a^(2/3)*C - (-b)^(2/3)*B*x - (-b)^(2/3)*C*x^2)/(a + b*x^3), x]
```

```
[Out] (2*Sqrt[3]*b^(1/3)*((-b)^(2/3) - (-b^2)^(1/3))*B + 2*a^(1/3)*b^(1/3)*C)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + (-2*b*((-b)^(2/3) + b^(2/3))*B + 2*a^(1/3)*(-b)^(1/3)*C)*Log[a^(1/3) + b^(1/3)*x] + ((-b)^(5/3)*B + b^(5/3)*B + 2*a^(1/3)*(-b)^(1/3)*b*C)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
```

$$*x^2] - 2*a^{(1/3)}*(-b)^{(2/3)}*(-b^2)^{(1/3)}*C*\text{Log}[a + b*x^3]/(-b^2)^{(1/3)}/(6*a^{(1/3)}*b)$$

Maple [B] time = 0.006, size = 345, normalized size = 3.9

$$\frac{B}{3b} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \sqrt[3]{a} \sqrt[3]{-b} \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{2C}{3b} a^{\frac{2}{3}} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{B}{6b} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \sqrt[3]{a} \sqrt[3]{-b} \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{C}{3b} a^{\frac{2}{3}} \ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^(1/3)*(-b)^(1/3)*B-2*a^(2/3)*C-(-b)^(2/3)*B*x-(-b)^(2/3)*C*x^2)/(b*x^3+a), x)

[Out] 1/3/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*a^(1/3)*(-b)^(1/3)*B-2/3*C*a^(2/3)/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/6/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*a^(1/3)*(-b)^(1/3)*B+1/3*C*a^(2/3)/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*a^(1/3)*(-b)^(1/3)*B-2/3*C*a^(2/3)/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/3*B*(-b)^(2/3)/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-1/6*B*(-b)^(2/3)/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-1/3*B*(-b)^(2/3)*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/3*C*(-b)^(2/3)/b*ln(b*x^3+a)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(1/3)*(-b)^(1/3)*B-2*a^(2/3)*C-(-b)^(2/3)*B*x-(-b)^(2/3)*C*x^2)/(b*x^3+a), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 9.57486, size = 1129, normalized size = 12.83

$$\left[\sqrt{\frac{1}{3}} b \sqrt{\frac{C^2 a (-b)^{\frac{1}{3}} - 2 B C a^{\frac{2}{3}} (-b)^{\frac{2}{3}} - B^2 a^{\frac{1}{3}} b}{ab}} \log \left(\frac{C^3 a^2 + B^3 ab - 2 (C^3 ab + B^3 b^2) x^3 - 3 (C^3 a + B^3 b) a^{\frac{2}{3}} (-b)^{\frac{1}{3}} x + 3 \sqrt{\frac{1}{3}} \left((2 B^2 b x^2 + C^2 a x + B C a) a^{\frac{2}{3}} (-b)^{\frac{2}{3}} + (2 C^2 a b x^2 - B^2 a^2 - 2 B C a b) a^{\frac{1}{3}} (-b)^{\frac{1}{3}} \right)}{bx^3 + a}} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*(-b)^(1/3)*B-2*a^(2/3)*C-(-b)^(2/3)*B*x-(-b)^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [(sqrt(1/3)*b*sqrt((C^2*a*(-b)^(1/3) - 2*B*C*a^(2/3)*(-b)^(2/3) - B^2*a^(1/3)*b)/(a*b))*log(-(C^3*a^2 + B^3*a*b - 2*(C^3*a*b + B^3*b^2)*x^3 - 3*(C^3*a + B^3*b)*a^(2/3)*(-b)^(1/3)*x + 3*sqrt(1/3)*((2*B^2*b*x^2 + C^2*a*x + B*C*a)*a^(2/3)*(-b)^(2/3) + (2*C^2*a*b*x^2 - B*C*a*b*x - B^2*a*b)*a^(1/3) + (2*B*C*a*b*x^2 - B^2*a*b*x + C^2*a^2)*(-b)^(1/3))*sqrt((C^2*a*(-b)^(1/3) - 2*B*C*a^(2/3)*(-b)^(2/3) - B^2*a^(1/3)*b)/(a*b)))/(b*x^3 + a)) - C*(-b)^(2/3)*log(b*x + a^(1/3)*(-b)^(2/3))/b, -(2*sqrt(1/3)*b*sqrt(-(C^2*a*(-b)^(1/3) - 2*B*C*a^(2/3)*(-b)^(2/3) - B^2*a^(1/3)*b)/(a*b))*arctan(sqrt(1/3)*((2*C^2*x + B*C)*a^(2/3)*(-b)^(2/3) - (2*B*C*b*x + B^2*b)*a^(1/3) - (2*B^2*b*x - C^2*a)*(-b)^(1/3))*sqrt(-(C^2*a*(-b)^(1/3) - 2*B*C*a^(2/3)*(-b)^(2/3) - B^2*a^(1/3)*b)/(a*b)))/(C^3*a + B^3*b)) + C*(-b)^(2/3)*log(b*x + a^(1/3)*(-b)^(2/3))/b]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialDivisionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)*(-b)**(1/3)*B-2*a**(2/3)*C-(-b)**(2/3)*B*x-(-b)**(2/3)*C*x**2)/(b*x**3+a),x)
```

```
[Out] Exception raised: PolynomialDivisionFailed
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*(-b)^(1/3)*B-2*a^(2/3)*C-(-b)^(2/3)*B*x-(-b)^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.42 \quad \int \frac{B^2 + BCx + C^2x^2}{-B^3 + C^3x^3} dx$$

Optimal. Leaf size=11

$$\frac{\log(B - Cx)}{C}$$

[Out] Log[B - C*x]/C

Rubi [A] time = 0.0105686, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1586, 31}

$$\frac{\log(B - Cx)}{C}$$

Antiderivative was successfully verified.

[In] Int[(B^2 + B*C*x + C^2*x^2)/(-B^3 + C^3*x^3), x]

[Out] Log[B - C*x]/C

Rule 1586

Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\int \frac{B^2 + BCx + C^2x^2}{-B^3 + C^3x^3} dx = \int \frac{1}{-B + Cx} dx = \frac{\log(B - Cx)}{C}$$

Mathematica [A] time = 0.0015161, size = 12, normalized size = 1.09

$$\frac{\log(Cx - B)}{C}$$

Antiderivative was successfully verified.

[In] Integrate[(B^2 + B*C*x + C^2*x^2)/(-B^3 + C^3*x^3), x]

[Out] Log[-B + C*x]/C

Maple [A] time = 0.039, size = 12, normalized size = 1.1

$$\frac{\ln(-Cx + B)}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C^2*x^2+B*C*x+B^2)/(C^3*x^3-B^3), x)

[Out] ln(-C*x+B)/C

Maxima [A] time = 0.99526, size = 16, normalized size = 1.45

$$\frac{\log(Cx - B)}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C^2*x^2+B*C*x+B^2)/(C^3*x^3-B^3), x, algorithm="maxima")

[Out] log(C*x - B)/C

Fricas [A] time = 0.918183, size = 22, normalized size = 2.

$$\frac{\log(Cx - B)}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C^2*x^2+B*C*x+B^2)/(C^3*x^3-B^3),x, algorithm="fricas")
```

```
[Out] log(C*x - B)/C
```

Sympy [A] time = 0.068666, size = 7, normalized size = 0.64

$$\frac{\log(-B + Cx)}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C**2*x**2+B*C*x+B**2)/(C**3*x**3-B**3),x)
```

```
[Out] log(-B + C*x)/C
```

Giac [A] time = 1.50249, size = 18, normalized size = 1.64

$$\frac{\log(|Cx - B|)}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C^2*x^2+B*C*x+B^2)/(C^3*x^3-B^3),x, algorithm="giac")
```

```
[Out] log(abs(C*x - B))/C
```


$$3.43 \quad \int \frac{a^{2/3}C - \sqrt[3]{a}\sqrt[3]{b}Cx + b^{2/3}Cx^2}{a + bx^3} dx$$

Optimal. Leaf size=21

$$\frac{C \log(\sqrt[3]{a} + \sqrt[3]{bx})}{\sqrt[3]{b}}$$

[Out] (C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)

Rubi [A] time = 0.0145199, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1586, 31}

$$\frac{C \log(\sqrt[3]{a} + \sqrt[3]{bx})}{\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[(a^(2/3)*C - a^(1/3)*b^(1/3)*C*x + b^(2/3)*C*x^2)/(a + b*x^3), x]

[Out] (C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)

Rule 1586

Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\int \frac{a^{2/3}C - \sqrt[3]{a}\sqrt[3]{b}Cx + b^{2/3}Cx^2}{a + bx^3} dx = \int \frac{1}{\frac{\sqrt[3]{a}}{C} + \frac{\sqrt[3]{bx}}{C}} dx$$

$$= \frac{C \log(\sqrt[3]{a} + \sqrt[3]{bx})}{\sqrt[3]{b}}$$

Mathematica [A] time = 0.0028395, size = 21, normalized size = 1.

$$\frac{C \log(\sqrt[3]{a} + \sqrt[3]{bx})}{\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Integrate[(a^(2/3)*C - a^(1/3)*b^(1/3)*C*x + b^(2/3)*C*x^2)/(a + b*x^3), x]

[Out] (C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)

Maple [B] time = 0.005, size = 218, normalized size = 10.4

$$\frac{C}{3b} a^{\frac{2}{3}} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{C}{6b} a^{\frac{2}{3}} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{C\sqrt{3}}{3b} a^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{C}{3} \sqrt[3]{a} \ln$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^(2/3)*C-a^(1/3)*b^(1/3)*C*x+b^(2/3)*C*x^2)/(b*x^3+a), x)

[Out] 1/3*C*a^(2/3)/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/6*C*a^(2/3)/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3*C*a^(2/3)/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/3*C*a^(1/3)/b^(2/3)/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-1/6*C*a^(1/3)/b^(2/3)/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-1/3*C*a^(1/3)/b^(2/3)*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/3*C/b^(1/3)*ln(b*x^3+a)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(2/3)*C-a^(1/3)*b^(1/3)*C*x+b^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.01843, size = 51, normalized size = 2.43

$$\frac{C \log\left(bx + a^{\frac{1}{3}}b^{\frac{2}{3}}\right)}{b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(2/3)*C-a^(1/3)*b^(1/3)*C*x+b^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="fricas")

[Out] C*log(b*x + a^(1/3)*b^(2/3))/b^(1/3)

Sympy [A] time = 0.311516, size = 20, normalized size = 0.95

$$\frac{C \log\left(\sqrt[3]{ab^{\frac{2}{3}}} + bx\right)}{\sqrt[3]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**(2/3)*C-a**(1/3)*b**(1/3)*C*x+b**(2/3)*C*x**2)/(b*x**3+a),x)

[Out] C*log(a**(1/3)*b**(2/3) + b*x)/b**(1/3)

Giac [A] time = 1.47152, size = 22, normalized size = 1.05

$$\frac{C \log\left(\left|b^{\frac{1}{3}}x + a^{\frac{1}{3}}\right|\right)}{b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(2/3)*C-a^(1/3)*b^(1/3)*C*x+b^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] C*log(abs(b^(1/3)*x + a^(1/3)))/b^(1/3)
```

$$3.44 \quad \int \frac{\sqrt[3]{\frac{a}{b}}B + 2\left(\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a + bx^3} dx$$

Optimal. Leaf size=71

$$\frac{C \log\left(\sqrt[3]{\frac{a}{b}} + x\right)}{b} - \frac{2\left(\frac{a}{b}\right)^{2/3} \left(C\sqrt[3]{\frac{a}{b}} + B\right) \tan^{-1}\left(\frac{1 - \frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}a}$$

[Out] $(-2*(a/b)^{(2/3)}*(B + (a/b)^{(1/3)}*C)*\text{ArcTan}[(1 - (2*x)/(a/b)^{(1/3)})/\text{Sqrt}[3]])/(\text{Sqrt}[3]*a) + (C*\text{Log}[(a/b)^{(1/3)} + x])/b$

Rubi [A] time = 0.0941567, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1867, 31, 617, 204}

$$\frac{C \log\left(\sqrt[3]{\frac{a}{b}} + x\right)}{b} - \frac{2\left(\frac{a}{b}\right)^{2/3} \left(C\sqrt[3]{\frac{a}{b}} + B\right) \tan^{-1}\left(\frac{1 - \frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}a}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a/b)^{(1/3)}*B + 2*(a/b)^{(2/3)}*C + B*x + C*x^2]/(a + b*x^3), x]$

[Out] $(-2*(a/b)^{(2/3)}*(B + (a/b)^{(1/3)}*C)*\text{ArcTan}[(1 - (2*x)/(a/b)^{(1/3)})/\text{Sqrt}[3]])/(\text{Sqrt}[3]*a) + (C*\text{Log}[(a/b)^{(1/3)} + x])/b$

Rule 1867

$\text{Int}[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] \rightarrow \text{With}[\{A = \text{Coeff}[P2, x, 0], B = \text{Coeff}[P2, x, 1], C = \text{Coeff}[P2, x, 2]\}, \text{With}[\{q = (a/b)^{(1/3)}\}, \text{Dist}[C/b, \text{Int}[1/(q + x), x], x] + \text{Dist}[(B + C*q)/b, \text{Int}[1/(q^2 - q*x + x^2), x], x]] /; \text{EqQ}[A - (a/b)^{(1/3)}*B - 2*(a/b)^{(2/3)}*C, 0]] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[P2, x, 2]$

Rule 31

$\text{Int}[(a_) + (b_)*(x_)^(-1), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}[\{a, b\}, x]$

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{\frac{a}{b}}B + 2\left(\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a + bx^3} dx = \frac{C \int \frac{1}{\sqrt[3]{\frac{a}{b}} + x} dx}{b} + \frac{\left(B + \sqrt[3]{\frac{a}{b}}C\right) \int \frac{1}{\left(\frac{a}{b}\right)^{2/3} - \sqrt[3]{\frac{a}{b}}x + x^2} dx}{b}$$

$$= \frac{C \log\left(\sqrt[3]{\frac{a}{b}} + x\right)}{b} + \left(2\left(\frac{a}{b}\right)^{2/3} \frac{B}{a} + \frac{C}{b}\right) \text{Subst}\left(\int \frac{1}{-3 - x^2} dx, x, 1 - \frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)$$

$$= -\frac{2\left(\frac{a}{b}\right)^{2/3} \frac{B}{a} + \frac{C}{b}}{\sqrt{3}} \tan^{-1}\left(\frac{1 - \frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right) + \frac{C \log\left(\sqrt[3]{\frac{a}{b}} + x\right)}{b}$$

Mathematica [B] time = 0.267454, size = 247, normalized size = 3.48

$$\frac{\sqrt[3]{b} \left(a^{2/3} B - \sqrt[3]{a} \sqrt[3]{b} \sqrt[3]{\frac{a}{b}} \left(2C \sqrt[3]{\frac{a}{b}} + B \right) \right) \log \left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2 \right) + 2 \sqrt[3]{b} \left(\sqrt[3]{a} \sqrt[3]{b} \sqrt[3]{\frac{a}{b}} \left(2C \sqrt[3]{\frac{a}{b}} + B \right) - a^{2/3} B \right) \log \left(\sqrt[3]{a} + \sqrt[3]{b} x \right)}{6ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a/b)^(1/3)*B + 2*(a/b)^(2/3)*C + B*x + C*x^2)/(a + b*x^3), x]
```

```
[Out] (2*Sqrt[3]*a^(1/3)*b^(1/3)*(a^(1/3)*B + (a/b)^(1/3)*b^(1/3)*(B + 2*(a/b)^(1/3)*C))*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] + 2*b^(1/3)*(-a^(2/3)*B + a^(1/3)*(a/b)^(1/3)*b^(1/3)*(B + 2*(a/b)^(1/3)*C))*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*(a^(2/3)*B - a^(1/3)*(a/b)^(1/3)*b^(1/3)*(B + 2*(a/b)^(1/3)*C))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*a*C*Log[a +
```

$b*x^3)/(6*a*b)$

Maple [A] time = 0.005, size = 121, normalized size = 1.7

$$\frac{2C}{3b} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) - \frac{C}{3b} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) + \frac{2C\sqrt{3}}{3b} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) + \frac{2B\sqrt{3}}{3b} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((1/b*a)^(1/3)*B+2*(1/b*a)^(2/3)*C+B*x+C*x^2)/(b*x^3+a), x)

[Out] $\frac{2}{3}C \ln(x + (1/b*a)^{1/3})/b - \frac{1}{3}C/b \ln(x^2 - (1/b*a)^{1/3}*x + (1/b*a)^{2/3}) + 2/3*C/b*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1)) + 2/3*B*3^{1/2}/b/(1/b*a)^{1/3}*\arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1)) + 1/3*C/b*\ln(b*x^3+a)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 5.69123, size = 948, normalized size = 13.35

$$\left[C \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + \sqrt{\frac{1}{3}} \sqrt{-\frac{2BCb\left(\frac{a}{b}\right)^{\frac{2}{3}} + B^2b\left(\frac{a}{b}\right)^{\frac{1}{3}} + C^2a}{a}} \log\left(\frac{C^3a^2 + B^3ab - 2(C^3ab + B^3b^2)x^3 + 3(C^3ab + B^3b^2)x\left(\frac{a}{b}\right)^{\frac{2}{3}} + 3\sqrt{\frac{1}{3}}(2BCabx^2 - B^2abx + C^2a^2)}}{b}\right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [(C*log(x + (a/b)^(1/3)) + sqrt(1/3)*sqrt(-(2*B*C*b*(a/b)^(2/3) + B^2*b*(a/b)^(1/3) + C^2*a)/a)*log(-(C^3*a^2 + B^3*a*b - 2*(C^3*a*b + B^3*b^2)*x^3 + 3*(C^3*a*b + B^3*b^2)*x*(a/b)^(2/3) + 3*sqrt(1/3)*(2*B*C*a*b*x^2 - B^2*a*b*x + C^2*a^2 - (2*B^2*b^2*x^2 + C^2*a*b*x + B*C*a*b)*(a/b)^(2/3) - (2*C^2*a*b*x^2 - B*C*a*b*x - B^2*a*b)*(a/b)^(1/3))*sqrt(-(2*B*C*b*(a/b)^(2/3) + B^2*b*(a/b)^(1/3) + C^2*a)/a))/b, (2*sqrt(1/3)*sqrt((2*B*C*b*(a/b)^(2/3) + B^2*b*(a/b)^(1/3) + C^2*a)/a)*arctan(sqrt(1/3)*(2*B^2*b*x - C^2*a + (2*C^2*b*x + B*C*b)*(a/b)^(2/3) - (2*B*C*b*x + B^2*b)*(a/b)^(1/3))*sqrt((2*B*C*b*(a/b)^(2/3) + B^2*b*(a/b)^(1/3) + C^2*a)/a)/(C^3*a + B^3*b)) + C*log(x + (a/b)^(1/3)))/b]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialDivisionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a/b)**(1/3)*B+2*(a/b)**(2/3)*C+B*x+C*x**2)/(b*x**3+a),x)
```

```
[Out] Exception raised: PolynomialDivisionFailed
```

Giac [B] time = 1.2488, size = 362, normalized size = 5.1

$$\frac{\left(Cb^2\left(-\frac{a}{b}\right)^{\frac{2}{3}} + Bb^2\left(-\frac{a}{b}\right)^{\frac{1}{3}} + (ab^2)^{\frac{1}{3}}Bb + 2(ab^2)^{\frac{2}{3}}C\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^2} + \frac{\sqrt{3}\left(\left(9(-a^2b^4)^{\frac{1}{3}}ab^2 - 27^{\frac{5}{6}}(-a^2b^4)^{\frac{5}{6}}\right)E\right)}{3ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorithm="giac")
```



```
[Out] -1/3*(C*b^2*(-a/b)^(2/3) + B*b^2*(-a/b)^(1/3) + (a*b^2)^(1/3)*B*b + 2*(a*b^2)^(2/3)*C)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2) + 1/54*sqrt(3)*((9*(-a^2*b^4)^(1/3)*a*b^2 - 27^(5/6)*(-a^2*b^4)^(5/6))*B + 18*(a^2*b^3 - sqrt(3)*sqrt(a^4*b^6)*i)*C)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^4) + 1/108*((27*(-a^2*b^4)^(1/3)*a*b^2 + 27^(5/6)*(-a^2*b^4)^(5/6))*B + 18*(3*a^2*b^3 - sqrt(3)*sqrt(a^4*b^6)*i)*C)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^4)
```

$$3.45 \quad \int \frac{\sqrt[3]{-\frac{a}{b}}B + 2\left(-\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a - bx^3} dx$$

Optimal. Leaf size=76

$$\frac{2\left(C\sqrt[3]{-\frac{a}{b}} + B\right)\tan^{-1}\left(\frac{1 - \frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b\sqrt[3]{-\frac{a}{b}}} - \frac{C\log\left(\sqrt[3]{-\frac{a}{b}} + x\right)}{b}$$

[Out] (2*(B + (-a/b))^(1/3)*C)*ArcTan[(1 - (2*x)/((-a/b))^(1/3))/Sqrt[3]]/(Sqrt[3]*(-a/b))^(1/3)*b - (C*Log[(-a/b))^(1/3) + x])/b

Rubi [A] time = 0.102125, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$, Rules used = {1867, 31, 617, 204}

$$\frac{2\left(C\sqrt[3]{-\frac{a}{b}} + B\right)\tan^{-1}\left(\frac{1 - \frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b\sqrt[3]{-\frac{a}{b}}} - \frac{C\log\left(\sqrt[3]{-\frac{a}{b}} + x\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[((-a/b))^(1/3)*B + 2*(-a/b))^(2/3)*C + B*x + C*x^2)/(a - b*x^3), x]

[Out] (2*(B + (-a/b))^(1/3)*C)*ArcTan[(1 - (2*x)/((-a/b))^(1/3))/Sqrt[3]]/(Sqrt[3]*(-a/b))^(1/3)*b - (C*Log[(-a/b))^(1/3) + x])/b

Rule 1867

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[C/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x] /; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^( -1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^( -1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{-\frac{a}{b}}B + 2\left(-\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a - bx^3} dx = -\frac{C \int \frac{1}{\sqrt[3]{-\frac{a}{b}}+x} dx}{b} - \frac{\left(B + \sqrt[3]{-\frac{a}{b}}C\right) \int \frac{1}{\left(-\frac{a}{b}\right)^{2/3} - \sqrt[3]{-\frac{a}{b}}x + x^2} dx}{b}$$

$$= -\frac{C \log\left(\sqrt[3]{-\frac{a}{b}} + x\right)}{b} - \frac{\left(2\left(\frac{B}{\sqrt[3]{-\frac{a}{b}}} + C\right)\right) \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 - \frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)}{b}$$

$$= \frac{2\left(\frac{B}{\sqrt[3]{-\frac{a}{b}}} + C\right) \tan^{-1}\left(\frac{1 - \frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b} - \frac{C \log\left(\sqrt[3]{-\frac{a}{b}} + x\right)}{b}$$

Mathematica [B] time = 0.177006, size = 288, normalized size = 3.79

$$\frac{\left(-a^{2/3}B - \sqrt[3]{a}\sqrt[3]{b}B\sqrt[3]{-\frac{a}{b}} - 2\sqrt[3]{a}\sqrt[3]{b}C\left(-\frac{a}{b}\right)^{2/3}\right) \log\left(a^{2/3} + \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{6ab^{2/3}} - \frac{\left(a^{2/3}B + \sqrt[3]{a}\sqrt[3]{b}B\sqrt[3]{-\frac{a}{b}} + 2\sqrt[3]{a}\sqrt[3]{b}C\left(-\frac{a}{b}\right)^{2/3}\right)}{3ab^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(((a/b))^(1/3)*B + 2*((a/b))^(2/3)*C + B*x + C*x^2)/(a - b*x^3)
,x]
```

```
[Out] -(((a^(2/3)*B - a^(1/3)*(-a/b)^(1/3)*b^(1/3)*B - 2*a^(1/3)*(-a/b)^(2/3)
*b^(1/3)*C)*ArcTan[(a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a*b
^(2/3))) - ((a^(2/3)*B + a^(1/3)*(-a/b)^(1/3)*b^(1/3)*B + 2*a^(1/3)*(-a/
b)^(2/3)*b^(1/3)*C)*Log[a^(1/3) - b^(1/3)*x]/(3*a*b^(2/3)) - (((-a^(2/3)*
B) - a^(1/3)*(-a/b)^(1/3)*b^(1/3)*B - 2*a^(1/3)*(-a/b)^(2/3)*b^(1/3)*C)
*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*a*b^(2/3)) - (C*Log[a -
b*x^3])/(3*b)
```

Maple [B] time = 0.006, size = 345, normalized size = 4.5

$$-\frac{2C}{3b} \left(-\frac{a}{b}\right)^{\frac{2}{3}} \ln\left(x - \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{B}{3b} \ln\left(x - \sqrt[3]{\frac{a}{b}}\right) \sqrt[3]{-\frac{a}{b}} \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{C}{3b} \left(-\frac{a}{b}\right)^{\frac{2}{3}} \ln\left(x^2 + \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{B}{6b} \ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/b*a)^(1/3)*B+2*((1/b*a)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a), x)
```

```
[Out] -2/3*C*(-1/b*a)^(2/3)/b/(1/b*a)^(2/3)*ln(x-(1/b*a)^(1/3))-1/3/b/(1/b*a)^(2/3)
*ln(x-(1/b*a)^(1/3))*(-1/b*a)^(1/3)*B+1/3*C*(-1/b*a)^(2/3)/b/(1/b*a)^(2/3)
*ln(x^2+(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/6/b/(1/b*a)^(2/3)*ln(x^2+(1/b*a)^(1/3)
*x+(1/b*a)^(2/3))*(-1/b*a)^(1/3)*B+2/3*C*(-1/b*a)^(2/3)/b/(1/b*a)^(2/3)
*3^(1/2)*arctan(1/3*(1+2/(1/b*a)^(1/3)*x)*3^(1/2))+1/3/b/(1/b*a)^(2/3)*3^(1/2)
*arctan(1/3*(1+2/(1/b*a)^(1/3)*x)*3^(1/2))*(-1/b*a)^(1/3)*B-1/3*B/b/(1/
b*a)^(1/3)*ln(x-(1/b*a)^(1/3))+1/6*B/b/(1/b*a)^(1/3)*ln(x^2+(1/b*a)^(1/3)*x
+(1/b*a)^(2/3))-1/3*B*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*(1+2/(1/b*a)^(1/3)
*x)*3^(1/2))-1/3*C/b*ln(b*x^3-a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1/b*a)^(1/3)*B+2*((1/b*a)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a), x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.77571, size = 972, normalized size = 12.79

$$\left[C \log \left(x + \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right) - \sqrt{\frac{1}{3}} \sqrt{\frac{2BCb \left(-\frac{a}{b} \right)^{\frac{2}{3}} + B^2b \left(-\frac{a}{b} \right)^{\frac{1}{3}} - C^2a}{a}} \log \left(\frac{C^3a^2 - B^3ab + 2(C^3ab - B^3b^2)x^3 - 3(C^3ab - B^3b^2)x \left(-\frac{a}{b} \right)^{\frac{2}{3}} + 3\sqrt{\frac{1}{3}}(2BCabx^2 - B^2abx - C^2a^2)}}{b} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorithm="fricas")

[Out] [-(C*log(x + (a/b)^(1/3)) - sqrt(1/3)*sqrt((2*B*C*b*(a/b)^(2/3) + B^2*b*(a/b)^(1/3) - C^2*a)/a)*log(-(C^3*a^2 - B^3*a*b + 2*(C^3*a*b - B^3*b^2)*x^3 - 3*(C^3*a*b - B^3*b^2)*x*(a/b)^(2/3) + 3*sqrt(1/3)*(2*B*C*a*b*x^2 - B^2*a*b*x - C^2*a^2 + (2*B^2*b^2*x^2 - C^2*a*b*x - B*C*a*b)*(a/b)^(2/3) - (2*C^2*a*b*x^2 - B*C*a*b*x - B^2*a*b)*(a/b)^(1/3))*sqrt((2*B*C*b*(a/b)^(2/3) + B^2*b*(a/b)^(1/3) - C^2*a)/a))/(b*x^3 - a))/b, -(2*sqrt(1/3)*sqrt(-(2*B*C*b*(a/b)^(2/3) + B^2*b*(a/b)^(1/3) - C^2*a)/a)*arctan(-sqrt(1/3)*(2*B^2*b*x + C^2*a + (2*C^2*b*x + B*C*b)*(a/b)^(2/3) - (2*B*C*b*x + B^2*b)*(a/b)^(1/3))*sqrt(-(2*B*C*b*(a/b)^(2/3) + B^2*b*(a/b)^(1/3) - C^2*a)/a)/(C^3*a - B^3*b)) + C*log(x + (a/b)^(1/3)))/b]

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialDivisionFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a/b)**(1/3)*B+2*(a/b)**(2/3)*C+B*x+C*x**2)/(-b*x**3+a),x)

[Out] Exception raised: PolynomialDivisionFailed

Giac [B] time = 1.10247, size = 351, normalized size = 4.62

$$\frac{\left(Cb^2\left(\frac{a}{b}\right)^{\frac{2}{3}} + Bb^2\left(\frac{a}{b}\right)^{\frac{1}{3}} + (-ab^2)^{\frac{1}{3}}Bb + 2(-ab^2)^{\frac{2}{3}}C\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^2} + \frac{\sqrt{3}\left(\left(9(-a^2b^4)^{\frac{1}{3}}ab^2 + 27^{\frac{5}{6}}(-a^2b^4)^{\frac{5}{6}}\right)B - \dots\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorithm="giac")

[Out] -1/3*(C*b^2*(a/b)^(2/3) + B*b^2*(a/b)^(1/3) + (-a*b^2)^(1/3)*B*b + 2*(-a*b^2)^(2/3)*C)*(a/b)^(1/3)*log(abs(x - (a/b)^(1/3)))/(a*b^2) + 1/54*sqrt(3)*((9*(-a^2*b^4)^(1/3)*a*b^2 + 27^(5/6)*(-a^2*b^4)^(5/6))*B - 18*(a^2*b^3 + sqrt(3)*sqrt(a^4*b^6)*i)*C)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^4) + 1/108*((27*(-a^2*b^4)^(1/3)*a*b^2 - 27^(5/6)*(-a^2*b^4)^(5/6))*B - 18*(3*a^2*b^3 + sqrt(3)*sqrt(a^4*b^6)*i)*C)*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^4)

$$3.46 \quad \int \frac{-\sqrt[3]{-\frac{a}{b}}B + 2\left(-\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a + bx^3} dx$$

Optimal. Leaf size=78

$$\frac{2\left(B - C\sqrt[3]{-\frac{a}{b}}\right) \tan^{-1}\left(\frac{\sqrt[3]{-\frac{a}{b}} + 1}{\sqrt{3}}\right)}{\sqrt{3}b\sqrt[3]{-\frac{a}{b}}} + \frac{C \log\left(\sqrt[3]{-\frac{a}{b}} - x\right)}{b}$$

[Out] (2*(B - (-a/b)^(1/3)*C)*ArcTan[(1 + (2*x)/(-a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*(-a/b)^(1/3)*b) + (C*Log[(-a/b)^(1/3) - x])/b

Rubi [A] time = 0.107623, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$, Rules used = {1869, 31, 617, 204}

$$\frac{2\left(B - C\sqrt[3]{-\frac{a}{b}}\right) \tan^{-1}\left(\frac{\sqrt[3]{-\frac{a}{b}} + 1}{\sqrt{3}}\right)}{\sqrt{3}b\sqrt[3]{-\frac{a}{b}}} + \frac{C \log\left(\sqrt[3]{-\frac{a}{b}} - x\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[(-((-a/b)^(1/3)*B) + 2*(-a/b)^(2/3)*C + B*x + C*x^2)/(a + b*x^3), x]

[Out] (2*(B - (-a/b)^(1/3)*C)*ArcTan[(1 + (2*x)/(-a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*(-a/b)^(1/3)*b) + (C*Log[(-a/b)^(1/3) - x])/b

Rule 1869

Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a/b)^(1/3)}, -Dist[C/b, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2), x], x]] /; EqQ[A + (-a/b)^(1/3)*B - 2*(-a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

Rule 31

```
Int[((a_) + (b_.)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^( -1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^( -1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{-\sqrt[3]{-\frac{a}{b}}B + 2\left(-\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a + bx^3} dx = -\frac{C \int \frac{1}{\sqrt[3]{-\frac{a}{b}}-x} dx}{b} + \frac{\left(B - \sqrt[3]{-\frac{a}{b}}C\right) \int \frac{1}{\left(-\frac{a}{b}\right)^{2/3} + \sqrt[3]{-\frac{a}{b}}x + x^2} dx}{b}$$

$$= \frac{C \log\left(\sqrt[3]{-\frac{a}{b}} - x\right)}{b} - \frac{\left(2\left(B - \sqrt[3]{-\frac{a}{b}}C\right)\right) \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 + \frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)}{\sqrt[3]{-\frac{a}{b}}b}$$

$$= \frac{2\left(B - \sqrt[3]{-\frac{a}{b}}C\right) \tan^{-1}\left(\frac{1 + \frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{-\frac{a}{b}}b} + \frac{C \log\left(\sqrt[3]{-\frac{a}{b}} - x\right)}{b}$$

Mathematica [B] time = 0.251451, size = 253, normalized size = 3.24

$$\frac{\sqrt[3]{b} \left(a^{2/3} B + \sqrt[3]{a} \sqrt[3]{b} \sqrt[3]{-\frac{a}{b}} \left(B - 2C \sqrt[3]{-\frac{a}{b}} \right) \right) \log \left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2 \right) - 2 \sqrt[3]{b} \left(a^{2/3} B + \sqrt[3]{a} \sqrt[3]{b} \sqrt[3]{-\frac{a}{b}} \left(B - 2C \sqrt[3]{-\frac{a}{b}} \right) \right) \log \left(\sqrt[3]{a} \right)}{6ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-((-a/b))^(1/3)*B) + 2*(-(a/b))^(2/3)*C + B*x + C*x^2)/(a + b*x
^3),x]
```



```
[Out] (2*sqrt[3]*a^(1/3)*b^(1/3)*(a^(1/3)*B + (-a/b)^(1/3)*b^(1/3)*(-B + 2*(-a/b)^(1/3)*C))*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(sqrt[3]*a^(1/3))] - 2*b^(1/3)*(a^(2/3)*B + a^(1/3)*(-a/b)^(1/3)*b^(1/3)*(B - 2*(-a/b)^(1/3)*C))*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*(a^(2/3)*B + a^(1/3)*(-a/b)^(1/3)*b^(1/3)*(B - 2*(-a/b)^(1/3)*C))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*a*C*Log[a + b*x^3]/(6*a*b)
```

Maple [B] time = 0.005, size = 340, normalized size = 4.4

$$\frac{2C}{3b} \left(-\frac{a}{b}\right)^{\frac{2}{3}} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{B}{3b} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \sqrt[3]{-\frac{a}{b}} \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{C}{3b} \left(-\frac{a}{b}\right)^{\frac{2}{3}} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{B}{6b} \ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(-1/b*a)^(1/3)*B+2*(-1/b*a)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x)
```

```
[Out] 2/3*C*(-1/b*a)^(2/3)/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/3/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*(-1/b*a)^(1/3)*B-1/3*C*(-1/b*a)^(2/3)/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/6/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*(-1/b*a)^(1/3)*B+2/3*C*(-1/b*a)^(2/3)/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/3/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*(-1/b*a)^(1/3)*B-1/3*B/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/6*B/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3*B*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/3*C/b*ln(b*x^3+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.75111, size = 968, normalized size = 12.41

$$\left[\frac{C \log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right) + \sqrt{\frac{1}{3}} \sqrt{-\frac{2BCb\left(-\frac{a}{b}\right)^{\frac{2}{3}} - B^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}} + C^2a}{a}} \log\left(\frac{C^3a^2 + B^3ab - 2(C^3ab + B^3b^2)x^3 + 3(C^3ab + B^3b^2)x\left(-\frac{a}{b}\right)^{\frac{2}{3}} + 3\sqrt{\frac{1}{3}}(2BCabx^2 - B^2abx + C^2a^2)}{C^3a^2 + B^3ab - 2(C^3ab + B^3b^2)x^3 + 3(C^3ab + B^3b^2)x\left(-\frac{a}{b}\right)^{\frac{2}{3}} + 3\sqrt{\frac{1}{3}}(2BCabx^2 - B^2abx + C^2a^2)}\right)}{b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-(-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorithm="fricas")

[Out] [(C*log(x - (-a/b)^(1/3)) + sqrt(1/3)*sqrt(-(2*B*C*b*(-a/b)^(2/3) - B^2*b*(-a/b)^(1/3) + C^2*a)/a)*log(-(C^3*a^2 + B^3*a*b - 2*(C^3*a*b + B^3*b^2)*x^3 + 3*(C^3*a*b + B^3*b^2)*x*(-a/b)^(2/3) + 3*sqrt(1/3)*(2*B*C*a*b*x^2 - B^2*a*b*x + C^2*a^2 - (2*B^2*b^2*x^2 + C^2*a*b*x + B*C*a*b)*(-a/b)^(2/3) + (2*C^2*a*b*x^2 - B*C*a*b*x - B^2*a*b)*(-a/b)^(1/3))*sqrt(-(2*B*C*b*(-a/b)^(2/3) - B^2*b*(-a/b)^(1/3) + C^2*a)/a))/(b*x^3 + a))/b, (2*sqrt(1/3)*sqrt((2*B*C*b*(-a/b)^(2/3) - B^2*b*(-a/b)^(1/3) + C^2*a)/a)*arctan(sqrt(1/3)*(2*B^2*b*x - C^2*a + (2*C^2*b*x + B*C*b)*(-a/b)^(2/3) + (2*B*C*b*x + B^2*b)*(-a/b)^(1/3))*sqrt((2*B*C*b*(-a/b)^(2/3) - B^2*b*(-a/b)^(1/3) + C^2*a)/a)/(C^3*a + B^3*b)) + C*log(x - (-a/b)^(1/3)))/b]

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialDivisionFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-(-a/b)**(1/3)*B+2*(-a/b)**(2/3)*C+B*x+C*x**2)/(b*x**3+a),x)

[Out] Exception raised: PolynomialDivisionFailed

Giac [A] time = 1.12964, size = 180, normalized size = 2.31

$$\frac{2\sqrt{3}\left(Cab + (-ab^2)^{\frac{2}{3}}B\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^2} - \frac{\left(Cb^2\left(-\frac{a}{b}\right)^{\frac{2}{3}} + Bb^2\left(-\frac{a}{b}\right)^{\frac{1}{3}} - (-ab^2)^{\frac{1}{3}}Bb + 2(-ab^2)^{\frac{2}{3}}C\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\right)}{3ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-(-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorithm="giac")

[Out] -2/3*sqrt(3)*(C*a*b + (-a*b^2)^(2/3)*B)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) - 1/3*(C*b^2*(-a/b)^(2/3) + B*b^2*(-a/b)^(1/3) - (-a*b^2)^(1/3)*B*b + 2*(-a*b^2)^(2/3)*C)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2)

$$3.47 \quad \int \frac{-\sqrt[3]{\frac{a}{b}}B + 2\left(\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a - bx^3} dx$$

Optimal. Leaf size=75

$$\frac{2\left(\frac{a}{b}\right)^{2/3}\left(B - C\sqrt[3]{\frac{a}{b}}\right)\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}a} - \frac{C\log\left(\sqrt[3]{\frac{a}{b}} - x\right)}{b}$$

[Out] $(-2*(a/b)^{(2/3)}*(B - (a/b)^{(1/3)}*C)*\text{ArcTan}[(1 + (2*x)/(a/b)^{(1/3)})/\text{Sqrt}[3]])/(\text{Sqrt}[3]*a) - (C*\text{Log}[(a/b)^{(1/3)} - x])/b$

Rubi [A] time = 0.104694, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1869, 31, 617, 204}

$$\frac{2\left(\frac{a}{b}\right)^{2/3}\left(B - C\sqrt[3]{\frac{a}{b}}\right)\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}a} - \frac{C\log\left(\sqrt[3]{\frac{a}{b}} - x\right)}{b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-((a/b)^{(1/3)}*B) + 2*(a/b)^{(2/3)}*C + B*x + C*x^2)/(a - b*x^3), x]$

[Out] $(-2*(a/b)^{(2/3)}*(B - (a/b)^{(1/3)}*C)*\text{ArcTan}[(1 + (2*x)/(a/b)^{(1/3)})/\text{Sqrt}[3]])/(\text{Sqrt}[3]*a) - (C*\text{Log}[(a/b)^{(1/3)} - x])/b$

Rule 1869

$\text{Int}[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] \rightarrow \text{With}[\{A = \text{Coeff}[P2, x, 0], B = \text{Coeff}[P2, x, 1], C = \text{Coeff}[P2, x, 2]\}, \text{With}[\{q = (-a/b)^{(1/3)}\}, -\text{Dist}[C/b, \text{Int}[1/(q - x), x], x] + \text{Dist}[(B - C*q)/b, \text{Int}[1/(q^2 + q*x + x^2), x], x]] \text{ /; EqQ}[A + (-a/b)^{(1/3)}*B - 2*(-a/b)^{(2/3)}*C, 0]] \text{ /; FreeQ}[\{a, b\}, x] \ \&\& \ \text{PolyQ}[P2, x, 2]$

Rule 31

$\text{Int}(((a_) + (b_)*(x_))^{(-1)}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] \text{ /; FreeQ}[\{a, b\}, x]$

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{-\sqrt[3]{\frac{a}{b}}B + 2\left(\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a - bx^3} dx = \frac{C \int \frac{1}{\sqrt[3]{\frac{a}{b}} - x} dx}{b} - \frac{\left(B - \sqrt[3]{\frac{a}{b}}C\right) \int \frac{1}{\left(\frac{a}{b}\right)^{2/3} + \sqrt[3]{\frac{a}{b}}x + x^2} dx}{b}$$

$$= -\frac{C \log\left(\sqrt[3]{\frac{a}{b}} - x\right)}{b} + \left(2\left(\frac{a}{b}\right)^{2/3}B - \frac{C}{b}\right) \text{Subst}\left(\int \frac{1}{-3 - x^2} dx, x, 1 + \frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)$$

$$= -\frac{2\left(\frac{a}{b}\right)^{2/3}B - \frac{C}{b} \tan^{-1}\left(\frac{1 + \frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{C \log\left(\sqrt[3]{\frac{a}{b}} - x\right)}{b}$$

Mathematica [B] time = 0.261048, size = 244, normalized size = 3.25

$$\sqrt[3]{b} \left(a^{2/3} B + \sqrt[3]{a} \sqrt[3]{b} \sqrt[3]{\frac{a}{b}} \left(2C \sqrt[3]{\frac{a}{b}} - B \right) \right) \log \left(a^{2/3} + \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2 \right) - 2 \sqrt[3]{b} \left(a^{2/3} B + \sqrt[3]{a} \sqrt[3]{b} \sqrt[3]{\frac{a}{b}} \left(2C \sqrt[3]{\frac{a}{b}} - B \right) \right) \log \left(\sqrt[3]{a} - \sqrt[3]{b} x \right)$$

6ab

Antiderivative was successfully verified.

```
[In] Integrate[(-(a/b)^(1/3)*B) + 2*(a/b)^(2/3)*C + B*x + C*x^2)/(a - b*x^3), x]
```

```
[Out] (-2*Sqrt[3]*a^(1/3)*b^(1/3)*(a^(1/3)*B + (a/b)^(1/3)*b^(1/3)*(B - 2*(a/b)^(1/3)*C))*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*b^(1/3)*(a^(2/3)*B + a^(1/3)*(a/b)^(1/3)*b^(1/3)*(-B + 2*(a/b)^(1/3)*C))*Log[a^(1/3) - b^(1/3)*x] + b^(1/3)*(a^(2/3)*B + a^(1/3)*(a/b)^(1/3)*b^(1/3)*(-B + 2*(a/b)^(1/3)*C))*Log[a^(1/3) - b^(1/3)*x]
```

$*C)) * \text{Log}[a^{(2/3)} + a^{(1/3)} * b^{(1/3)} * x + b^{(2/3)} * x^2] - 2 * a * C * \text{Log}[a - b * x^3] / (6 * a * b)$

Maple [A] time = 0.007, size = 124, normalized size = 1.7

$$-\frac{2C}{3b} \ln\left(x - \sqrt[3]{\frac{a}{b}}\right) + \frac{C}{3b} \ln\left(x^2 + \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) + \frac{2C\sqrt{3}}{3b} \arctan\left(\frac{\sqrt{3}}{3}\left(1 + 2x\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\right) - \frac{2B\sqrt{3}}{3b} \arctan\left(\frac{\sqrt{3}}{3}\left(1 + 2x\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1/b*a)^(1/3)*B+2*(1/b*a)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a), x)

[Out] -2/3*C/b*ln(x-(1/b*a)^(1/3))+1/3*C/b*ln(x^2+(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2/3*C*arctan(1/3*(1+2/(1/b*a)^(1/3)*x)*3^(1/2))/b*3^(1/2)-2/3*B*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*(1+2/(1/b*a)^(1/3)*x)*3^(1/2))-1/3*C/b*ln(b*x^3-a)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 5.99173, size = 952, normalized size = 12.69

$$\frac{C \log\left(x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) - \sqrt{\frac{1}{3}} \sqrt{\frac{2BCb\left(\frac{a}{b}\right)^{\frac{2}{3}} - B^2b\left(\frac{a}{b}\right)^{\frac{1}{3}} - C^2a}{a}} \log\left(\frac{C^3a^2 - B^3ab + 2(C^3ab - B^3b^2)x^3 - 3(C^3ab - B^3b^2)x\left(\frac{a}{b}\right)^{\frac{2}{3}} + 3\sqrt{\frac{1}{3}}(2BCabx^2 - B^2abx - C^2a^2)}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorithm="fricas")

[Out] [-(C*log(x - (a/b)^(1/3)) - sqrt(1/3)*sqrt((2*B*C*b*(a/b)^(2/3) - B^2*b*(a/b)^(1/3) - C^2*a)/a)*log(-(C^3*a^2 - B^3*a*b + 2*(C^3*a*b - B^3*b^2)*x^3 - 3*(C^3*a*b - B^3*b^2)*x*(a/b)^(2/3) + 3*sqrt(1/3)*(2*B*C*a*b*x^2 - B^2*a*b*x - C^2*a^2 + (2*B^2*b^2*x^2 - C^2*a*b*x - B*C*a*b)*(a/b)^(2/3) + (2*C^2*a*b*x^2 - B*C*a*b*x - B^2*a*b)*(a/b)^(1/3))*sqrt((2*B*C*b*(a/b)^(2/3) - B^2*b*(a/b)^(1/3) - C^2*a)/a))/(b*x^3 - a))/b, -(2*sqrt(1/3)*sqrt(-(2*B*C*b*(a/b)^(2/3) - B^2*b*(a/b)^(1/3) - C^2*a)/a)*arctan(-sqrt(1/3)*(2*B^2*b*x + C^2*a + (2*C^2*b*x + B*C*b)*(a/b)^(2/3) + (2*B*C*b*x + B^2*b)*(a/b)^(1/3))*sqrt(-(2*B*C*b*(a/b)^(2/3) - B^2*b*(a/b)^(1/3) - C^2*a)/a)/(C^3*a - B^3*b)) + C*log(x - (a/b)^(1/3)))/b]

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialDivisionFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a/b)**(1/3)*B+2*(a/b)**(2/3)*C+B*x+C*x**2)/(-b*x**3+a),x)

[Out] Exception raised: PolynomialDivisionFailed

Giac [A] time = 1.08892, size = 169, normalized size = 2.25

$$\frac{2\sqrt{3}\left(Cab - (ab^2)^{\frac{2}{3}}B\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^2} - \frac{\left(Cb^2\left(\frac{a}{b}\right)^{\frac{2}{3}} + Bb^2\left(\frac{a}{b}\right)^{\frac{1}{3}} - (ab^2)^{\frac{1}{3}}Bb + 2(ab^2)^{\frac{2}{3}}C\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorithm="giac")

[Out] 2/3*sqrt(3)*(C*a*b - (a*b^2)^(2/3)*B)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2) - 1/3*(C*b^2*(a/b)^(2/3) + B*b^2*(a/b)^(1/3) - (a*b^2)^(1/3)*B*b + 2*(a*b^2)^(2/3)*C)*(a/b)^(1/3)*log(abs(x - (a/b)^(1/3)))/(a*b^2)

$$3.48 \quad \int \frac{a+ax+cx^2}{1-x^3} dx$$

Optimal. Leaf size=32

$$\frac{1}{3}(a-c)\log(x^2+x+1) - \frac{1}{3}(2a+c)\log(1-x)$$

[Out] $-\frac{1}{3}(2a+c)\log(1-x) + \frac{1}{3}(a-c)\log(1+x+x^2)$

Rubi [A] time = 0.0337987, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$, Rules used = {1875, 31, 628}

$$\frac{1}{3}(a-c)\log(x^2+x+1) - \frac{1}{3}(2a+c)\log(1-x)$$

Antiderivative was successfully verified.

[In] Int[(a + a*x + c*x^2)/(1 - x^3), x]

[Out] $-\frac{1}{3}(2a+c)\log(1-x) + \frac{1}{3}(a-c)\log(1+x+x^2)$

Rule 1875

Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = $-(a/b)^{1/3}$ }, Dist[(q*(A + B*q + C*q^2))/(3*a), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q - C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 - b*A^3, 0] && NeQ[A + B*q + C*q^2, 0] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2] && LtQ[a/b, 0]

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}\int \frac{a + ax + cx^2}{1 - x^3} dx &= \frac{1}{3} \int \frac{a - c + (2a - 2c)x}{1 + x + x^2} dx + \frac{1}{3}(2a + c) \int \frac{1}{1 - x} dx \\ &= -\frac{1}{3}(2a + c) \log(1 - x) + \frac{1}{3}(a - c) \log(1 + x + x^2)\end{aligned}$$

Mathematica [A] time = 0.0121035, size = 31, normalized size = 0.97

$$\frac{1}{3} \left((a - c) \log(x^2 + x + 1) - (2a + c) \log(1 - x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*x + c*x^2)/(1 - x^3),x]

[Out] (-((2*a + c)*Log[1 - x]) + (a - c)*Log[1 + x + x^2])/3

Maple [A] time = 0.045, size = 36, normalized size = 1.1

$$-\frac{\ln(-1 + x)c}{3} - \frac{2 \ln(-1 + x)a}{3} + \frac{\ln(x^2 + x + 1)a}{3} - \frac{\ln(x^2 + x + 1)c}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+a*x+a)/(-x^3+1),x)

[Out] -1/3*ln(-1+x)*c-2/3*ln(-1+x)*a+1/3*ln(x^2+x+1)*a-1/3*ln(x^2+x+1)*c

Maxima [A] time = 1.44268, size = 35, normalized size = 1.09

$$\frac{1}{3} (a - c) \log(x^2 + x + 1) - \frac{1}{3} (2a + c) \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a*x+a)/(-x^3+1),x, algorithm="maxima")

[Out] $\frac{1}{3}(a - c)\log(x^2 + x + 1) - \frac{1}{3}(2a + c)\log(x - 1)$

Fricas [A] time = 1.27978, size = 77, normalized size = 2.41

$$\frac{1}{3}(a - c)\log(x^2 + x + 1) - \frac{1}{3}(2a + c)\log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+a*x+a)/(-x^3+1),x, algorithm="fricas")`

[Out] $\frac{1}{3}(a - c)\log(x^2 + x + 1) - \frac{1}{3}(2a + c)\log(x - 1)$

Sympy [A] time = 0.3104, size = 24, normalized size = 0.75

$$\frac{(a - c)\log(x^2 + x + 1)}{3} - \frac{(2a + c)\log(x - 1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+a*x+a)/(-x**3+1),x)`

[Out] $(a - c)\log(x^2 + x + 1)/3 - (2a + c)\log(x - 1)/3$

Giac [A] time = 1.064, size = 36, normalized size = 1.12

$$\frac{1}{3}(a - c)\log(x^2 + x + 1) - \frac{1}{3}(2a + c)\log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+a*x+a)/(-x^3+1),x, algorithm="giac")`

[Out] $\frac{1}{3}(a - c)\log(x^2 + x + 1) - \frac{1}{3}(2a + c)\log(\text{abs}(x - 1))$

$$3.49 \quad \int \frac{a+bx+cx^2}{1-x^3} dx$$

Optimal. Leaf size=55

$$\frac{1}{6} \log(x^2 + x + 1)(a + b - 2c) - \frac{1}{3} \log(1 - x)(a + b + c) + \frac{(a - b) \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] ((a - b)*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - ((a + b + c)*Log[1 - x])/3 + ((a + b - 2*c)*Log[1 + x + x^2])/6

Rubi [A] time = 0.0567216, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {1875, 31, 634, 618, 204, 628}

$$\frac{1}{6} \log(x^2 + x + 1)(a + b - 2c) - \frac{1}{3} \log(1 - x)(a + b + c) + \frac{(a - b) \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2)/(1 - x^3), x]

[Out] ((a - b)*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - ((a + b + c)*Log[1 - x])/3 + ((a + b - 2*c)*Log[1 + x + x^2])/6

Rule 1875

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-a/b)^(1/3)}, Dist[(q*(A + B*q + C*q^2))/(3*a), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q - C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 - b*A^3, 0] && NeQ[A + B*q + C*q^2, 0] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2] && LtQ[a/b, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(1-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[
(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[
(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[
2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[
Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{a + bx + cx^2}{1 - x^3} dx &= \frac{1}{3} \int \frac{2a - b - c + (a + b - 2c)x}{1 + x + x^2} dx + \frac{1}{3}(a + b + c) \int \frac{1}{1 - x} dx \\ &= -\frac{1}{3}(a + b + c) \log(1 - x) + \frac{1}{2}(a - b) \int \frac{1}{1 + x + x^2} dx + \frac{1}{6}(a + b - 2c) \int \frac{1 + 2x}{1 + x + x^2} dx \\ &= -\frac{1}{3}(a + b + c) \log(1 - x) + \frac{1}{6}(a + b - 2c) \log(1 + x + x^2) + (-a + b) \text{Subst} \left(\int \frac{1}{-3 - x^2} dx, x, 1 + x \right) \\ &= \frac{(a - b) \tan^{-1} \left(\frac{1 + 2x}{\sqrt{3}} \right)}{\sqrt{3}} - \frac{1}{3}(a + b + c) \log(1 - x) + \frac{1}{6}(a + b - 2c) \log(1 + x + x^2) \end{aligned}$$

Mathematica [A] time = 0.0333609, size = 62, normalized size = 1.13

$$\frac{1}{6} \left((a + b) \log(x^2 + x + 1) - 2(a + b) \log(1 - x) + 2\sqrt{3}(a - b) \tan^{-1} \left(\frac{2x + 1}{\sqrt{3}} \right) - 2c \log(1 - x^3) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2)/(1 - x^3),x]

[Out] (2*sqrt(3)*(a - b)*ArcTan[(1 + 2*x)/sqrt(3)] - 2*(a + b)*Log[1 - x] + (a + b)*Log[1 + x + x^2] - 2*c*Log[1 - x^3])/6

Maple [A] time = 0.043, size = 87, normalized size = 1.6

$$-\frac{\ln(-1+x)c}{3} - \frac{\ln(-1+x)b}{3} - \frac{\ln(-1+x)a}{3} + \frac{\ln(x^2+x+1)a}{6} + \frac{\ln(x^2+x+1)b}{6} - \frac{\ln(x^2+x+1)c}{3} + \frac{a\sqrt{3}}{3} \arctan\left(\frac{1+2x}{\sqrt{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)/(-x^3+1),x)

[Out] -1/3*ln(-1+x)*c-1/3*ln(-1+x)*b-1/3*ln(-1+x)*a+1/6*ln(x^2+x+1)*a+1/6*ln(x^2+x+1)*b-1/3*ln(x^2+x+1)*c+1/3*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*a-1/3*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*b

Maxima [A] time = 1.45421, size = 63, normalized size = 1.15

$$\frac{1}{3} \sqrt{3}(a-b) \arctan\left(\frac{1}{3} \sqrt{3}(2x+1)\right) + \frac{1}{6}(a+b-2c) \log(x^2+x+1) - \frac{1}{3}(a+b+c) \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)/(-x^3+1),x, algorithm="maxima")

[Out] 1/3*sqrt(3)*(a - b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*(a + b - 2*c)*log(x^2 + x + 1) - 1/3*(a + b + c)*log(x - 1)

Fricas [A] time = 1.34398, size = 158, normalized size = 2.87

$$\frac{1}{3} \sqrt{3}(a-b) \arctan\left(\frac{1}{3} \sqrt{3}(2x+1)\right) + \frac{1}{6}(a+b-2c) \log(x^2+x+1) - \frac{1}{3}(a+b+c) \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)/(-x^3+1),x, algorithm="fricas")

[Out] $\frac{1}{3}\sqrt{3}(a-b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}(a+b-2c)\log(x^2+x+1) - \frac{1}{3}(a+b+c)\log(x-1)$

Sympy [C] time = 0.891344, size = 323, normalized size = 5.87

$$\frac{(a+b+c)\log\left(x + \frac{a^2c - a^2(a+b+c) - 2ab^2 + bc^2 - 2bc(a+b+c) + b(a+b+c)^2}{a^3 - b^3}\right)}{3} - \left(-\frac{a}{6} - \frac{b}{6} + \frac{c}{3} - \frac{\sqrt{3}i(a-b)}{6}\right)\log\left(x + \frac{a^2c - 3a^2\left(-\frac{a}{6} - \frac{b}{6} + \frac{c}{3} - \frac{\sqrt{3}i(a-b)}{6}\right)}{a^3 - b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)/(-x**3+1),x)

[Out] $-(a+b+c)\log(x + (a**2*c - a**2*(a+b+c) - 2*a*b**2 + b*c**2 - 2*b*c*(a+b+c) + b*(a+b+c)**2)/(a**3 - b**3))/3 - (-a/6 - b/6 + c/3 - \sqrt{3}*I*(a-b)/6)*\log(x + (a**2*c - 3*a**2*(-a/6 - b/6 + c/3 - \sqrt{3}*I*(a-b)/6) - 2*a*b**2 + b*c**2 - 6*b*c*(-a/6 - b/6 + c/3 - \sqrt{3}*I*(a-b)/6) + 9*b*(-a/6 - b/6 + c/3 - \sqrt{3}*I*(a-b)/6)**2)/(a**3 - b**3)) - (-a/6 - b/6 + c/3 + \sqrt{3}*I*(a-b)/6)*\log(x + (a**2*c - 3*a**2*(-a/6 - b/6 + c/3 + \sqrt{3}*I*(a-b)/6) - 2*a*b**2 + b*c**2 - 6*b*c*(-a/6 - b/6 + c/3 + \sqrt{3}*I*(a-b)/6) + 9*b*(-a/6 - b/6 + c/3 + \sqrt{3}*I*(a-b)/6)**2)/(a**3 - b**3))$

Giac [A] time = 1.07136, size = 70, normalized size = 1.27

$$\frac{1}{3}(\sqrt{3}a - \sqrt{3}b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}(a+b-2c)\log(x^2+x+1) - \frac{1}{3}(a+b+c)\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)/(-x^3+1),x, algorithm="giac")

[Out] $\frac{1}{3}(\sqrt{3}a - \sqrt{3}b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}(a+b-2c)\log(x^2+x+1) - \frac{1}{3}(a+b+c)\log(\text{abs}(x-1))$

$$3.50 \quad \int \frac{1+x+x^2}{1-x^3} dx$$

Optimal. Leaf size=8

$$-\log(1-x)$$

[Out] -Log[1 - x]

Rubi [A] time = 0.0060494, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1586, 31}

$$-\log(1-x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^2)/(1 - x^3), x]

[Out] -Log[1 - x]

Rule 1586

Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p+q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\int \frac{1+x+x^2}{1-x^3} dx = \int \frac{1}{1-x} dx = -\log(1-x)$$

Mathematica [A] time = 0.0013289, size = 8, normalized size = 1.

$$-\log(1-x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + x^2)/(1 - x^3),x]

[Out] -Log[1 - x]

Maple [A] time = 0.001, size = 7, normalized size = 0.9

$$-\ln(-1 + x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+x+1)/(-x^3+1),x)

[Out] -ln(-1+x)

Maxima [A] time = 0.978024, size = 8, normalized size = 1.

$$-\log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)/(-x^3+1),x, algorithm="maxima")

[Out] -log(x - 1)

Fricas [A] time = 1.19605, size = 18, normalized size = 2.25

$$-\log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)/(-x^3+1),x, algorithm="fricas")

[Out] -log(x - 1)

Sympy [A] time = 0.072375, size = 5, normalized size = 0.62

$$-\log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+x+1)/(-x**3+1),x)

[Out] -log(x - 1)

Giac [A] time = 1.07858, size = 9, normalized size = 1.12

$$-\log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)/(-x^3+1),x, algorithm="giac")

[Out] -log(abs(x - 1))

$$3.51 \quad \int \frac{1-x+3x^2}{1-x^3} dx$$

Optimal. Leaf size=30

$$\frac{2 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \log(1-x^3)$$

[Out] (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - Log[1 - x^3]

Rubi [A] time = 0.0302431, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1871, 1586, 618, 204, 260}

$$\frac{2 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \log(1-x^3)$$

Antiderivative was successfully verified.

[In] Int[(1 - x + 3*x^2)/(1 - x^3), x]

[Out] (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - Log[1 - x^3]

Rule 1871

Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Dist[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

Rule 1586

Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p+q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

`x] && NeQ[b^2 - 4*a*c, 0]`

Rule 204

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

Rule 260

`Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]`

Rubi steps

$$\begin{aligned} \int \frac{1-x+3x^2}{1-x^3} dx &= 3 \int \frac{x^2}{1-x^3} dx + \int \frac{1-x}{1-x^3} dx \\ &= -\log(1-x^3) + \int \frac{1}{1+x+x^2} dx \\ &= -\log(1-x^3) - 2 \operatorname{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1+2x\right) \\ &= \frac{2 \tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \log(1-x^3) \end{aligned}$$

Mathematica [A] time = 0.008519, size = 30, normalized size = 1.

$$\frac{2 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \log(1-x^3)$$

Antiderivative was successfully verified.

`[In] Integrate[(1 - x + 3*x^2)/(1 - x^3), x]`

`[Out] (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - Log[1 - x^3]`

Maple [A] time = 0.005, size = 33, normalized size = 1.1

$$-\ln(-1+x) - \ln(x^2+x+1) + \frac{2\sqrt{3}}{3} \arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x^2-x+1)/(-x^3+1),x)`

[Out] $-\ln(-1+x)-\ln(x^2+x+1)+2/3*\arctan(1/3*(1+2*x)*3^{(1/2)})*3^{(1/2)}$

Maxima [A] time = 1.44615, size = 43, normalized size = 1.43

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\log(x^2+x+1)-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2-x+1)/(-x^3+1),x, algorithm="maxima")`

[Out] $2/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x + 1)) - \log(x^2 + x + 1) - \log(x - 1)$

Fricas [A] time = 1.23767, size = 101, normalized size = 3.37

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\log(x^2+x+1)-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2-x+1)/(-x^3+1),x, algorithm="fricas")`

[Out] $2/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x + 1)) - \log(x^2 + x + 1) - \log(x - 1)$

Sympy [A] time = 0.121207, size = 5, normalized size = 0.17

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x**2-x+1)/(-x**3+1),x)`

[Out] $-\log(x - 1)$

Giac [A] time = 1.06748, size = 45, normalized size = 1.5

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \log(x^2+x+1) - \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2-x+1)/(-x^3+1),x, algorithm="giac")`

[Out] $2/3*\text{sqrt}(3)*\text{arctan}(1/3*\text{sqrt}(3)*(2*x + 1)) - \log(x^2 + x + 1) - \log(\text{abs}(x - 1))$

$$3.52 \quad \int \frac{1+x+4x^2}{1-x^3} dx$$

Optimal. Leaf size=18

$$-\log(x^2 + x + 1) - 2\log(1 - x)$$

[Out] -2*Log[1 - x] - Log[1 + x + x^2]

Rubi [A] time = 0.0188959, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1875, 31, 628}

$$-\log(x^2 + x + 1) - 2\log(1 - x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x + 4*x^2)/(1 - x^3),x]

[Out] -2*Log[1 - x] - Log[1 + x + x^2]

Rule 1875

Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-(a/b))^(1/3)}, Dist[(q*(A + B*q + C*q^2))/(3*a), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q - C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 - b*A^3, 0] && NeQ[A + B*q + C*q^2, 0] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2] && LtQ[a/b, 0]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\int \frac{1+x+4x^2}{1-x^3} dx = \frac{1}{3} \int \frac{-3-6x}{1+x+x^2} dx + 2 \int \frac{1}{1-x} dx$$

$$= -2 \log(1-x) - \log(1+x+x^2)$$

Mathematica [A] time = 0.0050965, size = 18, normalized size = 1.

$$-\log(x^2 + x + 1) - 2 \log(1 - x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + 4*x^2)/(1 - x^3), x]

[Out] -2*Log[1 - x] - Log[1 + x + x^2]

Maple [A] time = 0.004, size = 17, normalized size = 0.9

$$-2 \ln(-1 + x) - \ln(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^2+x+1)/(-x^3+1), x)

[Out] -2*ln(-1+x)-ln(x^2+x+1)

Maxima [A] time = 1.44261, size = 22, normalized size = 1.22

$$-\log(x^2 + x + 1) - 2 \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2+x+1)/(-x^3+1), x, algorithm="maxima")

[Out] -log(x^2 + x + 1) - 2*log(x - 1)

Fricas [A] time = 1.4008, size = 46, normalized size = 2.56

$$-\log(x^2 + x + 1) - 2 \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2+x+1)/(-x^3+1),x, algorithm="fricas")

[Out] -log(x^2 + x + 1) - 2*log(x - 1)

Sympy [A] time = 0.112162, size = 15, normalized size = 0.83

$$-2 \log(x - 1) - \log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x**2+x+1)/(-x**3+1),x)

[Out] -2*log(x - 1) - log(x**2 + x + 1)

Giac [A] time = 1.07339, size = 23, normalized size = 1.28

$$-\log(x^2 + x + 1) - 2 \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2+x+1)/(-x^3+1),x, algorithm="giac")

[Out] -log(x^2 + x + 1) - 2*log(abs(x - 1))

3.53 $\int (a + bx^3)^3 (ac + adx + bcx^3 + bdx^4) dx$

Optimal. Leaf size=113

$$\frac{6}{7}a^2b^2cx^7 + \frac{3}{4}a^2b^2dx^8 + a^3bcx^4 + \frac{4}{5}a^3bdx^5 + a^4cx + \frac{1}{2}a^4dx^2 + \frac{2}{5}ab^3cx^{10} + \frac{4}{11}ab^3dx^{11} + \frac{1}{13}b^4cx^{13} + \frac{1}{14}b^4dx^{14}$$

[Out] $a^4cx + (a^4dx^2)/2 + a^3bcx^4 + (4a^3bdx^5)/5 + (6a^2b^2cx^7)/7 + (3a^2b^2dx^8)/4 + (2a^3bcx^{10})/5 + (4a^3bdx^{11})/11 + (b^4cx^{13})/13 + (b^4dx^{14})/14$

Rubi [A] time = 0.0986718, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {1850}

$$\frac{6}{7}a^2b^2cx^7 + \frac{3}{4}a^2b^2dx^8 + a^3bcx^4 + \frac{4}{5}a^3bdx^5 + a^4cx + \frac{1}{2}a^4dx^2 + \frac{2}{5}ab^3cx^{10} + \frac{4}{11}ab^3dx^{11} + \frac{1}{13}b^4cx^{13} + \frac{1}{14}b^4dx^{14}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*x^3)^3*(a*c + a*d*x + b*c*x^3 + b*d*x^4), x]$

[Out] $a^4cx + (a^4dx^2)/2 + a^3bcx^4 + (4a^3bdx^5)/5 + (6a^2b^2cx^7)/7 + (3a^2b^2dx^8)/4 + (2a^3bcx^{10})/5 + (4a^3bdx^{11})/11 + (b^4cx^{13})/13 + (b^4dx^{14})/14$

Rule 1850

$\text{Int}[(Pq_*)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[Pq*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, n\}, x \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ (\text{IGtQ}[p, 0] \ || \ \text{EqQ}[n, 1])$

Rubi steps

$$\begin{aligned} \int (a + bx^3)^3 (ac + adx + bcx^3 + bdx^4) dx &= \int (a^4c + a^4dx + 4a^3bcx^3 + 4a^3bdx^4 + 6a^2b^2cx^6 + 6a^2b^2dx^7 + 4ab^3cx^9 + 4a^4c \\ &= a^4cx + \frac{1}{2}a^4dx^2 + a^3bcx^4 + \frac{4}{5}a^3bdx^5 + \frac{6}{7}a^2b^2cx^7 + \frac{3}{4}a^2b^2dx^8 + \frac{2}{5}ab^3cx^{10} + \dots \end{aligned}$$

Mathematica [A] time = 0.0039217, size = 113, normalized size = 1.

$$\frac{6}{7}a^2b^2cx^7 + \frac{3}{4}a^2b^2dx^8 + a^3bcx^4 + \frac{4}{5}a^3bdx^5 + a^4cx + \frac{1}{2}a^4dx^2 + \frac{2}{5}ab^3cx^{10} + \frac{4}{11}ab^3dx^{11} + \frac{1}{13}b^4cx^{13} + \frac{1}{14}b^4dx^{14}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3)^3*(a*c + a*d*x + b*c*x^3 + b*d*x^4), x]

[Out] a^4*c*x + (a^4*d*x^2)/2 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5 + (6*a^2*b^2*c*x^7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a*b^3*c*x^10)/5 + (4*a*b^3*d*x^11)/11 + (b^4*c*x^13)/13 + (b^4*d*x^14)/14

Maple [A] time = 0.001, size = 98, normalized size = 0.9

$$a^4cx + \frac{a^4dx^2}{2} + a^3bcx^4 + \frac{4a^3bdx^5}{5} + \frac{6a^2b^2cx^7}{7} + \frac{3a^2b^2dx^8}{4} + \frac{2ab^3cx^{10}}{5} + \frac{4ab^3dx^{11}}{11} + \frac{b^4cx^{13}}{13} + \frac{b^4dx^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^3*(b*d*x^4+b*c*x^3+a*d*x+a*c), x)

[Out] a^4*c*x+1/2*a^4*d*x^2+a^3*b*c*x^4+4/5*a^3*b*d*x^5+6/7*a^2*b^2*c*x^7+3/4*a^2*b^2*d*x^8+2/5*a*b^3*c*x^10+4/11*a*b^3*d*x^11+1/13*b^4*c*x^13+1/14*b^4*d*x^14

Maxima [A] time = 0.985302, size = 131, normalized size = 1.16

$$\frac{1}{14}b^4dx^{14} + \frac{1}{13}b^4cx^{13} + \frac{4}{11}ab^3dx^{11} + \frac{2}{5}ab^3cx^{10} + \frac{3}{4}a^2b^2dx^8 + \frac{6}{7}a^2b^2cx^7 + \frac{4}{5}a^3bdx^5 + a^3bcx^4 + \frac{1}{2}a^4dx^2 + a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^3*(b*d*x^4+b*c*x^3+a*d*x+a*c), x, algorithm="maxima")

[Out] 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 4/11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 + 3/4*a^2*b^2*d*x^8 + 6/7*a^2*b^2*c*x^7 + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 + 1/2*a^4*d*x^2 + a^4*c*x

Fricas [A] time = 1.11782, size = 232, normalized size = 2.05

$$\frac{1}{14}x^{14}db^4 + \frac{1}{13}x^{13}cb^4 + \frac{4}{11}x^{11}db^3a + \frac{2}{5}x^{10}cb^3a + \frac{3}{4}x^8db^2a^2 + \frac{6}{7}x^7cb^2a^2 + \frac{4}{5}x^5dba^3 + x^4cba^3 + \frac{1}{2}x^2da^4 + xca^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^3*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas")

[Out] 1/14*x^14*d*b^4 + 1/13*x^13*c*b^4 + 4/11*x^11*d*b^3*a + 2/5*x^10*c*b^3*a + 3/4*x^8*d*b^2*a^2 + 6/7*x^7*c*b^2*a^2 + 4/5*x^5*d*b*a^3 + x^4*c*b*a^3 + 1/2*x^2*d*a^4 + x*c*a^4

Sympy [A] time = 0.087344, size = 117, normalized size = 1.04

$$a^4cx + \frac{a^4dx^2}{2} + a^3bcx^4 + \frac{4a^3bdx^5}{5} + \frac{6a^2b^2cx^7}{7} + \frac{3a^2b^2dx^8}{4} + \frac{2ab^3cx^{10}}{5} + \frac{4ab^3dx^{11}}{11} + \frac{b^4cx^{13}}{13} + \frac{b^4dx^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**3*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)

[Out] a**4*c*x + a**4*d*x**2/2 + a**3*b*c*x**4 + 4*a**3*b*d*x**5/5 + 6*a**2*b**2*c*x**7/7 + 3*a**2*b**2*d*x**8/4 + 2*a*b**3*c*x**10/5 + 4*a*b**3*d*x**11/11 + b**4*c*x**13/13 + b**4*d*x**14/14

Giac [A] time = 1.08494, size = 131, normalized size = 1.16

$$\frac{1}{14}b^4dx^{14} + \frac{1}{13}b^4cx^{13} + \frac{4}{11}ab^3dx^{11} + \frac{2}{5}ab^3cx^{10} + \frac{3}{4}a^2b^2dx^8 + \frac{6}{7}a^2b^2cx^7 + \frac{4}{5}a^3bdx^5 + a^3bcx^4 + \frac{1}{2}a^4dx^2 + a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^3*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")

[Out] 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 4/11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 + 3/4*a^2*b^2*d*x^8 + 6/7*a^2*b^2*c*x^7 + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 + 1/2*a^4*d*x^2 + a^4*c*x

$$3.54 \quad \int (a + bx^3)^2 (ac + adx + bcx^3 + bdx^4) dx$$

Optimal. Leaf size=88

$$\frac{3}{4}a^2bcx^4 + \frac{3}{5}a^2bdx^5 + a^3cx + \frac{1}{2}a^3dx^2 + \frac{3}{7}ab^2cx^7 + \frac{3}{8}ab^2dx^8 + \frac{1}{10}b^3cx^{10} + \frac{1}{11}b^3dx^{11}$$

[Out] $a^3cx + (a^3d*x^2)/2 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^5)/5 + (3*a*b^2*c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (b^3*c*x^{10})/10 + (b^3*d*x^{11})/11$

Rubi [A] time = 0.0602554, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {1850}

$$\frac{3}{4}a^2bcx^4 + \frac{3}{5}a^2bdx^5 + a^3cx + \frac{1}{2}a^3dx^2 + \frac{3}{7}ab^2cx^7 + \frac{3}{8}ab^2dx^8 + \frac{1}{10}b^3cx^{10} + \frac{1}{11}b^3dx^{11}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3)^2*(a*c + a*d*x + b*c*x^3 + b*d*x^4), x]

[Out] $a^3cx + (a^3d*x^2)/2 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^5)/5 + (3*a*b^2*c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (b^3*c*x^{10})/10 + (b^3*d*x^{11})/11$

Rule 1850

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand [Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int (a + bx^3)^2 (ac + adx + bcx^3 + bdx^4) dx &= \int (a^3c + a^3dx + 3a^2bcx^3 + 3a^2bdx^4 + 3ab^2cx^6 + 3ab^2dx^7 + b^3cx^9 + b^3dx^{11}) dx \\ &= a^3cx + \frac{1}{2}a^3dx^2 + \frac{3}{4}a^2bcx^4 + \frac{3}{5}a^2bdx^5 + \frac{3}{7}ab^2cx^7 + \frac{3}{8}ab^2dx^8 + \frac{1}{10}b^3cx^{10} + \frac{1}{11}b^3dx^{11} \end{aligned}$$

Mathematica [A] time = 0.0026354, size = 88, normalized size = 1.

$$\frac{3}{4}a^2bcx^4 + \frac{3}{5}a^2bdx^5 + a^3cx + \frac{1}{2}a^3dx^2 + \frac{3}{7}ab^2cx^7 + \frac{3}{8}ab^2dx^8 + \frac{1}{10}b^3cx^{10} + \frac{1}{11}b^3dx^{11}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3)^2*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]

[Out] a^3*c*x + (a^3*d*x^2)/2 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^5)/5 + (3*a*b^2*c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11

Maple [A] time = 0.001, size = 75, normalized size = 0.9

$$a^3cx + \frac{a^3dx^2}{2} + \frac{3a^2bcx^4}{4} + \frac{3a^2bdx^5}{5} + \frac{3ab^2cx^7}{7} + \frac{3ab^2dx^8}{8} + \frac{b^3cx^{10}}{10} + \frac{b^3dx^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^2*(b*d*x^4+b*c*x^3+a*d*x+a*c),x)

[Out] a^3*c*x+1/2*a^3*d*x^2+3/4*a^2*b*c*x^4+3/5*a^2*b*d*x^5+3/7*a*b^2*c*x^7+3/8*a*b^2*d*x^8+1/10*b^3*c*x^10+1/11*b^3*d*x^11

Maxima [A] time = 0.960131, size = 100, normalized size = 1.14

$$\frac{1}{11}b^3dx^{11} + \frac{1}{10}b^3cx^{10} + \frac{3}{8}ab^2dx^8 + \frac{3}{7}ab^2cx^7 + \frac{3}{5}a^2bdx^5 + \frac{3}{4}a^2bcx^4 + \frac{1}{2}a^3dx^2 + a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima")

[Out] 1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 3/8*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 3/5*a^2*b*d*x^5 + 3/4*a^2*b*c*x^4 + 1/2*a^3*d*x^2 + a^3*c*x

Fricas [A] time = 1.10836, size = 180, normalized size = 2.05

$$\frac{1}{11}x^{11}db^3 + \frac{1}{10}x^{10}cb^3 + \frac{3}{8}x^8db^2a + \frac{3}{7}x^7cb^2a + \frac{3}{5}x^5dba^2 + \frac{3}{4}x^4cba^2 + \frac{1}{2}x^2da^3 + xca^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas")

[Out] $\frac{1}{11}x^{11}db^3 + \frac{1}{10}x^{10}c^3b^3 + \frac{3}{8}x^8d^2b^2a + \frac{3}{7}x^7c^2b^2a + \frac{3}{5}x^5d^2ba^2 + \frac{3}{4}x^4c^2ba^2 + \frac{1}{2}x^2d^2a^3 + xca^3$

Sympy [A] time = 0.078548, size = 90, normalized size = 1.02

$$a^3cx + \frac{a^3dx^2}{2} + \frac{3a^2bcx^4}{4} + \frac{3a^2bdx^5}{5} + \frac{3ab^2cx^7}{7} + \frac{3ab^2dx^8}{8} + \frac{b^3cx^{10}}{10} + \frac{b^3dx^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**2*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)

[Out] $a**3*c*x + a**3*d*x**2/2 + 3*a**2*b*c*x**4/4 + 3*a**2*b*d*x**5/5 + 3*a*b**2*c*x**7/7 + 3*a*b**2*d*x**8/8 + b**3*c*x**10/10 + b**3*d*x**11/11$

Giac [A] time = 1.08721, size = 100, normalized size = 1.14

$$\frac{1}{11}b^3dx^{11} + \frac{1}{10}b^3cx^{10} + \frac{3}{8}ab^2dx^8 + \frac{3}{7}ab^2cx^7 + \frac{3}{5}a^2bdx^5 + \frac{3}{4}a^2bcx^4 + \frac{1}{2}a^3dx^2 + a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")

[Out] $\frac{1}{11}b^3d^2x^{11} + \frac{1}{10}b^3c^2x^{10} + \frac{3}{8}a^2b^2d^2x^8 + \frac{3}{7}a^2b^2c^2x^7 + \frac{3}{5}a^2b^2d^2x^5 + \frac{3}{4}a^2b^2c^2x^4 + \frac{1}{2}a^3d^2x^2 + a^3c^2x$

3.55 $\int (a + bx^3)(ac + adx + bcx^3 + bdx^4) dx$

Optimal. Leaf size=60

$$a^2cx + \frac{1}{2}a^2dx^2 + \frac{1}{2}abcx^4 + \frac{2}{5}abdx^5 + \frac{1}{7}b^2cx^7 + \frac{1}{8}b^2dx^8$$

[Out] $a^2c*x + (a^2*d*x^2)/2 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (b^2*c*x^7)/7 + (b^2*d*x^8)/8$

Rubi [A] time = 0.0380882, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1850}

$$a^2cx + \frac{1}{2}a^2dx^2 + \frac{1}{2}abcx^4 + \frac{2}{5}abdx^5 + \frac{1}{7}b^2cx^7 + \frac{1}{8}b^2dx^8$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3)*(a*c + a*d*x + b*c*x^3 + b*d*x^4), x]

[Out] $a^2c*x + (a^2*d*x^2)/2 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (b^2*c*x^7)/7 + (b^2*d*x^8)/8$

Rule 1850

Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int (a + bx^3)(ac + adx + bcx^3 + bdx^4) dx &= \int (a^2c + a^2dx + 2abcx^3 + 2abdx^4 + b^2cx^6 + b^2dx^7) dx \\ &= a^2cx + \frac{1}{2}a^2dx^2 + \frac{1}{2}abcx^4 + \frac{2}{5}abdx^5 + \frac{1}{7}b^2cx^7 + \frac{1}{8}b^2dx^8 \end{aligned}$$

Mathematica [A] time = 0.0021509, size = 60, normalized size = 1.

$$a^2cx + \frac{1}{2}a^2dx^2 + \frac{1}{2}abcx^4 + \frac{2}{5}abdx^5 + \frac{1}{7}b^2cx^7 + \frac{1}{8}b^2dx^8$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]

[Out] a^2*c*x + (a^2*d*x^2)/2 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (b^2*c*x^7)/7 + (b^2*d*x^8)/8

Maple [A] time = 0.001, size = 51, normalized size = 0.9

$$a^2cx + \frac{a^2dx^2}{2} + \frac{abcx^4}{2} + \frac{2abdx^5}{5} + \frac{b^2cx^7}{7} + \frac{b^2dx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x)

[Out] a^2*c*x+1/2*a^2*d*x^2+1/2*a*b*c*x^4+2/5*a*b*d*x^5+1/7*b^2*c*x^7+1/8*b^2*d*x^8

Maxima [A] time = 0.990225, size = 68, normalized size = 1.13

$$\frac{1}{8}b^2dx^8 + \frac{1}{7}b^2cx^7 + \frac{2}{5}abdx^5 + \frac{1}{2}abcx^4 + \frac{1}{2}a^2dx^2 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima")

[Out] 1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 2/5*a*b*d*x^5 + 1/2*a*b*c*x^4 + 1/2*a^2*d*x^2 + a^2*c*x

Fricas [A] time = 1.08037, size = 120, normalized size = 2.

$$\frac{1}{8}x^8db^2 + \frac{1}{7}x^7cb^2 + \frac{2}{5}x^5dba + \frac{1}{2}x^4cba + \frac{1}{2}x^2da^2 + xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas")

[Out] $\frac{1}{8}x^8db^2 + \frac{1}{7}x^7c^2b^2 + \frac{2}{5}x^5d^2ba + \frac{1}{2}x^4c^2ba + \frac{1}{2}x^2d^2a^2 + xca^2$

Sympy [A] time = 0.076342, size = 58, normalized size = 0.97

$$a^2cx + \frac{a^2dx^2}{2} + \frac{abcx^4}{2} + \frac{2abdx^5}{5} + \frac{b^2cx^7}{7} + \frac{b^2dx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)

[Out] $a^2cx + a^2dx^2/2 + abcx^4/2 + 2abdx^5/5 + b^2cx^7/7 + b^2dx^8/8$

Giac [A] time = 1.06269, size = 68, normalized size = 1.13

$$\frac{1}{8}b^2dx^8 + \frac{1}{7}b^2cx^7 + \frac{2}{5}abdx^5 + \frac{1}{2}abcx^4 + \frac{1}{2}a^2dx^2 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")

[Out] $\frac{1}{8}b^2dx^8 + \frac{1}{7}b^2cx^7 + \frac{2}{5}abdx^5 + \frac{1}{2}ab^2cx^4 + \frac{1}{2}a^2dx^2 + a^2cx$

$$3.56 \quad \int \frac{ac+adx+bcx^3+bdx^4}{a+bx^3} dx$$

Optimal. Leaf size=12

$$cx + \frac{dx^2}{2}$$

[Out] c*x + (d*x^2)/2

Rubi [A] time = 0.0138278, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {1586}

$$cx + \frac{dx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3), x]

[Out] c*x + (d*x^2)/2

Rule 1586

Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rubi steps

$$\begin{aligned} \int \frac{ac + adx + bcx^3 + bdx^4}{a + bx^3} dx &= \int (c + dx) dx \\ &= cx + \frac{dx^2}{2} \end{aligned}$$

Mathematica [A] time = 0.0007036, size = 12, normalized size = 1.

$$cx + \frac{dx^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3),x]

[Out] c*x + (d*x^2)/2

Maple [A] time = 0.002, size = 11, normalized size = 0.9

$$cx + \frac{dx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a),x)

[Out] c*x+1/2*d*x^2

Maxima [A] time = 0.949419, size = 14, normalized size = 1.17

$$\frac{1}{2} dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a),x, algorithm="maxima")

[Out] 1/2*d*x^2 + c*x

Fricas [A] time = 1.25331, size = 23, normalized size = 1.92

$$\frac{1}{2} dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a),x, algorithm="fricas")

[Out] $1/2*d*x^2 + c*x$

Sympy [A] time = 0.069176, size = 8, normalized size = 0.67

$$cx + \frac{dx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a), x)`

[Out] $c*x + d*x**2/2$

Giac [A] time = 1.06273, size = 14, normalized size = 1.17

$$\frac{1}{2}dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a), x, algorithm="giac")`

[Out] $1/2*d*x^2 + c*x$

$$3.57 \quad \int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^2} dx$$

Optimal. Leaf size=161

$$\frac{\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}} + \frac{\left(\sqrt[3]{bc} - \sqrt[3]{ad}\right) \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{3a^{2/3}b^{2/3}} - \frac{\left(\sqrt[3]{ad} + \sqrt[3]{bc}\right) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}$$

[Out] -(((b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(2/3)) - ((c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))

Rubi [A] time = 0.0979056, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1586, 1860, 31, 634, 617, 204, 628}

$$\frac{\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}} + \frac{\left(\sqrt[3]{bc} - \sqrt[3]{ad}\right) \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{3a^{2/3}b^{2/3}} - \frac{\left(\sqrt[3]{ad} + \sqrt[3]{bc}\right) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^2,x]

[Out] -(((b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(2/3)) - ((c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))

Rule 1586

Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 1860

Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*

s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^2} dx &= \int \frac{c + dx}{a + bx^3} dx \\
&= \frac{\int \frac{\sqrt[3]{a}(2\sqrt[3]{bc} + \sqrt[3]{ad}) + \sqrt[3]{b}(-\sqrt[3]{bc} + \sqrt[3]{ad})x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} dx}{3a^{2/3}\sqrt[3]{b}} + \frac{\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{bx}} dx}{3a^{2/3}} \\
&= \frac{\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{2/3}\sqrt[3]{b}} - \frac{(\sqrt[3]{bc} - \sqrt[3]{ad}) \int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} dx}{6a^{2/3}b^{2/3}} + \frac{1}{2} \left(\frac{c}{\sqrt[3]{a}} + \frac{d}{\sqrt[3]{b}}\right) \int \frac{1}{a^{2/3} + \sqrt[3]{a}\sqrt[3]{bx}} dx \\
&= \frac{\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{2/3}\sqrt[3]{b}} - \frac{(\sqrt[3]{bc} - \sqrt[3]{ad}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{6a^{2/3}b^{2/3}} + \frac{(\sqrt[3]{bc} + \sqrt[3]{ad})}{6a^{2/3}b^{2/3}} \\
&= -\frac{(\sqrt[3]{bc} + \sqrt[3]{ad}) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}} + \frac{\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{2/3}\sqrt[3]{b}} - \frac{(\sqrt[3]{bc} - \sqrt[3]{ad}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{6a^{2/3}b^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.0450009, size = 124, normalized size = 0.77

$$\frac{(\sqrt[3]{bc} - \sqrt[3]{ad}) \left(2 \log(\sqrt[3]{a} + \sqrt[3]{bx}) - \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)\right) - 2\sqrt{3}(\sqrt[3]{ad} + \sqrt[3]{bc}) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6a^{2/3}b^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^2,x]

[Out] (-2*Sqrt[3]*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + (b^(1/3)*c - a^(1/3)*d)*(2*Log[a^(1/3) + b^(1/3)*x] - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(2/3)*b^(2/3))

Maple [A] time = 0.002, size = 186, normalized size = 1.2

$$\frac{c}{3b} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{c}{6b} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{c\sqrt{3}}{3b} \arctan\left(\frac{\sqrt{3}}{3} \left(2x \frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{d}{3b} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^2,x)$

[Out] $\frac{1}{3}c/b/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})-1/6*c/b/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})+1/3*c/b/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))-1/3*d/b/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})+1/6*d/b/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})+1/3*d*3^{(1/2)}/b/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^2,x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

Fricas [C] time = 8.10478, size = 4590, normalized size = 28.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^2,x, \text{algorithm}="fricas")$

[Out]
$$-1/6*((1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^{(1/3)} - 2*(1/2)^{(2/3)}*c*d*(-I*\text{sqrt}(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^{(1/3)}))*\log(1/4*((1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^{(1/3)} - 2*(1/2)^{(2/3)}*c*d*(-I*\text{sqrt}(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^{(1/3)}))^2*a^2*b*d - 1/2*((1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^{(1/3)} - 2*(1/2)^{(2/3)}*c*d*(-I*\text{sqrt}(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^{(1/3)}))*a*b*c^2 + 2*a*c*d^2 + (b*c^3 + a*d^3)*x) + 1/12*((1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^{(1/3)} - 2*(1/2)^{(2/3)}*c*d*(-I*\text{sqrt}(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^{(1/3)})) + 3*\text{sqrt}(1/3)*\text{sqrt}(-((1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^{(1/3)}))$$


```
[Out] RootSum(27*_t**3*a**2*b**2 + 9*_t*a*b*c*d + a*d**3 - b*c**3, Lambda(_t, _t*
log(x + (9*_t**2*a**2*b*d + 3*_t*a*b*c**2 + 2*a*c*d**2)/(a*d**3 + b*c**3)))
)
```

Giac [A] time = 1.08031, size = 216, normalized size = 1.34

$$\frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}bc - \left(-ab^2\right)^{\frac{2}{3}}d\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^2} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}ab^3c + \left(-ab^2\right)^{\frac{2}{3}}d\right) \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*(d*(-a/b)^(1/3) + c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a + 1/3*sqrt(3)*((-a*b^2)^(1/3)*b*c - (-a*b^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) + 1/6*((-a*b^2)^(1/3)*a*b^3*c + (-a*b^2)^(2/3)*a*b^2*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^4)
```

$$3.58 \quad \int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^3} dx$$

Optimal. Leaf size=189

$$-\frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{18a^{5/3}b^{2/3}} + \frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{5/3}b^{2/3}} - \frac{(\sqrt[3]{ad} + 2\sqrt[3]{bc}) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} + \frac{x}{3a}$$

[Out] (x*(c + d*x))/(3*a*(a + b*x^3)) - ((2*b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(5/3)*b^(2/3)) + ((2*b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x]/(9*a^(5/3)*b^(2/3)) - ((2*b^(1/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(18*a^(5/3)*b^(2/3))

Rubi [A] time = 0.126884, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1586, 1855, 1860, 31, 634, 617, 204, 628}

$$-\frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{18a^{5/3}b^{2/3}} + \frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{5/3}b^{2/3}} - \frac{(\sqrt[3]{ad} + 2\sqrt[3]{bc}) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} + \frac{x}{3a}$$

Antiderivative was successfully verified.

[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^3,x]

[Out] (x*(c + d*x))/(3*a*(a + b*x^3)) - ((2*b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(5/3)*b^(2/3)) + ((2*b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x]/(9*a^(5/3)*b^(2/3)) - ((2*b^(1/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(18*a^(5/3)*b^(2/3))

Rule 1586

Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 1855

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
+ 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^3} dx &= \int \frac{c + dx}{(a + bx^3)^2} dx \\
&= \frac{x(c + dx)}{3a(a + bx^3)} - \frac{\int \frac{-2c - dx}{a + bx^3} dx}{3a} \\
&= \frac{x(c + dx)}{3a(a + bx^3)} - \frac{\int \frac{\sqrt[3]{a}(-4\sqrt[3]{bc} - \sqrt[3]{ad}) + \sqrt[3]{b}(2\sqrt[3]{bc} - \sqrt[3]{ad})x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} dx}{9a^{5/3}\sqrt[3]{b}} + \frac{(2c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{bx}} dx}{9a^{5/3}} \\
&= \frac{x(c + dx)}{3a(a + bx^3)} + \frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{5/3}b^{2/3}} - \frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} dx}{18a^{5/3}b^{2/3}} + \\
&= \frac{x(c + dx)}{3a(a + bx^3)} + \frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{5/3}b^{2/3}} - \frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3})}{18a^{5/3}b^{2/3}} \\
&= \frac{x(c + dx)}{3a(a + bx^3)} - \frac{(2\sqrt[3]{bc} + \sqrt[3]{ad}) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} + \frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{5/3}b^{2/3}} - \frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3})}{18a^{5/3}b^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.147975, size = 180, normalized size = 0.95

$$\frac{\frac{(a^{2/3}d - 2\sqrt[3]{a}\sqrt[3]{bc}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{b^{2/3}} + \frac{2(2\sqrt[3]{a}\sqrt[3]{bc} - a^{2/3}d) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{b^{2/3}} - \frac{2\sqrt{3}\sqrt[3]{a}(\sqrt[3]{ad} + 2\sqrt[3]{bc}) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{2/3}} + \frac{6ax(c + dx)}{a + bx^3}}{18a^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^3, x]

[Out] ((6*a*x*(c + d*x))/(a + b*x^3) - (2*sqrt[3]*a^(1/3)*(2*b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]])/b^(2/3) + (2*(2*a^(1/3)*b^(1/3)*c - a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((-2*a^(1/3)*b^(1/3)*c + a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(18*a^2)

Maple [A] time = 0.004, size = 238, normalized size = 1.3

$$\frac{cx}{3a(bx^3+a)} + \frac{2c}{9ab} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{c}{9ab} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{2c\sqrt{3}}{9ab} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^3,x)

[Out] 1/3*c*x/a/(b*x^3+a)+2/9*c/a/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/9*c/a/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2/9*c/a/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/3*d*x^2/a/(b*x^3+a)-1/9*d/a/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/18*d/a/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/9*d/a*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [C] time = 7.99868, size = 4961, normalized size = 26.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] 1/36*(12*d*x^2 - 2*(a*b*x^3 + a^2)*((1/2)^(1/3)*(I*sqrt(3) + 1))*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3))*log(1/4*((1/2)^(1/3)*(I*sqrt(3) + 1))*((8*b*c^3 + a*d^3)/(a^5*b^2))^(1/3))

Sympy [A] time = 1.11114, size = 105, normalized size = 0.56

$$\text{RootSum}\left(729t^3a^5b^2 + 54ta^2bcd + ad^3 - 8bc^3, \left(t \mapsto t \log\left(x + \frac{81t^2a^4bd + 36ta^2bc^2 + 4acd^2}{ad^3 + 8bc^3}\right)\right)\right) + \frac{cx + dx^2}{3a^2 + 3abx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**3,x)

[Out] RootSum(729*_t**3*a**5*b**2 + 54*_t*a**2*b*c*d + a*d**3 - 8*b*c**3, Lambda(_t, _t*log(x + (81*_t**2*a**4*b*d + 36*_t*a**2*b*c**2 + 4*a*c*d**2)/(a*d**3 + 8*b*c**3)))) + (c*x + d*x**2)/(3*a**2 + 3*a*b*x**3)

Giac [A] time = 1.08426, size = 252, normalized size = 1.33

$$\frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 2c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^2} + \frac{dx^2 + cx}{3(bx^3 + a)a} + \frac{\sqrt{3}\left(2\left(-ab^2\right)^{\frac{1}{3}}bc - \left(-ab^2\right)^{\frac{2}{3}}d\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^2b^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^3,x, algorithm="giac")

[Out] -1/9*(d*(-a/b)^(1/3) + 2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1/3*(d*x^2 + c*x)/((b*x^3 + a)*a) + 1/9*sqrt(3)*(2*(-a*b^2)^(1/3)*b*c - (-a*b^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^2) + 1/18*(2*(-a*b^2)^(1/3)*a*b^3*c + (-a*b^2)^(2/3)*a*b^2*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^4)

3.59 $\int (a + bx^3)^{3/2} (ac + adx + bcx^3 + bdx^4) dx$

Optimal. Leaf size=585

$$54 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} a^3 (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (1729 \sqrt[3]{bc} - 935 (1 - \sqrt{3}) \sqrt[3]{ad}) \text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{(1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}\right), -7 - \frac{323323 b^{2/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}}{323323 b^{2/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}}}{323323 b^{2/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}}}\right)$$

[Out] (810*a^3*d*Sqrt[a + b*x^3])/(1729*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (54*a^2*(1729*c*x + 935*d*x^2)*Sqrt[a + b*x^3])/323323 + (30*a*(247*c*x + 187*d*x^2)*(a + b*x^3)^(3/2))/46189 + (2*(19*c*x + 17*d*x^2)*(a + b*x^3)^(5/2))/323 - (405*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(10/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(1729*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (54*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^3*(1729*b^(1/3)*c - 935*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(323323*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 0.463832, antiderivative size = 585, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1852, 1853, 1878, 218, 1877}

$$54 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} a^3 (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (1729 \sqrt[3]{bc} - 935 (1 - \sqrt{3}) \sqrt[3]{ad}) F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx} + (1-\sqrt{3}) \sqrt[3]{a}}{\sqrt[3]{bx} + (1+\sqrt{3}) \sqrt[3]{a}}\right) \mid -7 - 4\sqrt{3}\right)$$

$$323323 b^{2/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3)^(3/2)*(a*c + a*d*x + b*c*x^3 + b*d*x^4), x]

[Out] (810*a^3*d*Sqrt[a + b*x^3])/(1729*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (54*a^2*(1729*c*x + 935*d*x^2)*Sqrt[a + b*x^3])/323323 + (30*a*(247*c

```
*x + 187*d*x^2)*(a + b*x^3)^(3/2))/46189 + (2*(19*c*x + 17*d*x^2)*(a + b*x^
3)^(5/2))/323 - (405*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(10/3)*d*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(2/3)*Sqrt[(a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3]) + (54*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^3*(1729*b^(1/3)*c - 935*(1 - Sqrt
[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*S
qrt[3]])/(323323*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1852

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_.))^p_], x_Symbol] := Int[PolynomialQuoti
ent[Pq, a + b*x^n, x]*(a + b*x^n)^(p + 1), x] /; FreeQ[{a, b, p}, x] && Pol
yQ[Pq, x] && IGtQ[n, 0] && GeQ[Expon[Pq, x], n] && EqQ[PolynomialRemainder[
Pq, a + b*x^n, x], 0]
```

Rule 1853

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_.))^p_], x_Symbol] := Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[(Coeff[Pq, x, i]*x^(i + 1))/(n*p + i + 1),
{i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*
x^i)/(n*p + i + 1), {i, 0, q}], x, x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
&& IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1878

```
Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)]], -7 - 4*Sqrt[3])]/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int (a + bx^3)^{3/2} (ac + adx + bcx^3 + bdx^4) dx &= \int (c + dx) (a + bx^3)^{5/2} dx \\
&= \frac{2}{323} (19cx + 17dx^2) (a + bx^3)^{5/2} + \frac{1}{2} (15a) \int \left(\frac{2c}{17} + \frac{2dx}{19} \right) (a + bx^3)^{3/2} dx \\
&= \frac{30a (247cx + 187dx^2) (a + bx^3)^{3/2}}{46189} + \frac{2}{323} (19cx + 17dx^2) (a + bx^3)^{5/2} + \frac{1}{4} \\
&= \frac{54a^2 (1729cx + 935dx^2) \sqrt{a + bx^3}}{323323} + \frac{30a (247cx + 187dx^2) (a + bx^3)^{3/2}}{46189} + \\
&= \frac{54a^2 (1729cx + 935dx^2) \sqrt{a + bx^3}}{323323} + \frac{30a (247cx + 187dx^2) (a + bx^3)^{3/2}}{46189} + \\
&= \frac{810a^3 d \sqrt{a + bx^3}}{1729b^{2/3} ((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})} + \frac{54a^2 (1729cx + 935dx^2) \sqrt{a + bx^3}}{323323} + \frac{30a}{46189}
\end{aligned}$$

Mathematica [C] time = 0.0398446, size = 78, normalized size = 0.13

$$\frac{a^2 x \sqrt{a + bx^3} \left(2c {}_2F_1 \left(-\frac{5}{2}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a} \right) + dx {}_2F_1 \left(-\frac{5}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^3}{a} \right) \right)}{2 \sqrt{\frac{bx^3}{a} + 1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(3/2)*(a*c + a*d*x + b*c*x^3 + b*d*x^4), x]
```

```
[Out] (a^2*x*sqrt[a + b*x^3]*(2*c*Hypergeometric2F1[-5/2, 1/3, 4/3, -((b*x^3)/a)]
+ d*x*Hypergeometric2F1[-5/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*sqrt[1 + (b*x^3
)/a])
```

Maple [B] time = 0.009, size = 1618, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c), x)
```

```
[Out] b*d*(2/19*b*x^8*(b*x^3+a)^(1/2)+44/247*a*x^5*(b*x^3+a)^(1/2)+54/1729/b*a^2*
x^2*(b*x^3+a)^(1/2)+72/1729*I/b^2*a^3*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-
b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2
)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(
1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I
*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)
-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)
/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(
1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1
/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b
*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/
2))))+b*c*(2/17*b*x^7*(b*x^3+a)^(1/2)+40/187*a*x^4*(b*x^3+a)^(1/2)+54/935/b
*a^2*x*(b*x^3+a)^(1/2)+36/935*I/b^2*a^3*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*
(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1
/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)
^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*
3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1
/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)
/b*(-b^2*a)^(1/3)))^(1/2))+a*d*(2/13*b*x^5*(b*x^3+a)^(1/2)+32/91*a*x^2*(b*
x^3+a)^(1/2)-18/91*I*a^2*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)
)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-
b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2
)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b
^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*
(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1
/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)
^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(
-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)
```

) / b * (-b^2*a)^(1/3)) * 3^(1/2) * b / (-b^2*a)^(1/3))^(1/2), (I*3^(1/2) / b * (-b^2*a)^(1/3) / (-3/2/b * (-b^2*a)^(1/3) + 1/2*I*3^(1/2) / b * (-b^2*a)^(1/3)))^(1/2))) + a * c * (2/11*b*x^4 * (b*x^3+a)^(1/2) + 28/55*a*x*(b*x^3+a)^(1/2) - 18/55*I*a^2*3^(1/2) / b * (-b^2*a)^(1/3) * (I*(x+1/2/b * (-b^2*a)^(1/3) - 1/2*I*3^(1/2) / b * (-b^2*a)^(1/3)) * 3^(1/2) * b / (-b^2*a)^(1/3))^(1/2) * ((x-1/b * (-b^2*a)^(1/3)) / (-3/2/b * (-b^2*a)^(1/3) + 1/2*I*3^(1/2) / b * (-b^2*a)^(1/3)))^(1/2) * (-I*(x+1/2/b * (-b^2*a)^(1/3) + 1/2*I*3^(1/2) / b * (-b^2*a)^(1/3)) * 3^(1/2) * b / (-b^2*a)^(1/3))^(1/2) / (b*x^3+a)^(1/2) * EllipticF(1/3*3^(1/2) * (I*(x+1/2/b * (-b^2*a)^(1/3) - 1/2*I*3^(1/2) / b * (-b^2*a)^(1/3)) * 3^(1/2) * b / (-b^2*a)^(1/3))^(1/2), (I*3^(1/2) / b * (-b^2*a)^(1/3) / (-3/2/b * (-b^2*a)^(1/3) + 1/2*I*3^(1/2) / b * (-b^2*a)^(1/3)))^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bdx^4 + bcx^3 + adx + ac)(bx^3 + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima")

[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*(b*x^3 + a)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2 dx^7 + b^2 cx^6 + 2 abdx^4 + 2 abcx^3 + a^2 dx + a^2 c\right)\sqrt{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas")

[Out] integral((b^2*d*x^7 + b^2*c*x^6 + 2*a*b*d*x^4 + 2*a*b*c*x^3 + a^2*d*x + a^2*c)*sqrt(b*x^3 + a), x)

Sympy [A] time = 5.44847, size = 265, normalized size = 0.45

$$\frac{a^{\frac{5}{2}}cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(-\frac{1}{2}, \frac{1}{3} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{5}{2}}dx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(-\frac{1}{2}, \frac{2}{3} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{2a^{\frac{3}{2}}bcx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(-\frac{1}{2}, \frac{4}{3} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{2a^{\frac{3}{2}}bdx^5\Gamma\left(\frac{5}{3}\right) {}_2F_1\left(-\frac{1}{2}, \frac{5}{3} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(3/2)*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)

[Out] a**(5/2)*c*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + a**(5/2)*d*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + 2*a**(3/2)*b*c*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + 2*a**(3/2)*b*d*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + sqrt(a)*b**2*c*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)*b**2*d*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bdx^4 + bcx^3 + adx + ac)(bx^3 + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")

[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*(b*x^3 + a)^(3/2), x)

3.60 $\int \sqrt{a + bx^3} (ac + adx + bcx^3 + bdx^4) dx$

Optimal. Leaf size=556

$$18 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} a^2 (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (91 \sqrt[3]{bc} - 55 (1 - \sqrt{3}) \sqrt[3]{ad}) \text{EllipticF}\left(\sin^{-1}\left(\frac{(1 - \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{(1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}\right), -7 - 4\sqrt{3}\right) \\ \frac{5005 b^{2/3} \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}}$$

[Out] (54*a^2*d*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (18*a*(91*c*x + 55*d*x^2)*Sqrt[a + b*x^3])/5005 + (2*(13*c*x + 11*d*x^2)*(a + b*x^3)^(3/2))/143 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(91*b^(1/3)*c - 55*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5005*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 0.327315, antiderivative size = 556, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1852, 1853, 1878, 218, 1877}

$$18 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} a^2 (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (91 \sqrt[3]{bc} - 55 (1 - \sqrt{3}) \sqrt[3]{ad}) F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx} + (1 - \sqrt{3}) \sqrt[3]{a}}{\sqrt[3]{bx} + (1 + \sqrt{3}) \sqrt[3]{a}}\right) \middle| -7 - 4\sqrt{3}\right) \\ \frac{5005 b^{2/3} \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x^3]*(a*c + a*d*x + b*c*x^3 + b*d*x^4), x]

[Out] (54*a^2*d*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (18*a*(91*c*x + 55*d*x^2)*Sqrt[a + b*x^3])/5005 + (2*(13*c*x + 11*d*x^2)


```

*(a + b*x^3)^(3/2))/143 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*d*(a^(1/3)
+ b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(2/3)*Sqrt
[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqr
t[a + b*x^3]) + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(91*b^(1/3)*c - 55*(1 - S
qrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(5005*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

```

Rule 1852

```

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Int[PolynomialQuoti
ent[Pq, a + b*x^n, x]*(a + b*x^n)^(p + 1), x] /; FreeQ[{a, b, p}, x] && Pol
yQ[Pq, x] && IGtQ[n, 0] && GeQ[Expon[Pq, x], n] && EqQ[PolynomialRemainder[
Pq, a + b*x^n, x], 0]

```

Rule 1853

```

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[(Coeff[Pq, x, i]*x^(i + 1))/(n*p + i + 1),
{i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*
x^i)/(n*p + i + 1), {i, 0, q}], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
&& IGtQ[(n - 1)/2, 0] && GtQ[p, 0]

```

Rule 1878

```

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]

```

Rule 218

```

Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b}, x] &
& PosQ[a]

```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)]], -7 - 4*Sqrt[3])]/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a + bx^3} (ac + adx + bcx^3 + bdx^4) dx &= \int (c + dx) (a + bx^3)^{3/2} dx \\
&= \frac{2}{143} (13cx + 11dx^2) (a + bx^3)^{3/2} + \frac{1}{2} (9a) \int \left(\frac{2c}{11} + \frac{2dx}{13} \right) \sqrt{a + bx^3} dx \\
&= \frac{18a (91cx + 55dx^2) \sqrt{a + bx^3}}{5005} + \frac{2}{143} (13cx + 11dx^2) (a + bx^3)^{3/2} + \frac{1}{4} (27a^2) \int \sqrt{a + bx^3} dx \\
&= \frac{18a (91cx + 55dx^2) \sqrt{a + bx^3}}{5005} + \frac{2}{143} (13cx + 11dx^2) (a + bx^3)^{3/2} + \frac{(27a^2 d) \int \sqrt{a + bx^3} dx}{143} \\
&= \frac{54a^2 d \sqrt{a + bx^3}}{91b^{2/3} \left((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx^3} \right)} + \frac{18a (91cx + 55dx^2) \sqrt{a + bx^3}}{5005} + \frac{2}{143} (13cx + 11dx^2) (a + bx^3)^{3/2}
\end{aligned}$$

Mathematica [C] time = 0.0317119, size = 76, normalized size = 0.14

$$\frac{ax\sqrt{a + bx^3} \left(2c {}_2F_1 \left(-\frac{3}{2}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a} \right) + dx {}_2F_1 \left(-\frac{3}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^3}{a} \right) \right)}{2\sqrt{\frac{bx^3}{a} + 1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3]*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] (a*x*Sqrt[a + b*x^3]*(2*c*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)] + d*x*Hypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[1 + (b*x^3)/a])
```



```
[In] integrate((b*x**3+a)**(1/2)*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

```
[Out] a**(3/2)*c*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(3/2)*d*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*c*x**4*gamma(4/3)*hyper(
(-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*b*d
*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*ga
mma(8/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bdx^4 + bcx^3 + adx + ac)\sqrt{bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a), x)
```

$$3.61 \quad \int \frac{ac+adx+bcx^3+bdx^4}{\sqrt{a+bx^3}} dx$$

Optimal. Leaf size=525

$$2^{3/4} \sqrt{2 + \sqrt{3}a} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (7\sqrt[3]{bc} - 5(1 - \sqrt{3}) \sqrt[3]{ad}) \text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{(1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}\right), -7 - 4\sqrt{3}\right) \\ \frac{35b^{2/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}}$$

[Out] (6*a*d*Sqrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*(7*c*x + 5*d*x^2)*Sqrt[a + b*x^3])/35 - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(7*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(7*b^(1/3)*c - 5*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(35*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 0.238771, antiderivative size = 525, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1586, 1853, 1878, 218, 1877}

$$2^{3/4} \sqrt{2 + \sqrt{3}a} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (7\sqrt[3]{bc} - 5(1 - \sqrt{3}) \sqrt[3]{ad}) F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx} + (1-\sqrt{3}) \sqrt[3]{a}}{\sqrt[3]{bx} + (1+\sqrt{3}) \sqrt[3]{a}}\right) \mid -7 - 4\sqrt{3}\right) \quad 3^4 \sqrt{3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

Antiderivative was successfully verified.

[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/Sqrt[a + b*x^3], x]

[Out] (6*a*d*Sqrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*(7*c*x + 5*d*x^2)*Sqrt[a + b*x^3])/35 - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]

$$\frac{1}{((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2} \text{EllipticE}\left[\text{ArcSin}\left[\frac{(1 - \sqrt{3})a^{1/3} + b^{1/3}x}{(1 + \sqrt{3})a^{1/3} + b^{1/3}x}\right], -7 - 4\sqrt{3}\right] / (7b^{2/3}\sqrt{(a^{1/3}(a^{1/3} + b^{1/3}x))}) / ((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2 \sqrt{a + b^3x^3} + (2 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}}) a^{1/3} (7b^{1/3}c - 5(1 - \sqrt{3})a^{1/3}d) (a^{1/3} + b^{1/3}x) \sqrt{(a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2)} / ((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2 \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1 - \sqrt{3})a^{1/3} + b^{1/3}x}{(1 + \sqrt{3})a^{1/3} + b^{1/3}x}\right], -7 - 4\sqrt{3}\right] / (35b^{2/3}\sqrt{(a^{1/3}(a^{1/3} + b^{1/3}x))}) / ((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2 \sqrt{a + b^3x^3}$$

Rule 1586

$$\text{Int}[(u.) \cdot (Px.)^{(p.)} \cdot (Qx.)^{(q.)}, x_Symbol] \rightarrow \text{Int}[u \cdot \text{PolynomialQuotient}[Px, Qx, x]^p \cdot Qx^{(p+q)}, x] /; \text{FreeQ}[q, x] \ \&\& \ \text{PolyQ}[Px, x] \ \&\& \ \text{PolyQ}[Qx, x] \ \&\& \ \text{EqQ}[\text{PolynomialRemainder}[Px, Qx, x], 0] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ \text{LtQ}[p, q, 0]$$

Rule 1853

$$\text{Int}[(Pq.) \cdot ((a.) + (b.) \cdot (x.)^{(n.)})^{(p.)}, x_Symbol] \rightarrow \text{Module}[\{q = \text{Expon}[Pq, x], i\}, \text{Simp}[(a + b \cdot x^n)^p \cdot \text{Sum}[(\text{Coeff}[Pq, x, i] \cdot x^{(i+1)}) / (n \cdot p + i + 1), \{i, 0, q\}], x] + \text{Dist}[a \cdot n \cdot p, \text{Int}[(a + b \cdot x^n)^{(p-1)} \cdot \text{Sum}[(\text{Coeff}[Pq, x, i] \cdot x^i) / (n \cdot p + i + 1), \{i, 0, q\}], x], x]] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{IGtQ}[(n-1)/2, 0] \ \&\& \ \text{GtQ}[p, 0]$$

Rule 1878

$$\text{Int}[(c.) + (d.) \cdot (x.) / \sqrt{(a.) + (b.) \cdot (x.)^3}, x_Symbol] \rightarrow \text{With}[\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Dist}[(c \cdot r - (1 - \sqrt{3}) \cdot d \cdot s) / r, \text{Int}[1 / \sqrt{a + b \cdot x^3}, x], x] + \text{Dist}[d/r, \text{Int}[(1 - \sqrt{3}) \cdot s + r \cdot x] / \sqrt{a + b \cdot x^3}, x], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{PosQ}[a] \ \&\& \ \text{NeQ}[b \cdot c^3 - 2 \cdot (5 - 3\sqrt{3}) \cdot a \cdot d^3, 0]$$

Rule 218

$$\text{Int}[1 / \sqrt{(a.) + (b.) \cdot (x.)^3}, x_Symbol] \rightarrow \text{With}[\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Simp}[(2 \cdot \sqrt{2 + \sqrt{3}}) \cdot (s + r \cdot x) \cdot \sqrt{(s^2 - r \cdot s \cdot x + r^2 \cdot x^2)} / ((1 + \sqrt{3}) \cdot s + r \cdot x)^2 \cdot \text{EllipticF}[\text{ArcSin}[(1 - \sqrt{3}) \cdot s + r \cdot x] / ((1 + \sqrt{3}) \cdot s + r \cdot x)], -7 - 4\sqrt{3}] / (3^{1/4} \cdot r \cdot \sqrt{a + b \cdot x^3}) \cdot \sqrt{(s \cdot (s + r \cdot x)) / ((1 + \sqrt{3}) \cdot s + r \cdot x)^2}], x]] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a]$$

Rule 1877

$$\text{Int}[(c.) + (d.) \cdot (x.) / \sqrt{(a.) + (b.) \cdot (x.)^3}, x_Symbol] \rightarrow \text{With}[\{r = \text{Numer}[\text{Simplify}[(1 - \sqrt{3}) \cdot d] / c], s = \text{Denom}[\text{Simplify}[(1 - \sqrt{3}) \cdot d] / c]$$

```

]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqr
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{ac + adx + bcx^3 + bdx^4}{\sqrt{a + bx^3}} dx &= \int (c + dx)\sqrt{a + bx^3} dx \\
&= \frac{2}{35} (7cx + 5dx^2) \sqrt{a + bx^3} + \frac{1}{2} (3a) \int \frac{\frac{2c}{5} + \frac{2dx}{7}}{\sqrt{a + bx^3}} dx \\
&= \frac{2}{35} (7cx + 5dx^2) \sqrt{a + bx^3} + \frac{(3ad) \int \frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt{a+bx^3}} dx}{7\sqrt[3]{b}} + \frac{1}{35} \left(3a \left(7c - \frac{5(1-\sqrt{3})\sqrt[3]{ad}}{\sqrt[3]{b}} \right) \right) \\
&= \frac{6ad\sqrt{a + bx^3}}{7b^{2/3} \left((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx} \right)} + \frac{2}{35} (7cx + 5dx^2) \sqrt{a + bx^3} - \frac{3^4\sqrt{3}\sqrt{2 - \sqrt{3}}a^{4/3}d \left(\sqrt[3]{a} + \sqrt[3]{b} \right)}{7}
\end{aligned}$$

Mathematica [C] time = 0.0294621, size = 75, normalized size = 0.14

$$\frac{x\sqrt{a + bx^3} \left(2c {}_2F_1 \left(-\frac{1}{2}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a} \right) + dx {}_2F_1 \left(-\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^3}{a} \right) \right)}{2\sqrt{\frac{bx^3}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/Sqrt[a + b*x^3],x]

[Out] (x*Sqrt[a + b*x^3]*(2*c*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^3)/a)] + d*x*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[1 + (b*x^3)/a])

Maple [B] time = 0.009, size = 1480, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^{(1/2)}, x)$

[Out]
$$b*d*(2/7/b*x^2*(b*x^3+a)^{(1/2)}+8/21*I/b^2*a*3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*EllipticE(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}+1/b*(-b^2*a)^{(1/3)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)})))+b*c*(2/5/b*x*(b*x^3+a)^{(1/2)}+4/15*I/b^2*a*3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/2)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}))-2/3*I*a*d*3^{(1/2)}/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*EllipticE(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}+1/b*(-b^2*a)^{(1/3)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}))-2/3*I*a*c*3^{(1/2)}/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/2)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}$$

$$2*a)^{(1/3)})^{(1/2)}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bdx^4 + bcx^3 + adx + ac}{\sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/sqrt(b*x^3 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx^3 + a}(dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^3 + a)*(d*x + c), x)

Sympy [A] time = 2.86097, size = 163, normalized size = 0.31

$$\frac{\sqrt{ac}x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a}dx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{bcx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{4}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)} + \frac{bdx^5\Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{5}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(1/2),x)

```
[Out] sqrt(a)*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(4/3)) + sqrt(a)*d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3
*exp_polar(I*pi)/a)/(3*gamma(5/3)) + b*c*x**4*gamma(4/3)*hyper((1/2, 4/3),
(7/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(7/3)) + b*d*x**5*gamma(5
/3)*hyper((1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(8/
3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bdx^4 + bcx^3 + adx + ac}{\sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/sqrt(b*x^3 + a), x)
```

$$3.62 \quad \int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}} dx$$

Optimal. Leaf size=490

$$2\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(\sqrt[3]{bc}-(1-\sqrt{3})\sqrt[3]{ad})\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)\sqrt[4]{3}\sqrt{2}$$

$$\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}$$

[Out] (2*d*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*c - (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 0.157961, antiderivative size = 490, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1586, 1878, 218, 1877}

$$2\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(\sqrt[3]{bc}-(1-\sqrt{3})\sqrt[3]{ad})F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}$$

$$\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}$$

Antiderivative was successfully verified.

[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(3/2), x]

[Out] (2*d*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elliptic

```

E[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*c - (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

```

Rule 1586

```

Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

```

Rule 1878

```

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

```

Rule 218

```

Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b}, x] && PosQ[a]

```

Rule 1877

```

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

```

Rubi steps


```

1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^
2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),
(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)
^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*
a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I
*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3)))^(1/2))))+b*c*(-2/3/b*x/((x^3+1/b*a)*b)^(1/2)-4/9*I/b^2*3^(1/2)*(-b^2
*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2
)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/
2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1
/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))
*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*
a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+a*d*(2/3*x^2/a/((x^3+1/b*
a)*b)^(1/2)+2/9*I/a*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2
*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*
a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I
*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)
^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2
*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b
*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3
)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*
a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(
-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/
(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+a*c*(2/3*x
/a/((x^3+1/b*a)*b)^(1/2)-2/9*I/a*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2
*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((
x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3
)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/
2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/
b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(
1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-
b^2*a)^(1/3)))^(1/2))

```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bdx^4 + bcx^3 + adx + ac}{(bx^3 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(3/2),x, algorithm="maxima"

)

[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx + c}{\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] integral((d*x + c)/sqrt(b*x^3 + a), x)

Sympy [A] time = 2.87228, size = 78, normalized size = 0.16

$$\frac{cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(3/2),x)

[Out] c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bdx^4 + bcx^3 + adx + ac}{(bx^3 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(3/2), x)
```

$$3.63 \quad \int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{5/2}} dx$$

Optimal. Leaf size=522

$$\frac{2\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}((1-\sqrt{3})\sqrt[3]{ad}+\sqrt[3]{bc})\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)\sqrt{2-\sqrt{3}}}{3^4\sqrt{3}ab^{2/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}} + \dots$$

```
[Out] (2*x*(c + d*x))/(3*a*Sqrt[a + b*x^3]) - (2*d*Sqrt[a + b*x^3])/(3*a*b^(2/3)*
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (Sqrt[2 - Sqrt[3]]*d*(a^(1/3) + b^(1
/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^(2/3)*b^(2/
3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*c + (1 - Sqrt[3])*a^(1
/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/
(3*3^(1/4)*a*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.248594, antiderivative size = 522, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1586, 1855, 1878, 218, 1877}

$$\frac{2\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}((1-\sqrt{3})\sqrt[3]{ad}+\sqrt[3]{bc})F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\sqrt{2-\sqrt{3}}d(\sqrt[3]{a}+\sqrt[3]{bx})}{3^4\sqrt{3}ab^{2/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}} + \dots$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(5/2), x]
```

```
[Out] (2*x*(c + d*x))/(3*a*Sqrt[a + b*x^3]) - (2*d*Sqrt[a + b*x^3])/(3*a*b^(2/3)*
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (Sqrt[2 - Sqrt[3]]*d*(a^(1/3) + b^(1
```

$$\begin{aligned} & /3)*x)*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2)/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)^2]*\text{EllipticE}[\text{ArcSin}[\frac{(1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x}{(1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x}], -7 - 4*\text{Sqrt}[3]]]/(3^{(3/4)}*a^{(2/3)}*b^{(2/3)}*\text{Sqrt}[(a^{(1/3)}*(a^{(1/3)} + b^{(1/3)}*x))/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)^2]*\text{Sqrt}[a + b*x^3]) + (2*\text{Sqrt}[2 + \text{Sqrt}[3]]*(b^{(1/3)}*c + (1 - \text{Sqrt}[3])*a^{(1/3)}*d)*(a^{(1/3)} + b^{(1/3)}*x)*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2)/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)^2]*\text{EllipticF}[\text{ArcSin}[\frac{(1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x}{(1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x}], -7 - 4*\text{Sqrt}[3]]]/(3*3^{(1/4)}*a*b^{(2/3)}*\text{Sqrt}[(a^{(1/3)}*(a^{(1/3)} + b^{(1/3)}*x))/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)^2]*\text{Sqrt}[a + b*x^3]) \end{aligned}$$

Rule 1586

$$\text{Int}[(u_)*(Px_)^{(p_)}*(Qx_)^{(q_)}, x_Symbol] \rightarrow \text{Int}[u*\text{PolynomialQuotient}[Px, Qx, x]^p*Qx^{(p+q)}, x] /; \text{FreeQ}[q, x] \ \&\& \ \text{PolyQ}[Px, x] \ \&\& \ \text{PolyQ}[Qx, x] \ \&\& \ \text{EqQ}[\text{PolynomialRemainder}[Px, Qx, x], 0] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ \text{LtQ}[p*q, 0]$$

Rule 1855

$$\text{Int}[(Pq_)*((a_)+(b_)*(x_)^{(n_}))^{(p_)}, x_Symbol] \rightarrow -\text{Simp}[(x*Pq*(a + b*x^n)^{(p+1)})/(a*n*(p+1)), x] + \text{Dist}[1/(a*n*(p+1)), \text{Int}[\text{ExpandToSum}[n*(p+1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^{(p+1)}, x], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{LtQ}[\text{Expon}[Pq, x], n - 1]$$

Rule 1878

$$\text{Int}[(c_)+(d_)*(x_)]/\text{Sqrt}[(a_)+(b_)*(x_)^3], x_Symbol] \rightarrow \text{With}[\{r = \text{Numerator}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Dist}[(c*r - (1 - \text{Sqrt}[3])*d*s)/r, \text{Int}[1/\text{Sqrt}[a + b*x^3], x], x] + \text{Dist}[d/r, \text{Int}[\frac{(1 - \text{Sqrt}[3])*s + r*x}{\text{Sqrt}[a + b*x^3]}, x], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{PosQ}[a] \ \&\& \ \text{NeQ}[b*c^3 - 2*(5 - 3*\text{Sqrt}[3])*a*d^3, 0]$$

Rule 218

$$\text{Int}[1/\text{Sqrt}[(a_)+(b_)*(x_)^3], x_Symbol] \rightarrow \text{With}[\{r = \text{Numerator}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Simp}[(2*\text{Sqrt}[2 + \text{Sqrt}[3]]*(s + r*x)*\text{Sqrt}[(s^2 - r*s*x + r^2*x^2)/((1 + \text{Sqrt}[3])*s + r*x)^2]*\text{EllipticF}[\text{ArcSin}[\frac{(1 - \text{Sqrt}[3])*s + r*x}{(1 + \text{Sqrt}[3])*s + r*x}], -7 - 4*\text{Sqrt}[3]]]/(3^{(1/4)}*r*\text{Sqrt}[a + b*x^3]*\text{Sqrt}[(s*(s + r*x))/((1 + \text{Sqrt}[3])*s + r*x)^2]), x]] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a]$$

Rule 1877

$$\text{Int}[(c_)+(d_)*(x_)]/\text{Sqrt}[(a_)+(b_)*(x_)^3], x_Symbol] \rightarrow \text{With}[\{r = \text{Numerator}[\text{Simplify}[\frac{(1 - \text{Sqrt}[3])*d}{c}], s = \text{Denom}[\text{Simplify}[\frac{(1 - \text{Sqrt}[3])*d}{c}]]\},$$

```

]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqr
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{5/2}} dx &= \int \frac{c + dx}{(a + bx^3)^{3/2}} dx \\
&= \frac{2x(c + dx)}{3a\sqrt{a + bx^3}} - \frac{2 \int \frac{-\frac{c}{2} + \frac{dx}{2}}{\sqrt{a + bx^3}} dx}{3a} \\
&= \frac{2x(c + dx)}{3a\sqrt{a + bx^3}} - \frac{d \int \frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt{a + bx^3}} dx}{3a\sqrt[3]{b}} + \frac{\left(c + \frac{(1-\sqrt{3})\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \int \frac{1}{\sqrt{a + bx^3}} dx}{3a} \\
&= \frac{2x(c + dx)}{3a\sqrt{a + bx^3}} - \frac{2d\sqrt{a + bx^3}}{3ab^{2/3}((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})} + \frac{\sqrt{2 - \sqrt{3}}d(\sqrt[3]{a} + \sqrt[3]{bx})}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}}}
\end{aligned}$$

Mathematica [C] time = 0.0510459, size = 96, normalized size = 0.18

$$\frac{x \left(2c\sqrt{\frac{bx^3}{a}} + {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a}\right) + 3dx\sqrt{\frac{bx^3}{a}} + {}_2F_1\left(\frac{2}{3}, \frac{3}{2}; \frac{5}{3}; -\frac{bx^3}{a}\right) + 4c \right)}{6a\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(5/2), x]
```

```
[Out] (x*(4*c + 2*c*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 3*d*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)]))/(6*a*Sqrt[a + b*x^3])
```

Maple [B] time = 0.009, size = 1662, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^{(5/2)}, x)$

[Out]
$$b*d*(-2/9*x^2/b^3*(b*x^3+a)^{(1/2)}/(x^3+1/b*a)^2+8/27/b*x^2/a/((x^3+1/b*a)*b)^{(1/2)}+8/81*I/b^2/a*3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}/(b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}*EllipticE(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}))+1/b*(-b^2*a)^{(1/3)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)})))+b*c*(-2/9*x/b^3*(b*x^3+a)^{(1/2)}/(x^3+1/b*a)^2+4/27/b*x/a/((x^3+1/b*a)*b)^{(1/2)}-4/81*I/b^2/a*3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}/(b*x^3+a)^{(1/2)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)})))+a*d*(2/9*x^2/a/b^2*(b*x^3+a)^{(1/2)}/(x^3+1/b*a)^2+10/27*x^2/a^2/((x^3+1/b*a)*b)^{(1/2)}+10/81*I/a^2*3^{(1/2)}/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}/(b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}*EllipticE(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}))+1/b*(-b^2*a)^{(1/3)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)})))+a*c*(2/9*x/a/b^2*(b*x^3+a)^{(1/2)}/(x^3+1/b*a)^2+14/27*x/a^2/((x^3+1/b*a)*b)^{(1/2)}-14/81*I/a^2*3^{(1/2)}/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}$$

$a^{1/3} + 1/2 * I * 3^{1/2} / b * (-b^2 * a)^{1/3} * 3^{1/2} * b / (-b^2 * a)^{1/3})^{1/2} / (b * x^3 + a)^{1/2} * \text{EllipticF}(1/3 * 3^{1/2} * (I * (x + 1/2 / b * (-b^2 * a)^{1/3} - 1/2 * I * 3^{1/2}) / b * (-b^2 * a)^{1/3}) * 3^{1/2} * b / (-b^2 * a)^{1/3})^{1/2}, (I * 3^{1/2} / b * (-b^2 * a)^{1/3} / (-3/2 / b * (-b^2 * a)^{1/3} + 1/2 * I * 3^{1/2} / b * (-b^2 * a)^{1/3})^{1/2})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bdx^4 + bcx^3 + adx + ac}{(bx^3 + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(5/2),x, algorithm="maxima")

[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + a}(dx + c)}{b^2x^6 + 2abx^3 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^3 + a)*(d*x + c)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)

Sympy [A] time = 12.033, size = 163, normalized size = 0.31

$$\frac{cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{5}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{2}{3}, \frac{5}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{bcx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{4}{3}, \frac{5}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{bdx^5\Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\frac{5}{3}, \frac{5}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(5/2),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 5/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 5/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(5/3)) + b*c*x**4*gamma(4/3)*hyper((4/3, 5/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(7/3)) + b*d*x**5*gamma(5/3)*hyper((5/3, 5/2), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(8/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bdx^4 + bcx^3 + adx + ac}{(bx^3 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(5/2), x)
```

$$3.64 \quad \int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{7/2}} dx$$

Optimal. Leaf size=554

$$\frac{2\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(5(1-\sqrt{3})\sqrt[3]{ad}+7\sqrt[3]{bc})\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^2b^{2/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}}$$

[Out] (2*x*(c + d*x))/(9*a*(a + b*x^3)^(3/2)) + (2*x*(7*c + 5*d*x))/(27*a^2*Sqrt[a + b*x^3]) - (10*d*Sqrt[a + b*x^3])/(27*a^2*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (5*Sqrt[2 - Sqrt[3]]*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(9*3^(3/4)*a^(5/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(7*b^(1/3)*c + 5*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(27*3^(1/4)*a^2*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 0.316835, antiderivative size = 554, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1586, 1855, 1878, 218, 1877}

$$\frac{2\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(5(1-\sqrt{3})\sqrt[3]{ad}+7\sqrt[3]{bc})F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^2b^{2/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(7/2), x]

[Out] (2*x*(c + d*x))/(9*a*(a + b*x^3)^(3/2)) + (2*x*(7*c + 5*d*x))/(27*a^2*Sqrt[a + b*x^3]) - (10*d*Sqrt[a + b*x^3])/(27*a^2*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (5*Sqrt[2 - Sqrt[3]]*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(9*3^(3/4)*a^(5/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(7*b^(1/3)*c + 5*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(27*3^(1/4)*a^2*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

$$\begin{aligned}
& + b^{(1/3)*x}) + (5*\text{Sqrt}[2 - \text{Sqrt}[3]]*d*(a^{(1/3)} + b^{(1/3)*x})*\text{Sqrt}[(a^{(2/3)} \\
& - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2]* \\
& \text{EllipticE}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})/((1 + \text{Sqrt}[3])*a^{(1/3)} \\
& + b^{(1/3)*x})], -7 - 4*\text{Sqrt}[3]])/(9*3^{(3/4)}*a^{(5/3)}*b^{(2/3)}*\text{Sqrt}[(a^{(1/3)}*(\\
& a^{(1/3)} + b^{(1/3)*x})/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2]*\text{Sqrt}[a + b*x^3 \\
&]) + (2*\text{Sqrt}[2 + \text{Sqrt}[3]]*(7*b^{(1/3)*c} + 5*(1 - \text{Sqrt}[3])*a^{(1/3)*d})*(a^{(1/3)} \\
&) + b^{(1/3)*x})*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 + \text{Sqrt}[\\
& 3])*a^{(1/3)} + b^{(1/3)*x})^2]*\text{EllipticF}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/ \\
& 3)*x})/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})], -7 - 4*\text{Sqrt}[3]])/(27*3^{(1/4)}*a^ \\
& 2*b^{(2/3)}*\text{Sqrt}[(a^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x})/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(\\
& 1/3)*x})^2]*\text{Sqrt}[a + b*x^3])
\end{aligned}$$

Rule 1586

$$\text{Int}[(u_.)*(Px_)^{(p_.)}*(Qx_)^{(q_.)}, x_Symbol] \rightarrow \text{Int}[u*\text{PolynomialQuotient}[Px, Qx, x]^p*Qx^{(p+q)}, x] /; \text{FreeQ}[q, x] \ \&\& \ \text{PolyQ}[Px, x] \ \&\& \ \text{PolyQ}[Qx, x] \ \&\& \ \text{EqQ}[\text{PolynomialRemainder}[Px, Qx, x], 0] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ \text{LtQ}[p*q, 0]$$

Rule 1855

$$\text{Int}[(Pq_)*((a_) + (b_.)*(x_)^{(n_.)})^{(p_)}, x_Symbol] \rightarrow -\text{Simp}[(x*Pq*(a + b*x^n)^{(p+1)})/(a*n*(p+1)), x] + \text{Dist}[1/(a*n*(p+1)), \text{Int}[\text{ExpandToSum}[n*(p+1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^{(p+1)}, x], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{LtQ}[\text{Expon}[Pq, x], n - 1]$$

Rule 1878

$$\text{Int}[(c_) + (d_.)*(x_)]/\text{Sqrt}[(a_) + (b_.)*(x_)^3], x_Symbol] \rightarrow \text{With}[\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Dist}[(c*r - (1 - \text{Sqrt}[3])*d*s)/r, \text{Int}[1/\text{Sqrt}[a + b*x^3], x], x] + \text{Dist}[d/r, \text{Int}[(1 - \text{Sqrt}[3])*s + r*x]/\text{Sqrt}[a + b*x^3], x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{PosQ}[a] \ \&\& \ \text{NeQ}[b*c^3 - 2*(5 - 3*\text{Sqrt}[3])*a*d^3, 0]$$

Rule 218

$$\text{Int}[1/\text{Sqrt}[(a_) + (b_.)*(x_)^3], x_Symbol] \rightarrow \text{With}[\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Simp}[(2*\text{Sqrt}[2 + \text{Sqrt}[3]]*(s + r*x)*\text{Sqrt}[(s^2 - r*s*x + r^2*x^2)/((1 + \text{Sqrt}[3])*s + r*x)^2]*\text{EllipticF}[\text{ArcSin}[(1 - \text{Sqrt}[3])*s + r*x]/((1 + \text{Sqrt}[3])*s + r*x)], -7 - 4*\text{Sqrt}[3]])/(3^{(1/4)}*r*\text{Sqrt}[a + b*x^3]*\text{Sqrt}[(s*(s + r*x))/((1 + \text{Sqrt}[3])*s + r*x)^2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a]$$

Rule 1877

$$\text{Int}[(c_) + (d_.)*(x_)]/\text{Sqrt}[(a_) + (b_.)*(x_)^3], x_Symbol] \rightarrow \text{With}[\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Dist}[(c*r - (1 - \text{Sqrt}[3])*d*s)/r, \text{Int}[1/\text{Sqrt}[a + b*x^3], x], x] + \text{Dist}[d/r, \text{Int}[(1 - \text{Sqrt}[3])*s + r*x]/\text{Sqrt}[a + b*x^3], x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{PosQ}[a]$$

```

umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]], Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + S
qrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{7/2}} dx &= \int \frac{c + dx}{(a + bx^3)^{5/2}} dx \\
&= \frac{2x(c + dx)}{9a(a + bx^3)^{3/2}} - \frac{2 \int \frac{\frac{7c}{2} - \frac{5dx}{2}}{(a + bx^3)^{3/2}} dx}{9a} \\
&= \frac{2x(c + dx)}{9a(a + bx^3)^{3/2}} + \frac{2x(7c + 5dx)}{27a^2\sqrt{a + bx^3}} + \frac{4 \int \frac{\frac{7c}{4} - \frac{5dx}{4}}{\sqrt{a + bx^3}} dx}{27a^2} \\
&= \frac{2x(c + dx)}{9a(a + bx^3)^{3/2}} + \frac{2x(7c + 5dx)}{27a^2\sqrt{a + bx^3}} - \frac{(5d) \int \frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx^3}}{\sqrt{a + bx^3}} dx}{27a^2\sqrt[3]{b}} + \frac{\left(7c + \frac{5(1-\sqrt{3})\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \int \frac{1}{\sqrt{a + bx^3}} dx}{27a^2} \\
&= \frac{2x(c + dx)}{9a(a + bx^3)^{3/2}} + \frac{2x(7c + 5dx)}{27a^2\sqrt{a + bx^3}} - \frac{10d\sqrt{a + bx^3}}{27a^2b^{2/3}((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx^3})} + \frac{5\sqrt{2 - \sqrt{3}}d(\sqrt[3]{a} + \sqrt[3]{bx^3})}{27a^2}
\end{aligned}$$

Mathematica [C] time = 0.0890753, size = 123, normalized size = 0.22

$$\frac{14cx(a + bx^3)\sqrt{\frac{bx^3}{a}} + {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a}\right) + 4cx(10a + 7bx^3) + 27dx^2(a + bx^3)\sqrt{\frac{bx^3}{a}} + {}_2F_1\left(\frac{2}{3}, \frac{5}{2}; \frac{5}{3}; -\frac{bx^3}{a}\right)}{54a^2(a + bx^3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(7/2), x]
```

```
[Out] (4*c*x*(10*a + 7*b*x^3) + 14*c*x*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 27*d*x^2*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 5/2, 5/3, -((b*x^3)/a)]/(54*a^2*(a + b*x^3)^(3/2))
```

Maple [B] time = 0.088, size = 1782, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(7/2), x)
```

```
[Out] b*d*(-2/15*x^2/b^4*(b*x^3+a)^(1/2)/(x^3+1/b*a)^3+8/135*x^2/a/b^3*(b*x^3+a)^(1/2)/(x^3+1/b*a)^2+8/81/b*x^2/a^2/((x^3+1/b*a)*b)^(1/2)+8/243*I/b^2/a^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^((1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^((1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^((1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^((1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+b*c*(-2/15*x/b^4*(b*x^3+a)^(1/2)/(x^3+1/b*a)^3+4/135*x/a/b^3*(b*x^3+a)^(1/2)/(x^3+1/b*a)^2+28/405/b*x/a^2/((x^3+1/b*a)*b)^(1/2)-28/1215*I/b^2/a^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^((1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^((1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^((1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+a*d*(2/15*x^2/a/b^3*(b*x^3+a)^(1/2)/(x^3+1/b*a)^3+22/135*x^2/a^2/b^2*(b*x^3+a)^(1/2)/(x^3+1/b*a)^2+22/81*x^2/a^3/((x^3+1/b*a)*b)^(1/2)+22/243*I/a^3*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^((1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^((1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^((1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))
```

)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+a*c*(2/15*x/a/b^3*(b*x^3+a)^(1/2)/(x^3+1/b*a)^3+26/135*x/a^2/b^2*(b*x^3+a)^(1/2)/(x^3+1/b*a)^2+182/405*x/a^3/((x^3+1/b*a)*b)^(1/2)-182/1215*I/a^3*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bdx^4 + bcx^3 + adx + ac}{(bx^3 + a)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(7/2),x, algorithm="maxima")

[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(7/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + a}(dx + c)}{b^3x^9 + 3ab^2x^6 + 3a^2bx^3 + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(7/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^3 + a)*(d*x + c)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*x^3 + a^3), x)

Sympy [A] time = 102.473, size = 163, normalized size = 0.29

$$\frac{cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{7}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{2}{3}, \frac{7}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{bcx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{4}{3}, \frac{7}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{7}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{bdx^5\Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\frac{5}{3}, \frac{7}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{7}{2}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(7/2),x)

[Out] c*x*gamma(1/3)*hyper((1/3, 7/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 7/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(5/3)) + b*c*x**4*gamma(4/3)*hyper((4/3, 7/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(7/2)*gamma(7/3)) + b*d*x**5*gamma(5/3)*hyper((5/3, 7/2), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(7/2)*gamma(8/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bdx^4 + bcx^3 + adx + ac}{(bx^3 + a)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(7/2),x, algorithm="giac")

[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(7/2), x)

$$3.65 \quad \int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{9/2}} dx$$

Optimal. Leaf size=581

$$2\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(55(1-\sqrt{3})\sqrt[3]{ad}+91\sqrt[3]{bc})\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)$$

$$\frac{405\sqrt[4]{3}a^3b^{2/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}}{81a^3b^{2/3}}$$

[Out] (2*x*(c + d*x))/(15*a*(a + b*x^3)^(5/2)) + (2*x*(13*c + 11*d*x))/(135*a^2*(a + b*x^3)^(3/2)) + (2*x*(91*c + 55*d*x))/(405*a^3*Sqrt[a + b*x^3]) - (22*d*Sqrt[a + b*x^3])/((81*a^3*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (11*Sqrt[2 - Sqrt[3]]*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticE[ArcSin[(1 - Sqrt[3])*a^(1/3) + b^(1/3)*x]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(27*3^(3/4)*a^(8/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(91*b^(1/3)*c + 55*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticF[ArcSin[(1 - Sqrt[3])*a^(1/3) + b^(1/3)*x]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(405*3^(1/4)*a^3*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*Sqrt[a + b*x^3])

Rubi [A] time = 0.409681, antiderivative size = 581, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1586, 1855, 1878, 218, 1877}

$$2\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(55(1-\sqrt{3})\sqrt[3]{ad}+91\sqrt[3]{bc})F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)$$

$$\frac{405\sqrt[4]{3}a^3b^{2/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}}{81a^3b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(9/2), x]

```
[Out] (2*x*(c + d*x))/(15*a*(a + b*x^3)^(5/2)) + (2*x*(13*c + 11*d*x))/(135*a^2*(
a + b*x^3)^(3/2)) + (2*x*(91*c + 55*d*x))/(405*a^3*Sqrt[a + b*x^3]) - (22*d
*Sqrt[a + b*x^3])/(81*a^3*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (1
1*Sqrt[2 - Sqrt[3]]*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[(
(1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(27*3^(3/4)*a^(8/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)
*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 +
Sqrt[3]]*(91*b^(1/3)*c + 55*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(405*3^(1/4)*a^3*b^(2/3)*Sqrt
[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqr
t[a + b*x^3])
```

Rule 1586

```
Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] := Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
+ 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1878

```
Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)]], -7 - 4*Sqrt[3])]/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{9/2}} dx &= \int \frac{c + dx}{(a + bx^3)^{7/2}} dx \\
 &= \frac{2x(c + dx)}{15a(a + bx^3)^{5/2}} - \frac{2 \int \frac{\frac{13c}{2} - \frac{11dx}{2}}{(a + bx^3)^{5/2}} dx}{15a} \\
 &= \frac{2x(c + dx)}{15a(a + bx^3)^{5/2}} + \frac{2x(13c + 11dx)}{135a^2(a + bx^3)^{3/2}} + \frac{4 \int \frac{\frac{91c}{4} + \frac{55dx}{4}}{(a + bx^3)^{3/2}} dx}{135a^2} \\
 &= \frac{2x(c + dx)}{15a(a + bx^3)^{5/2}} + \frac{2x(13c + 11dx)}{135a^2(a + bx^3)^{3/2}} + \frac{2x(91c + 55dx)}{405a^3\sqrt{a + bx^3}} - \frac{8 \int \frac{-\frac{91c}{8} + \frac{55dx}{8}}{\sqrt{a + bx^3}} dx}{405a^3} \\
 &= \frac{2x(c + dx)}{15a(a + bx^3)^{5/2}} + \frac{2x(13c + 11dx)}{135a^2(a + bx^3)^{3/2}} + \frac{2x(91c + 55dx)}{405a^3\sqrt{a + bx^3}} - \frac{(11d) \int \frac{(1 - \sqrt{3})\sqrt[3]{a + \sqrt[3]{bx^3}}}{\sqrt{a + bx^3}} dx}{81a^3\sqrt[3]{b}} + \dots \\
 &= \frac{2x(c + dx)}{15a(a + bx^3)^{5/2}} + \frac{2x(13c + 11dx)}{135a^2(a + bx^3)^{3/2}} + \frac{2x(91c + 55dx)}{405a^3\sqrt{a + bx^3}} - \frac{22d\sqrt{a + bx^3}}{81a^3b^{2/3}((1 + \sqrt{3})\sqrt[3]{a + \sqrt[3]{bx^3}})}
 \end{aligned}$$

Mathematica [C] time = 0.116788, size = 138, normalized size = 0.24

$$\frac{4cx(157a^2 + 221abx^3 + 91b^2x^6) + 182cx\sqrt{\frac{bx^3}{a} + 1}(a + bx^3)^2 {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a}\right) + 405dx^2\sqrt{\frac{bx^3}{a} + 1}(a + bx^3)^2 {}_2F_1\left(\frac{2}{3}, \frac{7}{2}; \dots\right)}{810a^3(a + bx^3)^{5/2}}$$

$$3+1/b*a)*b)^{(1/2)}+374/5103*I/a^4*3^{(1/2)}/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}/(b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*EllipticE(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)},(I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}))+1/b*(-b^2*a)^{(1/3)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)},(I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}))+a*c*(2/21*x/a/b^4*(b*x^3+a)^{(1/2)}/(x^3+1/b*a)^4+38/315*x/a^2/b^3*(b*x^3+a)^{(1/2)}/(x^3+1/b*a)^3+494/2835*x/a^3/b^2*(b*x^3+a)^{(1/2)}/(x^3+1/b*a)^2+494/1215*x/a^4/((x^3+1/b*a)*b)^{(1/2)}-494/3645*I/a^4*3^{(1/2)}/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}/(b*x^3+a)^{(1/2)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)},(I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}))^{(1/2)}))$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bdx^4 + bcx^3 + adx + ac}{(bx^3 + a)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(9/2),x, algorithm="maxima")

[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(9/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + a}(dx + c)}{b^4x^{12} + 4ab^3x^9 + 6a^2b^2x^6 + 4a^3bx^3 + a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(9/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*(d*x + c)/(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bdx^4 + bcx^3 + adx + ac}{(bx^3 + a)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(9/2), x)
```

$$3.66 \quad \int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}} dx$$

Optimal. Leaf size=590

$$2\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)(7\sqrt[3]{b}(5bc-2af)-5(1-\sqrt{3})\sqrt[3]{a})$$

$$35\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}$$

[Out] (2*e*Sqrt[a + b*x^3])/(3*b) + (2*f*x*Sqrt[a + b*x^3])/(5*b) + (2*g*x^2*Sqrt[a + b*x^3])/(7*b) + (2*(7*b*d - 4*a*g)*Sqrt[a + b*x^3])/(7*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(7*b*d - 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(7*b^(1/3)*(5*b*c - 2*a*f) - 5*(1 - Sqrt[3])*a^(1/3)*(7*b*d - 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(35*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 0.555599, antiderivative size = 590, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1888, 1886, 261, 1878, 218, 1877}

$$2\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)(7\sqrt[3]{b}(5bc-2af)-5(1-\sqrt{3})\sqrt[3]{a}(7bd-$$

$$35\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/Sqrt[a + b*x^3], x]

[Out] (2*e*Sqrt[a + b*x^3])/(3*b) + (2*f*x*Sqrt[a + b*x^3])/(5*b) + (2*g*x^2*Sqrt[a + b*x^3])/(7*b) + (2*(7*b*d - 4*a*g)*Sqrt[a + b*x^3])/(7*b^(5/3)*((1 + S

```

qrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(7*b*d -
4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(7*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(7*b^(1/3)*(5*b*c - 2*
a*f) - 5*(1 - Sqrt[3])*a^(1/3)*(7*b*d - 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(35*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3])

```

Rule 1888

```

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :=> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)]) /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]

```

Rule 1886

```

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :=> Dist[Coeff[Pq, x, n -
1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1

```

Rule 261

```

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :=> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
NeQ[p, -1]

```

Rule 1878

```

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :=> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]

```

Rule 218

```
Int[1/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4}{\sqrt{a + bx^3}} dx &= \frac{2gx^2\sqrt{a + bx^3}}{7b} + \frac{2 \int \frac{\frac{7bc}{2} + \frac{1}{2}(7bd - 4ag)x + \frac{7}{2}bex^2 + \frac{7}{2}bf x^3}{\sqrt{a + bx^3}} dx}{7b} \\
&= \frac{2fx\sqrt{a + bx^3}}{5b} + \frac{2gx^2\sqrt{a + bx^3}}{7b} + \frac{4 \int \frac{\frac{7}{4}b(5bc - 2af) + \frac{5}{4}b(7bd - 4ag)x + \frac{35}{4}b^2ex^2}{\sqrt{a + bx^3}} dx}{35b^2} \\
&= \frac{2fx\sqrt{a + bx^3}}{5b} + \frac{2gx^2\sqrt{a + bx^3}}{7b} + \frac{4 \int \frac{\frac{7}{4}b(5bc - 2af) + \frac{5}{4}b(7bd - 4ag)x}{\sqrt{a + bx^3}} dx}{35b^2} + e \int \frac{x^2}{\sqrt{a + bx^3}} dx \\
&= \frac{2e\sqrt{a + bx^3}}{3b} + \frac{2fx\sqrt{a + bx^3}}{5b} + \frac{2gx^2\sqrt{a + bx^3}}{7b} + \frac{(7bd - 4ag) \int \frac{(1 - \sqrt{3})^{\frac{3}{2}}\sqrt{a + \sqrt[3]{bx}}}{\sqrt{a + bx^3}} dx}{7b^{4/3}} + \frac{(7\sqrt{3})^{\frac{3}{2}}}{7b^{4/3}} \\
&= \frac{2e\sqrt{a + bx^3}}{3b} + \frac{2fx\sqrt{a + bx^3}}{5b} + \frac{2gx^2\sqrt{a + bx^3}}{7b} + \frac{2(7bd - 4ag)\sqrt{a + bx^3}}{7b^{5/3}((1 + \sqrt{3})^{\frac{3}{2}}\sqrt{a + \sqrt[3]{bx}})} - \frac{\sqrt[4]{3}\sqrt{2}}{7b^{5/3}((1 + \sqrt{3})^{\frac{3}{2}}\sqrt{a + \sqrt[3]{bx}})}
\end{aligned}$$

Mathematica [C] time = 0.121405, size = 135, normalized size = 0.23

$$\frac{42x\sqrt{\frac{bx^3}{a}} + 1(5bc - 2af) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a}\right) + 15x^2\sqrt{\frac{bx^3}{a}} + 1(7bd - 4ag) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^3}{a}\right) + 4(a + bx^3)(35e + 3x(7f + g))}{210b\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/Sqrt[a + b*x^3], x]

[Out] (4*(a + b*x^3)*(35*e + 3*x*(7*f + 5*g*x)) + 42*(5*b*c - 2*a*f)*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 15*(7*b*d - 4*a*g)*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(210*b*Sqrt[a + b*x^3])

Maple [B] time = 0.046, size = 1491, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(1/2), x)

[Out] g*(2/7/b*x^2*(b*x^3+a)^(1/2)+8/21*I/b^2*a^3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+f*(2/5/b*x*(b*x^3+a)^(1/2)+4/15*I/b^2*a^3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+2/3*e*(b*x^3+a)^(1/2)/b-2/3*I*d*3^(1/2)

$$\begin{aligned}
& 2)/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}) \\
& *3^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+ \\
& 1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+ \\
& 1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/2)} \\
& *((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*EllipticE(1/3*3^{(1/2)} \\
& *(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)}, \\
& (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)} \\
& +1/b*(-b^2*a)^{(1/3)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)}, \\
& (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)} \\
& -2/3*I*c*3^{(1/2)}/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)} \\
& *((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)} \\
& *(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/2)} \\
& *EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)}, \\
& (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)} \\
& +1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}
\end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^4 + fx^3 + ex^2 + dx + c}{\sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^3 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{gx^4 + fx^3 + ex^2 + dx + c}{\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^3 + a), x)

Sympy [A] time = 3.08759, size = 187, normalized size = 0.32

$$e \left(\begin{cases} \frac{x^3}{3\sqrt{a}} & \text{for } b = 0 \\ \frac{2\sqrt{a+bx^3}}{3b} & \text{otherwise} \end{cases} \right) + \frac{cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} + \frac{fx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{4}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)} + \frac{gx^5\Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{5}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)

[Out] e*Piecewise((x**3/(3*sqrt(a)), Eq(b, 0)), (2*sqrt(a + b*x**3)/(3*b), True)) + c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3)) + f*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(7/3)) + g*x**5*gamma(5/3)*hyper((1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(8/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^4 + fx^3 + ex^2 + dx + c}{\sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^3 + a), x)

$$3.67 \quad \int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}} dx$$

Optimal. Leaf size=594

$$2\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)(\sqrt[3]{b}(2af+bc)+(1-\sqrt{3})\sqrt[3]{a}(bd$$

$$3\sqrt[4]{3}ab^{5/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}$$

[Out] (2*x*(b*c - a*f + (b*d - a*g)*x + b*e*x^2))/(3*a*b*Sqrt[a + b*x^3]) - (2*e*Sqrt[a + b*x^3])/(3*a*b) - (2*(b*d - 4*a*g)*Sqrt[a + b*x^3])/(3*a*b^(5/3)*(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x) + (Sqrt[2 - Sqrt[3]]*(b*d - 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^(2/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*(b*c + 2*a*f) + (1 - Sqrt[3])*a^(1/3)*(b*d - 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 0.432719, antiderivative size = 594, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1858, 1886, 261, 1878, 218, 1877}

$$2\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)(\sqrt[3]{b}(2af+bc)+(1-\sqrt{3})\sqrt[3]{a}(bd-4ag)$$

$$3\sqrt[4]{3}ab^{5/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(3/2), x]

[Out] (2*x*(b*c - a*f + (b*d - a*g)*x + b*e*x^2))/(3*a*b*Sqrt[a + b*x^3]) - (2*e*Sqrt[a + b*x^3])/(3*a*b) - (2*(b*d - 4*a*g)*Sqrt[a + b*x^3])/(3*a*b^(5/3)*(

$$\begin{aligned} & (1 + \sqrt{3})a^{1/3} + b^{1/3}x) + (\sqrt{2 - \sqrt{3}})(bd - 4ag)(a^{1/3} + b^{1/3}x) \sqrt{(a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2)/((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2} \\ & \text{EllipticE}\left[\text{ArcSin}\left[\frac{(1 - \sqrt{3})a^{1/3} + b^{1/3}x}{(1 + \sqrt{3})a^{1/3} + b^{1/3}x}\right], -7 - 4\sqrt{3}\right] / (3^{3/4}a^{2/3}b^{5/3} \sqrt{(a^{1/3}(a^{1/3} + b^{1/3}x))/((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2} \sqrt{a + b^2x^3}) \\ & + (2\sqrt{2 + \sqrt{3}})(b^{1/3}(bc + 2af) + (1 - \sqrt{3})a^{1/3}(bd - 4ag))(a^{1/3} + b^{1/3}x) \sqrt{(a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2)/((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2} \\ & \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1 - \sqrt{3})a^{1/3} + b^{1/3}x}{(1 + \sqrt{3})a^{1/3} + b^{1/3}x}\right], -7 - 4\sqrt{3}\right] / (3^{3/4}ab^{5/3} \sqrt{(a^{1/3}(a^{1/3} + b^{1/3}x))/((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2} \sqrt{a + b^2x^3}) \end{aligned}$$

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[Coeff[Pq, x, n - 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]]/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]]]/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{c + dx + ex^2 + fx^3 + gx^4}{(a + bx^3)^{3/2}} dx &= \frac{2x(bc - af + (bd - ag)x + bex^2)}{3ab\sqrt{a + bx^3}} - \frac{2 \int \frac{-\frac{1}{2}b(bc+2af) + \frac{1}{2}b(bd-4ag)x + \frac{3}{2}b^2ex^2}{\sqrt{a+bx^3}} dx}{3ab^2} \\ &= \frac{2x(bc - af + (bd - ag)x + bex^2)}{3ab\sqrt{a + bx^3}} - \frac{2 \int \frac{-\frac{1}{2}b(bc+2af) + \frac{1}{2}b(bd-4ag)x}{\sqrt{a+bx^3}} dx}{3ab^2} - \frac{e \int \frac{x^2}{\sqrt{a+bx^3}} dx}{a} \\ &= \frac{2x(bc - af + (bd - ag)x + bex^2)}{3ab\sqrt{a + bx^3}} - \frac{2e\sqrt{a + bx^3}}{3ab} - \frac{(bd - 4ag) \int \frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt{a+bx^3}} dx}{3ab^{4/3}} + \frac{\sqrt[3]{2}}{\sqrt[3]{a}} \\ &= \frac{2x(bc - af + (bd - ag)x + bex^2)}{3ab\sqrt{a + bx^3}} - \frac{2e\sqrt{a + bx^3}}{3ab} - \frac{2(bd - 4ag)\sqrt{a + bx^3}}{3ab^{5/3}((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})} + \frac{\sqrt{2}}{\sqrt[3]{a}} \end{aligned}$$

Mathematica [C] time = 0.121357, size = 130, normalized size = 0.22

$$\frac{2x\sqrt{\frac{bx^3}{a}} + 1(2af + bc) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a}\right) + 3x^2\sqrt{\frac{bx^3}{a}} + 1(bd - 4ag) {}_2F_1\left(\frac{2}{3}, \frac{3}{2}; \frac{5}{3}; -\frac{bx^3}{a}\right) - 4a(e + x(f - 3gx)) + 4bcx}{6ab\sqrt{a + bx^3}}$$

$$3) - 1/2 * I * 3^{(1/2)} / b * (-b^2 * a)^{(1/3)} * 3^{(1/2)} * b / (-b^2 * a)^{(1/3)}^{(1/2)}, (I * 3^{(1/2)} / b * (-b^2 * a)^{(1/3)} / (-3/2 / b * (-b^2 * a)^{(1/3)} + 1/2 * I * 3^{(1/2)} / b * (-b^2 * a)^{(1/3)})^{(1/2)} + 1 / b * (-b^2 * a)^{(1/3)} * \text{EllipticF}(1/3 * 3^{(1/2)} * (I * (x + 1/2 / b * (-b^2 * a)^{(1/3)} - 1/2 * I * 3^{(1/2)} / b * (-b^2 * a)^{(1/3)}) * 3^{(1/2)} * b / (-b^2 * a)^{(1/3)}^{(1/2)}, (I * 3^{(1/2)} / b * (-b^2 * a)^{(1/3)} / (-3/2 / b * (-b^2 * a)^{(1/3)} + 1/2 * I * 3^{(1/2)} / b * (-b^2 * a)^{(1/3)})^{(1/2)})) + c * (2/3 * x / a / ((x^3 + 1/b * a) * b)^{(1/2)} - 2/9 * I / a * 3^{(1/2)} / b * (-b^2 * a)^{(1/3)} * (I * (x + 1/2 / b * (-b^2 * a)^{(1/3)} - 1/2 * I * 3^{(1/2)} / b * (-b^2 * a)^{(1/3)}) * 3^{(1/2)} * b / (-b^2 * a)^{(1/3)}^{(1/2)} * ((x - 1/b * (-b^2 * a)^{(1/3)}) / (-3/2 / b * (-b^2 * a)^{(1/3)} + 1/2 * I * 3^{(1/2)} / b * (-b^2 * a)^{(1/3)})^{(1/2)} * (-I * (x + 1/2 / b * (-b^2 * a)^{(1/3)} + 1/2 * I * 3^{(1/2)} / b * (-b^2 * a)^{(1/3)}) * 3^{(1/2)} * b / (-b^2 * a)^{(1/3)}^{(1/2)} / (b * x^3 + a)^{(1/2)} * \text{EllipticF}(1/3 * 3^{(1/2)} * (I * (x + 1/2 / b * (-b^2 * a)^{(1/3)} - 1/2 * I * 3^{(1/2)} / b * (-b^2 * a)^{(1/3)}) * 3^{(1/2)} * b / (-b^2 * a)^{(1/3)}^{(1/2)}, (I * 3^{(1/2)} / b * (-b^2 * a)^{(1/3)} / (-3/2 / b * (-b^2 * a)^{(1/3)} + 1/2 * I * 3^{(1/2)} / b * (-b^2 * a)^{(1/3)})^{(1/2)}))^{(1/2)}))$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^4 + fx^3 + ex^2 + dx + c}{(bx^3 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(gx^4 + fx^3 + ex^2 + dx + c) \sqrt{bx^3 + a}}{b^2x^6 + 2abx^3 + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)

Sympy [A] time = 18.5223, size = 189, normalized size = 0.32

$$e^{\left(\begin{cases} -\frac{2}{3b\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{x^3}{3a^{\frac{3}{2}}} & \text{otherwise} \end{cases} \right)} + \frac{cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{3}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{2}{3}, \frac{3}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{fx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{4}{3}, \frac{3}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)

[Out] e*Piecewise((-2/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**3/(3*a**(3/2)), True)) + c*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(5/3)) + f*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(7/3)) + g*x**5*gamma(5/3)*hyper((3/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(8/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^4 + fx^3 + ex^2 + dx + c}{(bx^3 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(3/2), x)

$$3.68 \quad \int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}} dx$$

Optimal. Leaf size=628

$$2\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)(\sqrt[3]{b}(2af+7bc)+(1-\sqrt{3})\sqrt[3]{a}(4$$

$$\frac{27\sqrt[4]{3}a^2b^{5/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}}$$

[Out] (2*x*(b*c - a*f + (b*d - a*g)*x + b*e*x^2))/(9*a*b*(a + b*x^3)^(3/2)) - (2*(5*b*d + 4*a*g)*Sqrt[a + b*x^3])/(27*a^2*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*(3*a*e - x*(7*b*c + 2*a*f + (5*b*d + 4*a*g)*x)))/(27*a^2*b*Sqrt[a + b*x^3]) + (Sqrt[2 - Sqrt[3]]*(5*b*d + 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*3^(3/4)*a^(5/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*(7*b*c + 2*a*f) + (1 - Sqrt[3])*a^(1/3)*(5*b*d + 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a^2*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 0.500377, antiderivative size = 628, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1858, 1854, 1878, 218, 1877}

$$2\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)(\sqrt[3]{b}(2af+7bc)+(1-\sqrt{3})\sqrt[3]{a}(4ag+5b$$

$$\frac{27\sqrt[4]{3}a^2b^{5/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(5/2), x]


```
[Out] (2*x*(b*c - a*f + (b*d - a*g)*x + b*e*x^2))/(9*a*b*(a + b*x^3)^(3/2)) - (2*(5*b*d + 4*a*g)*Sqrt[a + b*x^3])/(27*a^2*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*(3*a*e - x*(7*b*c + 2*a*f + (5*b*d + 4*a*g)*x)))/(27*a^2*b*Sqrt[a + b*x^3]) + (Sqrt[2 - Sqrt[3]]*(5*b*d + 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*3^(3/4)*a^(5/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*(7*b*c + 2*a*f) + (1 - Sqrt[3])*a^(1/3)*(5*b*d + 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a^2*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x] /; GeQ[q, n] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1})*(a + b*x^n)^(p + 1), x], x] /; q == n - 1 /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
```

```
s = Denom[Rt[b/a, 3]], Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{c + dx + ex^2 + fx^3 + gx^4}{(a + bx^3)^{5/2}} dx &= \frac{2x(bc - af + (bd - ag)x + bex^2)}{9ab(a + bx^3)^{3/2}} - \frac{2 \int \frac{-\frac{1}{2}b(7bc + 2af) - \frac{1}{2}b(5bd + 4ag)x - \frac{3}{2}b^2ex^2}{(a + bx^3)^{3/2}} dx}{9ab^2} \\ &= \frac{2x(bc - af + (bd - ag)x + bex^2)}{9ab(a + bx^3)^{3/2}} - \frac{2(3ae - x(7bc + 2af + (5bd + 4ag)x))}{27a^2b\sqrt{a + bx^3}} + \frac{4 \int \frac{\frac{1}{4}b(7bc + 2af)}{(a + bx^3)^{3/2}} dx}{9ab^2} \\ &= \frac{2x(bc - af + (bd - ag)x + bex^2)}{9ab(a + bx^3)^{3/2}} - \frac{2(3ae - x(7bc + 2af + (5bd + 4ag)x))}{27a^2b\sqrt{a + bx^3}} - \frac{(5bd + 4ag)}{27a^2b} \\ &= \frac{2x(bc - af + (bd - ag)x + bex^2)}{9ab(a + bx^3)^{3/2}} - \frac{2(5bd + 4ag)\sqrt{a + bx^3}}{27a^2b^{5/3}((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})} - \frac{2(3ae - x(7bc + 2af + (5bd + 4ag)x))}{27a^2b} \end{aligned}$$

Mathematica [C] time = 0.184539, size = 170, normalized size = 0.27

$$\frac{-4a^2(15e + x(5f + 27gx)) + 10x(a + bx^3)\sqrt{\frac{bx^3}{a}} + 1(2af + 7bc) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a}\right) + 40abx(5c + fx^3) + 27x^2(a + bx^3)}{270a^2b(a + bx^3)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(5/2),x]

[Out] $(140*b^2*c*x^4 + 40*a*b*x*(5*c + f*x^3) - 4*a^2*(15*e + x*(5*f + 27*g*x)) + 10*(7*b*c + 2*a*f)*x*(a + b*x^3)*\text{Sqrt}[1 + (b*x^3)/a]*\text{Hypergeometric2F1}[1/3, 1/2, 4/3, -((b*x^3)/a)] + 27*(5*b*d + 4*a*g)*x^2*(a + b*x^3)*\text{Sqrt}[1 + (b*x^3)/a]*\text{Hypergeometric2F1}[2/3, 5/2, 5/3, -((b*x^3)/a)]/(270*a^2*b*(a + b*x^3)^(3/2))$

Maple [B] time = 0.006, size = 1673, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(5/2),x)

[Out] $g*(-2/9*x^2/b^3*(b*x^3+a)^(1/2)/(x^3+1/b*a)^2+8/27/b*x^2/a/((x^3+1/b*a)*b)^(1/2)+8/81*I/b^2/a*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*\text{EllipticE}(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*\text{EllipticF}(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+f*(-2/9*x/b^3*(b*x^3+a)^(1/2)/(x^3+1/b*a)^2+4/27/b*x/a/((x^3+1/b*a)*b)^(1/2)-4/81*I/b^2/a*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*\text{EllipticF}(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))-2/9*e/b/(b*x^3+a)^(3/2)+d*(2/9*x^2/a/b^2*(b*x^3+a)^(1/2)/(x^3+1/b*a)^2+10/27*x^2/a^2/((x^3+1/b*a)*b)^(1/2)+10/81*I/a^2*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*$

$$\begin{aligned}
& 3))^{1/2} * (-I * (x + 1/2/b * (-b^2 * a)^{1/3}) + 1/2 * I * 3^{1/2} / b * (-b^2 * a)^{1/3}) * 3^{1/2} * b / (-b^2 * a)^{1/3})^{1/2} / (b * x^3 + a)^{1/2} * ((-3/2/b * (-b^2 * a)^{1/3}) + 1/2 * I * 3^{1/2} / b * (-b^2 * a)^{1/3}) * \text{EllipticE}(1/3 * 3^{1/2} * (I * (x + 1/2/b * (-b^2 * a)^{1/3}) - 1/2 * I * 3^{1/2} / b * (-b^2 * a)^{1/3}) * 3^{1/2} * b / (-b^2 * a)^{1/3})^{1/2}, (I * 3^{1/2} / b * (-b^2 * a)^{1/3} / (-3/2/b * (-b^2 * a)^{1/3}) + 1/2 * I * 3^{1/2} / b * (-b^2 * a)^{1/3}))^{1/2} \\
& + 1/b * (-b^2 * a)^{1/3} * \text{EllipticF}(1/3 * 3^{1/2} * (I * (x + 1/2/b * (-b^2 * a)^{1/3}) - 1/2 * I * 3^{1/2} / b * (-b^2 * a)^{1/3}) * 3^{1/2} * b / (-b^2 * a)^{1/3})^{1/2}, (I * 3^{1/2} / b * (-b^2 * a)^{1/3} / (-3/2/b * (-b^2 * a)^{1/3}) + 1/2 * I * 3^{1/2} / b * (-b^2 * a)^{1/3}))^{1/2} \\
&)) + c * (2/9 * x/a/b^2 * (b * x^3 + a)^{1/2} / (x^3 + 1/b * a)^2 + 14/27 * x/a^2 / ((x^3 + 1/b * a) * b)^{1/2} - 14/81 * I/a^2 * 3^{1/2} / b * (-b^2 * a)^{1/3} * (I * (x + 1/2/b * (-b^2 * a)^{1/3}) - 1/2 * I * 3^{1/2} / b * (-b^2 * a)^{1/3}) * 3^{1/2} * b / (-b^2 * a)^{1/3})^{1/2} * ((x - 1/b * (-b^2 * a)^{1/3}) / (-3/2/b * (-b^2 * a)^{1/3}) + 1/2 * I * 3^{1/2} / b * (-b^2 * a)^{1/3}))^{1/2} * (-I * (x + 1/2/b * (-b^2 * a)^{1/3}) + 1/2 * I * 3^{1/2} / b * (-b^2 * a)^{1/3}) * 3^{1/2} * b / (-b^2 * a)^{1/3})^{1/2} / (b * x^3 + a)^{1/2} * \text{EllipticF}(1/3 * 3^{1/2} * (I * (x + 1/2/b * (-b^2 * a)^{1/3}) - 1/2 * I * 3^{1/2} / b * (-b^2 * a)^{1/3}) * 3^{1/2} * b / (-b^2 * a)^{1/3})^{1/2}, (I * 3^{1/2} / b * (-b^2 * a)^{1/3} / (-3/2/b * (-b^2 * a)^{1/3}) + 1/2 * I * 3^{1/2} / b * (-b^2 * a)^{1/3}))^{1/2} \\
&))^{1/2}
\end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^4 + fx^3 + ex^2 + dx + c}{(bx^3 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(5/2),x, algorithm="maxima")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{b^3x^9 + 3ab^2x^6 + 3a^2bx^3 + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(5/2),x, algorithm="fricas")

[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*x^3 + a^3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(5/2), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^4 + fx^3 + ex^2 + dx + c}{(bx^3 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(5/2),x, algorithm="giac")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(5/2), x)

$$3.69 \quad \int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}} dx$$

Optimal. Leaf size=676

$$2\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)(7\sqrt[3]{b}(2af+13bc)+5(1-\sqrt{3})\sqrt[3]{a}(4ag$$

$$405\sqrt[4]{3}a^3b^{5/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}$$

[Out] (2*x*(b*c - a*f + (b*d - a*g)*x + b*e*x^2))/(15*a*b*(a + b*x^3)^(5/2)) + (2*x*(7*(13*b*c + 2*a*f) + 5*(11*b*d + 4*a*g)*x))/(405*a^3*b*Sqrt[a + b*x^3]) - (2*(11*b*d + 4*a*g)*Sqrt[a + b*x^3])/(81*a^3*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*(9*a*e - x*(13*b*c + 2*a*f + (11*b*d + 4*a*g)*x)))/(135*a^2*b*(a + b*x^3)^(3/2)) + (Sqrt[2 - Sqrt[3]]*(11*b*d + 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(3/4)*a^(8/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(7*b^(1/3)*(13*b*c + 2*a*f) + 5*(1 - Sqrt[3])*a^(1/3)*(11*b*d + 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(405*3^(1/4)*a^3*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 0.674968, antiderivative size = 676, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1858, 1854, 1855, 1878, 218, 1877}

$$2\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)(7\sqrt[3]{b}(2af+13bc)+5(1-\sqrt{3})\sqrt[3]{a}(4ag$$

$$405\sqrt[4]{3}a^3b^{5/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(7/2), x]

```
[Out] (2*x*(b*c - a*f + (b*d - a*g)*x + b*e*x^2))/(15*a*b*(a + b*x^3)^(5/2)) + (2
*x*(7*(13*b*c + 2*a*f) + 5*(11*b*d + 4*a*g)*x))/(405*a^3*b*Sqrt[a + b*x^3])
- (2*(11*b*d + 4*a*g)*Sqrt[a + b*x^3])/(81*a^3*b^(5/3)*((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)) - (2*(9*a*e - x*(13*b*c + 2*a*f + (11*b*d + 4*a*g)*x)))/(
135*a^2*b*(a + b*x^3)^(3/2)) + (Sqrt[2 - Sqrt[3]]*(11*b*d + 4*a*g)*(a^(1/3)
+ b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3
)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(3/4)*a^(
8/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(7*b^(1/3)*(13*b*c +
2*a*f) + 5*(1 - Sqrt[3])*a^(1/3)*(11*b*d + 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(405*3^(1/4)*a^3*b^(5/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3])
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a +
b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Module[{q = Expon[Pq,
x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1})*(a + b*x^n)^(p
+ 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
+ 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4}{(a + bx^3)^{7/2}} dx &= \frac{2x(bc - af + (bd - ag)x + bex^2)}{15ab(a + bx^3)^{5/2}} - \frac{2 \int \frac{-\frac{1}{2}b(13bc + 2af) - \frac{1}{2}b(11bd + 4ag)x - \frac{9}{2}b^2ex^2}{(a + bx^3)^{5/2}} dx}{15ab^2} \\
&= \frac{2x(bc - af + (bd - ag)x + bex^2)}{15ab(a + bx^3)^{5/2}} - \frac{2(9ae - x(13bc + 2af + (11bd + 4ag)x))}{135a^2b(a + bx^3)^{3/2}} + \frac{4 \int \frac{7}{4}}{15ab^2} \\
&= \frac{2x(bc - af + (bd - ag)x + bex^2)}{15ab(a + bx^3)^{5/2}} + \frac{2x(7(13bc + 2af) + 5(11bd + 4ag)x)}{405a^3b\sqrt{a + bx^3}} - \frac{2(9ae - x(13bc + 2af + (11bd + 4ag)x))}{135a^2b(a + bx^3)^{3/2}} \\
&= \frac{2x(bc - af + (bd - ag)x + bex^2)}{15ab(a + bx^3)^{5/2}} + \frac{2x(7(13bc + 2af) + 5(11bd + 4ag)x)}{405a^3b\sqrt{a + bx^3}} - \frac{2(9ae - x(13bc + 2af + (11bd + 4ag)x))}{135a^2b(a + bx^3)^{3/2}} \\
&= \frac{2x(bc - af + (bd - ag)x + bex^2)}{15ab(a + bx^3)^{5/2}} + \frac{2x(7(13bc + 2af) + 5(11bd + 4ag)x)}{405a^3b\sqrt{a + bx^3}} - \frac{2(11bd + 4ag)x}{81a^3b^{5/3}}
\end{aligned}$$

Mathematica [C] time = 0.253436, size = 196, normalized size = 0.29

$$\frac{44a^2bx(157c + 34fx^3) - 4a^3(297e + x(77f + 405gx)) + 44ab^2x^4(221c + 14fx^3) + 154x(a + bx^3)^2 \sqrt{\frac{bx^3}{a} + 1}(2af + 13bx^2)}{8910a^3b(a + bx^3)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(7/2), x]

[Out] (4004*b^3*c*x^7 + 44*a*b^2*x^4*(221*c + 14*f*x^3) + 44*a^2*b*x*(157*c + 34*f*x^3) - 4*a^3*(297*e + x*(77*f + 405*g*x)) + 154*(13*b*c + 2*a*f)*x*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 405*(11*b*d + 4*a*g)*x^2*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 7/2, 5/3, -((b*x^3)/a)])/(8910*a^3*b*(a + b*x^3)^(5/2))

Maple [B] time = 0.006, size = 1793, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^{(7/2)}, x)$

[Out]
$$g*(-2/15*x^2/b^4*(b*x^3+a)^{(1/2)}/(x^3+1/b*a)^3+8/135*x^2/a/b^3*(b*x^3+a)^{(1/2)}/(x^3+1/b*a)^2+8/81/b*x^2/a^2/((x^3+1/b*a)*b)^{(1/2)}+8/243*I/b^2/a^2*3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}*b/(-b^2*a)^{(1/3))^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)}*b/(-b^2*a)^{(1/3))^{(1/2)}/(b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*\text{EllipticE}(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)}*b/(-b^2*a)^{(1/3))^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}+1/b*(-b^2*a)^{(1/3)}*\text{EllipticF}(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)}*b/(-b^2*a)^{(1/3))^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)})))+f*(-2/15*x/b^4*(b*x^3+a)^{(1/2)}/(x^3+1/b*a)^3+4/135*x/a/b^3*(b*x^3+a)^{(1/2)}/(x^3+1/b*a)^2+28/405/b*x/a^2/((x^3+1/b*a)*b)^{(1/2)}-28/1215*I/b^2/a^2*3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)}*b/(-b^2*a)^{(1/3))^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)}*b/(-b^2*a)^{(1/3))^{(1/2)}/(b*x^3+a)^{(1/2)}*\text{EllipticF}(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)}*b/(-b^2*a)^{(1/3))^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}))-2/15*e/b/(b*x^3+a)^{(5/2)}+d*(2/15*x^2/a/b^3*(b*x^3+a)^{(1/2)}/(x^3+1/b*a)^3+22/135*x^2/a^2/b^2*(b*x^3+a)^{(1/2)}/(x^3+1/b*a)^2+22/81*x^2/a^3/((x^3+1/b*a)*b)^{(1/2)}+22/243*I/a^3*3^{(1/2)}/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)}*b/(-b^2*a)^{(1/3))^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)}*b/(-b^2*a)^{(1/3))^{(1/2)}/(b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*\text{EllipticE}(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)}*b/(-b^2*a)^{(1/3))^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}+1/b*(-b^2*a)^{(1/3)}*\text{EllipticF}(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)}*b/(-b^2*a)^{(1/3))^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)})))+c*(2/15*x/a/b^3*(b*x^3+a)^{(1/2)}/(x^3+1/b*a)^3+26/135*x/a^2/b^2*(b*x^3+a)^{(1/2)}/(x^3+1/b*a)^2+182/405*x/a^3/((x^3+1/b*a)*b)^{(1/2)}-182/1215*$$

$$\frac{I/a^3 3^{(1/2)}/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))})^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}/(b*x^3+a)^{(1/2)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))})^{(1/2))}}{}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^4 + fx^3 + ex^2 + dx + c}{(bx^3 + a)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(7/2),x, algorithm="maxima")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(7/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{b^4x^{12} + 4ab^3x^9 + 6a^2b^2x^6 + 4a^3bx^3 + a^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(7/2),x, algorithm="fricas")

[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(7/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^4 + fx^3 + ex^2 + dx + c}{(bx^3 + a)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(7/2),x, algorithm="giac")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(7/2), x)

3.70 $\int \frac{(a+bx)^2}{c+dx^3} dx$

Optimal. Leaf size=186

$$\frac{a(2b\sqrt[3]{c} - a\sqrt[3]{d}) \log(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2)}{6c^{2/3}d^{2/3}} - \frac{a(2b\sqrt[3]{c} - a\sqrt[3]{d}) \log(\sqrt[3]{c} + \sqrt[3]{dx})}{3c^{2/3}d^{2/3}} - \frac{a(a\sqrt[3]{d} + 2b\sqrt[3]{c}) \tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{dx}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{2/3}} +$$

[Out] $-\left(\frac{(a(2b\sqrt[3]{c} + a\sqrt[3]{d}))\text{ArcTan}\left[\frac{c^{1/3} - 2d^{1/3}x}{\sqrt{3}c^{1/3}}\right]}{\sqrt{3}c^{2/3}d^{2/3}} - \frac{(a(2b\sqrt[3]{c} - a\sqrt[3]{d}))\text{Log}\left[\frac{c^{1/3} + d^{1/3}x}{3c^{2/3}d^{2/3}}\right]}{3c^{2/3}d^{2/3}} + \frac{(a(2b\sqrt[3]{c} - a\sqrt[3]{d}))\text{Log}\left[\frac{c^{2/3} - c^{1/3}d^{1/3}x + d^{2/3}x^2}{6c^{2/3}d^{2/3}}\right]}{6c^{2/3}d^{2/3}} + \frac{b^2\text{Log}[c + d x^3]}{3d}\right)$

Rubi [A] time = 0.177635, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{a(2b\sqrt[3]{c} - a\sqrt[3]{d}) \log(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2)}{6c^{2/3}d^{2/3}} - \frac{a(2b\sqrt[3]{c} - a\sqrt[3]{d}) \log(\sqrt[3]{c} + \sqrt[3]{dx})}{3c^{2/3}d^{2/3}} - \frac{a(a\sqrt[3]{d} + 2b\sqrt[3]{c}) \tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{dx}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{2/3}} +$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*x)^2/(c + d*x^3), x]$

[Out] $-\left(\frac{(a(2b\sqrt[3]{c} + a\sqrt[3]{d}))\text{ArcTan}\left[\frac{c^{1/3} - 2d^{1/3}x}{\sqrt{3}c^{1/3}}\right]}{\sqrt{3}c^{2/3}d^{2/3}} - \frac{(a(2b\sqrt[3]{c} - a\sqrt[3]{d}))\text{Log}\left[\frac{c^{1/3} + d^{1/3}x}{3c^{2/3}d^{2/3}}\right]}{3c^{2/3}d^{2/3}} + \frac{(a(2b\sqrt[3]{c} - a\sqrt[3]{d}))\text{Log}\left[\frac{c^{2/3} - c^{1/3}d^{1/3}x + d^{2/3}x^2}{6c^{2/3}d^{2/3}}\right]}{6c^{2/3}d^{2/3}} + \frac{b^2\text{Log}[c + d x^3]}{3d}\right)$

Rule 1871

$\text{Int}[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] \rightarrow \text{With}[\{A = \text{Coeff}[P2, x, 0], B = \text{Coeff}[P2, x, 1], C = \text{Coeff}[P2, x, 2]\}, \text{Int}[(A + B*x)/(a + b*x^3), x] + \text{Dist}[C, \text{Int}[x^2/(a + b*x^3), x], x] /; \text{EqQ}[a*B^3 - b*A^3, 0] \|\ \text{!RationalQ}[a/b] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[P2, x, 2]$

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^n), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2}{c+dx^3} dx &= b^2 \int \frac{x^2}{c+dx^3} dx + \int \frac{a^2+2abx}{c+dx^3} dx \\
&= \frac{b^2 \log(c+dx^3)}{3d} + \frac{\int \frac{\sqrt[3]{c}(2ab\sqrt[3]{c}+2a^2\sqrt[3]{d})+(2ab\sqrt[3]{c}-a^2\sqrt[3]{d})\sqrt[3]{dx}}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{dx}+d^{2/3}x^2} dx}{3c^{2/3}\sqrt[3]{d}} - \frac{(2ab\sqrt[3]{c}-a^2\sqrt[3]{d}) \int \frac{1}{\sqrt[3]{c}+\sqrt[3]{dx}} dx}{3c^{2/3}\sqrt[3]{d}} \\
&= -\frac{a(2b\sqrt[3]{c}-a\sqrt[3]{d}) \log(\sqrt[3]{c}+\sqrt[3]{dx})}{3c^{2/3}d^{2/3}} + \frac{b^2 \log(c+dx^3)}{3d} + \frac{1}{2} \left(a \left(\frac{a}{\sqrt[3]{c}} + \frac{2b}{\sqrt[3]{d}} \right) \right) \int \frac{1}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{dx}+d^{2/3}x^2} \\
&= -\frac{a(2b\sqrt[3]{c}-a\sqrt[3]{d}) \log(\sqrt[3]{c}+\sqrt[3]{dx})}{3c^{2/3}d^{2/3}} + \frac{a(2b\sqrt[3]{c}-a\sqrt[3]{d}) \log(c^{2/3}-\sqrt[3]{c}\sqrt[3]{dx}+d^{2/3}x^2)}{6c^{2/3}d^{2/3}} + \frac{b^2 \log(c+dx^3)}{3d} \\
&= -\frac{a(2b\sqrt[3]{c}+a\sqrt[3]{d}) \tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{dx}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{2/3}} - \frac{a(2b\sqrt[3]{c}-a\sqrt[3]{d}) \log(\sqrt[3]{c}+\sqrt[3]{dx})}{3c^{2/3}d^{2/3}} + \frac{a(2b\sqrt[3]{c}-a\sqrt[3]{d}) \log(c^{2/3}-\sqrt[3]{c}\sqrt[3]{dx}+d^{2/3}x^2)}{6c^{2/3}d^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.0843078, size = 200, normalized size = 1.08

$$-\frac{(a^2\sqrt[3]{c}\sqrt[3]{d}-2abc^{2/3}) \log(c^{2/3}-\sqrt[3]{c}\sqrt[3]{dx}+d^{2/3}x^2)}{6cd^{2/3}} + \frac{(a^2\sqrt[3]{c}\sqrt[3]{d}-2abc^{2/3}) \log(\sqrt[3]{c}+\sqrt[3]{dx})}{3cd^{2/3}} + \frac{(a^2\sqrt[3]{c}\sqrt[3]{d}+2abc^{2/3}) \tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{dx}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}cd^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^2/(c + d*x^3), x]

[Out] $\left(\frac{2ab\sqrt[3]{c} + a^2\sqrt[3]{d}}{3\sqrt[3]{cd}} \operatorname{ArcTan}\left[\frac{-c^{1/3} + 2d^{1/3}x}{\sqrt{3}\sqrt[3]{cd}}\right] + \frac{(-2ab\sqrt[3]{c} + a^2\sqrt[3]{d}) \log(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2)}{6\sqrt[3]{cd}} - \frac{(-2ab\sqrt[3]{c} + a^2\sqrt[3]{d}) \log(\sqrt[3]{c} + \sqrt[3]{dx})}{3\sqrt[3]{cd}} + \frac{b^2 \log(c + dx^3)}{3d}\right)$

Maple [A] time = 0.002, size = 211, normalized size = 1.1

$$\frac{a^2}{3d} \ln\left(x + \sqrt[3]{\frac{c}{d}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} - \frac{a^2}{6d} \ln\left(x^2 - \sqrt[3]{\frac{c}{d}}x + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} + \frac{a^2\sqrt{3}}{3d} \arctan\left(\frac{\sqrt{3}}{3} \left(2x \frac{1}{\sqrt[3]{\frac{c}{d}}} - 1\right)\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} - \frac{2ab}{3d} \ln\left(x + \sqrt[3]{\frac{c}{d}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x^3+c),x)
```

```
[Out] 1/3*a^2/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6*a^2/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3*a^2/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))-2/3*a*b/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/3*a*b/d/(c/d)^(1/3)*3*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+2/3*a*b*3^(1/2)/d/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))+1/3*b^2*ln(d*x^3+c)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 8.10969, size = 10904, normalized size = 58.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/12*(2*(2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*b^2/d*d*log(2*b^5*c^2 + 7*a^3*b^2*c*d + 1/2*(2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*b^2/d)^2*b*c^2*d^2 + 1/2*(4*b^3*c^2*d - a^3*c*d^2)*(2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*b^2/d)
```


$$\begin{aligned}
& 2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2) \\
&)/(c^2*d^3))^{(1/3)} + (1/2)^{(1/3)}*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) \\
& 2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2) \\
&)/(c^2*d^3))^{(1/3)}*(I*\sqrt{3} + 1) - 2*b^2/d) + (8*a^2*b^3*c*d + a^5*d^2)*x \\
&) - (6*b^2 + (2*(1/2)^{(2/3)}*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2)))*(-I*\sqrt{3} \\
& t(3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b \\
& *d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^{(1/3)} + (1 \\
& /2)^{(1/3)}*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b \\
& *d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^{(1/3)}*(I*\sqrt{3} \\
& + 1) - 2*b^2/d)*d + 3*\sqrt{1/3}*d*\sqrt{-(4*b^4*c + 32*a^3*b*d + 4*(2 \\
& *(1/2)^{(2/3)}*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2)))*(-I*\sqrt{3} + 1)/(2*b^ \\
& 6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) \\
& + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^{(1/3)} + (1/2)^{(1/3)}*(2*b^ \\
& 6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) \\
& + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^{(1/3)}*(I*\sqrt{3} + 1) - 2 \\
& *b^2/d)*b^2*c*d + (2*(1/2)^{(2/3)}*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2)))*(- \\
& I*\sqrt{3} + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2* \\
& a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^{(1/3)} \\
& + (1/2)^{(1/3)}*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2* \\
& a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^{(1/3)} \\
& *(I*\sqrt{3} + 1) - 2*b^2/d)^2*c*d^2)/(c*d^2)))*\log(-2*b^5*c^2 - 7*a^3*b^2*c \\
& *d - 1/2*(2*(1/2)^{(2/3)}*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2)))*(-I*\sqrt{3} \\
& + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)* \\
& b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^{(1/3)} + (1/2)^{ \\
& (1/3)}*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)* \\
& b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^{(1/3)}*(I*\sqrt{3} \\
& + 1) - 2*b^2/d)^2*b*c^2*d^2 - 1/2*(4*b^3*c^2*d - a^3*c*d^2)*(2*(1/2)^{(2/ \\
& 3)}*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2)))*(-I*\sqrt{3} + 1)/(2*b^6/d^3 + (8 \\
& *b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^ \\
& 2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^{(1/3)} + (1/2)^{(1/3)}*(2*b^6/d^3 + (8 \\
& *b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^ \\
& 2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^{(1/3)}*(I*\sqrt{3} + 1) - 2*b^2/d) + \\
& 2*(8*a^2*b^3*c*d + a^5*d^2)*x + 3/2*\sqrt{1/3}*(2*b^3*c^2*d + a^3*c*d^2 + (2 \\
& *(1/2)^{(2/3)}*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2)))*(-I*\sqrt{3} + 1)/(2*b^ \\
& 6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) \\
& + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^{(1/3)} + (1/2)^{(1/3)}*(2*b^ \\
& 6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) \\
& + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^{(1/3)}*(I*\sqrt{3} + 1) - 2 \\
& *b^2/d)*b*c^2*d^2)*\sqrt{-(4*b^4*c + 32*a^3*b*d + 4*(2*(1/2)^{(2/3)}*(b^4/d^2 \\
& - (b^4*c + 2*a^3*b*d)/(c*d^2)))*(-I*\sqrt{3} + 1)/(2*b^6/d^3 + (8*b^3*c + a^3 \\
& *d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^ \\
& 3*c*d + a^6*d^2)/(c^2*d^3))^{(1/3)} + (1/2)^{(1/3)}*(2*b^6/d^3 + (8*b^3*c + a^3 \\
& *d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^ \\
& 3*c*d + a^6*d^2)/(c^2*d^3))^{(1/3)}*(I*\sqrt{3} + 1) - 2*b^2/d)*b^2*c*d + (2*(\\
& 1/2)^{(2/3)}*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2)))*(-I*\sqrt{3} + 1)/(2*b^6/
\end{aligned}$$

$$\begin{aligned}
& d^3 + (8b^3c + a^3d)a^3/(c^2d^2) - 3(b^4c + 2a^3bd)b^2/(cd^3) + \\
& (b^6c^2 - 2a^3b^3cd + a^6d^2)/(c^2d^3))^{(1/3)} + (1/2)^{(1/3)}(2b^6/ \\
& d^3 + (8b^3c + a^3d)a^3/(c^2d^2) - 3(b^4c + 2a^3bd)b^2/(cd^3) + \\
& (b^6c^2 - 2a^3b^3cd + a^6d^2)/(c^2d^3))^{(1/3)}(I\sqrt{3} + 1) - 2b \\
& ^2/d)^2cd^2)/(cd^2)) - (6b^2 + (2(1/2)^{(2/3)}(b^4/d^2 - (b^4c + 2a^ \\
& 3bd)/(cd^2))(-I\sqrt{3} + 1)/(2b^6/d^3 + (8b^3c + a^3d)a^3/(c^2d^ \\
& 2) - 3(b^4c + 2a^3bd)b^2/(cd^3) + (b^6c^2 - 2a^3b^3cd + a^6d^2 \\
&)/(c^2d^3))^{(1/3)} + (1/2)^{(1/3)}(2b^6/d^3 + (8b^3c + a^3d)a^3/(c^2d^ \\
& 2) - 3(b^4c + 2a^3bd)b^2/(cd^3) + (b^6c^2 - 2a^3b^3cd + a^6d^2 \\
&)/(c^2d^3))^{(1/3)}(I\sqrt{3} + 1) - 2b^2/d)*d - 3\sqrt{1/3}*d*\sqrt{-(4b^ \\
& 4c + 32a^3bd + 4(2(1/2)^{(2/3)}(b^4/d^2 - (b^4c + 2a^3bd)/(cd^2)) \\
& *(-I\sqrt{3} + 1)/(2b^6/d^3 + (8b^3c + a^3d)a^3/(c^2d^2) - 3(b^4c + \\
& 2a^3bd)b^2/(cd^3) + (b^6c^2 - 2a^3b^3cd + a^6d^2)/(c^2d^3))^{(1 \\
& /3)} + (1/2)^{(1/3)}(2b^6/d^3 + (8b^3c + a^3d)a^3/(c^2d^2) - 3(b^4c + \\
& 2a^3bd)b^2/(cd^3) + (b^6c^2 - 2a^3b^3cd + a^6d^2)/(c^2d^3))^{(1 \\
& /3)}(I\sqrt{3} + 1) - 2b^2/d)*b^2cd + (2(1/2)^{(2/3)}(b^4/d^2 - (b^4c + \\
& 2a^3bd)/(cd^2))(-I\sqrt{3} + 1)/(2b^6/d^3 + (8b^3c + a^3d)a^3/(c \\
& ^2d^2) - 3(b^4c + 2a^3bd)b^2/(cd^3) + (b^6c^2 - 2a^3b^3cd + a^ \\
& 6d^2)/(c^2d^3))^{(1/3)} + (1/2)^{(1/3)}(2b^6/d^3 + (8b^3c + a^3d)a^3/(c \\
& ^2d^2) - 3(b^4c + 2a^3bd)b^2/(cd^3) + (b^6c^2 - 2a^3b^3cd + a^ \\
& 6d^2)/(c^2d^3))^{(1/3)}(I\sqrt{3} + 1) - 2b^2/d)^2cd^2)/(cd^2)))*\log(- \\
& 2b^5c^2 - 7a^3b^2cd - 1/2(2(1/2)^{(2/3)}(b^4/d^2 - (b^4c + 2a^3bd) \\
& d)/(cd^2))(-I\sqrt{3} + 1)/(2b^6/d^3 + (8b^3c + a^3d)a^3/(c^2d^2) - \\
& 3(b^4c + 2a^3bd)b^2/(cd^3) + (b^6c^2 - 2a^3b^3cd + a^6d^2)/(c \\
& ^2d^3))^{(1/3)} + (1/2)^{(1/3)}(2b^6/d^3 + (8b^3c + a^3d)a^3/(c^2d^2) - \\
& 3(b^4c + 2a^3bd)b^2/(cd^3) + (b^6c^2 - 2a^3b^3cd + a^6d^2)/(c \\
& ^2d^3))^{(1/3)}(I\sqrt{3} + 1) - 2b^2/d)^2b^2cd^2 - 1/2(4b^3c^2d - \\
& a^3cd^2)*(2(1/2)^{(2/3)}(b^4/d^2 - (b^4c + 2a^3bd)/(cd^2))(-I\sqrt{3} \\
& (3) + 1)/(2b^6/d^3 + (8b^3c + a^3d)a^3/(c^2d^2) - 3(b^4c + 2a^3bd) \\
&)b^2/(cd^3) + (b^6c^2 - 2a^3b^3cd + a^6d^2)/(c^2d^3))^{(1/3)} + (1/2 \\
&)^{(1/3)}(2b^6/d^3 + (8b^3c + a^3d)a^3/(c^2d^2) - 3(b^4c + 2a^3bd) \\
&)b^2/(cd^3) + (b^6c^2 - 2a^3b^3cd + a^6d^2)/(c^2d^3))^{(1/3)}(I\sqrt{3} \\
& + 1) - 2b^2/d) + 2(8a^2b^3cd + a^5d^2)*x - 3/2\sqrt{1/3}(2b^3 \\
& *c^2d + a^3cd^2 + (2(1/2)^{(2/3)}(b^4/d^2 - (b^4c + 2a^3bd)/(cd^2)) \\
& *(-I\sqrt{3} + 1)/(2b^6/d^3 + (8b^3c + a^3d)a^3/(c^2d^2) - 3(b^4c + \\
& 2a^3bd)b^2/(cd^3) + (b^6c^2 - 2a^3b^3cd + a^6d^2)/(c^2d^3))^{(1 \\
& /3)} + (1/2)^{(1/3)}(2b^6/d^3 + (8b^3c + a^3d)a^3/(c^2d^2) - 3(b^4c + \\
& 2a^3bd)b^2/(cd^3) + (b^6c^2 - 2a^3b^3cd + a^6d^2)/(c^2d^3))^{(1 \\
& /3)}(I\sqrt{3} + 1) - 2b^2/d)*b^2cd^2)*\sqrt{-(4b^4c + 32a^3bd + 4(\\
& 2(1/2)^{(2/3)}(b^4/d^2 - (b^4c + 2a^3bd)/(cd^2))(-I\sqrt{3} + 1)/(2b \\
& ^6/d^3 + (8b^3c + a^3d)a^3/(c^2d^2) - 3(b^4c + 2a^3bd)b^2/(cd^3 \\
&) + (b^6c^2 - 2a^3b^3cd + a^6d^2)/(c^2d^3))^{(1/3)} + (1/2)^{(1/3)}(2b \\
& ^6/d^3 + (8b^3c + a^3d)a^3/(c^2d^2) - 3(b^4c + 2a^3bd)b^2/(cd^3 \\
&) + (b^6c^2 - 2a^3b^3cd + a^6d^2)/(c^2d^3))^{(1/3)}(I\sqrt{3} + 1) - \\
& 2b^2/d)*b^2cd + (2(1/2)^{(2/3)}(b^4/d^2 - (b^4c + 2a^3bd)/(cd^2))*
\end{aligned}$$

$$-I\sqrt{3} + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^{(1/3)} + (1/2)^{(1/3)}*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^{(1/3)}*(I\sqrt{3} + 1) - 2*b^2/d)^2*c*d^2)/(c*d^2)))/d$$

Sympy [A] time = 1.03391, size = 156, normalized size = 0.84

$$\text{RootSum}\left(27t^3c^2d^3 - 27t^2b^2c^2d^2 + t(18a^3bcd^2 + 9b^4c^2d) - a^6d^2 + 2a^3b^3cd - b^6c^2, \left(t \mapsto t \log\left(x + \frac{18t^2bc^2d^2 + 3ta^3cd}{a}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**2/(d*x**3+c),x)

[Out] RootSum(27*_t**3*c**2*d**3 - 27*_t**2*b**2*c**2*d**2 + _t*(18*a**3*b*c*d**2 + 9*b**4*c**2*d) - a**6*d**2 + 2*a**3*b**3*c*d - b**6*c**2, Lambda(_t, _t*log(x + (18*_t**2*b*c**2*d**2 + 3*_t*a**3*c*d**2 - 12*_t*b**3*c**2*d + 7*a**3*b**2*c*d + 2*b**5*c**2)/(a**5*d**2 + 8*a**2*b**3*c*d))))

Giac [A] time = 1.09442, size = 262, normalized size = 1.41

$$\frac{b^2 \log(|dx^3 + c|)}{3d} - \frac{\left(2abd\left(-\frac{c}{d}\right)^{\frac{1}{3}} + a^2d\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3cd} + \frac{\sqrt{3}\left(\left(-cd^2\right)^{\frac{1}{3}}a^2d - 2\left(-cd^2\right)^{\frac{2}{3}}ab\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^2/(d*x^3+c),x, algorithm="giac")

[Out] 1/3*b^2*log(abs(d*x^3 + c))/d - 1/3*(2*a*b*d*(-c/d)^(1/3) + a^2*d)*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(c*d) + 1/3*sqrt(3)*((-c*d^2)^(1/3)*a^2*d - 2*(-c*d^2)^(2/3)*a*b)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/(c*d^2) + 1/6*((-c*d^2)^(1/3)*a^2*c*d^3 + 2*(-c*d^2)^(2/3)*a*b*c*d^2)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(c^2*d^4)

$$3.71 \quad \int \frac{(a+bx)^3}{c+dx^3} dx$$

Optimal. Leaf size=222

$$\frac{(3a^2b\sqrt[3]{cd^2/3} + a^3(-d) + b^3c) \log(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2)}{6c^{2/3}d^{4/3}} - \frac{(3a^2b\sqrt[3]{cd^2/3} + a^3(-d) + b^3c) \log(\sqrt[3]{c} + \sqrt[3]{dx})}{3c^{2/3}d^{4/3}} + \frac{(-3a^2b\sqrt[3]{cd^2}}{}$$

[Out] (b^3*x)/d + ((b^3*c - 3*a^2*b*c^(1/3)*d^(2/3) - a^3*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))]/(Sqrt[3]*c^(2/3)*d^(4/3)) - ((b^3*c + 3*a^2*b*c^(1/3)*d^(2/3) - a^3*d)*Log[c^(1/3) + d^(1/3)*x]/(3*c^(2/3)*d^(4/3)) + ((b^3*c + 3*a^2*b*c^(1/3)*d^(2/3) - a^3*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]/(6*c^(2/3)*d^(4/3)) + (a*b^2*Log[c + d*x^3])/d

Rubi [A] time = 0.318566, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{(3a^2b\sqrt[3]{cd^2/3} + a^3(-d) + b^3c) \log(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2)}{6c^{2/3}d^{4/3}} - \frac{(3a^2b\sqrt[3]{cd^2/3} + a^3(-d) + b^3c) \log(\sqrt[3]{c} + \sqrt[3]{dx})}{3c^{2/3}d^{4/3}} + \frac{(-3a^2b\sqrt[3]{cd^2}}{}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^3/(c + d*x^3), x]

[Out] (b^3*x)/d + ((b^3*c - 3*a^2*b*c^(1/3)*d^(2/3) - a^3*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))]/(Sqrt[3]*c^(2/3)*d^(4/3)) - ((b^3*c + 3*a^2*b*c^(1/3)*d^(2/3) - a^3*d)*Log[c^(1/3) + d^(1/3)*x]/(3*c^(2/3)*d^(4/3)) + ((b^3*c + 3*a^2*b*c^(1/3)*d^(2/3) - a^3*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]/(6*c^(2/3)*d^(4/3)) + (a*b^2*Log[c + d*x^3])/d

Rule 1887

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Pq/(a + b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]

Rule 1871

Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di

st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

Rule 1860

Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]

Rule 31

Int[((a_) + (b_)*(x_))^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + bx)^3}{c + dx^3} dx &= \int \left(\frac{b^3}{d} - \frac{b^3c - a^3d - 3a^2bdx - 3ab^2dx^2}{d(c + dx^3)} \right) dx \\
 &= \frac{b^3x}{d} - \frac{\int \frac{b^3c - a^3d - 3a^2bdx - 3ab^2dx^2}{c + dx^3} dx}{d} \\
 &= \frac{b^3x}{d} + (3ab^2) \int \frac{x^2}{c + dx^3} dx - \frac{\int \frac{b^3c - a^3d - 3a^2bdx}{c + dx^3} dx}{d} \\
 &= \frac{b^3x}{d} + \frac{ab^2 \log(c + dx^3)}{d} - \frac{\int \frac{\sqrt[3]{c}(-3a^2b\sqrt[3]{cd} + 2\sqrt[3]{d}(b^3c - a^3d)) + \sqrt[3]{d}(-3a^2b\sqrt[3]{cd} - \sqrt[3]{d}(b^3c - a^3d))x}{c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2} dx}{3c^{2/3}d^{4/3}} - \frac{(b^3c + 3a^2b\sqrt[3]{cd}^{2/3} - a^3d) \int \frac{1}{c^{2/3}}}{3c^{2/3}d^{4/3}} \\
 &= \frac{b^3x}{d} - \frac{(b^3c + 3a^2b\sqrt[3]{cd}^{2/3} - a^3d) \log(\sqrt[3]{c} + \sqrt[3]{dx})}{3c^{2/3}d^{4/3}} + \frac{ab^2 \log(c + dx^3)}{d} - \frac{(b^3c - 3a^2b\sqrt[3]{cd}^{2/3} - a^3d) \int \frac{1}{c^{2/3}}}{2\sqrt[3]{cd}} \\
 &= \frac{b^3x}{d} - \frac{(b^3c + 3a^2b\sqrt[3]{cd}^{2/3} - a^3d) \log(\sqrt[3]{c} + \sqrt[3]{dx})}{3c^{2/3}d^{4/3}} + \frac{(b^3c + 3a^2b\sqrt[3]{cd}^{2/3} - a^3d) \log(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3})}{6c^{2/3}d^{4/3}} \\
 &= \frac{b^3x}{d} + \frac{(b^3c - 3a^2b\sqrt[3]{cd}^{2/3} - a^3d) \tan^{-1}\left(\frac{\sqrt[3]{c} - 2\sqrt[3]{dx}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{4/3}} - \frac{(b^3c + 3a^2b\sqrt[3]{cd}^{2/3} - a^3d) \log(\sqrt[3]{c} + \sqrt[3]{dx})}{3c^{2/3}d^{4/3}} + \frac{(b^3c - 3a^2b\sqrt[3]{cd}^{2/3} - a^3d) \log(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3})}{6c^{2/3}d^{4/3}}
 \end{aligned}$$

Mathematica [A] time = 0.170185, size = 214, normalized size = 0.96

$$\frac{(3a^2b\sqrt[3]{cd}^{2/3} + a^3(-d) + b^3c) \log(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2) - 2(3a^2b\sqrt[3]{cd}^{2/3} + a^3(-d) + b^3c) \log(\sqrt[3]{c} + \sqrt[3]{dx}) + 2\sqrt{3}(-3a^2b\sqrt[3]{cd}^{2/3} - a^3d) \log(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3})}{6c^{2/3}d^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^3/(c + d*x^3),x]

[Out] (6*b^3*c^(2/3)*d^(1/3)*x + 2*Sqrt[3]*(b^3*c - 3*a^2*b*c^(1/3)*d^(2/3) - a^3*d)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] - 2*(b^3*c + 3*a^2*b*c^(1/3)*d^(2/3) - a^3*d)*Log[c^(1/3) + d^(1/3)*x] + (b^3*c + 3*a^2*b*c^(1/3)*d^(2/3) - a^3*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2] + 6*a*b^2*c^(2/3)*d^(1/3)*x

$$3) * d^{(1/3)} * \text{Log}[c + d * x^3] / (6 * c^{(2/3)} * d^{(4/3)})$$

Maple [A] time = 0.005, size = 325, normalized size = 1.5

$$\frac{b^3 x}{d} + \frac{a^3}{3d} \ln\left(x + \sqrt[3]{\frac{c}{d}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} - \frac{b^3 c}{3d^2} \ln\left(x + \sqrt[3]{\frac{c}{d}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} - \frac{a^3}{6d} \ln\left(x^2 - \sqrt[3]{\frac{c}{d}} x + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} + \frac{b^3 c}{6d^2} \ln\left(x^2 - \sqrt[3]{\frac{c}{d}} x + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^3/(d*x^3+c), x)

[Out] $b^3 * x/d + 1/3 * d / (c/d)^{(2/3)} * \ln(x + (c/d)^{(1/3)}) * a^3 - 1/3 * d^2 / (c/d)^{(2/3)} * \ln(x + (c/d)^{(1/3)}) * b^3 * c - 1/6 * d / (c/d)^{(2/3)} * \ln(x^2 - (c/d)^{(1/3)} * x + (c/d)^{(2/3)}) * a^3 + 1/6 * d^2 / (c/d)^{(2/3)} * \ln(x^2 - (c/d)^{(1/3)} * x + (c/d)^{(2/3)}) * b^3 * c + 1/3 * d / (c/d)^{(2/3)} * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (2 / (c/d)^{(1/3)} * x - 1)) * a^3 - 1/3 * d^2 / (c/d)^{(2/3)} * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (2 / (c/d)^{(1/3)} * x - 1)) * b^3 * c - 1/d * a^2 * b / (c/d)^{(1/3)} * \ln(x + (c/d)^{(1/3)}) + 1/2 * d * a^2 * b / (c/d)^{(1/3)} * \ln(x^2 - (c/d)^{(1/3)} * x + (c/d)^{(2/3)}) + 1/d * a^2 * b * 3^{(1/2)} / (c/d)^{(1/3)} * \arctan(1/3 * 3^{(1/2)} * (2 / (c/d)^{(1/3)} * x - 1)) + a * b^2 * \ln(d * x^3 + c) / d$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^3/(d*x^3+c), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [C] time = 14.9484, size = 15101, normalized size = 68.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

$$\begin{aligned}
& *d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4)) \\
& ^{(1/3)} - 6*a*b^2/d + (1/2)^{(1/3)}*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b* \\
& d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/ \\
& (c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d \\
& ^4))^{(1/3)}*(I*sqrt(3) + 1))^{2*c*d^2)/(c*d^2)))*log(3*a*b^8*c^3 - 15*a^4*b^5 \\
& *c^2*d - 15*a^7*b^2*c*d^2 - 3/4*(6*(1/2)^{(2/3)}*(3*a^2*b^4/d^2 - (2*a^2*b^4*c \\
& + a^5*b*d)/(c*d^2)))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + \\
& a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9 \\
& *d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/ \\
& (c^2*d^4))^{(1/3)} - 6*a*b^2/d + (1/2)^{(1/3)}*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c \\
& + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - \\
& a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d \\
& ^3)/(c^2*d^4))^{(1/3)}*(I*sqrt(3) + 1))^{2*a^2*b*c^2*d^3 + 1/2*(b^6*c^3*d - 20 \\
& *a^3*b^3*c^2*d^2 + a^6*c*d^3)*(6*(1/2)^{(2/3)}*(3*a^2*b^4/d^2 - (2*a^2*b^4*c \\
& + a^5*b*d)/(c*d^2)))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^ \\
& 5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d \\
& ^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c \\
& ^2*d^4))^{(1/3)} - 6*a*b^2/d + (1/2)^{(1/3)}*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c \\
& + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a \\
& ^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3 \\
&)/(c^2*d^4))^{(1/3)}*(I*sqrt(3) + 1)) - 2*(b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6 \\
& *b^3*c*d^2 - a^9*d^3)*x + 3/4*sqrt(1/3)*(2*b^6*c^3*d + 14*a^3*b^3*c^2*d^2 + \\
& 2*a^6*c*d^3 + 3*(6*(1/2)^{(2/3)}*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(c \\
& *d^2)))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/ \\
& (c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) \\
& - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^{(1/3} \\
&) - 6*a*b^2/d + (1/2)^{(1/3)}*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a* \\
& b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2* \\
& d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^{ \\
& (1/3)}*(I*sqrt(3) + 1))*a^2*b*c^2*d^3)*sqrt((12*a^2*b^4*c - 48*a^5*b*d - 12* \\
& (6*(1/2)^{(2/3)}*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(c*d^2)))*(-I*sqrt(3) \\
&) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^ \\
& 3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a \\
& ^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^{(1/3)} - 6*a*b^2/d + (\\
& 1/2)^{(1/3)}*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^ \\
& 9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - \\
& 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^{(1/3)}*(I*sqrt(3) \\
& + 1))*a*b^2*c*d - (6*(1/2)^{(2/3)}*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(\\
& c*d^2)))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2 \\
& /(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4 \\
&) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^{(1/ \\
& 3)} - 6*a*b^2/d + (1/2)^{(1/3)}*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a \\
& *b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2 \\
& *d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4)) \\
& ^{(1/3)}*(I*sqrt(3) + 1))^{2*c*d^2)/(c*d^2)) + (18*a*b^2 + (6*(1/2)^{(2/3)}*(3*
\end{aligned}$$

$$\begin{aligned}
& a^2 b^4 / d^2 - (2 a^2 b^4 c + a^5 b d) / (c d^2) * (-I \sqrt{3} + 1) / (54 a^3 b^6 / d^3 - 27 * (2 a^2 b^4 c + a^5 b d) * a b^2 / (c d^3) - (b^9 c^3 - 3 a^3 b^6 c^2 d + 3 a^6 b^3 c d^2 - a^9 d^3) / (c^2 d^4) - (b^9 c^3 - 3 a^3 b^6 c^2 d - 24 a^6 b^3 c d^2 - a^9 d^3) / (c^2 d^4))^{1/3} - 6 a b^2 / d + (1/2)^{1/3} * (54 a^3 b^6 / d^3 - 27 * (2 a^2 b^4 c + a^5 b d) * a b^2 / (c d^3) - (b^9 c^3 - 3 a^3 b^6 c^2 d + 3 a^6 b^3 c d^2 - a^9 d^3) / (c^2 d^4) - (b^9 c^3 - 3 a^3 b^6 c^2 d - 24 a^6 b^3 c d^2 - a^9 d^3) / (c^2 d^4))^{1/3} * (I \sqrt{3} + 1) * d - 3 \sqrt{3} * d * \sqrt{3} * ((12 a^2 b^4 c - 48 a^5 b d - 12 * (6 * (1/2)^{2/3}) * (3 a^2 b^4 / d^2 - (2 a^2 b^4 c + a^5 b d) / (c d^2)) * (-I \sqrt{3} + 1) / (54 a^3 b^6 / d^3 - 27 * (2 a^2 b^4 c + a^5 b d) * a b^2 / (c d^3) - (b^9 c^3 - 3 a^3 b^6 c^2 d + 3 a^6 b^3 c d^2 - a^9 d^3) / (c^2 d^4) - (b^9 c^3 - 3 a^3 b^6 c^2 d - 24 a^6 b^3 c d^2 - a^9 d^3) / (c^2 d^4))^{1/3} - 6 a b^2 / d + (1/2)^{1/3} * (54 a^3 b^6 / d^3 - 27 * (2 a^2 b^4 c + a^5 b d) * a b^2 / (c d^3) - (b^9 c^3 - 3 a^3 b^6 c^2 d + 3 a^6 b^3 c d^2 - a^9 d^3) / (c^2 d^4) - (b^9 c^3 - 3 a^3 b^6 c^2 d - 24 a^6 b^3 c d^2 - a^9 d^3) / (c^2 d^4))^{1/3} * (I \sqrt{3} + 1)) * a b^2 * c * d - (6 * (1/2)^{2/3}) * (3 a^2 b^4 / d^2 - (2 a^2 b^4 c + a^5 b d) / (c d^2)) * (-I \sqrt{3} + 1) / (54 a^3 b^6 / d^3 - 27 * (2 a^2 b^4 c + a^5 b d) * a b^2 / (c d^3) - (b^9 c^3 - 3 a^3 b^6 c^2 d + 3 a^6 b^3 c d^2 - a^9 d^3) / (c^2 d^4) - (b^9 c^3 - 3 a^3 b^6 c^2 d - 24 a^6 b^3 c d^2 - a^9 d^3) / (c^2 d^4))^{1/3} - 6 a b^2 / d + (1/2)^{1/3} * (54 a^3 b^6 / d^3 - 27 * (2 a^2 b^4 c + a^5 b d) * a b^2 / (c d^3) - (b^9 c^3 - 3 a^3 b^6 c^2 d + 3 a^6 b^3 c d^2 - a^9 d^3) / (c^2 d^4) - (b^9 c^3 - 3 a^3 b^6 c^2 d - 24 a^6 b^3 c d^2 - a^9 d^3) / (c^2 d^4))^{1/3} * (I \sqrt{3} + 1))^{2 * c * d^2} / (c d^2)) * \log(3 a b^8 c^3 - 15 a^4 b^5 c^2 d - 15 a^7 b^2 c d^2 - 3/4 * (6 * (1/2)^{2/3}) * (3 a^2 b^4 / d^2 - (2 a^2 b^4 c + a^5 b d) / (c d^2)) * (-I \sqrt{3} + 1) / (54 a^3 b^6 / d^3 - 27 * (2 a^2 b^4 c + a^5 b d) * a b^2 / (c d^3) - (b^9 c^3 - 3 a^3 b^6 c^2 d + 3 a^6 b^3 c d^2 - a^9 d^3) / (c^2 d^4) - (b^9 c^3 - 3 a^3 b^6 c^2 d - 24 a^6 b^3 c d^2 - a^9 d^3) / (c^2 d^4))^{1/3} - 6 a b^2 / d + (1/2)^{1/3} * (54 a^3 b^6 / d^3 - 27 * (2 a^2 b^4 c + a^5 b d) * a b^2 / (c d^3) - (b^9 c^3 - 3 a^3 b^6 c^2 d + 3 a^6 b^3 c d^2 - a^9 d^3) / (c^2 d^4) - (b^9 c^3 - 3 a^3 b^6 c^2 d - 24 a^6 b^3 c d^2 - a^9 d^3) / (c^2 d^4))^{1/3} * (I \sqrt{3} + 1))^{2 * a^2 * b * c^2 * d^3} + 1/2 * (b^6 c^3 d - 20 a^3 b^3 c^2 d^2 + a^6 c d^3) * (6 * (1/2)^{2/3}) * (3 a^2 b^4 / d^2 - (2 a^2 b^4 c + a^5 b d) / (c d^2)) * (-I \sqrt{3} + 1) / (54 a^3 b^6 / d^3 - 27 * (2 a^2 b^4 c + a^5 b d) * a b^2 / (c d^3) - (b^9 c^3 - 3 a^3 b^6 c^2 d + 3 a^6 b^3 c d^2 - a^9 d^3) / (c^2 d^4) - (b^9 c^3 - 3 a^3 b^6 c^2 d - 24 a^6 b^3 c d^2 - a^9 d^3) / (c^2 d^4))^{1/3} - 6 a b^2 / d + (1/2)^{1/3} * (54 a^3 b^6 / d^3 - 27 * (2 a^2 b^4 c + a^5 b d) * a b^2 / (c d^3) - (b^9 c^3 - 3 a^3 b^6 c^2 d + 3 a^6 b^3 c d^2 - a^9 d^3) / (c^2 d^4) - (b^9 c^3 - 3 a^3 b^6 c^2 d - 24 a^6 b^3 c d^2 - a^9 d^3) / (c^2 d^4))^{1/3} * (I \sqrt{3} + 1)) - 2 * (b^9 c^3 - 3 a^3 b^6 c^2 d - 24 a^6 b^3 c d^2 - a^9 d^3) * x - 3/4 * \sqrt{3} * (1/3) * (2 b^6 c^3 d + 14 a^3 b^3 c^2 d^2 + 2 a^6 c d^3 + 3 * (6 * (1/2)^{2/3}) * (3 a^2 b^4 / d^2 - (2 a^2 b^4 c + a^5 b d) / (c d^2)) * (-I \sqrt{3} + 1) / (54 a^3 b^6 / d^3 - 27 * (2 a^2 b^4 c + a^5 b d) * a b^2 / (c d^3) - (b^9 c^3 - 3 a^3 b^6 c^2 d + 3 a^6 b^3 c d^2 - a^9 d^3) / (c^2 d^4) - (b^9 c^3 - 3 a^3 b^6 c^2 d - 24 a^6 b^3 c d^2 - a^9 d^3) / (c^2 d^4))^{1/3} - 6 a b^2 / d + (1/2)^{1/3} * (54 a^3 b^6 / d^3 - 27 * (2 a^2 b^4 c + a^5 b d) * a b^2 / (c d^3) - (b^9 c^3 - 3 a^3 b^6 c^2 d + 3 a^6 b^3 c d^2 - a^9 d^3) / (c^2 d^4) - (b^9 c^3 - 3 a^3 b^6 c^2 d - 24 a^6 b^3 c d^2 - a^9 d^3) / (c^2 d^4))^{1/3} * (I \sqrt{3} + 1))
\end{aligned}$$

$$\begin{aligned} &^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - \\ &24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^{(1/3)}*(I*sqrt(3) + 1))*a^2*b*c^2*d^3 \\ &)*sqrt(((12*a^2*b^4*c - 48*a^5*b*d - 12*(6*(1/2))^{(2/3)}*(3*a^2*b^4/d^2 - (2*a \\ &^2*b^4*c + a^5*b*d)/(c*d^2)))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b \\ &^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 \\ &- a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9 \\ &9*d^3)/(c^2*d^4))^{(1/3)} - 6*a*b^2/d + (1/2)^{(1/3)}*(54*a^3*b^6/d^3 - 27*(2*a \\ &^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c \\ &c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 \\ &- a^9*d^3)/(c^2*d^4))^{(1/3)}*(I*sqrt(3) + 1))*a*b^2*c*d - (6*(1/2))^{(2/3)}*(3* \\ &a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(c*d^2)))*(-I*sqrt(3) + 1)/(54*a^3*b^6 \\ &/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2* \\ &d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24* \\ &a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^{(1/3)} - 6*a*b^2/d + (1/2)^{(1/3)}*(54*a^3 \\ &*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6* \\ &c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - \\ &24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^{(1/3)}*(I*sqrt(3) + 1))^{2*c*d^2)/(c* \\ &d^2)))/d \end{aligned}$$

Sympy [A] time = 2.64378, size = 245, normalized size = 1.1

$$\frac{b^3x}{d} + \text{RootSum}\left(27t^3c^2d^4 - 81t^2ab^2c^2d^3 + t(27a^5bcd^3 + 54a^2b^4c^2d^2) - a^9d^3 + 3a^6b^3cd^2 - 3a^3b^6c^2d + b^9c^3, \left(t \mapsto t \log\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**3/(d*x**3+c),x)

[Out] b**3*x/d + RootSum(27*_t**3*c**2*d**4 - 81*_t**2*a*b**2*c**2*d**3 + *_t*(27*a**5*b*c*d**3 + 54*a**2*b**4*c**2*d**2) - a**9*d**3 + 3*a**6*b**3*c*d**2 - 3*a**3*b**6*c**2*d + b**9*c**3, Lambda(_t, _t*log(x + (27*_t**2*a**2*b*c**2*d**3 + 3*_t*a**6*c*d**3 - 60*_t*a**3*b**3*c**2*d**2 + 3*_t*b**6*c**3*d + 15*a**7*b**2*c*d**2 + 15*a**4*b**5*c**2*d - 3*a*b**8*c**3)/(a**9*d**3 + 24*a**6*b**3*c*d**2 + 3*a**3*b**6*c**2*d - b**9*c**3))))

Giac [A] time = 1.09219, size = 327, normalized size = 1.47

$$\frac{b^3x}{d} + \frac{ab^2 \log(|dx^3 + c|)}{d} - \frac{\sqrt{3}\left(\left(-cd^2\right)^{\frac{1}{3}} b^3c - \left(-cd^2\right)^{\frac{1}{3}} a^3d + 3\left(-cd^2\right)^{\frac{2}{3}} a^2b\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3cd^2} - \left(\left(-cd^2\right)^{\frac{1}{3}} b^3c - \left(-cd^2\right)^{\frac{1}{3}} a^3d + 3\left(-cd^2\right)^{\frac{2}{3}} a^2b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^3/(d*x^3+c),x, algorithm="giac")

[Out] $b^3x/d + a*b^2*\log(\text{abs}(d*x^3 + c))/d - 1/3*\text{sqrt}(3)*((-c*d^2)^{(1/3)}*b^3*c - (-c*d^2)^{(1/3)}*a^3*d + 3*(-c*d^2)^{(2/3)}*a^2*b)*\arctan(1/3*\text{sqrt}(3)*(2*x + (-c/d)^{(1/3)})/(-c/d)^{(1/3)})/(c*d^2) - 1/6*((-c*d^2)^{(1/3)}*b^3*c - (-c*d^2)^{(1/3)}*a^3*d - 3*(-c*d^2)^{(2/3)}*a^2*b)*\log(x^2 + x*(-c/d)^{(1/3)} + (-c/d)^{(2/3)})/(c*d^2) - 1/3*(3*a^2*b*d^3*(-c/d)^{(1/3)} - b^3*c*d^2 + a^3*d^3)*(-c/d)^{(1/3)}*\log(\text{abs}(x - (-c/d)^{(1/3)})/(c*d^3)$

$$3.72 \quad \int \frac{(a+bx)^4}{c+dx^3} dx$$

Optimal. Leaf size=282

$$\frac{(b\sqrt[3]{c}(b^3c-4a^3d)-\sqrt[3]{d}(4ab^3c-a^4d))\log(c^{2/3}-\sqrt[3]{c}\sqrt[3]{dx+d^{2/3}x^2})}{6c^{2/3}d^{5/3}} + \frac{(b\sqrt[3]{c}(b^3c-4a^3d)-\sqrt[3]{d}(4ab^3c-a^4d))\log(\sqrt[3]{c}+\sqrt[3]{dx+d^{2/3}x^2})}{3c^{2/3}d^{5/3}}$$

[Out] $(4*a*b^3*x)/d + (b^4*x^2)/(2*d) + ((b^4*c^{(4/3)} + 4*a*b^3*c*d^{(1/3)} - 4*a^3*b*c^{(1/3)}*d - a^4*d^{(4/3)})*ArcTan[(c^{(1/3)} - 2*d^{(1/3)}*x)/(Sqrt[3]*c^{(1/3)})])/(Sqrt[3]*c^{(2/3)}*d^{(5/3)}) + ((b*c^{(1/3)}*(b^3*c - 4*a^3*d) - d^{(1/3)}*(4*a*b^3*c - a^4*d))*Log[c^{(1/3)} + d^{(1/3)}*x]/(3*c^{(2/3)}*d^{(5/3)}) - ((b*c^{(1/3)}*(b^3*c - 4*a^3*d) - d^{(1/3)}*(4*a*b^3*c - a^4*d))*Log[c^{(2/3)} - c^{(1/3)}*d^{(1/3)}*x + d^{(2/3)}*x^2])/(6*c^{(2/3)}*d^{(5/3)}) + (2*a^2*b^2*Log[c + d*x^3])/d$

Rubi [A] time = 0.443201, antiderivative size = 280, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\left(-\frac{b\sqrt[3]{c}(b^3c-4a^3d)}{\sqrt[3]{d}} + a^4(-d) + 4ab^3c\right)\log(c^{2/3}-\sqrt[3]{c}\sqrt[3]{dx+d^{2/3}x^2})}{6c^{2/3}d^{4/3}} + \frac{(b\sqrt[3]{c}(b^3c-4a^3d)-\sqrt[3]{d}(4ab^3c-a^4d))\log(\sqrt[3]{c}+\sqrt[3]{dx+d^{2/3}x^2})}{3c^{2/3}d^{5/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^4/(c + d*x^3), x]

[Out] $(4*a*b^3*x)/d + (b^4*x^2)/(2*d) + ((b^4*c^{(4/3)} + 4*a*b^3*c*d^{(1/3)} - 4*a^3*b*c^{(1/3)}*d - a^4*d^{(4/3)})*ArcTan[(c^{(1/3)} - 2*d^{(1/3)}*x)/(Sqrt[3]*c^{(1/3)})])/(Sqrt[3]*c^{(2/3)}*d^{(5/3)}) + ((b*c^{(1/3)}*(b^3*c - 4*a^3*d) - d^{(1/3)}*(4*a*b^3*c - a^4*d))*Log[c^{(1/3)} + d^{(1/3)}*x]/(3*c^{(2/3)}*d^{(5/3)}) + ((4*a*b^3*c - a^4*d - (b*c^{(1/3)}*(b^3*c - 4*a^3*d))/d^{(1/3)})*Log[c^{(2/3)} - c^{(1/3)}*d^{(1/3)}*x + d^{(2/3)}*x^2])/(6*c^{(2/3)}*d^{(4/3)}) + (2*a^2*b^2*Log[c + d*x^3])/d$

Rule 1887

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a + b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]

Rule 1871

```
Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(n-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(n-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(n-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned}
 \int \frac{(a+bx)^4}{c+dx^3} dx &= \int \left(\frac{4ab^3}{d} + \frac{b^4x}{d} - \frac{4ab^3c - a^4d + b(b^3c - 4a^3d)x - 6a^2b^2dx^2}{d(c+dx^3)} \right) dx \\
 &= \frac{4ab^3x}{d} + \frac{b^4x^2}{2d} - \frac{\int \frac{4ab^3c - a^4d + b(b^3c - 4a^3d)x - 6a^2b^2dx^2}{c+dx^3} dx}{d} \\
 &= \frac{4ab^3x}{d} + \frac{b^4x^2}{2d} + (6a^2b^2) \int \frac{x^2}{c+dx^3} dx - \frac{\int \frac{4ab^3c - a^4d + b(b^3c - 4a^3d)x}{c+dx^3} dx}{d} \\
 &= \frac{4ab^3x}{d} + \frac{b^4x^2}{2d} + \frac{2a^2b^2 \log(c+dx^3)}{d} - \frac{\int \frac{\sqrt[3]{c}(b\sqrt[3]{c(b^3c-4a^3d)} + 2\sqrt[3]{d}(4ab^3c-a^4d)) + \sqrt[3]{d}(b\sqrt[3]{c(b^3c-4a^3d)} - \sqrt[3]{d}(4ab^3c-a^4d))}{c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx+d^{2/3}x^2}} dx}{3c^{2/3}d^{4/3}} \\
 &= \frac{4ab^3x}{d} + \frac{b^4x^2}{2d} - \frac{\left(4ab^3c - a^4d - \frac{b\sqrt[3]{c}(b^3c-4a^3d)}{\sqrt[3]{d}}\right) \log(\sqrt[3]{c} + \sqrt[3]{dx})}{3c^{2/3}d^{4/3}} + \frac{2a^2b^2 \log(c+dx^3)}{d} - \frac{(b^4c^{4/3} + 4ab^3c^{1/3}d)}{3c^{2/3}d^{4/3}} \\
 &= \frac{4ab^3x}{d} + \frac{b^4x^2}{2d} - \frac{\left(4ab^3c - a^4d - \frac{b\sqrt[3]{c}(b^3c-4a^3d)}{\sqrt[3]{d}}\right) \log(\sqrt[3]{c} + \sqrt[3]{dx})}{3c^{2/3}d^{4/3}} + \frac{\left(4ab^3c - a^4d - \frac{b\sqrt[3]{c}(b^3c-4a^3d)}{\sqrt[3]{d}}\right) \log(c^2 + dx^3)}{6c^{2/3}d^{4/3}} \\
 &= \frac{4ab^3x}{d} + \frac{b^4x^2}{2d} + \frac{(b^4c^{4/3} + 4ab^3c\sqrt[3]{d} - 4a^3b\sqrt[3]{cd} - a^4d^{4/3}) \tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{dx}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{5/3}} - \frac{\left(4ab^3c - a^4d - \frac{b\sqrt[3]{c}(b^3c-4a^3d)}{\sqrt[3]{d}}\right) \log(c^2 + dx^3)}{3c^{2/3}d^{4/3}}
 \end{aligned}$$

Mathematica [A] time = 0.244274, size = 277, normalized size = 0.98

$$\frac{\frac{(-4a^3b\sqrt[3]{cd+a^4d^{4/3}}-4ab^3c\sqrt[3]{d}+b^4c^{4/3}) \log(c^{2/3}-\sqrt[3]{c}\sqrt[3]{dx+d^{2/3}x^2})}{c^{2/3}} + \frac{2(-4a^3b\sqrt[3]{cd+a^4d^{4/3}}-4ab^3c\sqrt[3]{d}+b^4c^{4/3}) \log(\sqrt[3]{c}+\sqrt[3]{dx})}{c^{2/3}}}{6d^{5/3}} + \frac{2\sqrt{3}(-4a^3b\sqrt[3]{cd+a^4d^{4/3}}(-d^{4/3})+b^4c^{4/3})}{6d^{5/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^4/(c + d*x^3), x]

[Out] (24*a*b^3*d^(2/3)*x + 3*b^4*d^(2/3)*x^2 + (2*sqrt[3]*(b^4*c^(4/3) + 4*a*b^3*c*d^(1/3) - 4*a^3*b*c^(1/3)*d - a^4*d^(4/3))*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))])

$$\frac{1/3)/\text{Sqrt}[3]]/c^{(2/3)} + (2*(b^4*c^{(4/3)} - 4*a*b^3*c*d^{(1/3)} - 4*a^3*b*c^{(1/3)*d} + a^4*d^{(4/3)})*\text{Log}[c^{(1/3)} + d^{(1/3)*x}]/c^{(2/3)} - ((b^4*c^{(4/3)} - 4*a*b^3*c*d^{(1/3)} - 4*a^3*b*c^{(1/3)*d} + a^4*d^{(4/3)})*\text{Log}[c^{(2/3)} - c^{(1/3)*d}^{(1/3)*x} + d^{(2/3)*x^2}]/c^{(2/3)} + 12*a^2*b^2*d^{(2/3)*\text{Log}[c + d*x^3]})/(6*d^{(5/3)})$$

Maple [A] time = 0.003, size = 446, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x+a)^4/(d*x^3+c),x)`

[Out] $\frac{1}{2}b^4x^2/d + 4ab^3x/d + 1/3d/(c/d)^{(2/3)}\ln(x+(c/d)^{(1/3)})a^4 - 4/3d^2/(c/d)^{(2/3)}\ln(x+(c/d)^{(1/3)})ab^3c - 1/6d/(c/d)^{(2/3)}\ln(x^2-(c/d)^{(1/3)*x} + (c/d)^{(2/3)})a^4 + 2/3d^2/(c/d)^{(2/3)}\ln(x^2-(c/d)^{(1/3)*x} + (c/d)^{(2/3)})ab^3c + 1/3d/(c/d)^{(2/3)}3^{(1/2)}\arctan(1/3*3^{(1/2)}*(2/(c/d)^{(1/3)*x} - 1))a^4 - 4/3d^2/(c/d)^{(2/3)}3^{(1/2)}\arctan(1/3*3^{(1/2)}*(2/(c/d)^{(1/3)*x} - 1))ab^3c - 4/3d/(c/d)^{(1/3)}\ln(x+(c/d)^{(1/3)})a^3b + 1/3d^2/(c/d)^{(1/3)}\ln(x+(c/d)^{(1/3)})b^4c + 2/3d/(c/d)^{(1/3)}\ln(x^2-(c/d)^{(1/3)*x} + (c/d)^{(2/3)})a^3b - 1/6d^2/(c/d)^{(1/3)}\ln(x^2-(c/d)^{(1/3)*x} + (c/d)^{(2/3)})b^4c + 4/3d*3^{(1/2)}/(c/d)^{(1/3)}\arctan(1/3*3^{(1/2)}*(2/(c/d)^{(1/3)*x} - 1))a^3b - 1/3d^2*3^{(1/2)}/(c/d)^{(1/3)}\arctan(1/3*3^{(1/2)}*(2/(c/d)^{(1/3)*x} - 1))b^4c + 2a^2b^2\ln(d*x^3+c)/d$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)^4/(d*x^3+c),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [C] time = 57.8475, size = 18692, normalized size = 66.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^4/(d*x^3+c),x, algorithm="fricas")

[Out]
$$\frac{1}{12} \cdot (6b^4x^2 + 48ab^3x + 2 \cdot (12a^2b^2/d - 2 \cdot (1/2)^{(2/3)} \cdot (36a^4b^4/d^2 - (4ab^7c^2 + 19a^4b^4cd + 4a^7b^2d^2)/(cd^3))) \cdot (-I\sqrt{3} + 1) / (432a^6b^6/d^3 - 18 \cdot (4ab^7c^2 + 19a^4b^4cd + 4a^7b^2d^2) \cdot a^2b^2 / (cd^4) - (b^{12}c^4 + 52a^3b^9c^3d - 52a^9b^3cd^3 - a^{12}d^4) / (c^2d^5) + (b^{12}c^4 - 4a^3b^9c^3d + 6a^6b^6c^2d^2 - 4a^9b^3cd^3 + a^{12}d^4) / (c^2d^5))^{(1/3)} - (1/2)^{(1/3)} \cdot (432a^6b^6/d^3 - 18 \cdot (4ab^7c^2 + 19a^4b^4cd + 4a^7b^2d^2) \cdot a^2b^2 / (cd^4) - (b^{12}c^4 + 52a^3b^9c^3d - 52a^9b^3cd^3 - a^{12}d^4) / (c^2d^5) + (b^{12}c^4 - 4a^3b^9c^3d + 6a^6b^6c^2d^2 - 4a^9b^3cd^3 + a^{12}d^4) / (c^2d^5))^{(1/3)} \cdot (I\sqrt{3} + 1) \cdot \log(-8ab^{11}c^4 - 66a^4b^8c^3d + 48a^7b^5c^2d^2 + 26a^{10}b^2cd^3 - 1/4 \cdot (b^4c^3d^3 - 4a^3b^2c^2d^4) \cdot (12a^2b^2/d - 2 \cdot (1/2)^{(2/3)} \cdot (36a^4b^4/d^2 - (4ab^7c^2 + 19a^4b^4cd + 4a^7b^2d^2)/(cd^3))) \cdot (-I\sqrt{3} + 1) / (432a^6b^6/d^3 - 18 \cdot (4ab^7c^2 + 19a^4b^4cd + 4a^7b^2d^2) \cdot a^2b^2 / (cd^4) - (b^{12}c^4 + 52a^3b^9c^3d - 52a^9b^3cd^3 - a^{12}d^4) / (c^2d^5) + (b^{12}c^4 - 4a^3b^9c^3d + 6a^6b^6c^2d^2 - 4a^9b^3cd^3 + a^{12}d^4) / (c^2d^5))^{(1/3)} - (1/2)^{(1/3)} \cdot (432a^6b^6/d^3 - 18 \cdot (4ab^7c^2 + 19a^4b^4cd + 4a^7b^2d^2) \cdot a^2b^2 / (cd^4) - (b^{12}c^4 + 52a^3b^9c^3d - 52a^9b^3cd^3 - a^{12}d^4) / (c^2d^5) + (b^{12}c^4 - 4a^3b^9c^3d + 6a^6b^6c^2d^2 - 4a^9b^3cd^3 + a^{12}d^4) / (c^2d^5))^{(1/3)} \cdot (I\sqrt{3} + 1)^2 + 1/2 \cdot (28a^2b^6c^3d^2 - 56a^5b^3c^2d^3 + a^8cd^4) \cdot (12a^2b^2/d - 2 \cdot (1/2)^{(2/3)} \cdot (36a^4b^4/d^2 - (4ab^7c^2 + 19a^4b^4cd + 4a^7b^2d^2)/(cd^3))) \cdot (-I\sqrt{3} + 1) / (432a^6b^6/d^3 - 18 \cdot (4ab^7c^2 + 19a^4b^4cd + 4a^7b^2d^2) \cdot a^2b^2 / (cd^4) - (b^{12}c^4 + 52a^3b^9c^3d - 52a^9b^3cd^3 - a^{12}d^4) / (c^2d^5) + (b^{12}c^4 - 4a^3b^9c^3d + 6a^6b^6c^2d^2 - 4a^9b^3cd^3 + a^{12}d^4) / (c^2d^5))^{(1/3)} - (1/2)^{(1/3)} \cdot (432a^6b^6/d^3 - 18 \cdot (4ab^7c^2 + 19a^4b^4cd + 4a^7b^2d^2) \cdot a^2b^2 / (cd^4) - (b^{12}c^4 + 52a^3b^9c^3d - 52a^9b^3cd^3 - a^{12}d^4) / (c^2d^5) + (b^{12}c^4 - 4a^3b^9c^3d + 6a^6b^6c^2d^2 - 4a^9b^3cd^3 + a^{12}d^4) / (c^2d^5))^{(1/3)} \cdot (I\sqrt{3} + 1) - (b^{12}c^4 + 52a^3b^9c^3d - 52a^9b^3cd^3 - a^{12}d^4) \cdot x) + (36a^2b^2 - (12a^2b^2/d - 2 \cdot (1/2)^{(2/3)} \cdot (36a^4b^4/d^2 - (4ab^7c^2 + 19a^4b^4cd + 4a^7b^2d^2)/(cd^3))) \cdot (-I\sqrt{3} + 1) / (432a^6b^6/d^3 - 18 \cdot (4ab^7c^2 + 19a^4b^4cd + 4a^7b^2d^2) \cdot a^2b^2 / (cd^4) - (b^{12}c^4 + 52a^3b^9c^3d - 52a^9b^3cd^3 - a^{12}d^4) / (c^2d^5) + (b^{12}c^4 - 4a^3b^9c^3d + 6a^6b^6c^2d^2 - 4a^9b^3cd^3 + a^{12}d^4) / (c^2d^5))^{(1/3)} - (1/2)^{(1/3)} \cdot (432a^6b^6/d^3 - 18 \cdot (4ab^7c^2 + 19a^4b^4cd + 4a^7b^2d^2) \cdot a^2b^2 / (cd^4) - (b^{12}c^4 + 52a^3b^9c^3d - 52a^9b^3cd^3 - a^{12}d^4) / (c^2d^5) + (b^{12}c^4 - 4a^3b^9c^3d + 6a^6b^6c^2d^2 - 4a^9b^3cd^3 + a^{12}d^4) / (c^2d^5))^{(1/3)} \cdot (I\sqrt{3} + 1) \cdot d + 3\sqrt{3} \cdot (1/3) \cdot d \cdot \sqrt{3} \cdot (-64ab^7c^2 - 128a^4b^4cd + 64a^7b^2d^2 - 24 \cdot (12a^2b^2/d$$

$$\begin{aligned}
& - 2*(1/2)^{(2/3)}*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/c^3)) * (-I*\sqrt{3} + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/c^4) - (b^{12}*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^{12}*d^4)/c^2*d^5) + (b^{12}*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^{12}*d^4)/c^2*d^5)^{(1/3)} - (1/2)^{(1/3)}*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/c^4) - (b^{12}*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^{12}*d^4)/c^2*d^5) + (b^{12}*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^{12}*d^4)/c^2*d^5)^{(1/3)}*(I*\sqrt{3} + 1)) * a^2*b^2*c*d^2 + (12*a^2*b^2/d - 2*(1/2)^{(2/3)}*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/c^3)) * (-I*\sqrt{3} + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/c^4) - (b^{12}*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^{12}*d^4)/c^2*d^5) + (b^{12}*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^{12}*d^4)/c^2*d^5)^{(1/3)} - (1/2)^{(1/3)}*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/c^4) - (b^{12}*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^{12}*d^4)/c^2*d^5) + (b^{12}*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^{12}*d^4)/c^2*d^5)^{(1/3)}*(I*\sqrt{3} + 1))^2*c*d^3/c^3)) * \log(8*a*b^{11}*c^4 + 66*a^4*b^8*c^3*d - 48*a^7*b^5*c^2*d^2 - 26*a^{10}*b^2*c*d^3 + 1/4*(b^4*c^3*d^3 - 4*a^3*b*c^2*d^4)*(12*a^2*b^2/d - 2*(1/2)^{(2/3)}*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/c^3)) * (-I*\sqrt{3} + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/c^4) - (b^{12}*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^{12}*d^4)/c^2*d^5) + (b^{12}*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^{12}*d^4)/c^2*d^5)^{(1/3)} - (1/2)^{(1/3)}*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/c^4) - (b^{12}*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^{12}*d^4)/c^2*d^5) + (b^{12}*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^{12}*d^4)/c^2*d^5)^{(1/3)}*(I*\sqrt{3} + 1))^2 - 1/2*(28*a^2*b^6*c^3*d^2 - 56*a^5*b^3*c^2*d^3 + a^8*c*d^4)*(12*a^2*b^2/d - 2*(1/2)^{(2/3)}*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/c^3)) * (-I*\sqrt{3} + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/c^4) - (b^{12}*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^{12}*d^4)/c^2*d^5) + (b^{12}*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^{12}*d^4)/c^2*d^5)^{(1/3)} - (1/2)^{(1/3)}*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/c^4) - (b^{12}*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^{12}*d^4)/c^2*d^5) + (b^{12}*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^{12}*d^4)/c^2*d^5)^{(1/3)}*(I*\sqrt{3} + 1)) - 2*(b^{12}*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^{12}*d^4)*x + 3/4*\sqrt{1/3}*(20*a^2*b^6*c^3*d^2 + 32*a^5*b^3*c^2*d^3 + 2*a^8*c*d^4 + (b^4*c^3*d^3 - 4*a^3*b*c^2*d^4)*(12*a^2*b^2/d - 2*(1/2)^{(2/3)}*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/c^3)) * (-I*\sqrt{3} + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/c^4) - (b^{12}*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^{12}*d^4)/c^2*d^5) + (b^{12}*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^{12}*d^4)/c^2*d^5)^{(1/3)} - (1/2)^{(1/3)}*(432*
\end{aligned}$$

$$\begin{aligned} & a^6 b^6 / d^3 - 18(4 a^3 b^7 c^2 + 19 a^4 b^4 c d + 4 a^7 b d^2) a^2 b^2 / (c d^4) - (b^{12} c^4 + 52 a^3 b^9 c^3 d - 52 a^9 b^3 c d^3 - a^{12} d^4) / (c^2 d^5) \\ & + (b^{12} c^4 - 4 a^3 b^9 c^3 d + 6 a^6 b^6 c^2 d^2 - 4 a^9 b^3 c d^3 + a^{12} d^4) / (c^2 d^5)^{(1/3)} * (I \sqrt{3} + 1) * \sqrt{-(64 a^3 b^7 c^2 - 128 a^4 b^4 c d + 64 a^7 b d^2 - 24(12 a^2 b^2 / d - 2(1/2)^{(2/3)} * (36 a^4 b^4 / d^2 - (4 a^3 b^7 c^2 + 19 a^4 b^4 c d + 4 a^7 b d^2) / (c d^3)) * (-I \sqrt{3} + 1) / (432 a^6 b^6 / d^3 - 18(4 a^3 b^7 c^2 + 19 a^4 b^4 c d + 4 a^7 b d^2) a^2 b^2 / (c d^4) - (b^{12} c^4 + 52 a^3 b^9 c^3 d - 52 a^9 b^3 c d^3 - a^{12} d^4) / (c^2 d^5))^{(1/3)} - (1/2)^{(1/3)} * (432 a^6 b^6 / d^3 - 18(4 a^3 b^7 c^2 + 19 a^4 b^4 c d + 4 a^7 b d^2) a^2 b^2 / (c d^4) - (b^{12} c^4 + 52 a^3 b^9 c^3 d - 52 a^9 b^3 c d^3 - a^{12} d^4) / (c^2 d^5))^{(1/3)} * (I \sqrt{3} + 1)} \\ &) a^2 b^2 c d^2 + (12 a^2 b^2 / d - 2(1/2)^{(2/3)} * (36 a^4 b^4 / d^2 - (4 a^3 b^7 c^2 + 19 a^4 b^4 c d + 4 a^7 b d^2) / (c d^3)) * (-I \sqrt{3} + 1) / (432 a^6 b^6 / d^3 - 18(4 a^3 b^7 c^2 + 19 a^4 b^4 c d + 4 a^7 b d^2) a^2 b^2 / (c d^4) - (b^{12} c^4 + 52 a^3 b^9 c^3 d - 52 a^9 b^3 c d^3 - a^{12} d^4) / (c^2 d^5))^{(1/3)} - (1/2)^{(1/3)} * (432 a^6 b^6 / d^3 - 18(4 a^3 b^7 c^2 + 19 a^4 b^4 c d + 4 a^7 b d^2) a^2 b^2 / (c d^4) - (b^{12} c^4 + 52 a^3 b^9 c^3 d - 52 a^9 b^3 c d^3 - a^{12} d^4) / (c^2 d^5))^{(1/3)} * (I \sqrt{3} + 1)} \\ &)^2 c d^3 / (c d^3)) + (36 a^2 b^2 - (12 a^2 b^2 / d - 2(1/2)^{(2/3)} * (36 a^4 b^4 / d^2 - (4 a^3 b^7 c^2 + 19 a^4 b^4 c d + 4 a^7 b d^2) / (c d^3)) * (-I \sqrt{3} + 1) / (432 a^6 b^6 / d^3 - 18(4 a^3 b^7 c^2 + 19 a^4 b^4 c d + 4 a^7 b d^2) a^2 b^2 / (c d^4) - (b^{12} c^4 + 52 a^3 b^9 c^3 d - 52 a^9 b^3 c d^3 - a^{12} d^4) / (c^2 d^5))^{(1/3)} - (1/2)^{(1/3)} * (432 a^6 b^6 / d^3 - 18(4 a^3 b^7 c^2 + 19 a^4 b^4 c d + 4 a^7 b d^2) a^2 b^2 / (c d^4) - (b^{12} c^4 + 52 a^3 b^9 c^3 d - 52 a^9 b^3 c d^3 - a^{12} d^4) / (c^2 d^5))^{(1/3)} * (I \sqrt{3} + 1)} \\ &) * d - 3 \sqrt{3} * d * \sqrt{-(64 a^3 b^7 c^2 - 128 a^4 b^4 c d + 64 a^7 b d^2 - 24(12 a^2 b^2 / d - 2(1/2)^{(2/3)} * (36 a^4 b^4 / d^2 - (4 a^3 b^7 c^2 + 19 a^4 b^4 c d + 4 a^7 b d^2) / (c d^3)) * (-I \sqrt{3} + 1) / (432 a^6 b^6 / d^3 - 18(4 a^3 b^7 c^2 + 19 a^4 b^4 c d + 4 a^7 b d^2) a^2 b^2 / (c d^4) - (b^{12} c^4 + 52 a^3 b^9 c^3 d - 52 a^9 b^3 c d^3 - a^{12} d^4) / (c^2 d^5))^{(1/3)} - (1/2)^{(1/3)} * (432 a^6 b^6 / d^3 - 18(4 a^3 b^7 c^2 + 19 a^4 b^4 c d + 4 a^7 b d^2) a^2 b^2 / (c d^4) - (b^{12} c^4 + 52 a^3 b^9 c^3 d - 52 a^9 b^3 c d^3 - a^{12} d^4) / (c^2 d^5))^{(1/3)} * (I \sqrt{3} + 1)} \\ &) * a^2 b^2 c d^2 + (12 a^2 b^2 / d - 2(1/2)^{(2/3)} * (36 a^4 b^4 / d^2 - (4 a^3 b^7 c^2 + 19 a^4 b^4 c d + 4 a^7 b d^2) / (c d^3)) * (-I \sqrt{3} + 1) / (432 a^6 b^6 / d^3 - 18(4 a^3 b^7 c^2 + 19 a^4 b^4 c d + 4 a^7 b d^2) a^2 b^2 / (c d^4) - (b^{12} c^4 + 52 a^3 b^9 c^3 d - 52 a^9 b^3 c d^3 - a^{12} d^4) / (c^2 d^5))^{(1/3)} - (1/2)^{(1/3)} * (432 a^6 b^6 / d^3 - 18(4 a^3 b^7 c^2 + 19 a^4 b^4 c d + 4 a^7 b d^2) a^2 b^2 / (c d^4) - (b^{12} c^4 + 52 a^3 b^9 c^3 d - 52 a^9 b^3 c d^3 - a^{12} d^4) / (c^2 d^5))^{(1/3)} * (I \sqrt{3} + 1)} \end{aligned}$$

$$\begin{aligned}
& *b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^{12}*d^4)/(c^2*d^5)^{(1/3)} - (1/2)^{(1/3)}*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^{12}*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^{12}*d^4)/(c^2*d^5) + (b^{12}*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^{12}*d^4)/(c^2*d^5))^{(1/3)}*(I*\sqrt{3} + 1))^2*c*d^3/(c*d^3)) * \log(8*a*b^{11}*c^4 + 66*a^4*b^8*c^3*d - 48*a^7*b^5*c^2*d^2 - 26*a^{10}*b^2*c*d^3 + 1/4*(b^4*c^3*d^3 - 4*a^3*b*c^2*d^4)*(12*a^2*b^2/d - 2*(1/2)^{(2/3)}*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*\sqrt{3} + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^{12}*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^{12}*d^4)/(c^2*d^5) + (b^{12}*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^{12}*d^4)/(c^2*d^5))^{(1/3)} - (1/2)^{(1/3)}*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^{12}*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^{12}*d^4)/(c^2*d^5) + (b^{12}*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^{12}*d^4)/(c^2*d^5))^{(1/3)}*(I*\sqrt{3} + 1))^2 - 1/2*(28*a^2*b^6*c^3*d^2 - 56*a^5*b^3*c^2*d^3 + a^8*c*d^4)*(12*a^2*b^2/d - 2*(1/2)^{(2/3)}*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*\sqrt{3} + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^{12}*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^{12}*d^4)/(c^2*d^5) + (b^{12}*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^{12}*d^4)/(c^2*d^5))^{(1/3)} - (1/2)^{(1/3)}*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^{12}*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^{12}*d^4)/(c^2*d^5) + (b^{12}*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^{12}*d^4)/(c^2*d^5))^{(1/3)}*(I*\sqrt{3} + 1)) - 2*(b^{12}*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^{12}*d^4)*x - 3/4*\sqrt{1/3}*(20*a^2*b^6*c^3*d^2 + 32*a^5*b^3*c^2*d^3 + 2*a^8*c*d^4 + (b^4*c^3*d^3 - 4*a^3*b*c^2*d^4)*(12*a^2*b^2/d - 2*(1/2)^{(2/3)}*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*\sqrt{3} + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^{12}*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^{12}*d^4)/(c^2*d^5) + (b^{12}*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^{12}*d^4)/(c^2*d^5))^{(1/3)} - (1/2)^{(1/3)}*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^{12}*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^{12}*d^4)/(c^2*d^5) + (b^{12}*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^{12}*d^4)/(c^2*d^5))^{(1/3)}*(I*\sqrt{3} + 1))) * \sqrt{-(64*a*b^7*c^2 - 128*a^4*b^4*c*d + 64*a^7*b*d^2 - 24*(12*a^2*b^2/d - 2*(1/2)^{(2/3)}*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*\sqrt{3} + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^{12}*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^{12}*d^4)/(c^2*d^5) + (b^{12}*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^{12}*d^4)/(c^2*d^5))^{(1/3)} - (1/2)^{(1/3)}*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^{12}*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^{12}*d^4)/(c^2*d^5) + (b^{12}*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^{12}*d^4)/(c^2*d^5))^{(1/3)}*(I*\sqrt{3} + 1))}
\end{aligned}$$

$$\begin{aligned} & \left(\frac{36a^4b^4d^2}{d^2} - \frac{4a^7b^7c^2 + 19a^4b^4cd + 4a^7b^7d^2}{c^3d^3} \right) \frac{(-I\sqrt{3} + 1)}{(432a^6b^6d^3 - 18(4a^7b^7c^2 + 19a^4b^4cd + 4a^7b^7d^2))} \\ & - \frac{(b^{12}c^4 + 52a^3b^9c^3d - 52a^9b^3c^3d^3 - a^{12}d^4)}{(c^2d^5)} + \frac{(b^{12}c^4 - 4a^3b^9c^3d + 6a^6b^6c^2d^2 - 4a^9b^3c^3d^3 + a^{12}d^4)}{(c^2d^5)} \\ & - \frac{(1/2)^{1/3} \left(432a^6b^6d^3 - 18(4a^7b^7c^2 + 19a^4b^4cd + 4a^7b^7d^2) \right)}{(c^2d^5)^{1/3}} \\ & - \frac{(1/2)^{1/3} \left(432a^6b^6d^3 - 18(4a^7b^7c^2 + 19a^4b^4cd + 4a^7b^7d^2) \right)}{(c^2d^5)^{1/3}} \frac{(-I\sqrt{3} + 1)}{(c^2d^5)^{1/3}} \frac{1}{d} \end{aligned}$$

Sympy [A] time = 5.47972, size = 325, normalized size = 1.15

$$\frac{4ab^3x}{d} + \frac{b^4x^2}{2d} + \text{RootSum}\left(27t^3c^2d^5 - 162t^2a^2b^2c^2d^4 + t(36a^7bcd^4 + 171a^4b^4c^2d^3 + 36ab^7c^3d^2) - a^{12}d^4 + 4a^9b^3cd^3 - \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**4/(d*x**3+c),x)

[Out] $4a^4b^3x/d + b^4x^2/(2d) + \text{RootSum}(27_t^3c^2d^5 - 162_t^2a^2b^2c^2d^4 + \dots)$

Giac [A] time = 1.08479, size = 427, normalized size = 1.51

$$\frac{2a^2b^2 \log(|dx^3 + c|)}{d} + \frac{b^4dx^2 + 8ab^3dx}{2d^2} - \frac{\sqrt{3} \left(4(-cd^2)^{\frac{1}{3}} ab^3cd - (-cd^2)^{\frac{1}{3}} a^4d^2 - (-cd^2)^{\frac{2}{3}} b^4c + 4(-cd^2)^{\frac{2}{3}} a^3bd \right) \arctan}{3cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^4/(d*x^3+c),x, algorithm="giac")

```
[Out] 2*a^2*b^2*log(abs(d*x^3 + c))/d + 1/2*(b^4*d*x^2 + 8*a*b^3*d*x)/d^2 - 1/3*sqrt(3)*(4*(-c*d^2)^(1/3)*a*b^3*c*d - (-c*d^2)^(1/3)*a^4*d^2 - (-c*d^2)^(2/3)*b^4*c + 4*(-c*d^2)^(2/3)*a^3*b*d)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/(c*d^3) - 1/6*(4*(-c*d^2)^(1/3)*a*b^3*c*d - (-c*d^2)^(1/3)*a^4*d^2 + (-c*d^2)^(2/3)*b^4*c - 4*(-c*d^2)^(2/3)*a^3*b*d)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(c*d^3) + 1/3*(b^4*c*d^4*(-c/d)^(1/3) - 4*a^3*b*d^5*(-c/d)^(1/3) + 4*a*b^3*c*d^4 - a^4*d^5)*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(c*d^5)
```

$$3.73 \quad \int \frac{(a+bx+cx^2)^2}{d+ex^3} dx$$

Optimal. Leaf size=272

$$\frac{\log(d^{2/3} - \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2) (\sqrt[3]{e}(2bcd - a^2e) - \sqrt[3]{d}(c^2d - 2abe))}{6d^{2/3}e^{5/3}} - \frac{\log(\sqrt[3]{d} + \sqrt[3]{ex}) (\sqrt[3]{e}(2bcd - a^2e) - \sqrt[3]{d}(c^2d - 2abe))}{3d^{2/3}e^{5/3}}$$

[Out] $(2*b*c*x)/e + (c^2*x^2)/(2*e) + ((c^2*d^{4/3} + 2*b*c*d*e^{1/3} - a*(2*b*d^{1/3} + a*e^{1/3})*e)*\text{ArcTan}[(d^{1/3} - 2*e^{1/3}*x)/(\text{Sqrt}[3]*d^{1/3})])/(S\text{qrt}[3]*d^{2/3}*e^{5/3}) - ((e^{1/3}*(2*b*c*d - a^2*e) - d^{1/3}*(c^2*d - 2*a*b*e))*\text{Log}[d^{1/3} + e^{1/3}*x])/(3*d^{2/3}*e^{5/3}) + ((e^{1/3}*(2*b*c*d - a^2*e) - d^{1/3}*(c^2*d - 2*a*b*e))*\text{Log}[d^{2/3} - d^{1/3}*e^{1/3}*x + e^{2/3}*x^2])/(6*d^{2/3}*e^{5/3}) + ((b^2 + 2*a*c)*\text{Log}[d + e*x^3])/(3*e)$

Rubi [A] time = 0.490332, antiderivative size = 270, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log(d^{2/3} - \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2) \left(a^2(-e) - \frac{\sqrt[3]{d}(c^2d - 2abe)}{\sqrt[3]{e}} + 2bcd \right)}{6d^{2/3}e^{4/3}} - \frac{\log(\sqrt[3]{d} + \sqrt[3]{ex}) (\sqrt[3]{e}(2bcd - a^2e) - \sqrt[3]{d}(c^2d - 2abe))}{3d^{2/3}e^{5/3}} +$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2)^2/(d + e*x^3), x]

[Out] $(2*b*c*x)/e + (c^2*x^2)/(2*e) + ((c^2*d^{4/3} + 2*b*c*d*e^{1/3} - a*(2*b*d^{1/3} + a*e^{1/3})*e)*\text{ArcTan}[(d^{1/3} - 2*e^{1/3}*x)/(\text{Sqrt}[3]*d^{1/3})])/(S\text{qrt}[3]*d^{2/3}*e^{5/3}) - ((e^{1/3}*(2*b*c*d - a^2*e) - d^{1/3}*(c^2*d - 2*a*b*e))*\text{Log}[d^{1/3} + e^{1/3}*x])/(3*d^{2/3}*e^{5/3}) + ((2*b*c*d - a^2*e - (d^{1/3}*(c^2*d - 2*a*b*e))/e^{1/3})*\text{Log}[d^{2/3} - d^{1/3}*e^{1/3}*x + e^{2/3}*x^2])/(6*d^{2/3}*e^{4/3}) + ((b^2 + 2*a*c)*\text{Log}[d + e*x^3])/(3*e)$

Rule 1887

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Pq/(a + b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]

Rule 1871

```
Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```


Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx + cx^2)^2}{d + ex^3} dx &= \int \left(\frac{2bc}{e} + \frac{c^2x}{e} - \frac{2bcd - a^2e + (c^2d - 2abe)x - (b^2 + 2ac)ex^2}{e(d + ex^3)} \right) dx \\
&= \frac{2bcx}{e} + \frac{c^2x^2}{2e} - \frac{\int \frac{2bcd - a^2e + (c^2d - 2abe)x - (b^2 + 2ac)ex^2}{d + ex^3} dx}{e} \\
&= \frac{2bcx}{e} + \frac{c^2x^2}{2e} - (b^2 + 2ac) \int \frac{x^2}{d + ex^3} dx - \frac{\int \frac{2bcd - a^2e + (c^2d - 2abe)x}{d + ex^3} dx}{e} \\
&= \frac{2bcx}{e} + \frac{c^2x^2}{2e} + \frac{(b^2 + 2ac) \log(d + ex^3)}{3e} - \frac{\int \frac{\sqrt[3]{a}(2\sqrt[3]{e}(2bcd - a^2e) + \sqrt[3]{a}(c^2d - 2abe)) + \sqrt[3]{e}(-\sqrt[3]{e}(2bcd - a^2e) + \sqrt[3]{a}(c^2d - 2abe))}{d^{2/3} - \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2} dx}{3d^{2/3}e^{4/3}} \\
&= \frac{2bcx}{e} + \frac{c^2x^2}{2e} - \frac{\left(2bcd - a^2e - \frac{\sqrt[3]{d}(c^2d - 2abe)}{\sqrt[3]{e}}\right) \log(\sqrt[3]{d} + \sqrt[3]{ex})}{3d^{2/3}e^{4/3}} + \frac{(b^2 + 2ac) \log(d + ex^3)}{3e} - \frac{(c^2d^4)}{3d} \\
&= \frac{2bcx}{e} + \frac{c^2x^2}{2e} - \frac{\left(2bcd - a^2e - \frac{\sqrt[3]{d}(c^2d - 2abe)}{\sqrt[3]{e}}\right) \log(\sqrt[3]{d} + \sqrt[3]{ex})}{3d^{2/3}e^{4/3}} + \frac{\left(2bcd - a^2e - \frac{\sqrt[3]{d}(c^2d - 2abe)}{\sqrt[3]{e}}\right) \log(d + ex^3)}{6d^{2/3}e^{4/3}} \\
&= \frac{2bcx}{e} + \frac{c^2x^2}{2e} + \frac{(c^2d^{4/3} + 2bcd\sqrt[3]{e} - a(2b\sqrt[3]{d} + a\sqrt[3]{e})e) \tan^{-1}\left(\frac{\sqrt[3]{d} - 2\sqrt[3]{ex}}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{5/3}} - \frac{\left(2bcd - a^2e - \frac{\sqrt[3]{d}(c^2d - 2abe)}{\sqrt[3]{e}}\right) \log(d + ex^3)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.284016, size = 269, normalized size = 0.99

$$\frac{2e^{2/3}(2ac + b^2) \log(d + ex^3) - \frac{\log\left(d^{2/3} - \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2\right)\left(ae\left(a\sqrt[3]{e} - 2b\sqrt[3]{d}\right) - 2bcd\sqrt[3]{e} + c^2d^{4/3}\right)}{d^{2/3}}}{6e^{5/3}} + \frac{2 \log(\sqrt[3]{d} + \sqrt[3]{ex})\left(ae\left(a\sqrt[3]{e} - 2b\sqrt[3]{d}\right) - 2bcd\sqrt[3]{e} + c^2d^{4/3}\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2)^2/(d + e*x^3), x]

```
[Out] (12*b*c*e^(2/3)*x + 3*c^2*e^(2/3)*x^2 + (2*Sqrt[3]*(c*d^(2/3) - a*e^(2/3))*
(c*d^(2/3) + 2*b*d^(1/3)*e^(1/3) + a*e^(2/3))*ArcTan[(1 - (2*e^(1/3)*x)/d^(
1/3))/Sqrt[3]])/d^(2/3) + (2*(c^2*d^(4/3) - 2*b*c*d*e^(1/3) + a*(-2*b*d^(1/
3) + a*e^(1/3))*e)*Log[d^(1/3) + e^(1/3)*x])/d^(2/3) - ((c^2*d^(4/3) - 2*b*
c*d*e^(1/3) + a*(-2*b*d^(1/3) + a*e^(1/3))*e)*Log[d^(2/3) - d^(1/3)*e^(1/3)
*x + e^(2/3)*x^2])/d^(2/3) + 2*(b^2 + 2*a*c)*e^(2/3)*Log[d + e*x^3])/(6*e^(
5/3))
```

Maple [B] time = 0.004, size = 444, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(e*x^3+d),x)
```

```
[Out] 1/2*c^2*x^2/e+2*b*c*x/e+1/3/e/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*a^2-2/3/e^2/(d/
e)^(2/3)*ln(x+(d/e)^(1/3))*b*c*d-1/6/e/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/
e)^(2/3))*a^2+1/3/e^2/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*b*c*d+1
/3/e/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a^2-2/3/e^
2/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*b*c*d-2/3/e/(
d/e)^(1/3)*ln(x+(d/e)^(1/3))*a*b+1/3/e^2/(d/e)^(1/3)*ln(x+(d/e)^(1/3))*c^2*
d+1/3/e/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*a*b-1/6/e^2/(d/e)^(1/
3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*c^2*d+2/3/e*3^(1/2)/(d/e)^(1/3)*arctan
(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a*b-1/3/e^2*3^(1/2)/(d/e)^(1/3)*arctan(1/
3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*c^2*d+2/3/e*ln(e*x^3+d)*a*c+1/3/e*ln(e*x^3+d
)*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x^3+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```


$$\begin{aligned}
&) * d * e^3 / (d^2 * e^5)^{1/3} - 2 * (b^2 + 2 * a * c) / e * (b^2 + 2 * a * c) * d * e^2 + 4 * (b^4 \\
& - 12 * a * b^2 * c) * d * e / (d * e^3)) * \log(4 * b * c^5 * d^4 + (5 * b^4 * c^2 - 4 * a * b^2 * c^3 + \\
& 2 * a^2 * c^4) * d^3 * e - 2 * (a * b^5 - 2 * a^2 * b^3 * c + 4 * a^3 * b * c^2) * d^2 * e^2 - (7 * a^4 * b \\
& ^2 - 2 * a^5 * c) * d * e^3 + 1/4 * (c^2 * d^3 * e^3 - 2 * a * b * d^2 * e^4) * (2 * (1/2)^{2/3} * (-I * \\
& \text{sqrt}(3) + 1) * ((b^2 + 2 * a * c)^2 / e^2 - (2 * b * c^3 * d^2 + b^4 * d * e + 3 * a^2 * c^2 * d * e \\
& + 2 * a^3 * b * e^2) / (d * e^3)) / (2 * (b^2 + 2 * a * c)^3 / e^3 - 3 * (2 * b * c^3 * d^2 + b^4 * d * e + \\
& 3 * a^2 * c^2 * d * e + 2 * a^3 * b * e^2) * (b^2 + 2 * a * c) / (d * e^4) + (c^6 * d^4 - 2 * b^3 * c^3 * \\
& d^3 * e + b^6 * d^2 * e^2 + 9 * a^2 * b^2 * c^2 * d^2 * e^2 + 6 * a^4 * b * c * d * e^3 + a^6 * e^4 + 2 \\
& * (c^3 * d^2 * e^2 - b^3 * d * e^3) * a^3 + 6 * (b * c^4 * d^3 * e - b^4 * c * d^2 * e^2) * a) / (d^2 * e^5) - \\
& (c^6 * d^4 - a^6 * e^4 + 2 * (4 * b^3 * c^3 - 3 * a * b * c^4) * d^3 * e - 2 * (4 * a^3 * b^3 - \\
& 3 * a^4 * b * c) * d * e^3) / (d^2 * e^5)^{1/3} + (1/2)^{1/3} * (I * \text{sqrt}(3) + 1) * (2 * (b^2 + \\
& 2 * a * c)^3 / e^3 - 3 * (2 * b * c^3 * d^2 + b^4 * d * e + 3 * a^2 * c^2 * d * e + 2 * a^3 * b * e^2) * (b^2 \\
& + 2 * a * c) / (d * e^4) + (c^6 * d^4 - 2 * b^3 * c^3 * d^3 * e + b^6 * d^2 * e^2 + 9 * a^2 * b^2 * c^2 * \\
& 2 * d^2 * e^2 + 6 * a^4 * b * c * d * e^3 + a^6 * e^4 + 2 * (c^3 * d^2 * e^2 - b^3 * d * e^3) * a^3 + 6 \\
& * (b * c^4 * d^3 * e - b^4 * c * d^2 * e^2) * a) / (d^2 * e^5) - (c^6 * d^4 - a^6 * e^4 + 2 * (4 * b^3 * \\
& c^3 - 3 * a * b * c^4) * d^3 * e - 2 * (4 * a^3 * b^3 - 3 * a^4 * b * c) * d * e^3) / (d^2 * e^5)^{1/3} \\
& - 2 * (b^2 + 2 * a * c) / e)^2 + 1/2 * (a^4 * d * e^4 + 2 * (3 * b^2 * c^2 + 2 * a * c^3) * d^3 * e^2 \\
& - 4 * (a * b^3 + 3 * a^2 * b * c) * d^2 * e^3) * (2 * (1/2)^{2/3} * (-I * \text{sqrt}(3) + 1) * ((b^2 + 2 * \\
& a * c)^2 / e^2 - (2 * b * c^3 * d^2 + b^4 * d * e + 3 * a^2 * c^2 * d * e + 2 * a^3 * b * e^2) / (d * e^3)) \\
& / (2 * (b^2 + 2 * a * c)^3 / e^3 - 3 * (2 * b * c^3 * d^2 + b^4 * d * e + 3 * a^2 * c^2 * d * e + 2 * a^3 * \\
& b * e^2) * (b^2 + 2 * a * c) / (d * e^4) + (c^6 * d^4 - 2 * b^3 * c^3 * d^3 * e + b^6 * d^2 * e^2 + 9 \\
& * a^2 * b^2 * c^2 * d^2 * e^2 + 6 * a^4 * b * c * d * e^3 + a^6 * e^4 + 2 * (c^3 * d^2 * e^2 - b^3 * d * e \\
& ^3) * a^3 + 6 * (b * c^4 * d^3 * e - b^4 * c * d^2 * e^2) * a) / (d^2 * e^5) - (c^6 * d^4 - a^6 * e^4 \\
& + 2 * (4 * b^3 * c^3 - 3 * a * b * c^4) * d^3 * e - 2 * (4 * a^3 * b^3 - 3 * a^4 * b * c) * d * e^3) / (d^2 * \\
& e^5)^{1/3} + (1/2)^{1/3} * (I * \text{sqrt}(3) + 1) * (2 * (b^2 + 2 * a * c)^3 / e^3 - 3 * (2 * b * c \\
& ^3 * d^2 + b^4 * d * e + 3 * a^2 * c^2 * d * e + 2 * a^3 * b * e^2) * (b^2 + 2 * a * c) / (d * e^4) + (c^6 \\
& * d^4 - 2 * b^3 * c^3 * d^3 * e + b^6 * d^2 * e^2 + 9 * a^2 * b^2 * c^2 * d^2 * e^2 + 6 * a^4 * b * c * d \\
& * e^3 + a^6 * e^4 + 2 * (c^3 * d^2 * e^2 - b^3 * d * e^3) * a^3 + 6 * (b * c^4 * d^3 * e - b^4 * c * d \\
& ^2 * e^2) * a) / (d^2 * e^5) - (c^6 * d^4 - a^6 * e^4 + 2 * (4 * b^3 * c^3 - 3 * a * b * c^4) * d^3 * e \\
& - 2 * (4 * a^3 * b^3 - 3 * a^4 * b * c) * d * e^3) / (d^2 * e^5)^{1/3} - 2 * (b^2 + 2 * a * c) / e - \\
& 2 * (c^6 * d^4 - a^6 * e^4 + 2 * (4 * b^3 * c^3 - 3 * a * b * c^4) * d^3 * e - 2 * (4 * a^3 * b^3 - 3 * \\
& a^4 * b * c) * d * e^3) * x + 3/4 * \text{sqrt}(1/3) * (4 * a * b^3 * d^2 * e^3 + 2 * a^4 * d * e^4 + 2 * (3 * b^2 \\
& * c^2 - 2 * a * c^3) * d^3 * e^2 - (c^2 * d^3 * e^3 - 2 * a * b * d^2 * e^4) * (2 * (1/2)^{2/3} * (-I * \\
& \text{sqrt}(3) + 1) * ((b^2 + 2 * a * c)^2 / e^2 - (2 * b * c^3 * d^2 + b^4 * d * e + 3 * a^2 * c^2 * d * e \\
& + 2 * a^3 * b * e^2) / (d * e^3)) / (2 * (b^2 + 2 * a * c)^3 / e^3 - 3 * (2 * b * c^3 * d^2 + b^4 * d * e + \\
& 3 * a^2 * c^2 * d * e + 2 * a^3 * b * e^2) * (b^2 + 2 * a * c) / (d * e^4) + (c^6 * d^4 - 2 * b^3 * c^3 * \\
& d^3 * e + b^6 * d^2 * e^2 + 9 * a^2 * b^2 * c^2 * d^2 * e^2 + 6 * a^4 * b * c * d * e^3 + a^6 * e^4 + 2 \\
& * (c^3 * d^2 * e^2 - b^3 * d * e^3) * a^3 + 6 * (b * c^4 * d^3 * e - b^4 * c * d^2 * e^2) * a) / (d^2 * e^5) - \\
& (c^6 * d^4 - a^6 * e^4 + 2 * (4 * b^3 * c^3 - 3 * a * b * c^4) * d^3 * e - 2 * (4 * a^3 * b^3 - \\
& 3 * a^4 * b * c) * d * e^3) / (d^2 * e^5)^{1/3} + (1/2)^{1/3} * (I * \text{sqrt}(3) + 1) * (2 * (b^2 + \\
& 2 * a * c)^3 / e^3 - 3 * (2 * b * c^3 * d^2 + b^4 * d * e + 3 * a^2 * c^2 * d * e + 2 * a^3 * b * e^2) * (b^2 \\
& + 2 * a * c) / (d * e^4) + (c^6 * d^4 - 2 * b^3 * c^3 * d^3 * e + b^6 * d^2 * e^2 + 9 * a^2 * b^2 * c^2 * \\
& 2 * d^2 * e^2 + 6 * a^4 * b * c * d * e^3 + a^6 * e^4 + 2 * (c^3 * d^2 * e^2 - b^3 * d * e^3) * a^3 + 6 \\
& * (b * c^4 * d^3 * e - b^4 * c * d^2 * e^2) * a) / (d^2 * e^5) - (c^6 * d^4 - a^6 * e^4 + 2 * (4 * b^3 * \\
& c^3 - 3 * a * b * c^4) * d^3 * e - 2 * (4 * a^3 * b^3 - 3 * a^4 * b * c) * d * e^3) / (d^2 * e^5)^{1/3}
\end{aligned}$$

$$\begin{aligned}
& - 2*(b^2 + 2*a*c)/e)*\text{sqrt}(-(32*b*c^3*d^2 + 32*a^3*b*e^2 + (2*(1/2)^(2/3)* \\
& (-I*\text{sqrt}(3) + 1)*((b^2 + 2*a*c)^2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2* \\
& d*e + 2*a^3*b*e^2)/(d*e^3)))/(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d \\
& *e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3* \\
& c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 \\
& + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^ \\
& 2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^ \\
& 3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) + (1/2)^(1/3)*(I*\text{sqrt}(3) + 1)*(2*(b^ \\
& 2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)* \\
& (b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^ \\
& 2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 \\
& + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4 \\
& *b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(\\
& 1/3) - 2*(b^2 + 2*a*c)/e)^2*d*e^3 + 4*(2*(1/2)^(2/3)*(-I*\text{sqrt}(3) + 1)*((b^2 \\
& + 2*a*c)^2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d* \\
& e^3)))/(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2 \\
& *a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^ \\
& 2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^ \\
& 3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^ \\
& 6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/ \\
& (d^2*e^5))^(1/3) + (1/2)^(1/3)*(I*\text{sqrt}(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(\\
& 2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) \\
& + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4* \\
& b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^ \\
& 4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)* \\
& d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c) \\
& /e)*(b^2 + 2*a*c)*d*e^2 + 4*(b^4 - 12*a*b^2*c)*d*e)/(d*e^3))) + (6*b^2 + 12 \\
& *a*c + (2*(1/2)^(2/3)*(-I*\text{sqrt}(3) + 1)*((b^2 + 2*a*c)^2/e^2 - (2*b*c^3*d^2 \\
& + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3)))/(2*(b^2 + 2*a*c)^3/e^3 - \\
& 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^ \\
& 4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a \\
& ^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - \\
& b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^ \\
& 4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) + (1/2)^(1/3)* \\
& (I*\text{sqrt}(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c \\
& ^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + \\
& b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^ \\
& 2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^ \\
& 6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b* \\
& c)*d*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)/e)*e - 3*\text{sqrt}(1/3)*e*\text{sqrt}(-(32 \\
& *b*c^3*d^2 + 32*a^3*b*e^2 + (2*(1/2)^(2/3)*(-I*\text{sqrt}(3) + 1)*((b^2 + 2*a*c)^ \\
& 2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3)))/(2*(\\
& b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2) \\
&)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2* \\
& b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a
\end{aligned}$$

$$\begin{aligned}
&^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2* \\
&(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5)) \\
&^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 \\
&+ b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 \\
&- 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 \\
&+ a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2) \\
&2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2* \\
&(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^{(1/3)} - 2*(b^2 + 2*a*c)/e)^2*d*e^ \\
&3 + 4*(2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*((b^2 + 2*a*c)^2/e^2 - (2*b*c^3*d^2 + \\
&b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3)))/(2*(b^2 + 2*a*c)^3/e^3 - 3 \\
&)*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4 \\
&)) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4 \\
&b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - \\
&b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4) \\
&)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^{(1/3)} + (1/2)^{(1/3)}*(\\
&I*\sqrt{3} + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2 \\
&d^2*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b \\
&^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2 \\
&*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6 \\
&*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c \\
&)*d*e^3)/(d^2*e^5))^{(1/3)} - 2*(b^2 + 2*a*c)/e)*(b^2 + 2*a*c)*d*e^2 + 4*(b^4 \\
&- 12*a*b^2*c)*d*e)/(d*e^3))*\log(4*b*c^5*d^4 + (5*b^4*c^2 - 4*a*b^2*c^3 + \\
&2*a^2*c^4)*d^3*e - 2*(a*b^5 - 2*a^2*b^3*c + 4*a^3*b*c^2)*d^2*e^2 - (7*a^4*b \\
&^2 - 2*a^5*c)*d*e^3 + 1/4*(c^2*d^3*e^3 - 2*a*b*d^2*e^4)*(2*(1/2)^{(2/3)}*(-I* \\
&\sqrt{3} + 1)*((b^2 + 2*a*c)^2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e \\
&+ 2*a^3*b*e^2)/(d*e^3)))/(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + \\
&3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3* \\
&d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2 \\
&)*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^ \\
&5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - \\
&3*a^4*b*c)*d*e^3)/(d^2*e^5))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(2*(b^2 + \\
&2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 \\
&+ 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^ \\
&2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6 \\
&)*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3 \\
&*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^{(1/3)} \\
&- 2*(b^2 + 2*a*c)/e)^2 + 1/2*(a^4*d*e^4 + 2*(3*b^2*c^2 + 2*a*c^3)*d^3*e^2 \\
&- 4*(a*b^3 + 3*a^2*b*c)*d^2*e^3)*(2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*((b^2 + 2* \\
&a*c)^2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3)) \\
&)/(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3* \\
&b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9 \\
&a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e \\
&^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 \\
&+ 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2* \\
&e^5))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c
\end{aligned}$$

$$d^3e - 2(4a^3b^3 - 3a^4bc)d^3e^3 / (d^2e^5)^{1/3} - 2(b^2 + 2ac) / e * (b^2 + 2ac)d^2e^2 + 4(b^4 - 12ab^2c)d^2e / (d^3e^3) / e$$

Sympy [B] time = 8.7991, size = 546, normalized size = 2.01

$$\frac{2bcx}{e} + \frac{c^2x^2}{2e} + \text{RootSum}\left(27t^3d^2e^5 + t^2(-54acd^2e^4 - 27b^2d^2e^4) + t(18a^3bde^4 + 27a^2c^2d^2e^3 + 9b^4d^2e^3 + 18bc^3d^3e^2) - \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)**2/(e*x**3+d),x)

[Out] 2*b*c*x/e + c**2*x**2/(2*e) + RootSum(27*_t**3*d**2*e**5 + _t**2*(-54*a*c*d**2*e**4 - 27*b**2*d**2*e**4) + _t*(18*a**3*b*d*e**4 + 27*a**2*c**2*d**2*e**3 + 9*b**4*d**2*e**3 + 18*b*c**3*d**3*e**2) - a**6*e**4 - 6*a**4*b*c*d*e**3 + 2*a**3*b**3*d*e**3 - 2*a**3*c**3*d**2*e**2 - 9*a**2*b**2*c**2*d**2*e**2 + 6*a*b**4*c*d**2*e**2 - 6*a*b*c**4*d**3*e - b**6*d**2*e**2 + 2*b**3*c**3*d**3*e - c**6*d**4, Lambda(_t, _t*log(x + (18*_t**2*a*b*d**2*e**4 - 9*_t**2*c**2*d**3*e**3 + 3*_t*a**4*d*e**4 - 36*_t*a**2*b*c*d**2*e**3 - 12*_t*a*b**3*d**2*e**3 + 12*_t*a*c**3*d**3*e**2 + 18*_t*b**2*c**2*d**3*e**2 - 2*a**5*c*d*e**3 + 7*a**4*b**2*d*e**3 + 8*a**3*b*c**2*d**2*e**2 - 4*a**2*b**3*c*d**2*e**2 - 2*a**2*c**4*d**3*e + 2*a*b**5*d**2*e**2 + 4*a*b**2*c**3*d**3*e - 5*b**4*c**2*d**3*e - 4*b*c**5*d**4)/(a**6*e**4 - 6*a**4*b*c*d*e**3 + 8*a**3*b**3*d*e**3 + 6*a*b*c**4*d**3*e - 8*b**3*c**3*d**3*e - c**6*d**4))))

Giac [A] time = 1.07009, size = 383, normalized size = 1.41

$$\frac{1}{3} (b^2 + 2ac)e^{(-1)} \log(|x^3e + d|) - \frac{\sqrt{3} \left(2(-de^2)^{\frac{1}{3}} bcde - (-de^2)^{\frac{2}{3}} c^2d + 2(-de^2)^{\frac{2}{3}} abe - (-de^2)^{\frac{1}{3}} a^2e^2 \right) \arctan \left(\frac{\sqrt{3} (2x + (-de^2)^{\frac{1}{3}})}{3(-de^2)^{\frac{1}{3}}} \right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^2/(e*x^3+d),x, algorithm="giac")

[Out] 1/3*(b^2 + 2*a*c)*e^(-1)*log(abs(x^3*e + d)) - 1/3*sqrt(3)*(2*(-d*e^2)^(1/3))*b*c*d*e - (-d*e^2)^(2/3)*c^2*d + 2*(-d*e^2)^(2/3)*a*b*e - (-d*e^2)^(1/3)*a^2*e^2)*arctan(1/3*sqrt(3)*(2*x + (-d*e^(-1))^(1/3))/(-d*e^(-1))^(1/3))*e^

$$\begin{aligned}
& (-3)/d + 1/3*((-d*e^{(-1)})^{(1/3)}*c^2*d*e^4 + 2*b*c*d*e^4 - 2*(-d*e^{(-1)})^{(1/3)} \\
& *a*b*e^5 - a^2*e^5)*(-d*e^{(-1)})^{(1/3)}*e^{(-5)}*\log(\text{abs}(x - (-d*e^{(-1)})^{(1/3)} \\
&)))/d + 1/2*(c^2*x^2*e + 4*b*c*x*e)*e^{(-2)} - 1/6*(2*(-d*e^2)^{(1/3)}*b*c*d*e \\
& + (-d*e^2)^{(2/3)}*c^2*d - 2*(-d*e^2)^{(2/3)}*a*b*e - (-d*e^2)^{(1/3)}*a^2*e^2)*e \\
& ^{(-3)}*\log(x^2 + (-d*e^{(-1)})^{(1/3)}*x + (-d*e^{(-1)})^{(2/3)})/d
\end{aligned}$$

$$3.74 \quad \int \frac{(a+bx+cx^2)^3}{d+ex^3} dx$$

Optimal. Leaf size=416

$$\frac{\log(d^{2/3} - \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2) \left(3\sqrt[3]{de}^{2/3} (a^2(-b)e + ac^2d + b^2cd) - e(b^3d - a^3e) - 6abcde + c^3d^2\right)}{6d^{2/3}e^{7/3}} + \frac{\log(\sqrt[3]{d} + \sqrt[3]{ex}) (3\sqrt[3]{d} + \sqrt[3]{ex}) (3\sqrt[3]{d} + \sqrt[3]{ex})}{6d^{2/3}e^{7/3}}$$

[Out] -(((c^3*d - b^3*e - 6*a*b*c*e)*x)/e^2) + (3*c*(b^2 + a*c)*x^2)/(2*e) + (b*c^2*x^3)/e + (c^3*x^4)/(4*e) - ((c^3*d^2 - 3*b^2*c*d^(4/3)*e^(2/3) - 3*a*c^2*d^(4/3)*e^(2/3) - b^3*d*e - 6*a*b*c*d*e + 3*a^2*b*d^(1/3)*e^(5/3) + a^3*e^2)*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))]/(Sqrt[3]*d^(2/3)*e^(7/3)) + ((c^3*d^2 - 6*a*b*c*d*e - e*(b^3*d - a^3*e) + 3*d^(1/3)*e^(2/3)*(b^2*c*d + a*c^2*d - a^2*b*e))*Log[d^(1/3) + e^(1/3)*x]/(3*d^(2/3)*e^(7/3)) - ((c^3*d^2 - 6*a*b*c*d*e - e*(b^3*d - a^3*e) + 3*d^(1/3)*e^(2/3)*(b^2*c*d + a*c^2*d - a^2*b*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2]/(6*d^(2/3)*e^(7/3)) - ((b*c^2*d - a*b^2*e - a^2*c*e)*Log[d + e*x^3])/e^2

Rubi [A] time = 0.70029, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log(d^{2/3} - \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2) \left(3\sqrt[3]{de}^{2/3} (a^2(-b)e + ac^2d + b^2cd) - e(b^3d - a^3e) - 6abcde + c^3d^2\right)}{6d^{2/3}e^{7/3}} + \frac{\log(\sqrt[3]{d} + \sqrt[3]{ex}) (3\sqrt[3]{d} + \sqrt[3]{ex}) (3\sqrt[3]{d} + \sqrt[3]{ex})}{6d^{2/3}e^{7/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2)^3/(d + e*x^3), x]

[Out] -(((c^3*d - b^3*e - 6*a*b*c*e)*x)/e^2) + (3*c*(b^2 + a*c)*x^2)/(2*e) + (b*c^2*x^3)/e + (c^3*x^4)/(4*e) - ((c^3*d^2 - 3*b^2*c*d^(4/3)*e^(2/3) - 3*a*c^2*d^(4/3)*e^(2/3) - b^3*d*e - 6*a*b*c*d*e + 3*a^2*b*d^(1/3)*e^(5/3) + a^3*e^2)*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))]/(Sqrt[3]*d^(2/3)*e^(7/3)) + ((c^3*d^2 - 6*a*b*c*d*e - e*(b^3*d - a^3*e) + 3*d^(1/3)*e^(2/3)*(b^2*c*d + a*c^2*d - a^2*b*e))*Log[d^(1/3) + e^(1/3)*x]/(3*d^(2/3)*e^(7/3)) - ((c^3*d^2 - 6*a*b*c*d*e - e*(b^3*d - a^3*e) + 3*d^(1/3)*e^(2/3)*(b^2*c*d + a*c^2*d - a^2*b*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2]/(6*d^(2/3)*e^(7/3)) - ((b*c^2*d - a*b^2*e - a^2*c*e)*Log[d + e*x^3])/e^2

Rule 1887

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Pq/(a + b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]

Rule 1871

Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Dist[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

Rule 1860

Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx + cx^2)^3}{d + ex^3} dx &= \int \left(-\frac{c^3d - b^3e - 6abce}{e^2} + \frac{3c(b^2 + ac)x}{e} + \frac{3bc^2x^2}{e} + \frac{c^3x^3}{e} + \frac{c^3d^2 - 6abcde - e(b^3d - a^3e) - 3e}{e^2} \right) dx \\
&= -\frac{(c^3d - b^3e - 6abce)x}{e^2} + \frac{3c(b^2 + ac)x^2}{2e} + \frac{bc^2x^3}{e} + \frac{c^3x^4}{4e} + \frac{\int \frac{c^3d^2 - 6abcde - e(b^3d - a^3e) - 3e(b^2cd + ac^2d - a^3e)}{d + ex^3} dx}{e^2} \\
&= -\frac{(c^3d - b^3e - 6abce)x}{e^2} + \frac{3c(b^2 + ac)x^2}{2e} + \frac{bc^2x^3}{e} + \frac{c^3x^4}{4e} + \frac{\int \frac{c^3d^2 - 6abcde - e(b^3d - a^3e) - 3e(b^2cd + ac^2d - a^3e)}{d + ex^3} dx}{e^2} \\
&= -\frac{(c^3d - b^3e - 6abce)x}{e^2} + \frac{3c(b^2 + ac)x^2}{2e} + \frac{bc^2x^3}{e} + \frac{c^3x^4}{4e} - \frac{(bc^2d - ab^2e - a^2ce) \log(d + ex^3)}{e^2} \\
&= -\frac{(c^3d - b^3e - 6abce)x}{e^2} + \frac{3c(b^2 + ac)x^2}{2e} + \frac{bc^2x^3}{e} + \frac{c^3x^4}{4e} + \frac{(c^3d^2 - 6abcde - e(b^3d - a^3e) + 3e(b^2cd + ac^2d - a^3e)) \log(d + ex^3)}{e^2} \\
&= -\frac{(c^3d - b^3e - 6abce)x}{e^2} + \frac{3c(b^2 + ac)x^2}{2e} + \frac{bc^2x^3}{e} + \frac{c^3x^4}{4e} + \frac{(c^3d^2 - 6abcde - e(b^3d - a^3e) + 3e(b^2cd + ac^2d - a^3e)) \log(d + ex^3)}{e^2} \\
&= -\frac{(c^3d - b^3e - 6abce)x}{e^2} + \frac{3c(b^2 + ac)x^2}{2e} + \frac{bc^2x^3}{e} + \frac{c^3x^4}{4e} - \frac{(c^3d^2 - 3b^2cd^{4/3}e^{2/3} - 3ac^2d^{4/3}e^{2/3}) \log(d + ex^3)}{e^2}
\end{aligned}$$

Mathematica [A] time = 0.451074, size = 439, normalized size = 1.06

$$\frac{2 \log\left(d^{2/3} - \sqrt[3]{d} \sqrt[3]{ex + e^{2/3}x^2}\right) \left(-3a^2b \sqrt[3]{de^{5/3}} + a^3e^2 - 6abcde + 3ac^2d^{4/3}e^{2/3} + 3b^2cd^{4/3}e^{2/3} - b^3de + c^3d^2\right)}{d^{2/3}} + \frac{4 \log\left(\sqrt[3]{d} + \sqrt[3]{ex}\right) \left(-3a^2b \sqrt[3]{de^{5/3}} + a^3e^2 - 6abcde + 3ac^2d^{4/3}e^{2/3} + 3b^2cd^{4/3}e^{2/3} - b^3de + c^3d^2\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2)^3/(d + e*x^3),x]

[Out] $(12e^{1/3}(-(c^3d) + b^3e + 6a*b*c*e)*x + 18c*(b^2 + a*c)*e^{4/3}*x^2 + 12*b*c^2*e^{4/3}*x^3 + 3*c^3*e^{4/3}*x^4 - (4*\sqrt{3}*(c^3*d^2 - 3*a*c^2*d^{4/3})e^{2/3} + e*(-(b^3*d) + 3*a^2*b*d^{1/3})e^{2/3} + a^3*e) - 3*c*(b^2*d^{4/3})e^{2/3} + 2*a*b*d*e)*\text{ArcTan}[(1 - (2e^{1/3}*x)/d^{1/3})/\sqrt{3}]/d^{2/3} + (4*(c^3*d^2 + 3*b^2*c*d^{4/3})e^{2/3} + 3*a*c^2*d^{4/3})e^{2/3} - b^3*d*e - 6*a*b*c*d*e - 3*a^2*b*d^{1/3})e^{5/3} + a^3*e^2)*\text{Log}[d^{1/3} + e^{1/3}*x])/d^{2/3} - (2*(c^3*d^2 + 3*b^2*c*d^{4/3})e^{2/3} + 3*a*c^2*d^{4/3})e^{2/3} - b^3*d*e - 6*a*b*c*d*e - 3*a^2*b*d^{1/3})e^{5/3} + a^3*e^2)*\text{Log}[d^{2/3} - d^{1/3}*e^{1/3}*x + e^{2/3}*x^2])/d^{2/3} + 12e^{1/3}(-(b*c^2*d) + a*b^2*e + a^2*c*e)*\text{Log}[d + e*x^3)]/(12e^{7/3})$

Maple [B] time = 0.005, size = 837, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)^3/(e*x^3+d),x)

[Out] $-2/e^2/(d/e)^{2/3}*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(d/e)^{1/3}*x-1))*a*b*c*d + b*c^2*x^3/e - 1/6/e^3/(d/e)^{2/3}*\ln(x^2 - (d/e)^{1/3}*x + (d/e)^{2/3})*c^3*d^2 + 6/e*a*b*c*x^2/e^2/(d/e)^{2/3}*\ln(x + (d/e)^{1/3})*a*b*c*d + 1/e^2/(d/e)^{2/3}*\ln(x^2 - (d/e)^{1/3}*x + (d/e)^{2/3})*a*b*c*d - 1/e^2*3^{1/2}/(d/e)^{1/3}*\arctan(1/3*3^{1/2}*(2/(d/e)^{1/3}*x-1))*a*c^2*d - 1/e^2*3^{1/2}/(d/e)^{1/3}*\arctan(1/3*3^{1/2}*(2/(d/e)^{1/3}*x-1))*b^2*c*d + 1/e*b^3*x - 1/e^2*\ln(e*x^3+d)*b*c^2*d - 1/3/e^2/(d/e)^{2/3}*\ln(x + (d/e)^{1/3})*b^3*d + 1/3/e^3/(d/e)^{2/3}*\ln(x + (d/e)^{1/3})*c^3*d^2 + 1/6/e^2/(d/e)^{2/3}*\ln(x^2 - (d/e)^{1/3}*x + (d/e)^{2/3})*b^3*d - 1/e/(d/e)^{1/3}*\ln(x + (d/e)^{1/3})*a^2*b + 1/2/e/(d/e)^{1/3}*\ln(x^2 - (d/e)^{1/3})*x + (d/e)^{2/3})*a^2*b + 1/3/e/(d/e)^{2/3})*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(d/e)^{1/3}*x-1))*a^3 + 1/3/e/(d/e)^{2/3}*\ln(x + (d/e)^{1/3})*a^3 - 1/6/e/(d/e)^{2/3}*\ln(x^2 - (d/e)^{1/3}*x + (d/e)^{2/3})*a^3 + 3/2/e*x^2*b^2*c - 1/e^2*c^3*d*x + 3/2/e*x^2*a*c^2 + 1/e*\ln(e*x^3+d)*a^2*c + 1/e*\ln(e*x^3+d)*a*b^2 + 1/4*c^3*x^4/e + 1/e^3*(1/2)/(d/e)^{1/3}*\arctan(1/3*3^{1/2}*(2/(d/e)^{1/3}*x-1))*a^2*b + 1/e^2/(d/e)^{1/3}*\ln(x + (d/e)^{1/3})*b^2*c*d - 1/2/e^2/(d/e)^{1/3}*\ln(x^2 - (d/e)^{1/3}*x + (d/e)^{2/3})*a*c^2*d - 1/2/e^2/(d/e)^{1/3}*\ln(x^2 - (d/e)^{1/3}*x + (d/e)^{2/3})*b^2*c*d - 1/3/e^2/(d/e)^{2/3})*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(d/e)^{1/3}*x-1))*b^3*d + 1/3/e^3/(d/e)^{2/3})*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(d/e)^{1/3}*x-1))*c^3$

$$*d^2+1/e^2/(d/e)^{(1/3)}*\ln(x+(d/e)^{(1/3)})*a*c^2*d$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^3/(e*x^3+d),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^3/(e*x^3+d),x, algorithm="fricas")

[Out] Timed out

Sympy [B] time = 86.1001, size = 1314, normalized size = 3.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)**3/(e*x**3+d),x)

[Out] $b*c**2*x**3/e + c**3*x**4/(4*e) + \text{RootSum}(27*_t**3*d**2*e**7 + *_t**2*(-81*a**2*c*d**2*e**6 - 81*a*b**2*d**2*e**6 + 81*b*c**2*d**3*e**5) + *_t*(27*a**5*b*d*e**6 + 54*a**4*c**2*d**2*e**5 - 27*a**3*b**2*c*d**2*e**5 + 54*a**2*b**4*d**2*e**5 + 27*a**2*b*c**3*d**3*e**4 + 27*a*b**3*c**2*d**3*e**4 - 27*a*c**5*d**4*e**3 + 27*b**5*c*d**3*e**4 + 54*b**2*c**4*d**4*e**3) - a**9*e**6 - 9*a**7*b*c*d*e**5 + 3*a**6*b**3*d*e**5 - 3*a**6*c**3*d**2*e**4 - 27*a**5*b**2*c**2*d**2*e**4 + 18*a**4*b**4*c*d**2*e**4 - 18*a**4*b*c**4*d**3*e**3 - 3*$

```

a**3*b**6*d**2*e**4 - 21*a**3*b**3*c**3*d**3*e**3 - 3*a**3*c**6*d**4*e**2 +
27*a**2*b**5*c**2*d**3*e**3 - 27*a**2*b**2*c**5*d**4*e**2 - 9*a*b**7*c*d**
3*e**3 + 18*a*b**4*c**4*d**4*e**2 - 9*a*b*c**7*d**5*e + b**9*d**3*e**3 - 3*
b**6*c**3*d**4*e**2 + 3*b**3*c**6*d**5*e - c**9*d**6, Lambda(_t, _t*log(x +
(27*_t**2*a**2*b*d**2*e**6 - 27*_t**2*a*c**2*d**3*e**5 - 27*_t**2*b**2*c*d
**3*e**5 + 3*_t*a**6*d*e**6 - 90*_t*a**4*b*c*d**2*e**5 - 60*_t*a**3*b**3*d
**2*e**5 + 60*_t*a**3*c**3*d**3*e**4 + 270*_t*a**2*b**2*c**2*d**3*e**4 + 90*
_t*a*b**4*c*d**3*e**4 - 90*_t*a*b*c**4*d**4*e**3 + 3*_t*b**6*d**3*e**4 - 60
*_t*b**3*c**3*d**4*e**3 + 3*_t*c**6*d**5*e**2 - 3*a**8*c*d*e**5 + 15*a**7*b
**2*d*e**5 + 30*a**6*b*c**2*d**2*e**4 - 48*a**5*b**3*c*d**2*e**4 - 15*a**5*
c**4*d**3*e**3 + 15*a**4*b**5*d**2*e**4 - 15*a**4*b**2*c**3*d**3*e**3 - 15*
a**3*b**4*c**2*d**3*e**3 - 48*a**3*b*c**5*d**4*e**2 - 30*a**2*b**6*c*d**3*e
**3 + 15*a**2*b**3*c**4*d**4*e**2 + 15*a**2*c**7*d**5*e - 3*a*b**8*d**3*e**
3 - 48*a*b**5*c**3*d**4*e**2 - 30*a*b**2*c**6*d**5*e - 15*b**7*c**2*d**4*e
**2 - 15*b**4*c**5*d**5*e + 3*b*c**8*d**6)/(a**9*e**6 - 18*a**7*b*c*d*e**5 +
24*a**6*b**3*d*e**5 + 3*a**6*c**3*d**2*e**4 + 27*a**5*b**2*c**2*d**2*e**4
- 45*a**4*b**4*c*d**2*e**4 + 45*a**4*b*c**4*d**3*e**3 + 3*a**3*b**6*d**2*e
**4 - 60*a**3*b**3*c**3*d**3*e**3 - 24*a**3*c**6*d**4*e**2 - 27*a**2*b**5*c
**2*d**3*e**3 + 27*a**2*b**2*c**5*d**4*e**2 - 18*a*b**7*c*d**3*e**3 - 45*a*b
**4*c**4*d**4*e**2 - 18*a*b*c**7*d**5*e - b**9*d**3*e**3 - 24*b**6*c**3*d**
4*e**2 - 3*b**3*c**6*d**5*e + c**9*d**6))) + x**2*(3*a*c**2 + 3*b**2*c)/(2
*e) + x*(6*a*b*c*e + b**3*e - c**3*d)/e**2

```

Giac [A] time = 1.06767, size = 648, normalized size = 1.56

$$\sqrt{3} \left((-de^2)^{\frac{1}{3}} c^3 d^2 - (-de^2)^{\frac{1}{3}} b^3 de - 6 (-de^2)^{\frac{1}{3}} abcde + 3 (-de^2)^{\frac{2}{3}} b^2 cd + 3 (-de^2)^{\frac{2}{3}} b^2 cd + 3 (-de^2)^{\frac{2}{3}} b^2 cd \right) + \frac{- (bc^2 d - ab^2 e - a^2 ce) e^{(-2)} \log(|x^3 e + d|)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x^3+d),x, algorithm="giac")
```

```

[Out] -(b*c^2*d - a*b^2*e - a^2*c*e)*e^(-2)*log(abs(x^3*e + d)) + 1/3*sqrt(3)*((-
d*e^2)^(1/3)*c^3*d^2 - (-d*e^2)^(1/3)*b^3*d*e - 6*(-d*e^2)^(1/3)*a*b*c*d*e
+ 3*(-d*e^2)^(2/3)*b^2*c*d + 3*(-d*e^2)^(2/3)*a*c^2*d - 3*(-d*e^2)^(2/3)*a^
2*b*e + (-d*e^2)^(1/3)*a^3*e^2)*arctan(1/3*sqrt(3)*(2*x + (-d*e^(-1))^(1/3)
)/(-d*e^(-1))^(1/3))*e^(-3)/d - 1/3*(c^3*d^2*e^7 - 3*(-d*e^(-1))^(1/3)*b^2*
c*d*e^8 - 3*(-d*e^(-1))^(1/3)*a*c^2*d*e^8 - b^3*d*e^8 - 6*a*b*c*d*e^8 + 3*(
-d*e^(-1))^(1/3)*a^2*b*e^9 + a^3*e^9)*(-d*e^(-1))^(1/3)*e^(-9)*log(abs(x -
(-d*e^(-1))^(1/3)))/d + 1/4*(c^3*x^4*e^3 + 4*b*c^2*x^3*e^3 + 6*b^2*c*x^2*e^

```


$$\begin{aligned}
& 3 + 6*a*c^2*x^2*e^3 - 4*c^3*d*x*e^2 + 4*b^3*x*e^3 + 24*a*b*c*x*e^3)*e^{-4} \\
& + 1/6*((-d*e^2)^{(1/3)}*c^3*d^2 - (-d*e^2)^{(1/3)}*b^3*d*e - 6*(-d*e^2)^{(1/3)}*a \\
& *b*c*d*e - 3*(-d*e^2)^{(2/3)}*b^2*c*d - 3*(-d*e^2)^{(2/3)}*a*c^2*d + 3*(-d*e^2) \\
& ^{(2/3)}*a^2*b*e + (-d*e^2)^{(1/3)}*a^3*e^2)*e^{-3}*\log(x^2 + (-d*e^{-1})^{(1/3)} \\
& *x + (-d*e^{-1})^{(2/3)})/d
\end{aligned}$$

$$3.75 \quad \int \frac{(a+bx+cx^2)^4}{d+ex^3} dx$$

Optimal. Leaf size=645

$$\frac{\log\left(d^{2/3} - \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2\right)\left(\sqrt[3]{e}\left(-12a^2bcde + a^4e^2 - 4ab^3de + 4ac^3d^2 + 6b^2c^2d^2\right) + \sqrt[3]{d}\left(-4b\left(a^3e^2 + c^3d^2\right) + 6a^2c^2de + 12ab^2cde + b^4de\right)\right)}{6d^{2/3}e^{8/3}}$$

[Out] $(-2*(3*b^2*c^2*d + 2*a*c^3*d - 2*a*b^3*e - 6*a^2*b*c*e)*x)/e^2 - ((4*b*c^3*d - b^4*e - 12*a*b^2*c*e - 6*a^2*c^2*e)*x^2)/(2*e^2) - (c*(c^3*d - 4*b^3*e - 12*a*b*c*e)*x^3)/(3*e^2) + (c^2*(3*b^2 + 2*a*c)*x^4)/(2*e) + (4*b*c^3*x^5)/(5*e) + (c^4*x^6)/(6*e) - ((b*d^{(1/3)} + a*e^{(1/3)})*(4*c^3*d^2 + 6*c^2*(b*d^{(5/3)}*e^{(1/3)} - a*d^{(4/3)}*e^{(2/3)}) - 12*a*b*c*d*e - e*(b^3*d + 3*a*b^2*d^{(2/3)}*e^{(1/3)} - 3*a^2*b*d^{(1/3)}*e^{(2/3)} - a^3*e))*ArcTan[(d^{(1/3)} - 2*e^{(1/3)}*x)/(Sqrt[3]*d^{(1/3)})]/(Sqrt[3]*d^{(2/3)}*e^{(8/3)}) + ((e^{(1/3)}*(6*b^2*c^2*d^2 + 4*a*c^3*d^2 - 4*a*b^3*d*e - 12*a^2*b*c*d*e + a^4*e^2) + d^{(1/3)}*(b^4*d*e + 12*a*b^2*c*d*e + 6*a^2*c^2*d*e - 4*b*(c^3*d^2 + a^3*e^2)))*Log[d^{(1/3)} + e^{(1/3)}*x]/(3*d^{(2/3)}*e^{(8/3)}) - ((e^{(1/3)}*(6*b^2*c^2*d^2 + 4*a*c^3*d^2 - 4*a*b^3*d*e - 12*a^2*b*c*d*e + a^4*e^2) + d^{(1/3)}*(b^4*d*e + 12*a*b^2*c*d*e + 6*a^2*c^2*d*e - 4*b*(c^3*d^2 + a^3*e^2)))*Log[d^{(2/3)} - d^{(1/3)}*e^{(1/3)}*x + e^{(2/3)}*x^2]/(6*d^{(2/3)}*e^{(8/3)}) + ((c^4*d^2 - 12*a*b*c^2*d*e + 6*a^2*b^2*e^2 - 4*c*e*(b^3*d - a^3*e))*Log[d + e*x^3]/(3*e^3))$

Rubi [A] time = 1.09664, antiderivative size = 643, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(d^{2/3} - \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2\right)\left(\frac{\sqrt[3]{d}\left(-4b\left(a^3e^2 + c^3d^2\right) + 6a^2c^2de + 12ab^2cde + b^4de\right)}{\sqrt[3]{e}} - 12a^2bcde + a^4e^2 - 4ab^3de + 4ac^3d^2 + 6b^2c^2d^2\right)}{6d^{2/3}e^{7/3}} + \dots$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2)^4/(d + e*x^3), x]

[Out] $(-2*(3*b^2*c^2*d + 2*a*c^3*d - 2*a*b^3*e - 6*a^2*b*c*e)*x)/e^2 - ((4*b*c^3*d - b^4*e - 12*a*b^2*c*e - 6*a^2*c^2*e)*x^2)/(2*e^2) - (c*(c^3*d - 4*b^3*e - 12*a*b*c*e)*x^3)/(3*e^2) + (c^2*(3*b^2 + 2*a*c)*x^4)/(2*e) + (4*b*c^3*x^5)/(5*e) + (c^4*x^6)/(6*e) - ((b*d^{(1/3)} + a*e^{(1/3)})*(4*c^3*d^2 + 6*c^2*(b*d^{(5/3)}*e^{(1/3)} - a*d^{(4/3)}*e^{(2/3)}) - 12*a*b*c*d*e - e*(b^3*d + 3*a*b^2*d^{(2/3)}*e^{(1/3)} - 3*a^2*b*d^{(1/3)}*e^{(2/3)} - a^3*e))*ArcTan[(d^{(1/3)} - 2*e^{(1/3)}*x)/(Sqrt[3]*d^{(1/3)})]/(Sqrt[3]*d^{(2/3)}*e^{(8/3)}) + ((e^{(1/3)}*(6*b^2*c^2*d^2 + 4*a*c^3*d^2 - 4*a*b^3*d*e - 12*a^2*b*c*d*e + a^4*e^2) + d^{(1/3)}*(b^4*d*e + 12*a*b^2*c*d*e + 6*a^2*c^2*d*e - 4*b*(c^3*d^2 + a^3*e^2)))*Log[d^{(1/3)} + e^{(1/3)}*x]/(3*d^{(2/3)}*e^{(8/3)}) - ((e^{(1/3)}*(6*b^2*c^2*d^2 + 4*a*c^3*d^2 - 4*a*b^3*d*e - 12*a^2*b*c*d*e + a^4*e^2) + d^{(1/3)}*(b^4*d*e + 12*a*b^2*c*d*e + 6*a^2*c^2*d*e - 4*b*(c^3*d^2 + a^3*e^2)))*Log[d^{(2/3)} - d^{(1/3)}*e^{(1/3)}*x + e^{(2/3)}*x^2]/(6*d^{(2/3)}*e^{(8/3)}) + ((c^4*d^2 - 12*a*b*c^2*d*e + 6*a^2*b^2*e^2 - 4*c*e*(b^3*d - a^3*e))*Log[d + e*x^3]/(3*e^3))$

$$\begin{aligned} & (2/3)*e^{(1/3)} - 3*a^2*b*d^{(1/3)}*e^{(2/3)} - a^3*e) * \text{ArcTan}[(d^{(1/3)} - 2*e^{(1/3)}*x) / (\text{Sqrt}[3]*d^{(1/3)})] / (\text{Sqrt}[3]*d^{(2/3)}*e^{(8/3)}) + ((e^{(1/3)}*(6*b^2*c^2*d^2 + 4*a*c^3*d^2 - 4*a*b^3*d*e - 12*a^2*b*c*d*e + a^4*e^2) + d^{(1/3)}*(b^4*d*e + 12*a*b^2*c*d*e + 6*a^2*c^2*d*e - 4*b*(c^3*d^2 + a^3*e^2))) * \text{Log}[d^{(1/3)} + e^{(1/3)}*x] / (3*d^{(2/3)}*e^{(8/3)}) - ((6*b^2*c^2*d^2 + 4*a*c^3*d^2 - 4*a*b^3*d*e - 12*a^2*b*c*d*e + a^4*e^2 + (d^{(1/3)}*(b^4*d*e + 12*a*b^2*c*d*e + 6*a^2*c^2*d*e - 4*b*(c^3*d^2 + a^3*e^2))) / e^{(1/3)}) * \text{Log}[d^{(2/3)} - d^{(1/3)}*e^{(1/3)}*x + e^{(2/3)}*x^2] / (6*d^{(2/3)}*e^{(7/3)}) + ((c^4*d^2 - 12*a*b*c^2*d*e + 6*a^2*b^2*e^2 - 4*c*e*(b^3*d - a^3*e)) * \text{Log}[d + e*x^3]) / (3*e^3) \end{aligned}$$

Rule 1887

$\text{Int}[(\text{Pq}_)/((a_) + (b_)*(x_)^n)], x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\text{Pq}/(a + b*x^n), x], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PolyQ}[\text{Pq}, x] \ \&\& \ \text{IntegerQ}[n]$

Rule 1871

$\text{Int}[(\text{P2}_)/((a_) + (b_)*(x_)^3), x_Symbol] \rightarrow \text{With}\{A = \text{Coeff}[\text{P2}, x, 0], B = \text{Coeff}[\text{P2}, x, 1], C = \text{Coeff}[\text{P2}, x, 2]\}, \text{Int}[(A + B*x)/(a + b*x^3), x] + \text{Dist}[C, \text{Int}[x^2/(a + b*x^3), x], x] /; \text{EqQ}[a*B^3 - b*A^3, 0] \ \|\ \! \text{RationalQ}[a/b] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PolyQ}[\text{P2}, x, 2]$

Rule 1860

$\text{Int}[(A_ + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] \rightarrow \text{With}\{r = \text{Numerator}[\text{Rt}[a/b, 3]], s = \text{Denominator}[\text{Rt}[a/b, 3]]\}, -\text{Dist}[(r*(B*r - A*s))/(3*a*s), \text{Int}[1/(r + s*x), x], x] + \text{Dist}[r/(3*a*s), \text{Int}[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x] /; \text{FreeQ}\{a, b, A, B\}, x \ \&\& \ \text{NeQ}[a*B^3 - b*A^3, 0] \ \&\& \ \text{PosQ}[a/b]$

Rule 31

$\text{Int}[(a_ + (b_)*(x_))^{(-1)}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}\{a, b\}, x]$

Rule 634

$\text{Int}[(d_ + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] \rightarrow \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \! \text{NiceSqrtQ}[b^2 - 4*a*c]$

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx + cx^2)^4}{d + ex^3} dx &= \int \left(-\frac{2(3b^2c^2d + 2ac^3d - 2ab^3e - 6a^2bce)}{e^2} - \frac{(4bc^3d - b^4e - 12ab^2ce - 6a^2c^2e)x}{e^2} - \frac{c(c^3d - 4b^3e)}{e^2} \right) dx \\
&= -\frac{2(3b^2c^2d + 2ac^3d - 2ab^3e - 6a^2bce)x}{e^2} - \frac{(4bc^3d - b^4e - 12ab^2ce - 6a^2c^2e)x^2}{2e^2} - \frac{c(c^3d - 4b^3e)x^3}{3e^2} \\
&= -\frac{2(3b^2c^2d + 2ac^3d - 2ab^3e - 6a^2bce)x}{e^2} - \frac{(4bc^3d - b^4e - 12ab^2ce - 6a^2c^2e)x^2}{2e^2} - \frac{c(c^3d - 4b^3e)x^3}{3e^2} \\
&= -\frac{2(3b^2c^2d + 2ac^3d - 2ab^3e - 6a^2bce)x}{e^2} - \frac{(4bc^3d - b^4e - 12ab^2ce - 6a^2c^2e)x^2}{2e^2} - \frac{c(c^3d - 4b^3e)x^3}{3e^2} \\
&= -\frac{2(3b^2c^2d + 2ac^3d - 2ab^3e - 6a^2bce)x}{e^2} - \frac{(4bc^3d - b^4e - 12ab^2ce - 6a^2c^2e)x^2}{2e^2} - \frac{c(c^3d - 4b^3e)x^3}{3e^2} \\
&= -\frac{2(3b^2c^2d + 2ac^3d - 2ab^3e - 6a^2bce)x}{e^2} - \frac{(4bc^3d - b^4e - 12ab^2ce - 6a^2c^2e)x^2}{2e^2} - \frac{c(c^3d - 4b^3e)x^3}{3e^2} \\
&= -\frac{2(3b^2c^2d + 2ac^3d - 2ab^3e - 6a^2bce)x}{e^2} - \frac{(4bc^3d - b^4e - 12ab^2ce - 6a^2c^2e)x^2}{2e^2} - \frac{c(c^3d - 4b^3e)x^3}{3e^2}
\end{aligned}$$

Mathematica [A] time = 0.348562, size = 678, normalized size = 1.05

$$\frac{5 \log\left(d^{2/3} - \sqrt[3]{d} \sqrt[3]{ex + e^{2/3}x^2}\right) \left(-4b\left(3a^2cde^{4/3} + a^3\sqrt[3]{de^2 + c^3d^{7/3}}\right) + 6a^2c^2d^{4/3}e + a^4e^{7/3} + 6b^2\left(2acd^{4/3}e + c^2d^2\sqrt[3]{e}\right) - 4ab^3de^{4/3} + 4ac^3d^2\sqrt[3]{e} + b^4d^{4/3}e\right)}{d^{2/3}} + \frac{10 \log(d + ex^3)}{3e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2)^4/(d + e*x^3), x]

[Out] (60*e^(2/3)*(-3*b^2*c^2*d - 2*a*c^3*d + 2*a*b^3*e + 6*a^2*b*c*e)*x + 15*e^(2/3)*(-4*b*c^3*d + b^4*e + 12*a*b^2*c*e + 6*a^2*c^2*e)*x^2 + 10*c*e^(2/3)*(-c^3*d) + 4*b^3*e + 12*a*b*c*e)*x^3 + 15*c^2*(3*b^2 + 2*a*c)*e^(5/3)*x^4 + 24*b*c^3*e^(5/3)*x^5 + 5*c^4*e^(5/3)*x^6 + (10*sqrt[3]*(b*d^(1/3) + a*e^(1/3)))*(-4*c^3*d^2 + c^2*(-6*b*d^(5/3)*e^(1/3) + 6*a*d^(4/3)*e^(2/3)) + 12*a*b*c*d*e + e*(b^3*d + 3*a*b^2*d^(2/3)*e^(1/3) - 3*a^2*b*d^(1/3)*e^(2/3) - a^3*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/sqrt[3]]/d^(2/3) + (10*(4*a*c^3*d

$$\begin{aligned} &^2e^{(1/3)} + b^4d^{(4/3)}e + 6a^2c^2d^{(4/3)}e - 4ab^3de^{(4/3)} + a^4e^{(7/3)} \\ &+ 6b^2(c^2d^2e^{(1/3)} + 2ac^2d^{(4/3)}e) - 4b(c^3d^{(7/3)} + 3a^2c^2de^{(4/3)} \\ &+ a^3d^{(1/3)}e^2))\text{Log}[d^{(1/3)} + e^{(1/3)}x]/d^{(2/3)} - (5(4ac^3d^2e^{(1/3)} \\ &+ b^4d^{(4/3)}e + 6a^2c^2d^{(4/3)}e - 4ab^3de^{(4/3)} + a^4e^{(7/3)} + 6b^2(c^2d^2e^{(1/3)} \\ &+ 2ac^2d^{(4/3)}e) - 4b(c^3d^{(7/3)} + 3a^2c^2de^{(4/3)} + a^3d^{(1/3)}e^2))\text{Log}[d^{(2/3)} - d^{(1/3)}e^{(1/3)} \\ &*x + e^{(2/3)}x^2])/d^{(2/3)} + (10(c^4d^2 - 12ab^3c^2de + 6a^2b^2e^2 + 4c^2e(-b^3d) \\ &+ a^3e))\text{Log}[d + ex^3])/e^{(1/3)})/(30e^{(8/3)}) \end{aligned}$$

Maple [B] time = 0.007, size = 1339, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)^4/(e*x^3+d), x)`

[Out]
$$\begin{aligned} &-4/e^2/(d/e)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(d/e)^{(1/3)}*x-1))*a^2*b*c* \\ &d-4/e^2*3^{(1/2)}/(d/e)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(d/e)^{(1/3)}*x-1))*a*b^2*c \\ &*d-4/3/e^3/(d/e)^{(1/3)}*\ln(x+(d/e)^{(1/3)})*b*c^3*d^2-1/e^2/(d/e)^{(1/3)}*\ln(x^2 \\ &-(d/e)^{(1/3)}*x+(d/e)^{(2/3)})*a^2*c^2*d-4/3/e^2/(d/e)^{(2/3)}*\ln(x+(d/e)^{(1/3)}) \\ &*a*b^3*d+4/3/e^3/(d/e)^{(2/3)}*\ln(x+(d/e)^{(1/3)})*a*c^3*d^2+2/e^3/(d/e)^{(2/3)}* \\ &\ln(x+(d/e)^{(1/3)})*b^2*c^2*d^2+2/3/e^2/(d/e)^{(2/3)}*\ln(x^2-(d/e)^{(1/3)}*x+(d/e) \\ &)^{(2/3))*a*b^3*d-2/3/e^3/(d/e)^{(2/3)}*\ln(x^2-(d/e)^{(1/3)}*x+(d/e)^{(2/3)})*a*c^ \\ &3*d^2-4/e^2/(d/e)^{(2/3)}*\ln(x+(d/e)^{(1/3)})*a^2*b*c*d+2/e^2/(d/e)^{(2/3)}*\ln(x^ \\ &2-(d/e)^{(1/3)}*x+(d/e)^{(2/3)})*a^2*b*c*d-4/3/e^2/(d/e)^{(2/3)}*3^{(1/2)}*\arctan(1 \\ &/3*3^{(1/2)}*(2/(d/e)^{(1/3)}*x-1))*a*b^3*d+4/3/e^3/(d/e)^{(2/3)}*3^{(1/2)}*\arctan(\\ &1/3*3^{(1/2)}*(2/(d/e)^{(1/3)}*x-1))*a*c^3*d^2+2/e^3/(d/e)^{(2/3)}*3^{(1/2)}*\arctan \\ &(1/3*3^{(1/2)}*(2/(d/e)^{(1/3)}*x-1))*b^2*c^2*d^2+4/e^2/(d/e)^{(1/3)}*\ln(x+(d/e)^ \\ &(1/3))*a*b^2*c*d-2/e^2/(d/e)^{(1/3)}*\ln(x^2-(d/e)^{(1/3)}*x+(d/e)^{(2/3)})*a*b^2* \\ &c*d-2/e^2*3^{(1/2)}/(d/e)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(d/e)^{(1/3)}*x-1))*a^2*c \\ &^2*d+4/3/e^3*3^{(1/2)}/(d/e)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(d/e)^{(1/3)}*x-1))*b* \\ &c^3*d^2+4/e*x^3*a*b*c^2+6/e*x^2*a*b^2*c-2/e^2*x^2*b*c^3*d-4/3/e/(d/e)^{(1/3)} \\ &*\ln(x+(d/e)^{(1/3)})*a^3*b+1/3/e^2/(d/e)^{(1/3)}*\ln(x+(d/e)^{(1/3)})*b^4*d+2/3/e/ \\ &(d/e)^{(1/3)}*\ln(x^2-(d/e)^{(1/3)}*x+(d/e)^{(2/3)})*a^3*b+1/3/e/(d/e)^{(2/3)}*3^{(1/ \\ &2)}*\arctan(1/3*3^{(1/2)}*(2/(d/e)^{(1/3)}*x-1))*a^4-1/6/e^2/(d/e)^{(1/3)}*\ln(x^2-(\\ &d/e)^{(1/3)}*x+(d/e)^{(2/3)})*b^4*d-4/3/e^2*\ln(e*x^3+d)*b^3*c*d-6/e^2*b^2*c^2*d \\ &*x+1/2/e*x^2*b^4+12/e*a^2*b*c*x-4/e^2*a*c^3*d*x-1/e^3/(d/e)^{(2/3)}*\ln(x^2-(d \\ &/e)^{(1/3)}*x+(d/e)^{(2/3)})*b^2*c^2*d^2-4/e^2*\ln(e*x^3+d)*a*b*c^2*d-1/3/e^2*3^{(\\ &1/2)}/(d/e)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(d/e)^{(1/3)}*x-1))*b^4*d+4/3/e*3^{(1/ \\ &2)}/(d/e)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(d/e)^{(1/3)}*x-1))*a^3*b+1/6*c^4*x^6/e+ \\ &1/e*x^4*a*c^3+3/2/e*x^4*b^2*c^2+4/3/e*x^3*b^3*c-1/3/e^2*x^3*c^4*d+3/e*x^2*a \end{aligned}$$

$$\begin{aligned} &^2*c^2+4/e*a*b^3*x+4/3/e*\ln(e*x^3+d)*a^3*c+2/e*\ln(e*x^3+d)*a^2*b^2+1/3/e^3* \\ &\ln(e*x^3+d)*c^4*d^2+1/3/e/(d/e)^{(2/3)}*\ln(x+(d/e)^{(1/3)})*a^4-1/6/e/(d/e)^{(2/} \\ &3)*\ln(x^2-(d/e)^{(1/3)}*x+(d/e)^{(2/3)})*a^4+2/3/e^3/(d/e)^{(1/3)}*\ln(x^2-(d/e)^{(} \\ &1/3)*x+(d/e)^{(2/3)})*b*c^3*d^2+2/e^2/(d/e)^{(1/3)}*\ln(x+(d/e)^{(1/3)})*a^2*c^2*d \\ &+4/5*b*c^3*x^5/e \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^4/(e*x^3+d),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^4/(e*x^3+d),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)**4/(e*x**3+d),x)

[Out] Timed out

Giac [A] time = 1.08789, size = 1060, normalized size = 1.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^4/(e*x^3+d),x, algorithm="giac")

[Out] $\frac{1}{3}(c^4d^2 - 4b^3cd^2e - 12ab^2c^2d^2e + 6a^2b^2e^2 + 4a^3c^2e^2) \cdot e^{-3} \log(\text{abs}(x^3e + d)) + \frac{1}{3}\sqrt{3} \cdot (6(-de^2)^{1/3}b^2c^2d^2e + 4(-de^2)^{1/3}ac^3d^2e - 4(-de^2)^{2/3}b^2c^3d^2 + (-de^2)^{2/3}b^4d^2e + 12(-de^2)^{2/3}ab^2cd^2e + 6(-de^2)^{2/3}a^2c^2d^2e - 4(-de^2)^{1/3}ab^3d^2e^2 - 12(-de^2)^{1/3}a^2b^2cd^2e^2 - 4(-de^2)^{2/3}a^3b^2e^2 + (-de^2)^{1/3}a^4e^3) \cdot \arctan\left(\frac{1}{3}\sqrt{3}(2x + (-de^{-1})^{1/3}) / (-de^{-1})^{1/3}\right) \cdot e^{-4} / d - \frac{1}{3} \cdot (4(-de^{-1})^{1/3}b^2c^3d^2e^{11} + 6b^2c^2d^2e^{11} + 4ac^3d^2e^{11} - (-de^{-1})^{1/3}b^4d^2e^{12} - 12(-de^{-1})^{1/3}ab^2cd^2e^{12} - 6(-de^{-1})^{1/3}a^2c^2d^2e^{12} - 4ab^3d^2e^{12} - 12a^2b^2cd^2e^{12} + 4(-de^{-1})^{1/3}a^3b^2e^{13} + a^4e^{13}) \cdot (-de^{-1})^{1/3} \cdot e^{-13} \cdot \log(\text{abs}(x - (-de^{-1})^{1/3})) / d + \frac{1}{30} \cdot (5c^4x^6e^5 + 24b^2c^3x^5e^5 + 45b^2c^2x^4e^5 + 30ac^3x^4e^5 - 10c^4dx^3e^4 + 40b^3c^2x^3e^5 + 120ab^2c^2x^3e^5 - 60b^2c^3dx^2e^4 + 15b^4x^2e^5 + 180ab^2c^2x^2e^5 + 90a^2c^2x^2e^5 - 180b^2c^2d^2x^2e^4 - 120ac^3d^2x^2e^4 + 120ab^3x^2e^5 + 360a^2b^2cd^2x^2e^5) \cdot e^{-6} + \frac{1}{6} \cdot (6(-de^2)^{1/3}b^2c^2d^2e + 4(-de^2)^{1/3}ac^3d^2e + 4(-de^2)^{2/3}b^2c^3d^2 - (-de^2)^{2/3}b^4d^2e - 12(-de^2)^{2/3}ab^2c^2d^2e - 6(-de^2)^{2/3}a^2c^2d^2e - 4(-de^2)^{1/3}ab^3d^2e^2 - 12(-de^2)^{1/3}a^2b^2cd^2e^2 + 4(-de^2)^{2/3}a^3b^2e^2 + (-de^2)^{1/3}a^4e^3) \cdot e^{-4} \cdot \log(x^2 + (-de^{-1})^{1/3}x + (-de^{-1})^{2/3}) / d$

$$3.76 \quad \int \frac{2x^2+x^4}{1+x^3} dx$$

Optimal. Leaf size=43

$$\frac{x^2}{2} + \frac{1}{2} \log(x^2 - x + 1) + \log(x + 1) + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] $x^2/2 + \text{ArcTan}[(1 - 2x)/\text{Sqrt}[3]]/\text{Sqrt}[3] + \text{Log}[1 + x] + \text{Log}[1 - x + x^2]/2$

Rubi [A] time = 0.078164, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1593, 1887, 1874, 31, 634, 618, 204, 628}

$$\frac{x^2}{2} + \frac{1}{2} \log(x^2 - x + 1) + \log(x + 1) + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(2*x^2 + x^4)/(1 + x^3), x]

[Out] $x^2/2 + \text{ArcTan}[(1 - 2x)/\text{Sqrt}[3]]/\text{Sqrt}[3] + \text{Log}[1 + x] + \text{Log}[1 - x + x^2]/2$

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1887

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a + b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]

Rule 1874

Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[(q*(A - B*q + C*q^2))/(3*a), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q - C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*A^3, 0] && NeQ[A - B*q + C*q^2, 0] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

&& GtQ[a/b, 0]

Rule 31

Int[((a_) + (b_)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)²), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x²), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x²), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b² - 4*a*c, 0] && !NiceSqrtQ[b² - 4*a*c]

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)²)⁽⁻¹⁾, x_Symbol] := Dist[-2, Subst[Int[1/Simp[b² - 4*a*c - x², x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b² - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)²)⁽⁻¹⁾, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)²), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x², x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
\int \frac{2x^2 + x^4}{1 + x^3} dx &= \int \frac{x^2(2 + x^2)}{1 + x^3} dx \\
&= \int \left(x + \frac{x(-1 + 2x)}{1 + x^3} \right) dx \\
&= \frac{x^2}{2} + \int \frac{x(-1 + 2x)}{1 + x^3} dx \\
&= \frac{x^2}{2} + \frac{1}{3} \int \frac{-3 + 3x}{1 - x + x^2} dx + \int \frac{1}{1 + x} dx \\
&= \frac{x^2}{2} + \log(1 + x) - \frac{1}{2} \int \frac{1}{1 - x + x^2} dx + \frac{1}{2} \int \frac{-1 + 2x}{1 - x + x^2} dx \\
&= \frac{x^2}{2} + \log(1 + x) + \frac{1}{2} \log(1 - x + x^2) + \text{Subst} \left(\int \frac{1}{-3 - x^2} dx, x, -1 + 2x \right) \\
&= \frac{x^2}{2} - \frac{\tan^{-1} \left(\frac{-1 + 2x}{\sqrt{3}} \right)}{\sqrt{3}} + \log(1 + x) + \frac{1}{2} \log(1 - x + x^2)
\end{aligned}$$

Mathematica [A] time = 0.0124251, size = 54, normalized size = 1.26

$$\frac{1}{6} \left(3x^2 - \log(x^2 - x + 1) + 4 \log(x^3 + 1) + 2 \log(x + 1) - 2\sqrt{3} \tan^{-1} \left(\frac{2x - 1}{\sqrt{3}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(2*x^2 + x^4)/(1 + x^3), x]

[Out] (3*x^2 - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Log[1 + x] - Log[1 - x + x^2] + 4*Log[1 + x^3])/6

Maple [A] time = 0.005, size = 38, normalized size = 0.9

$$\frac{x^2}{2} + \frac{\ln(x^2 - x + 1)}{2} - \frac{\sqrt{3}}{3} \arctan \left(\frac{(2x - 1)\sqrt{3}}{3} \right) + \ln(1 + x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+2*x^2)/(x^3+1), x)

[Out] $\frac{1}{2}x^2 + \frac{1}{2}\ln(x^2 - x + 1) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x - 1)\right) + \ln(1 + x)$

Maxima [A] time = 1.40092, size = 50, normalized size = 1.16

$$\frac{1}{2}x^2 - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x - 1)\right) + \frac{1}{2}\log(x^2 - x + 1) + \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4+2*x^2)/(x^3+1),x, algorithm="maxima")`

[Out] $\frac{1}{2}x^2 - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x - 1)\right) + \frac{1}{2}\log(x^2 - x + 1) + \log(x + 1)$

Fricas [A] time = 1.55632, size = 120, normalized size = 2.79

$$\frac{1}{2}x^2 - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x - 1)\right) + \frac{1}{2}\log(x^2 - x + 1) + \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4+2*x^2)/(x^3+1),x, algorithm="fricas")`

[Out] $\frac{1}{2}x^2 - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x - 1)\right) + \frac{1}{2}\log(x^2 - x + 1) + \log(x + 1)$

Sympy [A] time = 0.123992, size = 44, normalized size = 1.02

$$\frac{x^2}{2} + \log(x + 1) + \frac{\log(x^2 - x + 1)}{2} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**4+2*x**2)/(x**3+1),x)`

[Out] $x^{**2}/2 + \log(x + 1) + \log(x^{**2} - x + 1)/2 - \sqrt{3}\operatorname{atan}(2*\sqrt{3}*x/3 - \sqrt{3}/3)/3$

Giac [A] time = 1.08713, size = 51, normalized size = 1.19

$$\frac{1}{2}x^2 - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{2}\log(x^2-x+1) + \log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+2*x^2)/(x^3+1),x, algorithm="giac")

[Out] 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
+ log(abs(x + 1))

$$3.77 \quad \int \frac{2x^2+x^4}{1-x^3} dx$$

Optimal. Leaf size=46

$$-\frac{x^2}{2} - \frac{1}{2} \log(x^2 + x + 1) - \log(1 - x) - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] $-x^2/2 - \text{ArcTan}[(1 + 2*x)/\text{Sqrt}[3]]/\text{Sqrt}[3] - \text{Log}[1 - x] - \text{Log}[1 + x + x^2]/2$

Rubi [A] time = 0.0838093, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {1593, 1887, 1875, 31, 634, 618, 204, 628}

$$-\frac{x^2}{2} - \frac{1}{2} \log(x^2 + x + 1) - \log(1 - x) - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2*x^2 + x^4)/(1 - x^3), x]$

[Out] $-x^2/2 - \text{ArcTan}[(1 + 2*x)/\text{Sqrt}[3]]/\text{Sqrt}[3] - \text{Log}[1 - x] - \text{Log}[1 + x + x^2]/2$

Rule 1593

$\text{Int}[(u_.)*((a_.)*(x_)^{(p_.)} + (b_.)*(x_)^{(q_.)})^{(n_.)}, x_Symbol] \rightarrow \text{Int}[u*x^{(n*p)}*(a + b*x^{(q-p)})^n, x] /;$ FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1887

$\text{Int}[(Pq_)/((a_) + (b_.)*(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[Pq/(a + b*x^n), x], x] /;$ FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]

Rule 1875

$\text{Int}[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] \rightarrow \text{With}[\{A = \text{Coeff}[P2, x, 0], B = \text{Coeff}[P2, x, 1], C = \text{Coeff}[P2, x, 2], q = -(a/b)^{(1/3)}\}, \text{Dist}[(q*(A + B*q + C*q^2))/(3*a), \text{Int}[1/(q - x), x], x] + \text{Dist}[q/(3*a), \text{Int}[(q*(2*A - B*q$

- C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 - b*A^3, 0] && NeQ[A + B*q + C*q^2, 0] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2] && LtQ[a/b, 0]

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^-1, x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
\int \frac{2x^2 + x^4}{1 - x^3} dx &= \int \frac{x^2(2 + x^2)}{1 - x^3} dx \\
&= \int \left(-x + \frac{x(1 + 2x)}{1 - x^3} \right) dx \\
&= -\frac{x^2}{2} + \int \frac{x(1 + 2x)}{1 - x^3} dx \\
&= -\frac{x^2}{2} + \frac{1}{3} \int \frac{-3 - 3x}{1 + x + x^2} dx + \int \frac{1}{1 - x} dx \\
&= -\frac{x^2}{2} - \log(1 - x) - \frac{1}{2} \int \frac{1}{1 + x + x^2} dx - \frac{1}{2} \int \frac{1 + 2x}{1 + x + x^2} dx \\
&= -\frac{x^2}{2} - \log(1 - x) - \frac{1}{2} \log(1 + x + x^2) + \text{Subst} \left(\int \frac{1}{-3 - x^2} dx, x, 1 + 2x \right) \\
&= -\frac{x^2}{2} - \frac{\tan^{-1} \left(\frac{1 + 2x}{\sqrt{3}} \right)}{\sqrt{3}} - \log(1 - x) - \frac{1}{2} \log(1 + x + x^2)
\end{aligned}$$

Mathematica [A] time = 0.0121212, size = 54, normalized size = 1.17

$$\frac{1}{6} \left(-3x^2 + \log(x^2 + x + 1) - 4 \log(1 - x^3) - 2 \log(1 - x) - 2\sqrt{3} \tan^{-1} \left(\frac{2x + 1}{\sqrt{3}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(2*x^2 + x^4)/(1 - x^3), x]

[Out] (-3*x^2 - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Log[1 - x] + Log[1 + x + x^2] - 4*Log[1 - x^3])/6

Maple [A] time = 0.005, size = 38, normalized size = 0.8

$$-\frac{x^2}{2} - \ln(-1 + x) - \frac{\ln(x^2 + x + 1)}{2} - \frac{\sqrt{3}}{3} \arctan \left(\frac{(1 + 2x)\sqrt{3}}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+2*x^2)/(-x^3+1), x)

[Out] $-1/2*x^2 - \ln(-1+x) - 1/2*\ln(x^2+x+1) - 1/3*\arctan(1/3*(1+2*x)*3^{(1/2)})*3^{(1/2)}$

Maxima [A] time = 1.41591, size = 50, normalized size = 1.09

$$-\frac{1}{2}x^2 - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{2}\log(x^2+x+1) - \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4+2*x^2)/(-x^3+1),x, algorithm="maxima")`

[Out] $-1/2*x^2 - 1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x + 1)) - 1/2*\log(x^2 + x + 1) - \log(x - 1)$

Fricas [A] time = 1.51551, size = 122, normalized size = 2.65

$$-\frac{1}{2}x^2 - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{2}\log(x^2+x+1) - \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4+2*x^2)/(-x^3+1),x, algorithm="fricas")`

[Out] $-1/2*x^2 - 1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x + 1)) - 1/2*\log(x^2 + x + 1) - \log(x - 1)$

Sympy [A] time = 0.120693, size = 46, normalized size = 1.

$$-\frac{x^2}{2} - \log(x-1) - \frac{\log(x^2+x+1)}{2} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**4+2*x**2)/(-x**3+1),x)`

[Out] $-x**2/2 - \log(x - 1) - \log(x**2 + x + 1)/2 - \sqrt{3}*\operatorname{atan}(2*\sqrt{3}*x/3 + \sqrt{3}/3)/3$

Giac [A] time = 1.0801, size = 51, normalized size = 1.11

$$-\frac{1}{2}x^2 - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{2}\log(x^2+x+1) - \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+2*x^2)/(-x^3+1),x, algorithm="giac")

[Out] -1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1)
- log(abs(x - 1))

$$3.78 \quad \int \frac{1-x+4x^3}{1+x^3} dx$$

Optimal. Leaf size=44

$$\frac{1}{3} \log(x^2 - x + 1) + 4x - \frac{2}{3} \log(x + 1) + \frac{4 \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] 4*x + (4*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3

Rubi [A] time = 0.0426502, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1887, 1860, 31, 634, 618, 204, 628}

$$\frac{1}{3} \log(x^2 - x + 1) + 4x - \frac{2}{3} \log(x + 1) + \frac{4 \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x + 4*x^3)/(1 + x^3), x]

[Out] 4*x + (4*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3

Rule 1887

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Pq/(a + b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]

Rule 1860

Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]

Rule 31

```
Int[((a_) + (b_.)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^( -1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^( -1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x+4x^3}{1+x^3} dx &= \int \left(4 - \frac{3+x}{1+x^3} \right) dx \\
&= 4x - \int \frac{3+x}{1+x^3} dx \\
&= 4x - \frac{1}{3} \int \frac{7-2x}{1-x+x^2} dx - \frac{2}{3} \int \frac{1}{1+x} dx \\
&= 4x - \frac{2}{3} \log(1+x) + \frac{1}{3} \int \frac{-1+2x}{1-x+x^2} dx - 2 \int \frac{1}{1-x+x^2} dx \\
&= 4x - \frac{2}{3} \log(1+x) + \frac{1}{3} \log(1-x+x^2) + 4 \operatorname{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x \right) \\
&= 4x + \frac{4 \tan^{-1} \left(\frac{1-2x}{\sqrt{3}} \right)}{\sqrt{3}} - \frac{2}{3} \log(1+x) + \frac{1}{3} \log(1-x+x^2)
\end{aligned}$$

Mathematica [A] time = 0.0078517, size = 44, normalized size = 1.

$$\frac{1}{3} \log(x^2 - x + 1) + 4x - \frac{2}{3} \log(x + 1) - \frac{4 \tan^{-1} \left(\frac{2x-1}{\sqrt{3}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x + 4*x^3)/(1 + x^3), x]

[Out] 4*x - (4*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3

Maple [A] time = 0.004, size = 38, normalized size = 0.9

$$4x + \frac{\ln(x^2 - x + 1)}{3} - \frac{4\sqrt{3}}{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right) - \frac{2\ln(1+x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^3-x+1)/(x^3+1), x)

[Out] 4*x+1/3*ln(x^2-x+1)-4/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-2/3*ln(1+x)

Maxima [A] time = 1.42637, size = 50, normalized size = 1.14

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+4x+\frac{1}{3}\log(x^2-x+1)-\frac{2}{3}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^3-x+1)/(x^3+1),x, algorithm="maxima")

[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 4*x + 1/3*log(x^2 - x + 1) - 2/3*log(x + 1)

Fricas [A] time = 1.54927, size = 122, normalized size = 2.77

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+4x+\frac{1}{3}\log(x^2-x+1)-\frac{2}{3}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^3-x+1)/(x^3+1),x, algorithm="fricas")

[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 4*x + 1/3*log(x^2 - x + 1) - 2/3*log(x + 1)

Sympy [A] time = 0.119879, size = 48, normalized size = 1.09

$$4x - \frac{2\log(x+1)}{3} + \frac{\log(x^2-x+1)}{3} - \frac{4\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x**3-x+1)/(x**3+1),x)

[Out] 4*x - 2*log(x + 1)/3 + log(x**2 - x + 1)/3 - 4*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3

Giac [A] time = 1.0711, size = 51, normalized size = 1.16

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + 4x + \frac{1}{3}\log(x^2 - x + 1) - \frac{2}{3}\log(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-x+1)/(x^3+1),x, algorithm="giac")
```

```
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 4*x + 1/3*log(x^2 - x + 1) - 2/3*log(abs(x + 1))
```

$$3.79 \quad \int \frac{1+\sqrt{3+x}}{\sqrt{1+x^3}} dx$$

Optimal. Leaf size=230

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right), -7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^3+1}} + \frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\right)}{\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^3+1}}$$

[Out] (2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])

Rubi [A] time = 0.0595062, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1878, 218, 1877}

$$\frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \middle| -7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^3+1}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\right)}{\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

[In] Int[(1 + Sqrt[3] + x)/Sqrt[1 + x^3], x]

[Out] (2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])

Rule 1878

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,


```
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :=> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]]/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :=> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((
1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1 + \sqrt{3} + x}{\sqrt{1 + x^3}} dx = (2\sqrt{3}) \int \frac{1}{\sqrt{1 + x^3}} dx + \int \frac{1 - \sqrt{3} + x}{\sqrt{1 + x^3}} dx$$

$$= \frac{2\sqrt{1 + x^3}}{1 + \sqrt{3} + x} - \frac{\sqrt[4]{3}\sqrt{2 - \sqrt{3}}(1 + x)\sqrt{\frac{1 - x + x^2}{(1 + \sqrt{3} + x)^2}} E\left(\sin^{-1}\left(\frac{1 - \sqrt{3} + x}{1 + \sqrt{3} + x}\right) \mid -7 - 4\sqrt{3}\right)}{\sqrt{\frac{1 + x}{(1 + \sqrt{3} + x)^2}} \sqrt{1 + x^3}} + \frac{4\sqrt[4]{3}\sqrt{2 + \sqrt{3}}(1 + x)}{\dots}$$

Mathematica [C] time = 0.0203391, size = 47, normalized size = 0.2

$$\frac{1}{2}x^2 {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -x^3\right) + (1 + \sqrt{3})x {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] + x)/Sqrt[1 + x^3], x]
```

[Out] $(1 + \sqrt{3})x \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, -x^3\right] + (x^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, -x^3\right])/2$

Maple [B] time = 0.013, size = 407, normalized size = 1.8

$$2 \frac{3/2 - i/2\sqrt{3}}{\sqrt{x^3 + 1}} \sqrt{\frac{1+x}{3/2 - i/2\sqrt{3}}} \sqrt{\frac{x - 1/2 - i/2\sqrt{3}}{-3/2 - i/2\sqrt{3}}} \sqrt{\frac{x - 1/2 + i/2\sqrt{3}}{-3/2 + i/2\sqrt{3}}} \left((-3/2 - i/2\sqrt{3}) \operatorname{EllipticE}\left(\sqrt{\frac{1+x}{3/2 - i/2\sqrt{3}}}, \sqrt{\frac{-3/2 + i/2\sqrt{3}}{-3/2 - i/2\sqrt{3}}}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+x+3^(1/2))/(x^3+1)^(1/2),x)`

[Out] $2*(3/2-1/2*I*3^{(1/2)})*((1+x)/(3/2-1/2*I*3^{(1/2)}))^{(1/2)}*((x-1/2-1/2*I*3^{(1/2)})/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)}*((x-1/2+1/2*I*3^{(1/2)})/(-3/2+1/2*I*3^{(1/2)}))^{(1/2)}/(x^3+1)^{(1/2)}*((-3/2-1/2*I*3^{(1/2)})*EllipticE(((1+x)/(3/2-1/2*I*3^{(1/2)}))^{(1/2)},((-3/2+1/2*I*3^{(1/2)})/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)}))+(1/2+1/2*I*3^{(1/2)})*EllipticF(((1+x)/(3/2-1/2*I*3^{(1/2)}))^{(1/2)},((-3/2+1/2*I*3^{(1/2)})/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)}))+2*(3/2-1/2*I*3^{(1/2)})*((1+x)/(3/2-1/2*I*3^{(1/2)}))^{(1/2)}*((x-1/2-1/2*I*3^{(1/2)})/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)}*((x-1/2+1/2*I*3^{(1/2)})/(-3/2+1/2*I*3^{(1/2)}))^{(1/2)}/(x^3+1)^{(1/2)}*EllipticF(((1+x)/(3/2-1/2*I*3^{(1/2)}))^{(1/2)},((-3/2+1/2*I*3^{(1/2)})/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)}))+2*3^{(1/2)}*(3/2-1/2*I*3^{(1/2)})*((1+x)/(3/2-1/2*I*3^{(1/2)}))^{(1/2)}*((x-1/2-1/2*I*3^{(1/2)})/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)}*((x-1/2+1/2*I*3^{(1/2)})/(-3/2+1/2*I*3^{(1/2)}))^{(1/2)}/(x^3+1)^{(1/2)}*EllipticF(((1+x)/(3/2-1/2*I*3^{(1/2)}))^{(1/2)},((-3/2+1/2*I*3^{(1/2)})/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)}))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x + \sqrt{3} + 1}{\sqrt{x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")`

[Out] `integrate((x + sqrt(3) + 1)/sqrt(x^3 + 1), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x + \sqrt{3} + 1}{\sqrt{x^3 + 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x*3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")

[Out] integral((x + sqrt(3) + 1)/sqrt(x^3 + 1), x)

Sympy [A] time = 1.63872, size = 92, normalized size = 0.4

$$\frac{x^2 \Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{2}{5}, \frac{2}{3} \right) x^3 e^{i\pi}}{3 \Gamma\left(\frac{5}{3}\right)} + \frac{x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{3}{4}, \frac{2}{3} \right) x^3 e^{i\pi}}{3 \Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3} x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{3}{4}, \frac{2}{3} \right) x^3 e^{i\pi}}{3 \Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x*3**(1/2))/(x**3+1)**(1/2),x)

[Out] x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x + \sqrt{3} + 1}{\sqrt{x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x*3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")

[Out] integrate((x + sqrt(3) + 1)/sqrt(x^3 + 1), x)

3.80 $\int \frac{1+\sqrt{3-x}}{\sqrt{1-x^3}} dx$

Optimal. Leaf size=257

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{(-x+\sqrt{3}+1)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right), -7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{(-x+\sqrt{3}+1)^2}}\sqrt{1-x^3}} - \frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{(-x+\sqrt{3}+1)^2}}}{\sqrt{\frac{1-x}{(-x+\sqrt{3}+1)^2}}\sqrt{1-x^3}}$$

[Out] $(-2*\text{Sqrt}[1-x^3])/(1+\text{Sqrt}[3]-x) + (3^{(1/4)}*\text{Sqrt}[2-\text{Sqrt}[3]]*(1-x)*\text{Sqrt}[(1+x+x^2)/(1+\text{Sqrt}[3]-x)^2]*\text{EllipticE}[\text{ArcSin}[(1-\text{Sqrt}[3]-x)/(1+\text{Sqrt}[3]-x)], -7-4*\text{Sqrt}[3]])/(\text{Sqrt}[(1-x)/(1+\text{Sqrt}[3]-x)^2]*\text{Sqrt}[1-x^3]) - (4*3^{(1/4)}*\text{Sqrt}[2+\text{Sqrt}[3]]*(1-x)*\text{Sqrt}[(1+x+x^2)/(1+\text{Sqrt}[3]-x)^2]*\text{EllipticF}[\text{ArcSin}[(1-\text{Sqrt}[3]-x)/(1+\text{Sqrt}[3]-x)], -7-4*\text{Sqrt}[3]])/(\text{Sqrt}[(1-x)/(1+\text{Sqrt}[3]-x)^2]*\text{Sqrt}[1-x^3])$

Rubi [A] time = 0.0669402, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1878, 218, 1877}

$$\frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{(-x+\sqrt{3}+1)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right) \middle| -7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{(-x+\sqrt{3}+1)^2}}\sqrt{1-x^3}} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{(-x+\sqrt{3}+1)^2}}E\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)\right)}{\sqrt{\frac{1-x}{(-x+\sqrt{3}+1)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1+\text{Sqrt}[3]-x)/\text{Sqrt}[1-x^3], x]$

[Out] $(-2*\text{Sqrt}[1-x^3])/(1+\text{Sqrt}[3]-x) + (3^{(1/4)}*\text{Sqrt}[2-\text{Sqrt}[3]]*(1-x)*\text{Sqrt}[(1+x+x^2)/(1+\text{Sqrt}[3]-x)^2]*\text{EllipticE}[\text{ArcSin}[(1-\text{Sqrt}[3]-x)/(1+\text{Sqrt}[3]-x)], -7-4*\text{Sqrt}[3]])/(\text{Sqrt}[(1-x)/(1+\text{Sqrt}[3]-x)^2]*\text{Sqrt}[1-x^3]) - (4*3^{(1/4)}*\text{Sqrt}[2+\text{Sqrt}[3]]*(1-x)*\text{Sqrt}[(1+x+x^2)/(1+\text{Sqrt}[3]-x)^2]*\text{EllipticF}[\text{ArcSin}[(1-\text{Sqrt}[3]-x)/(1+\text{Sqrt}[3]-x)], -7-4*\text{Sqrt}[3]])/(\text{Sqrt}[(1-x)/(1+\text{Sqrt}[3]-x)^2]*\text{Sqrt}[1-x^3])$

Rule 1878

$\text{Int}[(c_ + (d_)*(x_))/\text{Sqrt}[(a_ + (b_)*(x_)^3], x_Symbol] :> \text{With}\{r = \text{Numerator}[\text{Rt}[b/a, 3]], s = \text{Denominator}[\text{Rt}[b/a, 3]]\}, \text{Dist}[(c*r - (1 - \text{Sqrt}[3])*d*s)/r,$

```
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :=> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]]/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :=> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/
((1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1 + \sqrt{3} - x}{\sqrt{1 - x^3}} dx = (2\sqrt{3}) \int \frac{1}{\sqrt{1 - x^3}} dx + \int \frac{1 - \sqrt{3} - x}{\sqrt{1 - x^3}} dx$$

$$= -\frac{2\sqrt{1 - x^3}}{1 + \sqrt{3} - x} + \frac{\sqrt[4]{3}\sqrt{2 - \sqrt{3}}(1 - x)\sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right) \mid -7-4\sqrt{3}\right) - 4\sqrt[4]{3}\sqrt{2 + \sqrt{3}}(1 - x)}{\sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}}\sqrt{1 - x^3}}$$

Mathematica [C] time = 0.0157115, size = 43, normalized size = 0.17

$$(1 + \sqrt{3})x {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; x^3\right) - \frac{1}{2}x^2 {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] - x)/Sqrt[1 - x^3], x]
```

[Out] $(1 + \sqrt{3})x \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, x^3\right] - (x^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, x^3\right])/2$

Maple [A] time = 0.016, size = 368, normalized size = 1.4

$$\frac{2i}{3} \sqrt{3} \sqrt{i \left(x + \frac{1}{2} - \frac{i}{2} \sqrt{3}\right) \sqrt{3}} \sqrt{\frac{-1+x}{-\frac{3}{2} + \frac{i}{2} \sqrt{3}}} \sqrt{-i \left(x + \frac{1}{2} + \frac{i}{2} \sqrt{3}\right) \sqrt{3}} \left(-\frac{3}{2} + \frac{i}{2} \sqrt{3}\right) \operatorname{EllipticE}\left(\frac{\sqrt{3}}{3} \sqrt{i \left(x + \frac{1}{2} - \frac{i}{2} \sqrt{3}\right) \sqrt{3}}, \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1-x+3^(1/2))/(-x^3+1)^(1/2),x)`

[Out] $2/3 I^{3^{1/2}} (I(x+1/2-1/2 I^{3^{1/2}}) 3^{1/2})^{1/2} ((-1+x)/(-3/2+1/2 I^{3^{1/2}})^{1/2})^{1/2} (-I(x+1/2+1/2 I^{3^{1/2}}) 3^{1/2})^{1/2} / (-x^3+1)^{1/2} ((-3/2+1/2 I^{3^{1/2}}) \operatorname{EllipticE}(1/3 3^{1/2} (I(x+1/2-1/2 I^{3^{1/2}}) 3^{1/2})^{1/2}), (I^{3^{1/2}}/(-3/2+1/2 I^{3^{1/2}}))^{1/2}) + \operatorname{EllipticF}(1/3 3^{1/2} (I(x+1/2-1/2 I^{3^{1/2}}) 3^{1/2})^{1/2}), (I^{3^{1/2}}/(-3/2+1/2 I^{3^{1/2}}))^{1/2}) - 2/3 I^{3^{1/2}} (I(x+1/2-1/2 I^{3^{1/2}}) 3^{1/2})^{1/2} ((-1+x)/(-3/2+1/2 I^{3^{1/2}})^{1/2})^{1/2} (-I(x+1/2+1/2 I^{3^{1/2}}) 3^{1/2})^{1/2} / (-x^3+1)^{1/2} \operatorname{EllipticF}(1/3 3^{1/2} (I(x+1/2-1/2 I^{3^{1/2}}) 3^{1/2})^{1/2}), (I^{3^{1/2}}/(-3/2+1/2 I^{3^{1/2}}))^{1/2}) - 2 I (I(x+1/2-1/2 I^{3^{1/2}}) 3^{1/2})^{1/2} ((-1+x)/(-3/2+1/2 I^{3^{1/2}})^{1/2})^{1/2} (-I(x+1/2+1/2 I^{3^{1/2}}) 3^{1/2})^{1/2} / (-x^3+1)^{1/2} \operatorname{EllipticF}(1/3 3^{1/2} (I(x+1/2-1/2 I^{3^{1/2}}) 3^{1/2})^{1/2}), (I^{3^{1/2}}/(-3/2+1/2 I^{3^{1/2}}))^{1/2})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$- \int \frac{x - \sqrt{3} - 1}{\sqrt{-x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")`

[Out] `-integrate((x - sqrt(3) - 1)/sqrt(-x^3 + 1), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^3+1}(x-\sqrt{3}-1)}{x^3-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)/(x^3 - 1), x)

Sympy [A] time = 2.35611, size = 97, normalized size = 0.38

$$\frac{x^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{2}{5}, \frac{2}{3} \right) x^3 e^{2i\pi}}{3\Gamma\left(\frac{5}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{3}{4}, \frac{2}{3} \right) x^3 e^{2i\pi}}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{3}{4}, \frac{2}{3} \right) x^3 e^{2i\pi}}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x+3**(1/2))/(-x**3+1)**(1/2),x)

[Out] -x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamma(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")

[Out] integrate(-(x - sqrt(3) - 1)/sqrt(-x^3 + 1), x)

$$3.81 \quad \int \frac{1+\sqrt{3-x}}{\sqrt{-1+x^3}} dx$$

Optimal. Leaf size=144

$$\frac{2\sqrt{x^3-1}}{-x-\sqrt{3+1}} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{(-x-\sqrt{3+1})^2}}E\left(\sin^{-1}\left(\frac{-x+\sqrt{3+1}}{-x-\sqrt{3+1}}\right)\middle| -7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{(-x-\sqrt{3+1})^2}}\sqrt{x^3-1}}$$

[Out] (2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])

Rubi [A] time = 0.0270309, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$, Rules used = {1879}

$$\frac{2\sqrt{x^3-1}}{-x-\sqrt{3+1}} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{(-x-\sqrt{3+1})^2}}E\left(\sin^{-1}\left(\frac{-x+\sqrt{3+1}}{-x-\sqrt{3+1}}\right)\middle| -7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{(-x-\sqrt{3+1})^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

[In] Int[(1 + Sqrt[3] - x)/Sqrt[-1 + x^3], x]

[Out] (2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])

Rule 1879

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + Simp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(1 - Sqrt[3])*s + r*x]^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[3])*s + r*x)]/(1 - Sqrt[3])], x]


```
rt[3])*s + r*x)], -7 + 4*Sqrt[3]]/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1 + \sqrt{3} - x}{\sqrt{-1 + x^3}} dx = \frac{2\sqrt{-1 + x^3}}{1 - \sqrt{3} - x} - \frac{\sqrt[4]{3}\sqrt{2 + \sqrt{3}}(1 - x)\sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}}E\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right) \mid -7 + 4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}}\sqrt{-1 + x^3}}$$

Mathematica [C] time = 0.023749, size = 63, normalized size = 0.44

$$\frac{x\sqrt{1-x^3}\left(x {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; x^3\right) - 2(1 + \sqrt{3}) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; x^3\right)\right)}{2\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Sqrt[3] - x)/Sqrt[-1 + x^3], x]

[Out] -(x*Sqrt[1 - x^3]*(-2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] + x*Hypergeometric2F1[1/2, 2/3, 5/3, x^3]))/(2*Sqrt[-1 + x^3])

Maple [B] time = 0.013, size = 407, normalized size = 2.8

$$-2 \frac{-3/2 - i/2\sqrt{3}}{\sqrt{x^3-1}} \sqrt{\frac{-1+x}{-3/2 - i/2\sqrt{3}}} \sqrt{\frac{x+1/2 - i/2\sqrt{3}}{3/2 - i/2\sqrt{3}}} \sqrt{\frac{x+1/2 + i/2\sqrt{3}}{3/2 + i/2\sqrt{3}}} \left((3/2 - i/2\sqrt{3}) \text{EllipticE}\left(\sqrt{\frac{-1+x}{-3/2 - i/2\sqrt{3}}}, \sqrt{\frac{3/2 - i/2\sqrt{3}}{3/2 - i/2\sqrt{3}}}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-x+3^(1/2))/(x^3-1)^(1/2), x)

[Out] -2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*((3/2-1/2*I*3^(1/2))*EllipticE(((1-x)/(-3/2-1/2*I*3^(1/2)))^(1/2), ((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+(-1/2+1/2*I*3^(1/2))

$$I\sqrt{3} \operatorname{EllipticF}\left(\frac{-1+x}{-3/2-1/2I\sqrt{3}}\right)^{1/2}, \left(\frac{3/2+1/2I\sqrt{3}}{3/2-1/2I\sqrt{3}}\right)^{1/2} + 2\frac{-3/2-1/2I\sqrt{3}}{-3/2-1/2I\sqrt{3}} \left(\frac{-1+x}{-3/2-1/2I\sqrt{3}}\right)^{1/2} \left(\frac{x+1/2-1/2I\sqrt{3}}{3/2-1/2I\sqrt{3}}\right)^{1/2} \left(\frac{x+1/2+1/2I\sqrt{3}}{3/2+1/2I\sqrt{3}}\right)^{1/2} / (x^3-1)^{1/2} \operatorname{EllipticF}\left(\frac{-1+x}{-3/2-1/2I\sqrt{3}}\right)^{1/2}, \left(\frac{3/2+1/2I\sqrt{3}}{3/2-1/2I\sqrt{3}}\right)^{1/2} + 2\frac{3/2+1/2I\sqrt{3}}{3/2-1/2I\sqrt{3}} \left(\frac{-1+x}{-3/2-1/2I\sqrt{3}}\right)^{1/2} \left(\frac{x+1/2-1/2I\sqrt{3}}{3/2-1/2I\sqrt{3}}\right)^{1/2} \left(\frac{x+1/2+1/2I\sqrt{3}}{3/2+1/2I\sqrt{3}}\right)^{1/2} / (x^3-1)^{1/2} \operatorname{EllipticF}\left(\frac{-1+x}{-3/2-1/2I\sqrt{3}}\right)^{1/2}, \left(\frac{3/2+1/2I\sqrt{3}}{3/2-1/2I\sqrt{3}}\right)^{1/2}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x - \sqrt{3} - 1}{\sqrt{x^3 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x+sqrt(3))/(x^3-1)^(1/2),x, algorithm="maxima")

[Out] -integrate((x - sqrt(3) - 1)/sqrt(x^3 - 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{x - \sqrt{3} - 1}{\sqrt{x^3 - 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x+sqrt(3))/(x^3-1)^(1/2),x, algorithm="fricas")

[Out] integral(-(x - sqrt(3) - 1)/sqrt(x^3 - 1), x)

Sympy [A] time = 2.36226, size = 82, normalized size = 0.57

$$\frac{ix^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{2}{3} \middle| \frac{2}{3}; x^3\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{3} \middle| \frac{4}{3}; x^3\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{4}{3}; x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3)/(3*gamma(5/3)) - sqrt(3)*
I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) - I*x*gamma(1
/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x - \sqrt{3} - 1}{\sqrt{x^3 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x - sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

$$3.82 \quad \int \frac{1+\sqrt{3+x}}{\sqrt{-1-x^3}} dx$$

Optimal. Leaf size=135

$$\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x-\sqrt{3}+1)^2}}E\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)\middle| -7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{(x-\sqrt{3}+1)^2}}\sqrt{-x^3-1}} - \frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}$$

[Out] (-2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])

Rubi [A] time = 0.0327293, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$, Rules used = {1879}

$$\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x-\sqrt{3}+1)^2}}E\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)\middle| -7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{(x-\sqrt{3}+1)^2}}\sqrt{-x^3-1}} - \frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}$$

Antiderivative was successfully verified.

[In] Int[(1 + Sqrt[3] + x)/Sqrt[-1 - x^3], x]

[Out] (-2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])

Rule 1879

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + Simp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(1 - Sqrt[3])*s + r*x]^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[3])*s + r*x)]/(1 - Sqrt[3])], x]

```
rt[3])*s + r*x)], -7 + 4*Sqrt[3]]/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1 + \sqrt{3} + x}{\sqrt{-1 - x^3}} dx = -\frac{2\sqrt{-1 - x^3}}{1 - \sqrt{3} + x} + \frac{\sqrt[4]{3}\sqrt{2 + \sqrt{3}}(1 + x)\sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}}E\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\middle| -7 + 4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}}\sqrt{-1 - x^3}}$$

Mathematica [C] time = 0.0255676, size = 67, normalized size = 0.5

$$\frac{x\sqrt{x^3+1}\left(2(1+\sqrt{3}){}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -x^3\right) + x{}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -x^3\right)\right)}{2\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] + x)/Sqrt[-1 - x^3], x]
```

```
[Out] (x*Sqrt[1 + x^3]*(2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] +
x*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3]))/(2*Sqrt[-1 - x^3])
```

Maple [B] time = 0.011, size = 370, normalized size = 2.7

$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x - \frac{1}{2} - \frac{i}{2}\sqrt{3}\right)}\sqrt{3}\sqrt{\frac{1+x}{\frac{3}{2} + \frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x - \frac{1}{2} + \frac{i}{2}\sqrt{3}\right)}\sqrt{3}\left(\frac{3}{2} + \frac{i}{2}\sqrt{3}\right)\text{EllipticE}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x - \frac{1}{2} - \frac{i}{2}\sqrt{3}\right)}\sqrt{3}, \sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/(-x^3-1)^(1/2), x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*((3/2+1/2*I*3^(1/2))*EllipticE(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2), (I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2), (I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

$$\frac{1}{2}i\sqrt{3}^{(1/2)}\sqrt{3}^{(1/2)} \sqrt{(i\sqrt{3}^{(1/2)}/(3/2+1/2i\sqrt{3}^{(1/2)}))^{(1/2)}} - \frac{2}{3}i\sqrt{3}^{(1/2)} \sqrt{(i(x-1/2-1/2i\sqrt{3}^{(1/2)})\sqrt{3}^{(1/2)})^{(1/2)} \sqrt{((1+x)/(3/2+1/2i\sqrt{3}^{(1/2)}))^{(1/2)}}} \sqrt{(-i(x-1/2+1/2i\sqrt{3}^{(1/2)})\sqrt{3}^{(1/2)})^{(1/2)}/(-x^3-1)^{(1/2)}} \text{EllipticF}(1/3\sqrt{3}^{(1/2)} \sqrt{(i(x-1/2-1/2i\sqrt{3}^{(1/2)})\sqrt{3}^{(1/2)})^{(1/2)}}, (i\sqrt{3}^{(1/2)}/(3/2+1/2i\sqrt{3}^{(1/2)}))^{(1/2)}) - 2i \sqrt{(i(x-1/2-1/2i\sqrt{3}^{(1/2)})\sqrt{3}^{(1/2)})^{(1/2)} \sqrt{((1+x)/(3/2+1/2i\sqrt{3}^{(1/2)}))^{(1/2)}}} \sqrt{(-i(x-1/2+1/2i\sqrt{3}^{(1/2)})\sqrt{3}^{(1/2)})^{(1/2)}/(-x^3-1)^{(1/2)}} \text{EllipticF}(1/3\sqrt{3}^{(1/2)} \sqrt{(i(x-1/2-1/2i\sqrt{3}^{(1/2)})\sqrt{3}^{(1/2)})^{(1/2)}}, (i\sqrt{3}^{(1/2)}/(3/2+1/2i\sqrt{3}^{(1/2)}))^{(1/2)})$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x + \sqrt{3} + 1}{\sqrt{-x^3 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x+sqrt(3))/(-x^3-1)^(1/2),x, algorithm="maxima")

[Out] integrate((x + sqrt(3) + 1)/sqrt(-x^3 - 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^3-1}(x+\sqrt{3}+1)}{x^3+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x+sqrt(3))/(-x^3-1)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-x^3 - 1)*(x + sqrt(3) + 1)/(x^3 + 1), x)

Sympy [A] time = 1.65042, size = 99, normalized size = 0.73

$$\frac{ix^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{2}{3}; x^3 e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{4}{3}; x^3 e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{4}{3}; x^3 e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] -I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x + \sqrt{3} + 1}{\sqrt{-x^3 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

$$3.83 \quad \int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{\sqrt{a+bx^3}} dx$$

Optimal. Leaf size=468

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}}$$

[Out] (2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 0.120721, antiderivative size = 468, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1878, 218, 1877}

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[a + b*x^3], x]

[Out] (2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[Ar


```
cSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]]/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[
3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[
3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[
3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1878

```
Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt{a + bx^3}} dx = (2\sqrt{3}\sqrt[3]{a}) \int \frac{1}{\sqrt{a + bx^3}} dx + \int \frac{(1 - \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt{a + bx^3}} dx$$

$$= \frac{2\sqrt{a + bx^3}}{\sqrt[3]{b}((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})} - \frac{\sqrt[4]{3}\sqrt{2 - \sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} E\left(\sin^{-1}\left(\frac{(1 - \sqrt{3})}{(1 + \sqrt{3})}\right)\right)}{\sqrt[3]{b} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}}$$

Mathematica [C] time = 0.0556628, size = 90, normalized size = 0.19

$$\frac{x\sqrt{\frac{bx^3}{a} + 1} \left(2(1 + \sqrt{3}) \sqrt[3]{a} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a}\right) + \sqrt[3]{bx} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^3}{a}\right) \right)}{2\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[a + b*x^3], x]

[Out] (x*Sqrt[1 + (b*x^3)/a]*(2*(1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[a + b*x^3])

Maple [B] time = 0.068, size = 1003, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3+a)^(1/2), x)

[Out] -2/3*I/b^(2/3)*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2

$$\frac{1}{b^{1/3}(-b^2a)^{1/3} + 1/2 I 3^{1/2} / b^{1/3} (-b^2a)^{1/3}} \Big)^{1/2} + \frac{1}{b^{1/3}(-b^2a)^{1/3}} \text{EllipticF}\left(\frac{1}{3} 3^{1/2} \left(I \left(x + \frac{1}{2} \frac{1}{b^{1/3}(-b^2a)^{1/3}} - \frac{1}{2} I 3^{1/2} / b^{1/3} (-b^2a)^{1/3} \right) \right)^{1/2} \frac{b}{(-b^2a)^{1/3}} \Big)^{1/2}, \frac{I 3^{1/2} / b^{1/3} (-b^2a)^{1/3}}{(-3/2) b^{1/3} (-b^2a)^{1/3} + 1/2 I 3^{1/2} / b^{1/3} (-b^2a)^{1/3}} \Big)^{1/2} \right) - 2 I a^{1/3} / b^{1/3} (-b^2a)^{1/3} \left(I \left(x + \frac{1}{2} \frac{1}{b^{1/3}(-b^2a)^{1/3}} - \frac{1}{2} I 3^{1/2} / b^{1/3} (-b^2a)^{1/3} \right) \right)^{1/2} \frac{b}{(-b^2a)^{1/3}} \Big)^{1/2} \left(\frac{x - 1/b^{1/3}(-b^2a)^{1/3}}{(-3/2) b^{1/3} (-b^2a)^{1/3} + 1/2 I 3^{1/2} / b^{1/3} (-b^2a)^{1/3}} \right)^{1/2} \left(-I \left(x + \frac{1}{2} \frac{1}{b^{1/3}(-b^2a)^{1/3}} + \frac{1}{2} I 3^{1/2} / b^{1/3} (-b^2a)^{1/3} \right) \right)^{1/2} \frac{b}{(-b^2a)^{1/3}} \Big)^{1/2} \frac{1}{(b^3x^3 + a)^{1/2}} \text{EllipticF}\left(\frac{1}{3} 3^{1/2} \left(I \left(x + \frac{1}{2} \frac{1}{b^{1/3}(-b^2a)^{1/3}} - \frac{1}{2} I 3^{1/2} / b^{1/3} (-b^2a)^{1/3} \right) \right)^{1/2} \frac{b}{(-b^2a)^{1/3}} \Big)^{1/2}, \frac{I 3^{1/2} / b^{1/3} (-b^2a)^{1/3}}{(-3/2) b^{1/3} (-b^2a)^{1/3} + 1/2 I 3^{1/2} / b^{1/3} (-b^2a)^{1/3}} \right)^{1/2} \right) - 2/3 I a^{1/3} 3^{1/2} / b^{1/3} (-b^2a)^{1/3} \left(I \left(x + \frac{1}{2} \frac{1}{b^{1/3}(-b^2a)^{1/3}} - \frac{1}{2} I 3^{1/2} / b^{1/3} (-b^2a)^{1/3} \right) \right)^{1/2} \frac{b}{(-b^2a)^{1/3}} \Big)^{1/2} \left(\frac{x - 1/b^{1/3}(-b^2a)^{1/3}}{(-3/2) b^{1/3} (-b^2a)^{1/3} + 1/2 I 3^{1/2} / b^{1/3} (-b^2a)^{1/3}} \right)^{1/2} \left(-I \left(x + \frac{1}{2} \frac{1}{b^{1/3}(-b^2a)^{1/3}} + \frac{1}{2} I 3^{1/2} / b^{1/3} (-b^2a)^{1/3} \right) \right)^{1/2} \frac{b}{(-b^2a)^{1/3}} \Big)^{1/2} \frac{1}{(b^3x^3 + a)^{1/2}} \text{EllipticF}\left(\frac{1}{3} 3^{1/2} \left(I \left(x + \frac{1}{2} \frac{1}{b^{1/3}(-b^2a)^{1/3}} - \frac{1}{2} I 3^{1/2} / b^{1/3} (-b^2a)^{1/3} \right) \right)^{1/2} \frac{b}{(-b^2a)^{1/3}} \Big)^{1/2}, \frac{I 3^{1/2} / b^{1/3} (-b^2a)^{1/3}}{(-3/2) b^{1/3} (-b^2a)^{1/3} + 1/2 I 3^{1/2} / b^{1/3} (-b^2a)^{1/3}} \right)^{1/2} \right)$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(\sqrt{3} + 1)}{\sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(\sqrt{3} + 1)}{\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] integral((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 + a), x)

Sympy [A] time = 3.3471, size = 122, normalized size = 0.26

$$\frac{\sqrt[3]{bx^2}\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b*x**3+a)**(1/2),x)

[Out] b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(\sqrt{3} + 1)}{\sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="giac")

[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 + a), x)

$$3.84 \quad \int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}{\sqrt{a-bx^3}} dx$$

Optimal. Leaf size=481

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{bx})\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)+\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{bx})}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})^2}}\sqrt{a-bx^3}}$$

[Out] $(-2*\text{Sqrt}[a - b*x^3])/(b^{(1/3)}*((1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})) + (3^{(1/4)}*\text{Sqrt}[2 - \text{Sqrt}[3]]*a^{(1/3)}*(a^{(1/3)} - b^{(1/3)*x})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})^2]*\text{EllipticE}[\text{ArcSin}[\frac{(1 - \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x}}{(1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x}}], -7 - 4*\text{Sqrt}[3]])/(b^{(1/3)}*\text{Sqrt}[(a^{(1/3)}*(a^{(1/3)} - b^{(1/3)*x}))/((1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})^2]*\text{Sqrt}[a - b*x^3]) - (4*3^{(1/4)}*\text{Sqrt}[2 + \text{Sqrt}[3]]*a^{(1/3)}*(a^{(1/3)} - b^{(1/3)*x})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})^2]*\text{EllipticF}[\text{ArcSin}[\frac{(1 - \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x}}{(1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x}}], -7 - 4*\text{Sqrt}[3]])/(b^{(1/3)}*\text{Sqrt}[(a^{(1/3)}*(a^{(1/3)} - b^{(1/3)*x}))/((1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})^2]*\text{Sqrt}[a - b*x^3])$

Rubi [A] time = 0.138949, antiderivative size = 481, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {1878, 218, 1877}

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{bx})\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})^2}}F\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}\right)|-7-4\sqrt{3}\right)+\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{bx})}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})^2}}\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\frac{((1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})}{\text{Sqrt}[a - b*x^3]}, x]$

[Out] $(-2*\text{Sqrt}[a - b*x^3])/(b^{(1/3)}*((1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})) + (3^{(1/4)}*\text{Sqrt}[2 - \text{Sqrt}[3]]*a^{(1/3)}*(a^{(1/3)} - b^{(1/3)*x})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})^2]*\text{EllipticE}[\text{ArcSin}[\frac{(1 - \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x}}{(1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x}}], -7 - 4*\text{Sqrt}[3]])/(b^{(1/3)}*\text{Sqrt}[(a^{(1/3)}*(a^{(1/3)} - b^{(1/3)*x}))/((1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})^2]*\text{Sqrt}[a - b*x^3]) - (4*3^{(1/4)}*\text{Sqrt}[2 + \text{Sqrt}[3]]*a^{(1/3)}*(a^{(1/3)} - b^{(1/3)*x})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})^2]*\text{EllipticF}[\text{ArcSin}[\frac{(1 - \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x}}{(1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x}}], -7 - 4*\text{Sqrt}[3]])/(b^{(1/3)}*\text{Sqrt}[(a^{(1/3)}*(a^{(1/3)} - b^{(1/3)*x}))/((1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})^2]*\text{Sqrt}[a - b*x^3])$

```
rcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*
x)], -7 - 4*Sqrt[3]]/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + S
qrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3]) - (4*3^(1/4)*Sqrt[2 + Sqrt
[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)*EllipticF[ArcSin[((1 - Sqrt
[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt
[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
- b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 1878

```
Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2))/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]]/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx}}{\sqrt{a - bx^3}} dx = (2\sqrt{3}\sqrt[3]{a}) \int \frac{1}{\sqrt{a - bx^3}} dx + \int \frac{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx}}{\sqrt{a - bx^3}} dx$$

$$= -\frac{2\sqrt{a - bx^3}}{\sqrt[3]{b}((1 + \sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx})} + \frac{\sqrt[3]{3}\sqrt{2 - \sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a} - \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} + \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{((1 + \sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx})^2}} E\left(\sin^{-1}\left(\frac{1 - \sqrt{3}\sqrt[3]{a} - \sqrt[3]{bx}}{(1 + \sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx}}\right)\right)}{\sqrt[3]{b} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} - \sqrt[3]{bx})}{((1 + \sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx})^2}} \sqrt{a - bx^3}}$$

Mathematica [C] time = 0.0568437, size = 91, normalized size = 0.19

$$\frac{x\sqrt{1 - \frac{bx^3}{a}} \left(2(1 + \sqrt{3})\sqrt[3]{a} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \frac{bx^3}{a}\right) - \sqrt[3]{bx} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \frac{bx^3}{a}\right)\right)}{2\sqrt{a - bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[a - b*x^3], x]

[Out] (x*Sqrt[1 - (b*x^3)/a]*(2*(1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/2, 4/3, (b*x^3)/a] - b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a]))/(2*Sqrt[a - b*x^3])

Maple [B] time = 0.087, size = 949, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3+a)^(1/2), x)

[Out] -2/3*I/b^(2/3)*3^(1/2)*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)*((x-1/b*(b^2*a)^(1/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)*(I*(x+1/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*((-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2), -I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))^(1/2), I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))^(1/2), I*(x+1/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)

$$\begin{aligned} & /3) - 1/2 * I * 3^{(1/2)} / b * (b^2 * a)^{(1/3)})^{(1/2)} + 1/b * (b^2 * a)^{(1/3)} * \text{EllipticF}(1/3 * \\ & 3^{(1/2)} * (-I * (x + 1/2 / b * (b^2 * a)^{(1/3)} + 1/2 * I * 3^{(1/2)} / b * (b^2 * a)^{(1/3)}) * 3^{(1/2)} * b \\ & / (b^2 * a)^{(1/3)})^{(1/2)}, (-I * 3^{(1/2)} / b * (b^2 * a)^{(1/3)} / (-3/2 / b * (b^2 * a)^{(1/3)} - 1/2 \\ & * I * 3^{(1/2)} / b * (b^2 * a)^{(1/3)})^{(1/2)}) + 2 * I * a^{(1/3)} / b * (b^2 * a)^{(1/3)} * (-I * (x + 1/2 \\ & / b * (b^2 * a)^{(1/3)} + 1/2 * I * 3^{(1/2)} / b * (b^2 * a)^{(1/3)}) * 3^{(1/2)} * b / (b^2 * a)^{(1/3)})^{(1 \\ & / 2)} * ((x - 1 / b * (b^2 * a)^{(1/3)}) / (-3/2 / b * (b^2 * a)^{(1/3)} - 1/2 * I * 3^{(1/2)} / b * (b^2 * a)^{(1 \\ & / 3)}))^{(1/2)} * (I * (x + 1/2 / b * (b^2 * a)^{(1/3)} - 1/2 * I * 3^{(1/2)} / b * (b^2 * a)^{(1/3)}) * 3^{(1/2)} \\ &) * b / (b^2 * a)^{(1/3)})^{(1/2)} / (-b * x^3 + a)^{(1/2)} * \text{EllipticF}(1/3 * 3^{(1/2)} * (-I * (x + 1/2 / \\ & b * (b^2 * a)^{(1/3)} + 1/2 * I * 3^{(1/2)} / b * (b^2 * a)^{(1/3)}) * 3^{(1/2)} * b / (b^2 * a)^{(1/3)})^{(1/ \\ & 2)}, (-I * 3^{(1/2)} / b * (b^2 * a)^{(1/3)} / (-3/2 / b * (b^2 * a)^{(1/3)} - 1/2 * I * 3^{(1/2)} / b * (b^2 * a \\ &)^{(1/3)}))^{(1/2)} + 2/3 * I * a^{(1/3)} * 3^{(1/2)} / b * (b^2 * a)^{(1/3)} * (-I * (x + 1/2 / b * (b^2 * a) \\ & ^{(1/3)} + 1/2 * I * 3^{(1/2)} / b * (b^2 * a)^{(1/3)}) * 3^{(1/2)} * b / (b^2 * a)^{(1/3)})^{(1/2)} * ((x - 1 / \\ & b * (b^2 * a)^{(1/3)}) / (-3/2 / b * (b^2 * a)^{(1/3)} - 1/2 * I * 3^{(1/2)} / b * (b^2 * a)^{(1/3)}))^{(1/2)} \\ &) * (I * (x + 1/2 / b * (b^2 * a)^{(1/3)} - 1/2 * I * 3^{(1/2)} / b * (b^2 * a)^{(1/3)}) * 3^{(1/2)} * b / (b^2 * a \\ &)^{(1/3)})^{(1/2)} / (-b * x^3 + a)^{(1/2)} * \text{EllipticF}(1/3 * 3^{(1/2)} * (-I * (x + 1/2 / b * (b^2 * a)^{(1 \\ & / 3)} + 1/2 * I * 3^{(1/2)} / b * (b^2 * a)^{(1/3)}) * 3^{(1/2)} * b / (b^2 * a)^{(1/3)})^{(1/2)}, (-I * 3^{(\\ & 1/2)} / b * (b^2 * a)^{(1/3)} / (-3/2 / b * (b^2 * a)^{(1/3)} - 1/2 * I * 3^{(1/2)} / b * (b^2 * a)^{(1/3)}))^{(\\ & 1/2)} \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$- \int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}(\sqrt{3} + 1)}{\sqrt{-bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] -integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{-bx^3 + ab^{\frac{1}{3}}x} - \sqrt{-bx^3 + aa^{\frac{1}{3}}(\sqrt{3} + 1)}}{bx^3 - a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="f
ricas")

[Out] integral((sqrt(-b*x^3 + a)*b^(1/3)*x - sqrt(-b*x^3 + a)*a^(1/3)*(sqrt(3) +
1))/(b*x^3 - a), x)

Sympy [A] time = 3.50657, size = 128, normalized size = 0.27

$$-\frac{\sqrt[3]{bx^2}\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{bx^3 e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3 e^{2i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3 e^{2i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b*x**3+a)**(1/2),x)

[Out] -b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(2*I*pi
) / a) / (3*sqrt(a)*gamma(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3
*exp_polar(2*I*pi) / a) / (3*a**(1/6)*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper(
(1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi) / a) / (3*a**(1/6)*gamma(4/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}(\sqrt{3} + 1)}{\sqrt{-bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="g
iac")

[Out] integrate(-(b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 + a), x)

$$3.85 \quad \int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}{\sqrt{-a+bx^3}} dx$$

Optimal. Leaf size=271

$$\frac{2\sqrt{bx^3-a}}{\sqrt[3]{b}((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{bx})\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})^2}}E\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}\right)\middle| -7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{bx})}{((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})^2}}\sqrt{bx^3-a}}$$

[Out] (2*Sqrt[-a + b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]]/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2])*Sqrt[-a + b*x^3])

Rubi [A] time = 0.0657175, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {1879}

$$\frac{2\sqrt{bx^3-a}}{\sqrt[3]{b}((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{bx})\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})^2}}E\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}\right)\middle| -7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{bx})}{((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})^2}}\sqrt{bx^3-a}}$$

Antiderivative was successfully verified.

[In] Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[-a + b*x^3], x]

[Out] (2*Sqrt[-a + b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]]/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2])*Sqrt[-a + b*x^3])

Rule 1879

```
Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + Simp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x))/(1 - Sqrt[3])*s + r*x)^2])], x] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx}}{\sqrt{-a + bx^3}} dx = \frac{2\sqrt{-a + bx^3}}{\sqrt[3]{b}((1 - \sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx})} - \frac{\sqrt[4]{3}\sqrt{2 + \sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a} - \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} + \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{((1 - \sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx})^2}} E\left(\sin^{-1}\left(\frac{(1 + \sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx}}{(1 - \sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx}}\right)\right)}{\sqrt[3]{b} \sqrt{-\frac{\sqrt[3]{a}(\sqrt[3]{a} - \sqrt[3]{bx})}{((1 - \sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx})^2}} \sqrt{-a + bx^3}}$$

Mathematica [C] time = 0.0404672, size = 92, normalized size = 0.34

$$\frac{x\sqrt{1 - \frac{bx^3}{a}} \left(2(1 + \sqrt{3}) \sqrt[3]{a} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \frac{bx^3}{a}\right) - \sqrt[3]{bx} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \frac{bx^3}{a}\right) \right)}{2\sqrt{bx^3 - a}}$$

Antiderivative was successfully verified.

[In] Integrate[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[-a + b*x^3], x]

[Out] (x*Sqrt[1 - (b*x^3)/a]*(2*(1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/2, 4/3, (b*x^3)/a] - b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a]))/(2*Sqrt[-a + b*x^3])

Maple [B] time = 0.035, size = 952, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3-a)^(1/2), x)

```
[Out] -2/3*I/b^(2/3)*3^(1/2)*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)*((x-1/b*(b^2*a)^(1/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)*(I*(x+1/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*((-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2), (-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2))+1/b*(b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2), (-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2))+2*I*a^(1/3)/b*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)*((x-1/b*(b^2*a)^(1/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)*(I*(x+1/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2), (-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2))+2/3*I*a^(1/3)*3^(1/2)/b*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)*((x-1/b*(b^2*a)^(1/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)*(I*(x+1/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2), (-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}(\sqrt{3} + 1)}{\sqrt{bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(-\frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}(\sqrt{3} + 1)}{\sqrt{bx^3 - a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="fricas")

[Out] integral(-(b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 - a), x)

Sympy [A] time = 3.54667, size = 112, normalized size = 0.41

$$\frac{i^3 \sqrt{b} x^2 \Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3}{a}\right)}{3^{\frac{6}{3}}\sqrt{a}\Gamma\left(\frac{4}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3}{a}\right)}{3^{\frac{6}{3}}\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b*x**3-a)**(1/2),x)

[Out] I*b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(3*a**(1/6)*gamma(4/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(3*a**(1/6)*gamma(4/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}(\sqrt{3} + 1)}{\sqrt{bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="giac")

```
[Out] integrate(-(b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 - a), x)
```

$$3.86 \quad \int \frac{(1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt{-a-bx^3}} dx$$

Optimal. Leaf size=266

$$\frac{\sqrt[4]{3}\sqrt{2 + \sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{((1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} E\left(\sin^{-1}\left(\frac{\sqrt[3]{bx} + (1+\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx} + (1-\sqrt{3})\sqrt[3]{a}}\right) \mid -7 + 4\sqrt{3}\right)}{\sqrt[3]{b} \sqrt{-\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{-a - bx^3}} - \frac{2\sqrt{-a - bx^3}}{\sqrt[3]{b}((1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})}$$

[Out] $(-2*\text{Sqrt}[-a - b*x^3])/(b^{(1/3)}*((1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})) + (3^{(1/4)}*\text{Sqrt}[2 + \text{Sqrt}[3]]*a^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x})*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)})*b^{(1/3)*x} + b^{(2/3)*x^2}]/((1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2)*\text{EllipticE}[\text{ArcSin}[(1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x}/((1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})], -7 + 4*\text{Sqrt}[3]]/(b^{(1/3)}*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x}))/((1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2)]*\text{Sqrt}[-a - b*x^3])$

Rubi [A] time = 0.0618671, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {1879}

$$\frac{\sqrt[4]{3}\sqrt{2 + \sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{((1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} E\left(\sin^{-1}\left(\frac{\sqrt[3]{bx} + (1+\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx} + (1-\sqrt{3})\sqrt[3]{a}}\right) \mid -7 + 4\sqrt{3}\right)}{\sqrt[3]{b} \sqrt{-\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{-a - bx^3}} - \frac{2\sqrt{-a - bx^3}}{\sqrt[3]{b}((1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x}/\text{Sqrt}[-a - b*x^3], x]$

[Out] $(-2*\text{Sqrt}[-a - b*x^3])/(b^{(1/3)}*((1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})) + (3^{(1/4)}*\text{Sqrt}[2 + \text{Sqrt}[3]]*a^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x})*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)})*b^{(1/3)*x} + b^{(2/3)*x^2}]/((1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2)*\text{EllipticE}[\text{ArcSin}[(1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x}/((1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})], -7 + 4*\text{Sqrt}[3]]/(b^{(1/3)}*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x}))/((1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2)]*\text{Sqrt}[-a - b*x^3])$

Rule 1879

```

Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]

```

Rubi steps

$$\int \frac{(1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt{-a - bx^3}} dx = -\frac{2\sqrt{-a - bx^3}}{\sqrt[3]{b}((1 - \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})} + \frac{\sqrt[4]{3}\sqrt{2 + \sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{\sqrt[3]{b} \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{((1 - \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}}} E\left(\sin^{-1}\left(\frac{(1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{(1 - \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}\right)\right) - \frac{\sqrt[3]{b} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1 - \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}}}{\sqrt{-a - bx^3}}$$

Mathematica [C] time = 0.0434111, size = 93, normalized size = 0.35

$$\frac{x\sqrt{\frac{bx^3}{a} + 1} \left(2(1 + \sqrt{3}) \sqrt[3]{a} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a}\right) + \sqrt[3]{bx} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^3}{a}\right) \right)}{2\sqrt{-a - bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[-a - b*x^3], x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(2*(1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/2,
4/3, -((b*x^3)/a)] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/
a)]))/(2*Sqrt[-a - b*x^3])
```

Maple [B] time = 0.028, size = 1012, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3-a)^(1/2), x)
```



```
[Out] -2/3*I/b^(2/3)*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))-2*I*a^(1/3)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))-2/3*I*a^(1/3)*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(\sqrt{3} + 1)}{\sqrt{-bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{-bx^3 - ab^{\frac{1}{3}}x} + \sqrt{-bx^3 - aa^{\frac{1}{3}}(\sqrt{3} + 1)}}{bx^3 + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="fricas")

[Out] integral(-(sqrt(-b*x^3 - a)*b^(1/3)*x + sqrt(-b*x^3 - a)*a^(1/3)*(sqrt(3) + 1))/(b*x^3 + a), x)

Sympy [A] time = 3.53534, size = 129, normalized size = 0.48

$$\frac{i\sqrt[3]{bx^2}\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b*x**3-a)**(1/2),x)

[Out] -I*b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(\sqrt{3} + 1)}{\sqrt{-bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="giac")

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 - a), x)
```

$$3.87 \quad \int \frac{1 + \sqrt{3} + \sqrt[3]{\frac{b}{a}}x}{\sqrt{a + bx^3}} dx$$

Optimal. Leaf size=520

$$2\sqrt{2 + \sqrt{3}} \left((1 + \sqrt{3}) \sqrt[3]{b} - (1 - \sqrt{3}) \sqrt[3]{a} \sqrt[3]{\frac{b}{a}} \right) (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \text{EllipticF} \left(\sin^{-1} \left(\frac{(1 - \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{(1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}} \right), -7 - 4\sqrt{3} \right)$$

$$\frac{\sqrt[4]{3} b^{2/3} \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}}{\sqrt{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}}$$

[Out] (2*(b/a)^(1/3)*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(b/a)^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*((1 + Sqrt[3])*b^(1/3) - (1 - Sqrt[3])*a^(1/3)*(b/a)^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 0.213352, antiderivative size = 520, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {1878, 218, 1877}

$$2\sqrt{2 + \sqrt{3}} \left((1 + \sqrt{3}) \sqrt[3]{b} - (1 - \sqrt{3}) \sqrt[3]{a} \sqrt[3]{\frac{b}{a}} \right) (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} F \left(\sin^{-1} \left(\frac{\sqrt[3]{bx} + (1 - \sqrt{3}) \sqrt[3]{a}}{\sqrt[3]{bx} + (1 + \sqrt{3}) \sqrt[3]{a}} \right) \middle| -7 - 4\sqrt{3} \right) \sqrt[4]{3} b^{2/3}$$

$$\frac{\sqrt[4]{3} b^{2/3} \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}}{\sqrt{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[a + b*x^3], x]

[Out] (2*(b/a)^(1/3)*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(b/a)^(1/3)*(a^(1/3) + b^(1/3)*x)*S

```

qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*((1 + Sqrt[3])*b^(1/3) - (1 - Sqrt[3])*a^(1/3)*(b/a)^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

```

Rule 1878

```

Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

```

Rule 218

```

Int[1/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3])*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2)], x]] /; FreeQ[{a, b}, x] && PosQ[a]

```

Rule 1877

```

Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

```

Rubi steps

$$\int \frac{1 + \sqrt{3} + \sqrt[3]{\frac{b}{a}}x}{\sqrt{a + bx^3}} dx = \frac{\sqrt[3]{\frac{b}{a}} \int \frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt{a+bx^3}} dx}{\sqrt[3]{b}} + \left(1 + \sqrt{3} - \frac{(1-\sqrt{3})\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}} \right) \int \frac{1}{\sqrt{a + bx^3}} dx$$

$$= \frac{2\sqrt[3]{\frac{b}{a}}\sqrt{a + bx^3}}{b^{2/3}((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})} - \frac{\sqrt[4]{3}\sqrt{2 - \sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}(\sqrt[3]{a} + \sqrt[3]{bx})}{b^{2/3}\sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}}} E\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}}{(1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}\right)\right)$$

$$b^{2/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

Mathematica [C] time = 0.0484349, size = 89, normalized size = 0.17

$$\frac{x\sqrt{\frac{bx^3}{a} + 1} \left(2(1 + \sqrt{3}) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a}\right) + x\sqrt[3]{\frac{b}{a}} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^3}{a}\right) \right)}{2\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[a + b*x^3], x]

[Out] (x*Sqrt[1 + (b*x^3)/a]*(2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, -(b*x^3)/a]) + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -(b*x^3)/a])/(2*Sqrt[a + b*x^3])

Maple [B] time = 0.034, size = 1004, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2), x)

[Out] -2/3*I*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a

$$\begin{aligned} &)^{(1/3)/(-3/2/b*(-b^2*a)^{(1/3)+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}-2/3* \\ &I*(b/a)^{(1/3)*3^{(1/2)}/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))} \\ &)^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))} \\ &)^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)*3^{(1/2)*b/(-b^2*a)^{(1/3))} \\ &)^{(1/2)}/(b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}*EllipticE(1/3*3^{(1/2)*} \\ &I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}, \\ &(I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}+1/b*(-b^2*a)^{(1/3)} \\ &)*EllipticF(1/3*3^{(1/2)*}I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)*3^{(1/2)*b/(-b^2*a)^{(1/3))} \\ &)^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)})) \\ &-2*I/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)*3^{(1/2)*b/(-b^2*a)^{(1/3))} \\ &)^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)} \\ &)^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)*3^{(1/2)*b/(-b^2*a)^{(1/3))} \\ &)^{(1/2)}/(b*x^3+a)^{(1/2)}*EllipticF(1/3*3^{(1/2)*}I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)*3^{(1/2)*b/(-b^2*a)^{(1/3))} \\ &)^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)})) \\ &)^{(1/2)} \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \left(\frac{b}{a} \right)^{\frac{1}{3}} + \sqrt{3} + 1}{\sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(b*x^3 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{x \left(\frac{b}{a} \right)^{\frac{1}{3}} + \sqrt{3} + 1}{\sqrt{bx^3 + a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] integral((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(b*x^3 + a), x)

Sympy [A] time = 2.16748, size = 124, normalized size = 0.24

$$\frac{x^2 \sqrt[3]{\frac{b}{a}} \Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a} \Gamma\left(\frac{5}{3}\right)} + \frac{x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a} \Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3} x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a} \Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(b/a)**(1/3)*x+3**(1/2))/(b*x**3+a)**(1/2),x)

[Out] x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} + 1}{\sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="giac")

[Out] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(b*x^3 + a), x)

$$3.88 \quad \int \frac{1 + \sqrt{3} - \sqrt[3]{\frac{b}{a}}x}{\sqrt{a - bx^3}} dx$$

Optimal. Leaf size=533

$$2\sqrt{2 + \sqrt{3}} \left((1 + \sqrt{3}) \sqrt[3]{b} - (1 - \sqrt{3}) \sqrt[3]{a} \sqrt[3]{\frac{b}{a}} \right) (\sqrt[3]{a} - \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx})^2}} \text{EllipticF} \left(\sin^{-1} \left(\frac{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx}}{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx}} \right), -7 - 4\sqrt{3} \right) \\ \frac{\sqrt[4]{3} b^{2/3} \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} - \sqrt[3]{bx})}{((1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx})^2}} \sqrt{a - bx^3}}$$

[Out] $(-2*(b/a)^{(1/3)}*\text{Sqrt}[a - b*x^3])/(b^{(2/3)}*((1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)}*x)) + (3^{(1/4)}*\text{Sqrt}[2 - \text{Sqrt}[3]]*a^{(1/3)}*(b/a)^{(1/3)}*(a^{(1/3)} - b^{(1/3)}*x)*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2)/((1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)}*x)^2]*\text{EllipticE}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)}*x]/((1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)}*x)], -7 - 4*\text{Sqrt}[3]])/(b^{(2/3)}*\text{Sqrt}[(a^{(1/3)}*(a^{(1/3)} - b^{(1/3)}*x))/((1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)}*x)^2]*\text{Sqrt}[a - b*x^3]) - (2*\text{Sqrt}[2 + \text{Sqrt}[3]]*((1 + \text{Sqrt}[3])*b^{(1/3)} - (1 - \text{Sqrt}[3])*a^{(1/3)}*(b/a)^{(1/3)}*(a^{(1/3)} - b^{(1/3)}*x)*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2)/((1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)}*x)^2]*\text{EllipticF}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)}*x]/((1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)}*x)], -7 - 4*\text{Sqrt}[3]))/(3^{(1/4)}*b^{(2/3)}*\text{Sqrt}[(a^{(1/3)}*(a^{(1/3)} - b^{(1/3)}*x))/((1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)}*x)^2]*\text{Sqrt}[a - b*x^3])$

Rubi [A] time = 0.172301, antiderivative size = 533, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {1878, 218, 1877}

$$2\sqrt{2 + \sqrt{3}} \left((1 + \sqrt{3}) \sqrt[3]{b} - (1 - \sqrt{3}) \sqrt[3]{a} \sqrt[3]{\frac{b}{a}} \right) (\sqrt[3]{a} - \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx})^2}} F \left(\sin^{-1} \left(\frac{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx}}{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx}} \right), -7 - 4\sqrt{3} \right) \\ \frac{\sqrt[4]{3} b^{2/3} \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} - \sqrt[3]{bx})}{((1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx})^2}} \sqrt{a - bx^3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 + \text{Sqrt}[3] - (b/a)^{(1/3)}*x)/\text{Sqrt}[a - b*x^3], x]$

[Out] $(-2*(b/a)^{(1/3)}*\text{Sqrt}[a - b*x^3])/(b^{(2/3)}*((1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)}*x)) + (3^{(1/4)}*\text{Sqrt}[2 - \text{Sqrt}[3]]*a^{(1/3)}*(b/a)^{(1/3)}*(a^{(1/3)} - b^{(1/3)}*x)*$

$$\begin{aligned} & \text{Sqrt}[(a^{2/3} + a^{1/3}b^{1/3}x + b^{2/3}x^2)/((1 + \text{Sqrt}[3])a^{1/3} - b^{1/3}x)^2] * \text{EllipticE}[\text{ArcSin}[\frac{(1 - \text{Sqrt}[3])a^{1/3} - b^{1/3}x}{(1 + \text{Sqrt}[3])a^{1/3} - b^{1/3}x}], -7 - 4\text{Sqrt}[3]] / (b^{2/3} \text{Sqrt}[(a^{1/3}(a^{1/3} - b^{1/3}x))^2] * \text{Sqrt}[a - b^2x^3]) - \\ & (2\text{Sqrt}[2 + \text{Sqrt}[3]] * ((1 + \text{Sqrt}[3])b^{1/3} - (1 - \text{Sqrt}[3])a^{1/3}) * (b/a)^{1/3}) * (a^{1/3} - b^{1/3}x) * \text{Sqrt}[(a^{2/3} + a^{1/3}b^{1/3}x + b^{2/3}x^2) / ((1 + \text{Sqrt}[3])a^{1/3} - b^{1/3}x)^2] * \text{EllipticF}[\text{ArcSin}[\frac{(1 - \text{Sqrt}[3])a^{1/3} - b^{1/3}x}{(1 + \text{Sqrt}[3])a^{1/3} - b^{1/3}x}], -7 - 4\text{Sqrt}[3]] / (3^{1/4} * b^{2/3} * \text{Sqrt}[(a^{1/3}(a^{1/3} - b^{1/3}x)) / ((1 + \text{Sqrt}[3])a^{1/3} - b^{1/3}x)^2] * \text{Sqrt}[a - b^2x^3]) \end{aligned}$$

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2] * EllipticF[ArcSin[(((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x))], -7 - 4*Sqrt[3]]) / (3^(1/4)*r*Sqrt[a + b*x^3] * Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[(((1 - Sqrt[3])*d)/c)], s = Denom[Simplify[(((1 - Sqrt[3])*d)/c)]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3]) / (a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2) / ((1 + Sqrt[3])*s + r*x)^2] * EllipticE[ArcSin[(((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x))], -7 - 4*Sqrt[3]]) / (r^2*Sqrt[a + b*x^3] * Sqrt[(s*(s + r*x)) / ((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1 + \sqrt{3} - \sqrt[3]{\frac{b}{a}}x}{\sqrt{a - bx^3}} dx = \frac{\sqrt[3]{\frac{b}{a}} \int \frac{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx}}{\sqrt{a - bx^3}} dx}{\sqrt[3]{b}} - \left(-1 - \sqrt{3} + \frac{(1 - \sqrt{3}) \sqrt[3]{a} \sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}} \right) \int \frac{1}{\sqrt{a - bx^3}} dx$$

$$= -\frac{2 \sqrt[3]{\frac{b}{a}} \sqrt{a - bx^3}}{b^{2/3} ((1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx})} + \frac{\sqrt[4]{3} \sqrt{2 - \sqrt{3}} \sqrt[3]{a} \sqrt[3]{\frac{b}{a}} (\sqrt[3]{a} - \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx})^2}} E\left(\sin^{-1}\left(\frac{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx}}{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx}}\right)\right)}{b^{2/3} \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} - \sqrt[3]{bx})}{((1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx})^2}} \sqrt{a - bx^3}}$$

Mathematica [C] time = 0.044056, size = 89, normalized size = 0.17

$$\frac{x \sqrt{1 - \frac{bx^3}{a}} \left(x \sqrt[3]{\frac{b}{a}} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \frac{bx^3}{a}\right) - 2(1 + \sqrt{3}) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \frac{bx^3}{a}\right) \right)}{2\sqrt{a - bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[a - b*x^3], x]

[Out] -(x*Sqrt[1 - (b*x^3)/a]*(-2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, (b*x^3)/a] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a]))/(2*Sqrt[a - b*x^3])

Maple [B] time = 0.036, size = 950, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2)), x)

[Out] -2/3*I*(b/a)^(1/3)*3^(1/2)/b*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)*((x-1/b*(b^2*a)^(1/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)*(I*(x+1/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*((-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))

$$3)) * 3^{1/2} * b / (b^2 * a)^{1/3})^{1/2}, (-I * 3^{1/2} / b * (b^2 * a)^{1/3} / (-3/2 / b * (b^2 * a)^{1/3} - 1/2 * I * 3^{1/2} / b * (b^2 * a)^{1/3}))^{1/2} + 1 / b * (b^2 * a)^{1/3} * \text{EllipticF}(1/3 * 3^{1/2} * (-I * (x + 1/2 / b * (b^2 * a)^{1/3} + 1/2 * I * 3^{1/2} / b * (b^2 * a)^{1/3})) * 3^{1/2} * b / (b^2 * a)^{1/3})^{1/2}, (-I * 3^{1/2} / b * (b^2 * a)^{1/3} / (-3/2 / b * (b^2 * a)^{1/3} - 1/2 * I * 3^{1/2} / b * (b^2 * a)^{1/3}))^{1/2})) + 2 * I / b * (b^2 * a)^{1/3} * (-I * (x + 1/2 / b * (b^2 * a)^{1/3} + 1/2 * I * 3^{1/2} / b * (b^2 * a)^{1/3})) * 3^{1/2} * b / (b^2 * a)^{1/3})^{1/2} * ((x - 1 / b * (b^2 * a)^{1/3}) / (-3/2 / b * (b^2 * a)^{1/3} - 1/2 * I * 3^{1/2} / b * (b^2 * a)^{1/3}))^{1/2} * (I * (x + 1/2 / b * (b^2 * a)^{1/3} - 1/2 * I * 3^{1/2} / b * (b^2 * a)^{1/3})) * 3^{1/2} * b / (b^2 * a)^{1/3})^{1/2} / (-b * x^3 + a)^{1/2} * \text{EllipticF}(1/3 * 3^{1/2} * (-I * (x + 1/2 / b * (b^2 * a)^{1/3} + 1/2 * I * 3^{1/2} / b * (b^2 * a)^{1/3})) * 3^{1/2} * b / (b^2 * a)^{1/3})^{1/2}, (-I * 3^{1/2} / b * (b^2 * a)^{1/3} / (-3/2 / b * (b^2 * a)^{1/3} - 1/2 * I * 3^{1/2} / b * (b^2 * a)^{1/3}))^{1/2} + 2/3 * I * 3^{1/2} / b * (b^2 * a)^{1/3} * (-I * (x + 1/2 / b * (b^2 * a)^{1/3} + 1/2 * I * 3^{1/2} / b * (b^2 * a)^{1/3})) * 3^{1/2} * b / (b^2 * a)^{1/3})^{1/2} * ((x - 1 / b * (b^2 * a)^{1/3}) / (-3/2 / b * (b^2 * a)^{1/3} - 1/2 * I * 3^{1/2} / b * (b^2 * a)^{1/3}))^{1/2} * (I * (x + 1/2 / b * (b^2 * a)^{1/3} - 1/2 * I * 3^{1/2} / b * (b^2 * a)^{1/3})) * 3^{1/2} * b / (b^2 * a)^{1/3})^{1/2} / (-b * x^3 + a)^{1/2} * \text{EllipticF}(1/3 * 3^{1/2} * (-I * (x + 1/2 / b * (b^2 * a)^{1/3} + 1/2 * I * 3^{1/2} / b * (b^2 * a)^{1/3})) * 3^{1/2} * b / (b^2 * a)^{1/3})^{1/2}, (-I * 3^{1/2} / b * (b^2 * a)^{1/3} / (-3/2 / b * (b^2 * a)^{1/3} - 1/2 * I * 3^{1/2} / b * (b^2 * a)^{1/3}))^{1/2}))$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$- \int \frac{x \left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} - 1}{\sqrt{-bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] -integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(-b*x^3 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{-bx^3 + ax} \left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{-bx^3 + a}(\sqrt{3} + 1)}{bx^3 - a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] integral((sqrt(-b*x^3 + a)*x*(b/a)^(1/3) - sqrt(-b*x^3 + a)*(sqrt(3) + 1))/
(b*x^3 - a), x)

Sympy [A] time = 2.7697, size = 129, normalized size = 0.24

$$-\frac{x^2 \sqrt[3]{\frac{b}{a}} \Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{bx^3 e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} + \frac{x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3 e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3 e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(b/a)**(1/3)*x+3**(1/2))/(-b*x**3+a)**(1/2),x)

[Out] -x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(4/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x \left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} - 1}{\sqrt{-bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="giac")

[Out] integrate(-(x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(-b*x^3 + a), x)

$$3.89 \quad \int \frac{1 + \sqrt{3} - \sqrt[3]{\frac{b}{a}}x}{\sqrt{-a + bx^3}} dx$$

Optimal. Leaf size=256

$$\frac{2 \left(\frac{b}{a}\right)^{2/3} \sqrt{bx^3 - a}}{b \left(x \left(-\sqrt[3]{\frac{b}{a}}\right) - \sqrt{3} + 1\right)} - \frac{\sqrt[4]{3} \sqrt{2 + \sqrt{3}} \left(1 - x \sqrt[3]{\frac{b}{a}}\right) \sqrt{\frac{x^2 \left(\frac{b}{a}\right)^{2/3} + x \sqrt[3]{\frac{b}{a}} + 1}{\left(x \left(-\sqrt[3]{\frac{b}{a}}\right) - \sqrt{3} + 1\right)^2}} E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{b}{a}}x + \sqrt{3} + 1}{-\sqrt[3]{\frac{b}{a}}x - \sqrt{3} + 1}\right) \mid -7 + 4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}} \sqrt{\frac{1 - x \sqrt[3]{\frac{b}{a}}}{\left(x \left(-\sqrt[3]{\frac{b}{a}}\right) - \sqrt{3} + 1\right)^2}} \sqrt{bx^3 - a}}$$

[Out] (2*(b/a)^(2/3)*Sqrt[-a + b*x^3])/(b*(1 - Sqrt[3] - (b/a)^(1/3)*x)) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (b/a)^(1/3)*x)*Sqrt[(1 + (b/a)^(1/3)*x + (b/a)^(2/3)*x^2)/(1 - Sqrt[3] - (b/a)^(1/3)*x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (b/a)^(1/3)*x)/(1 - Sqrt[3] - (b/a)^(1/3)*x)], -7 + 4*Sqrt[3]])/((b/a)^(1/3)*Sqrt[-((1 - (b/a)^(1/3)*x)/(1 - Sqrt[3] - (b/a)^(1/3)*x)^2])*Sqrt[-a + b*x^3])

Rubi [A] time = 0.0831561, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.03$, Rules used = {1879}

$$\frac{2 \left(\frac{b}{a}\right)^{2/3} \sqrt{bx^3 - a}}{b \left(x \left(-\sqrt[3]{\frac{b}{a}}\right) - \sqrt{3} + 1\right)} - \frac{\sqrt[4]{3} \sqrt{2 + \sqrt{3}} \left(1 - x \sqrt[3]{\frac{b}{a}}\right) \sqrt{\frac{x^2 \left(\frac{b}{a}\right)^{2/3} + x \sqrt[3]{\frac{b}{a}} + 1}{\left(x \left(-\sqrt[3]{\frac{b}{a}}\right) - \sqrt{3} + 1\right)^2}} E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{b}{a}}x + \sqrt{3} + 1}{-\sqrt[3]{\frac{b}{a}}x - \sqrt{3} + 1}\right) \mid -7 + 4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}} \sqrt{\frac{1 - x \sqrt[3]{\frac{b}{a}}}{\left(x \left(-\sqrt[3]{\frac{b}{a}}\right) - \sqrt{3} + 1\right)^2}} \sqrt{bx^3 - a}}$$

Antiderivative was successfully verified.

[In] Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[-a + b*x^3], x]

[Out] (2*(b/a)^(2/3)*Sqrt[-a + b*x^3])/(b*(1 - Sqrt[3] - (b/a)^(1/3)*x)) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (b/a)^(1/3)*x)*Sqrt[(1 + (b/a)^(1/3)*x + (b/a)^(2/3)*x^2)/(1 - Sqrt[3] - (b/a)^(1/3)*x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (b/a)^(1/3)*x)/(1 - Sqrt[3] - (b/a)^(1/3)*x)], -7 + 4*Sqrt[3]])/((b/a)^(1/3)*Sqrt[-((1 - (b/a)^(1/3)*x)/(1 - Sqrt[3] - (b/a)^(1/3)*x)^2])*Sqrt[-a + b*x

^3])

Rule 1879

```
Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + Simp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x))/(1 - Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1 + \sqrt{3} - \sqrt[3]{\frac{b}{a}}x}{\sqrt{-a + bx^3}} dx = \frac{2\left(\frac{b}{a}\right)^{2/3} \sqrt{-a + bx^3}}{b\left(1 - \sqrt{3} - \sqrt[3]{\frac{b}{a}}x\right)} - \frac{\sqrt[4]{3}\sqrt{2 + \sqrt{3}}\left(1 - \sqrt[3]{\frac{b}{a}}x\right) \sqrt{\frac{1 + \sqrt[3]{\frac{b}{a}}x + \left(\frac{b}{a}\right)^{2/3}x^2}{\left(1 - \sqrt{3} - \sqrt[3]{\frac{b}{a}}x\right)^2}} E\left(\sin^{-1}\left(\frac{1 + \sqrt{3} - \sqrt[3]{\frac{b}{a}}x}{1 - \sqrt{3} - \sqrt[3]{\frac{b}{a}}x}\right) \mid -7 + 4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}} \sqrt{\frac{1 - \sqrt[3]{\frac{b}{a}}x}{\left(1 - \sqrt{3} - \sqrt[3]{\frac{b}{a}}x\right)^2}} \sqrt{-a + bx^3}}$$

Mathematica [C] time = 0.032436, size = 90, normalized size = 0.35

$$\frac{x\sqrt{1 - \frac{bx^3}{a}} \left(x\sqrt[3]{\frac{b}{a}} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \frac{bx^3}{a}\right) - 2(1 + \sqrt{3}) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \frac{bx^3}{a}\right) \right)}{2\sqrt{bx^3 - a}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[-a + b*x^3], x]

[Out] -(x*Sqrt[1 - (b*x^3)/a]*(-2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, (b*x^3)/a] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a]))/(2*Sqrt[-a + b*x^3])

Maple [B] time = 0.019, size = 953, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x)`

[Out]
$$\begin{aligned} & -\frac{2}{3}I*(b/a)^{(1/3)}*3^{(1/2)}/b*(b^2*a)^{(1/3)}*(-I*(x+1/2/b*(b^2*a)^{(1/3)}+1/2*I \\ & *3^{(1/2)}/b*(b^2*a)^{(1/3)})*3^{(1/2)}*b/(b^2*a)^{(1/3))^{(1/2)}*((x-1/b*(b^2*a)^{(1/3)}) \\ & /(-3/2/b*(b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(b^2*a)^{(1/3))^{(1/2)}*(I*(x+1/2/ \\ & b*(b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(b^2*a)^{(1/3)})*3^{(1/2)}*b/(b^2*a)^{(1/3))^{(1/2)} \\ & /((b*x^3-a)^{(1/2)}*((-3/2/b*(b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(b^2*a)^{(1/3))*E \\ & llipticE(1/3*3^{(1/2)}*(-I*(x+1/2/b*(b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(b^2*a)^{(1/3) \\ &))*3^{(1/2)}*b/(b^2*a)^{(1/3))^{(1/2)}, (-I*3^{(1/2)}/b*(b^2*a)^{(1/3)}/(-3/2/b*(b^2* \\ & a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(b^2*a)^{(1/3))^{(1/2)}+1/b*(b^2*a)^{(1/3)}*EllipticF \\ & (1/3*3^{(1/2)}*(-I*(x+1/2/b*(b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(b^2*a)^{(1/3)})*3^{(1/2)} \\ & *b/(b^2*a)^{(1/3))^{(1/2)}, (-I*3^{(1/2)}/b*(b^2*a)^{(1/3)}/(-3/2/b*(b^2*a)^{(1/3) \\ &)-1/2*I*3^{(1/2)}/b*(b^2*a)^{(1/3))^{(1/2)}+2*I/b*(b^2*a)^{(1/3)}*(-I*(x+1/2/b* \\ & (b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(b^2*a)^{(1/3)})*3^{(1/2)}*b/(b^2*a)^{(1/3))^{(1/2)} \\ & *((x-1/b*(b^2*a)^{(1/3)}/(-3/2/b*(b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(b^2*a)^{(1/3) \\ &))^{(1/2)}*(I*(x+1/2/b*(b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(b^2*a)^{(1/3)})*3^{(1/2)}*b \\ & /((b^2*a)^{(1/3))^{(1/2)}/(b*x^3-a)^{(1/2)}*EllipticF(1/3*3^{(1/2)}*(-I*(x+1/2/b*(b \\ & ^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(b^2*a)^{(1/3)})*3^{(1/2)}*b/(b^2*a)^{(1/3))^{(1/2)}, (\\ & -I*3^{(1/2)}/b*(b^2*a)^{(1/3)}/(-3/2/b*(b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(b^2*a)^{(1 \\ & /3))^{(1/2)}+2/3*I*3^{(1/2)}/b*(b^2*a)^{(1/3)}*(-I*(x+1/2/b*(b^2*a)^{(1/3)}+1/2*I \\ & *3^{(1/2)}/b*(b^2*a)^{(1/3)})*3^{(1/2)}*b/(b^2*a)^{(1/3))^{(1/2)}*((x-1/b*(b^2*a)^{(1 \\ & /3)))/(-3/2/b*(b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(b^2*a)^{(1/3))^{(1/2)}*(I*(x+1/2/ \\ & b*(b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(b^2*a)^{(1/3)})*3^{(1/2)}*b/(b^2*a)^{(1/3))^{(1/2)} \\ & /((b*x^3-a)^{(1/2)}*EllipticF(1/3*3^{(1/2)}*(-I*(x+1/2/b*(b^2*a)^{(1/3)}+1/2*I*3 \\ & ^{(1/2)}/b*(b^2*a)^{(1/3)})*3^{(1/2)}*b/(b^2*a)^{(1/3))^{(1/2)}, (-I*3^{(1/2)}/b*(b^2*a \\ &)^{(1/3)}/(-3/2/b*(b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(b^2*a)^{(1/3))^{(1/2)} \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x \left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} - 1}{\sqrt{bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="maxima")`

[Out] `-integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(b*x^3 - a), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(-\frac{x \left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} - 1}{\sqrt{bx^3 - a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="fricas")

[Out] integral(-(x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(b*x^3 - a), x)

Sympy [A] time = 2.75303, size = 114, normalized size = 0.45

$$\frac{ix^2 \sqrt[3]{\frac{b}{a}} \Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{bx^3}{a}\right)}{3\sqrt{a} \Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3}{a}\right)}{3\sqrt{a} \Gamma\left(\frac{4}{3}\right)} - \frac{ix \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3}{a}\right)}{3\sqrt{a} \Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(b/a)**(1/3)*x+3**(1/2))/(b*x**3-a)**(1/2),x)

[Out] I*x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(3*sqrt(a)*gamma(4/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(3*sqrt(a)*gamma(4/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x \left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} - 1}{\sqrt{bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(b*x^3 - a), x)
```

$$3.90 \quad \int \frac{1 + \sqrt{3} + \sqrt[3]{\frac{b}{a}}x}{\sqrt{-a - bx^3}} dx$$

Optimal. Leaf size=251

$$\frac{\sqrt[4]{3}\sqrt{2 + \sqrt{3}}\left(x\sqrt[3]{\frac{b}{a}} + 1\right) \sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3} - x\sqrt[3]{\frac{b}{a}} + 1}{\left(x\sqrt[3]{\frac{b}{a}} - \sqrt{3} + 1\right)^2}} E\left(\sin^{-1}\left(\frac{\sqrt[3]{\frac{b}{a}}x + \sqrt{3} + 1}{\sqrt[3]{\frac{b}{a}}x - \sqrt{3} + 1}\right) \middle| -7 + 4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}} \sqrt{\frac{x\sqrt[3]{\frac{b}{a}} + 1}{\left(x\sqrt[3]{\frac{b}{a}} - \sqrt{3} + 1\right)^2}} \sqrt{-a - bx^3}} - \frac{2\left(\frac{b}{a}\right)^{2/3} \sqrt{-a - bx^3}}{b\left(x\sqrt[3]{\frac{b}{a}} - \sqrt{3} + 1\right)}$$

[Out] $(-2*(b/a)^{(2/3)}*\text{Sqrt}[-a - b*x^3])/(b*(1 - \text{Sqrt}[3] + (b/a)^{(1/3)}*x)) + (3^{(1/4)}*\text{Sqrt}[2 + \text{Sqrt}[3]]*(1 + (b/a)^{(1/3)}*x)*\text{Sqrt}[(1 - (b/a)^{(1/3)}*x + (b/a)^{(2/3)}*x^2)/(1 - \text{Sqrt}[3] + (b/a)^{(1/3)}*x)^2]*\text{EllipticE}[\text{ArcSin}[(1 + \text{Sqrt}[3] + (b/a)^{(1/3)}*x)/(1 - \text{Sqrt}[3] + (b/a)^{(1/3)}*x)], -7 + 4*\text{Sqrt}[3]])/((b/a)^{(1/3)}*\text{Sqrt}[-((1 + (b/a)^{(1/3)}*x)/(1 - \text{Sqrt}[3] + (b/a)^{(1/3)}*x)^2])* \text{Sqrt}[-a - b*x^3])$

Rubi [A] time = 0.0644166, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.03$, Rules used = {1879}

$$\frac{\sqrt[4]{3}\sqrt{2 + \sqrt{3}}\left(x\sqrt[3]{\frac{b}{a}} + 1\right) \sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3} - x\sqrt[3]{\frac{b}{a}} + 1}{\left(x\sqrt[3]{\frac{b}{a}} - \sqrt{3} + 1\right)^2}} E\left(\sin^{-1}\left(\frac{\sqrt[3]{\frac{b}{a}}x + \sqrt{3} + 1}{\sqrt[3]{\frac{b}{a}}x - \sqrt{3} + 1}\right) \middle| -7 + 4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}} \sqrt{\frac{x\sqrt[3]{\frac{b}{a}} + 1}{\left(x\sqrt[3]{\frac{b}{a}} - \sqrt{3} + 1\right)^2}} \sqrt{-a - bx^3}} - \frac{2\left(\frac{b}{a}\right)^{2/3} \sqrt{-a - bx^3}}{b\left(x\sqrt[3]{\frac{b}{a}} - \sqrt{3} + 1\right)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 + \text{Sqrt}[3] + (b/a)^{(1/3)}*x)/\text{Sqrt}[-a - b*x^3], x]$

[Out] $(-2*(b/a)^{(2/3)}*\text{Sqrt}[-a - b*x^3])/(b*(1 - \text{Sqrt}[3] + (b/a)^{(1/3)}*x)) + (3^{(1/4)}*\text{Sqrt}[2 + \text{Sqrt}[3]]*(1 + (b/a)^{(1/3)}*x)*\text{Sqrt}[(1 - (b/a)^{(1/3)}*x + (b/a)^{(2/3)}*x^2)/(1 - \text{Sqrt}[3] + (b/a)^{(1/3)}*x)^2]*\text{EllipticE}[\text{ArcSin}[(1 + \text{Sqrt}[3] + (b/a)^{(1/3)}*x)/(1 - \text{Sqrt}[3] + (b/a)^{(1/3)}*x)], -7 + 4*\text{Sqrt}[3]])/((b/a)^{(1/3)}*\text{Sqrt}[-((1 + (b/a)^{(1/3)}*x)/(1 - \text{Sqrt}[3] + (b/a)^{(1/3)}*x)^2])* \text{Sqrt}[-a - b*x^3])$

x^3)

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + Simp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[3])*s + r*x)]], -7 + 4*Sqrt[3])]/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x))/(1 - Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1 + \sqrt{3} + \sqrt[3]{\frac{b}{a}}x}{\sqrt{-a - bx^3}} dx = -\frac{2\left(\frac{b}{a}\right)^{2/3} \sqrt{-a - bx^3}}{b\left(1 - \sqrt{3} + \sqrt[3]{\frac{b}{a}}x\right)} + \frac{\sqrt[4]{3}\sqrt{2 + \sqrt{3}}\left(1 + \sqrt[3]{\frac{b}{a}}x\right) \sqrt{\frac{1 - \sqrt[3]{\frac{b}{a}}x + \left(\frac{b}{a}\right)^{2/3}x^2}{\left(1 - \sqrt{3} + \sqrt[3]{\frac{b}{a}}x\right)^2}} E\left(\sin^{-1}\left(\frac{1 + \sqrt{3} + \sqrt[3]{\frac{b}{a}}x}{1 - \sqrt{3} + \sqrt[3]{\frac{b}{a}}x}\right)\right) - 7 + 4\sqrt{3}}{\sqrt[3]{\frac{b}{a}} \sqrt{\frac{1 + \sqrt[3]{\frac{b}{a}}x}{\left(1 - \sqrt{3} + \sqrt[3]{\frac{b}{a}}x\right)^2}} \sqrt{-a - bx^3}}$$

Mathematica [C] time = 0.0338203, size = 92, normalized size = 0.37

$$\frac{x\sqrt{\frac{bx^3}{a} + 1} \left(2(1 + \sqrt{3}) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a}\right) + x\sqrt[3]{\frac{b}{a}} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^3}{a}\right)\right)}{2\sqrt{-a - bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[-a - b*x^3], x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, -(b*x^3)/a]) + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -(b*x^3)/a])/(2*Sqrt[-a - b*x^3])
```

Maple [B] time = 0.012, size = 1013, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2), x)`

[Out]
$$\begin{aligned} & -2/3*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b* \\ & (-b^2*a)^{(1/3)})*3^{(1/2)*b}/(-b^2*a)^{(1/3))^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3 \\ & /2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)))^{(1/2)}*(-I*(x+1/2/b*(-b \\ & ^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b}/(-b^2*a)^{(1/3))^{(1/2)} \\ & /(-b*x^3-a)^{(1/2)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b}/(-b^2*a)^{(1/3))^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)))^{(1/2))}-2/3 \\ & *I*(b/a)^{(1/3)}*3^{(1/2)}/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b}/(-b^2*a)^{(1/3))^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)))^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b}/(-b^2*a)^{(1/3))^{(1/2)}/(-b*x^3-a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*EllipticE(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b}/(-b^2*a)^{(1/3))^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)))^{(1/2))}+1/b*(-b^2*a)^{(1/3)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b}/(-b^2*a)^{(1/3))^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)))^{(1/2)))-2*I/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b}/(-b^2*a)^{(1/3))^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)))^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b}/(-b^2*a)^{(1/3))^{(1/2)}/(-b*x^3-a)^{(1/2)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b}/(-b^2*a)^{(1/3))^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)))^{(1/2))} \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} + 1}{\sqrt{-bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2), x, algorithm="maxima")`

[Out] `integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(-b*x^3 - a), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{-bx^3 - ax} \left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{-bx^3 - a}(\sqrt{3} + 1)}{bx^3 + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2), x, algorithm="fricas")

[Out] integral(-(sqrt(-b*x^3 - a)*x*(b/a)^(1/3) + sqrt(-b*x^3 - a)*(sqrt(3) + 1)) / (b*x^3 + a), x)

Sympy [A] time = 2.27895, size = 131, normalized size = 0.52

$$\frac{ix^2 \sqrt[3]{\frac{b}{a}} \Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a} \Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3} ix \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a} \Gamma\left(\frac{4}{3}\right)} - \frac{ix \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a} \Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(b/a)**(1/3)*x+3**(1/2))/(-b*x**3-a)**(1/2), x)

[Out] -I*x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} + 1}{\sqrt{-bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(-b*x^3 - a), x)
```

3.91 $\int \frac{1-\sqrt{3+x}}{\sqrt{1+x^3}} dx$

Optimal. Leaf size=127

$$\frac{2\sqrt{x^3+1}}{x+\sqrt{3+1}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle| -7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^3+1}}$$

[Out] (2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])

Rubi [A] time = 0.0190498, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$, Rules used = {1877}

$$\frac{2\sqrt{x^3+1}}{x+\sqrt{3+1}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle| -7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

[In] Int[(1 - Sqrt[3] + x)/Sqrt[1 + x^3], x]

[Out] (2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])

Rule 1877

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(1 + Sqrt[3])*s + r*x]^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])


```
rt[3])*s + r*x)], -7 - 4*Sqrt[3]]/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1 - \sqrt{3} + x}{\sqrt{1 + x^3}} dx = \frac{2\sqrt{1 + x^3}}{1 + \sqrt{3} + x} - \frac{\sqrt[4]{3}\sqrt{2 - \sqrt{3}}(1 + x)\sqrt{\frac{1 - x + x^2}{(1 + \sqrt{3} + x)^2}} E\left(\sin^{-1}\left(\frac{1 - \sqrt{3} + x}{1 + \sqrt{3} + x}\right) \mid -7 - 4\sqrt{3}\right)}{\sqrt{\frac{1 + x}{(1 + \sqrt{3} + x)^2}} \sqrt{1 + x^3}}$$

Mathematica [C] time = 0.0206797, size = 49, normalized size = 0.39

$$\frac{1}{2}x^2 {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -x^3\right) + (1 - \sqrt{3})x {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] + x)/Sqrt[1 + x^3], x]
```

```
[Out] (1 - Sqrt[3])*x*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] + (x^2*Hypergeometri
c2F1[1/2, 2/3, 5/3, -x^3])/2
```

Maple [B] time = 0.013, size = 407, normalized size = 3.2

$$2 \frac{3/2 - i/2\sqrt{3}}{\sqrt{x^3 + 1}} \sqrt{\frac{1 + x}{3/2 - i/2\sqrt{3}}} \sqrt{\frac{x - 1/2 - i/2\sqrt{3}}{-3/2 - i/2\sqrt{3}}} \sqrt{\frac{x - 1/2 + i/2\sqrt{3}}{-3/2 + i/2\sqrt{3}}} \left((-3/2 - i/2\sqrt{3}) \text{EllipticE}\left(\sqrt{\frac{1 + x}{3/2 - i/2\sqrt{3}}}, \sqrt{\frac{-3/2 + i/2\sqrt{3}}{-3/2 - i/2\sqrt{3}}}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/(x^3+1)^(1/2), x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))
)^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))*EllipticE(((1+x)/(3/2-1/2*I*3^(
1/2)))^(1/2), ((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*
3^(1/2))*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2), ((-3/2+1/2*I*3^(1/2))/
```

$$\begin{aligned} & (-3/2-1/2*I*3^{(1/2)})^{(1/2)}+2*(3/2-1/2*I*3^{(1/2)})*((1+x)/(3/2-1/2*I*3^{(1/2)}))^{(1/2)}*((x-1/2-1/2*I*3^{(1/2)})/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)}*((x-1/2+1/2*I*3^{(1/2)})/(-3/2+1/2*I*3^{(1/2)}))^{(1/2)}/(x^3+1)^{(1/2)}*EllipticF(((1+x)/(3/2-1/2*I*3^{(1/2)}))^{(1/2)},((-3/2+1/2*I*3^{(1/2)})/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)})-2*3^{(1/2)}*(3/2-1/2*I*3^{(1/2)})*((1+x)/(3/2-1/2*I*3^{(1/2)}))^{(1/2)}*((x-1/2-1/2*I*3^{(1/2)})/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)}*((x-1/2+1/2*I*3^{(1/2)})/(-3/2+1/2*I*3^{(1/2)}))^{(1/2)}/(x^3+1)^{(1/2)}*EllipticF(((1+x)/(3/2-1/2*I*3^{(1/2)}))^{(1/2)},((-3/2+1/2*I*3^{(1/2)})/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)}) \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")

[Out] integrate((x - sqrt(3) + 1)/sqrt(x^3 + 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x - \sqrt{3} + 1}{\sqrt{x^3 + 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")

[Out] integral((x - sqrt(3) + 1)/sqrt(x^3 + 1), x)

Sympy [A] time = 1.59724, size = 92, normalized size = 0.72

$$\frac{x^2 \Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{1}{3}\right) x^3 e^{i\pi}}{3 \Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3} x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{2}{3}\right) x^3 e^{i\pi}}{3 \Gamma\left(\frac{4}{3}\right)} + \frac{x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{2}{3}\right) x^3 e^{i\pi}}{3 \Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

$$3.92 \quad \int \frac{1-\sqrt{3-x}}{\sqrt{1-x^3}} dx$$

Optimal. Leaf size=142

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{(-x+\sqrt{3}+1)^2}}E\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)\middle| -7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{(-x+\sqrt{3}+1)^2}}\sqrt{1-x^3}} - \frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1}$$

[Out] $(-2*\text{Sqrt}[1 - x^3])/(1 + \text{Sqrt}[3] - x) + (3^{(1/4)}*\text{Sqrt}[2 - \text{Sqrt}[3]]*(1 - x)*\text{Sqrt}[(1 + x + x^2)/(1 + \text{Sqrt}[3] - x)^2]*\text{EllipticE}[\text{ArcSin}[(1 - \text{Sqrt}[3] - x)/(1 + \text{Sqrt}[3] - x)], -7 - 4*\text{Sqrt}[3]])/(\text{Sqrt}[(1 - x)/(1 + \text{Sqrt}[3] - x)^2]*\text{Sqrt}[1 - x^3])$

Rubi [A] time = 0.0252626, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1877}

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{(-x+\sqrt{3}+1)^2}}E\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)\middle| -7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{(-x+\sqrt{3}+1)^2}}\sqrt{1-x^3}} - \frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 - \text{Sqrt}[3] - x)/\text{Sqrt}[1 - x^3], x]$

[Out] $(-2*\text{Sqrt}[1 - x^3])/(1 + \text{Sqrt}[3] - x) + (3^{(1/4)}*\text{Sqrt}[2 - \text{Sqrt}[3]]*(1 - x)*\text{Sqrt}[(1 + x + x^2)/(1 + \text{Sqrt}[3] - x)^2]*\text{EllipticE}[\text{ArcSin}[(1 - \text{Sqrt}[3] - x)/(1 + \text{Sqrt}[3] - x)], -7 - 4*\text{Sqrt}[3]])/(\text{Sqrt}[(1 - x)/(1 + \text{Sqrt}[3] - x)^2]*\text{Sqrt}[1 - x^3])$

Rule 1877

$\text{Int}[(c + (d \cdot x))/\text{Sqrt}[a + (b \cdot x)^3], x_Symbol] \rightarrow \text{With}[\{r = \text{Numerator}[\text{Simplify}[(1 - \text{Sqrt}[3]) \cdot d]/c], s = \text{Denominator}[\text{Simplify}[(1 - \text{Sqrt}[3]) \cdot d]/c]\}, \text{Simp}[(2 \cdot d \cdot s^3 \cdot \text{Sqrt}[a + b \cdot x^3])/(a \cdot r^2 \cdot ((1 + \text{Sqrt}[3]) \cdot s + r \cdot x)), x] - \text{Simp}[(3^{(1/4)} \cdot \text{Sqrt}[2 - \text{Sqrt}[3]] \cdot d \cdot s \cdot (s + r \cdot x) \cdot \text{Sqrt}[(s^2 - r \cdot s \cdot x + r^2 \cdot x^2)/(1 + \text{Sqrt}[3]) \cdot s + r \cdot x]^2] \cdot \text{EllipticE}[\text{ArcSin}[(1 - \text{Sqrt}[3]) \cdot s + r \cdot x]/((1 + \text{Sqrt}[3]) \cdot s + r \cdot x)])]$

```
rt[3])*s + r*x)], -7 - 4*Sqrt[3]]/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1 - \sqrt{3} - x}{\sqrt{1 - x^3}} dx = -\frac{2\sqrt{1 - x^3}}{1 + \sqrt{3} - x} + \frac{\sqrt[4]{3}\sqrt{2 - \sqrt{3}}(1 - x)\sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)\middle| -7 - 4\sqrt{3}\right)}{\sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}}\sqrt{1-x^3}}$$

Mathematica [C] time = 0.0162588, size = 45, normalized size = 0.32

$$(1 - \sqrt{3})x {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; x^3\right) - \frac{1}{2}x^2 {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] - x)/Sqrt[1 - x^3], x]
```

```
[Out] (1 - Sqrt[3])*x*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] - (x^2*Hypergeometric
2F1[1/2, 2/3, 5/3, x^3])/2
```

Maple [B] time = 0.011, size = 368, normalized size = 2.6

$$\frac{2i}{3}\sqrt{3}\sqrt{i\left(x + \frac{1}{2} - \frac{i}{2}\sqrt{3}\right)}\sqrt{3}\sqrt{\frac{-1+x}{-\frac{3}{2} + \frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x + \frac{1}{2} + \frac{i}{2}\sqrt{3}\right)}\sqrt{3}\left(-\frac{3}{2} + \frac{i}{2}\sqrt{3}\right)\text{EllipticE}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x + \frac{1}{2} - \frac{i}{2}\sqrt{3}\right)}\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x-3^(1/2))/(-x^3+1)^(1/2), x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*((-3
/2+1/2*I*3^(1/2))*EllipticE(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(
1/2), (I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+EllipticF(1/3*3^(1/2)*(I*(x+1/
2-1/2*I*3^(1/2))*3^(1/2))^(1/2), (I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))-2/
```

$$3 \cdot I \cdot 3^{1/2} \cdot (I \cdot (x + 1/2 - 1/2 \cdot I \cdot 3^{1/2}) \cdot 3^{1/2})^{1/2} \cdot ((-1+x)/(-3/2+1/2 \cdot I \cdot 3^{1/2}))^{1/2} \cdot (-I \cdot (x + 1/2 + 1/2 \cdot I \cdot 3^{1/2}) \cdot 3^{1/2})^{1/2} / (-x^3+1)^{1/2} \cdot \text{EllipticF}(1/3 \cdot 3^{1/2} \cdot (I \cdot (x + 1/2 - 1/2 \cdot I \cdot 3^{1/2}) \cdot 3^{1/2})^{1/2}, (I \cdot 3^{1/2}/(-3/2+1/2 \cdot I \cdot 3^{1/2}))^{1/2}) + 2 \cdot I \cdot (I \cdot (x + 1/2 - 1/2 \cdot I \cdot 3^{1/2}) \cdot 3^{1/2})^{1/2} \cdot ((-1+x)/(-3/2+1/2 \cdot I \cdot 3^{1/2}))^{1/2} \cdot (-I \cdot (x + 1/2 + 1/2 \cdot I \cdot 3^{1/2}) \cdot 3^{1/2})^{1/2} / (-x^3+1)^{1/2} \cdot \text{EllipticF}(1/3 \cdot 3^{1/2} \cdot (I \cdot (x + 1/2 - 1/2 \cdot I \cdot 3^{1/2}) \cdot 3^{1/2})^{1/2}, (I \cdot 3^{1/2}/(-3/2+1/2 \cdot I \cdot 3^{1/2}))^{1/2})$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$- \int \frac{x + \sqrt{3} - 1}{\sqrt{-x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")

[Out] -integrate((x + sqrt(3) - 1)/sqrt(-x^3 + 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^3 + 1}(x + \sqrt{3} - 1)}{x^3 - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(-x^3 + 1)*(x + sqrt(3) - 1)/(x^3 - 1), x)

Sympy [A] time = 2.41928, size = 97, normalized size = 0.68

$$\frac{x^2 \Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{2}{5}, \frac{2}{3} \right) x^3 e^{2i\pi}}{3 \Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3} x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{3}{4}, \frac{2}{3} \right) x^3 e^{2i\pi}}{3 \Gamma\left(\frac{4}{3}\right)} + \frac{x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{3}{4}, \frac{2}{3} \right) x^3 e^{2i\pi}}{3 \Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] -x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamma(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x + \sqrt{3} - 1}{\sqrt{-x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x + sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

3.93 $\int \frac{1-\sqrt{3-x}}{\sqrt{-1+x^3}} dx$

Optimal. Leaf size=264

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{(-x-\sqrt{3}+1)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right), 4\sqrt{3}-7\right)}{\sqrt{-\frac{1-x}{(-x-\sqrt{3}+1)^2}}\sqrt{x^3-1}} + \frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{(-x-\sqrt{3}+1)^2}}}{\sqrt{-\frac{1-x}{(-x-\sqrt{3}+1)^2}}}$$

[Out] (2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])

Rubi [A] time = 0.0540904, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1880, 219, 1879}

$$\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1} + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{(-x-\sqrt{3}+1)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right) \mid -7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{(-x-\sqrt{3}+1)^2}}\sqrt{x^3-1}} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{(-x-\sqrt{3}+1)^2}}E\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)\right)}{\sqrt{-\frac{1-x}{(-x-\sqrt{3}+1)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

[In] Int[(1 - Sqrt[3] - x)/Sqrt[-1 + x^3], x]

[Out] (2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])

Rule 1880

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numerator[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,


```
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :=> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]]]/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :=> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]]]/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1 - \sqrt{3} - x}{\sqrt{-1 + x^3}} dx = -\left((2\sqrt{3}) \int \frac{1}{\sqrt{-1 + x^3}} dx \right) + \int \frac{1 + \sqrt{3} - x}{\sqrt{-1 + x^3}} dx$$

$$= \frac{2\sqrt{-1 + x^3}}{1 - \sqrt{3} - x} - \frac{4\sqrt{3}\sqrt{2 + \sqrt{3}}(1 - x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} E\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right) \mid -7 + 4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1 + x^3}} + \frac{4\sqrt{3}\sqrt{2 - \sqrt{3}}(1 - x)}{\sqrt{-1 + x^3}}$$

Mathematica [C] time = 0.0261258, size = 63, normalized size = 0.24

$$\frac{x\sqrt{1-x^3} \left(2(\sqrt{3}-1) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; x^3\right) + x {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; x^3\right) \right)}{2\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - Sqrt[3] - x)/Sqrt[-1 + x^3],x]

[Out] $-(x\sqrt{1-x^3}*(2*(-1+\sqrt{3})*\text{Hypergeometric2F1}[1/3, 1/2, 4/3, x^3] + x*\text{Hypergeometric2F1}[1/2, 2/3, 5/3, x^3]))/(2*\sqrt{-1+x^3})$

Maple [A] time = 0.008, size = 407, normalized size = 1.5

$$-2 \frac{-3/2 - i/2\sqrt{3}}{\sqrt{x^3 - 1}} \sqrt{\frac{-1 + x}{-3/2 - i/2\sqrt{3}}} \sqrt{\frac{x + 1/2 - i/2\sqrt{3}}{3/2 - i/2\sqrt{3}}} \sqrt{\frac{x + 1/2 + i/2\sqrt{3}}{3/2 + i/2\sqrt{3}}} \left((3/2 - i/2\sqrt{3}) \text{EllipticE} \left(\sqrt{\frac{-1 + x}{-3/2 - i/2\sqrt{3}}}, \sqrt{\frac{3/2 - i/2\sqrt{3}}{3/2 + i/2\sqrt{3}}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-x-3^(1/2))/(x^3-1)^(1/2),x)

[Out] $-2*(-3/2-1/2*I*3^{(1/2)})*((-1+x)/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)}*((x+1/2-1/2*I*3^{(1/2)})/(3/2-1/2*I*3^{(1/2)}))^{(1/2)}*((x+1/2+1/2*I*3^{(1/2)})/(3/2+1/2*I*3^{(1/2)}))^{(1/2)}/(x^3-1)^{(1/2)}*((3/2-1/2*I*3^{(1/2)})*\text{EllipticE}(((1+x)/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)},((3/2+1/2*I*3^{(1/2)})/(3/2-1/2*I*3^{(1/2)}))^{(1/2)}))+(-1/2+1/2*I*3^{(1/2)})*\text{EllipticF}(((1+x)/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)},((3/2+1/2*I*3^{(1/2)})/(3/2-1/2*I*3^{(1/2)}))^{(1/2)}))+2*(-3/2-1/2*I*3^{(1/2)})*((-1+x)/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)}*((x+1/2-1/2*I*3^{(1/2)})/(3/2-1/2*I*3^{(1/2)}))^{(1/2)}*((x+1/2+1/2*I*3^{(1/2)})/(3/2+1/2*I*3^{(1/2)}))^{(1/2)}/(x^3-1)^{(1/2)}*\text{EllipticF}(((1+x)/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)},((3/2+1/2*I*3^{(1/2)})/(3/2-1/2*I*3^{(1/2)}))^{(1/2)})-2*3^{(1/2)}*(-3/2-1/2*I*3^{(1/2)})*((-1+x)/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)}*((x+1/2-1/2*I*3^{(1/2)})/(3/2-1/2*I*3^{(1/2)}))^{(1/2)}*((x+1/2+1/2*I*3^{(1/2)})/(3/2+1/2*I*3^{(1/2)}))^{(1/2)}/(x^3-1)^{(1/2)}*\text{EllipticF}(((1+x)/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)},((3/2+1/2*I*3^{(1/2)})/(3/2-1/2*I*3^{(1/2)}))^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x + \sqrt{3} - 1}{\sqrt{x^3 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")

[Out] -integrate((x + sqrt(3) - 1)/sqrt(x^3 - 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x + \sqrt{3} - 1}{\sqrt{x^3 - 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")

[Out] integral(-(x + sqrt(3) - 1)/sqrt(x^3 - 1), x)

Sympy [A] time = 2.43886, size = 82, normalized size = 0.31

$$\frac{ix^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{2}{5}, \frac{2}{3} \right) x^3}{3\Gamma\left(\frac{5}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{3}{4}, \frac{2}{3} \right) x^3}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{3}{4}, \frac{2}{3} \right) x^3}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x-3**(1/2))/(x**3-1)**(1/2),x)

[Out] I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3)/(3*gamma(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x + \sqrt{3} - 1}{\sqrt{x^3 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")

[Out] integrate(-(x + sqrt(3) - 1)/sqrt(x^3 - 1), x)

3.94 $\int \frac{1-\sqrt{3+x}}{\sqrt{-1-x^3}} dx$

Optimal. Leaf size=247

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x-\sqrt{3}+1)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right), 4\sqrt{3}-7\right)}{\sqrt{-\frac{x+1}{(x-\sqrt{3}+1)^2}}\sqrt{-x^3-1}} - \frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x-\sqrt{3}+1)^2}}E}{\sqrt{-\frac{x+1}{(x-\sqrt{3}+1)^2}}\sqrt{-x^3-1}}$$

[Out] $(-2*\text{Sqrt}[-1-x^3])/(1-\text{Sqrt}[3]+x) + (3^{(1/4)}*\text{Sqrt}[2+\text{Sqrt}[3]]*(1+x)*\text{Sqrt}[(1-x+x^2)/(1-\text{Sqrt}[3]+x)^2]*\text{EllipticE}[\text{ArcSin}[(1+\text{Sqrt}[3]+x)/(1-\text{Sqrt}[3]+x)], -7+4*\text{Sqrt}[3]])/(\text{Sqrt}[-((1+x)/(1-\text{Sqrt}[3]+x)^2)]*\text{Sqrt}[-1-x^3]) - (4*3^{(1/4)}*\text{Sqrt}[2-\text{Sqrt}[3]]*(1+x)*\text{Sqrt}[(1-x+x^2)/(1-\text{Sqrt}[3]+x)^2]*\text{EllipticF}[\text{ArcSin}[(1+\text{Sqrt}[3]+x)/(1-\text{Sqrt}[3]+x)], -7+4*\text{Sqrt}[3]])/(\text{Sqrt}[-((1+x)/(1-\text{Sqrt}[3]+x)^2)]*\text{Sqrt}[-1-x^3])$

Rubi [A] time = 0.0543306, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1880, 219, 1879}

$$\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1} - \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x-\sqrt{3}+1)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right) \middle| -7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{(x-\sqrt{3}+1)^2}}\sqrt{-x^3-1}} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x-\sqrt{3}+1)^2}}E\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)\right)}{\sqrt{-\frac{x+1}{(x-\sqrt{3}+1)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1-\text{Sqrt}[3]+x)/\text{Sqrt}[-1-x^3], x]$

[Out] $(-2*\text{Sqrt}[-1-x^3])/(1-\text{Sqrt}[3]+x) + (3^{(1/4)}*\text{Sqrt}[2+\text{Sqrt}[3]]*(1+x)*\text{Sqrt}[(1-x+x^2)/(1-\text{Sqrt}[3]+x)^2]*\text{EllipticE}[\text{ArcSin}[(1+\text{Sqrt}[3]+x)/(1-\text{Sqrt}[3]+x)], -7+4*\text{Sqrt}[3]])/(\text{Sqrt}[-((1+x)/(1-\text{Sqrt}[3]+x)^2)]*\text{Sqrt}[-1-x^3]) - (4*3^{(1/4)}*\text{Sqrt}[2-\text{Sqrt}[3]]*(1+x)*\text{Sqrt}[(1-x+x^2)/(1-\text{Sqrt}[3]+x)^2]*\text{EllipticF}[\text{ArcSin}[(1+\text{Sqrt}[3]+x)/(1-\text{Sqrt}[3]+x)], -7+4*\text{Sqrt}[3]])/(\text{Sqrt}[-((1+x)/(1-\text{Sqrt}[3]+x)^2)]*\text{Sqrt}[-1-x^3])$

Rule 1880

$\text{Int}[(c_ + (d_)*(x_))/\text{Sqrt}[(a_ + (b_)*(x_)^3], x_Symbol] := \text{With}[\{r = \text{Numerator}[\text{Rt}[b/a, 3]], s = \text{Denominator}[\text{Rt}[b/a, 3]]\}, \text{Dist}[(c*r - (1 + \text{Sqrt}[3])*d*s)/r,$

```
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]]]/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]]]/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1 - \sqrt{3} + x}{\sqrt{-1 - x^3}} dx = -\left((2\sqrt{3}) \int \frac{1}{\sqrt{-1 - x^3}} dx \right) + \int \frac{1 + \sqrt{3} + x}{\sqrt{-1 - x^3}} dx$$

$$= -\frac{2\sqrt{-1 - x^3}}{1 - \sqrt{3} + x} + \frac{\sqrt[4]{3}\sqrt{2 + \sqrt{3}}(1 + x)\sqrt{\frac{1 - x + x^2}{(1 - \sqrt{3} + x)^2}} E\left(\sin^{-1}\left(\frac{1 + \sqrt{3} + x}{1 - \sqrt{3} + x}\right) \mid -7 + 4\sqrt{3}\right)}{\sqrt{-\frac{1 + x}{(1 - \sqrt{3} + x)^2}}\sqrt{-1 - x^3}} - \frac{4\sqrt[4]{3}\sqrt{2 - \sqrt{3}}(1 + x)}{\sqrt{-\frac{1 + x}{(1 - \sqrt{3} + x)^2}}\sqrt{-1 - x^3}}$$

Mathematica [C] time = 0.0239983, size = 67, normalized size = 0.27

$$\frac{x\sqrt{x^3 + 1}\left(x {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -x^3\right) - 2(\sqrt{3} - 1) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -x^3\right)\right)}{2\sqrt{-x^3 - 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - Sqrt[3] + x)/Sqrt[-1 - x^3],x]

[Out] (x*Sqrt[1 + x^3]*(-2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] + x*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3]))/(2*Sqrt[-1 - x^3])

Maple [A] time = 0.008, size = 370, normalized size = 1.5

$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{3}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}\sqrt{3}\left(\frac{3}{2}+\frac{i}{2}\sqrt{3}\right)\text{EllipticE}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{3},\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x-3^(1/2))/(-x^3-1)^(1/2),x)

[Out]
$$\begin{aligned} & -2/3*I*3^{(1/2)}*(I*(x-1/2-1/2*I*3^{(1/2)})*3^{(1/2)})^{(1/2)}*((1+x)/(3/2+1/2*I*3^{(1/2)}))^{(1/2)} \\ & *(-I*(x-1/2+1/2*I*3^{(1/2)})*3^{(1/2)})^{(1/2)}/(-x^3-1)^{(1/2)}*((3/2+1/2*I*3^{(1/2)})* \\ & \text{EllipticE}(1/3*3^{(1/2)}*(I*(x-1/2-1/2*I*3^{(1/2)})*3^{(1/2)})^{(1/2)},(I*3^{(1/2)}/(3/2+1/2*I*3^{(1/2)}))^{(1/2)}) \\ & -\text{EllipticF}(1/3*3^{(1/2)}*(I*(x-1/2-1/2*I*3^{(1/2)})*3^{(1/2)})^{(1/2)},(I*3^{(1/2)}/(3/2+1/2*I*3^{(1/2)}))^{(1/2)}) \\ & -2/3*I*3^{(1/2)}*(I*(x-1/2-1/2*I*3^{(1/2)})*3^{(1/2)})^{(1/2)}*((1+x)/(3/2+1/2*I*3^{(1/2)}))^{(1/2)} \\ & *(-I*(x-1/2+1/2*I*3^{(1/2)})*3^{(1/2)})^{(1/2)}/(-x^3-1)^{(1/2)}*\text{EllipticF}(1/3*3^{(1/2)}*(I*(x-1/2-1/2*I*3^{(1/2)})*3^{(1/2)})^{(1/2)}, \\ & (I*3^{(1/2)}/(3/2+1/2*I*3^{(1/2)}))^{(1/2)})+2*I*(I*(x-1/2-1/2*I*3^{(1/2)})*3^{(1/2)})^{(1/2)}*((1+x)/(3/2+1/2*I*3^{(1/2)}))^{(1/2)} \\ & *(-I*(x-1/2+1/2*I*3^{(1/2)})*3^{(1/2)})^{(1/2)}/(-x^3-1)^{(1/2)}*\text{EllipticF}(1/3*3^{(1/2)}*(I*(x-1/2-1/2*I*3^{(1/2)})*3^{(1/2)})^{(1/2)}, \\ & (I*3^{(1/2)}/(3/2+1/2*I*3^{(1/2)}))^{(1/2)}) \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{-x^3 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")

[Out] integrate((x - sqrt(3) + 1)/sqrt(-x^3 - 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^3-1}(x-\sqrt{3}+1)}{x^3+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-x^3 - 1)*(x - sqrt(3) + 1)/(x^3 + 1), x)

Sympy [A] time = 1.6692, size = 97, normalized size = 0.39

$$-\frac{ix^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{2}{5}, \frac{2}{3} \right) x^3 e^{i\pi}}{3\Gamma\left(\frac{5}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{3}{4}, \frac{2}{3} \right) x^3 e^{i\pi}}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{3}{4}, \frac{2}{3} \right) x^3 e^{i\pi}}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x-3**(1/2))/(-x**3-1)**(1/2),x)

[Out] -I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{-x^3 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")

[Out] integrate((x - sqrt(3) + 1)/sqrt(-x^3 - 1), x)

$$3.95 \quad \int \frac{-1+\sqrt{3-x}}{\sqrt{1+x^3}} dx$$

Optimal. Leaf size=126

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle| -7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^3+1}} - \frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}$$

[Out] $(-2*\text{Sqrt}[1 + x^3])/(1 + \text{Sqrt}[3] + x) + (3^{(1/4)}*\text{Sqrt}[2 - \text{Sqrt}[3]]*(1 + x)*\text{Sqrt}[(1 - x + x^2)/(1 + \text{Sqrt}[3] + x)^2]*\text{EllipticE}[\text{ArcSin}[(1 - \text{Sqrt}[3] + x)/(1 + \text{Sqrt}[3] + x)], -7 - 4*\text{Sqrt}[3]])/(\text{Sqrt}[(1 + x)/(1 + \text{Sqrt}[3] + x)^2]*\text{Sqrt}[1 + x^3])$

Rubi [A] time = 0.0246693, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$, Rules used = {1877}

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle| -7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^3+1}} - \frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-1 + \text{Sqrt}[3] - x)/\text{Sqrt}[1 + x^3], x]$

[Out] $(-2*\text{Sqrt}[1 + x^3])/(1 + \text{Sqrt}[3] + x) + (3^{(1/4)}*\text{Sqrt}[2 - \text{Sqrt}[3]]*(1 + x)*\text{Sqrt}[(1 - x + x^2)/(1 + \text{Sqrt}[3] + x)^2]*\text{EllipticE}[\text{ArcSin}[(1 - \text{Sqrt}[3] + x)/(1 + \text{Sqrt}[3] + x)], -7 - 4*\text{Sqrt}[3]])/(\text{Sqrt}[(1 + x)/(1 + \text{Sqrt}[3] + x)^2]*\text{Sqrt}[1 + x^3])$

Rule 1877

$\text{Int}[((c_) + (d_)*(x_))/\text{Sqrt}[(a_) + (b_)*(x_)^3], x_Symbol] \rightarrow \text{With}[\{r = \text{Numerator}[\text{Simplify}[(1 - \text{Sqrt}[3])*d]/c], s = \text{Denominator}[\text{Simplify}[(1 - \text{Sqrt}[3])*d]/c]\}, \text{Simp}[(2*d*s^3*\text{Sqrt}[a + b*x^3])/(a*r^2*((1 + \text{Sqrt}[3])*s + r*x)), x] - \text{Simp}[(3^{(1/4)}*\text{Sqrt}[2 - \text{Sqrt}[3]]*d*s*(s + r*x)*\text{Sqrt}[(s^2 - r*s*x + r^2*x^2)/(1 + \text{Sqrt}[3])*s + r*x]^2]*\text{EllipticE}[\text{ArcSin}[(1 - \text{Sqrt}[3])*s + r*x]/((1 + \text{Sqrt}[3])*s + r*x)])$


```
rt[3])*s + r*x)], -7 - 4*Sqrt[3]]/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{-1 + \sqrt{3} - x}{\sqrt{1 + x^3}} dx = -\frac{2\sqrt{1 + x^3}}{1 + \sqrt{3} + x} + \frac{\sqrt[4]{3}\sqrt{2 - \sqrt{3}}(1 + x)\sqrt{\frac{1 - x + x^2}{(1 + \sqrt{3} + x)^2}}E\left(\sin^{-1}\left(\frac{1 - \sqrt{3} + x}{1 + \sqrt{3} + x}\right) \mid -7 - 4\sqrt{3}\right)}{\sqrt{\frac{1 + x}{(1 + \sqrt{3} + x)^2}}\sqrt{1 + x^3}}$$

Mathematica [C] time = 0.0153862, size = 47, normalized size = 0.37

$$(\sqrt{3} - 1)x {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -x^3\right) - \frac{1}{2}x^2 {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -x^3\right)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + Sqrt[3] - x)/Sqrt[1 + x^3], x]

[Out] (-1 + Sqrt[3])*x*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] - (x^2*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3])/2

Maple [B] time = 0.01, size = 407, normalized size = 3.2

$$-2 \frac{3/2 - i/2\sqrt{3}}{\sqrt{x^3 + 1}} \sqrt{\frac{1 + x}{3/2 - i/2\sqrt{3}}} \sqrt{\frac{x - 1/2 - i/2\sqrt{3}}{-3/2 - i/2\sqrt{3}}} \sqrt{\frac{x - 1/2 + i/2\sqrt{3}}{-3/2 + i/2\sqrt{3}}} \left((-3/2 - i/2\sqrt{3}) \text{EllipticE}\left(\sqrt{\frac{1 + x}{3/2 - i/2\sqrt{3}}}, \sqrt{\frac{-3/2 - i/2\sqrt{3}}{-3/2 + i/2\sqrt{3}}}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1-x+3^(1/2))/(x^3+1)^(1/2), x)

[Out] -2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))*EllipticE(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)), ((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*3^(1/2))*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2), ((-3/2+1/2*I*3^(1/2))

$$\begin{aligned} & /(-3/2-1/2*I*3^{(1/2)})^{(1/2)})-2*(3/2-1/2*I*3^{(1/2)})*((1+x)/(3/2-1/2*I*3^{(1/2)}))^{(1/2)}*((x-1/2-1/2*I*3^{(1/2)})/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)}*((x-1/2+1/2*I*3^{(1/2)})/(-3/2+1/2*I*3^{(1/2)}))^{(1/2)}/(x^3+1)^{(1/2)}*EllipticF(((1+x)/(3/2-1/2*I*3^{(1/2)}))^{(1/2)},((-3/2+1/2*I*3^{(1/2)})/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)})+2*3^{(1/2)}*(3/2-1/2*I*3^{(1/2)})*((1+x)/(3/2-1/2*I*3^{(1/2)}))^{(1/2)}*((x-1/2-1/2*I*3^{(1/2)})/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)}*((x-1/2+1/2*I*3^{(1/2)})/(-3/2+1/2*I*3^{(1/2)}))^{(1/2)}/(x^3+1)^{(1/2)}*EllipticF(((1+x)/(3/2-1/2*I*3^{(1/2)}))^{(1/2)},((-3/2+1/2*I*3^{(1/2)})/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)}) \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x - \sqrt{3} + 1}{\sqrt{x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")

[Out] -integrate((x - sqrt(3) + 1)/sqrt(x^3 + 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x - \sqrt{3} + 1}{\sqrt{x^3 + 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")

[Out] integral(-(x - sqrt(3) + 1)/sqrt(x^3 + 1), x)

Sympy [A] time = 2.30229, size = 92, normalized size = 0.73

$$\frac{x^2 \Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{1}{3} \right) x^3 e^{i\pi}}{3 \Gamma\left(\frac{5}{3}\right)} - \frac{x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{2}{3} \right) x^3 e^{i\pi}}{3 \Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3} x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{2}{3} \right) x^3 e^{i\pi}}{3 \Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] -x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(5/3)) - x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x - \sqrt{3} + 1}{\sqrt{x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x - sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

$$3.96 \quad \int \frac{-1+\sqrt{3+x}}{\sqrt{1-x^3}} dx$$

Optimal. Leaf size=143

$$\frac{2\sqrt{1-x^3}}{-x+\sqrt{3+1}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{(-x+\sqrt{3+1})^2}}E\left(\sin^{-1}\left(\frac{-x-\sqrt{3+1}}{-x+\sqrt{3+1}}\right)\middle| -7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{(-x+\sqrt{3+1})^2}}\sqrt{1-x^3}}$$

[Out] (2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])

Rubi [A] time = 0.024946, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$, Rules used = {1877}

$$\frac{2\sqrt{1-x^3}}{-x+\sqrt{3+1}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{(-x+\sqrt{3+1})^2}}E\left(\sin^{-1}\left(\frac{-x-\sqrt{3+1}}{-x+\sqrt{3+1}}\right)\middle| -7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{(-x+\sqrt{3+1})^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

[In] Int[(-1 + Sqrt[3] + x)/Sqrt[1 - x^3], x]

[Out] (2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])

Rule 1877

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(1 + Sqrt[3])*s + r*x]^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])

```
rt[3])*s + r*x)], -7 - 4*Sqrt[3]]/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{-1 + \sqrt{3} + x}{\sqrt{1 - x^3}} dx = \frac{2\sqrt{1 - x^3}}{1 + \sqrt{3} - x} - \frac{\sqrt[4]{3}\sqrt{2 - \sqrt{3}}(1 - x)\sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right) \mid -7 - 4\sqrt{3}\right)}{\sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}}\sqrt{1-x^3}}$$

Mathematica [C] time = 0.0110959, size = 43, normalized size = 0.3

$$\frac{1}{2}x\left(2(\sqrt{3}-1) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; x^3\right) + x {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; x^3\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + Sqrt[3] + x)/Sqrt[1 - x^3], x]

[Out] (x*(2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] + x*Hypergeometric2F1[1/2, 2/3, 5/3, x^3]))/2

Maple [B] time = 0.009, size = 368, normalized size = 2.6

$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x + \frac{1}{2} - \frac{i}{2}\sqrt{3}\right)}\sqrt{3}\sqrt{\frac{-1+x}{-\frac{3}{2} + \frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x + \frac{1}{2} + \frac{i}{2}\sqrt{3}\right)}\sqrt{3}\left(\left(-\frac{3}{2} + \frac{i}{2}\sqrt{3}\right)\text{EllipticE}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x + \frac{1}{2} - \frac{i}{2}\sqrt{3}\right)}\sqrt{\frac{-1+x}{-\frac{3}{2} + \frac{i}{2}\sqrt{3}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1+x+3^(1/2))/(-x^3+1)^(1/2), x)

[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*((-3/2+1/2*I*3^(1/2))*EllipticE(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2), (I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2), (I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+2

$$\frac{1}{3} I^3 \sqrt{3} \left(I \left(x + \frac{1}{2} - \frac{1}{2} I^3 \sqrt{3} \right) \sqrt{3} \right)^{1/2} \left(\frac{-1+x}{-3/2+1/2 I^3 \sqrt{3}} \right)^{1/2} \left(-I \left(x + \frac{1}{2} + \frac{1}{2} I^3 \sqrt{3} \right) \sqrt{3} \right)^{1/2} \left(-x^3+1 \right)^{1/2} \text{EllipticF} \left(\frac{1}{3} \sqrt{3} \left(I \left(x + \frac{1}{2} - \frac{1}{2} I^3 \sqrt{3} \right) \sqrt{3} \right)^{1/2}, \left(I^3 \sqrt{3} / \left(-3/2+1/2 I^3 \sqrt{3} \right) \right)^{1/2} \right) - 2 I \left(I \left(x + \frac{1}{2} - \frac{1}{2} I^3 \sqrt{3} \right) \sqrt{3} \right)^{1/2} \left(\frac{-1+x}{-3/2+1/2 I^3 \sqrt{3}} \right)^{1/2} \left(-I \left(x + \frac{1}{2} + \frac{1}{2} I^3 \sqrt{3} \right) \sqrt{3} \right)^{1/2} \left(-x^3+1 \right)^{1/2} \text{EllipticF} \left(\frac{1}{3} \sqrt{3} \left(I \left(x + \frac{1}{2} - \frac{1}{2} I^3 \sqrt{3} \right) \sqrt{3} \right)^{1/2}, \left(I^3 \sqrt{3} / \left(-3/2+1/2 I^3 \sqrt{3} \right) \right)^{1/2} \right)$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x + \sqrt{3} - 1}{\sqrt{-x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x+sqrt(3))/(-x^3+1)^(1/2),x, algorithm="maxima")

[Out] integrate((x + sqrt(3) - 1)/sqrt(-x^3 + 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(-\frac{\sqrt{-x^3 + 1}(x + \sqrt{3} - 1)}{x^3 - 1}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x+sqrt(3))/(-x^3+1)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-x^3 + 1)*(x + sqrt(3) - 1)/(x^3 - 1), x)

Sympy [A] time = 1.71918, size = 97, normalized size = 0.68

$$\frac{x^2 \Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{1}{3} \right) x^3 e^{2i\pi}}{3 \Gamma\left(\frac{5}{3}\right)} - \frac{x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{2}{3} \right) x^3 e^{2i\pi}}{3 \Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3} x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{2}{3} \right) x^3 e^{2i\pi}}{3 \Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamma(
5/3)) - x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*g
amma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(
2*I*pi))/(3*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x + \sqrt{3} - 1}{\sqrt{-x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

$$3.97 \quad \int \frac{-1+\sqrt{3+x}}{\sqrt{-1+x^3}} dx$$

Optimal. Leaf size=263

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{(-x-\sqrt{3}+1)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right), 4\sqrt{3}-7\right)}{\sqrt{-\frac{1-x}{(-x-\sqrt{3}+1)^2}}\sqrt{x^3-1}} - \frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{(-x-\sqrt{3}+1)^2}}}{\sqrt{-\frac{1-x}{(-x-\sqrt{3}+1)^2}}}$$

[Out] $(-2*\text{Sqrt}[-1 + x^3])/(1 - \text{Sqrt}[3] - x) + (3^{(1/4)}*\text{Sqrt}[2 + \text{Sqrt}[3]]*(1 - x)*\text{Sqrt}[(1 + x + x^2)/(1 - \text{Sqrt}[3] - x)^2]*\text{EllipticE}[\text{ArcSin}[(1 + \text{Sqrt}[3] - x)/(1 - \text{Sqrt}[3] - x)], -7 + 4*\text{Sqrt}[3]])/(\text{Sqrt}[-((1 - x)/(1 - \text{Sqrt}[3] - x)^2)]*\text{Sqrt}[-1 + x^3]) - (4*3^{(1/4)}*\text{Sqrt}[2 - \text{Sqrt}[3]]*(1 - x)*\text{Sqrt}[(1 + x + x^2)/(1 - \text{Sqrt}[3] - x)^2]*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3] - x)/(1 - \text{Sqrt}[3] - x)], -7 + 4*\text{Sqrt}[3]])/(\text{Sqrt}[-((1 - x)/(1 - \text{Sqrt}[3] - x)^2)]*\text{Sqrt}[-1 + x^3])$

Rubi [A] time = 0.0502606, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1880, 219, 1879}

$$\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1} - \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{(-x-\sqrt{3}+1)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right) \mid -7 + 4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{(-x-\sqrt{3}+1)^2}}\sqrt{x^3-1}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{(-x-\sqrt{3}+1)^2}}E\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right) \mid -7 + 4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{(-x-\sqrt{3}+1)^2}}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-1 + \text{Sqrt}[3] + x)/\text{Sqrt}[-1 + x^3], x]$

[Out] $(-2*\text{Sqrt}[-1 + x^3])/(1 - \text{Sqrt}[3] - x) + (3^{(1/4)}*\text{Sqrt}[2 + \text{Sqrt}[3]]*(1 - x)*\text{Sqrt}[(1 + x + x^2)/(1 - \text{Sqrt}[3] - x)^2]*\text{EllipticE}[\text{ArcSin}[(1 + \text{Sqrt}[3] - x)/(1 - \text{Sqrt}[3] - x)], -7 + 4*\text{Sqrt}[3]])/(\text{Sqrt}[-((1 - x)/(1 - \text{Sqrt}[3] - x)^2)]*\text{Sqrt}[-1 + x^3]) - (4*3^{(1/4)}*\text{Sqrt}[2 - \text{Sqrt}[3]]*(1 - x)*\text{Sqrt}[(1 + x + x^2)/(1 - \text{Sqrt}[3] - x)^2]*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3] - x)/(1 - \text{Sqrt}[3] - x)], -7 + 4*\text{Sqrt}[3]])/(\text{Sqrt}[-((1 - x)/(1 - \text{Sqrt}[3] - x)^2)]*\text{Sqrt}[-1 + x^3])$

Rule 1880

$\text{Int}[\frac{(c_*) + (d_*)(x_*)}{\text{Sqrt}[(a_*) + (b_*)(x_*)^3]}, x_Symbol] := \text{With}[\{r = \text{Numerator}[\text{Rt}[b/a, 3]], s = \text{Denominator}[\text{Rt}[b/a, 3]]\}, \text{Dist}[(c*r - (1 + \text{Sqrt}[3])*d*s)/r,$


```
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :=> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]]]/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :=> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]]]/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{-1 + \sqrt{3} + x}{\sqrt{-1 + x^3}} dx = (2\sqrt{3}) \int \frac{1}{\sqrt{-1 + x^3}} dx - \int \frac{1 + \sqrt{3} - x}{\sqrt{-1 + x^3}} dx$$

$$= -\frac{2\sqrt{-1 + x^3}}{1 - \sqrt{3} - x} + \frac{\sqrt[4]{3}\sqrt{2 + \sqrt{3}}(1 - x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} E\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right) \mid -7 + 4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1 + x^3}} - \frac{4\sqrt[4]{3}\sqrt{2 - \sqrt{3}}(1 - x)}{\sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1 + x^3}}$$

Mathematica [C] time = 0.0203051, size = 63, normalized size = 0.24

$$\frac{x\sqrt{1-x^3} \left(2(\sqrt{3}-1) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; x^3\right) + x {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; x^3\right) \right)}{2\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + Sqrt[3] + x)/Sqrt[-1 + x^3],x]

[Out] (x*Sqrt[1 - x^3]*(2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] + x*Hypergeometric2F1[1/2, 2/3, 5/3, x^3]))/(2*Sqrt[-1 + x^3])

Maple [A] time = 0.009, size = 407, normalized size = 1.6

$$2 \frac{-3/2 - i/2\sqrt{3}}{\sqrt{x^3 - 1}} \sqrt{\frac{-1 + x}{-3/2 - i/2\sqrt{3}}} \sqrt{\frac{x + 1/2 - i/2\sqrt{3}}{3/2 - i/2\sqrt{3}}} \sqrt{\frac{x + 1/2 + i/2\sqrt{3}}{3/2 + i/2\sqrt{3}}} \left((3/2 - i/2\sqrt{3}) \operatorname{EllipticE} \left(\sqrt{\frac{-1 + x}{-3/2 - i/2\sqrt{3}}}, \sqrt{\frac{3/2 + i/2\sqrt{3}}{3/2 - i/2\sqrt{3}}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1+x+3^(1/2))/(x^3-1)^(1/2),x)

[Out] 2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*((3/2-1/2*I*3^(1/2))*EllipticE(((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+(-1/2+1/2*I*3^(1/2))*EllipticF(((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)))-2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2*3^(1/2)*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x + \sqrt{3} - 1}{\sqrt{x^3 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")

[Out] integrate((x + sqrt(3) - 1)/sqrt(x^3 - 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x + \sqrt{3} - 1}{\sqrt{x^3 - 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")

[Out] integral((x + sqrt(3) - 1)/sqrt(x^3 - 1), x)

Sympy [A] time = 1.68934, size = 82, normalized size = 0.31

$$-\frac{ix^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{2}{5}, \frac{3}{3}\right) x^3}{3\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{3}{4}, \frac{2}{3}\right) x^3}{3\Gamma\left(\frac{4}{3}\right)} + \frac{ix\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{3}{4}, \frac{2}{3}\right) x^3}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x+3**(1/2))/(x**3-1)**(1/2),x)

[Out] -I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3)/(3*gamma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) + I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x + \sqrt{3} - 1}{\sqrt{x^3 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")

[Out] integrate((x + sqrt(3) - 1)/sqrt(x^3 - 1), x)

$$3.98 \quad \int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}} dx$$

Optimal. Leaf size=248

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x-\sqrt{3}+1)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{\sqrt{-\frac{x+1}{(x-\sqrt{3}+1)^2}}\sqrt{-x^3-1}} + \frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x-\sqrt{3}+1)^2}}E\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)\right)}{\sqrt{-\frac{x+1}{(x-\sqrt{3}+1)^2}}\sqrt{-x^3-1}}$$

[Out] (2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])

Rubi [A] time = 0.0519657, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1880, 219, 1879}

$$\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1} + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x-\sqrt{3}+1)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)\middle| -7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{(x-\sqrt{3}+1)^2}}\sqrt{-x^3-1}} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x-\sqrt{3}+1)^2}}E\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)\right)}{\sqrt{-\frac{x+1}{(x-\sqrt{3}+1)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

[In] Int[(-1 + Sqrt[3] - x)/Sqrt[-1 - x^3], x]

[Out] (2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])

Rule 1880

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numerator[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,

```
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :=> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]]]/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :=> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]]]/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{-1 + \sqrt{3} - x}{\sqrt{-1 - x^3}} dx = (2\sqrt{3}) \int \frac{1}{\sqrt{-1 - x^3}} dx - \int \frac{1 + \sqrt{3} + x}{\sqrt{-1 - x^3}} dx$$

$$= \frac{2\sqrt{-1 - x^3}}{1 - \sqrt{3} + x} - \frac{\sqrt[4]{3}\sqrt{2 + \sqrt{3}}(1 + x)\sqrt{\frac{1 - x + x^2}{(1 - \sqrt{3} + x)^2}} E\left(\sin^{-1}\left(\frac{1 + \sqrt{3} + x}{1 - \sqrt{3} + x}\right) \mid -7 + 4\sqrt{3}\right)}{\sqrt{-\frac{1 + x}{(1 - \sqrt{3} + x)^2}}\sqrt{-1 - x^3}} + \frac{4\sqrt[4]{3}\sqrt{2 - \sqrt{3}}(1 + x)}{\sqrt{-\frac{1 + x}{(1 - \sqrt{3} + x)^2}}\sqrt{-1 - x^3}}$$

Mathematica [C] time = 0.0187202, size = 67, normalized size = 0.27

$$\frac{x\sqrt{x^3 + 1} \left(x {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -x^3\right) - 2(\sqrt{3} - 1) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -x^3\right) \right)}{2\sqrt{-x^3 - 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + Sqrt[3] - x)/Sqrt[-1 - x^3], x]

[Out] -(x*Sqrt[1 + x^3]*(-2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] + x*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3]))/(2*Sqrt[-1 - x^3])

Maple [A] time = 0.011, size = 370, normalized size = 1.5

$$\frac{2i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\left(\frac{3}{2}+\frac{i}{2}\sqrt{3}\right)\text{EllipticE}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{3}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1-x+3^(1/2))/(-x^3-1)^(1/2), x)

[Out] 2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*((3/2+1/2*I*3^(1/2))*EllipticE(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2), (I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2), (I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2)))+2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2), (I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-2*I*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2), (I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x - \sqrt{3} + 1}{\sqrt{-x^3 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-x+3^(1/2))/(-x^3-1)^(1/2), x, algorithm="maxima")

[Out] -integrate((x - sqrt(3) + 1)/sqrt(-x^3 - 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^3-1}(x-\sqrt{3}+1)}{x^3+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(-x^3 - 1)*(x - sqrt(3) + 1)/(x^3 + 1), x)

Sympy [A] time = 2.39549, size = 97, normalized size = 0.39

$$\frac{ix^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}\left|\frac{2}{3}\right. x^3 e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}\left|\frac{4}{3}\right. x^3 e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{ix\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}\left|\frac{4}{3}\right. x^3 e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-x+3**(1/2))/(-x**3-1)**(1/2),x)

[Out] I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3)) + I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")

[Out] integrate(-(x - sqrt(3) + 1)/sqrt(-x^3 - 1), x)

$$3.99 \quad \int \frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt{a+bx^3}} dx$$

Optimal. Leaf size=256

$$\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} E\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right) \middle| -7-4\sqrt{3}\right)}{\sqrt[3]{b} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a+bx^3}}$$

[Out] (2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 0.0447983, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {1877}

$$\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} E\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right) \middle| -7-4\sqrt{3}\right)}{\sqrt[3]{b} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[a + b*x^3], x]

[Out] (2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rule 1877


```
Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(1 - \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt{a + bx^3}} dx = \frac{2\sqrt{a + bx^3}}{\sqrt[3]{b}((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})} - \frac{\sqrt[4]{3} \sqrt{2 - \sqrt{3}} \sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} E\left(\sin^{-1}\left(\frac{(1 - \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{(1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}\right)\right)}{\sqrt[3]{b} \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}}$$

Mathematica [C] time = 0.0623925, size = 90, normalized size = 0.35

$$\frac{x \sqrt{\frac{bx^3}{a} + 1} \left(\sqrt[3]{bx} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^3}{a}\right) - 2(\sqrt{3} - 1) \sqrt[3]{a} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a}\right) \right)}{2\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[a + b*x^3], x]

[Out] (x*Sqrt[1 + (b*x^3)/a]*(-2*(-1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/2, 4/3, -(b*x^3)/a] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -(b*x^3)/a]))/(2*Sqrt[a + b*x^3])

Maple [B] time = 0.038, size = 1003, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2), x)

```
[Out] -2/3*I/b^(2/3)*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3))*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+2*I*a^(1/3)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))-2/3*I*a^(1/3)*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}(\sqrt{3}-1)}{\sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}(\sqrt{3}-1)}{\sqrt{bx^3+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] integral((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 + a), x)

Sympy [A] time = 3.32725, size = 122, normalized size = 0.48

$$\frac{\sqrt[3]{bx^2}\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3+a)**(1/2),x)

[Out] b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}(\sqrt{3}-1)}{\sqrt{bx^3+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="giac")

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 + a), x)
```

$$3.100 \quad \int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}{\sqrt{a-bx^3}} dx$$

Optimal. Leaf size=263

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{bx})\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})^2}}E\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}\right)\middle| -7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})^2}}\sqrt{a-bx^3}} - \frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})}$$

[Out] $(-2*\text{Sqrt}[a - b*x^3])/(b^{(1/3)}*((1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})) + (3^{(1/4)}*\text{Sqrt}[2 - \text{Sqrt}[3]]*a^{(1/3)}*(a^{(1/3)} - b^{(1/3)*x})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})^2]*\text{EllipticE}[\text{ArcSin}[\frac{(1 - \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x}}{(1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x}}], -7 - 4*\text{Sqrt}[3]])/(b^{(1/3)}*\text{Sqrt}[(a^{(1/3)}*(a^{(1/3)} - b^{(1/3)*x})/((1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})^2]*\text{Sqrt}[a - b*x^3])$

Rubi [A] time = 0.0417543, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.027$, Rules used = {1877}

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{bx})\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})^2}}E\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}\right)\middle| -7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})^2}}\sqrt{a-bx^3}} - \frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\frac{((1 - \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})}{\text{Sqrt}[a - b*x^3]}, x]$

[Out] $(-2*\text{Sqrt}[a - b*x^3])/(b^{(1/3)}*((1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})) + (3^{(1/4)}*\text{Sqrt}[2 - \text{Sqrt}[3]]*a^{(1/3)}*(a^{(1/3)} - b^{(1/3)*x})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})^2]*\text{EllipticE}[\text{ArcSin}[\frac{(1 - \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x}}{(1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x}}], -7 - 4*\text{Sqrt}[3]])/(b^{(1/3)}*\text{Sqrt}[(a^{(1/3)}*(a^{(1/3)} - b^{(1/3)*x})/((1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})^2]*\text{Sqrt}[a - b*x^3])$

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numerator[Simplify[((1 - Sqrt[3])*d)/c]], s = Denominator[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)]], -7 - 4*Sqrt[3])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx}}{\sqrt{a - bx^3}} dx = -\frac{2\sqrt{a - bx^3}}{\sqrt[3]{b}((1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx})} + \frac{\sqrt[4]{3}\sqrt{2 - \sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a} - \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} + \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{((1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx})^2}} E\left(\sin^{-1}\left(\frac{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx}}{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx}}\right)\right)}{\sqrt[3]{b} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} - \sqrt[3]{bx})}{((1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx})^2}} \sqrt{a - bx^3}}$$

Mathematica [C] time = 0.0619548, size = 90, normalized size = 0.34

$$\frac{x\sqrt{1 - \frac{bx^3}{a}} \left(2(\sqrt{3} - 1) \sqrt[3]{a} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \frac{bx^3}{a}\right) + \sqrt[3]{bx} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \frac{bx^3}{a}\right) \right)}{2\sqrt{a - bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[a - b*x^3], x]

[Out] -(x*Sqrt[1 - (b*x^3)/a]*(2*(-1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/2, 4/3, (b*x^3)/a] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a])/(2*Sqrt[a - b*x^3])

Maple [B] time = 0.04, size = 949, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2), x)

```
[Out] -2/3*I/b^(2/3)*3^(1/2)*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)*((x-1/b*(b^2*a)^(1/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)*(I*(x+1/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*((-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2), (-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2))+1/b*(b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2), (-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2))-2*I*a^(1/3)/b*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)*((x-1/b*(b^2*a)^(1/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)*(I*(x+1/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2), (-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2))+2/3*I*a^(1/3)*3^(1/2)/b*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)*((x-1/b*(b^2*a)^(1/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)*(I*(x+1/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2), (-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(\sqrt{3}-1)}{\sqrt{-bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{-bx^3 + ab^{\frac{1}{3}}x} + \sqrt{-bx^3 + aa^{\frac{1}{3}}(\sqrt{3}-1)}}{bx^3 - a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] integral((sqrt(-b*x^3 + a)*b^(1/3)*x + sqrt(-b*x^3 + a)*a^(1/3)*(sqrt(3) - 1))/(b*x^3 - a), x)

Sympy [A] time = 3.55613, size = 128, normalized size = 0.49

$$\frac{\sqrt[3]{bx^2}\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{bx^3 e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3 e^{2i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3 e^{2i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3+a)**(1/2),x)

[Out] -b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*a**(1/6)*gamma(4/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*a**(1/6)*gamma(4/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(\sqrt{3}-1)}{\sqrt{-bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="giac")


```
[Out] integrate(-(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 + a), x)
```

$$3.101 \quad \int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}{\sqrt{-a+bx^3}} dx$$

Optimal. Leaf size=497

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{bx})\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}\right),4\sqrt{3}-7\right)\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{bx})}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{bx})}{((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})^2}}\sqrt{bx^3-a}}$$

[Out] (2*Sqrt[-a + b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2])*Sqrt[-a + b*x^3]) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2])*Sqrt[-a + b*x^3])

Rubi [A] time = 0.132815, antiderivative size = 497, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {1880, 219, 1879}

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{bx})\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})^2}}F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}\right)|-7+4\sqrt{3}\right)\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{bx})}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{bx})}{((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})^2}}\sqrt{bx^3-a}}$$

Antiderivative was successfully verified.

[In] Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[-a + b*x^3], x]

[Out] (2*Sqrt[-a + b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2])*Sqrt[-a + b*x^3]) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2])*Sqrt[-a + b*x^3])

```
rcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*
x)], -7 + 4*Sqrt[3]]/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 -
Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]]*Sqrt[-a + b*x^3]) + (4*3^(1/4)*Sqrt[2 -
Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 +
Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*
Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^
(1/3) - b^(1/3)*x)^2]]*Sqrt[-a + b*x^3])
```

Rule 1880

```
Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2))], x] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}{\sqrt{-a+bx^3}} dx = -\left((2\sqrt{3}\sqrt[3]{a}) \int \frac{1}{\sqrt{-a+bx^3}} dx\right) + \int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}{\sqrt{-a+bx^3}} dx$$

$$= \frac{2\sqrt{-a+bx^3}}{\sqrt[3]{b}((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{bx})\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})^2}} E\left(\sin^{-1}\left(\frac{(1+\sqrt{3})}{(1-\sqrt{3})}\right)\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{bx})}{((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})^2}}\sqrt{-a+bx^3}}$$

Mathematica [C] time = 0.0393337, size = 91, normalized size = 0.18

$$\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2(\sqrt{3}-1)\sqrt[3]{a}{}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \frac{bx^3}{a}\right) + \sqrt[3]{bx}{}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \frac{bx^3}{a}\right)\right)}{2\sqrt{bx^3-a}}$$

Antiderivative was successfully verified.

[In] Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[-a + b*x^3], x]

[Out] -(x*Sqrt[1 - (b*x^3)/a]*(2*(-1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/2, 4/3, (b*x^3)/a] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a])/(2*Sqrt[-a + b*x^3])

Maple [B] time = 0.014, size = 952, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2), x)

[Out] -2/3*I/b^(2/3)*3^(1/2)*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)*((x-1/b*(b^2*a)^(1/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)*(I*(x+1/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*((-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2), (-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)

$$\begin{aligned}
 & 3) - 1/2 * I * 3^{(1/2)} / b * (b^2 * a)^{(1/3)})^{(1/2)} + 1/b * (b^2 * a)^{(1/3)} * \text{EllipticF}(1/3 * 3 \\
 & ^{(1/2)} * (-I * (x + 1/2 / b * (b^2 * a)^{(1/3)} + 1/2 * I * 3^{(1/2)} / b * (b^2 * a)^{(1/3)}) * 3^{(1/2)} * b / \\
 & (b^2 * a)^{(1/3)})^{(1/2)}, (-I * 3^{(1/2)} / b * (b^2 * a)^{(1/3)} / (-3/2 / b * (b^2 * a)^{(1/3)} - 1/2 * \\
 & I * 3^{(1/2)} / b * (b^2 * a)^{(1/3)})^{(1/2)}) - 2 * I * a^{(1/3)} / b * (b^2 * a)^{(1/3)} * (-I * (x + 1/2 / \\
 & b * (b^2 * a)^{(1/3)} + 1/2 * I * 3^{(1/2)} / b * (b^2 * a)^{(1/3)}) * 3^{(1/2)} * b / (b^2 * a)^{(1/3)})^{(1/2)} \\
 & * ((x - 1 / b * (b^2 * a)^{(1/3)}) / (-3/2 / b * (b^2 * a)^{(1/3)} - 1/2 * I * 3^{(1/2)} / b * (b^2 * a)^{(1/3)}) \\
 &)^{(1/2)} * (I * (x + 1/2 / b * (b^2 * a)^{(1/3)} - 1/2 * I * 3^{(1/2)} / b * (b^2 * a)^{(1/3)}) * 3^{(1/2)} \\
 & * b / (b^2 * a)^{(1/3)})^{(1/2)} / (b * x^3 - a)^{(1/2)} * \text{EllipticF}(1/3 * 3^{(1/2)} * (-I * (x + 1/2 / b * \\
 & (b^2 * a)^{(1/3)} + 1/2 * I * 3^{(1/2)} / b * (b^2 * a)^{(1/3)}) * 3^{(1/2)} * b / (b^2 * a)^{(1/3)})^{(1/2)} \\
 & , (-I * 3^{(1/2)} / b * (b^2 * a)^{(1/3)} / (-3/2 / b * (b^2 * a)^{(1/3)} - 1/2 * I * 3^{(1/2)} / b * (b^2 * a)^{(1/3)}) \\
 &)^{(1/2)} + 2/3 * I * a^{(1/3)} * 3^{(1/2)} / b * (b^2 * a)^{(1/3)} * (-I * (x + 1/2 / b * (b^2 * a)^{(1/3)} + 1/2 * I * 3^{(1/2)} / b * (b^2 * a)^{(1/3)}) * 3^{(1/2)} * b / (b^2 * a)^{(1/3)})^{(1/2)} * ((x - 1 / b * \\
 & (b^2 * a)^{(1/3)}) / (-3/2 / b * (b^2 * a)^{(1/3)} - 1/2 * I * 3^{(1/2)} / b * (b^2 * a)^{(1/3)})^{(1/2)} * \\
 & (I * (x + 1/2 / b * (b^2 * a)^{(1/3)} - 1/2 * I * 3^{(1/2)} / b * (b^2 * a)^{(1/3)}) * 3^{(1/2)} * b / (b^2 * a)^{(1/3)})^{(1/2)} / (b * x^3 - a)^{(1/2)} * \text{EllipticF}(1/3 * 3^{(1/2)} * (-I * (x + 1/2 / b * (b^2 * a)^{(1/3)} + 1/2 * I * 3^{(1/2)} / b * (b^2 * a)^{(1/3)}) * 3^{(1/2)} * b / (b^2 * a)^{(1/3)})^{(1/2)}, (-I * 3^{(1/2)} / b * (b^2 * a)^{(1/3)} / (-3/2 / b * (b^2 * a)^{(1/3)} - 1/2 * I * 3^{(1/2)} / b * (b^2 * a)^{(1/3)})^{(1/2)}) \\
 & 2))
 \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$- \int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(\sqrt{3} - 1)}{\sqrt{bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="maxima")

[Out] -integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 - a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(-\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(\sqrt{3} - 1)}{\sqrt{bx^3 - a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="fricas")

[Out] integral(-(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 - a), x)

Sympy [A] time = 3.54467, size = 112, normalized size = 0.23

$$\frac{i\sqrt[3]{bx^2}\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3-a)**(1/2),x)

[Out] I*b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3*sqrt(a)*gamma(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(3*a**(1/6)*gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(3*a**(1/6)*gamma(4/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(\sqrt{3}-1)}{\sqrt{bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="giac")

[Out] integrate(-(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 - a), x)

$$3.102 \quad \int \frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt{-a-bx^3}} dx$$

Optimal. Leaf size=488

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} \operatorname{EllipticF}\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}\right), 4\sqrt{3}-7\right) \sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{\sqrt[3]{b} \sqrt{-\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{-a-bx^3}}$$

[Out] $(-2*\operatorname{Sqrt}[-a - b*x^3])/(b^{(1/3)}*((1 - \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})) + (3^{(1/4)}*\operatorname{Sqrt}[2 + \operatorname{Sqrt}[3]]*a^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x})*\operatorname{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 - \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[(1 + \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x}/((1 - \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})], -7 + 4*\operatorname{Sqrt}[3]])/(b^{(1/3)}*\operatorname{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x}))/((1 - \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2])*\operatorname{Sqrt}[-a - b*x^3]) - (4*3^{(1/4)}*\operatorname{Sqrt}[2 - \operatorname{Sqrt}[3]]*a^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x})*\operatorname{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 - \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[(1 + \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x}/((1 - \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})], -7 + 4*\operatorname{Sqrt}[3]])/(b^{(1/3)}*\operatorname{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x}))/((1 - \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2])*\operatorname{Sqrt}[-a - b*x^3])$

Rubi [A] time = 0.114932, antiderivative size = 488, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {1880, 219, 1879}

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}\right) \middle| -7 + 4\sqrt{3}\right) \sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{\sqrt[3]{b} \sqrt{-\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{-a-bx^3}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\frac{((1 - \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})}{\operatorname{Sqrt}[-a - b*x^3]}, x]$

[Out] $(-2*\operatorname{Sqrt}[-a - b*x^3])/(b^{(1/3)}*((1 - \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})) + (3^{(1/4)}*\operatorname{Sqrt}[2 + \operatorname{Sqrt}[3]]*a^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x})*\operatorname{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 - \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[(1 + \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x}/((1 - \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})], -7 + 4*\operatorname{Sqrt}[3]])/(b^{(1/3)}*\operatorname{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x}))/((1 - \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2])*\operatorname{Sqrt}[-a - b*x^3]) - (4*3^{(1/4)}*\operatorname{Sqrt}[2 - \operatorname{Sqrt}[3]]*a^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x})*\operatorname{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 - \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[\frac{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}], -7 + 4*\operatorname{Sqrt}[3]])/(b^{(1/3)}*\operatorname{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x}))/((1 - \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2])*\operatorname{Sqrt}[-a - b*x^3])$

```
ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)], -7 + 4*Sqrt[3]]/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)^2])*Sqrt[-a - b*x^3]) - (4*3^(1/4)*Sqrt[2 -
Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4
*Sqrt[3]]/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2])*Sqrt[-a - b*x^3])
```

Rule 1880

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2))], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2))], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt{-a-bx^3}} dx = -\left((2\sqrt{3}\sqrt[3]{a}) \int \frac{1}{\sqrt{-a-bx^3}} dx\right) + \int \frac{(1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt{-a-bx^3}} dx$$

$$= -\frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}((1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} E\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}\right)\right)}{\sqrt[3]{b} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{-a-bx^3}}$$

Mathematica [C] time = 0.0501872, size = 93, normalized size = 0.19

$$\frac{x\sqrt{\frac{bx^3}{a}} + 1 \left(\sqrt[3]{bx} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^3}{a}\right) - 2(\sqrt{3}-1)\sqrt[3]{a} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a}\right) \right)}{2\sqrt{-a-bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[-a - b*x^3], x]

[Out] (x*Sqrt[1 + (b*x^3)/a]*(-2*(-1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/2, 4/3, -(b*x^3)/a] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -(b*x^3)/a]))/(2*Sqrt[-a - b*x^3])

Maple [B] time = 0.017, size = 1012, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2), x)

[Out] -2/3*I/b^(2/3)*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/

$$\frac{2}{b}(-b^2a)^{1/3} + \frac{1}{2}I^3 \frac{1}{b}(-b^2a)^{1/3} \Big)^{1/2} + \frac{1}{b}(-b^2a)^{1/3} \text{EllipticF}\left(\frac{1}{3} \sqrt{3} \frac{1}{2} \left(I \left(x + \frac{1}{2} \frac{1}{b}(-b^2a)^{1/3} - \frac{1}{2} I^3 \frac{1}{b}(-b^2a)^{1/3} \right)^{1/2} \frac{b}{(-b^2a)^{1/3}} \right)^{1/2}, \frac{I^3 \frac{1}{b}(-b^2a)^{1/3}}{(-3/2/b} \right. \\
\left. \frac{1}{b}(-b^2a)^{1/3} + \frac{1}{2} I^3 \frac{1}{b}(-b^2a)^{1/3} \right)^{1/2} \Big) + 2I^3 a^{1/3} \frac{1}{b}(-b^2a)^{1/3} \left(I \left(x + \frac{1}{2} \frac{1}{b}(-b^2a)^{1/3} - \frac{1}{2} I^3 \frac{1}{b}(-b^2a)^{1/3} \right)^{1/2} \frac{b}{(-b^2a)^{1/3}} \right)^{1/2} \left(\frac{x-1/b}{(-b^2a)^{1/3}} \right) \frac{1}{(-3/2/b} \\
\frac{1}{b}(-b^2a)^{1/3} + \frac{1}{2} I^3 \frac{1}{b}(-b^2a)^{1/3} \Big)^{1/2} \left(-I \left(x + \frac{1}{2} \frac{1}{b}(-b^2a)^{1/3} + \frac{1}{2} I^3 \frac{1}{b}(-b^2a)^{1/3} \right)^{1/2} \frac{b}{(-b^2a)^{1/3}} \right)^{1/2} \frac{1}{(-b^2a)^{1/3}} \text{El} \\
\text{lipticF}\left(\frac{1}{3} \sqrt{3} \frac{1}{2} \left(I \left(x + \frac{1}{2} \frac{1}{b}(-b^2a)^{1/3} - \frac{1}{2} I^3 \frac{1}{b}(-b^2a)^{1/3} \right)^{1/2} \frac{b}{(-b^2a)^{1/3}} \right)^{1/2}, \frac{I^3 \frac{1}{b}(-b^2a)^{1/3}}{(-3/2/b} \right. \\
\left. \frac{1}{b}(-b^2a)^{1/3} + \frac{1}{2} I^3 \frac{1}{b}(-b^2a)^{1/3} \right)^{1/2} \Big) - \frac{2}{3} I^3 a^{1/3} \frac{1}{b}(-b^2a)^{1/3} \left(I \left(x + \frac{1}{2} \frac{1}{b}(-b^2a)^{1/3} - \frac{1}{2} I^3 \frac{1}{b}(-b^2a)^{1/3} \right)^{1/2} \frac{b}{(-b^2a)^{1/3}} \right)^{1/2} \left(\frac{x-1/b}{(-b^2a)^{1/3}} \right) \frac{1}{(-3/2/b} \\
\frac{1}{b}(-b^2a)^{1/3} + \frac{1}{2} I^3 \frac{1}{b}(-b^2a)^{1/3} \Big)^{1/2} \left(-I \left(x + \frac{1}{2} \frac{1}{b}(-b^2a)^{1/3} + \frac{1}{2} I^3 \frac{1}{b}(-b^2a)^{1/3} \right)^{1/2} \frac{b}{(-b^2a)^{1/3}} \right)^{1/2} \frac{1}{(-b^2a)^{1/3}} \text{E} \\
\text{llipticF}\left(\frac{1}{3} \sqrt{3} \frac{1}{2} \left(I \left(x + \frac{1}{2} \frac{1}{b}(-b^2a)^{1/3} - \frac{1}{2} I^3 \frac{1}{b}(-b^2a)^{1/3} \right)^{1/2} \frac{b}{(-b^2a)^{1/3}} \right)^{1/2}, \frac{I^3 \frac{1}{b}(-b^2a)^{1/3}}{(-3/2/b} \right. \\
\left. \frac{1}{b}(-b^2a)^{1/3} + \frac{1}{2} I^3 \frac{1}{b}(-b^2a)^{1/3} \right)^{1/2} \Big) \frac{1}{b}(-b^2a)^{1/3} \left(I \left(x + \frac{1}{2} \frac{1}{b}(-b^2a)^{1/3} - \frac{1}{2} I^3 \frac{1}{b}(-b^2a)^{1/3} \right)^{1/2} \frac{b}{(-b^2a)^{1/3}} \right)^{1/2} \left(\frac{x-1/b}{(-b^2a)^{1/3}} \right) \frac{1}{(-3/2/b} \\
\frac{1}{b}(-b^2a)^{1/3} + \frac{1}{2} I^3 \frac{1}{b}(-b^2a)^{1/3} \Big)^{1/2} \left(-I \left(x + \frac{1}{2} \frac{1}{b}(-b^2a)^{1/3} + \frac{1}{2} I^3 \frac{1}{b}(-b^2a)^{1/3} \right)^{1/2} \frac{b}{(-b^2a)^{1/3}} \right)^{1/2} \frac{1}{(-b^2a)^{1/3}} \Big)$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}(\sqrt{3}-1)}{\sqrt{-bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="maxima")

[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 - a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(-\frac{\sqrt{-bx^3 - ab^{\frac{1}{3}}x} - \sqrt{-bx^3 - aa^{\frac{1}{3}}(\sqrt{3}-1)}}{bx^3 + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="fricas")

[Out] integral(-(sqrt(-b*x^3 - a)*b^(1/3)*x - sqrt(-b*x^3 - a)*a^(1/3)*(sqrt(3) - 1))/(b*x^3 + a), x)

Sympy [A] time = 3.56258, size = 128, normalized size = 0.26

$$\frac{i\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3-a)**(1/2),x)

[Out] -I*b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}(\sqrt{3}-1)}{\sqrt{-bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="giac")

[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 - a), x)

$$3.103 \quad \int \frac{1 - \sqrt{3} + \sqrt[3]{\frac{b}{a}}x}{\sqrt{a + bx^3}} dx$$

Optimal. Leaf size=241

$$\frac{2\left(\frac{b}{a}\right)^{2/3} \sqrt{a + bx^3}}{b\left(x\sqrt[3]{\frac{b}{a}} + \sqrt{3} + 1\right)} - \frac{\sqrt[4]{3}\sqrt{2 - \sqrt{3}}\left(x\sqrt[3]{\frac{b}{a}} + 1\right) \sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3} - x\sqrt[3]{\frac{b}{a}} + 1}{\left(x\sqrt[3]{\frac{b}{a}} + \sqrt{3} + 1\right)^2}} E\left(\sin^{-1}\left(\frac{\sqrt[3]{\frac{b}{a}}x - \sqrt{3} + 1}{\sqrt[3]{\frac{b}{a}}x + \sqrt{3} + 1}\right) \middle| -7 - 4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}} \sqrt{\frac{x\sqrt[3]{\frac{b}{a}} + 1}{\left(x\sqrt[3]{\frac{b}{a}} + \sqrt{3} + 1\right)^2}} \sqrt{a + bx^3}}$$

[Out] (2*(b/a)^(2/3)*Sqrt[a + b*x^3])/(b*(1 + Sqrt[3] + (b/a)^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + (b/a)^(1/3)*x)*Sqrt[(1 - (b/a)^(1/3)*x + (b/a)^(2/3)*x^2]/(1 + Sqrt[3] + (b/a)^(1/3)*x)^2)*EllipticE[ArcSin[(1 - Sqrt[3] + (b/a)^(1/3)*x)/(1 + Sqrt[3] + (b/a)^(1/3)*x)], -7 - 4*Sqrt[3]]/((b/a)^(1/3)*Sqrt[(1 + (b/a)^(1/3)*x)/(1 + Sqrt[3] + (b/a)^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 0.0692954, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1877}

$$\frac{2\left(\frac{b}{a}\right)^{2/3} \sqrt{a + bx^3}}{b\left(x\sqrt[3]{\frac{b}{a}} + \sqrt{3} + 1\right)} - \frac{\sqrt[4]{3}\sqrt{2 - \sqrt{3}}\left(x\sqrt[3]{\frac{b}{a}} + 1\right) \sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3} - x\sqrt[3]{\frac{b}{a}} + 1}{\left(x\sqrt[3]{\frac{b}{a}} + \sqrt{3} + 1\right)^2}} E\left(\sin^{-1}\left(\frac{\sqrt[3]{\frac{b}{a}}x - \sqrt{3} + 1}{\sqrt[3]{\frac{b}{a}}x + \sqrt{3} + 1}\right) \middle| -7 - 4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}} \sqrt{\frac{x\sqrt[3]{\frac{b}{a}} + 1}{\left(x\sqrt[3]{\frac{b}{a}} + \sqrt{3} + 1\right)^2}} \sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[a + b*x^3], x]

[Out] (2*(b/a)^(2/3)*Sqrt[a + b*x^3])/(b*(1 + Sqrt[3] + (b/a)^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + (b/a)^(1/3)*x)*Sqrt[(1 - (b/a)^(1/3)*x + (b/a)^(2/3)*x^2]/(1 + Sqrt[3] + (b/a)^(1/3)*x)^2)*EllipticE[ArcSin[(1 - Sqrt[3] + (b/a)^(1/3)*x)/(1 + Sqrt[3] + (b/a)^(1/3)*x)], -7 - 4*Sqrt[3]]/((b/a)^(1/3)*Sqrt[(1 + (b/a)^(1/3)*x)/(1 + Sqrt[3] + (b/a)^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rule 1877

```
Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1 - \sqrt{3} + \sqrt[3]{\frac{b}{a}}x}{\sqrt{a + bx^3}} dx = \frac{2\left(\frac{b}{a}\right)^{2/3} \sqrt{a + bx^3}}{b\left(1 + \sqrt{3} + \sqrt[3]{\frac{b}{a}}x\right)} - \frac{\sqrt[4]{3}\sqrt{2 - \sqrt{3}}\left(1 + \sqrt[3]{\frac{b}{a}}x\right) \sqrt{\frac{1 - \sqrt[3]{\frac{b}{a}}x + \left(\frac{b}{a}\right)^{2/3}x^2}{\left(1 + \sqrt{3} + \sqrt[3]{\frac{b}{a}}x\right)^2}} E\left(\sin^{-1}\left(\frac{1 - \sqrt{3} + \sqrt[3]{\frac{b}{a}}x}{1 + \sqrt{3} + \sqrt[3]{\frac{b}{a}}x}\right)\right)}{\sqrt[3]{\frac{b}{a}} \sqrt{\frac{1 + \sqrt[3]{\frac{b}{a}}x}{\left(1 + \sqrt{3} + \sqrt[3]{\frac{b}{a}}x\right)^2}} \sqrt{a + bx^3}}$$

Mathematica [C] time = 0.0511168, size = 89, normalized size = 0.37

$$\frac{x\sqrt{\frac{bx^3}{a}} + 1 \left(x\sqrt[3]{\frac{b}{a}} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^3}{a}\right) - 2(\sqrt{3} - 1) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a}\right)\right)}{2\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[a + b*x^3], x]

[Out] (x*Sqrt[1 + (b*x^3)/a]*(-2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(2*Sqrt[a + b*x^3])

Maple [B] time = 0.03, size = 1004, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x)

[Out]
$$\begin{aligned} & -2/3*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b* \\ & (-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3 \\ & /2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}*(-I*(x+1/2/b*(-b \\ & ^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)} \\ & /((b*x^3+a)^{(1/2)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b* \\ & (-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a) \\ &)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)})-2/3* \\ & I*(b/a)^{(1/3)}*3^{(1/2)}/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b* \\ & (-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+ \\ & 1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b* \\ & (-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}/((b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))} \\ & *EllipticE(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2 \\ & *a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/ \\ & 2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)})+1/b*(-b^2*a)^{(1/3)} \\ & *EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a) \\ &)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/ \\ & b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)})))+2*I/b*(-b^2*a)^{(1/3)} \\ & *(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(- \\ & b^2*a)^{(1/3))}^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b* \\ & (-b^2*a)^{(1/3))}^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b* \\ & (-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}/((b*x^3+a)^{(1/2)}*EllipticF(1 \\ & /3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}, \\ & (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}) \\ &)^{(1/2)} \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} + 1}{\sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(b*x^3 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{x \left(\frac{b}{a} \right)^{\frac{1}{3}} - \sqrt{3} + 1}{\sqrt{bx^3 + a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] integral((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(b*x^3 + a), x)

Sympy [A] time = 2.17516, size = 124, normalized size = 0.51

$$\frac{x^2 \sqrt[3]{\frac{b}{a}} \Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a} \Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3} x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a} \Gamma\left(\frac{4}{3}\right)} + \frac{x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a} \Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(b/a)**(1/3)*x-3**(1/2))/(b*x**3+a)**(1/2),x)

[Out] x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \left(\frac{b}{a} \right)^{\frac{1}{3}} - \sqrt{3} + 1}{\sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="giac")

[Out] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(b*x^3 + a), x)

$$3.104 \quad \int \frac{1 - \sqrt{3} - \sqrt[3]{\frac{b}{a}}x}{\sqrt{a - bx^3}} dx$$

Optimal. Leaf size=248

$$\frac{\sqrt[4]{3}\sqrt{2 - \sqrt{3}}\left(1 - x\sqrt[3]{\frac{b}{a}}\right) \sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3} + x\sqrt[3]{\frac{b}{a}} + 1}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right) + \sqrt{3} + 1\right)^2}} E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{b}{a}}x - \sqrt{3} + 1}{-\sqrt[3]{\frac{b}{a}}x + \sqrt{3} + 1}\right) \middle| -7 - 4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}} \sqrt{\frac{1 - x\sqrt[3]{\frac{b}{a}}}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right) + \sqrt{3} + 1\right)^2}} \sqrt{a - bx^3}} - \frac{2\left(\frac{b}{a}\right)^{2/3} \sqrt{a - bx^3}}{b\left(x\left(-\sqrt[3]{\frac{b}{a}}\right) + \sqrt{3} + 1\right)}$$

[Out] $(-2*(b/a)^{(2/3)*Sqrt[a - b*x^3]})/(b*(1 + Sqrt[3] - (b/a)^{(1/3)*x})) + (3^{(1/4)*Sqrt[2 - Sqrt[3]]}*(1 - (b/a)^{(1/3)*x})*Sqrt[(1 + (b/a)^{(1/3)*x} + (b/a)^{(2/3)*x^2})/(1 + Sqrt[3] - (b/a)^{(1/3)*x})^2]*EllipticE[ArcSin[(1 - Sqrt[3] - (b/a)^{(1/3)*x})/(1 + Sqrt[3] - (b/a)^{(1/3)*x})], -7 - 4*Sqrt[3]])/((b/a)^{(1/3)*Sqrt[(1 - (b/a)^{(1/3)*x})/(1 + Sqrt[3] - (b/a)^{(1/3)*x})^2]*Sqrt[a - b*x^3])$

Rubi [A] time = 0.060878, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {1877}

$$\frac{\sqrt[4]{3}\sqrt{2 - \sqrt{3}}\left(1 - x\sqrt[3]{\frac{b}{a}}\right) \sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3} + x\sqrt[3]{\frac{b}{a}} + 1}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right) + \sqrt{3} + 1\right)^2}} E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{b}{a}}x - \sqrt{3} + 1}{-\sqrt[3]{\frac{b}{a}}x + \sqrt{3} + 1}\right) \middle| -7 - 4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}} \sqrt{\frac{1 - x\sqrt[3]{\frac{b}{a}}}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right) + \sqrt{3} + 1\right)^2}} \sqrt{a - bx^3}} - \frac{2\left(\frac{b}{a}\right)^{2/3} \sqrt{a - bx^3}}{b\left(x\left(-\sqrt[3]{\frac{b}{a}}\right) + \sqrt{3} + 1\right)}$$

Antiderivative was successfully verified.

[In] Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[a - b*x^3], x]

[Out] $(-2*(b/a)^{(2/3)*Sqrt[a - b*x^3]})/(b*(1 + Sqrt[3] - (b/a)^{(1/3)*x})) + (3^{(1/4)*Sqrt[2 - Sqrt[3]]}*(1 - (b/a)^{(1/3)*x})*Sqrt[(1 + (b/a)^{(1/3)*x} + (b/a)^{(2/3)*x^2})/(1 + Sqrt[3] - (b/a)^{(1/3)*x})^2]*EllipticE[ArcSin[(1 - Sqrt[3] - (b/a)^{(1/3)*x})/(1 + Sqrt[3] - (b/a)^{(1/3)*x})], -7 - 4*Sqrt[3]])/((b/a)^{(1/3)*Sqrt[(1 - (b/a)^{(1/3)*x})/(1 + Sqrt[3] - (b/a)^{(1/3)*x})^2]*Sqrt[a - b*x^3])$

Rule 1877


```
Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1 - \sqrt{3} - \sqrt[3]{\frac{b}{a}}x}{\sqrt{a - bx^3}} dx = -\frac{2\left(\frac{b}{a}\right)^{2/3} \sqrt{a - bx^3}}{b\left(1 + \sqrt{3} - \sqrt[3]{\frac{b}{a}}x\right)} + \frac{\sqrt[4]{3}\sqrt{2 - \sqrt{3}}\left(1 - \sqrt[3]{\frac{b}{a}}x\right) \sqrt{\frac{1 + \sqrt[3]{\frac{b}{a}}x + \left(\frac{b}{a}\right)^{2/3}x^2}{\left(1 + \sqrt{3} - \sqrt[3]{\frac{b}{a}}x\right)^2}} E\left(\sin^{-1}\left(\frac{1 - \sqrt{3} - \sqrt[3]{\frac{b}{a}}x}{1 + \sqrt{3} - \sqrt[3]{\frac{b}{a}}x}\right) \middle| -7 - 4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}} \sqrt{\frac{1 - \sqrt[3]{\frac{b}{a}}x}{\left(1 + \sqrt{3} - \sqrt[3]{\frac{b}{a}}x\right)^2}} \sqrt{a - bx^3}}$$

Mathematica [C] time = 0.0439258, size = 89, normalized size = 0.36

$$\frac{x\sqrt{1 - \frac{bx^3}{a}} \left(2(\sqrt{3} - 1) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \frac{bx^3}{a}\right) + x\sqrt[3]{\frac{b}{a}} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \frac{bx^3}{a}\right)\right)}{2\sqrt{a - bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[a - b*x^3], x]

[Out] -(x*Sqrt[1 - (b*x^3)/a]*(2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, (b*x^3)/a] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a]))/(2*Sqrt[a - b*x^3])

Maple [B] time = 0.033, size = 950, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x)`

[Out]
$$\begin{aligned} & -2/3*I*(b/a)^(1/3)*3^(1/2)/b*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I \\ & *3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)*((x-1/b*(b^2*a)^(1/3)) \\ & /(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)*(I*(x+1/2/ \\ & b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2) \\ & /(-b*x^3+a)^(1/2)*((-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*E \\ & llipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3)) \\ & *3^(1/2)*b/(b^2*a)^(1/3))^(1/2), (-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2 \\ & *a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2))+1/b*(b^2*a)^(1/3)*Elliptic \\ & F(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(\\ & 1/2)*b/(b^2*a)^(1/3))^(1/2), (-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/ \\ & 3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2))-2*I/b*(b^2*a)^(1/3)*(-I*(x+1/2/b \\ & *(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2) \\ &)*((x-1/b*(b^2*a)^(1/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3 \\ &)))^(1/2)*(I*(x+1/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)* \\ & b/(b^2*a)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b* \\ & (b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2) \\ & , (-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(\\ & 1/3)))^(1/2))+2/3*I*3^(1/2)/b*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2 \\ & *I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)*((x-1/b*(b^2*a)^(\\ & 1/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)*(I*(x+1/ \\ & 2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(\\ & 1/2)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2* \\ & I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2), (-I*3^(1/2)/b*(b^ \\ & 2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)) \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x \left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} - 1}{\sqrt{-bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="maxima")`

[Out] `-integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(-b*x^3 + a), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{-bx^3 + ax} \left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{-bx^3 + a}(\sqrt{3} - 1)}{bx^3 - a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] integral((sqrt(-b*x^3 + a)*x*(b/a)^(1/3) + sqrt(-b*x^3 + a)*(sqrt(3) - 1))/(b*x^3 - a), x)

Sympy [A] time = 2.77021, size = 129, normalized size = 0.52

$$\frac{x^2 \sqrt[3]{\frac{b}{a}} \Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{bx^3 e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3 e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3 e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(b/a)**(1/3)*x-3**(1/2))/(-b*x**3+a)**(1/2),x)

[Out] -x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(4/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x \left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} - 1}{\sqrt{-bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="giac")

```
[Out] integrate(-(x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(-b*x^3 + a), x)
```

$$3.105 \quad \int \frac{1 - \sqrt{3} - \sqrt[3]{\frac{b}{a}}x}{\sqrt{-a + bx^3}} dx$$

Optimal. Leaf size=549

$$2\sqrt{2 - \sqrt{3}} \left((1 - \sqrt{3}) \sqrt[3]{b} - (1 + \sqrt{3}) \sqrt[3]{a} \sqrt[3]{\frac{b}{a}} \right) (\sqrt[3]{a} - \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx})^2}} \text{EllipticF} \left(\sin^{-1} \left(\frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx}}{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx}} \right), 4\sqrt{3} - 7 \right)$$

$$\sqrt[4]{3} b^{2/3} \sqrt{-\frac{\sqrt[3]{a} (\sqrt[3]{a} - \sqrt[3]{bx})}{((1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx})^2}} \sqrt{bx^3 - a}$$

[Out] (2*(b/a)^(1/3)*Sqrt[-a + b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(b/a)^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]]/(b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3]) - (2*Sqrt[2 - Sqrt[3]]*((1 - Sqrt[3])*b^(1/3) - (1 + Sqrt[3])*a^(1/3)*(b/a)^(1/3))*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]]/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])

Rubi [A] time = 0.217697, antiderivative size = 549, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {1880, 219, 1879}

$$2\sqrt{2 - \sqrt{3}} \left((1 - \sqrt{3}) \sqrt[3]{b} - (1 + \sqrt{3}) \sqrt[3]{a} \sqrt[3]{\frac{b}{a}} \right) (\sqrt[3]{a} - \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx})^2}} F \left(\sin^{-1} \left(\frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx}}{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx}} \right), -7 + 4\sqrt{3} \right)$$

$$\sqrt[4]{3} b^{2/3} \sqrt{-\frac{\sqrt[3]{a} (\sqrt[3]{a} - \sqrt[3]{bx})}{((1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx})^2}} \sqrt{bx^3 - a}$$

Antiderivative was successfully verified.

[In] Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[-a + b*x^3], x]

[Out] (2*(b/a)^(1/3)*Sqrt[-a + b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(b/a)^(1/3)*(a^(1/3) - b^(1/3)*x)*

$$\text{Sqrt}[(a^{2/3} + a^{1/3}b^{1/3}x + b^{2/3}x^2)/((1 - \text{Sqrt}[3])a^{1/3} - b^{1/3}x)^2] * \text{EllipticE}[\text{ArcSin}[\frac{(1 + \text{Sqrt}[3])a^{1/3} - b^{1/3}x}{(1 - \text{Sqrt}[3])a^{1/3} - b^{1/3}x}], -7 + 4\text{Sqrt}[3]] / (b^{2/3} \text{Sqrt}[-(a^{1/3}(a^{1/3} - b^{1/3}x)) / ((1 - \text{Sqrt}[3])a^{1/3} - b^{1/3}x)^2]) * \text{Sqrt}[-a + b x^3] - (2\text{Sqrt}[2 - \text{Sqrt}[3]] * ((1 - \text{Sqrt}[3])b^{1/3} - (1 + \text{Sqrt}[3])a^{1/3})(b/a)^{1/3}) * (a^{1/3} - b^{1/3}x) * \text{Sqrt}[(a^{2/3} + a^{1/3}b^{1/3}x + b^{2/3}x^2) / ((1 - \text{Sqrt}[3])a^{1/3} - b^{1/3}x)^2] * \text{EllipticF}[\text{ArcSin}[\frac{(1 + \text{Sqrt}[3])a^{1/3} - b^{1/3}x}{(1 - \text{Sqrt}[3])a^{1/3} - b^{1/3}x}], -7 + 4\text{Sqrt}[3]] / (3^{1/4} b^{2/3} \text{Sqrt}[-(a^{1/3}(a^{1/3} - b^{1/3}x)) / ((1 - \text{Sqrt}[3])a^{1/3} - b^{1/3}x)^2]) * \text{Sqrt}[-a + b x^3])$$

Rule 1880

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2))], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + Simp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2))], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1 - \sqrt{3} - \sqrt[3]{\frac{b}{a}}x}{\sqrt{-a + bx^3}} dx = \frac{\sqrt[3]{\frac{b}{a}} \int \frac{(1+\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx}}{\sqrt{-a+bx^3}} dx}{\sqrt[3]{b}} - \left(-1 + \sqrt{3} + \frac{(1 + \sqrt{3}) \sqrt[3]{a} \sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}} \right) \int \frac{1}{\sqrt{-a + bx^3}} dx$$

$$= \frac{2\sqrt[3]{\frac{b}{a}}\sqrt{-a + bx^3}}{b^{2/3}((1 - \sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx})} - \frac{\sqrt[4]{3}\sqrt{2 + \sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}(\sqrt[3]{a} - \sqrt[3]{bx})\sqrt{\frac{a^{2/3} + \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{((1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx})^2}} E\left(\sin^{-1}\left(\frac{(1+\sqrt{3})}{(1-\sqrt{3})}\sqrt{\frac{a^{2/3} + \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{((1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx})^2}}\right)}{\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} - \sqrt[3]{bx})}{((1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx})^2}}\sqrt{-a + bx^3}}\right)}{b^{2/3}}$$

Mathematica [C] time = 0.0317337, size = 90, normalized size = 0.16

$$\frac{x\sqrt{1 - \frac{bx^3}{a}} \left(2(\sqrt{3} - 1) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \frac{bx^3}{a}\right) + x\sqrt[3]{\frac{b}{a}} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \frac{bx^3}{a}\right) \right)}{2\sqrt{bx^3 - a}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[-a + b*x^3], x]

[Out] -(x*Sqrt[1 - (b*x^3)/a]*(2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, (b*x^3)/a] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a]))/(2*Sqrt[-a + b*x^3])

Maple [B] time = 0.014, size = 953, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2)), x)

[Out] -2/3*I*(b/a)^(1/3)*3^(1/2)/b*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)*((x-1/b*(b^2*a)^(1/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)*(I*(x+1/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*((-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3)

$$\begin{aligned}
&)) * 3^{(1/2)} * b / (b^{2*a})^{(1/3)})^{(1/2)}, (-I * 3^{(1/2)} / b * (b^{2*a})^{(1/3)} / (-3/2 / b * (b^{2*} \\
& a)^{(1/3)} - 1/2 * I * 3^{(1/2)} / b * (b^{2*a})^{(1/3)})^{(1/2)}) + 1 / b * (b^{2*a})^{(1/3)} * \text{EllipticF} \\
& (1/3 * 3^{(1/2)} * (-I * (x + 1/2 / b * (b^{2*a})^{(1/3)} + 1/2 * I * 3^{(1/2)} / b * (b^{2*a})^{(1/3)}) * 3^{(1/} \\
& 2) * b / (b^{2*a})^{(1/3)})^{(1/2)}, (-I * 3^{(1/2)} / b * (b^{2*a})^{(1/3)} / (-3/2 / b * (b^{2*a})^{(1/3)} \\
& - 1/2 * I * 3^{(1/2)} / b * (b^{2*a})^{(1/3)})^{(1/2)}) - 2 * I / b * (b^{2*a})^{(1/3)} * (-I * (x + 1/2 / b * \\
& (b^{2*a})^{(1/3)} + 1/2 * I * 3^{(1/2)} / b * (b^{2*a})^{(1/3)}) * 3^{(1/2)} * b / (b^{2*a})^{(1/3)})^{(1/2)} \\
& * ((x - 1 / b * (b^{2*a})^{(1/3)}) / (-3/2 / b * (b^{2*a})^{(1/3)} - 1/2 * I * 3^{(1/2)} / b * (b^{2*a})^{(1/3)} \\
&))^{(1/2)} * (I * (x + 1/2 / b * (b^{2*a})^{(1/3)} - 1/2 * I * 3^{(1/2)} / b * (b^{2*a})^{(1/3)}) * 3^{(1/2)} * b \\
& / (b^{2*a})^{(1/3)})^{(1/2)} / (b * x^3 - a)^{(1/2)} * \text{EllipticF}(1/3 * 3^{(1/2)} * (-I * (x + 1/2 / b * (b^{2*} \\
& a)^{(1/3)} + 1/2 * I * 3^{(1/2)} / b * (b^{2*a})^{(1/3)}) * 3^{(1/2)} * b / (b^{2*a})^{(1/3)})^{(1/2)}, (\\
& -I * 3^{(1/2)} / b * (b^{2*a})^{(1/3)} / (-3/2 / b * (b^{2*a})^{(1/3)} - 1/2 * I * 3^{(1/2)} / b * (b^{2*a})^{(1/} \\
& 3)))^{(1/2)} + 2/3 * I * 3^{(1/2)} / b * (b^{2*a})^{(1/3)} * (-I * (x + 1/2 / b * (b^{2*a})^{(1/3)} + 1/2 * I \\
& * 3^{(1/2)} / b * (b^{2*a})^{(1/3)}) * 3^{(1/2)} * b / (b^{2*a})^{(1/3)})^{(1/2)} * ((x - 1 / b * (b^{2*a})^{(1/} \\
& 3))) / (-3/2 / b * (b^{2*a})^{(1/3)} - 1/2 * I * 3^{(1/2)} / b * (b^{2*a})^{(1/3)})^{(1/2)} * (I * (x + 1/2 / \\
& b * (b^{2*a})^{(1/3)} - 1/2 * I * 3^{(1/2)} / b * (b^{2*a})^{(1/3)}) * 3^{(1/2)} * b / (b^{2*a})^{(1/3)})^{(1/} \\
& 2) / (b * x^3 - a)^{(1/2)} * \text{EllipticF}(1/3 * 3^{(1/2)} * (-I * (x + 1/2 / b * (b^{2*a})^{(1/3)} + 1/2 * I * 3 \\
& ^{(1/2)} / b * (b^{2*a})^{(1/3)}) * 3^{(1/2)} * b / (b^{2*a})^{(1/3)})^{(1/2)}, (-I * 3^{(1/2)} / b * (b^{2*a} \\
&)^{(1/3)} / (-3/2 / b * (b^{2*a})^{(1/3)} - 1/2 * I * 3^{(1/2)} / b * (b^{2*a})^{(1/3)})^{(1/2)})
\end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$- \int \frac{x \left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} - 1}{\sqrt{bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2)),x, algorithm="maxima")

[Out] -integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(b*x^3 - a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(-\frac{x \left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} - 1}{\sqrt{bx^3 - a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="fricas")

[Out] integral(-(x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(b*x^3 - a), x)

Sympy [A] time = 2.731, size = 114, normalized size = 0.21

$$\frac{ix^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(b/a)**(1/3)*x-3**(1/2))/(b*x**3-a)**(1/2),x)

[Out] I*x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3*sqrt(a)*gamma(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(3*sqrt(a)*gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(3*sqrt(a)*gamma(4/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} - 1}{\sqrt{bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="giac")

[Out] integrate(-(x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(b*x^3 - a), x)

$$3.106 \quad \int \frac{1 - \sqrt{3} + \sqrt[3]{\frac{b}{a}}x}{\sqrt{-a - bx^3}} dx$$

Optimal. Leaf size=540

$$\frac{2\sqrt{2-\sqrt{3}}\left((1-\sqrt{3})\sqrt[3]{b} - (1+\sqrt{3})\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right), 4\sqrt{3}-7\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}\sqrt{-a-bx^3}}$$

[Out] $(-2*(b/a)^{(1/3)*\operatorname{Sqrt}[-a - b*x^3]})/(b^{(2/3)*((1 - \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})) + (3^{(1/4)*\operatorname{Sqrt}[2 + \operatorname{Sqrt}[3]]}*a^{(1/3)}*(b/a)^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x})*\operatorname{Sqrt}[(a^{(2/3)} - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 - \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[(1 + \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x}/((1 - \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})], -7 + 4*\operatorname{Sqrt}[3]])/(b^{(2/3)*\operatorname{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x})/((1 - \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2)]*\operatorname{Sqrt}[-a - b*x^3]) + (2*\operatorname{Sqrt}[2 - \operatorname{Sqrt}[3]]*((1 - \operatorname{Sqrt}[3])*b^{(1/3)} - (1 + \operatorname{Sqrt}[3])*a^{(1/3)}*(b/a)^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x})*\operatorname{Sqrt}[(a^{(2/3)} - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 - \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[(1 + \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x}/((1 - \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})], -7 + 4*\operatorname{Sqrt}[3]))/(3^{(1/4)*b^{(2/3)*\operatorname{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x})/((1 - \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2)]*\operatorname{Sqrt}[-a - b*x^3])$

Rubi [A] time = 0.16282, antiderivative size = 540, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {1880, 219, 1879}

$$\frac{2\sqrt{2-\sqrt{3}}\left((1-\sqrt{3})\sqrt[3]{b} - (1+\sqrt{3})\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}\right), -7 + 4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}\sqrt{-a-bx^3}} +$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(1 - \operatorname{Sqrt}[3] + (b/a)^{(1/3)*x})/\operatorname{Sqrt}[-a - b*x^3], x]$

[Out] $(-2*(b/a)^{(1/3)*\operatorname{Sqrt}[-a - b*x^3]})/(b^{(2/3)*((1 - \operatorname{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})) + (3^{(1/4)*\operatorname{Sqrt}[2 + \operatorname{Sqrt}[3]]}*a^{(1/3)}*(b/a)^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x})$

```
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]]/(b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2])*Sqrt[-a - b*x^3]) + (2*Sqrt[2 - Sqrt[3]]*((1 - Sqrt[3])*b^(1/3) - (1 + Sqrt[3])*a^(1/3)*(b/a)^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2])*Sqrt[-a - b*x^3))
```

Rule 1880

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3])*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2))], x] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + Simp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1 - \sqrt{3} + \sqrt[3]{\frac{b}{a}}x}{\sqrt{-a - bx^3}} dx = \frac{\sqrt[3]{\frac{b}{a}} \int \frac{(1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt{-a - bx^3}} dx}{\sqrt[3]{b}} + \left(1 - \sqrt{3} - \frac{(1 + \sqrt{3})\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}} \right) \int \frac{1}{\sqrt{-a - bx^3}} dx$$

$$= -\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{-a - bx^3}}{b^{2/3}((1 - \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})} + \frac{\sqrt[4]{3}\sqrt{2 + \sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}(\sqrt[3]{a} + \sqrt[3]{bx})\sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{((1 - \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} E\left(\sin^{-1}\left(\frac{1 + \sqrt{3}}{1 - \sqrt{3}}\right)\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1 - \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}}\sqrt{-a - bx^3}}$$

Mathematica [C] time = 0.0393996, size = 92, normalized size = 0.17

$$\frac{x\sqrt{\frac{bx^3}{a}} + 1 \left(x\sqrt[3]{\frac{b}{a}} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^3}{a}\right) - 2(\sqrt{3} - 1) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a}\right) \right)}{2\sqrt{-a - bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[-a - b*x^3], x]

[Out] (x*Sqrt[1 + (b*x^3)/a]*(-2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, -(b*x^3)/a]) + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -(b*x^3)/a])/(2*Sqrt[-a - b*x^3])

Maple [B] time = 0.016, size = 1013, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2), x)

[Out] -2/3*I*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*

$$\begin{aligned}
 & a^{1/3} / (-3/2/b*(-b^2*a)^{1/3} + 1/2*I*3^{1/2}/b*(-b^2*a)^{1/3})^{1/2} - 2/3 \\
 & * I * (b/a)^{1/3} * 3^{1/2} / b * (-b^2*a)^{1/3} * (I * (x + 1/2/b * (-b^2*a)^{1/3} - 1/2 * I * 3^{1/2} / b * (-b^2*a)^{1/3}) * 3^{1/2} * b / (-b^2*a)^{1/3})^{1/2} * ((x - 1/b * (-b^2*a)^{1/3}) / (-3/2/b * (-b^2*a)^{1/3} + 1/2 * I * 3^{1/2} / b * (-b^2*a)^{1/3}))^{1/2} * (-I * (x + 1/2/b * (-b^2*a)^{1/3} + 1/2 * I * 3^{1/2} / b * (-b^2*a)^{1/3}) * 3^{1/2} * b / (-b^2*a)^{1/3})^{1/2} / (-b * x^3 - a)^{1/2} * ((-3/2/b * (-b^2*a)^{1/3} + 1/2 * I * 3^{1/2} / b * (-b^2*a)^{1/3}) * \text{EllipticE}(1/3 * 3^{1/2} * (I * (x + 1/2/b * (-b^2*a)^{1/3} - 1/2 * I * 3^{1/2} / b * (-b^2*a)^{1/3}) * 3^{1/2} * b / (-b^2*a)^{1/3})^{1/2}, (I * 3^{1/2} / b * (-b^2*a)^{1/3} / (-3/2/b * (-b^2*a)^{1/3} + 1/2 * I * 3^{1/2} / b * (-b^2*a)^{1/3}))^{1/2}) + 1/b * (-b^2*a)^{1/3} * \text{EllipticF}(1/3 * 3^{1/2} * (I * (x + 1/2/b * (-b^2*a)^{1/3} - 1/2 * I * 3^{1/2} / b * (-b^2*a)^{1/3}) * 3^{1/2} * b / (-b^2*a)^{1/3})^{1/2}, (I * 3^{1/2} / b * (-b^2*a)^{1/3} / (-3/2/b * (-b^2*a)^{1/3} + 1/2 * I * 3^{1/2} / b * (-b^2*a)^{1/3}))^{1/2})) + 2 * I / b * (-b^2*a)^{1/3} * (I * (x + 1/2/b * (-b^2*a)^{1/3} - 1/2 * I * 3^{1/2} / b * (-b^2*a)^{1/3}) * 3^{1/2} * b / (-b^2*a)^{1/3})^{1/2} * ((x - 1/b * (-b^2*a)^{1/3}) / (-3/2/b * (-b^2*a)^{1/3} + 1/2 * I * 3^{1/2} / b * (-b^2*a)^{1/3}))^{1/2} * (-I * (x + 1/2/b * (-b^2*a)^{1/3} + 1/2 * I * 3^{1/2} / b * (-b^2*a)^{1/3}) * 3^{1/2} * b / (-b^2*a)^{1/3})^{1/2} / (-b * x^3 - a)^{1/2} * \text{EllipticF}(1/3 * 3^{1/2} * (I * (x + 1/2/b * (-b^2*a)^{1/3} - 1/2 * I * 3^{1/2} / b * (-b^2*a)^{1/3}) * 3^{1/2} * b / (-b^2*a)^{1/3})^{1/2}, (I * 3^{1/2} / b * (-b^2*a)^{1/3} / (-3/2/b * (-b^2*a)^{1/3} + 1/2 * I * 3^{1/2} / b * (-b^2*a)^{1/3}))^{1/2}))
 \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} + 1}{\sqrt{-bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="maxima")

[Out] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(-b*x^3 - a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(-\frac{\sqrt{-bx^3 - ax} \left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{-bx^3 - a}(\sqrt{3} - 1)}{bx^3 + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="fricas")

[Out] integral(-(sqrt(-b*x^3 - a)*x*(b/a)^(1/3) - sqrt(-b*x^3 - a)*(sqrt(3) - 1)) / (b*x^3 + a), x)

Sympy [A] time = 2.29273, size = 129, normalized size = 0.24

$$-\frac{ix^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(b/a)**(1/3)*x-3**(1/2))/(-b*x**3-a)**(1/2),x)

[Out] -I*x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} + 1}{\sqrt{-bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="giac")

[Out] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(-b*x^3 - a), x)

$$3.107 \quad \int \frac{c+dx}{\sqrt{a+bx^3}} dx$$

Optimal. Leaf size=490

$$\frac{2\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(\sqrt[3]{bc}-(1-\sqrt{3})\sqrt[3]{ad})\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)\sqrt[4]{3}\sqrt{2-\sqrt{3}}}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}}$$

[Out] (2*d*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*c - (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 0.147741, antiderivative size = 490, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1878, 218, 1877}

$$\frac{2\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(\sqrt[3]{bc}-(1-\sqrt{3})\sqrt[3]{ad})F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right)\middle| -7-4\sqrt{3}\right)\sqrt[4]{3}\sqrt{2-\sqrt{3}}}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)/Sqrt[a + b*x^3], x]

[Out] (2*d*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*c - (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

```

3)*x]], -7 - 4*Sqrt[3]]/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*
(b^(1/3)*c - (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)], -7 - 4*Sqrt[3]]]/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

```

Rule 1878

```

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]

```

Rule 218

```

Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]]/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]

```

Rule 1877

```

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/
(1 + Sqrt[3])*s + r*x]^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + S
qrt[3])*s + r*x)], -7 - 4*Sqrt[3]]]/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

```

Rubi steps

$$\int \frac{c + dx}{\sqrt{a + bx^3}} dx = \frac{d \int \frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt{a+bx^3}} dx}{\sqrt[3]{b}} + \left(c - \frac{(1-\sqrt{3})\sqrt[3]{ad}}{\sqrt[3]{b}} \right) \int \frac{1}{\sqrt{a + bx^3}} dx$$

$$= \frac{2d\sqrt{a + bx^3}}{b^{2/3} \left((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx} \right)} - \frac{\sqrt[4]{3} \sqrt{2 - \sqrt{3}} \sqrt[3]{ad} \left(\sqrt[3]{a} + \sqrt[3]{bx} \right) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{\left((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx} \right)^2}} E \left(\sin^{-1} \left(\frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}} \right) \right)}{b^{2/3} \sqrt{\frac{\sqrt[3]{a} \left(\sqrt[3]{a} + \sqrt[3]{bx} \right)}{\left((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx} \right)^2}} \sqrt{a + bx^3}}$$

Mathematica [C] time = 0.029043, size = 75, normalized size = 0.15

$$\frac{x\sqrt{\frac{bx^3}{a} + 1} \left(2c {}_2F_1 \left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a} \right) + dx {}_2F_1 \left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^3}{a} \right) \right)}{2\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/Sqrt[a + b*x^3], x]

[Out] (x*Sqrt[1 + (b*x^3)/a]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + d*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[a + b*x^3])

Maple [A] time = 0.003, size = 720, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)/(b*x^3+a)^(1/2), x)

[Out]
$$-2/3 * I * d * 3^{(1/2)} / b * (-b^2 * a)^{(1/3)} * (I * (x + 1/2 / b * (-b^2 * a)^{(1/3)} - 1/2 * I * 3^{(1/2)} / b * (-b^2 * a)^{(1/3)}) * 3^{(1/2)} * b / (-b^2 * a)^{(1/3)} - 1/2 * I * 3^{(1/2)} / b * (-b^2 * a)^{(1/3)}) / (-3/2 / b * (-b^2 * a)^{(1/3)} + 1/2 * I * 3^{(1/2)} / b * (-b^2 * a)^{(1/3)})^{(1/2)} * (-I * (x + 1/2 / b * (-b^2 * a)^{(1/3)} + 1/2 * I * 3^{(1/2)} / b * (-b^2 * a)^{(1/3)}) * 3^{(1/2)} * b / (-b^2 * a)^{(1/3)} - 1/2 * I * 3^{(1/2)} / b * (-b^2 * a)^{(1/3)}) / (b * x^3 + a)^{(1/2)} * ((-3/2 / b * (-b^2 * a)^{(1/3)} + 1/2 * I * 3^{(1/2)} / b * (-b^2 * a)^{(1/3)}) * EllipticE(1/3 * 3^{(1/2)} * (I * (x + 1/2 / b * (-b^2 * a)^{(1/3)} - 1/2 * I * 3^{(1/2)} / b * (-b^2 * a)^{(1/3)}) * 3^{(1/2)} * b / (-b^2 * a)^{(1/3)} - 1/2 * I * 3^{(1/2)} / b * (-b^2 * a)^{(1/3)}) / (-3/2 / b * (-b^2 * a)^{(1/3)} + 1/2 * I * 3^{(1/2)} / b * (-b^2 * a)^{(1/3)})^{(1/2)} + 1 / b * (-b^2 * a)^{(1/3)} * El$$

```

lipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))
)*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b
^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))-2/3*I*c*3^(1/2)/b*(-b^
2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/
2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1
/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(
1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)
)*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2
*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))

```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] integrate((d*x + c)/sqrt(b*x^3 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx + c}{\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] integral((d*x + c)/sqrt(b*x^3 + a), x)

Sympy [A] time = 1.62566, size = 78, normalized size = 0.16

$$\frac{cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**3+a)**(1/2),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(b*x^3 + a), x)
```

$$3.108 \quad \int \frac{c+dx}{\sqrt{a-bx^3}} dx$$

Optimal. Leaf size=503

$$2\sqrt{2+\sqrt{3}}(\sqrt[3]{a}-\sqrt[3]{bx})\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})^2}}((1-\sqrt{3})\sqrt[3]{ad}+\sqrt[3]{bc})\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)\sqrt[3]{3}\sqrt{a-bx^3}}$$

$$\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})^2}}\sqrt{a-bx^3}$$

[Out] (2*d*Sqrt[a - b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3]) - (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*c + (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])

Rubi [A] time = 0.148506, antiderivative size = 503, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1878, 218, 1877}

$$2\sqrt{2+\sqrt{3}}(\sqrt[3]{a}-\sqrt[3]{bx})\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})^2}}((1-\sqrt{3})\sqrt[3]{ad}+\sqrt[3]{bc})F\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}\right)|-7-4\sqrt{3}\right)\sqrt[3]{3}\sqrt{2-\sqrt{3}}\sqrt{a-bx^3}}$$

$$\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})^2}}\sqrt{a-bx^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)/Sqrt[a - b*x^3],x]

[Out] (2*d*Sqrt[a - b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3]) - (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*c + (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])

$$\frac{(3x)^{-7-4\sqrt{3}}}{(b^{2/3}\sqrt{(a^{1/3}(a^{1/3}-b^{1/3}x))((1+\sqrt{3})a^{1/3}-b^{1/3}x)^2}\sqrt{a-bx^3}) - (2\sqrt{2+\sqrt{3}}(b^{1/3}c+(1-\sqrt{3})a^{1/3}d)(a^{1/3}-b^{1/3}x)\sqrt{(a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2))((1+\sqrt{3})a^{1/3}-b^{1/3}x)^2}\text{EllipticF}[\text{ArcSin}[\frac{(1-\sqrt{3})a^{1/3}-b^{1/3}x}{(1+\sqrt{3})a^{1/3}-b^{1/3}x}], -7-4\sqrt{3}])}{(3^{1/4}b^{2/3}\sqrt{(a^{1/3}(a^{1/3}-b^{1/3}x))((1+\sqrt{3})a^{1/3}-b^{1/3}x)^2}\sqrt{a-bx^3})}$$

Rule 1878

$$\text{Int}[\frac{(c_+)(d_+)(x_+)}{\sqrt{(a_+)(b_+)(x_+)^3}}, x_Symbol] \rightarrow \text{With}[\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Dist}[\frac{c_+r - (1-\sqrt{3})d_+s}{r}, \text{Int}[1/\sqrt{a+b_+x^3}, x], x] + \text{Dist}[d_+/r, \text{Int}[\frac{(1-\sqrt{3})s+r_+x}{\sqrt{a+b_+x^3}}, x], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{PosQ}[a] \&\& \text{NeQ}[b_+c^3 - 2(5-3\sqrt{3})a_+d^3, 0]$$

Rule 218

$$\text{Int}[1/\sqrt{(a_+)(b_+)(x_+)^3}, x_Symbol] \rightarrow \text{With}[\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Simp}[\frac{(2\sqrt{2+\sqrt{3}}(s+r_+x)\sqrt{(s^2-r_+s_+x+r_+^2x^2))((1+\sqrt{3})s+r_+x)^2}\text{EllipticF}[\text{ArcSin}[\frac{(1-\sqrt{3})s+r_+x}{(1+\sqrt{3})s+r_+x}], -7-4\sqrt{3}])}{(3^{1/4}r_+\sqrt{a+b_+x^3})\sqrt{(s(s+r_+x))((1+\sqrt{3})s+r_+x)^2}}, x]] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[a]$$

Rule 1877

$$\text{Int}[\frac{(c_+)(d_+)(x_+)}{\sqrt{(a_+)(b_+)(x_+)^3}}, x_Symbol] \rightarrow \text{With}[\{r = \text{Numer}[\text{Simplify}[\frac{(1-\sqrt{3})d_+}{c_+}], s = \text{Denom}[\text{Simplify}[\frac{(1-\sqrt{3})d_+}{c_+}]]\}, \text{Simp}[\frac{2d_+s^3\sqrt{a+b_+x^3}}{(a_+r^2((1+\sqrt{3})s+r_+x))}, x] - \text{Simp}[\frac{3^{1/4}\sqrt{2-\sqrt{3}}d_+s(s+r_+x)\sqrt{(s^2-r_+s_+x+r_+^2x^2))((1+\sqrt{3})s+r_+x)^2}\text{EllipticE}[\text{ArcSin}[\frac{(1-\sqrt{3})s+r_+x}{(1+\sqrt{3})s+r_+x}], -7-4\sqrt{3}])}{(r^2\sqrt{a+b_+x^3})\sqrt{(s(s+r_+x))((1+\sqrt{3})s+r_+x)^2}}, x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{PosQ}[a] \&\& \text{EqQ}[b_+c^3 - 2(5-3\sqrt{3})a_+d^3, 0]$$

Rubi steps

$$\int \frac{c + dx}{\sqrt{a - bx^3}} dx = -\frac{d \int \frac{(1-\sqrt{3})\sqrt[3]{a-\sqrt[3]{bx}}}{\sqrt{a-bx^3}} dx}{\sqrt[3]{b}} - \left(-c - \frac{(1-\sqrt{3})\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \int \frac{1}{\sqrt{a-bx^3}} dx$$

$$= \frac{2d\sqrt{a-bx^3}}{b^{2/3}\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{ad}\left(\sqrt[3]{a}-\sqrt[3]{bx}\right) \sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}\right)^2}} E\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}\right)\right)}{b^{2/3} \sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{bx}\right)}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}\right)^2}} \sqrt{a-bx^3}}$$

Mathematica [C] time = 0.0354163, size = 75, normalized size = 0.15

$$\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2c {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \frac{bx^3}{a}\right) + dx {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \frac{bx^3}{a}\right)\right)}{2\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/Sqrt[a - b*x^3], x]

[Out] (x*Sqrt[1 - (b*x^3)/a]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, (b*x^3)/a] + d*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a]))/(2*Sqrt[a - b*x^3])

Maple [A] time = 0.004, size = 681, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)/(-b*x^3+a)^(1/2), x)

[Out] $\frac{2}{3}I*d^{3^{1/2}}/b*(b^2*a)^{1/3}*(-I*(x+1/2/b*(b^2*a)^{1/3})+1/2*I*3^{1/2}/b*(b^2*a)^{1/3})*3^{1/2}*b/(b^2*a)^{1/3})^{1/2}*((x-1/b*(b^2*a)^{1/3})/(-3/2/b*(b^2*a)^{1/3}-1/2*I*3^{1/2}/b*(b^2*a)^{1/3}))^{1/2}*(I*(x+1/2/b*(b^2*a)^{1/3})-1/2*I*3^{1/2}/b*(b^2*a)^{1/3})*3^{1/2}*b/(b^2*a)^{1/3})^{1/2}/(-b*x^3+a)^{1/2}*((-3/2/b*(b^2*a)^{1/3}-1/2*I*3^{1/2}/b*(b^2*a)^{1/3})*EllipticE(1/3*3^{1/2}*(-I*(x+1/2/b*(b^2*a)^{1/3})+1/2*I*3^{1/2}/b*(b^2*a)^{1/3})*3^{1/2}*b/(b^2*a)^{1/3})^{1/2}, (-I*3^{1/2}/b*(b^2*a)^{1/3}/(-3/2/b*(b^2*a)^{1/3}-1/2*I*3^{1/2}/b*(b^2*a)^{1/3}))^{1/2})+1/b*(b^2*a)^{1/3}*EllipticF(1/3*3^{1/2}*(-I*(x+1/2/b*(b^2*a)^{1/3})+1/2*I*3^{1/2}/b*(b^2*a)^{1/3})*3^{1/2}*b/(b^2*a)^{1/3})^{1/2}$

$$2) * (-I * (x + 1/2/b * (b^2 * a)^{1/3}) + 1/2 * I * 3^{1/2} / b * (b^2 * a)^{1/3}) * 3^{1/2} * b / (b^2 * a)^{1/3})^{1/2}, (-I * 3^{1/2} / b * (b^2 * a)^{1/3} / (-3/2/b * (b^2 * a)^{1/3} - 1/2 * I * 3^{1/2} / b * (b^2 * a)^{1/3}))^{1/2})) + 2/3 * I * c * 3^{1/2} / b * (b^2 * a)^{1/3} * (-I * (x + 1/2/b * (b^2 * a)^{1/3}) + 1/2 * I * 3^{1/2} / b * (b^2 * a)^{1/3}) * 3^{1/2} * b / (b^2 * a)^{1/3})^{1/2} * ((x - 1/b * (b^2 * a)^{1/3}) / (-3/2/b * (b^2 * a)^{1/3} - 1/2 * I * 3^{1/2} / b * (b^2 * a)^{1/3}))^{1/2} * (I * (x + 1/2/b * (b^2 * a)^{1/3}) - 1/2 * I * 3^{1/2} / b * (b^2 * a)^{1/3}) * 3^{1/2} * b / (b^2 * a)^{1/3})^{1/2} / (-b * x^3 + a)^{1/2} * \text{EllipticF}(1/3 * 3^{1/2} * (-I * (x + 1/2/b * (b^2 * a)^{1/3}) + 1/2 * I * 3^{1/2} / b * (b^2 * a)^{1/3}) * 3^{1/2} * b / (b^2 * a)^{1/3})^{1/2}, (-I * 3^{1/2} / b * (b^2 * a)^{1/3} / (-3/2/b * (b^2 * a)^{1/3} - 1/2 * I * 3^{1/2} / b * (b^2 * a)^{1/3}))^{1/2}))^{1/2}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{-bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] integrate((d*x + c)/sqrt(-b*x^3 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-bx^3 + a}(dx + c)}{bx^3 - a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-b*x^3 + a)*(d*x + c)/(b*x^3 - a), x)

Sympy [A] time = 1.72232, size = 82, normalized size = 0.16

$$\frac{cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3 e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{bx^3 e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x**3+a)**(1/2),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{-bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(-b*x^3 + a), x)
```


$$3.109 \quad \int \frac{c+dx}{\sqrt{-a+bx^3}} dx$$

Optimal. Leaf size=515

$$\frac{2\sqrt{2-\sqrt{3}}(\sqrt[3]{a}-\sqrt[3]{bx})\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})^2}}((1+\sqrt{3})\sqrt[3]{ad}+\sqrt[3]{bc})\text{EllipticF}\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}\right),4\sqrt{3}-7\right)\sqrt[4]{3}\sqrt{2+\sqrt{3}}}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{bx})}{((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})^2}}\sqrt{bx^3-a}} +$$

[Out] $(-2*d*\text{Sqrt}[-a + b*x^3])/(b^{(2/3)}*((1 - \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})) + (3^{(1/4)}*\text{Sqrt}[2 + \text{Sqrt}[3]]*a^{(1/3)}*d*(a^{(1/3)} - b^{(1/3)*x})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 - \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})^2]*\text{EllipticE}[\text{ArcSin}[(1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x}/((1 - \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})], -7 + 4*\text{Sqrt}[3]])/(b^{(2/3)}*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} - b^{(1/3)*x}))/((1 - \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})^2])* \text{Sqrt}[-a + b*x^3]) - (2*\text{Sqrt}[2 - \text{Sqrt}[3]]*(b^{(1/3)*c} + (1 + \text{Sqrt}[3])*a^{(1/3)*d}*(a^{(1/3)} - b^{(1/3)*x})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 - \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})^2]*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x}/((1 - \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})], -7 + 4*\text{Sqrt}[3]])/(3^{(1/4)}*b^{(2/3)}*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} - b^{(1/3)*x}))/((1 - \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})^2])* \text{Sqrt}[-a + b*x^3])$

Rubi [A] time = 0.163478, antiderivative size = 515, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1880, 219, 1879}

$$\frac{2\sqrt{2-\sqrt{3}}(\sqrt[3]{a}-\sqrt[3]{bx})\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})^2}}((1+\sqrt{3})\sqrt[3]{ad}+\sqrt[3]{bc})F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx}}\right),-7+4\sqrt{3}\right)\sqrt[4]{3}\sqrt{2+\sqrt{3}}}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{bx})}{((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx})^2}}\sqrt{bx^3-a}} +$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)/\text{Sqrt}[-a + b*x^3], x]$

[Out] $(-2*d*\text{Sqrt}[-a + b*x^3])/(b^{(2/3)}*((1 - \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})) + (3^{(1/4)}*\text{Sqrt}[2 + \text{Sqrt}[3]]*a^{(1/3)}*d*(a^{(1/3)} - b^{(1/3)*x})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 - \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})^2]*\text{EllipticE}[\text{ArcSin}[(1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x}/((1 - \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})], -7 + 4*\text{Sqrt}[3]])/(b^{(2/3)}*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} - b^{(1/3)*x}))/((1 - \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})^2])* \text{Sqrt}[-a + b*x^3]) - (2*\text{Sqrt}[2 - \text{Sqrt}[3]]*(b^{(1/3)*c} + (1 + \text{Sqrt}[3])*a^{(1/3)*d}*(a^{(1/3)} - b^{(1/3)*x})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 - \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})^2]*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x}/((1 - \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})], -7 + 4*\text{Sqrt}[3]])/(3^{(1/4)}*b^{(2/3)}*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} - b^{(1/3)*x}))/((1 - \text{Sqrt}[3])*a^{(1/3)} - b^{(1/3)*x})^2])* \text{Sqrt}[-a + b*x^3])$

$$\frac{1/3*x)], -7 + 4*\sqrt{3}}{(b^{2/3}*\sqrt{-((a^{1/3}*(a^{1/3} - b^{1/3}*x))/((1 - \sqrt{3})*a^{1/3} - b^{1/3}*x)^2)}*\sqrt{-a + b*x^3}) - (2*\sqrt{2 - \sqrt{3}}*(b^{1/3}*c + (1 + \sqrt{3})*a^{1/3}*d)*(a^{1/3} - b^{1/3}*x)*\sqrt{(a^{2/3} + a^{1/3}*b^{1/3}*x + b^{2/3}*x^2)/((1 - \sqrt{3})*a^{1/3} - b^{1/3}*x)^2})*\text{EllipticF}[\text{ArcSin}[\frac{(1 + \sqrt{3})*a^{1/3} - b^{1/3}*x}{(1 - \sqrt{3})*a^{1/3} - b^{1/3}*x}], -7 + 4*\sqrt{3}])/(3^{1/4}*b^{2/3}*\sqrt{-((a^{1/3}*(a^{1/3} - b^{1/3}*x))/((1 - \sqrt{3})*a^{1/3} - b^{1/3}*x)^2)}*\sqrt{-a + b*x^3})$$

Rule 1880

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^{1/4}*r*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2))], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + Simp[(3^{1/4}*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2))], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c + dx}{\sqrt{-a + bx^3}} dx = -\frac{d \int \frac{(1+\sqrt{3})\sqrt[3]{a-\sqrt[3]{bx}}}{\sqrt{-a+bx^3}} dx}{\sqrt[3]{b}} - \left(-c - \frac{(1+\sqrt{3})\sqrt[3]{ad}}{\sqrt[3]{b}} \right) \int \frac{1}{\sqrt{-a + bx^3}} dx$$

$$= -\frac{2d\sqrt{-a + bx^3}}{b^{2/3}((1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx})} + \frac{\sqrt[4]{3}\sqrt{2 + \sqrt{3}}\sqrt[3]{ad}(\sqrt[3]{a} - \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} + \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{((1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx})^2}} E\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}}{(1-\sqrt{3})\sqrt[3]{a}}\right)\right)}{b^{2/3} \sqrt{-\frac{\sqrt[3]{a}(\sqrt[3]{a} - \sqrt[3]{bx})}{((1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx})^2}} \sqrt{-a + bx^3}}$$

Mathematica [C] time = 0.0246097, size = 76, normalized size = 0.15

$$\frac{x\sqrt{1 - \frac{bx^3}{a}} \left(2c {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \frac{bx^3}{a}\right) + dx {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \frac{bx^3}{a}\right) \right)}{2\sqrt{bx^3 - a}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/Sqrt[-a + b*x^3], x]

[Out] (x*Sqrt[1 - (b*x^3)/a]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, (b*x^3)/a] + d*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a]))/(2*Sqrt[-a + b*x^3])

Maple [A] time = 0.004, size = 683, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)/(b*x^3-a)^(1/2), x)

[Out] $\frac{2}{3} I d \sqrt[3]{1/2} / b * (b^2 a)^{1/3} * (-I * (x + 1/2 / b * (b^2 a)^{1/3}) + 1/2 * I \sqrt[3]{1/2} / b * (b^2 a)^{1/3}) * \sqrt[3]{1/2} * b / (b^2 a)^{1/3} / (-3/2 / b * (b^2 a)^{1/3} - 1/2 * I \sqrt[3]{1/2} / b * (b^2 a)^{1/3})^{1/2} * (I * (x + 1/2 / b * (b^2 a)^{1/3}) - 1/2 * I \sqrt[3]{1/2} / b * (b^2 a)^{1/3}) * \sqrt[3]{1/2} * b / (b^2 a)^{1/3} / (b x^3 - a)^{1/2} * ((-3/2 / b * (b^2 a)^{1/3} - 1/2 * I \sqrt[3]{1/2} / b * (b^2 a)^{1/3}) * \text{EllipticE}(1/3 \sqrt[3]{1/2} * (-I * (x + 1/2 / b * (b^2 a)^{1/3}) + 1/2 * I \sqrt[3]{1/2} / b * (b^2 a)^{1/3}) * \sqrt[3]{1/2} * b / (b^2 a)^{1/3})^{1/2}, (-I \sqrt[3]{1/2} / b * (b^2 a)^{1/3} / (-3/2 / b * (b^2 a)^{1/3} - 1/2 * I \sqrt[3]{1/2} / b * (b^2 a)^{1/3}))^{1/2}) + 1 / b * (b^2 a)^{1/3} * \text{EllipticF}(1/3 \sqrt[3]{1/2}$

)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2), (-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)))+2/3*I*c*3^(1/2)/b*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2))*((x-1/b*(b^2*a)^(1/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)*(I*(x+1/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2), (-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x^3-a)^(1/2),x, algorithm="maxima")

[Out] integrate((d*x + c)/sqrt(b*x^3 - a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx + c}{\sqrt{bx^3 - a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x^3-a)^(1/2),x, algorithm="fricas")

[Out] integral((d*x + c)/sqrt(b*x^3 - a), x)

Sympy [A] time = 1.70066, size = 73, normalized size = 0.14

$$\frac{icx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} - \frac{idx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{2}{3}, \frac{2}{3} \middle| \frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**3-a)**(1/2),x)
```

```
[Out] -I*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(3*sqrt(a)*gamma(4/3)
) - I*d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3*sqrt(a)*gamm
a(5/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(b*x^3 - a), x)
```

$$3.110 \quad \int \frac{c+dx}{\sqrt{-a-bx^3}} dx$$

Optimal. Leaf size=508

$$\frac{2\sqrt{2-\sqrt{3}}(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}} (\sqrt[3]{bc} - (1+\sqrt{3})\sqrt[3]{ad}) \text{EllipticF}\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right), 4\sqrt{3}-7\right) \sqrt[3]{3}\sqrt{2+\sqrt{3}}}{\sqrt[3]{3}b^{2/3} \sqrt{-\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}} \sqrt{-a-bx^3}} + \dots$$

[Out] $(-2*d*\text{Sqrt}[-a - b*x^3])/(b^{(2/3)}*((1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})) + (3^{(1/4)}*\text{Sqrt}[2 + \text{Sqrt}[3]]*a^{(1/3)}*d*(a^{(1/3)} + b^{(1/3)*x})*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2]*\text{EllipticE}[\text{ArcSin}[(1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x}/((1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})], -7 + 4*\text{Sqrt}[3]])/(b^{(2/3)}*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x}))/((1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2])* \text{Sqrt}[-a - b*x^3]) + (2*\text{Sqrt}[2 - \text{Sqrt}[3]]*(b^{(1/3)}*c - (1 + \text{Sqrt}[3])*a^{(1/3)}*d)*(a^{(1/3)} + b^{(1/3)*x})*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2]*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x}/((1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})], -7 + 4*\text{Sqrt}[3]])/(3^{(1/4)}*b^{(2/3)}*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x}))/((1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2])* \text{Sqrt}[-a - b*x^3])$

Rubi [A] time = 0.153626, antiderivative size = 508, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$, Rules used = {1880, 219, 1879}

$$\frac{2\sqrt{2-\sqrt{3}}(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}} (\sqrt[3]{bc} - (1+\sqrt{3})\sqrt[3]{ad}) F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}\right) \middle| -7 + 4\sqrt{3}\right) \sqrt[3]{3}\sqrt{2+\sqrt{3}}}{\sqrt[3]{3}b^{2/3} \sqrt{-\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}} \sqrt{-a-bx^3}} + \dots$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)/\text{Sqrt}[-a - b*x^3], x]$

[Out] $(-2*d*\text{Sqrt}[-a - b*x^3])/(b^{(2/3)}*((1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})) + (3^{(1/4)}*\text{Sqrt}[2 + \text{Sqrt}[3]]*a^{(1/3)}*d*(a^{(1/3)} + b^{(1/3)*x})*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2]*\text{EllipticE}[\text{ArcSin}[(1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x}/((1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})], -7 + 4*\text{Sqrt}[3]])/(b^{(2/3)}*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x}))/((1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2])* \text{Sqrt}[-a - b*x^3]) + (2*\text{Sqrt}[2 - \text{Sqrt}[3]]*(b^{(1/3)}*c - (1 + \text{Sqrt}[3])*a^{(1/3)}*d)*(a^{(1/3)} + b^{(1/3)*x})*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2]*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x}/((1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})], -7 + 4*\text{Sqrt}[3]])/(3^{(1/4)}*b^{(2/3)}*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x}))/((1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2])* \text{Sqrt}[-a - b*x^3])$

$$\frac{1/3*x]], -7 + 4*\text{Sqrt}[3]]/(b^{2/3}*\text{Sqrt}[-((a^{1/3}*(a^{1/3} + b^{1/3}*x))/((1 - \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2)]*\text{Sqrt}[-a - b*x^3]) + (2*\text{Sqrt}[2 - \text{Sqrt}[3]]*(b^{1/3}*c - (1 + \text{Sqrt}[3])*a^{1/3}*d)*(a^{1/3} + b^{1/3}*x)*\text{Sqrt}[(a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2)/((1 - \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2])* \text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)/((1 - \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)], -7 + 4*\text{Sqrt}[3]]/(3^{1/4}*b^{2/3}*\text{Sqrt}[-((a^{1/3}*(a^{1/3} + b^{1/3}*x))/((1 - \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2)]*\text{Sqrt}[-a - b*x^3])$$

Rule 1880

```
Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2])*EllipticF[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]]/(3^{1/4}*r*Sqrt[a + b*x^3])*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2))], x] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + Simp[(3^{1/4}*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2])*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]]/(r^2*Sqrt[a + b*x^3])*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c + dx}{\sqrt{-a - bx^3}} dx = \frac{d \int \frac{(1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt{-a - bx^3}} dx}{\sqrt[3]{b}} + \left(c - \frac{(1 + \sqrt{3}) \sqrt[3]{ad}}{\sqrt[3]{b}} \right) \int \frac{1}{\sqrt{-a - bx^3}} dx$$

$$= -\frac{2d\sqrt{-a - bx^3}}{b^{2/3} \left((1 - \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx} \right)} + \frac{\sqrt[4]{3} \sqrt{2 + \sqrt{3}} \sqrt[3]{ad} \left(\sqrt[3]{a} + \sqrt[3]{bx} \right) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{\left((1 - \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx} \right)^2}} E \left(\sin^{-1} \left(\frac{(1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{(1 - \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}} \right) \right)}{b^{2/3} \sqrt{-\frac{\sqrt[3]{a} \left(\sqrt[3]{a} + \sqrt[3]{bx} \right)}{\left((1 - \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx} \right)^2}} \sqrt{-a - bx^3}}$$

Mathematica [C] time = 0.025052, size = 78, normalized size = 0.15

$$\frac{x \sqrt{\frac{bx^3}{a} + 1} \left(2c {}_2F_1 \left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a} \right) + dx {}_2F_1 \left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^3}{a} \right) \right)}{2\sqrt{-a - bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/Sqrt[-a - b*x^3], x]

[Out] (x*Sqrt[1 + (b*x^3)/a]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + d*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[-a - b*x^3])

Maple [A] time = 0.005, size = 726, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)/(-b*x^3-a)^(1/2), x)

[Out] -2/3*I*d*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*E


```

llipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1
/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-
b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))-2/3*I*c*3^(1/2)/b*(-b
^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1
/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(
1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b
^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))

```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{-bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-b*x^3-a)^(1/2),x, algorithm="maxima")

[Out] integrate((d*x + c)/sqrt(-b*x^3 - a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-bx^3 - a}(dx + c)}{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-b*x^3-a)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-b*x^3 - a)*(d*x + c)/(b*x^3 + a), x)

Sympy [A] time = 1.72232, size = 83, normalized size = 0.16

$$\frac{icx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \left| \frac{bx^3 e^{i\pi}}{a} \right. \right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} - \frac{idx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \left| \frac{bx^3 e^{i\pi}}{a} \right. \right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x**3-a)**(1/2),x)
```

```
[Out] -I*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) - I*d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{-bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(-b*x^3 - a), x)
```

$$3.111 \quad \int \frac{c+dx}{\sqrt{1+x^3}} dx$$

Optimal. Leaf size=246

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}(c-(1-\sqrt{3})d)\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right), -7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^3+1}} + \frac{2d\sqrt{x^3+1}}{x+\sqrt{3}+1} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d(x+1)\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}}{\sqrt{x^3+1}}$$

[Out] (2*d*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (2*Sqrt[2 + Sqrt[3]]*(c - (1 - Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])

Rubi [A] time = 0.0810052, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1878, 218, 1877}

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}(c-(1-\sqrt{3})d)F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \middle| -7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^3+1}} + \frac{2d\sqrt{x^3+1}}{x+\sqrt{3}+1} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d(x+1)\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}}{\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)/Sqrt[1 + x^3], x]

[Out] (2*d*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (2*Sqrt[2 + Sqrt[3]]*(c - (1 - Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c + dx}{\sqrt{1 + x^3}} dx = d \int \frac{1 - \sqrt{3} + x}{\sqrt{1 + x^3}} dx + (c - (1 - \sqrt{3})d) \int \frac{1}{\sqrt{1 + x^3}} dx$$

$$= \frac{2d\sqrt{1 + x^3}}{1 + \sqrt{3} + x} - \frac{\sqrt[4]{3}\sqrt{2 - \sqrt{3}}d(1 + x)\sqrt{\frac{1 - x + x^2}{(1 + \sqrt{3} + x)^2}}E\left(\sin^{-1}\left(\frac{1 - \sqrt{3} + x}{1 + \sqrt{3} + x}\right) \mid -7 - 4\sqrt{3}\right)}{\sqrt{\frac{1 + x}{(1 + \sqrt{3} + x)^2}}\sqrt{1 + x^3}} + \frac{2\sqrt{2 + \sqrt{3}}(c - (1 - \sqrt{3})d)}{\sqrt{1 + x^3}}$$

Mathematica [C] time = 0.0102201, size = 42, normalized size = 0.17

$$cx {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -x^3\right) + \frac{1}{2}dx^2 {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -x^3\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/Sqrt[1 + x^3],x]

[Out] c*x*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] + (d*x^2*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3])/2

Maple [A] time = 0.004, size = 291, normalized size = 1.2

$$2 \frac{d(3/2 - i/2\sqrt{3})}{\sqrt{x^3 + 1}} \sqrt{\frac{1+x}{3/2 - i/2\sqrt{3}}} \sqrt{\frac{x - 1/2 - i/2\sqrt{3}}{-3/2 - i/2\sqrt{3}}} \sqrt{\frac{x - 1/2 + i/2\sqrt{3}}{-3/2 + i/2\sqrt{3}}} \left((-3/2 - i/2\sqrt{3}) \text{EllipticE} \left(\sqrt{\frac{1+x}{3/2 - i/2\sqrt{3}}}, \sqrt{\frac{-3/2 - i/2\sqrt{3}}{-3/2 - i/2\sqrt{3}}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)/(x^3+1)^(1/2),x)

[Out] 2*d*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))*EllipticE(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*3^(1/2))*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))+2*c*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(x^3+1)^(1/2),x, algorithm="maxima")

[Out] integrate((d*x + c)/sqrt(x^3 + 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx+c}{\sqrt{x^3+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(x^3+1)^(1/2),x, algorithm="fricas")

[Out] integral((d*x + c)/sqrt(x^3 + 1), x)

Sympy [A] time = 1.38533, size = 61, normalized size = 0.25

$$\frac{cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{4}{3}, x^3 e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{2}{3}, \frac{2}{3} \middle| \frac{5}{3}, x^3 e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(x**3+1)**(1/2),x)

[Out] c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(5/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{\sqrt{x^3+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(x^3+1)^(1/2),x, algorithm="giac")

[Out] integrate((d*x + c)/sqrt(x^3 + 1), x)

$$3.112 \quad \int \frac{c+dx}{\sqrt{1-x^3}} dx$$

Optimal. Leaf size=271

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{(-x+\sqrt{3}+1)^2}}(c-\sqrt{3}d+d)\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right), -7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{(-x+\sqrt{3}+1)^2}}\sqrt{1-x^3}} + \frac{2d\sqrt{1-x^3}}{-x+\sqrt{3}+1} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d}{\sqrt{1-x^3}}$$

[Out] (2*d*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) - (2*Sqrt[2 + Sqrt[3]]*(c + d - Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])

Rubi [A] time = 0.0873074, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1878, 218, 1877}

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{(-x+\sqrt{3}+1)^2}}(c-\sqrt{3}d+d)F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right) \middle| -7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{(-x+\sqrt{3}+1)^2}}\sqrt{1-x^3}} + \frac{2d\sqrt{1-x^3}}{-x+\sqrt{3}+1} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d(1-x)}{\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)/Sqrt[1 - x^3], x]

[Out] (2*d*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) - (2*Sqrt[2 + Sqrt[3]]*(c + d - Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c + dx}{\sqrt{1-x^3}} dx = -\left(d \int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}} dx\right) + (c+d-\sqrt{3}d) \int \frac{1}{\sqrt{1-x^3}} dx$$

$$= \frac{2d\sqrt{1-x^3}}{1+\sqrt{3}-x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d(1-x)\sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)\middle| -7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}}\sqrt{1-x^3}} - \frac{2\sqrt{2+\sqrt{3}}(c+d-\sqrt{3}d)}{\sqrt{1-x^3}}$$

Mathematica [C] time = 0.0093198, size = 38, normalized size = 0.14

$$cx {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; x^3\right) + \frac{1}{2}dx^2 {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; x^3\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/Sqrt[1 - x^3],x]

[Out] c*x*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] + (d*x^2*Hypergeometric2F1[1/2, 2/3, 5/3, x^3])/2

Maple [A] time = 0.005, size = 267, normalized size = 1.

$$-\frac{2i}{3}d\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{3}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}\sqrt{3}\left(-\frac{3}{2}+\frac{i}{2}\sqrt{3}\right)\text{EllipticE}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)/(-x^3+1)^(1/2),x)

[Out]
$$\begin{aligned} & -2/3*I*d*3^{(1/2)}*(I*(x+1/2-1/2*I*3^{(1/2)})*3^{(1/2)})^{(1/2)}*((-1+x)/(-3/2+1/2*I*3^{(1/2)}))^{(1/2)}*(-I*(x+1/2+1/2*I*3^{(1/2)})*3^{(1/2)})^{(1/2)}/(-x^3+1)^{(1/2)}* \\ & (-3/2+1/2*I*3^{(1/2)})*EllipticE(1/3*3^{(1/2)}*(I*(x+1/2-1/2*I*3^{(1/2)})*3^{(1/2)})^{(1/2)},(I*3^{(1/2)}/(-3/2+1/2*I*3^{(1/2)}))^{(1/2)})+EllipticF(1/3*3^{(1/2)}*(I*(x+1/2-1/2*I*3^{(1/2)})*3^{(1/2)})^{(1/2)},(I*3^{(1/2)}/(-3/2+1/2*I*3^{(1/2)}))^{(1/2)}) \\ & -2/3*I*c*3^{(1/2)}*(I*(x+1/2-1/2*I*3^{(1/2)})*3^{(1/2)})^{(1/2)}*((-1+x)/(-3/2+1/2*I*3^{(1/2)}))^{(1/2)}*(-I*(x+1/2+1/2*I*3^{(1/2)})*3^{(1/2)})^{(1/2)}/(-x^3+1)^{(1/2)}* \\ & EllipticF(1/3*3^{(1/2)}*(I*(x+1/2-1/2*I*3^{(1/2)})*3^{(1/2)})^{(1/2)},(I*3^{(1/2)}/(-3/2+1/2*I*3^{(1/2)}))^{(1/2)}) \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{-x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-x^3+1)^(1/2),x, algorithm="maxima")

[Out] integrate((d*x + c)/sqrt(-x^3 + 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^3+1}(dx+c)}{x^3-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-x^3+1)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-x^3 + 1)*(d*x + c)/(x^3 - 1), x)

Sympy [A] time = 1.4608, size = 65, normalized size = 0.24

$$\frac{cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{4}{3} \middle| x^3 e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{2}{3}, \frac{2}{3} \middle| \frac{5}{3} \middle| x^3 e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-x**3+1)**(1/2),x)

[Out] c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamma(5/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{\sqrt{-x^3+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-x^3+1)^(1/2),x, algorithm="giac")

[Out] integrate((d*x + c)/sqrt(-x^3 + 1), x)

$$3.113 \quad \int \frac{c+dx}{\sqrt{-1+x^3}} dx$$

Optimal. Leaf size=275

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{(-x-\sqrt{3}+1)^2}}(c+\sqrt{3}d+d)\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{(-x-\sqrt{3}+1)^2}}\sqrt{x^3-1}} - \frac{2d\sqrt{x^3-1}}{-x-\sqrt{3}+1} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}d(1-x)}{\sqrt{x^3-1}}$$

[Out] $(-2*d*\operatorname{Sqrt}[-1+x^3])/(1-\operatorname{Sqrt}[3]-x) + (3^{1/4}*\operatorname{Sqrt}[2+\operatorname{Sqrt}[3]]*d*(1-x)*\operatorname{Sqrt}[(1+x+x^2)/(1-\operatorname{Sqrt}[3]-x)^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[(1+\operatorname{Sqrt}[3]-x)/(1-\operatorname{Sqrt}[3]-x)],-7+4*\operatorname{Sqrt}[3]])/(\operatorname{Sqrt}[-((1-x)/(1-\operatorname{Sqrt}[3]-x)^2)]*\operatorname{Sqrt}[-1+x^3]) - (2*\operatorname{Sqrt}[2-\operatorname{Sqrt}[3]]*(c+d+\operatorname{Sqrt}[3]*d)*(1-x)*\operatorname{Sqrt}[(1+x+x^2)/(1-\operatorname{Sqrt}[3]-x)^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[(1+\operatorname{Sqrt}[3]-x)/(1-\operatorname{Sqrt}[3]-x)],-7+4*\operatorname{Sqrt}[3]])/(3^{1/4}*\operatorname{Sqrt}[-((1-x)/(1-\operatorname{Sqrt}[3]-x)^2)]*\operatorname{Sqrt}[-1+x^3])$

Rubi [A] time = 0.0873031, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1880, 219, 1879}

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{(-x-\sqrt{3}+1)^2}}(c+\sqrt{3}d+d)F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)\middle| -7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{(-x-\sqrt{3}+1)^2}}\sqrt{x^3-1}} - \frac{2d\sqrt{x^3-1}}{-x-\sqrt{3}+1} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}d(1-x)}{\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c+d*x)/\operatorname{Sqrt}[-1+x^3],x]$

[Out] $(-2*d*\operatorname{Sqrt}[-1+x^3])/(1-\operatorname{Sqrt}[3]-x) + (3^{1/4}*\operatorname{Sqrt}[2+\operatorname{Sqrt}[3]]*d*(1-x)*\operatorname{Sqrt}[(1+x+x^2)/(1-\operatorname{Sqrt}[3]-x)^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[(1+\operatorname{Sqrt}[3]-x)/(1-\operatorname{Sqrt}[3]-x)],-7+4*\operatorname{Sqrt}[3]])/(\operatorname{Sqrt}[-((1-x)/(1-\operatorname{Sqrt}[3]-x)^2)]*\operatorname{Sqrt}[-1+x^3]) - (2*\operatorname{Sqrt}[2-\operatorname{Sqrt}[3]]*(c+d+\operatorname{Sqrt}[3]*d)*(1-x)*\operatorname{Sqrt}[(1+x+x^2)/(1-\operatorname{Sqrt}[3]-x)^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[(1+\operatorname{Sqrt}[3]-x)/(1-\operatorname{Sqrt}[3]-x)],-7+4*\operatorname{Sqrt}[3]])/(3^{1/4}*\operatorname{Sqrt}[-((1-x)/(1-\operatorname{Sqrt}[3]-x)^2)]*\operatorname{Sqrt}[-1+x^3])$

Rule 1880

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + Simp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c + dx}{\sqrt{-1 + x^3}} dx = -\left(d \int \frac{1 + \sqrt{3} - x}{\sqrt{-1 + x^3}} dx\right) + (c + d + \sqrt{3}d) \int \frac{1}{\sqrt{-1 + x^3}} dx$$

$$= -\frac{2d\sqrt{-1 + x^3}}{1 - \sqrt{3} - x} + \frac{\sqrt[4]{3}\sqrt{2 + \sqrt{3}}d(1 - x)\sqrt{\frac{1+x^2}{(1-\sqrt{3}-x)^2}}E\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right) \mid -7 + 4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}}\sqrt{-1 + x^3}} - \frac{2\sqrt{2 - \sqrt{3}}(c + d)}{\sqrt{-1 + x^3}}$$

Mathematica [C] time = 0.021755, size = 58, normalized size = 0.21

$$\frac{x\sqrt{1-x^3}\left(2c {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; x^3\right) + dx {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; x^3\right)\right)}{2\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/Sqrt[-1 + x^3],x]

[Out] (x*Sqrt[1 - x^3]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] + d*x*Hypergeometric2F1[1/2, 2/3, 5/3, x^3]))/(2*Sqrt[-1 + x^3])

Maple [A] time = 0.004, size = 291, normalized size = 1.1

$$2 \frac{d(-3/2 - i/2\sqrt{3})}{\sqrt{x^3 - 1}} \sqrt{\frac{-1 + x}{-3/2 - i/2\sqrt{3}}} \sqrt{\frac{x + 1/2 - i/2\sqrt{3}}{3/2 - i/2\sqrt{3}}} \sqrt{\frac{x + 1/2 + i/2\sqrt{3}}{3/2 + i/2\sqrt{3}}} \left((3/2 - i/2\sqrt{3}) \text{EllipticE} \left(\sqrt{\frac{-1 + x}{-3/2 - i/2\sqrt{3}}}, \sqrt{\frac{3}{3/2 - i/2\sqrt{3}}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)/(x^3-1)^(1/2),x)

[Out] 2*d*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*((3/2-1/2*I*3^(1/2))*EllipticE(((x+1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))+(-1/2+1/2*I*3^(1/2))*EllipticF(((x+1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))+2*c*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x+1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{x^3 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(x^3-1)^(1/2),x, algorithm="maxima")

[Out] integrate((d*x + c)/sqrt(x^3 - 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx + c}{\sqrt{x^3 - 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(x^3-1)^(1/2),x, algorithm="fricas")

[Out] integral((d*x + c)/sqrt(x^3 - 1), x)

Sympy [A] time = 1.42982, size = 56, normalized size = 0.2

$$-\frac{icx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{4}{3}, x^3\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{idx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{2}{3}, \frac{2}{3} \middle| \frac{5}{3}, x^3\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(x**3-1)**(1/2),x)

[Out] -I*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) - I*d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3)/(3*gamma(5/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{x^3 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(x^3-1)^(1/2),x, algorithm="giac")

[Out] integrate((d*x + c)/sqrt(x^3 - 1), x)

$$3.114 \quad \int \frac{c+dx}{\sqrt{-1-x^3}} dx$$

Optimal. Leaf size=261

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x-\sqrt{3}+1)^2}}(c-(1+\sqrt{3})d)\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right), 4\sqrt{3}-7\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{(x-\sqrt{3}+1)^2}}\sqrt{-x^3-1}} - \frac{2d\sqrt{-x^3-1}}{x-\sqrt{3}+1} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}d(x+1)}{\sqrt{-x^3-1}}$$

[Out] $(-2*d*\operatorname{Sqrt}[-1-x^3])/(1-\operatorname{Sqrt}[3]+x) + (3^{1/4}*\operatorname{Sqrt}[2+\operatorname{Sqrt}[3]]*d*(1+x)*\operatorname{Sqrt}[(1-x+x^2)/(1-\operatorname{Sqrt}[3]+x)^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[(1+\operatorname{Sqrt}[3]+x)/(1-\operatorname{Sqrt}[3]+x)], -7+4*\operatorname{Sqrt}[3]])/(\operatorname{Sqrt}[-((1+x)/(1-\operatorname{Sqrt}[3]+x)^2)]*\operatorname{Sqrt}[-1-x^3]) + (2*\operatorname{Sqrt}[2-\operatorname{Sqrt}[3]]*(c-(1+\operatorname{Sqrt}[3])*d)*(1+x)*\operatorname{Sqrt}[(1-x+x^2)/(1-\operatorname{Sqrt}[3]+x)^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[(1+\operatorname{Sqrt}[3]+x)/(1-\operatorname{Sqrt}[3]+x)], -7+4*\operatorname{Sqrt}[3]])/(3^{1/4}*\operatorname{Sqrt}[-((1+x)/(1-\operatorname{Sqrt}[3]+x)^2)]*\operatorname{Sqrt}[-1-x^3])$

Rubi [A] time = 0.0777453, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1880, 219, 1879}

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x-\sqrt{3}+1)^2}}(c-(1+\sqrt{3})d)F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right) \middle| -7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{(x-\sqrt{3}+1)^2}}\sqrt{-x^3-1}} - \frac{2d\sqrt{-x^3-1}}{x-\sqrt{3}+1} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}d(x+1)\sqrt{-x^3-1}}{\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c+d*x)/\operatorname{Sqrt}[-1-x^3], x]$

[Out] $(-2*d*\operatorname{Sqrt}[-1-x^3])/(1-\operatorname{Sqrt}[3]+x) + (3^{1/4}*\operatorname{Sqrt}[2+\operatorname{Sqrt}[3]]*d*(1+x)*\operatorname{Sqrt}[(1-x+x^2)/(1-\operatorname{Sqrt}[3]+x)^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[(1+\operatorname{Sqrt}[3]+x)/(1-\operatorname{Sqrt}[3]+x)], -7+4*\operatorname{Sqrt}[3]])/(\operatorname{Sqrt}[-((1+x)/(1-\operatorname{Sqrt}[3]+x)^2)]*\operatorname{Sqrt}[-1-x^3]) + (2*\operatorname{Sqrt}[2-\operatorname{Sqrt}[3]]*(c-(1+\operatorname{Sqrt}[3])*d)*(1+x)*\operatorname{Sqrt}[(1-x+x^2)/(1-\operatorname{Sqrt}[3]+x)^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[(1+\operatorname{Sqrt}[3]+x)/(1-\operatorname{Sqrt}[3]+x)], -7+4*\operatorname{Sqrt}[3]])/(3^{1/4}*\operatorname{Sqrt}[-((1+x)/(1-\operatorname{Sqrt}[3]+x)^2)]*\operatorname{Sqrt}[-1-x^3])$

Rule 1880

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + Simp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c + dx}{\sqrt{-1 - x^3}} dx = d \int \frac{1 + \sqrt{3} + x}{\sqrt{-1 - x^3}} dx + (c - (1 + \sqrt{3})d) \int \frac{1}{\sqrt{-1 - x^3}} dx$$

$$= -\frac{2d\sqrt{-1 - x^3}}{1 - \sqrt{3} + x} + \frac{\sqrt[4]{3}\sqrt{2 + \sqrt{3}}d(1 + x)\sqrt{\frac{1 - x^2}{(1 - \sqrt{3} + x)^2}}E\left(\sin^{-1}\left(\frac{1 + \sqrt{3} + x}{1 - \sqrt{3} + x}\right) \mid -7 + 4\sqrt{3}\right)}{\sqrt{-\frac{1 + x}{(1 - \sqrt{3} + x)^2}}\sqrt{-1 - x^3}} + \frac{2\sqrt{2 - \sqrt{3}}(c - (1 + \sqrt{3})d)}{\sqrt{-1 - x^3}}$$

Mathematica [C] time = 0.0220398, size = 62, normalized size = 0.24

$$\frac{x\sqrt{x^3 + 1} \left(2c {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -x^3\right) + dx {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -x^3\right) \right)}{2\sqrt{-x^3 - 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/Sqrt[-1 - x^3],x]

[Out] (x*Sqrt[1 + x^3]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] + d*x*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3]))/(2*Sqrt[-1 - x^3])

Maple [A] time = 0.004, size = 269, normalized size = 1.

$$-\frac{2i}{3}d\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{3}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}\sqrt{3}\left(\frac{3}{2}+\frac{i}{2}\sqrt{3}\right)\text{EllipticE}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{3},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)/(-x^3-1)^(1/2),x)

[Out] $-2/3*I*d*3^{(1/2)}*(I*(x-1/2-1/2*I*3^{(1/2)})*3^{(1/2)})^{(1/2)}*((1+x)/(3/2+1/2*I*3^{(1/2)}))^{(1/2)}*(-I*(x-1/2+1/2*I*3^{(1/2)})*3^{(1/2)})^{(1/2)}/(-x^3-1)^{(1/2)}*((3/2+1/2*I*3^{(1/2)})*\text{EllipticE}(1/3*3^{(1/2)}*(I*(x-1/2-1/2*I*3^{(1/2)})*3^{(1/2)})^{(1/2)},(I*3^{(1/2)}/(3/2+1/2*I*3^{(1/2)}))^{(1/2)})-\text{EllipticF}(1/3*3^{(1/2)}*(I*(x-1/2-1/2*I*3^{(1/2)})*3^{(1/2)})^{(1/2)},(I*3^{(1/2)}/(3/2+1/2*I*3^{(1/2)}))^{(1/2)}))-2/3*I*c*3^{(1/2)}*(I*(x-1/2-1/2*I*3^{(1/2)})*3^{(1/2)})^{(1/2)}*((1+x)/(3/2+1/2*I*3^{(1/2)}))^{(1/2)}*(-I*(x-1/2+1/2*I*3^{(1/2)})*3^{(1/2)})^{(1/2)}/(-x^3-1)^{(1/2)}*\text{EllipticF}(1/3*3^{(1/2)}*(I*(x-1/2-1/2*I*3^{(1/2)})*3^{(1/2)})^{(1/2)},(I*3^{(1/2)}/(3/2+1/2*I*3^{(1/2)}))^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{-x^3 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-x^3-1)^(1/2),x, algorithm="maxima")

[Out] integrate((d*x + c)/sqrt(-x^3 - 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^3-1}(dx+c)}{x^3+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-x^3-1)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-x^3 - 1)*(d*x + c)/(x^3 + 1), x)

Sympy [A] time = 1.44814, size = 66, normalized size = 0.25

$$-\frac{icx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}\left|\frac{4}{3}\right|x^3 e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{idx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{2}{3}, \frac{2}{3}\left|\frac{5}{3}\right|x^3 e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-x**3-1)**(1/2),x)

[Out] -I*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3)) - I*d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(5/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{\sqrt{-x^3-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-x^3-1)^(1/2),x, algorithm="giac")

[Out] integrate((d*x + c)/sqrt(-x^3 - 1), x)

$$3.115 \quad \int \frac{c+dx}{a-bx^4} dx$$

Optimal. Leaf size=87

$$\frac{c \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}} + \frac{c \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

[Out] (c*ArcTan[(b^(1/4)*x)/a^(1/4)]/(2*a^(3/4)*b^(1/4)) + (c*ArcTanh[(b^(1/4)*x)/a^(1/4)]/(2*a^(3/4)*b^(1/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]))

Rubi [A] time = 0.0654591, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1876, 212, 208, 205, 275}

$$\frac{c \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}} + \frac{c \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)/(a - b*x^4), x]

[Out] (c*ArcTan[(b^(1/4)*x)/a^(1/4)]/(2*a^(3/4)*b^(1/4)) + (c*ArcTanh[(b^(1/4)*x)/a^(1/4)]/(2*a^(3/4)*b^(1/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]))

Rule 1876

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x] /; FreeQ[{a, b}, x] && !GtQ

[a/b, 0]

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx}{a - bx^4} dx &= \int \left(\frac{c}{a - bx^4} + \frac{dx}{a - bx^4} \right) dx \\
&= c \int \frac{1}{a - bx^4} dx + d \int \frac{x}{a - bx^4} dx \\
&= \frac{c \int \frac{1}{\sqrt{a} - \sqrt{bx^2}} dx}{2\sqrt{a}} + \frac{c \int \frac{1}{\sqrt{a} + \sqrt{bx^2}} dx}{2\sqrt{a}} + \frac{1}{2} d \text{Subst} \left(\int \frac{1}{a - bx^2} dx, x, x^2 \right) \\
&= \frac{c \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2a^{3/4} \sqrt[4]{b}} + \frac{c \tanh^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2a^{3/4} \sqrt[4]{b}} + \frac{d \tanh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.0353374, size = 134, normalized size = 1.54

$$\frac{-\left(\sqrt[4]{ad} + \sqrt[4]{bc}\right) \log\left(\sqrt[4]{a} - \sqrt[4]{bx}\right) + \sqrt[4]{bc} \log\left(\sqrt[4]{a} + \sqrt[4]{bx}\right) + 2\sqrt[4]{bc} \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) + \sqrt[4]{ad} \log\left(\sqrt{a} + \sqrt{bx^2}\right) - \sqrt[4]{ad} \log\left(\sqrt[4]{a} + \sqrt[4]{bx}\right)}{4a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a - b*x^4), x]
```

[Out] $(2*b^{(1/4)}*c*ArcTan[(b^{(1/4)}*x)/a^{(1/4)}] - (b^{(1/4)}*c + a^{(1/4)}*d)*Log[a^{(1/4)} - b^{(1/4)}*x] + b^{(1/4)}*c*Log[a^{(1/4)} + b^{(1/4)}*x] - a^{(1/4)}*d*Log[a^{(1/4)} + b^{(1/4)}*x] + a^{(1/4)}*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(4*a^{(3/4)}*Sqrt[b])$

Maple [A] time = 0.004, size = 101, normalized size = 1.2

$$\frac{c}{4a} \sqrt[4]{\frac{a}{b}} \ln \left(\left(x + \sqrt[4]{\frac{a}{b}} \right) \left(x - \sqrt[4]{\frac{a}{b}} \right)^{-1} \right) + \frac{c}{2a} \sqrt[4]{\frac{a}{b}} \arctan \left(x \frac{1}{\sqrt[4]{\frac{a}{b}}} \right) - \frac{d}{4} \ln \left(\left(-a + x^2 \sqrt{ab} \right) \left(-a - x^2 \sqrt{ab} \right)^{-1} \right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)/(-b*x^4+a),x)`

[Out] $1/4*c*(1/b*a)^{(1/4)}/a*\ln((x+(1/b*a)^{(1/4)})/(x-(1/b*a)^{(1/4)}))+1/2*c*(1/b*a)^{(1/4)}/a*\arctan(x/(1/b*a)^{(1/4)})-1/4*d/(a*b)^{(1/2)}*\ln((-a+x^2*(a*b)^{(1/2)})/(-a-x^2*(a*b)^{(1/2)}))$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)/(-b*x^4+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)/(-b*x^4+a),x, algorithm="fricas")`

[Out] Timed out

Sympy [A] time = 0.770489, size = 126, normalized size = 1.45

$$-\text{RootSum}\left(256t^4a^3b^2 - 32t^2a^2bd^2 - 16tabc^2d + ad^4 - bc^4, \left(t \mapsto t \log\left(x + \frac{-128t^3a^3bd^2 + 16t^2a^2bc^2d + 8ta^2d^4 - 4tabc^2d}{4acd^4 + bc^5}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-b*x**4+a),x)

[Out] -RootSum(256*_t**4*a**3*b**2 - 32*_t**2*a**2*b*d**2 - 16*_t*a*b*c**2*d + a*d**4 - b*c**4, Lambda(_t, _t*log(x + (-128*_t**3*a**3*b*d**2 + 16*_t**2*a**2*b*c**2*d + 8*_t*a**2*d**4 - 4*_t*a*b*c**4 + 5*a*c**2*d**3)/(4*a*c*d**4 + b*c**5))))

Giac [B] time = 1.09158, size = 304, normalized size = 3.49

$$\frac{\sqrt{2}(-ab^3)^{\frac{1}{4}}c \log\left(x^2 + \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{8ab} - \frac{\sqrt{2}(-ab^3)^{\frac{1}{4}}c \log\left(x^2 - \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{8ab} + \frac{\sqrt{2}\left(\sqrt{2}\sqrt{-abbd} + (-ab^3)^{\frac{1}{4}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-b*x^4+a),x, algorithm="giac")

[Out] 1/8*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b) - 1/8*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b) + 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b*d + (-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^2) + 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b*d + (-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^2)

$$3.116 \quad \int \frac{c+dx}{a+bx^4} dx$$

Optimal. Leaf size=219

$$-\frac{c \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{c \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}\sqrt[4]{b}} - \frac{c \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{c \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} +$$

[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - (c*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) + (c*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) - (c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)) + (c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4))

Rubi [A] time = 0.170693, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$, Rules used = {1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$-\frac{c \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{c \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}\sqrt[4]{b}} - \frac{c \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{c \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} +$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)/(a + b*x^4), x]

[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - (c*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) + (c*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) - (c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)) + (c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4))

Rule 1876

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```


$\wedge k], x] /; k \neq 1] /; \text{FreeQ}[\{a, b, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[m]$

Rule 205

$\text{Int}[\frac{(a_ + (b_)*(x_)^2)^{-1}}{a, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b]$

Rubi steps

$$\begin{aligned} \int \frac{c + dx}{a + bx^4} dx &= \int \left(\frac{c}{a + bx^4} + \frac{dx}{a + bx^4} \right) dx \\ &= c \int \frac{1}{a + bx^4} dx + d \int \frac{x}{a + bx^4} dx \\ &= \frac{c \int \frac{\sqrt{a} - \sqrt{bx^2}}{a + bx^4} dx}{2\sqrt{a}} + \frac{c \int \frac{\sqrt{a} + \sqrt{bx^2}}{a + bx^4} dx}{2\sqrt{a}} + \frac{1}{2} d \text{Subst} \left(\int \frac{1}{a + bx^2} dx, x, x^2 \right) \\ &= \frac{d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{b}} + \frac{c \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{b}} + x^2} dx}{4\sqrt{a}\sqrt{b}} + \frac{c \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{b}} + x^2} dx}{4\sqrt{a}\sqrt{b}} - \frac{c \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}} + 2x}{-\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{b}} - x^2} dx}{4\sqrt{2}a^{3/4}\sqrt[4]{b}} - \frac{c \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}} - 2x}{-\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{b}} - x^2} dx}{4\sqrt{2}a^{3/4}\sqrt[4]{b}} \\ &= \frac{d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{b}} - \frac{c \log \left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2} \right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{c \log \left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2} \right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{c \text{Subst} \left(\int \frac{1}{-1 + bx^2} dx, x, x^2 \right)}{2\sqrt{a}\sqrt{b}} \\ &= \frac{d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{b}} - \frac{c \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{c \tan^{-1} \left(1 + \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} - \frac{c \log \left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2} \right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{c \log \left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2} \right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}} \end{aligned}$$

Mathematica [A] time = 0.0819112, size = 184, normalized size = 0.84

$$\frac{-2 \left(2\sqrt[4]{ad} + \sqrt{2}\sqrt[4]{bc} \right) \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} \right) + 2 \left(\sqrt{2}\sqrt[4]{bc} - 2\sqrt[4]{ad} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + 1 \right) + \sqrt{2}\sqrt[4]{bc} \left(\log \left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2} \right) - \log \left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} - \sqrt{bx^2} \right) \right)}{8a^{3/4}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/(a + b*x^4), x]

[Out] (-2*(Sqrt[2]*b^(1/4)*c + 2*a^(1/4)*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*b^(1/4)*c - 2*a^(1/4)*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*b^(1/4)*c*(-Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[

$b] * x^2] + \text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2] * a^{(1/4)} * b^{(1/4)} * x + \text{Sqrt}[b] * x^2]) / (8 * a^{(3/4)} * \text{Sqrt}[b])$

Maple [A] time = 0.003, size = 151, normalized size = 0.7

$$\frac{c\sqrt{2}}{8a} \sqrt[4]{\frac{a}{b}} \ln \left(\left(x^2 + \sqrt[4]{\frac{a}{b}} x \sqrt{2} + \sqrt{\frac{a}{b}} \right) \left(x^2 - \sqrt[4]{\frac{a}{b}} x \sqrt{2} + \sqrt{\frac{a}{b}} \right)^{-1} \right) + \frac{c\sqrt{2}}{4a} \sqrt[4]{\frac{a}{b}} \arctan \left(x \sqrt{2} \frac{1}{\sqrt[4]{\frac{a}{b}}} + 1 \right) + \frac{c\sqrt{2}}{4a} \sqrt[4]{\frac{a}{b}} \arctan \left(x \sqrt{2} \frac{1}{\sqrt[4]{\frac{a}{b}}} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)/(b*x^4+a),x)`

[Out] $\frac{1}{8} * c * (1/b * a)^{(1/4)} / a * 2^{(1/2)} * \ln((x^2 + (1/b * a)^{(1/4)} * x * 2^{(1/2)} + (1/b * a)^{(1/2)}) / (x^2 - (1/b * a)^{(1/4)} * x * 2^{(1/2)} + (1/b * a)^{(1/2)})) + 1/4 * c * (1/b * a)^{(1/4)} / a * 2^{(1/2)} * \arctan(2^{(1/2)} / (1/b * a)^{(1/4)} * x + 1) + 1/4 * c * (1/b * a)^{(1/4)} / a * 2^{(1/2)} * \arctan(2^{(1/2)} / (1/b * a)^{(1/4)} * x - 1) + 1/2 * d / (a * b)^{(1/2)} * \arctan(x^2 * (b/a)^{(1/2)})$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)/(b*x^4+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)/(b*x^4+a),x, algorithm="fricas")`

[Out] Timed out

Sympy [A] time = 0.758778, size = 124, normalized size = 0.57

$$\text{RootSum}\left(256t^4a^3b^2 + 32t^2a^2bd^2 - 16tabc^2d + ad^4 + bc^4, \left(t \mapsto t \log\left(x + \frac{-128t^3a^3bd^2 - 16t^2a^2bc^2d - 8ta^2d^4 - 4tabc^2d}{4acd^4 - bc^5}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x**4+a),x)

[Out] RootSum(256*_t**4*a**3*b**2 + 32*_t**2*a**2*b*d**2 - 16*_t*a*b*c**2*d + a*d**4 + b*c**4, Lambda(_t, _t*log(x + (-128*_t**3*a**3*b*d**2 - 16*_t**2*a**2*b*c**2*d - 8*_t*a**2*d**4 - 4*_t*a*b*c**4 + 5*a*c**2*d**3)/(4*a*c*d**4 - b*c**5))))

Giac [A] time = 1.08081, size = 288, normalized size = 1.32

$$\frac{\sqrt{2}(ab^3)^{\frac{1}{4}}c \log\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8ab} - \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}c \log\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8ab} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{abbd} - (ab^3)^{\frac{1}{4}}bc\right) \arctan\left(\frac{x + \sqrt{\frac{a}{b}}}{x - \sqrt{\frac{a}{b}}}\right)}{4ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x^4+a),x, algorithm="giac")

[Out] 1/8*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b) - 1/8*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b*d - (a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^2) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b*d - (a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^2)

$$3.117 \quad \int \frac{c+dx}{(a-bx^4)^2} dx$$

Optimal. Leaf size=110

$$\frac{3c \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}} + \frac{3c \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} + \frac{x(c+dx)}{4a(a-bx^4)}$$

[Out] (x*(c + d*x))/(4*a*(a - b*x^4)) + (3*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(1/4)) + (3*c*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(1/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b])

Rubi [A] time = 0.0816042, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1855, 1876, 212, 208, 205, 275}

$$\frac{3c \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}} + \frac{3c \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} + \frac{x(c+dx)}{4a(a-bx^4)}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)/(a - b*x^4)^2, x]

[Out] (x*(c + d*x))/(4*a*(a - b*x^4)) + (3*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(1/4)) + (3*c*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(1/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b])

Rule 1855

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] & & PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]

Rule 1876

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,

0] && Expon[Pq, x] < n

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{c+dx}{(a-bx^4)^2} dx &= \frac{x(c+dx)}{4a(a-bx^4)} - \frac{\int \frac{-3c-2dx}{a-bx^4} dx}{4a} \\
&= \frac{x(c+dx)}{4a(a-bx^4)} - \frac{\int \left(-\frac{3c}{a-bx^4} - \frac{2dx}{a-bx^4} \right) dx}{4a} \\
&= \frac{x(c+dx)}{4a(a-bx^4)} + \frac{(3c) \int \frac{1}{a-bx^4} dx}{4a} + \frac{d \int \frac{x}{a-bx^4} dx}{2a} \\
&= \frac{x(c+dx)}{4a(a-bx^4)} + \frac{(3c) \int \frac{1}{\sqrt{a}-\sqrt{bx^2}} dx}{8a^{3/2}} + \frac{(3c) \int \frac{1}{\sqrt{a}+\sqrt{bx^2}} dx}{8a^{3/2}} + \frac{d \operatorname{Subst} \left(\int \frac{1}{a-bx^2} dx, x, x^2 \right)}{4a} \\
&= \frac{x(c+dx)}{4a(a-bx^4)} + \frac{3c \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{8a^{7/4} \sqrt[4]{b}} + \frac{3c \tanh^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{8a^{7/4} \sqrt[4]{b}} + \frac{d \tanh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{4a^{3/2} \sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.160481, size = 168, normalized size = 1.53

$$\frac{\frac{4ax(c+dx)}{a-bx^4} - \frac{(3\sqrt[4]{a}\sqrt[4]{bc}+2\sqrt{ad})\log\left(\frac{\sqrt[4]{a}-\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{\sqrt{b}} + \frac{(3\sqrt[4]{a}\sqrt[4]{bc}-2\sqrt{ad})\log\left(\frac{\sqrt[4]{a}+\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{\sqrt{b}} + \frac{6\sqrt[4]{ac}\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}} + \frac{2\sqrt{ad}\log(\sqrt{a}+\sqrt{bx^2})}{\sqrt{b}}}{16a^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/(a - b*x^4)^2, x]

[Out] ((4*a*x*(c + d*x))/(a - b*x^4) + (6*a^(1/4)*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/b^(1/4) - ((3*a^(1/4)*b^(1/4)*c + 2*Sqrt[a]*d)*Log[a^(1/4) - b^(1/4)*x])/Sqrt[b] + ((3*a^(1/4)*b^(1/4)*c - 2*Sqrt[a]*d)*Log[a^(1/4) + b^(1/4)*x])/Sqrt[b] + (2*Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(16*a^2)

Maple [A] time = 0.004, size = 142, normalized size = 1.3

$$-\frac{cx}{4a(bx^4-a)} + \frac{3c}{16a^2} \sqrt[4]{\frac{a}{b}} \ln \left(\left(x + \sqrt[4]{\frac{a}{b}} \right) \left(x - \sqrt[4]{\frac{a}{b}} \right)^{-1} \right) + \frac{3c}{8a^2} \sqrt[4]{\frac{a}{b}} \arctan \left(x \frac{1}{\sqrt[4]{\frac{a}{b}}} \right) - \frac{dx^2}{4a(bx^4-a)} - \frac{d}{8a} \ln \left(\left(-a + x^2 \sqrt{ab} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)/(-b*x^4+a)^2,x)`

[Out]
$$-1/4*c*x/a/(b*x^4-a)+3/16*c/a^2*(1/b*a)^{1/4}*ln((x+(1/b*a)^{1/4})/(x-(1/b*a)^{1/4}))+3/8*c/a^2*(1/b*a)^{1/4}*arctan(x/(1/b*a)^{1/4})-1/4*d*x^2/a/(b*x^4-a)-1/8*d/a/(a*b)^{1/2}*ln((-a+x^2*(a*b)^{1/2})/(-a-x^2*(a*b)^{1/2}))$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")`

[Out] Timed out

Sympy [A] time = 1.27555, size = 155, normalized size = 1.41

$$\text{RootSum}\left(65536t^4a^7b^2 - 2048t^2a^4bd^2 + 1152ta^2bc^2d + 16ad^4 - 81bc^4, \left(t \mapsto t \log\left(x + \frac{32768t^3a^6bd^2 + 4608t^2a^4bc^2d}{192ac}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)/(-b*x**4+a)**2,x)`

[Out]
$$\text{RootSum}(65536*_t^{**4}*a^{**7}*b^{**2} - 2048*_t^{**2}*a^{**4}*b*d^{**2} + 1152*_t*a^{**2}*b*c^{**2}*d + 16*a*d^{**4} - 81*b*c^{**4}, \text{Lambda}(_t, _t*\log(x + (32768*_t^{**3}*a^{**6}*b*d^{**2}$$

+ 4608*_t**2*a**4*b*c**2*d - 512*_t*a**3*d**4 + 1296*_t*a**2*b*c**4 + 360*a*c**2*d**3)/(192*a*c*d**4 + 243*b*c**5)))) - (c*x + d*x**2)/(-4*a**2 + 4*a*b*x**4)

Giac [B] time = 1.09117, size = 343, normalized size = 3.12

$$\frac{3\sqrt{2}(-ab^3)^{\frac{1}{4}}c\log\left(x^2 + \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{32a^2b} - \frac{3\sqrt{2}(-ab^3)^{\frac{1}{4}}c\log\left(x^2 - \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{32a^2b} - \frac{dx^2 + cx}{4(bx^4 - a)a} - \frac{\sqrt{2}\left(2\sqrt{2}\right)}{4(bx^4 - a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")

[Out] 3/32*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b) - 3/32*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b) - 1/4*(d*x^2 + c*x)/((b*x^4 - a)*a) - 1/16*sqrt(2)*(2*sqrt(2)*sqrt(-a*b)*b*d - 3*(-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^2) - 1/16*sqrt(2)*(2*sqrt(2)*sqrt(-a*b)*b*d - 3*(-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^2)

$$3.118 \quad \int \frac{c+dx}{(a+bx^4)^2} dx$$

Optimal. Leaf size=241

$$\frac{3c \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{3c \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{b}} - \frac{3c \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{3c \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + \sqrt{2}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}$$

[Out] (x*(c + d*x))/(4*a*(a + b*x^4)) + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b]) - (3*c*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(1/4)) + (3*c*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(1/4)) - (3*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(1/4)) + (3*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(1/4))

Rubi [A] time = 0.201599, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {1855, 1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$\frac{3c \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{3c \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{b}} - \frac{3c \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{3c \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + \sqrt{2}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)/(a + b*x^4)^2, x]

[Out] (x*(c + d*x))/(4*a*(a + b*x^4)) + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b]) - (3*c*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(1/4)) + (3*c*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(1/4)) - (3*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(1/4)) + (3*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(1/4))

Rule 1855

Int[(Pq_)*((a_) + (b_)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &

& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]

Rule 1876

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 1165

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1162

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p_, x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx}{(a+bx^4)^2} dx &= \frac{x(c+dx)}{4a(a+bx^4)} - \frac{\int \frac{-3c-2dx}{a+bx^4} dx}{4a} \\
&= \frac{x(c+dx)}{4a(a+bx^4)} - \frac{\int \left(-\frac{3c}{a+bx^4} - \frac{2dx}{a+bx^4} \right) dx}{4a} \\
&= \frac{x(c+dx)}{4a(a+bx^4)} + \frac{(3c) \int \frac{1}{a+bx^4} dx}{4a} + \frac{d \int \frac{x}{a+bx^4} dx}{2a} \\
&= \frac{x(c+dx)}{4a(a+bx^4)} + \frac{(3c) \int \frac{\sqrt{a}-\sqrt{bx^2}}{a+bx^4} dx}{8a^{3/2}} + \frac{(3c) \int \frac{\sqrt{a}+\sqrt{bx^2}}{a+bx^4} dx}{8a^{3/2}} + \frac{d \text{Subst} \left(\int \frac{1}{a+bx^2} dx, x, x^2 \right)}{4a} \\
&= \frac{x(c+dx)}{4a(a+bx^4)} + \frac{d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{4a^{3/2} \sqrt{b}} + \frac{(3c) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2} \sqrt[4]{ax}}{\sqrt[4]{b}} + x^2} dx}{16a^{3/2} \sqrt{b}} + \frac{(3c) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2} \sqrt[4]{ax}}{\sqrt[4]{b}} + x^2} dx}{16a^{3/2} \sqrt{b}} - \frac{(3c) \int \frac{\frac{\sqrt{2} \sqrt[4]{a}}{\sqrt[4]{b}} + 2x}{-\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2} \sqrt[4]{ax}}{\sqrt[4]{b}}} dx}{16\sqrt{2}a^{7/4} \sqrt[4]{b}} \\
&= \frac{x(c+dx)}{4a(a+bx^4)} + \frac{d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{4a^{3/2} \sqrt{b}} - \frac{3c \log \left(\sqrt{a} - \sqrt{2} \sqrt[4]{a} \sqrt[4]{bx} + \sqrt{bx^2} \right)}{16\sqrt{2}a^{7/4} \sqrt[4]{b}} + \frac{3c \log \left(\sqrt{a} + \sqrt{2} \sqrt[4]{a} \sqrt[4]{bx} + \sqrt{bx^2} \right)}{16\sqrt{2}a^{7/4} \sqrt[4]{b}} \\
&= \frac{x(c+dx)}{4a(a+bx^4)} + \frac{d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{4a^{3/2} \sqrt{b}} - \frac{3c \tan^{-1} \left(1 - \frac{\sqrt{2} \sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{8\sqrt{2}a^{7/4} \sqrt[4]{b}} + \frac{3c \tan^{-1} \left(1 + \frac{\sqrt{2} \sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{8\sqrt{2}a^{7/4} \sqrt[4]{b}} - \frac{3c \log \left(\sqrt{a} - \sqrt{2} \sqrt[4]{a} \sqrt[4]{bx} \right)}{16\sqrt{2}a^{7/4} \sqrt[4]{b}}
\end{aligned}$$

Mathematica [A] time = 0.204034, size = 224, normalized size = 0.93

$$\frac{8a^{3/4}x(c+dx)}{a+bx^4} - \frac{2\left(4\sqrt[4]{ad}+3\sqrt{2}\sqrt[4]{bc}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{\sqrt{b}} + \frac{2\left(3\sqrt{2}\sqrt[4]{bc}-4\sqrt[4]{ad}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}+1\right)}{\sqrt{b}} - \frac{3\sqrt{2}c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}+\sqrt{a}+\sqrt{bx^2}\right)}{\sqrt[4]{b}} + \frac{3\sqrt{2}c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}+\sqrt{a}+\sqrt{bx^2}\right)}{\sqrt[4]{b}}$$

$$32a^{7/4}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/(a + b*x^4)^2, x]

[Out] ((8*a^(3/4)*x*(c + d*x))/(a + b*x^4) - (2*(3*Sqrt[2]*b^(1/4)*c + 4*a^(1/4)*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/Sqrt[b] + (2*(3*Sqrt[2]*b^(1/4)*c - 4*a^(1/4)*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/Sqrt[b] - (3*Sqrt[2]*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4) + (3*Sqrt[2]*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4))

$/(32*a^{(7/4)})$

Maple [A] time = 0.004, size = 188, normalized size = 0.8

$$\frac{cx}{4a(bx^4+a)} + \frac{3c\sqrt{2}}{32a^2} \sqrt[4]{\frac{a}{b}} \ln\left(\left(x^2 + \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x^2 - \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) + \frac{3c\sqrt{2}}{16a^2} \sqrt[4]{\frac{a}{b}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) + \frac{3c\sqrt{2}}{16a^2} \sqrt[4]{\frac{a}{b}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)/(b*x^4+a)^2,x)

[Out] $\frac{1}{4}c*x/a/(b*x^4+a) + \frac{3}{32}c/a^2*(1/b*a)^{(1/4)}*2^{(1/2)}*\ln((x^2+(1/b*a)^{(1/4)}*x*2^{(1/2)}+(1/b*a)^{(1/2)})/(x^2-(1/b*a)^{(1/4)}*x*2^{(1/2)}+(1/b*a)^{(1/2)}))+ \frac{3}{16}c/a^2*(1/b*a)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)}*x+1) + \frac{3}{16}c/a^2*(1/b*a)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)}*x-1) + \frac{1}{4}d*x^2/a/(b*x^4+a) + \frac{1}{4}d/a/(a*b)^{(1/2)}*\arctan(x^2*(b/a)^{(1/2)})$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")

[Out] Timed out

Sympy [A] time = 1.25806, size = 155, normalized size = 0.64

$$\text{RootSum}\left(65536t^4a^7b^2 + 2048t^2a^4bd^2 - 1152ta^2bc^2d + 16ad^4 + 81bc^4, \left(t \mapsto t \log\left(x + \frac{-32768t^3a^6bd^2 - 4608t^2a^4bc^2d}{192aca}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x**4+a)**2,x)

[Out] RootSum(65536*_t**4*a**7*b**2 + 2048*_t**2*a**4*b*d**2 - 1152*_t*a**2*b*c**2*d + 16*a*d**4 + 81*b*c**4, Lambda(_t, _t*log(x + (-32768*_t**3*a**6*b*d**2 - 4608*_t**2*a**4*b*c**2*d - 512*_t*a**3*d**4 - 1296*_t*a**2*b*c**4 + 360*a*c**2*d**3)/(192*a*c*d**4 - 243*b*c**5)))) + (c*x + d*x**2)/(4*a**2 + 4*a*b*x**4)

Giac [A] time = 1.07802, size = 321, normalized size = 1.33

$$\frac{3\sqrt{2}(ab^3)^{\frac{1}{4}}c\log\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32a^2b} - \frac{3\sqrt{2}(ab^3)^{\frac{1}{4}}c\log\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32a^2b} + \frac{dx^2 + cx}{4(bx^4 + a)a} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{abbd} + \dots\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x^4+a)^2,x, algorithm="giac")

[Out] 3/32*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b) - 3/32*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b) + 1/4*(d*x^2 + c*x)/((b*x^4 + a)*a) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b*d + 3*(a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4)))/(a/b)^(1/4))/(a^2*b^2) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b*d + 3*(a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4)))/(a/b)^(1/4))/(a^2*b^2)

$$3.119 \quad \int \frac{c+dx}{(a-bx^4)^3} dx$$

Optimal. Leaf size=136

$$\frac{x(7c+6dx)}{32a^2(a-bx^4)} + \frac{21c \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}} + \frac{21c \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}} + \frac{3d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} + \frac{x(c+dx)}{8a(a-bx^4)^2}$$

[Out] (x*(c + d*x))/(8*a*(a - b*x^4)^2) + (x*(7*c + 6*d*x))/(32*a^2*(a - b*x^4)) + (21*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(1/4)) + (21*c*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(1/4)) + (3*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b])

Rubi [A] time = 0.110073, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1855, 1876, 212, 208, 205, 275}

$$\frac{x(7c+6dx)}{32a^2(a-bx^4)} + \frac{21c \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}} + \frac{21c \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}} + \frac{3d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} + \frac{x(c+dx)}{8a(a-bx^4)^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)/(a - b*x^4)^3, x]

[Out] (x*(c + d*x))/(8*a*(a - b*x^4)^2) + (x*(7*c + 6*d*x))/(32*a^2*(a - b*x^4)) + (21*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(1/4)) + (21*c*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(1/4)) + (3*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b])

Rule 1855

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] & & PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b),
2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx}{(a-bx^4)^3} dx &= \frac{x(c+dx)}{8a(a-bx^4)^2} - \frac{\int \frac{-7c-6dx}{(a-bx^4)^2} dx}{8a} \\
&= \frac{x(c+dx)}{8a(a-bx^4)^2} + \frac{x(7c+6dx)}{32a^2(a-bx^4)} + \frac{\int \frac{21c+12dx}{a-bx^4} dx}{32a^2} \\
&= \frac{x(c+dx)}{8a(a-bx^4)^2} + \frac{x(7c+6dx)}{32a^2(a-bx^4)} + \frac{\int \left(\frac{21c}{a-bx^4} + \frac{12dx}{a-bx^4} \right) dx}{32a^2} \\
&= \frac{x(c+dx)}{8a(a-bx^4)^2} + \frac{x(7c+6dx)}{32a^2(a-bx^4)} + \frac{(21c) \int \frac{1}{a-bx^4} dx}{32a^2} + \frac{(3d) \int \frac{x}{a-bx^4} dx}{8a^2} \\
&= \frac{x(c+dx)}{8a(a-bx^4)^2} + \frac{x(7c+6dx)}{32a^2(a-bx^4)} + \frac{(21c) \int \frac{1}{\sqrt{a}-\sqrt{bx^2}} dx}{64a^{5/2}} + \frac{(21c) \int \frac{1}{\sqrt{a}+\sqrt{bx^2}} dx}{64a^{5/2}} + \frac{(3d) \text{Subst} \left(\int \frac{1}{a-bx^2} dx \right)}{16a^2} \\
&= \frac{x(c+dx)}{8a(a-bx^4)^2} + \frac{x(7c+6dx)}{32a^2(a-bx^4)} + \frac{21c \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{64a^{11/4} \sqrt[4]{b}} + \frac{21c \tanh^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{64a^{11/4} \sqrt[4]{b}} + \frac{3d \tanh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{16a^{5/2} \sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.146495, size = 193, normalized size = 1.42

$$\frac{16a^2 x(c+dx)}{(a-bx^4)^2} + \frac{4ax(7c+6dx)}{a-bx^4} - \frac{3(7\sqrt[4]{a}\sqrt[4]{bc}+4\sqrt{ad})\log(\sqrt[4]{a}-\sqrt[4]{bx})}{\sqrt{b}} + \frac{3(7\sqrt[4]{a}\sqrt[4]{bc}-4\sqrt{ad})\log(\sqrt[4]{a}+\sqrt[4]{bx})}{\sqrt{b}} + \frac{42\sqrt[4]{ac}\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}} + \frac{12\sqrt{ad}\log(\sqrt{a}+\sqrt{bx^2})}{\sqrt{b}}$$

$128a^3$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/(a - b*x^4)^3, x]

[Out] ((16*a^2*x*(c + d*x))/(a - b*x^4)^2 + (4*a*x*(7*c + 6*d*x))/(a - b*x^4) + (42*a^(1/4)*c*ArcTan[(b^(1/4)*x)/a^(1/4)]/b^(1/4) - (3*(7*a^(1/4)*b^(1/4)*c + 4*Sqrt[a]*d)*Log[a^(1/4) - b^(1/4)*x])/Sqrt[b] + (3*(7*a^(1/4)*b^(1/4)*c - 4*Sqrt[a]*d)*Log[a^(1/4) + b^(1/4)*x])/Sqrt[b] + (12*Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(128*a^3)

Maple [A] time = 0.006, size = 180, normalized size = 1.3

$$\frac{cx}{8a(bx^4 - a)^2} - \frac{7cx}{32a^2(bx^4 - a)} + \frac{21c}{128a^3} \sqrt[4]{\frac{a}{b}} \ln \left(\left(x + \sqrt[4]{\frac{a}{b}} \right) \left(x - \sqrt[4]{\frac{a}{b}} \right)^{-1} \right) + \frac{21c}{64a^3} \sqrt[4]{\frac{a}{b}} \arctan \left(x \frac{1}{\sqrt[4]{\frac{a}{b}}} \right) + \frac{dx^2}{8a(bx^4 - a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)/(-b*x^4+a)^3,x)

[Out] 1/8*c*x/a/(b*x^4-a)^2-7/32*c/a^2*x/(b*x^4-a)+21/128*c/a^3*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))+21/64*c/a^3*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))+1/8*d*x^2/a/(b*x^4-a)^2-3/16*d/a^2*x^2/(b*x^4-a)-3/32*d/a^2/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [A] time = 2.1943, size = 194, normalized size = 1.43

$$-\text{RootSum} \left(268435456t^4a^{11}b^2 - 4718592t^2a^6bd^2 - 2709504ta^3bc^2d + 20736ad^4 - 194481bc^4, \left(t \mapsto t \log \left(x + \frac{-671088}{\dots} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-b*x**4+a)**3,x)

[Out] -RootSum(268435456*_t**4*a**11*b**2 - 4718592*_t**2*a**6*b*d**2 - 2709504*_t*a**3*b*c**2*d + 20736*a*d**4 - 194481*b*c**4, Lambda(_t, _t*log(x + (-67108864*_t**3*a**9*b*d**2 + 9633792*_t**2*a**6*b*c**2*d + 589824*_t*a**4*d**4 - 2765952*_t*a**3*b*c**4 + 423360*a*c**2*d**3)/(193536*a*c*d**4 + 453789*b*c**5)))) - (-11*a*c*x - 10*a*d*x**2 + 7*b*c*x**5 + 6*b*d*x**6)/(32*a**4 - 64*a**3*b*x**4 + 32*a**2*b**2*x**8)

Giac [B] time = 1.08903, size = 367, normalized size = 2.7

$$\frac{21\sqrt{2}(-ab^3)^{\frac{1}{4}}c\log\left(x^2 + \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{256a^3b} - \frac{21\sqrt{2}(-ab^3)^{\frac{1}{4}}c\log\left(x^2 - \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{256a^3b} + \frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{-abbd}\right)}{256a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")

[Out] 21/256*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b) - 21/256*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(-a*b)*b*d + 7*(-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^2) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(-a*b)*b*d + 7*(-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^2) - 1/32*(6*b*d*x^6 + 7*b*c*x^5 - 10*a*d*x^2 - 11*a*c*x)/((b*x^4 - a)^2*a^2)

$$3.120 \quad \int \frac{c+dx}{(a+bx^4)^3} dx$$

Optimal. Leaf size=266

$$\frac{x(7c + 6dx)}{32a^2(a + bx^4)} - \frac{21c \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{128\sqrt{2}a^{11/4}\sqrt[4]{b}} + \frac{21c \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{128\sqrt{2}a^{11/4}\sqrt[4]{b}} - \frac{21c \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}} + \dots$$

[Out] (x*(c + d*x))/(8*a*(a + b*x^4)^2) + (x*(7*c + 6*d*x))/(32*a^2*(a + b*x^4)) + (3*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b]) - (21*c*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(1/4)) + (21*c*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(1/4)) - (21*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(1/4)) + (21*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(1/4))

Rubi [A] time = 0.230498, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {1855, 1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$\frac{x(7c + 6dx)}{32a^2(a + bx^4)} - \frac{21c \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{128\sqrt{2}a^{11/4}\sqrt[4]{b}} + \frac{21c \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{128\sqrt{2}a^{11/4}\sqrt[4]{b}} - \frac{21c \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}} + \dots$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)/(a + b*x^4)^3, x]

[Out] (x*(c + d*x))/(8*a*(a + b*x^4)^2) + (x*(7*c + 6*d*x))/(32*a^2*(a + b*x^4)) + (3*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b]) - (21*c*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(1/4)) + (21*c*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(1/4)) - (21*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(1/4)) + (21*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(1/4))

Rule 1855

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p

+ 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] & PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]

Rule 1876

Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 211

Int[((a_) + (b_)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx}{(a+bx^4)^3} dx &= \frac{x(c+dx)}{8a(a+bx^4)^2} - \frac{\int \frac{-7c-6dx}{(a+bx^4)^2} dx}{8a} \\
&= \frac{x(c+dx)}{8a(a+bx^4)^2} + \frac{x(7c+6dx)}{32a^2(a+bx^4)} + \frac{\int \frac{21c+12dx}{a+bx^4} dx}{32a^2} \\
&= \frac{x(c+dx)}{8a(a+bx^4)^2} + \frac{x(7c+6dx)}{32a^2(a+bx^4)} + \frac{\int \left(\frac{21c}{a+bx^4} + \frac{12dx}{a+bx^4} \right) dx}{32a^2} \\
&= \frac{x(c+dx)}{8a(a+bx^4)^2} + \frac{x(7c+6dx)}{32a^2(a+bx^4)} + \frac{(21c) \int \frac{1}{a+bx^4} dx}{32a^2} + \frac{(3d) \int \frac{x}{a+bx^4} dx}{8a^2} \\
&= \frac{x(c+dx)}{8a(a+bx^4)^2} + \frac{x(7c+6dx)}{32a^2(a+bx^4)} + \frac{(21c) \int \frac{\sqrt{a}-\sqrt{bx^2}}{a+bx^4} dx}{64a^{5/2}} + \frac{(21c) \int \frac{\sqrt{a}+\sqrt{bx^2}}{a+bx^4} dx}{64a^{5/2}} + \frac{(3d) \text{Subst} \left(\int \frac{1}{a+bx^2} dx \right)}{16a^2} \\
&= \frac{x(c+dx)}{8a(a+bx^4)^2} + \frac{x(7c+6dx)}{32a^2(a+bx^4)} + \frac{3d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{16a^{5/2}\sqrt{b}} + \frac{(21c) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{b}} + x^2} dx}{128a^{5/2}\sqrt{b}} + \frac{(21c) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{b}} + x^2} dx}{128a^{5/2}\sqrt{b}} \\
&= \frac{x(c+dx)}{8a(a+bx^4)^2} + \frac{x(7c+6dx)}{32a^2(a+bx^4)} + \frac{3d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{16a^{5/2}\sqrt{b}} - \frac{21c \log \left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2} \right)}{128\sqrt{2}a^{11/4}\sqrt[4]{b}} + \frac{21c \log \left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2} \right)}{128\sqrt{2}a^{11/4}\sqrt[4]{b}} \\
&= \frac{x(c+dx)}{8a(a+bx^4)^2} + \frac{x(7c+6dx)}{32a^2(a+bx^4)} + \frac{3d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{16a^{5/2}\sqrt{b}} - \frac{21c \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}} + \frac{21c \tan^{-1} \left(1 + \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}}
\end{aligned}$$

Mathematica [A] time = 0.207356, size = 249, normalized size = 0.94

$$\frac{32a^{7/4}x(c+dx)}{(a+bx^4)^2} + \frac{8a^{3/4}x(7c+6dx)}{a+bx^4} - \frac{6(8\sqrt[4]{ad}+7\sqrt{2}\sqrt[4]{bc})\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{\sqrt{b}} + \frac{6(7\sqrt{2}\sqrt[4]{bc}-8\sqrt[4]{ad})\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}+1\right)}{\sqrt{b}} - \frac{21\sqrt{2}c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}+\sqrt{a}+\sqrt{bx^2}\right)}{\sqrt[4]{b}}$$

$$256a^{11/4}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/(a + b*x^4)^3,x]

```
[Out] ((32*a^(7/4)*x*(c + d*x))/(a + b*x^4)^2 + (8*a^(3/4)*x*(7*c + 6*d*x))/(a +
b*x^4) - (6*(7*Sqrt[2]*b^(1/4)*c + 8*a^(1/4)*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*x)/a^(1/4)]/Sqrt[b] + (6*(7*Sqrt[2]*b^(1/4)*c - 8*a^(1/4)*d)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*x)/a^(1/4)]/Sqrt[b] - (21*Sqrt[2]*c*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4) + (21*Sqrt[2]*c*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4))/(256*a^(11/4))
```

Maple [A] time = 0.006, size = 222, normalized size = 0.8

$$\frac{cx}{8a(bx^4+a)^2} + \frac{7cx}{32a^2(bx^4+a)} + \frac{21c\sqrt{2}\sqrt[4]{a}}{256a^3\sqrt{b}} \ln\left(\left(x^2 + \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x^2 - \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) + \frac{21c\sqrt{2}\sqrt[4]{a}}{128a^3\sqrt{b}} \arctan\left(\frac{x^2 + \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}}{x^2 - \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^4+a)^3,x)
```

```
[Out] 1/8*c*x/a/(b*x^4+a)^2+7/32*c/a^2*x/(b*x^4+a)+21/256*c/a^3*(1/b*a)^(1/4)*2^(
1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(
1/2)+(1/b*a)^(1/2)))+21/128*c/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/
b*a)^(1/4)*x+1)+21/128*c/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(
1/4)*x-1)+1/8*d*x^2/a/(b*x^4+a)^2+3/16*d/a^2*x^2/(b*x^4+a)+3/16*d/a^2/(a*b)
^(1/2)*arctan(x^2*(b/a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [A] time = 2.15468, size = 192, normalized size = 0.72

RootSum($268435456t^4a^{11}b^2 + 4718592t^2a^6bd^2 - 2709504ta^3bc^2d + 20736ad^4 + 194481bc^4$, $t \mapsto t \log\left(x + \frac{-671088}{\dots}\right)$)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x**4+a)**3,x)

[Out] RootSum($268435456*_t^{**4}*a^{**11}*b^{**2} + 4718592*_t^{**2}*a^{**6}*b*d^{**2} - 2709504*_t^{**3}*b*c^{**2}*d + 20736*a*d^{**4} + 194481*b*c^{**4}$, Lambda($_t, _t*\log(x + (-67108864*_t^{**3}*a^{**9}*b*d^{**2} - 9633792*_t^{**2}*a^{**6}*b*c^{**2}*d - 589824*_t*a^{**4}*d^{**4} - 2765952*_t*a^{**3}*b*c^{**4} + 423360*a*c^{**2}*d^{**3})/(193536*a*c*d^{**4} - 453789*b*c^{**5}))$)) + $(11*a*c*x + 10*a*d*x^{**2} + 7*b*c*x^{**5} + 6*b*d*x^{**6})/(32*a^{**4} + 64*a^{**3}*b*x^{**4} + 32*a^{**2}*b^{**2}*x^{**8})$)

Giac [A] time = 1.08334, size = 346, normalized size = 1.3

$$\frac{21\sqrt{2}(ab^3)^{\frac{1}{4}}c\log\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{256a^3b} - \frac{21\sqrt{2}(ab^3)^{\frac{1}{4}}c\log\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{256a^3b} + \frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{abbd} + 7(ab^3)^{\frac{1}{4}}\right)}{128a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x^4+a)^3,x, algorithm="giac")

[Out] $\frac{21}{256}\sqrt{2}*(a*b^3)^{(1/4)}*c*\log(x^2 + \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(a^3*b) - \frac{21}{256}\sqrt{2}*(a*b^3)^{(1/4)}*c*\log(x^2 - \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(a^3*b) + \frac{3}{128}\sqrt{2}*(4*\sqrt{2}*\sqrt{a*b}*b*d + 7*(a*b^3)^{(1/4)}*b*c)*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(a/b)^{(1/4)})/(a/b)^{(1/4)})/(a^3*b^2) + \frac{3}{128}\sqrt{2}*(4*\sqrt{2}*\sqrt{a*b}*b*d + 7*(a*b^3)^{(1/4)}*b*c)*\arctan($

$$\frac{1}{2}\sqrt{2}(2x - \sqrt{2}(a/b)^{1/4})/(a/b)^{1/4}/(a^3b^2) + \frac{1}{32}(6bx^6 + 7bcx^5 + 10ad^2x^2 + 11acx)/(bx^4 + a)^2a^2$$

$$3.121 \quad \int \frac{c+dx}{(a-bx^4)^4} dx$$

Optimal. Leaf size=162

$$\frac{x(77c + 60dx)}{384a^3(a - bx^4)} + \frac{x(11c + 10dx)}{96a^2(a - bx^4)^2} + \frac{77c \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}} + \frac{77c \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}} + \frac{5d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} + \frac{x(c + dx)}{12a(a - bx^4)^3}$$

[Out] (x*(c + d*x))/(12*a*(a - b*x^4)^3) + (x*(11*c + 10*d*x))/(96*a^2*(a - b*x^4)^2) + (x*(77*c + 60*d*x))/(384*a^3*(a - b*x^4)) + (77*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(1/4)) + (77*c*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(1/4)) + (5*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b])

Rubi [A] time = 0.130489, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1855, 1876, 212, 208, 205, 275}

$$\frac{x(77c + 60dx)}{384a^3(a - bx^4)} + \frac{x(11c + 10dx)}{96a^2(a - bx^4)^2} + \frac{77c \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}} + \frac{77c \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}} + \frac{5d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} + \frac{x(c + dx)}{12a(a - bx^4)^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)/(a - b*x^4)^4, x]

[Out] (x*(c + d*x))/(12*a*(a - b*x^4)^3) + (x*(11*c + 10*d*x))/(96*a^2*(a - b*x^4)^2) + (x*(77*c + 60*d*x))/(384*a^3*(a - b*x^4)) + (77*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(1/4)) + (77*c*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(1/4)) + (5*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b])

Rule 1855

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] & & PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b),
2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx}{(a-bx^4)^4} dx &= \frac{x(c+dx)}{12a(a-bx^4)^3} - \frac{\int \frac{-11c-10dx}{(a-bx^4)^3} dx}{12a} \\
&= \frac{x(c+dx)}{12a(a-bx^4)^3} + \frac{x(11c+10dx)}{96a^2(a-bx^4)^2} + \frac{\int \frac{77c+60dx}{(a-bx^4)^2} dx}{96a^2} \\
&= \frac{x(c+dx)}{12a(a-bx^4)^3} + \frac{x(11c+10dx)}{96a^2(a-bx^4)^2} + \frac{x(77c+60dx)}{384a^3(a-bx^4)} - \frac{\int \frac{-231c-120dx}{a-bx^4} dx}{384a^3} \\
&= \frac{x(c+dx)}{12a(a-bx^4)^3} + \frac{x(11c+10dx)}{96a^2(a-bx^4)^2} + \frac{x(77c+60dx)}{384a^3(a-bx^4)} - \frac{\int \left(-\frac{231c}{a-bx^4} - \frac{120dx}{a-bx^4}\right) dx}{384a^3} \\
&= \frac{x(c+dx)}{12a(a-bx^4)^3} + \frac{x(11c+10dx)}{96a^2(a-bx^4)^2} + \frac{x(77c+60dx)}{384a^3(a-bx^4)} + \frac{(77c) \int \frac{1}{a-bx^4} dx}{128a^3} + \frac{(5d) \int \frac{x}{a-bx^4} dx}{16a^3} \\
&= \frac{x(c+dx)}{12a(a-bx^4)^3} + \frac{x(11c+10dx)}{96a^2(a-bx^4)^2} + \frac{x(77c+60dx)}{384a^3(a-bx^4)} + \frac{(77c) \int \frac{1}{\sqrt{a}-\sqrt{bx^2}} dx}{256a^{7/2}} + \frac{(77c) \int \frac{1}{\sqrt{a}+\sqrt{bx^2}} dx}{256a^{7/2}} + \dots \\
&= \frac{x(c+dx)}{12a(a-bx^4)^3} + \frac{x(11c+10dx)}{96a^2(a-bx^4)^2} + \frac{x(77c+60dx)}{384a^3(a-bx^4)} + \frac{77c \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}} + \frac{77c \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}} + \dots
\end{aligned}$$

Mathematica [A] time = 0.177661, size = 217, normalized size = 1.34

$$\frac{\frac{128a^3x(c+dx)}{(a-bx^4)^3} + \frac{16a^2x(11c+10dx)}{(a-bx^4)^2} + \frac{4ax(77c+60dx)}{a-bx^4} - \frac{3(77\sqrt[4]{a}\sqrt[4]{bc}+40\sqrt{ad})\log(\sqrt[4]{a}-\sqrt[4]{bx})}{\sqrt{b}} + \frac{3(77\sqrt[4]{a}\sqrt[4]{bc}-40\sqrt{ad})\log(\sqrt[4]{a}+\sqrt[4]{bx})}{\sqrt{b}} + \frac{462\sqrt[4]{ac}\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}}}{1536a^4}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/(a - b*x^4)^4, x]

[Out] ((128*a^3*x*(c + d*x))/(a - b*x^4)^3 + (16*a^2*x*(11*c + 10*d*x))/(a - b*x^4)^2 + (4*a*x*(77*c + 60*d*x))/(a - b*x^4) + (462*a^(1/4)*c*ArcTan[(b^(1/4)*x)/a^(1/4)]/b^(1/4) - (3*(77*a^(1/4)*b^(1/4)*c + 40*sqrt[a]*d)*Log[a^(1/4) - b^(1/4)*x])/sqrt[b] + (3*(77*a^(1/4)*b^(1/4)*c - 40*sqrt[a]*d)*Log[a^(1/4) + b^(1/4)*x])/sqrt[b] + (120*sqrt[a]*d*Log[sqrt[a] + sqrt[b]*x^2])/sqrt

[b])/(1536*a^4)

Maple [A] time = 0.013, size = 177, normalized size = 1.1

$$\frac{1}{(bx^4 - a)^3} \left(-\frac{5b^2 dx^{10}}{32a^3} - \frac{77b^2 cx^9}{384a^3} + \frac{5bdx^6}{12a^2} + \frac{33bcx^5}{64a^2} - \frac{11dx^2}{32a} - \frac{51cx}{128a} \right) + \frac{77c}{512a^4} \sqrt[4]{\frac{a}{b}} \ln \left(\left(x + \sqrt[4]{\frac{a}{b}} \right) \left(x - \sqrt[4]{\frac{a}{b}} \right)^{-1} \right) + \frac{77c}{512a^4} \sqrt[4]{\frac{a}{b}} \arctan \left(\frac{x}{\sqrt[4]{\frac{a}{b}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)/(-b*x^4+a)^4,x)

[Out] (-5/32*d/a^3*b^2*x^10-77/384*c/a^3*b^2*x^9+5/12/a^2*d*b*x^6+33/64/a^2*c*b*x^5-11/32*d/a*x^2-51/128*c/a*x)/(b*x^4-a)^3+77/512/a^4*c*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))+77/256/a^4*c*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))-5/64/a^3*d/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")

[Out] Timed out

Sympy [A] time = 6.64574, size = 231, normalized size = 1.43

RootSum $\left(68719476736t^4a^{15}b^2 - 838860800t^2a^8bd^2 + 485703680ta^4bc^2d + 2560000ad^4 - 35153041bc^4, \left(t \mapsto t \log\left(\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-b*x**4+a)**4,x)

[Out] RootSum(68719476736*_t**4*a**15*b**2 - 838860800*_t**2*a**8*b*d**2 + 485703680*_t*a**4*b*c**2*d + 2560000*a*d**4 - 35153041*b*c**4, Lambda(_t, _t*log(x + (429496729600*_t**3*a**12*b*d**2 + 62170071040*_t**2*a**8*b*c**2*d - 2621440000*_t*a**5*d**4 + 17998356992*_t*a**4*b*c**4 + 1897280000*a*c**2*d**3)/(788480000*a*c*d**4 + 2706784157*b*c**5)))) - (153*a**2*c*x + 132*a**2*d*x**2 - 198*a*b*c*x**5 - 160*a*b*d*x**6 + 77*b**2*c*x**9 + 60*b**2*d*x**10)/(-384*a**6 + 1152*a**5*b*x**4 - 1152*a**4*b**2*x**8 + 384*a**3*b**3*x**12)

Giac [B] time = 1.0728, size = 400, normalized size = 2.47

$$\frac{77\sqrt{2}(-ab^3)^{\frac{1}{4}}c \log\left(x^2 + \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{1024a^4b} - \frac{77\sqrt{2}(-ab^3)^{\frac{1}{4}}c \log\left(x^2 - \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{1024a^4b} - \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{-abbd} - \dots\right)}{1024a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")

[Out] 77/1024*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^4*b) - 77/1024*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^4*b) - 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b*d - 77*(-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^4*b^2) - 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b*d - 77*(-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^4*b^2) - 1/384*(60*b^2*d*x^10 + 77*b^2*c*x^9 - 160*a*b*d*x^6 - 198*a*b*c*x^5 + 132*a^2*d*x^2 + 153*a^2*c*x)/((b*x^4 - a)^3*a^3)

$$3.122 \quad \int \frac{c+dx}{(a+bx^4)^4} dx$$

Optimal. Leaf size=291

$$\frac{x(77c + 60dx)}{384a^3 (a + bx^4)} + \frac{x(11c + 10dx)}{96a^2 (a + bx^4)^2} - \frac{77c \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{512\sqrt{2}a^{15/4}\sqrt[4]{b}} + \frac{77c \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{512\sqrt{2}a^{15/4}\sqrt[4]{b}} - \frac{77c \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} - \sqrt{a} - \sqrt{bx^2}}\right)}{512\sqrt{2}a^{15/4}\sqrt[4]{b}}$$

[Out] (x*(c + d*x))/(12*a*(a + b*x^4)^3) + (x*(11*c + 10*d*x))/(96*a^2*(a + b*x^4)^2) + (x*(77*c + 60*d*x))/(384*a^3*(a + b*x^4)) + (5*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/(32*a^(7/2)*Sqrt[b]) - (77*c*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(1/4)) + (77*c*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(1/4)) - (77*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(1/4)) + (77*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(1/4))

Rubi [A] time = 0.267577, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {1855, 1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$\frac{x(77c + 60dx)}{384a^3 (a + bx^4)} + \frac{x(11c + 10dx)}{96a^2 (a + bx^4)^2} - \frac{77c \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{512\sqrt{2}a^{15/4}\sqrt[4]{b}} + \frac{77c \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{512\sqrt{2}a^{15/4}\sqrt[4]{b}} - \frac{77c \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} - \sqrt{a} - \sqrt{bx^2}}\right)}{512\sqrt{2}a^{15/4}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)/(a + b*x^4)^4, x]

[Out] (x*(c + d*x))/(12*a*(a + b*x^4)^3) + (x*(11*c + 10*d*x))/(96*a^2*(a + b*x^4)^2) + (x*(77*c + 60*d*x))/(384*a^3*(a + b*x^4)) + (5*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/(32*a^(7/2)*Sqrt[b]) - (77*c*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(1/4)) + (77*c*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(1/4)) - (77*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(1/4)) + (77*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(1/4))

Rule 1855


```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
+ 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))]/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rule 211

```
Int[((a_) + (b_)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])) /; Free
```

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 204

$\text{Int}[\{(a_) + (b_.) \cdot (x_)^2\}^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2] \cdot x] / \text{Rt}[-a, 2]] / (\text{Rt}[-a, 2] \cdot \text{Rt}[-b, 2]), x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 275

$\text{Int}[(x_)^{(m_.)} \cdot \{(a_) + (b_.) \cdot (x_)^{(n_.)}\}^{(p_)}, x_Symbol] \rightarrow \text{With}[\{k = \text{GCD}[m + 1, n]\}, \text{Dist}[1/k, \text{Subst}[\text{Int}[x^{(m+1)/k - 1} \cdot (a + b \cdot x^{(n/k)})^p], x], x, x^{k}], x] \ /; \ k \neq 1] \ /; \ \text{FreeQ}[\{a, b, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[m]$

Rule 205

$\text{Int}[\{(a_) + (b_.) \cdot (x_)^2\}^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2] \cdot \text{ArcTan}[x / \text{Rt}[a/b, 2]]) / a, x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b]$

Rubi steps

$$\begin{aligned}
\int \frac{c+dx}{(a+bx^4)^4} dx &= \frac{x(c+dx)}{12a(a+bx^4)^3} - \frac{\int \frac{-11c-10dx}{(a+bx^4)^3} dx}{12a} \\
&= \frac{x(c+dx)}{12a(a+bx^4)^3} + \frac{x(11c+10dx)}{96a^2(a+bx^4)^2} + \frac{\int \frac{77c+60dx}{(a+bx^4)^2} dx}{96a^2} \\
&= \frac{x(c+dx)}{12a(a+bx^4)^3} + \frac{x(11c+10dx)}{96a^2(a+bx^4)^2} + \frac{x(77c+60dx)}{384a^3(a+bx^4)} - \frac{\int \frac{-231c-120dx}{a+bx^4} dx}{384a^3} \\
&= \frac{x(c+dx)}{12a(a+bx^4)^3} + \frac{x(11c+10dx)}{96a^2(a+bx^4)^2} + \frac{x(77c+60dx)}{384a^3(a+bx^4)} - \frac{\int \left(-\frac{231c}{a+bx^4} - \frac{120dx}{a+bx^4} \right) dx}{384a^3} \\
&= \frac{x(c+dx)}{12a(a+bx^4)^3} + \frac{x(11c+10dx)}{96a^2(a+bx^4)^2} + \frac{x(77c+60dx)}{384a^3(a+bx^4)} + \frac{(77c) \int \frac{1}{a+bx^4} dx}{128a^3} + \frac{(5d) \int \frac{x}{a+bx^4} dx}{16a^3} \\
&= \frac{x(c+dx)}{12a(a+bx^4)^3} + \frac{x(11c+10dx)}{96a^2(a+bx^4)^2} + \frac{x(77c+60dx)}{384a^3(a+bx^4)} + \frac{(77c) \int \frac{\sqrt{a}-\sqrt{bx^2}}{a+bx^4} dx}{256a^{7/2}} + \frac{(77c) \int \frac{\sqrt{a}+\sqrt{bx^2}}{a+bx^4} dx}{256a^{7/2}} + \\
&= \frac{x(c+dx)}{12a(a+bx^4)^3} + \frac{x(11c+10dx)}{96a^2(a+bx^4)^2} + \frac{x(77c+60dx)}{384a^3(a+bx^4)} + \frac{5d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{32a^{7/2}\sqrt{b}} + \frac{(77c) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{b}} + x^2} dx}{512a^{7/2}\sqrt{b}} \\
&= \frac{x(c+dx)}{12a(a+bx^4)^3} + \frac{x(11c+10dx)}{96a^2(a+bx^4)^2} + \frac{x(77c+60dx)}{384a^3(a+bx^4)} + \frac{5d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{32a^{7/2}\sqrt{b}} - \frac{77c \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx})}{512\sqrt{2}a^{15/4}\sqrt[4]{b}} \\
&= \frac{x(c+dx)}{12a(a+bx^4)^3} + \frac{x(11c+10dx)}{96a^2(a+bx^4)^2} + \frac{x(77c+60dx)}{384a^3(a+bx^4)} + \frac{5d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{32a^{7/2}\sqrt{b}} - \frac{77c \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{256\sqrt{2}a^{15/4}\sqrt[4]{b}} +
\end{aligned}$$

Mathematica [A] time = 0.284213, size = 274, normalized size = 0.94

$$\frac{256a^{11/4}x(c+dx)}{(a+bx^4)^3} + \frac{32a^{7/4}x(11c+10dx)}{(a+bx^4)^2} + \frac{8a^{3/4}x(77c+60dx)}{a+bx^4} - \frac{6(80\sqrt[4]{ad}+77\sqrt{2}\sqrt[4]{bc})\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{\sqrt{b}} + \frac{6(77\sqrt{2}\sqrt[4]{bc}-80\sqrt[4]{ad})\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}+1\right)}{\sqrt{b}} - \frac{231c \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx})}{512\sqrt{2}a^{15/4}\sqrt[4]{b}}$$

3072a^{15/4}

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/(a + b*x^4)^4, x]

[Out]
$$\frac{(256a^{11/4}x(c + dx))/(a + b^4x^4)^3 + (32a^{7/4}x(11c + 10dx))/(a + b^4x^4)^2 + (8a^{3/4}x(77c + 60dx))/(a + b^4x^4) - (6(77\sqrt{2}b^{1/4}c + 80a^{1/4}d)\text{ArcTan}[1 - (\sqrt{2}b^{1/4}x)/a^{1/4}])/\sqrt{b} + (6(77\sqrt{2}b^{1/4}c - 80a^{1/4}d)\text{ArcTan}[1 + (\sqrt{2}b^{1/4}x)/a^{1/4}])/\sqrt{b} - (231\sqrt{2}c\text{Log}[\sqrt{a} - \sqrt{2}a^{1/4}b^{1/4}x + \sqrt{b}x^2])/b^{1/4} + (231\sqrt{2}c\text{Log}[\sqrt{a} + \sqrt{2}a^{1/4}b^{1/4}x + \sqrt{b}x^2])/b^{1/4}}{(3072a^{15/4})}$$

Maple [A] time = 0.014, size = 225, normalized size = 0.8

$$\frac{1}{(bx^4 + a)^3} \left(\frac{5b^2dx^{10}}{32a^3} + \frac{77b^2cx^9}{384a^3} + \frac{5b^2dx^6}{12a^2} + \frac{33bcx^5}{64a^2} + \frac{11dx^2}{32a} + \frac{51cx}{128a} \right) + \frac{77c\sqrt{2}}{1024a^4} \sqrt[4]{\frac{a}{b}} \ln \left(\left(x^2 + \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}} \right) \left(x^2 - \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)/(b*x^4+a)^4, x)

[Out]
$$\frac{(5/32*d/a^3*b^2*x^{10} + 77/384*c/a^3*b^2*x^9 + 5/12/a^2*d*b*x^6 + 33/64/a^2*c*b*x^5 + 11/32*d/a*x^2 + 51/128*c/a*x)/(b*x^4+a)^3 + 77/1024/a^4*c*(1/b*a)^{1/4}*2^{1/2}*\ln((x^2+(1/b*a)^{1/4}*x*2^{1/2}+(1/b*a)^{1/2}))/((x^2-(1/b*a)^{1/4}*x*2^{1/2}+(1/b*a)^{1/2})) + 77/512/a^4*c*(1/b*a)^{1/4}*2^{1/2}*arctan(2^{1/2}/(1/b*a)^{1/4}*x+1) + 77/512/a^4*c*(1/b*a)^{1/4}*2^{1/2}*arctan(2^{1/2}/(1/b*a)^{1/4}*x-1) + 5/32/a^3*d/(a*b)^{1/2}*arctan(x^2*(b/a)^{1/2})$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x^4+a)^4, x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")

[Out] Timed out

Sympy [A] time = 6.6255, size = 231, normalized size = 0.79

RootSum(68719476736t⁴a¹⁵b² + 838860800t²a⁸bd² - 485703680ta⁴bc²d + 2560000ad⁴ + 35153041bc⁴, (t ↦ t log(

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x**4+a)**4,x)

[Out] RootSum(68719476736*_t**4*a**15*b**2 + 838860800*_t**2*a**8*b*d**2 - 485703680*_t*a**4*b*c**2*d + 2560000*a*d**4 + 35153041*b*c**4, Lambda(_t, _t*log(x + (-429496729600*_t**3*a**12*b*d**2 - 62170071040*_t**2*a**8*b*c**2*d - 2621440000*_t*a**5*d**4 - 17998356992*_t*a**4*b*c**4 + 1897280000*a*c**2*d**3)/(788480000*a*c*d**4 - 2706784157*b*c**5)))) + (153*a**2*c*x + 132*a**2*d*x**2 + 198*a*b*c*x**5 + 160*a*b*d*x**6 + 77*b**2*c*x**9 + 60*b**2*d*x**10)/(384*a**6 + 1152*a**5*b*x**4 + 1152*a**4*b**2*x**8 + 384*a**3*b**3*x**12)

Giac [A] time = 1.07999, size = 378, normalized size = 1.3

$$\frac{77\sqrt{2}(ab^3)^{\frac{1}{4}}c\log\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{1024a^4b} - \frac{77\sqrt{2}(ab^3)^{\frac{1}{4}}c\log\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{1024a^4b} + \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{abbd} + 77(ab^3)\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x^4+a)^4,x, algorithm="giac")

```
[Out] 77/1024*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b)
)/(a^4*b) - 77/1024*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(a/b)^(1/4)
+ sqrt(a/b))/(a^4*b) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b*d + 77*(a*b^3
)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a
^4*b^2) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b*d + 77*(a*b^3)^(1/4)*b*c)*a
rctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^2) + 1/38
4*(60*b^2*d*x^10 + 77*b^2*c*x^9 + 160*a*b*d*x^6 + 198*a*b*c*x^5 + 132*a^2*d
*x^2 + 153*a^2*c*x)/((b*x^4 + a)^3*a^3)
```

$$3.123 \quad \int \frac{c+dx}{1-x^4} dx$$

Optimal. Leaf size=24

$$\frac{1}{2}c \tan^{-1}(x) + \frac{1}{2}c \tanh^{-1}(x) + \frac{1}{2}d \tanh^{-1}(x^2)$$

[Out] (c*ArcTan[x])/2 + (c*ArcTanh[x])/2 + (d*ArcTanh[x^2])/2

Rubi [A] time = 0.0180047, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1876, 212, 206, 203, 275}

$$\frac{1}{2}c \tan^{-1}(x) + \frac{1}{2}c \tanh^{-1}(x) + \frac{1}{2}d \tanh^{-1}(x^2)$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)/(1 - x^4), x]

[Out] (c*ArcTan[x])/2 + (c*ArcTanh[x])/2 + (d*ArcTanh[x^2])/2

Rule 1876

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \int \frac{c + dx}{1 - x^4} dx &= \int \left(\frac{c}{1 - x^4} + \frac{dx}{1 - x^4} \right) dx \\ &= c \int \frac{1}{1 - x^4} dx + d \int \frac{x}{1 - x^4} dx \\ &= \frac{1}{2}c \int \frac{1}{1 - x^2} dx + \frac{1}{2}c \int \frac{1}{1 + x^2} dx + \frac{1}{2}d \operatorname{Subst} \left(\int \frac{1}{1 - x^2} dx, x, x^2 \right) \\ &= \frac{1}{2}c \tan^{-1}(x) + \frac{1}{2}c \tanh^{-1}(x) + \frac{1}{2}d \tanh^{-1}(x^2) \end{aligned}$$

Mathematica [A] time = 0.0147418, size = 42, normalized size = 1.75

$$\frac{1}{4} \left(-(c + d) \log(1 - x) + c \log(x + 1) + 2c \tan^{-1}(x) + d \log(x^2 + 1) - d \log(x + 1) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(1 - x^4), x]
```

```
[Out] (2*c*ArcTan[x] - (c + d)*Log[1 - x] + c*Log[1 + x] - d*Log[1 + x] + d*Log[1 + x^2])/4
```

Maple [B] time = 0.004, size = 44, normalized size = 1.8

$$-\frac{\ln(-1 + x)c}{4} - \frac{\ln(-1 + x)d}{4} + \frac{\ln(1 + x)c}{4} - \frac{\ln(1 + x)d}{4} + \frac{d \ln(x^2 + 1)}{4} + \frac{c \arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)/(-x^4+1),x)`

[Out] $-1/4*\ln(-1+x)*c-1/4*\ln(-1+x)*d+1/4*\ln(1+x)*c-1/4*\ln(1+x)*d+1/4*d*\ln(x^2+1)+1/2*c*\arctan(x)$

Maxima [A] time = 1.4422, size = 47, normalized size = 1.96

$$\frac{1}{2}c \arctan(x) + \frac{1}{4}d \log(x^2 + 1) + \frac{1}{4}(c - d) \log(x + 1) - \frac{1}{4}(c + d) \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)/(-x^4+1),x, algorithm="maxima")`

[Out] $1/2*c*\arctan(x) + 1/4*d*\log(x^2 + 1) + 1/4*(c - d)*\log(x + 1) - 1/4*(c + d)*\log(x - 1)$

Fricas [A] time = 1.29288, size = 119, normalized size = 4.96

$$\frac{1}{2}c \arctan(x) + \frac{1}{4}d \log(x^2 + 1) + \frac{1}{4}(c - d) \log(x + 1) - \frac{1}{4}(c + d) \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)/(-x^4+1),x, algorithm="fricas")`

[Out] $1/2*c*\arctan(x) + 1/4*d*\log(x^2 + 1) + 1/4*(c - d)*\log(x + 1) - 1/4*(c + d)*\log(x - 1)$

Sympy [C] time = 0.459214, size = 313, normalized size = 13.04

$$\frac{(c - d) \log\left(x + \frac{c^4(c-d)+5c^2d^3+c^2d(c-d)^2-2d^4(c-d)+2d^2(c-d)^3}{c^5+4cd^4}\right)}{4} - \frac{(c + d) \log\left(x + \frac{-c^4(c+d)+5c^2d^3+c^2d(c+d)^2+2d^4(c+d)-2d^2(c+d)^3}{c^5+4cd^4}\right)}{4} - \left(\frac{ic}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-x**4+1),x)

[Out] (c - d)*log(x + (c**4*(c - d) + 5*c**2*d**3 + c**2*d*(c - d)**2 - 2*d**4*(c - d) + 2*d**2*(c - d)**3)/(c**5 + 4*c*d**4))/4 - (c + d)*log(x + (-c**4*(c + d) + 5*c**2*d**3 + c**2*d*(c + d)**2 + 2*d**4*(c + d) - 2*d**2*(c + d)**3)/(c**5 + 4*c*d**4))/4 - (-I*c/4 - d/4)*log(x + (-4*c**4*(-I*c/4 - d/4) + 5*c**2*d**3 + 16*c**2*d*(-I*c/4 - d/4)**2 + 8*d**4*(-I*c/4 - d/4) - 128*d**2*(-I*c/4 - d/4)**3)/(c**5 + 4*c*d**4)) - (I*c/4 - d/4)*log(x + (-4*c**4*(I*c/4 - d/4) + 5*c**2*d**3 + 16*c**2*d*(I*c/4 - d/4)**2 + 8*d**4*(I*c/4 - d/4) - 128*d**2*(I*c/4 - d/4)**3)/(c**5 + 4*c*d**4))

Giac [B] time = 1.05346, size = 50, normalized size = 2.08

$$\frac{1}{2}c \arctan(x) + \frac{1}{4}d \log(x^2 + 1) + \frac{1}{4}(c - d) \log(|x + 1|) - \frac{1}{4}(c + d) \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-x^4+1),x, algorithm="giac")

[Out] 1/2*c*arctan(x) + 1/4*d*log(x^2 + 1) + 1/4*(c - d)*log(abs(x + 1)) - 1/4*(c + d)*log(abs(x - 1))

3.124 $\int \frac{c+dx}{1+x^4} dx$

Optimal. Leaf size=98

$$-\frac{c \log(x^2 - \sqrt{2}x + 1)}{4\sqrt{2}} + \frac{c \log(x^2 + \sqrt{2}x + 1)}{4\sqrt{2}} - \frac{c \tan^{-1}(1 - \sqrt{2}x)}{2\sqrt{2}} + \frac{c \tan^{-1}(\sqrt{2}x + 1)}{2\sqrt{2}} + \frac{1}{2}d \tan^{-1}(x^2)$$

[Out] (d*ArcTan[x^2])/2 - (c*ArcTan[1 - Sqrt[2]*x])/(2*Sqrt[2]) + (c*ArcTan[1 + Sqrt[2]*x])/(2*Sqrt[2]) - (c*Log[1 - Sqrt[2]*x + x^2])/(4*Sqrt[2]) + (c*Log[1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])

Rubi [A] time = 0.0672962, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {1876, 211, 1165, 628, 1162, 617, 204, 275, 203}

$$-\frac{c \log(x^2 - \sqrt{2}x + 1)}{4\sqrt{2}} + \frac{c \log(x^2 + \sqrt{2}x + 1)}{4\sqrt{2}} - \frac{c \tan^{-1}(1 - \sqrt{2}x)}{2\sqrt{2}} + \frac{c \tan^{-1}(\sqrt{2}x + 1)}{2\sqrt{2}} + \frac{1}{2}d \tan^{-1}(x^2)$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)/(1 + x^4),x]

[Out] (d*ArcTan[x^2])/2 - (c*ArcTan[1 - Sqrt[2]*x])/(2*Sqrt[2]) + (c*ArcTan[1 + Sqrt[2]*x])/(2*Sqrt[2]) - (c*Log[1 - Sqrt[2]*x + x^2])/(4*Sqrt[2]) + (c*Log[1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])

Rule 1876

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))]/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1 /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx}{1 + x^4} dx &= \int \left(\frac{c}{1 + x^4} + \frac{dx}{1 + x^4} \right) dx \\
&= c \int \frac{1}{1 + x^4} dx + d \int \frac{x}{1 + x^4} dx \\
&= \frac{1}{2}c \int \frac{1 - x^2}{1 + x^4} dx + \frac{1}{2}c \int \frac{1 + x^2}{1 + x^4} dx + \frac{1}{2}d \operatorname{Subst} \left(\int \frac{1}{1 + x^2} dx, x, x^2 \right) \\
&= \frac{1}{2}d \tan^{-1}(x^2) + \frac{1}{4}c \int \frac{1}{1 - \sqrt{2}x + x^2} dx + \frac{1}{4}c \int \frac{1}{1 + \sqrt{2}x + x^2} dx - \frac{c \int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2} dx}{4\sqrt{2}} - \frac{c \int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2} dx}{4\sqrt{2}} \\
&= \frac{1}{2}d \tan^{-1}(x^2) - \frac{c \log(1 - \sqrt{2}x + x^2)}{4\sqrt{2}} + \frac{c \log(1 + \sqrt{2}x + x^2)}{4\sqrt{2}} + \frac{c \operatorname{Subst} \left(\int \frac{1}{-1-x^2} dx, x, 1 - \sqrt{2}x \right)}{2\sqrt{2}} - \frac{c \operatorname{Subst} \left(\int \frac{1}{-1-x^2} dx, x, 1 + \sqrt{2}x \right)}{2\sqrt{2}} \\
&= \frac{1}{2}d \tan^{-1}(x^2) - \frac{c \tan^{-1}(1 - \sqrt{2}x)}{2\sqrt{2}} + \frac{c \tan^{-1}(1 + \sqrt{2}x)}{2\sqrt{2}} - \frac{c \log(1 - \sqrt{2}x + x^2)}{4\sqrt{2}} + \frac{c \log(1 + \sqrt{2}x + x^2)}{4\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.067398, size = 99, normalized size = 1.01

$$\frac{1}{4} \left(- \left(\sqrt[4]{-1}c + id \right) \log \left(\sqrt[4]{-1} - x \right) + \left(-(-1)^{3/4}c + id \right) \log \left((-1)^{3/4} - x \right) + \left(\sqrt[4]{-1}c - id \right) \log \left(x + \sqrt[4]{-1} \right) + \left((-1)^{3/4}c + id \right) \log \left(x + (-1)^{3/4} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/(1 + x^4), x]

[Out] (-((-1)^(1/4)*c + I*d)*Log[(-1)^(1/4) - x]) + (-((-1)^(3/4)*c) + I*d)*Log[(-1)^(3/4) - x] + ((-1)^(1/4)*c - I*d)*Log[(-1)^(1/4) + x] + ((-1)^(3/4)*c + I*d)*Log[(-1)^(3/4) + x])/4

Maple [A] time = 0.004, size = 68, normalized size = 0.7

$$\frac{c \arctan(1 + x\sqrt{2})\sqrt{2}}{4} + \frac{c \arctan(-1 + x\sqrt{2})\sqrt{2}}{4} + \frac{c\sqrt{2}}{8} \ln \left(\frac{1 + x^2 + x\sqrt{2}}{1 + x^2 - x\sqrt{2}} \right) + \frac{d \arctan(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)/(x^4+1), x)

[Out] $\frac{1}{4}c \arctan(1+x\sqrt{2}^{(1/2)})\sqrt{2}^{(1/2)} + \frac{1}{4}c \arctan(-1+x\sqrt{2}^{(1/2)})\sqrt{2}^{(1/2)} + \frac{1}{8}c\sqrt{2}^{(1/2)} \ln\left(\frac{1+x\sqrt{2}+x\sqrt{2}^{(1/2)}}{1+x\sqrt{2}-x\sqrt{2}^{(1/2)}}\right) + \frac{1}{2}d \arctan(x\sqrt{2})$

Maxima [A] time = 1.53606, size = 116, normalized size = 1.18

$\frac{1}{8}\sqrt{2}c \log(x^2 + \sqrt{2}x + 1) - \frac{1}{8}\sqrt{2}c \log(x^2 - \sqrt{2}x + 1) + \frac{1}{4}(\sqrt{2}c - 2d) \arctan\left(\frac{1}{2}\sqrt{2}(2x + \sqrt{2})\right) + \frac{1}{4}(\sqrt{2}c + 2d) \arctan\left(\frac{1}{2}\sqrt{2}(2x - \sqrt{2})\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(x^4+1),x, algorithm="maxima")

[Out] $\frac{1}{8}\sqrt{2}c \log(x^2 + \sqrt{2}x + 1) - \frac{1}{8}\sqrt{2}c \log(x^2 - \sqrt{2}x + 1) + \frac{1}{4}(\sqrt{2}c - 2d) \arctan(1/2\sqrt{2}(2x + \sqrt{2})) + \frac{1}{4}(\sqrt{2}c + 2d) \arctan(1/2\sqrt{2}(2x - \sqrt{2}))$

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(x^4+1),x, algorithm="fricas")

[Out] Timed out

Sympy [A] time = 0.411487, size = 83, normalized size = 0.85

$\text{RootSum}\left(256t^4 + 32t^2d^2 - 16tc^2d + c^4 + d^4, \left(t \mapsto t \log\left(x + \frac{128t^3d^2 + 16t^2c^2d + 4tc^4 + 8td^4 - 5c^2d^3}{c^5 - 4cd^4}\right)\right)\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(x**4+1),x)

[Out] $\text{RootSum}(256*_t**4 + 32*_t**2*d**2 - 16*_t*c**2*d + c**4 + d**4, \text{Lambda}(_t, _t*\log(x + (128*_t**3*d**2 + 16*_t**2*c**2*d + 4*_t*c**4 + 8*_t*d**4 - 5*c**5 - 4*c*d**4))))$

$*2*d**3)/(c**5 - 4*c*d**4))))$

Giac [A] time = 1.06388, size = 116, normalized size = 1.18

$$\frac{1}{8} \sqrt{2} c \log(x^2 + \sqrt{2}x + 1) - \frac{1}{8} \sqrt{2} c \log(x^2 - \sqrt{2}x + 1) + \frac{1}{4} (\sqrt{2}c - 2d) \arctan\left(\frac{1}{2} \sqrt{2}(2x + \sqrt{2})\right) + \frac{1}{4} (\sqrt{2}c + 2d) \arctan\left(\frac{1}{2} \sqrt{2}(2x - \sqrt{2})\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(x^4+1),x, algorithm="giac")

[Out] 1/8*sqrt(2)*c*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*c*log(x^2 - sqrt(2)*x + 1) + 1/4*(sqrt(2)*c - 2*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*(sqrt(2)*c + 2*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)))

$$3.125 \quad \int \frac{c+dx+ex^2}{a-bx^4} dx$$

Optimal. Leaf size=116

$$\frac{(\sqrt{bc} - \sqrt{ae}) \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{(\sqrt{ae} + \sqrt{bc}) \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

[Out] ((Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)]/(2*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)]/(2*a^(3/4)*b^(3/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b])

Rubi [A] time = 0.0954763, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1876, 275, 208, 1167, 205}

$$\frac{(\sqrt{bc} - \sqrt{ae}) \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{(\sqrt{ae} + \sqrt{bc}) \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/(a - b*x^4), x]

[Out] ((Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)]/(2*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)]/(2*a^(3/4)*b^(3/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b])

Rule 1876

Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 275

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1167

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[-(a*c)]

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{c + dx + ex^2}{a - bx^4} dx &= \int \left(\frac{dx}{a - bx^4} + \frac{c + ex^2}{a - bx^4} \right) dx \\ &= d \int \frac{x}{a - bx^4} dx + \int \frac{c + ex^2}{a - bx^4} dx \\ &= \frac{1}{2} d \operatorname{Subst} \left(\int \frac{1}{a - bx^2} dx, x, x^2 \right) + \frac{1}{2} \left(-\frac{\sqrt{bc}}{\sqrt{a}} + e \right) \int \frac{1}{-\sqrt{a}\sqrt{b} - bx^2} dx + \frac{1}{2} \left(\frac{\sqrt{bc}}{\sqrt{a}} + e \right) \int \frac{1}{\sqrt{a}\sqrt{b} - bx^2} dx \\ &= \frac{(\sqrt{bc} - \sqrt{ae}) \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2a^{3/4}b^{3/4}} + \frac{(\sqrt{bc} + \sqrt{ae}) \tanh^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2a^{3/4}b^{3/4}} + \frac{d \tanh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{b}} \end{aligned}$$

Mathematica [A] time = 0.050743, size = 187, normalized size = 1.61

$$\frac{-\log(\sqrt[4]{a} - \sqrt[4]{bx}) (\sqrt[4]{a}\sqrt[4]{bd} + \sqrt{ae} + \sqrt{bc}) + 2(\sqrt{bc} - \sqrt{ae}) \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right) + \sqrt{bc} \log(\sqrt[4]{a} + \sqrt[4]{bx}) + \sqrt[4]{a}\sqrt[4]{bd} \log(\sqrt{a} + \sqrt{b})}{4a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(a - b*x^4), x]

[Out] (2*(Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c + a^(1/4)*b^(1/4)*d + Sqrt[a]*e)*Log[a^(1/4) - b^(1/4)*x] + Sqrt[b]*c*Log[a^(1/4)

$+ b^{(1/4)*x}] - a^{(1/4)*b^{(1/4)*d*Log[a^{(1/4)} + b^{(1/4)*x}] + Sqrt[a]*e*Log[a^{(1/4)} + b^{(1/4)*x}] + a^{(1/4)*b^{(1/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2]}/(4*a^{(3/4)*b^{(3/4)})}$

Maple [B] time = 0.002, size = 161, normalized size = 1.4

$$\frac{c}{4a} \sqrt[4]{\frac{a}{b}} \ln \left(\left(x + \sqrt[4]{\frac{a}{b}} \right) \left(x - \sqrt[4]{\frac{a}{b}} \right)^{-1} \right) + \frac{c}{2a} \sqrt[4]{\frac{a}{b}} \arctan \left(x \frac{1}{\sqrt[4]{\frac{a}{b}}} \right) - \frac{d}{4} \ln \left(\left(-a + x^2 \sqrt{ab} \right) \left(-a - x^2 \sqrt{ab} \right)^{-1} \right) \frac{1}{\sqrt{ab}} - \frac{e}{2b} \arctan$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)/(-b*x^4+a),x)

[Out] $\frac{1}{4} * c * (1/b * a)^{(1/4)} / a * \ln((x + (1/b * a)^{(1/4)}) / (x - (1/b * a)^{(1/4)})) + 1/2 * c * (1/b * a)^{(1/4)} / a * \arctan(x / (1/b * a)^{(1/4)}) - 1/4 * d / (a * b)^{(1/2)} * \ln((-a + x^2 * (a * b)^{(1/2)}) / (-a - x^2 * (a * b)^{(1/2)})) - 1/2 * e / b / (1/b * a)^{(1/4)} * \arctan(x / (1/b * a)^{(1/4)}) + 1/4 * e / b / (1/b * a)^{(1/4)} * \ln((x + (1/b * a)^{(1/4)}) / (x - (1/b * a)^{(1/4)}))$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")

[Out] Timed out

Sympy [B] time = 5.49401, size = 471, normalized size = 4.06

$$-\text{RootSum}\left(256t^4a^3b^3 + t^2(-64a^2b^2ce - 32a^2b^2d^2) + t(-16a^2bde^2 - 16ab^2c^2d) - a^2e^4 + 2abc^2e^2 - 4abcd^2e + abd^4 - t\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/(-b*x**4+a),x)

[Out] $-\text{RootSum}(256*_t^{**4}*a^{**3}*b^{**3} + *_t^{**2}*(-64*a^{**2}*b^{**2}*c*e - 32*a^{**2}*b^{**2}*d^{**2}) + *_t*(-16*a^{**2}*b*d*e^{**2} - 16*a*b^{**2}*c^{**2}*d) - a^{**2}*e^{**4} + 2*a*b*c^{**2}*e^{**2} - 4*a*b*c*d^{**2}*e + a*b*d^{**4} - b^{**2}*c^{**4}, \text{Lambda}(_t, *_t*\log(x + (-64*_t^{**3}*a^{**4}*b^{**2}*e^{**3} - 64*_t^{**3}*a^{**3}*b^{**3}*c^{**2}*e + 128*_t^{**3}*a^{**3}*b^{**3}*c*d^{**2} + 48*_t^{**2}*a^{**3}*b^{**2}*c*d*e^{**2} - 32*_t^{**2}*a^{**3}*b^{**2}*d^{**3}*e - 16*_t^{**2}*a^{**2}*b^{**3}*c^{**3}*d + 12*_t*a^{**3}*b*c*e^{**4} + 12*_t*a^{**3}*b*d^{**2}*e^{**3} + 16*_t*a^{**2}*b^{**2}*c^{**3}*e^{**2} - 36*_t*a^{**2}*b^{**2}*c^{**2}*d^{**2}*e - 8*_t*a^{**2}*b^{**2}*c*d^{**4} + 4*_t*a*b^{**3}*c^{**5} + 3*a^{**3}*d*e^{**5} - 5*a^{**2}*b*c*d^{**3}*e^{**2} + 2*a^{**2}*b*d^{**5}*e + 5*a*b^{**2}*c^{**4}*d*e - 5*a*b^{**2}*c^{**3}*d^{**3}))/ (a^{**3}*e^{**6} + a^{**2}*b*c^{**2}*e^{**4} - 8*a^{**2}*b*c*d^{**2}*e^{**3} + 4*a^{**2}*b*d^{**4}*e^{**2} - a*b^{**2}*c^{**4}*e^{**2} + 8*a*b^{**2}*c^{**3}*d^{**2}*e - 4*a*b^{**2}*c^{**2}*d^{**4} - b^{**3}*c^{**6}))$

Giac [B] time = 1.08251, size = 396, normalized size = 3.41

$$\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-abb^2d} - (-ab^3)^{\frac{1}{4}}b^2c - (-ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{-abb^2d} - (-ab^3)^{\frac{1}{4}}b^2c - (-ab^3)^{\frac{3}{4}}e\right)}{4ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")

[Out] $-1/4*\sqrt{2}*(\sqrt{2}*\sqrt{-a*b}*b^2*d - (-a*b^3)^{(1/4)}*b^2*c - (-a*b^3)^{(3/4)}*e)*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(-a/b)^{(1/4)})/(-a/b)^{(1/4)})/(a*b^3) - 1/4*\sqrt{2}*(\sqrt{2}*\sqrt{-a*b}*b^2*d - (-a*b^3)^{(1/4)}*b^2*c - (-a*b^3)^{(3/4)}*e)*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(-a/b)^{(1/4)})/(-a/b)^{(1/4)})/(a*b^3) + 1/8*\sqrt{2}*((-a*b^3)^{(1/4)}*b^2*c - (-a*b^3)^{(3/4)}*e)*\log(x^2 + \sqrt{2}*\sqrt{-a*b})$

$$(2) * x * (-a/b)^{1/4} + \sqrt{-a/b}) / (a * b^3) - 1/8 * \sqrt{2} * ((-a * b^3)^{1/4} * b^2 * c - (-a * b^3)^{3/4} * e) * \log(x^2 - \sqrt{2} * x * (-a/b)^{1/4} + \sqrt{-a/b}) / (a * b^3)$$

3.126 $\int \frac{c+dx+ex^2}{a+bx^4} dx$

Optimal. Leaf size=277

$$\frac{(\sqrt{bc} - \sqrt{ae}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{(\sqrt{bc} - \sqrt{ae}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} - \frac{(\sqrt{ae} + \sqrt{bc}) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}}{2\sqrt{2}a^{3/4}b^{3/4}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}$$

[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4))

Rubi [A] time = 0.196624, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$, Rules used = {1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(\sqrt{bc} - \sqrt{ae}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{(\sqrt{bc} - \sqrt{ae}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} - \frac{(\sqrt{ae} + \sqrt{bc}) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}}{2\sqrt{2}a^{3/4}b^{3/4}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/(a + b*x^4), x]

[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4))

Rule 1876

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,

0] && Expon[Pq, x] < n

Rule 275

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1168

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],

x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{c + dx + ex^2}{a + bx^4} dx &= \int \left(\frac{dx}{a + bx^4} + \frac{c + ex^2}{a + bx^4} \right) dx \\
 &= d \int \frac{x}{a + bx^4} dx + \int \frac{c + ex^2}{a + bx^4} dx \\
 &= \frac{1}{2} d \operatorname{Subst} \left(\int \frac{1}{a + bx^2} dx, x, x^2 \right) + \frac{\left(\frac{\sqrt{bc}}{\sqrt{a}} - e \right) \int \frac{\sqrt{a}\sqrt{b-bx^2}}{a+bx^4} dx}{2b} + \frac{\left(\frac{\sqrt{bc}}{\sqrt{a}} + e \right) \int \frac{\sqrt{a}\sqrt{b+bx^2}}{a+bx^4} dx}{2b} \\
 &= \frac{d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{b}} + \frac{\left(\frac{\sqrt{bc}}{\sqrt{a}} + e \right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt{b}} + x^2} dx}{4b} + \frac{\left(\frac{\sqrt{bc}}{\sqrt{a}} + e \right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt{b}} + x^2} dx}{4b} - \frac{(\sqrt{bc} - \sqrt{ae}) \int \frac{1}{\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2}} dx}{4\sqrt{2}a^{3/4}b^{3/4}} \\
 &= \frac{d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{b}} - \frac{(\sqrt{bc} - \sqrt{ae}) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{(\sqrt{bc} - \sqrt{ae}) \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} \\
 &= \frac{d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{b}} - \frac{(\sqrt{bc} + \sqrt{ae}) \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt{a}} \right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{(\sqrt{bc} + \sqrt{ae}) \tan^{-1} \left(1 + \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt{a}} \right)}{2\sqrt{2}a^{3/4}b^{3/4}} - \frac{(\sqrt{bc} - \sqrt{ae}) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}}
 \end{aligned}$$

Mathematica [A] time = 0.100156, size = 229, normalized size = 0.83

$$\frac{-2 \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt{a}} \right) (2\sqrt[4]{a}\sqrt[4]{bd} + \sqrt{2}\sqrt{ae} + \sqrt{2}\sqrt{bc}) + 2 \tan^{-1} \left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt{a}} + 1 \right) (-2\sqrt[4]{a}\sqrt[4]{bd} + \sqrt{2}\sqrt{ae} + \sqrt{2}\sqrt{bc}) - \sqrt{2} (\sqrt{bc} - \sqrt{ae}) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2})}{8a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(a + b*x^4), x]

```
[Out] (-2*(Sqrt[2]*Sqrt[b]*c + 2*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*Sqrt[b]*c - 2*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - Sqrt[2]*(Sqrt[b]*c - Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(8*a^(3/4)*b^(3/4))
```

Maple [A] time = 0.003, size = 280, normalized size = 1.

$$\frac{c\sqrt{2}}{8a} \sqrt[4]{\frac{a}{b}} \ln \left(\left(x^2 + \sqrt[4]{\frac{a}{b}} x \sqrt{2} + \sqrt{\frac{a}{b}} \right) \left(x^2 - \sqrt[4]{\frac{a}{b}} x \sqrt{2} + \sqrt{\frac{a}{b}} \right)^{-1} \right) + \frac{c\sqrt{2}}{4a} \sqrt[4]{\frac{a}{b}} \arctan \left(x \sqrt{2} \frac{1}{\sqrt[4]{\frac{a}{b}}} + 1 \right) + \frac{c\sqrt{2}}{4a} \sqrt[4]{\frac{a}{b}} \arctan \left(x \sqrt{2} \frac{1}{\sqrt[4]{\frac{a}{b}}} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^4+a),x)
```

```
[Out] 1/8*c*(1/b*a)^(1/4)/a*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/4*c*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/4*c*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+1/2*d/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+1/8*e/b/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/4*e/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/4*e/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")

[Out] Timed out

Sympy [A] time = 5.46942, size = 466, normalized size = 1.68

RootSum($256t^4a^3b^3 + t^2(64a^2b^2ce + 32a^2b^2d^2) + t(16a^2bde^2 - 16ab^2c^2d) + a^2e^4 + 2abc^2e^2 - 4abcd^2e + abd^4 + b^2c^4$,

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/(b*x**4+a),x)

[Out] RootSum($256*_t**4*a**3*b**3 + _t**2*(64*a**2*b**2*c*e + 32*a**2*b**2*d**2) + _t*(16*a**2*b*d*e**2 - 16*a*b**2*c**2*d) + a**2*e**4 + 2*a*b*c**2*e**2 - 4*a*b*c*d**2*e + a*b*d**4 + b**2*c**4, \text{Lambda}(_t, _t*\log(x + (64*_t**3*a**4*b**2*e**3 - 64*_t**3*a**3*b**3*c**2*e + 128*_t**3*a**3*b**3*c*d**2 + 48*_t**2*a**3*b**2*c*d*e**2 - 32*_t**2*a**3*b**2*d**3*e + 16*_t**2*a**2*b**3*c**3*d + 12*_t*a**3*b*c*e**4 + 12*_t*a**3*b*d**2*e**3 - 16*_t*a**2*b**2*c**3*e**2 + 36*_t*a**2*b**2*c**2*d**2*e + 8*_t*a**2*b**2*c*d**4 + 4*_t*a*b**3*c**5 + 3*a**3*d*e**5 + 5*a**2*b*c*d**3*e**2 - 2*a**2*b*d**5*e + 5*a*b**2*c**4*d*e - 5*a*b**2*c**3*d**3)/(a**3*e**6 - a**2*b*c**2*e**4 + 8*a**2*b*c*d**2*e**3 - 4*a**2*b*d**4*e**2 - a*b**2*c**4*e**2 + 8*a*b**2*c**3*d**2*e - 4*a*b**2*c**2*d**4 + b**3*c**6))$)

Giac [A] time = 1.08677, size = 371, normalized size = 1.34

$$\frac{\sqrt{2}\left(\sqrt{2}\sqrt{abb^2d} - (ab^3)^{\frac{1}{4}}b^2c - (ab^3)^{\frac{3}{4}}e\right) \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{abb^2d} - (ab^3)^{\frac{1}{4}}b^2c - (ab^3)^{\frac{3}{4}}e\right) \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")

```
[Out] -1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d - (a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)
*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) - 1
/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d - (a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e
)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/8
*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(
1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/
4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

$$3.127 \quad \int \frac{c+dx+ex^2}{(a-bx^4)^2} dx$$

Optimal. Leaf size=146

$$\frac{(3\sqrt{bc} - \sqrt{ae}) \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}} + \frac{(\sqrt{ae} + 3\sqrt{bc}) \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} + \frac{x(c+dx+ex^2)}{4a(a-bx^4)}$$

[Out] (x*(c + d*x + e*x^2))/(4*a*(a - b*x^4)) + ((3*Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b])

Rubi [A] time = 0.127515, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1855, 1876, 275, 208, 1167, 205}

$$\frac{(3\sqrt{bc} - \sqrt{ae}) \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}} + \frac{(\sqrt{ae} + 3\sqrt{bc}) \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} + \frac{x(c+dx+ex^2)}{4a(a-bx^4)}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/(a - b*x^4)^2,x]

[Out] (x*(c + d*x + e*x^2))/(4*a*(a - b*x^4)) + ((3*Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b])

Rule 1855

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] & & PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2}{(a - bx^4)^2} dx &= \frac{x(c + dx + ex^2)}{4a(a - bx^4)} - \frac{\int \frac{-3c - 2dx - ex^2}{a - bx^4} dx}{4a} \\
&= \frac{x(c + dx + ex^2)}{4a(a - bx^4)} - \frac{\int \left(-\frac{2dx}{a - bx^4} + \frac{-3c - ex^2}{a - bx^4} \right) dx}{4a} \\
&= \frac{x(c + dx + ex^2)}{4a(a - bx^4)} - \frac{\int \frac{-3c - ex^2}{a - bx^4} dx}{4a} + \frac{d \int \frac{x}{a - bx^4} dx}{2a} \\
&= \frac{x(c + dx + ex^2)}{4a(a - bx^4)} + \frac{d \operatorname{Subst} \left(\int \frac{1}{a - bx^2} dx, x, x^2 \right)}{4a} - \frac{\left(\frac{3\sqrt{bc}}{\sqrt{a}} - e \right) \int \frac{1}{-\sqrt{a}\sqrt{b} - bx^2} dx}{8a} + \frac{(3\sqrt{bc} + \sqrt{ae}) \int \frac{1}{\sqrt{a}\sqrt{b} + bx^2} dx}{8a^{3/2}} \\
&= \frac{x(c + dx + ex^2)}{4a(a - bx^4)} + \frac{(3\sqrt{bc} - \sqrt{ae}) \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{8a^{7/4}b^{3/4}} + \frac{(3\sqrt{bc} + \sqrt{ae}) \tanh^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{8a^{7/4}b^{3/4}} + \frac{d \tanh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{4a^{3/2}\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.201098, size = 211, normalized size = 1.45

$$\frac{-\frac{\log\left(\sqrt[4]{a}-\sqrt[4]{bx}\right)\left(a^{3/4}e+3\sqrt[4]{a}\sqrt{bc}+2\sqrt{a}\sqrt[4]{bd}\right)}{b^{3/4}}+\frac{\log\left(\sqrt[4]{a}+\sqrt[4]{bx}\right)\left(a^{3/4}e+3\sqrt[4]{a}\sqrt{bc}-2\sqrt{a}\sqrt[4]{bd}\right)}{b^{3/4}}-\frac{2\sqrt[4]{a}\left(\sqrt{ae}-3\sqrt{bc}\right)\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{b^{3/4}}+\frac{4ax(c+dx+ex)}{a-bx^4}+\frac{2\sqrt{ad}\log\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}}{16a^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(a - b*x^4)^2, x]

[Out] $\left(\frac{4ax(c + dx + ex)}{a - bx^4} - (2a^{1/4}(-3\sqrt{b}c + \sqrt{a}e) \operatorname{ArcTan}[\frac{b^{1/4}x}{a^{1/4}}])/b^{3/4} - ((3a^{1/4}\sqrt{b}c + 2\sqrt{a}e) \operatorname{ArcTan}[\frac{b^{1/4}x}{a^{1/4}}])/b^{3/4} + ((3a^{1/4}\sqrt{b}c - 2\sqrt{a}e) \operatorname{Log}[a^{1/4} - b^{1/4}x])/b^{3/4} + ((3a^{1/4}\sqrt{b}c + 2\sqrt{a}e) \operatorname{Log}[a^{1/4} + b^{1/4}x])/b^{3/4} + (2\sqrt{a}d \operatorname{Log}[\frac{\sqrt{bx^2}}{\sqrt{a}}])/4a^{3/2}\sqrt{b}\right)/16a^2$

Maple [B] time = 0.003, size = 228, normalized size = 1.6

$$-\frac{cx}{4a(bx^4 - a)} + \frac{3c}{16a^2} \sqrt[4]{\frac{a}{b}} \ln\left(\left(x + \sqrt[4]{\frac{a}{b}}\right)\left(x - \sqrt[4]{\frac{a}{b}}\right)^{-1}\right) + \frac{3c}{8a^2} \sqrt[4]{\frac{a}{b}} \arctan\left(x \frac{1}{\sqrt[4]{\frac{a}{b}}}\right) - \frac{dx^2}{4a(bx^4 - a)} - \frac{d}{8a} \ln\left(\left(-a + x^2\sqrt{b}\right)\left(-a - x^2\sqrt{b}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d*x+c)/(-b*x^4+a)^2,x)`

[Out]
$$-1/4*c*x/a/(b*x^4-a)+3/16*c/a^2*(1/b*a)^{(1/4)}*\ln((x+(1/b*a)^{(1/4)})/(x-(1/b*a)^{(1/4)}))+3/8*c/a^2*(1/b*a)^{(1/4)}*\arctan(x/(1/b*a)^{(1/4)})-1/4*d*x^2/a/(b*x^4-a)-1/8*d/a/(a*b)^{(1/2)}*\ln((-a+x^2*(a*b)^{(1/2)})/(-a-x^2*(a*b)^{(1/2)}))-1/4*e*x^3/a/(b*x^4-a)-1/8*e/a/b/(1/b*a)^{(1/4)}*\arctan(x/(1/b*a)^{(1/4)})+1/16*e/a/b/(1/b*a)^{(1/4)}*\ln((x+(1/b*a)^{(1/4)})/(x-(1/b*a)^{(1/4)}))$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")`

[Out] Timed out

Sympy [B] time = 7.2781, size = 507, normalized size = 3.47

$$\text{RootSum}\left(65536t^4a^7b^3 + t^2(-3072a^4b^2ce - 2048a^4b^2d^2) + t(128a^3bde^2 + 1152a^2b^2c^2d) - a^2e^4 + 18abc^2e^2 - 48abcd^2e\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/(-b*x**4+a)**2,x)

[Out] RootSum(65536*_t**4*a**7*b**3 + _t**2*(-3072*a**4*b**2*c*e - 2048*a**4*b**2*d**2) + _t*(128*a**3*b*d*e**2 + 1152*a**2*b**2*c**2*d) - a**2*e**4 + 18*a*b*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 - 81*b**2*c**4, Lambda(_t, _t*log(x + (4096*_t**3*a**7*b**2*e**3 + 36864*_t**3*a**6*b**3*c**2*e - 98304*_t**3*a**6*b**3*c*d**2 + 4608*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5*b**2*d**3*e - 13824*_t**2*a**4*b**3*c**3*d - 144*_t*a**4*b*c*e**4 - 192*_t*a**4*b*d**2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*d**2*e + 1536*_t*a**3*b**2*c*d**4 - 3888*_t*a**2*b**3*c**5 + 6*a**3*d*e**5 - 120*a**2*b*c*d**3*e**2 + 64*a**2*b*d**5*e + 810*a*b**2*c**4*d*e - 1080*a*b**2*c**3*d**3)/(a**3*e**6 + 9*a**2*b*c**2*e**4 - 96*a**2*b*c*d**2*e**3 + 64*a**2*b*d**4*e**2 - 81*a*b**2*c**4*e**2 + 864*a*b**2*c**3*d**2*e - 576*a*b**2*c**2*d**4 - 729*b**3*c**6)))) - (c*x + d*x**2 + e*x**3)/(-4*a**2 + 4*a*b*x**4)

Giac [B] time = 1.08587, size = 440, normalized size = 3.01

$$\frac{x^3 e + dx^2 + cx}{4(bx^4 - a)a} + \frac{\sqrt{2} \left(2\sqrt{2}\sqrt{-abb^2d} + 3(-ab^3)^{\frac{1}{4}}b^2c + (-ab^3)^{\frac{3}{4}}e \right) \arctan \left(\frac{\sqrt{2} \left(2x + \sqrt{2} \left(-\frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left(-\frac{a}{b} \right)^{\frac{1}{4}}} \right)}{16a^2b^3} + \frac{\sqrt{2} \left(2\sqrt{2}\sqrt{-abb^2d} + 3 \right)}{16a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")

[Out] -1/4*(x^3*e + d*x^2 + c*x)/((b*x^4 - a)*a) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(-a*b)*b^2*d + 3*(-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(-a*b)*b^2*d + 3*(-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^3)

$$3.128 \quad \int \frac{c+dx+ex^2}{(a+bx^4)^2} dx$$

Optimal. Leaf size=308

$$\frac{(3\sqrt{bc} - \sqrt{ae}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{16\sqrt{2}a^{7/4}b^{3/4}} + \frac{(3\sqrt{bc} - \sqrt{ae}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{16\sqrt{2}a^{7/4}b^{3/4}} - \frac{(\sqrt{ae} + 3\sqrt{bc}) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} - \sqrt{a} - \sqrt{bx^2}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}}$$

[Out] (x*(c + d*x + e*x^2))/(4*a*(a + b*x^4)) + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b]) - ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(3/4)) - ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(3/4))

Rubi [A] time = 0.254637, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(3\sqrt{bc} - \sqrt{ae}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{16\sqrt{2}a^{7/4}b^{3/4}} + \frac{(3\sqrt{bc} - \sqrt{ae}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{16\sqrt{2}a^{7/4}b^{3/4}} - \frac{(\sqrt{ae} + 3\sqrt{bc}) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} - \sqrt{a} - \sqrt{bx^2}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/(a + b*x^4)^2, x]

[Out] (x*(c + d*x + e*x^2))/(4*a*(a + b*x^4)) + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b]) - ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(3/4)) - ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(3/4))

Rule 1855

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p

+ 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] & PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]

Rule 1876

Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 275

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1168

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2}{(a + bx^4)^2} dx &= \frac{x(c + dx + ex^2)}{4a(a + bx^4)} - \frac{\int \frac{-3c - 2dx - ex^2}{a + bx^4} dx}{4a} \\
&= \frac{x(c + dx + ex^2)}{4a(a + bx^4)} - \frac{\int \left(-\frac{2dx}{a + bx^4} + \frac{-3c - ex^2}{a + bx^4} \right) dx}{4a} \\
&= \frac{x(c + dx + ex^2)}{4a(a + bx^4)} - \frac{\int \frac{-3c - ex^2}{a + bx^4} dx}{4a} + \frac{d \int \frac{x}{a + bx^4} dx}{2a} \\
&= \frac{x(c + dx + ex^2)}{4a(a + bx^4)} + \frac{d \operatorname{Subst} \left(\int \frac{1}{a + bx^2} dx, x, x^2 \right)}{4a} + \frac{\left(\frac{3\sqrt{bc}}{\sqrt{a}} - e \right) \int \frac{\sqrt{a}\sqrt{b} - bx^2}{a + bx^4} dx}{8ab} + \frac{\left(\frac{3\sqrt{bc}}{\sqrt{a}} + e \right) \int \frac{\sqrt{a}\sqrt{b} + bx^2}{a + bx^4} dx}{8ab} \\
&= \frac{x(c + dx + ex^2)}{4a(a + bx^4)} + \frac{d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{4a^{3/2}\sqrt{b}} + \frac{\left(\frac{3\sqrt{bc}}{\sqrt{a}} + e \right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{b}} + x^2} dx}{16ab} + \frac{\left(\frac{3\sqrt{bc}}{\sqrt{a}} - e \right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{b}} + x^2} dx}{16ab} \\
&= \frac{x(c + dx + ex^2)}{4a(a + bx^4)} + \frac{d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{4a^{3/2}\sqrt{b}} - \frac{(3\sqrt{bc} - \sqrt{ae}) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2})}{16\sqrt{2}a^{7/4}b^{3/4}} + \frac{(3\sqrt{bc} - \sqrt{ae}) \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2})}{16\sqrt{2}a^{7/4}b^{3/4}} \\
&= \frac{x(c + dx + ex^2)}{4a(a + bx^4)} + \frac{d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{4a^{3/2}\sqrt{b}} - \frac{(3\sqrt{bc} + \sqrt{ae}) \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{8\sqrt{2}a^{7/4}b^{3/4}} + \frac{(3\sqrt{bc} + \sqrt{ae}) \tan^{-1} \left(1 + \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{8\sqrt{2}a^{7/4}b^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.330297, size = 305, normalized size = 0.99

$$\frac{\sqrt{2}(a^{3/4}e - 3\sqrt[4]{a}\sqrt{bc}) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{b^{3/4}} + \frac{\sqrt{2}(3\sqrt[4]{a}\sqrt{bc} - a^{3/4}e) \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{b^{3/4}} - \frac{2\sqrt[4]{a} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \left(4\sqrt[4]{a}\sqrt[4]{bd} + \sqrt{2}\sqrt{ae} + 3\sqrt{2}\sqrt{bc}\right)}{32a^2 b^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(a + b*x^4)^2, x]

[Out] ((8*a*x*(c + x*(d + e*x)))/(a + b*x^4) - (2*a^(1/4)*(3*Sqrt[2]*Sqrt[b]*c + 4*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (2*a^(1/4)*(3*Sqrt[2]*Sqrt[b]*c - 4*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (Sqrt[2]*(-3*a^(1/4)*Sqrt[b]*c + a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4) + (Sqrt[2]*(3*a^(1/4)*Sqrt[b]*c - a^(3/4)*e)*Log[

$\text{Sqrt}[a] + \text{Sqrt}[2] * a^{(1/4)} * b^{(1/4)} * x + \text{Sqrt}[b] * x^2) / b^{(3/4)} / (32 * a^2)$

Maple [A] time = 0.004, size = 344, normalized size = 1.1

$$\frac{cx}{4a(bx^4+a)} + \frac{3c\sqrt{2}^4\sqrt{a}}{32a^2\sqrt{b}} \ln\left(\left(x^2 + \sqrt{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x^2 - \sqrt{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) + \frac{3c\sqrt{2}^4\sqrt{a}}{16a^2\sqrt{b}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt{\frac{a}{b}}} + 1\right) + \frac{3c}{16a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d*x+c)/(b*x^4+a)^2,x)`

[Out] $\frac{1}{4} * c * x / a / (b * x^4 + a) + 3 / 32 * c / a^2 * (1 / b * a)^{(1/4)} * 2^{(1/2)} * \ln((x^2 + (1 / b * a)^{(1/4)} * x * 2^{(1/2)} + (1 / b * a)^{(1/2)}) / (x^2 - (1 / b * a)^{(1/4)} * x * 2^{(1/2)} + (1 / b * a)^{(1/2)})) + 3 / 16 * c / a^2 * (1 / b * a)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (1 / b * a)^{(1/4)} * x + 1) + 3 / 16 * c / a^2 * (1 / b * a)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (1 / b * a)^{(1/4)} * x - 1) + 1 / 4 * d * x^2 / a / (b * x^4 + a) + 1 / 4 * d / a / (a * b)^{(1/2)} * \arctan(x^2 * (b / a)^{(1/2)}) + 1 / 4 * e * x^3 / a / (b * x^4 + a) + 1 / 32 * e / a / b / (1 / b * a)^{(1/4)} * 2^{(1/2)} * \ln((x^2 - (1 / b * a)^{(1/4)} * x * 2^{(1/2)} + (1 / b * a)^{(1/2)}) / (x^2 + (1 / b * a)^{(1/4)} * x * 2^{(1/2)} + (1 / b * a)^{(1/2)})) + 1 / 16 * e / a / b / (1 / b * a)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (1 / b * a)^{(1/4)} * x + 1) + 1 / 16 * e / a / b / (1 / b * a)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (1 / b * a)^{(1/4)} * x - 1)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")

[Out] Timed out

Sympy [A] time = 7.28354, size = 505, normalized size = 1.64

RootSum($65536t^4a^7b^3 + t^2(3072a^4b^2ce + 2048a^4b^2d^2) + t(128a^3bde^2 - 1152a^2b^2c^2d) + a^2e^4 + 18abc^2e^2 - 48abcd^2e$)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/(b*x**4+a)**2,x)

[Out] RootSum($65536*_t**4*a**7*b**3 + _t**2*(3072*a**4*b**2*c*e + 2048*a**4*b**2*d**2) + _t*(128*a**3*b*d*e**2 - 1152*a**2*b**2*c**2*d) + a**2*e**4 + 18*a*b*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 + 81*b**2*c**4, \text{Lambda}(_t, _t \log(x + (4096*_t**3*a**7*b**2*e**3 - 36864*_t**3*a**6*b**3*c**2*e + 98304*_t**3*a**6*b**3*c*d**2 + 4608*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5*b**2*d**3*e + 13824*_t**2*a**4*b**3*c**3*d + 144*_t*a**4*b*c*e**4 + 192*_t*a**4*b*d**2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*d**2*e + 1536*_t*a**3*b**2*c*d**4 + 3888*_t*a**2*b**3*c**5 + 6*a**3*d*e**5 + 120*a**2*b*c*d**3*e**2 - 64*a**2*b*d**5*e + 810*a*b**2*c**4*d*e - 1080*a*b**2*c**3*d**3)/(a**3*e**6 - 9*a**2*b*c**2*e**4 + 96*a**2*b*c*d**2*e**3 - 64*a**2*b*d**4*e**2 - 81*a*b**2*c**4*e**2 + 864*a*b**2*c**3*d**2*e - 576*a*b**2*c**2*d**4 + 729*b**3*c**6)))) + (c*x + d*x**2 + e*x**3)/(4*a**2 + 4*a*b*x**4)$)

Giac [A] time = 1.09527, size = 413, normalized size = 1.34

$$\frac{x^3e + dx^2 + cx}{4(bx^4 + a)a} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{abb^2d} + 3(ab^3)^{\frac{1}{4}}b^2c + (ab^3)^{\frac{3}{4}}e\right) \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2b^3} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{abb^2d} + 3(ab^3)^{\frac{1}{4}}b^2c + (ab^3)^{\frac{3}{4}}e\right)}{4a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")

[Out] $1/4*(x^3*e + d*x^2 + c*x)/((b*x^4 + a)*a) + 1/16*\text{sqrt}(2)*(2*\text{sqrt}(2)*\text{sqrt}(a*b)*b^2*d + 3*(a*b^3)^{(1/4)}*b^2*c + (a*b^3)^{(3/4)}*e)*\text{arctan}(1/2*\text{sqrt}(2)*(2*x$

$$\begin{aligned}
& + \sqrt{2} * (a/b)^{(1/4)} / (a/b)^{(1/4)} / (a^2 * b^3) + 1/16 * \sqrt{2} * (2 * \sqrt{2} * \sqrt{a * b} * b^2 * d + 3 * (a * b^3)^{(1/4)} * b^2 * c + (a * b^3)^{(3/4)} * e) * \arctan(1/2 * \sqrt{2} * (2 * x - \sqrt{2} * (a/b)^{(1/4)} / (a/b)^{(1/4)} / (a^2 * b^3) + 1/32 * \sqrt{2} * (3 * (a * b^3)^{(1/4)} * b^2 * c - (a * b^3)^{(3/4)} * e) * \log(x^2 + \sqrt{2} * x * (a/b)^{(1/4)} + \sqrt{a/b})) / (a^2 * b^3) - 1/32 * \sqrt{2} * (3 * (a * b^3)^{(1/4)} * b^2 * c - (a * b^3)^{(3/4)} * e) * \log(x^2 - \sqrt{2} * x * (a/b)^{(1/4)} + \sqrt{a/b})) / (a^2 * b^3)
\end{aligned}$$

$$3.129 \quad \int \frac{c+dx+ex^2}{(a-bx^4)^3} dx$$

Optimal. Leaf size=179

$$\frac{(21\sqrt{bc} - 5\sqrt{ae}) \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}} + \frac{(5\sqrt{ae} + 21\sqrt{bc}) \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}} + \frac{x(7c + 6dx + 5ex^2)}{32a^2(a - bx^4)} + \frac{3d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} + \frac{x(c + dx)}{8a(a - bx^4)}$$

[Out] (x*(c + d*x + e*x^2))/(8*a*(a - b*x^4)^2) + (x*(7*c + 6*d*x + 5*e*x^2))/(32*a^2*(a - b*x^4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(3/4)) + (3*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b])

Rubi [A] time = 0.167009, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1855, 1876, 275, 208, 1167, 205}

$$\frac{(21\sqrt{bc} - 5\sqrt{ae}) \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}} + \frac{(5\sqrt{ae} + 21\sqrt{bc}) \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}} + \frac{x(7c + 6dx + 5ex^2)}{32a^2(a - bx^4)} + \frac{3d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} + \frac{x(c + dx)}{8a(a - bx^4)}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/(a - b*x^4)^3, x]

[Out] (x*(c + d*x + e*x^2))/(8*a*(a - b*x^4)^2) + (x*(7*c + 6*d*x + 5*e*x^2))/(32*a^2*(a - b*x^4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(3/4)) + (3*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b])

Rule 1855

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] & & PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2}{(a - bx^4)^3} dx &= \frac{x(c + dx + ex^2)}{8a(a - bx^4)^2} - \frac{\int \frac{-7c - 6dx - 5ex^2}{(a - bx^4)^2} dx}{8a} \\
&= \frac{x(c + dx + ex^2)}{8a(a - bx^4)^2} + \frac{x(7c + 6dx + 5ex^2)}{32a^2(a - bx^4)} + \frac{\int \frac{21c + 12dx + 5ex^2}{a - bx^4} dx}{32a^2} \\
&= \frac{x(c + dx + ex^2)}{8a(a - bx^4)^2} + \frac{x(7c + 6dx + 5ex^2)}{32a^2(a - bx^4)} + \frac{\int \left(\frac{12dx}{a - bx^4} + \frac{21c + 5ex^2}{a - bx^4} \right) dx}{32a^2} \\
&= \frac{x(c + dx + ex^2)}{8a(a - bx^4)^2} + \frac{x(7c + 6dx + 5ex^2)}{32a^2(a - bx^4)} + \frac{\int \frac{21c + 5ex^2}{a - bx^4} dx}{32a^2} + \frac{(3d) \int \frac{x}{a - bx^4} dx}{8a^2} \\
&= \frac{x(c + dx + ex^2)}{8a(a - bx^4)^2} + \frac{x(7c + 6dx + 5ex^2)}{32a^2(a - bx^4)} + \frac{(3d) \text{Subst} \left(\int \frac{1}{a - bx^2} dx, x, x^2 \right)}{16a^2} - \frac{\left(\frac{21\sqrt{bc}}{\sqrt{a}} - 5e \right) \int \frac{1}{-\sqrt{a}\sqrt{b-bx^4}} dx}{64a^2} \\
&= \frac{x(c + dx + ex^2)}{8a(a - bx^4)^2} + \frac{x(7c + 6dx + 5ex^2)}{32a^2(a - bx^4)} + \frac{\left(\frac{21\sqrt{bc}}{\sqrt{a}} - 5e \right) \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{64a^{9/4}b^{3/4}} + \frac{(21\sqrt{bc} + 5\sqrt{ae}) \tanh^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{64a^{11/4}b^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.213492, size = 244, normalized size = 1.36

$$\frac{\frac{\log\left(\sqrt[4]{a} - \sqrt[4]{bx}\right)\left(5a^{3/4}e + 21\sqrt[4]{a}\sqrt{bc} + 12\sqrt{a}\sqrt[4]{bd}\right)}{b^{3/4}} + \frac{\log\left(\sqrt[4]{a} + \sqrt[4]{bx}\right)\left(5a^{3/4}e + 21\sqrt[4]{a}\sqrt{bc} - 12\sqrt{a}\sqrt[4]{bd}\right)}{b^{3/4}} + \frac{16a^2x(c + x(d + ex))}{(a - bx^4)^2} + \frac{2\sqrt[4]{a}(21\sqrt{bc} - 5\sqrt{ae}) \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{b^{3/4}}}{128a^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(a - b*x^4)^3, x]

[Out] ((16*a^2*x*(c + x*(d + e*x)))/(a - b*x^4)^2 + (4*a*x*(7*c + x*(6*d + 5*e*x)))/(a - b*x^4) + (2*a^(1/4)*(21*sqrt[b]*c - 5*sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/b^(3/4) - ((21*a^(1/4)*sqrt[b]*c + 12*sqrt[a]*b^(1/4)*d + 5*a^(3/4)*e)*Log[a^(1/4) - b^(1/4)*x])/b^(3/4) + ((21*a^(1/4)*sqrt[b]*c - 12*sqrt[a]*b^(1/4)*d + 5*a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4) + (12*sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(128*a^3)

Maple [B] time = 0.005, size = 286, normalized size = 1.6

$$\frac{cx}{8a(bx^4 - a)^2} - \frac{7cx}{32a^2(bx^4 - a)} + \frac{21c}{128a^3} \sqrt[4]{\frac{a}{b}} \ln \left(\left(x + \sqrt[4]{\frac{a}{b}} \right) \left(x - \sqrt[4]{\frac{a}{b}} \right)^{-1} \right) + \frac{21c}{64a^3} \sqrt[4]{\frac{a}{b}} \arctan \left(x \frac{1}{\sqrt[4]{\frac{a}{b}}} \right) + \frac{dx^2}{8a(bx^4 - a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)/(-b*x^4+a)^3,x)

[Out] 1/8*c*x/a/(b*x^4-a)^2-7/32*c/a^2*x/(b*x^4-a)+21/128*c/a^3*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))+21/64*c/a^3*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))+1/8*d*x^2/a/(b*x^4-a)^2-3/16*d/a^2*x^2/(b*x^4-a)-3/32*d/a^2/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))+1/8*e*x^3/a/(b*x^4-a)^2-5/32*e/a^2*x^3/(b*x^4-a)-5/64*e/a^2/b/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))+5/128*e/a^2/b/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [B] time = 10.2057, size = 563, normalized size = 3.15

$$-\text{RootSum}\left(268435456t^4a^{11}b^3 + t^2(-6881280a^6b^2ce - 4718592a^6b^2d^2) + t(-153600a^4bde^2 - 2709504a^3b^2c^2d) - 62\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/(-b*x**4+a)**3,x)

[Out] -RootSum(268435456*_t**4*a**11*b**3 + _t**2*(-6881280*a**6*b**2*c*e - 4718592*a**6*b**2*d**2) + _t*(-153600*a**4*b*d*e**2 - 2709504*a**3*b**2*c**2*d) - 625*a**2*e**4 + 22050*a*b*c**2*e**2 - 60480*a*b*c*d**2*e + 20736*a*b*d**4 - 194481*b**2*c**4, Lambda(_t, _t*log(x + (-262144000*_t**3*a**10*b**2*e**3 - 4624220160*_t**3*a**9*b**3*c**2*e + 12683575296*_t**3*a**9*b**3*c*d**2 + 309657600*_t**2*a**7*b**2*c*d*e**2 - 283115520*_t**2*a**7*b**2*d**3*e - 1820786688*_t**2*a**6*b**3*c**3*d + 5040000*_t*a**5*b*c*e**4 + 6912000*_t*a**5*b*d**2*e**3 + 118540800*_t*a**4*b**2*c**3*e**2 - 365783040*_t*a**4*b**2*c**2*d**2*e - 111476736*_t*a**4*b**2*c*d**4 + 522764928*_t*a**3*b**3*c**5 + 112500*a**3*d*e**5 - 4536000*a**2*b*c*d**3*e**2 + 2488320*a**2*b*d**5*e + 58344300*a*b**2*c**4*d*e - 80015040*a*b**2*c**3*d**3)/(15625*a**3*e**6 + 275625*a**2*b*c**2*e**4 - 3024000*a**2*b*c*d**2*e**3 + 2073600*a**2*b*d**4*e**2 - 4862025*a*b**2*c**4*e**2 + 53343360*a*b**2*c**3*d**2*e - 36578304*a*b**2*c**2*d**4 - 85766121*b**3*c**6)))) - (-11*a*c*x - 10*a*d*x**2 - 9*a*e*x**3 + 7*b*c*x**5 + 6*b*d*x**6 + 5*b*e*x**7)/(32*a**4 - 64*a**3*b*x**4 + 32*a**2*b**2*x**8)

Giac [B] time = 1.10124, size = 481, normalized size = 2.69

$$\frac{5bx^7e + 6bdx^6 + 7bcx^5 - 9ax^3e - 10adx^2 - 11acx}{32(bx^4 - a)^2a^2} - \frac{\sqrt{2}\left(12\sqrt{2}\sqrt{-abb^2d} - 21(-ab^3)^{\frac{1}{4}}b^2c - 5(-ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\sqrt{-abb^2d} - 21(-ab^3)^{\frac{1}{4}}b^2c - 5(-ab^3)^{\frac{3}{4}}e}{128a^3b^3}\right)}{128a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")

[Out] -1/32*(5*b*x^7*e + 6*b*d*x^6 + 7*b*c*x^5 - 9*a*x^3*e - 10*a*d*x^2 - 11*a*c*x)/((b*x^4 - a)^2*a^2) - 1/128*sqrt(2)*(12*sqrt(2)*sqrt(-a*b)*b^2*d - 21*(-a*b^3)^(1/4)*b^2*c - 5*(-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*

$$\begin{aligned}
& (-a/b)^{(1/4)} / (-a/b)^{(1/4)} / (a^3 b^3) - 1/128 \sqrt{2} * (12 \sqrt{2} \sqrt{-a*b} \\
& * b^2 * d - 21 * (-a*b^3)^{(1/4)} * b^2 * c - 5 * (-a*b^3)^{(3/4)} * e) * \arctan(1/2 \sqrt{2} * \\
& (2*x - \sqrt{2}) * (-a/b)^{(1/4)} / (-a/b)^{(1/4)} / (a^3 b^3) + 1/256 \sqrt{2} * (21 * (- \\
& a*b^3)^{(1/4)} * b^2 * c - 5 * (-a*b^3)^{(3/4)} * e) * \log(x^2 + \sqrt{2} * x * (-a/b)^{(1/4)} + \\
& \sqrt{-a/b}) / (a^3 b^3) - 1/256 \sqrt{2} * (21 * (-a*b^3)^{(1/4)} * b^2 * c - 5 * (-a*b^3 \\
&)^{(3/4)} * e) * \log(x^2 - \sqrt{2} * x * (-a/b)^{(1/4)} + \sqrt{-a/b}) / (a^3 b^3)
\end{aligned}$$

$$3.130 \quad \int \frac{c+dx+ex^2}{(a+bx^4)^3} dx$$

Optimal. Leaf size=341

$$-\frac{(21\sqrt{bc} - 5\sqrt{ae}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{128\sqrt{2}a^{11/4}b^{3/4}} + \frac{(21\sqrt{bc} - 5\sqrt{ae}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{128\sqrt{2}a^{11/4}b^{3/4}} - \frac{(5\sqrt{ae} + 21\sqrt{bc})}{64\sqrt{2}a^{11/4}b^{3/4}}$$

[Out] (x*(c + d*x + e*x^2))/(8*a*(a + b*x^4)^2) + (x*(7*c + 6*d*x + 5*e*x^2))/(32*a^2*(a + b*x^4)) + (3*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b]) - ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(3/4)) - ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(3/4))

Rubi [A] time = 0.310719, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{(21\sqrt{bc} - 5\sqrt{ae}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{128\sqrt{2}a^{11/4}b^{3/4}} + \frac{(21\sqrt{bc} - 5\sqrt{ae}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{128\sqrt{2}a^{11/4}b^{3/4}} - \frac{(5\sqrt{ae} + 21\sqrt{bc})}{64\sqrt{2}a^{11/4}b^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/(a + b*x^4)^3,x]

[Out] (x*(c + d*x + e*x^2))/(8*a*(a + b*x^4)^2) + (x*(7*c + 6*d*x + 5*e*x^2))/(32*a^2*(a + b*x^4)) + (3*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b]) - ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(3/4)) - ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(3/4))

Rule 1855

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
+ 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))]/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2}{(a + bx^4)^3} dx &= \frac{x(c + dx + ex^2)}{8a(a + bx^4)^2} - \frac{\int \frac{-7c - 6dx - 5ex^2}{(a + bx^4)^2} dx}{8a} \\
&= \frac{x(c + dx + ex^2)}{8a(a + bx^4)^2} + \frac{x(7c + 6dx + 5ex^2)}{32a^2(a + bx^4)} + \frac{\int \frac{21c + 12dx + 5ex^2}{a + bx^4} dx}{32a^2} \\
&= \frac{x(c + dx + ex^2)}{8a(a + bx^4)^2} + \frac{x(7c + 6dx + 5ex^2)}{32a^2(a + bx^4)} + \frac{\int \left(\frac{12dx}{a + bx^4} + \frac{21c + 5ex^2}{a + bx^4} \right) dx}{32a^2} \\
&= \frac{x(c + dx + ex^2)}{8a(a + bx^4)^2} + \frac{x(7c + 6dx + 5ex^2)}{32a^2(a + bx^4)} + \frac{\int \frac{21c + 5ex^2}{a + bx^4} dx}{32a^2} + \frac{(3d) \int \frac{x}{a + bx^4} dx}{8a^2} \\
&= \frac{x(c + dx + ex^2)}{8a(a + bx^4)^2} + \frac{x(7c + 6dx + 5ex^2)}{32a^2(a + bx^4)} + \frac{(3d) \text{Subst} \left(\int \frac{1}{a + bx^2} dx, x, x^2 \right)}{16a^2} + \frac{\left(\frac{21\sqrt{bc}}{\sqrt{a}} - 5e \right) \int \frac{\sqrt{a}\sqrt{b} - bx^2}{a + bx^4}}{64a^2b} \\
&= \frac{x(c + dx + ex^2)}{8a(a + bx^4)^2} + \frac{x(7c + 6dx + 5ex^2)}{32a^2(a + bx^4)} + \frac{3d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{16a^{5/2}\sqrt{b}} - \frac{\left(\frac{21\sqrt{bc}}{\sqrt{a}} - 5e \right) \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt{b}} + 2x}{-\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt{b}} - x^2} dx}{128\sqrt{2}a^{9/4}b^{3/4}}}{128\sqrt{2}a^{9/4}b^{3/4}} - \frac{\left(\frac{21\sqrt{bc}}{\sqrt{a}} - 5e \right) \int \frac{\sqrt{a}\sqrt{b} - bx^2}{a + bx^4}}{64a^2b} \\
&= \frac{x(c + dx + ex^2)}{8a(a + bx^4)^2} + \frac{x(7c + 6dx + 5ex^2)}{32a^2(a + bx^4)} + \frac{3d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{16a^{5/2}\sqrt{b}} - \frac{\left(\frac{21\sqrt{bc}}{\sqrt{a}} - 5e \right) \log \left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a}\sqrt{bx^2} \right)}{128\sqrt{2}a^{9/4}b^{3/4}} \\
&= \frac{x(c + dx + ex^2)}{8a(a + bx^4)^2} + \frac{x(7c + 6dx + 5ex^2)}{32a^2(a + bx^4)} + \frac{3d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{16a^{5/2}\sqrt{b}} - \frac{(21\sqrt{bc} + 5\sqrt{ae}) \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt{a}} \right)}{64\sqrt{2}a^{11/4}b^{3/4}} + \frac{\left(\frac{21\sqrt{bc}}{\sqrt{a}} - 5e \right) \int \frac{\sqrt{a}\sqrt{b} - bx^2}{a + bx^4}}{64a^2b}
\end{aligned}$$

Mathematica [A] time = 0.306739, size = 337, normalized size = 0.99

$$\frac{\sqrt{2}(5a^{3/4}e - 21\sqrt[4]{a}\sqrt{bc}) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{b^{3/4}} + \frac{\sqrt{2}(21\sqrt[4]{a}\sqrt{bc} - 5a^{3/4}e) \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{b^{3/4}} + \frac{32a^2x(c + x(d + ex))}{(a + bx^4)^2} - \frac{2\sqrt[4]{a} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt{a}}\right)(24\sqrt{2}a^{11/4}b^{3/4} - (21\sqrt{bc} + 5\sqrt{ae}) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt{a}}\right))}{256a^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(a + b*x^4)^3,x]


```
[Out] ((32*a^2*x*(c + x*(d + e*x)))/(a + b*x^4)^2 + (8*a*x*(7*c + x*(6*d + 5*e*x)))/(a + b*x^4) - (2*a^(1/4)*(21*Sqrt[2]*Sqrt[b]*c + 24*a^(1/4)*b^(1/4)*d + 5*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/b^(3/4) + (2*a^(1/4)*(21*Sqrt[2]*Sqrt[b]*c - 24*a^(1/4)*b^(1/4)*d + 5*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/b^(3/4) + (Sqrt[2]*(-21*a^(1/4)*Sqrt[b]*c + 5*a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4) + (Sqrt[2]*(21*a^(1/4)*Sqrt[b]*c - 5*a^(3/4)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(256*a^3)
```

Maple [A] time = 0.006, size = 396, normalized size = 1.2

$$\frac{cx}{8a(bx^4+a)^2} + \frac{7cx}{32a^2(bx^4+a)} + \frac{21c\sqrt{2}}{256a^3} \sqrt[4]{\frac{a}{b}} \ln\left(\left(x^2 + \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x^2 - \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) + \frac{21c\sqrt{2}}{128a^3} \sqrt[4]{\frac{a}{b}} \arctan\left(\frac{x^2 + \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}}{x^2 - \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^4+a)^3,x)
```

```
[Out] 1/8*c*x/a/(b*x^4+a)^2+7/32*c/a^2*x/(b*x^4+a)+21/256*c/a^3*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+21/128*c/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+21/128*c/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+1/8*d*x^2/a/(b*x^4+a)^2+3/16*d/a^2*x^2/(b*x^4+a)+3/16*d/a^2/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+1/8*e*x^3/a/(b*x^4+a)^2+5/32*e/a^2*x^3/(b*x^4+a)+5/256*e/a^2/b/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+5/128*e/a^2/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+5/128*e/a^2/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [A] time = 10.127, size = 558, normalized size = 1.64

$$\text{RootSum}\left(268435456t^4a^{11}b^3 + t^2(6881280a^6b^2ce + 4718592a^6b^2d^2) + t(153600a^4bde^2 - 2709504a^3b^2c^2d) + 625a^2e^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/(b*x**4+a)**3,x)

[Out] RootSum(268435456*_t**4*a**11*b**3 + _t**2*(6881280*a**6*b**2*c*e + 4718592*a**6*b**2*d**2) + _t*(153600*a**4*b*d*e**2 - 2709504*a**3*b**2*c**2*d) + 625*a**2*e**4 + 22050*a*b*c**2*e**2 - 60480*a*b*c*d**2*e + 20736*a*b*d**4 + 194481*b**2*c**4, Lambda(_t, _t*log(x + (262144000*_t**3*a**10*b**2*e**3 - 4624220160*_t**3*a**9*b**3*c**2*e + 12683575296*_t**3*a**9*b**3*c*d**2 + 309657600*_t**2*a**7*b**2*c*d*e**2 - 283115520*_t**2*a**7*b**2*d**3*e + 1820786688*_t**2*a**6*b**3*c**3*d + 5040000*_t*a**5*b*c*e**4 + 6912000*_t*a**5*b*d**2*e**3 - 118540800*_t*a**4*b**2*c**3*e**2 + 365783040*_t*a**4*b**2*c**2*d**2*e + 111476736*_t*a**4*b**2*c*d**4 + 522764928*_t*a**3*b**3*c**5 + 112500*a**3*d*e**5 + 4536000*a**2*b*c*d**3*e**2 - 2488320*a**2*b*d**5*e + 58344300*a*b**2*c**4*d*e - 80015040*a*b**2*c**3*d**3)/(15625*a**3*e**6 - 275625*a**2*b*c**2*e**4 + 3024000*a**2*b*c*d**2*e**3 - 2073600*a**2*b*d**4*e**2 - 4862025*a*b**2*c**4*e**2 + 53343360*a*b**2*c**3*d**2*e - 36578304*a*b**2*c**2*d**4 + 85766121*b**3*c**6))) + (11*a*c*x + 10*a*d*x**2 + 9*a*e*x**3 + 7*b*c*x**5 + 6*b*d*x**6 + 5*b*e*x**7)/(32*a**4 + 64*a**3*b*x**4 + 32*a**2*b**2*x**8)

Giac [A] time = 1.09774, size = 454, normalized size = 1.33

$$\frac{5bx^7e + 6bdx^6 + 7bcx^5 + 9ax^3e + 10adx^2 + 11acx}{32(bx^4 + a)^2 a^2} + \frac{\sqrt{2} \left(12\sqrt{2}\sqrt{ab}b^2d + 21(ab^3)^{\frac{1}{4}}b^2c + 5(ab^3)^{\frac{3}{4}}e \right) \arctan \left(\frac{\sqrt{2}(2x + \sqrt{2}\sqrt{a/b})}{2\sqrt{a/b}} \right)}{128a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")

[Out] 1/32*(5*b*x^7*e + 6*b*d*x^6 + 7*b*c*x^5 + 9*a*x^3*e + 10*a*d*x^2 + 11*a*c*x) / ((b*x^4 + a)^2*a^2) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b^3)^(1/4)*b^2*c + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4)))/(a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b^3)^(1/4)*b^2*c + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4)))/(a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c - 5*(a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c - 5*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3)

$$3.131 \quad \int \frac{c+dx+ex^2}{(a-bx^4)^4} dx$$

Optimal. Leaf size=211

$$\frac{(77\sqrt{bc} - 15\sqrt{ae}) \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}} + \frac{(15\sqrt{ae} + 77\sqrt{bc}) \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}} + \frac{x(77c + 60dx + 45ex^2)}{384a^3(a - bx^4)} + \frac{x(11c + 10dx + 9ex^2)}{96a^2(a - bx^4)^2} +$$

[Out] (x*(c + d*x + e*x^2))/(12*a*(a - b*x^4)^3) + (x*(11*c + 10*d*x + 9*e*x^2))/(96*a^2*(a - b*x^4)^2) + (x*(77*c + 60*d*x + 45*e*x^2))/(384*a^3*(a - b*x^4)) + ((77*sqrt[b]*c - 15*sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(3/4)) + ((77*sqrt[b]*c + 15*sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(3/4)) + (5*d*ArcTanh[(sqrt[b]*x^2)/sqrt[a]])/(32*a^(7/2)*sqrt[b])

Rubi [A] time = 0.211313, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1855, 1876, 275, 208, 1167, 205}

$$\frac{(77\sqrt{bc} - 15\sqrt{ae}) \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}} + \frac{(15\sqrt{ae} + 77\sqrt{bc}) \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}} + \frac{x(77c + 60dx + 45ex^2)}{384a^3(a - bx^4)} + \frac{x(11c + 10dx + 9ex^2)}{96a^2(a - bx^4)^2} +$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/(a - b*x^4)^4, x]

[Out] (x*(c + d*x + e*x^2))/(12*a*(a - b*x^4)^3) + (x*(11*c + 10*d*x + 9*e*x^2))/(96*a^2*(a - b*x^4)^2) + (x*(77*c + 60*d*x + 45*e*x^2))/(384*a^3*(a - b*x^4)) + ((77*sqrt[b]*c - 15*sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(3/4)) + ((77*sqrt[b]*c + 15*sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(3/4)) + (5*d*ArcTanh[(sqrt[b]*x^2)/sqrt[a]])/(32*a^(7/2)*sqrt[b])

Rule 1855

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &

& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]

Rule 1876

Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 275

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1167

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[-(a*c)]

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2}{(a - bx^4)^4} dx &= \frac{x(c + dx + ex^2)}{12a(a - bx^4)^3} - \frac{\int \frac{-11c - 10dx - 9ex^2}{(a - bx^4)^3} dx}{12a} \\
&= \frac{x(c + dx + ex^2)}{12a(a - bx^4)^3} + \frac{x(11c + 10dx + 9ex^2)}{96a^2(a - bx^4)^2} + \frac{\int \frac{77c + 60dx + 45ex^2}{(a - bx^4)^2} dx}{96a^2} \\
&= \frac{x(c + dx + ex^2)}{12a(a - bx^4)^3} + \frac{x(11c + 10dx + 9ex^2)}{96a^2(a - bx^4)^2} + \frac{x(77c + 60dx + 45ex^2)}{384a^3(a - bx^4)} - \frac{\int \frac{-231c - 120dx - 45ex^2}{a - bx^4} dx}{384a^3} \\
&= \frac{x(c + dx + ex^2)}{12a(a - bx^4)^3} + \frac{x(11c + 10dx + 9ex^2)}{96a^2(a - bx^4)^2} + \frac{x(77c + 60dx + 45ex^2)}{384a^3(a - bx^4)} - \frac{\int \left(-\frac{120dx}{a - bx^4} + \frac{-231c - 45ex^2}{a - bx^4} \right) dx}{384a^3} \\
&= \frac{x(c + dx + ex^2)}{12a(a - bx^4)^3} + \frac{x(11c + 10dx + 9ex^2)}{96a^2(a - bx^4)^2} + \frac{x(77c + 60dx + 45ex^2)}{384a^3(a - bx^4)} - \frac{\int \frac{-231c - 45ex^2}{a - bx^4} dx}{384a^3} + \frac{(5d) \int \frac{x}{a - bx^4} dx}{16a^3} \\
&= \frac{x(c + dx + ex^2)}{12a(a - bx^4)^3} + \frac{x(11c + 10dx + 9ex^2)}{96a^2(a - bx^4)^2} + \frac{x(77c + 60dx + 45ex^2)}{384a^3(a - bx^4)} + \frac{(5d) \text{Subst} \left(\int \frac{1}{a - bx^2} dx, x, x^2 \right)}{32a^3} \\
&= \frac{x(c + dx + ex^2)}{12a(a - bx^4)^3} + \frac{x(11c + 10dx + 9ex^2)}{96a^2(a - bx^4)^2} + \frac{x(77c + 60dx + 45ex^2)}{384a^3(a - bx^4)} + \frac{(77\sqrt{bc} - 15\sqrt{ae}) \tan^{-1} \left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}} \right)}{256a^{15/4}b^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.258105, size = 276, normalized size = 1.31

$$\frac{3 \log \left(\sqrt[4]{a} - \sqrt[4]{bx} \right) \left(15a^{3/4}e + 77\sqrt[4]{a}\sqrt{bc} + 40\sqrt{a}\sqrt[4]{bd} \right)}{b^{3/4}} + \frac{3 \log \left(\sqrt[4]{a} + \sqrt[4]{bx} \right) \left(15a^{3/4}e + 77\sqrt[4]{a}\sqrt{bc} - 40\sqrt{a}\sqrt[4]{bd} \right)}{b^{3/4}} + \frac{128a^3x(c + x(d + ex))}{(a - bx^4)^3} + \frac{16a^2x(11c + x(10d + 9ex))}{(a - bx^4)^2} + \frac{(77\sqrt{bc} - 15\sqrt{ae}) \tan^{-1} \left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}} \right)}{256a^{15/4}b^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(a - b*x^4)^4, x]

[Out] ((128*a^3*x*(c + x*(d + e*x)))/(a - b*x^4)^3 + (4*a*x*(77*c + 15*x*(4*d + 3*e*x)))/(a - b*x^4) + (16*a^2*x*(11*c + x*(10*d + 9*e*x)))/(a - b*x^4)^2 + (6*a^(1/4)*(77*sqrt[b]*c - 15*sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)]/b^(3/4) - (3*(77*a^(1/4)*sqrt[b]*c + 40*sqrt[a]*b^(1/4)*d + 15*a^(3/4)*e)*Log[a^(1/4) - b^(1/4)*x])/b^(3/4) + (3*(77*a^(1/4)*sqrt[b]*c - 40*sqrt[a]*b^(1/4)

$*d + 15*a^{(3/4)*e}*Log[a^{(1/4)} + b^{(1/4)*x}]/b^{(3/4)} + (120*sqrt[a]*d*Log[sqrt[a] + sqrt[b]*x^2])/sqrt[b]/(1536*a^4)$

Maple [A] time = 0.013, size = 274, normalized size = 1.3

$$\frac{1}{(bx^4 - a)^3} \left(-\frac{15b^2ex^{11}}{128a^3} - \frac{5b^2dx^{10}}{32a^3} - \frac{77b^2cx^9}{384a^3} + \frac{21bex^7}{64a^2} + \frac{5bdx^6}{12a^2} + \frac{33bcx^5}{64a^2} - \frac{113ex^3}{384a} - \frac{11dx^2}{32a} - \frac{51cx}{128a} \right) + \frac{77c}{512a^4} \sqrt[4]{\frac{a}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)/(-b*x^4+a)^4,x)

[Out] $(-15/128*e/a^3*b^2*x^{11}-5/32*d/a^3*b^2*x^{10}-77/384*c/a^3*b^2*x^9+21/64/a^2*b*e*x^7+5/12/a^2*d*b*x^6+33/64/a^2*c*b*x^5-113/384/a*e*x^3-11/32*d/a*x^2-51/128*c/a*x)/(b*x^4-a)^3+77/512/a^4*c*(1/b*a)^{(1/4)}*\ln((x+(1/b*a)^{(1/4)})/(x-(1/b*a)^{(1/4)}))+77/256/a^4*c*(1/b*a)^{(1/4)}*\arctan(x/(1/b*a)^{(1/4)})-5/64/a^3*d/(a*b)^{(1/2)}*\ln((-a+x^2*(a*b)^{(1/2)})/(-a-x^2*(a*b)^{(1/2)}))-15/256/a^3*e/b/(1/b*a)^{(1/4)}*\arctan(x/(1/b*a)^{(1/4)})+15/512/a^3*e/b/(1/b*a)^{(1/4)}*\ln((x+(1/b*a)^{(1/4)})/(x-(1/b*a)^{(1/4)}))$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")

[Out] Timed out

Sympy [B] time = 16.4793, size = 612, normalized size = 2.9

RootSum(68719476736t^4a^15b^3 + t^2(-1211105280a^8b^2ce - 838860800a^8b^2d^2) + t(18432000a^5bde^2 + 485703680a^4b^2c

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/(-b*x**4+a)**4,x)

[Out] RootSum(68719476736*_t**4*a**15*b**3 + _t**2*(-1211105280*a**8*b**2*c*e - 838860800*a**8*b**2*d**2) + _t*(18432000*a**5*b*d*e**2 + 485703680*a**4*b**2*c**2*d) - 50625*a**2*e**4 + 2668050*a*b*c**2*e**2 - 7392000*a*b*c*d**2*e + 2560000*a*b*d**4 - 35153041*b**2*c**4, Lambda(_t, _t*log(x + (452984832000*_t**3*a**13*b**2*e**3 + 11936653639680*_t**3*a**12*b**3*c**2*e - 33071248179200*_t**3*a**12*b**3*c*d**2 + 544997376000*_t**2*a**9*b**2*c*d*e**2 - 50331648000*_t**2*a**9*b**2*d**3*e - 4787095470080*_t**2*a**8*b**3*c**3*d - 5987520000*_t*a**6*b*c*e**4 - 8294400000*_t*a**6*b*d**2*e**3 - 210370406400*_t*a**5*b**2*c**3*e**2 + 655699968000*_t*a**5*b**2*c**2*d**2*e + 20185088000*_t*a**5*b**2*c*d**4 - 1385873488384*_t*a**4*b**3*c**5 + 91125000*a**3*d*e**5 - 5544000000*a**2*b*c*d**3*e**2 + 3072000000*a**2*b*d**5*e + 105459123000*a*b**2*c**4*d*e - 146090560000*a*b**2*c**3*d**3)/(11390625*a**3*e**6 + 300155625*a**2*b*c**2*e**4 - 3326400000*a**2*b*c*d**2*e**3 + 2304000000*a**2*b*d**4*e**2 - 7909434225*a*b**2*c**4*e**2 + 87654336000*a*b**2*c**3*d**2*e - 60712960000*a*b**2*c**2*d**4 - 208422380089*b**3*c**6))) - (153*a**2*c*x + 132*a**2*d*x**2 + 113*a**2*e*x**3 - 198*a*b*c*x**5 - 160*a*b*d*x**6 - 126*a*b*e*x**7 + 77*b**2*c*x**9 + 60*b**2*d*x**10 + 45*b**2*e*x**11)/(-384*a**6 + 1152*a**5*b*x**4 - 1152*a**4*b**2*x**8 + 384*a**3*b**3*x**12)

Giac [B] time = 1.0982, size = 531, normalized size = 2.52

$$\frac{\sqrt{2} \left(40 \sqrt{2} \sqrt{-abb^2d} + 77 (-ab^3)^{\frac{1}{4}} b^2c + 15 (-ab^3)^{\frac{3}{4}} e \right) \arctan \left(\frac{\sqrt{2} \left(2x + \sqrt{2} \left(-\frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left(-\frac{a}{b} \right)^{\frac{1}{4}}} \right)}{512 a^4 b^3} + \frac{\sqrt{2} \left(40 \sqrt{2} \sqrt{-abb^2d} + 77 (-ab^3)^{\frac{1}{4}} b^2c \right)}{512 a^4 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")`

[Out]
$$\frac{1}{512}\sqrt{2}\left(40\sqrt{2}\sqrt{-ab}b^2d + 77(-ab^3)^{1/4}b^2c + 15(-ab^3)^{3/4}e\right)\arctan\left(\frac{1}{2}\sqrt{2}(2x + \sqrt{2})(-a/b)^{1/4}\right)/(-a/b)^{1/4} + \frac{1}{512}\sqrt{2}\left(40\sqrt{2}\sqrt{-ab}b^2d + 77(-ab^3)^{1/4}b^2c + 15(-ab^3)^{3/4}e\right)\arctan\left(\frac{1}{2}\sqrt{2}(2x - \sqrt{2})(-a/b)^{1/4}\right)/(-a/b)^{1/4} + \frac{1}{1024}\sqrt{2}\left(77(-ab^3)^{1/4}b^2c - 15(-ab^3)^{3/4}e\right)\log(x^2 + \sqrt{2}x(-a/b)^{1/4} + \sqrt{-a/b})/(-a/b)^{1/4} - \frac{1}{1024}\sqrt{2}\left(77(-ab^3)^{1/4}b^2c - 15(-ab^3)^{3/4}e\right)\log(x^2 - \sqrt{2}x(-a/b)^{1/4} + \sqrt{-a/b})/(-a/b)^{1/4} - \frac{1}{384}\left(45b^2x^{11}e + 60b^2dx^{10} + 77b^2cx^9 - 126abx^7e - 160abd^2x^6 - 198abc^2x^5 + 113a^2x^3e + 132a^2dx^2 + 153a^2cx\right)/((bx^4 - a)^3a^3)$$

$$3.132 \quad \int \frac{c+dx+ex^2}{(a+bx^4)^4} dx$$

Optimal. Leaf size=372

$$-\frac{(77\sqrt{bc} - 15\sqrt{ae}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{512\sqrt{2}a^{15/4}b^{3/4}} + \frac{(77\sqrt{bc} - 15\sqrt{ae}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{512\sqrt{2}a^{15/4}b^{3/4}} - \frac{(15\sqrt{ae} + 77\sqrt{bc})}{256}$$

[Out] (x*(c + d*x + e*x^2))/(12*a*(a + b*x^4)^3) + (x*(11*c + 10*d*x + 9*e*x^2))/(96*a^2*(a + b*x^4)^2) + (x*(77*c + 60*d*x + 45*e*x^2))/(384*a^3*(a + b*x^4)) + (5*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b]) - ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(3/4)) - ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(3/4))

Rubi [A] time = 0.379463, antiderivative size = 372, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{(77\sqrt{bc} - 15\sqrt{ae}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{512\sqrt{2}a^{15/4}b^{3/4}} + \frac{(77\sqrt{bc} - 15\sqrt{ae}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{512\sqrt{2}a^{15/4}b^{3/4}} - \frac{(15\sqrt{ae} + 77\sqrt{bc})}{256}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/(a + b*x^4)^4, x]

[Out] (x*(c + d*x + e*x^2))/(12*a*(a + b*x^4)^3) + (x*(11*c + 10*d*x + 9*e*x^2))/(96*a^2*(a + b*x^4)^2) + (x*(77*c + 60*d*x + 45*e*x^2))/(384*a^3*(a + b*x^4)) + (5*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b]) - ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(3/4)) - ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(3/4))

Rule 1855

Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] & & PolyQ[Pq, x] & & IGtQ[n, 0] & & LtQ[p, -1] & & LtQ[Expon[Pq, x], n - 1]

Rule 1876

Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii])*x^(n/2))]/(a + b*x^n), {ii, 0, n/2 - 1}}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] & & PolyQ[Pq, x] & & IGtQ[n/2, 0] & & Expon[Pq, x] < n

Rule 275

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] & & IGtQ[n, 0] & & IntegerQ[m]

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] & & PosQ[a/b]

Rule 1168

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] & & NeQ[c*d^2 + a*e^2, 0] & & NeQ[c*d^2 - a*e^2, 0] & & NegQ[-(a*c)]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] & & EqQ[c*d^2 - a*e^2, 0] & & PosQ[d*e]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] & & (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 204

$\text{Int}[\frac{(a_1 + (b_1)x^2)^{-1}}{(a_2 + (c_2)x^2)}, x_Symbol] \ :> \ -\text{Simp}[\frac{\text{ArcTan}[\frac{\text{Rt}[-b, 2]x}{\text{Rt}[-a, 2]}}{\text{Rt}[-a, 2] \ \text{Rt}[-b, 2]}], x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 1165

$\text{Int}[\frac{(d_1 + (e_1)x^2)}{(a_1 + (c_1)x^4)}, x_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[\frac{-2d}{e}, 2]\}, \ \text{Dist}[e/(2cq), \ \text{Int}[\frac{q - 2x}{\text{Simp}[d/e + qx - x^2, x]}, x], \ x] \ + \ \text{Dist}[e/(2cq), \ \text{Int}[\frac{q + 2x}{\text{Simp}[d/e - qx - x^2, x]}, x], \ x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2d^2 - ae^2, 0] \ \&\& \ \text{NegQ}[de]$

Rule 628

$\text{Int}[\frac{(d_1 + (e_1)x)}{(a_1 + (b_1)x + (c_1)x^2)}, x_Symbol] \ :> \ \text{Simp}[\frac{d \ \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]}{b}, x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2cd - be, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2}{(a + bx^4)^4} dx &= \frac{x(c + dx + ex^2)}{12a(a + bx^4)^3} - \frac{\int \frac{-11c - 10dx - 9ex^2}{(a + bx^4)^3} dx}{12a} \\
&= \frac{x(c + dx + ex^2)}{12a(a + bx^4)^3} + \frac{x(11c + 10dx + 9ex^2)}{96a^2(a + bx^4)^2} + \frac{\int \frac{77c + 60dx + 45ex^2}{(a + bx^4)^2} dx}{96a^2} \\
&= \frac{x(c + dx + ex^2)}{12a(a + bx^4)^3} + \frac{x(11c + 10dx + 9ex^2)}{96a^2(a + bx^4)^2} + \frac{x(77c + 60dx + 45ex^2)}{384a^3(a + bx^4)} - \frac{\int \frac{-231c - 120dx - 45ex^2}{a + bx^4} dx}{384a^3} \\
&= \frac{x(c + dx + ex^2)}{12a(a + bx^4)^3} + \frac{x(11c + 10dx + 9ex^2)}{96a^2(a + bx^4)^2} + \frac{x(77c + 60dx + 45ex^2)}{384a^3(a + bx^4)} - \frac{\int \left(-\frac{120dx}{a + bx^4} + \frac{-231c - 45ex^2}{a + bx^4} \right) dx}{384a^3} \\
&= \frac{x(c + dx + ex^2)}{12a(a + bx^4)^3} + \frac{x(11c + 10dx + 9ex^2)}{96a^2(a + bx^4)^2} + \frac{x(77c + 60dx + 45ex^2)}{384a^3(a + bx^4)} - \frac{\int \frac{-231c - 45ex^2}{a + bx^4} dx}{384a^3} + \frac{(5d) \int \frac{1}{a} dx}{16} \\
&= \frac{x(c + dx + ex^2)}{12a(a + bx^4)^3} + \frac{x(11c + 10dx + 9ex^2)}{96a^2(a + bx^4)^2} + \frac{x(77c + 60dx + 45ex^2)}{384a^3(a + bx^4)} + \frac{(5d) \text{Subst} \left(\int \frac{1}{a + bx^2} dx, x, \sqrt{a} \right)}{32a^3} \\
&= \frac{x(c + dx + ex^2)}{12a(a + bx^4)^3} + \frac{x(11c + 10dx + 9ex^2)}{96a^2(a + bx^4)^2} + \frac{x(77c + 60dx + 45ex^2)}{384a^3(a + bx^4)} + \frac{5d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{32a^{7/2} \sqrt{b}} + \frac{\left(\frac{77\sqrt{bc}}{\sqrt{a}} \right)}{16} \\
&= \frac{x(c + dx + ex^2)}{12a(a + bx^4)^3} + \frac{x(11c + 10dx + 9ex^2)}{96a^2(a + bx^4)^2} + \frac{x(77c + 60dx + 45ex^2)}{384a^3(a + bx^4)} + \frac{5d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{32a^{7/2} \sqrt{b}} - \frac{(77\sqrt{bc})}{16} \\
&= \frac{x(c + dx + ex^2)}{12a(a + bx^4)^3} + \frac{x(11c + 10dx + 9ex^2)}{96a^2(a + bx^4)^2} + \frac{x(77c + 60dx + 45ex^2)}{384a^3(a + bx^4)} + \frac{5d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{32a^{7/2} \sqrt{b}} - \frac{(77\sqrt{bc})}{16}
\end{aligned}$$

Mathematica [A] time = 0.394003, size = 369, normalized size = 0.99

$$\frac{3\sqrt{2}(15a^{3/4}e - 77\sqrt[4]{a}\sqrt{bc}) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx+\sqrt{a}+\sqrt{bx^2}}\right)}{b^{3/4}} + \frac{3\sqrt{2}(77\sqrt[4]{a}\sqrt{bc} - 15a^{3/4}e) \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx+\sqrt{a}+\sqrt{bx^2}}\right)}{b^{3/4}} + \frac{256a^3x(c+x(d+ex))}{(a+bx^4)^3} + \frac{32a^2x(11c+x(10d+ex))}{(a+bx^4)^2}$$

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Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(a + b*x^4)^4,x]

[Out] ((256*a^3*x*(c + x*(d + e*x)))/(a + b*x^4)^3 + (8*a*x*(77*c + 15*x*(4*d + 3*e*x)))/(a + b*x^4) + (32*a^2*x*(11*c + x*(10*d + 9*e*x)))/(a + b*x^4)^2 - (6*a^(1/4)*(77*Sqrt[2]*Sqrt[b]*c + 80*a^(1/4)*b^(1/4)*d + 15*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/b^(3/4) + (6*a^(1/4)*(77*Sqrt[2]*Sqrt[b]*c - 80*a^(1/4)*b^(1/4)*d + 15*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/b^(3/4) + (3*Sqrt[2]*(-77*a^(1/4)*Sqrt[b]*c + 15*a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/b^(3/4) + (3*Sqrt[2]*(77*a^(1/4)*Sqrt[b]*c - 15*a^(3/4)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/b^(3/4))/(3072*a^4)

Maple [A] time = 0.013, size = 394, normalized size = 1.1

$$\frac{1}{(bx^4 + a)^3} \left(\frac{15b^2ex^{11}}{128a^3} + \frac{5b^2dx^{10}}{32a^3} + \frac{77b^2cx^9}{384a^3} + \frac{21bex^7}{64a^2} + \frac{5bdx^6}{12a^2} + \frac{33bcx^5}{64a^2} + \frac{113ex^3}{384a} + \frac{11dx^2}{32a} + \frac{51cx}{128a} \right) + \frac{77c\sqrt{2}\sqrt[4]{a}}{1024a^4}\sqrt[4]{\frac{a}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)/(b*x^4+a)^4,x)

[Out] (15/128*e/a^3*b^2*x^11+5/32*d/a^3*b^2*x^10+77/384*c/a^3*b^2*x^9+21/64/a^2*b*e*x^7+5/12/a^2*d*b*x^6+33/64/a^2*c*b*x^5+113/384/a*e*x^3+11/32*d/a*x^2+51/128*c/a*x)/(b*x^4+a)^3+77/1024/a^4*c*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))) +77/512/a^4*c*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+77/512/a^4*c*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+5/32/a^3*d/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+15/1024/a^3*e/b/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))) +15/512/a^3*e/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+15/512/a^3*e/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 16.3421, size = 610, normalized size = 1.64

$$\text{RootSum}\left(68719476736t^4a^{15}b^3 + t^2\left(1211105280a^8b^2ce + 838860800a^8b^2d^2\right) + t\left(18432000a^5bde^2 - 485703680a^4b^2c\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] RootSum(68719476736*_t**4*a**15*b**3 + _t**2*(1211105280*a**8*b**2*c*e + 838860800*a**8*b**2*d**2) + _t*(18432000*a**5*b*d*e**2 - 485703680*a**4*b**2*c**2*d) + 50625*a**2*e**4 + 2668050*a*b*c**2*e**2 - 7392000*a*b*c*d**2*e + 2560000*a*b*d**4 + 35153041*b**2*c**4, Lambda(_t, _t*log(x + (45298483200*_t**3*a**13*b**2*e**3 - 11936653639680*_t**3*a**12*b**3*c**2*e + 33071248179200*_t**3*a**12*b**3*c*d**2 + 544997376000*_t**2*a**9*b**2*c*d*e**2 - 50331648000*_t**2*a**9*b**2*d**3*e + 4787095470080*_t**2*a**8*b**3*c**3*d + 5987520000*_t*a**6*b*c*e**4 + 8294400000*_t*a**6*b*d**2*e**3 - 210370406400*_t*a**5*b**2*c**3*e**2 + 655699968000*_t*a**5*b**2*c**2*d**2*e + 20185088000*_t*a**5*b**2*c*d**4 + 1385873488384*_t*a**4*b**3*c**5 + 91125000*a**3*d*e**5 + 5544000000*a**2*b*c*d**3*e**2 - 3072000000*a**2*b*d**5*e + 105459123000*a*b**2*c**4*d*e - 146090560000*a*b**2*c**3*d**3)/(11390625*a**3*e**6 - 300155625*a**2*b*c**2*e**4 + 3326400000*a**2*b*c*d**2*e**3 - 2304000000*a**2*b*d**4*e**2 - 7909434225*a*b**2*c**4*e**2 + 87654336000*a*b**2*c**3*d**2*e - 60712960000*a*b**2*c**2*d**4 + 208422380089*b**3*c**6))) + (153*a**2*c*x + 132*a**2*d*x**2 + 113*a**2*e*x**3 + 198*a*b*c*x**5 + 160*a*b*d*x**6 + 1
```

$26*a*b*e*x**7 + 77*b**2*c*x**9 + 60*b**2*d*x**10 + 45*b**2*e*x**11)/(384*a*
*6 + 1152*a**5*b*x**4 + 1152*a**4*b**2*x**8 + 384*a**3*b**3*x**12)$

Giac [A] time = 1.08912, size = 504, normalized size = 1.35

$$\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{abb^2d} + 77(ab^3)^{\frac{1}{4}}b^2c + 15(ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512a^4b^3} + \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{abb^2d} + 77(ab^3)^{\frac{1}{4}}b^2c + 15(ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512a^4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")

[Out] $\frac{1}{512}\sqrt{2}\left(40\sqrt{2}\sqrt{abb^2d} + 77(a*b^3)^{\frac{1}{4}}b^2c + 15(a*b^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)\right)\left(\frac{1}{a^4b^3} + \frac{1}{1024}\sqrt{2}\left(77(a*b^3)^{\frac{1}{4}}b^2c - 15(a*b^3)^{\frac{3}{4}}e\right)\log\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{a/b}\right)\right) - \frac{1}{1024}\sqrt{2}\left(77(a*b^3)^{\frac{1}{4}}b^2c - 15(a*b^3)^{\frac{3}{4}}e\right)\log\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{a/b}\right)\left(\frac{1}{a^4b^3} + \frac{1}{384}\left(45b^2*x^{11}e + 60b^2*d*x^{10} + 77b^2*c*x^9 + 126a*b*x^7e + 160a*b*d*x^6 + 198a*b*c*x^5 + 113a^2*x^3e + 132a^2*d*x^2 + 153a^2*c*x\right)\right)\left(\frac{1}{(b*x^4 + a)^3a^3}\right)$

3.133 $\int a(e + fx^4)^2 dx$

Optimal. Leaf size=28

$$ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9$$

[Out] $a*e^{2*x} + (2*a*e*f*x^5)/5 + (a*f^2*x^9)/9$

Rubi [A] time = 0.0113591, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {12, 194}

$$ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9$$

Antiderivative was successfully verified.

[In] Int[a*(e + f*x^4)^2,x]

[Out] $a*e^{2*x} + (2*a*e*f*x^5)/5 + (a*f^2*x^9)/9$

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int a(e + fx^4)^2 dx &= a \int (e + fx^4)^2 dx \\ &= a \int (e^2 + 2efx^4 + f^2x^8) dx \\ &= ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 \end{aligned}$$

Mathematica [A] time = 0.0017667, size = 27, normalized size = 0.96

$$a \left(e^2 x + \frac{2}{5} e f x^5 + \frac{f^2 x^9}{9} \right)$$

Antiderivative was successfully verified.

[In] Integrate[a*(e + f*x^4)^2,x]

[Out] a*(e^2*x + (2*e*f*x^5)/5 + (f^2*x^9)/9)

Maple [A] time = 0.04, size = 24, normalized size = 0.9

$$a \left(\frac{f^2 x^9}{9} + \frac{2 e f x^5}{5} + e^2 x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a*(f*x^4+e)^2,x)

[Out] a*(1/9*f^2*x^9+2/5*e*f*x^5+e^2*x)

Maxima [A] time = 0.955677, size = 34, normalized size = 1.21

$$\frac{1}{45} (5 f^2 x^9 + 18 e f x^5 + 45 e^2 x) a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*(f*x^4+e)^2,x, algorithm="maxima")

[Out] 1/45*(5*f^2*x^9 + 18*e*f*x^5 + 45*e^2*x)*a

Fricas [A] time = 1.05641, size = 55, normalized size = 1.96

$$\frac{1}{9} x^9 f^2 a + \frac{2}{5} x^5 f e a + x e^2 a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a*(f*x^4+e)^2,x, algorithm="fricas")`

[Out] `1/9*x^9*f^2*a + 2/5*x^5*f*e*a + x*e^2*a`

Sympy [A] time = 0.059508, size = 27, normalized size = 0.96

$$ae^2x + \frac{2aefx^5}{5} + \frac{af^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a*(f*x**4+e)**2,x)`

[Out] `a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9`

Giac [A] time = 1.06919, size = 34, normalized size = 1.21

$$\frac{1}{45} (5 f^2 x^9 + 18 f x^5 e + 45 x e^2) a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a*(f*x^4+e)^2,x, algorithm="giac")`

[Out] `1/45*(5*f^2*x^9 + 18*f*x^5*e + 45*x*e^2)*a`

3.134 $\int bx(e + fx^4)^2 dx$

Optimal. Leaf size=33

$$\frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10}$$

[Out] (b*e^2*x^2)/2 + (b*e*f*x^6)/3 + (b*f^2*x^10)/10

Rubi [A] time = 0.0143123, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {12, 270}

$$\frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10}$$

Antiderivative was successfully verified.

[In] Int[b*x*(e + f*x^4)^2,x]

[Out] (b*e^2*x^2)/2 + (b*e*f*x^6)/3 + (b*f^2*x^10)/10

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int bx(e + fx^4)^2 dx &= b \int x(e + fx^4)^2 dx \\ &= b \int (e^2x + 2efx^5 + f^2x^9) dx \\ &= \frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} \end{aligned}$$

Mathematica [A] time = 0.001375, size = 32, normalized size = 0.97

$$b \left(\frac{e^2 x^2}{2} + \frac{1}{3} e f x^6 + \frac{f^2 x^{10}}{10} \right)$$

Antiderivative was successfully verified.

[In] Integrate[b*x*(e + f*x^4)^2,x]

[Out] b*((e^2*x^2)/2 + (e*f*x^6)/3 + (f^2*x^10)/10)

Maple [A] time = 0.039, size = 27, normalized size = 0.8

$$b \left(\frac{f^2 x^{10}}{10} + \frac{e f x^6}{3} + \frac{e^2 x^2}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(b*x*(f*x^4+e)^2,x)

[Out] b*(1/10*f^2*x^10+1/3*e*f*x^6+1/2*e^2*x^2)

Maxima [A] time = 1.05494, size = 36, normalized size = 1.09

$$\frac{1}{30} (3 f^2 x^{10} + 10 e f x^6 + 15 e^2 x^2) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b*x*(f*x^4+e)^2,x, algorithm="maxima")

[Out] 1/30*(3*f^2*x^10 + 10*e*f*x^6 + 15*e^2*x^2)*b

Fricas [A] time = 1.06542, size = 66, normalized size = 2.

$$\frac{1}{10} x^{10} f^2 b + \frac{1}{3} x^6 f e b + \frac{1}{2} x^2 e^2 b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b*x*(f*x^4+e)^2,x, algorithm="fricas")

[Out] 1/10*x^10*f^2*b + 1/3*x^6*f*e*b + 1/2*x^2*e^2*b

Sympy [A] time = 0.061032, size = 29, normalized size = 0.88

$$\frac{be^2x^2}{2} + \frac{befx^6}{3} + \frac{bf^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b*x*(f*x**4+e)**2,x)

[Out] b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10

Giac [A] time = 1.04414, size = 36, normalized size = 1.09

$$\frac{1}{30} (3 f^2 x^{10} + 10 f x^6 e + 15 x^2 e^2) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b*x*(f*x^4+e)^2,x, algorithm="giac")

[Out] 1/30*(3*f^2*x^10 + 10*f*x^6*e + 15*x^2*e^2)*b

3.135 $\int (a + bx)(e + fx^4)^2 dx$

Optimal. Leaf size=60

$$ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 + \frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10}$$

[Out] $a*e^{2*x} + (b*e^{2*x^2})/2 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (a*f^{2*x^9})/9 + (b*f^{2*x^{10}})/10$

Rubi [A] time = 0.0624218, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1850}

$$ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 + \frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)*(e + f*x^4)^2,x]

[Out] $a*e^{2*x} + (b*e^{2*x^2})/2 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (a*f^{2*x^9})/9 + (b*f^{2*x^{10}})/10$

Rule 1850

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int (a + bx)(e + fx^4)^2 dx &= \int (ae^2 + be^2x + 2aefx^4 + 2befx^5 + af^2x^8 + bf^2x^9) dx \\ &= ae^2x + \frac{1}{2}be^2x^2 + \frac{2}{5}aefx^5 + \frac{1}{3}befx^6 + \frac{1}{9}af^2x^9 + \frac{1}{10}bf^2x^{10} \end{aligned}$$

Mathematica [A] time = 0.0019781, size = 60, normalized size = 1.

$$ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 + \frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)*(e + f*x^4)^2,x]

[Out] a*e^2*x + (b*e^2*x^2)/2 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10

Maple [A] time = 0.039, size = 51, normalized size = 0.9

$$ae^2x + \frac{be^2x^2}{2} + \frac{2aefx^5}{5} + \frac{befx^6}{3} + \frac{af^2x^9}{9} + \frac{bf^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)*(f*x^4+e)^2,x)

[Out] a*e^2*x+1/2*b*e^2*x^2+2/5*a*e*f*x^5+1/3*b*e*f*x^6+1/9*a*f^2*x^9+1/10*b*f^2*x^10

Maxima [A] time = 0.981534, size = 68, normalized size = 1.13

$$\frac{1}{10}bf^2x^{10} + \frac{1}{9}af^2x^9 + \frac{1}{3}befx^6 + \frac{2}{5}aefx^5 + \frac{1}{2}be^2x^2 + ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(f*x^4+e)^2,x, algorithm="maxima")

[Out] 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/3*b*e*f*x^6 + 2/5*a*e*f*x^5 + 1/2*b*e^2*x^2 + a*e^2*x

Fricas [A] time = 1.00017, size = 123, normalized size = 2.05

$$\frac{1}{10}x^{10}f^2b + \frac{1}{9}x^9f^2a + \frac{1}{3}x^6feb + \frac{2}{5}x^5fea + \frac{1}{2}x^2e^2b + xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(f*x^4+e)^2,x, algorithm="fricas")

[Out] $\frac{1}{10}x^{10}f^2b + \frac{1}{9}x^9f^2a + \frac{1}{3}x^6f^2e + \frac{2}{5}x^5f^2ea + \frac{1}{2}x^2e^2b + xe^2a$

Sympy [A] time = 0.066282, size = 58, normalized size = 0.97

$$ae^2x + \frac{2aefx^5}{5} + \frac{af^2x^9}{9} + \frac{be^2x^2}{2} + \frac{befx^6}{3} + \frac{bf^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(f*x**4+e)**2,x)

[Out] $a e^{**2} x + 2 a e f x^{**5} / 5 + a f^{**2} x^{**9} / 9 + b e^{**2} x^{**2} / 2 + b e f x^{**6} / 3 + b f^{**2} x^{**10} / 10$

Giac [A] time = 1.05481, size = 68, normalized size = 1.13

$$\frac{1}{10}bf^2x^{10} + \frac{1}{9}af^2x^9 + \frac{1}{3}bf^2x^6e + \frac{2}{5}afx^5e + \frac{1}{2}bx^2e^2 + axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(f*x^4+e)^2,x, algorithm="giac")

[Out] $\frac{1}{10}b*f^2*x^{10} + \frac{1}{9}a*f^2*x^9 + \frac{1}{3}b*f*x^6*e + \frac{2}{5}a*f*x^5*e + \frac{1}{2}b*x^2*e^2 + a*x*e^2$

3.136 $\int cx^2 (e + fx^4)^2 dx$

Optimal. Leaf size=33

$$\frac{1}{3}ce^2x^3 + \frac{2}{7}cef x^7 + \frac{1}{11}cf^2x^{11}$$

[Out] (c*e^2*x^3)/3 + (2*c*e*f*x^7)/7 + (c*f^2*x^11)/11

Rubi [A] time = 0.0132649, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {12, 270}

$$\frac{1}{3}ce^2x^3 + \frac{2}{7}cef x^7 + \frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

[In] Int[c*x^2*(e + f*x^4)^2,x]

[Out] (c*e^2*x^3)/3 + (2*c*e*f*x^7)/7 + (c*f^2*x^11)/11

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int cx^2 (e + fx^4)^2 dx &= c \int x^2 (e + fx^4)^2 dx \\ &= c \int (e^2x^2 + 2efx^6 + f^2x^{10}) dx \\ &= \frac{1}{3}ce^2x^3 + \frac{2}{7}cef x^7 + \frac{1}{11}cf^2x^{11} \end{aligned}$$

Mathematica [A] time = 0.0011796, size = 33, normalized size = 1.

$$\frac{1}{3}ce^2x^3 + \frac{2}{7}cef x^7 + \frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

[In] Integrate[c*x^2*(e + f*x^4)^2,x]

[Out] (c*e^2*x^3)/3 + (2*c*e*f*x^7)/7 + (c*f^2*x^11)/11

Maple [A] time = 0.041, size = 27, normalized size = 0.8

$$c \left(\frac{f^2x^{11}}{11} + \frac{2efx^7}{7} + \frac{e^2x^3}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(c*x^2*(f*x^4+e)^2,x)

[Out] c*(1/11*f^2*x^11+2/7*e*f*x^7+1/3*e^2*x^3)

Maxima [A] time = 0.986278, size = 36, normalized size = 1.09

$$\frac{1}{231} (21 f^2 x^{11} + 66 e f x^7 + 77 e^2 x^3) c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(c*x^2*(f*x^4+e)^2,x, algorithm="maxima")

[Out] 1/231*(21*f^2*x^11 + 66*e*f*x^7 + 77*e^2*x^3)*c

Fricas [A] time = 1.05338, size = 66, normalized size = 2.

$$\frac{1}{11}x^{11}f^2c + \frac{2}{7}x^7fec + \frac{1}{3}x^3e^2c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(c*x^2*(f*x^4+e)^2,x, algorithm="fricas")`

[Out] $1/11*x^{11}*f^2*c + 2/7*x^7*f*e*c + 1/3*x^3*e^2*c$

Sympy [A] time = 0.059401, size = 31, normalized size = 0.94

$$\frac{ce^2x^3}{3} + \frac{2cef x^7}{7} + \frac{cf^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(c*x**2*(f*x**4+e)**2,x)`

[Out] $c*e**2*x**3/3 + 2*c*e*f*x**7/7 + c*f**2*x**11/11$

Giac [A] time = 1.08192, size = 36, normalized size = 1.09

$$\frac{1}{231} (21 f^2 x^{11} + 66 f x^7 e + 77 x^3 e^2) c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(c*x^2*(f*x^4+e)^2,x, algorithm="giac")`

[Out] $1/231*(21*f^2*x^{11} + 66*f*x^7*e + 77*x^3*e^2)*c$

3.137 $\int (a + cx^2)(e + fx^4)^2 dx$

Optimal. Leaf size=60

$$ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 + \frac{1}{3}ce^2x^3 + \frac{2}{7}cef x^7 + \frac{1}{11}cf^2x^{11}$$

[Out] $a e^{2x} + (c e^{2x^3})/3 + (2 a e f x^5)/5 + (2 c e f x^7)/7 + (a f^2 x^9)/9 + (c f^2 x^{11})/11$

Rubi [A] time = 0.0266964, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1154}

$$ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 + \frac{1}{3}ce^2x^3 + \frac{2}{7}cef x^7 + \frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^2)*(e + f*x^4)^2,x]

[Out] $a e^{2x} + (c e^{2x^3})/3 + (2 a e f x^5)/5 + (2 c e f x^7)/7 + (a f^2 x^9)/9 + (c f^2 x^{11})/11$

Rule 1154

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int (a + cx^2)(e + fx^4)^2 dx &= \int (ae^2 + ce^2x^2 + 2aefx^4 + 2cef x^6 + af^2x^8 + cf^2x^{10}) dx \\ &= ae^2x + \frac{1}{3}ce^2x^3 + \frac{2}{5}aefx^5 + \frac{2}{7}cef x^7 + \frac{1}{9}af^2x^9 + \frac{1}{11}cf^2x^{11} \end{aligned}$$

Mathematica [A] time = 0.0022818, size = 60, normalized size = 1.

$$ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 + \frac{1}{3}ce^2x^3 + \frac{2}{7}cef x^7 + \frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^2)*(e + f*x^4)^2,x]

[Out] a*e^2*x + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9 + (c*f^2*x^11)/11

Maple [A] time = 0.041, size = 51, normalized size = 0.9

$$ae^2x + \frac{ce^2x^3}{3} + \frac{2aefx^5}{5} + \frac{2cef x^7}{7} + \frac{af^2x^9}{9} + \frac{cf^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+a)*(f*x^4+e)^2,x)

[Out] a*e^2*x+1/3*c*e^2*x^3+2/5*a*e*f*x^5+2/7*c*e*f*x^7+1/9*a*f^2*x^9+1/11*c*f^2*x^11

Maxima [A] time = 0.974872, size = 68, normalized size = 1.13

$$\frac{1}{11}cf^2x^{11} + \frac{1}{9}af^2x^9 + \frac{2}{7}cef x^7 + \frac{2}{5}aef x^5 + \frac{1}{3}ce^2x^3 + ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)*(f*x^4+e)^2,x, algorithm="maxima")

[Out] 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 2/7*c*e*f*x^7 + 2/5*a*e*f*x^5 + 1/3*c*e^2*x^3 + a*e^2*x

Fricas [A] time = 1.0562, size = 123, normalized size = 2.05

$$\frac{1}{11}x^{11}f^2c + \frac{1}{9}x^9f^2a + \frac{2}{7}x^7fec + \frac{2}{5}x^5fea + \frac{1}{3}x^3e^2c + xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)*(f*x^4+e)^2,x, algorithm="fricas")

[Out] $1/11*x^{11}*f^2*c + 1/9*x^9*f^2*a + 2/7*x^7*f*e*c + 2/5*x^5*f*e*a + 1/3*x^3*e^2*c + x*e^2*a$

Sympy [A] time = 0.069098, size = 60, normalized size = 1.

$$ae^2x + \frac{2aefx^5}{5} + \frac{af^2x^9}{9} + \frac{ce^2x^3}{3} + \frac{2cef x^7}{7} + \frac{cf^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+a)*(f*x**4+e)**2,x)

[Out] $a*e^{**2}*x + 2*a*e*f*x^{**5}/5 + a*f^{**2}*x^{**9}/9 + c*e^{**2}*x^{**3}/3 + 2*c*e*f*x^{**7}/7 + c*f^{**2}*x^{**11}/11$

Giac [A] time = 1.04692, size = 68, normalized size = 1.13

$$\frac{1}{11}cf^2x^{11} + \frac{1}{9}af^2x^9 + \frac{2}{7}cfx^7e + \frac{2}{5}afx^5e + \frac{1}{3}cx^3e^2 + axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)*(f*x^4+e)^2,x, algorithm="giac")

[Out] $1/11*c*f^2*x^{11} + 1/9*a*f^2*x^9 + 2/7*c*f*x^7*e + 2/5*a*f*x^5*e + 1/3*c*x^3*e^2 + a*x*e^2$

3.138 $\int (bx + cx^2)(e + fx^4)^2 dx$

Optimal. Leaf size=65

$$\frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{1}{3}ce^2x^3 + \frac{2}{7}cef x^7 + \frac{1}{11}cf^2x^{11}$$

[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (b*f^2*x^10)/10 + (c*f^2*x^11)/11

Rubi [A] time = 0.0975165, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1593, 1620}

$$\frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{1}{3}ce^2x^3 + \frac{2}{7}cef x^7 + \frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2)*(e + f*x^4)^2,x]

[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (b*f^2*x^10)/10 + (c*f^2*x^11)/11

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1620

Int[(Px_)*((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[Expon[Px, x], 2]

Rubi steps

$$\begin{aligned}
\int (bx + cx^2)(e + fx^4)^2 dx &= \int x(b + cx)(e + fx^4)^2 dx \\
&= \int (be^2x + ce^2x^2 + 2befx^5 + 2cefx^6 + bf^2x^9 + cf^2x^{10}) dx \\
&= \frac{1}{2}be^2x^2 + \frac{1}{3}ce^2x^3 + \frac{1}{3}befx^6 + \frac{2}{7}cefx^7 + \frac{1}{10}bf^2x^{10} + \frac{1}{11}cf^2x^{11}
\end{aligned}$$

Mathematica [A] time = 0.0025941, size = 65, normalized size = 1.

$$\frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{1}{3}ce^2x^3 + \frac{2}{7}cefx^7 + \frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2)*(e + f*x^4)^2,x]

[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (b*f^2*x^10)/10 + (c*f^2*x^11)/11

Maple [A] time = 0.04, size = 54, normalized size = 0.8

$$\frac{be^2x^2}{2} + \frac{ce^2x^3}{3} + \frac{befx^6}{3} + \frac{2cefx^7}{7} + \frac{bf^2x^{10}}{10} + \frac{cf^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x)*(f*x^4+e)^2,x)

[Out] 1/2*b*e^2*x^2+1/3*c*e^2*x^3+1/3*b*e*f*x^6+2/7*c*e*f*x^7+1/10*b*f^2*x^10+1/11*c*f^2*x^11

Maxima [A] time = 1.01028, size = 72, normalized size = 1.11

$$\frac{1}{11}cf^2x^{11} + \frac{1}{10}bf^2x^{10} + \frac{2}{7}cefx^7 + \frac{1}{3}befx^6 + \frac{1}{3}ce^2x^3 + \frac{1}{2}be^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="maxima")

[Out] $\frac{1}{11}c*f^2*x^{11} + \frac{1}{10}b*f^2*x^{10} + \frac{2}{7}c*e*f*x^7 + \frac{1}{3}b*e*f*x^6 + \frac{1}{3}c*e^2*x^3 + \frac{1}{2}b*e^2*x^2$

Fricas [A] time = 1.08661, size = 134, normalized size = 2.06

$$\frac{1}{11}x^{11}f^2c + \frac{1}{10}x^{10}f^2b + \frac{2}{7}x^7fec + \frac{1}{3}x^6feb + \frac{1}{3}x^3e^2c + \frac{1}{2}x^2e^2b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="fricas")

[Out] $\frac{1}{11}x^{11}f^2c + \frac{1}{10}x^{10}f^2b + \frac{2}{7}x^7f^*e*c + \frac{1}{3}x^6f^*e*b + \frac{1}{3}x^3e^2*c + \frac{1}{2}x^2e^2*b$

Sympy [A] time = 0.068121, size = 61, normalized size = 0.94

$$\frac{be^2x^2}{2} + \frac{befx^6}{3} + \frac{bf^2x^{10}}{10} + \frac{ce^2x^3}{3} + \frac{2cef x^7}{7} + \frac{cf^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x)*(f*x**4+e)**2,x)

[Out] $b*e^{**2}*x^{**2}/2 + b*e*f*x^{**6}/3 + b*f^{**2}*x^{**10}/10 + c*e^{**2}*x^{**3}/3 + 2*c*e*f*x^{**7}/7 + c*f^{**2}*x^{**11}/11$

Giac [A] time = 1.0566, size = 72, normalized size = 1.11

$$\frac{1}{11}cf^2x^{11} + \frac{1}{10}bf^2x^{10} + \frac{2}{7}cfx^7e + \frac{1}{3}bfx^6e + \frac{1}{3}cx^3e^2 + \frac{1}{2}bx^2e^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="giac")

[Out] $\frac{1}{11}c*f^2*x^{11} + \frac{1}{10}b*f^2*x^{10} + \frac{2}{7}c*f*x^7*e + \frac{1}{3}b*f*x^6*e + \frac{1}{3}c*x^3*e^2 + \frac{1}{2}b*x^2*e^2$

3.139 $\int (a + bx + cx^2)(e + fx^4)^2 dx$

Optimal. Leaf size=92

$$ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 + \frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{1}{3}ce^2x^3 + \frac{2}{7}cefx^7 + \frac{1}{11}cf^2x^{11}$$

[Out] $a e^2 x + (b e^2 x^2)/2 + (c e^2 x^3)/3 + (2 a e f x^5)/5 + (b e f x^6)/3 + (2 c e f x^7)/7 + (a f^2 x^9)/9 + (b f^2 x^{10})/10 + (c f^2 x^{11})/11$

Rubi [A] time = 0.0531848, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$, Rules used = {1657}

$$ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 + \frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{1}{3}ce^2x^3 + \frac{2}{7}cefx^7 + \frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2)*(e + f*x^4)^2,x]

[Out] $a e^2 x + (b e^2 x^2)/2 + (c e^2 x^3)/3 + (2 a e f x^5)/5 + (b e f x^6)/3 + (2 c e f x^7)/7 + (a f^2 x^9)/9 + (b f^2 x^{10})/10 + (c f^2 x^{11})/11$

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (a + bx + cx^2)(e + fx^4)^2 dx &= \int (ae^2 + be^2x + ce^2x^2 + 2aefx^4 + 2befx^5 + 2cefx^6 + af^2x^8 + bf^2x^9 + cf^2x^{10}) dx \\ &= ae^2x + \frac{1}{2}be^2x^2 + \frac{1}{3}ce^2x^3 + \frac{2}{5}aefx^5 + \frac{1}{3}befx^6 + \frac{2}{7}cefx^7 + \frac{1}{9}af^2x^9 + \frac{1}{10}bf^2x^{10} + \frac{1}{11}cf^2x^{11} \end{aligned}$$

Mathematica [A] time = 0.0033933, size = 92, normalized size = 1.

$$ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 + \frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{1}{3}ce^2x^3 + \frac{2}{7}cefx^7 + \frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2)*(e + f*x^4)^2,x]

[Out] $a*e^{2*x} + (b*e^{2*x^2})/2 + (c*e^{2*x^3})/3 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9 + (b*f^2*x^{10})/10 + (c*f^2*x^{11})/11$

Maple [A] time = 0.041, size = 77, normalized size = 0.8

$$ae^2x + \frac{be^2x^2}{2} + \frac{ce^2x^3}{3} + \frac{2aefx^5}{5} + \frac{befx^6}{3} + \frac{2cefx^7}{7} + \frac{af^2x^9}{9} + \frac{bf^2x^{10}}{10} + \frac{cf^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)*(f*x^4+e)^2,x)

[Out] $a*e^{2*x} + 1/2*b*e^{2*x^2} + 1/3*c*e^{2*x^3} + 2/5*a*e*f*x^5 + 1/3*b*e*f*x^6 + 2/7*c*e*f*x^7 + 1/9*a*f^2*x^9 + 1/10*b*f^2*x^{10} + 1/11*c*f^2*x^{11}$

Maxima [A] time = 0.981105, size = 103, normalized size = 1.12

$$\frac{1}{11}cf^2x^{11} + \frac{1}{10}bf^2x^{10} + \frac{1}{9}af^2x^9 + \frac{2}{7}cefx^7 + \frac{1}{3}befx^6 + \frac{2}{5}aefx^5 + \frac{1}{3}ce^2x^3 + \frac{1}{2}be^2x^2 + ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)*(f*x^4+e)^2,x, algorithm="maxima")

[Out] $1/11*c*f^2*x^{11} + 1/10*b*f^2*x^{10} + 1/9*a*f^2*x^9 + 2/7*c*e*f*x^7 + 1/3*b*e*f*x^6 + 2/5*a*e*f*x^5 + 1/3*c*e^2*x^3 + 1/2*b*e^2*x^2 + a*e^2*x$

Fricas [A] time = 1.12252, size = 190, normalized size = 2.07

$$\frac{1}{11}x^{11}f^2c + \frac{1}{10}x^{10}f^2b + \frac{1}{9}x^9f^2a + \frac{2}{7}x^7fec + \frac{1}{3}x^6feb + \frac{2}{5}x^5fea + \frac{1}{3}x^3e^2c + \frac{1}{2}x^2e^2b + xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)*(f*x^4+e)^2,x, algorithm="fricas")

[Out] $1/11*x^{11}*f^2*c + 1/10*x^{10}*f^2*b + 1/9*x^9*f^2*a + 2/7*x^7*f*e*c + 1/3*x^6*f*e*b + 2/5*x^5*f*e*a + 1/3*x^3*e^2*c + 1/2*x^2*e^2*b + x*e^2*a$

Sympy [A] time = 0.069704, size = 90, normalized size = 0.98

$$ae^2x + \frac{2aefx^5}{5} + \frac{af^2x^9}{9} + \frac{be^2x^2}{2} + \frac{befx^6}{3} + \frac{bf^2x^{10}}{10} + \frac{ce^2x^3}{3} + \frac{2cef x^7}{7} + \frac{cf^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)*(f*x**4+e)**2,x)

[Out] $a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10 + c*e**2*x**3/3 + 2*c*e*f*x**7/7 + c*f**2*x**11/11$

Giac [A] time = 1.05879, size = 103, normalized size = 1.12

$$\frac{1}{11}cf^2x^{11} + \frac{1}{10}bf^2x^{10} + \frac{1}{9}af^2x^9 + \frac{2}{7}cfx^7e + \frac{1}{3}bfx^6e + \frac{2}{5}afx^5e + \frac{1}{3}cx^3e^2 + \frac{1}{2}bx^2e^2 + axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)*(f*x^4+e)^2,x, algorithm="giac")

[Out] $1/11*c*f^2*x^{11} + 1/10*b*f^2*x^{10} + 1/9*a*f^2*x^9 + 2/7*c*f*x^7*e + 1/3*b*f*x^6*e + 2/5*a*f*x^5*e + 1/3*c*x^3*e^2 + 1/2*b*x^2*e^2 + a*x*e^2$

$$3.140 \quad \int dx^3 (e + fx^4)^2 dx$$

Optimal. Leaf size=17

$$\frac{d(e + fx^4)^3}{12f}$$

[Out] (d*(e + f*x^4)^3)/(12*f)

Rubi [A] time = 0.0050034, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {12, 261}

$$\frac{d(e + fx^4)^3}{12f}$$

Antiderivative was successfully verified.

[In] Int[d*x^3*(e + f*x^4)^2,x]

[Out] (d*(e + f*x^4)^3)/(12*f)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int dx^3 (e + fx^4)^2 dx &= d \int x^3 (e + fx^4)^2 dx \\ &= \frac{d(e + fx^4)^3}{12f} \end{aligned}$$

Mathematica [A] time = 0.0008476, size = 33, normalized size = 1.94

$$\frac{1}{4}de^2x^4 + \frac{1}{4}defx^8 + \frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

[In] Integrate[d*x^3*(e + f*x^4)^2,x]

[Out] (d*e^2*x^4)/4 + (d*e*f*x^8)/4 + (d*f^2*x^12)/12

Maple [A] time = 0.039, size = 27, normalized size = 1.6

$$d\left(\frac{f^2x^{12}}{12} + \frac{efx^8}{4} + \frac{e^2x^4}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(d*x^3*(f*x^4+e)^2,x)

[Out] d*(1/12*f^2*x^12+1/4*e*f*x^8+1/4*e^2*x^4)

Maxima [A] time = 1.01359, size = 20, normalized size = 1.18

$$\frac{(fx^4 + e)^3 d}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d*x^3*(f*x^4+e)^2,x, algorithm="maxima")

[Out] 1/12*(f*x^4 + e)^3*d/f

Fricas [A] time = 1.05472, size = 66, normalized size = 3.88

$$\frac{1}{12}x^{12}f^2d + \frac{1}{4}x^8fed + \frac{1}{4}x^4e^2d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(d*x^3*(f*x^4+e)^2,x, algorithm="fricas")`

[Out] $1/12*x^{12}*f^2*d + 1/4*x^8*f*e*d + 1/4*x^4*e^2*d$

Sympy [B] time = 0.059406, size = 29, normalized size = 1.71

$$\frac{de^2x^4}{4} + \frac{defx^8}{4} + \frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(d*x**3*(f*x**4+e)**2,x)`

[Out] $d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12$

Giac [A] time = 1.06087, size = 22, normalized size = 1.29

$$\frac{(fx^4 + e)^3 d}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(d*x^3*(f*x^4+e)^2,x, algorithm="giac")`

[Out] $1/12*(f*x^4 + e)^3*d/f$

3.141 $\int (a + dx^3)(e + fx^4)^2 dx$

Optimal. Leaf size=45

$$ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 + \frac{d(e + fx^4)^3}{12f}$$

[Out] $a e^2 x + (2 a e f x^5) / 5 + (a f^2 x^9) / 9 + (d (e + f x^4)^3) / (12 f)$

Rubi [A] time = 0.0190194, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1582, 12, 194}

$$ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 + \frac{d(e + fx^4)^3}{12f}$$

Antiderivative was successfully verified.

[In] Int[(a + d*x^3)*(e + f*x^4)^2,x]

[Out] $a e^2 x + (2 a e f x^5) / 5 + (a f^2 x^9) / 9 + (d (e + f x^4)^3) / (12 f)$

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(Coeff[Px, x, n - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a + b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
 \int (a + dx^3)(e + fx^4)^2 dx &= \frac{d(e + fx^4)^3}{12f} + \int a(e + fx^4)^2 dx \\
 &= \frac{d(e + fx^4)^3}{12f} + a \int (e + fx^4)^2 dx \\
 &= \frac{d(e + fx^4)^3}{12f} + a \int (e^2 + 2efx^4 + f^2x^8) dx \\
 &= ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 + \frac{d(e + fx^4)^3}{12f}
 \end{aligned}$$

Mathematica [A] time = 0.0022259, size = 60, normalized size = 1.33

$$ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 + \frac{1}{4}de^2x^4 + \frac{1}{4}defx^8 + \frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

[In] Integrate[(a + d*x^3)*(e + f*x^4)^2,x]

[Out] a*e^2*x + (d*e^2*x^4)/4 + (2*a*e*f*x^5)/5 + (d*e*f*x^8)/4 + (a*f^2*x^9)/9 + (d*f^2*x^12)/12

Maple [A] time = 0.038, size = 51, normalized size = 1.1

$$\frac{df^2x^{12}}{12} + \frac{af^2x^9}{9} + \frac{defx^8}{4} + \frac{2aefx^5}{5} + \frac{de^2x^4}{4} + ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+a)*(f*x^4+e)^2,x)

[Out] 1/12*d*f^2*x^12+1/9*a*f^2*x^9+1/4*d*e*f*x^8+2/5*a*e*f*x^5+1/4*d*e^2*x^4+a*e^2*x

Maxima [A] time = 0.992621, size = 68, normalized size = 1.51

$$\frac{1}{12}df^2x^{12} + \frac{1}{9}af^2x^9 + \frac{1}{4}defx^8 + \frac{2}{5}aefx^5 + \frac{1}{4}de^2x^4 + ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+a)*(f*x^4+e)^2,x, algorithm="maxima")

[Out] 1/12*d*f^2*x^12 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 2/5*a*e*f*x^5 + 1/4*d*e^2*x^4 + a*e^2*x

Fricas [A] time = 1.04397, size = 123, normalized size = 2.73

$$\frac{1}{12}x^{12}f^2d + \frac{1}{9}x^9f^2a + \frac{1}{4}x^8fed + \frac{2}{5}x^5fea + \frac{1}{4}x^4e^2d + xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+a)*(f*x^4+e)^2,x, algorithm="fricas")

[Out] 1/12*x^12*f^2*d + 1/9*x^9*f^2*a + 1/4*x^8*f*e*d + 2/5*x^5*f*e*a + 1/4*x^4*e^2*d + x*e^2*a

Sympy [A] time = 0.064955, size = 58, normalized size = 1.29

$$ae^2x + \frac{2aefx^5}{5} + \frac{af^2x^9}{9} + \frac{de^2x^4}{4} + \frac{defx^8}{4} + \frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+a)*(f*x**4+e)**2,x)

[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12

Giac [A] time = 1.07443, size = 68, normalized size = 1.51

$$\frac{1}{12}df^2x^{12} + \frac{1}{9}af^2x^9 + \frac{1}{4}dfx^8e + \frac{2}{5}afx^5e + \frac{1}{4}dx^4e^2 + axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+a)*(f*x^4+e)^2,x, algorithm="giac")

[Out] 1/12*d*f^2*x^12 + 1/9*a*f^2*x^9 + 1/4*d*f*x^8*e + 2/5*a*f*x^5*e + 1/4*d*x^4*e^2 + a*x*e^2

3.142 $\int (bx + dx^3)(e + fx^4)^2 dx$

Optimal. Leaf size=50

$$\frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{d(e + fx^4)^3}{12f}$$

[Out] (b*e^2*x^2)/2 + (b*e*f*x^6)/3 + (b*f^2*x^10)/10 + (d*(e + f*x^4)^3)/(12*f)

Rubi [A] time = 0.0198986, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1582, 12, 270}

$$\frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{d(e + fx^4)^3}{12f}$$

Antiderivative was successfully verified.

[In] Int[(b*x + d*x^3)*(e + f*x^4)^2,x]

[Out] (b*e^2*x^2)/2 + (b*e*f*x^6)/3 + (b*f^2*x^10)/10 + (d*(e + f*x^4)^3)/(12*f)

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
- 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int (bx + dx^3)(e + fx^4)^2 dx &= \frac{d(e + fx^4)^3}{12f} + \int bx(e + fx^4)^2 dx \\
&= \frac{d(e + fx^4)^3}{12f} + b \int x(e + fx^4)^2 dx \\
&= \frac{d(e + fx^4)^3}{12f} + b \int (e^2x + 2efx^5 + f^2x^9) dx \\
&= \frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{d(e + fx^4)^3}{12f}
\end{aligned}$$

Mathematica [A] time = 0.002989, size = 65, normalized size = 1.3

$$\frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{1}{4}de^2x^4 + \frac{1}{4}defx^8 + \frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + d*x^3)*(e + f*x^4)^2,x]

[Out] (b*e^2*x^2)/2 + (d*e^2*x^4)/4 + (b*e*f*x^6)/3 + (d*e*f*x^8)/4 + (b*f^2*x^10)/10 + (d*f^2*x^12)/12

Maple [A] time = 0.042, size = 54, normalized size = 1.1

$$\frac{df^2x^{12}}{12} + \frac{bf^2x^{10}}{10} + \frac{defx^8}{4} + \frac{befx^6}{3} + \frac{de^2x^4}{4} + \frac{be^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+b*x)*(f*x^4+e)^2,x)

[Out] 1/12*d*f^2*x^12+1/10*b*f^2*x^10+1/4*d*e*f*x^8+1/3*b*e*f*x^6+1/4*d*e^2*x^4+1/2*b*e^2*x^2

Maxima [A] time = 1.00185, size = 72, normalized size = 1.44

$$\frac{1}{12}df^2x^{12} + \frac{1}{10}bf^2x^{10} + \frac{1}{4}defx^8 + \frac{1}{3}befx^6 + \frac{1}{4}de^2x^4 + \frac{1}{2}be^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+b*x)*(f*x^4+e)^2,x, algorithm="maxima")

[Out] 1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/4*d*e*f*x^8 + 1/3*b*e*f*x^6 + 1/4*d*e^2*x^4 + 1/2*b*e^2*x^2

Fricas [A] time = 1.02542, size = 134, normalized size = 2.68

$$\frac{1}{12}x^{12}f^2d + \frac{1}{10}x^{10}f^2b + \frac{1}{4}x^8fed + \frac{1}{3}x^6feb + \frac{1}{4}x^4e^2d + \frac{1}{2}x^2e^2b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+b*x)*(f*x^4+e)^2,x, algorithm="fricas")

[Out] 1/12*x^12*f^2*d + 1/10*x^10*f^2*b + 1/4*x^8*f*e*d + 1/3*x^6*f*e*b + 1/4*x^4*e^2*d + 1/2*x^2*e^2*b

Sympy [A] time = 0.065855, size = 60, normalized size = 1.2

$$\frac{be^2x^2}{2} + \frac{befx^6}{3} + \frac{bf^2x^{10}}{10} + \frac{de^2x^4}{4} + \frac{defx^8}{4} + \frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+b*x)*(f*x**4+e)**2,x)

[Out] b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10 + d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12

Giac [A] time = 1.05355, size = 72, normalized size = 1.44

$$\frac{1}{12} df^2x^{12} + \frac{1}{10} bf^2x^{10} + \frac{1}{4} dfx^8e + \frac{1}{3} bfx^6e + \frac{1}{4} dx^4e^2 + \frac{1}{2} bx^2e^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+b*x)*(f*x^4+e)^2,x, algorithm="giac")

[Out] 1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/4*d*f*x^8*e + 1/3*b*f*x^6*e + 1/4*d*x^4*e^2 + 1/2*b*x^2*e^2

3.143 $\int (a + bx + dx^3)(e + fx^4)^2 dx$

Optimal. Leaf size=77

$$ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 + \frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{d(e + fx^4)^3}{12f}$$

[Out] a*e^2*x + (b*e^2*x^2)/2 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (d*(e + f*x^4)^3)/(12*f)

Rubi [A] time = 0.0511904, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {1582, 1850}

$$ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 + \frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{d(e + fx^4)^3}{12f}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + d*x^3)*(e + f*x^4)^2,x]

[Out] a*e^2*x + (b*e^2*x^2)/2 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (d*(e + f*x^4)^3)/(12*f)

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(Coeff[Px, x, n - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a + b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int (a + bx + dx^3)(e + fx^4)^2 dx &= \frac{d(e + fx^4)^3}{12f} + \int (a + bx)(e + fx^4)^2 dx \\
&= \frac{d(e + fx^4)^3}{12f} + \int (ae^2 + be^2x + 2aefx^4 + 2befx^5 + af^2x^8 + bf^2x^9) dx \\
&= ae^2x + \frac{1}{2}be^2x^2 + \frac{2}{5}aefx^5 + \frac{1}{3}befx^6 + \frac{1}{9}af^2x^9 + \frac{1}{10}bf^2x^{10} + \frac{d(e + fx^4)^3}{12f}
\end{aligned}$$

Mathematica [A] time = 0.0033786, size = 92, normalized size = 1.19

$$ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 + \frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{1}{4}de^2x^4 + \frac{1}{4}defx^8 + \frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + d*x^3)*(e + f*x^4)^2,x]

[Out] a*e^2*x + (b*e^2*x^2)/2 + (d*e^2*x^4)/4 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (d*e*f*x^8)/4 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (d*f^2*x^12)/12

Maple [A] time = 0.04, size = 77, normalized size = 1.

$$\frac{df^2x^{12}}{12} + \frac{bf^2x^{10}}{10} + \frac{af^2x^9}{9} + \frac{defx^8}{4} + \frac{befx^6}{3} + \frac{2aefx^5}{5} + \frac{de^2x^4}{4} + \frac{be^2x^2}{2} + ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+b*x+a)*(f*x^4+e)^2,x)

[Out] 1/12*d*f^2*x^12+1/10*b*f^2*x^10+1/9*a*f^2*x^9+1/4*d*e*f*x^8+1/3*b*e*f*x^6+2/5*a*e*f*x^5+1/4*d*e^2*x^4+1/2*b*e^2*x^2+a*e^2*x

Maxima [A] time = 1.00375, size = 103, normalized size = 1.34

$$\frac{1}{12}df^2x^{12} + \frac{1}{10}bf^2x^{10} + \frac{1}{9}af^2x^9 + \frac{1}{4}defx^8 + \frac{1}{3}befx^6 + \frac{2}{5}aefx^5 + \frac{1}{4}de^2x^4 + \frac{1}{2}be^2x^2 + ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+b*x+a)*(f*x^4+e)^2,x, algorithm="maxima")

[Out] $\frac{1}{12}d*f^2*x^{12} + \frac{1}{10}b*f^2*x^{10} + \frac{1}{9}a*f^2*x^9 + \frac{1}{4}d*e*f*x^8 + \frac{1}{3}b*e*f*x^6 + \frac{2}{5}a*e*f*x^5 + \frac{1}{4}d*e^2*x^4 + \frac{1}{2}b*e^2*x^2 + a*e^2*x$

Fricas [A] time = 1.09603, size = 190, normalized size = 2.47

$$\frac{1}{12}x^{12}f^2d + \frac{1}{10}x^{10}f^2b + \frac{1}{9}x^9f^2a + \frac{1}{4}x^8fed + \frac{1}{3}x^6feb + \frac{2}{5}x^5fea + \frac{1}{4}x^4e^2d + \frac{1}{2}x^2e^2b + xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+b*x+a)*(f*x^4+e)^2,x, algorithm="fricas")

[Out] $\frac{1}{12}x^{12}f^2d + \frac{1}{10}x^{10}f^2b + \frac{1}{9}x^9f^2a + \frac{1}{4}x^8f*e*d + \frac{1}{3}x^6*f*e*b + \frac{2}{5}x^5*f*e*a + \frac{1}{4}x^4*e^2*d + \frac{1}{2}x^2*e^2*b + x*e^2*a$

Sympy [A] time = 0.068377, size = 88, normalized size = 1.14

$$ae^2x + \frac{2aefx^5}{5} + \frac{af^2x^9}{9} + \frac{be^2x^2}{2} + \frac{befx^6}{3} + \frac{bf^2x^{10}}{10} + \frac{de^2x^4}{4} + \frac{defx^8}{4} + \frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+b*x+a)*(f*x**4+e)**2,x)

[Out] $a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10 + d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12$

Giac [A] time = 1.05752, size = 103, normalized size = 1.34

$$\frac{1}{12}df^2x^{12} + \frac{1}{10}bf^2x^{10} + \frac{1}{9}af^2x^9 + \frac{1}{4}dfx^8e + \frac{1}{3}bfx^6e + \frac{2}{5}afx^5e + \frac{1}{4}dx^4e^2 + \frac{1}{2}bx^2e^2 + axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x+a)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/4*d*f*x^8*e + 1/3*b*f*x^6*e + 2/5*a*f*x^5*e + 1/4*d*x^4*e^2 + 1/2*b*x^2*e^2 + a*x*e^2
```

3.144 $\int (cx^2 + dx^3)(e + fx^4)^2 dx$

Optimal. Leaf size=50

$$\frac{1}{3}ce^2x^3 + \frac{2}{7}cef x^7 + \frac{1}{11}cf^2x^{11} + \frac{d(e + fx^4)^3}{12f}$$

[Out] (c*e^2*x^3)/3 + (2*c*e*f*x^7)/7 + (c*f^2*x^11)/11 + (d*(e + f*x^4)^3)/(12*f)

Rubi [A] time = 0.0234746, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1582, 12, 270}

$$\frac{1}{3}ce^2x^3 + \frac{2}{7}cef x^7 + \frac{1}{11}cf^2x^{11} + \frac{d(e + fx^4)^3}{12f}$$

Antiderivative was successfully verified.

[In] Int[(c*x^2 + d*x^3)*(e + f*x^4)^2,x]

[Out] (c*e^2*x^3)/3 + (2*c*e*f*x^7)/7 + (c*f^2*x^11)/11 + (d*(e + f*x^4)^3)/(12*f)

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(Coeff[Px, x, n - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a + b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int (cx^2 + dx^3)(e + fx^4)^2 dx &= \frac{d(e + fx^4)^3}{12f} + \int cx^2(e + fx^4)^2 dx \\ &= \frac{d(e + fx^4)^3}{12f} + c \int x^2(e + fx^4)^2 dx \\ &= \frac{d(e + fx^4)^3}{12f} + c \int (e^2x^2 + 2efx^6 + f^2x^{10}) dx \\ &= \frac{1}{3}ce^2x^3 + \frac{2}{7}cef x^7 + \frac{1}{11}cf^2x^{11} + \frac{d(e + fx^4)^3}{12f} \end{aligned}$$

Mathematica [A] time = 0.0028124, size = 65, normalized size = 1.3

$$\frac{1}{3}ce^2x^3 + \frac{2}{7}cef x^7 + \frac{1}{11}cf^2x^{11} + \frac{1}{4}de^2x^4 + \frac{1}{4}def x^8 + \frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (2*c*e*f*x^7)/7 + (d*e*f*x^8)/4 + (c*f^2*x^11)/11 + (d*f^2*x^12)/12
```

Maple [A] time = 0.039, size = 54, normalized size = 1.1

$$\frac{df^2x^{12}}{12} + \frac{cf^2x^{11}}{11} + \frac{defx^8}{4} + \frac{2cef x^7}{7} + \frac{de^2x^4}{4} + \frac{ce^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2)*(f*x^4+e)^2,x)
```

[Out] $\frac{1}{12}d*f^2*x^{12} + \frac{1}{11}c*f^2*x^{11} + \frac{1}{4}d*e*f*x^8 + \frac{2}{7}c*e*f*x^7 + \frac{1}{4}d*e^2*x^4 + \frac{1}{3}c*e^2*x^3$

Maxima [A] time = 1.0436, size = 72, normalized size = 1.44

$$\frac{1}{12}df^2x^{12} + \frac{1}{11}cf^2x^{11} + \frac{1}{4}defx^8 + \frac{2}{7}cef x^7 + \frac{1}{4}de^2x^4 + \frac{1}{3}ce^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3+c*x^2)*(f*x^4+e)^2,x, algorithm="maxima")`

[Out] $\frac{1}{12}d*f^2*x^{12} + \frac{1}{11}c*f^2*x^{11} + \frac{1}{4}d*e*f*x^8 + \frac{2}{7}c*e*f*x^7 + \frac{1}{4}d*e^2*x^4 + \frac{1}{3}c*e^2*x^3$

Fricas [A] time = 1.08882, size = 134, normalized size = 2.68

$$\frac{1}{12}x^{12}f^2d + \frac{1}{11}x^{11}f^2c + \frac{1}{4}x^8fed + \frac{2}{7}x^7fec + \frac{1}{4}x^4e^2d + \frac{1}{3}x^3e^2c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3+c*x^2)*(f*x^4+e)^2,x, algorithm="fricas")`

[Out] $\frac{1}{12}*x^{12}*f^2*d + \frac{1}{11}*x^{11}*f^2*c + \frac{1}{4}*x^8*f*e*d + \frac{2}{7}*x^7*f*e*c + \frac{1}{4}*x^4*e^2*d + \frac{1}{3}*x^3*e^2*c$

Sympy [A] time = 0.066073, size = 61, normalized size = 1.22

$$\frac{ce^2x^3}{3} + \frac{2cef x^7}{7} + \frac{cf^2x^{11}}{11} + \frac{de^2x^4}{4} + \frac{defx^8}{4} + \frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**3+c*x**2)*(f*x**4+e)**2,x)`

[Out] $c*e**2*x**3/3 + 2*c*e*f*x**7/7 + c*f**2*x**11/11 + d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12$

Giac [A] time = 1.05582, size = 72, normalized size = 1.44

$$\frac{1}{12}df^2x^{12} + \frac{1}{11}cf^2x^{11} + \frac{1}{4}dfx^8e + \frac{2}{7}cfx^7e + \frac{1}{4}dx^4e^2 + \frac{1}{3}cx^3e^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2)*(f*x^4+e)^2,x, algorithm="giac")

[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/4*d*f*x^8*e + 2/7*c*f*x^7*e + 1/4*d*x^4*e^2 + 1/3*c*x^3*e^2

$$3.145 \quad \int (a + cx^2 + dx^3)(e + fx^4)^2 dx$$

Optimal. Leaf size=77

$$ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 + \frac{1}{3}ce^2x^3 + \frac{2}{7}cef x^7 + \frac{1}{11}cf^2x^{11} + \frac{d(e + fx^4)^3}{12f}$$

[Out] a*e^2*x + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9 + (c*f^2*x^11)/11 + (d*(e + f*x^4)^3)/(12*f)

Rubi [A] time = 0.04152, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1582, 1154}

$$ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 + \frac{1}{3}ce^2x^3 + \frac{2}{7}cef x^7 + \frac{1}{11}cf^2x^{11} + \frac{d(e + fx^4)^3}{12f}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^2 + d*x^3)*(e + f*x^4)^2,x]

[Out] a*e^2*x + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9 + (c*f^2*x^11)/11 + (d*(e + f*x^4)^3)/(12*f)

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(Coeff[Px, x, n - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a + b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}
\int (a + cx^2 + dx^3)(e + fx^4)^2 dx &= \frac{d(e + fx^4)^3}{12f} + \int (a + cx^2)(e + fx^4)^2 dx \\
&= \frac{d(e + fx^4)^3}{12f} + \int (ae^2 + ce^2x^2 + 2aefx^4 + 2cefx^6 + af^2x^8 + cf^2x^{10}) dx \\
&= ae^2x + \frac{1}{3}ce^2x^3 + \frac{2}{5}aefx^5 + \frac{2}{7}cefx^7 + \frac{1}{9}af^2x^9 + \frac{1}{11}cf^2x^{11} + \frac{d(e + fx^4)^3}{12f}
\end{aligned}$$

Mathematica [A] time = 0.0038174, size = 92, normalized size = 1.19

$$ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 + \frac{1}{3}ce^2x^3 + \frac{2}{7}cefx^7 + \frac{1}{11}cf^2x^{11} + \frac{1}{4}de^2x^4 + \frac{1}{4}defx^8 + \frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^2 + d*x^3)*(e + f*x^4)^2,x]

[Out] a*e^2*x + (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (d*e*f*x^8)/4 + (a*f^2*x^9)/9 + (c*f^2*x^11)/11 + (d*f^2*x^12)/12

Maple [A] time = 0.039, size = 77, normalized size = 1.

$$\frac{df^2x^{12}}{12} + \frac{cf^2x^{11}}{11} + \frac{af^2x^9}{9} + \frac{defx^8}{4} + \frac{2cefx^7}{7} + \frac{2aefx^5}{5} + \frac{de^2x^4}{4} + \frac{ce^2x^3}{3} + ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+c*x^2+a)*(f*x^4+e)^2,x)

[Out] 1/12*d*f^2*x^12+1/11*c*f^2*x^11+1/9*a*f^2*x^9+1/4*d*e*f*x^8+2/7*c*e*f*x^7+2/5*a*e*f*x^5+1/4*d*e^2*x^4+1/3*c*e^2*x^3+a*e^2*x

Maxima [A] time = 0.942446, size = 103, normalized size = 1.34

$$\frac{1}{12}df^2x^{12} + \frac{1}{11}cf^2x^{11} + \frac{1}{9}af^2x^9 + \frac{1}{4}defx^8 + \frac{2}{7}cefx^7 + \frac{2}{5}aefx^5 + \frac{1}{4}de^2x^4 + \frac{1}{3}ce^2x^3 + ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+a)*(f*x^4+e)^2,x, algorithm="maxima")

[Out] $\frac{1}{12}d*f^2*x^{12} + \frac{1}{11}c*f^2*x^{11} + \frac{1}{9}a*f^2*x^9 + \frac{1}{4}d*e*f*x^8 + \frac{2}{7}c*e*f*x^7 + \frac{2}{5}a*e*f*x^5 + \frac{1}{4}d*e^2*x^4 + \frac{1}{3}c*e^2*x^3 + a*e^2*x$

Fricas [A] time = 0.98629, size = 190, normalized size = 2.47

$$\frac{1}{12}x^{12}f^2d + \frac{1}{11}x^{11}f^2c + \frac{1}{9}x^9f^2a + \frac{1}{4}x^8fed + \frac{2}{7}x^7fec + \frac{2}{5}x^5fea + \frac{1}{4}x^4e^2d + \frac{1}{3}x^3e^2c + xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+a)*(f*x^4+e)^2,x, algorithm="fricas")

[Out] $\frac{1}{12}x^{12}f^2d + \frac{1}{11}x^{11}f^2c + \frac{1}{9}x^9f^2a + \frac{1}{4}x^8f*e*d + \frac{2}{7}x^7*f*e*c + \frac{2}{5}x^5*f*e*a + \frac{1}{4}x^4*e^2*d + \frac{1}{3}x^3*e^2*c + x*e^2*a$

Sympy [A] time = 0.068376, size = 90, normalized size = 1.17

$$ae^2x + \frac{2aefx^5}{5} + \frac{af^2x^9}{9} + \frac{ce^2x^3}{3} + \frac{2cef x^7}{7} + \frac{cf^2x^{11}}{11} + \frac{de^2x^4}{4} + \frac{defx^8}{4} + \frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+c*x**2+a)*(f*x**4+e)**2,x)

[Out] $a*e^{**2}*x + 2*a*e*f*x^{**5}/5 + a*f^{**2}*x^{**9}/9 + c*e^{**2}*x^{**3}/3 + 2*c*e*f*x^{**7}/7 + c*f^{**2}*x^{**11}/11 + d*e^{**2}*x^{**4}/4 + d*e*f*x^{**8}/4 + d*f^{**2}*x^{**12}/12$

Giac [A] time = 1.05124, size = 103, normalized size = 1.34

$$\frac{1}{12}df^2x^{12} + \frac{1}{11}cf^2x^{11} + \frac{1}{9}af^2x^9 + \frac{1}{4}dfx^8e + \frac{2}{7}cfx^7e + \frac{2}{5}afx^5e + \frac{1}{4}dx^4e^2 + \frac{1}{3}cx^3e^2 + axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+a)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 1/4*d*f*x^8*e + 2/7*c*f*x^7*e + 2/5*a*f*x^5*e + 1/4*d*x^4*e^2 + 1/3*c*x^3*e^2 + a*x*e^2
```

3.146 $\int (bx + cx^2 + dx^3)(e + fx^4)^2 dx$

Optimal. Leaf size=82

$$\frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{1}{3}ce^2x^3 + \frac{2}{7}cef x^7 + \frac{1}{11}cf^2x^{11} + \frac{d(e + fx^4)^3}{12f}$$

[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (b*f^2*x^10)/10 + (c*f^2*x^11)/11 + (d*(e + f*x^4)^3)/(12*f)

Rubi [A] time = 0.0835611, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1582, 1593, 1620}

$$\frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{1}{3}ce^2x^3 + \frac{2}{7}cef x^7 + \frac{1}{11}cf^2x^{11} + \frac{d(e + fx^4)^3}{12f}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2 + d*x^3)*(e + f*x^4)^2,x]

[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (b*f^2*x^10)/10 + (c*f^2*x^11)/11 + (d*(e + f*x^4)^3)/(12*f)

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(Coeff[Px, x, n - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a + b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}
\int (bx + cx^2 + dx^3)(e + fx^4)^2 dx &= \frac{d(e + fx^4)^3}{12f} + \int (bx + cx^2)(e + fx^4)^2 dx \\
&= \frac{d(e + fx^4)^3}{12f} + \int x(b + cx)(e + fx^4)^2 dx \\
&= \frac{d(e + fx^4)^3}{12f} + \int (be^2x + ce^2x^2 + 2befx^5 + 2cefx^6 + bf^2x^9 + cf^2x^{10}) dx \\
&= \frac{1}{2}be^2x^2 + \frac{1}{3}ce^2x^3 + \frac{1}{3}befx^6 + \frac{2}{7}cefx^7 + \frac{1}{10}bf^2x^{10} + \frac{1}{11}cf^2x^{11} + \frac{d(e + fx^4)^3}{12f}
\end{aligned}$$

Mathematica [A] time = 0.0041537, size = 97, normalized size = 1.18

$$\frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{1}{3}ce^2x^3 + \frac{2}{7}cefx^7 + \frac{1}{11}cf^2x^{11} + \frac{1}{4}de^2x^4 + \frac{1}{4}defx^8 + \frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2 + d*x^3)*(e + f*x^4)^2,x]

[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (d*e*f*x^8)/4 + (b*f^2*x^10)/10 + (c*f^2*x^11)/11 + (d*f^2*x^12)/12

Maple [A] time = 0.04, size = 80, normalized size = 1.

$$\frac{df^2x^{12}}{12} + \frac{cf^2x^{11}}{11} + \frac{bf^2x^{10}}{10} + \frac{defx^8}{4} + \frac{2cefx^7}{7} + \frac{befx^6}{3} + \frac{de^2x^4}{4} + \frac{ce^2x^3}{3} + \frac{be^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^3+c*x^2+b*x)*(f*x^4+e)^2,x)`

[Out] $\frac{1}{12}d*f^2*x^{12} + \frac{1}{11}c*f^2*x^{11} + \frac{1}{10}b*f^2*x^{10} + \frac{1}{4}d*e*f*x^8 + \frac{2}{7}c*e*f*x^7 + \frac{1}{3}b*e*f*x^6 + \frac{1}{4}d*e^2*x^4 + \frac{1}{3}c*e^2*x^3 + \frac{1}{2}b*e^2*x^2$

Maxima [A] time = 0.942487, size = 107, normalized size = 1.3

$$\frac{1}{12}df^2x^{12} + \frac{1}{11}cf^2x^{11} + \frac{1}{10}bf^2x^{10} + \frac{1}{4}defx^8 + \frac{2}{7}cef x^7 + \frac{1}{3}befx^6 + \frac{1}{4}de^2x^4 + \frac{1}{3}ce^2x^3 + \frac{1}{2}be^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3+c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="maxima")`

[Out] $\frac{1}{12}d*f^2*x^{12} + \frac{1}{11}c*f^2*x^{11} + \frac{1}{10}b*f^2*x^{10} + \frac{1}{4}d*e*f*x^8 + \frac{2}{7}c*e*f*x^7 + \frac{1}{3}b*e*f*x^6 + \frac{1}{4}d*e^2*x^4 + \frac{1}{3}c*e^2*x^3 + \frac{1}{2}b*e^2*x^2$

Fricas [A] time = 1.0713, size = 201, normalized size = 2.45

$$\frac{1}{12}x^{12}f^2d + \frac{1}{11}x^{11}f^2c + \frac{1}{10}x^{10}f^2b + \frac{1}{4}x^8fed + \frac{2}{7}x^7fec + \frac{1}{3}x^6feb + \frac{1}{4}x^4e^2d + \frac{1}{3}x^3e^2c + \frac{1}{2}x^2e^2b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3+c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="fricas")`

[Out] $\frac{1}{12}*x^{12}*f^2*d + \frac{1}{11}*x^{11}*f^2*c + \frac{1}{10}*x^{10}*f^2*b + \frac{1}{4}*x^8*f*e*d + \frac{2}{7}*x^7*f*e*c + \frac{1}{3}*x^6*f*e*b + \frac{1}{4}*x^4*e^2*d + \frac{1}{3}*x^3*e^2*c + \frac{1}{2}*x^2*e^2*b$

Sympy [A] time = 0.070572, size = 92, normalized size = 1.12

$$\frac{be^2x^2}{2} + \frac{befx^6}{3} + \frac{bf^2x^{10}}{10} + \frac{ce^2x^3}{3} + \frac{2cef x^7}{7} + \frac{cf^2x^{11}}{11} + \frac{de^2x^4}{4} + \frac{defx^8}{4} + \frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**3+c*x**2+b*x)*(f*x**4+e)**2,x)`

[Out] $b e^{2x^2/2} + b e^{f x^6/3} + b f^{2x^{10}/10} + c e^{2x^3/3} + 2 c e^{f x^7/7} + c f^{2x^{11}/11} + d e^{2x^4/4} + d e^{f x^8/4} + d f^{2x^{12}/12}$

Giac [A] time = 1.06003, size = 107, normalized size = 1.3

$$\frac{1}{12} d f^2 x^{12} + \frac{1}{11} c f^2 x^{11} + \frac{1}{10} b f^2 x^{10} + \frac{1}{4} d f x^8 e + \frac{2}{7} c f x^7 e + \frac{1}{3} b f x^6 e + \frac{1}{4} d x^4 e^2 + \frac{1}{3} c x^3 e^2 + \frac{1}{2} b x^2 e^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3+c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="giac")`

[Out] $1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/4*d*f*x^8*e + 2/7*c*f*x^7*e + 1/3*b*f*x^6*e + 1/4*d*x^4*e^2 + 1/3*c*x^3*e^2 + 1/2*b*x^2*e^2$

3.147 $\int (c + dx + ex^2 + fx^3)(a + bx^4)^2 dx$

Optimal. Leaf size=109

$$a^2cx + \frac{1}{2}a^2dx^2 + \frac{1}{3}a^2ex^3 + \frac{2}{5}abcx^5 + \frac{1}{3}abdx^6 + \frac{2}{7}abex^7 + \frac{f(a+bx^4)^3}{12b} + \frac{1}{9}b^2cx^9 + \frac{1}{10}b^2dx^{10} + \frac{1}{11}b^2ex^{11}$$

[Out] $a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (2*a*b*c*x^5)/5 + (a*b*d*x^6)/3 + (2*a*b*e*x^7)/7 + (b^2*c*x^9)/9 + (b^2*d*x^{10})/10 + (b^2*e*x^{11})/11 + (f*(a + b*x^4)^3)/(12*b)$

Rubi [A] time = 0.0804812, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$, Rules used = {1582, 1657}

$$a^2cx + \frac{1}{2}a^2dx^2 + \frac{1}{3}a^2ex^3 + \frac{2}{5}abcx^5 + \frac{1}{3}abdx^6 + \frac{2}{7}abex^7 + \frac{f(a+bx^4)^3}{12b} + \frac{1}{9}b^2cx^9 + \frac{1}{10}b^2dx^{10} + \frac{1}{11}b^2ex^{11}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]

[Out] $a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (2*a*b*c*x^5)/5 + (a*b*d*x^6)/3 + (2*a*b*e*x^7)/7 + (b^2*c*x^9)/9 + (b^2*d*x^{10})/10 + (b^2*e*x^{11})/11 + (f*(a + b*x^4)^3)/(12*b)$

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(Coeff[Px, x, n - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_)] /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a + b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[Expand Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
```

, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (c + dx + ex^2 + fx^3)(a + bx^4)^2 dx &= \frac{f(a + bx^4)^3}{12b} + \int (c + dx + ex^2)(a + bx^4)^2 dx \\ &= \frac{f(a + bx^4)^3}{12b} + \int (a^2c + a^2dx + a^2ex^2 + 2abcx^4 + 2abdx^5 + 2abex^6 + b^2cx^8 + \\ &= a^2cx + \frac{1}{2}a^2dx^2 + \frac{1}{3}a^2ex^3 + \frac{2}{5}abcx^5 + \frac{1}{3}abdx^6 + \frac{2}{7}abex^7 + \frac{1}{9}b^2cx^9 + \frac{1}{10}b^2dx^{10} + \frac{1}{11}b^2ex^{11} + \frac{1}{12}b^2fx^{12} \end{aligned}$$

Mathematica [A] time = 0.0063405, size = 124, normalized size = 1.14

$$a^2cx + \frac{1}{2}a^2dx^2 + \frac{1}{3}a^2ex^3 + \frac{1}{4}a^2fx^4 + \frac{2}{5}abcx^5 + \frac{1}{3}abdx^6 + \frac{2}{7}abex^7 + \frac{1}{4}abfx^8 + \frac{1}{9}b^2cx^9 + \frac{1}{10}b^2dx^{10} + \frac{1}{11}b^2ex^{11} + \frac{1}{12}b^2fx^{12}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]

[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (a^2*f*x^4)/4 + (2*a*b*c*x^5)/5 + (a*b*d*x^6)/3 + (2*a*b*e*x^7)/7 + (a*b*f*x^8)/4 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10 + (b^2*e*x^11)/11 + (b^2*f*x^12)/12

Maple [A] time = 0., size = 103, normalized size = 0.9

$$\frac{b^2fx^{12}}{12} + \frac{b^2ex^{11}}{11} + \frac{b^2dx^{10}}{10} + \frac{b^2cx^9}{9} + \frac{fabx^8}{4} + \frac{2abex^7}{7} + \frac{abdx^6}{3} + \frac{2abcx^5}{5} + \frac{fa^2x^4}{4} + \frac{a^2ex^3}{3} + \frac{a^2dx^2}{2} + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x)

[Out] 1/12*b^2*f*x^12+1/11*b^2*e*x^11+1/10*b^2*d*x^10+1/9*b^2*c*x^9+1/4*f*a*b*x^8+2/7*a*b*e*x^7+1/3*a*b*d*x^6+2/5*a*b*c*x^5+1/4*f*a^2*x^4+1/3*a^2*e*x^3+1/2*a^2*d*x^2+a^2*c*x

Maxima [A] time = 0.932908, size = 138, normalized size = 1.27

$$\frac{1}{12} b^2 f x^{12} + \frac{1}{11} b^2 e x^{11} + \frac{1}{10} b^2 d x^{10} + \frac{1}{9} b^2 c x^9 + \frac{1}{4} a b f x^8 + \frac{2}{7} a b e x^7 + \frac{1}{3} a b d x^6 + \frac{2}{5} a b c x^5 + \frac{1}{4} a^2 f x^4 + \frac{1}{3} a^2 e x^3 + \frac{1}{2} a^2 d x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="maxima")

[Out] 1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a*b*f*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4 + 1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x

Fricas [A] time = 1.15396, size = 258, normalized size = 2.37

$$\frac{1}{12} x^{12} f b^2 + \frac{1}{11} x^{11} e b^2 + \frac{1}{10} x^{10} d b^2 + \frac{1}{9} x^9 c b^2 + \frac{1}{4} x^8 f b a + \frac{2}{7} x^7 e b a + \frac{1}{3} x^6 d b a + \frac{2}{5} x^5 c b a + \frac{1}{4} x^4 f a^2 + \frac{1}{3} x^3 e a^2 + \frac{1}{2} x^2 d a^2 + x c a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="fricas")

[Out] 1/12*x^12*f*b^2 + 1/11*x^11*e*b^2 + 1/10*x^10*d*b^2 + 1/9*x^9*c*b^2 + 1/4*x^8*f*b*a + 2/7*x^7*e*b*a + 1/3*x^6*d*b*a + 2/5*x^5*c*b*a + 1/4*x^4*f*a^2 + 1/3*x^3*e*a^2 + 1/2*x^2*d*a^2 + x*c*a^2

Sympy [A] time = 0.073329, size = 121, normalized size = 1.11

$$a^2 c x + \frac{a^2 d x^2}{2} + \frac{a^2 e x^3}{3} + \frac{a^2 f x^4}{4} + \frac{2 a b c x^5}{5} + \frac{a b d x^6}{3} + \frac{2 a b e x^7}{7} + \frac{a b f x^8}{4} + \frac{b^2 c x^9}{9} + \frac{b^2 d x^{10}}{10} + \frac{b^2 e x^{11}}{11} + \frac{b^2 f x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**2,x)

[Out] a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + a**2*f*x**4/4 + 2*a*b*c*x**5/5 + a*b*d*x**6/3 + 2*a*b*e*x**7/7 + a*b*f*x**8/4 + b**2*c*x**9/9 + b**2*d*x**10/10 + b**2*e*x**11/11 + b**2*f*x**12/12

Giac [A] time = 1.05891, size = 142, normalized size = 1.3

$$\frac{1}{12} b^2 f x^{12} + \frac{1}{11} b^2 x^{11} e + \frac{1}{10} b^2 d x^{10} + \frac{1}{9} b^2 c x^9 + \frac{1}{4} a b f x^8 + \frac{2}{7} a b x^7 e + \frac{1}{3} a b d x^6 + \frac{2}{5} a b c x^5 + \frac{1}{4} a^2 f x^4 + \frac{1}{3} a^2 x^3 e + \frac{1}{2} a^2 d x^2 + a^2 c x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="giac")

[Out] 1/12*b^2*f*x^12 + 1/11*b^2*x^11*e + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a*b*f*x^8 + 2/7*a*b*x^7*e + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4 + 1/3*a^2*x^3*e + 1/2*a^2*d*x^2 + a^2*c*x

3.148 $\int (c + dx + ex^2 + fx^3)(a + bx^4)^3 dx$

Optimal. Leaf size=151

$$\frac{3}{5}a^2bcx^5 + \frac{1}{2}a^2bdx^6 + \frac{3}{7}a^2bex^7 + a^3cx + \frac{1}{2}a^3dx^2 + \frac{1}{3}a^3ex^3 + \frac{1}{3}ab^2cx^9 + \frac{3}{10}ab^2dx^{10} + \frac{3}{11}ab^2ex^{11} + \frac{f(a + bx^4)^4}{16b} + \frac{1}{13}b^3c$$

[Out] $a^3c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^5)/5 + (a^2*b*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x^{10})/10 + (3*a*b^2*e*x^{11})/11 + (b^3*c*x^{13})/13 + (b^3*d*x^{14})/14 + (b^3*e*x^{15})/15 + (f*(a + b*x^4)^4)/(16*b)$

Rubi [A] time = 0.105837, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$, Rules used = {1582, 1657}

$$\frac{3}{5}a^2bcx^5 + \frac{1}{2}a^2bdx^6 + \frac{3}{7}a^2bex^7 + a^3cx + \frac{1}{2}a^3dx^2 + \frac{1}{3}a^3ex^3 + \frac{1}{3}ab^2cx^9 + \frac{3}{10}ab^2dx^{10} + \frac{3}{11}ab^2ex^{11} + \frac{f(a + bx^4)^4}{16b} + \frac{1}{13}b^3c$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]

[Out] $a^3c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^5)/5 + (a^2*b*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x^{10})/10 + (3*a*b^2*e*x^{11})/11 + (b^3*c*x^{13})/13 + (b^3*d*x^{14})/14 + (b^3*e*x^{15})/15 + (f*(a + b*x^4)^4)/(16*b)$

Rule 1582

Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(Coeff[Px, x, n - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a + b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]

Rule 1657

```
Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned} \int (c + dx + ex^2 + fx^3)(a + bx^4)^3 dx &= \frac{f(a + bx^4)^4}{16b} + \int (c + dx + ex^2)(a + bx^4)^3 dx \\ &= \frac{f(a + bx^4)^4}{16b} + \int (a^3c + a^3dx + a^3ex^2 + 3a^2bcx^4 + 3a^2bdx^5 + 3a^2bex^6 + 3ab^2cx^7 \\ &\quad + a^3cx + \frac{1}{2}a^3dx^2 + \frac{1}{3}a^3ex^3 + \frac{3}{5}a^2bcx^5 + \frac{1}{2}a^2bdx^6 + \frac{3}{7}a^2bex^7 + \frac{1}{3}ab^2cx^9 + \frac{3}{10}ab^2dx^{10} + \frac{3}{11}ab^2ex^{11} \end{aligned}$$

Mathematica [A] time = 0.0042748, size = 180, normalized size = 1.19

$$\frac{3}{5}a^2bcx^5 + \frac{1}{2}a^2bdx^6 + \frac{3}{7}a^2bex^7 + \frac{3}{8}a^2bfx^8 + a^3cx + \frac{1}{2}a^3dx^2 + \frac{1}{3}a^3ex^3 + \frac{1}{4}a^3fx^4 + \frac{1}{3}ab^2cx^9 + \frac{3}{10}ab^2dx^{10} + \frac{3}{11}ab^2ex^{11} -$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (a^3*f*x^4)/4 + (3*a^2*b*c*x^5)/5
+ (a^2*b*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (3*a^2*b*f*x^8)/8 + (a*b^2*c*x^9)/
3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2*e*x^11)/11 + (a*b^2*f*x^12)/4 + (b^3*c*x
^13)/13 + (b^3*d*x^14)/14 + (b^3*e*x^15)/15 + (b^3*f*x^16)/16
```

Maple [A] time = 0.001, size = 151, normalized size = 1.

$$\frac{b^3fx^{16}}{16} + \frac{b^3ex^{15}}{15} + \frac{b^3dx^{14}}{14} + \frac{b^3cx^{13}}{13} + \frac{ab^2fx^{12}}{4} + \frac{3ab^2ex^{11}}{11} + \frac{3ab^2dx^{10}}{10} + \frac{ab^2cx^9}{3} + \frac{3fba^2x^8}{8} + \frac{3a^2bex^7}{7} + \frac{a^2bdx^6}{2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x)
```

```
[Out] 1/16*b^3*f*x^16+1/15*b^3*e*x^15+1/14*b^3*d*x^14+1/13*b^3*c*x^13+1/4*a*b^2*f
*x^12+3/11*a*b^2*e*x^11+3/10*a*b^2*d*x^10+1/3*a*b^2*c*x^9+3/8*f*b*a^2*x^8+3
```

$$/7*a^2*b*e*x^7+1/2*a^2*b*d*x^6+3/5*a^2*b*c*x^5+1/4*f*a^3*x^4+1/3*a^3*e*x^3+1/2*a^3*d*x^2+a^3*c*x$$

Maxima [A] time = 0.950192, size = 203, normalized size = 1.34

$$\frac{1}{16} b^3 f x^{16} + \frac{1}{15} b^3 e x^{15} + \frac{1}{14} b^3 d x^{14} + \frac{1}{13} b^3 c x^{13} + \frac{1}{4} a b^2 f x^{12} + \frac{3}{11} a b^2 e x^{11} + \frac{3}{10} a b^2 d x^{10} + \frac{1}{3} a b^2 c x^9 + \frac{3}{8} a^2 b f x^8 + \frac{3}{7} a^2 b e x^7 + \frac{3}{5} a^2 b d x^6 + \frac{1}{4} a^2 b c x^5 + \frac{1}{3} a^3 f x^4 + \frac{1}{2} a^3 e x^3 + \frac{1}{2} a^3 d x^2 + a^3 c x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="maxima")

[Out] 1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/4*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x

Fricas [A] time = 0.924338, size = 375, normalized size = 2.48

$$\frac{1}{16} x^{16} f b^3 + \frac{1}{15} x^{15} e b^3 + \frac{1}{14} x^{14} d b^3 + \frac{1}{13} x^{13} c b^3 + \frac{1}{4} x^{12} f b^2 a + \frac{3}{11} x^{11} e b^2 a + \frac{3}{10} x^{10} d b^2 a + \frac{1}{3} x^9 c b^2 a + \frac{3}{8} x^8 f b a^2 + \frac{3}{7} x^7 e b a^2 + \frac{3}{5} x^6 d b a^2 + \frac{1}{4} x^5 c b a^2 + \frac{1}{3} x^4 f a^3 + \frac{1}{2} x^3 e a^3 + \frac{1}{2} x^2 d a^3 + x c a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="fricas")

[Out] 1/16*x^16*f*b^3 + 1/15*x^15*e*b^3 + 1/14*x^14*d*b^3 + 1/13*x^13*c*b^3 + 1/4*x^12*f*b^2*a + 3/11*x^11*e*b^2*a + 3/10*x^10*d*b^2*a + 1/3*x^9*c*b^2*a + 3/8*x^8*f*b*a^2 + 3/7*x^7*e*b*a^2 + 1/2*x^6*d*b*a^2 + 3/5*x^5*c*b*a^2 + 1/4*x^4*f*a^3 + 1/3*x^3*e*a^3 + 1/2*x^2*d*a^3 + x*c*a^3

Sympy [A] time = 0.084647, size = 180, normalized size = 1.19

$$a^3 c x + \frac{a^3 d x^2}{2} + \frac{a^3 e x^3}{3} + \frac{a^3 f x^4}{4} + \frac{3 a^2 b c x^5}{5} + \frac{a^2 b d x^6}{2} + \frac{3 a^2 b e x^7}{7} + \frac{3 a^2 b f x^8}{8} + \frac{a b^2 c x^9}{3} + \frac{3 a b^2 d x^{10}}{10} + \frac{3 a b^2 e x^{11}}{11} + \frac{a b^2 f x^{12}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**3,x)

[Out] a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + a**3*f*x**4/4 + 3*a**2*b*c*x**5/5 + a**2*b*d*x**6/2 + 3*a**2*b*e*x**7/7 + 3*a**2*b*f*x**8/8 + a*b**2*c*x**9/3 + 3*a*b**2*d*x**10/10 + 3*a*b**2*e*x**11/11 + a*b**2*f*x**12/4 + b**3*c*x**13/13 + b**3*d*x**14/14 + b**3*e*x**15/15 + b**3*f*x**16/16

Giac [A] time = 1.05389, size = 208, normalized size = 1.38

$$\frac{1}{16} b^3 f x^{16} + \frac{1}{15} b^3 x^{15} e + \frac{1}{14} b^3 d x^{14} + \frac{1}{13} b^3 c x^{13} + \frac{1}{4} a b^2 f x^{12} + \frac{3}{11} a b^2 x^{11} e + \frac{3}{10} a b^2 d x^{10} + \frac{1}{3} a b^2 c x^9 + \frac{3}{8} a^2 b f x^8 + \frac{3}{7} a^2 b d x^7 + \frac{3}{7} a^2 b e x^6 + \frac{3}{5} a^2 b c x^5 + \frac{1}{4} a^3 f x^4 + \frac{1}{3} a^3 x^3 e + \frac{1}{2} a^3 d x^2 + a^3 c x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="giac")

[Out] 1/16*b^3*f*x^16 + 1/15*b^3*x^15*e + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/4*a*b^2*f*x^12 + 3/11*a*b^2*x^11*e + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3/8*a^2*b*f*x^8 + 3/7*a^2*b*x^7*e + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*a^3*f*x^4 + 1/3*a^3*x^3*e + 1/2*a^3*d*x^2 + a^3*c*x

$$3.149 \quad \int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^2} dx$$

Optimal. Leaf size=155

$$\frac{(3\sqrt{bc} - \sqrt{ae}) \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}} + \frac{(\sqrt{ae} + 3\sqrt{bc}) \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} + \frac{af + bx(c + dx + ex^2)}{4ab(a - bx^4)}$$

[Out] (a*f + b*x*(c + d*x + e*x^2))/(4*a*b*(a - b*x^4)) + ((3*Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b])

Rubi [A] time = 0.118335, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1854, 1876, 275, 208, 1167, 205}

$$\frac{(3\sqrt{bc} - \sqrt{ae}) \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}} + \frac{(\sqrt{ae} + 3\sqrt{bc}) \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} + \frac{af + bx(c + dx + ex^2)}{4ab(a - bx^4)}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^2,x]

[Out] (a*f + b*x*(c + d*x + e*x^2))/(4*a*b*(a - b*x^4)) + ((3*Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b])

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}](a + b*x^n)^(p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1167

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[-(a*c)]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3}{(a - bx^4)^2} dx &= \frac{af + bx(c + dx + ex^2)}{4ab(a - bx^4)} - \frac{\int \frac{-3c - 2dx - ex^2}{a - bx^4} dx}{4a} \\
&= \frac{af + bx(c + dx + ex^2)}{4ab(a - bx^4)} - \frac{\int \left(-\frac{2dx}{a - bx^4} + \frac{-3c - ex^2}{a - bx^4} \right) dx}{4a} \\
&= \frac{af + bx(c + dx + ex^2)}{4ab(a - bx^4)} - \frac{\int \frac{-3c - ex^2}{a - bx^4} dx}{4a} + \frac{d \int \frac{x}{a - bx^4} dx}{2a} \\
&= \frac{af + bx(c + dx + ex^2)}{4ab(a - bx^4)} + \frac{d \text{Subst} \left(\int \frac{1}{a - bx^2} dx, x, x^2 \right)}{4a} - \frac{\left(\frac{3\sqrt{bc}}{\sqrt{a}} - e \right) \int \frac{1}{-\sqrt{a}\sqrt{b} - bx^2} dx}{8a} + \frac{(3\sqrt{bc} + \dots)}{4} \\
&= \frac{af + bx(c + dx + ex^2)}{4ab(a - bx^4)} + \frac{(3\sqrt{bc} - \sqrt{ae}) \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{8a^{7/4}b^{3/4}} + \frac{(3\sqrt{bc} + \sqrt{ae}) \tanh^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{8a^{7/4}b^{3/4}} + \frac{d \tan^{-1} \left(\frac{x}{\sqrt[4]{a}} \right)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.139363, size = 220, normalized size = 1.42

$$\frac{-\sqrt[4]{b} \log(\sqrt[4]{a} - \sqrt[4]{bx}) (a^{3/4}e + 3\sqrt[4]{a}\sqrt{bc} + 2\sqrt{a}\sqrt[4]{bd}) + \sqrt[4]{b} \log(\sqrt[4]{a} + \sqrt[4]{bx}) (a^{3/4}e + 3\sqrt[4]{a}\sqrt{bc} - 2\sqrt{a}\sqrt[4]{bd}) + \frac{4a(af + bx(c + dx + ex^2))}{a - bx^4}}{16a^2b}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^2,x]

[Out] ((4*a*(a*f + b*x*(c + x*(d + e*x)))/(a - b*x^4) - 2*a^(1/4)*b^(1/4)*(-3*sqrt[b]*c + sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)] - b^(1/4)*(3*a^(1/4)*sqrt[b]*c + 2*sqrt[a]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) - b^(1/4)*x] + b^(1/4)*(3*a^(1/4)*sqrt[b]*c - 2*sqrt[a]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x] + 2*sqrt[a]*sqrt[b]*d*Log[sqrt[a] + sqrt[b]*x^2])/(16*a^2*b)

Maple [B] time = 0.003, size = 248, normalized size = 1.6

$$-\frac{cx}{4a(bx^4 - a)} + \frac{3c}{16a^2} \sqrt[4]{\frac{a}{b}} \ln \left(\left(x + \sqrt[4]{\frac{a}{b}} \right) \left(x - \sqrt[4]{\frac{a}{b}} \right)^{-1} \right) + \frac{3c}{8a^2} \sqrt[4]{\frac{a}{b}} \arctan \left(x \frac{1}{\sqrt[4]{\frac{a}{b}}} \right) - \frac{dx^2}{4a(bx^4 - a)} - \frac{d}{8a} \ln \left(\left(-a + x^2 \sqrt{ab} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x)`

[Out]
$$-1/4*c*x/a/(b*x^4-a)+3/16*c/a^2*(1/b*a)^{1/4}*ln((x+(1/b*a)^{1/4})/(x-(1/b*a)^{1/4}))+3/8*c/a^2*(1/b*a)^{1/4}*arctan(x/(1/b*a)^{1/4})-1/4*d*x^2/a/(b*x^4-a)-1/8*d/a/(a*b)^{1/2}*ln((-a+x^2*(a*b)^{1/2})/(-a-x^2*(a*b)^{1/2}))-1/4*e*x^3/a/(b*x^4-a)-1/8*e/a/b/(1/b*a)^{1/4}*arctan(x/(1/b*a)^{1/4})+1/16*e/a/b/(1/b*a)^{1/4}*ln((x+(1/b*a)^{1/4})/(x-(1/b*a)^{1/4}))-1/4*f*x^4/a/(b*x^4-a)$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")`

[Out] Timed out

Sympy [B] time = 15.1272, size = 518, normalized size = 3.34

$$\text{RootSum}\left(65536t^4a^7b^3 + t^2(-3072a^4b^2ce - 2048a^4b^2d^2) + t(128a^3bde^2 + 1152a^2b^2c^2d) - a^2e^4 + 18abc^2e^2 - 48abcd^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)`

```
[Out] RootSum(65536*_t**4*a**7*b**3 + _t**2*(-3072*a**4*b**2*c*e - 2048*a**4*b**2
*d**2) + _t*(128*a**3*b*d*e**2 + 1152*a**2*b**2*c**2*d) - a**2*e**4 + 18*a*
b*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 - 81*b**2*c**4, Lambda(_t, _t*log(x + (4096*_t**3*a**7*b**2*e**3 + 36864*_t**3*a**6*b**3*c**2*e - 98304*_t
**3*a**6*b**3*c*d**2 + 4608*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5*b**2
*d**3*e - 13824*_t**2*a**4*b**3*c**3*d - 144*_t*a**4*b*c*e**4 - 192*_t*a**4
*b*d**2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*d**2*e
+ 1536*_t*a**3*b**2*c*d**4 - 3888*_t*a**2*b**3*c**5 + 6*a**3*d*e**5 - 120*a
**2*b*c*d**3*e**2 + 64*a**2*b*d**5*e + 810*a*b**2*c**4*d*e - 1080*a*b**2*c*
*3*d**3)/(a**3*e**6 + 9*a**2*b*c**2*e**4 - 96*a**2*b*c*d**2*e**3 + 64*a**2*
b*d**4*e**2 - 81*a*b**2*c**4*e**2 + 864*a*b**2*c**3*d**2*e - 576*a*b**2*c**
2*d**4 - 729*b**3*c**6)))) - (a*f + b*c*x + b*d*x**2 + b*e*x**3)/(-4*a**2*b
+ 4*a*b**2*x**4)
```

Giac [B] time = 1.0842, size = 452, normalized size = 2.92

$$-\frac{bx^3e + bdx^2 + bcx + af}{4(bx^4 - a)ab} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{-abb^2d} + 3(-ab^3)^{\frac{1}{4}}b^2c + (-ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2b^3} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{-abb^2d} + 3(-ab^3)^{\frac{1}{4}}b^2c + (-ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*(b*x^3*e + b*d*x^2 + b*c*x + a*f)/((b*x^4 - a)*a*b) + 1/16*sqrt(2)*(2*
sqrt(2)*sqrt(-a*b)*b^2*d + 3*(-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(3/4)*e)*arcta
n(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^3) + 1/16*s
qrt(2)*(2*sqrt(2)*sqrt(-a*b)*b^2*d + 3*(-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(3/4
)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^3
) + 1/32*sqrt(2)*(3*(-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(3/4)*e)*log(x^2 + sqrt
(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(-a*b^3)^(1/4)
*b^2*c - (-a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(
a^2*b^3)
```

$$3.150 \quad \int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^3} dx$$

Optimal. Leaf size=188

$$\frac{(21\sqrt{bc} - 5\sqrt{ae}) \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}} + \frac{(5\sqrt{ae} + 21\sqrt{bc}) \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}} + \frac{x(7c + 6dx + 5ex^2)}{32a^2(a - bx^4)} + \frac{3d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} + \frac{af + bx}{8ab}$$

[Out] $(x*(7*c + 6*d*x + 5*e*x^2))/(32*a^2*(a - b*x^4)) + (a*f + b*x*(c + d*x + e*x^2))/(8*a*b*(a - b*x^4)^2) + ((21*sqrt[b]*c - 5*sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(3/4)) + ((21*sqrt[b]*c + 5*sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(3/4)) + (3*d*ArcTanh[(sqrt[b]*x^2)/sqrt[a]])/(16*a^(5/2)*sqrt[b])$

Rubi [A] time = 0.154601, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1854, 1855, 1876, 275, 208, 1167, 205}

$$\frac{(21\sqrt{bc} - 5\sqrt{ae}) \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}} + \frac{(5\sqrt{ae} + 21\sqrt{bc}) \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}} + \frac{x(7c + 6dx + 5ex^2)}{32a^2(a - bx^4)} + \frac{3d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} + \frac{af + bx}{8ab}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^3, x]

[Out] $(x*(7*c + 6*d*x + 5*e*x^2))/(32*a^2*(a - b*x^4)) + (a*f + b*x*(c + d*x + e*x^2))/(8*a*b*(a - b*x^4)^2) + ((21*sqrt[b]*c - 5*sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(3/4)) + ((21*sqrt[b]*c + 5*sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(3/4)) + (3*d*ArcTanh[(sqrt[b]*x^2)/sqrt[a]])/(16*a^(5/2)*sqrt[b])$

Rule 1854

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]]*(a + b*x^n)^(p + 1), x], x] /; q == n - 1 /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,

0] && LtQ[p, -1]

Rule 1855

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]

Rule 1876

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1167

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[-(a*c)]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3}{(a - bx^4)^3} dx &= \frac{af + bx(c + dx + ex^2)}{8ab(a - bx^4)^2} - \frac{\int \frac{-7c - 6dx - 5ex^2}{(a - bx^4)^2} dx}{8a} \\
&= \frac{x(7c + 6dx + 5ex^2)}{32a^2(a - bx^4)} + \frac{af + bx(c + dx + ex^2)}{8ab(a - bx^4)^2} + \frac{\int \frac{21c + 12dx + 5ex^2}{a - bx^4} dx}{32a^2} \\
&= \frac{x(7c + 6dx + 5ex^2)}{32a^2(a - bx^4)} + \frac{af + bx(c + dx + ex^2)}{8ab(a - bx^4)^2} + \frac{\int \left(\frac{12dx}{a - bx^4} + \frac{21c + 5ex^2}{a - bx^4} \right) dx}{32a^2} \\
&= \frac{x(7c + 6dx + 5ex^2)}{32a^2(a - bx^4)} + \frac{af + bx(c + dx + ex^2)}{8ab(a - bx^4)^2} + \frac{\int \frac{21c + 5ex^2}{a - bx^4} dx}{32a^2} + \frac{(3d) \int \frac{x}{a - bx^4} dx}{8a^2} \\
&= \frac{x(7c + 6dx + 5ex^2)}{32a^2(a - bx^4)} + \frac{af + bx(c + dx + ex^2)}{8ab(a - bx^4)^2} + \frac{(3d) \text{Subst} \left(\int \frac{1}{a - bx^2} dx, x, x^2 \right)}{16a^2} - \frac{\left(\frac{21\sqrt{bc}}{\sqrt{a}} - 5 \right)}{16a^2} \\
&= \frac{x(7c + 6dx + 5ex^2)}{32a^2(a - bx^4)} + \frac{af + bx(c + dx + ex^2)}{8ab(a - bx^4)^2} + \frac{\left(\frac{21\sqrt{bc}}{\sqrt{a}} - 5e \right) \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{64a^{9/4}b^{3/4}} + \frac{(21\sqrt{bc} + 5\sqrt{a})}{64a^2}
\end{aligned}$$

Mathematica [A] time = 0.218945, size = 253, normalized size = 1.35

$$\frac{\frac{\log\left(\sqrt[4]{a} - \sqrt[4]{bx}\right)\left(5a^{3/4}e + 21\sqrt[4]{a}\sqrt{bc} + 12\sqrt{a}\sqrt[4]{bd}\right)}{b^{3/4}} + \frac{\log\left(\sqrt[4]{a} + \sqrt[4]{bx}\right)\left(5a^{3/4}e + 21\sqrt[4]{a}\sqrt{bc} - 12\sqrt{a}\sqrt[4]{bd}\right)}{b^{3/4}}}{128a^3} + \frac{16a^2(af + bx(c + x(dx + ex)))}{b(a - bx^4)^2} + \frac{2\sqrt[4]{a}(21\sqrt{bc} - 5\sqrt{a})\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{b^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^3, x]

[Out] ((4*a*x*(7*c + x*(6*d + 5*e*x)))/(a - b*x^4) + (16*a^2*(a*f + b*x*(c + x*(d + e*x))))/(b*(a - b*x^4)^2) + (2*a^(1/4)*(21*sqrt[b]*c - 5*sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)]/b^(3/4) - ((21*a^(1/4)*sqrt[b]*c + 12*sqrt[a]*b^(1/4)*d + 5*a^(3/4)*e)*Log[a^(1/4) - b^(1/4)*x])/b^(3/4) + ((21*a^(1/4)*sqrt[b]*c - 12*sqrt[a]*b^(1/4)*d + 5*a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4) + (12*sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(128*a^3)

Maple [B] time = 0.006, size = 326, normalized size = 1.7

$$\frac{cx}{8a(bx^4 - a)^2} - \frac{7cx}{32a^2(bx^4 - a)} + \frac{21c}{128a^3} \sqrt[4]{\frac{a}{b}} \ln\left(\left(x + \sqrt[4]{\frac{a}{b}}\right)\left(x - \sqrt[4]{\frac{a}{b}}\right)^{-1}\right) + \frac{21c}{64a^3} \sqrt[4]{\frac{a}{b}} \arctan\left(x \frac{1}{\sqrt[4]{\frac{a}{b}}}\right) + \frac{dx^2}{8a(bx^4 - a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x)

[Out] 1/8*c*x/a/(b*x^4-a)^2-7/32*c/a^2*x/(b*x^4-a)+21/128*c/a^3*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))+21/64*c/a^3*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))+1/8*d*x^2/a/(b*x^4-a)^2-3/16*d/a^2*x^2/(b*x^4-a)-3/32*d/a^2/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))+1/8*e*x^3/a/(b*x^4-a)^2-5/32*e/a^2*x^3/(b*x^4-a)-5/64*e/a^2/b/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))+5/128*e/a^2/b/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))+1/8*f*x^4/a/(b*x^4-a)^2-1/8*f/a^2*x^4/(b*x^4-a)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [B] time = 66.1839, size = 583, normalized size = 3.1

$$-\text{RootSum}\left(268435456t^4a^{11}b^3 + t^2(-6881280a^6b^2ce - 4718592a^6b^2d^2) + t(-153600a^4bde^2 - 2709504a^3b^2c^2d) - 62\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)

[Out] -RootSum(268435456*_t**4*a**11*b**3 + _t**2*(-6881280*a**6*b**2*c*e - 4718592*a**6*b**2*d**2) + _t*(-153600*a**4*b*d*e**2 - 2709504*a**3*b**2*c**2*d) - 625*a**2*e**4 + 22050*a*b*c**2*e**2 - 60480*a*b*c*d**2*e + 20736*a*b*d**4 - 194481*b**2*c**4, Lambda(_t, _t*log(x + (-262144000*_t**3*a**10*b**2*e**3 - 4624220160*_t**3*a**9*b**3*c**2*e + 12683575296*_t**3*a**9*b**3*c*d**2 + 309657600*_t**2*a**7*b**2*c*d*e**2 - 283115520*_t**2*a**7*b**2*d**3*e - 1820786688*_t**2*a**6*b**3*c**3*d + 5040000*_t*a**5*b*c*e**4 + 6912000*_t*a**5*b*d**2*e**3 + 118540800*_t*a**4*b**2*c**3*e**2 - 365783040*_t*a**4*b**2*c**2*d**2*e - 111476736*_t*a**4*b**2*c*d**4 + 522764928*_t*a**3*b**3*c**5 + 112500*a**3*d*e**5 - 4536000*a**2*b*c*d**3*e**2 + 2488320*a**2*b*d**5*e + 58344300*a*b**2*c**4*d*e - 80015040*a*b**2*c**3*d**3)/(15625*a**3*e**6 + 275625*a**2*b*c**2*e**4 - 3024000*a**2*b*c*d**2*e**3 + 2073600*a**2*b*d**4*e**2 - 4862025*a*b**2*c**4*e**2 + 53343360*a*b**2*c**3*d**2*e - 36578304*a*b**2*c**2*d**4 - 85766121*b**3*c**6)))) - (-4*a**2*f - 11*a*b*c*x - 10*a*b*d*x**2 - 9*a*b*e*x**3 + 7*b**2*c*x**5 + 6*b**2*d*x**6 + 5*b**2*e*x**7)/(32*a**4*b - 64*a**3*b**2*x**4 + 32*a**2*b**3*x**8)

Giac [B] time = 1.08412, size = 505, normalized size = 2.69

$$\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{-abb^2d} - 21(-ab^3)^{\frac{1}{4}}b^2c - 5(-ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128a^3b^3} - \frac{\sqrt{2}\left(12\sqrt{2}\sqrt{-abb^2d} - 21(-ab^3)^{\frac{1}{4}}b^2c\right)}{128a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")

[Out] -1/128*sqrt(2)*(12*sqrt(2)*sqrt(-a*b)*b^2*d - 21*(-a*b^3)^(1/4)*b^2*c - 5*(-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^3) - 1/128*sqrt(2)*(12*sqrt(2)*sqrt(-a*b)*b^2*d - 21*(-a*b^3)^(1/4)*b^2*c - 5*(-a*b^3)^(3/4)*e)

$$\begin{aligned} & /4)*b^2*c - 5*(-a*b^3)^{(3/4)*e}*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^{(1/4)))/(-a/b)^{(1/4)))/(a^3*b^3) + 1/256*sqrt(2)*(21*(-a*b^3)^{(1/4)*b^2*c - 5*(-a*b^3)^{(3/4)*e}*log(x^2 + sqrt(2)*x*(-a/b)^{(1/4) + sqrt(-a/b)))/(a^3*b^3) - \\ & 1/256*sqrt(2)*(21*(-a*b^3)^{(1/4)*b^2*c - 5*(-a*b^3)^{(3/4)*e}*log(x^2 - sqrt(2)*x*(-a/b)^{(1/4) + sqrt(-a/b)))/(a^3*b^3) - 1/32*(5*b^2*x^7*e + 6*b^2*d*x^6 + 7*b^2*c*x^5 - 9*a*b*x^3*e - 10*a*b*d*x^2 - 11*a*b*c*x - 4*a^2*f)/((b*x^4 - a)^2*a^2*b) \end{aligned}$$

$$3.151 \quad \int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4} dx$$

Optimal. Leaf size=220

$$\frac{(77\sqrt{bc} - 15\sqrt{ae}) \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}} + \frac{(15\sqrt{ae} + 77\sqrt{bc}) \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}} + \frac{x(11c + 10dx + 9ex^2)}{96a^2(a - bx^4)^2} + \frac{x(77c + 60dx + 45ex^2)}{384a^3(a - bx^4)}$$

[Out] (x*(11*c + 10*d*x + 9*e*x^2))/(96*a^2*(a - b*x^4)^2) + (x*(77*c + 60*d*x + 45*e*x^2))/(384*a^3*(a - b*x^4)) + (a*f + b*x*(c + d*x + e*x^2))/(12*a*b*(a - b*x^4)^3) + ((77*sqrt[b]*c - 15*sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(3/4)) + ((77*sqrt[b]*c + 15*sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(3/4)) + (5*d*ArcTanh[(sqrt[b]*x^2)/sqrt[a]])/(32*a^(7/2)*sqrt[b])

Rubi [A] time = 0.188727, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1854, 1855, 1876, 275, 208, 1167, 205}

$$\frac{(77\sqrt{bc} - 15\sqrt{ae}) \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}} + \frac{(15\sqrt{ae} + 77\sqrt{bc}) \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}} + \frac{x(11c + 10dx + 9ex^2)}{96a^2(a - bx^4)^2} + \frac{x(77c + 60dx + 45ex^2)}{384a^3(a - bx^4)}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^4, x]

[Out] (x*(11*c + 10*d*x + 9*e*x^2))/(96*a^2*(a - b*x^4)^2) + (x*(77*c + 60*d*x + 45*e*x^2))/(384*a^3*(a - b*x^4)) + (a*f + b*x*(c + d*x + e*x^2))/(12*a*b*(a - b*x^4)^3) + ((77*sqrt[b]*c - 15*sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(3/4)) + ((77*sqrt[b]*c + 15*sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(3/4)) + (5*d*ArcTanh[(sqrt[b]*x^2)/sqrt[a]])/(32*a^(7/2)*sqrt[b])

Rule 1854

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq, x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int

$$\text{Int}[\text{Sum}[(n*(p + 1) + i + 1)*\text{Coeff}[\text{Pq}, x, i]*x^i, \{i, 0, q - 1\}](a + b*x^n)^{(p + 1)}, x], x] /; q == n - 1 /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[\text{Pq}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[p, -1]$$

Rule 1855

$$\text{Int}[(\text{Pq}_-)((a_-) + (b_-)*(x_-)^{(n_-}))^{(p_-)}, x_Symbol] := -\text{Simp}[(x*\text{Pq}*(a + b*x^n)^{(p + 1)})/(a*n*(p + 1)), x] + \text{Dist}[1/(a*n*(p + 1)), \text{Int}[\text{ExpandToSum}[n*(p + 1)*\text{Pq} + D[x*\text{Pq}, x], x](a + b*x^n)^{(p + 1)}, x], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[\text{Pq}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[p, -1] \&\& \text{LtQ}[\text{Expon}[\text{Pq}, x], n - 1]$$

Rule 1876

$$\text{Int}[(\text{Pq}_-)/((a_-) + (b_-)*(x_-)^{(n_-)}), x_Symbol] := \text{With}[\{v = \text{Sum}[(x^{ii}*(\text{Coeff}[\text{Pq}, x, ii] + \text{Coeff}[\text{Pq}, x, n/2 + ii]*x^{(n/2)}))/(a + b*x^n), \{ii, 0, n/2 - 1\}]\}, \text{Int}[v, x] /; \text{SumQ}[v]] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[\text{Pq}, x] \&\& \text{IGtQ}[n/2, 0] \&\& \text{Expon}[\text{Pq}, x] < n$$

Rule 275

$$\text{Int}[(x_-)^{(m_-)}*((a_-) + (b_-)*(x_-)^{(n_-}))^{(p_-)}, x_Symbol] := \text{With}[\{k = \text{GCD}[m + 1, n]\}, \text{Dist}[1/k, \text{Subst}[\text{Int}[x^{((m + 1)/k - 1)}*(a + b*x^{(n/k)})^p], x], x, x^k], x] /; k != 1 /; \text{FreeQ}[\{a, b, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[m]$$

Rule 208

$$\text{Int}[(a_-) + (b_-)*(x_-)^2)^{-1}, x_Symbol] := \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{NegQ}[a/b]$$

Rule 1167

$$\text{Int}[(d_-) + (e_-)*(x_-)^2)/((a_-) + (c_-)*(x_-)^4), x_Symbol] := \text{With}[\{q = \text{Rt}[-(a*c), 2]\}, \text{Dist}[e/2 + (c*d)/(2*q), \text{Int}[1/(-q + c*x^2), x], x] + \text{Dist}[e/2 - (c*d)/(2*q), \text{Int}[1/(q + c*x^2), x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{NeQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[-(a*c)]$$

Rule 205

$$\text{Int}[(a_-) + (b_-)*(x_-)^2)^{-1}, x_Symbol] := \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[a/b]$$

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3}{(a - bx^4)^4} dx &= \frac{af + bx(c + dx + ex^2)}{12ab(a - bx^4)^3} - \frac{\int \frac{-11c - 10dx - 9ex^2}{(a - bx^4)^3} dx}{12a} \\
&= \frac{x(11c + 10dx + 9ex^2)}{96a^2(a - bx^4)^2} + \frac{af + bx(c + dx + ex^2)}{12ab(a - bx^4)^3} + \frac{\int \frac{77c + 60dx + 45ex^2}{(a - bx^4)^2} dx}{96a^2} \\
&= \frac{x(11c + 10dx + 9ex^2)}{96a^2(a - bx^4)^2} + \frac{x(77c + 60dx + 45ex^2)}{384a^3(a - bx^4)} + \frac{af + bx(c + dx + ex^2)}{12ab(a - bx^4)^3} - \frac{\int \frac{-231c - 120dx - 9ex^2}{(a - bx^4)} dx}{384a^3} \\
&= \frac{x(11c + 10dx + 9ex^2)}{96a^2(a - bx^4)^2} + \frac{x(77c + 60dx + 45ex^2)}{384a^3(a - bx^4)} + \frac{af + bx(c + dx + ex^2)}{12ab(a - bx^4)^3} - \frac{\int \left(\frac{120dx}{a - bx^4} + \frac{-231c}{a - bx^4} \right) dx}{384a^3} \\
&= \frac{x(11c + 10dx + 9ex^2)}{96a^2(a - bx^4)^2} + \frac{x(77c + 60dx + 45ex^2)}{384a^3(a - bx^4)} + \frac{af + bx(c + dx + ex^2)}{12ab(a - bx^4)^3} - \frac{\int \frac{-231c - 45ex^2}{a - bx^4} dx}{384a^3} \\
&= \frac{x(11c + 10dx + 9ex^2)}{96a^2(a - bx^4)^2} + \frac{x(77c + 60dx + 45ex^2)}{384a^3(a - bx^4)} + \frac{af + bx(c + dx + ex^2)}{12ab(a - bx^4)^3} + \frac{(5d) \text{Subst} \left(\int \frac{120dx}{a - bx^4} \right)}{384a^3} \\
&= \frac{x(11c + 10dx + 9ex^2)}{96a^2(a - bx^4)^2} + \frac{x(77c + 60dx + 45ex^2)}{384a^3(a - bx^4)} + \frac{af + bx(c + dx + ex^2)}{12ab(a - bx^4)^3} + \frac{(77\sqrt{bc} - 15\sqrt{a})}{256a^3}
\end{aligned}$$

Mathematica [A] time = 0.307724, size = 286, normalized size = 1.3

$$\frac{3 \log\left(\sqrt[4]{a} - \sqrt[4]{bx}\right)\left(15a^{3/4}e + 77\sqrt[4]{a}\sqrt{bc} + 40\sqrt{a}\sqrt[4]{bd}\right)}{b^{3/4}} + \frac{3 \log\left(\sqrt[4]{a} + \sqrt[4]{bx}\right)\left(15a^{3/4}e + 77\sqrt[4]{a}\sqrt{bc} - 40\sqrt{a}\sqrt[4]{bd}\right)}{b^{3/4}} - \frac{128a^3(af + bx(c + x(d + ex)))}{b(bx^4 - a)^3} + \frac{16a^2x(11c + x(10d + 9e))}{(a - bx^4)^2}$$

1536a⁴

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^4, x]

[Out] ((4*a*x*(77*c + 15*x*(4*d + 3*e*x)))/(a - b*x^4) + (16*a^2*x*(11*c + x*(10*d + 9*e*x)))/(a - b*x^4)^2 - (128*a^3*(a*f + b*x*(c + x*(d + e*x))))/(b*(a + b*x^4)^3) + (6*a^(1/4)*(77*sqrt[b]*c - 15*sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)]/b^(3/4) - (3*(77*a^(1/4)*sqrt[b]*c + 40*sqrt[a]*b^(1/4)*d + 15*a^(3/4)*e)*Log[a^(1/4) - b^(1/4)*x])/b^(3/4) + (3*(77*a^(1/4)*sqrt[b]*c - 40*

$\text{Sqrt}[a] * b^{(1/4)} * d + 15 * a^{(3/4)} * e * \text{Log}[a^{(1/4)} + b^{(1/4)} * x] / b^{(3/4)} + (120 * \text{Sqrt}[a] * d * \text{Log}[\text{Sqrt}[a] + \text{Sqrt}[b] * x^2]) / \text{Sqrt}[b] / (1536 * a^4)$

Maple [A] time = 0.012, size = 280, normalized size = 1.3

$$\frac{1}{(bx^4 - a)^3} \left(-\frac{15b^2ex^{11}}{128a^3} - \frac{5b^2dx^{10}}{32a^3} - \frac{77b^2cx^9}{384a^3} + \frac{21bex^7}{64a^2} + \frac{5bdx^6}{12a^2} + \frac{33bcx^5}{64a^2} - \frac{113ex^3}{384a} - \frac{11dx^2}{32a} - \frac{51cx}{128a} - \frac{f}{12b} \right) + \frac{77c}{512a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x)`

[Out] $(-15/128 * e / a^3 * b^2 * x^{11} - 5/32 * d / a^3 * b^2 * x^{10} - 77/384 * c / a^3 * b^2 * x^9 + 21/64 / a^2 * b * e * x^7 + 5/12 / a^2 * d * b * x^6 + 33/64 / a^2 * c * b * x^5 - 113/384 / a * e * x^3 - 11/32 * d / a * x^2 - 51/128 * c / a * x - 1/12 * f / b) / (b * x^4 - a)^3 + 77/512 / a^4 * c * (1/b * a)^{(1/4)} * \ln((x + (1/b * a)^{(1/4)}) / (x - (1/b * a)^{(1/4)})) + 77/256 / a^4 * c * (1/b * a)^{(1/4)} * \arctan(x / (1/b * a)^{(1/4)}) - 5/64 / a^3 * d / (a * b)^{(1/2)} * \ln((-a + x^2 * (a * b)^{(1/2)}) / (-a - x^2 * (a * b)^{(1/2)})) - 15/256 / a^3 * e / b / (1/b * a)^{(1/4)} * \arctan(x / (1/b * a)^{(1/4)}) + 15/512 / a^3 * e / b / (1/b * a)^{(1/4)} * \ln((x + (1/b * a)^{(1/4)}) / (x - (1/b * a)^{(1/4)}))$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)

[Out] Timed out

Giac [B] time = 1.08736, size = 555, normalized size = 2.52

$$\frac{\sqrt{2} \left(40 \sqrt{2} \sqrt{-abb^2d} + 77 (-ab^3)^{\frac{1}{4}} b^2c + 15 (-ab^3)^{\frac{3}{4}} e \right) \arctan \left(\frac{\sqrt{2} \left(2x + \sqrt{2} \left(\frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left(\frac{a}{b} \right)^{\frac{1}{4}}} \right)}{512 a^4 b^3} + \frac{\sqrt{2} \left(40 \sqrt{2} \sqrt{-abb^2d} + 77 (-ab^3)^{\frac{1}{4}} b^2c + 15 (-ab^3)^{\frac{3}{4}} e \right) \arctan \left(\frac{\sqrt{2} \left(2x - \sqrt{2} \left(\frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left(\frac{a}{b} \right)^{\frac{1}{4}}} \right)}{512 a^4 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")

[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b^2*d + 77*(-a*b^3)^(1/4)*b^2*c + 15*(-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b^2*d + 77*(-a*b^3)^(1/4)*b^2*c + 15*(-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(-a*b^3)^(1/4)*b^2*c - 15*(-a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^4*b^3) - 1/1024*sqrt(2)*(77*(-a*b^3)^(1/4)*b^2*c - 15*(-a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^4*b^3) - 1/384*(45*b^3*x^11*e + 60*b^3*d*x^10 + 77*b^3*c*x^9 - 126*a*b^2*x^7*e - 160*a*b^2*d*x^6 - 198*a*b^2*c*x^5 + 113*a^2*b*x^3*e + 132*a^2*b*d*x^2 + 153*a^2*b*c*x + 32*a^3*f)/((b*x^4 - a)^3*a^3*b)

$$3.152 \quad \int \frac{a}{2+3x^4} dx$$

Optimal. Leaf size=101

$$-\frac{a \log(3x^2 - 6^{3/4}x + \sqrt{6})}{8\sqrt[4]{6}} + \frac{a \log(3x^2 + 6^{3/4}x + \sqrt{6})}{8\sqrt[4]{6}} - \frac{a \tan^{-1}(1 - \sqrt[4]{6}x)}{4\sqrt[4]{6}} + \frac{a \tan^{-1}(\sqrt[4]{6}x + 1)}{4\sqrt[4]{6}}$$

[Out] $-(a*\text{ArcTan}[1 - 6^{(1/4)}*x])/(4*6^{(1/4)}) + (a*\text{ArcTan}[1 + 6^{(1/4)}*x])/(4*6^{(1/4)}) - (a*\text{Log}[\text{Sqrt}[6] - 6^{(3/4)}*x + 3*x^2])/(8*6^{(1/4)}) + (a*\text{Log}[\text{Sqrt}[6] + 6^{(3/4)}*x + 3*x^2])/(8*6^{(1/4)})$

Rubi [A] time = 0.0960809, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {12, 211, 1165, 628, 1162, 617, 204}

$$-\frac{a \log(3x^2 - 6^{3/4}x + \sqrt{6})}{8\sqrt[4]{6}} + \frac{a \log(3x^2 + 6^{3/4}x + \sqrt{6})}{8\sqrt[4]{6}} - \frac{a \tan^{-1}(1 - \sqrt[4]{6}x)}{4\sqrt[4]{6}} + \frac{a \tan^{-1}(\sqrt[4]{6}x + 1)}{4\sqrt[4]{6}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[a/(2 + 3*x^4), x]$

[Out] $-(a*\text{ArcTan}[1 - 6^{(1/4)}*x])/(4*6^{(1/4)}) + (a*\text{ArcTan}[1 + 6^{(1/4)}*x])/(4*6^{(1/4)}) - (a*\text{Log}[\text{Sqrt}[6] - 6^{(3/4)}*x + 3*x^2])/(8*6^{(1/4)}) + (a*\text{Log}[\text{Sqrt}[6] + 6^{(3/4)}*x + 3*x^2])/(8*6^{(1/4)})$

Rule 12

$\text{Int}[(a_*)*(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)*(v_)] /; \text{FreeQ}[b, x]$

Rule 211

$\text{Int}[((a_*) + (b_*)*(x_)^4)^{-1}, x_Symbol] \rightarrow \text{With}[\{r = \text{Numerator}[\text{Rt}[a/b, 2]], s = \text{Denominator}[\text{Rt}[a/b, 2]]\}, \text{Dist}[1/(2*r), \text{Int}[(r - s*x^2)/(a + b*x^4), x], x] + \text{Dist}[1/(2*r), \text{Int}[(r + s*x^2)/(a + b*x^4), x], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ (\text{GtQ}[a/b, 0] \ || \ (\text{PosQ}[a/b] \ \&\& \ \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, a]] \ \&\& \ \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, b]]))$

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Free
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a}{2+3x^4} dx &= a \int \frac{1}{2+3x^4} dx \\
&= \frac{a \int \frac{\sqrt{2}-\sqrt{3}x^2}{2+3x^4} dx}{2\sqrt{2}} + \frac{a \int \frac{\sqrt{2}+\sqrt{3}x^2}{2+3x^4} dx}{2\sqrt{2}} \\
&= \frac{a \int \frac{1}{\sqrt{\frac{2}{3}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}} dx}{4\sqrt{6}} + \frac{a \int \frac{1}{\sqrt{\frac{2}{3}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}} dx}{4\sqrt{6}} - \frac{a \int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}} dx}{8\sqrt{6}} - \frac{a \int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}} dx}{8\sqrt{6}} \\
&= -\frac{a \log(\sqrt{6}-6^{3/4}x+3x^2)}{8\sqrt{6}} + \frac{a \log(\sqrt{6}+6^{3/4}x+3x^2)}{8\sqrt{6}} + \frac{a \text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1-\sqrt[4]{6}x\right)}{4\sqrt{6}} - \frac{a \text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1+\sqrt[4]{6}x\right)}{4\sqrt{6}} \\
&= -\frac{a \tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt{6}} + \frac{a \tan^{-1}\left(1+\sqrt[4]{6}x\right)}{4\sqrt{6}} - \frac{a \log(\sqrt{6}-6^{3/4}x+3x^2)}{8\sqrt{6}} + \frac{a \log(\sqrt{6}+6^{3/4}x+3x^2)}{8\sqrt{6}}
\end{aligned}$$

Mathematica [A] time = 0.0290976, size = 78, normalized size = 0.77

$$\frac{a\left(-\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)+\log\left(\sqrt{6}x^2+2\sqrt[4]{6}x+2\right)-2\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\tan^{-1}\left(\sqrt[4]{6}x+1\right)\right)}{8\sqrt[4]{6}}$$

Antiderivative was successfully verified.

[In] Integrate[a/(2 + 3*x^4), x]

[Out] (a*(-2*ArcTan[1 - 6^(1/4)*x] + 2*ArcTan[1 + 6^(1/4)*x] - Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] + Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))/(8*6^(1/4))

Maple [A] time = 0.043, size = 114, normalized size = 1.1

$$\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24} \arctan\left(\frac{\sqrt{2}\sqrt{3}6^{3/4}x}{6}-1\right) + \frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{48} \ln\left(\left(x^2 + \frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3} + \frac{\sqrt{6}}{3}\right)\left(x^2 - \frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3} + \frac{\sqrt{6}}{3}\right)^{-1}\right) + \frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a/(3*x^4+2), x)

[Out] 1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/48*a*3^(1/2)*6^(1/4)*2^(1/2)*ln((x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)

))/((x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))+1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)

Maxima [A] time = 1.477, size = 166, normalized size = 1.64

$$\frac{1}{48} \left(2 \cdot 3^{\frac{3}{4}} 2^{\frac{3}{4}} \arctan \left(\frac{1}{6} \cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}} \left(2 \sqrt{3} x + 3^{\frac{1}{4}} 2^{\frac{3}{4}} \right) \right) + 2 \cdot 3^{\frac{3}{4}} 2^{\frac{3}{4}} \arctan \left(\frac{1}{6} \cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}} \left(2 \sqrt{3} x - 3^{\frac{1}{4}} 2^{\frac{3}{4}} \right) \right) + 3^{\frac{3}{4}} 2^{\frac{3}{4}} \log \left(\sqrt{3} x^2 + 3^{\frac{1}{4}} 2^{\frac{3}{4}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a/(3*x^4+2),x, algorithm="maxima")

[Out] 1/48*(2*3^(3/4)*2^(3/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 2*3^(3/4)*2^(3/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4))) + 3^(3/4)*2^(3/4)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) - 3^(3/4)*2^(3/4)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)))*a

Fricas [B] time = 1.66575, size = 882, normalized size = 8.73

$$-\frac{1}{48} \cdot 24^{\frac{3}{4}} \sqrt{2} (a^4)^{\frac{1}{4}} \arctan \left(\frac{4a^3 + 2 \cdot 24^{\frac{1}{4}} \sqrt{2} (a^4)^{\frac{3}{4}} x - 24^{\frac{1}{4}} \sqrt{2} \sqrt{\frac{1}{3}} (a^4)^{\frac{3}{4}} \sqrt{\frac{12a^2x^2 + 24^{\frac{3}{4}} \sqrt{2} (a^4)^{\frac{1}{4}} ax + 4\sqrt{6}\sqrt{a^4}}{a^2}}}{4a^3} \right) - \frac{1}{48} \cdot 24^{\frac{3}{4}} \sqrt{2} (a^4)^{\frac{1}{4}} \arctan \left(\frac{4a^3 - 2 \cdot 24^{\frac{1}{4}} \sqrt{2} (a^4)^{\frac{3}{4}} x + 24^{\frac{1}{4}} \sqrt{2} \sqrt{\frac{1}{3}} (a^4)^{\frac{3}{4}} \sqrt{\frac{12a^2x^2 + 24^{\frac{3}{4}} \sqrt{2} (a^4)^{\frac{1}{4}} ax + 4\sqrt{6}\sqrt{a^4}}{a^2}}}{4a^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a/(3*x^4+2),x, algorithm="fricas")

[Out] -1/48*24^(3/4)*sqrt(2)*(a^4)^(1/4)*arctan(-1/4*(4*a^3 + 2*24^(1/4)*sqrt(2)*(a^4)^(3/4)*x - 24^(1/4)*sqrt(2)*sqrt(1/3)*(a^4)^(3/4)*sqrt((12*a^2*x^2 + 24^(3/4)*sqrt(2)*(a^4)^(1/4)*a*x + 4*sqrt(6)*sqrt(a^4))/a^2))/a^3 - 1/48*24^(3/4)*sqrt(2)*(a^4)^(1/4)*arctan(1/4*(4*a^3 - 2*24^(1/4)*sqrt(2)*(a^4)^(3/4)*x + 24^(1/4)*sqrt(2)*sqrt(1/3)*(a^4)^(3/4)*sqrt((12*a^2*x^2 - 24^(3/4)*sqrt(2)*(a^4)^(1/4)*a*x + 4*sqrt(6)*sqrt(a^4))/a^2))/a^3 + 1/192*24^(3/4)*sqrt(2)*(a^4)^(1/4)*log(12*a^2*x^2 + 24^(3/4)*sqrt(2)*(a^4)^(1/4)*a*x + 4*sqrt(6)*sqrt(a^4)) - 1/192*24^(3/4)*sqrt(2)*(a^4)^(1/4)*log(12*a^2*x^2 - 24^(3/4)*sqrt(2)*(a^4)^(1/4)*a*x + 4*sqrt(6)*sqrt(a^4))

Sympy [A] time = 0.359027, size = 88, normalized size = 0.87

$$a \left(-\frac{6^{\frac{3}{4}} \log\left(x^2 - \frac{6^{\frac{3}{4}}x}{3} + \frac{\sqrt{6}}{3}\right)}{48} + \frac{6^{\frac{3}{4}} \log\left(x^2 + \frac{6^{\frac{3}{4}}x}{3} + \frac{\sqrt{6}}{3}\right)}{48} + \frac{6^{\frac{3}{4}} \operatorname{atan}\left(\sqrt[4]{6}x - 1\right)}{24} + \frac{6^{\frac{3}{4}} \operatorname{atan}\left(\sqrt[4]{6}x + 1\right)}{24} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a/(3*x**4+2),x)

[Out] a*(-6**(3/4)*log(x**2 - 6**(3/4)*x/3 + sqrt(6)/3)/48 + 6**(3/4)*log(x**2 + 6**(3/4)*x/3 + sqrt(6)/3)/48 + 6**(3/4)*atan(6**(1/4)*x - 1)/24 + 6**(3/4)*atan(6**(1/4)*x + 1)/24)

Giac [A] time = 1.11121, size = 131, normalized size = 1.3

$$\frac{1}{48} \left(2 \cdot 6^{\frac{3}{4}} \arctan\left(\frac{3}{4} \sqrt{2} \left(\frac{2}{3}\right)^{\frac{3}{4}} \left(2x + \sqrt{2} \left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + 2 \cdot 6^{\frac{3}{4}} \arctan\left(\frac{3}{4} \sqrt{2} \left(\frac{2}{3}\right)^{\frac{3}{4}} \left(2x - \sqrt{2} \left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + 6^{\frac{3}{4}} \log\left(x^2 + \sqrt{2} \left(\frac{2}{3}\right)^{\frac{1}{4}} x + \sqrt{2/3}\right) - 6^{\frac{3}{4}} \log\left(x^2 - \sqrt{2} \left(\frac{2}{3}\right)^{\frac{1}{4}} x + \sqrt{2/3}\right) \right) a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a/(3*x^4+2),x, algorithm="giac")

[Out] 1/48*(2*6^(3/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))) + 2*6^(3/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 6^(3/4)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 6^(3/4)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)))*a

$$3.153 \quad \int \frac{bx}{2+3x^4} dx$$

Optimal. Leaf size=22

$$\frac{b \tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{6}}$$

[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6])

Rubi [A] time = 0.0130876, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {12, 275, 203}

$$\frac{b \tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[(b*x)/(2 + 3*x^4),x]

[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{bx}{2+3x^4} dx &= b \int \frac{x}{2+3x^4} dx \\ &= \frac{1}{2} b \text{Subst} \left(\int \frac{1}{2+3x^2} dx, x, x^2 \right) \\ &= \frac{b \tan^{-1} \left(\sqrt{\frac{3}{2}} x^2 \right)}{2\sqrt{6}} \end{aligned}$$

Mathematica [A] time = 0.0090014, size = 22, normalized size = 1.

$$\frac{b \tan^{-1} \left(\sqrt{\frac{3}{2}} x^2 \right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x)/(2 + 3*x^4), x]

[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6])

Maple [A] time = 0.04, size = 16, normalized size = 0.7

$$\frac{b\sqrt{6}}{12} \arctan\left(\frac{x^2\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(b*x/(3*x^4+2), x)

[Out] 1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)

Maxima [A] time = 1.42433, size = 20, normalized size = 0.91

$$\frac{1}{12} \sqrt{6} b \arctan\left(\frac{1}{2} \sqrt{6} x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b*x/(3*x^4+2),x, algorithm="maxima")

[Out] 1/12*sqrt(6)*b*arctan(1/2*sqrt(6)*x^2)

Fricas [A] time = 1.49792, size = 54, normalized size = 2.45

$$\frac{1}{12} \sqrt{6} b \arctan\left(\frac{1}{2} \sqrt{6} x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b*x/(3*x^4+2),x, algorithm="fricas")

[Out] 1/12*sqrt(6)*b*arctan(1/2*sqrt(6)*x^2)

Sympy [A] time = 0.089657, size = 19, normalized size = 0.86

$$\frac{\sqrt{6} b \operatorname{atan}\left(\frac{\sqrt{6} x^2}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b*x/(3*x**4+2),x)

[Out] sqrt(6)*b*atan(sqrt(6)*x**2/2)/12

Giac [A] time = 1.08175, size = 20, normalized size = 0.91

$$\frac{1}{12} \sqrt{6} b \arctan\left(\frac{1}{2} \sqrt{6} x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b*x/(3*x^4+2),x, algorithm="giac")

```
[Out] 1/12*sqrt(6)*b*arctan(1/2*sqrt(6)*x^2)
```

$$3.154 \quad \int \frac{a+bx}{2+3x^4} dx$$

Optimal. Leaf size=123

$$-\frac{a \log(3x^2 - 6^{3/4}x + \sqrt{6})}{8\sqrt[4]{6}} + \frac{a \log(3x^2 + 6^{3/4}x + \sqrt{6})}{8\sqrt[4]{6}} - \frac{a \tan^{-1}(1 - \sqrt[4]{6}x)}{4\sqrt[4]{6}} + \frac{a \tan^{-1}(\sqrt[4]{6}x + 1)}{4\sqrt[4]{6}} + \frac{b \tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}$$

[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (a*ArcTan[1 - 6^(1/4)*x])/(4*6^(1/4)) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/4)) - (a*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(1/4))

Rubi [A] time = 0.102932, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$, Rules used = {1876, 211, 1165, 628, 1162, 617, 204, 275, 203}

$$-\frac{a \log(3x^2 - 6^{3/4}x + \sqrt{6})}{8\sqrt[4]{6}} + \frac{a \log(3x^2 + 6^{3/4}x + \sqrt{6})}{8\sqrt[4]{6}} - \frac{a \tan^{-1}(1 - \sqrt[4]{6}x)}{4\sqrt[4]{6}} + \frac{a \tan^{-1}(\sqrt[4]{6}x + 1)}{4\sqrt[4]{6}} + \frac{b \tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/(2 + 3*x^4), x]

[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (a*ArcTan[1 - 6^(1/4)*x])/(4*6^(1/4)) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/4)) - (a*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(1/4))

Rule 1876

Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 211

Int[((a_) + (b_)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&

AtomQ[SplitProduct[SumBaseQ, b]])

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 275

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1 /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a

, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{a+bx}{2+3x^4} dx &= \int \left(\frac{a}{2+3x^4} + \frac{bx}{2+3x^4} \right) dx \\
 &= a \int \frac{1}{2+3x^4} dx + b \int \frac{x}{2+3x^4} dx \\
 &= \frac{a \int \frac{\sqrt{2}-\sqrt{3}x^2}{2+3x^4} dx}{2\sqrt{2}} + \frac{a \int \frac{\sqrt{2}+\sqrt{3}x^2}{2+3x^4} dx}{2\sqrt{2}} + \frac{1}{2} b \operatorname{Subst} \left(\int \frac{1}{2+3x^2} dx, x, x^2 \right) \\
 &= \frac{b \tan^{-1} \left(\sqrt{\frac{3}{2}} x^2 \right)}{2\sqrt{6}} + \frac{a \int \frac{1}{\sqrt{\frac{2}{3}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}} dx}{4\sqrt{6}} + \frac{a \int \frac{1}{\sqrt{\frac{2}{3}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}} dx}{4\sqrt{6}} - \frac{a \int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}} dx}{8\sqrt[4]{6}} - \frac{a \int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}} dx}{8\sqrt[4]{6}} \\
 &= \frac{b \tan^{-1} \left(\sqrt{\frac{3}{2}} x^2 \right)}{2\sqrt{6}} - \frac{a \log \left(\sqrt{6} - 6^{3/4}x + 3x^2 \right)}{8\sqrt[4]{6}} + \frac{a \log \left(\sqrt{6} + 6^{3/4}x + 3x^2 \right)}{8\sqrt[4]{6}} + \frac{a \operatorname{Subst} \left(\int \frac{1}{-1-x^2} dx, x, 1-x^2 \right)}{4\sqrt[4]{6}} \\
 &= \frac{b \tan^{-1} \left(\sqrt{\frac{3}{2}} x^2 \right)}{2\sqrt{6}} - \frac{a \tan^{-1} \left(1 - \sqrt[4]{6}x \right)}{4\sqrt[4]{6}} + \frac{a \tan^{-1} \left(1 + \sqrt[4]{6}x \right)}{4\sqrt[4]{6}} - \frac{a \log \left(\sqrt{6} - 6^{3/4}x + 3x^2 \right)}{8\sqrt[4]{6}} + \frac{a \log \left(\sqrt{6} + 6^{3/4}x + 3x^2 \right)}{8\sqrt[4]{6}}
 \end{aligned}$$

Mathematica [A] time = 0.0492228, size = 107, normalized size = 0.87

$$\frac{-2 \left(\sqrt[4]{6}a + 2b \right) \tan^{-1} \left(1 - \sqrt[4]{6}x \right) + 2 \left(\sqrt[4]{6}a - 2b \right) \tan^{-1} \left(\sqrt[4]{6}x + 1 \right) + \sqrt[4]{6}a \left(\log \left(\sqrt{6}x^2 + 2\sqrt[4]{6}x + 2 \right) - \log \left(\sqrt{6}x^2 - 2\sqrt[4]{6}x + 2 \right) \right)}{8\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)/(2 + 3*x^4), x]

[Out] (-2*(6^(1/4)*a + 2*b)*ArcTan[1 - 6^(1/4)*x] + 2*(6^(1/4)*a - 2*b)*ArcTan[1 + 6^(1/4)*x] + 6^(1/4)*a*(-Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] + Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))/(8*Sqrt[6])

Maple [A] time = 0.003, size = 129, normalized size = 1.1

$$\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24} \arctan \left(\frac{\sqrt{2}\sqrt{36^{\frac{3}{4}}x}}{6} - 1 \right) + \frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{48} \ln \left(\left(x^2 + \frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3} + \frac{\sqrt{6}}{3} \right) \left(x^2 - \frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3} + \frac{\sqrt{6}}{3} \right)^{-1} \right) + \frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x+a)/(3*x^4+2),x)`

[Out] $\frac{1}{24}a3^{1/2}6^{1/4}2^{1/2}\arctan\left(\frac{1}{6}2^{1/2}3^{1/2}6^{3/4}x-1\right)+\frac{1}{48}a3^{1/2}6^{1/4}2^{1/2}\ln\left(\frac{x^2+1/33^{1/2}6^{1/4}x2^{1/2}+1/36^{1/2}}{x^2-1/33^{1/2}6^{1/4}x2^{1/2}+1/36^{1/2}}\right)+\frac{1}{24}a3^{1/2}6^{1/4}2^{1/2}\arctan\left(\frac{1}{6}2^{1/2}3^{1/2}6^{3/4}x+1\right)+\frac{1}{12}b\arctan\left(\frac{1}{2}x2^{1/2}6^{1/2}\right)6^{1/2}$

Maxima [A] time = 1.48593, size = 198, normalized size = 1.61

$\frac{1}{48} \cdot 3^{\frac{3}{4}} 2^{\frac{3}{4}} a \log\left(\sqrt{3}x^2 + 3^{\frac{1}{4}} 2^{\frac{3}{4}} x + \sqrt{2}\right) - \frac{1}{48} \cdot 3^{\frac{3}{4}} 2^{\frac{3}{4}} a \log\left(\sqrt{3}x^2 - 3^{\frac{1}{4}} 2^{\frac{3}{4}} x + \sqrt{2}\right) + \frac{1}{24} \sqrt{3} \left(3^{\frac{1}{4}} 2^{\frac{3}{4}} a - 2\sqrt{2}b\right) \arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)/(3*x^4+2),x, algorithm="maxima")`

[Out] $\frac{1}{48}3^{3/4}2^{3/4}a\log(\sqrt{3}x^2 + 3^{1/4}2^{3/4}x + \sqrt{2}) - \frac{1}{48}3^{3/4}2^{3/4}a\log(\sqrt{3}x^2 - 3^{1/4}2^{3/4}x + \sqrt{2}) + \frac{1}{24}\sqrt{3}(3^{1/4}2^{3/4}a - 2\sqrt{2}b)\arctan\left(\frac{1}{6}3^{3/4}2^{3/4}\right) + \frac{1}{24}\sqrt{3}(3^{1/4}2^{3/4}a + 2\sqrt{2}b)\arctan\left(\frac{1}{6}3^{3/4}2^{3/4}\right) + \frac{1}{24}\sqrt{3}(3^{1/4}2^{3/4}a - 2\sqrt{2}b)\arctan\left(\frac{1}{6}3^{3/4}2^{3/4}\right) + \frac{1}{24}\sqrt{3}(3^{1/4}2^{3/4}a + 2\sqrt{2}b)\arctan\left(\frac{1}{6}3^{3/4}2^{3/4}\right)$

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)/(3*x^4+2),x, algorithm="fricas")`

[Out] Timed out

Sympy [A] time = 0.454476, size = 88, normalized size = 0.72

$\text{RootSum}\left(18432t^4 + 384t^2b^2 - 96ta^2b + 3a^4 + 2b^4, \left(t \mapsto t \log\left(x + \frac{3072t^3b^2 + 192t^2a^2b + 24ta^4 + 32tb^4 - 10a^2b^3}{3a^5 - 8ab^4}\right)\right)\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)/(3*x**4+2),x)

[Out] RootSum(18432*_t**4 + 384*_t**2*b**2 - 96*_t*a**2*b + 3*a**4 + 2*b**4, Lambda da(_t, _t*log(x + (3072*_t**3*b**2 + 192*_t**2*a**2*b + 24*_t*a**4 + 32*_t*b**4 - 10*a**2*b**3)/(3*a**5 - 8*a*b**4))))

Giac [A] time = 1.09838, size = 155, normalized size = 1.26

$$\frac{1}{48} \cdot 6^{\frac{3}{4}} a \log \left(x^2 + \sqrt{2} \left(\frac{2}{3} \right)^{\frac{1}{4}} x + \sqrt{\frac{2}{3}} \right) - \frac{1}{48} \cdot 6^{\frac{3}{4}} a \log \left(x^2 - \sqrt{2} \left(\frac{2}{3} \right)^{\frac{1}{4}} x + \sqrt{\frac{2}{3}} \right) + \frac{1}{24} \left(6^{\frac{3}{4}} a - 2 \sqrt{6} b \right) \arctan \left(\frac{3}{4} \sqrt{2} \left(\frac{2}{3} \right)^{\frac{3}{4}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)/(3*x^4+2),x, algorithm="giac")

[Out] 1/48*6^(3/4)*a*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*6^(3/4)*a*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/24*(6^(3/4)*a - 2*sqrt(6)*b)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*sqrt(6)*b)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4)))

3.155 $\int \frac{cx^2}{2+3x^4} dx$

Optimal. Leaf size=101

$$\frac{c \log(3x^2 - 6^{3/4}x + \sqrt{6})}{4 \cdot 6^{3/4}} - \frac{c \log(3x^2 + 6^{3/4}x + \sqrt{6})}{4 \cdot 6^{3/4}} - \frac{c \tan^{-1}(1 - \sqrt[4]{6}x)}{2 \cdot 6^{3/4}} + \frac{c \tan^{-1}(\sqrt[4]{6}x + 1)}{2 \cdot 6^{3/4}}$$

[Out] $-(c \cdot \text{ArcTan}[1 - 6^{(1/4)} \cdot x]) / (2 \cdot 6^{(3/4)}) + (c \cdot \text{ArcTan}[1 + 6^{(1/4)} \cdot x]) / (2 \cdot 6^{(3/4)}) + (c \cdot \text{Log}[\text{Sqrt}[6] - 6^{(3/4)} \cdot x + 3 \cdot x^2]) / (4 \cdot 6^{(3/4)}) - (c \cdot \text{Log}[\text{Sqrt}[6] + 6^{(3/4)} \cdot x + 3 \cdot x^2]) / (4 \cdot 6^{(3/4)})$

Rubi [A] time = 0.0765918, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {12, 297, 1162, 617, 204, 1165, 628}

$$\frac{c \log(3x^2 - 6^{3/4}x + \sqrt{6})}{4 \cdot 6^{3/4}} - \frac{c \log(3x^2 + 6^{3/4}x + \sqrt{6})}{4 \cdot 6^{3/4}} - \frac{c \tan^{-1}(1 - \sqrt[4]{6}x)}{2 \cdot 6^{3/4}} + \frac{c \tan^{-1}(\sqrt[4]{6}x + 1)}{2 \cdot 6^{3/4}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c \cdot x^2) / (2 + 3 \cdot x^4), x]$

[Out] $-(c \cdot \text{ArcTan}[1 - 6^{(1/4)} \cdot x]) / (2 \cdot 6^{(3/4)}) + (c \cdot \text{ArcTan}[1 + 6^{(1/4)} \cdot x]) / (2 \cdot 6^{(3/4)}) + (c \cdot \text{Log}[\text{Sqrt}[6] - 6^{(3/4)} \cdot x + 3 \cdot x^2]) / (4 \cdot 6^{(3/4)}) - (c \cdot \text{Log}[\text{Sqrt}[6] + 6^{(3/4)} \cdot x + 3 \cdot x^2]) / (4 \cdot 6^{(3/4)})$

Rule 12

$\text{Int}[(a_)(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{Match}[\text{Q}[u, (b_)(v_)] /; \text{FreeQ}[b, x]]$

Rule 297

$\text{Int}[(x_)^2 / ((a_)(b_)(x_)^4), x_Symbol] \rightarrow \text{With}[\{r = \text{Numerator}[\text{Rt}[a/b, 2]], s = \text{Denominator}[\text{Rt}[a/b, 2]]\}, \text{Dist}[1 / (2 \cdot s), \text{Int}[(r + s \cdot x^2) / (a + b \cdot x^4), x], x] - \text{Dist}[1 / (2 \cdot s), \text{Int}[(r - s \cdot x^2) / (a + b \cdot x^4), x], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ (\text{GtQ}[a/b, 0] \ || \ (\text{PosQ}[a/b] \ \&\& \ \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, a]] \ \& \ \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, b]]))$

Rule 1162


```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{cx^2}{2+3x^4} dx &= c \int \frac{x^2}{2+3x^4} dx \\
&= -\frac{c \int \frac{\sqrt{2}-\sqrt{3}x^2}{2+3x^4} dx}{2\sqrt{3}} + \frac{c \int \frac{\sqrt{2}+\sqrt{3}x^2}{2+3x^4} dx}{2\sqrt{3}} \\
&= \frac{1}{12}c \int \frac{1}{\sqrt{\frac{2}{3}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}} dx + \frac{1}{12}c \int \frac{1}{\sqrt{\frac{2}{3}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}} dx + \frac{c \int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}} dx}{4 \cdot 6^{3/4}} + \frac{c \int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}} dx}{4 \cdot 6^{3/4}} \\
&= \frac{c \log(\sqrt{6}-6^{3/4}x+3x^2)}{4 \cdot 6^{3/4}} - \frac{c \log(\sqrt{6}+6^{3/4}x+3x^2)}{4 \cdot 6^{3/4}} + \frac{c \operatorname{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1-\sqrt[4]{6}x\right)}{2 \cdot 6^{3/4}} - \frac{c \operatorname{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1+\sqrt[4]{6}x\right)}{2 \cdot 6^{3/4}} \\
&= -\frac{c \tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2 \cdot 6^{3/4}} + \frac{c \tan^{-1}\left(1+\sqrt[4]{6}x\right)}{2 \cdot 6^{3/4}} + \frac{c \log(\sqrt{6}-6^{3/4}x+3x^2)}{4 \cdot 6^{3/4}} - \frac{c \log(\sqrt{6}+6^{3/4}x+3x^2)}{4 \cdot 6^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.0147119, size = 78, normalized size = 0.77

$$\frac{c \left(\log(\sqrt{6}x^2 - 2\sqrt[4]{6}x + 2) - \log(\sqrt{6}x^2 + 2\sqrt[4]{6}x + 2) - 2 \tan^{-1}\left(1 - \sqrt[4]{6}x\right) + 2 \tan^{-1}\left(\sqrt[4]{6}x + 1\right) \right)}{4 \cdot 6^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c*x^2)/(2 + 3*x^4), x]

[Out] (c*(-2*ArcTan[1 - 6^(1/4)*x] + 2*ArcTan[1 + 6^(1/4)*x] + Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] - Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))/(4*6^(3/4))

Maple [A] time = 0.041, size = 114, normalized size = 1.1

$$\frac{c\sqrt{36^{\frac{3}{4}}}\sqrt{2}}{72} \arctan\left(\frac{\sqrt{2}\sqrt{36^{\frac{3}{4}}}x}{6} - 1\right) + \frac{c\sqrt{36^{\frac{3}{4}}}\sqrt{2}}{144} \ln\left(\left(x^2 - \frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3} + \frac{\sqrt{6}}{3}\right)\left(x^2 + \frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3} + \frac{\sqrt{6}}{3}\right)^{-1}\right) + \frac{c\sqrt{36^{\frac{3}{4}}}\sqrt{2}}{72} \arctan\left(\frac{\sqrt{2}\sqrt{36^{\frac{3}{4}}}x}{6} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(c*x^2/(3*x^4+2), x)

[Out] $\frac{1}{72}c^{\frac{1}{2}}3^{\frac{1}{2}}6^{\frac{3}{4}}2^{\frac{1}{2}}\arctan\left(\frac{1}{6}2^{\frac{1}{2}}3^{\frac{1}{2}}6^{\frac{3}{4}}x-1\right)+\frac{1}{14}4c^{\frac{1}{2}}3^{\frac{1}{2}}6^{\frac{3}{4}}2^{\frac{1}{2}}\ln\left(\frac{x^2-1/3\cdot 3^{\frac{1}{2}}6^{\frac{1}{4}}x\cdot 2^{\frac{1}{2}}+1/3\cdot 6^{\frac{1}{2}}\right)}{(x^2+1/3\cdot 3^{\frac{1}{2}}6^{\frac{1}{4}}x\cdot 2^{\frac{1}{2}}+1/3\cdot 6^{\frac{1}{2}})}+\frac{1}{72}c^{\frac{1}{2}}3^{\frac{1}{2}}6^{\frac{3}{4}}2^{\frac{1}{2}}\arctan\left(\frac{1}{6}2^{\frac{1}{2}}3^{\frac{1}{2}}6^{\frac{3}{4}}x+1\right)$

Maxima [A] time = 1.45521, size = 166, normalized size = 1.64

$$\frac{1}{24}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)\right)+2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)-3^{\frac{1}{4}}2^{\frac{1}{4}}\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(c*x^2/(3*x^4+2),x, algorithm="maxima")`

[Out] $\frac{1}{24}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)\right)+2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)-3^{\frac{1}{4}}2^{\frac{1}{4}}\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)+3^{\frac{1}{4}}2^{\frac{1}{4}}\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)*c$

Fricas [B] time = 1.44386, size = 869, normalized size = 8.6

$$-\frac{1}{108}\cdot 54^{\frac{3}{4}}\sqrt{2}(c^4)^{\frac{1}{4}}\arctan\left(\frac{54^{\frac{3}{4}}\sqrt{2}(c^4)^{\frac{1}{4}}x-54^{\frac{3}{4}}\sqrt{2}\sqrt{\frac{1}{3}}(c^4)^{\frac{1}{4}}\sqrt{\frac{3c^3x^2+54^{\frac{1}{4}}\sqrt{2}(c^4)^{\frac{3}{4}}x+\sqrt{6}\sqrt{c^4}c}}{c^3}}+18c}{18c}\right)-\frac{1}{108}\cdot 54^{\frac{3}{4}}\sqrt{2}(c^4)^{\frac{1}{4}}\arctan\left(\frac{54^{\frac{3}{4}}\sqrt{2}(c^4)^{\frac{1}{4}}x+54^{\frac{3}{4}}\sqrt{2}\sqrt{\frac{1}{3}}(c^4)^{\frac{1}{4}}\sqrt{\frac{3c^3x^2+54^{\frac{1}{4}}\sqrt{2}(c^4)^{\frac{3}{4}}x+\sqrt{6}\sqrt{c^4}c}}{c^3}}+18c}{18c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(c*x^2/(3*x^4+2),x, algorithm="fricas")`

[Out] $-1/108\cdot 54^{\frac{3}{4}}\sqrt{2}(c^4)^{\frac{1}{4}}\arctan\left(\frac{-1/18\cdot(54^{\frac{3}{4}}\sqrt{2}(c^4)^{\frac{1}{4}}x-54^{\frac{3}{4}}\sqrt{2}\sqrt{\frac{1}{3}}(c^4)^{\frac{1}{4}}\sqrt{\frac{3c^3x^2+54^{\frac{1}{4}}\sqrt{2}(c^4)^{\frac{3}{4}}x+\sqrt{6}\sqrt{c^4}c}}{c^3}}+18c)}{18c}\right)+1/108\cdot 54^{\frac{3}{4}}\sqrt{2}(c^4)^{\frac{1}{4}}\arctan\left(\frac{-1/18\cdot(54^{\frac{3}{4}}\sqrt{2}(c^4)^{\frac{1}{4}}x+54^{\frac{3}{4}}\sqrt{2}\sqrt{\frac{1}{3}}(c^4)^{\frac{1}{4}}\sqrt{\frac{3c^3x^2+54^{\frac{1}{4}}\sqrt{2}(c^4)^{\frac{3}{4}}x+\sqrt{6}\sqrt{c^4}c}}{c^3}}+18c)}{18c}\right)-1/432\cdot 54^{\frac{3}{4}}\sqrt{2}(c^4)^{\frac{1}{4}}\log\left(\frac{9c^3x^2+3\cdot 54^{\frac{1}{4}}\sqrt{2}(c^4)^{\frac{3}{4}}x+3\sqrt{6}\sqrt{c^4}c}{9c^3x^2-3\cdot 54^{\frac{1}{4}}\sqrt{2}(c^4)^{\frac{3}{4}}x+3\sqrt{6}\sqrt{c^4}c}\right)$

$$(2)*(c^4)^{(3/4)}*x + 3*\sqrt{6}*\sqrt{c^4}*c$$

Sympy [A] time = 0.361851, size = 88, normalized size = 0.87

$$c \left(\frac{\sqrt[4]{6} \log \left(x^2 - \frac{6^{3/4}x}{3} + \frac{\sqrt{6}}{3} \right)}{24} - \frac{\sqrt[4]{6} \log \left(x^2 + \frac{6^{3/4}x}{3} + \frac{\sqrt{6}}{3} \right)}{24} + \frac{\sqrt[4]{6} \operatorname{atan}(\sqrt[4]{6}x - 1)}{12} + \frac{\sqrt[4]{6} \operatorname{atan}(\sqrt[4]{6}x + 1)}{12} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(c*x**2/(3*x**4+2),x)

[Out] c*(6**(1/4)*log(x**2 - 6**(3/4)*x/3 + sqrt(6)/3)/24 - 6**(1/4)*log(x**2 + 6**(3/4)*x/3 + sqrt(6)/3)/24 + 6**(1/4)*atan(6**(1/4)*x - 1)/12 + 6**(1/4)*atan(6**(1/4)*x + 1)/12)

Giac [A] time = 1.0839, size = 131, normalized size = 1.3

$$\frac{1}{24} \left(2 \cdot 6^{1/4} \arctan \left(\frac{3}{4} \sqrt{2} \left(\frac{2}{3} \right)^{3/4} \left(2x + \sqrt{2} \left(\frac{2}{3} \right)^{1/4} \right) \right) + 2 \cdot 6^{1/4} \arctan \left(\frac{3}{4} \sqrt{2} \left(\frac{2}{3} \right)^{3/4} \left(2x - \sqrt{2} \left(\frac{2}{3} \right)^{1/4} \right) \right) - 6^{1/4} \log \left(x^2 + \sqrt{2} \left(\frac{2}{3} \right)^{1/4} x + \sqrt{2/3} \right) + 6^{1/4} \log \left(x^2 - \sqrt{2} \left(\frac{2}{3} \right)^{1/4} x + \sqrt{2/3} \right) \right) * c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(c*x^2/(3*x^4+2),x, algorithm="giac")

[Out] 1/24*(2*6^(1/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))) + 2*6^(1/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) - 6^(1/4)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 6^(1/4)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)))*c

$$3.156 \quad \int \frac{a+cx^2}{2+3x^4} dx$$

Optimal. Leaf size=141

$$-\frac{(\sqrt{6a-2c}) \log(3x^2 - 6^{3/4}x + \sqrt{6})}{8 \cdot 6^{3/4}} + \frac{(\sqrt{6a-2c}) \log(3x^2 + 6^{3/4}x + \sqrt{6})}{8 \cdot 6^{3/4}} - \frac{(\sqrt{6a+2c}) \tan^{-1}(1 - \sqrt[4]{6}x)}{4 \cdot 6^{3/4}} + \frac{(\sqrt{6a+2c}) \tan^{-1}(1 + \sqrt[4]{6}x)}{4 \cdot 6^{3/4}}$$

```
[Out] -((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)
*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4)) - ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3
/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x +
3*x^2])/(8*6^(3/4))
```

Rubi [A] time = 0.0977998, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1168, 1162, 617, 204, 1165, 628}

$$-\frac{(\sqrt{6a-2c}) \log(3x^2 - 6^{3/4}x + \sqrt{6})}{8 \cdot 6^{3/4}} + \frac{(\sqrt{6a-2c}) \log(3x^2 + 6^{3/4}x + \sqrt{6})}{8 \cdot 6^{3/4}} - \frac{(\sqrt{6a+2c}) \tan^{-1}(1 - \sqrt[4]{6}x)}{4 \cdot 6^{3/4}} + \frac{(\sqrt{6a+2c}) \tan^{-1}(1 + \sqrt[4]{6}x)}{4 \cdot 6^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/(2 + 3*x^4), x]
```

```
[Out] -((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)
*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4)) - ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3
/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x +
3*x^2])/(8*6^(3/4))
```

Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
```

& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{a + cx^2}{2 + 3x^4} dx &= \frac{1}{12} (\sqrt{6a} - 2c) \int \frac{\sqrt{6} - 3x^2}{2 + 3x^4} dx + \frac{1}{12} (\sqrt{6a} + 2c) \int \frac{\sqrt{6} + 3x^2}{2 + 3x^4} dx \\ &= -\frac{(\sqrt{6a} - 2c) \int \frac{\frac{2^{3/4} + 2x}{\sqrt[4]{3}}}{-\sqrt{\frac{2}{3} - \frac{2^{3/4}x}{\sqrt[4]{3}}} - x^2} dx}{8 \cdot 6^{3/4}} - \frac{(\sqrt{6a} - 2c) \int \frac{\frac{2^{3/4} - 2x}{\sqrt[4]{3}}}{-\sqrt{\frac{2}{3} + \frac{2^{3/4}x}{\sqrt[4]{3}}} - x^2} dx}{8 \cdot 6^{3/4}} + \frac{1}{24} (\sqrt{6a} + 2c) \int \frac{1}{\sqrt{\frac{2}{3} - \frac{2^{3/4}x}{\sqrt[4]{3}}} + x^2} dx \\ &= -\frac{(\sqrt{6a} - 2c) \log(\sqrt{6} - 6^{3/4}x + 3x^2)}{8 \cdot 6^{3/4}} + \frac{(\sqrt{6a} - 2c) \log(\sqrt{6} + 6^{3/4}x + 3x^2)}{8 \cdot 6^{3/4}} + \frac{(\sqrt{6a} + 2c) \text{Subst}\left(\int \frac{1}{-1-x^2}\right)}{4 \cdot 6^{3/4}} \\ &= -\frac{(\sqrt{6a} + 2c) \tan^{-1}(1 - \sqrt[4]{6}x)}{4 \cdot 6^{3/4}} + \frac{(\sqrt{6a} + 2c) \tan^{-1}(1 + \sqrt[4]{6}x)}{4 \cdot 6^{3/4}} - \frac{(\sqrt{6a} - 2c) \log(\sqrt{6} - 6^{3/4}x + 3x^2)}{8 \cdot 6^{3/4}} + \dots \end{aligned}$$

Mathematica [A] time = 0.0505038, size = 113, normalized size = 0.8

$$\frac{-\left(\sqrt{6}a - 2c\right)\left(\log\left(\sqrt{6}x^2 - 2\sqrt[4]{6}x + 2\right) - \log\left(\sqrt{6}x^2 + 2\sqrt[4]{6}x + 2\right)\right) - 2\left(\sqrt{6}a + 2c\right)\tan^{-1}\left(1 - \sqrt[4]{6}x\right) + 2\left(\sqrt{6}a + 2c\right)\tan^{-1}\left(1 + \sqrt[4]{6}x\right)}{8 \cdot 6^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^2)/(2 + 3*x^4), x]

[Out] $(-2*(\text{Sqrt}[6]*a + 2*c)*\text{ArcTan}[1 - 6^{(1/4)}*x] + 2*(\text{Sqrt}[6]*a + 2*c)*\text{ArcTan}[1 + 6^{(1/4)}*x] - (\text{Sqrt}[6]*a - 2*c)*(\text{Log}[2 - 2*6^{(1/4)}*x + \text{Sqrt}[6]*x^2] - \text{Log}[2 + 2*6^{(1/4)}*x + \text{Sqrt}[6]*x^2]))/(8*6^{(3/4)})$

Maple [B] time = 0.002, size = 226, normalized size = 1.6

$$\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24} \arctan\left(\frac{\sqrt{2}\sqrt[3]{36^{\frac{3}{4}}x}}{6} - 1\right) + \frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{48} \ln\left(\left(x^2 + \frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3} + \frac{\sqrt{6}}{3}\right)\left(x^2 - \frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3} + \frac{\sqrt{6}}{3}\right)^{-1}\right) + \frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+a)/(3*x^4+2), x)

[Out] $1/24*a*3^{(1/2)}*6^{(1/4)}*2^{(1/2)}*\arctan(1/6*2^{(1/2)}*3^{(1/2)}*6^{(3/4)}*x-1)+1/48*a*3^{(1/2)}*6^{(1/4)}*2^{(1/2)}*\ln((x^2+1/3*3^{(1/2)}*6^{(1/4)}*x*2^{(1/2)}+1/3*6^{(1/2)})/(x^2-1/3*3^{(1/2)}*6^{(1/4)}*x*2^{(1/2)}+1/3*6^{(1/2)}))+1/24*a*3^{(1/2)}*6^{(1/4)}*2^{(1/2)}*\arctan(1/6*2^{(1/2)}*3^{(1/2)}*6^{(3/4)}*x+1)+1/72*c*3^{(1/2)}*6^{(3/4)}*2^{(1/2)}*\arctan(1/6*2^{(1/2)}*3^{(1/2)}*6^{(3/4)}*x-1)+1/144*c*3^{(1/2)}*6^{(3/4)}*2^{(1/2)}*\ln((x^2-1/3*3^{(1/2)}*6^{(1/4)}*x*2^{(1/2)}+1/3*6^{(1/2)})/(x^2+1/3*3^{(1/2)}*6^{(1/4)}*x*2^{(1/2)}+1/3*6^{(1/2)}))+1/72*c*3^{(1/2)}*6^{(3/4)}*2^{(1/2)}*\arctan(1/6*2^{(1/2)}*3^{(1/2)}*6^{(3/4)}*x+1)$

Maxima [A] time = 1.45568, size = 225, normalized size = 1.6

$$\frac{1}{24} \cdot 3^{\frac{1}{4}} 2^{\frac{3}{4}} (\sqrt{3}a + \sqrt{2}c) \arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}} (2\sqrt{3}x + 3^{\frac{1}{4}} 2^{\frac{3}{4}})\right) + \frac{1}{24} \cdot 3^{\frac{1}{4}} 2^{\frac{3}{4}} (\sqrt{3}a + \sqrt{2}c) \arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}} (2\sqrt{3}x - 3^{\frac{1}{4}} 2^{\frac{3}{4}})\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)/(3*x^4+2), x, algorithm="maxima")

[Out] $\frac{1}{24}3^{1/4}2^{3/4}(\sqrt{3}a + \sqrt{2}c)\arctan(1/63^{3/4}2^{1/4}(2\sqrt{3}x + 3^{1/4}2^{3/4})) + \frac{1}{24}3^{1/4}2^{3/4}(\sqrt{3}a + \sqrt{2}c)\arctan(1/63^{3/4}2^{1/4}(2\sqrt{3}x - 3^{1/4}2^{3/4})) + \frac{1}{48}3^{1/4}2^{3/4}(\sqrt{3}a - \sqrt{2}c)\log(\sqrt{3}x^2 + 3^{1/4}2^{3/4}x + \sqrt{2}) - \frac{1}{48}3^{1/4}2^{3/4}(\sqrt{3}a - \sqrt{2}c)\log(\sqrt{3}x^2 - 3^{1/4}2^{3/4}x + \sqrt{2})$

Fricas [B] time = 1.6198, size = 5063, normalized size = 35.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+a)/(3*x^4+2),x, algorithm="fricas")`

[Out] $\frac{1}{144}(2\sqrt{6}\sqrt{2}(54a^4 + 72a^2c^2 + 24c^4)^{3/4}\sqrt{9a^4 - 12a^2c^2 + 4c^4}\sqrt{(9a^4 + 12a^2c^2 + 4c^4 + 2\sqrt{54a^4 + 72a^2c^2 + 24c^4})ac}/(9a^4 - 12a^2c^2 + 4c^4))\arctan(-1/12(\sqrt{2}\sqrt{1/3}(54a^4 + 72a^2c^2 + 24c^4)^{3/4}(\sqrt{6}\sqrt{54a^4 + 72a^2c^2 + 24c^4})\sqrt{9a^4 - 12a^2c^2 + 4c^4})a - 2\sqrt{6}\sqrt{9a^4 - 12a^2c^2 + 4c^4})(3a^2c + 2c^3))\sqrt{(9a^4 + 12a^2c^2 + 4c^4 + 2\sqrt{54a^4 + 72a^2c^2 + 24c^4})ac}/(9a^4 - 12a^2c^2 + 4c^4))\sqrt{((3(9a^4 + 12a^2c^2 + 4c^4)x^2 + \sqrt{2}(54a^4 + 72a^2c^2 + 24c^4)^{1/4})(\sqrt{54a^4 + 72a^2c^2 + 24c^4})cx - 3(3a^3 + 2ac^2)x)*\sqrt{(9a^4 + 12a^2c^2 + 4c^4 + 2\sqrt{54a^4 + 72a^2c^2 + 24c^4})ac}}/(9a^4 - 12a^2c^2 + 4c^4)) + \sqrt{54a^4 + 72a^2c^2 + 24c^4}(3a^2 + 2c^2))/(9a^4 + 12a^2c^2 + 4c^4) - \sqrt{2}(54a^4 + 72a^2c^2 + 24c^4)^{3/4}(\sqrt{6}\sqrt{54a^4 + 72a^2c^2 + 24c^4})\sqrt{9a^4 - 12a^2c^2 + 4c^4})ax - 2\sqrt{6}\sqrt{9a^4 - 12a^2c^2 + 4c^4})(3a^2c + 2c^3)x*\sqrt{(9a^4 + 12a^2c^2 + 4c^4 + 2\sqrt{54a^4 + 72a^2c^2 + 24c^4})ac}}/(9a^4 - 12a^2c^2 + 4c^4)) + 2\sqrt{6}\sqrt{54a^4 + 72a^2c^2 + 24c^4}(9a^4 + 12a^2c^2 + 4c^4)\sqrt{9a^4 - 12a^2c^2 + 4c^4}}/(81a^8 + 108a^6c^2 - 48a^2c^6 - 16c^8)) + 2\sqrt{6}\sqrt{2}(54a^4 + 72a^2c^2 + 24c^4)^{3/4}\sqrt{9a^4 - 12a^2c^2 + 4c^4})\sqrt{(9a^4 + 12a^2c^2 + 4c^4 + 2\sqrt{54a^4 + 72a^2c^2 + 24c^4})ac}}/(9a^4 - 12a^2c^2 + 4c^4))\arctan(-1/12(\sqrt{2}\sqrt{1/3}(54a^4 + 72a^2c^2 + 24c^4)^{3/4}(\sqrt{6}\sqrt{54a^4 + 72a^2c^2 + 24c^4})\sqrt{9a^4 - 12a^2c^2 + 4c^4})a - 2\sqrt{6}\sqrt{9a^4 - 12a^2c^2 + 4c^4})(3a^2c + 2c^3))\sqrt{(9a^4 + 12a^2c^2 + 4c^4 + 2\sqrt{54a^4 + 72a^2c^2 + 24c^4})ac}}/(9a^4 - 12a^2c^2 + 4c^4))\sqrt{((3(9a^4 + 12a^2c^2 + 4c^4)x^2 - \sqrt{2}(54a^4 + 72a^2c^2 + 24c^4)^{1/4})(\sqrt{54a^4 + 72a^2c^2 + 24c^4})cx - 3(3a^3 + 2ac^2)x)*\sqrt{(9a^4 + 12a^2c^2 + 4c^4 + 2\sqrt{54a^4 + 72a^2c^2 + 24c^4})ac}}/(9a^4 - 12a^2c^2 + 4c^4))$

$$\begin{aligned}
& 2\sqrt{54a^4 + 72a^2c^2 + 24c^4}ac / (9a^4 - 12a^2c^2 + 4c^4) + \\
& \sqrt{54a^4 + 72a^2c^2 + 24c^4}(3a^2 + 2c^2) / (9a^4 + 12a^2c^2 + 4c^4) - \\
& \sqrt{2}(54a^4 + 72a^2c^2 + 24c^4)^{3/4}(\sqrt{6}\sqrt{54a^4 + 72a^2c^2 + 24c^4} \\
& \sqrt{9a^4 - 12a^2c^2 + 4c^4}ax - 2\sqrt{6}\sqrt{9a^4 - 12a^2c^2 + 4c^4}(3a^2c + 2c^3)x) \\
& \sqrt{(9a^4 + 12a^2c^2 + 4c^4 + 2\sqrt{54a^4 + 72a^2c^2 + 24c^4}ac) / (9a^4 - 12a^2c^2 + 4c^4)} - \\
& 2\sqrt{6}\sqrt{54a^4 + 72a^2c^2 + 24c^4}(9a^4 + 12a^2c^2 + 4c^4)\sqrt{9a^4 - 12a^2c^2 + 4c^4} \\
& / (81a^8 + 108a^6c^2 - 48a^2c^6 - 16c^8) - 3\sqrt{2}(54a^4 + 72a^2c^2 + 24c^4)^{1/4}(9a^4 + 12a^2c^2 + 4c^4 - \\
& 2\sqrt{54a^4 + 72a^2c^2 + 24c^4}ac)\sqrt{(9a^4 + 12a^2c^2 + 4c^4 + 2\sqrt{54a^4 + 72a^2c^2 + 24c^4}ac) / (9a^4 - 12a^2c^2 + 4c^4)} \\
& \log(3(9a^4 + 12a^2c^2 + 4c^4)x^2 + \sqrt{2}(54a^4 + 72a^2c^2 + 24c^4)^{1/4}(\sqrt{54a^4 + 72a^2c^2 + 24c^4}cx - 3(3a^3 + 2ac^2)x) \\
& \sqrt{(9a^4 + 12a^2c^2 + 4c^4 + 2\sqrt{54a^4 + 72a^2c^2 + 24c^4}ac) / (9a^4 - 12a^2c^2 + 4c^4)}) + \\
& \sqrt{54a^4 + 72a^2c^2 + 24c^4}(3a^2 + 2c^2) + 3\sqrt{2}(54a^4 + 72a^2c^2 + 24c^4)^{1/4}(9a^4 + 12a^2c^2 + 4c^4 - 2\sqrt{54a^4 + 72a^2c^2 + 24c^4}ac) \\
& \sqrt{(9a^4 + 12a^2c^2 + 4c^4 + 2\sqrt{54a^4 + 72a^2c^2 + 24c^4}ac) / (9a^4 - 12a^2c^2 + 4c^4)} \log(3(9a^4 + 12a^2c^2 + 4c^4)x^2 - \sqrt{2}(54a^4 + 72a^2c^2 + 24c^4)^{1/4}(\sqrt{54a^4 + 72a^2c^2 + 24c^4}cx - 3(3a^3 + 2ac^2)x) \\
& \sqrt{(9a^4 + 12a^2c^2 + 4c^4 + 2\sqrt{54a^4 + 72a^2c^2 + 24c^4}ac) / (9a^4 - 12a^2c^2 + 4c^4)}) + \sqrt{54a^4 + 72a^2c^2 + 24c^4}(3a^2 + 2c^2) / (9a^4 + 12a^2c^2 + 4c^4)
\end{aligned}$$

Sympy [A] time = 0.364615, size = 68, normalized size = 0.48

$$\text{RootSum}\left(55296t^4 + 2304t^2ac + 9a^4 + 12a^2c^2 + 4c^4, \left(t \mapsto t \log\left(x + \frac{-4608t^3c + 72ta^3 - 144tac^2}{9a^4 - 4c^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+a)/(3*x**4+2),x)

[Out] RootSum(55296*_t**4 + 2304*_t**2*a*c + 9*a**4 + 12*a**2*c**2 + 4*c**4, Lambda(_t, _t*log(x + (-4608*_t**3*c + 72*_t*a**3 - 144*_t*a*c**2)/(9*a**4 - 4*c**4))))

Giac [A] time = 1.10751, size = 177, normalized size = 1.26

$$\frac{1}{24} \left(6^{\frac{3}{4}} a + 2 \cdot 6^{\frac{1}{4}} c \right) \arctan \left(\frac{3}{4} \sqrt{2} \left(\frac{2}{3} \right)^{\frac{3}{4}} \left(2x + \sqrt{2} \left(\frac{2}{3} \right)^{\frac{1}{4}} \right) \right) + \frac{1}{24} \left(6^{\frac{3}{4}} a + 2 \cdot 6^{\frac{1}{4}} c \right) \arctan \left(\frac{3}{4} \sqrt{2} \left(\frac{2}{3} \right)^{\frac{3}{4}} \left(2x - \sqrt{2} \left(\frac{2}{3} \right)^{\frac{1}{4}} \right) \right) + \frac{1}{48} \left(6^{\frac{3}{4}} a - 2 \cdot 6^{\frac{1}{4}} c \right) \log \left(x^2 - \sqrt{2} \left(\frac{2}{3} \right)^{\frac{1}{4}} x + \sqrt{2/3} \right) - \frac{1}{48} \left(6^{\frac{3}{4}} a - 2 \cdot 6^{\frac{1}{4}} c \right) \log \left(x^2 + \sqrt{2} \left(\frac{2}{3} \right)^{\frac{1}{4}} x + \sqrt{2/3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)/(3*x^4+2),x, algorithm="giac")

[Out] 1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(3/4)*a - 2*6^(1/4)*c)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*(6^(3/4)*a - 2*6^(1/4)*c)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))

$$3.157 \quad \int \frac{bx+cx^2}{2+3x^4} dx$$

Optimal. Leaf size=123

$$\frac{b \tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} + \frac{c \log(3x^2 - 6^{3/4}x + \sqrt{6})}{4 \cdot 6^{3/4}} - \frac{c \log(3x^2 + 6^{3/4}x + \sqrt{6})}{4 \cdot 6^{3/4}} - \frac{c \tan^{-1}(1 - \sqrt[4]{6}x)}{2 \cdot 6^{3/4}} + \frac{c \tan^{-1}(\sqrt[4]{6}x + 1)}{2 \cdot 6^{3/4}}$$

[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (c*ArcTan[1 - 6^(1/4)*x])/(2*6^(3/4)) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/4)) + (c*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(4*6^(3/4))

Rubi [A] time = 0.117672, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1593, 1831, 275, 203, 297, 1162, 617, 204, 1165, 628}

$$\frac{b \tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} + \frac{c \log(3x^2 - 6^{3/4}x + \sqrt{6})}{4 \cdot 6^{3/4}} - \frac{c \log(3x^2 + 6^{3/4}x + \sqrt{6})}{4 \cdot 6^{3/4}} - \frac{c \tan^{-1}(1 - \sqrt[4]{6}x)}{2 \cdot 6^{3/4}} + \frac{c \tan^{-1}(\sqrt[4]{6}x + 1)}{2 \cdot 6^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2)/(2 + 3*x^4), x]

[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (c*ArcTan[1 - 6^(1/4)*x])/(2*6^(3/4)) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/4)) + (c*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(4*6^(3/4))

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1831

Int[((Pq_)*((c_.)*(x_)^(m_.)))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii])*x^(n/2))]/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

$\text{eQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

Rule 628

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x_Symbol] \ :> \ \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rubi steps

$$\begin{aligned} \int \frac{bx + cx^2}{2 + 3x^4} dx &= \int \frac{x(b + cx)}{2 + 3x^4} dx \\ &= \int \left(\frac{bx}{2 + 3x^4} + \frac{cx^2}{2 + 3x^4} \right) dx \\ &= b \int \frac{x}{2 + 3x^4} dx + c \int \frac{x^2}{2 + 3x^4} dx \\ &= \frac{1}{2} b \text{Subst} \left(\int \frac{1}{2 + 3x^2} dx, x, x^2 \right) - \frac{c \int \frac{\sqrt{2} - \sqrt{3}x^2}{2 + 3x^4} dx}{2\sqrt{3}} + \frac{c \int \frac{\sqrt{2} + \sqrt{3}x^2}{2 + 3x^4} dx}{2\sqrt{3}} \\ &= \frac{b \tan^{-1} \left(\sqrt{\frac{3}{2}} x^2 \right)}{2\sqrt{6}} + \frac{1}{12} c \int \frac{1}{\sqrt{\frac{2}{3}} - \frac{2^{3/4}x}{\sqrt{3}} + x^2} dx + \frac{1}{12} c \int \frac{1}{\sqrt{\frac{2}{3}} + \frac{2^{3/4}x}{\sqrt{3}} + x^2} dx + \frac{c \int \frac{\frac{2^{3/4}}{\sqrt{3}} + 2x}{-\sqrt{\frac{2}{3}} - \frac{2^{3/4}x}{\sqrt{3}} - x^2} dx}{4 \cdot 6^{3/4}} + \dots \\ &= \frac{b \tan^{-1} \left(\sqrt{\frac{3}{2}} x^2 \right)}{2\sqrt{6}} + \frac{c \log(\sqrt{6} - 6^{3/4}x + 3x^2)}{4 \cdot 6^{3/4}} - \frac{c \log(\sqrt{6} + 6^{3/4}x + 3x^2)}{4 \cdot 6^{3/4}} + \frac{c \text{Subst} \left(\int \frac{1}{-1-x^2} dx, x, 1 - \dots \right)}{2 \cdot 6^{3/4}} \\ &= \frac{b \tan^{-1} \left(\sqrt{\frac{3}{2}} x^2 \right)}{2\sqrt{6}} - \frac{c \tan^{-1}(1 - \sqrt[4]{6}x)}{2 \cdot 6^{3/4}} + \frac{c \tan^{-1}(1 + \sqrt[4]{6}x)}{2 \cdot 6^{3/4}} + \frac{c \log(\sqrt{6} - 6^{3/4}x + 3x^2)}{4 \cdot 6^{3/4}} - \frac{c \log(\sqrt{6} + \dots)}{4 \cdot 6^{3/4}} \end{aligned}$$

Mathematica [A] time = 0.0436847, size = 99, normalized size = 0.8

$$\frac{-2(\sqrt[4]{6}b + c) \tan^{-1}(1 - \sqrt[4]{6}x) + 2(c - \sqrt[4]{6}b) \tan^{-1}(\sqrt[4]{6}x + 1) + c \log(\sqrt{6}x^2 - 2\sqrt[4]{6}x + 2) - c \log(\sqrt{6}x^2 + 2\sqrt[4]{6}x + 2)}{4 \cdot 6^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2)/(2 + 3*x^4), x]

[Out] $(-2*(6^{(1/4)}*b + c)*\text{ArcTan}[1 - 6^{(1/4)}*x] + 2*(-(6^{(1/4)}*b) + c)*\text{ArcTan}[1 + 6^{(1/4)}*x] + c*\text{Log}[2 - 2*6^{(1/4)}*x + \text{Sqrt}[6]*x^2] - c*\text{Log}[2 + 2*6^{(1/4)}*x + \text{Sqrt}[6]*x^2])/(4*6^{(3/4)})$

Maple [A] time = 0.022, size = 129, normalized size = 1.1

$$\frac{b\sqrt{6}}{12} \arctan\left(\frac{x^2\sqrt{6}}{2}\right) + \frac{c\sqrt{36^{\frac{3}{4}}}\sqrt{2}}{72} \arctan\left(\frac{\sqrt{2}\sqrt{36^{\frac{3}{4}}}x}{6} - 1\right) + \frac{c\sqrt{36^{\frac{3}{4}}}\sqrt{2}}{144} \ln\left(\left(x^2 - \frac{\sqrt{3}\sqrt{6}x\sqrt{2}}{3} + \frac{\sqrt{6}}{3}\right)\left(x^2 + \frac{\sqrt{3}\sqrt{6}x\sqrt{2}}{3} + \frac{\sqrt{6}}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x)/(3*x^4+2),x)`

[Out] $1/12*b*\arctan(1/2*x^2*6^{(1/2)})*6^{(1/2)}+1/72*c*3^{(1/2)}*6^{(3/4)}*2^{(1/2)}*\arctan(1/6*2^{(1/2)}*3^{(1/2)}*6^{(3/4)}*x-1)+1/144*c*3^{(1/2)}*6^{(3/4)}*2^{(1/2)}*\ln((x^2-1/3*3^{(1/2)}*6^{(1/4)}*x*2^{(1/2)}+1/3*6^{(1/2)})/(x^2+1/3*3^{(1/2)}*6^{(1/4)}*x*2^{(1/2)}+1/3*6^{(1/2)}))+1/72*c*3^{(1/2)}*6^{(3/4)}*2^{(1/2)}*\arctan(1/6*2^{(1/2)}*3^{(1/2)}*6^{(3/4)}*x+1)$

Maxima [A] time = 1.45964, size = 198, normalized size = 1.61

$$\frac{1}{24} \sqrt{2} \left(3^{\frac{1}{4}} 2^{\frac{3}{4}} c - 2\sqrt{3}b\right) \arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}} \left(2\sqrt{3}x + 3^{\frac{1}{4}} 2^{\frac{3}{4}}\right)\right) + \frac{1}{24} \sqrt{2} \left(3^{\frac{1}{4}} 2^{\frac{3}{4}} c + 2\sqrt{3}b\right) \arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}} \left(2\sqrt{3}x - 3^{\frac{1}{4}} 2^{\frac{3}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x)/(3*x^4+2),x, algorithm="maxima")`

[Out] $1/24*\text{sqrt}(2)*(3^{(1/4)}*2^{(3/4)}*c - 2*\text{sqrt}(3)*b)*\arctan(1/6*3^{(3/4)}*2^{(1/4)}*(2*\text{sqrt}(3)*x + 3^{(1/4)}*2^{(3/4)})) + 1/24*\text{sqrt}(2)*(3^{(1/4)}*2^{(3/4)}*c + 2*\text{sqrt}(3)*b)*\arctan(1/6*3^{(3/4)}*2^{(1/4)}*(2*\text{sqrt}(3)*x - 3^{(1/4)}*2^{(3/4)})) - 1/24*3^{(1/4)}*2^{(1/4)}*c*\log(\text{sqrt}(3)*x^2 + 3^{(1/4)}*2^{(3/4)}*x + \text{sqrt}(2)) + 1/24*3^{(1/4)}*2^{(1/4)}*c*\log(\text{sqrt}(3)*x^2 - 3^{(1/4)}*2^{(3/4)}*x + \text{sqrt}(2))$

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)/(3*x^4+2),x, algorithm="fricas")

[Out] Timed out

Sympy [A] time = 0.458956, size = 85, normalized size = 0.69

RootSum $\left(27648t^4 + 576t^2b^2 + 96tbc^2 + 3b^4 + 2c^4, \left(t \mapsto t \log\left(x + \frac{-1152t^3c^2 + 288t^2b^3 - 36tb^2c^2 + 3b^5 - 3bc^4}{6b^4c - c^5}\right)\right)\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x)/(3*x**4+2),x)

[Out] RootSum(27648*_t**4 + 576*_t**2*b**2 + 96*_t*b*c**2 + 3*b**4 + 2*c**4, Lambda(_t, _t*log(x + (-1152*_t**3*c**2 + 288*_t**2*b**3 - 36*_t*b**2*c**2 + 3*b**5 - 3*b*c**4)/(6*b**4*c - c**5))))

Giac [A] time = 1.11023, size = 154, normalized size = 1.25

$-\frac{1}{24} \cdot 6^{\frac{1}{4}}c \log\left(x^2 + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right) + \frac{1}{24} \cdot 6^{\frac{1}{4}}c \log\left(x^2 - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right) - \frac{1}{12}(\sqrt{6}b - 6^{\frac{1}{4}}c) \arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)/(3*x^4+2),x, algorithm="giac")

[Out] -1/24*6^(1/4)*c*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/24*6^(1/4)*c*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/12*(sqrt(6)*b - 6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/12*(sqrt(6)*b + 6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4)))

$$3.158 \quad \int \frac{a+bx+cx^2}{2+3x^4} dx$$

Optimal. Leaf size=163

$$-\frac{(\sqrt{6a-2c})\log(3x^2-6^{3/4}x+\sqrt{6})}{8\ 6^{3/4}} + \frac{(\sqrt{6a-2c})\log(3x^2+6^{3/4}x+\sqrt{6})}{8\ 6^{3/4}} - \frac{(\sqrt{6a+2c})\tan^{-1}(1-\sqrt[4]{6}x)}{4\ 6^{3/4}} + \frac{(\sqrt{6a+2c})\tan^{-1}(1+\sqrt[4]{6}x)}{4\ 6^{3/4}}$$

[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - ((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4)) - ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(3/4))

Rubi [A] time = 0.12368, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$, Rules used = {1876, 275, 203, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{(\sqrt{6a-2c})\log(3x^2-6^{3/4}x+\sqrt{6})}{8\ 6^{3/4}} + \frac{(\sqrt{6a-2c})\log(3x^2+6^{3/4}x+\sqrt{6})}{8\ 6^{3/4}} - \frac{(\sqrt{6a+2c})\tan^{-1}(1-\sqrt[4]{6}x)}{4\ 6^{3/4}} + \frac{(\sqrt{6a+2c})\tan^{-1}(1+\sqrt[4]{6}x)}{4\ 6^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2)/(2 + 3*x^4), x]

[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - ((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4)) - ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(3/4))

Rule 1876

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}], Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x

$\wedge k], x] /; k \neq 1] /; \text{FreeQ}[\{a, b, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[m]$

Rule 203

$\text{Int}[\{(a_)+(b_)*(x_)^2\}^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTan}[\text{Rt}[b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 1168

$\text{Int}[\{(d_)+(e_)*(x_)^2\}/\{(a_)+(c_)*(x_)^4\}, x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \text{Dist}[(d*q - a*e)/(2*a*c), \text{Int}[(q - c*x^2)/(a + c*x^4), x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[-(a*c)]$

Rule 1162

$\text{Int}[\{(d_)+(e_)*(x_)^2\}/\{(a_)+(c_)*(x_)^4\}, x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[(2*d)/e, 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[d*e]$

Rule 617

$\text{Int}[\{(a_)+(b_)*(x_)+(c_)*(x_)^2\}^{-1}, x_Symbol] \rightarrow \text{With}[\{q = 1 - 4*S\text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4*a*c])] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 204

$\text{Int}[\{(a_)+(b_)*(x_)^2\}^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2]*x]/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 1165

$\text{Int}[\{(d_)+(e_)*(x_)^2\}/\{(a_)+(c_)*(x_)^4\}, x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-(2*d)/e, 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + bx + cx^2}{2 + 3x^4} dx &= \int \left(\frac{bx}{2 + 3x^4} + \frac{a + cx^2}{2 + 3x^4} \right) dx \\
&= b \int \frac{x}{2 + 3x^4} dx + \int \frac{a + cx^2}{2 + 3x^4} dx \\
&= \frac{1}{2} b \operatorname{Subst} \left(\int \frac{1}{2 + 3x^2} dx, x, x^2 \right) + \frac{1}{12} (\sqrt{6}a - 2c) \int \frac{\sqrt{6} - 3x^2}{2 + 3x^4} dx + \frac{1}{12} (\sqrt{6}a + 2c) \int \frac{\sqrt{6} + 3x^2}{2 + 3x^4} dx \\
&= \frac{b \tan^{-1} \left(\sqrt{\frac{3}{2}} x^2 \right)}{2\sqrt{6}} - \frac{(\sqrt{6}a - 2c) \int \frac{\frac{2^{3/4}}{\sqrt[4]{3}} + 2x}{-\sqrt{\frac{2}{3} - \frac{2^{3/4}x}{\sqrt[4]{3}}} - x^2} dx}{8 \cdot 6^{3/4}} - \frac{(\sqrt{6}a - 2c) \int \frac{\frac{2^{3/4}}{\sqrt[4]{3}} - 2x}{-\sqrt{\frac{2}{3} + \frac{2^{3/4}x}{\sqrt[4]{3}}} - x^2} dx}{8 \cdot 6^{3/4}} + \frac{1}{24} (\sqrt{6}a + 2c) \\
&= \frac{b \tan^{-1} \left(\sqrt{\frac{3}{2}} x^2 \right)}{2\sqrt{6}} - \frac{(\sqrt{6}a - 2c) \log(\sqrt{6} - 6^{3/4}x + 3x^2)}{8 \cdot 6^{3/4}} + \frac{(\sqrt{6}a - 2c) \log(\sqrt{6} + 6^{3/4}x + 3x^2)}{8 \cdot 6^{3/4}} + \frac{(\sqrt{6}a + 2c) \log(\sqrt{6}x^2 - 2\sqrt[4]{6}x + 2)}{8 \cdot 6^{3/4}} \\
&= \frac{b \tan^{-1} \left(\sqrt{\frac{3}{2}} x^2 \right)}{2\sqrt{6}} - \frac{(\sqrt{6}a + 2c) \tan^{-1}(1 - \sqrt[4]{6}x)}{4 \cdot 6^{3/4}} + \frac{(\sqrt{6}a + 2c) \tan^{-1}(1 + \sqrt[4]{6}x)}{4 \cdot 6^{3/4}} - \frac{(\sqrt{6}a - 2c) \log(\sqrt{6}x^2 - 2\sqrt[4]{6}x + 2)}{8 \cdot 6^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.0713316, size = 129, normalized size = 0.79

$$\frac{-2 \tan^{-1} \left(1 - \sqrt[4]{6}x \right) \left(\sqrt{6}a + 2 \left(\sqrt[4]{6}b + c \right) \right) + 2 \tan^{-1} \left(\sqrt[4]{6}x + 1 \right) \left(\sqrt{6}a - 2 \sqrt[4]{6}b + 2c \right) - \left(\sqrt{6}a - 2c \right) \left(\log \left(\sqrt{6}x^2 - 2 \sqrt[4]{6}x + 2 \right) \right)}{8 \cdot 6^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2)/(2 + 3*x^4), x]

[Out] (-2*(Sqrt[6]*a + 2*(6^(1/4)*b + c))*ArcTan[1 - 6^(1/4)*x] + 2*(Sqrt[6]*a - 2*6^(1/4)*b + 2*c)*ArcTan[1 + 6^(1/4)*x] - (Sqrt[6]*a - 2*c)*(Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] - Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))/(8*6^(3/4))

Maple [B] time = 0.003, size = 241, normalized size = 1.5

$$\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24} \arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6} - 1\right) + \frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{48} \ln\left(\left(x^2 + \frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3} + \frac{\sqrt{6}}{3}\right)\left(x^2 - \frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3} + \frac{\sqrt{6}}{3}\right)^{-1}\right) + \frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)/(3*x^4+2),x)

[Out] 1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/48*a*3^(1/2)*6^(1/4)*2^(1/2)*ln((x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2))/(x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))+1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)+1/72*c*3^(1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/144*c*3^(1/2)*6^(3/4)*2^(1/2)*ln((x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2))/(x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))+1/72*c*3^(1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)

Maxima [A] time = 1.44739, size = 252, normalized size = 1.55

$$\frac{1}{48} \cdot 3^{\frac{1}{4}} 2^{\frac{3}{4}} (\sqrt{3}a - \sqrt{2}c) \log\left(\sqrt{3}x^2 + 3^{\frac{1}{4}} 2^{\frac{3}{4}} x + \sqrt{2}\right) - \frac{1}{48} \cdot 3^{\frac{1}{4}} 2^{\frac{3}{4}} (\sqrt{3}a - \sqrt{2}c) \log\left(\sqrt{3}x^2 - 3^{\frac{1}{4}} 2^{\frac{3}{4}} x + \sqrt{2}\right) + \frac{1}{24} \left(3^{\frac{3}{4}} 2^{\frac{3}{4}} a - \sqrt{3}x^2 + \sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)/(3*x^4+2),x, algorithm="maxima")

[Out] 1/48*3^(1/4)*2^(3/4)*(sqrt(3)*a - sqrt(2)*c)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) - 1/48*3^(1/4)*2^(3/4)*(sqrt(3)*a - sqrt(2)*c)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/24*(3^(3/4)*2^(3/4)*a - 2*sqrt(3)*sqrt(2)*b + 2*3^(1/4)*2^(1/4)*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/24*(3^(3/4)*2^(3/4)*a + 2*sqrt(3)*sqrt(2)*b + 2*3^(1/4)*2^(1/4)*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)/(3*x^4+2),x, algorithm="fricas")

[Out] Timed out

Sympy [B] time = 3.07534, size = 292, normalized size = 1.79

RootSum($55296t^4 + t^2(2304ac + 1152b^2) + t(-288a^2b + 192bc^2) + 9a^4 + 12a^2c^2 - 24ab^2c + 6b^4 + 4c^4$, ($t \mapsto t \log(x +$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)/(3*x**4+2),x)

[Out] RootSum(55296*_t**4 + _t**2*(2304*a*c + 1152*b**2) + _t*(-288*a**2*b + 192*b*c**2) + 9*a**4 + 12*a**2*c**2 - 24*a*b**2*c + 6*b**4 + 4*c**4, Lambda(_t, _t*log(x + (-13824*_t**3*a**2*c + 27648*_t**3*a*b**2 + 9216*_t**3*c**3 + 1728*_t**2*a**3*b + 3456*_t**2*a*b*c**2 - 2304*_t**2*b**3*c + 216*_t*a**5 - 576*_t*a**3*c**2 + 1296*_t*a**2*b**2*c + 288*_t*a*b**4 + 288*_t*a*c**4 + 288*_t*b**2*c**3 + 90*a**4*b*c - 90*a**3*b**3 + 60*a*b**3*c**2 - 24*b**5*c + 24*b*c**5)/(27*a**6 - 18*a**4*c**2 + 144*a**3*b**2*c - 72*a**2*b**4 - 12*a**2*c**4 + 96*a*b**2*c**3 - 48*b**4*c**2 + 8*c**6))))

Giac [A] time = 1.11269, size = 193, normalized size = 1.18

$\frac{1}{24} \left(6^{\frac{3}{4}}a - 2\sqrt{6}b + 2 \cdot 6^{\frac{1}{4}}c \right) \arctan \left(\frac{3}{4} \sqrt{2} \left(\frac{2}{3} \right)^{\frac{3}{4}} \left(2x + \sqrt{2} \left(\frac{2}{3} \right)^{\frac{1}{4}} \right) \right) + \frac{1}{24} \left(6^{\frac{3}{4}}a + 2\sqrt{6}b + 2 \cdot 6^{\frac{1}{4}}c \right) \arctan \left(\frac{3}{4} \sqrt{2} \left(\frac{2}{3} \right)^{\frac{3}{4}} \left(2x - \sqrt{2} \left(\frac{2}{3} \right)^{\frac{1}{4}} \right) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)/(3*x^4+2),x, algorithm="giac")

[Out] $\frac{1}{24} \cdot (6^{\frac{3}{4}}a - 2\sqrt{6}b + 2 \cdot 6^{\frac{1}{4}}c) \cdot \arctan\left(\frac{3}{4}\sqrt{2} \cdot \left(\frac{2}{3}\right)^{\frac{3}{4}} \cdot (2x + \sqrt{2} \cdot \left(\frac{2}{3}\right)^{\frac{1}{4}})\right) + \frac{1}{24} \cdot (6^{\frac{3}{4}}a + 2\sqrt{6}b + 2 \cdot 6^{\frac{1}{4}}c) \cdot \arctan\left(\frac{3}{4}\sqrt{2} \cdot \left(\frac{2}{3}\right)^{\frac{3}{4}} \cdot (2x - \sqrt{2} \cdot \left(\frac{2}{3}\right)^{\frac{1}{4}})\right) + \frac{1}{48} \cdot (6^{\frac{3}{4}}a - 2 \cdot 6^{\frac{1}{4}}c) \cdot \log(x^2 + \sqrt{2} \cdot \left(\frac{2}{3}\right)^{\frac{1}{4}} \cdot x + \sqrt{2/3}) - \frac{1}{48} \cdot (6^{\frac{3}{4}}a - 2 \cdot 6^{\frac{1}{4}}c) \cdot \log(x^2 - \sqrt{2} \cdot \left(\frac{2}{3}\right)^{\frac{1}{4}} \cdot x + \sqrt{2/3})$

$$3.159 \quad \int \frac{dx^3}{2+3x^4} dx$$

Optimal. Leaf size=13

$$\frac{1}{12}d \log(3x^4 + 2)$$

[Out] (d*Log[2 + 3*x^4])/12

Rubi [A] time = 0.0037983, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {12, 260}

$$\frac{1}{12}d \log(3x^4 + 2)$$

Antiderivative was successfully verified.

[In] Int[(d*x^3)/(2 + 3*x^4),x]

[Out] (d*Log[2 + 3*x^4])/12

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned} \int \frac{dx^3}{2+3x^4} dx &= d \int \frac{x^3}{2+3x^4} dx \\ &= \frac{1}{12}d \log(2+3x^4) \end{aligned}$$

Mathematica [A] time = 0.0028887, size = 13, normalized size = 1.

$$\frac{1}{12} d \log(3x^4 + 2)$$

Antiderivative was successfully verified.

[In] Integrate[(d*x^3)/(2 + 3*x^4), x]

[Out] (d*Log[2 + 3*x^4])/12

Maple [A] time = 0.039, size = 12, normalized size = 0.9

$$\frac{d \ln(3x^4 + 2)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(d*x^3/(3*x^4+2), x)

[Out] 1/12*d*ln(3*x^4+2)

Maxima [A] time = 0.943257, size = 15, normalized size = 1.15

$$\frac{1}{12} d \log(3x^4 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d*x^3/(3*x^4+2), x, algorithm="maxima")

[Out] 1/12*d*log(3*x^4 + 2)

Fricas [A] time = 1.39175, size = 31, normalized size = 2.38

$$\frac{1}{12} d \log(3x^4 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(d*x^3/(3*x^4+2),x, algorithm="fricas")`

[Out] $1/12*d*log(3*x^4 + 2)$

Sympy [A] time = 0.076402, size = 10, normalized size = 0.77

$$\frac{d \log(3x^4 + 2)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(d*x**3/(3*x**4+2),x)`

[Out] $d*log(3*x**4 + 2)/12$

Giac [A] time = 1.07502, size = 15, normalized size = 1.15

$$\frac{1}{12} d \log(3x^4 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(d*x^3/(3*x^4+2),x, algorithm="giac")`

[Out] $1/12*d*log(3*x^4 + 2)$

3.160 $\int \frac{a+dx^3}{2+3x^4} dx$

Optimal. Leaf size=114

$$-\frac{a \log(3x^2 - 6^{3/4}x + \sqrt{6})}{8\sqrt[4]{6}} + \frac{a \log(3x^2 + 6^{3/4}x + \sqrt{6})}{8\sqrt[4]{6}} - \frac{a \tan^{-1}(1 - \sqrt[4]{6}x)}{4\sqrt[4]{6}} + \frac{a \tan^{-1}(\sqrt[4]{6}x + 1)}{4\sqrt[4]{6}} + \frac{1}{12}d \log(3x^4 + 2)$$

[Out] $-(a*\text{ArcTan}[1 - 6^{(1/4)}*x])/(4*6^{(1/4)}) + (a*\text{ArcTan}[1 + 6^{(1/4)}*x])/(4*6^{(1/4)}) - (a*\text{Log}[\text{Sqrt}[6] - 6^{(3/4)}*x + 3*x^2])/(8*6^{(1/4)}) + (a*\text{Log}[\text{Sqrt}[6] + 6^{(3/4)}*x + 3*x^2])/(8*6^{(1/4)}) + (d*\text{Log}[2 + 3*x^4])/12$

Rubi [A] time = 0.0991563, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1876, 211, 1165, 628, 1162, 617, 204, 260}

$$-\frac{a \log(3x^2 - 6^{3/4}x + \sqrt{6})}{8\sqrt[4]{6}} + \frac{a \log(3x^2 + 6^{3/4}x + \sqrt{6})}{8\sqrt[4]{6}} - \frac{a \tan^{-1}(1 - \sqrt[4]{6}x)}{4\sqrt[4]{6}} + \frac{a \tan^{-1}(\sqrt[4]{6}x + 1)}{4\sqrt[4]{6}} + \frac{1}{12}d \log(3x^4 + 2)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + d*x^3)/(2 + 3*x^4), x]$

[Out] $-(a*\text{ArcTan}[1 - 6^{(1/4)}*x])/(4*6^{(1/4)}) + (a*\text{ArcTan}[1 + 6^{(1/4)}*x])/(4*6^{(1/4)}) - (a*\text{Log}[\text{Sqrt}[6] - 6^{(3/4)}*x + 3*x^2])/(8*6^{(1/4)}) + (a*\text{Log}[\text{Sqrt}[6] + 6^{(3/4)}*x + 3*x^2])/(8*6^{(1/4)}) + (d*\text{Log}[2 + 3*x^4])/12$

Rule 1876

$\text{Int}[(\text{Pq}_)/((a_) + (b_)*(x_)^{(n_)}), x_Symbol] \rightarrow \text{With}[\{v = \text{Sum}[(x^{ii}*(\text{Coeff}[\text{Pq}, x, ii] + \text{Coeff}[\text{Pq}, x, n/2 + ii]*x^{(n/2)}))]/(a + b*x^n), \{ii, 0, n/2 - 1\}], \text{Int}[v, x] /; \text{SumQ}[v] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[\text{Pq}, x] \&\& \text{IGtQ}[n/2, 0] \&\& \text{Expon}[\text{Pq}, x] < n$

Rule 211

$\text{Int}(((a_) + (b_)*(x_)^4)^{-1}, x_Symbol] \rightarrow \text{With}[\{r = \text{Numerator}[\text{Rt}[a/b, 2]], s = \text{Denominator}[\text{Rt}[a/b, 2]]\}, \text{Dist}[1/(2*r), \text{Int}[(r - s*x^2)/(a + b*x^4), x], x] + \text{Dist}[1/(2*r), \text{Int}[(r + s*x^2)/(a + b*x^4), x], x] /; \text{FreeQ}[\{a, b\}, x] \&\& (\text{GtQ}[a/b, 0] || (\text{PosQ}[a/b] \&\& \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, a]] \&\& \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, b]]))$

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned}
\int \frac{a + dx^3}{2 + 3x^4} dx &= \int \left(\frac{a}{2 + 3x^4} + \frac{dx^3}{2 + 3x^4} \right) dx \\
&= a \int \frac{1}{2 + 3x^4} dx + d \int \frac{x^3}{2 + 3x^4} dx \\
&= \frac{1}{12} d \log(2 + 3x^4) + \frac{a \int \frac{\sqrt{2} - \sqrt{3}x^2}{2 + 3x^4} dx}{2\sqrt{2}} + \frac{a \int \frac{\sqrt{2} + \sqrt{3}x^2}{2 + 3x^4} dx}{2\sqrt{2}} \\
&= \frac{1}{12} d \log(2 + 3x^4) + \frac{a \int \frac{1}{\sqrt{\frac{2}{3} - \frac{2^{3/4}x}{\sqrt[4]{3}} + x^2}} dx}{4\sqrt{6}} + \frac{a \int \frac{1}{\sqrt{\frac{2}{3} + \frac{2^{3/4}x}{\sqrt[4]{3}} + x^2}} dx}{4\sqrt{6}} - \frac{a \int \frac{\frac{2^{3/4} + 2x}{\sqrt[4]{3}}}{-\sqrt{\frac{2}{3} - \frac{2^{3/4}x}{\sqrt[4]{3}} - x^2}} dx}{8\sqrt[4]{6}} - \frac{a \int \frac{\frac{2^{3/4} - 2x}{\sqrt[4]{3}}}{-\sqrt{\frac{2}{3} + \frac{2^{3/4}x}{\sqrt[4]{3}} - x^2}} dx}{8\sqrt[4]{6}} \\
&= -\frac{a \log(\sqrt{6} - 6^{3/4}x + 3x^2)}{8\sqrt[4]{6}} + \frac{a \log(\sqrt{6} + 6^{3/4}x + 3x^2)}{8\sqrt[4]{6}} + \frac{1}{12} d \log(2 + 3x^4) + \frac{a \text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1 - \sqrt[4]{6}x\right)}{4\sqrt[4]{6}} \\
&= -\frac{a \tan^{-1}\left(1 - \sqrt[4]{6}x\right)}{4\sqrt[4]{6}} + \frac{a \tan^{-1}\left(1 + \sqrt[4]{6}x\right)}{4\sqrt[4]{6}} - \frac{a \log(\sqrt{6} - 6^{3/4}x + 3x^2)}{8\sqrt[4]{6}} + \frac{a \log(\sqrt{6} + 6^{3/4}x + 3x^2)}{8\sqrt[4]{6}} + \frac{1}{12} d
\end{aligned}$$

Mathematica [A] time = 0.031317, size = 108, normalized size = 0.95

$$\frac{1}{48} \left(-6^{3/4} a \log(\sqrt{6}x^2 - 2\sqrt[4]{6}x + 2) + 6^{3/4} a \log(\sqrt{6}x^2 + 2\sqrt[4]{6}x + 2) - 2 \cdot 6^{3/4} a \tan^{-1}\left(1 - \sqrt[4]{6}x\right) + 2 \cdot 6^{3/4} a \tan^{-1}\left(\sqrt[4]{6}x + 1\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + d*x^3)/(2 + 3*x^4), x]

[Out] (-2*6^(3/4)*a*ArcTan[1 - 6^(1/4)*x] + 2*6^(3/4)*a*ArcTan[1 + 6^(1/4)*x] - 6^(3/4)*a*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] + 6^(3/4)*a*Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2] + 4*d*Log[2 + 3*x^4])/48

Maple [A] time = 0.043, size = 125, normalized size = 1.1

$$\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24} \arctan\left(\frac{\sqrt{2}\sqrt{36^{\frac{3}{4}}x}}{6} - 1\right) + \frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{48} \ln\left(\left(x^2 + \frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3} + \frac{\sqrt{6}}{3}\right)\left(x^2 - \frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3} + \frac{\sqrt{6}}{3}\right)^{-1}\right) + \frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+a)/(3*x^4+2), x)

[Out] $\frac{1}{24}a^3 \cdot 3^{1/2} \cdot 6^{1/4} \cdot 2^{1/2} \cdot \arctan\left(\frac{1}{6} \cdot 2^{1/2} \cdot 3^{1/2} \cdot 6^{3/4} \cdot x - 1\right) + \frac{1}{48} \cdot a^3 \cdot 3^{1/2} \cdot 6^{1/4} \cdot 2^{1/2} \cdot \ln\left(\frac{(x^2 + 1/3 \cdot 3^{1/2} \cdot 6^{1/4} \cdot x \cdot 2^{1/2} + 1/3 \cdot 6^{1/2})}{(x^2 - 1/3 \cdot 3^{1/2} \cdot 6^{1/4} \cdot x \cdot 2^{1/2} + 1/3 \cdot 6^{1/2})}\right) + \frac{1}{24}a^3 \cdot 3^{1/2} \cdot 6^{1/4} \cdot 2^{1/2} \cdot \arctan\left(\frac{1}{6} \cdot 2^{1/2} \cdot 3^{1/2} \cdot 6^{3/4} \cdot x + 1\right) + \frac{1}{12}d \cdot \ln(3 \cdot x^4 + 2)$

Maxima [A] time = 1.45189, size = 201, normalized size = 1.76

$$\frac{1}{24} \cdot 3^{\frac{3}{4}} 2^{\frac{3}{4}} a \arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}} \left(2\sqrt{3}x + 3^{\frac{1}{4}} 2^{\frac{3}{4}}\right)\right) + \frac{1}{24} \cdot 3^{\frac{3}{4}} 2^{\frac{3}{4}} a \arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}} \left(2\sqrt{3}x - 3^{\frac{1}{4}} 2^{\frac{3}{4}}\right)\right) + \frac{1}{144} \cdot 3^{\frac{3}{4}} 2^{\frac{3}{4}} \left(2 \cdot 3^{\frac{1}{4}} 2^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3+a)/(3*x^4+2),x, algorithm="maxima")`

[Out] $\frac{1}{24} \cdot 3^{3/4} \cdot 2^{3/4} \cdot a \cdot \arctan\left(\frac{1}{6} \cdot 3^{3/4} \cdot 2^{1/4} \cdot (2\sqrt{3}x + 3^{1/4} \cdot 2^{3/4})\right) + \frac{1}{24} \cdot 3^{3/4} \cdot 2^{3/4} \cdot a \cdot \arctan\left(\frac{1}{6} \cdot 3^{3/4} \cdot 2^{1/4} \cdot (2\sqrt{3}x - 3^{1/4} \cdot 2^{3/4})\right) + \frac{1}{144} \cdot 3^{3/4} \cdot 2^{3/4} \cdot (2 \cdot 3^{1/4} \cdot 2^{1/4} \cdot d + 3a) \cdot \log(\sqrt{3}x^2 + 3^{1/4} \cdot 2^{3/4} \cdot x + \sqrt{2}) + \frac{1}{144} \cdot 3^{3/4} \cdot 2^{3/4} \cdot (2 \cdot 3^{1/4} \cdot 2^{1/4} \cdot d - 3a) \cdot \log(\sqrt{3}x^2 - 3^{1/4} \cdot 2^{3/4} \cdot x + \sqrt{2})$

Fricas [B] time = 1.61018, size = 1041, normalized size = 9.13

$$4 \cdot 6^{\frac{1}{4}} \sqrt{3} \sqrt{2} (a^4)^{\frac{1}{4}} a^4 \arctan\left(-\frac{6^{\frac{3}{4}} \sqrt{3} \sqrt{2} (a^4)^{\frac{3}{4}} a^4 x - 6^{\frac{3}{4}} \sqrt{3} \sqrt{2} \sqrt{\frac{1}{3}} (a^4)^{\frac{3}{4}} a^4 \sqrt{\frac{3a^2x^2 + 6^{\frac{1}{4}} \sqrt{3} \sqrt{2} (a^4)^{\frac{1}{4}} ax + \sqrt{6} \sqrt{a^4}}{a^2}} + 6a^7}{6a^7}}\right) + 4 \cdot 6^{\frac{1}{4}} \sqrt{3} \sqrt{2} (a^4)^{\frac{1}{4}} a^4 \arctan\left(\frac{6^{\frac{3}{4}} \sqrt{3} \sqrt{2} (a^4)^{\frac{3}{4}} a^4 x + 6^{\frac{3}{4}} \sqrt{3} \sqrt{2} \sqrt{\frac{1}{3}} (a^4)^{\frac{3}{4}} a^4 \sqrt{\frac{3a^2x^2 + 6^{\frac{1}{4}} \sqrt{3} \sqrt{2} (a^4)^{\frac{1}{4}} ax + \sqrt{6} \sqrt{a^4}}{a^2}} + 6a^7}{6a^7}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3+a)/(3*x^4+2),x, algorithm="fricas")`

[Out] $-\frac{1}{48} \cdot (4 \cdot 6^{1/4} \cdot \sqrt{3} \cdot \sqrt{2} \cdot (a^4)^{1/4} \cdot a^4 \cdot \arctan\left(-\frac{1}{6} \cdot (6^{3/4} \cdot \sqrt{3} \cdot \sqrt{2} \cdot (a^4)^{3/4} \cdot a^4 \cdot x - 6^{3/4} \cdot \sqrt{3} \cdot \sqrt{2} \cdot \sqrt{1/3} \cdot (a^4)^{3/4} \cdot a^4 \cdot \sqrt{\frac{3a^2x^2 + 6^{1/4} \cdot \sqrt{3} \cdot \sqrt{2} \cdot (a^4)^{1/4} \cdot ax + \sqrt{6} \cdot \sqrt{a^4}}{a^2}} + 6a^7}{a^7}\right) + 6a^7)/a^7) + 4 \cdot 6^{1/4} \cdot \sqrt{3} \cdot \sqrt{2} \cdot (a^4)^{1/4} \cdot a^4 \cdot \arctan\left(-\frac{1}{6} \cdot (6^{3/4} \cdot \sqrt{3} \cdot \sqrt{2} \cdot (a^4)^{3/4} \cdot a^4 \cdot x - 6^{3/4} \cdot \sqrt{3} \cdot \sqrt{2} \cdot \sqrt{1/3} \cdot (a^4)^{3/4} \cdot a^4 \cdot \sqrt{\frac{3a^2x^2 + 6^{1/4} \cdot \sqrt{3} \cdot \sqrt{2} \cdot (a^4)^{1/4} \cdot ax + \sqrt{6} \cdot \sqrt{a^4}}{a^2}} - 6a^7}{a^7}\right) - (6^{1/4} \cdot \sqrt{3} \cdot \sqrt{2} \cdot (a^4)^{1/4} \cdot a^4 \cdot \sqrt{2} \cdot (a^4)^{1/4} \cdot a^4 + 4 \cdot a^4 \cdot d) \cdot \log(3a^2x^2 + 6^{1/4} \cdot \sqrt{3} \cdot \sqrt{2} \cdot (a^4)^{1/4} \cdot a^4 \cdot \sqrt{2} \cdot (a^4)^{1/4} \cdot a^4 + 4 \cdot a^4 \cdot d) \cdot \log(3a^2x^2 + 6^{1/4} \cdot \sqrt{3} \cdot \sqrt{2} \cdot (a^4)^{1/4} \cdot a^4 \cdot \sqrt{2} \cdot (a^4)^{1/4} \cdot a^4 - 6a^7)/a^7$

$$4)^{(1/4)} * a * x + \sqrt{6} * \sqrt{a^4}) + (6^{(1/4)} * \sqrt{3} * \sqrt{2}) * (a^4)^{(1/4)} * a^4 - 4 * a^4 * d) * \log(3 * a^2 * x^2 - 6^{(1/4)} * \sqrt{3} * \sqrt{2}) * (a^4)^{(1/4)} * a * x + \sqrt{6} * \sqrt{a^4})) / a^4$$

Sympy [A] time = 0.300339, size = 51, normalized size = 0.45

$$\text{RootSum}\left(165888t^4 - 55296t^3d + 6912t^2d^2 - 384td^3 + 27a^4 + 8d^4, \left(t \mapsto t \log\left(x + \frac{24t - 2d}{3a}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+a)/(3*x**4+2),x)

[Out] RootSum(165888*_t**4 - 55296*_t**3*d + 6912*_t**2*d**2 - 384*_t*d**3 + 27*a**4 + 8*d**4, Lambda(_t, _t*log(x + (24*_t - 2*d)/(3*a))))

Giac [A] time = 1.13125, size = 147, normalized size = 1.29

$$\frac{1}{24} \cdot 6^{\frac{3}{4}} a \arctan\left(\frac{3}{4} \sqrt{2} \left(\frac{2}{3}\right)^{\frac{3}{4}} \left(2x + \sqrt{2} \left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{24} \cdot 6^{\frac{3}{4}} a \arctan\left(\frac{3}{4} \sqrt{2} \left(\frac{2}{3}\right)^{\frac{3}{4}} \left(2x - \sqrt{2} \left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{48} (6^{\frac{3}{4}} a + 4d) \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+a)/(3*x^4+2),x, algorithm="giac")

[Out] 1/24*6^(3/4)*a*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/24*6^(3/4)*a*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(3/4)*a + 4*d)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*(6^(3/4)*a - 4*d)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))

$$3.161 \quad \int \frac{bx+dx^3}{2+3x^4} dx$$

Optimal. Leaf size=36

$$\frac{b \tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} + \frac{1}{12}d \log(3x^4 + 2)$$

[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) + (d*Log[2 + 3*x^4])/12

Rubi [A] time = 0.0314551, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1593, 1248, 635, 203, 260}

$$\frac{b \tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} + \frac{1}{12}d \log(3x^4 + 2)$$

Antiderivative was successfully verified.

[In] Int[(b*x + d*x^3)/(2 + 3*x^4), x]

[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) + (d*Log[2 + 3*x^4])/12

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1248

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned} \int \frac{bx + dx^3}{2 + 3x^4} dx &= \int \frac{x(b + dx^2)}{2 + 3x^4} dx \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{b + dx}{2 + 3x^2} dx, x, x^2 \right) \\ &= \frac{1}{2} b \text{Subst} \left(\int \frac{1}{2 + 3x^2} dx, x, x^2 \right) + \frac{1}{2} d \text{Subst} \left(\int \frac{x}{2 + 3x^2} dx, x, x^2 \right) \\ &= \frac{b \tan^{-1} \left(\sqrt{\frac{3}{2}} x^2 \right)}{2\sqrt{6}} + \frac{1}{12} d \log(2 + 3x^4) \end{aligned}$$

Mathematica [C] time = 0.0304642, size = 65, normalized size = 1.81

$$\frac{1}{24} (2d + i\sqrt{6}b) \log(\sqrt{6} - 3ix^2) + \frac{1}{24} (2d - i\sqrt{6}b) \log(\sqrt{6} + 3ix^2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + d*x^3)/(2 + 3*x^4), x]
```

```
[Out] ((I*Sqrt[6]*b + 2*d)*Log[Sqrt[6] - (3*I)*x^2])/24 + (((-I)*Sqrt[6]*b + 2*d)*Log[Sqrt[6] + (3*I)*x^2])/24
```

Maple [A] time = 0.042, size = 28, normalized size = 0.8

$$\frac{d \ln(3x^4 + 2)}{12} + \frac{b\sqrt{6}}{12} \arctan\left(\frac{x^2\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^3+b*x)/(3*x^4+2),x)`

[Out] $1/12*d*\ln(3*x^4+2)+1/12*b*\arctan(1/2*x^2*6^{(1/2)})*6^{(1/2)}$

Maxima [B] time = 1.46961, size = 153, normalized size = 4.25

$$-\frac{1}{12} \sqrt{3} \sqrt{2} b \arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}} \left(2 \sqrt{3} x + 3^{\frac{1}{4}} 2^{\frac{3}{4}}\right)\right) + \frac{1}{12} \sqrt{3} \sqrt{2} b \arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}} \left(2 \sqrt{3} x - 3^{\frac{1}{4}} 2^{\frac{3}{4}}\right)\right) + \frac{1}{12} d \log\left(\sqrt{3} x^2 + 3^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3+b*x)/(3*x^4+2),x, algorithm="maxima")`

[Out] $-1/12*\sqrt{3}*\sqrt{2}*b*\arctan(1/6*3^{(3/4)}*2^{(1/4)}*(2*\sqrt{3}*x + 3^{(1/4)}*2^{(3/4)})) + 1/12*\sqrt{3}*\sqrt{2}*b*\arctan(1/6*3^{(3/4)}*2^{(1/4)}*(2*\sqrt{3}*x - 3^{(1/4)}*2^{(3/4)})) + 1/12*d*\log(\sqrt{3}*x^2 + 3^{(1/4)}*2^{(3/4)}*x + \sqrt{3}) + 1/12*d*\log(\sqrt{3}*x^2 - 3^{(1/4)}*2^{(3/4)}*x + \sqrt{3})$

Fricas [A] time = 1.47641, size = 86, normalized size = 2.39

$$\frac{1}{12} \sqrt{6} b \arctan\left(\frac{1}{2} \sqrt{6} x^2\right) + \frac{1}{12} d \log(3 x^4 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3+b*x)/(3*x^4+2),x, algorithm="fricas")`

[Out] $1/12*\sqrt{6}*b*\arctan(1/2*\sqrt{6}*x^2) + 1/12*d*\log(3*x^4 + 2)$

Sympy [C] time = 0.305611, size = 53, normalized size = 1.47

$$\left(-\frac{\sqrt{6}ib}{24} + \frac{d}{12}\right) \log\left(x^2 - \frac{\sqrt{6}i}{3}\right) + \left(\frac{\sqrt{6}ib}{24} + \frac{d}{12}\right) \log\left(x^2 + \frac{\sqrt{6}i}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+b*x)/(3*x**4+2),x)
```

```
[Out] (-sqrt(6)*I*b/24 + d/12)*log(x**2 - sqrt(6)*I/3) + (sqrt(6)*I*b/24 + d/12)*
log(x**2 + sqrt(6)*I/3)
```

Giac [B] time = 1.08778, size = 126, normalized size = 3.5

$$-\frac{1}{12} \sqrt{6} b \arctan \left(\frac{3}{4} \sqrt{2} \left(\frac{2}{3} \right)^{\frac{3}{4}} \left(2x + \sqrt{2} \left(\frac{2}{3} \right)^{\frac{1}{4}} \right) \right) + \frac{1}{12} \sqrt{6} b \arctan \left(\frac{3}{4} \sqrt{2} \left(\frac{2}{3} \right)^{\frac{3}{4}} \left(2x - \sqrt{2} \left(\frac{2}{3} \right)^{\frac{1}{4}} \right) \right) + \frac{1}{12} d \log \left(x^2 + \sqrt{2} \left(\frac{2}{3} \right)^{\frac{1}{4}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(6)*b*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4)))
+ 1/12*sqrt(6)*b*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4)
)) + 1/12*d*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/12*d*log(x^2 -
sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```


3.162 $\int \frac{a+bx+dx^3}{2+3x^4} dx$

Optimal. Leaf size=136

$$-\frac{a \log(3x^2 - 6^{3/4}x + \sqrt{6})}{8\sqrt[4]{6}} + \frac{a \log(3x^2 + 6^{3/4}x + \sqrt{6})}{8\sqrt[4]{6}} - \frac{a \tan^{-1}(1 - \sqrt[4]{6}x)}{4\sqrt[4]{6}} + \frac{a \tan^{-1}(\sqrt[4]{6}x + 1)}{4\sqrt[4]{6}} + \frac{b \tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} +$$

[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (a*ArcTan[1 - 6^(1/4)*x])/(4*6^(1/4)) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/4)) - (a*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (d*Log[2 + 3*x^4])/12

Rubi [A] time = 0.117721, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.55$, Rules used = {1876, 211, 1165, 628, 1162, 617, 204, 1248, 635, 203, 260}

$$-\frac{a \log(3x^2 - 6^{3/4}x + \sqrt{6})}{8\sqrt[4]{6}} + \frac{a \log(3x^2 + 6^{3/4}x + \sqrt{6})}{8\sqrt[4]{6}} - \frac{a \tan^{-1}(1 - \sqrt[4]{6}x)}{4\sqrt[4]{6}} + \frac{a \tan^{-1}(\sqrt[4]{6}x + 1)}{4\sqrt[4]{6}} + \frac{b \tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} +$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + d*x^3)/(2 + 3*x^4), x]

[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (a*ArcTan[1 - 6^(1/4)*x])/(4*6^(1/4)) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/4)) - (a*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (d*Log[2 + 3*x^4])/12

Rule 1876

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b

}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 1165

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1162

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1248

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + bx + dx^3}{2 + 3x^4} dx &= \int \left(\frac{a}{2 + 3x^4} + \frac{x(b + dx^2)}{2 + 3x^4} \right) dx \\
&= a \int \frac{1}{2 + 3x^4} dx + \int \frac{x(b + dx^2)}{2 + 3x^4} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{b + dx}{2 + 3x^2} dx, x, x^2 \right) + \frac{a \int \frac{\sqrt{2} - \sqrt{3}x^2}{2 + 3x^4} dx}{2\sqrt{2}} + \frac{a \int \frac{\sqrt{2} + \sqrt{3}x^2}{2 + 3x^4} dx}{2\sqrt{2}} \\
&= \frac{a \int \frac{1}{\sqrt{\frac{2}{3} - \frac{2^{3/4}x}{4\sqrt{3}} + x^2}} dx}{4\sqrt{6}} + \frac{a \int \frac{1}{\sqrt{\frac{2}{3} + \frac{2^{3/4}x}{4\sqrt{3}} + x^2}} dx}{4\sqrt{6}} - \frac{a \int \frac{\frac{2^{3/4}}{4\sqrt{3}} + 2x}{-\sqrt{\frac{2}{3} - \frac{2^{3/4}x}{4\sqrt{3}} - x^2}} dx}{8\sqrt[4]{6}} - \frac{a \int \frac{\frac{2^{3/4}}{4\sqrt{3}} - 2x}{-\sqrt{\frac{2}{3} + \frac{2^{3/4}x}{4\sqrt{3}} - x^2}} dx}{8\sqrt[4]{6}} + \frac{1}{2} b \text{Subst} \\
&= \frac{b \tan^{-1} \left(\sqrt{\frac{3}{2}} x^2 \right)}{2\sqrt{6}} - \frac{a \log \left(\sqrt{6} - 6^{3/4} x + 3x^2 \right)}{8\sqrt[4]{6}} + \frac{a \log \left(\sqrt{6} + 6^{3/4} x + 3x^2 \right)}{8\sqrt[4]{6}} + \frac{1}{12} d \log \left(2 + 3x^4 \right) + \frac{a}{12} \\
&= \frac{b \tan^{-1} \left(\sqrt{\frac{3}{2}} x^2 \right)}{2\sqrt{6}} - \frac{a \tan^{-1} \left(1 - \sqrt[4]{6} x \right)}{4\sqrt[4]{6}} + \frac{a \tan^{-1} \left(1 + \sqrt[4]{6} x \right)}{4\sqrt[4]{6}} - \frac{a \log \left(\sqrt{6} - 6^{3/4} x + 3x^2 \right)}{8\sqrt[4]{6}} + \frac{a \log \left(\sqrt{6} + 6^{3/4} x + 3x^2 \right)}{8\sqrt[4]{6}}
\end{aligned}$$

Mathematica [A] time = 0.0584346, size = 128, normalized size = 0.94

$$\frac{1}{48} \left(-2\sqrt{6} \left(\sqrt[4]{6} a + 2b \right) \tan^{-1} \left(1 - \sqrt[4]{6} x \right) + 2\sqrt{6} \left(\sqrt[4]{6} a - 2b \right) \tan^{-1} \left(\sqrt[4]{6} x + 1 \right) - 6^{3/4} a \log \left(\sqrt{6} x^2 - 2\sqrt[4]{6} x + 2 \right) + 6^{3/4} a \log \left(\sqrt{6} x^2 + 2\sqrt[4]{6} x + 2 \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + d*x^3)/(2 + 3*x^4),x]

[Out] (-2*Sqrt[6]*(6^(1/4)*a + 2*b)*ArcTan[1 - 6^(1/4)*x] + 2*Sqrt[6]*(6^(1/4)*a - 2*b)*ArcTan[1 + 6^(1/4)*x] - 6^(3/4)*a*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] + 6^(3/4)*a*Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2] + 4*d*Log[2 + 3*x^4])/48

Maple [A] time = 0.041, size = 140, normalized size = 1.

$$\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24} \arctan\left(\frac{\sqrt{2}\sqrt[3]{36^{\frac{3}{4}}x}}{6} - 1\right) + \frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{48} \ln\left(\left(x^2 + \frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3} + \frac{\sqrt{6}}{3}\right)\left(x^2 - \frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3} + \frac{\sqrt{6}}{3}\right)^{-1}\right) + \frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+b*x+a)/(3*x^4+2),x)

[Out] 1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/48*a*3^(1/2)*6^(1/4)*2^(1/2)*ln((x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2))/(x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))+1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)+1/12*d*ln(3*x^4+2)

Maxima [A] time = 1.45488, size = 231, normalized size = 1.7

$$\frac{1}{144} \cdot 3^{\frac{3}{4}} 2^{\frac{3}{4}} \left(2 \cdot 3^{\frac{1}{4}} 2^{\frac{1}{4}} d + 3a\right) \log\left(\sqrt{3}x^2 + 3^{\frac{1}{4}} 2^{\frac{3}{4}} x + \sqrt{2}\right) + \frac{1}{144} \cdot 3^{\frac{3}{4}} 2^{\frac{3}{4}} \left(2 \cdot 3^{\frac{1}{4}} 2^{\frac{1}{4}} d - 3a\right) \log\left(\sqrt{3}x^2 - 3^{\frac{1}{4}} 2^{\frac{3}{4}} x + \sqrt{2}\right) + \frac{1}{24} \sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+b*x+a)/(3*x^4+2),x, algorithm="maxima")

[Out] 1/144*3^(3/4)*2^(3/4)*(2*3^(1/4)*2^(1/4)*d + 3*a)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/144*3^(3/4)*2^(3/4)*(2*3^(1/4)*2^(1/4)*d - 3*a)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/24*sqrt(3)*(3^(1/4)*2^(3/4)*a - 2*sqrt(2)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/24*sqrt(3)*(3^(1/4)*2^(3/4)*a + 2*sqrt(2)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+b*x+a)/(3*x^4+2),x, algorithm="fricas")

[Out] Timed out

Sympy [A] time = 1.18933, size = 199, normalized size = 1.46

RootSum(165888t⁴ - 55296t³d + t²(3456b² + 6912d²) + t(-864a²b - 576b²d - 384d³) + 27a⁴ + 72a²bd + 18b⁴ + 2

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+b*x+a)/(3*x**4+2),x)

[Out] RootSum(165888*_t**4 - 55296*_t**3*d + _t**2*(3456*b**2 + 6912*d**2) + _t*(-864*a**2*b - 576*b**2*d - 384*d**3) + 27*a**4 + 72*a**2*b*d + 18*b**4 + 24*b**2*d**2 + 8*d**4, Lambda(_t, _t*log(x + (27648*_t**3*b**2 + 1728*_t**2*a**2*b - 6912*_t**2*b**2*d + 216*_t*a**4 - 288*_t*a**2*b*d + 288*_t*b**4 + 576*_t*b**2*d**2 - 18*a**4*d - 90*a**2*b**3 + 12*a**2*b*d**2 - 24*b**4*d - 16*b**2*d**3)/(27*a**5 - 72*a*b**4))))

Giac [A] time = 1.10922, size = 169, normalized size = 1.24

$$\frac{1}{24} \left(6^{\frac{3}{4}} a - 2\sqrt{6}b \right) \arctan \left(\frac{3}{4} \sqrt{2} \left(\frac{2}{3} \right)^{\frac{3}{4}} \left(2x + \sqrt{2} \left(\frac{2}{3} \right)^{\frac{1}{4}} \right) \right) + \frac{1}{24} \left(6^{\frac{3}{4}} a + 2\sqrt{6}b \right) \arctan \left(\frac{3}{4} \sqrt{2} \left(\frac{2}{3} \right)^{\frac{3}{4}} \left(2x - \sqrt{2} \left(\frac{2}{3} \right)^{\frac{1}{4}} \right) \right) + \frac{1}{48} (6^{\frac{3}{4}} a + 4d) \log(x^2 + \sqrt{2}x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+b*x+a)/(3*x^4+2),x, algorithm="giac")

[Out] 1/24*(6^(3/4)*a - 2*sqrt(6)*b)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2))*(2/3)^(1/4)) + 1/24*(6^(3/4)*a + 2*sqrt(6)*b)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4)) + 1/48*(6^(3/4)*a + 4*d)*log(x^2 + sqrt(2)x + 1)

$$\begin{aligned} &)*(2/3)^{(1/4)}*x + \text{sqrt}(2/3)) - 1/48*(6^{(3/4)}*a - 4*d)*\log(x^2 - \text{sqrt}(2)*(2/ \\ & 3)^{(1/4)}*x + \text{sqrt}(2/3)) \end{aligned}$$

$$3.163 \quad \int \frac{cx^2 + dx^3}{2 + 3x^4} dx$$

Optimal. Leaf size=114

$$\frac{c \log(3x^2 - 6^{3/4}x + \sqrt{6})}{4 \cdot 6^{3/4}} - \frac{c \log(3x^2 + 6^{3/4}x + \sqrt{6})}{4 \cdot 6^{3/4}} - \frac{c \tan^{-1}(1 - \sqrt[4]{6}x)}{2 \cdot 6^{3/4}} + \frac{c \tan^{-1}(\sqrt[4]{6}x + 1)}{2 \cdot 6^{3/4}} + \frac{1}{12} d \log(3x^4 + 2)$$

[Out] $-(c \cdot \text{ArcTan}[1 - 6^{(1/4)} \cdot x]) / (2 \cdot 6^{(3/4)}) + (c \cdot \text{ArcTan}[1 + 6^{(1/4)} \cdot x]) / (2 \cdot 6^{(3/4)}) + (c \cdot \text{Log}[\text{Sqrt}[6] - 6^{(3/4)} \cdot x + 3 \cdot x^2]) / (4 \cdot 6^{(3/4)}) - (c \cdot \text{Log}[\text{Sqrt}[6] + 6^{(3/4)} \cdot x + 3 \cdot x^2]) / (4 \cdot 6^{(3/4)}) + (d \cdot \text{Log}[2 + 3 \cdot x^4]) / 12$

Rubi [A] time = 0.120906, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1593, 1831, 297, 1162, 617, 204, 1165, 628, 260}

$$\frac{c \log(3x^2 - 6^{3/4}x + \sqrt{6})}{4 \cdot 6^{3/4}} - \frac{c \log(3x^2 + 6^{3/4}x + \sqrt{6})}{4 \cdot 6^{3/4}} - \frac{c \tan^{-1}(1 - \sqrt[4]{6}x)}{2 \cdot 6^{3/4}} + \frac{c \tan^{-1}(\sqrt[4]{6}x + 1)}{2 \cdot 6^{3/4}} + \frac{1}{12} d \log(3x^4 + 2)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c \cdot x^2 + d \cdot x^3) / (2 + 3 \cdot x^4), x]$

[Out] $-(c \cdot \text{ArcTan}[1 - 6^{(1/4)} \cdot x]) / (2 \cdot 6^{(3/4)}) + (c \cdot \text{ArcTan}[1 + 6^{(1/4)} \cdot x]) / (2 \cdot 6^{(3/4)}) + (c \cdot \text{Log}[\text{Sqrt}[6] - 6^{(3/4)} \cdot x + 3 \cdot x^2]) / (4 \cdot 6^{(3/4)}) - (c \cdot \text{Log}[\text{Sqrt}[6] + 6^{(3/4)} \cdot x + 3 \cdot x^2]) / (4 \cdot 6^{(3/4)}) + (d \cdot \text{Log}[2 + 3 \cdot x^4]) / 12$

Rule 1593

$\text{Int}[(u_.) * ((a_.) * (x_.)^{(p_.)} + (b_.) * (x_.)^{(q_.)})^{(n_.)}, x_Symbol] \rightarrow \text{Int}[u * x^{(n * p)} * (a + b * x^{(q - p)})^n, x] /;$ FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1831

$\text{Int}[(Pq_.) * ((c_.) * (x_.)^{(m_.)}) / ((a_.) + (b_.) * (x_.)^{(n_.)}), x_Symbol] \rightarrow \text{With}[\{v = \text{Sum}[(c * x)^{(m + ii)} * (\text{Coeff}[Pq, x, ii] + \text{Coeff}[Pq, x, n/2 + ii]) * x^{(n/2)}] / (c^{ii} * (a + b * x^n)), \{ii, 0, n/2 - 1\}\}, \text{Int}[v, x] /;$ SumQ[v] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```


Rubi steps

$$\begin{aligned}
\int \frac{cx^2 + dx^3}{2 + 3x^4} dx &= \int \frac{x^2(c + dx)}{2 + 3x^4} dx \\
&= \int \left(\frac{cx^2}{2 + 3x^4} + \frac{dx^3}{2 + 3x^4} \right) dx \\
&= c \int \frac{x^2}{2 + 3x^4} dx + d \int \frac{x^3}{2 + 3x^4} dx \\
&= \frac{1}{12} d \log(2 + 3x^4) - \frac{c \int \frac{\sqrt{2} - \sqrt{3}x^2}{2 + 3x^4} dx}{2\sqrt{3}} + \frac{c \int \frac{\sqrt{2} + \sqrt{3}x^2}{2 + 3x^4} dx}{2\sqrt{3}} \\
&= \frac{1}{12} d \log(2 + 3x^4) + \frac{1}{12} c \int \frac{1}{\sqrt{\frac{2}{3} - \frac{2^{3/4}x}{\sqrt{3}} + x^2}} dx + \frac{1}{12} c \int \frac{1}{\sqrt{\frac{2}{3} + \frac{2^{3/4}x}{\sqrt{3}} + x^2}} dx + \frac{c \int \frac{\frac{2^{3/4}}{\sqrt{3}} + 2x}{-\sqrt{\frac{2}{3} - \frac{2^{3/4}x}{\sqrt{3}} - x^2}} dx}{4 \cdot 6^{3/4}} + \dots \\
&= \frac{c \log(\sqrt{6} - 6^{3/4}x + 3x^2)}{4 \cdot 6^{3/4}} - \frac{c \log(\sqrt{6} + 6^{3/4}x + 3x^2)}{4 \cdot 6^{3/4}} + \frac{1}{12} d \log(2 + 3x^4) + \frac{c \operatorname{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1\right)}{2 \cdot 6^{3/4}} \\
&= -\frac{c \tan^{-1}\left(1 - \sqrt[4]{6}x\right)}{2 \cdot 6^{3/4}} + \frac{c \tan^{-1}\left(1 + \sqrt[4]{6}x\right)}{2 \cdot 6^{3/4}} + \frac{c \log(\sqrt{6} - 6^{3/4}x + 3x^2)}{4 \cdot 6^{3/4}} - \frac{c \log(\sqrt{6} + 6^{3/4}x + 3x^2)}{4 \cdot 6^{3/4}} + \dots
\end{aligned}$$

Mathematica [A] time = 0.0276467, size = 108, normalized size = 0.95

$$\frac{1}{24} \left(\sqrt[4]{6}c \log(\sqrt{6}x^2 - 2\sqrt[4]{6}x + 2) - \sqrt[4]{6}c \log(\sqrt{6}x^2 + 2\sqrt[4]{6}x + 2) - 2\sqrt[4]{6}c \tan^{-1}\left(1 - \sqrt[4]{6}x\right) + 2\sqrt[4]{6}c \tan^{-1}\left(\sqrt[4]{6}x + 1\right) + 2d \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c*x^2 + d*x^3)/(2 + 3*x^4), x]

[Out] $(-2 \cdot 6^{1/4} \cdot c \cdot \operatorname{ArcTan}[1 - 6^{1/4} \cdot x] + 2 \cdot 6^{1/4} \cdot c \cdot \operatorname{ArcTan}[1 + 6^{1/4} \cdot x] + 6^{1/4} \cdot c \cdot \operatorname{Log}[2 - 2 \cdot 6^{1/4} \cdot x + \operatorname{Sqrt}[6] \cdot x^2] - 6^{1/4} \cdot c \cdot \operatorname{Log}[2 + 2 \cdot 6^{1/4} \cdot x + \operatorname{Sqrt}[6] \cdot x^2] + 2 \cdot d \cdot \operatorname{Log}[2 + 3 \cdot x^4]) / 24$

Maple [A] time = 0.001, size = 125, normalized size = 1.1

$$\frac{c\sqrt{36^{\frac{3}{4}}}\sqrt{2}}{72} \arctan\left(\frac{\sqrt{2}\sqrt{36^{\frac{3}{4}}x}}{6} - 1\right) + \frac{c\sqrt{36^{\frac{3}{4}}}\sqrt{2}}{144} \ln\left(\left(x^2 - \frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3} + \frac{\sqrt{6}}{3}\right)\left(x^2 + \frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3} + \frac{\sqrt{6}}{3}\right)^{-1}\right) + \frac{c\sqrt{36^{\frac{3}{4}}}\sqrt{2}}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^3+c*x^2)/(3*x^4+2),x)`

[Out] $\frac{1}{72}c^{3/2}6^{3/4}2^{1/2}\arctan\left(\frac{1}{6}2^{1/2}3^{1/2}6^{3/4}x-1\right)+\frac{1}{14}4c^{3/2}6^{3/4}2^{1/2}\ln\left(\frac{x^2-1/33^{1/2}6^{1/4}x^2+1/36^{1/2}}{x^2+1/33^{1/2}6^{1/4}x^2+1/36^{1/2}}\right)+\frac{1}{72}c^{3/2}6^{3/4}2^{1/2}\arctan\left(\frac{1}{6}2^{1/2}3^{1/2}6^{3/4}x+1\right)+\frac{1}{12}d\ln(3x^4+2)$

Maxima [A] time = 1.45013, size = 205, normalized size = 1.8

$$\frac{1}{72} \cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}} \left(3^{\frac{1}{4}} 2^{\frac{3}{4}} d - \sqrt{3}c \right) \log \left(\sqrt{3}x^2 + 3^{\frac{1}{4}} 2^{\frac{3}{4}} x + \sqrt{2} \right) + \frac{1}{72} \cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}} \left(3^{\frac{1}{4}} 2^{\frac{3}{4}} d + \sqrt{3}c \right) \log \left(\sqrt{3}x^2 - 3^{\frac{1}{4}} 2^{\frac{3}{4}} x + \sqrt{2} \right) + \frac{1}{12} \cdot 3^{\frac{1}{4}} 2^{\frac{1}{4}} c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3+c*x^2)/(3*x^4+2),x, algorithm="maxima")`

[Out] $\frac{1}{72}3^{3/4}2^{1/4}(3^{1/4}2^{3/4}d - \sqrt{3}c)\log(\sqrt{3}x^2 + 3^{1/4}2^{3/4}x + \sqrt{2}) + \frac{1}{72}3^{3/4}2^{1/4}(3^{1/4}2^{3/4}d + \sqrt{3}c)\log(\sqrt{3}x^2 - 3^{1/4}2^{3/4}x + \sqrt{2}) + \frac{1}{12}3^{1/4}2^{1/4}c\arctan\left(\frac{1}{6}3^{3/4}2^{1/4}(2\sqrt{3}x + 3^{1/4}2^{3/4})\right) + \frac{1}{12}3^{1/4}2^{1/4}c\arctan\left(\frac{1}{6}3^{3/4}2^{1/4}(2\sqrt{3}x - 3^{1/4}2^{3/4})\right)$

Fricas [B] time = 1.64733, size = 743, normalized size = 6.52

$$4 \cdot 6^{\frac{1}{4}} (c^4)^{\frac{1}{4}} c^4 \arctan \left(\frac{c^5 + 6^{\frac{1}{4}} (c^4)^{\frac{5}{4}} x - 6^{\frac{1}{4}} \sqrt{\frac{1}{3}} (c^4)^{\frac{5}{4}} \sqrt{\frac{3c^3x^2 + 6^{\frac{3}{4}} (c^4)^{\frac{3}{4}} x + \sqrt{6} \sqrt{c^4} c}{c^3}}}{c^5} \right) + 4 \cdot 6^{\frac{1}{4}} (c^4)^{\frac{1}{4}} c^4 \arctan \left(\frac{c^5 - 6^{\frac{1}{4}} (c^4)^{\frac{5}{4}} x + 6^{\frac{1}{4}} \sqrt{\frac{1}{3}} (c^4)^{\frac{5}{4}} \sqrt{\frac{3c^3x^2 - 6^{\frac{3}{4}} (c^4)^{\frac{3}{4}} x + \sqrt{6} \sqrt{c^4} c}{c^3}}}{c^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3+c*x^2)/(3*x^4+2),x, algorithm="fricas")`

[Out] $-\frac{1}{24}(4 \cdot 6^{1/4} (c^4)^{1/4} c^4 \arctan(-c^5 + 6^{1/4} (c^4)^{5/4} x - 6^{1/4} \sqrt{1/3} (c^4)^{5/4} \sqrt{(3c^3x^2 + 6^{3/4} (c^4)^{3/4} x + \sqrt{6} \sqrt{c^4} c) / c^3}) / c^5) + 4 \cdot 6^{1/4} (c^4)^{1/4} c^4 \arctan((c^5 - 6^{1/4} (c^4)^{5/4} x + 6^{1/4} \sqrt{1/3} (c^4)^{5/4} \sqrt{(3c^3x^2 - 6^{3/4} (c^4)^{3/4} x + \sqrt{6} \sqrt{c^4} c) / c^3}) / c^5)$

$$\begin{aligned} & \sqrt[4]{6} \sqrt[4]{c} \sqrt[4]{c^3} / c^5 - (2c^4 d - 6^{1/4} (c^4)^{1/4} c^4) \log(3c^3 x^2 + 6^{3/4} (c^4)^{3/4} x + \sqrt{6} \sqrt[4]{c} c) - (2c^4 d + 6^{1/4} (c^4)^{1/4} c^4) \log(3c^3 x^2 - 6^{3/4} (c^4)^{3/4} x + \sqrt{6} \sqrt[4]{c} c) / c^4 \end{aligned}$$

Sympy [A] time = 0.273372, size = 70, normalized size = 0.61

$$\text{RootSum}\left(41472t^4 - 13824t^3d + 1728t^2d^2 - 96td^3 + 3c^4 + 2d^4, \left(t \mapsto t \log\left(x + \frac{3456t^3 - 864t^2d + 72td^2 - 2d^3}{3c^3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+c*x**2)/(3*x**4+2),x)

[Out] RootSum(41472*_t**4 - 13824*_t**3*d + 1728*_t**2*d**2 - 96*_t*d**3 + 3*c**4 + 2*d**4, Lambda(_t, _t*log(x + (3456*_t**3 - 864*_t**2*d + 72*_t*d**2 - 2*d**3)/(3*c**3))))

Giac [A] time = 1.1086, size = 147, normalized size = 1.29

$$\frac{1}{12} \cdot 6^{1/4} c \arctan\left(\frac{3}{4} \sqrt{2} \left(\frac{2}{3}\right)^{3/4} \left(2x + \sqrt{2} \left(\frac{2}{3}\right)^{1/4}\right)\right) + \frac{1}{12} \cdot 6^{1/4} c \arctan\left(\frac{3}{4} \sqrt{2} \left(\frac{2}{3}\right)^{3/4} \left(2x - \sqrt{2} \left(\frac{2}{3}\right)^{1/4}\right)\right) - \frac{1}{24} (6^{1/4} c - 2d) \log\left(x + \sqrt{2} \left(\frac{2}{3}\right)^{1/4}\right) + \frac{1}{24} (6^{1/4} c + 2d) \log\left(x - \sqrt{2} \left(\frac{2}{3}\right)^{1/4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2)/(3*x^4+2),x, algorithm="giac")

[Out] 1/12*6^(1/4)*c*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/12*6^(1/4)*c*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) - 1/24*(6^(1/4)*c - 2*d)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/24*(6^(1/4)*c + 2*d)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))

$$3.164 \quad \int \frac{a+cx^2+dx^3}{2+3x^4} dx$$

Optimal. Leaf size=154

$$-\frac{(\sqrt{6a-2c})\log(3x^2-6^{3/4}x+\sqrt{6})}{8\ 6^{3/4}} + \frac{(\sqrt{6a-2c})\log(3x^2+6^{3/4}x+\sqrt{6})}{8\ 6^{3/4}} - \frac{(\sqrt{6a+2c})\tan^{-1}(1-\sqrt[4]{6}x)}{4\ 6^{3/4}} + \frac{(\sqrt{6a+2c})\tan^{-1}(1+\sqrt[4]{6}x)}{4\ 6^{3/4}}$$

[Out] -((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4)) - ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + (d*Log[2 + 3*x^4])/12

Rubi [A] time = 0.117019, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1876, 260, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{(\sqrt{6a-2c})\log(3x^2-6^{3/4}x+\sqrt{6})}{8\ 6^{3/4}} + \frac{(\sqrt{6a-2c})\log(3x^2+6^{3/4}x+\sqrt{6})}{8\ 6^{3/4}} - \frac{(\sqrt{6a+2c})\tan^{-1}(1-\sqrt[4]{6}x)}{4\ 6^{3/4}} + \frac{(\sqrt{6a+2c})\tan^{-1}(1+\sqrt[4]{6}x)}{4\ 6^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^2 + d*x^3)/(2 + 3*x^4), x]

[Out] -((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4)) - ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + (d*Log[2 + 3*x^4])/12

Rule 1876

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + cx^2 + dx^3}{2 + 3x^4} dx &= \int \left(\frac{dx^3}{2 + 3x^4} + \frac{a + cx^2}{2 + 3x^4} \right) dx \\
&= d \int \frac{x^3}{2 + 3x^4} dx + \int \frac{a + cx^2}{2 + 3x^4} dx \\
&= \frac{1}{12} d \log(2 + 3x^4) + \frac{1}{12} (\sqrt{6}a - 2c) \int \frac{\sqrt{6} - 3x^2}{2 + 3x^4} dx + \frac{1}{12} (\sqrt{6}a + 2c) \int \frac{\sqrt{6} + 3x^2}{2 + 3x^4} dx \\
&= \frac{1}{12} d \log(2 + 3x^4) - \frac{(\sqrt{6}a - 2c) \int \frac{\frac{2^{3/4}}{\sqrt{3}} + 2x}{-\sqrt{\frac{2}{3} - \frac{2^{3/4}x}{\sqrt{3}} - x^2}} dx}{8 \cdot 6^{3/4}} - \frac{(\sqrt{6}a + 2c) \int \frac{\frac{2^{3/4}}{\sqrt{3}} - 2x}{-\sqrt{\frac{2}{3} + \frac{2^{3/4}x}{\sqrt{3}} - x^2}} dx}{8 \cdot 6^{3/4}} + \frac{1}{24} (\sqrt{6}a + 2c) \log(2 + 3x^4) \\
&= -\frac{(\sqrt{6}a - 2c) \log(\sqrt{6} - 6^{3/4}x + 3x^2)}{8 \cdot 6^{3/4}} + \frac{(\sqrt{6}a - 2c) \log(\sqrt{6} + 6^{3/4}x + 3x^2)}{8 \cdot 6^{3/4}} + \frac{1}{12} d \log(2 + 3x^4) + \\
&= -\frac{(\sqrt{6}a + 2c) \tan^{-1}(1 - \sqrt[4]{6}x)}{4 \cdot 6^{3/4}} + \frac{(\sqrt{6}a + 2c) \tan^{-1}(1 + \sqrt[4]{6}x)}{4 \cdot 6^{3/4}} - \frac{(\sqrt{6}a - 2c) \log(\sqrt{6} - 6^{3/4}x + 3x^2)}{8 \cdot 6^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.092163, size = 148, normalized size = 0.96

$$\frac{1}{48} \left(-\sqrt[4]{6} (\sqrt{6}a - 2c) \log(\sqrt{6}x^2 - 2\sqrt[4]{6}x + 2) + \sqrt[4]{6} (\sqrt{6}a - 2c) \log(\sqrt{6}x^2 + 2\sqrt[4]{6}x + 2) - 2\sqrt[4]{6} (\sqrt{6}a + 2c) \tan^{-1}(1 - \sqrt[4]{6}x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^2 + d*x^3)/(2 + 3*x^4), x]

[Out] $(-2 \cdot 6^{1/4} \cdot (\text{Sqrt}[6] \cdot a + 2 \cdot c) \cdot \text{ArcTan}[1 - 6^{1/4} \cdot x] + 2 \cdot 6^{1/4} \cdot (\text{Sqrt}[6] \cdot a + 2 \cdot c) \cdot \text{ArcTan}[1 + 6^{1/4} \cdot x] - 6^{1/4} \cdot (\text{Sqrt}[6] \cdot a - 2 \cdot c) \cdot \text{Log}[2 - 2 \cdot 6^{1/4} \cdot x + \text{Sqrt}[6] \cdot x^2] + 6^{1/4} \cdot (\text{Sqrt}[6] \cdot a - 2 \cdot c) \cdot \text{Log}[2 + 2 \cdot 6^{1/4} \cdot x + \text{Sqrt}[6] \cdot x^2] + 4 \cdot d \cdot \text{Log}[2 + 3 \cdot x^4]) / 48$

Maple [B] time = 0.041, size = 237, normalized size = 1.5

$$\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24} \arctan\left(\frac{\sqrt{2}\sqrt{36^{\frac{3}{4}}x}}{6} - 1\right) + \frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{48} \ln\left(\left(x^2 + \frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3} + \frac{\sqrt{6}}{3}\right)\left(x^2 - \frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3} + \frac{\sqrt{6}}{3}\right)^{-1}\right) + \frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^3+c*x^2+a)/(3*x^4+2),x)`

[Out] $\frac{1}{24}a3^{1/2}6^{1/4}2^{1/2}\arctan\left(\frac{1}{6}2^{1/2}3^{1/2}6^{3/4}x-1\right)+\frac{1}{48}a3^{1/2}6^{1/4}2^{1/2}\ln\left(\frac{x^2+1/33^{1/2}6^{1/4}x2^{1/2}+1/36^{1/2}}{x^2-1/33^{1/2}6^{1/4}x2^{1/2}+1/36^{1/2}}\right)+\frac{1}{24}a3^{1/2}6^{1/4}2^{1/2}\arctan\left(\frac{1}{6}2^{1/2}3^{1/2}6^{3/4}x+1\right)+\frac{1}{72}c3^{1/2}6^{3/4}2^{1/2}\arctan\left(\frac{1}{6}2^{1/2}3^{1/2}6^{3/4}x-1\right)+\frac{1}{144}c3^{1/2}6^{3/4}2^{1/2}\ln\left(\frac{x^2-1/33^{1/2}6^{1/4}x2^{1/2}+1/36^{1/2}}{x^2+1/33^{1/2}6^{1/4}x2^{1/2}+1/36^{1/2}}\right)+\frac{1}{72}c3^{1/2}6^{3/4}2^{1/2}\arctan\left(\frac{1}{6}2^{1/2}3^{1/2}6^{3/4}x+1\right)+\frac{1}{12}d\ln(3x^4+2)$

Maxima [A] time = 1.45753, size = 263, normalized size = 1.71

$$-\frac{1}{144} \cdot 3^{\frac{3}{4}} 2^{\frac{3}{4}} \left(\sqrt{3}\sqrt{2}c - 2 \cdot 3^{\frac{1}{4}} 2^{\frac{1}{4}} d - 3a \right) \log\left(\sqrt{3}x^2 + 3^{\frac{1}{4}} 2^{\frac{3}{4}} x + \sqrt{2}\right) + \frac{1}{144} \cdot 3^{\frac{3}{4}} 2^{\frac{3}{4}} \left(\sqrt{3}\sqrt{2}c + 2 \cdot 3^{\frac{1}{4}} 2^{\frac{1}{4}} d - 3a \right) \log\left(\sqrt{3}x^2 - 3^{\frac{1}{4}} 2^{\frac{3}{4}} x + \sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3+c*x^2+a)/(3*x^4+2),x, algorithm="maxima")`

[Out] $-1/144 \cdot 3^{3/4} 2^{3/4} (\sqrt{3}\sqrt{2}c - 2 \cdot 3^{1/4} 2^{1/4} d - 3a) \log(\sqrt{3}x^2 + 3^{1/4} 2^{3/4} x + \sqrt{2}) + 1/144 \cdot 3^{3/4} 2^{3/4} (\sqrt{3}\sqrt{2}c + 2 \cdot 3^{1/4} 2^{1/4} d - 3a) \log(\sqrt{3}x^2 - 3^{1/4} 2^{3/4} x + \sqrt{2}) + 1/72 \sqrt{3} (3 \cdot 3^{1/4} 2^{3/4} a + 2 \cdot 3^{3/4} 2^{1/4} c) \arctan\left(\frac{1}{6} 3^{3/4} 2^{1/4} (2\sqrt{3}x + 3^{1/4} 2^{3/4})\right) + 1/72 \sqrt{3} (3 \cdot 3^{1/4} 2^{3/4} a + 2 \cdot 3^{3/4} 2^{1/4} c) \arctan\left(\frac{1}{6} 3^{3/4} 2^{1/4} (2\sqrt{3}x - 3^{1/4} 2^{3/4})\right)$

Fricas [B] time = 1.95114, size = 5162, normalized size = 33.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3+c*x^2+a)/(3*x^4+2),x, algorithm="fricas")`

[Out] $\frac{1}{144} (2\sqrt{6}\sqrt{2}) (54a^4 + 72a^2c^2 + 24c^4)^{3/4} \sqrt{9a^4 - 12a^2c^2 + 4c^4} \sqrt{(9a^4 + 12a^2c^2 + 4c^4 + 2\sqrt{54a^4 + 72a^2c^2 + 24c^4})ac} / (9a^4 - 12a^2c^2 + 4c^4) \arctan\left(-\frac{1}{12}(\sqrt{2})s\right)$

$$\begin{aligned}
& \text{qrt}(1/3)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^{(3/4)}*(\text{sqrt}(6)*\text{sqrt}(54*a^4 + 72*a^2*c^2 \\
& *c^2 + 24*c^4)*\text{sqrt}(9*a^4 - 12*a^2*c^2 + 4*c^4)*a - 2*\text{sqrt}(6)*\text{sqrt}(9*a^4 - \\
& 12*a^2*c^2 + 4*c^4)*(3*a^2*c + 2*c^3))*\text{sqrt}((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2 \\
& *\text{sqrt}(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4))*\text{sqrt} \\
& ((3*(9*a^4 + 12*a^2*c^2 + 4*c^4)*x^2 + \text{sqrt}(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4) \\
&)^{(1/4)}*(\text{sqrt}(54*a^4 + 72*a^2*c^2 + 24*c^4)*c*x - 3*(3*a^3 + 2*a*c^2)*x)*\text{s} \\
& \text{qrt}((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*\text{sqrt}(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c) \\
& / (9*a^4 - 12*a^2*c^2 + 4*c^4)) + \text{sqrt}(54*a^4 + 72*a^2*c^2 + 24*c^4)*(3*a^2 \\
& + 2*c^2))/(9*a^4 + 12*a^2*c^2 + 4*c^4)) - \text{sqrt}(2)*(54*a^4 + 72*a^2*c^2 + 24 \\
& *c^4)^{(3/4)}*(\text{sqrt}(6)*\text{sqrt}(54*a^4 + 72*a^2*c^2 + 24*c^4)*\text{sqrt}(9*a^4 - 12*a^2 \\
& *c^2 + 4*c^4)*a*x - 2*\text{sqrt}(6)*\text{sqrt}(9*a^4 - 12*a^2*c^2 + 4*c^4)*(3*a^2*c + 2 \\
& *c^3)*x)*\text{sqrt}((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*\text{sqrt}(54*a^4 + 72*a^2*c^2 + 24 \\
& *c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) + 2*\text{sqrt}(6)*\text{sqrt}(54*a^4 + 72*a^2*c \\
& ^2 + 24*c^4)*(9*a^4 + 12*a^2*c^2 + 4*c^4)*\text{sqrt}(9*a^4 - 12*a^2*c^2 + 4*c^4) \\
& / (81*a^8 + 108*a^6*c^2 - 48*a^2*c^6 - 16*c^8)) + 2*\text{sqrt}(6)*\text{sqrt}(2)*(54*a^4 \\
& + 72*a^2*c^2 + 24*c^4)^{(3/4)}*\text{sqrt}(9*a^4 - 12*a^2*c^2 + 4*c^4)*\text{sqrt}((9*a^4 + \\
& 12*a^2*c^2 + 4*c^4 + 2*\text{sqrt}(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12 \\
& *a^2*c^2 + 4*c^4))*\text{arctan}(-1/12*(\text{sqrt}(2)*\text{sqrt}(1/3))*(54*a^4 + 72*a^2*c^2 + 2 \\
& 4*c^4)^{(3/4)}*(\text{sqrt}(6)*\text{sqrt}(54*a^4 + 72*a^2*c^2 + 24*c^4)*\text{sqrt}(9*a^4 - 12*a^ \\
& 2*c^2 + 4*c^4)*a - 2*\text{sqrt}(6)*\text{sqrt}(9*a^4 - 12*a^2*c^2 + 4*c^4)*(3*a^2*c + 2 \\
& *c^3))*\text{sqrt}((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*\text{sqrt}(54*a^4 + 72*a^2*c^2 + 24*c^ \\
& 4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4))*\text{sqrt}((3*(9*a^4 + 12*a^2*c^2 + 4*c^4)* \\
& x^2 - \text{sqrt}(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^{(1/4)}*(\text{sqrt}(54*a^4 + 72*a^2*c^ \\
& 2 + 24*c^4)*c*x - 3*(3*a^3 + 2*a*c^2)*x)*\text{sqrt}((9*a^4 + 12*a^2*c^2 + 4*c^4 + \\
& 2*\text{sqrt}(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) + \\
& \text{sqrt}(54*a^4 + 72*a^2*c^2 + 24*c^4)*(3*a^2 + 2*c^2))/(9*a^4 + 12*a^2*c^2 + 4 \\
& *c^4)) - \text{sqrt}(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^{(3/4)}*(\text{sqrt}(6)*\text{sqrt}(54*a^4 \\
& + 72*a^2*c^2 + 24*c^4)*\text{sqrt}(9*a^4 - 12*a^2*c^2 + 4*c^4)*a*x - 2*\text{sqrt}(6)*\text{sqr} \\
& \text{t}(9*a^4 - 12*a^2*c^2 + 4*c^4)*(3*a^2*c + 2*c^3)*x)*\text{sqrt}((9*a^4 + 12*a^2*c^2 \\
& + 4*c^4 + 2*\text{sqrt}(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + \\
& 4*c^4)) - 2*\text{sqrt}(6)*\text{sqrt}(54*a^4 + 72*a^2*c^2 + 24*c^4)*(9*a^4 + 12*a^2*c^2 \\
& + 4*c^4)*\text{sqrt}(9*a^4 - 12*a^2*c^2 + 4*c^4)/(81*a^8 + 108*a^6*c^2 - 48*a^2*c \\
& ^6 - 16*c^8)) - 3*(\text{sqrt}(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^{(1/4)}*(9*a^4 + 12 \\
& *a^2*c^2 + 4*c^4 - 2*\text{sqrt}(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)*\text{sqrt}((9*a^4 + \\
& 12*a^2*c^2 + 4*c^4 + 2*\text{sqrt}(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12* \\
& a^2*c^2 + 4*c^4)) - 4*(9*a^4 + 12*a^2*c^2 + 4*c^4)*d)*\log(3*(9*a^4 + 12*a^2 \\
& *c^2 + 4*c^4)*x^2 + \text{sqrt}(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^{(1/4)}*(\text{sqrt}(54*a \\
& ^4 + 72*a^2*c^2 + 24*c^4)*c*x - 3*(3*a^3 + 2*a*c^2)*x)*\text{sqrt}((9*a^4 + 12*a^2 \\
& *c^2 + 4*c^4 + 2*\text{sqrt}(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^ \\
& 2 + 4*c^4)) + \text{sqrt}(54*a^4 + 72*a^2*c^2 + 24*c^4)*(3*a^2 + 2*c^2)) + 3*(\text{sqrt} \\
& (2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^{(1/4)}*(9*a^4 + 12*a^2*c^2 + 4*c^4 - 2*\text{sqr} \\
& \text{t}(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)*\text{sqrt}((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2* \\
& \text{sqrt}(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) + 4*(\\
& 9*a^4 + 12*a^2*c^2 + 4*c^4)*d)*\log(3*(9*a^4 + 12*a^2*c^2 + 4*c^4)*x^2 - \text{sqr} \\
& \text{t}(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^{(1/4)}*(\text{sqrt}(54*a^4 + 72*a^2*c^2 + 24*c^
\end{aligned}$$

$$4)*c*x - 3*(3*a^3 + 2*a*c^2)*x)*\sqrt{(9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*\sqrt{54*a^4 + 72*a^2*c^2 + 24*c^4})*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) + \sqrt{(54*a^4 + 72*a^2*c^2 + 24*c^4)*(3*a^2 + 2*c^2)))/(9*a^4 + 12*a^2*c^2 + 4*c^4)}$$

Sympy [A] time = 1.07802, size = 148, normalized size = 0.96

$$\text{RootSum}\left(165888t^4 - 55296t^3d + t^2(6912ac + 6912d^2) + t(-1152acd - 384d^3) + 27a^4 + 36a^2c^2 + 48acd^2 + 12c^4 + 8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+c*x**2+a)/(3*x**4+2),x)

[Out] RootSum(165888*_t**4 - 55296*_t**3*d + _t**2*(6912*a*c + 6912*d**2) + _t*(-1152*a*c*d - 384*d**3) + 27*a**4 + 36*a**2*c**2 + 48*a*c*d**2 + 12*c**4 + 8*d**4, Lambda(_t, _t*log(x + (-13824*_t**3*c + 3456*_t**2*c*d + 216*_t*a**3 - 432*_t*a*c**2 - 288*_t*c*d**2 - 18*a**3*d + 36*a*c**2*d + 8*c*d**3)/(27*a**4 - 12*c**4))))

Giac [A] time = 1.10467, size = 185, normalized size = 1.2

$$\frac{1}{24} \left(6^{\frac{3}{4}}a + 2 \cdot 6^{\frac{1}{4}}c\right) \arctan\left(\frac{3}{4} \sqrt{2} \left(\frac{2}{3}\right)^{\frac{3}{4}} \left(2x + \sqrt{2} \left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{24} \left(6^{\frac{3}{4}}a + 2 \cdot 6^{\frac{1}{4}}c\right) \arctan\left(\frac{3}{4} \sqrt{2} \left(\frac{2}{3}\right)^{\frac{3}{4}} \left(2x - \sqrt{2} \left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+a)/(3*x^4+2),x, algorithm="giac")

[Out] 1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2))*(2/3)^(1/4)) + 1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(3/4)*a - 2*6^(1/4)*c + 4*d)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*(6^(3/4)*a - 2*6^(1/4)*c - 4*d)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))

$$3.165 \quad \int \frac{bx+cx^2+dx^3}{2+3x^4} dx$$

Optimal. Leaf size=136

$$\frac{b \tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} + \frac{c \log\left(3x^2 - 6^{3/4}x + \sqrt{6}\right)}{4 \cdot 6^{3/4}} - \frac{c \log\left(3x^2 + 6^{3/4}x + \sqrt{6}\right)}{4 \cdot 6^{3/4}} - \frac{c \tan^{-1}\left(1 - \sqrt[4]{6}x\right)}{2 \cdot 6^{3/4}} + \frac{c \tan^{-1}\left(\sqrt[4]{6}x + 1\right)}{2 \cdot 6^{3/4}} + \frac{1}{12}d$$

[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (c*ArcTan[1 - 6^(1/4)*x])/(2*6^(3/4)) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/4)) + (c*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) + (d*Log[2 + 3*x^4])/12

Rubi [A] time = 0.139748, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {1594, 1831, 297, 1162, 617, 204, 1165, 628, 1248, 635, 203, 260}

$$\frac{b \tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} + \frac{c \log\left(3x^2 - 6^{3/4}x + \sqrt{6}\right)}{4 \cdot 6^{3/4}} - \frac{c \log\left(3x^2 + 6^{3/4}x + \sqrt{6}\right)}{4 \cdot 6^{3/4}} - \frac{c \tan^{-1}\left(1 - \sqrt[4]{6}x\right)}{2 \cdot 6^{3/4}} + \frac{c \tan^{-1}\left(\sqrt[4]{6}x + 1\right)}{2 \cdot 6^{3/4}} + \frac{1}{12}d$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2 + d*x^3)/(2 + 3*x^4), x]

[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (c*ArcTan[1 - 6^(1/4)*x])/(2*6^(3/4)) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/4)) + (c*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) + (d*Log[2 + 3*x^4])/12

Rule 1594

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1831

Int[((Pq_)*((c_)*(x_)^(m_)))/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v] /; FreeQ[{

a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 297

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] & AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 1162

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1165

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
  :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
  [{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{bx + cx^2 + dx^3}{2 + 3x^4} dx &= \int \frac{x(b + cx + dx^2)}{2 + 3x^4} dx \\
&= \int \left(\frac{cx^2}{2 + 3x^4} + \frac{x(b + dx^2)}{2 + 3x^4} \right) dx \\
&= c \int \frac{x^2}{2 + 3x^4} dx + \int \frac{x(b + dx^2)}{2 + 3x^4} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{b + dx}{2 + 3x^2} dx, x, x^2 \right) - \frac{c \int \frac{\sqrt{2} - \sqrt{3}x^2}{2 + 3x^4} dx}{2\sqrt{3}} + \frac{c \int \frac{\sqrt{2} + \sqrt{3}x^2}{2 + 3x^4} dx}{2\sqrt{3}} \\
&= \frac{1}{2} b \text{Subst} \left(\int \frac{1}{2 + 3x^2} dx, x, x^2 \right) + \frac{1}{12} c \int \frac{1}{\sqrt{\frac{2}{3}} - \frac{2^{3/4}x}{\sqrt{3}} + x^2} dx + \frac{1}{12} c \int \frac{1}{\sqrt{\frac{2}{3}} + \frac{2^{3/4}x}{\sqrt{3}} + x^2} dx + \dots \\
&= \frac{b \tan^{-1} \left(\sqrt{\frac{3}{2}} x^2 \right)}{2\sqrt{6}} + \frac{c \log(\sqrt{6} - 6^{3/4}x + 3x^2)}{4 \cdot 6^{3/4}} - \frac{c \log(\sqrt{6} + 6^{3/4}x + 3x^2)}{4 \cdot 6^{3/4}} + \frac{1}{12} d \log(2 + 3x^4) + \dots \\
&= \frac{b \tan^{-1} \left(\sqrt{\frac{3}{2}} x^2 \right)}{2\sqrt{6}} - \frac{c \tan^{-1}(1 - \sqrt[4]{6}x)}{2 \cdot 6^{3/4}} + \frac{c \tan^{-1}(1 + \sqrt[4]{6}x)}{2 \cdot 6^{3/4}} + \frac{c \log(\sqrt{6} - 6^{3/4}x + 3x^2)}{4 \cdot 6^{3/4}} - \frac{c \log(\sqrt{6} + 6^{3/4}x + 3x^2)}{4 \cdot 6^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.0625569, size = 125, normalized size = 0.92

$$\frac{1}{24} \left(-2\sqrt[4]{6} (\sqrt[4]{6}b + c) \tan^{-1} \left(1 - \sqrt[4]{6}x \right) + 2\sqrt[4]{6} (c - \sqrt[4]{6}b) \tan^{-1} \left(\sqrt[4]{6}x + 1 \right) + \sqrt[4]{6}c \log(\sqrt{6}x^2 - 2\sqrt[4]{6}x + 2) - \sqrt[4]{6}c \log(\sqrt{6}x^2 + 2\sqrt[4]{6}x + 2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2 + d*x^3)/(2 + 3*x^4), x]

[Out] $(-2 \cdot 6^{1/4} \cdot (6^{1/4} \cdot b + c) \cdot \text{ArcTan}[1 - 6^{1/4} \cdot x] + 2 \cdot 6^{1/4} \cdot (c - 6^{1/4} \cdot b) \cdot \text{ArcTan}[1 + 6^{1/4} \cdot x] + 6^{1/4} \cdot c \cdot \text{Log}[2 - 2 \cdot 6^{1/4} \cdot x + \text{Sqrt}[6] \cdot x^2] - 6^{1/4} \cdot c \cdot \text{Log}[2 + 2 \cdot 6^{1/4} \cdot x + \text{Sqrt}[6] \cdot x^2] + 2 \cdot d \cdot \text{Log}[2 + 3 \cdot x^4]) / 24$

Maple [A] time = 0.046, size = 140, normalized size = 1.

$$\frac{b\sqrt{6}}{12} \arctan\left(\frac{x^2\sqrt{6}}{2}\right) + \frac{c\sqrt{36^{\frac{3}{4}}}\sqrt{2}}{72} \arctan\left(\frac{\sqrt{2}\sqrt{36^{\frac{3}{4}}}x}{6} - 1\right) + \frac{c\sqrt{36^{\frac{3}{4}}}\sqrt{2}}{144} \ln\left(\left(x^2 - \frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3} + \frac{\sqrt{6}}{3}\right)\left(x^2 + \frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^3+c*x^2+b*x)/(3*x^4+2),x)`

[Out] $\frac{1}{12}b \arctan\left(\frac{1}{2}x^2\sqrt{6}\right)\sqrt{6} + \frac{1}{72}c\sqrt{3}\sqrt{6}^{3/4}2^{1/2} \arctan\left(\frac{1}{6}2^{1/2}\sqrt{3}\sqrt{6}^{3/4}x-1\right) + \frac{1}{144}c\sqrt{3}\sqrt{6}^{3/4}2^{1/2} \ln\left(\frac{x^2-1/3\sqrt{3}\sqrt{6}^{1/4}x\sqrt{2}+1/3\sqrt{6}^{1/2}}{x^2+1/3\sqrt{3}\sqrt{6}^{1/4}x\sqrt{2}+1/3\sqrt{6}^{1/2}}\right) + \frac{1}{72}c\sqrt{3}\sqrt{6}^{3/4}2^{1/2} \arctan\left(\frac{1}{6}2^{1/2}\sqrt{3}\sqrt{6}^{3/4}x+1\right) + \frac{1}{12}d \ln(3x^4+2)$

Maxima [A] time = 1.45402, size = 235, normalized size = 1.73

$$\frac{1}{72} \sqrt{3} \sqrt{2} \left(3^{\frac{3}{4}} 2^{\frac{3}{4}} c - 6b\right) \arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}} \left(2\sqrt{3}x + 3^{\frac{1}{4}} 2^{\frac{3}{4}}\right)\right) + \frac{1}{72} \sqrt{3} \sqrt{2} \left(3^{\frac{3}{4}} 2^{\frac{3}{4}} c + 6b\right) \arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}} \left(2\sqrt{3}x - 3^{\frac{1}{4}} 2^{\frac{3}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3+c*x^2+b*x)/(3*x^4+2),x, algorithm="maxima")`

[Out] $\frac{1}{72} \sqrt{3} \sqrt{2} (3^{3/4} 2^{3/4} c - 6b) \arctan\left(\frac{1}{6} 3^{3/4} 2^{1/4} (2\sqrt{3}x + 3^{1/4} 2^{3/4})\right) + \frac{1}{72} \sqrt{3} \sqrt{2} (3^{3/4} 2^{3/4} c + 6b) \arctan\left(\frac{1}{6} 3^{3/4} 2^{1/4} (2\sqrt{3}x - 3^{1/4} 2^{3/4})\right) + \frac{1}{72} 3^{3/4} 2^{1/4} (3^{1/4} 2^{3/4} d - \sqrt{3}c) \log(\sqrt{3}x^2 + 3^{1/4} 2^{3/4} x + \sqrt{2}) + \frac{1}{72} 3^{3/4} 2^{1/4} (3^{1/4} 2^{3/4} d + \sqrt{3}c) \log(\sqrt{3}x^2 - 3^{1/4} 2^{3/4} x + \sqrt{2})$

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3+c*x^2+b*x)/(3*x^4+2),x, algorithm="fricas")`

[Out] Timed out

Sympy [A] time = 1.1706, size = 189, normalized size = 1.39

$$\text{RootSum}\left(82944t^4 - 27648t^3d + t^2(1728b^2 + 3456d^2) + t(-288b^2d + 288bc^2 - 192d^3) + 9b^4 + 12b^2d^2 - 24bc^2d + 6c^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+c*x**2+b*x)/(3*x**4+2),x)

[Out] RootSum(82944*_t**4 - 27648*_t**3*d + _t**2*(1728*b**2 + 3456*d**2) + _t*(-288*b**2*d + 288*b*c**2 - 192*d**3) + 9*b**4 + 12*b**2*d**2 - 24*b*c**2*d + 6*c**4 + 4*d**4, Lambda(_t, _t*log(x + (-3456*_t**3*c**2 + 864*_t**2*b**3 + 864*_t**2*c**2*d - 144*_t*b**3*d - 108*_t*b**2*c**2 - 72*_t*c**2*d**2 + 9*b**5 + 6*b**3*d**2 + 9*b**2*c**2*d - 9*b*c**4 + 2*c**2*d**3)/(18*b**4*c - 3*c**5))))

Giac [A] time = 1.10254, size = 167, normalized size = 1.23

$$-\frac{1}{12}\left(\sqrt{6b} - 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{12}\left(\sqrt{6b} + 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) - \frac{1}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+b*x)/(3*x^4+2),x, algorithm="giac")

[Out] -1/12*(sqrt(6)*b - 6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/12*(sqrt(6)*b + 6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) - 1/24*(6^(1/4)*c - 2*d)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/24*(6^(1/4)*c + 2*d)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))

$$3.166 \quad \int \frac{a+bx+cx^2+dx^3}{2+3x^4} dx$$

Optimal. Leaf size=176

$$-\frac{(\sqrt{6a-2c})\log(3x^2-6^{3/4}x+\sqrt{6})}{8\ 6^{3/4}} + \frac{(\sqrt{6a-2c})\log(3x^2+6^{3/4}x+\sqrt{6})}{8\ 6^{3/4}} - \frac{(\sqrt{6a+2c})\tan^{-1}(1-\sqrt[4]{6}x)}{4\ 6^{3/4}} + \frac{(\sqrt{6a+2c})\tan^{-1}(1+\sqrt[4]{6}x)}{4\ 6^{3/4}}$$

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - ((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4)) - ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + (d*Log[2 + 3*x^4])/12
```

Rubi [A] time = 0.143739, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.44$, Rules used = {1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 203, 260}

$$-\frac{(\sqrt{6a-2c})\log(3x^2-6^{3/4}x+\sqrt{6})}{8\ 6^{3/4}} + \frac{(\sqrt{6a-2c})\log(3x^2+6^{3/4}x+\sqrt{6})}{8\ 6^{3/4}} - \frac{(\sqrt{6a+2c})\tan^{-1}(1-\sqrt[4]{6}x)}{4\ 6^{3/4}} + \frac{(\sqrt{6a+2c})\tan^{-1}(1+\sqrt[4]{6}x)}{4\ 6^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2 + d*x^3)/(2 + 3*x^4), x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - ((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4)) - ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + (d*Log[2 + 3*x^4])/12
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 1168


```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol]
:= Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned}
 \int \frac{a + bx + cx^2 + dx^3}{2 + 3x^4} dx &= \int \left(\frac{a + cx^2}{2 + 3x^4} + \frac{x(b + dx^2)}{2 + 3x^4} \right) dx \\
 &= \int \frac{a + cx^2}{2 + 3x^4} dx + \int \frac{x(b + dx^2)}{2 + 3x^4} dx \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{b + dx}{2 + 3x^2} dx, x, x^2 \right) + \frac{1}{12} (\sqrt{6}a - 2c) \int \frac{\sqrt{6} - 3x^2}{2 + 3x^4} dx + \frac{1}{12} (\sqrt{6}a + 2c) \int \frac{\sqrt{6} + 3x^2}{2 + 3x^4} dx \\
 &= \frac{1}{2} b \text{Subst} \left(\int \frac{1}{2 + 3x^2} dx, x, x^2 \right) - \frac{(\sqrt{6}a - 2c) \int \frac{\frac{2^{3/4} + 2x}{\sqrt[4]{3}}}{-\sqrt{\frac{2}{3} - \frac{2^{3/4}x}{\sqrt[4]{3}} - x^2}} dx}{8 \cdot 6^{3/4}} - \frac{(\sqrt{6}a - 2c) \int \frac{\frac{2^{3/4} - 2x}{\sqrt[4]{3}}}{-\sqrt{\frac{2}{3} + \frac{2^{3/4}x}{\sqrt[4]{3}} - x^2}} dx}{8 \cdot 6^{3/4}} \\
 &= \frac{b \tan^{-1} \left(\sqrt{\frac{3}{2}} x^2 \right)}{2\sqrt{6}} - \frac{(\sqrt{6}a - 2c) \log(\sqrt{6} - 6^{3/4}x + 3x^2)}{8 \cdot 6^{3/4}} + \frac{(\sqrt{6}a - 2c) \log(\sqrt{6} + 6^{3/4}x + 3x^2)}{8 \cdot 6^{3/4}} \\
 &= \frac{b \tan^{-1} \left(\sqrt{\frac{3}{2}} x^2 \right)}{2\sqrt{6}} - \frac{(\sqrt{6}a + 2c) \tan^{-1} \left(1 - \sqrt[4]{6}x \right)}{4 \cdot 6^{3/4}} + \frac{(\sqrt{6}a + 2c) \tan^{-1} \left(1 + \sqrt[4]{6}x \right)}{4 \cdot 6^{3/4}} - \frac{(\sqrt{6}a - 2c) \log(\sqrt{6} - 6^{3/4}x + 3x^2)}{8 \cdot 6^{3/4}} + \frac{(\sqrt{6}a - 2c) \log(\sqrt{6} + 6^{3/4}x + 3x^2)}{8 \cdot 6^{3/4}}
 \end{aligned}$$

Mathematica [A] time = 0.104701, size = 164, normalized size = 0.93

$$\frac{1}{48} \left(-2\sqrt[4]{6} \tan^{-1} \left(1 - \sqrt[4]{6}x \right) \left(\sqrt{6}a + 2 \left(\sqrt[4]{6}bx + c \right) \right) + 2\sqrt[4]{6} \tan^{-1} \left(\sqrt[4]{6}x + 1 \right) \left(\sqrt{6}a - 2\sqrt[4]{6}bx + 2c \right) - \sqrt[4]{6} \left(\sqrt{6}a - 2c \right) \log \left(\sqrt{6}x^2 \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2 + d*x^3)/(2 + 3*x^4), x]

[Out] $(-2*6^{1/4}*(\text{Sqrt}[6]*a + 2*(6^{1/4}*b + c))*\text{ArcTan}[1 - 6^{1/4}*x] + 2*6^{1/4}*(\text{Sqrt}[6]*a - 2*6^{1/4}*b + 2*c)*\text{ArcTan}[1 + 6^{1/4}*x] - 6^{1/4}*(\text{Sqrt}[6]*a - 2*c)*\text{Log}[2 - 2*6^{1/4}*x + \text{Sqrt}[6]*x^2] + 6^{1/4}*(\text{Sqrt}[6]*a - 2*c)*\text{Log}[2 + 2*6^{1/4}*x + \text{Sqrt}[6]*x^2] + 4*d*\text{Log}[2 + 3*x^4])/48$

Maple [A] time = 0.003, size = 252, normalized size = 1.4

$$\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24} \arctan\left(\frac{\sqrt{2}\sqrt[3]{36^{\frac{3}{4}}x}}{6} - 1\right) + \frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{48} \ln\left(\left(x^2 + \frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3} + \frac{\sqrt{6}}{3}\right)\left(x^2 - \frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3} + \frac{\sqrt{6}}{3}\right)^{-1}\right) + \frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+c*x^2+b*x+a)/(3*x^4+2), x)

[Out] $1/24*a*3^{1/2}*6^{1/4}*2^{1/2}*\arctan(1/6*2^{1/2}*3^{1/2}*6^{3/4}*x-1)+1/48*a*3^{1/2}*6^{1/4}*2^{1/2}*\ln((x^2+1/3*3^{1/2}*6^{1/4}*x*2^{1/2}+1/3*6^{1/2}))/((x^2-1/3*3^{1/2}*6^{1/4}*x*2^{1/2}+1/3*6^{1/2}))+1/24*a*3^{1/2}*6^{1/4}*2^{1/2}*\arctan(1/6*2^{1/2}*3^{1/2}*6^{3/4}*x+1)+1/12*b*\arctan(1/2*x^2*6^{1/2})*6^{1/2}+1/72*c*3^{1/2}*6^{3/4}*2^{1/2}*\arctan(1/6*2^{1/2}*3^{1/2}*6^{3/4}*x-1)+1/144*c*3^{1/2}*6^{3/4}*2^{1/2}*\ln((x^2-1/3*3^{1/2}*6^{1/4}*x*2^{1/2}+1/3*6^{1/2}))/((x^2+1/3*3^{1/2}*6^{1/4}*x*2^{1/2}+1/3*6^{1/2}))+1/72*c*3^{1/2}*6^{3/4}*2^{1/2}*\arctan(1/6*2^{1/2}*3^{1/2}*6^{3/4}*x+1)+1/12*d*\ln(3*x^4+2)$

Maxima [A] time = 1.48142, size = 279, normalized size = 1.59

$$-\frac{1}{144} \cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\sqrt{2}c - 2 \cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d - 3a\right) \log\left(\sqrt{3}x^2 + 3^{\frac{1}{4}}2^{\frac{3}{4}}x + \sqrt{2}\right) + \frac{1}{144} \cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\sqrt{2}c + 2 \cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d - 3a\right) \log\left(\sqrt{3}x^2 - 3^{\frac{1}{4}}2^{\frac{3}{4}}x + \sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+b*x+a)/(3*x^4+2), x, algorithm="maxima")

[Out] $-1/144*3^{3/4}*2^{3/4}*(\text{sqrt}(3)*\text{sqrt}(2)*c - 2*3^{1/4}*2^{1/4}*d - 3*a)*\log(\text{sqrt}(3)*x^2 + 3^{1/4}*2^{3/4}*x + \text{sqrt}(2)) + 1/144*3^{3/4}*2^{3/4}*(\text{sqrt}(3)$

$$\begin{aligned} & \sqrt{2}c + 2 \cdot 3^{1/4} \cdot 2^{1/4} \cdot d - 3a \cdot \log(\sqrt{3} \cdot x^2 - 3^{1/4} \cdot 2^{3/4} \cdot x \\ & + \sqrt{2}) + 1/72 \cdot \sqrt{3} \cdot (3 \cdot 3^{1/4} \cdot 2^{3/4} \cdot a + 2 \cdot 3^{3/4} \cdot 2^{1/4} \cdot c - 6 \cdot \sqrt{2} \cdot b) \cdot \arctan(1/6 \cdot 3^{3/4} \cdot 2^{1/4} \cdot (2 \cdot \sqrt{3} \cdot x + 3^{1/4} \cdot 2^{3/4})) + 1/7 \\ & 2 \cdot \sqrt{3} \cdot (3 \cdot 3^{1/4} \cdot 2^{3/4} \cdot a + 2 \cdot 3^{3/4} \cdot 2^{1/4} \cdot c + 6 \cdot \sqrt{2} \cdot b) \cdot \arctan(\\ & 1/6 \cdot 3^{3/4} \cdot 2^{1/4} \cdot (2 \cdot \sqrt{3} \cdot x - 3^{1/4} \cdot 2^{3/4})) \end{aligned}$$

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+b*x+a)/(3*x^4+2),x, algorithm="fricas")

[Out] Timed out

Sympy [B] time = 6.53729, size = 580, normalized size = 3.3

$$\text{RootSum}\left(165888t^4 - 55296t^3d + t^2(6912ac + 3456b^2 + 6912d^2) + t(-864a^2b - 1152acd - 576b^2d + 576bc^2 - 384d^3)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+c*x**2+b*x+a)/(3*x**4+2),x)

[Out] RootSum(165888*_t**4 - 55296*_t**3*d + _t**2*(6912*a*c + 3456*b**2 + 6912*d**2) + _t*(-864*a**2*b - 1152*a*c*d - 576*b**2*d + 576*b*c**2 - 384*d**3) + 27*a**4 + 72*a**2*b*d + 36*a**2*c**2 - 72*a*b**2*c + 48*a*c*d**2 + 18*b**4 + 24*b**2*d**2 - 48*b*c**2*d + 12*c**4 + 8*d**4, Lambda(_t, _t*log(x + (-41472*_t**3*a**2*c + 82944*_t**3*a*b**2 + 27648*_t**3*c**3 + 5184*_t**2*a**3*b + 10368*_t**2*a**2*c*d - 20736*_t**2*a*b**2*d + 10368*_t**2*a*b*c**2 - 6912*_t**2*b**3*c - 6912*_t**2*c**3*d + 648*_t*a**5 - 864*_t*a**3*b*d - 1728*_t*a**3*c**2 + 3888*_t*a**2*b**2*c - 864*_t*a**2*c*d**2 + 864*_t*a*b**4 + 1728*_t*a*b**2*d**2 - 1728*_t*a*b*c**2*d + 864*_t*a*c**4 + 1152*_t*b**3*c*d + 864*_t*b**2*c**3 + 576*_t*c**3*d**2 - 54*a**5*d + 270*a**4*b*c - 270*a**3*b**3 + 36*a**3*b*d**2 + 144*a**3*c**2*d - 324*a**2*b**2*c*d + 24*a**2*c*d**3 - 72*a*b**4*d + 180*a*b**3*c**2 - 48*a*b**2*d**3 + 72*a*b*c**2*d**2 - 72*a*c**4*d - 72*b**5*c - 48*b**3*c*d**2 - 72*b**2*c**3*d + 72*b*c**5 - 16*c**3*d**3)/(81*a**6 - 54*a**4*c**2 + 432*a**3*b**2*c - 216*a**2*b**4 - 36*a

$*2*c**4 + 288*a*b**2*c**3 - 144*b**4*c**2 + 24*c**6))))$

Giac [A] time = 1.1216, size = 201, normalized size = 1.14

$$\frac{1}{24} \left(6^{\frac{3}{4}}a - 2\sqrt{6}b + 2 \cdot 6^{\frac{1}{4}}c \right) \arctan \left(\frac{3}{4} \sqrt{2} \left(\frac{2}{3} \right)^{\frac{3}{4}} \left(2x + \sqrt{2} \left(\frac{2}{3} \right)^{\frac{1}{4}} \right) \right) + \frac{1}{24} \left(6^{\frac{3}{4}}a + 2\sqrt{6}b + 2 \cdot 6^{\frac{1}{4}}c \right) \arctan \left(\frac{3}{4} \sqrt{2} \left(\frac{2}{3} \right)^{\frac{3}{4}} \left(2x - \sqrt{2} \left(\frac{2}{3} \right)^{\frac{1}{4}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+b*x+a)/(3*x^4+2),x, algorithm="giac")

[Out] $1/24*(6^{(3/4)*a} - 2*\text{sqrt}(6)*b + 2*6^{(1/4)*c})*\arctan(3/4*\text{sqrt}(2)*(2/3)^{(3/4)}*(2*x + \text{sqrt}(2)*(2/3)^{(1/4)})) + 1/24*(6^{(3/4)*a} + 2*\text{sqrt}(6)*b + 2*6^{(1/4)*c})*\arctan(3/4*\text{sqrt}(2)*(2/3)^{(3/4)}*(2*x - \text{sqrt}(2)*(2/3)^{(1/4)})) + 1/48*(6^{(3/4)*a} - 2*6^{(1/4)*c} + 4*d)*\log(x^2 + \text{sqrt}(2)*(2/3)^{(1/4)*x} + \text{sqrt}(2/3)) - 1/48*(6^{(3/4)*a} - 2*6^{(1/4)*c} - 4*d)*\log(x^2 - \text{sqrt}(2)*(2/3)^{(1/4)*x} + \text{sqrt}(2/3))$

$$3.167 \quad \int \frac{1+x+x^2+x^3}{1-x^4} dx$$

Optimal. Leaf size=8

$$-\log(1-x)$$

[Out] -Log[1 - x]

Rubi [A] time = 0.0076716, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1586, 31}

$$-\log(1-x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^2 + x^3)/(1 - x^4), x]

[Out] -Log[1 - x]

Rule 1586

Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p+q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\int \frac{1+x+x^2+x^3}{1-x^4} dx = \int \frac{1}{1-x} dx = -\log(1-x)$$

Mathematica [A] time = 0.0009808, size = 8, normalized size = 1.

$$-\log(1-x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + x^2 + x^3)/(1 - x^4),x]

[Out] -Log[1 - x]

Maple [A] time = 0., size = 7, normalized size = 0.9

$$-\ln(-1 + x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x^2+x+1)/(-x^4+1),x)

[Out] -ln(-1+x)

Maxima [A] time = 0.953607, size = 8, normalized size = 1.

$$-\log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="maxima")

[Out] -log(x - 1)

Fricas [A] time = 1.48741, size = 18, normalized size = 2.25

$$-\log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="fricas")

[Out] -log(x - 1)

Sympy [A] time = 0.05629, size = 5, normalized size = 0.62

$$-\log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+x**2+x+1)/(-x**4+1),x)

[Out] -log(x - 1)

Giac [A] time = 1.0498, size = 9, normalized size = 1.12

$$-\log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="giac")

[Out] -log(abs(x - 1))

$$3.168 \quad \int \frac{1+x+x^2+x^3}{1+x^4} dx$$

Optimal. Leaf size=53

$$\frac{1}{4} \log(x^4 + 1) + \frac{1}{2} \tan^{-1}(x^2) - \frac{\tan^{-1}(1 - \sqrt{2}x)}{\sqrt{2}} + \frac{\tan^{-1}(\sqrt{2}x + 1)}{\sqrt{2}}$$

[Out] ArcTan[x^2]/2 - ArcTan[1 - Sqrt[2]*x]/Sqrt[2] + ArcTan[1 + Sqrt[2]*x]/Sqrt[2] + Log[1 + x^4]/4

Rubi [A] time = 0.0418564, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1876, 1162, 617, 204, 1248, 635, 203, 260}

$$\frac{1}{4} \log(x^4 + 1) + \frac{1}{2} \tan^{-1}(x^2) - \frac{\tan^{-1}(1 - \sqrt{2}x)}{\sqrt{2}} + \frac{\tan^{-1}(\sqrt{2}x + 1)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^2 + x^3)/(1 + x^4), x]

[Out] ArcTan[x^2]/2 - ArcTan[1 - Sqrt[2]*x]/Sqrt[2] + ArcTan[1 + Sqrt[2]*x]/Sqrt[2] + Log[1 + x^4]/4

Rule 1876

Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
:= Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x+x^2+x^3}{1+x^4} dx &= \int \left(\frac{1+x^2}{1+x^4} + \frac{x(1+x^2)}{1+x^4} \right) dx \\
&= \int \frac{1+x^2}{1+x^4} dx + \int \frac{x(1+x^2)}{1+x^4} dx \\
&= \frac{1}{2} \int \frac{1}{1-\sqrt{2}x+x^2} dx + \frac{1}{2} \int \frac{1}{1+\sqrt{2}x+x^2} dx + \frac{1}{2} \text{Subst} \left(\int \frac{1+x}{1+x^2} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, x^2 \right) + \frac{1}{2} \text{Subst} \left(\int \frac{x}{1+x^2} dx, x, x^2 \right) + \frac{\text{Subst} \left(\int \frac{1}{-1-x^2} dx, x, 1-\sqrt{2}x \right)}{\sqrt{2}} \\
&= \frac{1}{2} \tan^{-1}(x^2) - \frac{\tan^{-1}(1-\sqrt{2}x)}{\sqrt{2}} + \frac{\tan^{-1}(1+\sqrt{2}x)}{\sqrt{2}} + \frac{1}{4} \log(1+x^4)
\end{aligned}$$

Mathematica [A] time = 0.0351092, size = 50, normalized size = 0.94

$$\frac{1}{4} \left(\log(x^4 + 1) - 2(1 + \sqrt{2}) \tan^{-1}(1 - \sqrt{2}x) + 2(\sqrt{2} - 1) \tan^{-1}(\sqrt{2}x + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + x^2 + x^3)/(1 + x^4), x]

[Out] (-2*(1 + Sqrt[2])*ArcTan[1 - Sqrt[2]*x] + 2*(-1 + Sqrt[2])*ArcTan[1 + Sqrt[2]*x] + Log[1 + x^4])/4

Maple [B] time = 0.003, size = 102, normalized size = 1.9

$$\frac{\arctan(-1 + x\sqrt{2})\sqrt{2}}{2} + \frac{\sqrt{2}}{8} \ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right) + \frac{\arctan(1+x\sqrt{2})\sqrt{2}}{2} + \frac{\arctan(x^2)}{2} + \frac{\sqrt{2}}{8} \ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right) + \frac{1}{4} \ln(1+x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x^2+x+1)/(x^4+1), x)

[Out] 1/2*arctan(-1+x*2^(1/2))*2^(1/2)+1/8*2^(1/2)*ln((1+x^2+x*2^(1/2))/(1+x^2-x*2^(1/2)))+1/2*arctan(1+x*2^(1/2))*2^(1/2)+1/2*arctan(x^2)+1/8*2^(1/2)*ln((1+x^2-x*2^(1/2))/(1+x^2+x*2^(1/2)))+1/4*ln(x^4+1)

Maxima [A] time = 1.43715, size = 103, normalized size = 1.94

$$-\frac{1}{4}\sqrt{2}(\sqrt{2}-2)\arctan\left(\frac{1}{2}\sqrt{2}(2x+\sqrt{2})\right)+\frac{1}{4}\sqrt{2}(\sqrt{2}+2)\arctan\left(\frac{1}{2}\sqrt{2}(2x-\sqrt{2})\right)+\frac{1}{4}\log(x^2+\sqrt{2}x+1)+\frac{1}{4}\log(x^2-\sqrt{2}x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+x+1)/(x^4+1),x, algorithm="maxima")

[Out] -1/4*sqrt(2)*(sqrt(2) - 2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*(sqrt(2) + 2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 1/4*log(x^2 + sqrt(2)*x + 1) + 1/4*log(x^2 - sqrt(2)*x + 1)

Fricas [B] time = 1.56056, size = 440, normalized size = 8.3

$$-\sqrt{-2\sqrt{2}+3}\arctan\left(\sqrt{x^2+\sqrt{2}x+1}(\sqrt{2}+2)\sqrt{-2\sqrt{2}+3}-\left(\sqrt{2}(x+1)+2x+1\right)\sqrt{-2\sqrt{2}+3}\right)+\sqrt{2\sqrt{2}+3}\arctan\left(\sqrt{x^2+\sqrt{2}x+1}(\sqrt{2}-2)\sqrt{2\sqrt{2}+3}-\left(\sqrt{2}(x+1)+2x+1\right)\sqrt{2\sqrt{2}+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+x+1)/(x^4+1),x, algorithm="fricas")

[Out] -sqrt(-2*sqrt(2) + 3)*arctan(sqrt(x^2 + sqrt(2)*x + 1)*(sqrt(2) + 2)*sqrt(-2*sqrt(2) + 3) - (sqrt(2)*(x + 1) + 2*x + 1)*sqrt(-2*sqrt(2) + 3)) + sqrt(2*sqrt(2) + 3)*arctan(-sqrt(2)*(x + 1) - sqrt(x^2 - sqrt(2)*x + 1)*(sqrt(2) - 2) - 2*x - 1)*sqrt(2*sqrt(2) + 3)) + 1/4*log(x^2 + sqrt(2)*x + 1) + 1/4*log(x^2 - sqrt(2)*x + 1)

Sympy [A] time = 0.324015, size = 73, normalized size = 1.38

$$\frac{\log(x^2 - \sqrt{2}x + 1)}{4} + \frac{\log(x^2 + \sqrt{2}x + 1)}{4} + 2\left(\frac{1}{4} + \frac{\sqrt{2}}{4}\right)\operatorname{atan}\left(\sqrt{2}x - 1\right) + 2\left(-\frac{1}{4} + \frac{\sqrt{2}}{4}\right)\operatorname{atan}\left(\sqrt{2}x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+x**2+x+1)/(x**4+1),x)

```
[Out] log(x**2 - sqrt(2)*x + 1)/4 + log(x**2 + sqrt(2)*x + 1)/4 + 2*(1/4 + sqrt(2)
)/4)*atan(sqrt(2)*x - 1) + 2*(-1/4 + sqrt(2)/4)*atan(sqrt(2)*x + 1)
```

Giac [A] time = 1.06195, size = 95, normalized size = 1.79

$$\frac{1}{2}(\sqrt{2}-1)\arctan\left(\frac{1}{2}\sqrt{2}(2x+\sqrt{2})\right) + \frac{1}{2}(\sqrt{2}+1)\arctan\left(\frac{1}{2}\sqrt{2}(2x-\sqrt{2})\right) + \frac{1}{4}\log(x^2+\sqrt{2}x+1) + \frac{1}{4}\log(x^2-\sqrt{2}x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2) - 1)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/2*(sqrt(2) + 1)*a
rctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 1/4*log(x^2 + sqrt(2)*x + 1) + 1/4*log
(x^2 - sqrt(2)*x + 1)
```

$$3.169 \quad \int \frac{1+x+x^2+x^3}{a-bx^4} dx$$

Optimal. Leaf size=124

$$-\frac{(\sqrt{a}-\sqrt{b})\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{(\sqrt{a}+\sqrt{b})\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} - \frac{\log(a-bx^4)}{4b} + \frac{\tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

[Out] -((Sqrt[a] - Sqrt[b])*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) + ((Sqrt[a] + Sqrt[b])*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) + ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]) - Log[a - b*x^4]/(4*b)

Rubi [A] time = 0.0870212, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$, Rules used = {1876, 1167, 205, 208, 1248, 635, 260}

$$-\frac{(\sqrt{a}-\sqrt{b})\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{(\sqrt{a}+\sqrt{b})\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} - \frac{\log(a-bx^4)}{4b} + \frac{\tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^2 + x^3)/(a - b*x^4), x]

[Out] -((Sqrt[a] - Sqrt[b])*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) + ((Sqrt[a] + Sqrt[b])*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) + ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]) - Log[a - b*x^4]/(4*b)

Rule 1876

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 1167

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[

$c*d^2 - a*e^2, 0]$ && PosQ[-(a*c)]

Rule 205

$\text{Int}[\frac{(a_1 + (b_1)x^2)^{-1}}{a}, x]$ /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

$\text{Int}[\frac{(a_1 + (b_1)x^2)^{-1}}{a}, x]$ /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1248

$\text{Int}[(x_1)((d_1) + (e_1)x^2)^{q_1}((a_1) + (c_1)x^4)^{p_1}, x]$ /; FreeQ[{a, c, d, e, p, q}, x]

Rule 635

$\text{Int}[\frac{(d_1) + (e_1)x}{(a_1) + (c_1)x^2}, x]$ /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 260

$\text{Int}[\frac{x^m}{(a_1) + (b_1)x^n}, x]$ /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned}
\int \frac{1+x+x^2+x^3}{a-bx^4} dx &= \int \left(\frac{1+x^2}{a-bx^4} + \frac{x(1+x^2)}{a-bx^4} \right) dx \\
&= \int \frac{1+x^2}{a-bx^4} dx + \int \frac{x(1+x^2)}{a-bx^4} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1+x}{a-bx^2} dx, x, x^2 \right) + \frac{1}{2} \left(1 - \frac{\sqrt{b}}{\sqrt{a}} \right) \int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2} dx + \frac{1}{2} \left(1 + \frac{\sqrt{b}}{\sqrt{a}} \right) \int \frac{1}{\sqrt{a}\sqrt{b}-bx^2} dx \\
&= -\frac{(\sqrt{a}-\sqrt{b}) \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2a^{3/4}b^{3/4}} + \frac{(\sqrt{a}+\sqrt{b}) \tanh^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2a^{3/4}b^{3/4}} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{a-bx^2} dx, x, x^2 \right) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{a-bx^2} dx, x, x^2 \right) \\
&= -\frac{(\sqrt{a}-\sqrt{b}) \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2a^{3/4}b^{3/4}} + \frac{(\sqrt{a}+\sqrt{b}) \tanh^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2a^{3/4}b^{3/4}} + \frac{\tanh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{b}} - \frac{\log(a-bx^4)}{4b}
\end{aligned}$$

Mathematica [A] time = 0.0499505, size = 203, normalized size = 1.64

$$-\frac{(a^{3/4} + \sqrt{a}\sqrt[4]{b} + \sqrt[4]{a}\sqrt{b}) \log(\sqrt[4]{a} - \sqrt[4]{bx})}{4ab^{3/4}} - \frac{(-a^{3/4} + \sqrt{a}\sqrt[4]{b} - \sqrt[4]{a}\sqrt{b}) \log(\sqrt[4]{a} + \sqrt[4]{bx})}{4ab^{3/4}} + \frac{(\sqrt[4]{a}\sqrt{b} - a^{3/4}) \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2ab^{3/4}} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{a-bx^2} dx, x, x^2 \right) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{a-bx^2} dx, x, x^2 \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + x^2 + x^3)/(a - b*x^4), x]

[Out] $((-a^{3/4} + a^{1/4} \sqrt{b}) \text{ArcTan}[(b^{1/4} x)/a^{1/4}]) / (2 a b^{3/4}) - ((a^{3/4} + \sqrt{a} b^{1/4} + a^{1/4} \sqrt{b}) \text{Log}[a^{1/4} - b^{1/4} x]) / (4 a b^{3/4}) - ((-a^{3/4} + \sqrt{a} b^{1/4} - a^{1/4} \sqrt{b}) \text{Log}[a^{1/4} + b^{1/4} x]) / (4 a b^{3/4}) + \text{Log}[\sqrt{a} + \sqrt{b} x^2] / (4 \sqrt{a} \sqrt{b}) - \text{Log}[a - b x^4] / (4 b)$

Maple [B] time = 0.004, size = 171, normalized size = 1.4

$$\frac{1}{4a} \sqrt[4]{\frac{a}{b}} \ln \left(\left(x + \sqrt[4]{\frac{a}{b}} \right) \left(x - \sqrt[4]{\frac{a}{b}} \right)^{-1} \right) + \frac{1}{2a} \sqrt[4]{\frac{a}{b}} \arctan \left(x \frac{1}{\sqrt[4]{\frac{a}{b}}} \right) - \frac{1}{4} \ln \left(\left(-a + x^2 \sqrt{ab} \right) \left(-a - x^2 \sqrt{ab} \right)^{-1} \right) \frac{1}{\sqrt{ab}} - \frac{1}{2b} \arctan \left(x \frac{1}{\sqrt{b}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3+x^2+x+1)/(-b*x^4+a),x)`

[Out] $\frac{1}{4} \cdot \frac{1}{b \cdot a}^{1/4} / a \cdot \ln\left(\frac{x + (1/b \cdot a)^{1/4}}{x - (1/b \cdot a)^{1/4}}\right) + \frac{1}{2} \cdot \frac{1}{b \cdot a}^{1/4} / a \cdot \arctan\left(\frac{x}{(1/b \cdot a)^{1/4}}\right) - \frac{1}{4} / (a \cdot b)^{1/2} \cdot \ln\left(\frac{-a + x^2 \cdot (a \cdot b)^{1/2}}{-a - x^2 \cdot (a \cdot b)^{1/2}}\right) - \frac{1}{2} / b / (1/b \cdot a)^{1/4} \cdot \arctan\left(\frac{x}{(1/b \cdot a)^{1/4}}\right) + \frac{1}{4} / b / (1/b \cdot a)^{1/4} \cdot \ln\left(\frac{x + (1/b \cdot a)^{1/4}}{x - (1/b \cdot a)^{1/4}}\right) - \frac{1}{4} / b \cdot \ln(b \cdot x^4 - a)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+x^2+x+1)/(-b*x^4+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+x^2+x+1)/(-b*x^4+a),x, algorithm="fricas")`

[Out] Timed out

Sympy [A] time = 1.02863, size = 187, normalized size = 1.51

$-\text{RootSum}\left(256t^4a^3b^4 - 256t^3a^3b^3 + t^2(96a^3b^2 - 96a^2b^3) + t(-16a^3b + 32a^2b^2 - 16ab^3) + a^3 - 3a^2b + 3ab^2 - b^3, (t\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**3+x**2+x+1)/(-b*x**4+a),x)`

```
[Out] -RootSum(256*_t**4*a**3*b**4 - 256*_t**3*a**3*b**3 + _t**2*(96*a**3*b**2 -
96*a**2*b**3) + _t*(-16*a**3*b + 32*a**2*b**2 - 16*a*b**3) + a**3 - 3*a**2*
b + 3*a*b**2 - b**3, Lambda(_t, _t*log(x + (-64*_t**3*a**3*b**3 + 48*_t**2*
a**3*b**2 + 16*_t**2*a**2*b**3 - 12*_t*a**3*b + 16*_t*a**2*b**2 - 4*_t*a*b
**3 + a**3 - 2*a**2*b + a*b**2)/(a**2*b - 2*a*b**2 + b**3))))
```

Giac [B] time = 1.09042, size = 392, normalized size = 3.16

$$-\frac{\log(|bx^4 - a|)}{4b} + \frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2 - \sqrt{2}\sqrt{-ab^3b} + \left(-ab^3\right)^{\frac{3}{4}}\right) \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3} + \frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2 + \sqrt{2}\sqrt{-ab^3b} + \left(-ab^3\right)^{\frac{3}{4}}\right) \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*log(abs(b*x^4 - a))/b + 1/4*sqrt(2)*((-a*b^3)^(1/4)*b^2 - sqrt(2)*sqrt
(-a*b^3)*b + (-a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4)
)/(-a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((-a*b^3)^(1/4)*b^2 + sqrt(2)*sqrt(-a
*b^3)*b + (-a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(
-a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2 - (-a*b^3)^(3/4))*lo
g(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^3) - 1/8*sqrt(2)*((-a*b^3
)^(1/4)*b^2 - (-a*b^3)^(3/4))*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b)
)/(a*b^3)
```

$$3.170 \quad \int \frac{1+x+x^2+x^3}{a+bx^4} dx$$

Optimal. Leaf size=277

$$\frac{(\sqrt{a}-\sqrt{b})\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}+\sqrt{a}+\sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} - \frac{(\sqrt{a}-\sqrt{b})\log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}+\sqrt{a}+\sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} - \frac{(\sqrt{a}+\sqrt{b})\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}$$

[Out] ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[a] + Sqrt[b])*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[a] + Sqrt[b])*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[a] - Sqrt[b])*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[a] - Sqrt[b])*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + Log[a + b*x^4]/(4*b)

Rubi [A] time = 0.195501, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 205, 260}

$$\frac{(\sqrt{a}-\sqrt{b})\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}+\sqrt{a}+\sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} - \frac{(\sqrt{a}-\sqrt{b})\log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}+\sqrt{a}+\sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} - \frac{(\sqrt{a}+\sqrt{b})\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^2 + x^3)/(a + b*x^4), x]

[Out] ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[a] + Sqrt[b])*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[a] + Sqrt[b])*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[a] - Sqrt[b])*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[a] - Sqrt[b])*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + Log[a + b*x^4]/(4*b)

Rule 1876

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,

0] && Expon[Pq, x] < n

Rule 1168

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1162

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1165

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1248

```
Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol]
  :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1+x+x^2+x^3}{a+bx^4} dx &= \int \left(\frac{1+x^2}{a+bx^4} + \frac{x(1+x^2)}{a+bx^4} \right) dx \\
 &= \int \frac{1+x^2}{a+bx^4} dx + \int \frac{x(1+x^2)}{a+bx^4} dx \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{1+x}{a+bx^2} dx, x, x^2 \right) - \frac{\left(1 - \frac{\sqrt{b}}{\sqrt{a}}\right) \int \frac{\sqrt{a}\sqrt{b-bx^2}}{a+bx^4} dx}{2b} + \frac{\left(1 + \frac{\sqrt{b}}{\sqrt{a}}\right) \int \frac{\sqrt{a}\sqrt{b+bx^2}}{a+bx^4} dx}{2b} \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{a+bx^2} dx, x, x^2 \right) + \frac{1}{2} \text{Subst} \left(\int \frac{x}{a+bx^2} dx, x, x^2 \right) + \frac{\left(1 + \frac{\sqrt{b}}{\sqrt{a}}\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt{b}} + x^2} dx}{4b} \\
 &= \frac{\tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{b}} + \frac{(\sqrt{a} - \sqrt{b}) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} - \frac{(\sqrt{a} - \sqrt{b}) \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx})}{4\sqrt{2}a^{3/4}b^{3/4}} \\
 &= \frac{\tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{b}} - \frac{(\sqrt{a} + \sqrt{b}) \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt{a}} \right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{(\sqrt{a} + \sqrt{b}) \tan^{-1} \left(1 + \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt{a}} \right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{(\sqrt{a} - \sqrt{b}) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} - \frac{(\sqrt{a} - \sqrt{b}) \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx})}{4\sqrt{2}a^{3/4}b^{3/4}}
 \end{aligned}$$

Mathematica [A] time = 0.188188, size = 283, normalized size = 1.02

$$\sqrt{2}\sqrt[4]{b}(a^{3/4} - \sqrt[4]{a}\sqrt{b})\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{bx^2}) + \sqrt{2}\sqrt[4]{b}(\sqrt[4]{a}\sqrt{b} - a^{3/4})\log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{bx^2}) + 2a\log(a + \sqrt{bx^2})$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + x^2 + x^3)/(a + b*x^4), x]

[Out] (-2*a^(1/4)*(Sqrt[2]*Sqrt[a] + 2*a^(1/4)*b^(1/4) + Sqrt[2]*Sqrt[b])*b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*(Sqrt[2]*Sqrt[a] - 2*a^(1/4)*b^(1/4) + Sqrt[2]*Sqrt[b])*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(a^(3/4) - a^(1/4)*Sqrt[b])*b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(-a^(3/4) + a^(1/4)*Sqrt[b])*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 2*a*Log[a + b*x^4])/(8*a*b)

Maple [A] time = 0.003, size = 286, normalized size = 1.

$$\frac{\sqrt{2}}{8a}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x^2 + \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x^2 - \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) + \frac{\sqrt{2}}{4a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) + \frac{\sqrt{2}}{4a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x^2+x+1)/(b*x^4+a), x)

[Out] 1/8*(1/b*a)^(1/4)/a*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/4*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/4*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+1/2/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+1/8/b/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/4/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/4/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+1/4*ln(b*x^4+a)/b

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+x^2+x+1)/(b*x^4+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+x^2+x+1)/(b*x^4+a),x, algorithm="fricas")`

[Out] Timed out

Sympy [A] time = 0.997206, size = 187, normalized size = 0.68

$\text{RootSum}\left(256t^4a^3b^4 - 256t^3a^3b^3 + t^2(96a^3b^2 + 96a^2b^3) + t(-16a^3b - 32a^2b^2 - 16ab^3) + a^3 + 3a^2b + 3ab^2 + b^3, (t \mapsto \dots)\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**3+x**2+x+1)/(b*x**4+a),x)`

[Out] $\text{RootSum}(256_t**4a**3b**4 - 256_t**3a**3b**3 + _t**2(96a**3b**2 + 96a**2b**3) + _t*(-16a**3b - 32a**2b**2 - 16a*b**3) + a**3 + 3a**2b + 3a*b**2 + b**3, \text{Lambda}(_t, _t*\log(x + (64_t**3a**3b**3 - 48_t**2a**3b**2 + 16_t**2a**2b**3 + 12_t*a**3b + 16_t*a**2b**2 + 4_t*a*b**3 - a**3 - 2a**2b - a*b**2)/(a**2b + 2a*b**2 + b**3))))$

Giac [A] time = 1.06264, size = 365, normalized size = 1.32

$$\frac{\log(|bx^4 + a|)}{4b} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2 - \sqrt{2}\sqrt{ab^3b} + (ab^3)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2 + \sqrt{2}\sqrt{ab^3b} + (ab^3)^{\frac{3}{4}}\right)}{4ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(b*x^4 + a))/b + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2 - sqrt(2)*sqrt(a
*b^3)*b + (a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/
b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2 + sqrt(2)*sqrt(a*b^3)*b
+ (a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4)
)/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2 - (a*b^3)^(3/4))*log(x^2 + sqrt(
2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2 - (a
*b^3)^(3/4))*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```


$$3.171 \quad \int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4} dx$$

Optimal. Leaf size=148

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(-\sqrt{a}\sqrt{be}+ag+bc)}{2a^{3/4}b^{5/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(\sqrt{a}\sqrt{be}+ag+bc)}{2a^{3/4}b^{5/4}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{f \log(a-bx^4)}{4b} - \frac{gx}{b}$$

[Out] -((g*x)/b) + ((b*c - Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(5/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - (f*Log[a - b*x^4])/(4*b)

Rubi [A] time = 0.203053, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {1885, 1248, 635, 208, 260, 1887, 1167, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(-\sqrt{a}\sqrt{be}+ag+bc)}{2a^{3/4}b^{5/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(\sqrt{a}\sqrt{be}+ag+bc)}{2a^{3/4}b^{5/4}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{f \log(a-bx^4)}{4b} - \frac{gx}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4), x]

[Out] -((g*x)/b) + ((b*c - Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(5/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - (f*Log[a - b*x^4])/(4*b)

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*a + b*x^n]^p, {j, 0, n/2 - 1}, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
```

[{a, c, d, e, p, q}, x]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 1887

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Pq/(a + b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]

Rule 1167

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[-(a*c)]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4}{a - bx^4} dx &= \int \left(\frac{x(d + fx^2)}{a - bx^4} + \frac{c + ex^2 + gx^4}{a - bx^4} \right) dx \\
&= \int \frac{x(d + fx^2)}{a - bx^4} dx + \int \frac{c + ex^2 + gx^4}{a - bx^4} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{d + fx}{a - bx^2} dx, x, x^2 \right) + \int \left(-\frac{g}{b} + \frac{bc + ag + bex^2}{b(a - bx^4)} \right) dx \\
&= -\frac{gx}{b} + \frac{\int \frac{bc+ag+bex^2}{a-bx^4} dx}{b} + \frac{1}{2} d \text{Subst} \left(\int \frac{1}{a - bx^2} dx, x, x^2 \right) + \frac{1}{2} f \text{Subst} \left(\int \frac{x}{a - bx^2} dx, x, x^2 \right) \\
&= -\frac{gx}{b} + \frac{d \tanh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{b}} - \frac{f \log(a - bx^4)}{4b} + \frac{1}{2} \left(e - \frac{bc + ag}{\sqrt{a}\sqrt{b}} \right) \int \frac{1}{-\sqrt{a}\sqrt{b} - bx^2} dx + \frac{1}{2} \int \frac{x}{a - bx^2} dx \\
&= -\frac{gx}{b} + \frac{(bc - \sqrt{a}\sqrt{be} + ag) \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2a^{3/4}b^{5/4}} + \frac{(bc + \sqrt{a}\sqrt{be} + ag) \tanh^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2a^{3/4}b^{5/4}} + \frac{d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.0812388, size = 249, normalized size = 1.68

$$-a^{3/4} \sqrt[4]{b} f \log(a - bx^4) - 4a^{3/4} \sqrt[4]{b} gx - \log(\sqrt[4]{a} - \sqrt[4]{bx}) (\sqrt[4]{ab^{3/4}} d + \sqrt{a}\sqrt{be} + ag + bc) + \sqrt[4]{ab^{3/4}} d \log(\sqrt{a} + \sqrt{bx^2}) - \sqrt[4]{a}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4),x]

[Out] $(-4*a^{(3/4)}*b^{(1/4)}*g*x + 2*(b*c - \text{Sqrt}[a]*\text{Sqrt}[b]*e + a*g)*\text{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}] - (b*c + a^{(1/4)}*b^{(3/4)}*d + \text{Sqrt}[a]*\text{Sqrt}[b]*e + a*g)*\text{Log}[a^{(1/4)} - b^{(1/4)}*x] + b*c*\text{Log}[a^{(1/4)} + b^{(1/4)}*x] - a^{(1/4)}*b^{(3/4)}*d*\text{Log}[a^{(1/4)} + b^{(1/4)}*x] + \text{Sqrt}[a]*\text{Sqrt}[b]*e*\text{Log}[a^{(1/4)} + b^{(1/4)}*x] + a*g*\text{Log}[a^{(1/4)} + b^{(1/4)}*x] + a^{(1/4)}*b^{(3/4)}*d*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[b]*x^2] - a^{(3/4)}*b^{(1/4)}*f*\text{Log}[a - b*x^4])/(4*a^{(3/4)}*b^{(5/4)})$

Maple [B] time = 0.005, size = 244, normalized size = 1.7

$$-\frac{gx}{b} + \frac{g}{2b} \sqrt[4]{\frac{a}{b}} \arctan\left(x \frac{1}{\sqrt[4]{\frac{a}{b}}}\right) + \frac{c}{2a} \sqrt[4]{\frac{a}{b}} \arctan\left(x \frac{1}{\sqrt[4]{\frac{a}{b}}}\right) + \frac{g}{4b} \sqrt[4]{\frac{a}{b}} \ln\left(\left(x + \sqrt[4]{\frac{a}{b}}\right)\left(x - \sqrt[4]{\frac{a}{b}}\right)^{-1}\right) + \frac{c}{4a} \sqrt[4]{\frac{a}{b}} \ln\left(\left(x + \sqrt[4]{\frac{a}{b}}\right)\left(x - \sqrt[4]{\frac{a}{b}}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x)
```

```
[Out] -g*x/b+1/2/b*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))*g+1/2*c*(1/b*a)^(1/4)/a*
arctan(x/(1/b*a)^(1/4))+1/4/b*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)
^(1/4)))*g+1/4*c*(1/b*a)^(1/4)/a*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))-1/
4*d/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))-1/2*e/b/(1/b*
a)^(1/4)*arctan(x/(1/b*a)^(1/4))+1/4*e/b/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))
/(x-(1/b*a)^(1/4)))-1/4/b*f*ln(b*x^4-a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] time = 156.777, size = 2394, normalized size = 16.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)

[Out]
$$-\text{RootSum}(256*_t^{*4}*a^{*3}*b^{*5} - 256*_t^{*3}*a^{*3}*b^{*4}*f + *_t^{*2}*(-64*a^{*3}*b^{*3}*e*g + 96*a^{*3}*b^{*3}*f^{*2} - 64*a^{*2}*b^{*4}*c*e - 32*a^{*2}*b^{*4}*d^{*2}) + *_t*(-16*a^{*3}*b^{*2}*d*g^{*2} + 32*a^{*3}*b^{*2}*e*f*g - 16*a^{*3}*b^{*2}*f^{*3} - 32*a^{*2}*b^{*3}*c*d*g + 32*a^{*2}*b^{*3}*c*e*f + 16*a^{*2}*b^{*3}*d^{*2}*f - 16*a^{*2}*b^{*3}*d*e^{*2} - 16*a*b^{*4}*c^{*2}*d) - a^{*4}*g^{*4} - 4*a^{*3}*b*c*g^{*3} + 4*a^{*3}*b*d*f*g^{*2} + 2*a^{*3}*b*e^{*2}*g^{*2} - 4*a^{*3}*b*e*f^{*2}*g + a^{*3}*b*f^{*4} - 6*a^{*2}*b^{*2}*c^{*2}*g^{*2} + 8*a^{*2}*b^{*2}*c*d*f*g + 4*a^{*2}*b^{*2}*c*e^{*2}*g - 4*a^{*2}*b^{*2}*c*e*f^{*2} - 4*a^{*2}*b^{*2}*d^{*2}*e*g - 2*a^{*2}*b^{*2}*d^{*2}*f^{*2} + 4*a^{*2}*b^{*2}*d*e^{*2}*f - a^{*2}*b^{*2}*e^{*4} - 4*a*b^{*3}*c^{*3}*g + 4*a*b^{*3}*c^{*2}*d*f + 2*a*b^{*3}*c^{*2}*e^{*2} - 4*a*b^{*3}*c*d^{*2}*e + a*b^{*3}*d^{*4} - b^{*4}*c^{*4}, \text{Lambda}(*_t, *_t*\log(x + (64*_t^{*3}*a^{*5}*b^{*4}*e*g^{*2} + 128*_t^{*3}*a^{*4}*b^{*5}*c*e*g - 128*_t^{*3}*a^{*4}*b^{*5}*d^{*2}*g + 64*_t^{*3}*a^{*4}*b^{*5}*e^{*3} + 64*_t^{*3}*a^{*3}*b^{*6}*c^{*2}*e - 128*_t^{*3}*a^{*3}*b^{*6}*c*d^{*2} + 16*_t^{*2}*a^{*5}*b^{*3}*d*g^{*3} - 48*_t^{*2}*a^{*5}*b^{*3}*e*f*g^{*2} + 48*_t^{*2}*a^{*4}*b^{*4}*c*d*g^{*2} - 96*_t^{*2}*a^{*4}*b^{*4}*c*e*f*g + 96*_t^{*2}*a^{*4}*b^{*4}*d^{*2}*f*g - 48*_t^{*2}*a^{*4}*b^{*4}*d*e^{*2}*g - 48*_t^{*2}*a^{*4}*b^{*4}*e^{*3}*f + 48*_t^{*2}*a^{*3}*b^{*5}*c^{*2}*d*g - 48*_t^{*2}*a^{*3}*b^{*5}*c^{*2}*e*f + 96*_t^{*2}*a^{*3}*b^{*5}*c*d^{*2}*f - 48*_t^{*2}*a^{*3}*b^{*5}*c*d*e^{*2} + 32*_t^{*2}*a^{*3}*b^{*5}*d^{*3}*e + 16*_t^{*2}*a^{*2}*b^{*6}*c^{*3}*d - 4*_t*a^{*6}*b*g^{*5} - 20*_t*a^{*5}*b^{*2}*c*g^{*4} - 8*_t*a^{*5}*b^{*2}*d*f*g^{*3} - 16*_t*a^{*5}*b^{*2}*e^{*2}*g^{*3} + 12*_t*a^{*5}*b^{*2}*e*f^{*2}*g^{*2} - 40*_t*a^{*4}*b^{*3}*c^{*2}*g^{*3} - 24*_t*a^{*4}*b^{*3}*c*d*f*g^{*2} - 48*_t*a^{*4}*b^{*3}*c*e^{*2}*g^{*2} + 24*_t*a^{*4}*b^{*3}*c*e*f^{*2}*g + 36*_t*a^{*4}*b^{*3}*d^{*2}*e*g^{*2} - 24*_t*a^{*4}*b^{*3}*d^{*2}*f^{*2}*g + 24*_t*a^{*4}*b^{*3}*d*e^{*2}*f*g - 12*_t*a^{*4}*b^{*3}*e^{*4}*g + 12*_t*a^{*4}*b^{*3}*e^{*3}*f^{*2} - 40*_t*a^{*3}*b^{*4}*c^{*3}*g^{*2} - 24*_t*a^{*3}*b^{*4}*c^{*2}*d*f*g - 48*_t*a^{*3}*b^{*4}*c^{*2}*e^{*2}*g + 12*_t*a^{*3}*b^{*4}*c^{*2}*e*f^{*2} + 72*_t*a^{*3}*b^{*4}*c*d^{*2}*e*g - 24*_t*a^{*3}*b^{*4}*c*d^{*2}*f^{*2} + 24*_t*a^{*3}*b^{*4}*c*d*e^{*2}*f - 12*_t*a^{*3}*b^{*4}*c*e^{*4} + 8*_t*a^{*3}*b^{*4}*d^{*4}*g - 16*_t*a^{*3}*b^{*4}*d^{*3}*e*f - 12*_t*a^{*3}*b^{*4}*d^{*2}*e^{*3} - 20*_t*a^{*2}*b^{*5}*c^{*4}*g - 8*_t*a^{*2}*b^{*5}*c^{*3}*d*f - 16*_t*a^{*2}*b^{*5}*c^{*3}*e^{*2} + 36*_t*a^{*2}*b^{*5}*c^{*2}*d^{*2}*e + 8*_t*a^{*2}*b^{*5}*c*d^{*4} - 4*_t*a*b^{*6}*c^{*5} + a^{*6}*f*g^{*5} + 5*a^{*5}*b*c*f*g^{*4} - 5*a^{*5}*b*d*e*g^{*4} + a^{*5}*b*d*f^{*2}*g^{*3} + 4*a^{*5}*b*e^{*2}*f*g^{*3} - a^{*5}*b*e*f^{*3}*g^{*2} + 10*a^{*4}*b^{*2}*c^{*2}*f*g^{*3} - 20*a^{*4}*b^{*2}*c*d*e*g^{*3} + 3*a^{*4}*b^{*2}*c*d*f^{*2}*g^{*2} + 12*a^{*4}*b^{*2}*c*e^{*2}*f*g^{*2} - 2*a^{*4}*b^{*2}*c*e*f^{*3}*g + 5*a^{*4}*b^{*2}*d^{*3}*g^{*3} - 9*a^{*4}*b^{*2}*d^{*2}*e*f*g^{*2} + 2*a^{*4}*b^{*2}*d^{*2}*f^{*3}*g - 3*a^{*4}*b^{*2}*d*e^{*2}*f^{*2}*g + 3*a^{*4}*b^{*2}*e^{*4}*f*g - a^{*4}*b^{*2}*e^{*3}*f^{*3} + 10*a^{*3}*b^{*3}*c^{*3}*f*g^{*2} - 30*a^{*3}*b^{*3}*c^{*2}*d*e*g^{*2} + 3*a^{*3}*b^{*3}*c^{*2}*d*f^{*2}*g + 12*a^{*3}*b^{*3}*c^{*2}*e^{*2}*f*g - a^{*3}*b^{*3}*c^{*2}*e*f^{*3} + 15*a^{*3}*b^{*3}*c*d^{*3}*g^{*2} - 18*a^{*3}*b^{*3}*c*d^{*2}*e*f*g + 2*a^{*3}*b^{*3}*c*d^{*2}*f^{*3} - 3*a^{*3}*b^{*3}*c*d*e^{*2}*f^{*2} + 3*a^{*3}*b^{*3}*c*e^{*4}*f - 2*a^{*3}*b^{*3}*d^{*4}*f*g + 5*a^{*3}*b^{*3}*d^{*3}*e^{*2}*g + 2*a^{*3}*b^{*3}*d^{*3}*e*f^{*2} + 3*a^{*3}*b^{*3}*d^{*2}*e^{*3}*f - 3*a^{*3}*b^{*3}*d*e^{*5} + 5*a^{*2}*b^{*4}*c^{*4}*f*g - 20*a^{*2}*b^{*4}*c^{*3}*d*e*g + a^{*2}*b^{*4}*c^{*3}*d*f^{*2} + 4*a^{*2}*b^{*4}*c^{*3}*e^{*2}*f + 15*a^{*2}*b^{*4}*c^{*2}*d^{*3}*g - 9*a^{*2}*b^{*4}*c^{*2}*d^{*2}*e*f - 2*a^{*2}*b^{*4}*c*d^{*4}*f + 5*a^{*2}*b^{*4}*c*d^{*3}*e^{*2} - 2*a^{*2}*b^{*4}*d^{*5}*e + a*b^{*5}$$

```

c**5*f - 5*a*b**5*c**4*d*e + 5*a*b**5*c**3*d**3)/(a**6*g**6 + 6*a**5*b*c*g*
*5 + a**5*b**e**2*g**4 + 15*a**4*b**2*c**2*g**4 + 4*a**4*b**2*c**e**2*g**3 -
8*a**4*b**2*d**2*e*g**3 - a**4*b**2*e**4*g**2 + 20*a**3*b**3*c**3*g**3 + 6*
a**3*b**3*c**2*e**2*g**2 - 24*a**3*b**3*c*d**2*e*g**2 - 2*a**3*b**3*c**e**4*
g + 4*a**3*b**3*d**4*g**2 + 8*a**3*b**3*d**2*e**3*g - a**3*b**3*e**6 + 15*a
**2*b**4*c**4*g**2 + 4*a**2*b**4*c**3*e**2*g - 24*a**2*b**4*c**2*d**2*e*g -
a**2*b**4*c**2*e**4 + 8*a**2*b**4*c*d**4*g + 8*a**2*b**4*c*d**2*e**3 - 4*a
**2*b**4*d**4*e**2 + 6*a*b**5*c**5*g + a*b**5*c**4*e**2 - 8*a*b**5*c**3*d**
2*e + 4*a*b**5*c**2*d**4 + b**6*c**6)))) - g*x/b

```

Giac [B] time = 1.08823, size = 539, normalized size = 3.64

$$\frac{gx}{b} - \frac{f \log(|bx^4 - a|)}{4b} + \frac{\sqrt{2} \left(\sqrt{2} \sqrt{-ab} b^4 d + (-ab^3)^{\frac{1}{4}} ab^4 c + (-ab^3)^{\frac{1}{4}} a^2 b^3 g + (-ab^3)^{\frac{3}{4}} ab^2 e \right) \arctan \left(\frac{\sqrt{2} \left(2x + \sqrt{2} \left(-\frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left(-\frac{a}{b} \right)^{\frac{1}{4}}} \right)}{4 a^2 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

```

[Out] -g*x/b - 1/4*f*log(abs(b*x^4 - a))/b + 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*a*b^
4*d + (-a*b^3)^(1/4)*a*b^4*c + (-a*b^3)^(1/4)*a^2*b^3*g + (-a*b^3)^(3/4)*a*
b^2*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b
^5) + 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*a*b^4*d + (-a*b^3)^(1/4)*a*b^4*c + (-
a*b^3)^(1/4)*a^2*b^3*g + (-a*b^3)^(3/4)*a*b^2*e)*arctan(1/2*sqrt(2)*(2*x -
sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^5) + 1/8*sqrt(2)*((-a*b^3)^(1/4)
*a*b^4*c + (-a*b^3)^(1/4)*a^2*b^3*g - (-a*b^3)^(3/4)*a*b^2*e)*log(x^2 + sqr
t(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^5) - 1/8*sqrt(2)*((-a*b^3)^(1/4)*a
*b^4*c + (-a*b^3)^(1/4)*a^2*b^3*g - (-a*b^3)^(3/4)*a*b^2*e)*log(x^2 - sqrt(
2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^5)

```

$$3.172 \quad \int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2} dx$$

Optimal. Leaf size=172

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(-\sqrt{a}\sqrt{be}-ag+3bc)}{8a^{7/4}b^{5/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(\sqrt{a}\sqrt{be}-ag+3bc)}{8a^{7/4}b^{5/4}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} + \frac{x(ag+bc+bdx+bx^2)}{4ab(a-bx^4)}$$

[Out] (x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a - b*x^4)) + ((3*b*c - Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(5/4)) + ((3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(5/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b])

Rubi [A] time = 0.164898, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {1858, 1876, 275, 208, 1167, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(-\sqrt{a}\sqrt{be}-ag+3bc)}{8a^{7/4}b^{5/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(\sqrt{a}\sqrt{be}-ag+3bc)}{8a^{7/4}b^{5/4}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} + \frac{x(ag+bc+bdx+bx^2)}{4ab(a-bx^4)}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^2,x]

[Out] (x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a - b*x^4)) + ((3*b*c - Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(5/4)) + ((3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(5/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b])

Rule 1858

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4}{(a - bx^4)^2} dx &= \frac{x(bc + ag + bdx + bex^2 + bfx^3)}{4ab(a - bx^4)} + \frac{\int \frac{3bc - ag + 2bdx + bex^2}{a - bx^4} dx}{4ab} \\
&= \frac{x(bc + ag + bdx + bex^2 + bfx^3)}{4ab(a - bx^4)} + \frac{\int \left(\frac{2bdx}{a - bx^4} + \frac{3bc - ag + bex^2}{a - bx^4} \right) dx}{4ab} \\
&= \frac{x(bc + ag + bdx + bex^2 + bfx^3)}{4ab(a - bx^4)} + \frac{\int \frac{3bc - ag + bex^2}{a - bx^4} dx}{4ab} + \frac{d \int \frac{x}{a - bx^4} dx}{2a} \\
&= \frac{x(bc + ag + bdx + bex^2 + bfx^3)}{4ab(a - bx^4)} + \frac{d \operatorname{Subst} \left(\int \frac{1}{a - bx^2} dx, x, x^2 \right)}{4a} - \frac{(3bc - \sqrt{a}\sqrt{be} - ag)}{8a^{3/2}\sqrt{b}} \\
&= \frac{x(bc + ag + bdx + bex^2 + bfx^3)}{4ab(a - bx^4)} + \frac{(3bc - \sqrt{a}\sqrt{be} - ag) \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{8a^{7/4}b^{5/4}} + \frac{(3bc + \sqrt{a}\sqrt{be} + ag)}{8a^{3/2}\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.314004, size = 221, normalized size = 1.28

$$\frac{4a^{3/4} \sqrt[4]{b(a(f+gx)+bx(c+x(d+ex)))}}{a-bx^4} - \log\left(\sqrt[4]{a} - \sqrt[4]{bx}\right) \left(2\sqrt[4]{ab^3}d + \sqrt{a}\sqrt{be} - ag + 3bc\right) + \log\left(\sqrt[4]{a} + \sqrt[4]{bx}\right) \left(-2\sqrt[4]{ab^3}d + \sqrt{a}\sqrt{be} + ag\right)$$

$$16a^{7/4}b^{5/4}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^2,x]

[Out] ((4*a^(3/4)*b^(1/4)*(a*(f + g*x) + b*x*(c + x*(d + e*x))))/(a - b*x^4) - 2*(-3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (3*b*c + 2*a^(1/4)*b^(3/4)*d + Sqrt[a]*Sqrt[b]*e - a*g)*Log[a^(1/4) - b^(1/4)*x] + (3*b*c - 2*a^(1/4)*b^(3/4)*d + Sqrt[a]*Sqrt[b]*e - a*g)*Log[a^(1/4) + b^(1/4)*x] + 2*a^(1/4)*b^(3/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(16*a^(7/4)*b^(5/4))

Maple [B] time = 0.008, size = 289, normalized size = 1.7

$$\frac{1}{bx^4 - a} \left(-\frac{ex^3}{4a} - \frac{dx^2}{4a} - \frac{(ag + bc)x}{4ab} - \frac{f}{4b} \right) - \frac{g}{8ab} \sqrt[4]{\frac{a}{b}} \arctan\left(x \frac{1}{\sqrt[4]{\frac{a}{b}}}\right) + \frac{3c}{8a^2} \sqrt[4]{\frac{a}{b}} \arctan\left(x \frac{1}{\sqrt[4]{\frac{a}{b}}}\right) - \frac{g}{16ab} \sqrt[4]{\frac{a}{b}} \ln\left(\left(x + \sqrt[4]{\frac{a}{b}}\right) \frac{x + \sqrt[4]{\frac{a}{b}}}{x - \sqrt[4]{\frac{a}{b}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x)$

[Out] $(-1/4/a*e*x^3-1/4*d/a*x^2-1/4*(a*g+b*c)/a/b*x-1/4*f/b)/(b*x^4-a)-1/8/b/a*(1/b*a)^{1/4}*\arctan(x/(1/b*a)^{1/4})*g+3/8*c/a^2*(1/b*a)^{1/4}*\arctan(x/(1/b*a)^{1/4})-1/16/b/a*(1/b*a)^{1/4}*\ln((x+(1/b*a)^{1/4})/(x-(1/b*a)^{1/4}))*g+3/16*c/a^2*(1/b*a)^{1/4}*\ln((x+(1/b*a)^{1/4})/(x-(1/b*a)^{1/4}))-1/8*d/a/(a*b)^{1/2}*\ln((-a+x^2*(a*b)^{1/2})/(-a-x^2*(a*b)^{1/2}))-1/8*e/a/b/(1/b*a)^{1/4}*\arctan(x/(1/b*a)^{1/4})+1/16*e/a/b/(1/b*a)^{1/4}*\ln((x+(1/b*a)^{1/4})/(x-(1/b*a)^{1/4}))$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, \text{algorithm}="fricas")$

[Out] Timed out

Sympy [B] time = 169.346, size = 1406, normalized size = 8.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)$

```
[Out] RootSum(65536*_t**4*a**7*b**5 + _t**2*(1024*a**5*b**3*e*g - 3072*a**4*b**4*
c*e - 2048*a**4*b**4*d**2) + _t*(128*a**4*b**2*d*g**2 - 768*a**3*b**3*c*d*g
+ 128*a**3*b**3*d*e**2 + 1152*a**2*b**4*c**2*d) - a**4*g**4 + 12*a**3*b*c*
g**3 + 2*a**3*b*e**2*g**2 - 54*a**2*b**2*c**2*g**2 - 12*a**2*b**2*c*e**2*g
+ 16*a**2*b**2*d**2*e*g - a**2*b**2*e**4 + 108*a*b**3*c**3*g + 18*a*b**3*c*
*2*e**2 - 48*a*b**3*c*d**2*e + 16*a*b**3*d**4 - 81*b**4*c**4, Lambda(_t, _t
*log(x + (-4096*_t**3*a**8*b**4*e*g**2 + 24576*_t**3*a**7*b**5*c*e*g - 3276
8*_t**3*a**7*b**5*d**2*g - 4096*_t**3*a**7*b**5*e**3 - 36864*_t**3*a**6*b**
6*c**2*e + 98304*_t**3*a**6*b**6*c*d**2 - 512*_t**2*a**7*b**3*d*g**3 + 4608
*_t**2*a**6*b**4*c*d*g**2 + 1536*_t**2*a**6*b**4*d*e**2*g - 13824*_t**2*a**
5*b**5*c**2*d*g - 4608*_t**2*a**5*b**5*c*d*e**2 + 4096*_t**2*a**5*b**5*d**3
*e + 13824*_t**2*a**4*b**6*c**3*d - 16*_t*a**7*b*g**5 + 240*_t*a**6*b**2*c*
g**4 - 64*_t*a**6*b**2*e**2*g**3 - 1440*_t*a**5*b**3*c**2*g**3 + 576*_t*a**
5*b**3*c*e**2*g**2 - 576*_t*a**5*b**3*d**2*e*g**2 - 48*_t*a**5*b**3*e**4*g
+ 4320*_t*a**4*b**4*c**3*g**2 - 1728*_t*a**4*b**4*c**2*e**2*g + 3456*_t*a**
4*b**4*c*d**2*e*g + 144*_t*a**4*b**4*c*e**4 + 512*_t*a**4*b**4*d**4*g + 192
*_t*a**4*b**4*d**2*e**3 - 6480*_t*a**3*b**5*c**4*g + 1728*_t*a**3*b**5*c**3
*e**2 - 5184*_t*a**3*b**5*c**2*d**2*e - 1536*_t*a**3*b**5*c*d**4 + 3888*_t*
a**2*b**6*c**5 - 10*a**5*b*d*e*g**4 + 120*a**4*b**2*c*d*e*g**3 - 40*a**4*b*
*2*d**3*g**3 - 540*a**3*b**3*c**2*d*e*g**2 + 360*a**3*b**3*c*d**3*g**2 - 40
*a**3*b**3*d**3*e**2*g - 6*a**3*b**3*d*e**5 + 1080*a**2*b**4*c**3*d*e*g - 1
080*a**2*b**4*c**2*d**3*g + 120*a**2*b**4*c*d**3*e**2 - 64*a**2*b**4*d**5*e
- 810*a*b**5*c**4*d*e + 1080*a*b**5*c**3*d**3)/(a**6*g**6 - 18*a**5*b*c*g*
*5 + a**5*b*e**2*g**4 + 135*a**4*b**2*c**2*g**4 - 12*a**4*b**2*c*e**2*g**3
+ 32*a**4*b**2*d**2*e*g**3 - a**4*b**2*e**4*g**2 - 540*a**3*b**3*c**3*g**3
+ 54*a**3*b**3*c**2*e**2*g**2 - 288*a**3*b**3*c*d**2*e*g**2 + 6*a**3*b**3*c
*e**4*g + 64*a**3*b**3*d**4*g**2 - 32*a**3*b**3*d**2*e**3*g - a**3*b**3*e**
6 + 1215*a**2*b**4*c**4*g**2 - 108*a**2*b**4*c**3*e**2*g + 864*a**2*b**4*c*
*2*d**2*e*g - 9*a**2*b**4*c**2*e**4 - 384*a**2*b**4*c*d**4*g + 96*a**2*b**4
*c*d**2*e**3 - 64*a**2*b**4*d**4*e**2 - 1458*a*b**5*c**5*g + 81*a*b**5*c**4
*e**2 - 864*a*b**5*c**3*d**2*e + 576*a*b**5*c**2*d**4 + 729*b**6*c**6)))) -
(a*f + b*d*x**2 + b*e*x**3 + x*(a*g + b*c))/(-4*a**2*b + 4*a*b**2*x**4)
```

Giac [B] time = 1.08497, size = 528, normalized size = 3.07

$$\frac{bx^3e + bdx^2 + bcx + agx + af}{4(bx^4 - a)ab} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{-abb^2d} + 3(-ab^3)^{\frac{1}{4}}b^2c - (-ab^3)^{\frac{1}{4}}abg + (-ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/4*(b*x^3*e + b*d*x^2 + b*c*x + a*g*x + a*f)/((b*x^4 - a)*a*b) + 1/16*\sqrt{2}*(2*\sqrt{2}*\sqrt{-a*b}*b^2*d + 3*(-a*b^3)^{1/4}*b^2*c - (-a*b^3)^{1/4}*a*b*g + (-a*b^3)^{3/4}*e)*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(-a/b)^{1/4})/(-a/b)^{1/4})/(a^2*b^3) \\ & + 1/16*\sqrt{2}*(2*\sqrt{2}*\sqrt{-a*b}*b^2*d + 3*(-a*b^3)^{1/4}*b^2*c - (-a*b^3)^{1/4}*a*b*g + (-a*b^3)^{3/4}*e)*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(-a/b)^{1/4})/(-a/b)^{1/4})/(a^2*b^3) \\ & + 1/32*\sqrt{2}*(3*(-a*b^3)^{1/4}*b^2*c - (-a*b^3)^{1/4}*a*b*g - (-a*b^3)^{3/4}*e)*\log(x^2 + \sqrt{2}*x*(-a/b)^{1/4} + \sqrt{-a/b})/(a^2*b^3) - 1/32*\sqrt{2}*(3*(-a*b^3)^{1/4}*b^2*c - (-a*b^3)^{1/4}*a*b*g - (-a*b^3)^{3/4}*e)*\log(x^2 - \sqrt{2}*x*(-a/b)^{1/4} + \sqrt{-a/b})/(a^2*b^3) \end{aligned}$$

$$3.173 \quad \int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3} dx$$

Optimal. Leaf size=221

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(-5\sqrt{a}\sqrt{be}-3ag+21bc)}{64a^{11/4}b^{5/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(5\sqrt{a}\sqrt{be}-3ag+21bc)}{64a^{11/4}b^{5/4}} + \frac{x(-ag+7bc+6bdx+5bex^2)+4a}{32a^2b(a-bx^4)}$$

[Out] (x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a - b*x^4)^2) + (4*a*f + x*(7*b*c - a*g + 6*b*d*x + 5*b*e*x^2))/(32*a^2*b*(a - b*x^4)) + ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(5/4)) + (3*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b])

Rubi [A] time = 0.263298, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1858, 1854, 1876, 275, 208, 1167, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(-5\sqrt{a}\sqrt{be}-3ag+21bc)}{64a^{11/4}b^{5/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(5\sqrt{a}\sqrt{be}-3ag+21bc)}{64a^{11/4}b^{5/4}} + \frac{x(-ag+7bc+6bdx+5bex^2)+4a}{32a^2b(a-bx^4)}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^3,x]

[Out] (x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a - b*x^4)^2) + (4*a*f + x*(7*b*c - a*g + 6*b*d*x + 5*b*e*x^2))/(32*a^2*b*(a - b*x^4)) + ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(5/4)) + (3*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b])

Rule 1858

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D

```
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1})*(a + b*x^n)^(p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4}{(a - bx^4)^3} dx &= \frac{x(bc + ag + bdx + bex^2 + bfx^3)}{8ab(a - bx^4)^2} + \frac{\int \frac{7bc - ag + 6bdx + 5bex^2 + 4bfx^3}{(a - bx^4)^2} dx}{8ab} \\
&= \frac{x(bc + ag + bdx + bex^2 + bfx^3)}{8ab(a - bx^4)^2} + \frac{4af + x(7bc - ag + 6bdx + 5bex^2)}{32a^2b(a - bx^4)} - \frac{\int \frac{-3(7bc - ag)}{a - bx^4} dx}{32a^2} \\
&= \frac{x(bc + ag + bdx + bex^2 + bfx^3)}{8ab(a - bx^4)^2} + \frac{4af + x(7bc - ag + 6bdx + 5bex^2)}{32a^2b(a - bx^4)} - \frac{\int \left(-\frac{12bdx}{a - bx^4}\right) dx}{32a^2} \\
&= \frac{x(bc + ag + bdx + bex^2 + bfx^3)}{8ab(a - bx^4)^2} + \frac{4af + x(7bc - ag + 6bdx + 5bex^2)}{32a^2b(a - bx^4)} - \frac{\int \frac{-3(7bc - ag)}{a - bx^4} dx}{32a^2} \\
&= \frac{x(bc + ag + bdx + bex^2 + bfx^3)}{8ab(a - bx^4)^2} + \frac{4af + x(7bc - ag + 6bdx + 5bex^2)}{32a^2b(a - bx^4)} + \frac{(3d) \text{Subst}}{32a^2} \\
&= \frac{x(bc + ag + bdx + bex^2 + bfx^3)}{8ab(a - bx^4)^2} + \frac{4af + x(7bc - ag + 6bdx + 5bex^2)}{32a^2b(a - bx^4)} + \frac{(21bc - 5\sqrt{ab^3})}{32a^2}
\end{aligned}$$

Mathematica [A] time = 0.491858, size = 263, normalized size = 1.19

$$\frac{4a^{3/4} \sqrt[4]{b} (a^2(4f + 3gx) + abx(11c + x(10d + 9ex + gx^3)) - b^2x^5(7c + x(6d + 5ex)))}{(a - bx^4)^2} - \log\left(\sqrt[4]{a} - \sqrt[4]{bx}\right) (12\sqrt[4]{ab^3}d + 5\sqrt{a}\sqrt{be} - 3ag + 21bc) + \log\left(\frac{128a^{11/4}b^{5/4}}{128}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^3, x]

[Out] ((4*a^(3/4)*b^(1/4)*(a^2*(4*f + 3*g*x) - b^2*x^5*(7*c + x*(6*d + 5*e*x))) + a*b*x*(11*c + x*(10*d + 9*e*x + g*x^3)))/(a - b*x^4)^2 + 2*(21*b*c - 5*sqrt[a]*sqrt[b]*e - 3*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (21*b*c + 12*a^(1/4)*b^(3/4)*d + 5*sqrt[a]*sqrt[b]*e - 3*a*g)*Log[a^(1/4) - b^(1/4)*x] + (21*b*c - 12*a^(1/4)*b^(3/4)*d + 5*sqrt[a]*sqrt[b]*e - 3*a*g)*Log[a^(1/4) + b^(1/4)*x] + 12*a^(1/4)*b^(3/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2]/(128*a^(11/4)*b^(5/4))

Maple [A] time = 0.011, size = 328, normalized size = 1.5

$$-\frac{1}{(bx^4 - a)^2} \left(\frac{5bex^7}{32a^2} + \frac{3bdx^6}{16a^2} - \frac{(ag - 7bc)x^5}{32a^2} - \frac{9ex^3}{32a} - \frac{5dx^2}{16a} - \frac{(3ag + 11bc)x}{32ab} - \frac{f}{8b} \right) - \frac{3g}{64ba^2} \sqrt[4]{a} \sqrt{b} \arctan \left(x \frac{1}{\sqrt[4]{a} \sqrt{b}} \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x)`

[Out] $-(5/32/a^2*b*e*x^7+3/16/a^2*d*b*x^6-1/32*(a*g-7*b*c)/a^2*x^5-9/32/a*e*x^3-5/16*d/a*x^2-1/32*(3*a*g+11*b*c)/a/b*x-1/8*f/b)/(b*x^4-a)^2-3/64/b/a^2*(1/b*a)^{(1/4)}*\arctan(x/(1/b*a)^{(1/4)})*g+21/64*c/a^3*(1/b*a)^{(1/4)}*\arctan(x/(1/b*a)^{(1/4)})-3/128/b/a^2*(1/b*a)^{(1/4)}*\ln((x+(1/b*a)^{(1/4)})/(x-(1/b*a)^{(1/4)}))*g+21/128*c/a^3*(1/b*a)^{(1/4)}*\ln((x+(1/b*a)^{(1/4)})/(x-(1/b*a)^{(1/4)}))-3/32*d/a^2/(a*b)^{(1/2)}*\ln((-a+x^2*(a*b)^{(1/2)})/(-a-x^2*(a*b)^{(1/2)}))-5/64*e/a^2/b/(1/b*a)^{(1/4)}*\arctan(x/(1/b*a)^{(1/4)})+5/128*e/a^2/b/(1/b*a)^{(1/4)}*\ln((x+(1/b*a)^{(1/4)})/(x-(1/b*a)^{(1/4)}))$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")`

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)

[Out] Timed out

Giac [B] time = 1.08181, size = 595, normalized size = 2.69

$$\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{-abb^2d} - 21(-ab^3)^{\frac{1}{4}}b^2c + 3(-ab^3)^{\frac{1}{4}}abg - 5(-ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right) + \sqrt{2}\left(12\sqrt{2}\sqrt{-abb^2d} - 21(-ab^3)^{\frac{1}{4}}b^2c + 3(-ab^3)^{\frac{1}{4}}abg - 5(-ab^3)^{\frac{3}{4}}e\right)}{128a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/128*\sqrt{2}*(12*\sqrt{2}*\sqrt{-a*b}*b^2*d - 21*(-a*b^3)^{(1/4)}*b^2*c + 3*(-a*b^3)^{(1/4)}*a*b*g - 5*(-a*b^3)^{(3/4)}*e)*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2} \\ & *(-a/b)^{(1/4)})/(-a/b)^{(1/4)})/(a^3*b^3) - 1/128*\sqrt{2}*(12*\sqrt{2}*\sqrt{-a*b} \\ & *b^2*d - 21*(-a*b^3)^{(1/4)}*b^2*c + 3*(-a*b^3)^{(1/4)}*a*b*g - 5*(-a*b^3)^{(3/4)}*e)*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2} \\ & *(-a/b)^{(1/4)})/(-a/b)^{(1/4)})/(a^3*b^3) + 1/256*\sqrt{2}*(21*(-a*b^3)^{(1/4)}*b^2*c - 3*(-a*b^3)^{(1/4)}*a*b*g - 5*(-a*b^3)^{(3/4)}*e)*\log(x^2 + \sqrt{2} \\ & *x*(-a/b)^{(1/4)} + \sqrt{-a/b})/(a^3*b^3) - 1/256*\sqrt{2}*(21*(-a*b^3)^{(1/4)}*b^2*c - 3*(-a*b^3)^{(1/4)}*a*b*g - 5*(-a*b^3)^{(3/4)}*e)*\log(x^2 - \sqrt{2} \\ & *x*(-a/b)^{(1/4)} + \sqrt{-a/b})/(a^3*b^3) - 1/32 \\ & *(5*b^2*x^7*e + 6*b^2*d*x^6 + 7*b^2*c*x^5 - a*b*g*x^5 - 9*a*b*x^3*e - 10*a*b*d*x^2 - 11*a*b*c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 - a)^2*a^2*b) \end{aligned}$$

$$3.174 \quad \int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4} dx$$

Optimal. Leaf size=266

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(-15\sqrt{a}\sqrt{be}-7ag+77bc)}{256a^{15/4}b^{5/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(15\sqrt{a}\sqrt{be}-7ag+77bc)}{256a^{15/4}b^{5/4}} + \frac{x(-ag+11bc+10bdx+9bex^2)}{96a^2b(a-bx^4)^2}$$

[Out] (x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a - b*x^4)^3) + (x*(7*(11*b*c - a*g) + 60*b*d*x + 45*b*e*x^2))/(384*a^3*b*(a - b*x^4)) + (8*a*f + x*(11*b*c - a*g + 10*b*d*x + 9*b*e*x^2))/(96*a^2*b*(a - b*x^4)^2) + ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(5/4)) + ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(5/4)) + (5*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b])

Rubi [A] time = 0.320272, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {1858, 1854, 1855, 1876, 275, 208, 1167, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(-15\sqrt{a}\sqrt{be}-7ag+77bc)}{256a^{15/4}b^{5/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(15\sqrt{a}\sqrt{be}-7ag+77bc)}{256a^{15/4}b^{5/4}} + \frac{x(-ag+11bc+10bdx+9bex^2)}{96a^2b(a-bx^4)^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^4, x]

[Out] (x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a - b*x^4)^3) + (x*(7*(11*b*c - a*g) + 60*b*d*x + 45*b*e*x^2))/(384*a^3*b*(a - b*x^4)) + (8*a*f + x*(11*b*c - a*g + 10*b*d*x + 9*b*e*x^2))/(96*a^2*b*(a - b*x^4)^2) + ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(5/4)) + ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(5/4)) + (5*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b])

Rule 1858

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,

```
x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x], Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x] /; GeQ[q, n] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1})*(a + b*x^n)^(p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
```

$c*d^2 - a*e^2, 0]$ && PosQ[-(a*c)]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned}
 \int \frac{c + dx + ex^2 + fx^3 + gx^4}{(a - bx^4)^4} dx &= \frac{x(bc + ag + bdx + bex^2 + bfx^3)}{12ab(a - bx^4)^3} + \frac{\int \frac{11bc - ag + 10bdx + 9bex^2 + 8bfx^3}{(a - bx^4)^3} dx}{12ab} \\
 &= \frac{x(bc + ag + bdx + bex^2 + bfx^3)}{12ab(a - bx^4)^3} + \frac{8af + x(11bc - ag + 10bdx + 9bex^2)}{96a^2b(a - bx^4)^2} - \frac{\int \frac{-7(11bc - ag + 10bdx + 9bex^2 + 8bfx^3)}{(a - bx^4)^3} dx}{96} \\
 &= \frac{x(bc + ag + bdx + bex^2 + bfx^3)}{12ab(a - bx^4)^3} + \frac{x(7(11bc - ag) + 60bdx + 45bex^2)}{384a^3b(a - bx^4)} + \frac{8af + x(11bc - ag + 10bdx + 9bex^2)}{96} \\
 &= \frac{x(bc + ag + bdx + bex^2 + bfx^3)}{12ab(a - bx^4)^3} + \frac{x(7(11bc - ag) + 60bdx + 45bex^2)}{384a^3b(a - bx^4)} + \frac{8af + x(11bc - ag + 10bdx + 9bex^2)}{96} \\
 &= \frac{x(bc + ag + bdx + bex^2 + bfx^3)}{12ab(a - bx^4)^3} + \frac{x(7(11bc - ag) + 60bdx + 45bex^2)}{384a^3b(a - bx^4)} + \frac{8af + x(11bc - ag + 10bdx + 9bex^2)}{96} \\
 &= \frac{x(bc + ag + bdx + bex^2 + bfx^3)}{12ab(a - bx^4)^3} + \frac{x(7(11bc - ag) + 60bdx + 45bex^2)}{384a^3b(a - bx^4)} + \frac{8af + x(11bc - ag + 10bdx + 9bex^2)}{96} \\
 &= \frac{x(bc + ag + bdx + bex^2 + bfx^3)}{12ab(a - bx^4)^3} + \frac{x(7(11bc - ag) + 60bdx + 45bex^2)}{384a^3b(a - bx^4)} + \frac{8af + x(11bc - ag + 10bdx + 9bex^2)}{96}
 \end{aligned}$$

Mathematica [A] time = 0.323845, size = 313, normalized size = 1.18

$$\frac{128a^{11/4} \sqrt[4]{b}(a(f+gx)+bx(c+x(d+ex)))}{(a-bx^4)^3} + \frac{16a^{7/4} \sqrt[4]{bx}(-ag+11bc+bx(10d+9ex))}{(a-bx^4)^2} + \frac{4a^{3/4} \sqrt[4]{bx}(-7ag+77bc+15bx(4d+3ex))}{a-bx^4} - 3 \log(\sqrt[4]{a} - \sqrt[4]{bx}) (40 \sqrt[4]{ab^3})$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^4, x]

[Out]
$$\begin{aligned} & ((4*a^{3/4}*b^{1/4}*x*(77*b*c - 7*a*g + 15*b*x*(4*d + 3*e*x)))/(a - b*x^4) \\ & + (16*a^{7/4}*b^{1/4}*x*(11*b*c - a*g + b*x*(10*d + 9*e*x)))/(a - b*x^4)^2 \\ & + (128*a^{11/4}*b^{1/4}*(a*(f + g*x) + b*x*(c + x*(d + e*x))))/(a - b*x^4)^3 \\ & + 6*(77*b*c - 15*\text{Sqrt}[a]*\text{Sqrt}[b]*e - 7*a*g)*\text{ArcTan}[(b^{1/4}*x)/a^{1/4}] - \\ & 3*(77*b*c + 40*a^{1/4}*b^{3/4}*d + 15*\text{Sqrt}[a]*\text{Sqrt}[b]*e - 7*a*g)*\text{Log}[a^{1/4} \\ & - b^{1/4}*x] + 3*(77*b*c - 40*a^{1/4}*b^{3/4}*d + 15*\text{Sqrt}[a]*\text{Sqrt}[b]*e - \\ & 7*a*g)*\text{Log}[a^{1/4} + b^{1/4}*x] + 120*a^{1/4}*b^{3/4}*d*\text{Log}[\text{Sqrt}[a] + \text{Sqrt} \\ & [b]*x^2)]/(1536*a^{15/4}*b^{5/4}) \end{aligned}$$

Maple [A] time = 0.012, size = 368, normalized size = 1.4

$$\frac{1}{(bx^4 - a)^3} \left(-\frac{15b^2ex^{11}}{128a^3} - \frac{5b^2dx^{10}}{32a^3} + \frac{(7ag - 77bc)bx^9}{384a^3} + \frac{21bex^7}{64a^2} + \frac{5bdx^6}{12a^2} - \frac{(3ag - 33bc)x^5}{64a^2} - \frac{113ex^3}{384a} - \frac{11dx^2}{32a} - \frac{7}{32a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4, x)

[Out]
$$\begin{aligned} & (-15/128*e/a^3*b^2*x^{11}-5/32*d/a^3*b^2*x^{10}+7/384*(a*g-11*b*c)/a^3*b*x^9+21 \\ & /64/a^2*b*e*x^7+5/12/a^2*d*b*x^6-3/64/a^2*(a*g-11*b*c)*x^5-113/384/a*e*x^3- \\ & 11/32*d/a*x^2-1/128*(7*a*g+51*b*c)/a/b*x-1/12*f/b)/(b*x^4-a)^3-7/512/b/a^3* \\ & (1/b*a)^{1/4}*ln((x+(1/b*a)^{1/4})/(x-(1/b*a)^{1/4}))*g+77/512/a^4*c*(1/b*a)^{1/4} \\ & *ln((x+(1/b*a)^{1/4})/(x-(1/b*a)^{1/4}))-7/256/b/a^3*(1/b*a)^{1/4}*a \\ & rctan(x/(1/b*a)^{1/4})*g+77/256/a^4*c*(1/b*a)^{1/4}*arctan(x/(1/b*a)^{1/4}) \\ & -5/64/a^3*d/(a*b)^{1/2}*ln((-a+x^2*(a*b)^{1/2})/(-a-x^2*(a*b)^{1/2}))-15/25 \\ & 6/a^3*e/b/(1/b*a)^{1/4}*arctan(x/(1/b*a)^{1/4})+15/512/a^3*e/b/(1/b*a)^{1/4} \\ & *ln((x+(1/b*a)^{1/4})/(x-(1/b*a)^{1/4})) \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4, x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)

[Out] Timed out

Giac [B] time = 1.09935, size = 662, normalized size = 2.49

$$\frac{\sqrt{2} \left(40 \sqrt{2} \sqrt{-abb^2d} + 77 (-ab^3)^{\frac{1}{4}} b^2c - 7 (-ab^3)^{\frac{1}{4}} abg + 15 (-ab^3)^{\frac{3}{4}} e \right) \arctan \left(\frac{\sqrt{2} \left(2x + \sqrt{2} \left(-\frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left(-\frac{a}{b} \right)^{\frac{1}{4}}} \right) + \sqrt{2} \left(40 \sqrt{2} \sqrt{-abb^2d} + \dots \right)}{512 a^4 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")

[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b^2*d + 77*(-a*b^3)^(1/4)*b^2*c - 7*(-a*b^3)^(1/4)*a*b*g + 15*(-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*

$$\begin{aligned}
& b)b^2d + 77*(-ab^3)^{1/4}b^2c - 7*(-ab^3)^{1/4}abg + 15*(-ab^3)^{3/4}e) \arctan(1/2\sqrt{2}(2x - \sqrt{2})(-a/b)^{1/4})/(-a/b)^{1/4})/(a^4b^3) \\
& + 1/1024\sqrt{2}(77*(-ab^3)^{1/4}b^2c - 7*(-ab^3)^{1/4}abg - 15*(-ab^3)^{3/4}e) \log(x^2 + \sqrt{2}x(-a/b)^{1/4} + \sqrt{-a/b})/(a^4b^3) \\
&) - 1/1024\sqrt{2}(77*(-ab^3)^{1/4}b^2c - 7*(-ab^3)^{1/4}abg - 15*(-ab^3)^{3/4}e) \log(x^2 - \sqrt{2}x(-a/b)^{1/4} + \sqrt{-a/b})/(a^4b^3) - \\
& 1/384(45b^3x^{11}e + 60b^3d*x^{10} + 77b^3c*x^9 - 7a*b^2*g*x^9 - 126a*b^2*x^7e - 160a*b^2*d*x^6 - 198a*b^2*c*x^5 + 18a^2*b*g*x^5 + 113a^2*b*x^3e + 132a^2*b*d*x^2 + 153a^2*b*c*x + 21a^3*g*x + 32a^3*f)/(b*x^4 - a)^3a^3b)
\end{aligned}$$

$$3.175 \quad \int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4} dx$$

Optimal. Leaf size=319

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-\sqrt{a}\sqrt{be} - ag + bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-\sqrt{a}\sqrt{be} - ag + bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}} - \frac{\tan^{-1}\left(1\right)}{1}$$

[Out] (g*x)/b + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) - ((b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + (f*Log[a + b*x^4])/(4*b)

Rubi [A] time = 0.350505, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {1885, 1248, 635, 205, 260, 1887, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-\sqrt{a}\sqrt{be} - ag + bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-\sqrt{a}\sqrt{be} - ag + bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}} - \frac{\tan^{-1}\left(1\right)}{1}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4), x]

[Out] (g*x)/b + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) - ((b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + (f*Log[a + b*x^4])/(4*b)

Rule 1885

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (

$2*(q - j)/n + 1\}*(a + b*x^n)^p, \{j, 0, n/2 - 1\}, x] /; \text{FreeQ}\{a, b, p\}, x] \&\& \text{PolyQ}\{Pq, x\} \&\& \text{IGtQ}\{n/2, 0\} \&\& \text{!PolyQ}\{Pq, x^{(n/2)}\}$

Rule 1248

$\text{Int}[(x_*)*((d_*) + (e_*)*(x_*)^2)^{(q_*)}*((a_*) + (c_*)*(x_*)^4)^{(p_*)}, x_Symbol] \text{:>} \text{Dist}[1/2, \text{Subst}[\text{Int}[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; \text{FreeQ}\{a, c, d, e, p, q\}, x]$

Rule 635

$\text{Int}[((d_*) + (e_*)*(x_*)^2)/((a_*) + (c_*)*(x_*)^2), x_Symbol] \text{:>} \text{Dist}[d, \text{Int}[1/(a + c*x^2), x], x] + \text{Dist}[e, \text{Int}[x/(a + c*x^2), x], x] /; \text{FreeQ}\{a, c, d, e\}, x] \&\& \text{!NiceSqrtQ}[-(a*c)]$

Rule 205

$\text{Int}[((a_*) + (b_*)*(x_*)^2)^{-1}, x_Symbol] \text{:>} \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b]$

Rule 260

$\text{Int}[(x_*)^{(m_*)}/((a_*) + (b_*)*(x_*)^n), x_Symbol] \text{:>} \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]]/(b*n), x] /; \text{FreeQ}\{a, b, m, n\}, x] \&\& \text{EqQ}[m, n - 1]$

Rule 1887

$\text{Int}[(Pq_*)/((a_*) + (b_*)*(x_*)^n), x_Symbol] \text{:>} \text{Int}[\text{ExpandIntegrand}[Pq/(a + b*x^n), x], x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PolyQ}\{Pq, x\} \&\& \text{IntegerQ}[n]$

Rule 1168

$\text{Int}[((d_*) + (e_*)*(x_*)^2)/((a_*) + (c_*)*(x_*)^4), x_Symbol] \text{:>} \text{With}\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \text{Dist}[(d*q - a*e)/(2*a*c), \text{Int}[(q - c*x^2)/(a + c*x^4), x], x] /; \text{FreeQ}\{a, c, d, e\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{NeQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[-(a*c)]$

Rule 1162

$\text{Int}[((d_*) + (e_*)*(x_*)^2)/((a_*) + (c_*)*(x_*)^4), x_Symbol] \text{:>} \text{With}\{q = \text{Rt}[(2*d)/e, 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x] /; \text{FreeQ}\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[d*e]$

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4}{a + bx^4} dx &= \int \left(\frac{x(d + fx^2)}{a + bx^4} + \frac{c + ex^2 + gx^4}{a + bx^4} \right) dx \\
&= \int \frac{x(d + fx^2)}{a + bx^4} dx + \int \frac{c + ex^2 + gx^4}{a + bx^4} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{d + fx}{a + bx^2} dx, x, x^2 \right) + \int \left(\frac{g}{b} + \frac{bc - ag + bex^2}{b(a + bx^4)} \right) dx \\
&= \frac{gx}{b} + \frac{\int \frac{bc - ag + bex^2}{a + bx^4} dx}{b} + \frac{1}{2} d \text{Subst} \left(\int \frac{1}{a + bx^2} dx, x, x^2 \right) + \frac{1}{2} f \text{Subst} \left(\int \frac{x}{a + bx^2} dx, x, x^2 \right) \\
&= \frac{gx}{b} + \frac{d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{b}} + \frac{f \log(a + bx^4)}{4b} + \frac{(bc - \sqrt{a}\sqrt{be} - ag) \int \frac{\sqrt{a}\sqrt{b-bx^2}}{a + bx^4} dx}{2\sqrt{ab}^{3/2}} + \frac{(bc + \sqrt{a}\sqrt{be} - ag) \int \frac{\sqrt{a}\sqrt{b-bx^2}}{a + bx^4} dx}{2\sqrt{ab}^{3/2}} \\
&= \frac{gx}{b} + \frac{d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{b}} + \frac{f \log(a + bx^4)}{4b} - \frac{(bc - \sqrt{a}\sqrt{be} - ag) \int \frac{\frac{\sqrt{2}\sqrt[4]{a} + 2x}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt{b}} - x^2}{4\sqrt{2}a^{3/4}b^{5/4}} dx}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{(bc + \sqrt{a}\sqrt{be} - ag) \int \frac{\sqrt{a}\sqrt{b-bx^2}}{a + bx^4} dx}{2\sqrt{2}a^{3/4}b^{5/4}} \\
&= \frac{gx}{b} + \frac{d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{b}} - \frac{(bc - \sqrt{a}\sqrt{be} - ag) \log \left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2} \right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{(bc - \sqrt{a}\sqrt{be} - ag) \int \frac{\sqrt{a}\sqrt{b-bx^2}}{a + bx^4} dx}{2\sqrt{2}a^{3/4}b^{5/4}} \\
&= \frac{gx}{b} + \frac{d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{b}} - \frac{(bc + \sqrt{a}\sqrt{be} - ag) \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt{a}} \right)}{2\sqrt{2}a^{3/4}b^{5/4}} + \frac{(bc + \sqrt{a}\sqrt{be} - ag) \int \frac{\sqrt{a}\sqrt{b-bx^2}}{a + bx^4} dx}{2\sqrt{2}a^{3/4}b^{5/4}}
\end{aligned}$$

Mathematica [A] time = 0.318302, size = 311, normalized size = 0.97

$$2a^{3/4}\sqrt[4]{b}f \log(a + bx^4) + 8a^{3/4}\sqrt[4]{b}gx - 2 \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt{a}} \right) \left(2\sqrt[4]{ab}^{3/4}d + \sqrt{2}\sqrt{a}\sqrt{be} - \sqrt{2}ag + \sqrt{2}bc \right) + 2 \tan^{-1} \left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt{a}} \right) \left(2\sqrt[4]{ab}^{3/4}d + \sqrt{2}\sqrt{a}\sqrt{be} - \sqrt{2}ag + \sqrt{2}bc \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4),x]

[Out] (8*a^(3/4)*b^(1/4)*g*x - 2*(Sqrt[2]*b*c + 2*a^(1/4)*b^(3/4)*d + Sqrt[2]*Sqrt[a]*Sqrt[b]*e - Sqrt[2]*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*b*c - 2*a^(1/4)*b^(3/4)*d + Sqrt[2]*Sqrt[a]*Sqrt[b]*e - Sqrt[2]*a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-b*c) + Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]

]*(b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 2*a^(3/4)*b^(1/4)*f*Log[a + b*x^4])/(8*a^(3/4)*b^(5/4))

Maple [A] time = 0.005, size = 429, normalized size = 1.3

$$\frac{gx}{b} - \frac{\sqrt{2}g}{4b} \sqrt[4]{\frac{a}{b}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) + \frac{c\sqrt{2}}{4a} \sqrt[4]{\frac{a}{b}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) - \frac{\sqrt{2}g}{4b} \sqrt[4]{\frac{a}{b}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right) + \frac{c\sqrt{2}}{4a} \sqrt[4]{\frac{a}{b}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x)

[Out] g*x/b-1/4/b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*g+1/4*c*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)-1/4/b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*g+1/4*c*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)-1/8/b*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*g+1/8*c*(1/b*a)^(1/4)/a*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/2*d/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+1/8*e/b/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/4*e/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/4*e/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+1/4*f*ln(b*x^4+a)/b

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] time = 158.354, size = 2392, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**5 - 256*_t**3*a**3*b**4*f + _t**2*(-64*a**3*b**3*
e*g + 96*a**3*b**3*f**2 + 64*a**2*b**4*c*e + 32*a**2*b**4*d**2) + _t*(-16*a
**3*b**2*d*g**2 + 32*a**3*b**2*e*f*g - 16*a**3*b**2*f**3 + 32*a**2*b**3*c*d
*g - 32*a**2*b**3*c*e*f - 16*a**2*b**3*d**2*f + 16*a**2*b**3*d*e**2 - 16*a*
b**4*c**2*d) + a**4*g**4 - 4*a**3*b*c*g**3 + 4*a**3*b*d*f*g**2 + 2*a**3*b*e
**2*g**2 - 4*a**3*b*e*f**2*g + a**3*b*f**4 + 6*a**2*b**2*c**2*g**2 - 8*a**2
*b**2*c*d*f*g - 4*a**2*b**2*c*e**2*g + 4*a**2*b**2*c*e*f**2 + 4*a**2*b**2*d
**2*e*g + 2*a**2*b**2*d**2*f**2 - 4*a**2*b**2*d*e**2*f + a**2*b**2*e**4 - 4
*a*b**3*c**3*g + 4*a*b**3*c**2*d*f + 2*a*b**3*c**2*e**2 - 4*a*b**3*c*d**2*e
+ a*b**3*d**4 + b**4*c**4, Lambda(_t, _t*log(x + (-64*_t**3*a**5*b**4*e*g*
*2 + 128*_t**3*a**4*b**5*c*e*g - 128*_t**3*a**4*b**5*d**2*g + 64*_t**3*a**4
*b**5*e**3 - 64*_t**3*a**3*b**6*c**2*e + 128*_t**3*a**3*b**6*c*d**2 - 16*_t
**2*a**5*b**3*d*g**3 + 48*_t**2*a**5*b**3*e*f*g**2 + 48*_t**2*a**4*b**4*c*d
*g**2 - 96*_t**2*a**4*b**4*c*e*f*g + 96*_t**2*a**4*b**4*d**2*f*g - 48*_t**2
*a**4*b**4*d*e**2*g - 48*_t**2*a**4*b**4*e**3*f - 48*_t**2*a**3*b**5*c**2*d
*g + 48*_t**2*a**3*b**5*c**2*e*f - 96*_t**2*a**3*b**5*c*d**2*f + 48*_t**2*a
**3*b**5*c*d*e**2 - 32*_t**2*a**3*b**5*d**3*e + 16*_t**2*a**2*b**6*c**3*d -
4*_t*a**6*b*g**5 + 20*_t*a**5*b**2*c*g**4 + 8*_t*a**5*b**2*d*f*g**3 + 16*_
t*a**5*b**2*e**2*g**3 - 12*_t*a**5*b**2*e*f**2*g**2 - 40*_t*a**4*b**3*c**2*
g**3 - 24*_t*a**4*b**3*c*d*f*g**2 - 48*_t*a**4*b**3*c*e**2*g**2 + 24*_t*a**
4*b**3*c*e*f**2*g + 36*_t*a**4*b**3*d**2*e*g**2 - 24*_t*a**4*b**3*d**2*f**2
*g + 24*_t*a**4*b**3*d*e**2*f*g - 12*_t*a**4*b**3*e**4*g + 12*_t*a**4*b**3*
e**3*f**2 + 40*_t*a**3*b**4*c**3*g**2 + 24*_t*a**3*b**4*c**2*d*f*g + 48*_t*
a**3*b**4*c**2*e**2*g - 12*_t*a**3*b**4*c**2*e*f**2 - 72*_t*a**3*b**4*c*d**
2*e*g + 24*_t*a**3*b**4*c*d**2*f**2 - 24*_t*a**3*b**4*c*d*e**2*f + 12*_t*a*
**3*b**4*c*e**4 - 8*_t*a**3*b**4*d**4*g + 16*_t*a**3*b**4*d**3*e*f + 12*_t*a
**3*b**4*d**2*e**3 - 20*_t*a**2*b**5*c**4*g - 8*_t*a**2*b**5*c**3*d*f - 16*
_t*a**2*b**5*c**3*e**2 + 36*_t*a**2*b**5*c**2*d**2*e + 8*_t*a**2*b**5*c*d**
```

```

4 + 4*_t*a*b**6*c**5 + a**6*f*g**5 - 5*a**5*b*c*f*g**4 + 5*a**5*b*d*e*g**4
- a**5*b*d*f**2*g**3 - 4*a**5*b*e**2*f*g**3 + a**5*b*e*f**3*g**2 + 10*a**4*
b**2*c**2*f*g**3 - 20*a**4*b**2*c*d*e*g**3 + 3*a**4*b**2*c*d*f**2*g**2 + 12
*a**4*b**2*c*e**2*f*g**2 - 2*a**4*b**2*c*e*f**3*g + 5*a**4*b**2*d**3*g**3 -
9*a**4*b**2*d**2*e*f*g**2 + 2*a**4*b**2*d**2*f**3*g - 3*a**4*b**2*d*e**2*f
**2*g + 3*a**4*b**2*e**4*f*g - a**4*b**2*e**3*f**3 - 10*a**3*b**3*c**3*f*g*
*2 + 30*a**3*b**3*c**2*d*e*g**2 - 3*a**3*b**3*c**2*d*f**2*g - 12*a**3*b**3*
c**2*e**2*f*g + a**3*b**3*c**2*e*f**3 - 15*a**3*b**3*c*d**3*g**2 + 18*a**3*
b**3*c*d**2*e*f*g - 2*a**3*b**3*c*d**2*f**3 + 3*a**3*b**3*c*d*e**2*f**2 - 3
*a**3*b**3*c*e**4*f + 2*a**3*b**3*d**4*f*g - 5*a**3*b**3*d**3*e**2*g - 2*a*
**3*b**3*d**3*e*f**2 - 3*a**3*b**3*d**2*e**3*f + 3*a**3*b**3*d*e**5 + 5*a**2
*b**4*c**4*f*g - 20*a**2*b**4*c**3*d*e*g + a**2*b**4*c**3*d*f**2 + 4*a**2*b
**4*c**3*e**2*f + 15*a**2*b**4*c**2*d**3*g - 9*a**2*b**4*c**2*d**2*e*f - 2*
a**2*b**4*c*d**4*f + 5*a**2*b**4*c*d**3*e**2 - 2*a**2*b**4*d**5*e - a*b**5*
c**5*f + 5*a*b**5*c**4*d*e - 5*a*b**5*c**3*d**3)/(a**6*g**6 - 6*a**5*b*c*g*
*5 - a**5*b*e**2*g**4 + 15*a**4*b**2*c**2*g**4 + 4*a**4*b**2*c*e**2*g**3 -
8*a**4*b**2*d**2*e*g**3 - a**4*b**2*e**4*g**2 - 20*a**3*b**3*c**3*g**3 - 6*
a**3*b**3*c**2*e**2*g**2 + 24*a**3*b**3*c*d**2*e*g**2 + 2*a**3*b**3*c*e**4*
g - 4*a**3*b**3*d**4*g**2 - 8*a**3*b**3*d**2*e**3*g + a**3*b**3*e**6 + 15*a
**2*b**4*c**4*g**2 + 4*a**2*b**4*c**3*e**2*g - 24*a**2*b**4*c**2*d**2*e*g -
a**2*b**4*c**2*e**4 + 8*a**2*b**4*c*d**4*g + 8*a**2*b**4*c*d**2*e**3 - 4*a
**2*b**4*d**4*e**2 - 6*a*b**5*c**5*g - a*b**5*c**4*e**2 + 8*a*b**5*c**3*d**
2*e - 4*a*b**5*c**2*d**4 + b**6*c**6)))) + g*x/b

```

Giac [A] time = 1.10431, size = 459, normalized size = 1.44

$$\frac{gx}{b} + \frac{f \log(|bx^4 + a|)}{4b} + \frac{\sqrt{2} \left(\sqrt{2} \sqrt{abb^2d} + (ab^3)^{\frac{1}{4}} b^2c - (ab^3)^{\frac{1}{4}} abg + (ab^3)^{\frac{3}{4}} e \right) \arctan \left(\frac{\sqrt{2} \left(2x + \sqrt{2} \left(\frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left(\frac{a}{b} \right)^{\frac{1}{4}}} \right)}{4ab^3} + \frac{\sqrt{2} \left(\sqrt{2} \sqrt{abb^2d} \right)}{4ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")

[Out] g*x/b + 1/4*f*log(abs(b*x^4 + a))/b + 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d + (a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d + (a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)

$$3)^{(1/4)} * b^2 * c - (a * b^3)^{(1/4)} * a * b * g - (a * b^3)^{(3/4)} * e * \log(x^2 - \sqrt{2} * x \\ * (a/b)^{(1/4)} + \sqrt{a/b}) / (a * b^3)$$

$$3.176 \quad \int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2} dx$$

Optimal. Leaf size=341

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-\sqrt{a}\sqrt{be} + ag + 3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-\sqrt{a}\sqrt{be} + ag + 3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}} - \frac{\tan^{-1}}{16\sqrt{2}a^{7/4}b^{5/4}}$$

[Out] (x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a + b*x^4)) + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/(4*a^(3/2)*Sqrt[b]) - ((3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) + ((3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) - ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/4)) + ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/4))

Rubi [A] time = 0.305006, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1858, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-\sqrt{a}\sqrt{be} + ag + 3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-\sqrt{a}\sqrt{be} + ag + 3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}} - \frac{\tan^{-1}}{16\sqrt{2}a^{7/4}b^{5/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^2, x]

[Out] (x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a + b*x^4)) + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/(4*a^(3/2)*Sqrt[b]) - ((3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) + ((3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) - ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/4)) + ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/4))

Rule 1858


```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a +
b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))]/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4}{(a + bx^4)^2} dx &= \frac{x(bc - ag + bdx + bex^2 + bfx^3)}{4ab(a + bx^4)} - \frac{\int \frac{-3bc - ag - 2bdx - bex^2}{a + bx^4} dx}{4ab} \\
&= \frac{x(bc - ag + bdx + bex^2 + bfx^3)}{4ab(a + bx^4)} - \frac{\int \left(-\frac{2bdx}{a + bx^4} + \frac{-3bc - ag - bex^2}{a + bx^4} \right) dx}{4ab} \\
&= \frac{x(bc - ag + bdx + bex^2 + bfx^3)}{4ab(a + bx^4)} - \frac{\int \frac{-3bc - ag - bex^2}{a + bx^4} dx}{4ab} + \frac{d \int \frac{x}{a + bx^4} dx}{2a} \\
&= \frac{x(bc - ag + bdx + bex^2 + bfx^3)}{4ab(a + bx^4)} + \frac{d \operatorname{Subst} \left(\int \frac{1}{a + bx^2} dx, x, x^2 \right)}{4a} + \frac{(3bc - \sqrt{a}\sqrt{be} + ag)}{8a^{3/2}b} \\
&= \frac{x(bc - ag + bdx + bex^2 + bfx^3)}{4ab(a + bx^4)} + \frac{d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{4a^{3/2}\sqrt{b}} - \frac{(3bc - \sqrt{a}\sqrt{be} + ag) \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt{b}} - \frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}}{\sqrt{b}}}{\sqrt{b}} dx}{16\sqrt{2}a^{7/4}b^{5/4}} \\
&= \frac{x(bc - ag + bdx + bex^2 + bfx^3)}{4ab(a + bx^4)} + \frac{d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{4a^{3/2}\sqrt{b}} - \frac{(3bc - \sqrt{a}\sqrt{be} + ag) \log(\sqrt{a} - \dots)}{16\sqrt{2}a^{7/4}b^{5/4}} \\
&= \frac{x(bc - ag + bdx + bex^2 + bfx^3)}{4ab(a + bx^4)} + \frac{d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{4a^{3/2}\sqrt{b}} - \frac{(3bc + \sqrt{a}\sqrt{be} + ag) \tan^{-1} \left(1 - \dots \right)}{8\sqrt{2}a^{7/4}b^{5/4}}
\end{aligned}$$

Mathematica [A] time = 0.196124, size = 319, normalized size = 0.94

$$-\frac{8a^{3/4}\sqrt[4]{b}(a(f+gx)-bx(c+x(d+ex)))}{a+bx^4} - 2 \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} \right) (4\sqrt[4]{ab}^{3/4}d + \sqrt{2}\sqrt{a}\sqrt{be} + \sqrt{2}ag + 3\sqrt{2}bc) + 2 \tan^{-1} \left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + 1 \right) (-\dots)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^2, x]

[Out] ((-8*a^(3/4)*b^(1/4)*(a*(f + g*x) - b*x*(c + x*(d + e*x)))/(a + b*x^4) - 2*(3*Sqrt[2]*b*c + 4*a^(1/4)*b^(3/4)*d + Sqrt[2]*Sqrt[a]*Sqrt[b]*e + Sqrt[2]*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(3*Sqrt[2]*b*c - 4*a^(1/4)*b^(3/4)*d + Sqrt[2]*Sqrt[a]*Sqrt[b]*e + Sqrt[2]*a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(3*b*c - Sqrt[a]*Sqr

$t[b]*e + a*g)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*x + \text{Sqrt}[b]*x^2]/(32*a^{(7/4)}*b^{(5/4)})$

Maple [A] time = 0.009, size = 482, normalized size = 1.4

$$\frac{1}{bx^4 + a} \left(\frac{ex^3}{4a} + \frac{dx^2}{4a} - \frac{(ag - bc)x}{4ab} - \frac{f}{4b} \right) + \frac{\sqrt{2}g}{16ab} \sqrt[4]{\frac{a}{b}} \arctan \left(x\sqrt{2} \frac{1}{\sqrt[4]{\frac{a}{b}}} + 1 \right) + \frac{3c\sqrt{2}}{16a^2} \sqrt[4]{\frac{a}{b}} \arctan \left(x\sqrt{2} \frac{1}{\sqrt[4]{\frac{a}{b}}} + 1 \right) + \frac{\sqrt{2}g}{16ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x)`

[Out] $(1/4/a*e*x^3+1/4*d/a*x^2-1/4*(a*g-b*c)/a/b*x-1/4*f/b)/(b*x^4+a)+1/16/b/a*(1/b*a)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)}*x+1)*g+3/16*c/a^2*(1/b*a)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)}*x+1)+1/16/b/a*(1/b*a)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)}*x-1)*g+3/16*c/a^2*(1/b*a)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)}*x-1)+1/32/b/a*(1/b*a)^{(1/4)}*2^{(1/2)}*\ln((x^2+(1/b*a)^{(1/4)}*x*2^{(1/2)}+(1/b*a)^{(1/2)})/(x^2-(1/b*a)^{(1/4)}*x*2^{(1/2)}+(1/b*a)^{(1/2)}))*g+3/32*c/a^2*(1/b*a)^{(1/4)}*2^{(1/2)}*\ln((x^2+(1/b*a)^{(1/4)}*x*2^{(1/2)}+(1/b*a)^{(1/2)})/(x^2-(1/b*a)^{(1/4)}*x*2^{(1/2)}+(1/b*a)^{(1/2)}))+1/4*d/a/(a*b)^{(1/2)}*\arctan(x^2*(b/a)^{(1/2)})+1/32*e/a/b/(1/b*a)^{(1/4)}*2^{(1/2)}*\ln((x^2-(1/b*a)^{(1/4)}*x*2^{(1/2)}+(1/b*a)^{(1/2)})/(x^2+(1/b*a)^{(1/4)}*x*2^{(1/2)}+(1/b*a)^{(1/2)}))+1/16*e/a/b/(1/b*a)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)}*x+1)+1/16*e/a/b/(1/b*a)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)}*x-1)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] time = 170.756, size = 1406, normalized size = 4.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**5 + _t**2*(1024*a**5*b**3*e*g + 3072*a**4*b**4*c*e + 2048*a**4*b**4*d**2) + _t*(-128*a**4*b**2*d*g**2 - 768*a**3*b**3*c*d*g + 128*a**3*b**3*d*e**2 - 1152*a**2*b**4*c**2*d) + a**4*g**4 + 12*a**3*b*c*g**3 + 2*a**3*b*e**2*g**2 + 54*a**2*b**2*c**2*g**2 + 12*a**2*b**2*c*e**2*g - 16*a**2*b**2*d**2*e*g + a**2*b**2*e**4 + 108*a*b**3*c**3*g + 18*a*b**3*c**2*e**2 - 48*a*b**3*c*d**2*e + 16*a*b**3*d**4 + 81*b**4*c**4, Lambda(_t, _t*log(x + (-4096*_t**3*a**8*b**4*e*g**2 - 24576*_t**3*a**7*b**5*c*e*g + 32768*_t**3*a**7*b**5*d**2*g + 4096*_t**3*a**7*b**5*e**3 - 36864*_t**3*a**6*b**6*c**2*e + 98304*_t**3*a**6*b**6*c*d**2 + 512*_t**2*a**7*b**3*d*g**3 + 4608*_t**2*a**6*b**4*c*d*g**2 + 1536*_t**2*a**6*b**4*d*e**2*g + 13824*_t**2*a**5*b**5*c**2*d*g + 4608*_t**2*a**5*b**5*c*d*e**2 - 4096*_t**2*a**5*b**5*d**3*e + 13824*_t**2*a**4*b**6*c**3*d + 16*_t*a**7*b*g**5 + 240*_t*a**6*b**2*c*g**4 - 64*_t*a**6*b**2*e**2*g**3 + 1440*_t*a**5*b**3*c**2*g**3 - 576*_t*a**5*b**3*c*e**2*g**2 + 576*_t*a**5*b**3*d**2*e*g**2 + 48*_t*a**5*b**3*e**4*g + 4320*_t*a**4*b**4*c**3*g**2 - 1728*_t*a**4*b**4*c**2*e**2*g + 3456*_t*a**4*b**4*c*d**2*e*g + 144*_t*a**4*b**4*c*e**4 + 512*_t*a**4*b**4*d**4*g + 192*_t*a**4*b**4*d**2*e**3 + 6480*_t*a**3*b**5*c**4*g - 1728*_t*a**3*b**5*c**3*e**2 + 5184*_t*a**3*b**5*c**2*d**2*e + 1536*_t*a**3*b**5*c*d**4 + 3888*_t*a**2*b**6*c**5 + 10*a**5*b*d*e*g**4 + 120*a**4*b**2*c*d*e*g**3 - 40*a**4*b**2*d**3*g**3 + 540*a**3*b**3*c**2*d*e*g**2 - 360*a**3*b**3*c*d**3*g**2 + 40*a**3*b**3*d**3*e**2*g + 6*a**3*b**3*d*e**5 + 1080*a**2*b**4*c**3*d*e*g - 1080*a**2*b**4*c**2*d**3*g + 120*a**2*b**4*c*d**3*e**2 - 64*a**2*b**4*d**5*e + 810*a*b**5*c**4*d*e - 1080*a*b**5*c**3*d**3)/(a**6*g**6 + 18*a**5*b*c*g**5 - a**5*b*e**2*g**4 + 135*a**4*b**2*c**2*g**4 - 12*a**4*b**2*c*e**2*g**3 + 32*a**4*b**2*d**2*e*g**3 - a**4*b**2*e**4*g**2 + 540*a**3*b**3*c**3*g**3 - 54*a**3*b**3*c**2*e**2*g**2 + 288*a**3*b**3*c*d**2*e*g**2 - 6*a**3*b**3*c*e**4*g - 64*a**3*b**3*d**4*g**2 + 32*a**3*b**3*d**2*e**3*g + a**3*b**3*e**6 + 1215*a**2*b**4*c**4*g**2 - 108*a**2*b**4*c**3*e**2*g + 864*a**2*b**4*c
```

```

**2*d**2*e*g - 9*a**2*b**4*c**2*e**4 - 384*a**2*b**4*c*d**4*g + 96*a**2*b**
4*c*d**2*e**3 - 64*a**2*b**4*d**4*e**2 + 1458*a*b**5*c**5*g - 81*a*b**5*c**
4*e**2 + 864*a*b**5*c**3*d**2*e - 576*a*b**5*c**2*d**4 + 729*b**6*c**6))))
+ (-a*f + b*d*x**2 + b*e*x**3 + x*(-a*g + b*c))/(4*a**2*b + 4*a*b**2*x**4)

```

Giac [A] time = 1.07941, size = 493, normalized size = 1.45

$$\frac{bx^3e + bdx^2 + bcx - agx - af}{4(bx^4 + a)ab} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{abb^2d} + 3(ab^3)^{\frac{1}{4}}b^2c + (ab^3)^{\frac{1}{4}}abg + (ab^3)^{\frac{3}{4}}e\right) \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2b^3} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```

[Out] 1/4*(b*x^3*e + b*d*x^2 + b*c*x - a*g*x - a*f)/((b*x^4 + a)*a*b) + 1/16*sqrt
(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*
g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(
1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)
*b^2*c + (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - s
qrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*
b^2*c + (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1
/4) + sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(
1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))
/(a^2*b^3)

```

$$3.177 \quad \int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3} dx$$

Optimal. Leaf size=394

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-5\sqrt{a}\sqrt{be} + 3ag + 21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-5\sqrt{a}\sqrt{be} + 3ag + 21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}}$$

[Out] $(x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a + b*x^4)^2) - (4*a*f - x*(7*b*c + a*g + 6*b*d*x + 5*b*e*x^2))/(32*a^2*b*(a + b*x^4)) + (3*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b]) - ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(5/4)) - ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(5/4)) + ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(5/4))$

Rubi [A] time = 0.438579, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {1858, 1854, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-5\sqrt{a}\sqrt{be} + 3ag + 21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-5\sqrt{a}\sqrt{be} + 3ag + 21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^3,x]

[Out] $(x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a + b*x^4)^2) - (4*a*f - x*(7*b*c + a*g + 6*b*d*x + 5*b*e*x^2))/(32*a^2*b*(a + b*x^4)) + (3*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b]) - ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(5/4)) - ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(5/4)) + ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(5/4))$

$\text{qrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*x + \text{Sqrt}[b]*x^2)/(128*\text{Sqrt}[2]*a^{(11/4)}*b^{(5/4)})$

Rule 1858

$\text{Int}[(Pq_)*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] := \text{With}[\{q = \text{Expon}[Pq, x]\}, \text{Module}[\{Q = \text{PolynomialQuotient}[b^{(\text{Floor}[(q - 1)/n] + 1)*Pq}, a + b*x^n, x], R = \text{PolynomialRemainder}[b^{(\text{Floor}[(q - 1)/n] + 1)*Pq}, a + b*x^n, x]\}, \text{Dist}[1/(a*n*(p + 1)*b^{(\text{Floor}[(q - 1)/n] + 1)}), \text{Int}[(a + b*x^n)^{(p + 1)}*\text{ExpandToSum}[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - \text{Simp}[(x*R*(a + b*x^n)^{(p + 1)})/(a*n*(p + 1)*b^{(\text{Floor}[(q - 1)/n] + 1)}), x]] /; \text{GeQ}[q, n]] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[p, -1]$

Rule 1854

$\text{Int}[(Pq_)*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] := \text{Module}[\{q = \text{Expon}[Pq, x], i\}, \text{Simp}[(a*\text{Coeff}[Pq, x, q] - b*x*\text{ExpandToSum}[Pq - \text{Coeff}[Pq, x, q]*x^q, x])*(a + b*x^n)^{(p + 1)})/(a*b*n*(p + 1)), x] + \text{Dist}[1/(a*n*(p + 1)), \text{Int}[\text{Sum}[(n*(p + 1) + i + 1)*\text{Coeff}[Pq, x, i]*x^i, \{i, 0, q - 1\})*(a + b*x^n)^{(p + 1)}, x], x] /; q == n - 1] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[p, -1]$

Rule 1876

$\text{Int}[(Pq_)/((a_) + (b_)*(x_)^{(n_)})], x_Symbol] := \text{With}[\{v = \text{Sum}[(x^{ii}*(\text{Coeff}[Pq, x, ii] + \text{Coeff}[Pq, x, n/2 + ii]*x^{(n/2)}))]/(a + b*x^n), \{ii, 0, n/2 - 1\}\}, \text{Int}[v, x] /; \text{SumQ}[v]] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[n/2, 0] \&\& \text{Expon}[Pq, x] < n$

Rule 275

$\text{Int}[(x_)^{(m_)*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] := \text{With}[\{k = \text{GCD}[m + 1, n]\}, \text{Dist}[1/k, \text{Subst}[\text{Int}[x^{((m + 1)/k - 1)*(a + b*x^{(n/k)})^p}, x], x, x^k], x] /; k != 1] /; \text{FreeQ}[\{a, b, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[m]$

Rule 205

$\text{Int}(((a_) + (b_)*(x_)^2)^{(-1)}, x_Symbol] := \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[a/b]$

Rule 1168

$\text{Int}(((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := \text{With}[\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \text{Dist}[(d*q - a*e)/(2*a*c), \text{Int}[(q - c*x^2)/(a + c*x^4), x], x]] /; \text{FreeQ}[\{a,$

c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4}{(a + bx^4)^3} dx &= \frac{x(bc - ag + bdx + bex^2 + bfx^3)}{8ab(a + bx^4)^2} - \frac{\int \frac{-7bc - ag - 6bdx - 5bex^2 - 4bfx^3}{(a + bx^4)^2} dx}{8ab} \\
&= \frac{x(bc - ag + bdx + bex^2 + bfx^3)}{8ab(a + bx^4)^2} - \frac{4af - x(7bc + ag + 6bdx + 5bex^2)}{32a^2b(a + bx^4)} + \frac{\int \frac{-3(-7bc - ag) + 12bdx}{a + bx^4} dx}{32a^2b} \\
&= \frac{x(bc - ag + bdx + bex^2 + bfx^3)}{8ab(a + bx^4)^2} - \frac{4af - x(7bc + ag + 6bdx + 5bex^2)}{32a^2b(a + bx^4)} + \frac{\int \left(\frac{12bdx}{a + bx^4} + \frac{-3(-7bc - ag)}{a + bx^4}\right) dx}{32a^2b} \\
&= \frac{x(bc - ag + bdx + bex^2 + bfx^3)}{8ab(a + bx^4)^2} - \frac{4af - x(7bc + ag + 6bdx + 5bex^2)}{32a^2b(a + bx^4)} + \frac{\int \frac{-3(-7bc - ag) + 12bdx}{a + bx^4} dx}{32a^2b} \\
&= \frac{x(bc - ag + bdx + bex^2 + bfx^3)}{8ab(a + bx^4)^2} - \frac{4af - x(7bc + ag + 6bdx + 5bex^2)}{32a^2b(a + bx^4)} + \frac{(3d) \text{Subst}\left(\int \frac{\sqrt{b}}{\sqrt{a + bx^4}} dx\right)}{16a^{5/2}\sqrt{b}} \\
&= \frac{x(bc - ag + bdx + bex^2 + bfx^3)}{8ab(a + bx^4)^2} - \frac{4af - x(7bc + ag + 6bdx + 5bex^2)}{32a^2b(a + bx^4)} + \frac{3d \tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a + bx^4}}\right)}{16a^{5/2}\sqrt{b}} \\
&= \frac{x(bc - ag + bdx + bex^2 + bfx^3)}{8ab(a + bx^4)^2} - \frac{4af - x(7bc + ag + 6bdx + 5bex^2)}{32a^2b(a + bx^4)} + \frac{3d \tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a + bx^4}}\right)}{16a^{5/2}\sqrt{b}} \\
&= \frac{x(bc - ag + bdx + bex^2 + bfx^3)}{8ab(a + bx^4)^2} - \frac{4af - x(7bc + ag + 6bdx + 5bex^2)}{32a^2b(a + bx^4)} + \frac{3d \tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a + bx^4}}\right)}{16a^{5/2}\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.277805, size = 366, normalized size = 0.93

$$-\frac{32a^{7/4}\sqrt[4]{b(a(f+gx)-bx(c+x(d+ex)))}}{(a+bx^4)^2} + \frac{8a^{3/4}\sqrt[4]{bx(ag+7bc+bx(6d+5ex))}}{a+bx^4} - 2 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \left(24\sqrt[4]{ab}b^{3/4}d + 5\sqrt{2}\sqrt{a}\sqrt{b}e + 3\sqrt{2}ag + 21\sqrt{2}bx\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^3, x]

[Out] ((8*a^(3/4)*b^(1/4)*x*(7*b*c + a*g + b*x*(6*d + 5*e*x)))/(a + b*x^4) - (32*a^(7/4)*b^(1/4)*(a*(f + g*x) - b*x*(c + x*(d + e*x)))/(a + b*x^4)^2 - 2*(2

$$\frac{1*\sqrt{2}*b*c + 24*a^{(1/4)}*b^{(3/4)}*d + 5*\sqrt{2}*\sqrt{a}*\sqrt{b}*e + 3*\sqrt{2}*a*g*\text{ArcTan}[1 - (\sqrt{2}*b^{(1/4)}*x)/a^{(1/4)}] + 2*(21*\sqrt{2}*b*c - 24*a^{(1/4)}*b^{(3/4)}*d + 5*\sqrt{2}*\sqrt{a}*\sqrt{b}*e + 3*\sqrt{2}*a*g*\text{ArcTan}[1 + (\sqrt{2}*b^{(1/4)}*x)/a^{(1/4)}] + \sqrt{2}*(-21*b*c + 5*\sqrt{a}*\sqrt{b}*e - 3*a*g)*\text{Log}[\sqrt{a} - \sqrt{2}*a^{(1/4)}*b^{(1/4)}*x + \sqrt{b}*x^2] + \sqrt{2}*(21*b*c - 5*\sqrt{a}*\sqrt{b}*e + 3*a*g)*\text{Log}[\sqrt{a} + \sqrt{2}*a^{(1/4)}*b^{(1/4)}*x + \sqrt{b}*x^2])}{(256*a^{(11/4)}*b^{(5/4)})}$$

Maple [A] time = 0.01, size = 519, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x)`

[Out] $(5/32/a^2*b*e*x^7+3/16/a^2*d*b*x^6+1/32*(a*g+7*b*c)/a^2*x^5+9/32/a*e*x^3+5/16*d/a*x^2-1/32*(3*a*g-11*b*c)/a/b*x-1/8*f/b)/(b*x^4+a)^2+3/128/b/a^2*(1/b*a)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)}*x+1)*g+21/128*c/a^3*(1/b*a)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)}*x+1)+3/128/b/a^2*(1/b*a)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)}*x-1)*g+21/128*c/a^3*(1/b*a)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)}*x-1)+3/256/b/a^2*(1/b*a)^{(1/4)}*2^{(1/2)}*\ln((x^2+(1/b*a)^{(1/4)}*x*2^{(1/2)}+(1/b*a)^{(1/2)})/(x^2-(1/b*a)^{(1/4)}*x*2^{(1/2)}+(1/b*a)^{(1/2)}))*g+21/256*c/a^3*(1/b*a)^{(1/4)}*2^{(1/2)}*\ln((x^2+(1/b*a)^{(1/4)}*x*2^{(1/2)}+(1/b*a)^{(1/2)})/(x^2-(1/b*a)^{(1/4)}*x*2^{(1/2)}+(1/b*a)^{(1/2)}))+3/16*d/a^2/(a*b)^{(1/2)}*\arctan(x^2*(b/a)^{(1/2)})+5/256*e/a^2/b/(1/b*a)^{(1/4)}*2^{(1/2)}*\ln((x^2-(1/b*a)^{(1/4)}*x*2^{(1/2)}+(1/b*a)^{(1/2)})/(x^2+(1/b*a)^{(1/4)}*x*2^{(1/2)}+(1/b*a)^{(1/2)}))+5/128*e/a^2/b/(1/b*a)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)}*x+1)+5/128*e/a^2/b/(1/b*a)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)}*x-1)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)

[Out] Timed out

Giac [A] time = 1.09804, size = 562, normalized size = 1.43

$$\frac{\sqrt{2} \left(12 \sqrt{2} \sqrt{ab} b^2 d + 21 (ab^3)^{\frac{1}{4}} b^2 c + 3 (ab^3)^{\frac{1}{4}} abg + 5 (ab^3)^{\frac{3}{4}} e \right) \arctan \left(\frac{\sqrt{2} \left(2x + \sqrt{2} \left(\frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left(\frac{a}{b} \right)^{\frac{1}{4}}} \right)}{128 a^3 b^3} + \frac{\sqrt{2} \left(12 \sqrt{2} \sqrt{ab} b^2 d + 21 (ab^3)^{\frac{1}{4}} b^2 c + 3 (ab^3)^{\frac{1}{4}} abg + 5 (ab^3)^{\frac{3}{4}} e \right)}{128 a^3 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")

[Out] 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d

$$\begin{aligned}
& + 21*(a*b^3)^{(1/4)}*b^2*c + 3*(a*b^3)^{(1/4)}*a*b*g + 5*(a*b^3)^{(3/4)}*e)*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a/b)^{(1/4)})/(a/b)^{(1/4)})/(a^3*b^3) + 1/256*\sqrt{2}*(21*(a*b^3)^{(1/4)}*b^2*c + 3*(a*b^3)^{(1/4)}*a*b*g - 5*(a*b^3)^{(3/4)}*e)*\log(x^2 + \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(a^3*b^3) - 1/256*\sqrt{2}*(21*(a*b^3)^{(1/4)}*b^2*c + 3*(a*b^3)^{(1/4)}*a*b*g - 5*(a*b^3)^{(3/4)}*e)*\log(x^2 - \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(a^3*b^3) + 1/32*(5*b^2*x^7*e + 6*b^2*d*x^6 + 7*b^2*c*x^5 + a*b*g*x^5 + 9*a*b*x^3*e + 10*a*b*d*x^2 + 11*a*b*c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 + a)^2*a^2*b)
\end{aligned}$$

$$3.178 \quad \int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4} dx$$

Optimal. Leaf size=437

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-15\sqrt{a}\sqrt{be} + 7ag + 77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-15\sqrt{a}\sqrt{be} + 7ag + 77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}}$$

```
[Out] (x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a + b*x^4)^3) + (x*(7*(11*b*c + a*g) + 60*b*d*x + 45*b*e*x^2))/(384*a^3*b*(a + b*x^4)) - (8*a*f - x*(11*b*c + a*g + 10*b*d*x + 9*b*e*x^2))/(96*a^2*b*(a + b*x^4)^2) + (5*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b]) - ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(5/4)) + ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(5/4)) - ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(5/4)) + ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(5/4))
```

Rubi [A] time = 0.531377, antiderivative size = 437, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {1858, 1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-15\sqrt{a}\sqrt{be} + 7ag + 77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-15\sqrt{a}\sqrt{be} + 7ag + 77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^4, x]
```

```
[Out] (x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a + b*x^4)^3) + (x*(7*(11*b*c + a*g) + 60*b*d*x + 45*b*e*x^2))/(384*a^3*b*(a + b*x^4)) - (8*a*f - x*(11*b*c + a*g + 10*b*d*x + 9*b*e*x^2))/(96*a^2*b*(a + b*x^4)^2) + (5*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b]) - ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(5/4)) + ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(5/4)) - ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(5/4)) + ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(5/4))
```

$$\frac{t[a]*\text{Sqrt}[b]*e + 7*a*g*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*x + \text{Sqrt}[b]*x^2]}{(512*\text{Sqrt}[2]*a^{(15/4)}*b^{(5/4)})} + \frac{((77*b*c - 15*\text{Sqrt}[a]*\text{Sqrt}[b]*e + 7*a*g)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*x + \text{Sqrt}[b]*x^2])}{(512*\text{Sqrt}[2]*a^{(15/4)}*b^{(5/4)})}$$

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1})*(a + b*x^n)^(p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1168

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1162

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1165

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d},

Mathematica [A] time = 0.366398, size = 411, normalized size = 0.94

$$\frac{-\frac{256a^{11/4}\sqrt[4]{b(a(f+gx)-bx(c+x(d+ex)))}}{(a+bx^4)^3} + \frac{32a^{7/4}\sqrt[4]{bx(ag+11bc+bx(10d+9ex))}}{(a+bx^4)^2} + \frac{8a^{3/4}\sqrt[4]{bx(7ag+77bc+15bx(4d+3ex))}}{a+bx^4} - 6 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{(80\sqrt[4]{a})}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^4, x]

[Out] ((8*a^(3/4)*b^(1/4)*x*(77*b*c + 7*a*g + 15*b*x*(4*d + 3*e*x)))/(a + b*x^4) + (32*a^(7/4)*b^(1/4)*x*(11*b*c + a*g + b*x*(10*d + 9*e*x)))/(a + b*x^4)^2 - (256*a^(11/4)*b^(1/4)*(a*(f + g*x) - b*x*(c + x*(d + e*x)))/(a + b*x^4)^3 - 6*(77*sqrt[2]*b*c + 80*a^(1/4)*b^(3/4)*d + 15*sqrt[2]*sqrt[a]*sqrt[b]*e + 7*sqrt[2]*a*g)*ArcTan[1 - (sqrt[2]*b^(1/4)*x)/a^(1/4)] + 6*(77*sqrt[2]*b*c - 80*a^(1/4)*b^(3/4)*d + 15*sqrt[2]*sqrt[a]*sqrt[b]*e + 7*sqrt[2]*a*g)*ArcTan[1 + (sqrt[2]*b^(1/4)*x)/a^(1/4)] - 3*sqrt[2]*(77*b*c - 15*sqrt[a]*sqrt[b]*e + 7*a*g)*Log[sqrt[a] - sqrt[2]*a^(1/4)*b^(1/4)*x + sqrt[b]*x^2] + 3*sqrt[2]*(77*b*c - 15*sqrt[a]*sqrt[b]*e + 7*a*g)*Log[sqrt[a] + sqrt[2]*a^(1/4)*b^(1/4)*x + sqrt[b]*x^2])/(3072*a^(15/4)*b^(5/4))

Maple [A] time = 0.014, size = 560, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4, x)

[Out] (15/128*e/a^3*b^2*x^11+5/32*d/a^3*b^2*x^10+7/384*(a*g+11*b*c)/a^3*b*x^9+21/64/a^2*b*e*x^7+5/12/a^2*d*b*x^6+3/64/a^2*(a*g+11*b*c)*x^5+113/384/a*e*x^3+1/32*d/a*x^2-1/128*(7*a*g-51*b*c)/a/b*x-1/12*f/b)/(b*x^4+a)^3+7/512/b/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*g+77/512/a^4*c*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+7/512/b/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*g+77/512/a^4*c*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+7/1024/b/a^3*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*g+77/1024/a^4*c*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+5/32/a^3*d/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+15/1024/a^3*e/b/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+15/512/a^3*e/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2))

$$\frac{1}{(b*a)^{1/4}*x+1} + \frac{15}{512} \frac{e}{a^3} \frac{1}{b} \frac{1}{(b*a)^{1/4}} * 2^{1/2} * \arctan\left(\frac{2^{1/2}}{(b*a)^{1/4}*x-1}\right)$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)

[Out] Timed out

Giac [A] time = 1.09294, size = 629, normalized size = 1.44

$$\frac{\sqrt{2} \left(40 \sqrt{2} \sqrt{ab} b^2 d + 77 (ab^3)^{\frac{1}{4}} b^2 c + 7 (ab^3)^{\frac{1}{4}} abg + 15 (ab^3)^{\frac{3}{4}} e \right) \arctan \left(\frac{\sqrt{2} \left(2x + \sqrt{2} \left(\frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left(\frac{a}{b} \right)^{\frac{1}{4}}} \right)}{512 a^4 b^3} + \frac{\sqrt{2} \left(40 \sqrt{2} \sqrt{ab} b^2 d + 77 (ab^3)^{\frac{1}{4}} b^2 c + 7 (ab^3)^{\frac{1}{4}} abg + 15 (ab^3)^{\frac{3}{4}} e \right) \arctan \left(\frac{\sqrt{2} \left(2x - \sqrt{2} \left(\frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left(\frac{a}{b} \right)^{\frac{1}{4}}} \right)}{512 a^4 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")

[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(45*b^3*x^11*e + 60*b^3*d*x^10 + 77*b^3*c*x^9 + 7*a*b^2*g*x^9 + 126*a*b^2*x^7*e + 160*a*b^2*d*x^6 + 198*a*b^2*c*x^5 + 18*a^2*b*g*x^5 + 113*a^2*b*x^3*e + 132*a^2*b*d*x^2 + 153*a^2*b*c*x - 21*a^3*g*x - 32*a^3*f)/((b*x^4 + a)^3*a^3*b)

$$3.179 \quad \int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3} dx$$

Optimal. Leaf size=11

$$-\frac{1}{4}(1-x)^4$$

[Out] $-(1-x)^4/4$

Rubi [A] time = 0.0125012, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1586, 32}

$$-\frac{1}{4}(1-x)^4$$

Antiderivative was successfully verified.

[In] Int[(1 - x^4)^3/(1 + x + x^2 + x^3)^3,x]

[Out] $-(1-x)^4/4$

Rule 1586

Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] :> Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p+q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m+1)/(b*(m+1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3} dx = \int (1-x)^3 dx$$

$$= -\frac{1}{4}(1-x)^4$$

Mathematica [A] time = 0.0013102, size = 9, normalized size = 0.82

$$-\frac{1}{4}(x-1)^4$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^4)^3/(1 + x + x^2 + x^3)^3,x]

[Out] -(-1 + x)^4/4

Maple [A] time = 0.002, size = 8, normalized size = 0.7

$$-\frac{(-1+x)^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+1)^3/(x^3+x^2+x+1)^3,x)

[Out] -1/4*(-1+x)^4

Maxima [B] time = 0.964346, size = 20, normalized size = 1.82

$$-\frac{1}{4}x^4 + x^3 - \frac{3}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)^3/(x^3+x^2+x+1)^3,x, algorithm="maxima")

[Out] $-1/4*x^4 + x^3 - 3/2*x^2 + x$

Fricas [B] time = 1.72758, size = 41, normalized size = 3.73

$$-\frac{1}{4}x^4 + x^3 - \frac{3}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^4+1)^3/(x^3+x^2+x+1)^3,x, algorithm="fricas")`

[Out] $-1/4*x^4 + x^3 - 3/2*x^2 + x$

Sympy [B] time = 0.075679, size = 15, normalized size = 1.36

$$-\frac{x^4}{4} + x^3 - \frac{3x^2}{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x**4+1)**3/(x**3+x**2+x+1)**3,x)`

[Out] $-x**4/4 + x**3 - 3*x**2/2 + x$

Giac [B] time = 1.10327, size = 20, normalized size = 1.82

$$-\frac{1}{4}x^4 + x^3 - \frac{3}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^4+1)^3/(x^3+x^2+x+1)^3,x, algorithm="giac")`

[Out] $-1/4*x^4 + x^3 - 3/2*x^2 + x$

$$3.180 \quad \int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2} dx$$

Optimal. Leaf size=11

$$-\frac{1}{3}(1-x)^3$$

[Out] $-(1-x)^3/3$

Rubi [A] time = 0.0122637, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1586, 32}

$$-\frac{1}{3}(1-x)^3$$

Antiderivative was successfully verified.

[In] `Int[(1 - x^4)^2/(1 + x + x^2 + x^3)^2,x]`

[Out] $-(1-x)^3/3$

Rule 1586

`Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]`

Rule 32

`Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]`

Rubi steps

$$\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2} dx = \int (1-x)^2 dx$$

$$= -\frac{1}{3}(1-x)^3$$

Mathematica [A] time = 0.0005153, size = 14, normalized size = 1.27

$$\frac{x^3}{3} - x^2 + x$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^4)^2/(1 + x + x^2 + x^3)^2,x]

[Out] x - x^2 + x^3/3

Maple [A] time = 0.001, size = 8, normalized size = 0.7

$$\frac{(-1+x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+1)^2/(x^3+x^2+x+1)^2,x)

[Out] 1/3*(-1+x)^3

Maxima [A] time = 0.939669, size = 16, normalized size = 1.45

$$\frac{1}{3}x^3 - x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)^2/(x^3+x^2+x+1)^2,x, algorithm="maxima")

[Out] $\frac{1}{3}x^3 - x^2 + x$

Fricas [A] time = 1.68165, size = 26, normalized size = 2.36

$$\frac{1}{3}x^3 - x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^4+1)^2/(x^3+x^2+x+1)^2,x, algorithm="fricas")`

[Out] $\frac{1}{3}x^3 - x^2 + x$

Sympy [A] time = 0.066356, size = 8, normalized size = 0.73

$$\frac{x^3}{3} - x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x**4+1)**2/(x**3+x**2+x+1)**2,x)`

[Out] $x^{**3}/3 - x^{**2} + x$

Giac [A] time = 1.06477, size = 16, normalized size = 1.45

$$\frac{1}{3}x^3 - x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^4+1)^2/(x^3+x^2+x+1)^2,x, algorithm="giac")`

[Out] $\frac{1}{3}x^3 - x^2 + x$

$$3.181 \quad \int \frac{1-x^4}{1+x+x^2+x^3} dx$$

Optimal. Leaf size=9

$$x - \frac{x^2}{2}$$

[Out] x - x^2/2

Rubi [A] time = 0.0086773, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1586}

$$x - \frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^4)/(1 + x + x^2 + x^3), x]

[Out] x - x^2/2

Rule 1586

Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rubi steps

$$\begin{aligned} \int \frac{1-x^4}{1+x+x^2+x^3} dx &= \int (1-x) dx \\ &= x - \frac{x^2}{2} \end{aligned}$$

Mathematica [A] time = 0.0004395, size = 9, normalized size = 1.

$$x - \frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^4)/(1 + x + x^2 + x^3),x]

[Out] x - x^2/2

Maple [A] time = 0.001, size = 8, normalized size = 0.9

$$x - \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+1)/(x^3+x^2+x+1),x)

[Out] x-1/2*x^2

Maxima [A] time = 0.93638, size = 9, normalized size = 1.

$$-\frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/(x^3+x^2+x+1),x, algorithm="maxima")

[Out] -1/2*x^2 + x

Fricas [A] time = 1.67103, size = 19, normalized size = 2.11

$$-\frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/(x^3+x^2+x+1),x, algorithm="fricas")

[Out] $-1/2*x^2 + x$

Sympy [A] time = 0.057147, size = 5, normalized size = 0.56

$$-\frac{x^2}{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x**4+1)/(x**3+x**2+x+1),x)`

[Out] $-x**2/2 + x$

Giac [A] time = 1.0496, size = 9, normalized size = 1.

$$-\frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^4+1)/(x^3+x^2+x+1),x, algorithm="giac")`

[Out] $-1/2*x^2 + x$

$$3.182 \quad \int \frac{1+x+x^2+x^3}{1-x^4} dx$$

Optimal. Leaf size=8

$$-\log(1-x)$$

[Out] -Log[1 - x]

Rubi [A] time = 0.0074175, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1586, 31}

$$-\log(1-x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^2 + x^3)/(1 - x^4), x]

[Out] -Log[1 - x]

Rule 1586

Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p+q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\int \frac{1+x+x^2+x^3}{1-x^4} dx = \int \frac{1}{1-x} dx = -\log(1-x)$$

Mathematica [A] time = 0.0008638, size = 8, normalized size = 1.

$$-\log(1-x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + x^2 + x^3)/(1 - x^4), x]

[Out] -Log[1 - x]

Maple [A] time = 0., size = 7, normalized size = 0.9

$$-\ln(-1 + x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x^2+x+1)/(-x^4+1), x)

[Out] -ln(-1+x)

Maxima [A] time = 0.943333, size = 8, normalized size = 1.

$$-\log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+x+1)/(-x^4+1), x, algorithm="maxima")

[Out] -log(x - 1)

Fricas [A] time = 1.70181, size = 18, normalized size = 2.25

$$-\log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+x+1)/(-x^4+1), x, algorithm="fricas")

[Out] -log(x - 1)

Sympy [A] time = 0.056644, size = 5, normalized size = 0.62

$$-\log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+x**2+x+1)/(-x**4+1),x)

[Out] -log(x - 1)

Giac [A] time = 1.05278, size = 9, normalized size = 1.12

$$-\log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="giac")

[Out] -log(abs(x - 1))

$$3.183 \quad \int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2} dx$$

Optimal. Leaf size=7

$$\frac{1}{1-x}$$

[Out] (1 - x)^(-1)

Rubi [A] time = 0.015811, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1586, 32}

$$\frac{1}{1-x}$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^2 + x^3)^2/(1 - x^4)^2,x]

[Out] (1 - x)^(-1)

Rule 1586

Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] :> Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2} dx = \int \frac{1}{(1-x)^2} dx$$

$$= \frac{1}{1-x}$$

Mathematica [A] time = 0.0008222, size = 7, normalized size = 1.

$$-\frac{1}{x-1}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + x^2 + x^3)^2/(1 - x^4)^2,x]

[Out] -(-1 + x)^(-1)

Maple [A] time = 0.001, size = 8, normalized size = 1.1

$$-(-1+x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x^2+x+1)^2/(-x^4+1)^2,x)

[Out] -1/(-1+x)

Maxima [A] time = 0.960074, size = 9, normalized size = 1.29

$$-\frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+x+1)^2/(-x^4+1)^2,x, algorithm="maxima")

[Out] $-1/(x - 1)$

Fricas [A] time = 1.66394, size = 16, normalized size = 2.29

$$-\frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+x^2+x+1)^2/(-x^4+1)^2,x, algorithm="fricas")`

[Out] $-1/(x - 1)$

Sympy [A] time = 0.080331, size = 5, normalized size = 0.71

$$-\frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**3+x**2+x+1)**2/(-x**4+1)**2,x)`

[Out] $-1/(x - 1)$

Giac [A] time = 1.05921, size = 9, normalized size = 1.29

$$-\frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+x^2+x+1)^2/(-x^4+1)^2,x, algorithm="giac")`

[Out] $-1/(x - 1)$

$$3.184 \quad \int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3} dx$$

Optimal. Leaf size=11

$$\frac{1}{2(1-x)^2}$$

[Out] 1/(2*(1 - x)^2)

Rubi [A] time = 0.0181356, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1586, 32}

$$\frac{1}{2(1-x)^2}$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^2 + x^3)^3/(1 - x^4)^3,x]

[Out] 1/(2*(1 - x)^2)

Rule 1586

Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3} dx = \int \frac{1}{(1-x)^3} dx$$

$$= \frac{1}{2(1-x)^2}$$

Mathematica [A] time = 0.0013108, size = 9, normalized size = 0.82

$$\frac{1}{2(x-1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + x^2 + x^3)^3/(1 - x^4)^3,x]

[Out] 1/(2*(-1 + x)^2)

Maple [A] time = 0.002, size = 8, normalized size = 0.7

$$\frac{1}{2(-1+x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x^2+x+1)^3/(-x^4+1)^3,x)

[Out] 1/2/(-1+x)^2

Maxima [A] time = 0.936703, size = 16, normalized size = 1.45

$$\frac{1}{2(x^2-2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+x+1)^3/(-x^4+1)^3,x, algorithm="maxima")

[Out] $1/2/(x^2 - 2*x + 1)$

Fricas [A] time = 1.66084, size = 28, normalized size = 2.55

$$\frac{1}{2(x^2 - 2x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+x^2+x+1)^3/(-x^4+1)^3,x, algorithm="fricas")`

[Out] $1/2/(x^2 - 2*x + 1)$

Sympy [A] time = 0.096645, size = 10, normalized size = 0.91

$$\frac{1}{2x^2 - 4x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**3+x**2+x+1)**3/(-x**4+1)**3,x)`

[Out] $1/(2*x**2 - 4*x + 2)$

Giac [A] time = 1.06814, size = 9, normalized size = 0.82

$$\frac{1}{2(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+x^2+x+1)^3/(-x^4+1)^3,x, algorithm="giac")`

[Out] $1/2/(x - 1)^2$

$$3.185 \quad \int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4} dx$$

Optimal. Leaf size=11

$$\frac{1}{3(1-x)^3}$$

[Out] 1/(3*(1 - x)^3)

Rubi [A] time = 0.0193226, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1586, 32}

$$\frac{1}{3(1-x)^3}$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^2 + x^3)^4/(1 - x^4)^4,x]

[Out] 1/(3*(1 - x)^3)

Rule 1586

Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] :> Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 32

Int[((a_) + (b_)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4} dx = \int \frac{1}{(1-x)^4} dx$$

$$= \frac{1}{3(1-x)^3}$$

Mathematica [A] time = 0.0010176, size = 9, normalized size = 0.82

$$-\frac{1}{3(x-1)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + x^2 + x^3)^4/(1 - x^4)^4,x]

[Out] -1/(3*(-1 + x)^3)

Maple [A] time = 0., size = 8, normalized size = 0.7

$$-\frac{1}{3(-1+x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x^2+x+1)^4/(-x^4+1)^4,x)

[Out] -1/3/(-1+x)^3

Maxima [B] time = 0.941818, size = 23, normalized size = 2.09

$$-\frac{1}{3(x^3-3x^2+3x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+x+1)^4/(-x^4+1)^4,x, algorithm="maxima")

[Out] $-1/3/(x^3 - 3x^2 + 3x - 1)$

Fricas [B] time = 1.69488, size = 41, normalized size = 3.73

$$-\frac{1}{3(x^3 - 3x^2 + 3x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+x^2+x+1)^4/(-x^4+1)^4,x, algorithm="fricas")`

[Out] $-1/3/(x^3 - 3x^2 + 3x - 1)$

Sympy [B] time = 0.102664, size = 17, normalized size = 1.55

$$-\frac{1}{3x^3 - 9x^2 + 9x - 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**3+x**2+x+1)**4/(-x**4+1)**4,x)`

[Out] $-1/(3x^3 - 9x^2 + 9x - 3)$

Giac [A] time = 1.05314, size = 9, normalized size = 0.82

$$-\frac{1}{3(x-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+x^2+x+1)^4/(-x^4+1)^4,x, algorithm="giac")`

[Out] $-1/3/(x - 1)^3$

$$3.186 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a-bx^4} dx$$

Optimal. Leaf size=165

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(-\sqrt{a}\sqrt{be}+ag+bc)}{2a^{3/4}b^{5/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(\sqrt{a}\sqrt{be}+ag+bc)}{2a^{3/4}b^{5/4}} + \frac{(ah+bd)\tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2\sqrt{ab}^{3/2}} - \frac{f\log(a-bx^4)}{4b} - \frac{g}{b}$$

[Out] $-\left(\frac{g*x}{b}\right) - \frac{(h*x^2)}{(2*b)} + \frac{((b*c - \text{Sqrt}[a]*\text{Sqrt}[b]*e + a*g)*\text{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}])}{(2*a^{(3/4)}*b^{(5/4)})} + \frac{((b*c + \text{Sqrt}[a]*\text{Sqrt}[b]*e + a*g)*\text{ArcTanh}[(b^{(1/4)}*x)/a^{(1/4)}])}{(2*a^{(3/4)}*b^{(5/4)})} + \frac{((b*d + a*h)*\text{ArcTanh}[(\text{Sqrt}[b]*x^2)/\text{Sqrt}[a]])}{(2*\text{Sqrt}[a]*b^{(3/2)})} - \frac{(f*\text{Log}[a - b*x^4])}{(4*b)}$

Rubi [A] time = 0.260628, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1885, 1887, 1167, 205, 208, 1819, 1810, 635, 260}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(-\sqrt{a}\sqrt{be}+ag+bc)}{2a^{3/4}b^{5/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(\sqrt{a}\sqrt{be}+ag+bc)}{2a^{3/4}b^{5/4}} + \frac{(ah+bd)\tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2\sqrt{ab}^{3/2}} - \frac{f\log(a-bx^4)}{4b} - \frac{g}{b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4), x]$

[Out] $-\left(\frac{g*x}{b}\right) - \frac{(h*x^2)}{(2*b)} + \frac{((b*c - \text{Sqrt}[a]*\text{Sqrt}[b]*e + a*g)*\text{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}])}{(2*a^{(3/4)}*b^{(5/4)})} + \frac{((b*c + \text{Sqrt}[a]*\text{Sqrt}[b]*e + a*g)*\text{ArcTanh}[(b^{(1/4)}*x)/a^{(1/4)}])}{(2*a^{(3/4)}*b^{(5/4)})} + \frac{((b*d + a*h)*\text{ArcTanh}[(\text{Sqrt}[b]*x^2)/\text{Sqrt}[a]])}{(2*\text{Sqrt}[a]*b^{(3/2)})} - \frac{(f*\text{Log}[a - b*x^4])}{(4*b)}$

Rule 1885

$\text{Int}[(Pq_)*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Module}[\{q = \text{Expon}[Pq, x], j, k\}, \text{Int}[\text{Sum}[x^j*\text{Sum}[\text{Coeff}[Pq, x, j + (k*n)/2]*x^{((k*n)/2)}, \{k, 0, (2*(q - j))/n + 1\}]*\text{Int}[(a + b*x^n)^p, \{j, 0, n/2 - 1\}], x]] /; \text{FreeQ}[\{a, b, p\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[n/2, 0] \&\& \text{!PolyQ}[Pq, x^{(n/2)}]$

Rule 1887

$\text{Int}[(Pq_)/((a_) + (b_)*(x_)^{(n_)})], x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[Pq/(a + b*x^n), x], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IntegerQ}[n]$

Rule 1167

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[-(a*c)]

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1819

Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]

Rule 1810

Int[(Pq_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 635

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{a - bx^4} dx &= \int \left(\frac{c + ex^2 + gx^4}{a - bx^4} + \frac{x(d + fx^2 + hx^4)}{a - bx^4} \right) dx \\
&= \int \frac{c + ex^2 + gx^4}{a - bx^4} dx + \int \frac{x(d + fx^2 + hx^4)}{a - bx^4} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{d + fx + hx^2}{a - bx^2} dx, x, x^2 \right) + \int \left(-\frac{g}{b} + \frac{bc + ag + bex^2}{b(a - bx^4)} \right) dx \\
&= -\frac{gx}{b} + \frac{1}{2} \text{Subst} \left(\int \left(-\frac{h}{b} + \frac{bd + ah + bfx}{b(a - bx^2)} \right) dx, x, x^2 \right) + \frac{\int \frac{bc + ag + bex^2}{a - bx^4} dx}{b} \\
&= -\frac{gx}{b} - \frac{hx^2}{2b} + \frac{\text{Subst} \left(\int \frac{bd + ah + bfx}{a - bx^2} dx, x, x^2 \right)}{2b} + \frac{1}{2} \left(e - \frac{bc + ag}{\sqrt{a}\sqrt{b}} \right) \int \frac{1}{-\sqrt{a}\sqrt{b} - bx^2} dx \\
&= -\frac{gx}{b} - \frac{hx^2}{2b} + \frac{(bc - \sqrt{a}\sqrt{b}e + ag) \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2a^{3/4}b^{5/4}} + \frac{(bc + \sqrt{a}\sqrt{b}e + ag) \tanh^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2a^{3/4}b^{5/4}} \\
&= -\frac{gx}{b} - \frac{hx^2}{2b} + \frac{(bc - \sqrt{a}\sqrt{b}e + ag) \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2a^{3/4}b^{5/4}} + \frac{(bc + \sqrt{a}\sqrt{b}e + ag) \tanh^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2a^{3/4}b^{5/4}}
\end{aligned}$$

Mathematica [A] time = 0.257312, size = 256, normalized size = 1.55

$$-\log(\sqrt[4]{a} - \sqrt[4]{bx}) (a^{5/4}h + \sqrt{a}b^{3/4}e + \sqrt[4]{abd} + a\sqrt[4]{bg} + b^{5/4}c) + \log(\sqrt[4]{a} + \sqrt[4]{bx}) (a^{5/4}(-h) + \sqrt{a}b^{3/4}e - \sqrt[4]{abd} + a\sqrt[4]{bg} + b^{5/4}c)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4),x]

[Out] (-4*a^(3/4)*Sqrt[b]*g*x - 2*a^(3/4)*Sqrt[b]*h*x^2 + 2*b^(1/4)*(b*c - Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (b^(5/4)*c + a^(1/4)*b*d + Sqrt[a]*b^(3/4)*e + a*b^(1/4)*g + a^(5/4)*h)*Log[a^(1/4) - b^(1/4)*x] + (b^(5/4)*c - a^(1/4)*b*d + Sqrt[a]*b^(3/4)*e + a*b^(1/4)*g - a^(5/4)*h)*Log[a^(1/4) + b^(1/4)*x] + a^(1/4)*(b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2] - a^(3/4)*Sqrt[b]*f*Log[a - b*x^4])/(4*a^(3/4)*b^(3/2))

Maple [B] time = 0.045, size = 296, normalized size = 1.8

$$-\frac{hx^2}{2b} - \frac{gx}{b} + \frac{g}{2b} \sqrt[4]{a} \arctan\left(x \frac{1}{\sqrt[4]{b}}\right) + \frac{c}{2a} \sqrt[4]{a} \arctan\left(x \frac{1}{\sqrt[4]{b}}\right) + \frac{g}{4b} \sqrt[4]{a} \ln\left(\left(x + \sqrt[4]{\frac{a}{b}}\right)\left(x - \sqrt[4]{\frac{a}{b}}\right)^{-1}\right) + \frac{c}{4a} \sqrt[4]{a} \ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a), x)`

[Out]
$$-1/2*h*x^2/b - g*x/b + 1/2/b*(1/b*a)^{(1/4)}*\arctan(x/(1/b*a)^{(1/4)})*g + 1/2*c*(1/b*a)^{(1/4)}/a*\arctan(x/(1/b*a)^{(1/4)}) + 1/4/b*(1/b*a)^{(1/4)}*\ln((x+(1/b*a)^{(1/4)})/(x-(1/b*a)^{(1/4)})) * g + 1/4*c*(1/b*a)^{(1/4)}/a*\ln((x+(1/b*a)^{(1/4)})/(x-(1/b*a)^{(1/4)})) - 1/4/b/(a*b)^{(1/2)}*\ln((-a+x^2*(a*b)^{(1/2)})/(-a-x^2*(a*b)^{(1/2)})) * a * h - 1/4*d/(a*b)^{(1/2)}*\ln((-a+x^2*(a*b)^{(1/2)})/(-a-x^2*(a*b)^{(1/2)})) - 1/2*e/b/(1/b*a)^{(1/4)}*\arctan(x/(1/b*a)^{(1/4)}) + 1/4*e/b/(1/b*a)^{(1/4)}*\ln((x+(1/b*a)^{(1/4)})/(x-(1/b*a)^{(1/4)})) - 1/4/b*f*\ln(b*x^4-a)$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a), x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a), x, algorithm="fricas")`

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)

[Out] Timed out

Giac [B] time = 1.08378, size = 539, normalized size = 3.27

$$-\frac{f \log(|bx^4 - a|)}{4b} - \frac{bhx^2 + 2bgx}{2b^2} + \frac{\sqrt{2} \left(\sqrt{2} \sqrt{-abb^2d} - \sqrt{2} \sqrt{-ababh} + (-ab^3)^{\frac{1}{4}} b^2c + (-ab^3)^{\frac{1}{4}} abg + (-ab^3)^{\frac{3}{4}} e \right) \arctan \left(\frac{\sqrt{2} \sqrt{-abb^2d} - \sqrt{2} \sqrt{-ababh} + (-ab^3)^{\frac{1}{4}} b^2c + (-ab^3)^{\frac{1}{4}} abg + (-ab^3)^{\frac{3}{4}} e}{\sqrt{2} \sqrt{-abb^2d} - \sqrt{2} \sqrt{-ababh} + (-ab^3)^{\frac{1}{4}} b^2c + (-ab^3)^{\frac{1}{4}} abg + (-ab^3)^{\frac{3}{4}} e} \right)}{4ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")

[Out]
$$-1/4*f*\log(\text{abs}(b*x^4 - a))/b - 1/2*(b*h*x^2 + 2*b*g*x)/b^2 + 1/4*\text{sqrt}(2)*(s$$

$$\text{qrt}(2)*\text{sqrt}(-a*b)*b^2*d - \text{sqrt}(2)*\text{sqrt}(-a*b)*a*b*h + (-a*b^3)^{(1/4)}*b^2*c +$$

$$(-a*b^3)^{(1/4)}*a*b*g + (-a*b^3)^{(3/4)}*e)*\text{arctan}(1/2*\text{sqrt}(2)*(2*x + \text{sqrt}(2)$$

$$*(-a/b)^{(1/4)})/(-a/b)^{(1/4)})/(a*b^3) + 1/4*\text{sqrt}(2)*(s$$

$$\text{qrt}(2)*\text{sqrt}(-a*b)*b^2*d - \text{sqrt}(2)*\text{sqrt}(-a*b)*a*b*h + (-a*b^3)^{(1/4)}*b^2*c + (-a*b^3)^{(1/4)}*a*b*g$$

$$+ (-a*b^3)^{(3/4)}*e)*\text{arctan}(1/2*\text{sqrt}(2)*(2*x - \text{sqrt}(2)*(-a/b)^{(1/4)})/(-a/b)^{(1/4)})/(a*b^3) + 1/8*\text{sqrt}(2)*((-a*b^3)^{(1/4)}*b^2*c + (-a*b^3)^{(1/4)}*a*b*g -$$

$$(-a*b^3)^{(3/4)}*e)*\log(x^2 + \text{sqrt}(2)*x*(-a/b)^{(1/4)} + \text{sqrt}(-a/b))/(a*b^3) -$$

$$1/8*\text{sqrt}(2)*((-a*b^3)^{(1/4)}*b^2*c + (-a*b^3)^{(1/4)}*a*b*g - (-a*b^3)^{(3/4)}*e)*\log(x^2 - \text{sqrt}(2)*x*(-a/b)^{(1/4)} + \text{sqrt}(-a/b))/(a*b^3)$$

$$3.187 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a-bx^4} dx$$

Optimal. Leaf size=188

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b(ag+bc)}}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{ab^{7/4}}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b(ag+bc)}}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{ab^{7/4}}} + \frac{(ah+bd)\tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2\sqrt{ab^{3/2}}} - \frac{f\log(a-bx^4)}{4b}$$

[Out] -((g*x)/b) - (h*x^2)/(2*b) - (i*x^3)/(3*b) - ((b*e - (Sqrt[b]*(b*c + a*g))/Sqrt[a] + a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*b^(7/4)) + ((b*e + (Sqrt[b]*(b*c + a*g))/Sqrt[a] + a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*b^(7/4)) + ((b*d + a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - (f*Log[a - b*x^4])/(4*b)

Rubi [A] time = 0.326583, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.22$, Rules used = {1885, 1819, 1810, 635, 208, 260, 1887, 1167, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b(ag+bc)}}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{ab^{7/4}}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b(ag+bc)}}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{ab^{7/4}}} + \frac{(ah+bd)\tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2\sqrt{ab^{3/2}}} - \frac{f\log(a-bx^4)}{4b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4), x]

[Out] -((g*x)/b) - (h*x^2)/(2*b) - (i*x^3)/(3*b) - ((b*e - (Sqrt[b]*(b*c + a*g))/Sqrt[a] + a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*b^(7/4)) + ((b*e + (Sqrt[b]*(b*c + a*g))/Sqrt[a] + a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*b^(7/4)) + ((b*d + a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - (f*Log[a - b*x^4])/(4*b)

Rule 1885

Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*a + b*x^n]^p, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]

Rule 1819

Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]

Rule 1810

Int[(Pq_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 635

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 1887

Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Pq/(a + b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]

Rule 1167

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[-(a*c)]

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + 187x^6}{a - bx^4} dx &= \int \left(\frac{x(d + fx^2 + hx^4)}{a - bx^4} + \frac{c + ex^2 + gx^4 + 187x^6}{a - bx^4} \right) dx \\
&= \int \frac{x(d + fx^2 + hx^4)}{a - bx^4} dx + \int \frac{c + ex^2 + gx^4 + 187x^6}{a - bx^4} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{d + fx + hx^2}{a - bx^2} dx, x, x^2 \right) + \int \left(-\frac{g}{b} - \frac{187x^2}{b} + \frac{bc + ag + (187a + be - \sqrt{b(bc+ag)})}{b(a - bx^2)} \right) dx \\
&= -\frac{gx}{b} - \frac{187x^3}{3b} + \frac{1}{2} \text{Subst} \left(\int \left(-\frac{h}{b} + \frac{bd + ah + bfx}{b(a - bx^2)} \right) dx, x, x^2 \right) + \frac{\int \frac{bc+ag}{a-bx^2} dx}{2} \\
&= -\frac{gx}{b} - \frac{hx^2}{2b} - \frac{187x^3}{3b} + \frac{\text{Subst} \left(\int \frac{bd+ah+bfx}{a-bx^2} dx, x, x^2 \right)}{2b} + \frac{(187a + be - \sqrt{b(bc+ag)}) \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2\sqrt[4]{ab^7/4}} + \frac{(187a + be + \sqrt{b(bc+ag)}) \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2\sqrt[4]{ab^7/4}} \\
&= -\frac{gx}{b} - \frac{hx^2}{2b} - \frac{187x^3}{3b} - \frac{(187a + be - \frac{\sqrt{b(bc+ag)}}{\sqrt{a}}) \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2\sqrt[4]{ab^7/4}} + \frac{(187a + be + \frac{\sqrt{b(bc+ag)}}{\sqrt{a}}) \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2\sqrt[4]{ab^7/4}}
\end{aligned}$$

Mathematica [A] time = 0.375655, size = 301, normalized size = 1.6

$$\frac{3 \log \left(\frac{\sqrt[4]{a} - \sqrt[4]{bx}}{\sqrt[4]{a} + \sqrt[4]{bx}} \right) \left(a^{5/4} \sqrt[4]{bh+a^{3/2}i} + \sqrt[4]{ab^{5/4}d + \sqrt{abe+a}\sqrt{bg+b^{3/2}c}} \right)}{a^{3/4}} + \frac{3 \log \left(\frac{\sqrt[4]{a} + \sqrt[4]{bx}}{\sqrt[4]{a} - \sqrt[4]{bx}} \right) \left(-a^{5/4} \sqrt[4]{bh+a^{3/2}i} - \sqrt[4]{ab^{5/4}d + \sqrt{abe+a}\sqrt{bg+b^{3/2}c}} \right)}{a^{3/4}} + \frac{6 \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right) (a^{3/2} + \sqrt{b(bc+ag)})}{12b^{7/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4), x]

[Out] $(-12*b^{(3/4)}*g*x - 6*b^{(3/4)}*h*x^2 - 4*b^{(3/4)}*i*x^3 + (6*(b^{(3/2)}*c - \text{Sqrt}[a]*b*e + a*\text{Sqrt}[b]*g - a^{(3/2)}*i)*\text{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}])/a^{(3/4)} - (3*(b^{(3/2)}*c + a^{(1/4)}*b^{(5/4)}*d + \text{Sqrt}[a]*b*e + a*\text{Sqrt}[b]*g + a^{(5/4)}*b^{(1/4)}*h + a^{(3/2)}*i)*\text{Log}[a^{(1/4)} - b^{(1/4)}*x])/a^{(3/4)} + (3*(b^{(3/2)}*c - a^{(1/4)}*b^{(5/4)}*d + \text{Sqrt}[a]*b*e + a*\text{Sqrt}[b]*g - a^{(5/4)}*b^{(1/4)}*h + a^{(3/2)}*i)*\text{Log}[a^{(1/4)} + b^{(1/4)}*x])/a^{(3/4)} + (3*b^{(1/4)}*(b*d + a*h)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[b]*x^2])/ \text{Sqrt}[a] - 3*b^{(3/4)}*f*\text{Log}[a - b*x^4])/(12*b^{(7/4)})$

Maple [B] time = 0.045, size = 367, normalized size = 2.

$$-\frac{ix^3}{3b} - \frac{hx^2}{2b} - \frac{gx}{b} + \frac{g}{2b} \sqrt[4]{\frac{a}{b}} \arctan\left(x \frac{1}{\sqrt[4]{\frac{a}{b}}}\right) + \frac{c}{2a} \sqrt[4]{\frac{a}{b}} \arctan\left(x \frac{1}{\sqrt[4]{\frac{a}{b}}}\right) + \frac{g}{4b} \sqrt[4]{\frac{a}{b}} \ln\left(\left(x + \sqrt[4]{\frac{a}{b}}\right)\left(x - \sqrt[4]{\frac{a}{b}}\right)^{-1}\right) + \frac{c}{4a} \sqrt[4]{\frac{a}{b}} \ln\left(\left(x + \sqrt[4]{\frac{a}{b}}\right)\left(x - \sqrt[4]{\frac{a}{b}}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a), x)

[Out]
$$-1/3*i*x^3/b - 1/2*h*x^2/b - g*x/b + 1/2/b*(1/b*a)^{(1/4)}*\arctan(x/(1/b*a)^{(1/4)})*g + 1/2*c*(1/b*a)^{(1/4)}/a*\arctan(x/(1/b*a)^{(1/4)}) + 1/4/b*(1/b*a)^{(1/4)}*\ln((x+(1/b*a)^{(1/4)})/(x-(1/b*a)^{(1/4)})) * g + 1/4*c*(1/b*a)^{(1/4)}/a*\ln((x+(1/b*a)^{(1/4)})/(x-(1/b*a)^{(1/4)})) - 1/4/b/(a*b)^{(1/2)}*\ln((-a+x^2*(a*b)^{(1/2)})/(-a-x^2*(a*b)^{(1/2)})) * a*h - 1/4*d/(a*b)^{(1/2)}*\ln((-a+x^2*(a*b)^{(1/2)})/(-a-x^2*(a*b)^{(1/2)})) - 1/2/b^2/(1/b*a)^{(1/4)}*\arctan(x/(1/b*a)^{(1/4)}) * a*i - 1/2*e/b/(1/b*a)^{(1/4)}*\arctan(x/(1/b*a)^{(1/4)}) + 1/4/b^2/(1/b*a)^{(1/4)}*\ln((x+(1/b*a)^{(1/4)})/(x-(1/b*a)^{(1/4)})) * a*i + 1/4*e/b/(1/b*a)^{(1/4)}*\ln((x+(1/b*a)^{(1/4)})/(x-(1/b*a)^{(1/4)})) - 1/4/b*f*\ln(b*x^4-a)$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a), x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)

[Out] Timed out

Giac [B] time = 1.08977, size = 807, normalized size = 4.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")

[Out]
$$\begin{aligned} & \frac{1}{8}i(2\sqrt{2})(-ab^3)^{3/4}\arctan\left(\frac{1}{2}\sqrt{2}(2x + \sqrt{2})(-a/b)^{1/4}\right) / (-a/b)^{1/4} / b^4 - \sqrt{2}(-ab^3)^{3/4}\log(x^2 + \sqrt{2}x(-a/b)^{1/4} + \sqrt{-a/b}) / b^4 \\ & + \frac{1}{8}i(2\sqrt{2})(-ab^3)^{3/4}\arctan\left(\frac{1}{2}\sqrt{2}(2x - \sqrt{2})(-a/b)^{1/4}\right) / (-a/b)^{1/4} / b^4 + \sqrt{2}(-ab^3)^{3/4}\log(x^2 - \sqrt{2}x(-a/b)^{1/4} + \sqrt{-a/b}) / b^4 \\ & - \frac{1}{4}f\log(\text{abs}(bx^4 - a)) / b + \frac{1}{4}\sqrt{2}(\sqrt{2}\sqrt{-ab}b^2d - \sqrt{2}\sqrt{-ab}abh + (-ab^3)^{1/4}b^2c + (-ab^3)^{1/4}abg + (-ab^3)^{3/4}e)\arctan\left(\frac{1}{2}\sqrt{2}(2x + \sqrt{2})(-a/b)^{1/4}\right) / (-a/b)^{1/4} / (ab^3) \\ & + \frac{1}{4}\sqrt{2}(\sqrt{2}\sqrt{-ab}b^2d - \sqrt{2}\sqrt{-ab}abh + (-ab^3)^{1/4}b^2c + (-ab^3)^{1/4}abg + (-ab^3)^{3/4}e)\arctan\left(\frac{1}{2}\sqrt{2}(2x - \sqrt{2})(-a/b)^{1/4}\right) / (-a/b)^{1/4} / (ab^3) \\ & + \frac{1}{8}\sqrt{2}((-ab^3)^{1/4}b^2c + (-ab^3)^{1/4}abg - (-ab^3)^{3/4}e)\log(x^2 + \sqrt{2}x(-a/b)^{1/4} + \sqrt{-a/b}) / (ab^3) \\ & - \frac{1}{8}\sqrt{2}((-ab^3)^{1/4}b^2c + (-ab^3)^{1/4}abg - (-ab^3)^{3/4}e)\log(x^2 - \sqrt{2}x(-a/b)^{1/4} + \sqrt{-a/b}) / (ab^3) \\ & - \frac{1}{6}(2b^2ix^3 + 3b^2hx^2 + 6b^2gx) / b^3 \end{aligned}$$

$$3.188 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a-bx^4} dx$$

Optimal. Leaf size=205

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b(ag+bc)}}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{ab^7/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b(ag+bc)}}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{ab^7/4}} + \frac{(ah+bd)\tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2\sqrt{ab^{3/2}}} - \frac{(aj+bf)\log(a-bx^4)}{4b^2}$$

[Out] $-\left(\frac{g*x}{b}\right) - \frac{h*x^2}{2*b} - \frac{i*x^3}{3*b} - \frac{j*x^4}{4*b} - \left(\frac{b*e - \left(\sqrt{b}\right)*(b*c + a*g)}{\sqrt{a}} + a*i\right)*\text{ArcTan}\left[\frac{b^{1/4}*x}{a^{1/4}}\right] / \left(2*a^{1/4}*b^{7/4}\right) + \left(\frac{b*e + \left(\sqrt{b}\right)*(b*c + a*g)}{\sqrt{a}} + a*i\right)*\text{ArcTanh}\left[\frac{b^{1/4}*x}{a^{1/4}}\right] / \left(2*a^{1/4}*b^{7/4}\right) + \left(\frac{b*d + a*h}{\sqrt{a}}\right)*\text{ArcTanh}\left[\frac{\sqrt{b}*x^2}{\sqrt{a}}\right] / \left(2*\sqrt{a}*b^{3/2}\right) - \left(\frac{b*f + a*j}{4*b^2}\right)*\text{Log}\left[a - b*x^4\right]$

Rubi [A] time = 0.313444, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.196$, Rules used = {1885, 1887, 1167, 205, 208, 1819, 1810, 635, 260}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b(ag+bc)}}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{ab^7/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b(ag+bc)}}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{ab^7/4}} + \frac{(ah+bd)\tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2\sqrt{ab^{3/2}}} - \frac{(aj+bf)\log(a-bx^4)}{4b^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}\left[\frac{c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7}{a - b*x^4}, x\right]$

[Out] $-\left(\frac{g*x}{b}\right) - \frac{h*x^2}{2*b} - \frac{i*x^3}{3*b} - \frac{j*x^4}{4*b} - \left(\frac{b*e - \left(\sqrt{b}\right)*(b*c + a*g)}{\sqrt{a}} + a*i\right)*\text{ArcTan}\left[\frac{b^{1/4}*x}{a^{1/4}}\right] / \left(2*a^{1/4}*b^{7/4}\right) + \left(\frac{b*e + \left(\sqrt{b}\right)*(b*c + a*g)}{\sqrt{a}} + a*i\right)*\text{ArcTanh}\left[\frac{b^{1/4}*x}{a^{1/4}}\right] / \left(2*a^{1/4}*b^{7/4}\right) + \left(\frac{b*d + a*h}{\sqrt{a}}\right)*\text{ArcTanh}\left[\frac{\sqrt{b}*x^2}{\sqrt{a}}\right] / \left(2*\sqrt{a}*b^{3/2}\right) - \left(\frac{b*f + a*j}{4*b^2}\right)*\text{Log}\left[a - b*x^4\right]$

Rule 1885

$\text{Int}\left[\left(\frac{Pq}{a} + \frac{b}{x}\right)^p, x\right] := \text{Module}\left[\{q = \text{Expon}[Pq, x], j, k\}, \text{Int}\left[\text{Sum}\left[x^j * \text{Sum}\left[\text{Coeff}[Pq, x, j + (k*n)/2] * x^{(k*n)/2}, \{k, 0, (2*(q - j))/n + 1\}\right] * (a + b*x^n)^p, \{j, 0, n/2 - 1\}\right], x\right] /; \text{FreeQ}\left[\{a, b, p\}, x\right] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[n/2, 0] \&\& \text{!PolyQ}[Pq, x^{(n/2)}]$

Rule 1887

$\text{Int}[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[Pq/(a + b*x^n), x], x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{IntegerQ}[n]$

Rule 1167

$\text{Int}[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[-(a*c), 2]\}, \text{Dist}[e/2 + (c*d)/(2*q), \text{Int}[1/(-q + c*x^2), x], x] + \text{Dist}[e/2 - (c*d)/(2*q), \text{Int}[1/(q + c*x^2), x], x] /; \text{FreeQ}\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[-(a*c)]$

Rule 205

$\text{Int}(((a_) + (b_)*(x_)^2)^{-1}), x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b]$

Rule 208

$\text{Int}(((a_) + (b_)*(x_)^2)^{-1}), x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

Rule 1819

$\text{Int}[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] \rightarrow \text{Dist}[1/(m + 1), \text{Subst}[\text{Int}[\text{SubstFor}[x^{(m + 1)}, Pq, x]*(a + b*x^{\text{Simplify}[n/(m + 1)])}]^p, x], x, x^{(m + 1)}], x] /; \text{FreeQ}\{a, b, m, n, p\}, x] \ \&\& \ \text{NeQ}[m, -1] \ \&\& \ \text{IGtQ}[\text{Simplify}[n/(m + 1)], 0] \ \&\& \ \text{PolyQ}[Pq, x^{(m + 1)}]$

Rule 1810

$\text{Int}[(Pq_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[Pq*(a + b*x^2)^p, x], x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{IGtQ}[p, -2]$

Rule 635

$\text{Int}(((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] \rightarrow \text{Dist}[d, \text{Int}[1/(a + c*x^2), x], x] + \text{Dist}[e, \text{Int}[x/(a + c*x^2), x], x] /; \text{FreeQ}\{a, c, d, e\}, x] \ \&\& \ \text{!NiceSqrtQ}[-(a*c)]$

Rule 260

$\text{Int}[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]]/(b*n), x] /; \text{FreeQ}\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + 188x^6 + jx^7}{a - bx^4} dx &= \int \left(\frac{c + ex^2 + gx^4 + 188x^6}{a - bx^4} + \frac{x(d + fx^2 + hx^4 + jx^6)}{a - bx^4} \right) dx \\
&= \int \frac{c + ex^2 + gx^4 + 188x^6}{a - bx^4} dx + \int \frac{x(d + fx^2 + hx^4 + jx^6)}{a - bx^4} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{d + fx + hx^2 + jx^3}{a - bx^2} dx, x, x^2 \right) + \int \left(-\frac{g}{b} - \frac{188x^2}{b} + \frac{bc}{b} \right) dx \\
&= -\frac{gx}{b} - \frac{188x^3}{3b} + \frac{1}{2} \text{Subst} \left(\int \left(-\frac{h}{b} - \frac{jx}{b} + \frac{bd + ah + (bf + aj)x}{b(a - bx^2)} \right) dx, x, x^2 \right) \\
&= -\frac{gx}{b} - \frac{hx^2}{2b} - \frac{188x^3}{3b} - \frac{jx^4}{4b} + \frac{\text{Subst} \left(\int \frac{bd + ah + (bf + aj)x}{a - bx^2} dx, x, x^2 \right)}{2b} + \frac{bcx}{b} \\
&= -\frac{gx}{b} - \frac{hx^2}{2b} - \frac{188x^3}{3b} - \frac{jx^4}{4b} - \frac{\left(188a + be - \frac{\sqrt{b(bc + ag)}}{\sqrt{a}} \right) \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2\sqrt[4]{ab}^{7/4}} \\
&= -\frac{gx}{b} - \frac{hx^2}{2b} - \frac{188x^3}{3b} - \frac{jx^4}{4b} - \frac{\left(188a + be - \frac{\sqrt{b(bc + ag)}}{\sqrt{a}} \right) \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2\sqrt[4]{ab}^{7/4}}
\end{aligned}$$

Mathematica [A] time = 0.351059, size = 318, normalized size = 1.55

$$\frac{3 \log \left(\sqrt[4]{a} - \sqrt[4]{bx} \right) \left(a^{5/4} \sqrt[4]{bh + a^{3/2}i} + \sqrt[4]{ab^{5/4}d + \sqrt{abe + a}\sqrt{bg + b^{3/2}c}} \right)}{a^{3/4}} + \frac{3 \log \left(\sqrt[4]{a} + \sqrt[4]{bx} \right) \left(-a^{5/4} \sqrt[4]{bh + a^{3/2}i} - \sqrt[4]{ab^{5/4}d + \sqrt{abe + a}\sqrt{bg + b^{3/2}c}} \right)}{a^{3/4}} + \frac{6 \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right) \left(a^{3/2}(-i) \right)}{a^3}$$

$$12b^{7/4}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4), x]

[Out] (-12*b^(3/4)*g*x - 6*b^(3/4)*h*x^2 - 4*b^(3/4)*i*x^3 - 3*b^(3/4)*j*x^4 + (6*(b^(3/2)*c - Sqrt[a]*b*e + a*Sqrt[b]*g - a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)]/a^(3/4) - (3*(b^(3/2)*c + a^(1/4)*b^(5/4)*d + Sqrt[a]*b*e + a*Sqrt[b]*g + a^(5/4)*b^(1/4)*h + a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x]/a^(3/4) + (3*(b^(3/2)*c - a^(1/4)*b^(5/4)*d + Sqrt[a]*b*e + a*Sqrt[b]*g - a^(5/4)*b^(1/4)*h + a^(3/2)*i)*Log[a^(1/4) + b^(1/4)*x]/a^(3/4) + (3*b^(1/4)*(b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[a] - (3*(b*f + a*j)*Log[a - b*x^4])/b^(1/4))/(12*b^(7/4))

Maple [B] time = 0.045, size = 393, normalized size = 1.9

$$-\frac{jx^4}{4b} - \frac{ix^3}{3b} - \frac{hx^2}{2b} - \frac{gx}{b} + \frac{g}{2b} \sqrt[4]{\frac{a}{b}} \arctan\left(x \frac{1}{\sqrt[4]{\frac{a}{b}}}\right) + \frac{c}{2a} \sqrt[4]{\frac{a}{b}} \arctan\left(x \frac{1}{\sqrt[4]{\frac{a}{b}}}\right) + \frac{g}{4b} \sqrt[4]{\frac{a}{b}} \ln\left(\left(x + \sqrt[4]{\frac{a}{b}}\right)\left(x - \sqrt[4]{\frac{a}{b}}\right)^{-1}\right) + \frac{c}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x)

[Out]
$$-1/4*j*x^4/b - 1/3*i*x^3/b - 1/2*h*x^2/b - g*x/b + 1/2/b*(1/b*a)^{(1/4)}*\arctan(x/(1/b*a)^{(1/4)})*g + 1/2*c*(1/b*a)^{(1/4)}/a*\arctan(x/(1/b*a)^{(1/4)}) + 1/4/b*(1/b*a)^{(1/4)}*\ln((x+(1/b*a)^{(1/4)})/(x-(1/b*a)^{(1/4)})) *g + 1/4*c*(1/b*a)^{(1/4)}/a*\ln((x+(1/b*a)^{(1/4)})/(x-(1/b*a)^{(1/4)})) - 1/4/b/(a*b)^{(1/2)}*\ln((-a+x^2*(a*b)^{(1/2)})/(-a-x^2*(a*b)^{(1/2)})) *a*h - 1/4*d/(a*b)^{(1/2)}*\ln((-a+x^2*(a*b)^{(1/2)})/(-a-x^2*(a*b)^{(1/2)})) - 1/2/b^2/(1/b*a)^{(1/4)}*\arctan(x/(1/b*a)^{(1/4)}) *a*i - 1/2*e/b/(1/b*a)^{(1/4)}*\arctan(x/(1/b*a)^{(1/4)}) + 1/4/b^2/(1/b*a)^{(1/4)}*\ln((x+(1/b*a)^{(1/4)})/(x-(1/b*a)^{(1/4)})) *a*i + 1/4*e/b/(1/b*a)^{(1/4)}*\ln((x+(1/b*a)^{(1/4)})/(x-(1/b*a)^{(1/4)})) - 1/4/b^2*\ln(b*x^4-a) *a*j - 1/4/b*f*\ln(b*x^4-a)$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.10172, size = 828, normalized size = 4.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4)))/(-a/b)^(1/4))/b^4 - sqrt(2)*(-a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/b^4 + 1/8*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4)))/(-a/b)^(1/4))/b^4 + sqrt(2)*(-a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/b^4 - 1/4*(b*f + a*j)*log(abs(b*x^4 - a))/b^2 + 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*d - sqrt(2)*sqrt(-a*b)*a*b*h + (-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(1/4)*a*b*g + (-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4)))/(-a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*d - sqrt(2)*sqrt(-a*b)*a*b*h + (-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(1/4)*a*b*g + (-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4)))/(-a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(1/4)*a*b*g - (-a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^3) - 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(1/4)*a*b*g - (-a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^3) - 1/12*(3*b^3*j*x^4 + 4*b^3*i*x^3 + 6*b^3*h*x^2 + 12*b^3*
```


$g*x)/b^4$

$$3.189 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^4} dx$$

Optimal. Leaf size=337

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-\sqrt{a}\sqrt{be} - ag + bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-\sqrt{a}\sqrt{be} - ag + bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}} - \tan^{-1}\left(1\right)$$

[Out] (g*x)/b + (h*x^2)/(2*b) + ((b*d - a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - ((b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) - ((b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + (f*Log[a + b*x^4])/(4*b)

Rubi [A] time = 0.399356, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {1885, 1887, 1168, 1162, 617, 204, 1165, 628, 1819, 1810, 635, 205, 260}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-\sqrt{a}\sqrt{be} - ag + bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-\sqrt{a}\sqrt{be} - ag + bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}} - \tan^{-1}\left(1\right)$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4), x]

[Out] (g*x)/b + (h*x^2)/(2*b) + ((b*d - a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - ((b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) - ((b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + (f*Log[a + b*x^4])/(4*b)

Rule 1885

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (

$2*(q - j)/n + 1](a + b*x^n)^p, \{j, 0, n/2 - 1\}, x] /; \text{FreeQ}\{a, b, p\}, x] \&\& \text{PolyQ}\{Pq, x\} \&\& \text{IGtQ}\{n/2, 0\} \&\& \text{!PolyQ}\{Pq, x^{(n/2)}\}$

Rule 1887

$\text{Int}[(Pq_)/((a_) + (b_)*(x_)^{(n_)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[Pq/(a + b*x^n), x], x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PolyQ}\{Pq, x\} \&\& \text{IntegerQ}\{n\}$

Rule 1168

$\text{Int}[(d_ + (e_)*(x_)^2)/(a_ + (c_)*(x_)^4), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \text{Dist}[(d*q - a*e)/(2*a*c), \text{Int}[(q - c*x^2)/(a + c*x^4), x], x] /; \text{FreeQ}\{a, c, d, e\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{NeQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[-(a*c)]$

Rule 1162

$\text{Int}[(d_ + (e_)*(x_)^2)/(a_ + (c_)*(x_)^4), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[(2*d)/e, 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x] /; \text{FreeQ}\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[d*e]$

Rule 617

$\text{Int}[(a_ + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{With}\{q = 1 - 4*S\text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \|\ \text{!RationalQ}[b^2 - 4*a*c]) /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 204

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2]*x]/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{LtQ}[a, 0] \|\ \text{LtQ}[b, 0])$

Rule 1165

$\text{Int}[(d_ + (e_)*(x_)^2)/(a_ + (c_)*(x_)^4), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[-2*d)/e, 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x] /; \text{FreeQ}\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[d*e]$

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1819

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/(m
+ 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{a + bx^4} dx &= \int \left(\frac{c + ex^2 + gx^4}{a + bx^4} + \frac{x(d + fx^2 + hx^4)}{a + bx^4} \right) dx \\
&= \int \frac{c + ex^2 + gx^4}{a + bx^4} dx + \int \frac{x(d + fx^2 + hx^4)}{a + bx^4} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{d + fx + hx^2}{a + bx^2} dx, x, x^2 \right) + \int \left(\frac{g}{b} + \frac{bc - ag + bex^2}{b(a + bx^4)} \right) dx \\
&= \frac{gx}{b} + \frac{1}{2} \text{Subst} \left(\int \left(\frac{h}{b} + \frac{bd - ah + bfx}{b(a + bx^2)} \right) dx, x, x^2 \right) + \frac{\int \frac{bc - ag + bex^2}{a + bx^4} dx}{b} \\
&= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{\text{Subst} \left(\int \frac{bd - ah + bfx}{a + bx^2} dx, x, x^2 \right)}{2b} + \frac{(bc - \sqrt{a}\sqrt{be} - ag) \int \frac{\sqrt{a}\sqrt{b} - bx^2}{a + bx^4} dx}{2\sqrt{ab}^{3/2}} \\
&= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{1}{2} f \text{Subst} \left(\int \frac{x}{a + bx^2} dx, x, x^2 \right) - \frac{(bc - \sqrt{a}\sqrt{be} - ag) \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt{b}}}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{b}}} dx}{4\sqrt{2}a^{3/4}b^{5/4}} \\
&= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{(bd - ah) \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{2\sqrt{ab}^{3/2}} - \frac{(bc - \sqrt{a}\sqrt{be} - ag) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx})}{4\sqrt{2}a^{3/4}b^{5/4}} \\
&= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{(bd - ah) \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{2\sqrt{ab}^{3/2}} - \frac{(bc + \sqrt{a}\sqrt{be} - ag) \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt{a}} \right)}{2\sqrt{2}a^{3/4}b^{5/4}} +
\end{aligned}$$

Mathematica [A] time = 0.324556, size = 342, normalized size = 1.01

$$-2 \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt{a}} \right) (-2a^{5/4}h + \sqrt{2}\sqrt{ab}^{3/4}e + 2\sqrt[4]{abd} - \sqrt{2}a\sqrt[4]{bg} + \sqrt{2}b^{5/4}c) + 2 \tan^{-1} \left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt{a}} + 1 \right) (2a^{5/4}h + \sqrt{2}\sqrt{ab}^{3/4}e + 2\sqrt[4]{abd} - \sqrt{2}a\sqrt[4]{bg} + \sqrt{2}b^{5/4}c)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4), x]

[Out] (-2*(Sqrt[2]*b^(5/4)*c + 2*a^(1/4)*b*d + Sqrt[2]*Sqrt[a]*b^(3/4)*e - Sqrt[2]*a*b^(1/4)*g - 2*a^(5/4)*h)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*b^(5/4)*c - 2*a^(1/4)*b*d + Sqrt[2]*Sqrt[a]*b^(3/4)*e - Sqrt[2]*a*b^(1/4)*g + 2*a^(5/4)*h)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + b^(1/4)*(Sqrt[2]*(-b*c) + Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a]

+ Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 2*a^(3/4)*b^(1/4)*(2*x*(2*g + h*x) + f*Log[a + b*x^4])))/(8*a^(3/4)*b^(3/2))

Maple [A] time = 0.003, size = 462, normalized size = 1.4

$$\frac{hx^2}{2b} + \frac{gx}{b} - \frac{\sqrt{2}g}{4b} \sqrt[4]{\frac{a}{b}} \arctan\left(x\sqrt{2} \frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) + \frac{c\sqrt{2}}{4a} \sqrt[4]{\frac{a}{b}} \arctan\left(x\sqrt{2} \frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) - \frac{\sqrt{2}g}{4b} \sqrt[4]{\frac{a}{b}} \arctan\left(x\sqrt{2} \frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right) + \frac{c\sqrt{2}}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a), x)

[Out] 1/2*h*x^2/b+g*x/b-1/4/b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*g+1/4*c*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)-1/4/b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*g+1/4*c*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)-1/8/b*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*g+1/8*c*(1/b*a)^(1/4)/a*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))-1/2/b/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))*a*h+1/2*d/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+1/8*e/b/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/4*e/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/4*e/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+1/4*f*ln(b*x^4+a)/b

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)

[Out] Timed out

Giac [A] time = 1.08285, size = 506, normalized size = 1.5

$$\frac{f \log(|bx^4 + a|)}{4b} + \frac{bhx^2 + 2bgx}{2b^2} + \frac{\sqrt{2} \left(\sqrt{2} \sqrt{abb^2d} + \sqrt{2} \sqrt{ababh} + (ab^3)^{\frac{1}{4}} b^2c - (ab^3)^{\frac{1}{4}} abg + (ab^3)^{\frac{3}{4}} e \right) \arctan \left(\frac{\sqrt{2} (2x + \sqrt{a/b})}{2 \sqrt{a/b}} \right)}{4ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")

[Out] $\frac{1}{4} f \log(\text{abs}(b*x^4 + a))/b + \frac{1}{2} (b*h*x^2 + 2*b*g*x)/b^2 + \frac{1}{4} \sqrt{2} (\sqrt{2} \sqrt{a*b} * b^2*d + \sqrt{2} \sqrt{a*b} * a*b*h + (a*b^3)^{(1/4)} * b^2*c - (a*b^3)^{(1/4)} * a*b*g + (a*b^3)^{(3/4)} * e) * \arctan(1/2 * \sqrt{2} * (2*x + \sqrt{2} * (a/b)^{(1/4)}) / (a/b)^{(1/4)}) / (a*b^3) + \frac{1}{4} \sqrt{2} (\sqrt{2} \sqrt{a*b} * b^2*d + \sqrt{2} \sqrt{a*b} * a*b*h + (a*b^3)^{(1/4)} * b^2*c - (a*b^3)^{(1/4)} * a*b*g + (a*b^3)^{(3/4)} * e) * \arctan(1/2 * \sqrt{2} * (2*x - \sqrt{2} * (a/b)^{(1/4)}) / (a/b)^{(1/4)}) / (a*b^3) + \frac{1}{8} \sqrt{2} ((a*b^3)^{(1/4)} * b^2*c - (a*b^3)^{(1/4)} * a*b*g - (a*b^3)^{(3/4)} * e) * \log(x^2 + \sqrt{2} * x * (a/b)^{(1/4)} + \sqrt{a/b}) / (a*b^3) - \frac{1}{8} \sqrt{2} ((a*b^3)^{(1/4)} * b^2*c - (a*b^3)^{(1/4)} * a*b*g - (a*b^3)^{(3/4)} * e) * \log(x^2 - \sqrt{2} * x * (a/b)^{(1/4)} + \sqrt{a/b}) / (a*b^3)$

$$3.190 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a+bx^4} dx$$

Optimal. Leaf size=384

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(\sqrt{b}(bc-ag) - \sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(\sqrt{b}(bc-ag) - \sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}}$$

[Out] (g*x)/b + (h*x^2)/(2*b) + (i*x^3)/(3*b) + ((b*d - a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - ((Sqrt[b]*(b*c - a*g) + Sqrt[a]*(b*e - a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) + Sqrt[a]*(b*e - a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(7/4)) - ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(b*e - a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(b*e - a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(7/4)) + (f*Log[a + b*x^4])/(4*b)

Rubi [A] time = 0.565177, antiderivative size = 384, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 13, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {1885, 1819, 1810, 635, 205, 260, 1887, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(\sqrt{b}(bc-ag) - \sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(\sqrt{b}(bc-ag) - \sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4), x]

[Out] (g*x)/b + (h*x^2)/(2*b) + (i*x^3)/(3*b) + ((b*d - a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - ((Sqrt[b]*(b*c - a*g) + Sqrt[a]*(b*e - a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) + Sqrt[a]*(b*e - a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(7/4)) - ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(b*e - a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(b*e - a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(7/4)) + (f*Log[a + b*x^4])/(4*b)

Rule 1885


```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq,
x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, p},
x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 1819

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/(m
+ 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
```

c)]

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + 190x^6}{a + bx^4} dx &= \int \left(\frac{x(d + fx^2 + hx^4)}{a + bx^4} + \frac{c + ex^2 + gx^4 + 190x^6}{a + bx^4} \right) dx \\
&= \int \frac{x(d + fx^2 + hx^4)}{a + bx^4} dx + \int \frac{c + ex^2 + gx^4 + 190x^6}{a + bx^4} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{d + fx + hx^2}{a + bx^2} dx, x, x^2 \right) + \int \left(\frac{g}{b} + \frac{190x^2}{b} + \frac{bc - ag - (190a - be - \sqrt{b(bc - ag)})}{b(a + bx^2)} \right) dx \\
&= \frac{gx}{b} + \frac{190x^3}{3b} + \frac{1}{2} \text{Subst} \left(\int \left(\frac{h}{b} + \frac{bd - ah + bfx}{b(a + bx^2)} \right) dx, x, x^2 \right) + \frac{\int \frac{bc - ag - (190a - be - \sqrt{b(bc - ag)})}{b(a + bx^2)} dx}{2} \\
&= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{190x^3}{3b} + \frac{\text{Subst} \left(\int \frac{bd - ah + bfx}{a + bx^2} dx, x, x^2 \right) - \left(190a - be - \frac{\sqrt{b(bc - ag)}}{\sqrt{a}} \right)}{2b} \\
&= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{190x^3}{3b} + \frac{1}{2} f \text{Subst} \left(\int \frac{x}{a + bx^2} dx, x, x^2 \right) - \frac{\left(190a - be - \frac{\sqrt{b(bc - ag)}}{\sqrt{a}} \right)}{2\sqrt{ab^{3/2}}} \\
&= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{190x^3}{3b} + \frac{(bd - ah) \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right) - \left(190a - be + \frac{\sqrt{b(bc - ag)}}{\sqrt{a}} \right)}{2\sqrt{ab^{3/2}}} \\
&= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{190x^3}{3b} + \frac{(bd - ah) \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right) + \left(190a - be - \frac{\sqrt{b(bc - ag)}}{\sqrt{a}} \right)}{2\sqrt{2}\sqrt[4]{ab^3}}
\end{aligned}$$

Mathematica [A] time = 0.306906, size = 427, normalized size = 1.11

$$\frac{6 \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt{a}} \right) \left(2a^{5/4}\sqrt[4]{bh} + \sqrt{2}a^{3/2}i - 2\sqrt[4]{ab^{5/4}}d - \sqrt{2}\sqrt{abe} + \sqrt{2}a\sqrt{bg} - \sqrt{2}b^{3/2}c \right)}{a^{3/4}} + \frac{6 \tan^{-1} \left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt{a}} + 1 \right) \left(2a^{5/4}\sqrt[4]{bh} - \sqrt{2}a^{3/2}i - 2\sqrt[4]{ab^{5/4}}d + \sqrt{2}\sqrt{abe} - \sqrt{2}a\sqrt{bg} + \sqrt{2}b^{3/2}c \right)}{a^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4), x]

[Out] (24*b^(3/4)*g*x + 12*b^(3/4)*h*x^2 + 8*b^(3/4)*i*x^3 + (6*(-(Sqrt[2]*b^(3/2))*c) - 2*a^(1/4)*b^(5/4)*d - Sqrt[2]*Sqrt[a]*b*e + Sqrt[2]*a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h + Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) + (6*(Sqrt[2]*b^(3/2))*c - 2*a^(1/4)*b^(5/4)*d + Sqrt[2]*Sqrt[a]*b*e - Sqrt[2]*a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h - Sqrt[2]*a^(3/2)*i)*ArcT

$$\frac{\operatorname{atan}\left(\frac{\sqrt{2}b^{1/4}x}{a^{1/4}}\right)/a^{3/4} - (3\sqrt{2}(b^{3/2}c - \sqrt{a}b^*e - a\sqrt{b}g + a^{3/2}i)\operatorname{Log}[\sqrt{a} - \sqrt{2}a^{1/4}b^{1/4}x + \sqrt{b}x^2])/a^{3/4} + (3\sqrt{2}(b^{3/2}c - \sqrt{a}b^*e - a\sqrt{b}g + a^{3/2}i)\operatorname{Log}[\sqrt{a} + \sqrt{2}a^{1/4}b^{1/4}x + \sqrt{b}x^2])/a^{3/4} + 6b^{3/4}f\operatorname{Log}[a + b^*x^4])}{(24b^{7/4})}$$

Maple [B] time = 0.004, size = 603, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x)`

[Out] $\frac{1}{3}i x^3/b + \frac{1}{2}h x^2/b + g x/b - \frac{1}{4}b^{1/4} \operatorname{arctan}\left(\frac{2^{1/2}}{b^{1/4}x+1}\right) + \frac{1}{4}c \operatorname{arctan}\left(\frac{2^{1/2}}{b^{1/4}x+1}\right) - \frac{1}{4}b^{1/4} \operatorname{arctan}\left(\frac{2^{1/2}}{b^{1/4}x-1}\right) + \frac{1}{4}c \operatorname{arctan}\left(\frac{2^{1/2}}{b^{1/4}x-1}\right) - \frac{1}{8}b^{1/4} \operatorname{arctan}\left(\frac{2^{1/2}}{b^{1/4}x-1}\right) + \frac{1}{4} \operatorname{arctan}\left(\frac{2^{1/2}}{b^{1/4}x-1}\right) \ln\left(\frac{x^2+(1/b^*a)^{1/4}x+2^{1/2}}{x^2-(1/b^*a)^{1/4}x+2^{1/2}}\right) + \frac{1}{8}c \operatorname{arctan}\left(\frac{2^{1/2}}{b^{1/4}x-1}\right) \ln\left(\frac{x^2+(1/b^*a)^{1/4}x+2^{1/2}}{x^2-(1/b^*a)^{1/4}x+2^{1/2}}\right) + \frac{1}{8}c \operatorname{arctan}\left(\frac{2^{1/2}}{b^{1/4}x-1}\right) \ln\left(\frac{x^2+(1/b^*a)^{1/4}x+2^{1/2}}{x^2-(1/b^*a)^{1/4}x+2^{1/2}}\right) - \frac{1}{2}b^{1/4} \operatorname{arctan}\left(\frac{2^{1/2}}{b^{1/4}x-1}\right) \operatorname{arctan}\left(\frac{2^{1/2}}{b^{1/4}x-1}\right) + \frac{1}{8}b^{1/4} \operatorname{arctan}\left(\frac{2^{1/2}}{b^{1/4}x-1}\right) \operatorname{arctan}\left(\frac{2^{1/2}}{b^{1/4}x-1}\right) \ln\left(\frac{x^2+(1/b^*a)^{1/4}x+2^{1/2}}{x^2-(1/b^*a)^{1/4}x+2^{1/2}}\right) + \frac{1}{8}e/b \operatorname{arctan}\left(\frac{2^{1/2}}{b^{1/4}x+1}\right) + \frac{1}{4}e/b \operatorname{arctan}\left(\frac{2^{1/2}}{b^{1/4}x+1}\right) - \frac{1}{4}b^{1/4} \operatorname{arctan}\left(\frac{2^{1/2}}{b^{1/4}x+1}\right) \operatorname{arctan}\left(\frac{2^{1/2}}{b^{1/4}x+1}\right) + \frac{1}{4}e/b \operatorname{arctan}\left(\frac{2^{1/2}}{b^{1/4}x+1}\right) \operatorname{arctan}\left(\frac{2^{1/2}}{b^{1/4}x+1}\right) \ln(b^*x^4+a)/b$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")`

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)`

[Out] Timed out

Giac [A] time = 1.09652, size = 759, normalized size = 1.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")`

[Out]
$$-1/8*i*(2*\sqrt{2}*(a*b^3)^{3/4}*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(a/b)^{1/4}))/ (a/b)^{1/4})/b^4 - \sqrt{2}*(a*b^3)^{3/4}*\log(x^2 + \sqrt{2}*x*(a/b)^{1/4} + \sqrt{a/b})/b^4 - 1/8*i*(2*\sqrt{2}*(a*b^3)^{3/4}*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a/b)^{1/4}))/ (a/b)^{1/4})/b^4 + \sqrt{2}*(a*b^3)^{3/4}*\log(x^2 - \sqrt{2}*x*(a/b)^{1/4} + \sqrt{a/b})/b^4 + 1/4*f*\log(\text{abs}(b*x^4 + a))/b + 1/$$

$$\begin{aligned}
& 4\sqrt{2}*(\sqrt{2}*\sqrt{a*b}*b^2*d + \sqrt{2}*\sqrt{a*b}*a*b*h + (a*b^3)^{(1/4)} \\
&)*b^2*c - (a*b^3)^{(1/4)}*a*b*g + (a*b^3)^{(3/4)}*e)*\arctan(1/2*\sqrt{2}*(2*x + \\
& \sqrt{2}*(a/b)^{(1/4)})/(a/b)^{(1/4)})/(a*b^3) + 1/4*\sqrt{2}*(\sqrt{2}*\sqrt{a*b}* \\
& b^2*d + \sqrt{2}*\sqrt{a*b}*a*b*h + (a*b^3)^{(1/4)}*b^2*c - (a*b^3)^{(1/4)}*a*b*g \\
& + (a*b^3)^{(3/4)}*e)*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a/b)^{(1/4)})/(a/b)^{(1/4)})/(a*b^3) \\
& + 1/8*\sqrt{2}*((a*b^3)^{(1/4)}*b^2*c - (a*b^3)^{(1/4)}*a*b*g - (a*b^3)^{(3/4)}*e)*\log(x^2 + \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(a*b^3) - 1/8*\sqrt{2} \\
& *((a*b^3)^{(1/4)}*b^2*c - (a*b^3)^{(1/4)}*a*b*g - (a*b^3)^{(3/4)}*e)*\log(x^2 - \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(a*b^3) + 1/6*(2*b^2*i*x^3 + 3*b^2*h*x^2 + 6*b^2*g*x)/b^3
\end{aligned}$$

$$3.191 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a+bx^4} dx$$

Optimal. Leaf size=402

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(\sqrt{b}(bc-ag) - \sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(\sqrt{b}(bc-ag) - \sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}}$$

[Out] (g*x)/b + (h*x^2)/(2*b) + (i*x^3)/(3*b) + (j*x^4)/(4*b) + ((b*d - a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - ((Sqrt[b]*(b*c - a*g) + Sqrt[a]*(b*e - a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) + Sqrt[a]*(b*e - a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(7/4)) - ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(b*e - a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(b*e - a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(7/4)) + ((b*f - a*j)*Log[a + b*x^4])/(4*b^2)

Rubi [A] time = 0.566919, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 13, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.289$, Rules used = {1885, 1887, 1168, 1162, 617, 204, 1165, 628, 1819, 1810, 635, 205, 260}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(\sqrt{b}(bc-ag) - \sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(\sqrt{b}(bc-ag) - \sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4), x]

[Out] (g*x)/b + (h*x^2)/(2*b) + (i*x^3)/(3*b) + (j*x^4)/(4*b) + ((b*d - a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - ((Sqrt[b]*(b*c - a*g) + Sqrt[a]*(b*e - a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) + Sqrt[a]*(b*e - a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(7/4)) - ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(b*e - a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(b*e - a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(7/4)) + ((b*f - a*j)*Log[a + b*x^4])/(4*b^2)

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq,
x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, p},
x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[
e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```


$\text{eQ}\{a, c, d, e, x\} \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

Rule 628

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x_Symbol] \ :> \ \text{Simp}[\frac{d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]}{b}, x] \ /; \ \text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 1819

$\text{Int}[(Pq_)*(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x_Symbol] \ :> \ \text{Dist}[1/(m + 1), \text{Subst}[\text{Int}[\text{SubstFor}[x^{(m + 1)}, Pq, x]*(a + b*x^{\text{Simplify}[n/(m + 1)])}]^p, x], x, x^{(m + 1)}], x] \ /; \ \text{FreeQ}\{a, b, m, n, p\}, x\} \ \&\& \ \text{NeQ}[m, -1] \ \&\& \ \text{IGtQ}[\text{Simplify}[n/(m + 1)], 0] \ \&\& \ \text{PolyQ}[Pq, x^{(m + 1)}]$

Rule 1810

$\text{Int}[(Pq_)*((a_.) + (b_.)*(x_.)^2)^{(p_.)}, x_Symbol] \ :> \ \text{Int}[\text{ExpandIntegrand}[Pq*(a + b*x^2)^p, x], x] \ /; \ \text{FreeQ}\{a, b\}, x\} \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{IGtQ}[p, -2]$

Rule 635

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (c_.)*(x_.)^2}, x_Symbol] \ :> \ \text{Dist}[d, \text{Int}[1/(a + c*x^2), x], x] + \text{Dist}[e, \text{Int}[x/(a + c*x^2), x], x] \ /; \ \text{FreeQ}\{a, c, d, e\}, x\} \ \&\& \ \text{!NiceSqrtQ}[-(a*c)]$

Rule 205

$\text{Int}[\frac{(a_.) + (b_.)*(x_.)^2)^{-1}}{x_Symbol}, x_Symbol] \ :> \ \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] \ /; \ \text{FreeQ}\{a, b\}, x\} \ \&\& \ \text{PosQ}[a/b]$

Rule 260

$\text{Int}[\frac{(x_.)^{(m_.)}}{(a_.) + (b_.)*(x_.)^{(n_.)}}, x_Symbol] \ :> \ \text{Simp}[\frac{\text{Log}[\text{RemoveContent}[a + b*x^n, x]]}{(b*n)}, x] \ /; \ \text{FreeQ}\{a, b, m, n\}, x\} \ \&\& \ \text{EqQ}[m, n - 1]$

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + 191x^6 + jx^7}{a + bx^4} dx &= \int \left(\frac{c + ex^2 + gx^4 + 191x^6}{a + bx^4} + \frac{x(d + fx^2 + hx^4 + jx^6)}{a + bx^4} \right) dx \\
&= \int \frac{c + ex^2 + gx^4 + 191x^6}{a + bx^4} dx + \int \frac{x(d + fx^2 + hx^4 + jx^6)}{a + bx^4} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{d + fx + hx^2 + jx^3}{a + bx^2} dx, x, x^2 \right) + \int \left(\frac{g}{b} + \frac{191x^2}{b} + \frac{bc -}{b} \right) dx, x \\
&= \frac{gx}{b} + \frac{191x^3}{3b} + \frac{1}{2} \text{Subst} \left(\int \left(\frac{h}{b} + \frac{jx}{b} + \frac{bd - ah + (bf - aj)x}{b(a + bx^2)} \right) dx, x, x^2 \right) \\
&= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{191x^3}{3b} + \frac{jx^4}{4b} + \frac{\text{Subst} \left(\int \frac{bd - ah + (bf - aj)x}{a + bx^2} dx, x, x^2 \right)}{2b} - \frac{(191a - be - \frac{\sqrt{b(bc - ag)}}{\sqrt{a}}) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{b}} + x^2}}{4b^2} \\
&= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{191x^3}{3b} + \frac{jx^4}{4b} + \frac{(bd - ah) \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{2\sqrt{ab}^{3/2}} - \frac{(191a - be + \frac{\sqrt{b(bc - ag)}}{\sqrt{a}}) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{b}} + x^2}}{4b^2} \\
&= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{191x^3}{3b} + \frac{jx^4}{4b} + \frac{(bd - ah) \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{2\sqrt{ab}^{3/2}} + \frac{(191a - be - \frac{\sqrt{b(bc - ag)}}{\sqrt{a}}) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{b}} + x^2}}{4b^2}
\end{aligned}$$

Mathematica [A] time = 0.308731, size = 445, normalized size = 1.11

$$\frac{6 \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} \right) \left(2a^{5/4}\sqrt[4]{bh} + \sqrt{2}a^{3/2}i - 2\sqrt[4]{ab}^{5/4}d - \sqrt{2}\sqrt{abe} + \sqrt{2}a\sqrt{bg} - \sqrt{2}b^{3/2}c \right)}{a^{3/4}} + \frac{6 \tan^{-1} \left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + 1 \right) \left(2a^{5/4}\sqrt[4]{bh} - \sqrt{2}a^{3/2}i - 2\sqrt[4]{ab}^{5/4}d + \sqrt{2}\sqrt{abe} - \sqrt{2}a\sqrt{bg} + \sqrt{2}b^{3/2}c \right)}{a^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4), x]

[Out] (24*b^(3/4)*g*x + 12*b^(3/4)*h*x^2 + 8*b^(3/4)*i*x^3 + 6*b^(3/4)*j*x^4 + (6*(-(Sqrt[2]*b^(3/2)*c) - 2*a^(1/4)*b^(5/4)*d - Sqrt[2]*Sqrt[a]*b*e + Sqrt[2]*a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h + Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) + (6*(Sqrt[2]*b^(3/2)*c - 2*a^(1/4)*b^(5/4)*d + Sqrt[2]*Sqrt[a]*b*e - Sqrt[2]*a*Sqrt[b]*g - 2*a^(5/4)*b^(1/4)*h + Sqrt[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4)

$$d + \sqrt{2} \sqrt{a} b e - \sqrt{2} a \sqrt{b} g + 2 a^{5/4} b^{1/4} h - \sqrt{2} a^{3/2} i \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} b^{1/4} x}{a^{1/4}}\right] / a^{3/4} - (3 \sqrt{2} (b^{3/2} c - \sqrt{a} b e - a \sqrt{b} g + a^{3/2} i) \operatorname{Log}[\sqrt{a} - \sqrt{2} a^{1/4} b^{1/4} x + \sqrt{b} x^2]) / a^{3/4} + (3 \sqrt{2} (b^{3/2} c - \sqrt{a} b e - a \sqrt{b} g + a^{3/2} i) \operatorname{Log}[\sqrt{a} + \sqrt{2} a^{1/4} b^{1/4} x + \sqrt{b} x^2]) / a^{3/4} + (6 (b f - a j) \operatorname{Log}[a + b x^4]) / b^{1/4} / (24 b^{7/4})$$

Maple [B] time = 0.006, size = 627, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}((j x^7 + i x^6 + h x^5 + g x^4 + f x^3 + e x^2 + d x + c) / (b x^4 + a), x)$

[Out] $\frac{1}{4} j x^4 / b + \frac{1}{3} i x^3 / b + \frac{1}{2} h x^2 / b + g x / b - \frac{1}{4} b (1/b a)^{1/4} 2^{1/2} \arctan(2^{1/2} / (1/b a)^{1/4} x + 1) g + \frac{1}{4} c (1/b a)^{1/4} / a 2^{1/2} \arctan(2^{1/2} / (1/b a)^{1/4} x + 1) - \frac{1}{4} b (1/b a)^{1/4} 2^{1/2} \arctan(2^{1/2} / (1/b a)^{1/4} x - 1) g + \frac{1}{4} c (1/b a)^{1/4} / a 2^{1/2} \arctan(2^{1/2} / (1/b a)^{1/4} x - 1) - \frac{1}{8} b (1/b a)^{1/4} 2^{1/2} \ln((x^2 + (1/b a)^{1/4} x 2^{1/2} + (1/b a)^{1/2}) / (x^2 - (1/b a)^{1/4} x 2^{1/2} + (1/b a)^{1/2})) g + \frac{1}{8} c (1/b a)^{1/4} / a 2^{1/2} \ln((x^2 + (1/b a)^{1/4} x 2^{1/2} + (1/b a)^{1/2}) / (x^2 - (1/b a)^{1/4} x 2^{1/2} + (1/b a)^{1/2})) + \frac{1}{2} b / (a b)^{1/2} \arctan(x^2 (b/a)^{1/2}) a h + \frac{1}{2} d / (a b)^{1/2} \arctan(x^2 (b/a)^{1/2}) - \frac{1}{4} b^2 / (1/b a)^{1/4} 2^{1/2} \arctan(2^{1/2} / (1/b a)^{1/4} x + 1) a i + \frac{1}{4} e / b / (1/b a)^{1/4} 2^{1/2} \arctan(2^{1/2} / (1/b a)^{1/4} x + 1) - \frac{1}{4} b^2 / (1/b a)^{1/4} 2^{1/2} \arctan(2^{1/2} / (1/b a)^{1/4} x - 1) a i + \frac{1}{4} e / b / (1/b a)^{1/4} 2^{1/2} \arctan(2^{1/2} / (1/b a)^{1/4} x - 1) - \frac{1}{8} b^2 / (1/b a)^{1/4} 2^{1/2} \ln((x^2 - (1/b a)^{1/4} x 2^{1/2} + (1/b a)^{1/2}) / (x^2 + (1/b a)^{1/4} x 2^{1/2} + (1/b a)^{1/2})) a i + \frac{1}{8} e / b / (1/b a)^{1/4} 2^{1/2} \ln((x^2 - (1/b a)^{1/4} x 2^{1/2} + (1/b a)^{1/2}) / (x^2 + (1/b a)^{1/4} x 2^{1/2} + (1/b a)^{1/2})) - \frac{1}{4} b^2 \ln(b x^4 + a) a j + \frac{1}{4} f \ln(b x^4 + a) / b$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}((j x^7 + i x^6 + h x^5 + g x^4 + f x^3 + e x^2 + d x + c) / (b x^4 + a), x, \operatorname{algorithm} = "maxima")$

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)

[Out] Timed out

Giac [A] time = 1.0966, size = 780, normalized size = 1.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")

[Out]
$$-1/8*i*(2*\sqrt{2}*(a*b^3)^{3/4}*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(a/b)^{1/4}))/ (a/b)^{1/4})/b^4 - \sqrt{2}*(a*b^3)^{3/4}*\log(x^2 + \sqrt{2}*x*(a/b)^{1/4} + \sqrt{a/b})/b^4 - 1/8*i*(2*\sqrt{2}*(a*b^3)^{3/4}*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a/b)^{1/4}))/ (a/b)^{1/4})/b^4 + \sqrt{2}*(a*b^3)^{3/4}*\log(x^2 - \sqrt{2}*x*(a/b)^{1/4} + \sqrt{a/b})/b^4 + 1/4*(b*f - a*j)*\log(\text{abs}(b*x^4 +$$

$$\begin{aligned}
& a)/b^2 + 1/4*\sqrt{2}*(\sqrt{2}*\sqrt{a*b}*b^2*d + \sqrt{2}*\sqrt{a*b}*a*b*h + \\
& (a*b^3)^{(1/4)}*b^2*c - (a*b^3)^{(1/4)}*a*b*g + (a*b^3)^{(3/4)}*e)*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(a/b)^{(1/4)})/(a/b)^{(1/4)})/(a*b^3) + 1/4*\sqrt{2}*(\sqrt{2} \\
&)*\sqrt{a*b}*b^2*d + \sqrt{2}*\sqrt{a*b}*a*b*h + (a*b^3)^{(1/4)}*b^2*c - (a*b^3)^{(1/4)}*a*b*g + (a*b^3)^{(3/4)}*e)*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a/b)^{(1/4)})/(a/b)^{(1/4)})/(a*b^3) + 1/8*\sqrt{2}*((a*b^3)^{(1/4)}*b^2*c - (a*b^3)^{(1/4)} \\
& *a*b*g - (a*b^3)^{(3/4)}*e)*\log(x^2 + \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(a*b^3) - 1/8*\sqrt{2}*((a*b^3)^{(1/4)}*b^2*c - (a*b^3)^{(1/4)}*a*b*g - (a*b^3)^{(3/4)} \\
&)*e)*\log(x^2 - \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(a*b^3) + 1/12*(3*b^3*j*x^4 + 4*b^3*i*x^3 + 6*b^3*h*x^2 + 12*b^3*g*x)/b^4
\end{aligned}$$

$$3.192 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^2} dx$$

Optimal. Leaf size=184

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(-\sqrt{a}\sqrt{be}-ag+3bc)}{8a^{7/4}b^{5/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(\sqrt{a}\sqrt{be}-ag+3bc)}{8a^{7/4}b^{5/4}} + \frac{(bd-ah)\tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}} + \frac{x(ax+bd)+a}{4ab}$$

[Out] (x*(b*c + a*g + (b*d + a*h)*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a - b*x^4)) + ((3*b*c - Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(5/4)) + ((3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(5/4)) + ((b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2))

Rubi [A] time = 0.203857, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1858, 1876, 275, 208, 1167, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(-\sqrt{a}\sqrt{be}-ag+3bc)}{8a^{7/4}b^{5/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(\sqrt{a}\sqrt{be}-ag+3bc)}{8a^{7/4}b^{5/4}} + \frac{(bd-ah)\tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}} + \frac{x(ax+bd)+a}{4ab}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^2,x]

[Out] (x*(b*c + a*g + (b*d + a*h)*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a - b*x^4)) + ((3*b*c - Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(5/4)) + ((3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(5/4)) + ((b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2))

Rule 1858

Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]

;/ FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]

Rule 1876

Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 275

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1167

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[-(a*c)]

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{(a - bx^4)^2} dx &= \frac{x(bc + ag + (bd + ah)x + bex^2 + bfx^3)}{4ab(a - bx^4)} - \frac{\int \frac{-b(3bc-ag) - 2b(bd-ah)x - b^2ex^2}{a-bx^4} dx}{4ab^2} \\
&= \frac{x(bc + ag + (bd + ah)x + bex^2 + bfx^3)}{4ab(a - bx^4)} - \frac{\int \left(-\frac{2b(bd-ah)x}{a-bx^4} + \frac{-b(3bc-ag) - b^2ex^2}{a-bx^4} \right) dx}{4ab^2} \\
&= \frac{x(bc + ag + (bd + ah)x + bex^2 + bfx^3)}{4ab(a - bx^4)} - \frac{\int \frac{-b(3bc-ag) - b^2ex^2}{a-bx^4} dx}{4ab^2} + \frac{(bd - ah) \int \frac{x}{a-bx^4} dx}{2ab} \\
&= \frac{x(bc + ag + (bd + ah)x + bex^2 + bfx^3)}{4ab(a - bx^4)} - \frac{(3bc - \sqrt{a}\sqrt{be} - ag) \int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2} dx}{8a^{3/2}\sqrt{b}} + \\
&= \frac{x(bc + ag + (bd + ah)x + bex^2 + bfx^3)}{4ab(a - bx^4)} + \frac{(3bc - \sqrt{a}\sqrt{be} - ag) \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{5/4}} +
\end{aligned}$$

Mathematica [A] time = 0.163229, size = 257, normalized size = 1.4

$$\frac{\log\left(\sqrt[4]{a} - \sqrt[4]{bx}\right)\left(2a^{5/4}h - \sqrt{ab^{3/4}}e - 2\sqrt[4]{abd} + a\sqrt[4]{bg} - 3b^{5/4}c\right) + \log\left(\sqrt[4]{a} + \sqrt[4]{bx}\right)\left(2a^{5/4}h + \sqrt{ab^{3/4}}e - 2\sqrt[4]{abd} - a\sqrt[4]{bg} + 3b^{5/4}c\right)}{16a^{7/4}b^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^2,x]

[Out] ((4*a^(3/4)*Sqrt[b]*(b*x*(c + x*(d + e*x)) + a*(f + x*(g + h*x))))/(a - b*x^4) - 2*b^(1/4)*(-3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] + (-3*b^(5/4)*c - 2*a^(1/4)*b*d - Sqrt[a]*b^(3/4)*e + a*b^(1/4)*g + 2*a^(5/4)*h)*Log[a^(1/4) - b^(1/4)*x] + (3*b^(5/4)*c - 2*a^(1/4)*b*d + Sqrt[a]*b^(3/4)*e - a*b^(1/4)*g + 2*a^(5/4)*h)*Log[a^(1/4) + b^(1/4)*x] - 2*a^(1/4)*(-b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2]/(16*a^(7/4)*b^(3/2))

Maple [B] time = 0.008, size = 340, normalized size = 1.9

$$\frac{1}{bx^4 - a} \left(-\frac{ex^3}{4a} - \frac{(ah + bd)x^2}{4ab} - \frac{(ag + bc)x}{4ab} - \frac{f}{4b} \right) - \frac{g}{8ab} \sqrt[4]{\frac{a}{b}} \arctan\left(x \sqrt[4]{\frac{a}{b}}\right) + \frac{3c}{8a^2} \sqrt[4]{\frac{a}{b}} \arctan\left(x \sqrt[4]{\frac{a}{b}}\right) - \frac{g}{16ab} \sqrt[4]{\frac{a}{b}} \ln\left(\dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x)$

[Out] $(-1/4/a*e*x^3-1/4*(a*h+b*d)/a/b*x^2-1/4*(a*g+b*c)/a/b*x-1/4*f/b)/(b*x^4-a)-1/8/b/a*(1/b*a)^{(1/4)}*\arctan(x/(1/b*a)^{(1/4)})*g+3/8*c/a^2*(1/b*a)^{(1/4)}*\arctan(x/(1/b*a)^{(1/4)})-1/16/b/a*(1/b*a)^{(1/4)}*\ln((x+(1/b*a)^{(1/4)})/(x-(1/b*a)^{(1/4)}))*g+3/16*c/a^2*(1/b*a)^{(1/4)}*\ln((x+(1/b*a)^{(1/4)})/(x-(1/b*a)^{(1/4)}))+1/8/b/(a*b)^{(1/2)}*\ln((-a+x^2*(a*b)^{(1/2)})/(-a-x^2*(a*b)^{(1/2)}))*h-1/8*d/a/(a*b)^{(1/2)}*\ln((-a+x^2*(a*b)^{(1/2)})/(-a-x^2*(a*b)^{(1/2)}))-1/8*e/a/b/(1/b*a)^{(1/4)}*\arctan(x/(1/b*a)^{(1/4)})+1/16*e/a/b/(1/b*a)^{(1/4)}*\ln((x+(1/b*a)^{(1/4)})/(x-(1/b*a)^{(1/4)}))$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, \text{algorithm}=\text{"maxima"})$

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, \text{algorithm}=\text{"fricas"})$

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)

[Out] Timed out

Giac [B] time = 1.07615, size = 574, normalized size = 3.12

$$\frac{bx^3e + bdx^2 + ahx^2 + bcx + agx + af}{4(bx^4 - a)ab} - \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{-abb^2d} - 2\sqrt{2}\sqrt{-ababh} - 3(-ab^3)^{\frac{1}{4}}b^2c + (-ab^3)^{\frac{1}{4}}abg - (-ab^3)^{\frac{3}{4}}e\right)}{16a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/4*(b*x^3*e + b*d*x^2 + a*h*x^2 + b*c*x + a*g*x + a*f)/((b*x^4 - a)*a*b) \\ & - 1/16*\sqrt{2}*(2*\sqrt{2}*\sqrt{-a*b}*b^2*d - 2*\sqrt{2}*\sqrt{-a*b}*a*b*h - 3 \\ & *(-a*b^3)^{(1/4)}*b^2*c + (-a*b^3)^{(1/4)}*a*b*g - (-a*b^3)^{(3/4)}*e)*\arctan(1/2 \\ & *\sqrt{2}*(2*x + \sqrt{2}*(-a/b)^{(1/4)})/(-a/b)^{(1/4)})/(a^2*b^3) - 1/16*\sqrt{2} \\ & *(2*\sqrt{2}*\sqrt{-a*b}*b^2*d - 2*\sqrt{2}*\sqrt{-a*b}*a*b*h - 3*(-a*b^3)^{(1/4)} \\ & *b^2*c + (-a*b^3)^{(1/4)}*a*b*g - (-a*b^3)^{(3/4)}*e)*\arctan(1/2*\sqrt{2}*(2*x \\ & - \sqrt{2}*(-a/b)^{(1/4)})/(-a/b)^{(1/4)})/(a^2*b^3) + 1/32*\sqrt{2}*(3*(-a*b^3) \\ & ^{(1/4)}*b^2*c - (-a*b^3)^{(1/4)}*a*b*g - (-a*b^3)^{(3/4)}*e)*\log(x^2 + \sqrt{2}*x \\ & *(-a/b)^{(1/4)} + \sqrt{-a/b})/(a^2*b^3) - 1/32*\sqrt{2}*(3*(-a*b^3)^{(1/4)}*b^2* \\ & c - (-a*b^3)^{(1/4)}*a*b*g - (-a*b^3)^{(3/4)}*e)*\log(x^2 - \sqrt{2}*x*(-a/b)^{(1/4)} \\ & + \sqrt{-a/b})/(a^2*b^3) \end{aligned}$$

$$3.193 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^2} dx$$

Optimal. Leaf size=203

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b(3bc-ag)}}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b(3bc-ag)}}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}} + \frac{(bd-ah)\tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}} + \frac{x(ax+bx^2)}{a-bx^4}$$

[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + b*f*x^3))/(4*a*b*(a - b*x^4)) - ((b*e - (Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*e + (Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2))

Rubi [A] time = 0.27428, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.146$, Rules used = {1858, 1876, 275, 208, 1167, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b(3bc-ag)}}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b(3bc-ag)}}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}} + \frac{(bd-ah)\tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}} + \frac{x(ax+bx^2)}{a-bx^4}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^2,x]

[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + b*f*x^3))/(4*a*b*(a - b*x^4)) - ((b*e - (Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*e + (Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2))

Rule 1858

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x] /; GeQ[q, n]]

/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]

Rule 1876

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1167

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[-(a*c)]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + 193x^6}{(a - bx^4)^2} dx &= \frac{x(bc + ag + (bd + ah)x + (193a + be)x^2 + bfx^3)}{4ab(a - bx^4)} - \int \frac{-b(3bc-ag) - 2b(bd-ah)}{a-bx^4} dx \\
&= \frac{x(bc + ag + (bd + ah)x + (193a + be)x^2 + bfx^3)}{4ab(a - bx^4)} - \int \left(-\frac{2b(bd-ah)x}{a-bx^4} + \frac{-b(3bc-ag)}{a-bx^4} \right) dx \\
&= \frac{x(bc + ag + (bd + ah)x + (193a + be)x^2 + bfx^3)}{4ab(a - bx^4)} - \frac{\int \frac{-b(3bc-ag) + b(579a - be)}{a-bx^4} dx}{4ab^2} \\
&= \frac{x(bc + ag + (bd + ah)x + (193a + be)x^2 + bfx^3)}{4ab(a - bx^4)} - \frac{(579a - be - \frac{\sqrt{b}(3b^2c)}{\sqrt{a}})}{8ab^2} \\
&= \frac{x(bc + ag + (bd + ah)x + (193a + be)x^2 + bfx^3)}{4ab(a - bx^4)} + \frac{(579a - be + \frac{\sqrt{b}(3b^2c)}{\sqrt{a}})}{8a^{5/2}b^2}
\end{aligned}$$

Mathematica [A] time = 0.221807, size = 302, normalized size = 1.49

$$\frac{4a^{3/4}b^{3/4}(af+x(g+x(h+ix))+bx(c+x(d+ex)))}{a-bx^4} + \log\left(\sqrt[4]{a} - \sqrt[4]{bx}\right)\left(2a^{5/4}\sqrt[4]{bh} + 3a^{3/2}i - 2\sqrt[4]{ab}b^{5/4}d - \sqrt{abe} + a\sqrt{bg} - 3b^{3/2}c\right) + \log\left(\sqrt[4]{a} + \sqrt[4]{bx}\right)\left(2a^{5/4}\sqrt[4]{bh} + 3a^{3/2}i - 2\sqrt[4]{ab}b^{5/4}d - \sqrt{abe} + a\sqrt{bg} - 3b^{3/2}c\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^2,x]

[Out] (((4*a^(3/4)*b^(3/4)*(b*x*(c + x*(d + e*x)) + a*(f + x*(g + x*(h + i*x)))))/(a - b*x^4) + 2*(3*b^(3/2)*c - Sqrt[a]*b*e - a*Sqrt[b]*g + 3*a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)] + (-3*b^(3/2)*c - 2*a^(1/4)*b^(5/4)*d - Sqrt[a]*b*e + a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h + 3*a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x] + (3*b^(3/2)*c - 2*a^(1/4)*b^(5/4)*d + Sqrt[a]*b*e - a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h - 3*a^(3/2)*i)*Log[a^(1/4) + b^(1/4)*x] - 2*a^(1/4)*b^(1/4)*(-b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(16*a^(7/4)*b^(7/4))

Maple [B] time = 0.009, size = 409, normalized size = 2.

$$\frac{1}{bx^4 - a} \left(-\frac{(ai + be)x^3}{4ab} - \frac{(ah + bd)x^2}{4ab} - \frac{(ag + bc)x}{4ab} - \frac{f}{4b} \right) - \frac{g}{8ab} \sqrt[4]{\frac{a}{b}} \arctan\left(x \sqrt[4]{\frac{1}{ab}}\right) + \frac{3c}{8a^2} \sqrt[4]{\frac{a}{b}} \arctan\left(x \sqrt[4]{\frac{1}{ab}}\right) - \frac{g}{16ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x)$

[Out] $(-1/4*(a*i+b*e)/a/b*x^3-1/4*(a*h+b*d)/a/b*x^2-1/4*(a*g+b*c)/a/b*x-1/4*f/b)/(b*x^4-a)-1/8/b/a*(1/b*a)^{1/4}*\arctan(x/(1/b*a)^{1/4})*g+3/8*c/a^2*(1/b*a)^{1/4}*\arctan(x/(1/b*a)^{1/4})-1/16/b/a*(1/b*a)^{1/4}*\ln((x+(1/b*a)^{1/4})/(x-(1/b*a)^{1/4}))*g+3/16*c/a^2*(1/b*a)^{1/4}*\ln((x+(1/b*a)^{1/4})/(x-(1/b*a)^{1/4}))+1/8/b/(a*b)^{1/2}*\ln((-a+x^2*(a*b)^{1/2})/(-a-x^2*(a*b)^{1/2}))*h-1/8*d/a/(a*b)^{1/2}*\ln((-a+x^2*(a*b)^{1/2})/(-a-x^2*(a*b)^{1/2}))+3/8/b^2/(1/b*a)^{1/4}*\arctan(x/(1/b*a)^{1/4})*i-1/8*e/a/b/(1/b*a)^{1/4}*\arctan(x/(1/b*a)^{1/4})-3/16/b^2/(1/b*a)^{1/4}*\ln((x+(1/b*a)^{1/4})/(x-(1/b*a)^{1/4}))*i+1/16*e/a/b/(1/b*a)^{1/4}*\ln((x+(1/b*a)^{1/4})/(x-(1/b*a)^{1/4}))$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, \text{algorithm}="fricas")$

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)

[Out] Timed out

Giac [B] time = 1.09331, size = 848, normalized size = 4.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")

[Out]
$$\begin{aligned} & -3/32*i*(2*\sqrt{2})*(-a*b^3)^{3/4}*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2})*(-a/b)^{1/4})/(-a/b)^{1/4})/(a*b^4) - \sqrt{2})*(-a*b^3)^{3/4}*\log(x^2 + \sqrt{2})*x*(\\ & -a/b)^{1/4} + \sqrt{-a/b})/(a*b^4)) - 3/32*i*(2*\sqrt{2})*(-a*b^3)^{3/4}*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2})*(-a/b)^{1/4})/(-a/b)^{1/4})/(a*b^4) + \sqrt{2})* \\ & (-a*b^3)^{3/4}*\log(x^2 - \sqrt{2})*x*(-a/b)^{1/4} + \sqrt{-a/b})/(a*b^4)) - 1/ \\ & 4*(a*i*x^3 + b*x^3*e + b*d*x^2 + a*h*x^2 + b*c*x + a*g*x + a*f)/((b*x^4 - a) \\ &)*a*b) - 1/16*\sqrt{2}*(2*\sqrt{2})*\sqrt{-a*b}*b^2*d - 2*\sqrt{2})*\sqrt{-a*b}*a \\ & b*h - 3*(-a*b^3)^{1/4}*b^2*c + (-a*b^3)^{1/4}*a*b*g - (-a*b^3)^{3/4}*e)*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2})*(-a/b)^{1/4})/(-a/b)^{1/4})/(a^2*b^3) - 1/16 \\ & *\sqrt{2}*(2*\sqrt{2})*\sqrt{-a*b}*b^2*d - 2*\sqrt{2})*\sqrt{-a*b}*a*b*h - 3*(-a*b \\ & ^3)^{1/4}*b^2*c + (-a*b^3)^{1/4}*a*b*g - (-a*b^3)^{3/4}*e)*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2})*(-a/b)^{1/4})/(-a/b)^{1/4})/(a^2*b^3) + 1/32*\sqrt{2}*(3*(\\ & -a*b^3)^{1/4}*b^2*c - (-a*b^3)^{1/4}*a*b*g - (-a*b^3)^{3/4}*e)*\log(x^2 + \sqrt{2})*x*(-a/b)^{1/4} + \sqrt{-a/b})/(a^2*b^3) - 1/32*\sqrt{2}*(3*(-a*b^3)^{1/4} \\ &)*b^2*c - (-a*b^3)^{1/4}*a*b*g - (-a*b^3)^{3/4}*e)*\log(x^2 - \sqrt{2})*x*(-a/b)^{1/4} + \sqrt{-a/b})/(a^2*b^3) \end{aligned}$$

$$3.194 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^2} dx$$

Optimal. Leaf size=225

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b(3bc-ag)}}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b(3bc-ag)}}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}} + \frac{(bd-ah)\tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}} + \frac{j\log(a-bx^4)}{4b^2}$$

[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + (b*f + a*j)*x^3))/(4*a*b*(a - b*x^4)) - ((b*e - (Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*e + (Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2)) + (j*Log[a - b*x^4])/(4*b^2)

Rubi [A] time = 0.310455, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1858, 1876, 1167, 205, 208, 1248, 635, 260}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b(3bc-ag)}}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b(3bc-ag)}}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}} + \frac{(bd-ah)\tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}} + \frac{j\log(a-bx^4)}{4b^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^2, x]

[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + (b*f + a*j)*x^3))/(4*a*b*(a - b*x^4)) - ((b*e - (Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*e + (Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2)) + (j*Log[a - b*x^4])/(4*b^2)

Rule 1858

Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D


```

ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a +
b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]

```

Rule 1876

```

Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))]/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n

```

Rule 1167

```

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]

```

Rule 205

```

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

```

Rule 208

```

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

Rule 1248

```

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol]
:= Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]

```

Rule 635

```

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]

```

Rule 260

```

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + 194x^6 + jx^7}{(a - bx^4)^2} dx &= \frac{x(bc + ag + (bd + ah)x + (194a + be)x^2 + (bf + aj)x^3)}{4ab(a - bx^4)} - \int \frac{-b(3bc)}{(a - bx^4)^2} dx \\
&= \frac{x(bc + ag + (bd + ah)x + (194a + be)x^2 + (bf + aj)x^3)}{4ab(a - bx^4)} - \int \frac{-b(3bc)}{(a - bx^4)^2} dx \\
&= \frac{x(bc + ag + (bd + ah)x + (194a + be)x^2 + (bf + aj)x^3)}{4ab(a - bx^4)} - \int \frac{-b(3bc)}{(a - bx^4)^2} dx \\
&= \frac{x(bc + ag + (bd + ah)x + (194a + be)x^2 + (bf + aj)x^3)}{4ab(a - bx^4)} - \text{Subst} \left(\frac{-b(3bc)}{(a - bx^4)^2}, x, \sqrt[4]{a - bx^4} \right) \\
&= \frac{x(bc + ag + (bd + ah)x + (194a + be)x^2 + (bf + aj)x^3)}{4ab(a - bx^4)} + \frac{(582a - 194b)}{16b^2} \\
&= \frac{x(bc + ag + (bd + ah)x + (194a + be)x^2 + (bf + aj)x^3)}{4ab(a - bx^4)} + \frac{(582a - 194b)}{16b^2}
\end{aligned}$$

Mathematica [A] time = 0.239555, size = 338, normalized size = 1.5

$$\frac{4(a^{2j+ab}(f+x(g+x(h+ix))) + b^2x(c+x(d+ex)))}{a(a-bx^4)} + \frac{\sqrt[4]{b} \log\left(\frac{\sqrt[4]{a}-\sqrt[4]{bx}}{\sqrt[4]{a}+\sqrt[4]{bx}}\right) \left(2a^{5/4} \sqrt[4]{bh} + 3a^{3/2}i - 2\sqrt[4]{ab^{5/4}}d - \sqrt{abe} + a\sqrt{bg} - 3b^{3/2}c\right)}{a^{7/4}} + \frac{\sqrt[4]{b} \log\left(\frac{\sqrt[4]{a}+\sqrt[4]{bx}}{\sqrt[4]{a}-\sqrt[4]{bx}}\right) \left(2a^{5/4} \sqrt[4]{bh} - 3a^{3/2}i - 2\sqrt[4]{ab^{5/4}}d - \sqrt{abe} + a\sqrt{bg} - 3b^{3/2}c\right)}{a^{7/4}}$$

16b²

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^2, x]

[Out] ((4*(a^2*j + b^2*x*(c + x*(d + e*x))) + a*b*(f + x*(g + x*(h + i*x))))/(a*(a - b*x^4)) + (2*b^(1/4)*(3*b^(3/2)*c - Sqrt[a]*b*e - a*Sqrt[b]*g + 3*a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)]/a^(7/4) + (b^(1/4)*(-3*b^(3/2)*c - 2*a^(1/4)*b^(5/4)*d - Sqrt[a]*b*e + a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h + 3*a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x]/a^(7/4) + (b^(1/4)*(3*b^(3/2)*c - 2*a^(1/4)*

$$b^{5/4}d + \sqrt{a}be - a\sqrt{b}g + 2a^{5/4}b^{1/4}h - 3a^{3/2}i) \cdot \frac{\log[a^{1/4} + b^{1/4}x]}{a^{7/4}} + \frac{(2\sqrt{b}(bd - ah) \cdot \log[\sqrt{a} + \sqrt{b}x^2])}{a^{3/2}} + 4j \cdot \frac{\log[a - bx^4]}{(16b^2)}$$

Maple [B] time = 0.011, size = 431, normalized size = 1.9

$$\frac{1}{bx^4 - a} \left(-\frac{(ai + be)x^3}{4ab} - \frac{(ah + bd)x^2}{4ab} - \frac{(ag + bc)x}{4ab} - \frac{aj + bf}{4b^2} \right) - \frac{g}{8ab} \sqrt[4]{\frac{a}{b}} \arctan \left(x \frac{1}{\sqrt[4]{\frac{a}{b}}} \right) + \frac{3c}{8a^2} \sqrt[4]{\frac{a}{b}} \arctan \left(x \frac{1}{\sqrt[4]{\frac{a}{b}}} \right) -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x)

[Out] $(-1/4*(a*i+b*e)/a/b*x^3-1/4*(a*h+b*d)/a/b*x^2-1/4*(a*g+b*c)/a/b*x-1/4*(a*j+b*f)/b^2)/(b*x^4-a)-1/8/b/a*(1/b*a)^{1/4}*\arctan(x/(1/b*a)^{1/4})*g+3/8*c/a^{1/4}*(1/b*a)^{1/4}*\arctan(x/(1/b*a)^{1/4})-1/16/b/a*(1/b*a)^{1/4}*\ln((x+(1/b*a)^{1/4})/(x-(1/b*a)^{1/4}))*g+3/16*c/a^{1/4}*(1/b*a)^{1/4}*\ln((x+(1/b*a)^{1/4})/(x-(1/b*a)^{1/4}))+1/8/b/(a*b)^{1/2}*\ln((-a+x^2*(a*b)^{1/2})/(-a-x^2*(a*b)^{1/2}))*h-1/8*d/a/(a*b)^{1/2}*\ln((-a+x^2*(a*b)^{1/2})/(-a-x^2*(a*b)^{1/2}))+3/8/b^2/(1/b*a)^{1/4}*\arctan(x/(1/b*a)^{1/4})*i-1/8*e/a/b/(1/b*a)^{1/4}*\arctan(x/(1/b*a)^{1/4})-3/16/b^2/(1/b*a)^{1/4}*\ln((x+(1/b*a)^{1/4})/(x-(1/b*a)^{1/4}))*i+1/16*e/a/b/(1/b*a)^{1/4}*\ln((x+(1/b*a)^{1/4})/(x-(1/b*a)^{1/4}))+1/4/b^2*j*\ln(b*x^4-a)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")
```

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

[Out] Timed out

Giac [B] time = 1.11957, size = 884, normalized size = 3.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -3/32*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^4) - sqrt(2)*(-a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^4) - 3/32*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^4) + sqrt(2)*(-a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^4) + 1/4*j*log(abs(b*x^4 - a))/b^2 - 1/4*((a*i + b*e)*x^3 + (b*d + a*h)*x^2 + (b*c + a*g)*x + (a*b*f + a^2*j)/b)/((b*x^4 - a)*a*b) - 1/16*sqrt(2)*(2*sqrt(2)*sqrt(-a*b)*b^2*d - 2*sqrt(2)*sqrt(-a*b)*a*b*h - 3*(-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(1/4)*a*b*g - (-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^3) - 1/16*sqrt(2)*(2*sqrt(2)*sqrt(-a*b)*b^2*d - 2*sqrt(2)*sqrt(-a*b)*a*b*h - 3*(-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(1/4)*a*b*g - (-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^3)
```

$$\begin{aligned}
& -a/b)^{(1/4)}/(a^2*b^3) + 1/32*\sqrt{2}*(3*(-a*b^3)^{(1/4)}*b^2*c - (-a*b^3)^{(1/4)}*a*b*g - (-a*b^3)^{(3/4)}*e)*\log(x^2 + \sqrt{2}*x*(-a/b)^{(1/4)} + \sqrt{-a/b})/(a^2*b^3) \\
& - 1/32*\sqrt{2}*(3*(-a*b^3)^{(1/4)}*b^2*c - (-a*b^3)^{(1/4)}*a*b*g - (-a*b^3)^{(3/4)}*e)*\log(x^2 - \sqrt{2}*x*(-a/b)^{(1/4)} + \sqrt{-a/b})/(a^2*b^3)
\end{aligned}$$

$$3.195 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^2} dx$$

Optimal. Leaf size=353

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-\sqrt{a}\sqrt{be} + ag + 3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-\sqrt{a}\sqrt{be} + ag + 3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}} - \frac{\tan^{-1}}{16\sqrt{2}a^{7/4}b^{5/4}}$$

[Out] (x*(b*c - a*g + (b*d - a*h)*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a + b*x^4)) + (b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/(4*a^(3/2)*b^(3/2)) - ((3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(8*Sqrt[2]*a^(7/4)*b^(5/4)) + ((3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(8*Sqrt[2]*a^(7/4)*b^(5/4)) - ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/4)) + ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/4))

Rubi [A] time = 0.340409, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1858, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-\sqrt{a}\sqrt{be} + ag + 3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-\sqrt{a}\sqrt{be} + ag + 3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}} - \frac{\tan^{-1}}{16\sqrt{2}a^{7/4}b^{5/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^2,x]

[Out] (x*(b*c - a*g + (b*d - a*h)*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a + b*x^4)) + (b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/(4*a^(3/2)*b^(3/2)) - ((3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(8*Sqrt[2]*a^(7/4)*b^(5/4)) + ((3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(8*Sqrt[2]*a^(7/4)*b^(5/4)) - ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/4)) + ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/4))

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a +
b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))]/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{(a + bx^4)^2} dx &= \frac{x(bc - ag + (bd - ah)x + bex^2 + bfx^3)}{4ab(a + bx^4)} - \frac{\int \frac{-b(3bc+ag)-2b(bd+ah)x-b^2ex^2}{a+bx^4} dx}{4ab^2} \\
&= \frac{x(bc - ag + (bd - ah)x + bex^2 + bfx^3)}{4ab(a + bx^4)} - \frac{\int \left(\frac{-2b(bd+ah)x}{a+bx^4} + \frac{-b(3bc+ag)-b^2ex^2}{a+bx^4} \right) dx}{4ab^2} \\
&= \frac{x(bc - ag + (bd - ah)x + bex^2 + bfx^3)}{4ab(a + bx^4)} - \frac{\int \frac{-b(3bc+ag)-b^2ex^2}{a+bx^4} dx}{4ab^2} + \frac{(bd + ah) \int \frac{1}{a}}{2ab} \\
&= \frac{x(bc - ag + (bd - ah)x + bex^2 + bfx^3)}{4ab(a + bx^4)} + \frac{(3bc - \sqrt{a}\sqrt{be} + ag) \int \frac{\sqrt{a}\sqrt{b-bx^2}}{a+bx^4} dx}{8a^{3/2}b^{3/2}} + \frac{(bd + ah) \int \frac{1}{a}}{2ab} \\
&= \frac{x(bc - ag + (bd - ah)x + bex^2 + bfx^3)}{4ab(a + bx^4)} + \frac{(bd + ah) \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{4a^{3/2}b^{3/2}} - \frac{(3bc - \sqrt{a}\sqrt{be} + ag)}{8a^{3/2}b^{3/2}} \\
&= \frac{x(bc - ag + (bd - ah)x + bex^2 + bfx^3)}{4ab(a + bx^4)} + \frac{(bd + ah) \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{4a^{3/2}b^{3/2}} - \frac{(3bc - \sqrt{a}\sqrt{be} + ag)}{8a^{3/2}b^{3/2}} \\
&= \frac{x(bc - ag + (bd - ah)x + bex^2 + bfx^3)}{4ab(a + bx^4)} + \frac{(bd + ah) \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{4a^{3/2}b^{3/2}} - \frac{(3bc + \sqrt{a}\sqrt{be} - ag)}{8a^{3/2}b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.241713, size = 359, normalized size = 1.02

$$-2 \tan^{-1} \left(1 - \frac{\sqrt{2} \sqrt[4]{bx}}{\sqrt[4]{a}} \right) (4a^{5/4}h + \sqrt{2}\sqrt{ab}^{3/4}e + 4\sqrt[4]{abd} + \sqrt{2}a\sqrt[4]{b}g + 3\sqrt{2}b^{5/4}c) + 2 \tan^{-1} \left(\frac{\sqrt{2} \sqrt[4]{bx}}{\sqrt[4]{a}} + 1 \right) (-4a^{5/4}h + \sqrt{2}\sqrt{ab}^{3/4}e + 4\sqrt[4]{abd} + \sqrt{2}a\sqrt[4]{b}g + 3\sqrt{2}b^{5/4}c)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^2,x]

[Out] ((-8*a^(3/4)*Sqrt[b]*(-(b*x*(c + x*(d + e*x))) + a*(f + x*(g + h*x))))/(a + b*x^4) - 2*(3*Sqrt[2]*b^(5/4)*c + 4*a^(1/4)*b*d + Sqrt[2]*Sqrt[a]*b^(3/4)*e + Sqrt[2]*a*b^(1/4)*g + 4*a^(5/4)*h)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(3*Sqrt[2]*b^(5/4)*c - 4*a^(1/4)*b*d + Sqrt[2]*Sqrt[a]*b^(3/4)*e + Sqrt[2]*a*b^(1/4)*g - 4*a^(5/4)*h)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*b^(1/4)*(-3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] - Sqrt[2]*

$$a^{1/4}b^{1/4}x + \text{Sqrt}[b]*x^2] + \text{Sqrt}[2]*b^{1/4}*(3*b*c - \text{Sqrt}[a]*\text{Sqrt}[b]*e + a*g)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{1/4}*b^{1/4}x + \text{Sqrt}[b]*x^2)]/(32*a^{7/4}*b^{3/2})$$

Maple [A] time = 0.009, size = 515, normalized size = 1.5

$$\frac{1}{bx^4 + a} \left(\frac{ex^3}{4a} - \frac{(ah - bd)x^2}{4ab} - \frac{(ag - bc)x}{4ab} - \frac{f}{4b} \right) + \frac{\sqrt{2}g}{16ab} \sqrt[4]{\frac{a}{b}} \arctan \left(x\sqrt{2} \frac{1}{\sqrt[4]{\frac{a}{b}}} + 1 \right) + \frac{3c\sqrt{2}}{16a^2} \sqrt[4]{\frac{a}{b}} \arctan \left(x\sqrt{2} \frac{1}{\sqrt[4]{\frac{a}{b}}} + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x)

[Out] (1/4/a*e*x^3-1/4*(a*h-b*d)/a/b*x^2-1/4*(a*g-b*c)/a/b*x-1/4*f/b)/(b*x^4+a)+1/16/b/a*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*g+3/16*c/a^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/16/b/a*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*g+3/16*c/a^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+1/32/b/a*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*g+3/32*c/a^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/4/b/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))*h+1/4*d/a/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+1/32*e/a/b/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/16*e/a/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/16*e/a/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)

[Out] Timed out

Giac [A] time = 1.10694, size = 537, normalized size = 1.52

$$\frac{bx^3e + bdx^2 - ahx^2 + bcx - agx - af}{4(bx^4 + a)ab} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{abb^2d} + 2\sqrt{2}\sqrt{ababh} + 3(ab^3)^{\frac{1}{4}}b^2c + (ab^3)^{\frac{1}{4}}abg + (ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{2x + \sqrt{2}(a/b)^{1/4}}{(a/b)^{1/4}}\right)}{16a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")

[Out] 1/4*(b*x^3*e + b*d*x^2 - a*h*x^2 + b*c*x - a*g*x - a*f)/((b*x^4 + a)*a*b) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 2*sqrt(2)*sqrt(a*b)*a*b*h + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 2*sqrt(2)*sqrt(a*b)*a*b*h + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3)

$$\begin{aligned}
& t(2)*\sqrt{a*b}*b^2*d + 2*\sqrt{2}*\sqrt{a*b}*a*b*h + 3*(a*b^3)^{(1/4)}*b^2*c + \\
& (a*b^3)^{(1/4)}*a*b*g + (a*b^3)^{(3/4)}*e)*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a \\
& /b)^{(1/4)})/(a/b)^{(1/4)})/(a^2*b^3) + 1/32*\sqrt{2}*(3*(a*b^3)^{(1/4)}*b^2*c + (\\
& a*b^3)^{(1/4)}*a*b*g - (a*b^3)^{(3/4)}*e)*\log(x^2 + \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{ \\
& t(a/b)})/(a^2*b^3) - 1/32*\sqrt{2}*(3*(a*b^3)^{(1/4)}*b^2*c + (a*b^3)^{(1/4)}*a*b \\
& *g - (a*b^3)^{(3/4)}*e)*\log(x^2 - \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(a^2*b^3 \\
&)
\end{aligned}$$

$$3.196 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^2} dx$$

Optimal. Leaf size=395

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(\sqrt{b}(ag+3bc) - \sqrt{a}(3ai+be)\right)}{16\sqrt{2}a^{7/4}b^{7/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(\sqrt{b}(ag+3bc) - \sqrt{a}(3ai+be)\right)}{16\sqrt{2}a^{7/4}b^{7/4}}$$

[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + b*f*x^3))/(4*a*b*(a + b*x^4)) + ((b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2)) - ((Sqrt[b]*(3*b*c + a*g) + Sqrt[a]*(b*e + 3*a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(7/4)) + ((Sqrt[b]*(3*b*c + a*g) + Sqrt[a]*(b*e + 3*a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(7/4)) - ((Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(7/4)) + ((Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(7/4))

Rubi [A] time = 0.492387, antiderivative size = 395, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1858, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(\sqrt{b}(ag+3bc) - \sqrt{a}(3ai+be)\right)}{16\sqrt{2}a^{7/4}b^{7/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(\sqrt{b}(ag+3bc) - \sqrt{a}(3ai+be)\right)}{16\sqrt{2}a^{7/4}b^{7/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^2,x]

[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + b*f*x^3))/(4*a*b*(a + b*x^4)) + ((b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2)) - ((Sqrt[b]*(3*b*c + a*g) + Sqrt[a]*(b*e + 3*a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(7/4)) + ((Sqrt[b]*(3*b*c + a*g) + Sqrt[a]*(b*e + 3*a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(7/4)) - ((Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(7/4)) + ((Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(7/4))

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a +
b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))]/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + 196x^6}{(a + bx^4)^2} dx &= \frac{x(bc - ag + (bd - ah)x - (196a - be)x^2 + bfx^3)}{4ab(a + bx^4)} - \frac{\int \frac{-b(3bc+ag)-2b(bd+ah)x}{a+bx^4}}{4ab^2} \\
&= \frac{x(bc - ag + (bd - ah)x - (196a - be)x^2 + bfx^3)}{4ab(a + bx^4)} - \frac{\int \left(-\frac{2b(bd+ah)x}{a+bx^4} + \frac{-b(3bc+ag)}{a+bx^4} \right)}{4ab^2} \\
&= \frac{x(bc - ag + (bd - ah)x - (196a - be)x^2 + bfx^3)}{4ab(a + bx^4)} - \frac{\int \frac{-b(3bc+ag)-b(588a+be)}{a+bx^4}}{4ab^2} \\
&= \frac{x(bc - ag + (bd - ah)x - (196a - be)x^2 + bfx^3)}{4ab(a + bx^4)} - \frac{\left(588a + be - \frac{\sqrt{b}(3bc+ag)}{\sqrt{a}} \right)}{8ab} \\
&= \frac{x(bc - ag + (bd - ah)x - (196a - be)x^2 + bfx^3)}{4ab(a + bx^4)} + \frac{(bd + ah) \tan^{-1} \left(\frac{\sqrt{bx}}{\sqrt{a}} \right)}{4a^{3/2}b^{3/2}} \\
&= \frac{x(bc - ag + (bd - ah)x - (196a - be)x^2 + bfx^3)}{4ab(a + bx^4)} + \frac{(bd + ah) \tan^{-1} \left(\frac{\sqrt{bx}}{\sqrt{a}} \right)}{4a^{3/2}b^{3/2}} \\
&= \frac{x(bc - ag + (bd - ah)x - (196a - be)x^2 + bfx^3)}{4ab(a + bx^4)} + \frac{(bd + ah) \tan^{-1} \left(\frac{\sqrt{bx}}{\sqrt{a}} \right)}{4a^{3/2}b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.344446, size = 415, normalized size = 1.05

$$-\frac{8a^{3/4}b^{3/4}(a(f+x(g+x(h+ix)))-bx(c+x(d+ex)))}{a+bx^4} - 2 \tan^{-1} \left(1 - \frac{\sqrt{2} \sqrt[4]{bx}}{\sqrt{a}} \right) (4a^{5/4} \sqrt[4]{bh} + 3\sqrt{2}a^{3/2}i + 4\sqrt[4]{ab}b^{5/4}d + \sqrt{2}\sqrt{abe} + \sqrt{2}a\sqrt{bg} + 3\sqrt{2}a\sqrt{bh})$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^2,x
]

[Out] ((-8*a^(3/4)*b^(3/4)*(-(b*x*(c + x*(d + e*x))) + a*(f + x*(g + x*(h + i*x)))))/(a + b*x^4) - 2*(3*Sqrt[2]*b^(3/2)*c + 4*a^(1/4)*b^(5/4)*d + Sqrt[2]*Sqrt[a]*b*e + Sqrt[2]*a*Sqrt[b]*g + 4*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(3*Sqrt[2]*b^(3/2)*c - 4*a^(1/4)*b^(5/4)*d + Sqrt[2]*Sqrt[a]*b*e + Sqrt[2]*a*Sqrt[b]*g - 4*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i)

$$4) * h + 3 * \sqrt{2} * a^{(3/2)} * i * \text{ArcTan}[1 + (\sqrt{2} * b^{(1/4)} * x) / a^{(1/4)}] + \sqrt{2} * (-3 * b^{(3/2)} * c + \sqrt{a} * b * e - a * \sqrt{b} * g + 3 * a^{(3/2)} * i) * \text{Log}[\sqrt{a} - \sqrt{2} * a^{(1/4)} * b^{(1/4)} * x + \sqrt{b} * x^2] + \sqrt{2} * (3 * b^{(3/2)} * c - \sqrt{a} * b * e + a * \sqrt{b} * g - 3 * a^{(3/2)} * i) * \text{Log}[\sqrt{a} + \sqrt{2} * a^{(1/4)} * b^{(1/4)} * x + \sqrt{b} * x^2] / (32 * a^{(7/4)} * b^{(7/4)})$$

Maple [B] time = 0.01, size = 654, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x)`

[Out] $(-1/4 * (a * i - b * e) / a / b * x^3 - 1/4 * (a * h - b * d) / a / b * x^2 - 1/4 * (a * g - b * c) / a / b * x - 1/4 * f / b) / (b * x^4 + a) + 1/16 / b / a * (1 / b * a)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (1 / b * a)^{(1/4)} * x + 1) * g + 3/16 * c / a^2 * (1 / b * a)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (1 / b * a)^{(1/4)} * x + 1) + 1/16 / b / a * (1 / b * a)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (1 / b * a)^{(1/4)} * x - 1) * g + 3/16 * c / a^2 * (1 / b * a)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (1 / b * a)^{(1/4)} * x - 1) + 1/32 / b / a * (1 / b * a)^{(1/4)} * 2^{(1/2)} * \ln((x^2 + (1 / b * a)^{(1/4)} * x * 2^{(1/2)} + (1 / b * a)^{(1/2)}) / (x^2 - (1 / b * a)^{(1/4)} * x * 2^{(1/2)} + (1 / b * a)^{(1/2)})) * g + 3/32 * c / a^2 * (1 / b * a)^{(1/4)} * 2^{(1/2)} * \ln((x^2 + (1 / b * a)^{(1/4)} * x * 2^{(1/2)} + (1 / b * a)^{(1/2)}) / (x^2 - (1 / b * a)^{(1/4)} * x * 2^{(1/2)} + (1 / b * a)^{(1/2)})) + 1/4 / b / (a * b)^{(1/2)} * \arctan(x^2 * (b / a)^{(1/2)}) * h + 1/4 * d / a / (a * b)^{(1/2)} * \arctan(x^2 * (b / a)^{(1/2)}) + 3/32 / b^2 / (1 / b * a)^{(1/4)} * 2^{(1/2)} * \ln((x^2 - (1 / b * a)^{(1/4)} * x * 2^{(1/2)} + (1 / b * a)^{(1/2)}) / (x^2 + (1 / b * a)^{(1/4)} * x * 2^{(1/2)} + (1 / b * a)^{(1/2)})) * i + 1/32 * e / a / b / (1 / b * a)^{(1/4)} * 2^{(1/2)} * \ln((x^2 - (1 / b * a)^{(1/4)} * x * 2^{(1/2)} + (1 / b * a)^{(1/2)}) / (x^2 + (1 / b * a)^{(1/4)} * x * 2^{(1/2)} + (1 / b * a)^{(1/2)})) + 3/16 / b^2 / (1 / b * a)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (1 / b * a)^{(1/4)} * x + 1) * i + 1/16 * e / a / b / (1 / b * a)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (1 / b * a)^{(1/4)} * x + 1) + 3/16 / b^2 / (1 / b * a)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (1 / b * a)^{(1/4)} * x - 1) * i + 1/16 * e / a / b / (1 / b * a)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (1 / b * a)^{(1/4)} * x - 1)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)

[Out] Timed out

Giac [A] time = 1.09558, size = 795, normalized size = 2.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")

[Out]
$$\frac{3}{32}i(2\sqrt{2})(a^3b)^{3/4}\arctan\left(\frac{1}{2}\sqrt{2}(2x + \sqrt{2})(a/b)^{1/4}\right)/(a/b)^{1/4} - \sqrt{2}(a^3b)^{3/4}\log(x^2 + \sqrt{2}x(a/b)^{1/4} + \sqrt{a/b})/(a^4b^4) + \frac{3}{32}i(2\sqrt{2})(a^3b)^{3/4}\arctan\left(\frac{1}{2}\sqrt{2}(2x - \sqrt{2})(a/b)^{1/4}\right)/(a/b)^{1/4} + \sqrt{2}(a^3b)^{3/4}\log(x^2 - \sqrt{2}x(a/b)^{1/4} + \sqrt{a/b})/(a^4b^4) - \frac{1}{4}(a^3ix^3 -$$

$$\begin{aligned}
& b*x^3*e - b*d*x^2 + a*h*x^2 - b*c*x + a*g*x + a*f)/((b*x^4 + a)*a*b) + 1/16 \\
& *sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 2*sqrt(2)*sqrt(a*b)*a*b*h + 3*(a*b^3) \\
& ^{(1/4)}*b^2*c + (a*b^3)^{(1/4)}*a*b*g + (a*b^3)^{(3/4)}*e)*arctan(1/2*sqrt(2)*(2 \\
& *x + sqrt(2)*(a/b)^{(1/4)})/(a/b)^{(1/4)})/(a^2*b^3) + 1/16*sqrt(2)*(2*sqrt(2)* \\
& sqrt(a*b)*b^2*d + 2*sqrt(2)*sqrt(a*b)*a*b*h + 3*(a*b^3)^{(1/4)}*b^2*c + (a*b^ \\
& 3)^{(1/4)}*a*b*g + (a*b^3)^{(3/4)}*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^{(\\
& 1/4)})/(a/b)^{(1/4)})/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^{(1/4)}*b^2*c + (a*b^3 \\
&)^{(1/4)}*a*b*g - (a*b^3)^{(3/4)}*e)*log(x^2 + sqrt(2)*x*(a/b)^{(1/4)} + sqrt(a/b \\
&))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^{(1/4)}*b^2*c + (a*b^3)^{(1/4)}*a*b*g - \\
& (a*b^3)^{(3/4)}*e)*log(x^2 - sqrt(2)*x*(a/b)^{(1/4)} + sqrt(a/b))/(a^2*b^3)
\end{aligned}$$

$$3.197 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^2} dx$$

Optimal. Leaf size=417

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(\sqrt{b}(ag+3bc) - \sqrt{a}(3ai+be)\right)}{16\sqrt{2}a^{7/4}b^{7/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(\sqrt{b}(ag+3bc) - \sqrt{a}(3ai+be)\right)}{16\sqrt{2}a^{7/4}b^{7/4}}$$

[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + (b*f - a*j)*x^3))/(4*a*b*(a + b*x^4)) + ((b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2)) - ((Sqrt[b]*(3*b*c + a*g) + Sqrt[a]*(b*e + 3*a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(7/4)) + ((Sqrt[b]*(3*b*c + a*g) + Sqrt[a]*(b*e + 3*a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(7/4)) - ((Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(7/4)) + ((Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(7/4)) + (j*Log[a + b*x^4])/(4*b^2)

Rubi [A] time = 0.53602, antiderivative size = 417, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1858, 1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 205, 260}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(\sqrt{b}(ag+3bc) - \sqrt{a}(3ai+be)\right)}{16\sqrt{2}a^{7/4}b^{7/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(\sqrt{b}(ag+3bc) - \sqrt{a}(3ai+be)\right)}{16\sqrt{2}a^{7/4}b^{7/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^2, x]

[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + (b*f - a*j)*x^3))/(4*a*b*(a + b*x^4)) + ((b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2)) - ((Sqrt[b]*(3*b*c + a*g) + Sqrt[a]*(b*e + 3*a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(7/4)) + ((Sqrt[b]*(3*b*c + a*g) + Sqrt[a]*(b*e + 3*a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(7/4)) - ((Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(7/4)) + ((Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(7/4)) + (j*Log[a + b*x^4])/(4*b^2)

$2] * a^{(1/4)} * b^{(1/4)} * x + \text{Sqrt}[b * x^2]) / (16 * \text{Sqrt}[2] * a^{(7/4)} * b^{(7/4)}) + (j * \text{Log}[a + b * x^4]) / (4 * b^2)$

Rule 1858

$\text{Int}[(Pq_)*((a_)+(b_)*(x_)^{(n_}))^{(p_)}, x_Symbol] := \text{With}[\{q = \text{Expon}[Pq, x]\}, \text{Module}[\{Q = \text{PolynomialQuotient}[b^{(\text{Floor}[(q - 1)/n] + 1)*Pq}, a + b * x^n, x], R = \text{PolynomialRemainder}[b^{(\text{Floor}[(q - 1)/n] + 1)*Pq}, a + b * x^n, x]\}, \text{Dist}[1/(a * n * (p + 1) * b^{(\text{Floor}[(q - 1)/n] + 1)}), \text{Int}[(a + b * x^n)^{(p + 1)} * \text{ExpandToSum}[a * n * (p + 1) * Q + n * (p + 1) * R + D[x * R, x], x], x] - \text{Simp}[(x * R * (a + b * x^n)^{(p + 1)}) / (a * n * (p + 1) * b^{(\text{Floor}[(q - 1)/n] + 1)}), x]] /; \text{GeQ}[q, n]] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[p, -1]$

Rule 1876

$\text{Int}[(Pq_)/((a_)+(b_)*(x_)^{(n_)}), x_Symbol] := \text{With}[\{v = \text{Sum}[(x^{ii} * (\text{Coeff}[Pq, x, ii] + \text{Coeff}[Pq, x, n/2 + ii] * x^{(n/2)})) / (a + b * x^n), \{ii, 0, n/2 - 1\}]\}, \text{Int}[v, x] /; \text{SumQ}[v]] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[n/2, 0] \&\& \text{Expon}[Pq, x] < n$

Rule 1168

$\text{Int}[((d_)+(e_)*(x_)^2)/((a_)+(c_)*(x_)^4), x_Symbol] := \text{With}[\{q = \text{Rt}[a * c, 2]\}, \text{Dist}[(d * q + a * e) / (2 * a * c), \text{Int}[(q + c * x^2) / (a + c * x^4), x], x] + \text{Dist}[(d * q - a * e) / (2 * a * c), \text{Int}[(q - c * x^2) / (a + c * x^4), x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{NeQ}[c * d^2 + a * e^2, 0] \&\& \text{NeQ}[c * d^2 - a * e^2, 0] \&\& \text{NegQ}[-(a * c)]$

Rule 1162

$\text{Int}[((d_)+(e_)*(x_)^2)/((a_)+(c_)*(x_)^4), x_Symbol] := \text{With}[\{q = \text{Rt}[(2 * d) / e, 2]\}, \text{Dist}[e / (2 * c), \text{Int}[1 / \text{Simp}[d / e + q * x + x^2, x], x], x] + \text{Dist}[e / (2 * c), \text{Int}[1 / \text{Simp}[d / e - q * x + x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{EqQ}[c * d^2 - a * e^2, 0] \&\& \text{PosQ}[d * e]$

Rule 617

$\text{Int}[(a_)+(b_)*(x_)+(c_)*(x_)^2)^{(-1)}, x_Symbol] := \text{With}[\{q = 1 - 4 * \text{Simplify}[(a * c) / b^2]\}, \text{Dist}[-2 / b, \text{Subst}[\text{Int}[1 / (q - x^2), x], x, 1 + (2 * c * x) / b], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] || !\text{RationalQ}[b^2 - 4 * a * c])] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4 * a * c, 0]$

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1248

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]

Rule 635

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + 197x^6 + jx^7}{(a + bx^4)^2} dx &= \frac{x(bc - ag + (bd - ah)x - (197a - be)x^2 + (bf - aj)x^3)}{4ab(a + bx^4)} - \int \frac{-b(3b}{ \\
&= \frac{x(bc - ag + (bd - ah)x - (197a - be)x^2 + (bf - aj)x^3)}{4ab(a + bx^4)} - \int \left(\frac{-b(3b}{ \\
&= \frac{x(bc - ag + (bd - ah)x - (197a - be)x^2 + (bf - aj)x^3)}{4ab(a + bx^4)} - \int \frac{-b(3b}{ \\
&= \frac{x(bc - ag + (bd - ah)x - (197a - be)x^2 + (bf - aj)x^3)}{4ab(a + bx^4)} - \text{Subst} \\
&= \frac{x(bc - ag + (bd - ah)x - (197a - be)x^2 + (bf - aj)x^3)}{4ab(a + bx^4)} + \frac{(591a -}{ \\
&= \frac{x(bc - ag + (bd - ah)x - (197a - be)x^2 + (bf - aj)x^3)}{4ab(a + bx^4)} + \frac{(bd + a}{ \\
&= \frac{x(bc - ag + (bd - ah)x - (197a - be)x^2 + (bf - aj)x^3)}{4ab(a + bx^4)} + \frac{(bd + a}{
\end{aligned}$$

Mathematica [A] time = 0.328068, size = 460, normalized size = 1.1

$$\frac{8(a^2j - ab(f + x(g + x(h + ix))) + b^2x(c + x(d + ex)))}{a(a + bx^4)} - \frac{2\sqrt[4]{b} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \left(4a^{5/4}\sqrt[4]{bh} + 3\sqrt{2}a^{3/2}i + 4\sqrt[4]{ab}^{5/4}d + \sqrt{2}\sqrt{abe} + \sqrt{2}a\sqrt{bg} + 3\sqrt{2}b^{3/2}c\right)}{a^{7/4}} + \frac{2\sqrt[4]{b} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{a^{7/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^2, x]

[Out] ((8*(a^2*j + b^2*x*(c + x*(d + e*x)) - a*b*(f + x*(g + x*(h + i*x))))/(a*(a + b*x^4)) - (2*b^(1/4)*(3*Sqrt[2]*b^(3/2)*c + 4*a^(1/4)*b^(5/4)*d + Sqrt[2]*Sqrt[a]*b*e + Sqrt[2]*a*Sqrt[b]*g + 4*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (2*b^(1/4)*(3*Sqr

$$t[2]*b^{(3/2)*c} - 4*a^{(1/4)*b^{(5/4)*d} + \text{Sqrt}[2]*\text{Sqrt}[a]*b*e + \text{Sqrt}[2]*a*\text{Sqrt}[b]*g - 4*a^{(5/4)*b^{(1/4)*h} + 3*\text{Sqrt}[2]*a^{(3/2)*i}*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)*x})/a^{(1/4)})]/a^{(7/4)} + (\text{Sqrt}[2]*b^{(1/4)*(-3*b^{(3/2)*c} + \text{Sqrt}[a]*b*e - a*\text{Sqrt}[b]*g + 3*a^{(3/2)*i})*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)*b^{(1/4)*x} + \text{Sqrt}[b]*x^2)]/a^{(7/4)} + (\text{Sqrt}[2]*b^{(1/4)*(3*b^{(3/2)*c} - \text{Sqrt}[a]*b*e + a*\text{Sqrt}[b]*g - 3*a^{(3/2)*i})*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)*b^{(1/4)*x} + \text{Sqrt}[b]*x^2)]/a^{(7/4)} + 8*j*\text{Log}[a + b*x^4)]/(32*b^2)$$

Maple [B] time = 0.011, size = 675, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2, x)$

[Out] $(-1/4*(a*i-b*e)/a/b*x^3-1/4*(a*h-b*d)/a/b*x^2-1/4*(a*g-b*c)/a/b*x+1/4*(a*j-b*f)/b^2)/(b*x^4+a)+1/16/b/a*(1/b*a)^{(1/4)*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)*x-1}*g+3/16*c/a^2*(1/b*a)^{(1/4)*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)*x-1})+1/32/b/a*(1/b*a)^{(1/4)*2^{(1/2)}*\ln((x^2+(1/b*a)^{(1/4)*x*2^{(1/2)}+(1/b*a)^{(1/2)})/(x^2-(1/b*a)^{(1/4)*x*2^{(1/2)}+(1/b*a)^{(1/2)})})*g+3/32*c/a^2*(1/b*a)^{(1/4)*2^{(1/2)}*\ln((x^2+(1/b*a)^{(1/4)*x*2^{(1/2)}+(1/b*a)^{(1/2)})/(x^2-(1/b*a)^{(1/4)*x*2^{(1/2)}+(1/b*a)^{(1/2)})}))+1/16/b/a*(1/b*a)^{(1/4)*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)*x+1}*g+3/16*c/a^2*(1/b*a)^{(1/4)*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)*x+1})+1/4/b/(a*b)^{(1/2)}*\arctan(x^2*(b/a)^{(1/2)})*h+1/4*d/a/(a*b)^{(1/2)}*\arctan(x^2*(b/a)^{(1/2)}))+3/32/b^2/(1/b*a)^{(1/4)*2^{(1/2)}*\ln((x^2-(1/b*a)^{(1/4)*x*2^{(1/2)}+(1/b*a)^{(1/2)})/(x^2+(1/b*a)^{(1/4)*x*2^{(1/2)}+(1/b*a)^{(1/2)})})*i+1/32*e/a/b/(1/b*a)^{(1/4)*2^{(1/2)}*\ln((x^2-(1/b*a)^{(1/4)*x*2^{(1/2)}+(1/b*a)^{(1/2)})/(x^2+(1/b*a)^{(1/4)*x*2^{(1/2)}+(1/b*a)^{(1/2)})}))+3/16/b^2/(1/b*a)^{(1/4)*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)*x-1}*i+1/16*e/a/b/(1/b*a)^{(1/4)*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)*x-1})+3/16/b^2/(1/b*a)^{(1/4)*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)*x+1}*i+1/16*e/a/b/(1/b*a)^{(1/4)*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)*x+1})+1/4*j*\ln(b*x^4+a)/b^2$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11231, size = 833, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 3/32*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4)))/(a/b)^(1/4))/(a*b^4) - sqrt(2)*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4) + 3/32*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4)))/(a/b)^(1/4))/(a*b^4) + sqrt(2)*(a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4) + 1/4*j*log(abs(b*x^4 + a))/b^2 - 1/4*((a*i - b*e)*x^3 - (b*d - a*h)*x^2 - (b*c - a*g)*x + (a*b*f - a^2*j)/b)/((b*x^4 + a)*a*b) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 2*sqrt(2)*sqrt(a*b)*a*b*h + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4)))/(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 2*sqrt(2)*sqrt(a*b)*a*b*h + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4)))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3)
```

$$3.198 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^3} dx$$

Optimal. Leaf size=241

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(-5\sqrt{a}\sqrt{be}-3ag+21bc)}{64a^{11/4}b^{5/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(5\sqrt{a}\sqrt{be}-3ag+21bc)}{64a^{11/4}b^{5/4}} + \frac{(3bd-ah)\tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}} + \frac{x(2x(3$$

[Out] (x*(b*c + a*g + (b*d + a*h)*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a - b*x^4)^2) + (4*a*f + x*(7*b*c - a*g + 2*(3*b*d - a*h)*x + 5*b*e*x^2))/(32*a^2*b*(a - b*x^4)) + ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/ (64*a^(11/4)*b^(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/ (64*a^(11/4)*b^(5/4)) + ((3*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2))

Rubi [A] time = 0.338917, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {1858, 1854, 1876, 275, 208, 1167, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(-5\sqrt{a}\sqrt{be}-3ag+21bc)}{64a^{11/4}b^{5/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(5\sqrt{a}\sqrt{be}-3ag+21bc)}{64a^{11/4}b^{5/4}} + \frac{(3bd-ah)\tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}} + \frac{x(2x(3$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^3,x]

[Out] (x*(b*c + a*g + (b*d + a*h)*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a - b*x^4)^2) + (4*a*f + x*(7*b*c - a*g + 2*(3*b*d - a*h)*x + 5*b*e*x^2))/(32*a^2*b*(a - b*x^4)) + ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/ (64*a^(11/4)*b^(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/ (64*a^(11/4)*b^(5/4)) + ((3*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2))

Rule 1858

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D

```
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1})*(a + b*x^n)^(p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{(a - bx^4)^3} dx &= \frac{x(bc + ag + (bd + ah)x + bex^2 + bfx^3)}{8ab(a - bx^4)^2} - \frac{\int \frac{-b(7bc - ag) - 2b(3bd - ah)x - 5b^2ex^2 - 4b^2fx^3}{(a - bx^4)^2} dx}{8ab^2} \\
&= \frac{x(bc + ag + (bd + ah)x + bex^2 + bfx^3)}{8ab(a - bx^4)^2} + \frac{4af + x(7bc - ag + 2(3bd - ah)x + 3b^2ex^2 + 2b^2fx^3)}{32a^2b(a - bx^4)} \\
&= \frac{x(bc + ag + (bd + ah)x + bex^2 + bfx^3)}{8ab(a - bx^4)^2} + \frac{4af + x(7bc - ag + 2(3bd - ah)x + 3b^2ex^2 + 2b^2fx^3)}{32a^2b(a - bx^4)} \\
&= \frac{x(bc + ag + (bd + ah)x + bex^2 + bfx^3)}{8ab(a - bx^4)^2} + \frac{4af + x(7bc - ag + 2(3bd - ah)x + 3b^2ex^2 + 2b^2fx^3)}{32a^2b(a - bx^4)} \\
&= \frac{x(bc + ag + (bd + ah)x + bex^2 + bfx^3)}{8ab(a - bx^4)^2} + \frac{4af + x(7bc - ag + 2(3bd - ah)x + 3b^2ex^2 + 2b^2fx^3)}{32a^2b(a - bx^4)} \\
&= \frac{x(bc + ag + (bd + ah)x + bex^2 + bfx^3)}{8ab(a - bx^4)^2} + \frac{4af + x(7bc - ag + 2(3bd - ah)x + 3b^2ex^2 + 2b^2fx^3)}{32a^2b(a - bx^4)}
\end{aligned}$$

Mathematica [A] time = 0.28654, size = 309, normalized size = 1.28

$$\log(\sqrt[4]{a} - \sqrt[4]{bx}) (4a^{5/4}h - 5\sqrt{ab^{3/4}}e - 12\sqrt[4]{abd} + 3a\sqrt[4]{bg} - 21b^{5/4}c) + \log(\sqrt[4]{a} + \sqrt[4]{bx}) (4a^{5/4}h + 5\sqrt{ab^{3/4}}e - 12\sqrt[4]{abd} - 3a$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^3,x]

[Out] ((4*a^(3/4)*Sqrt[b]*x*(7*b*c + b*x*(6*d + 5*e*x)) - a*(g + 2*h*x))/(a - b*x^4) + (16*a^(7/4)*Sqrt[b]*(b*x*(c + x*(d + e*x)) + a*(f + x*(g + h*x)))/(a - b*x^4)^2 + 2*b^(1/4)*(21*b*c - 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] + (-21*b^(5/4)*c - 12*a^(1/4)*b*d - 5*Sqrt[a]*b^(3/4)*e + 3*a*b^(1/4)*g + 4*a^(5/4)*h)*Log[a^(1/4) - b^(1/4)*x] + (21*b^(5/4)*c - 12*a^(1/4)*b*d + 5*Sqrt[a]*b^(3/4)*e - 3*a*b^(1/4)*g + 4*a^(5/4)*h)*Log[a^(1/4) + b^(1/4)*x] - 4*a^(1/4)*(-3*b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2]/(128*a^(11/4)*b^(3/2))

Maple [A] time = 0.011, size = 389, normalized size = 1.6

$$-\frac{1}{(bx^4 - a)^2} \left(\frac{5bex^7}{32a^2} - \frac{(ah - 3bd)x^6}{16a^2} - \frac{(ag - 7bc)x^5}{32a^2} - \frac{9ex^3}{32a} - \frac{(ah + 5bd)x^2}{16ab} - \frac{(3ag + 11bc)x}{32ab} - \frac{f}{8b} \right) - \frac{3g}{64ba^2} \sqrt[4]{\frac{a}{b}} \arctan\left(\frac{x}{\sqrt[4]{\frac{a}{b}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x)

[Out] $-(5/32/a^2*b*e*x^7-1/16*(a*h-3*b*d)/a^2*x^6-1/32*(a*g-7*b*c)/a^2*x^5-9/32/a$
 $*e*x^3-1/16*(a*h+5*b*d)/a/b*x^2-1/32*(3*a*g+11*b*c)/a/b*x-1/8*f/b)/(b*x^4-a$
 $)^2-3/64/b/a^2*(1/b*a)^(1/4)*\arctan(x/(1/b*a)^(1/4))*g+21/64*c/a^3*(1/b*a)^($
 $1/4)*\arctan(x/(1/b*a)^(1/4))-3/128/b/a^2*(1/b*a)^(1/4)*\ln((x+(1/b*a)^(1/4)$
 $)/(x-(1/b*a)^(1/4)))*g+21/128*c/a^3*(1/b*a)^(1/4)*\ln((x+(1/b*a)^(1/4))/(x-$
 $(1/b*a)^(1/4)))+1/32/b/a/(a*b)^(1/2)*\ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^($
 $1/2)))*h-3/32*d/a^2/(a*b)^(1/2)*\ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)$
 $))-5/64*e/a^2/b/(1/b*a)^(1/4)*\arctan(x/(1/b*a)^(1/4))+5/128*e/a^2/b/(1/b*a)$
 $^(1/4)*\ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)

[Out] Timed out

Giac [B] time = 1.12877, size = 656, normalized size = 2.72

$$\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{-abb^2d} - 4\sqrt{2}\sqrt{-ababh} + 21(-ab^3)^{\frac{1}{4}}b^2c - 3(-ab^3)^{\frac{1}{4}}abg + 5(-ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right) + \sqrt{2}\left(\dots\right)}{128a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")

[Out] 1/128*sqrt(2)*(12*sqrt(2)*sqrt(-a*b)*b^2*d - 4*sqrt(2)*sqrt(-a*b)*a*b*h + 21*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g + 5*(-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(-a*b)*b^2*d - 4*sqrt(2)*sqrt(-a*b)*a*b*h + 21*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g + 5*(-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g - 5*(-a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^3) - 1/256*sqrt(2)*(21*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g - 5*(-a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^3) - 1/32*(5*b^2*x^7*e + 6*b^2*d*x^6 - 2*a*b*h*x^6 + 7*b^2*c*x^5 - a*b*g*x^5 - 9*a*b*x^3*e - 10*a*b*d*x^2 - 2*a^2*h*x^2 - 11*a*b*c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 - a)^2*a^2*b)

$$3.199 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^3} dx$$

Optimal. Leaf size=268

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\left(-\frac{3\sqrt{b(7bc-ag)}}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\left(\frac{3\sqrt{b(7bc-ag)}}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}} + \frac{(3bd-ah)\tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}} + \frac{x(2x(3$$

[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + b*f*x^3))/(8*a*b*(a - b*x^4)^2) + (4*a*f + x*(7*b*c - a*g + 2*(3*b*d - a*h)*x + (5*b*e - 3*a*i)*x^2))/(32*a^2*b*(a - b*x^4)) - ((5*b*e - (3*sqrt[b]*(7*b*c - a*g))/sqrt[a] - 3*a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(9/4)*b^(7/4)) + ((5*b*e + (3*sqrt[b]*(7*b*c - a*g))/sqrt[a] - 3*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(64*a^(9/4)*b^(7/4)) + ((3*b*d - a*h)*ArcTanh[(sqrt[b]*x^2)/sqrt[a]])/(16*a^(5/2)*b^(3/2))

Rubi [A] time = 0.434678, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {1858, 1854, 1876, 275, 208, 1167, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\left(-\frac{3\sqrt{b(7bc-ag)}}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\left(\frac{3\sqrt{b(7bc-ag)}}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}} + \frac{(3bd-ah)\tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}} + \frac{x(2x(3$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^3,x]

[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + b*f*x^3))/(8*a*b*(a - b*x^4)^2) + (4*a*f + x*(7*b*c - a*g + 2*(3*b*d - a*h)*x + (5*b*e - 3*a*i)*x^2))/(32*a^2*b*(a - b*x^4)) - ((5*b*e - (3*sqrt[b]*(7*b*c - a*g))/sqrt[a] - 3*a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(9/4)*b^(7/4)) + ((5*b*e + (3*sqrt[b]*(7*b*c - a*g))/sqrt[a] - 3*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(64*a^(9/4)*b^(7/4)) + ((3*b*d - a*h)*ArcTanh[(sqrt[b]*x^2)/sqrt[a]])/(16*a^(5/2)*b^(3/2))

Rule 1858

Int[(Pq)*(a_) + (b_.)*(x_)^(n_.)]^(p_), x_Symbol] :> With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,


```

x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x], Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x] /; GeQ[q, n] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]

```

Rule 1854

```

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*a + b*x^n)^(p + 1), x], x] /; q == n - 1 /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]

```

Rule 1876

```

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

```

Rule 275

```

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1 /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

```

Rule 208

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

Rule 1167

```

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[-(a*c)]

```

Rule 205

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + 199x^6}{(a - bx^4)^3} dx &= \frac{x(bc + ag + (bd + ah)x + (199a + be)x^2 + bfx^3)}{8ab(a - bx^4)^2} - \int \frac{-b(7bc - ag) - 2b(3bd - ah)}{(a - bx^4)^3} dx \\
&= \frac{x(bc + ag + (bd + ah)x + (199a + be)x^2 + bfx^3)}{8ab(a - bx^4)^2} + \frac{4af + x(7bc - ag + 3bd - ah)}{8ab(a - bx^4)^2} \\
&= \frac{x(bc + ag + (bd + ah)x + (199a + be)x^2 + bfx^3)}{8ab(a - bx^4)^2} + \frac{4af + x(7bc - ag + 3bd - ah)}{8ab(a - bx^4)^2} \\
&= \frac{x(bc + ag + (bd + ah)x + (199a + be)x^2 + bfx^3)}{8ab(a - bx^4)^2} + \frac{4af + x(7bc - ag + 3bd - ah)}{8ab(a - bx^4)^2} \\
&= \frac{x(bc + ag + (bd + ah)x + (199a + be)x^2 + bfx^3)}{8ab(a - bx^4)^2} + \frac{4af + x(7bc - ag + 3bd - ah)}{8ab(a - bx^4)^2} \\
&= \frac{x(bc + ag + (bd + ah)x + (199a + be)x^2 + bfx^3)}{8ab(a - bx^4)^2} + \frac{4af + x(7bc - ag + 3bd - ah)}{8ab(a - bx^4)^2}
\end{aligned}$$

Mathematica [A] time = 0.352227, size = 359, normalized size = 1.34

$$\frac{16a^{7/4}b^{3/4}(a(f+x(g+x(h+ix)))+bx(c+x(d+ex)))}{(a-bx^4)^2} - \frac{4a^{3/4}b^{3/4}x(a(g+x(2h+3ix))-b(7c+x(6d+5ex)))}{a-bx^4} + \log\left(\sqrt[4]{a} - \sqrt[4]{bx}\right)\left(4a^{5/4}\sqrt[4]{bh} + 3a^{3/2}i - 12\sqrt[4]{ab^5}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^3, x]

[Out] ((-4*a^(3/4)*b^(3/4)*x*(-(b*(7*c + x*(6*d + 5*e*x))) + a*(g + x*(2*h + 3*i*x))))/(a - b*x^4) + (16*a^(7/4)*b^(3/4)*(b*x*(c + x*(d + e*x)) + a*(f + x*(g + x*(h + i*x))))/(a - b*x^4)^2 + 2*(21*b^(3/2)*c - 5*Sqrt[a]*b*e - 3*a*Sqrt[b]*g + 3*a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)] + (-21*b^(3/2)*c - 12*a^(1/4)*b^(5/4)*d - 5*Sqrt[a]*b*e + 3*a*Sqrt[b]*g + 4*a^(5/4)*b^(1/4)*h + 3*a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x] + (21*b^(3/2)*c - 12*a^(1/4)*b^(5/4)*d + 5*Sqrt[a]*b*e - 3*a*Sqrt[b]*g + 4*a^(5/4)*b^(1/4)*h - 3*a^(3/2)*i)*Log[a^

$(1/4) + b^{(1/4)} * x] - 4 * a^{(1/4)} * b^{(1/4)} * (-3 * b * d + a * h) * \text{Log}[\text{Sqrt}[a] + \text{Sqrt}[b] * x^2] / (128 * a^{(11/4)} * b^{(7/4)})$

Maple [B] time = 0.011, size = 472, normalized size = 1.8

$$\frac{1}{(bx^4 - a)^2} \left(\frac{(3ai - 5be)x^7}{32a^2} - \frac{(ah - 3bd)x^6}{16a^2} - \frac{(ag - 7bc)x^5}{32a^2} - \frac{(ai + 9be)x^3}{32ab} - \frac{(ah + 5bd)x^2}{16ab} - \frac{(3ag + 11bc)x}{32ab} - \frac{f}{8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x)`

[Out] $-(1/32 * (3 * a * i - 5 * b * e) / a^2 * x^7 - 1/16 * (a * h - 3 * b * d) / a^2 * x^6 - 1/32 * (a * g - 7 * b * c) / a^2 * x^5 - 1/32 * (a * i + 9 * b * e) / a / b * x^3 - 1/16 * (a * h + 5 * b * d) / a / b * x^2 - 1/32 * (3 * a * g + 11 * b * c) / a / b * x - 1/8 * f / b) / (b * x^4 - a)^2 - 3/64 * b / a^2 * (1/b * a)^{(1/4)} * \arctan(x / (1/b * a)^{(1/4)}) * g + 21/64 * c / a^3 * (1/b * a)^{(1/4)} * \arctan(x / (1/b * a)^{(1/4)}) - 3/128 * b / a^2 * (1/b * a)^{(1/4)} * \ln((x + (1/b * a)^{(1/4)}) / (x - (1/b * a)^{(1/4)})) * g + 21/128 * c / a^3 * (1/b * a)^{(1/4)} * \ln((x + (1/b * a)^{(1/4)}) / (x - (1/b * a)^{(1/4)})) + 1/32 * b / a / (a * b)^{(1/2)} * \ln((-a + x^2 * (a * b)^{(1/2)}) / (-a - x^2 * (a * b)^{(1/2)})) * h - 3/32 * d / a^2 / (a * b)^{(1/2)} * \ln((-a + x^2 * (a * b)^{(1/2)}) / (-a - x^2 * (a * b)^{(1/2)})) + 3/64 * b^2 / a / (1/b * a)^{(1/4)} * \arctan(x / (1/b * a)^{(1/4)}) * i - 5/64 * e / a^2 / b / (1/b * a)^{(1/4)} * \arctan(x / (1/b * a)^{(1/4)}) - 3/128 * b^2 / a / (1/b * a)^{(1/4)} * \ln((x + (1/b * a)^{(1/4)}) / (x - (1/b * a)^{(1/4)})) * i + 5/128 * e / a^2 / b / (1/b * a)^{(1/4)} * \ln((x + (1/b * a)^{(1/4)}) / (x - (1/b * a)^{(1/4)}))$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.12048, size = 942, normalized size = 3.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="
giac")
```

```
[Out] -3/256*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)
^(1/4))/(-a/b)^(1/4))/(a^2*b^4) - sqrt(2)*(-a*b^3)^(3/4)*log(x^2 + sqrt(2)*
x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^4) - 3/256*i*(2*sqrt(2)*(-a*b^3)^(3/4)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^4) +
sqrt(2)*(-a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*
b^4) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(-a*b)*b^2*d - 4*sqrt(2)*sqrt(-a*b)*a
*b*h + 21*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g + 5*(-a*b^3)^(3/4)*
e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^3)
+ 1/128*sqrt(2)*(12*sqrt(2)*sqrt(-a*b)*b^2*d - 4*sqrt(2)*sqrt(-a*b)*a*b*h +
21*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g + 5*(-a*b^3)^(3/4)*e)*arc
tan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^3) + 1/25
6*sqrt(2)*(21*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g - 5*(-a*b^3)^(3
/4)*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^3) - 1/256*sqrt
```

$$\begin{aligned}
& t(2) * (21 * (-a * b^3)^{(1/4)} * b^2 * c - 3 * (-a * b^3)^{(1/4)} * a * b * g - 5 * (-a * b^3)^{(3/4)} * e) \\
& * \log(x^2 - \sqrt{2} * x * (-a/b)^{(1/4)} + \sqrt{-a/b}) / (a^3 * b^3) + 1/32 * (3 * a * b * i * \\
& x^7 - 5 * b^2 * x^7 * e - 6 * b^2 * d * x^6 + 2 * a * b * h * x^6 - 7 * b^2 * c * x^5 + a * b * g * x^5 + a \\
& ^2 * i * x^3 + 9 * a * b * x^3 * e + 10 * a * b * d * x^2 + 2 * a^2 * h * x^2 + 11 * a * b * c * x + 3 * a^2 * g * \\
& x + 4 * a^2 * f) / ((b * x^4 - a)^2 * a^2 * b)
\end{aligned}$$

$$3.200 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^3} dx$$

Optimal. Leaf size=285

$$\frac{x(b(7bc-ag) + 2bx(3bd-ah) + bx^2(5be-3ai)) + 4a(bf-aj)}{32a^2b^2(a-bx^4)} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\left(-\frac{3\sqrt{b(7bc-ag)}}{\sqrt{a}} - 3ai + 5be\right)}{64a^{9/4}b^{7/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{1}$$

[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + (b*f + a*j)*x^3))/(8*a*b*(a - b*x^4)^2) + (4*a*(b*f - a*j) + x*(b*(7*b*c - a*g) + 2*b*(3*b*d - a*h)*x + b*(5*b*e - 3*a*i)*x^2))/(32*a^2*b^2*(a - b*x^4)) - ((5*b*e - (3*Sqrt[b]*(7*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(9/4)*b^(7/4)) + ((5*b*e + (3*Sqrt[b]*(7*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(64*a^(9/4)*b^(7/4)) + ((3*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2))

Rubi [A] time = 0.391448, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1858, 1854, 1876, 275, 208, 1167, 205}

$$\frac{x(b(7bc-ag) + 2bx(3bd-ah) + bx^2(5be-3ai)) + 4a(bf-aj)}{32a^2b^2(a-bx^4)} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\left(-\frac{3\sqrt{b(7bc-ag)}}{\sqrt{a}} - 3ai + 5be\right)}{64a^{9/4}b^{7/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{1}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^3, x]

[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + (b*f + a*j)*x^3))/(8*a*b*(a - b*x^4)^2) + (4*a*(b*f - a*j) + x*(b*(7*b*c - a*g) + 2*b*(3*b*d - a*h)*x + b*(5*b*e - 3*a*i)*x^2))/(32*a^2*b^2*(a - b*x^4)) - ((5*b*e - (3*Sqrt[b]*(7*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(9/4)*b^(7/4)) + ((5*b*e + (3*Sqrt[b]*(7*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(64*a^(9/4)*b^(7/4)) + ((3*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2))

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a +
b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Module[{q = Expon[Pq,
x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1})*(a + b*x^n)^(p
+ 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + 200x^6 + jx^7}{(a - bx^4)^3} dx &= \frac{x(bc + ag + (bd + ah)x + (200a + be)x^2 + (bf + aj)x^3)}{8ab(a - bx^4)^2} - \int \frac{-b(7bc}{ \\ &= \frac{x(bc + ag + (bd + ah)x + (200a + be)x^2 + (bf + aj)x^3)}{8ab(a - bx^4)^2} + \frac{4a(bf -}{ \\ &= \frac{x(bc + ag + (bd + ah)x + (200a + be)x^2 + (bf + aj)x^3)}{8ab(a - bx^4)^2} + \frac{4a(bf -}{ \\ &= \frac{x(bc + ag + (bd + ah)x + (200a + be)x^2 + (bf + aj)x^3)}{8ab(a - bx^4)^2} + \frac{4a(bf -}{ \\ &= \frac{x(bc + ag + (bd + ah)x + (200a + be)x^2 + (bf + aj)x^3)}{8ab(a - bx^4)^2} + \frac{4a(bf -}{ \\ &= \frac{x(bc + ag + (bd + ah)x + (200a + be)x^2 + (bf + aj)x^3)}{8ab(a - bx^4)^2} + \frac{4a(bf -}{ \end{aligned}$$

Mathematica [A] time = 0.277334, size = 380, normalized size = 1.33

$$\frac{16a^{7/4}(a^2j+ab(f+x(g+x(h+ix)))+b^2x(c+x(d+ex)))}{(a-bx^4)^2} - \frac{4a^{3/4}(8a^2j+abx(g+x(2h+3ix))-b^2x(7c+x(6d+5ex)))}{a-bx^4} + \sqrt[4]{b} \log(\sqrt[4]{a} - \sqrt[4]{bx})(4a^{5/4}\sqrt[4]{bh} + 3a^{3/2})$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^3,x]

[Out] ((-4*a^(3/4)*(8*a^2*j - b^2*x*(7*c + x*(6*d + 5*e*x)) + a*b*x*(g + x*(2*h + 3*i*x))))/(a - b*x^4) + (16*a^(7/4)*(a^2*j + b^2*x*(c + x*(d + e*x)) + a*b*(f + x*(g + x*(h + i*x))))/(a - b*x^4)^2 + 2*b^(1/4)*(21*b^(3/2)*c - 5*Sq


```
rt[a]*b*e - 3*a*Sqrt[b]*g + 3*a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)] + b^(1/4)*(-21*b^(3/2)*c - 12*a^(1/4)*b^(5/4)*d - 5*Sqrt[a]*b*e + 3*a*Sqrt[b]*g + 4*a^(5/4)*b^(1/4)*h + 3*a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x] + b^(1/4)*(21*b^(3/2)*c - 12*a^(1/4)*b^(5/4)*d + 5*Sqrt[a]*b*e - 3*a*Sqrt[b]*g + 4*a^(5/4)*b^(1/4)*h - 3*a^(3/2)*i)*Log[a^(1/4) + b^(1/4)*x] - 4*a^(1/4)*Sqrt[b]*(-3*b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2)]/(128*a^(11/4)*b^2)
```

Maple [B] time = 0.012, size = 488, normalized size = 1.7

$$-\frac{1}{(bx^4 - a)^2} \left(\frac{(3ai - 5be)x^7}{32a^2} - \frac{(ah - 3bd)x^6}{16a^2} - \frac{(ag - 7bc)x^5}{32a^2} - \frac{jx^4}{4b} - \frac{(ai + 9be)x^3}{32ab} - \frac{(ah + 5bd)x^2}{16ab} - \frac{(3ag + 11bc)}{32ab} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x)
```

```
[Out] -(-1/32*(3*a*i-5*b*e)/a^2*x^7-1/16*(a*h-3*b*d)/a^2*x^6-1/32*(a*g-7*b*c)/a^2*x^5-1/4*j*x^4/b-1/32*(a*i+9*b*e)/a/b*x^3-1/16*(a*h+5*b*d)/a/b*x^2-1/32*(3*a*g+11*b*c)/a/b*x+1/8*(a*j-b*f)/b^2)/(b*x^4-a)^2-3/64/b/a^2*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))*g+21/64*c/a^3*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))-3/128/b/a^2*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))*g+21/128*c/a^3*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))+1/32/b/a/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))*h-3/32*d/a^2/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))+3/64/b^2/a/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))*i-5/64*e/a^2/b/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))-3/128/b^2/a/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))*i+5/128*e/a^2/b/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorith="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3, x)

[Out] Timed out

Giac [B] time = 1.12438, size = 986, normalized size = 3.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")

[Out]
$$-3/256*i*(2*\sqrt{2}*(-a*b^3)^{3/4}*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(-a/b)^{1/4}))/(-a/b)^{1/4})/(a^2*b^4) - \sqrt{2}*(-a*b^3)^{3/4}*\log(x^2 + \sqrt{2}*x*(-a/b)^{1/4} + \sqrt{-a/b})/(a^2*b^4) - 3/256*i*(2*\sqrt{2}*(-a*b^3)^{3/4}*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(-a/b)^{1/4}))/(-a/b)^{1/4})/(a^2*b^4) + \sqrt{2}*(-a*b^3)^{3/4}*\log(x^2 - \sqrt{2}*x*(-a/b)^{1/4} + \sqrt{-a/b})/(a^2*b^4) + 1/128*\sqrt{2}*(12*\sqrt{2}*\sqrt{-a*b}*b^2*d - 4*\sqrt{2}*\sqrt{-a*b})*a$$

$$\begin{aligned}
& *b*h + 21*(-a*b^3)^{(1/4)}*b^2*c - 3*(-a*b^3)^{(1/4)}*a*b*g + 5*(-a*b^3)^{(3/4)}* \\
& e)*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2})*(-a/b)^{(1/4)))/(-a/b)^{(1/4)))/(a^3*b^3) \\
& + 1/128*\sqrt{2}*(12*\sqrt{2}*\sqrt{-a*b}*b^2*d - 4*\sqrt{2}*\sqrt{-a*b}*a*b*h + \\
& 21*(-a*b^3)^{(1/4)}*b^2*c - 3*(-a*b^3)^{(1/4)}*a*b*g + 5*(-a*b^3)^{(3/4)}*e)*\arcc \\
& \tan(1/2*\sqrt{2}*(2*x - \sqrt{2})*(-a/b)^{(1/4)))/(-a/b)^{(1/4)))/(a^3*b^3) + 1/25 \\
& 6*\sqrt{2}*(21*(-a*b^3)^{(1/4)}*b^2*c - 3*(-a*b^3)^{(1/4)}*a*b*g - 5*(-a*b^3)^{(3 \\
& /4)*e)*\log(x^2 + \sqrt{2}*x*(-a/b)^{(1/4)} + \sqrt{-a/b}))/ (a^3*b^3) - 1/256*\sqrt{ \\
& t(2)*(21*(-a*b^3)^{(1/4)}*b^2*c - 3*(-a*b^3)^{(1/4)}*a*b*g - 5*(-a*b^3)^{(3/4)*e \\
&)*\log(x^2 - \sqrt{2}*x*(-a/b)^{(1/4)} + \sqrt{-a/b}))/ (a^3*b^3) + 1/32*(3*a*b^2* \\
& i*x^7 - 5*b^3*x^7*e - 6*b^3*d*x^6 + 2*a*b^2*h*x^6 - 7*b^3*c*x^5 + a*b^2*g*x \\
& ^5 + 8*a^2*b*j*x^4 + a^2*b*i*x^3 + 9*a*b^2*x^3*e + 10*a*b^2*d*x^2 + 2*a^2*b \\
& *h*x^2 + 11*a*b^2*c*x + 3*a^2*b*g*x + 4*a^2*b*f - 4*a^3*j)/((b*x^4 - a)^2*a \\
& ^2*b^2)
\end{aligned}$$

$$3.201 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^3} dx$$

Optimal. Leaf size=413

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-5\sqrt{a}\sqrt{be} + 3ag + 21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-5\sqrt{a}\sqrt{be} + 3ag + 21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}}$$

[Out] (x*(b*c - a*g + (b*d - a*h)*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a + b*x^4)^2) - (4*a*f - x*(7*b*c + a*g + 2*(3*b*d + a*h)*x + 5*b*e*x^2))/(32*a^2*b*(a + b*x^4)) + ((3*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2)) - ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(5/4)) - ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(5/4)) + ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(5/4))

Rubi [A] time = 0.48642, antiderivative size = 413, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {1858, 1854, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-5\sqrt{a}\sqrt{be} + 3ag + 21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-5\sqrt{a}\sqrt{be} + 3ag + 21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^3, x]

[Out] (x*(b*c - a*g + (b*d - a*h)*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a + b*x^4)^2) - (4*a*f - x*(7*b*c + a*g + 2*(3*b*d + a*h)*x + 5*b*e*x^2))/(32*a^2*b*(a + b*x^4)) + ((3*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2)) - ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(5/4)) - ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(5/4)) + ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(5/4))

$$\frac{[a] \sqrt{b} e + 3a g \log[\sqrt{a} + \sqrt{2} a^{1/4} b^{1/4} x + \sqrt{b} x^2]}{(128 \sqrt{2} a^{11/4} b^{5/4})}$$

Rule 1858

$$\text{Int}[(Pq_*)((a_) + (b_*)(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{With}[\{q = \text{Expon}[Pq, x]\}, \text{Module}[\{Q = \text{PolynomialQuotient}[b^{(\text{Floor}[(q - 1)/n] + 1)*Pq}, a + b*x^n, x], R = \text{PolynomialRemainder}[b^{(\text{Floor}[(q - 1)/n] + 1)*Pq}, a + b*x^n, x]\}, \text{Dist}[1/(a*n*(p + 1)*b^{(\text{Floor}[(q - 1)/n] + 1)}), \text{Int}[(a + b*x^n)^{(p + 1)} \text{ExpandToSum}[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - \text{Simp}[(x*R*(a + b*x^n)^{(p + 1)})/(a*n*(p + 1)*b^{(\text{Floor}[(q - 1)/n] + 1)}), x]] /; \text{GeQ}[q, n]] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[p, -1]$$

Rule 1854

$$\text{Int}[(Pq_*)((a_) + (b_*)(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Module}[\{q = \text{Expon}[Pq, x], i\}, \text{Simp}[(a*\text{Coeff}[Pq, x, q] - b*x*\text{ExpandToSum}[Pq - \text{Coeff}[Pq, x, q]*x^q, x])*(a + b*x^n)^{(p + 1)})/(a*b*n*(p + 1)), x] + \text{Dist}[1/(a*n*(p + 1)), \text{Int}[\text{Sum}[(n*(p + 1) + i + 1)*\text{Coeff}[Pq, x, i]*x^i, \{i, 0, q - 1\}](a + b*x^n)^{(p + 1)}, x], x] /; q == n - 1] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[p, -1]$$

Rule 1876

$$\text{Int}[(Pq_)/((a_) + (b_*)(x_)^{(n_)})], x_Symbol] \rightarrow \text{With}[\{v = \text{Sum}[(x^{ii}*(\text{Coeff}[Pq, x, ii] + \text{Coeff}[Pq, x, n/2 + ii]*x^{(n/2)}))]/(a + b*x^n), \{ii, 0, n/2 - 1\}]\}, \text{Int}[v, x] /; \text{SumQ}[v] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[n/2, 0] \&\& \text{Expon}[Pq, x] < n]$$

Rule 275

$$\text{Int}[(x_)^{(m_)*((a_) + (b_*)(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{With}[\{k = \text{GCD}[m + 1, n]\}, \text{Dist}[1/k, \text{Subst}[\text{Int}[x^{((m + 1)/k - 1)*(a + b*x^{(n/k)})^p}, x], x, x^k], x] /; k != 1] /; \text{FreeQ}[\{a, b, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[m]$$

Rule 205

$$\text{Int}[(a_ + (b_*)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[a/b]$$

Rule 1168

$$\text{Int}[(d_ + (e_*)(x_)^2)/((a_) + (c_*)(x_)^4), x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \text{Dist}[(d*q - a*e)/(2*a*c), \text{Int}[(q - c*x^2)/(a + c*x^4), x], x]] /; \text{FreeQ}[\{a,$$

c, d, e, x && $\text{NeQ}[c*d^2 + a*e^2, 0]$ && $\text{NeQ}[c*d^2 - a*e^2, 0]$ && $\text{NegQ}[-(a*c)]$

Rule 1162

$\text{Int}[\frac{(d_.) + (e_.)(x_)^2}{(a_) + (c_.)(x_)^4}, x_Symbol] := \text{With}[\{q = \text{Rt}[(2*d)/e, 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /; \text{FreeQ}\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[d*e]$

Rule 617

$\text{Int}[\frac{(a_) + (b_.)(x_) + (c_.)(x_)^2}{(a_) + (b_.)(x_) + (c_.)(x_)^2}^{-1}, x_Symbol] := \text{With}[\{q = 1 - 4*S\text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \|\| \text{!RationalQ}[b^2 - 4*a*c])] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 204

$\text{Int}[\frac{(a_) + (b_.)(x_)^2}{(a_) + (b_.)(x_)^2}^{-1}, x_Symbol] := -\text{Simp}[\text{ArcTan}[\frac{\text{Rt}[-b, 2]*x}{\text{Rt}[-a, 2]}] / (\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{LtQ}[a, 0] \|\| \text{LtQ}[b, 0])$

Rule 1165

$\text{Int}[\frac{(d_.) + (e_.)(x_)^2}{(a_) + (c_.)(x_)^4}, x_Symbol] := \text{With}[\{q = \text{Rt}[-(2*d)/e, 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] /; \text{FreeQ}\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[d*e]$

Rule 628

$\text{Int}[\frac{(d_.) + (e_.)(x_)}{(a_.) + (b_.)(x_) + (c_.)(x_)^2}, x_Symbol] := \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

Rubi steps

$$\begin{aligned} \int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{(a + bx^4)^3} dx &= \frac{x(bc - ag + (bd - ah)x + bex^2 + bfx^3)}{8ab(a + bx^4)^2} - \frac{\int \frac{-b(7bc+ag)-2b(3bd+ah)x-5b^2ex^2-4b^2fx^3}{(a+bx^4)^2} dx}{8ab^2} \\ &= \frac{x(bc - ag + (bd - ah)x + bex^2 + bfx^3)}{8ab(a + bx^4)^2} - \frac{4af - x(7bc + ag + 2(3bd + ah)x + bex^2 + bfx^3)}{32a^2b(a + bx^4)} \\ &= \frac{x(bc - ag + (bd - ah)x + bex^2 + bfx^3)}{8ab(a + bx^4)^2} - \frac{4af - x(7bc + ag + 2(3bd + ah)x + bex^2 + bfx^3)}{32a^2b(a + bx^4)} \\ &= \frac{x(bc - ag + (bd - ah)x + bex^2 + bfx^3)}{8ab(a + bx^4)^2} - \frac{4af - x(7bc + ag + 2(3bd + ah)x + bex^2 + bfx^3)}{32a^2b(a + bx^4)} \\ &= \frac{x(bc - ag + (bd - ah)x + bex^2 + bfx^3)}{8ab(a + bx^4)^2} - \frac{4af - x(7bc + ag + 2(3bd + ah)x + bex^2 + bfx^3)}{32a^2b(a + bx^4)} \\ &= \frac{x(bc - ag + (bd - ah)x + bex^2 + bfx^3)}{8ab(a + bx^4)^2} - \frac{4af - x(7bc + ag + 2(3bd + ah)x + bex^2 + bfx^3)}{32a^2b(a + bx^4)} \\ &= \frac{x(bc - ag + (bd - ah)x + bex^2 + bfx^3)}{8ab(a + bx^4)^2} - \frac{4af - x(7bc + ag + 2(3bd + ah)x + bex^2 + bfx^3)}{32a^2b(a + bx^4)} \\ &= \frac{x(bc - ag + (bd - ah)x + bex^2 + bfx^3)}{8ab(a + bx^4)^2} - \frac{4af - x(7bc + ag + 2(3bd + ah)x + bex^2 + bfx^3)}{32a^2b(a + bx^4)} \\ &= \frac{x(bc - ag + (bd - ah)x + bex^2 + bfx^3)}{8ab(a + bx^4)^2} - \frac{4af - x(7bc + ag + 2(3bd + ah)x + bex^2 + bfx^3)}{32a^2b(a + bx^4)} \\ &= \frac{x(bc - ag + (bd - ah)x + bex^2 + bfx^3)}{8ab(a + bx^4)^2} - \frac{4af - x(7bc + ag + 2(3bd + ah)x + bex^2 + bfx^3)}{32a^2b(a + bx^4)} \end{aligned}$$

Mathematica [A] time = 0.364447, size = 411, normalized size = 1.

$$-2 \tan^{-1} \left(1 - \frac{\sqrt{2} \sqrt[4]{bx}}{\sqrt[4]{a}} \right) (8a^{5/4}h + 5\sqrt{2}\sqrt{ab}^{3/4}e + 24\sqrt[4]{abd} + 3\sqrt{2}a\sqrt[4]{bg} + 21\sqrt{2}b^{5/4}c) + 2 \tan^{-1} \left(\frac{\sqrt{2} \sqrt[4]{bx}}{\sqrt[4]{a}} + 1 \right) (-8a^{5/4}h + 5\sqrt{2}\sqrt{ab}^{3/4}e + 24\sqrt[4]{abd} + 3\sqrt{2}a\sqrt[4]{bg} + 21\sqrt{2}b^{5/4}c)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^3,x]
```

```
[Out] ((8*a^(3/4)*Sqrt[b]*x*(7*b*c + b*x*(6*d + 5*e*x) + a*(g + 2*h*x)))/(a + b*x^4) - (32*a^(7/4)*Sqrt[b]*(-(b*x*(c + x*(d + e*x))) + a*(f + x*(g + h*x))))
```

$$\frac{\begin{aligned} &/(a + b*x^4)^2 - 2*(21*\sqrt{2}*b^{(5/4)*c} + 24*a^{(1/4)*b*d} + 5*\sqrt{2}*\sqrt{a} \\ &*b^{(3/4)*e} + 3*\sqrt{2}*a*b^{(1/4)*g} + 8*a^{(5/4)*h})*\text{ArcTan}[1 - (\sqrt{2}*b^{(1/4)*x}/a^{(1/4)})] \\ &+ 2*(21*\sqrt{2}*b^{(5/4)*c} - 24*a^{(1/4)*b*d} + 5*\sqrt{2}*\sqrt{a} \\ &*b^{(3/4)*e} + 3*\sqrt{2}*a*b^{(1/4)*g} - 8*a^{(5/4)*h})*\text{ArcTan}[1 + (\sqrt{2}*b^{(1/4)*x}/a^{(1/4)})] \\ &+ \sqrt{2}*b^{(1/4)*(-21*b*c} + 5*\sqrt{a}*\sqrt{b}*e - 3*a*g) \\ &)*\text{Log}[\sqrt{a} - \sqrt{2}*a^{(1/4)*b^{(1/4)*x} + \sqrt{b}*x^2}] + \sqrt{2}*b^{(1/4)*} \\ &(21*b*c - 5*\sqrt{a}*\sqrt{b}*e + 3*a*g)*\text{Log}[\sqrt{a} + \sqrt{2}*a^{(1/4)*b^{(1/4)*} \\ &)*x + \sqrt{b}*x^2)]/(256*a^{(11/4)*b^{(3/2)})} \end{aligned}}$$

Maple [A] time = 0.012, size = 561, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x)`

[Out]
$$\begin{aligned} &(5/32/a^2*b*e*x^7+1/16*(a*h+3*b*d)/a^2*x^6+1/32*(a*g+7*b*c)/a^2*x^5+9/32/a* \\ &e*x^3-1/16*(a*h-5*b*d)/a/b*x^2-1/32*(3*a*g-11*b*c)/a/b*x-1/8*f/b)/(b*x^4+a) \\ &^2+3/128/b/a^2*(1/b*a)^{(1/4)*2^{(1/2)*\arctan(2^{(1/2)/(1/b*a)^{(1/4)*x+1}*g+21} \\ &/128*c/a^3*(1/b*a)^{(1/4)*2^{(1/2)*\arctan(2^{(1/2)/(1/b*a)^{(1/4)*x+1}+3/128/b/ \\ &a^2*(1/b*a)^{(1/4)*2^{(1/2)*\arctan(2^{(1/2)/(1/b*a)^{(1/4)*x-1}*g+21/128*c/a^3* \\ &(1/b*a)^{(1/4)*2^{(1/2)*\arctan(2^{(1/2)/(1/b*a)^{(1/4)*x-1}+3/256/b/a^2*(1/b*a) \\ &)^{(1/4)*2^{(1/2)*\ln((x^2+(1/b*a)^{(1/4)*x*2^{(1/2)+(1/b*a)^{(1/2)})/(x^2-(1/b*a) \\ &)^{(1/4)*x*2^{(1/2)+(1/b*a)^{(1/2)})}*g+21/256*c/a^3*(1/b*a)^{(1/4)*2^{(1/2)*\ln((x^2+ \\ &(1/b*a)^{(1/4)*x*2^{(1/2)+(1/b*a)^{(1/2)})/(x^2-(1/b*a)^{(1/4)*x*2^{(1/2)+(1/b* \\ &a)^{(1/2)})})+1/16/b/a/(a*b)^{(1/2)*\arctan(x^2*(b/a)^{(1/2))*h+3/16*d/a^2/(a*b) \\ &)^{(1/2)*\arctan(x^2*(b/a)^{(1/2))+5/256*e/a^2/b/(1/b*a)^{(1/4)*2^{(1/2)*\ln((x^2-(1/b*a) \\ &)^{(1/4)*x*2^{(1/2)+(1/b*a)^{(1/2)})/(x^2+(1/b*a)^{(1/4)*x*2^{(1/2)+(1/b*a) \\ &)^{(1/2)})})+5/128*e/a^2/b/(1/b*a)^{(1/4)*2^{(1/2)*\arctan(2^{(1/2)/(1/b*a)^{(1/4)*x+1} \\ &)+5/128*e/a^2/b/(1/b*a)^{(1/4)*2^{(1/2)*\arctan(2^{(1/2)/(1/b*a)^{(1/4)*x-1} \end{aligned}}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)

[Out] Timed out

Giac [A] time = 1.10831, size = 620, normalized size = 1.5

$$\frac{\sqrt{2} \left(12 \sqrt{2} \sqrt{ab} b^2 d + 4 \sqrt{2} \sqrt{ab} abh + 21 (ab^3)^{\frac{1}{4}} b^2 c + 3 (ab^3)^{\frac{1}{4}} abg + 5 (ab^3)^{\frac{3}{4}} e \right) \arctan \left(\frac{\sqrt{2} \left(2x + \sqrt{2} \left(\frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left(\frac{a}{b} \right)^{\frac{1}{4}}} \right)}{128 a^3 b^3} + \frac{\sqrt{2} \left(12 \sqrt{2} \sqrt{ab} b^2 d + 4 \sqrt{2} \sqrt{ab} abh + 21 (ab^3)^{\frac{1}{4}} b^2 c + 3 (ab^3)^{\frac{1}{4}} abg + 5 (ab^3)^{\frac{3}{4}} e \right)}{128 a^3 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")

[Out] 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h + 21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2

$$\begin{aligned}
& \sqrt{2} \cdot (2x + \sqrt{2} \cdot (a/b)^{1/4}) / (a/b)^{1/4} / (a^3 b^3) + 1/128 \sqrt{2} \\
& \cdot (12 \sqrt{2} \sqrt{a \cdot b} \cdot b^2 d + 4 \sqrt{2} \sqrt{a \cdot b} \cdot a \cdot b \cdot h + 21 \cdot (a \cdot b^3)^{1/4} \\
& \cdot b^2 c + 3 \cdot (a \cdot b^3)^{1/4} \cdot a \cdot b \cdot g + 5 \cdot (a \cdot b^3)^{3/4} \cdot e) \cdot \arctan(1/2 \sqrt{2} \cdot (2x \\
& - \sqrt{2} \cdot (a/b)^{1/4}) / (a/b)^{1/4} / (a^3 b^3) + 1/256 \sqrt{2} \cdot (21 \cdot (a \cdot b^3)^{1/4} \\
& \cdot b^2 c + 3 \cdot (a \cdot b^3)^{1/4} \cdot a \cdot b \cdot g - 5 \cdot (a \cdot b^3)^{3/4} \cdot e) \cdot \log(x^2 + \sqrt{2} \cdot \\
& x \cdot (a/b)^{1/4} + \sqrt{a/b}) / (a^3 b^3) - 1/256 \sqrt{2} \cdot (21 \cdot (a \cdot b^3)^{1/4} \cdot b^2 \cdot \\
& c + 3 \cdot (a \cdot b^3)^{1/4} \cdot a \cdot b \cdot g - 5 \cdot (a \cdot b^3)^{3/4} \cdot e) \cdot \log(x^2 - \sqrt{2} \cdot x \cdot (a/b)^{1/4} \\
& + \sqrt{a/b}) / (a^3 b^3) + 1/32 \cdot (5 \cdot b^2 \cdot x^7 \cdot e + 6 \cdot b^2 \cdot d \cdot x^6 + 2 \cdot a \cdot b \cdot h \cdot x^6 \\
& + 7 \cdot b^2 \cdot c \cdot x^5 + a \cdot b \cdot g \cdot x^5 + 9 \cdot a \cdot b \cdot x^3 \cdot e + 10 \cdot a \cdot b \cdot d \cdot x^2 - 2 \cdot a^2 \cdot h \cdot x^2 + 11 \cdot a \\
& \cdot b \cdot c \cdot x - 3 \cdot a^2 \cdot g \cdot x - 4 \cdot a^2 \cdot f) / ((b \cdot x^4 + a)^2 \cdot a^2 \cdot b)
\end{aligned}$$

$$3.202 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^3} dx$$

Optimal. Leaf size=463

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(3\sqrt{b}(ag+7bc) - \sqrt{a}(3ai+5be)\right)}{128\sqrt{2}a^{11/4}b^{7/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(3\sqrt{b}(ag+7bc) - \sqrt{a}(3ai+5be)\right)}{128\sqrt{2}a^{11/4}b^{7/4}}$$

[Out] $(x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + b*f*x^3))/(8*a*b*(a + b*x^4)^2) - (4*a*f - x*(7*b*c + a*g + 2*(3*b*d + a*h)*x + (5*b*e + 3*a*i)*x^2))/(32*a^2*b*(a + b*x^4)) + ((3*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2)) - ((3*Sqrt[b]*(7*b*c + a*g) + Sqrt[a]*(5*b*e + 3*a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(7/4)) + ((3*Sqrt[b]*(7*b*c + a*g) + Sqrt[a]*(5*b*e + 3*a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(7/4)) - ((3*Sqrt[b]*(7*b*c + a*g) - Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(7/4)) + ((3*Sqrt[b]*(7*b*c + a*g) - Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(7/4))$

Rubi [A] time = 0.685778, antiderivative size = 463, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {1858, 1854, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(3\sqrt{b}(ag+7bc) - \sqrt{a}(3ai+5be)\right)}{128\sqrt{2}a^{11/4}b^{7/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(3\sqrt{b}(ag+7bc) - \sqrt{a}(3ai+5be)\right)}{128\sqrt{2}a^{11/4}b^{7/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^3,x]

[Out] $(x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + b*f*x^3))/(8*a*b*(a + b*x^4)^2) - (4*a*f - x*(7*b*c + a*g + 2*(3*b*d + a*h)*x + (5*b*e + 3*a*i)*x^2))/(32*a^2*b*(a + b*x^4)) + ((3*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2)) - ((3*Sqrt[b]*(7*b*c + a*g) + Sqrt[a]*(5*b*e + 3*a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(7/4)) + ((3*Sqrt[b]*(7*b*c + a*g) + Sqrt[a]*(5*b*e + 3*a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(7/4)) - ((3*Sqrt[b]*(7*b*c + a*g) - Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(7/4)) + ((3*Sqrt[b]*(7*b*c + a*g) - Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(7/4))$

$$\frac{\sqrt{a}*(5*b*e + 3*a*i)*\text{Log}[\sqrt{a} - \sqrt{2}*a^{(1/4)}*b^{(1/4)}*x + \sqrt{b}*x^2]}{(128*\sqrt{2}*a^{(11/4)}*b^{(7/4)}) + ((3*\sqrt{b}*(7*b*c + a*g) - \sqrt{a}*(5*b*e + 3*a*i))*\text{Log}[\sqrt{a} + \sqrt{2}*a^{(1/4)}*b^{(1/4)}*x + \sqrt{b}*x^2])}{128*\sqrt{2}*a^{(11/4)}*b^{(7/4)}}$$
Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a +
b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq,
x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1})*(a + b*x^n)^(p
+ 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + 202x^6}{(a + bx^4)^3} dx &= \frac{x(bc - ag + (bd - ah)x - (202a - be)x^2 + bfx^3)}{8ab(a + bx^4)^2} - \frac{\int \frac{-b(7bc+ag)-2b(3bd+ah)}{(a+bx^4)^2} dx}{8ab} \\
&= \frac{x(bc - ag + (bd - ah)x - (202a - be)x^2 + bfx^3)}{8ab(a + bx^4)^2} - \frac{4af - x(7bc + ag + 3bd + ah)}{8ab} \\
&= \frac{x(bc - ag + (bd - ah)x - (202a - be)x^2 + bfx^3)}{8ab(a + bx^4)^2} - \frac{4af - x(7bc + ag + 3bd + ah)}{8ab} \\
&= \frac{x(bc - ag + (bd - ah)x - (202a - be)x^2 + bfx^3)}{8ab(a + bx^4)^2} - \frac{4af - x(7bc + ag + 3bd + ah)}{8ab} \\
&= \frac{x(bc - ag + (bd - ah)x - (202a - be)x^2 + bfx^3)}{8ab(a + bx^4)^2} - \frac{4af - x(7bc + ag + 3bd + ah)}{8ab} \\
&= \frac{x(bc - ag + (bd - ah)x - (202a - be)x^2 + bfx^3)}{8ab(a + bx^4)^2} - \frac{4af - x(7bc + ag + 3bd + ah)}{8ab} \\
&= \frac{x(bc - ag + (bd - ah)x - (202a - be)x^2 + bfx^3)}{8ab(a + bx^4)^2} - \frac{4af - x(7bc + ag + 3bd + ah)}{8ab} \\
&= \frac{x(bc - ag + (bd - ah)x - (202a - be)x^2 + bfx^3)}{8ab(a + bx^4)^2} - \frac{4af - x(7bc + ag + 3bd + ah)}{8ab} \\
&= \frac{x(bc - ag + (bd - ah)x - (202a - be)x^2 + bfx^3)}{8ab(a + bx^4)^2} - \frac{4af - x(7bc + ag + 3bd + ah)}{8ab} \\
&= \frac{x(bc - ag + (bd - ah)x - (202a - be)x^2 + bfx^3)}{8ab(a + bx^4)^2} - \frac{4af - x(7bc + ag + 3bd + ah)}{8ab} \\
&= \frac{x(bc - ag + (bd - ah)x - (202a - be)x^2 + bfx^3)}{8ab(a + bx^4)^2} - \frac{4af - x(7bc + ag + 3bd + ah)}{8ab}
\end{aligned}$$

Mathematica [A] time = 0.530798, size = 473, normalized size = 1.02

$$\frac{-\frac{32a^{7/4}b^{3/4}(a(f+x(g+x(h+ix)))-bx(c+x(d+ex)))}{(a+bx^4)^2} + \frac{8a^{3/4}b^{3/4}x(ag+ax(2h+3ix)+7bc+bx(6d+5ex))}{a+bx^4}}{8ab} - 2 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right) (8a^{5/4}\sqrt[4]{bh} + 3\sqrt{2}a^{3/2}i + \dots)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^3,x
]

```
[Out] ((8*a^(3/4)*b^(3/4)*x*(7*b*c + a*g + b*x*(6*d + 5*e*x) + a*x*(2*h + 3*i*x))
)/(a + b*x^4) - (32*a^(7/4)*b^(3/4)*(-(b*x*(c + x*(d + e*x))) + a*(f + x*(g
+ x*(h + i*x)))))/(a + b*x^4)^2 - 2*(21*Sqrt[2]*b^(3/2)*c + 24*a^(1/4)*b^(
5/4)*d + 5*Sqrt[2]*Sqrt[a]*b*e + 3*Sqrt[2]*a*Sqrt[b]*g + 8*a^(5/4)*b^(1/4)*
h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(21*Sq
rt[2]*b^(3/2)*c - 24*a^(1/4)*b^(5/4)*d + 5*Sqrt[2]*Sqrt[a]*b*e + 3*Sqrt[2]*
a*Sqrt[b]*g - 8*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2
]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-21*b^(3/2)*c + 5*Sqrt[a]*b*e - 3*a*Sqrt[b
]*g + 3*a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] +
Sqrt[2]*(21*b^(3/2)*c - 5*Sqrt[a]*b*e + 3*a*Sqrt[b]*g - 3*a^(3/2)*i)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2)]/(256*a^(11/4)*b^(7/4))
```

Maple [A] time = 0.011, size = 716, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x)
```

```
[Out] (1/32*(3*a*i+5*b*e)/a^2*x^7+1/16*(a*h+3*b*d)/a^2*x^6+1/32*(a*g+7*b*c)/a^2*x
^5-1/32*(a*i-9*b*e)/a/b*x^3-1/16*(a*h-5*b*d)/a/b*x^2-1/32*(3*a*g-11*b*c)/a/
b*x-1/8*f/b)/(b*x^4+a)^2+3/128/b/a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
1/b*a)^(1/4)*x+1)*g+21/128*c/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*
a)^(1/4)*x+1)+3/128/b/a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4
)*x-1)*g+21/128*c/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-
1)+3/256/b/a^2*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a
)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*g+21/256*c/a^3*(1/b*a
)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a
)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/16/b/a/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2
))*h+3/16*d/a^2/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+3/256/b^2/a/(1/b*a)^(1/
4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4
)*x*2^(1/2)+(1/b*a)^(1/2)))*i+5/256*e/a^2/b/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-
(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)
^(1/2)))+3/128/b^2/a/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1
)*i+5/128*e/a^2/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+3/
128/b^2/a/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*i+5/128*e
/a^2/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12375, size = 892, normalized size = 1.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 3/256*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) - sqrt(2)*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4)) + 3/256*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + sqrt(2)*(a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4)) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h + 21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h + 21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) + 1/32*(3*a*b*i*x^7 + 5*b^2*x^7*e + 6*b^2*d*x^6 + 2*a*b*h*x^6 + 7*b^2*c*x^5 + a*b*g*x^5 - a^2*i*x^3 + 9*a*b*x^3*e + 10*a*b*d*x^2 - 2*a^2*h*x^2 + 11*a*b*c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 + a)^2*a^2*b)
```

$$3.203 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^3} dx$$

Optimal. Leaf size=480

$$\frac{4a(aj+bf) - x(b(ag+7bc) + 2bx(ah+3bd) + bx^2(3ai+5be))}{32a^2b^2(a+bx^4)} - \frac{\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})(3\sqrt{b}(ag+7bc) - \sqrt{a})}{128\sqrt{2}a^{11/4}b^{7/4}}$$

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + (b*f - a*j)*x^3))/(8*a*b*(a + b*x^4)^2) - (4*a*(b*f + a*j) - x*(b*(7*b*c + a*g) + 2*b*(3*b*d + a*h)*x + b*(5*b*e + 3*a*i)*x^2))/(32*a^2*b^2*(a + b*x^4)) + ((3*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/(16*a^(5/2)*b^(3/2)) - ((3*Sqrt[b]*(7*b*c + a*g) + Sqrt[a]*(5*b*e + 3*a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(64*Sqrt[2]*a^(11/4)*b^(7/4)) + ((3*Sqrt[b]*(7*b*c + a*g) + Sqrt[a]*(5*b*e + 3*a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(64*Sqrt[2]*a^(11/4)*b^(7/4)) - ((3*Sqrt[b]*(7*b*c + a*g) - Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(7/4)) + ((3*Sqrt[b]*(7*b*c + a*g) - Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(7/4))
```

Rubi [A] time = 0.665761, antiderivative size = 480, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.244$, Rules used = {1858, 1854, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{4a(aj+bf) - x(b(ag+7bc) + 2bx(ah+3bd) + bx^2(3ai+5be))}{32a^2b^2(a+bx^4)} - \frac{\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})(3\sqrt{b}(ag+7bc) - \sqrt{a})}{128\sqrt{2}a^{11/4}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^3, x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + (b*f - a*j)*x^3))/(8*a*b*(a + b*x^4)^2) - (4*a*(b*f + a*j) - x*(b*(7*b*c + a*g) + 2*b*(3*b*d + a*h)*x + b*(5*b*e + 3*a*i)*x^2))/(32*a^2*b^2*(a + b*x^4)) + ((3*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/(16*a^(5/2)*b^(3/2)) - ((3*Sqrt[b]*(7*b*c + a*g) + Sqrt[a]*(5*b*e + 3*a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(64*Sqrt[2]*a^(11/4)*b^(7/4)) + ((3*Sqrt[b]*(7*b*c + a*g) + Sqrt[a]*(5*b*e + 3*a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(64*Sqrt[2]*a^(11/4)*b^(7/4)) - ((3*Sqrt[b]*(7*b*c + a*g) - Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(7/4)) + ((3*Sqrt[b]*(7*b*c + a*g) - Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(7/4))
```

```

)) * ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(64*Sqrt[2]*a^(11/4)*b^(7/4)) -
((3*Sqrt[b]*(7*b*c + a*g) - Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(7/4)) + ((3*Sqr
t[b]*(7*b*c + a*g) - Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(7/4))

```

Rule 1858

```

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a +
b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]

```

Rule 1854

```

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Module[{q = Expon[Pq,
x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]]*(a + b*x^n)^(p
+ 1), x], x] /; q == n - 1 /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
0] && LtQ[p, -1]

```

Rule 1876

```

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii])*x^(n/2))]/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n

```

Rule 275

```

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1 /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

```

Rule 205

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + 203x^6 + jx^7}{(a + bx^4)^3} dx &= \frac{x(bc - ag + (bd - ah)x - (203a - be)x^2 + (bf - aj)x^3)}{8ab(a + bx^4)^2} - \int \frac{-b(7b}{ \\
&= \frac{x(bc - ag + (bd - ah)x - (203a - be)x^2 + (bf - aj)x^3)}{8ab(a + bx^4)^2} - \frac{4a(bf}{ \\
&= \frac{x(bc - ag + (bd - ah)x - (203a - be)x^2 + (bf - aj)x^3)}{8ab(a + bx^4)^2} - \frac{4a(bf}{ \\
&= \frac{x(bc - ag + (bd - ah)x - (203a - be)x^2 + (bf - aj)x^3)}{8ab(a + bx^4)^2} - \frac{4a(bf}{ \\
&= \frac{x(bc - ag + (bd - ah)x - (203a - be)x^2 + (bf - aj)x^3)}{8ab(a + bx^4)^2} - \frac{4a(bf}{ \\
&= \frac{x(bc - ag + (bd - ah)x - (203a - be)x^2 + (bf - aj)x^3)}{8ab(a + bx^4)^2} - \frac{4a(bf}{ \\
&= \frac{x(bc - ag + (bd - ah)x - (203a - be)x^2 + (bf - aj)x^3)}{8ab(a + bx^4)^2} - \frac{4a(bf}{ \\
&= \frac{x(bc - ag + (bd - ah)x - (203a - be)x^2 + (bf - aj)x^3)}{8ab(a + bx^4)^2} - \frac{4a(bf}{ \\
&= \frac{x(bc - ag + (bd - ah)x - (203a - be)x^2 + (bf - aj)x^3)}{8ab(a + bx^4)^2} - \frac{4a(bf}{
\end{aligned}$$

Mathematica [A] time = 0.429171, size = 500, normalized size = 1.04

$$\frac{32a^{7/4}(a^{2j-ab(f+x(g+x(h+ix)))}+b^2x(c+x(d+ex)))}{(a+bx^4)^2} + \frac{8a^{3/4}(-8a^2j+abx(g+x(2h+3ix))+b^2x(7c+x(6d+5ex)))}{a+bx^4} - 2\sqrt[4]{b} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right) (8a^{5/4}\sqrt[4]{bh} +$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^3, x]

```
[Out] ((8*a^(3/4)*(-8*a^2*j + b^2*x*(7*c + x*(6*d + 5*e*x)) + a*b*x*(g + x*(2*h + 3*i*x))))/(a + b*x^4) + (32*a^(7/4)*(a^2*j + b^2*x*(c + x*(d + e*x)) - a*b*(f + x*(g + x*(h + i*x))))/(a + b*x^4)^2 - 2*b^(1/4)*(21*Sqrt[2]*b^(3/2)*c + 24*a^(1/4)*b^(5/4)*d + 5*Sqrt[2]*Sqrt[a]*b*e + 3*Sqrt[2]*a*Sqrt[b]*g + 8*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*b^(1/4)*(21*Sqrt[2]*b^(3/2)*c - 24*a^(1/4)*b^(5/4)*d + 5*Sqrt[2]*Sqrt[a]*b*e + 3*Sqrt[2]*a*Sqrt[b]*g - 8*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*b^(1/4)*(-21*b^(3/2)*c + 5*Sqrt[a]*b*e - 3*a*Sqrt[b]*g + 3*a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*(21*b^(3/2)*c - 5*Sqrt[a]*b*e + 3*a*Sqrt[b]*g - 3*a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(256*a^(11/4)*b^2)
```

Maple [A] time = 0.013, size = 731, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x)
```

```
[Out] (1/32*(3*a*i+5*b*e)/a^2*x^7+1/16*(a*h+3*b*d)/a^2*x^6+1/32*(a*g+7*b*c)/a^2*x^5-1/4*j*x^4/b-1/32*(a*i-9*b*e)/a/b*x^3-1/16*(a*h-5*b*d)/a/b*x^2-1/32*(3*a*g-11*b*c)/a/b*x-1/8*(a*j+b*f)/b^2)/(b*x^4+a)^2+3/128/b/a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*g+21/128*c/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+3/128/b/a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*g+21/128*c/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+3/256/b/a^2*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*g+21/256*c/a^3*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/16/b/a/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))*h+3/16*d/a^2/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+3/256/b^2/a/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*i+5/256*e/a^2/b/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+3/128/b^2/a/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*i+5/128*e/a^2/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+3/128/b^2/a/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*i+5/128*e/a^2/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11724, size = 936, normalized size = 1.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 3/256*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4)))/(a/b)^(1/4))/(a^2*b^4) - sqrt(2)*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4) + 3/256*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4)))/(a/b)^(1/4))/(a^2*b^4) + sqrt(2)*(a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h + 21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4)))/(a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h + 21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4)))/(a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) + 1/32*(3*a*b^2*i*x^7 + 5*b^3*x^7*e + 6*b^3*d*x^6 + 2*a*b^2*h*x^6 + 7*b^3*c*x^5 + a*b^2*g*x^5 - 8*a^2*b*j*x^4 - a^2*b*i*x^3 + 9*a*b^2*x^3*e + 10*a*b^2*d*x^2 - 2*a^2*b*h*x^2 + 11*a*b^2*c*x - 3*a^2*b*g*x - 4*a^2*b*f - 4*a^3*j)/((b*x^4 + a)^2*a^2*b^2)
```


$$3.204 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^4} dx$$

Optimal. Leaf size=293

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(-15\sqrt{a}\sqrt{be}-7ag+77bc)}{256a^{15/4}b^{5/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(15\sqrt{a}\sqrt{be}-7ag+77bc)}{256a^{15/4}b^{5/4}} + \frac{(5bd-ah)\tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}} + \frac{x(2x^2+ax+b)}{12ab(a-bx^4)^3} + \frac{x(7(11bc-ag)+12(5bd-ah)x+45be^2)}{384a^3b(a-bx^4)} + \frac{8af+x(11bc-ag+2(5bd-ah)x+9be^2)}{96a^2b(a-bx^4)^2} + \frac{((77bc-15\sqrt{a}\sqrt{b})e-7ag)\text{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right]}{(256a^{15/4}b^{5/4})} + \frac{((77bc+15\sqrt{a}\sqrt{b})e-7ag)\text{ArcTanh}\left[\frac{b^{1/4}x}{a^{1/4}}\right]}{(256a^{15/4}b^{5/4})} + \frac{(5bd-ah)\text{ArcTanh}\left[\frac{\sqrt{b}x^2}{\sqrt{a}}\right]}{(32a^{7/2}b^{3/2})}$$

[Out] (x*(b*c + a*g + (b*d + a*h)*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a - b*x^4)^3) + (x*(7*(11*b*c - a*g) + 12*(5*b*d - a*h)*x + 45*b*e*x^2))/(384*a^3*b*(a - b*x^4)) + (8*a*f + x*(11*b*c - a*g + 2*(5*b*d - a*h)*x + 9*b*e*x^2))/(96*a^2*b*(a - b*x^4)^2) + ((77*b*c - 15*sqrt[a]*sqrt[b]*e - 7*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(5/4)) + ((77*b*c + 15*sqrt[a]*sqrt[b]*e - 7*a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(5/4)) + ((5*b*d - a*h)*ArcTanh[(sqrt[b]*x^2)/sqrt[a]])/(32*a^(7/2)*b^(3/2))

Rubi [A] time = 0.430552, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1858, 1854, 1855, 1876, 275, 208, 1167, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(-15\sqrt{a}\sqrt{be}-7ag+77bc)}{256a^{15/4}b^{5/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)(15\sqrt{a}\sqrt{be}-7ag+77bc)}{256a^{15/4}b^{5/4}} + \frac{(5bd-ah)\tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}} + \frac{x(2x^2+ax+b)}{12ab(a-bx^4)^3} + \frac{x(7(11bc-ag)+12(5bd-ah)x+45be^2)}{384a^3b(a-bx^4)} + \frac{8af+x(11bc-ag+2(5bd-ah)x+9be^2)}{96a^2b(a-bx^4)^2} + \frac{((77bc-15\sqrt{a}\sqrt{b})e-7ag)\text{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right]}{(256a^{15/4}b^{5/4})} + \frac{((77bc+15\sqrt{a}\sqrt{b})e-7ag)\text{ArcTanh}\left[\frac{b^{1/4}x}{a^{1/4}}\right]}{(256a^{15/4}b^{5/4})} + \frac{(5bd-ah)\text{ArcTanh}\left[\frac{\sqrt{b}x^2}{\sqrt{a}}\right]}{(32a^{7/2}b^{3/2})}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^4, x]

[Out] (x*(b*c + a*g + (b*d + a*h)*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a - b*x^4)^3) + (x*(7*(11*b*c - a*g) + 12*(5*b*d - a*h)*x + 45*b*e*x^2))/(384*a^3*b*(a - b*x^4)) + (8*a*f + x*(11*b*c - a*g + 2*(5*b*d - a*h)*x + 9*b*e*x^2))/(96*a^2*b*(a - b*x^4)^2) + ((77*b*c - 15*sqrt[a]*sqrt[b]*e - 7*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(5/4)) + ((77*b*c + 15*sqrt[a]*sqrt[b]*e - 7*a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(5/4)) + ((5*b*d - a*h)*ArcTanh[(sqrt[b]*x^2)/sqrt[a]])/(32*a^(7/2)*b^(3/2))

Rule 1858

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,

```
x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x], Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x] /; GeQ[q, n] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]]*(a + b*x^n)^(p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
```

$c*d^2 - a*e^2, 0]$ && PosQ[-(a*c)]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{(a - bx^4)^4} dx = \frac{x(bc + ag + (bd + ah)x + bex^2 + bfx^3)}{12ab(a - bx^4)^3} - \frac{\int \frac{-b(11bc - ag) - 2b(5bd - ah)x - 9b^2ex^2 - 8b^2fx^3}{(a - bx^4)^3} dx}{12ab^2}$$

$$= \frac{x(bc + ag + (bd + ah)x + bex^2 + bfx^3)}{12ab(a - bx^4)^3} + \frac{8af + x(11bc - ag + 2(5bd - ah)x + 4c)}{96a^2b(a - bx^4)^2}$$

$$= \frac{x(bc + ag + (bd + ah)x + bex^2 + bfx^3)}{12ab(a - bx^4)^3} + \frac{x(7(11bc - ag) + 12(5bd - ah)x + 4c)}{384a^3b(a - bx^4)}$$

$$= \frac{x(bc + ag + (bd + ah)x + bex^2 + bfx^3)}{12ab(a - bx^4)^3} + \frac{x(7(11bc - ag) + 12(5bd - ah)x + 4c)}{384a^3b(a - bx^4)}$$

$$= \frac{x(bc + ag + (bd + ah)x + bex^2 + bfx^3)}{12ab(a - bx^4)^3} + \frac{x(7(11bc - ag) + 12(5bd - ah)x + 4c)}{384a^3b(a - bx^4)}$$

$$= \frac{x(bc + ag + (bd + ah)x + bex^2 + bfx^3)}{12ab(a - bx^4)^3} + \frac{x(7(11bc - ag) + 12(5bd - ah)x + 4c)}{384a^3b(a - bx^4)}$$

$$= \frac{x(bc + ag + (bd + ah)x + bex^2 + bfx^3)}{12ab(a - bx^4)^3} + \frac{x(7(11bc - ag) + 12(5bd - ah)x + 4c)}{384a^3b(a - bx^4)}$$

Mathematica [A] time = 0.39516, size = 360, normalized size = 1.23

$$-3 \log(\sqrt[4]{a} - \sqrt[4]{bx}) (-8a^{5/4}h + 15\sqrt{ab^3/4}e + 40\sqrt[4]{abd} - 7a\sqrt[4]{bg} + 77b^{5/4}c) + 3 \log(\sqrt[4]{a} + \sqrt[4]{bx}) (8a^{5/4}h + 15\sqrt{ab^3/4}e - 40\sqrt[4]{abd} + 7a\sqrt[4]{bg} - 77b^{5/4}c)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^4, x]

[Out] ((4*a^(3/4)*Sqrt[b]*x*(77*b*c - 7*a*g + 60*b*d*x - 12*a*h*x + 45*b*e*x^2))/(a - b*x^4) + (16*a^(7/4)*Sqrt[b]*x*(11*b*c + b*x*(10*d + 9*e*x) - a*(g + 2*h*x)))/(a - b*x^4)^2 + (128*a^(11/4)*Sqrt[b]*(b*x*(c + x*(d + e*x)) + a*(f + x*(g + h*x)))/(a - b*x^4)^3 + 6*b^(1/4)*(77*b*c - 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] - 3*(77*b^(5/4)*c + 40*a^(1/4)*b*d + 15*Sqrt[a]*b^(3/4)*e - 7*a*b^(1/4)*g - 8*a^(5/4)*h)*Log[a^(1/4) - b^(1/4)*x] + 3*(77*b^(5/4)*c - 40*a^(1/4)*b*d + 15*Sqrt[a]*b^(3/4)*e - 7*a*b^(1/4)*g + 8*a^(5/4)*h)*Log[a^(1/4) + b^(1/4)*x] - 24*a^(1/4)*(-5*b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2)]/(1536*a^(15/4)*b^(3/2))

Maple [A] time = 0.013, size = 434, normalized size = 1.5

$$\frac{1}{(bx^4 - a)^3} \left(-\frac{15b^2ex^{11}}{128a^3} + \frac{(ah - 5bd)bx^{10}}{32a^3} + \frac{(7ag - 77bc)bx^9}{384a^3} + \frac{21bex^7}{64a^2} - \frac{(ah - 5bd)x^6}{12a^2} - \frac{(3ag - 33bc)x^5}{64a^2} - \frac{113ex^3}{384a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4, x)

[Out] (-15/128*e/a^3*b^2*x^11+1/32*(a*h-5*b*d)/a^3*b*x^10+7/384*(a*g-11*b*c)/a^3*b*x^9+21/64/a^2*b*e*x^7-1/12/a^2*(a*h-5*b*d)*x^6-3/64/a^2*(a*g-11*b*c)*x^5-113/384/a*e*x^3-1/32*(a*h+11*b*d)/a/b*x^2-1/128*(7*a*g+51*b*c)/a/b*x-1/12*f/b)/(b*x^4-a)^3-7/256/b/a^3*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))*g+77/256/a^4*c*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))-7/512/b/a^3*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))*g+77/512/a^4*c*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))+1/64/b/a^2/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))*h-5/64/a^3*d/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))-15/256/a^3*e/b/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))+15/512/a^3*e/b/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)

[Out] Timed out

Giac [B] time = 1.08415, size = 738, normalized size = 2.52

$$\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{-abb^2d} - 8\sqrt{2}\sqrt{-ababh} - 77(-ab^3)^{\frac{1}{4}}b^2c + 7(-ab^3)^{\frac{1}{4}}abg - 15(-ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512a^4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")

```
[Out] -1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b^2*d - 8*sqrt(2)*sqrt(-a*b)*a*b*h -
77*(-a*b^3)^(1/4)*b^2*c + 7*(-a*b^3)^(1/4)*a*b*g - 15*(-a*b^3)^(3/4)*e)*arc
tan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^4*b^3) - 1/51
2*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b^2*d - 8*sqrt(2)*sqrt(-a*b)*a*b*h - 77*(-
a*b^3)^(1/4)*b^2*c + 7*(-a*b^3)^(1/4)*a*b*g - 15*(-a*b^3)^(3/4)*e)*arctan(1
/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^4*b^3) + 1/1024*sq
rt(2)*(77*(-a*b^3)^(1/4)*b^2*c - 7*(-a*b^3)^(1/4)*a*b*g - 15*(-a*b^3)^(3/4)
*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^4*b^3) - 1/1024*sqrt(
2)*(77*(-a*b^3)^(1/4)*b^2*c - 7*(-a*b^3)^(1/4)*a*b*g - 15*(-a*b^3)^(3/4)*e)
*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^4*b^3) - 1/384*(45*b^3*x
^11*e + 60*b^3*d*x^10 - 12*a*b^2*h*x^10 + 77*b^3*c*x^9 - 7*a*b^2*g*x^9 - 12
6*a*b^2*x^7*e - 160*a*b^2*d*x^6 + 32*a^2*b*h*x^6 - 198*a*b^2*c*x^5 + 18*a^2
*b*g*x^5 + 113*a^2*b*x^3*e + 132*a^2*b*d*x^2 + 12*a^3*h*x^2 + 153*a^2*b*c*x
+ 21*a^3*g*x + 32*a^3*f)/((b*x^4 - a)^3*a^3*b)
```

$$3.205 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^4} dx$$

Optimal. Leaf size=331

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}} - 5(3be - ai)\right)}{256a^{13/4}b^{7/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}} - 5ai + 15be\right)}{256a^{13/4}b^{7/4}} + \frac{(5bd - ah)\tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}} + x(2$$

[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + b*f*x^3))/(12*a*b*(a - b*x^4)^3) + (x*(7*(11*b*c - a*g) + 12*(5*b*d - a*h)*x + 15*(3*b*e - a*i)*x^2))/(384*a^3*b*(a - b*x^4)) + (8*a*f + x*(11*b*c - a*g + 2*(5*b*d - a*h)*x + 3*(3*b*e - a*i)*x^2))/(96*a^2*b*(a - b*x^4)^2) + (((7*Sqrt[b]*(11*b*c - a*g))/Sqrt[a] - 5*(3*b*e - a*i))*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(13/4)*b^(7/4)) + ((15*b*e + (7*Sqrt[b]*(11*b*c - a*g))/Sqrt[a] - 5*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(13/4)*b^(7/4)) + ((5*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2))

Rubi [A] time = 0.567183, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {1858, 1854, 1855, 1876, 275, 208, 1167, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}} - 5(3be - ai)\right)}{256a^{13/4}b^{7/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}} - 5ai + 15be\right)}{256a^{13/4}b^{7/4}} + \frac{(5bd - ah)\tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}} + x(2$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^4, x]

[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + b*f*x^3))/(12*a*b*(a - b*x^4)^3) + (x*(7*(11*b*c - a*g) + 12*(5*b*d - a*h)*x + 15*(3*b*e - a*i)*x^2))/(384*a^3*b*(a - b*x^4)) + (8*a*f + x*(11*b*c - a*g + 2*(5*b*d - a*h)*x + 3*(3*b*e - a*i)*x^2))/(96*a^2*b*(a - b*x^4)^2) + (((7*Sqrt[b]*(11*b*c - a*g))/Sqrt[a] - 5*(3*b*e - a*i))*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(13/4)*b^(7/4)) + ((15*b*e + (7*Sqrt[b]*(11*b*c - a*g))/Sqrt[a] - 5*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(13/4)*b^(7/4)) + ((5*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2))

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a +
b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Module[{q = Expon[Pq,
x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]]*(a + b*x^n)^(p
+ 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
+ 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167


```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + 205x^6}{(a - bx^4)^4} dx = \frac{x(bc + ag + (bd + ah)x + (205a + be)x^2 + bfx^3)}{12ab(a - bx^4)^3} - \frac{\int \frac{-b(11bc - ag) - 2b(5bd - ag)x - b^2x^2}{(a - bx^4)^4} dx}{12ab(a - bx^4)^3}$$

$$= \frac{x(bc + ag + (bd + ah)x + (205a + be)x^2 + bfx^3)}{12ab(a - bx^4)^3} + \frac{8af + x(11bc - ag)}{12ab(a - bx^4)^3}$$

$$= \frac{x(bc + ag + (bd + ah)x + (205a + be)x^2 + bfx^3)}{12ab(a - bx^4)^3} + \frac{x(7(11bc - ag) + 8a)}{12ab(a - bx^4)^3}$$

$$= \frac{x(bc + ag + (bd + ah)x + (205a + be)x^2 + bfx^3)}{12ab(a - bx^4)^3} + \frac{x(7(11bc - ag) + 8a)}{12ab(a - bx^4)^3}$$

$$= \frac{x(bc + ag + (bd + ah)x + (205a + be)x^2 + bfx^3)}{12ab(a - bx^4)^3} + \frac{x(7(11bc - ag) + 8a)}{12ab(a - bx^4)^3}$$

$$= \frac{x(bc + ag + (bd + ah)x + (205a + be)x^2 + bfx^3)}{12ab(a - bx^4)^3} + \frac{x(7(11bc - ag) + 8a)}{12ab(a - bx^4)^3}$$

$$= \frac{x(bc + ag + (bd + ah)x + (205a + be)x^2 + bfx^3)}{12ab(a - bx^4)^3} + \frac{x(7(11bc - ag) + 8a)}{12ab(a - bx^4)^3}$$

Mathematica [A] time = 0.368419, size = 422, normalized size = 1.27

$$\frac{128a^3b^{3/4}(af + x(g + x(h + ix))) + bx(c + x(d + ex))}{(a - bx^4)^3} - \frac{16a^2b^{3/4}x(a(g + x(2h + 3ix)) - b(11c + x(10d + 9ex)))}{(a - bx^4)^2} + 3\sqrt[4]{a} \log(\sqrt[4]{a} - \sqrt[4]{bx}) (8a^{5/4}\sqrt[4]{bh} + 5a^{3/2}i -$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^4,x
]
```

```
[Out] ((-4*a*b^(3/4)*x*(-77*b*c + 7*a*g - 15*b*x*(4*d + 3*e*x) + 3*a*x*(4*h + 5*i
*x)))/(a - b*x^4) - (16*a^2*b^(3/4)*x*(-(b*(11*c + x*(10*d + 9*e*x))) + a*(
g + x*(2*h + 3*i*x)))/(a - b*x^4)^2 + (128*a^3*b^(3/4)*(b*x*(c + x*(d + e*
x)) + a*(f + x*(g + x*(h + i*x))))/(a - b*x^4)^3 + 6*a^(1/4)*(77*b^(3/2)*c
- 15*Sqrt[a]*b*e - 7*a*Sqrt[b]*g + 5*a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)
] + 3*a^(1/4)*(-77*b^(3/2)*c - 40*a^(1/4)*b^(5/4)*d - 15*Sqrt[a]*b*e + 7*a*
Sqrt[b]*g + 8*a^(5/4)*b^(1/4)*h + 5*a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x] - 3
*a^(1/4)*(-77*b^(3/2)*c + 40*a^(1/4)*b^(5/4)*d - 15*Sqrt[a]*b*e + 7*a*Sqrt[
b]*g - 8*a^(5/4)*b^(1/4)*h + 5*a^(3/2)*i)*Log[a^(1/4) + b^(1/4)*x] - 24*Sqr
t[a]*b^(1/4)*(-5*b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2)]/(1536*a^4*b^(7/4))
```

Maple [A] time = 0.015, size = 522, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x)
```

```
[Out] (5/128*(a*i-3*b*e)/a^3*b*x^11+1/32*(a*h-5*b*d)/a^3*b*x^10+7/384*(a*g-11*b*c
)/a^3*b*x^9-7/64/a^2*(a*i-3*b*e)*x^7-1/12/a^2*(a*h-5*b*d)*x^6-3/64/a^2*(a*g
-11*b*c)*x^5-1/384*(5*a*i+113*b*e)/a/b*x^3-1/32*(a*h+11*b*d)/a/b*x^2-1/128*
(7*a*g+51*b*c)/a/b*x-1/12*f/b)/(b*x^4-a)^3-7/256/b/a^3*(1/b*a)^(1/4)*arctan
(x/(1/b*a)^(1/4))*g+77/256/a^4*c*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))-7/51
2/b/a^3*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))*g+77/512/a^4*
c*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))+1/64/b/a^2/(a*b)^(1
/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))*h-5/64/a^3*d/(a*b)^(1/2)*
ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))+5/256/b^2/a^2/(1/b*a)^(1/4)*a
rctan(x/(1/b*a)^(1/4))*i-15/256/a^3*e/b/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4
))-5/512/b^2/a^2/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))*i+15
/512/a^3*e/b/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.10286, size = 1044, normalized size = 3.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] -5/1024*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^4) - sqrt(2)*(-a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^4) - 5/1024*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^4) + sqrt(2)*(-a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^4) - 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b^2*d - 8*sqrt(2)*sqrt(-a*b)*a*b*h - 77*(-a*b^3)^(1/4)*b^2*c + 7*(-a*b^3)^(1/4)*a*b*g - 15*(-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^4*b^3) - 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b^2*d - 8*sqrt(2)*sqrt(-a*b)*a*b*h - 77*(-a*b^3)^(1/4)*b^2*c + 7*(-a*b^3)^(1/4)*a*b*g - 15*(-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(-a*b^3)^(1/4)*b^2*c - 7*(-a*b^3)^(1/4)*a*b*g - 15*(-a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^4*b^3) - 1/1024*sqrt(2)*(77*(-a*b^3)^(1/4)*b^2*c - 7*(-a*b^3)^(1/4)*a*b*g - 15*(-a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^4*b^3) + 1/384*(15*a*b^2*i*x^11 - 45*b^3*x^11*e - 60*b^3*d*x^10 + 12*a*b^2*h*x^10 - 77*b^3*c*x^9 + 7*a*b^2*g*x^9 - 42*a^2*b*i*x^7 + 126*a*b^2*x^7*e + 160*a*b^2*d*x^6 - 32*a^2*b*h*x^6 + 198*a*b^2*c*x^5 - 18*a^2*b*g*x^5 - 5*a^3*i*x^3 - 113*a^2*b*x^3*e - 132*a^2*b*d*x^2 - 12*a^3*h*x^2 - 153*a^2*b*c*x - 21*a^3*g*x - 32*a^3*f)/((b*x^4 - a)^3*a^3*b)
```

$$3.206 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^4} dx$$

Optimal. Leaf size=349

$$\frac{x(b(11bc-ag) + 2bx(5bd-ah) + 3bx^2(3be-ai)) + 4a(2bf-aj)}{96a^2b^2(a-bx^4)^2} + \frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b(11bc-ag)}}{\sqrt{a}} - 5(3be-ai)\right)}{256a^{13/4}b^{7/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\left(\frac{7\sqrt{b(11bc-ag)}}{\sqrt{a}} - 5(3be-ai)\right)}{256a^{13/4}b^{7/4}}$$

[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + (b*f + a*j)*x^3))/(12*a*b*(a - b*x^4)^3) + (x*(7*(11*b*c - a*g) + 12*(5*b*d - a*h)*x + 15*(3*b*e - a*i)*x^2))/(384*a^3*b*(a - b*x^4)) + (4*a*(2*b*f - a*j) + x*(b*(11*b*c - a*g) + 2*b*(5*b*d - a*h)*x + 3*b*(3*b*e - a*i)*x^2))/(96*a^2*b^2*(a - b*x^4)^2) + (((7*sqrt[b]*(11*b*c - a*g))/sqrt[a] - 5*(3*b*e - a*i))*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(13/4)*b^(7/4)) + ((15*b*e + (7*sqrt[b]*(11*b*c - a*g))/sqrt[a] - 5*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(13/4)*b^(7/4)) + ((5*b*d - a*h)*ArcTanh[(sqrt[b]*x^2)/sqrt[a]])/(32*a^(7/2)*b^(3/2))

Rubi [A] time = 0.523832, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1858, 1854, 1855, 1876, 275, 208, 1167, 205}

$$\frac{x(b(11bc-ag) + 2bx(5bd-ah) + 3bx^2(3be-ai)) + 4a(2bf-aj)}{96a^2b^2(a-bx^4)^2} + \frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b(11bc-ag)}}{\sqrt{a}} - 5(3be-ai)\right)}{256a^{13/4}b^{7/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\left(\frac{7\sqrt{b(11bc-ag)}}{\sqrt{a}} - 5(3be-ai)\right)}{256a^{13/4}b^{7/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^4, x]

[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + (b*f + a*j)*x^3))/(12*a*b*(a - b*x^4)^3) + (x*(7*(11*b*c - a*g) + 12*(5*b*d - a*h)*x + 15*(3*b*e - a*i)*x^2))/(384*a^3*b*(a - b*x^4)) + (4*a*(2*b*f - a*j) + x*(b*(11*b*c - a*g) + 2*b*(5*b*d - a*h)*x + 3*b*(3*b*e - a*i)*x^2))/(96*a^2*b^2*(a - b*x^4)^2) + (((7*sqrt[b]*(11*b*c - a*g))/sqrt[a] - 5*(3*b*e - a*i))*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(13/4)*b^(7/4)) + ((15*b*e + (7*sqrt[b]*(11*b*c - a*g))/sqrt[a] - 5*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(13/4)*b^(7/4)) + ((5*b*d - a*h)*ArcTanh[(sqrt[b]*x^2)/sqrt[a]])/(32*a^(7/2)*b^(3/2))

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a +
b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Module[{q = Expon[Pq,
x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1})*(a + b*x^n)^(p
+ 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
+ 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + 206x^6 + jx^7}{(a - bx^4)^4} dx = \frac{x(bc + ag + (bd + ah)x + (206a + be)x^2 + (bf + aj)x^3)}{12ab(a - bx^4)^3} - \frac{\int^{-b(1}}{\dots}$$

$$= \frac{x(bc + ag + (bd + ah)x + (206a + be)x^2 + (bf + aj)x^3)}{12ab(a - bx^4)^3} + \frac{4a(2b}{\dots}$$

$$= \frac{x(bc + ag + (bd + ah)x + (206a + be)x^2 + (bf + aj)x^3)}{12ab(a - bx^4)^3} + \frac{x(7(1}}{\dots}$$

$$= \frac{x(bc + ag + (bd + ah)x + (206a + be)x^2 + (bf + aj)x^3)}{12ab(a - bx^4)^3} + \frac{x(7(1}}{\dots}$$

$$= \frac{x(bc + ag + (bd + ah)x + (206a + be)x^2 + (bf + aj)x^3)}{12ab(a - bx^4)^3} + \frac{x(7(1}}{\dots}$$

$$= \frac{x(bc + ag + (bd + ah)x + (206a + be)x^2 + (bf + aj)x^3)}{12ab(a - bx^4)^3} + \frac{x(7(1}}{\dots}$$

$$= \frac{x(bc + ag + (bd + ah)x + (206a + be)x^2 + (bf + aj)x^3)}{12ab(a - bx^4)^3} + \frac{x(7(1}}{\dots}$$

Mathematica [A] time = 0.36775, size = 439, normalized size = 1.26

$$\frac{128a^3(a^2j + ab(f + x(g + x(h + ix))) + b^2x(c + x(d + ex)))}{(a - bx^4)^3} - \frac{16a^2(12a^2j + abx(g + x(2h + 3ix)) - b^2x(11c + x(10d + 9ex)))}{(a - bx^4)^2} + 3\sqrt[4]{a}\sqrt[4]{b}\log(\sqrt[4]{a} - \sqrt[4]{bx})(8a^{5/4}\sqrt[4]{bh})$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^4, x]

[Out]
$$\frac{\begin{aligned} &((-4*a*b*x*(-77*b*c + 7*a*g - 15*b*x*(4*d + 3*e*x) + 3*a*x*(4*h + 5*i*x)))/ \\ &(a - b*x^4) - (16*a^2*(12*a^2*j - b^2*x*(11*c + x*(10*d + 9*e*x)) + a*b*x*(\\ &g + x*(2*h + 3*i*x))))/(a - b*x^4)^2 + (128*a^3*(a^2*j + b^2*x*(c + x*(d + \\ &e*x)) + a*b*(f + x*(g + x*(h + i*x))))/(a - b*x^4)^3 + 6*a^{1/4}*b^{1/4}* \\ &77*b^{3/2}*c - 15*\text{Sqrt}[a]*b*e - 7*a*\text{Sqrt}[b]*g + 5*a^{3/2}*i)*\text{ArcTan}[(b^{1/4} \\ &)*x/a^{1/4}] + 3*a^{1/4}*b^{1/4}*(-77*b^{3/2}*c - 40*a^{1/4}*b^{5/4}*d - 1 \\ &5*\text{Sqrt}[a]*b*e + 7*a*\text{Sqrt}[b]*g + 8*a^{5/4}*b^{1/4}*h + 5*a^{3/2}*i)*\text{Log}[a^{1/4} \\ &- b^{1/4}*x] + 3*a^{1/4}*b^{1/4}*(77*b^{3/2}*c - 40*a^{1/4}*b^{5/4}*d + \\ &15*\text{Sqrt}[a]*b*e - 7*a*\text{Sqrt}[b]*g + 8*a^{5/4}*b^{1/4}*h - 5*a^{3/2}*i)*\text{Log}[a^{1/4} \\ &+ b^{1/4}*x] - 24*\text{Sqrt}[a]*\text{Sqrt}[b]*(-5*b*d + a*h)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[b] \\ &]*x^2] \end{aligned}}{(1536*a^4*b^2)}$$

Maple [A] time = 0.015, size = 538, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4, x)

[Out]
$$\begin{aligned} &(5/128*(a*i-3*b*e)/a^3*b*x^{11}+1/32*(a*h-5*b*d)/a^3*b*x^{10}+7/384*(a*g-11*b*c \\ &)/a^3*b*x^9-7/64/a^2*(a*i-3*b*e)*x^7-1/12/a^2*(a*h-5*b*d)*x^6-3/64/a^2*(a*g \\ &-11*b*c)*x^5-1/8*j*x^4/b-1/384*(5*a*i+113*b*e)/a/b*x^3-1/32*(a*h+11*b*d)/a/ \\ &b*x^2-1/128*(7*a*g+51*b*c)/a/b*x+1/24*(a*j-2*b*f)/b^2)/(b*x^4-a)^3-7/256/b/ \\ &a^3*(1/b*a)^{1/4}*\arctan(x/(1/b*a)^{1/4})*g+77/256/a^4*c*(1/b*a)^{1/4}*\arctan \\ &(x/(1/b*a)^{1/4})-7/512/b/a^3*(1/b*a)^{1/4}*\ln((x+(1/b*a)^{1/4})/(x-(1/b*a)^{1/4})) \\ &)*g+77/512/a^4*c*(1/b*a)^{1/4}*\ln((x+(1/b*a)^{1/4})/(x-(1/b*a)^{1/4})) \\ &)+1/64/b/a^2/(a*b)^{1/2}*\ln((-a+x^2*(a*b)^{1/2})/(-a-x^2*(a*b)^{1/2}))*h \\ &-5/64/a^3*d/(a*b)^{1/2}*\ln((-a+x^2*(a*b)^{1/2})/(-a-x^2*(a*b)^{1/2}))+5/256 \\ &/b^2/a^2/(1/b*a)^{1/4}*\arctan(x/(1/b*a)^{1/4})*i-15/256/a^3*e/b/(1/b*a)^{1/4} \\ &)*\arctan(x/(1/b*a)^{1/4})-5/512/b^2/a^2/(1/b*a)^{1/4}*\ln((x+(1/b*a)^{1/4}) \\ &/(x-(1/b*a)^{1/4}))*i+15/512/a^3*e/b/(1/b*a)^{1/4}*\ln((x+(1/b*a)^{1/4})/(x- \\ &(1/b*a)^{1/4})) \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)

[Out] Timed out

Giac [B] time = 1.11209, size = 1087, normalized size = 3.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] -5/1024*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4)))/(-a/b)^(1/4))/(a^3*b^4) - sqrt(2)*(-a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^4) - 5/1024*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4)))/(-a/b)^(1/4))/(a^3*b^4) + sqrt(2)*(-a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^4) - 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b^2*d - 8*sqrt(2)*sqrt(-a*b)*a*b*h - 77*(-a*b^3)^(1/4)*b^2*c + 7*(-a*b^3)^(1/4)*a*b*g - 15*(-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4)))/(-a/b)^(1/4))/(a^4*b^3) - 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b^2*d - 8*sqrt(2)*sqrt(-a*b)*a*b*h - 77*(-a*b^3)^(1/4)*b^2*c + 7*(-a*b^3)^(1/4)*a*b*g - 15*(-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4)))/(-a/b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(-a*b^3)^(1/4)*b^2*c - 7*(-a*b^3)^(1/4)*a*b*g - 15*(-a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^4*b^3) - 1/1024*sqrt(2)*(77*(-a*b^3)^(1/4)*b^2*c - 7*(-a*b^3)^(1/4)*a*b*g - 15*(-a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^4*b^3) + 1/384*(15*a*b^3*i*x^11 - 45*b^4*x^11*e - 60*b^4*d*x^10 + 12*a*b^3*h*x^10 - 77*b^4*c*x^9 + 7*a*b^3*g*x^9 - 42*a^2*b^2*i*x^7 + 126*a*b^3*x^7*e + 160*a*b^3*d*x^6 - 32*a^2*b^2*h*x^6 + 198*a*b^3*c*x^5 - 18*a^2*b^2*g*x^5 - 48*a^3*b*j*x^4 - 5*a^3*b*i*x^3 - 113*a^2*b^2*x^3*e - 132*a^2*b^2*d*x^2 - 12*a^3*b*h*x^2 - 153*a^2*b^2*c*x - 21*a^3*b*g*x - 32*a^3*b*f + 16*a^4*j)/((b*x^4 - a)^3*a^3*b^2)
```

$$3.207 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^4} dx$$

Optimal. Leaf size=462

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-15\sqrt{a}\sqrt{be} + 7ag + 77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-15\sqrt{a}\sqrt{be} + 7ag + 77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}}$$

[Out] (x*(b*c - a*g + (b*d - a*h)*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a + b*x^4)^3) + (x*(7*(11*b*c + a*g) + 12*(5*b*d + a*h)*x + 45*b*e*x^2))/(384*a^3*b*(a + b*x^4)) - (8*a*f - x*(11*b*c + a*g + 2*(5*b*d + a*h)*x + 9*b*e*x^2))/(96*a^2*b*(a + b*x^4)^2) + ((5*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2)) - ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(5/4)) + ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(5/4)) - ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(5/4)) + ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(5/4))

Rubi [A] time = 0.618692, antiderivative size = 462, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {1858, 1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-15\sqrt{a}\sqrt{be} + 7ag + 77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(-15\sqrt{a}\sqrt{be} + 7ag + 77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^4, x]

[Out] (x*(b*c - a*g + (b*d - a*h)*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a + b*x^4)^3) + (x*(7*(11*b*c + a*g) + 12*(5*b*d + a*h)*x + 45*b*e*x^2))/(384*a^3*b*(a + b*x^4)) - (8*a*f - x*(11*b*c + a*g + 2*(5*b*d + a*h)*x + 9*b*e*x^2))/(96*a^2*b*(a + b*x^4)^2) + ((5*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2)) - ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(5/4)) + ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(5/4)) - ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(5/4)) + ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(5/4))

$$a^{(15/4)}*b^{(5/4)} - ((77*b*c - 15*\text{Sqrt}[a]*\text{Sqrt}[b]*e + 7*a*g)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*x + \text{Sqrt}[b]*x^2])/(512*\text{Sqrt}[2]*a^{(15/4)}*b^{(5/4)}) + ((77*b*c - 15*\text{Sqrt}[a]*\text{Sqrt}[b]*e + 7*a*g)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*x + \text{Sqrt}[b]*x^2])/(512*\text{Sqrt}[2]*a^{(15/4)}*b^{(5/4)})$$
Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1})*(a + b*x^n)^(p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1168

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d},

Mathematica [A] time = 0.461843, size = 461, normalized size = 1.

$$-6 \tan^{-1} \left(1 - \frac{\sqrt{2} \sqrt[4]{bx}}{\sqrt[4]{a}} \right) \left(16a^{5/4}h + 15\sqrt{2}\sqrt{ab}^{3/4}e + 80\sqrt[4]{abd} + 7\sqrt{2}a\sqrt[4]{bg} + 77\sqrt{2}b^{5/4}c \right) + 6 \tan^{-1} \left(\frac{\sqrt{2} \sqrt[4]{bx}}{\sqrt[4]{a}} + 1 \right) \left(-16a^{5/4}h + \dots \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^4, x]

[Out] ((8*a^(3/4)*Sqrt[b]*x*(77*b*c + 7*a*g + 60*b*d*x + 12*a*h*x + 45*b*e*x^2))/ (a + b*x^4) + (32*a^(7/4)*Sqrt[b]*x*(11*b*c + b*x*(10*d + 9*e*x) + a*(g + 2 *h*x)))/(a + b*x^4)^2 - (256*a^(11/4)*Sqrt[b]*(-(b*x*(c + x*(d + e*x))) + a *(f + x*(g + h*x))))/(a + b*x^4)^3 - 6*(77*Sqrt[2]*b^(5/4)*c + 80*a^(1/4)*b *d + 15*Sqrt[2]*Sqrt[a]*b^(3/4)*e + 7*Sqrt[2]*a*b^(1/4)*g + 16*a^(5/4)*h)*A rcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 6*(77*Sqrt[2]*b^(5/4)*c - 80*a^(1/ 4)*b*d + 15*Sqrt[2]*Sqrt[a]*b^(3/4)*e + 7*Sqrt[2]*a*b^(1/4)*g - 16*a^(5/4)* h)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 3*Sqrt[2]*b^(1/4)*(77*b*c - 15 *Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 3*Sqrt[2]*b^(1/4)*(77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqr t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2)]/(3072*a^(15/4)*b^(3/2))

Maple [A] time = 0.014, size = 607, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4, x)

[Out] (15/128*e/a^3*b^2*x^11+1/32*(a*h+5*b*d)/a^3*b*x^10+7/384*(a*g+11*b*c)/a^3*b *x^9+21/64/a^2*b*e*x^7+1/12/a^2*(a*h+5*b*d)*x^6+3/64/a^2*(a*g+11*b*c)*x^5+1 13/384/a*e*x^3-1/32*(a*h-11*b*d)/a/b*x^2-1/128*(7*a*g-51*b*c)/a/b*x-1/12*f/ b)/(b*x^4+a)^3+7/512/b/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/ 4)*x+1)*g+77/512/a^4*c*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x +1)+7/512/b/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*g+7 7/512/a^4*c*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+7/1024/ b/a^3*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/ (x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*g+77/1024/a^4*c*(1/b*a)^(1/4)* 2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x *2^(1/2)+(1/b*a)^(1/2)))+1/32/b/a^2/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))*h+5 /32/a^3*d/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+15/1024/a^3*e/b/(1/b*a)^(1/4)

$$*2^{(1/2)}*\ln((x^2-(1/b*a)^{(1/4)}*x*2^{(1/2)}+(1/b*a)^{(1/2)})/(x^2+(1/b*a)^{(1/4)}*x*2^{(1/2)}+(1/b*a)^{(1/2)}))+15/512/a^3*e/b/(1/b*a)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)}*x+1)+15/512/a^3*e/b/(1/b*a)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)}*x-1)$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] Timed out
```


Giac [A] time = 1.09323, size = 703, normalized size = 1.52

$$\frac{\sqrt{2} \left(40 \sqrt{2} \sqrt{ab} b^2 d + 8 \sqrt{2} \sqrt{ab} abh + 77 (ab^3)^{\frac{1}{4}} b^2 c + 7 (ab^3)^{\frac{1}{4}} abg + 15 (ab^3)^{\frac{3}{4}} e \right) \arctan \left(\frac{\sqrt{2} \left(2x + \sqrt{2} \left(\frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left(\frac{a}{b} \right)^{\frac{1}{4}}} \right)}{512 a^4 b^3} + \frac{\sqrt{2} \left(40 \sqrt{2} \sqrt{ab} b^2 d + 8 \sqrt{2} \sqrt{ab} abh + 77 (ab^3)^{\frac{1}{4}} b^2 c + 7 (ab^3)^{\frac{1}{4}} abg + 15 (ab^3)^{\frac{3}{4}} e \right) \arctan \left(\frac{\sqrt{2} \left(2x + \sqrt{2} \left(\frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left(\frac{a}{b} \right)^{\frac{1}{4}}} \right)}{512 a^4 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")

[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b*h + 77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b*h + 77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(45*b^3*x^11*e + 60*b^3*d*x^10 + 12*a*b^2*h*x^10 + 77*b^3*c*x^9 + 7*a*b^2*g*x^9 + 126*a*b^2*x^7*e + 160*a*b^2*d*x^6 + 32*a^2*b*h*x^6 + 198*a*b^2*c*x^5 + 18*a^2*b*g*x^5 + 113*a^2*b*x^3*e + 132*a^2*b*d*x^2 - 12*a^3*h*x^2 + 153*a^2*b*c*x - 21*a^3*g*x - 32*a^3*f)/((b*x^4 + a)^3*a^3*b)

$$3.208 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^4} dx$$

Optimal. Leaf size=516

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(7\sqrt{b}(ag+11bc) - 5\sqrt{a}(ai+3be)\right)}{512\sqrt{2}a^{15/4}b^{7/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(7\sqrt{b}(ag+11bc) - 5\sqrt{a}(ai+3be)\right)}{512\sqrt{2}a^{15/4}b^{7/4}}$$

[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + b*f*x^3))/(12*a*b*(a + b*x^4)^3) + (x*(7*(11*b*c + a*g) + 12*(5*b*d + a*h)*x + 15*(3*b*e + a*i)*x^2))/(384*a^3*b*(a + b*x^4)) - (8*a*f - x*(11*b*c + a*g + 2*(5*b*d + a*h)*x + 3*(3*b*e + a*i)*x^2))/(96*a^2*b*(a + b*x^4)^2) + ((5*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2)) - ((7*Sqrt[b]*(11*b*c + a*g) + 5*Sqrt[a]*(3*b*e + a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(7/4)) + ((7*Sqrt[b]*(11*b*c + a*g) + 5*Sqrt[a]*(3*b*e + a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(7/4)) - ((7*Sqrt[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e + a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(7/4)) + ((7*Sqrt[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e + a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(7/4))

Rubi [A] time = 0.850028, antiderivative size = 516, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {1858, 1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(7\sqrt{b}(ag+11bc) - 5\sqrt{a}(ai+3be)\right)}{512\sqrt{2}a^{15/4}b^{7/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)\left(7\sqrt{b}(ag+11bc) - 5\sqrt{a}(ai+3be)\right)}{512\sqrt{2}a^{15/4}b^{7/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^4, x]

[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + b*f*x^3))/(12*a*b*(a + b*x^4)^3) + (x*(7*(11*b*c + a*g) + 12*(5*b*d + a*h)*x + 15*(3*b*e + a*i)*x^2))/(384*a^3*b*(a + b*x^4)) - (8*a*f - x*(11*b*c + a*g + 2*(5*b*d + a*h)*x + 3*(3*b*e + a*i)*x^2))/(96*a^2*b*(a + b*x^4)^2) + ((5*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2)) - ((7*Sqrt[b]*(11*b*c + a*g) + 5*Sqrt[a]*(3*b*e + a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]

$$*a^{(15/4)}*b^{(7/4)} + ((7*\text{Sqrt}[b]*(11*b*c + a*g) + 5*\text{Sqrt}[a]*(3*b*e + a*i))*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*x)/a^{(1/4)}])/(256*\text{Sqrt}[2]*a^{(15/4)}*b^{(7/4)}) - ((7*\text{Sqrt}[b]*(11*b*c + a*g) - 5*\text{Sqrt}[a]*(3*b*e + a*i))*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*x + \text{Sqrt}[b]*x^2])/(512*\text{Sqrt}[2]*a^{(15/4)}*b^{(7/4)}) + ((7*\text{Sqrt}[b]*(11*b*c + a*g) - 5*\text{Sqrt}[a]*(3*b*e + a*i))*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*x + \text{Sqrt}[b]*x^2])/(512*\text{Sqrt}[2]*a^{(15/4)}*b^{(7/4)})$$
Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q, x])*(a + b*x^n)^(p + 1)/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1})*(a + b*x^n)^(p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

$x^k, x] /; k \neq 1] /; \text{FreeQ}\{a, b, p, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[m]$

Rule 205

$\text{Int}[(a + (b \cdot x)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2] \cdot \text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}\{a, b, x\} \&\& \text{PosQ}[a/b]$

Rule 1168

$\text{Int}[(d + (e \cdot x)^2)/((a + (c \cdot x)^4), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[a \cdot c, 2]\}, \text{Dist}[(d \cdot q + a \cdot e)/(2 \cdot a \cdot c), \text{Int}[(q + c \cdot x^2)/(a + c \cdot x^4), x], x] + \text{Dist}[(d \cdot q - a \cdot e)/(2 \cdot a \cdot c), \text{Int}[(q - c \cdot x^2)/(a + c \cdot x^4), x], x] /; \text{FreeQ}\{a, c, d, e, x\} \&\& \text{NeQ}[c \cdot d^2 + a \cdot e^2, 0] \&\& \text{NeQ}[c \cdot d^2 - a \cdot e^2, 0] \&\& \text{NegQ}[-(a \cdot c)]$

Rule 1162

$\text{Int}[(d + (e \cdot x)^2)/((a + (c \cdot x)^4), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[(2 \cdot d)/e, 2]\}, \text{Dist}[e/(2 \cdot c), \text{Int}[1/\text{Simp}[d/e + q \cdot x + x^2, x], x], x] + \text{Dist}[e/(2 \cdot c), \text{Int}[1/\text{Simp}[d/e - q \cdot x + x^2, x], x], x] /; \text{FreeQ}\{a, c, d, e, x\} \&\& \text{EqQ}[c \cdot d^2 - a \cdot e^2, 0] \&\& \text{PosQ}[d \cdot e]$

Rule 617

$\text{Int}[(a + (b \cdot x) + (c \cdot x)^2)^{-1}, x_Symbol] \rightarrow \text{With}\{q = 1 - 4 \cdot S\text{implify}[(a \cdot c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2 \cdot c \cdot x)/b], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \mid \mid \text{!RationalQ}[b^2 - 4 \cdot a \cdot c]) /; \text{FreeQ}\{a, b, c, x\} \&\& \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0]$

Rule 204

$\text{Int}[(a + (b \cdot x)^2)^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2] \cdot x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2] \cdot \text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b, x\} \&\& \text{PosQ}[a/b] \&\& (\text{LtQ}[a, 0] \mid \mid \text{LtQ}[b, 0])$

Rule 1165

$\text{Int}[(d + (e \cdot x)^2)/((a + (c \cdot x)^4), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[(-2 \cdot d)/e, 2]\}, \text{Dist}[e/(2 \cdot c \cdot q), \text{Int}[(q - 2 \cdot x)/\text{Simp}[d/e + q \cdot x - x^2, x], x], x] + \text{Dist}[e/(2 \cdot c \cdot q), \text{Int}[(q + 2 \cdot x)/\text{Simp}[d/e - q \cdot x - x^2, x], x], x] /; \text{FreeQ}\{a, c, d, e, x\} \&\& \text{EqQ}[c \cdot d^2 - a \cdot e^2, 0] \&\& \text{NegQ}[d \cdot e]$

Rule 628

Mathematica [A] time = 0.702842, size = 530, normalized size = 1.03

$$\frac{-256a^{11/4}b^{3/4}(a(f+x(g+x(h+ix)))-bx(c+x(d+ex)))}{(a+bx^4)^3} + \frac{32a^{7/4}b^{3/4}x(ag+ax(2h+3ix)+11bc+bx(10d+9ex))}{(a+bx^4)^2} + \frac{8a^{3/4}b^{3/4}x(7ag+3ax(4h+5ix)+77bc+15bx(4d+3ex))}{a+bx^4}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^4, x]

[Out] ((32*a^(7/4)*b^(3/4)*x*(11*b*c + a*g + b*x*(10*d + 9*e*x) + a*x*(2*h + 3*i*x)))/(a + b*x^4)^2 + (8*a^(3/4)*b^(3/4)*x*(77*b*c + 7*a*g + 15*b*x*(4*d + 3*e*x) + 3*a*x*(4*h + 5*i*x)))/(a + b*x^4) - (256*a^(11/4)*b^(3/4)*(-b*x*(c + x*(d + e*x))) + a*(f + x*(g + x*(h + i*x))))/(a + b*x^4)^3 - 6*(77*sqrt[2]*b^(3/2)*c + 80*a^(1/4)*b^(5/4)*d + 15*sqrt[2]*sqrt[a]*b*e + 7*sqrt[2]*a*sqrt[b]*g + 16*a^(5/4)*b^(1/4)*h + 5*sqrt[2]*a^(3/2)*i)*ArcTan[1 - (sqrt[2]*b^(1/4)*x)/a^(1/4)] + 6*(77*sqrt[2]*b^(3/2)*c - 80*a^(1/4)*b^(5/4)*d + 15*sqrt[2]*sqrt[a]*b*e + 7*sqrt[2]*a*sqrt[b]*g - 16*a^(5/4)*b^(1/4)*h + 5*sqrt[2]*a^(3/2)*i)*ArcTan[1 + (sqrt[2]*b^(1/4)*x)/a^(1/4)] + 3*sqrt[2]*(-77*b^(3/2)*c + 15*sqrt[a]*b*e - 7*a*sqrt[b]*g + 5*a^(3/2)*i)*Log[sqrt[a] - sqrt[2]*a^(1/4)*b^(1/4)*x + sqrt[b]*x^2] + 3*sqrt[2]*(77*b^(3/2)*c - 15*sqrt[a]*b*e + 7*a*sqrt[b]*g - 5*a^(3/2)*i)*Log[sqrt[a] + sqrt[2]*a^(1/4)*b^(1/4)*x + sqrt[b]*x^2])/(3072*a^(15/4)*b^(7/4))

Maple [A] time = 0.014, size = 767, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4, x)

[Out] (5/128*(a*i+3*b*e)/a^3*b*x^11+1/32*(a*h+5*b*d)/a^3*b*x^10+7/384*(a*g+11*b*c)/a^3*b*x^9+7/64*(a*i+3*b*e)/a^2*x^7+1/12/a^2*(a*h+5*b*d)*x^6+3/64/a^2*(a*g+11*b*c)*x^5-1/384*(5*a*i-113*b*e)/a/b*x^3-1/32*(a*h-11*b*d)/a/b*x^2-1/128*(7*a*g-51*b*c)/a/b*x-1/12*f/b)/(b*x^4+a)^3+7/512/b/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*g+77/512/a^4*c*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+7/512/b/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*g+77/512/a^4*c*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+7/1024/b/a^3*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*g+

$$\begin{aligned} & 77/1024/a^4*c*(1/b*a)^{(1/4)}*2^{(1/2)}*\ln((x^2+(1/b*a)^{(1/4)}*x*2^{(1/2)}+(1/b*a)^{(1/2)})/(x^2-(1/b*a)^{(1/4)}*x*2^{(1/2)}+(1/b*a)^{(1/2)}))+1/32/b/a^2/(a*b)^{(1/2)} \\ & *arctan(x^2*(b/a)^{(1/2)})*h+5/32/a^3*d/(a*b)^{(1/2)}*arctan(x^2*(b/a)^{(1/2)})+5/512/b^2/a^2/(1/b*a)^{(1/4)}*2^{(1/2)}*arctan(2^{(1/2)}/(1/b*a)^{(1/4)}*x+1)*i+15/512/a^3*e/b/(1/b*a)^{(1/4)}*2^{(1/2)}*arctan(2^{(1/2)}/(1/b*a)^{(1/4)}*x+1)+5/512/b^2/a^2/(1/b*a)^{(1/4)}*2^{(1/2)}*arctan(2^{(1/2)}/(1/b*a)^{(1/4)}*x-1)*i+15/512/a^3*e/b/(1/b*a)^{(1/4)}*2^{(1/2)}*arctan(2^{(1/2)}/(1/b*a)^{(1/4)}*x-1)+5/1024/b^2/a^2/(1/b*a)^{(1/4)}*2^{(1/2)}*ln((x^2-(1/b*a)^{(1/4)}*x*2^{(1/2)}+(1/b*a)^{(1/2)})/(x^2+(1/b*a)^{(1/4)}*x*2^{(1/2)}+(1/b*a)^{(1/2)}))*i+15/1024/a^3*e/b/(1/b*a)^{(1/4)}*2^{(1/2)}*ln((x^2-(1/b*a)^{(1/4)}*x*2^{(1/2)}+(1/b*a)^{(1/2)})/(x^2+(1/b*a)^{(1/4)}*x*2^{(1/2)}+(1/b*a)^{(1/2)})) \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10734, size = 992, normalized size = 1.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 5/1024*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4)))/(a/b)^(1/4))/(a^3*b^4) - sqrt(2)*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^4) + 5/1024*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4)))/(a/b)^(1/4))/(a^3*b^4) + sqrt(2)*(a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^4) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b*h + 77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4)))/(a/b)^(1/4))/(a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b*h + 77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4)))/(a/b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(15*a*b^2*i*x^11 + 45*b^3*x^11*e + 60*b^3*d*x^10 + 12*a*b^2*h*x^10 + 77*b^3*c*x^9 + 7*a*b^2*g*x^9 + 42*a^2*b*i*x^7 + 126*a*b^2*x^7*e + 160*a*b^2*d*x^6 + 32*a^2*b*h*x^6 + 198*a*b^2*c*x^5 + 18*a^2*b*g*x^5 - 5*a^3*i*x^3 + 113*a^2*b*x^3*e + 132*a^2*b*d*x^2 - 12*a^3*h*x^2 + 153*a^2*b*c*x - 21*a^3*g*x - 32*a^3*f)/(b*x^4 + a)^3*a^3*b)
```


$$3.209 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^4} dx$$

Optimal. Leaf size=534

$$\frac{4a(aj+2bf) - x(b(ag+11bc) + 2bx(ah+5bd) + 3bx^2(ai+3be))}{96a^2b^2(a+bx^4)^2} - \frac{\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})(7\sqrt{b}(ag+11bc))}{512\sqrt{2}a^{15/4}b^{7/4}}$$

[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + (b*f - a*j)*x^3))/(12*a*b*(a + b*x^4)^3) + (x*(7*(11*b*c + a*g) + 12*(5*b*d + a*h)*x + 15*(3*b*e + a*i)*x^2))/(384*a^3*b*(a + b*x^4)) - (4*a*(2*b*f + a*j) - x*(b*(11*b*c + a*g) + 2*b*(5*b*d + a*h)*x + 3*b*(3*b*e + a*i)*x^2))/(96*a^2*b^2*(a + b*x^4)^2) + ((5*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2)) - ((7*Sqrt[b]*(11*b*c + a*g) + 5*Sqrt[a]*(3*b*e + a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(7/4)) + ((7*Sqrt[b]*(11*b*c + a*g) + 5*Sqrt[a]*(3*b*e + a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(7/4)) - ((7*Sqrt[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e + a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(7/4)) + ((7*Sqrt[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e + a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(7/4))

Rubi [A] time = 0.824189, antiderivative size = 534, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1858, 1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{4a(aj+2bf) - x(b(ag+11bc) + 2bx(ah+5bd) + 3bx^2(ai+3be))}{96a^2b^2(a+bx^4)^2} - \frac{\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})(7\sqrt{b}(ag+11bc))}{512\sqrt{2}a^{15/4}b^{7/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^4, x]

[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + (b*f - a*j)*x^3))/(12*a*b*(a + b*x^4)^3) + (x*(7*(11*b*c + a*g) + 12*(5*b*d + a*h)*x + 15*(3*b*e + a*i)*x^2))/(384*a^3*b*(a + b*x^4)) - (4*a*(2*b*f + a*j) - x*(b*(11*b*c + a*g) + 2*b*(5*b*d + a*h)*x + 3*b*(3*b*e + a*i)*x^2))/(96*a^2*b^2*(a + b*x^4)^2)

$$\begin{aligned} &) + ((5*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2)) - ((\\ & 7*Sqrt[b]*(11*b*c + a*g) + 5*Sqrt[a]*(3*b*e + a*i))*ArcTan[1 - (Sqrt[2]*b^(\\ & 1/4)*x)/a^(1/4)]/(256*Sqrt[2]*a^(15/4)*b^(7/4)) + ((7*Sqrt[b]*(11*b*c + a* \\ & g) + 5*Sqrt[a]*(3*b*e + a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(256 \\ & *Sqrt[2]*a^(15/4)*b^(7/4)) - ((7*Sqrt[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e \\ & + a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2 \\ &]*a^(15/4)*b^(7/4)) + ((7*Sqrt[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e + a*i)) \\ & *Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15 \\ & /4)*b^(7/4)) \end{aligned}$$

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a +
b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Module[{q = Expon[Pq,
x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]* (a + b*x^n)^(p
+ 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
+ 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rule 275

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1168

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Mathematica [A] time = 0.553262, size = 555, normalized size = 1.04

$$\frac{256a^{11/4}(a^{2j}-ab(f+x(g+x(h+ix))))+b^2x(c+x(d+ex))}{(a+bx^4)^3} - \frac{32a^{7/4}(12a^{2j}-abx(g+x(2h+3ix))-b^2x(11c+x(10d+9ex)))}{(a+bx^4)^2} - 6\sqrt[4]{b} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right) (16a^{5/4}\sqrt[4]{b})$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^4, x]

[Out] ((8*a^(3/4)*b*x*(77*b*c + 7*a*g + 15*b*x*(4*d + 3*e*x) + 3*a*x*(4*h + 5*i*x)))/(a + b*x^4) - (32*a^(7/4)*(12*a^2*j - b^2*x*(11*c + x*(10*d + 9*e*x)) - a*b*x*(g + x*(2*h + 3*i*x)))/(a + b*x^4)^2 + (256*a^(11/4)*(a^2*j + b^2*x*(c + x*(d + e*x)) - a*b*(f + x*(g + x*(h + i*x))))/(a + b*x^4)^3 - 6*b^(1/4)*(77*Sqrt[2]*b^(3/2)*c + 80*a^(1/4)*b^(5/4)*d + 15*Sqrt[2]*Sqrt[a]*b*e + 7*Sqrt[2]*a*Sqrt[b]*g + 16*a^(5/4)*b^(1/4)*h + 5*Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 6*b^(1/4)*(77*Sqrt[2]*b^(3/2)*c - 80*a^(1/4)*b^(5/4)*d + 15*Sqrt[2]*Sqrt[a]*b*e + 7*Sqrt[2]*a*Sqrt[b]*g - 16*a^(5/4)*b^(1/4)*h + 5*Sqrt[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 3*Sqrt[2]*b^(1/4)*(-77*b^(3/2)*c + 15*Sqrt[a]*b*e - 7*a*Sqrt[b]*g + 5*a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 3*Sqrt[2]*b^(1/4)*(77*b^(3/2)*c - 15*Sqrt[a]*b*e + 7*a*Sqrt[b]*g - 5*a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(3072*a^(15/4)*b^2)

Maple [A] time = 0.014, size = 783, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x)

[Out] (5/128*(a*i+3*b*e)/a^3*b*x^11+1/32*(a*h+5*b*d)/a^3*b*x^10+7/384*(a*g+11*b*c)/a^3*b*x^9+7/64*(a*i+3*b*e)/a^2*x^7+1/12/a^2*(a*h+5*b*d)*x^6+3/64/a^2*(a*g+11*b*c)*x^5-1/8*j*x^4/b-1/384*(5*a*i-113*b*e)/a/b*x^3-1/32*(a*h-11*b*d)/a/b*x^2-1/128*(7*a*g-51*b*c)/a/b*x-1/24*(a*j+2*b*f)/b^2)/(b*x^4+a)^3+7/512/b/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*g+77/512/a^4*c*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+7/512/b/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*g+77/512/a^4*c*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+7/1024/b/a^3*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2))

$$\begin{aligned} & ((1/2)+(1/b*a)^{(1/2)})) * g + 77/1024/a^4 * c * (1/b*a)^{(1/4)} * 2^{(1/2)} * \ln((x^2+(1/b*a)^{(1/4)} * x * 2^{(1/2)} + (1/b*a)^{(1/2)}) / (x^2 - (1/b*a)^{(1/4)} * x * 2^{(1/2)} + (1/b*a)^{(1/2)})) \\ & + 1/32/b/a^2 / (a*b)^{(1/2)} * \arctan(x^2 * (b/a)^{(1/2)}) * h + 5/32/a^3 * d / (a*b)^{(1/2)} * \arctan(x^2 * (b/a)^{(1/2)}) + 5/512/b^2/a^2 / (1/b*a)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (1/b*a)^{(1/4)} * x + 1) * i \\ & + 15/512/a^3 * e / b / (1/b*a)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (1/b*a)^{(1/4)} * x + 1) + 5/512/b^2/a^2 / (1/b*a)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (1/b*a)^{(1/4)} * x - 1) * i \\ & + 15/512/a^3 * e / b / (1/b*a)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (1/b*a)^{(1/4)} * x - 1) + 5/1024/b^2/a^2 / (1/b*a)^{(1/4)} * 2^{(1/2)} * \ln((x^2 - (1/b*a)^{(1/4)} * x * 2^{(1/2)} + (1/b*a)^{(1/2)}) / (x^2 + (1/b*a)^{(1/4)} * x * 2^{(1/2)} + (1/b*a)^{(1/2)})) * i \\ & + 15/1024/a^3 * e / b / (1/b*a)^{(1/4)} * 2^{(1/2)} * \ln((x^2 - (1/b*a)^{(1/4)} * x * 2^{(1/2)} + (1/b*a)^{(1/2)}) / (x^2 + (1/b*a)^{(1/4)} * x * 2^{(1/2)} + (1/b*a)^{(1/2)})) \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10909, size = 1035, normalized size = 1.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 5/1024*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4)))/(a/b)^(1/4))/(a^3*b^4) - sqrt(2)*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^4) + 5/1024*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4)))/(a/b)^(1/4))/(a^3*b^4) + sqrt(2)*(a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^4) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b*h + 77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4)))/(a/b)^(1/4))/(a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b*h + 77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4)))/(a/b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(15*a*b^3*i*x^11 + 45*b^4*x^11*e + 60*b^4*d*x^10 + 12*a*b^3*h*x^10 + 77*b^4*c*x^9 + 7*a*b^3*g*x^9 + 42*a^2*b^2*i*x^7 + 126*a*b^3*x^7*e + 160*a*b^3*d*x^6 + 32*a^2*b^2*h*x^6 + 198*a*b^3*c*x^5 + 18*a^2*b^2*g*x^5 - 48*a^3*b*j*x^4 - 5*a^3*b*i*x^3 + 113*a^2*b^2*x^3*e + 132*a^2*b^2*d*x^2 - 12*a^3*b*h*x^2 + 153*a^2*b^2*c*x - 21*a^3*b*g*x - 32*a^3*b*f - 16*a^4*j)/((b*x^4 + a)^3*a^3*b^2)
```

$$3.210 \quad \int \frac{c+dx}{\sqrt{a+bx^4}} dx$$

Optimal. Leaf size=121

$$\frac{c(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}$$

[Out] (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) + (c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[a + b*x^4])

Rubi [A] time = 0.0678343, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1885, 220, 275, 217, 206}

$$\frac{c(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)/Sqrt[a + b*x^4], x]

[Out] (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) + (c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[a + b*x^4])

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]* (a + b*x^n)^p, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2]]*EllipticF[2*ArcTan[q*x]
```


, 1/2))/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{c + dx}{\sqrt{a + bx^4}} dx &= \int \left(\frac{c}{\sqrt{a + bx^4}} + \frac{dx}{\sqrt{a + bx^4}} \right) dx \\
 &= c \int \frac{1}{\sqrt{a + bx^4}} dx + d \int \frac{x}{\sqrt{a + bx^4}} dx \\
 &= \frac{c(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}} + \frac{1}{2} d \operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}} dx, x, x^2\right) \\
 &= \frac{c(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}} + \frac{1}{2} d \operatorname{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{x^2}{\sqrt{a+bx^4}}\right) \\
 &= \frac{d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} + \frac{c(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}}
 \end{aligned}$$

Mathematica [C] time = 0.0469559, size = 79, normalized size = 0.65

$$\frac{cx\sqrt{\frac{bx^4}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{bx^4}{a}\right)}{\sqrt{a+bx^4}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{bx^4}}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/Sqrt[a + b*x^4], x]

[Out] (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]]/(2*Sqrt[b])) + (c*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -(b*x^4)/a])/Sqrt[a + b*x^4]

Maple [C] time = 0.073, size = 96, normalized size = 0.8

$$\frac{d}{2} \ln\left(x^2\sqrt{b} + \sqrt{bx^4 + a}\right) \frac{1}{\sqrt{b}} + c \sqrt{1 - ix^2\sqrt{b} \frac{1}{\sqrt{a}}} \sqrt{1 + ix^2\sqrt{b} \frac{1}{\sqrt{a}}} \text{EllipticF}\left(x \sqrt{i\sqrt{b} \frac{1}{\sqrt{a}}}, i\right) \frac{1}{\sqrt{i\sqrt{b} \frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)/(b*x^4+a)^(1/2), x)

[Out] 1/2*d*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2)+c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2), I)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{bx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x^4+a)^(1/2), x, algorithm="maxima")

[Out] integrate((d*x + c)/sqrt(b*x^4 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx + c}{\sqrt{bx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")

[Out] integral((d*x + c)/sqrt(b*x^4 + a), x)

Sympy [C] time = 1.84028, size = 61, normalized size = 0.5

$$\frac{d \operatorname{asinh}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{cx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x**4+a)**(1/2),x)

[Out] d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{bx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate((d*x + c)/sqrt(b*x^4 + a), x)

$$3.211 \quad \int \frac{c+dx}{\sqrt{a-bx^4}} dx$$

Optimal. Leaf size=87

$$\frac{\sqrt[4]{ac}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), -1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}} + \frac{d \tan^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}$$

[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]])/(2*Sqrt[b]) + (a^(1/4)*c*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(1/4)*Sqrt[a - b*x^4])

Rubi [A] time = 0.0621967, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1885, 224, 221, 275, 217, 203}

$$\frac{\sqrt[4]{ac}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle| -1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}} + \frac{d \tan^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)/Sqrt[a - b*x^4], x]

[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]])/(2*Sqrt[b]) + (a^(1/4)*c*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(1/4)*Sqrt[a - b*x^4])

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]* (a + b*x^n)^p, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := Dist[Sqrt[1 + (b*x^4)/a]/Sqrt[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[b/a] && !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := Simp[EllipticF[ArcSin[(Rt[-b,
4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p, x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx}{\sqrt{a - bx^4}} dx &= \int \left(\frac{c}{\sqrt{a - bx^4}} + \frac{dx}{\sqrt{a - bx^4}} \right) dx \\
&= c \int \frac{1}{\sqrt{a - bx^4}} dx + d \int \frac{x}{\sqrt{a - bx^4}} dx \\
&= \frac{1}{2} d \operatorname{Subst} \left(\int \frac{1}{\sqrt{a - bx^2}} dx, x, x^2 \right) + \frac{\left(c \sqrt{1 - \frac{bx^4}{a}} \right) \int \frac{1}{\sqrt{1 - \frac{bx^4}{a}}} dx}{\sqrt{a - bx^4}} \\
&= \frac{\sqrt[4]{ac} \sqrt{1 - \frac{bx^4}{a}} F \left(\sin^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right) \middle| -1 \right)}{\sqrt[4]{b} \sqrt{a - bx^4}} + \frac{1}{2} d \operatorname{Subst} \left(\int \frac{1}{1 + bx^2} dx, x, \frac{x^2}{\sqrt{a - bx^4}} \right) \\
&= \frac{d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a - bx^4}} \right)}{2\sqrt{b}} + \frac{\sqrt[4]{ac} \sqrt{1 - \frac{bx^4}{a}} F \left(\sin^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right) \middle| -1 \right)}{\sqrt[4]{b} \sqrt{a - bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.0472358, size = 81, normalized size = 0.93

$$\frac{cx\sqrt{1-\frac{bx^4}{a}} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; \frac{bx^4}{a}\right)}{\sqrt{a-bx^4}} + \frac{d \tan^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/Sqrt[a - b*x^4], x]

[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]])/(2*Sqrt[b]) + (c*x*Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (b*x^4)/a])/Sqrt[a - b*x^4]

Maple [A] time = 0.015, size = 90, normalized size = 1.

$$\frac{d}{2} \arctan\left(x^2\sqrt{b}\frac{1}{\sqrt{-bx^4+a}}\right) \frac{1}{\sqrt{b}} + c\sqrt{1-x^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{\sqrt{b}\frac{1}{\sqrt{a}}}, i\right) \frac{1}{\sqrt{\sqrt{b}\frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{-bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)/(-b*x^4+a)^(1/2), x)

[Out] 1/2*d*arctan(x^2*b^(1/2)/(-b*x^4+a)^(1/2))/b^(1/2)+c/(1/a^(1/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2), I)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{-bx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-b*x^4+a)^(1/2), x, algorithm="maxima")

[Out] integrate((d*x + c)/sqrt(-b*x^4 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-bx^4 + a}(dx + c)}{bx^4 - a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-b*x^4+a)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-b*x^4 + a)*(d*x + c)/(b*x^4 - a), x)

Sympy [A] time = 1.96213, size = 97, normalized size = 1.11

$$d \left(\left(\begin{array}{l} -\frac{i \operatorname{acosh}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2\sqrt{b}} \\ \frac{\operatorname{asin}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2\sqrt{b}} \end{array} \right. \begin{array}{l} \text{for } \frac{|bx^4|}{|a|} > 1 \\ \text{otherwise} \end{array} \right) + \frac{cx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{bx^4 e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-b*x**4+a)**(1/2),x)

[Out] d*Piecewise((-I*acosh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), Abs(b*x**4)/Abs(a) > 1), (asin(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), True)) + c*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(5/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{-bx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-b*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate((d*x + c)/sqrt(-b*x^4 + a), x)

$$3.212 \quad \int \frac{c+dx}{\sqrt{-a+bx^4}} dx$$

Optimal. Leaf size=89

$$\frac{\sqrt[4]{ac}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), -1\right)}{\sqrt[4]{b}\sqrt{bx^4-a}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{bx^4-a}}\right)}{2\sqrt{b}}$$

[Out] (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[-a + b*x^4]])/(2*Sqrt[b]) + (a^(1/4)*c*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(1/4)*Sqrt[-a + b*x^4])

Rubi [A] time = 0.0599722, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1885, 224, 221, 275, 217, 206}

$$\frac{\sqrt[4]{ac}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle| -1\right)}{\sqrt[4]{b}\sqrt{bx^4-a}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{bx^4-a}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)/Sqrt[-a + b*x^4], x]

[Out] (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[-a + b*x^4]])/(2*Sqrt[b]) + (a^(1/4)*c*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(1/4)*Sqrt[-a + b*x^4])

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := Dist[Sqrt[1 + (b*x^4)/a]/Sqrt[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[b/a] && !GtQ[a, 0]
```


Rule 221

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := Simp[EllipticF[ArcSin[(Rt[-b, 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p, x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{c + dx}{\sqrt{-a + bx^4}} dx &= \int \left(\frac{c}{\sqrt{-a + bx^4}} + \frac{dx}{\sqrt{-a + bx^4}} \right) dx \\
 &= c \int \frac{1}{\sqrt{-a + bx^4}} dx + d \int \frac{x}{\sqrt{-a + bx^4}} dx \\
 &= \frac{1}{2} d \operatorname{Subst} \left(\int \frac{1}{\sqrt{-a + bx^2}} dx, x, x^2 \right) + \frac{\left(c \sqrt{1 - \frac{bx^4}{a}} \right) \int \frac{1}{\sqrt{1 - \frac{bx^4}{a}}} dx}{\sqrt{-a + bx^4}} \\
 &= \frac{\sqrt[4]{ac} \sqrt{1 - \frac{bx^4}{a}} F \left(\sin^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right) \middle| -1 \right)}{\sqrt[4]{b} \sqrt{-a + bx^4}} + \frac{1}{2} d \operatorname{Subst} \left(\int \frac{1}{1 - bx^2} dx, x, \frac{x^2}{\sqrt{-a + bx^4}} \right) \\
 &= \frac{d \tanh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{-a + bx^4}} \right)}{2\sqrt{b}} + \frac{\sqrt[4]{ac} \sqrt{1 - \frac{bx^4}{a}} F \left(\sin^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right) \middle| -1 \right)}{\sqrt[4]{b} \sqrt{-a + bx^4}}
 \end{aligned}$$

Mathematica [C] time = 0.0420012, size = 83, normalized size = 0.93

$$\frac{cx\sqrt{1-\frac{bx^4}{a}} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; \frac{bx^4}{a}\right)}{\sqrt{bx^4-a}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{bx^4-a}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/Sqrt[-a + b*x^4], x]

[Out] (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[-a + b*x^4]])/(2*Sqrt[b]) + (c*x*Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (b*x^4)/a])/Sqrt[-a + b*x^4]

Maple [A] time = 0.019, size = 95, normalized size = 1.1

$$\frac{d}{2} \ln\left(x^2\sqrt{b} + \sqrt{bx^4 - a}\right) \frac{1}{\sqrt{b}} + c \sqrt{1 + x^2\sqrt{b} \frac{1}{\sqrt{a}}} \sqrt{1 - x^2\sqrt{b} \frac{1}{\sqrt{a}}} \text{EllipticF}\left(x \sqrt{-\sqrt{b} \frac{1}{\sqrt{a}}}, i\right) \frac{1}{\sqrt{-\sqrt{b} \frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{bx^4 - a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)/(b*x^4-a)^(1/2), x)

[Out] 1/2*d*ln(x^2*b^(1/2)+(b*x^4-a)^(1/2))/b^(1/2)+c/(-1/a^(1/2)*b^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)/(b*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/2)*b^(1/2))^(1/2), I)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{bx^4 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x^4-a)^(1/2), x, algorithm="maxima")

[Out] integrate((d*x + c)/sqrt(b*x^4 - a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx + c}{\sqrt{bx^4 - a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)/(b*x^4-a)^(1/2),x, algorithm="fricas")`

[Out] `integral((d*x + c)/sqrt(b*x^4 - a), x)`

Sympy [A] time = 1.96492, size = 92, normalized size = 1.03

$$d \left(\begin{array}{l} \frac{\operatorname{acosh}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2\sqrt{b}} \quad \text{for } \frac{|bx^4|}{|a|} > 1 \\ i \operatorname{asin}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right) \\ -\frac{i \operatorname{asin}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2\sqrt{b}} \quad \text{otherwise} \end{array} \right) - \frac{icx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{bx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)/(b*x**4-a)**(1/2),x)`

[Out] `d*Piecewise((acosh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), Abs(b*x**4)/Abs(a) > 1), (-I*asin(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), True)) - I*c*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**4/a)/(4*sqrt(a)*gamma(5/4))`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{bx^4 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)/(b*x^4-a)^(1/2),x, algorithm="giac")`

[Out] `integrate((d*x + c)/sqrt(b*x^4 - a), x)`

$$3.213 \quad \int \frac{c+dx}{\sqrt{-a-bx^4}} dx$$

Optimal. Leaf size=127

$$\frac{c(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{-a-bx^4}} + \frac{d \tan^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{-a-bx^4}}\right)}{2\sqrt{b}}$$

[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[-a - b*x^4]])/(2*Sqrt[b]) + (c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[-a - b*x^4])

Rubi [A] time = 0.0721643, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1885, 220, 275, 217, 203}

$$\frac{c(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{-a-bx^4}} + \frac{d \tan^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{-a-bx^4}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)/Sqrt[-a - b*x^4], x]

[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[-a - b*x^4]])/(2*Sqrt[b]) + (c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[-a - b*x^4])

Rule 1885

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*a + b*x^n]^p, {j, 0, n/2 - 1}, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]

, 1/2))/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p, x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{c + dx}{\sqrt{-a - bx^4}} dx &= \int \left(\frac{c}{\sqrt{-a - bx^4}} + \frac{dx}{\sqrt{-a - bx^4}} \right) dx \\
 &= c \int \frac{1}{\sqrt{-a - bx^4}} dx + d \int \frac{x}{\sqrt{-a - bx^4}} dx \\
 &= \frac{c(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{-a - bx^4}} + \frac{1}{2} d \operatorname{Subst}\left(\int \frac{1}{\sqrt{-a - bx^2}} dx, x, x^2\right) \\
 &= \frac{c(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{-a - bx^4}} + \frac{1}{2} d \operatorname{Subst}\left(\int \frac{1}{1 + bx^2} dx, x, \frac{x^2}{\sqrt{-a - bx^4}}\right) \\
 &= \frac{d \tan^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{-a - bx^4}}\right)}{2\sqrt{b}} + \frac{c(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{-a - bx^4}}
 \end{aligned}$$

Mathematica [C] time = 0.0451096, size = 85, normalized size = 0.67

$$\frac{cx\sqrt{\frac{bx^4}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{bx^4}{a}\right)}{\sqrt{-a - bx^4}} + \frac{d \tan^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{-a - bx^4}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/Sqrt[-a - b*x^4], x]

[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[-a - b*x^4]])/(2*Sqrt[b]) + (c*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -(b*x^4)/a])/Sqrt[-a - b*x^4]

Maple [C] time = 0.018, size = 101, normalized size = 0.8

$$\frac{d}{2} \arctan\left(x^2\sqrt{b}\frac{1}{\sqrt{-bx^4 - a}}\right) \frac{1}{\sqrt{b}} + c\sqrt{1 + ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1 - ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{-i\sqrt{b}\frac{1}{\sqrt{a}}}, i\right) \frac{1}{\sqrt{-i\sqrt{b}\frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{-bx^4 - a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)/(-b*x^4-a)^(1/2), x)

[Out] 1/2*d*arctan(x^2*b^(1/2)/(-b*x^4-a)^(1/2))/b^(1/2)+c/(-I/a^(1/2)*b^(1/2))^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(-b*x^4-a)^(1/2)*EllipticF(x*(-I/a^(1/2)*b^(1/2))^(1/2), I)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{-bx^4 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-b*x^4-a)^(1/2), x, algorithm="maxima")

[Out] integrate((d*x + c)/sqrt(-b*x^4 - a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-bx^4 - a}(dx + c)}{bx^4 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-b*x^4-a)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-b*x^4 - a)*(d*x + c)/(b*x^4 + a), x)

Sympy [C] time = 1.96542, size = 66, normalized size = 0.52

$$-\frac{id \operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} - \frac{icx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-b*x**4-a)**(1/2),x)

[Out] -I*d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) - I*c*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{-bx^4 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(-b*x^4-a)^(1/2),x, algorithm="giac")

[Out] integrate((d*x + c)/sqrt(-b*x^4 - a), x)

$$3.214 \quad \int \frac{c+dx+ex^2}{\sqrt{a+bx^4}} dx$$

Optimal. Leaf size=257

$$\frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} \left(\frac{\sqrt{bc}}{\sqrt{a}} + e \right) \text{EllipticF} \left(2 \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right), \frac{1}{2} \right)}{2b^{3/4}\sqrt{a+bx^4}} - \frac{\sqrt[4]{ae}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E \left(2 \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right) \right)}{b^{3/4}\sqrt{a+bx^4}}$$

[Out] (e*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (a^(1/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*((Sqrt[b]*c)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])

Rubi [A] time = 0.12442, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1885, 275, 217, 206, 1198, 220, 1196}

$$\frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} \left(\frac{\sqrt{bc}}{\sqrt{a}} + e \right) F \left(2 \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right) \middle| \frac{1}{2} \right)}{2b^{3/4}\sqrt{a+bx^4}} - \frac{\sqrt[4]{ae}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E \left(2 \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right) \middle| \frac{1}{2} \right)}{b^{3/4}\sqrt{a+bx^4}} + \dots$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/Sqrt[a + b*x^4], x]

[Out] (e*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (a^(1/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*((Sqrt[b]*c)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])

Rule 1885

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, p},

$x] \&\& \text{PolyQ}[\text{Pq}, x] \&\& \text{IGtQ}[n/2, 0] \&\& \text{!PolyQ}[\text{Pq}, x^{(n/2)}]$

Rule 275

$\text{Int}[(x_)^{(m_.)} * ((a_) + (b_.) * (x_)^{(n_)})^{(p_)}, x_Symbol] \text{ :> With}[\{k = \text{GCD}[m + 1, n]\}, \text{Dist}[1/k, \text{Subst}[\text{Int}[x^{((m + 1)/k - 1) * (a + b * x^{(n/k)})^p}, x], x, x^k], x] \text{ /; } k \neq 1] \text{ /; FreeQ}[\{a, b, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[m]$

Rule 217

$\text{Int}[1/\text{Sqrt}[(a_) + (b_.) * (x_)^2], x_Symbol] \text{ :> Subst}[\text{Int}[1/(1 - b * x^2), x], x, x/\text{Sqrt}[a + b * x^2]] \text{ /; FreeQ}[\{a, b\}, x] \&\& \text{!GtQ}[a, 0]$

Rule 206

$\text{Int}[(a_) + (b_.) * (x_)^2]^{-1}, x_Symbol] \text{ :> Simp}[(1 * \text{ArcTanh}[(\text{Rt}[-b, 2] * x) / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] * \text{Rt}[-b, 2]), x] \text{ /; FreeQ}[\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \text{ || LtQ}[b, 0])$

Rule 1198

$\text{Int}[(d_) + (e_.) * (x_)^2] / \text{Sqrt}[(a_) + (c_.) * (x_)^4], x_Symbol] \text{ :> With}[\{q = \text{Rt}[c/a, 2]\}, \text{Dist}[(e + d * q) / q, \text{Int}[1/\text{Sqrt}[a + c * x^4], x], x] - \text{Dist}[e/q, \text{Int}[(1 - q * x^2) / \text{Sqrt}[a + c * x^4], x], x] \text{ /; NeQ}[e + d * q, 0] \text{ /; FreeQ}[\{a, c, d, e\}, x] \&\& \text{PosQ}[c/a]$

Rule 220

$\text{Int}[1/\text{Sqrt}[(a_) + (b_.) * (x_)^4], x_Symbol] \text{ :> With}[\{q = \text{Rt}[b/a, 4]\}, \text{Simp}[(1 + q^2 * x^2) * \text{Sqrt}[(a + b * x^4) / (a * (1 + q^2 * x^2)^2)] * \text{EllipticF}[2 * \text{ArcTan}[q * x], 1/2]) / (2 * q * \text{Sqrt}[a + b * x^4]), x] \text{ /; FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[b/a]$

Rule 1196

$\text{Int}[(d_) + (e_.) * (x_)^2] / \text{Sqrt}[(a_) + (c_.) * (x_)^4], x_Symbol] \text{ :> With}[\{q = \text{Rt}[c/a, 4]\}, -\text{Simp}[(d * x * \text{Sqrt}[a + c * x^4]) / (a * (1 + q^2 * x^2)), x] + \text{Simp}[(d * (1 + q^2 * x^2) * \text{Sqrt}[(a + c * x^4) / (a * (1 + q^2 * x^2)^2)] * \text{EllipticE}[2 * \text{ArcTan}[q * x], 1/2]) / (q * \text{Sqrt}[a + c * x^4]), x] \text{ /; EqQ}[e + d * q^2, 0] \text{ /; FreeQ}[\{a, c, d, e\}, x] \&\& \text{PosQ}[c/a]$

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2}{\sqrt{a + bx^4}} dx &= \int \left(\frac{dx}{\sqrt{a + bx^4}} + \frac{c + ex^2}{\sqrt{a + bx^4}} \right) dx \\
&= d \int \frac{x}{\sqrt{a + bx^4}} dx + \int \frac{c + ex^2}{\sqrt{a + bx^4}} dx \\
&= \frac{1}{2} d \operatorname{Subst} \left(\int \frac{1}{\sqrt{a + bx^2}} dx, x, x^2 \right) - \frac{(\sqrt{ae}) \int \frac{1 - \frac{\sqrt{bx^2}}{\sqrt{a}}}{\sqrt{a + bx^4}} dx}{\sqrt{b}} + \left(c + \frac{\sqrt{ae}}{\sqrt{b}} \right) \int \frac{1}{\sqrt{a + bx^4}} dx \\
&= \frac{ex\sqrt{a + bx^4}}{\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} - \frac{\sqrt[4]{ae}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a + bx^4}{(\sqrt{a} + \sqrt{bx^2})^2}} E \left(2 \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right) \middle| \frac{1}{2} \right)}{b^{3/4}\sqrt{a + bx^4}} + \frac{(\sqrt{bc} + \sqrt{ae})(\sqrt{a} + \sqrt{bx^2})}{2\sqrt{b}} \\
&= \frac{ex\sqrt{a + bx^4}}{\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{d \tanh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a + bx^4}} \right)}{2\sqrt{b}} - \frac{\sqrt[4]{ae}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a + bx^4}{(\sqrt{a} + \sqrt{bx^2})^2}} E \left(2 \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right) \middle| \frac{1}{2} \right)}{b^{3/4}\sqrt{a + bx^4}} + \frac{(\sqrt{bc} + \sqrt{ae})(\sqrt{a} + \sqrt{bx^2})}{2\sqrt{b}}
\end{aligned}$$

Mathematica [C] time = 0.103442, size = 131, normalized size = 0.51

$$\frac{cx\sqrt{\frac{bx^4}{a} + 1} {}_2F_1 \left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{bx^4}{a} \right)}{\sqrt{a + bx^4}} + \frac{d \tanh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a + bx^4}} \right)}{2\sqrt{b}} + \frac{ex^3 \sqrt{\frac{bx^4}{a} + 1} {}_2F_1 \left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -\frac{bx^4}{a} \right)}{3\sqrt{a + bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/Sqrt[a + b*x^4], x]

[Out] (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]]/(2*Sqrt[b])) + (c*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -(b*x^4)/a])/Sqrt[a + b*x^4] + (e*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -(b*x^4)/a])/(3*Sqrt[a + b*x^4])

Maple [C] time = 0.005, size = 193, normalized size = 0.8

$$ie\sqrt{a}\sqrt{1 - ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1 + ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\left(\operatorname{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}, i\right) - \operatorname{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}, i\right)\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4 + a}}\frac{1}{\sqrt{b}} + \frac{d}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)`

[Out] $I * e * a^{1/2} / (I/a^{1/2} * b^{1/2})^{1/2} * (1 - I/a^{1/2} * b^{1/2} * x^2)^{1/2} * (1 + I/a^{1/2} * b^{1/2} * x^2)^{1/2} / (b * x^4 + a)^{1/2} / b^{1/2} * (\text{EllipticF}(x * (I/a^{1/2}) * b^{1/2})^{1/2}, I) - \text{EllipticE}(x * (I/a^{1/2}) * b^{1/2})^{1/2}, I) + 1/2 * d * \ln(x^2 * b^{1/2} + (b * x^4 + a)^{1/2}) / b^{1/2} + c / (I/a^{1/2} * b^{1/2})^{1/2} * (1 - I/a^{1/2} * b^{1/2} * x^2)^{1/2} * (1 + I/a^{1/2} * b^{1/2} * x^2)^{1/2} / (b * x^4 + a)^{1/2} * \text{EllipticF}(x * (I/a^{1/2}) * b^{1/2})^{1/2}, I)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{\sqrt{bx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")`

[Out] `integrate((e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2 + dx + c}{\sqrt{bx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")`

[Out] `integral((e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)`

Sympy [C] time = 2.15659, size = 102, normalized size = 0.4

$$\frac{d \operatorname{asinh}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{cx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{ex^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{3}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)

[Out] d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{\sqrt{bx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate((e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)

$$3.215 \quad \int \frac{ag - bgx^4}{(a + bx^4)^{3/2}} dx$$

Optimal. Leaf size=14

$$\frac{gx}{\sqrt{a + bx^4}}$$

[Out] (g*x)/Sqrt[a + b*x^4]

Rubi [A] time = 0.0055367, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {383}

$$\frac{gx}{\sqrt{a + bx^4}}$$

Antiderivative was successfully verified.

[In] Int[(a*g - b*g*x^4)/(a + b*x^4)^(3/2), x]

[Out] (g*x)/Sqrt[a + b*x^4]

Rule 383

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> S imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]

Rubi steps

$$\int \frac{ag - bgx^4}{(a + bx^4)^{3/2}} dx = \frac{gx}{\sqrt{a + bx^4}}$$

Mathematica [A] time = 0.0086365, size = 14, normalized size = 1.

$$\frac{gx}{\sqrt{a + bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*g - b*g*x^4)/(a + b*x^4)^(3/2),x]

[Out] (g*x)/Sqrt[a + b*x^4]

Maple [A] time = 0.043, size = 13, normalized size = 0.9

$$gx \frac{1}{\sqrt{bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b*g*x^4+a*g)/(b*x^4+a)^(3/2),x)

[Out] g*x/(b*x^4+a)^(1/2)

Maxima [A] time = 1.05881, size = 16, normalized size = 1.14

$$\frac{gx}{\sqrt{bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*g*x^4+a*g)/(b*x^4+a)^(3/2),x, algorithm="maxima")

[Out] g*x/sqrt(b*x^4 + a)

Fricas [A] time = 1.3868, size = 28, normalized size = 2.

$$\frac{gx}{\sqrt{bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*g*x^4+a*g)/(b*x^4+a)^(3/2),x, algorithm="fricas")

[Out] $g*x/\sqrt{b*x^4 + a}$

Sympy [C] time = 7.55679, size = 80, normalized size = 5.71

$$\frac{gx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{3}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} - \frac{bgx^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\frac{5}{4}, \frac{3}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*g*x**4+a*g)/(b*x**4+a)**(3/2),x)`

[Out] $g*x*\gamma(1/4)*\text{hyper}((1/4, 3/2), (5/4,), b*x**4*\exp_polar(I*pi)/a)/(4*\sqrt{a}*\gamma(5/4)) - b*g*x**5*\gamma(5/4)*\text{hyper}((5/4, 3/2), (9/4,), b*x**4*\exp_polar(I*pi)/a)/(4*a**(3/2)*\gamma(9/4))$

Giac [A] time = 1.07768, size = 16, normalized size = 1.14

$$\frac{gx}{\sqrt{bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*g*x^4+a*g)/(b*x^4+a)^(3/2),x, algorithm="giac")`

[Out] $g*x/\sqrt{b*x^4 + a}$

$$3.216 \quad \int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}} dx$$

Optimal. Leaf size=29

$$\frac{2agx + ex^2}{2a\sqrt{a + bx^4}}$$

[Out] (2*a*g*x + e*x^2)/(2*a*Sqrt[a + b*x^4])

Rubi [A] time = 0.0230818, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {1856}

$$\frac{2agx + ex^2}{2a\sqrt{a + bx^4}}$$

Antiderivative was successfully verified.

[In] Int[(a*g + e*x - b*g*x^4)/(a + b*x^4)^(3/2),x]

[Out] (2*a*g*x + e*x^2)/(2*a*Sqrt[a + b*x^4])

Rule 1856

Int[(P4_)/((a_) + (b_.)*(x_)^4)^(3/2), x_Symbol] := With[{d = Coeff[P4, x, 0], e = Coeff[P4, x, 1], f = Coeff[P4, x, 3], g = Coeff[P4, x, 4]}, -Simp[(a*f + 2*a*g*x - b*e*x^2)/(2*a*b*Sqrt[a + b*x^4]), x] /; EqQ[b*d + a*g, 0] /; FreeQ[{a, b}, x] && PolyQ[P4, x, 4] && EqQ[Coeff[P4, x, 2], 0]

Rubi steps

$$\int \frac{ag + ex - bgx^4}{(a + bx^4)^{3/2}} dx = \frac{2agx + ex^2}{2a\sqrt{a + bx^4}}$$

Mathematica [A] time = 0.0368414, size = 27, normalized size = 0.93

$$\frac{x(2ag + ex)}{2a\sqrt{a + bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*g + e*x - b*g*x^4)/(a + b*x^4)^(3/2), x]

[Out] (x*(2*a*g + e*x))/(2*a*Sqrt[a + b*x^4])

Maple [A] time = 0.042, size = 24, normalized size = 0.8

$$\frac{x(2ag + ex)}{2a} \frac{1}{\sqrt{bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b*g*x^4+a*g+e*x)/(b*x^4+a)^(3/2), x)

[Out] 1/2*x*(2*a*g+e*x)/(b*x^4+a)^(1/2)/a

Maxima [A] time = 1.07298, size = 34, normalized size = 1.17

$$\frac{2agx + ex^2}{2\sqrt{bx^4 + aa}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*g*x^4+a*g+e*x)/(b*x^4+a)^(3/2), x, algorithm="maxima")

[Out] 1/2*(2*a*g*x + e*x^2)/(sqrt(b*x^4 + a)*a)

Fricas [A] time = 1.34993, size = 74, normalized size = 2.55

$$\frac{\sqrt{bx^4 + a}(2agx + ex^2)}{2(abx^4 + a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*g*x^4+a*g+e*x)/(b*x^4+a)^(3/2), x, algorithm="fricas")

[Out] $\frac{1}{2}\sqrt{bx^4 + a}(2agx + ex^2)/(abx^4 + a^2)$

Sympy [C] time = 11.0955, size = 104, normalized size = 3.59

$$\frac{gx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{3}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} - \frac{bgx^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\frac{5}{4}, \frac{3}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{ex^2}{2a^{\frac{3}{2}}\sqrt{1 + \frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*g*x**4+a*g+e*x)/(b*x**4+a)**(3/2),x)

[Out] $g*x*\gamma(1/4)*\text{hyper}((1/4, 3/2), (5/4,), b*x**4*\exp_polar(I*\pi)/a)/(4*\sqrt{a}*\gamma(5/4)) - b*g*x**5*\gamma(5/4)*\text{hyper}((5/4, 3/2), (9/4,), b*x**4*\exp_polar(I*\pi)/a)/(4*a**(3/2)*\gamma(9/4)) + e*x**2/(2*a**(3/2)*\sqrt{1 + b*x**4/a})$

Giac [A] time = 1.0737, size = 43, normalized size = 1.48

$$\frac{x\left(\frac{2g}{a^2b^4} + \frac{xe}{a^3b^4}\right)}{64\sqrt{bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*g*x^4+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="giac")

[Out] $-1/64*x*(2g/(a^2*b^4) + x*e/(a^3*b^4))/\sqrt{b*x^4 + a}$

$$3.217 \quad \int \frac{ag+fx^3-bgx^4}{(a+bx^4)^{3/2}} dx$$

Optimal. Leaf size=25

$$-\frac{f-2bgx}{2b\sqrt{a+bx^4}}$$

[Out] $-(f - 2*b*g*x)/(2*b*Sqrt[a + b*x^4])$

Rubi [A] time = 0.0282155, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1856}

$$-\frac{f-2bgx}{2b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a*g + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2), x]$

[Out] $-(f - 2*b*g*x)/(2*b*Sqrt[a + b*x^4])$

Rule 1856

$\text{Int}[(P4_)/((a_)+(b_)*(x_)^4)^(3/2), x_Symbol] := \text{With}[\{d = \text{Coeff}[P4, x, 0], e = \text{Coeff}[P4, x, 1], f = \text{Coeff}[P4, x, 3], g = \text{Coeff}[P4, x, 4]\}, -\text{Simp}[(a*f + 2*a*g*x - b*e*x^2)/(2*a*b*Sqrt[a + b*x^4]), x] /; \text{EqQ}[b*d + a*g, 0]] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[P4, x, 4] \&\& \text{EqQ}[\text{Coeff}[P4, x, 2], 0]$

Rubi steps

$$\int \frac{ag+fx^3-bgx^4}{(a+bx^4)^{3/2}} dx = -\frac{f-2bgx}{2b\sqrt{a+bx^4}}$$

Mathematica [A] time = 0.0366762, size = 27, normalized size = 1.08

$$\frac{2bgx-f}{2b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*g + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]

[Out] (-f + 2*b*g*x)/(2*b*Sqrt[a + b*x^4])

Maple [A] time = 0.043, size = 24, normalized size = 1.

$$\frac{2bgx - f}{2b} \frac{1}{\sqrt{bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b*g*x^4+f*x^3+a*g)/(b*x^4+a)^(3/2),x)

[Out] 1/2*(2*b*g*x-f)/b/(b*x^4+a)^(1/2)

Maxima [A] time = 1.08228, size = 31, normalized size = 1.24

$$\frac{2bgx - f}{2\sqrt{bx^4 + ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*g*x^4+f*x^3+a*g)/(b*x^4+a)^(3/2),x, algorithm="maxima")

[Out] 1/2*(2*b*g*x - f)/(sqrt(b*x^4 + a)*b)

Fricas [A] time = 1.50489, size = 69, normalized size = 2.76

$$\frac{\sqrt{bx^4 + a}(2bgx - f)}{2(b^2x^4 + ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*g*x^4+f*x^3+a*g)/(b*x^4+a)^(3/2),x, algorithm="fricas")

[Out] $\frac{1}{2}\sqrt{bx^4 + a}(2bgx - f)/(b^2x^4 + a^2b)$

Sympy [A] time = 10.4613, size = 109, normalized size = 4.36

$$f \left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases} \right) + \frac{gx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{3}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} - \frac{bgx^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\frac{5}{4}, \frac{3}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*g*x**4+f*x**3+a*g)/(b*x**4+a)**(3/2), x)`

[Out] `f*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), True)) + g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) - b*g*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4))`

Giac [A] time = 1.08497, size = 30, normalized size = 1.2

$$\frac{2gx - \frac{f}{b}}{2\sqrt{bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*g*x^4+f*x^3+a*g)/(b*x^4+a)^(3/2), x, algorithm="giac")`

[Out] $\frac{1}{2}(2gx - f/b)/\sqrt{bx^4 + a}$

$$3.218 \quad \int \frac{ag+ex+fx^3-bgx^4}{(a+bx^4)^{3/2}} dx$$

Optimal. Leaf size=38

$$-\frac{-2abgx + af - bex^2}{2ab\sqrt{a + bx^4}}$$

[Out] $-(a*f - 2*a*b*g*x - b*e*x^2)/(2*a*b*\text{Sqrt}[a + b*x^4])$

Rubi [A] time = 0.0298362, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {1856}

$$-\frac{-2abgx + af - bex^2}{2ab\sqrt{a + bx^4}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a*g + e*x + f*x^3 - b*g*x^4)/(a + b*x^4)^{(3/2)}, x]$

[Out] $-(a*f - 2*a*b*g*x - b*e*x^2)/(2*a*b*\text{Sqrt}[a + b*x^4])$

Rule 1856

$\text{Int}[(P4_)/((a_)+(b_)*(x_)^4)^{(3/2)}, x_Symbol] := \text{With}[\{d = \text{Coeff}[P4, x, 0], e = \text{Coeff}[P4, x, 1], f = \text{Coeff}[P4, x, 3], g = \text{Coeff}[P4, x, 4]\}, -\text{Simp}[(a*f + 2*a*g*x - b*e*x^2)/(2*a*b*\text{Sqrt}[a + b*x^4]), x] /; \text{EqQ}[b*d + a*g, 0] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[P4, x, 4] \&\& \text{EqQ}[\text{Coeff}[P4, x, 2], 0]$

Rubi steps

$$\int \frac{ag + ex + fx^3 - bgx^4}{(a + bx^4)^{3/2}} dx = -\frac{af - 2abgx - bex^2}{2ab\sqrt{a + bx^4}}$$

Mathematica [A] time = 0.0415902, size = 38, normalized size = 1.

$$\frac{2abgx - af + bex^2}{2ab\sqrt{a + bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*g + e*x + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]

[Out] $(-(a*f) + 2*a*b*g*x + b*e*x^2)/(2*a*b*\text{Sqrt}[a + b*x^4])$

Maple [A] time = 0.042, size = 35, normalized size = 0.9

$$\frac{2abgx + bex^2 - af}{2ab} \frac{1}{\sqrt{bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b*g*x^4+f*x^3+a*g+e*x)/(b*x^4+a)^(3/2),x)

[Out] $1/2*(2*a*b*g*x+b*e*x^2-a*f)/a/b/(b*x^4+a)^(1/2)$

Maxima [A] time = 1.07834, size = 59, normalized size = 1.55

$$\frac{\sqrt{bx^4 + a}(2abgx + bex^2 - af)}{2(ab^2x^4 + a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*g*x^4+f*x^3+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="maxima")

[Out] $1/2*\text{sqrt}(b*x^4 + a)*(2*a*b*g*x + b*e*x^2 - a*f)/(a*b^2*x^4 + a^2*b)$

Fricas [A] time = 1.30205, size = 93, normalized size = 2.45

$$\frac{\sqrt{bx^4 + a}(2abgx + bex^2 - af)}{2(ab^2x^4 + a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*g*x^4+f*x^3+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="fricas")

[Out] $\frac{1}{2}\sqrt{bx^4 + a}(2abgx + bex^2 - af)/(ab^2x^4 + a^2b)$

Sympy [A] time = 13.7758, size = 133, normalized size = 3.5

$$f \left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases} \right) + \frac{gx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{3}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} - \frac{bgx^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\frac{5}{4}, \frac{3}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{ex^2}{2a^{\frac{3}{2}}\sqrt{1 + \frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*g*x**4+f*x**3+a*g+e*x)/(b*x**4+a)**(3/2),x)`

[Out] `f*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), True)) + g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) - b*g*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4)) + e*x**2/(2*a**(3/2)*sqrt(1 + b*x**4/a))`

Giac [A] time = 1.09686, size = 42, normalized size = 1.11

$$\frac{\left(2g + \frac{xe}{a}\right)x - \frac{f}{b}}{2\sqrt{bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*g*x^4+f*x^3+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="giac")`

[Out] $\frac{1}{2} * ((2*g + x*e/a)*x - f/b) / \sqrt{bx^4 + a}$

$$3.219 \quad \int \frac{-1+x^4}{(1+x^4)^{3/2}} dx$$

Optimal. Leaf size=12

$$-\frac{x}{\sqrt{x^4+1}}$$

[Out] -(x/Sqrt[1 + x^4])

Rubi [A] time = 0.0031383, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {383}

$$-\frac{x}{\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^4)/(1 + x^4)^(3/2), x]

[Out] -(x/Sqrt[1 + x^4])

Rule 383

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
 imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
 b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]

Rubi steps

$$\int \frac{-1+x^4}{(1+x^4)^{3/2}} dx = -\frac{x}{\sqrt{1+x^4}}$$

Mathematica [A] time = 0.0050631, size = 12, normalized size = 1.

$$-\frac{x}{\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^4)/(1 + x^4)^(3/2),x]

[Out] -(x/Sqrt[1 + x^4])

Maple [A] time = 0.004, size = 11, normalized size = 0.9

$$-x \frac{1}{\sqrt{x^4 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4-1)/(x^4+1)^(3/2),x)

[Out] -x/(x^4+1)^(1/2)

Maxima [A] time = 1.54444, size = 14, normalized size = 1.17

$$-\frac{x}{\sqrt{x^4 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-1)/(x^4+1)^(3/2),x, algorithm="maxima")

[Out] -x/sqrt(x^4 + 1)

Fricas [A] time = 1.30916, size = 24, normalized size = 2.

$$-\frac{x}{\sqrt{x^4 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-1)/(x^4+1)^(3/2),x, algorithm="fricas")

[Out] $-x/\sqrt{x^4 + 1}$

Sympy [C] time = 2.61457, size = 58, normalized size = 4.83

$$\frac{x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\frac{5}{4}, \frac{3}{2} \middle| \frac{5}{4}, x^4 e^{i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)} - \frac{x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{3}{2} \middle| \frac{5}{4}, x^4 e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**4-1)/(x**4+1)**(3/2),x)`

[Out] `x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), x**4*exp_polar(I*pi))/(4*gamma(9/4)) - x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), x**4*exp_polar(I*pi))/(4*gamma(5/4))`

Giac [A] time = 1.06654, size = 14, normalized size = 1.17

$$-\frac{x}{\sqrt{x^4 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4-1)/(x^4+1)^(3/2),x, algorithm="giac")`

[Out] $-x/\sqrt{x^4 + 1}$

$$3.220 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}} dx$$

Optimal. Leaf size=385

$$\frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right) \left(\frac{5\sqrt{b}(3bc-ag)}{\sqrt{a}} - 9ai + 15be\right)}{30b^{7/4}\sqrt{a+bx^4}} + \frac{(2bd - ah) \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right)}{4b^{3/2}} + \frac{x\sqrt{a+bx^4}}{5b^{3/2}}$$

[Out] (f*Sqrt[a + b*x^4])/(2*b) + (g*x*Sqrt[a + b*x^4])/(3*b) + (h*x^2*Sqrt[a + b*x^4])/(4*b) + (i*x^3*Sqrt[a + b*x^4])/(5*b) + ((5*b*e - 3*a*i)*x*Sqrt[a + b*x^4])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) - (a^(1/4)*(5*b*e - 3*a*i)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(15*b*e + (5*Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 9*a*i)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(30*b^(7/4)*Sqrt[a + b*x^4])

Rubi [A] time = 0.423748, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1885, 1819, 1815, 641, 217, 206, 1888, 1198, 220, 1196}

$$\frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right) \left(\frac{5\sqrt{b}(3bc-ag)}{\sqrt{a}} - 9ai + 15be\right)}{30b^{7/4}\sqrt{a+bx^4}} + \frac{(2bd - ah) \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right)}{4b^{3/2}} + \frac{x\sqrt{a+bx^4}}{5b^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/Sqrt[a + b*x^4], x]

[Out] (f*Sqrt[a + b*x^4])/(2*b) + (g*x*Sqrt[a + b*x^4])/(3*b) + (h*x^2*Sqrt[a + b*x^4])/(4*b) + (i*x^3*Sqrt[a + b*x^4])/(5*b) + ((5*b*e - 3*a*i)*x*Sqrt[a + b*x^4])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) - (a^(1/4)*(5*b*e - 3*a*i)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(15*b*e + (5*Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 9*a*i)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(30*b^(7/4)*Sqrt[a + b*x^4])

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq,
x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*a + b*x^n]^p, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, p},
x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 1819

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/(m
+ 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/
Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
```

)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]

Rule 1198

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + 220x^6}{\sqrt{a + bx^4}} dx &= \int \left(\frac{x(d + fx^2 + hx^4)}{\sqrt{a + bx^4}} + \frac{c + ex^2 + gx^4 + 220x^6}{\sqrt{a + bx^4}} \right) dx \\
&= \int \frac{x(d + fx^2 + hx^4)}{\sqrt{a + bx^4}} dx + \int \frac{c + ex^2 + gx^4 + 220x^6}{\sqrt{a + bx^4}} dx \\
&= \frac{44x^3\sqrt{a + bx^4}}{b} + \frac{1}{2} \text{Subst} \left(\int \frac{d + fx + hx^2}{\sqrt{a + bx^2}} dx, x, x^2 \right) + \frac{\int \frac{5bc - 5(132a - b)}{\sqrt{a + bx^4}} dx}{5b} \\
&= \frac{gx\sqrt{a + bx^4}}{3b} + \frac{hx^2\sqrt{a + bx^4}}{4b} + \frac{44x^3\sqrt{a + bx^4}}{b} + \frac{\int \frac{5b(3bc - ag) - 15b(132a - b)}{\sqrt{a + bx^4}} dx}{15b^2} \\
&= \frac{f\sqrt{a + bx^4}}{2b} + \frac{gx\sqrt{a + bx^4}}{3b} + \frac{hx^2\sqrt{a + bx^4}}{4b} + \frac{44x^3\sqrt{a + bx^4}}{b} + \frac{(\sqrt{a + bx^4})(132a - 3bc)}{b^2} \\
&= \frac{f\sqrt{a + bx^4}}{2b} + \frac{gx\sqrt{a + bx^4}}{3b} + \frac{hx^2\sqrt{a + bx^4}}{4b} + \frac{44x^3\sqrt{a + bx^4}}{b} - \frac{(132a - 3bc)\sqrt{a + bx^4}}{b^2} \\
&= \frac{f\sqrt{a + bx^4}}{2b} + \frac{gx\sqrt{a + bx^4}}{3b} + \frac{hx^2\sqrt{a + bx^4}}{4b} + \frac{44x^3\sqrt{a + bx^4}}{b} - \frac{(132a - 3bc)\sqrt{a + bx^4}}{b^2}
\end{aligned}$$

Mathematica [C] time = 0.223516, size = 281, normalized size = 0.73

$$-20\sqrt{bx}\sqrt{\frac{bx^4}{a}} + 1(ag - 3bc) {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{bx^4}{a}\right) + 30bd\sqrt{a + bx^4} \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a + bx^4}}\right) + 4\sqrt{bx^3}\sqrt{\frac{bx^4}{a}} + 1(5be - 3ai) {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -\frac{bx^4}{a}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/Sqrt[a + b*x^4], x]

[Out] (30*a*Sqrt[b]*f + 20*a*Sqrt[b]*g*x + 15*a*Sqrt[b]*h*x^2 + 12*a*Sqrt[b]*i*x^3 + 30*b^(3/2)*f*x^4 + 20*b^(3/2)*g*x^5 + 15*b^(3/2)*h*x^6 + 12*b^(3/2)*i*x^7 + 30*b*d*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 15*a*h*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 20*Sqrt[b]*(-3*b*c + a*g)*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -(b*x^4)/a]) + 4*Sqrt[b]*(5*b*e - 3*a*i)*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -(b*x^4)/a])/(60*b^(3/2)*Sqrt[a + b*x^4])

Maple [C] time = 0.017, size = 516, normalized size = 1.3

$$\frac{ix^3}{5b} \sqrt{bx^4 + a} - \frac{3i}{5} ia^{\frac{3}{2}} \sqrt{1 - ix^2 \sqrt{b} \frac{1}{\sqrt{a}}} \sqrt{1 + ix^2 \sqrt{b} \frac{1}{\sqrt{a}}} \text{EllipticF} \left(x \sqrt{i \sqrt{b} \frac{1}{\sqrt{a}}}, i \right) b^{-\frac{3}{2}} \frac{1}{\sqrt{i \sqrt{b} \frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{bx^4 + a}} + \frac{3i}{5} ia^{\frac{3}{2}} \sqrt{1 - ix^2 \sqrt{b} \frac{1}{\sqrt{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)`

[Out] $\frac{1}{5} i x^3 (b x^4 + a)^{1/2} / b - \frac{3}{5} I i / b^{3/2} a^{3/2} / (I/a^{1/2} b^{1/2})^{1/2} (1 - I/a^{1/2} b^{1/2} x^2)^{1/2} * (1 + I/a^{1/2} b^{1/2} x^2)^{1/2} / (b x^4 + a)^{1/2} * \text{EllipticF}(x * (I/a^{1/2} b^{1/2})^{1/2}, I) + \frac{3}{5} I i / b^{3/2} a^{3/2} / (I/a^{1/2} b^{1/2})^{1/2} (1 - I/a^{1/2} b^{1/2} x^2)^{1/2} * (1 + I/a^{1/2} b^{1/2} x^2)^{1/2} / (b x^4 + a)^{1/2} * \text{EllipticE}(x * (I/a^{1/2} b^{1/2})^{1/2}, I) + \frac{1}{4} h x^2 * (b x^4 + a)^{1/2} / b - \frac{1}{4} h a / b^{3/2} * \ln(x^2 * b^{1/2} + (b x^4 + a)^{1/2}) + \frac{1}{3} g x * (b x^4 + a)^{1/2} / b - \frac{1}{3} g / b * a / (I/a^{1/2} b^{1/2})^{1/2} (1 - I/a^{1/2} b^{1/2} x^2)^{1/2} * (1 + I/a^{1/2} b^{1/2} x^2)^{1/2} / (b x^4 + a)^{1/2} * \text{EllipticF}(x * (I/a^{1/2} b^{1/2})^{1/2}, I) + \frac{1}{2} f * (b x^4 + a)^{1/2} / b + I * e * a^{1/2} / (I/a^{1/2} b^{1/2})^{1/2} (1 - I/a^{1/2} b^{1/2} x^2)^{1/2} * (1 + I/a^{1/2} b^{1/2} x^2)^{1/2} / (b x^4 + a)^{1/2} / b^{1/2} * (\text{EllipticF}(x * (I/a^{1/2} b^{1/2})^{1/2}, I) - \text{EllipticE}(x * (I/a^{1/2} b^{1/2})^{1/2}, I)) + \frac{1}{2} d * \ln(x^2 * b^{1/2} + (b x^4 + a)^{1/2}) / b^{1/2} + c / (I/a^{1/2} b^{1/2})^{1/2} (1 - I/a^{1/2} b^{1/2} x^2)^{1/2} * (1 + I/a^{1/2} b^{1/2} x^2)^{1/2} / (b x^4 + a)^{1/2} * \text{EllipticF}(x * (I/a^{1/2} b^{1/2})^{1/2}, I)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ix^6 + hx^5 + gx^4 + fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")`

[Out] `integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ix^6 + hx^5 + gx^4 + fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")

[Out] integral((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)

Sympy [A] time = 5.73435, size = 260, normalized size = 0.68

$$\frac{\sqrt{ah}x^2\sqrt{1+\frac{bx^4}{a}}}{4b} - \frac{ah \operatorname{asinh}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}} + f \left(\left(\frac{x^4}{4\sqrt{a}} \right) \text{ for } b = 0 \right) + \frac{d \operatorname{asinh}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{cx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{ex^3\Gamma\left(\frac{3}{4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)

[Out] sqrt(a)*h*x**2*sqrt(1 + b*x**4/a)/(4*b) - a*h*asinh(sqrt(b)*x**2/sqrt(a))/(4*b**(3/2)) + f*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)/(2*b), True)) + d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + g*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + i*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(11/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ix^6 + hx^5 + gx^4 + fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)
```

$$3.221 \quad \int \frac{1+x}{1+x^5} dx$$

Optimal. Leaf size=109

$$-\frac{1}{5}\sqrt[5]{-1}\left(1 + \sqrt[5]{-1}\right)\log\left(\sqrt[5]{-1} - x\right) + \frac{1}{5}(-1)^{4/5}\left(1 - (-1)^{4/5}\right)\log\left(-x - (-1)^{4/5}\right) + \frac{1}{5}(-1)^{2/5}\left(1 - (-1)^{2/5}\right)\log\left(x + (-1)^{2/5}\right) -$$

[Out] $-\left((-1)^{(1/5)}*(1 + (-1)^{(1/5)})*\text{Log}[(-1)^{(1/5)} - x]\right)/5 + \left((-1)^{(4/5)}*(1 - (-1)^{(4/5)})*\text{Log}[-(-1)^{(4/5)} - x]\right)/5 + \left((-1)^{(2/5)}*(1 - (-1)^{(2/5)})*\text{Log}[(-1)^{(2/5)} + x]\right)/5 - \left((-1)^{(3/5)}*(1 + (-1)^{(3/5)})*\text{Log}[-(-1)^{(3/5)} + x]\right)/5$

Rubi [A] time = 0.0641871, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1586, 2068}

$$-\frac{1}{5}\sqrt[5]{-1}\left(1 + \sqrt[5]{-1}\right)\log\left(\sqrt[5]{-1} - x\right) + \frac{1}{5}(-1)^{4/5}\left(1 - (-1)^{4/5}\right)\log\left(-x - (-1)^{4/5}\right) + \frac{1}{5}(-1)^{2/5}\left(1 - (-1)^{2/5}\right)\log\left(x + (-1)^{2/5}\right) -$$

Antiderivative was successfully verified.

[In] Int[(1 + x)/(1 + x^5), x]

[Out] $-\left((-1)^{(1/5)}*(1 + (-1)^{(1/5)})*\text{Log}[(-1)^{(1/5)} - x]\right)/5 + \left((-1)^{(4/5)}*(1 - (-1)^{(4/5)})*\text{Log}[-(-1)^{(4/5)} - x]\right)/5 + \left((-1)^{(2/5)}*(1 - (-1)^{(2/5)})*\text{Log}[(-1)^{(2/5)} + x]\right)/5 - \left((-1)^{(3/5)}*(1 + (-1)^{(3/5)})*\text{Log}[-(-1)^{(3/5)} + x]\right)/5$

Rule 1586

Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] :> Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 2068

Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1], c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[1/a^(3*p), Int[ExpandIntegrand[1/((a - b*x)^p/(a^5 - b^5*x^5)^p), x], x], x] /; NeQ[a, 0] && EqQ[c, b^2/a] && EqQ[d, b^3/a^2] && EqQ[e, b^4/a^3] /; FreeQ[p, x] && PolyQ[P4, x, 4] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{1+x}{1+x^5} dx &= \int \frac{1}{1-x+x^2-x^3+x^4} dx \\ &= \int \left(\frac{-1+(-1)^{4/5}}{5(-1+\sqrt[5]{-1}x)} + \frac{-1-(-1)^{3/5}}{5(-1-(-1)^{2/5}x)} + \frac{-1+(-1)^{2/5}}{5(-1+(-1)^{3/5}x)} + \frac{-1-\sqrt[5]{-1}}{5(-1-(-1)^{4/5}x)} \right) dx \\ &= -\frac{1}{5}\sqrt[5]{-1}(1+\sqrt[5]{-1})\log(\sqrt[5]{-1}-x) + \frac{1}{5}(-1)^{4/5}(1-(-1)^{4/5})\log(-(-1)^{4/5}-x) + \frac{1}{5}(-1)^{2/5}(1-(-1)^{2/5})\log(\end{aligned}$$

Mathematica [C] time = 0.0098981, size = 51, normalized size = 0.47

$$\text{RootSum}\left[\#1^4 - \#1^3 + \#1^2 - \#1 + 1 \&, \frac{\log(x - \#1)}{4\#1^3 - 3\#1^2 + 2\#1 - 1} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x)/(1 + x^5), x]

[Out] RootSum[1 - #1 + #1^2 - #1^3 + #1^4 & , Log[x - #1]/(-1 + 2*#1 - 3*#1^2 + 4*#1^3) &]

Maple [B] time = 0.019, size = 173, normalized size = 1.6

$$-\frac{\sqrt{5} \ln(-x\sqrt{5} + 2x^2 - x + 2)}{10} + \frac{1}{\sqrt{10 - 2\sqrt{5}}} \arctan\left(\frac{-\sqrt{5} + 4x - 1}{\sqrt{10 - 2\sqrt{5}}}\right) + \frac{\sqrt{5}}{5\sqrt{10 - 2\sqrt{5}}} \arctan\left(\frac{-\sqrt{5} + 4x - 1}{\sqrt{10 - 2\sqrt{5}}}\right) + \frac{\sqrt{5} \ln}{\sqrt{10 - 2\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x)/(x^5+1), x)

[Out] -1/10*5^(1/2)*ln(-x*5^(1/2)+2*x^2-x+2)+1/(10-2*5^(1/2))^(1/2)*arctan((-5^(1/2)+4*x-1)/(10-2*5^(1/2))^(1/2))+1/5/(10-2*5^(1/2))^(1/2)*arctan((-5^(1/2)+4*x-1)/(10-2*5^(1/2))^(1/2))*5^(1/2)+1/10*5^(1/2)*ln(x*5^(1/2)+2*x^2-x+2)+1/(10+2*5^(1/2))^(1/2)*arctan((5^(1/2)+4*x-1)/(10+2*5^(1/2))^(1/2))-1/5/(10+2*5^(1/2))^(1/2)*arctan((5^(1/2)+4*x-1)/(10+2*5^(1/2))^(1/2))*5^(1/2)

$$\begin{aligned} & 2/25*\sqrt{5} - 1/5))^2 - 12)*(\sqrt{5} - 5*\sqrt{-2/25*\sqrt{5} - 1/5}) - 5/4* \\ & \sqrt{-3/100*(\sqrt{5} + 5*\sqrt{-2/25*\sqrt{5} - 1/5}))^2 - 1/50*(\sqrt{5} + 5* \\ & \sqrt{-2/25*\sqrt{5} - 1/5})*(\sqrt{5} - 5*\sqrt{-2/25*\sqrt{5} - 1/5}) - 3/100*(\\ & \sqrt{5} - 5*\sqrt{-2/25*\sqrt{5} - 1/5}))^2*((3*\sqrt{5} + 15*\sqrt{-2/25*\sqrt{5} \\ & 5) - 1/5) + 8)*(\sqrt{5} - 5*\sqrt{-2/25*\sqrt{5} - 1/5})) + 8*\sqrt{5} + 40*\sqrt{ \\ & t(-2/25*\sqrt{5} - 1/5) + 36) + 22*x + 9/2*\sqrt{5} + 45/2*\sqrt{-2/25*\sqrt{5} \\ & - 1/5} + 2) \end{aligned}$$

Sympy [A] time = 0.372474, size = 36, normalized size = 0.33

$$\text{RootSum}\left(125t^4 - 5t + 1, \left(t \mapsto t \log\left(\frac{375t^3}{11} + \frac{100t^2}{11} + \frac{45t}{11} + x - \frac{14}{11}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x**5+1),x)

[Out] RootSum(125*_t**4 - 5*_t + 1, Lambda(_t, _t*log(375*_t**3/11 + 100*_t**2/11 + 45*_t/11 + x - 14/11)))

Giac [A] time = 1.10738, size = 136, normalized size = 1.25

$$\frac{1}{5} \sqrt{-2\sqrt{5}+5} \arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right) + \frac{1}{5} \sqrt{2\sqrt{5}+5} \arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right) - \frac{1}{10} \sqrt{5} \log\left(x^2 - \frac{1}{2}x(\sqrt{5}+1) + 1\right) + \frac{1}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x^5+1),x, algorithm="giac")

[Out] 1/5*sqrt(-2*sqrt(5) + 5)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10)) + 1/5*sqrt(2*sqrt(5) + 5)*arctan((4*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 10)) - 1/10*sqrt(5)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) + 1/10*sqrt(5)*log(x^2 + 1/2*x*(sqrt(5) - 1) + 1)

$$3.222 \quad \int \frac{1-x}{1-x^5} dx$$

Optimal. Leaf size=109

$$-\frac{1}{5}(-1)^{2/5} (1 - (-1)^{2/5}) \log((-1)^{2/5} - x) + \frac{1}{5}(-1)^{3/5} (1 + (-1)^{3/5}) \log(-x - (-1)^{3/5}) + \frac{1}{5}\sqrt[5]{-1} (1 + \sqrt[5]{-1}) \log(x + \sqrt[5]{-1}) -$$

[Out] $-\frac{1}{5}(-1)^{2/5}(1 - (-1)^{2/5})\text{Log}[(-1)^{2/5} - x]/5 + \frac{1}{5}(-1)^{3/5}(1 + (-1)^{3/5})\text{Log}[-(-1)^{3/5} - x]/5 + \frac{1}{5}(-1)^{1/5}(1 + (-1)^{1/5})\text{Log}[(-1)^{1/5} + x]/5 - \frac{1}{5}(-1)^{4/5}(1 - (-1)^{4/5})\text{Log}[-(-1)^{4/5} + x]/5$

Rubi [A] time = 0.0407916, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1586, 2068}

$$-\frac{1}{5}(-1)^{2/5} (1 - (-1)^{2/5}) \log((-1)^{2/5} - x) + \frac{1}{5}(-1)^{3/5} (1 + (-1)^{3/5}) \log(-x - (-1)^{3/5}) + \frac{1}{5}\sqrt[5]{-1} (1 + \sqrt[5]{-1}) \log(x + \sqrt[5]{-1}) -$$

Antiderivative was successfully verified.

[In] Int[(1 - x)/(1 - x^5), x]

[Out] $-\frac{1}{5}(-1)^{2/5}(1 - (-1)^{2/5})\text{Log}[(-1)^{2/5} - x]/5 + \frac{1}{5}(-1)^{3/5}(1 + (-1)^{3/5})\text{Log}[-(-1)^{3/5} - x]/5 + \frac{1}{5}(-1)^{1/5}(1 + (-1)^{1/5})\text{Log}[(-1)^{1/5} + x]/5 - \frac{1}{5}(-1)^{4/5}(1 - (-1)^{4/5})\text{Log}[-(-1)^{4/5} + x]/5$

Rule 1586

Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] :> Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 2068

Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1], c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[1/a^(3*p), Int[ExpandIntegrand[1/((a - b*x)^p/(a^5 - b^5*x^5)^p), x], x], x] /; NeQ[a, 0] && EqQ[c, b^2/a] && EqQ[d, b^3/a^2] && EqQ[e, b^4/a^3] /; FreeQ[p, x] && PolyQ[P4, x, 4] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{1-x}{1-x^5} dx &= \int \frac{1}{1+x+x^2+x^3+x^4} dx \\ &= \int \left(\frac{1-(-1)^{4/5}}{5(1+\sqrt[5]{-1}x)} + \frac{1+(-1)^{3/5}}{5(1-(-1)^{2/5}x)} + \frac{1-(-1)^{2/5}}{5(1+(-1)^{3/5}x)} + \frac{1+\sqrt[5]{-1}}{5(1-(-1)^{4/5}x)} \right) dx \\ &= -\frac{1}{5}(-1)^{2/5}(1-(-1)^{2/5}) \log((-1)^{2/5}-x) + \frac{1}{5}(-1)^{3/5}(1+(-1)^{3/5}) \log(-(-1)^{3/5}-x) + \frac{1}{5}\sqrt[5]{-1}(1+\sqrt[5]{-1}) \log \end{aligned}$$

Mathematica [C] time = 0.0087526, size = 47, normalized size = 0.43

$$\text{RootSum} \left[\#1^4 + \#1^3 + \#1^2 + \#1 + 1 \&, \frac{\log(x - \#1)}{4\#1^3 + 3\#1^2 + 2\#1 + 1} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x)/(1 - x^5), x]

[Out] RootSum[1 + #1 + #1^2 + #1^3 + #1^4 & , Log[x - #1]/(1 + 2*#1 + 3*#1^2 + 4*#1^3) &]

Maple [B] time = 0.017, size = 169, normalized size = 1.6

$$-\frac{\sqrt{5} \ln(-x\sqrt{5} + 2x^2 + x + 2)}{10} + \frac{1}{\sqrt{10 + 2\sqrt{5}}} \arctan\left(\frac{1 + 4x - \sqrt{5}}{\sqrt{10 + 2\sqrt{5}}}\right) - \frac{\sqrt{5}}{5\sqrt{10 + 2\sqrt{5}}} \arctan\left(\frac{1 + 4x - \sqrt{5}}{\sqrt{10 + 2\sqrt{5}}}\right) + \frac{\sqrt{5} \ln(x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-x)/(-x^5+1), x)

[Out] -1/10*5^(1/2)*ln(-x*5^(1/2)+2*x^2+x+2)+1/(10+2*5^(1/2))^(1/2)*arctan((1+4*x-5^(1/2))/(10+2*5^(1/2))^(1/2))-1/5/(10+2*5^(1/2))^(1/2)*arctan((1+4*x-5^(1/2))/(10+2*5^(1/2))^(1/2))*5^(1/2)+1/10*5^(1/2)*ln(x*5^(1/2)+2*x^2+x+2)+1/(10-2*5^(1/2))^(1/2)*arctan((1+4*x+5^(1/2))/(10-2*5^(1/2))^(1/2))+1/5/(10-2*5^(1/2))^(1/2)*arctan((1+4*x+5^(1/2))/(10-2*5^(1/2))^(1/2))*5^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-1}{x^5-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)/(-x^5+1),x, algorithm="maxima")

[Out] integrate((x - 1)/(x^5 - 1), x)

Fricas [B] time = 8.55768, size = 2493, normalized size = 22.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)/(-x^5+1),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/10*(\sqrt{5} - \sqrt{2*\sqrt{5} - 5})*\log(3/8*(\sqrt{5} + \sqrt{2*\sqrt{5} - 5}))^3 + 1/8*(3*\sqrt{5} + 3*\sqrt{2*\sqrt{5} - 5} - 8)*(\sqrt{5} - \sqrt{2*\sqrt{5} - 5})^2 + 3/8*((\sqrt{5} + \sqrt{2*\sqrt{5} - 5})^2 - 12)*(\sqrt{5} - \sqrt{2*\sqrt{5} - 5}) + 11*x - 1 - 1/10*(\sqrt{5} + \sqrt{2*\sqrt{5} - 5})*\log(-3/8*(\sqrt{5} + \sqrt{2*\sqrt{5} - 5}))^3 - (\sqrt{5} + \sqrt{2*\sqrt{5} - 5})^2 + 11*x - 9/2*\sqrt{5} - 9/2*\sqrt{2*\sqrt{5} - 5} + 14) + 1/10*(\sqrt{5} + 5*\sqrt{-3/100*(\sqrt{5} + \sqrt{2*\sqrt{5} - 5}))^2 - 1/50*(\sqrt{5} + \sqrt{2*\sqrt{5} - 5})*(\sqrt{5} - \sqrt{2*\sqrt{5} - 5}) - 3/100*(\sqrt{5} - \sqrt{2*\sqrt{5} - 5})^2) * \log(-1/8*(3*\sqrt{5} + 3*\sqrt{2*\sqrt{5} - 5} - 8)*(\sqrt{5} - \sqrt{2*\sqrt{5} - 5})^2 + (\sqrt{5} + \sqrt{2*\sqrt{5} - 5})^2 - 3/8*((\sqrt{5} + \sqrt{2*\sqrt{5} - 5})^2 - 12)*(\sqrt{5} - \sqrt{2*\sqrt{5} - 5}) + 5/4*\sqrt{-3/100*(\sqrt{5} + \sqrt{2*\sqrt{5} - 5}))^2 - 1/50*(\sqrt{5} + \sqrt{2*\sqrt{5} - 5})*(\sqrt{5} - \sqrt{2*\sqrt{5} - 5}) - 3/100*(\sqrt{5} - \sqrt{2*\sqrt{5} - 5})^2)*((3*\sqrt{5} + 3*\sqrt{2*\sqrt{5} - 5} - 8)*(\sqrt{5} - \sqrt{2*\sqrt{5} - 5}) - 8*\sqrt{5} - 8*\sqrt{2*\sqrt{5} - 5} + 36) + 22*x + 9/2*\sqrt{5} + 9/2*\sqrt{2*\sqrt{5} - 5} - 2) + 1/10*(\sqrt{5} - 5*\sqrt{-3/100*(\sqrt{5} + \sqrt{2*\sqrt{5} - 5}))^2 - 1/50*(\sqrt{5} + \sqrt{2*\sqrt{5} - 5})*(\sqrt{5} - \sqrt{2*\sqrt{5} - 5}) - 3/100*(\sqrt{5} - \sqrt{2*\sqrt{5} - 5})^2) * \log(-1/8*(3*\sqrt{5} + 3*\sqrt{2*\sqrt{5} - 5} - 8)*(\sqrt{5} - \sqrt{2*\sqrt{5} - 5})^2 + (\sqrt{5} + \sqrt{2*\sqrt{5} - 5})^2 - 3/8*((\sqrt{5} + \sqrt{2*\sqrt{5} - 5})^2 - 12)*(\sqrt{5} - \sqrt{2*\sqrt{5} - 5}) - 5/4*\sqrt{-3/100*(\sqrt{5} + \sqrt{2*\sqrt{5} - 5}))^2 - 1/50*(\sqrt{5} + \sqrt{2*\sqrt{5} - 5})*(\sqrt{5} - \sqrt{2*\sqrt{5} - 5}) - 3/100*(\sqrt{5} - \sqrt{2*\sqrt{5} - 5})^2) * ((3*\sqrt{5} + 3*\sqrt{2*\sqrt{5} - 5} - 8)*(\sqrt{5} - \sqrt{2*\sqrt{5} - 5}) - 8*\sqrt{5} - 8*\sqrt{2*\sqrt{5} - 5} + 36) + 22*x + 9/2*\sqrt{5} + 9/2*\sqrt{2*\sqrt{5} - 5} - 2) + 1/10*(\sqrt{5} - 5*\sqrt{-3/100*(\sqrt{5} + \sqrt{2*\sqrt{5} - 5}))^2 - 1/50*(\sqrt{5} + \sqrt{2*\sqrt{5} - 5})*(\sqrt{5} - \sqrt{2*\sqrt{5} - 5}) - 3/100*(\sqrt{5} - \sqrt{2*\sqrt{5} - 5})^2) * \log(-1/8*(3*\sqrt{5} + 3*\sqrt{2*\sqrt{5} - 5} - 8)*(\sqrt{5} - \sqrt{2*\sqrt{5} - 5})^2 + (\sqrt{5} + \sqrt{2*\sqrt{5} - 5})^2 - 3/8*((\sqrt{5} + \sqrt{2*\sqrt{5} - 5})^2 - 12)*(\sqrt{5} - \sqrt{2*\sqrt{5} - 5}) - 5/4*\sqrt{-3/100*(\sqrt{5} + \sqrt{2*\sqrt{5} - 5}))^2 - 1/50*(\sqrt{5} + \sqrt{2*\sqrt{5} - 5})*(\sqrt{5} - \sqrt{2*\sqrt{5} - 5}) - 3/100*(\sqrt{5} - \sqrt{2*\sqrt{5} - 5})^2) * ((3*\sqrt{5} + 3*\sqrt{2*\sqrt{5} - 5} - 8)*(\sqrt{5} - \sqrt{2*\sqrt{5} - 5}) - 8*\sqrt{5} - 8*\sqrt{2*\sqrt{5} - 5} + 36) + 22*x + 9/2*\sqrt{5} + 9/2*\sqrt{2*\sqrt{5} - 5} - 2) \end{aligned}$$

$$(5) - \sqrt{2\sqrt{5} - 5} - 8\sqrt{5} - 8\sqrt{2\sqrt{5} - 5} + 36 + 22x + 9/2\sqrt{5} + 9/2\sqrt{2\sqrt{5} - 5} - 2)$$

Sympy [A] time = 0.372795, size = 36, normalized size = 0.33

$$\text{RootSum}\left(125t^4 + 5t + 1, \left(t \mapsto t \log\left(\frac{375t^3}{11} - \frac{100t^2}{11} + \frac{45t}{11} + x + \frac{14}{11}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)/(-x**5+1),x)

[Out] RootSum(125*_t**4 + 5*_t + 1, Lambda(_t, _t*log(375*_t**3/11 - 100*_t**2/11 + 45*_t/11 + x + 14/11)))

Giac [A] time = 1.09347, size = 136, normalized size = 1.25

$$\frac{1}{5} \sqrt{-2\sqrt{5} + 5} \arctan\left(\frac{4x - \sqrt{5} + 1}{\sqrt{2\sqrt{5} + 10}}\right) + \frac{1}{5} \sqrt{2\sqrt{5} + 5} \arctan\left(\frac{4x + \sqrt{5} + 1}{\sqrt{-2\sqrt{5} + 10}}\right) + \frac{1}{10} \sqrt{5} \log\left(x^2 + \frac{1}{2}x(\sqrt{5} + 1) + 1\right) - \frac{1}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)/(-x^5+1),x, algorithm="giac")

[Out] 1/5*sqrt(-2*sqrt(5) + 5)*arctan((4*x - sqrt(5) + 1)/sqrt(2*sqrt(5) + 10)) + 1/5*sqrt(2*sqrt(5) + 5)*arctan((4*x + sqrt(5) + 1)/sqrt(-2*sqrt(5) + 10)) + 1/10*sqrt(5)*log(x^2 + 1/2*x*(sqrt(5) + 1) + 1) - 1/10*sqrt(5)*log(x^2 - 1/2*x*(sqrt(5) - 1) + 1)

$$3.223 \quad \int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{a+bx^3} dx$$

Optimal. Leaf size=208

$$\frac{x^9(a^2be + a^3(-f) - ab^2d + b^3c)}{9b^4} - \frac{ax^6(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^5} + \frac{a^2x^3(a^2be + a^3(-f) - ab^2d + b^3c)}{3b^6} - \frac{a^3 \log(a + bx^3)}{3b^7}$$

[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(3*b^6) - (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^6)/(6*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9)/(9*b^4) + ((b^2*d - a*b*e + a^2*f)*x^12)/(12*b^3) + ((b*e - a*f)*x^15)/(15*b^2) + (f*x^18)/(18*b) - (a^3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*b^7)

Rubi [A] time = 0.315686, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1821, 1620}

$$\frac{x^9(a^2be + a^3(-f) - ab^2d + b^3c)}{9b^4} - \frac{ax^6(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^5} + \frac{a^2x^3(a^2be + a^3(-f) - ab^2d + b^3c)}{3b^6} - \frac{a^3 \log(a + bx^3)}{3b^7}$$

Antiderivative was successfully verified.

[In] Int[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3), x]

[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(3*b^6) - (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^6)/(6*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9)/(9*b^4) + ((b^2*d - a*b*e + a^2*f)*x^12)/(12*b^3) + ((b*e - a*f)*x^15)/(15*b^2) + (f*x^18)/(18*b) - (a^3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*b^7)

Rule 1821

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p_.], x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1620

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x]

, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[Expon[Px, x], 2]

Rubi steps

$$\begin{aligned} \int \frac{x^{11}(c + dx^3 + ex^6 + fx^9)}{a + bx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^3(c + dx + ex^2 + fx^3)}{a + bx} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(-\frac{a^2(-b^3c + ab^2d - a^2be + a^3f)}{b^6} + \frac{a(-b^3c + ab^2d - a^2be + a^3f)x}{b^5} + \frac{(b^3c - ab^2d + a^2be)}{9b^4} \right) dx, x, x^3 \right) \\ &= \frac{a^2(b^3c - ab^2d + a^2be - a^3f)x^3}{3b^6} - \frac{a(b^3c - ab^2d + a^2be - a^3f)x^6}{6b^5} + \frac{(b^3c - ab^2d + a^2be)x^9}{9b^4} \end{aligned}$$

Mathematica [A] time = 0.0902507, size = 187, normalized size = 0.9

$$\frac{bx^3(5a^2b^3(12c + 6dx^3 + 4ex^6 + 3fx^9) - 10a^3b^2(6d + 3ex^3 + 2fx^6) + 30a^4b(2e + fx^3) - 60a^5f - ab^4x^3(30c + 20dx^3 + 10ex^6 + 3fx^9))}{180b^7}$$

Antiderivative was successfully verified.

[In] Integrate[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]

[Out] (b*x^3*(-60*a^5*f + 30*a^4*b*(2*e + f*x^3) - 10*a^3*b^2*(6*d + 3*e*x^3 + 2*f*x^6) + 5*a^2*b^3*(12*c + 6*d*x^3 + 4*e*x^6 + 3*f*x^9) + b^5*x^6*(20*c + 15*d*x^3 + 12*e*x^6 + 10*f*x^9) - a*b^4*x^3*(30*c + 20*d*x^3 + 15*e*x^6 + 12*f*x^9)) + 60*a^3*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a + b*x^3])/(180*b^7)

Maple [A] time = 0.005, size = 266, normalized size = 1.3

$$\frac{fx^{18}}{18b} - \frac{x^{15}af}{15b^2} + \frac{x^{15}e}{15b} + \frac{x^{12}a^2f}{12b^3} - \frac{x^{12}ae}{12b^2} + \frac{x^{12}d}{12b} - \frac{x^9a^3f}{9b^4} + \frac{x^9a^2e}{9b^3} - \frac{x^9ad}{9b^2} + \frac{x^9c}{9b} + \frac{x^6a^4f}{6b^5} - \frac{a^3ex^6}{6b^4} + \frac{a^2dx^6}{6b^3} - \frac{acx^6}{6b^2} - \frac{a^5f}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)

[Out] 1/18*f*x^18/b-1/15/b^2*x^15*a*f+1/15/b*x^15*e+1/12/b^3*x^12*a^2*f-1/12/b^2*x^12*a*e+1/12/b*x^12*d-1/9/b^4*x^9*a^3*f+1/9/b^3*x^9*a^2*e-1/9/b^2*x^9*a*d

$$\frac{1}{9}bx^9c + \frac{1}{6}b^5x^6a^4f - \frac{1}{6}b^4x^6a^3e + \frac{1}{6}b^3x^6a^2d - \frac{1}{6}b^2x^6a^2c - \frac{1}{3}b^6a^5fx^3 + \frac{1}{3}b^5a^4ex^3 - \frac{1}{3}b^4a^3dx^3 + \frac{1}{3}b^3a^2cx^3 + \frac{1}{3}a^6/b^7 \ln(bx^3+a) \cdot f - \frac{1}{3}a^5/b^6 \ln(bx^3+a) \cdot e + \frac{1}{3}a^4/b^5 \ln(bx^3+a) \cdot d - \frac{1}{3}a^3/b^4 \ln(bx^3+a) \cdot c$$

Maxima [A] time = 0.956447, size = 282, normalized size = 1.36

$$\frac{10b^5fx^{18} + 12(b^5e - ab^4f)x^{15} + 15(b^5d - ab^4e + a^2b^3f)x^{12} + 20(b^5c - ab^4d + a^2b^3e - a^3b^2f)x^9 - 30(ab^4c - a^2b^3d + a^3b^2e - a^4b^1f)x^6 + 60(a^2b^3c - a^3b^2d + a^4b^1e - a^5b^0f)x^3}{180b^6} - \frac{1}{3} \frac{(a^3b^3c - a^4b^2d + a^5b^1e - a^6b^0f) \log(bx^3 + a)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹*(f*x⁹+e*x⁶+d*x³+c)/(b*x³+a),x, algorithm="maxima")

[Out] 1/180*(10*b⁵*f*x¹⁸ + 12*(b⁵*e - a*b⁴*f)*x¹⁵ + 15*(b⁵*d - a*b⁴*e + a²*b³*f)*x¹² + 20*(b⁵*c - a*b⁴*d + a²*b³*e - a³*b²*f)*x⁹ - 30*(a*b⁴*c - a²*b³*d + a³*b²*e - a⁴*b*f)*x⁶ + 60*(a²*b³*c - a³*b²*d + a⁴*b*e - a⁵*f)*x³)/b⁶ - 1/3*(a³*b³*c - a⁴*b²*d + a⁵*b*e - a⁶*f)*log(b*x³ + a)/b⁷

Fricas [A] time = 1.27343, size = 429, normalized size = 2.06

$$\frac{10b^6fx^{18} + 12(b^6e - ab^5f)x^{15} + 15(b^6d - ab^5e + a^2b^4f)x^{12} + 20(b^6c - ab^5d + a^2b^4e - a^3b^3f)x^9 - 30(ab^5c - a^2b^4d + a^3b^3e - a^4b^2f)x^6 + 60(a^2b^4c - a^3b^3d + a^4b^2e - a^5b^1f)x^3 - 60(a^3b^3c - a^4b^2d + a^5b^1e - a^6b^0f) \log(bx^3 + a)}{180b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹*(f*x⁹+e*x⁶+d*x³+c)/(b*x³+a),x, algorithm="fricas")

[Out] 1/180*(10*b⁶*f*x¹⁸ + 12*(b⁶*e - a*b⁵*f)*x¹⁵ + 15*(b⁶*d - a*b⁵*e + a²*b⁴*f)*x¹² + 20*(b⁶*c - a*b⁵*d + a²*b⁴*e - a³*b³*f)*x⁹ - 30*(a*b⁵*c - a²*b⁴*d + a³*b³*e - a⁴*b²*f)*x⁶ + 60*(a²*b⁴*c - a³*b³*d + a⁴*b²*e - a⁵*b*f)*x³ - 60*(a³*b³*c - a⁴*b²*d + a⁵*b*e - a⁶*f)*log(b*x³ + a))/b⁷

Sympy [A] time = 1.16717, size = 192, normalized size = 0.92

$$\frac{a^3(a^3f - a^2be + ab^2d - b^3c) \log(a + bx^3)}{3b^7} + \frac{fx^{18}}{18b} - \frac{x^{15}(af - be)}{15b^2} + \frac{x^{12}(a^2f - abe + b^2d)}{12b^3} - \frac{x^9(a^3f - a^2be + ab^2d - b^3c)}{9b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**11*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)

[Out] $a^{**3}*(a^{**3}*f - a^{**2}*b*e + a*b^{**2}*d - b^{**3}*c)*\log(a + b*x^{**3})/(3*b^{**7}) + f*x^{**18}/(18*b) - x^{**15}*(a*f - b*e)/(15*b^{**2}) + x^{**12}*(a^{**2}*f - a*b*e + b^{**2}*d)/(12*b^{**3}) - x^{**9}*(a^{**3}*f - a^{**2}*b*e + a*b^{**2}*d - b^{**3}*c)/(9*b^{**4}) + x^{**6}*(a^{**4}*f - a^{**3}*b*e + a^{**2}*b^{**2}*d - a*b^{**3}*c)/(6*b^{**5}) - x^{**3}*(a^{**5}*f - a^{**4}*b*e + a^{**3}*b^{**2}*d - a^{**2}*b^{**3}*c)/(3*b^{**6})$

Giac [A] time = 1.05679, size = 332, normalized size = 1.6

$$\frac{10b^5fx^{18} - 12ab^4fx^{15} + 12b^5x^{15}e + 15b^5dx^{12} + 15a^2b^3fx^{12} - 15ab^4x^{12}e + 20b^5cx^9 - 20ab^4dx^9 - 20a^3b^2fx^9 + 20a^2b^3e}{180b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")

[Out] $\frac{1}{180}*(10*b^5*f*x^{18} - 12*a*b^4*f*x^{15} + 12*b^5*x^{15}*e + 15*b^5*d*x^{12} + 15*a^2*b^3*f*x^{12} - 15*a*b^4*x^{12}*e + 20*b^5*c*x^9 - 20*a*b^4*d*x^9 - 20*a^3*b^2*f*x^9 + 20*a^2*b^3*x^9*e - 30*a*b^4*c*x^6 + 30*a^2*b^3*d*x^6 + 30*a^4*b*f*x^6 - 30*a^3*b^2*x^6*e + 60*a^2*b^3*c*x^3 - 60*a^3*b^2*d*x^3 - 60*a^5*f*x^3 + 60*a^4*b*x^3*e)/b^6 - 1/3*(a^3*b^3*c - a^4*b^2*d - a^6*f + a^5*b*e)*\log(\text{abs}(b*x^3 + a))/b^7$

$$3.224 \quad \int \frac{x^8(c+dx^3+ex^6+fx^9)}{a+bx^3} dx$$

Optimal. Leaf size=170

$$\frac{x^6(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^4} - \frac{ax^3(a^2be + a^3(-f) - ab^2d + b^3c)}{3b^5} + \frac{a^2 \log(a + bx^3)(a^2be + a^3(-f) - ab^2d + b^3c)}{3b^6} +$$

[Out] $-(a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(3*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^6)/(6*b^4) + ((b^2*d - a*b*e + a^2*f)*x^9)/(9*b^3) + ((b*e - a*f)*x^{12})/(12*b^2) + (f*x^{15})/(15*b) + (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[a + b*x^3])/(3*b^6)$

Rubi [A] time = 0.242332, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1821, 1620}

$$\frac{x^6(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^4} - \frac{ax^3(a^2be + a^3(-f) - ab^2d + b^3c)}{3b^5} + \frac{a^2 \log(a + bx^3)(a^2be + a^3(-f) - ab^2d + b^3c)}{3b^6} +$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3), x]$

[Out] $-(a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(3*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^6)/(6*b^4) + ((b^2*d - a*b*e + a^2*f)*x^9)/(9*b^3) + ((b*e - a*f)*x^{12})/(12*b^2) + (f*x^{15})/(15*b) + (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[a + b*x^3])/(3*b^6)$

Rule 1821

$\text{Int}[(Pq_)*(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*\text{SubstFor}[x^n, Pq, x]*(a + b*x)^p, x], x, x^n], x] /;$ FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1620

$\text{Int}[(Px_)*((a_.) + (b_.)*(x_))^{(m_.)}*((c_.) + (d_.)*(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[Px*(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E

xpon[Px, x], 2]

Rubi steps

$$\begin{aligned} \int \frac{x^8 (c + dx^3 + ex^6 + fx^9)}{a + bx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2 (c + dx + ex^2 + fx^3)}{a + bx} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{a(-b^3c + ab^2d - a^2be + a^3f)}{b^5} + \frac{(b^3c - ab^2d + a^2be - a^3f)x}{b^4} + \frac{(b^2d - abe + a^2f)x^2}{b^3} \right) dx, x, x^3 \right) \\ &= -\frac{a(b^3c - ab^2d + a^2be - a^3f)x^3}{3b^5} + \frac{(b^3c - ab^2d + a^2be - a^3f)x^6}{6b^4} + \frac{(b^2d - abe + a^2f)x^9}{9b^3} \end{aligned}$$

Mathematica [A] time = 0.0659395, size = 154, normalized size = 0.91

$$\frac{bx^3(10a^2b^2(6d + 3ex^3 + 2fx^6) - 30a^3b(2e + fx^3) + 60a^4f - 5ab^3(12c + 6dx^3 + 4ex^6 + 3fx^9) + b^4x^3(30c + 20dx^3 + 15ex^6 + 12fx^9)) - 60a^2(-b^3c) + a^2b^2d - a^2b^2e + a^3f}{180b^6} \text{Log}[a + b^3x^3]$$

Antiderivative was successfully verified.

[In] Integrate[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]

[Out] (b*x^3*(60*a^4*f - 30*a^3*b*(2*e + f*x^3) + 10*a^2*b^2*(6*d + 3*e*x^3 + 2*f*x^6) - 5*a*b^3*(12*c + 6*d*x^3 + 4*e*x^6 + 3*f*x^9) + b^4*x^3*(30*c + 20*d*x^3 + 15*e*x^6 + 12*f*x^9)) - 60*a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a + b*x^3])/(180*b^6)

Maple [A] time = 0.004, size = 218, normalized size = 1.3

$$\frac{fx^{15}}{15b} - \frac{x^{12}af}{12b^2} + \frac{x^{12}e}{12b} + \frac{x^9a^2f}{9b^3} - \frac{x^9ae}{9b^2} + \frac{x^9d}{9b} - \frac{a^3fx^6}{6b^4} + \frac{a^2ex^6}{6b^3} - \frac{adx^6}{6b^2} + \frac{x^6c}{6b} + \frac{a^4fx^3}{3b^5} - \frac{a^3ex^3}{3b^4} + \frac{a^2dx^3}{3b^3} - \frac{acx^3}{3b^2} - \frac{a^5 \ln(a + b^3x^3)}{180b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)

[Out] 1/15*f*x^15/b-1/12/b^2*x^12*a*f+1/12/b*x^12*e+1/9/b^3*x^9*a^2*f-1/9/b^2*x^9*a*e+1/9/b*x^9*d-1/6/b^4*x^6*a^3*f+1/6/b^3*x^6*a^2*e-1/6/b^2*x^6*a*d+1/6/b*x^6*c+1/3/b^5*a^4*f*x^3-1/3/b^4*a^3*e*x^3+1/3/b^3*a^2*d*x^3-1/3/b^2*a*c*x^3

$$-1/3*a^5/b^6*\ln(b*x^3+a)*f+1/3*a^4/b^5*\ln(b*x^3+a)*e-1/3*a^3/b^4*\ln(b*x^3+a)*d+1/3*a^2/b^3*\ln(b*x^3+a)*c$$

Maxima [A] time = 0.95531, size = 228, normalized size = 1.34

$$\frac{12b^4fx^{15} + 15(b^4e - ab^3f)x^{12} + 20(b^4d - ab^3e + a^2b^2f)x^9 + 30(b^4c - ab^3d + a^2b^2e - a^3bf)x^6 - 60(ab^3c - a^2b^2d + a^3be - a^4f)x^3}{180b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")

[Out] 1/180*(12*b^4*f*x^15 + 15*(b^4*e - a*b^3*f)*x^12 + 20*(b^4*d - a*b^3*e + a^2*b^2*f)*x^9 + 30*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^6 - 60*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^3)/b^5 + 1/3*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*log(b*x^3 + a)/b^6

Fricas [A] time = 1.234, size = 351, normalized size = 2.06

$$\frac{12b^5fx^{15} + 15(b^5e - ab^4f)x^{12} + 20(b^5d - ab^4e + a^2b^3f)x^9 + 30(b^5c - ab^4d + a^2b^3e - a^3b^2f)x^6 - 60(ab^4c - a^2b^3d + a^3be - a^4f)x^3}{180b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")

[Out] 1/180*(12*b^5*f*x^15 + 15*(b^5*e - a*b^4*f)*x^12 + 20*(b^5*d - a*b^4*e + a^2*b^3*f)*x^9 + 30*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^6 - 60*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^3 + 60*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*log(b*x^3 + a))/b^6

Sympy [A] time = 1.13667, size = 155, normalized size = 0.91

$$-\frac{a^2(a^3f - a^2be + ab^2d - b^3c)\log(a + bx^3)}{3b^6} + \frac{fx^{15}}{15b} - \frac{x^{12}(af - be)}{12b^2} + \frac{x^9(a^2f - abe + b^2d)}{9b^3} - \frac{x^6(a^3f - a^2be + ab^2d - b^3c)}{6b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)

[Out] $-a^{**2}*(a^{**3}*f - a^{**2}*b*e + a*b^{**2}*d - b^{**3}*c)*\log(a + b*x^{**3})/(3*b^{**6}) + f*x^{**15}/(15*b) - x^{**12}*(a*f - b*e)/(12*b^{**2}) + x^{**9}*(a^{**2}*f - a*b*e + b^{**2}*d)/(9*b^{**3}) - x^{**6}*(a^{**3}*f - a^{**2}*b*e + a*b^{**2}*d - b^{**3}*c)/(6*b^{**4}) + x^{**3}*(a^{**4}*f - a^{**3}*b*e + a^{**2}*b^{**2}*d - a*b^{**3}*c)/(3*b^{**5})$

Giac [A] time = 1.06393, size = 266, normalized size = 1.56

$$\frac{12b^4fx^{15} - 15ab^3fx^{12} + 15b^4x^{12}e + 20b^4dx^9 + 20a^2b^2fx^9 - 20ab^3x^9e + 30b^4cx^6 - 30ab^3dx^6 - 30a^3bfx^6 + 30a^2b^2x^6}{180b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")

[Out] $1/180*(12*b^4*f*x^{15} - 15*a*b^3*f*x^{12} + 15*b^4*x^{12}*e + 20*b^4*d*x^9 + 20*a^2*b^2*f*x^9 - 20*a*b^3*x^9*e + 30*b^4*c*x^6 - 30*a*b^3*d*x^6 - 30*a^3*b*f*x^6 + 30*a^2*b^2*x^6*e - 60*a*b^3*c*x^3 + 60*a^2*b^2*d*x^3 + 60*a^4*f*x^3 - 60*a^3*b*x^3*e)/b^5 + 1/3*(a^2*b^3*c - a^3*b^2*d - a^5*f + a^4*b*e)*\log(a bs(b*x^3 + a))/b^6$

$$3.225 \quad \int \frac{x^5(c+dx^3+ex^6+fx^9)}{a+bx^3} dx$$

Optimal. Leaf size=132

$$\frac{x^3(a^2be + a^3(-f) - ab^2d + b^3c)}{3b^4} - \frac{a \log(a + bx^3)(a^2be + a^3(-f) - ab^2d + b^3c)}{3b^5} + \frac{x^6(a^2f - abe + b^2d)}{6b^3} + \frac{x^9(be - af)}{9b^2}$$

[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(3*b^4) + ((b^2*d - a*b*e + a^2*f)*x^6)/(6*b^3) + ((b*e - a*f)*x^9)/(9*b^2) + (f*x^12)/(12*b) - (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*b^5)

Rubi [A] time = 0.183319, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1821, 1620}

$$\frac{x^3(a^2be + a^3(-f) - ab^2d + b^3c)}{3b^4} - \frac{a \log(a + bx^3)(a^2be + a^3(-f) - ab^2d + b^3c)}{3b^5} + \frac{x^6(a^2f - abe + b^2d)}{6b^3} + \frac{x^9(be - af)}{9b^2}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]

[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(3*b^4) + ((b^2*d - a*b*e + a^2*f)*x^6)/(6*b^3) + ((b*e - a*f)*x^9)/(9*b^2) + (f*x^12)/(12*b) - (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*b^5)

Rule 1821

Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1620

Int[(Px_)*((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[Expon[Px, x], 2]

Rubi steps

$$\int \frac{x^5 (c + dx^3 + ex^6 + fx^9)}{a + bx^3} dx = \frac{1}{3} \text{Subst} \left(\int \frac{x (c + dx + ex^2 + fx^3)}{a + bx} dx, x, x^3 \right)$$

$$= \frac{1}{3} \text{Subst} \left(\int \left(\frac{b^3c - ab^2d + a^2be - a^3f}{b^4} + \frac{(b^2d - abe + a^2f)x}{b^3} + \frac{(be - af)x^2}{b^2} + \frac{fx^3}{b} + \frac{a}{b^4} \right) dx, x, x^3 \right)$$

$$= \frac{(b^3c - ab^2d + a^2be - a^3f)x^3}{3b^4} + \frac{(b^2d - abe + a^2f)x^6}{6b^3} + \frac{(be - af)x^9}{9b^2} + \frac{fx^{12}}{12b} + \frac{a(b^3c - ab^2d + a^2be - a^3f)}{3b^4}$$

Mathematica [A] time = 0.052875, size = 119, normalized size = 0.9

$$\frac{bx^3 (6a^2b(2e + fx^3) - 12a^3f - 2ab^2(6d + 3ex^3 + 2fx^6) + b^3(12c + 6dx^3 + 4ex^6 + 3fx^9)) + 12a \log(a + bx^3) (-a^2be + a^3f)}{36b^5}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3), x]

[Out] (b*x^3*(-12*a^3*f + 6*a^2*b*(2*e + f*x^3) - 2*a*b^2*(6*d + 3*e*x^3 + 2*f*x^6) + b^3*(12*c + 6*d*x^3 + 4*e*x^6 + 3*f*x^9)) + 12*a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a + b*x^3])/(36*b^5)

Maple [A] time = 0.003, size = 170, normalized size = 1.3

$$\frac{fx^{12}}{12b} - \frac{x^9af}{9b^2} + \frac{x^9e}{9b} + \frac{a^2fx^6}{6b^3} - \frac{aex^6}{6b^2} + \frac{dx^6}{6b} - \frac{a^3fx^3}{3b^4} + \frac{a^2ex^3}{3b^3} - \frac{adx^3}{3b^2} + \frac{cx^3}{3b} + \frac{a^4 \ln(bx^3 + a)f}{3b^5} - \frac{a^3 \ln(bx^3 + a)e}{3b^4} + \frac{a^2 \ln(bx^3 + a)d}{3b^3} + \frac{a \ln(bx^3 + a)c}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a), x)

[Out] 1/12*f*x^12/b-1/9/b^2*x^9*a*f+1/9/b*x^9*e+1/6/b^3*x^6*a^2*f-1/6/b^2*x^6*a*e+1/6/b*x^6*d-1/3/b^4*a^3*f*x^3+1/3/b^3*a^2*e*x^3-1/3/b^2*a*d*x^3+1/3/b*c*x^3+1/3*a^4/b^5*ln(b*x^3+a)*f-1/3*a^3/b^4*ln(b*x^3+a)*e+1/3*a^2/b^3*ln(b*x^3+a)*d-1/3*a/b^2*ln(b*x^3+a)*c

Maxima [A] time = 0.953117, size = 174, normalized size = 1.32

$$\frac{3b^3fx^{12} + 4(b^3e - ab^2f)x^9 + 6(b^3d - ab^2e + a^2bf)x^6 + 12(b^3c - ab^2d + a^2be - a^3f)x^3}{36b^4} - \frac{(ab^3c - a^2b^2d + a^3be - a^4f)}{3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")

[Out] 1/36*(3*b^3*f*x^12 + 4*(b^3*e - a*b^2*f)*x^9 + 6*(b^3*d - a*b^2*e + a^2*b*f)*x^6 + 12*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/b^4 - 1/3*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*log(b*x^3 + a)/b^5

Fricas [A] time = 1.27328, size = 267, normalized size = 2.02

$$\frac{3b^4fx^{12} + 4(b^4e - ab^3f)x^9 + 6(b^4d - ab^3e + a^2b^2f)x^6 + 12(b^4c - ab^3d + a^2b^2e - a^3bf)x^3 - 12(ab^3c - a^2b^2d + a^3be - a^4f)}{36b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")

[Out] 1/36*(3*b^4*f*x^12 + 4*(b^4*e - a*b^3*f)*x^9 + 6*(b^4*d - a*b^3*e + a^2*b^2*f)*x^6 + 12*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^3 - 12*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*log(b*x^3 + a))/b^5

Sympy [A] time = 1.08249, size = 117, normalized size = 0.89

$$\frac{a(a^3f - a^2be + ab^2d - b^3c) \log(a + bx^3)}{3b^5} + \frac{fx^{12}}{12b} - \frac{x^9(af - be)}{9b^2} + \frac{x^6(a^2f - abe + b^2d)}{6b^3} - \frac{x^3(a^3f - a^2be + ab^2d - b^3c)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)

[Out] a*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*x**3)/(3*b**5) + f*x**12/(12*b) - x**9*(a*f - b*e)/(9*b**2) + x**6*(a**2*f - a*b*e + b**2*d)/(6*b**3) - x**3*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*b**4)

Giac [A] time = 1.06881, size = 200, normalized size = 1.52

$$\frac{3b^3fx^{12} - 4ab^2fx^9 + 4b^3x^9e + 6b^3dx^6 + 6a^2bfx^6 - 6ab^2x^6e + 12b^3cx^3 - 12ab^2dx^3 - 12a^3fx^3 + 12a^2bx^3e}{36b^4} - \frac{(ab^3c - a^4f + a^3be)}{b^5} \log(\text{abs}(bx^3 + a))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")

[Out] 1/36*(3*b^3*f*x^12 - 4*a*b^2*f*x^9 + 4*b^3*x^9*e + 6*b^3*d*x^6 + 6*a^2*b*f*x^6 - 6*a*b^2*x^6*e + 12*b^3*c*x^3 - 12*a*b^2*d*x^3 - 12*a^3*f*x^3 + 12*a^2*b*x^3*e)/b^4 - 1/3*(a*b^3*c - a^2*b^2*d - a^4*f + a^3*b*e)*log(abs(b*x^3 + a))/b^5

$$3.226 \quad \int \frac{x^2(c+dx^3+ex^6+fx^9)}{a+bx^3} dx$$

Optimal. Leaf size=96

$$\frac{\log(a+bx^3)(a^2be+a^3(-f)-ab^2d+b^3c)}{3b^4} + \frac{x^3(a^2f-abe+b^2d)}{3b^3} + \frac{x^6(be-af)}{6b^2} + \frac{fx^9}{9b}$$

[Out] $((b^2*d - a*b*e + a^2*f)*x^3)/(3*b^3) + ((b*e - a*f)*x^6)/(6*b^2) + (f*x^9)/(9*b) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[a + b*x^3])/(3*b^4)$

Rubi [A] time = 0.139912, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1819, 1850}

$$\frac{\log(a+bx^3)(a^2be+a^3(-f)-ab^2d+b^3c)}{3b^4} + \frac{x^3(a^2f-abe+b^2d)}{3b^3} + \frac{x^6(be-af)}{6b^2} + \frac{fx^9}{9b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3), x]$

[Out] $((b^2*d - a*b*e + a^2*f)*x^3)/(3*b^3) + ((b*e - a*f)*x^6)/(6*b^2) + (f*x^9)/(9*b) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[a + b*x^3])/(3*b^4)$

Rule 1819

$\text{Int}[(Pq_)*(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/(m + 1), \text{Subst}[\text{Int}[\text{SubstFor}[x^{(m + 1)}, Pq, x]*(a + b*x^{\text{Simplify}[n/(m + 1)])}]]^p, x], x, x^{(m + 1)}, x] /;$ FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^{(m + 1)}]

Rule 1850

$\text{Int}[(Pq_)*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[Pq*(a + b*x^n)^p, x], x] /;$ FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int \frac{x^2(c + dx^3 + ex^6 + fx^9)}{a + bx^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{c + dx + ex^2 + fx^3}{a + bx} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{b^2d - abe + a^2f}{b^3} + \frac{(be - af)x}{b^2} + \frac{fx^2}{b} + \frac{b^3c - ab^2d + a^2be - a^3f}{b^3(a + bx)} \right) dx, x, x^3 \right) \\ &= \frac{(b^2d - abe + a^2f)x^3}{3b^3} + \frac{(be - af)x^6}{6b^2} + \frac{fx^9}{9b} + \frac{(b^3c - ab^2d + a^2be - a^3f) \log(a + bx^3)}{3b^4} \end{aligned}$$

Mathematica [A] time = 0.0430123, size = 88, normalized size = 0.92

$$\frac{6 \log(a + bx^3) (a^2be + a^3(-f) - ab^2d + b^3c) + bx^3 (6a^2f - 3ab(2e + fx^3) + b^2(6d + 3ex^3 + 2fx^6))}{18b^4}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3), x]

[Out] (b*x^3*(6*a^2*f - 3*a*b*(2*e + f*x^3) + b^2*(6*d + 3*e*x^3 + 2*f*x^6)) + 6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(18*b^4)

Maple [A] time = 0.003, size = 124, normalized size = 1.3

$$\frac{fx^9}{9b} - \frac{x^6af}{6b^2} + \frac{ex^6}{6b} + \frac{a^2fx^3}{3b^3} - \frac{aex^3}{3b^2} + \frac{dx^3}{3b} - \frac{\ln(bx^3 + a)a^3f}{3b^4} + \frac{\ln(bx^3 + a)a^2e}{3b^3} - \frac{\ln(bx^3 + a)ad}{3b^2} + \frac{c \ln(bx^3 + a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a), x)

[Out] 1/9*f*x^9/b-1/6/b^2*x^6*a*f+1/6/b*x^6*e+1/3/b^3*a^2*f*x^3-1/3/b^2*a*e*x^3+1/3/b*d*x^3-1/3/b^4*ln(b*x^3+a)*a^3*f+1/3/b^3*ln(b*x^3+a)*a^2*e-1/3/b^2*ln(b*x^3+a)*a*d+1/3*c*ln(b*x^3+a)/b

Maxima [A] time = 0.948623, size = 123, normalized size = 1.28

$$\frac{2b^2fx^9 + 3(b^2e - abf)x^6 + 6(b^2d - abe + a^2f)x^3}{18b^3} + \frac{(b^3c - ab^2d + a^2be - a^3f) \log(bx^3 + a)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")`

[Out] $\frac{1}{18}*(2*b^2*f*x^9 + 3*(b^2*e - a*b*f)*x^6 + 6*(b^2*d - a*b*e + a^2*f)*x^3)/b^3 + \frac{1}{3}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\log(b*x^3 + a)/b^4$

Fricas [A] time = 1.19505, size = 190, normalized size = 1.98

$$\frac{2b^3fx^9 + 3(b^3e - ab^2f)x^6 + 6(b^3d - ab^2e + a^2bf)x^3 + 6(b^3c - ab^2d + a^2be - a^3f)\log(bx^3 + a)}{18b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")`

[Out] $\frac{1}{18}*(2*b^3*f*x^9 + 3*(b^3*e - a*b^2*f)*x^6 + 6*(b^3*d - a*b^2*e + a^2*b*f)*x^3 + 6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\log(b*x^3 + a))/b^4$

Sympy [A] time = 1.00665, size = 83, normalized size = 0.86

$$\frac{fx^9}{9b} - \frac{x^6(af - be)}{6b^2} + \frac{x^3(a^2f - abe + b^2d)}{3b^3} - \frac{(a^3f - a^2be + ab^2d - b^3c)\log(a + bx^3)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)`

[Out] $f*x**9/(9*b) - x**6*(a*f - b*e)/(6*b**2) + x**3*(a**2*f - a*b*e + b**2*d)/(3*b**3) - (a**3*f - a**2*b*e + a*b**2*d - b**3*c)*\log(a + b*x**3)/(3*b**4)$

Giac [A] time = 1.06947, size = 136, normalized size = 1.42

$$\frac{2b^2fx^9 - 3abfx^6 + 3b^2x^6e + 6b^2dx^3 + 6a^2fx^3 - 6abx^3e}{18b^3} + \frac{(b^3c - ab^2d - a^3f + a^2be)\log(|bx^3 + a|)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/18*(2*b^2*f*x^9 - 3*a*b*f*x^6 + 3*b^2*x^6*e + 6*b^2*d*x^3 + 6*a^2*f*x^3 -  
6*a*b*x^3*e)/b^3 + 1/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*log(abs(b*x^3 +  
a))/b^4
```

$$3.227 \quad \int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)} dx$$

Optimal. Leaf size=80

$$-\frac{\log(a+bx^3)(a^2be+a^3(-f)-ab^2d+b^3c)}{3ab^3} + \frac{x^3(be-af)}{3b^2} + \frac{c \log(x)}{a} + \frac{fx^6}{6b}$$

[Out] ((b*e - a*f)*x^3)/(3*b^2) + (f*x^6)/(6*b) + (c*Log[x])/a - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a*b^3)

Rubi [A] time = 0.120113, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1821, 1620}

$$-\frac{\log(a+bx^3)(a^2be+a^3(-f)-ab^2d+b^3c)}{3ab^3} + \frac{x^3(be-af)}{3b^2} + \frac{c \log(x)}{a} + \frac{fx^6}{6b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)),x]

[Out] ((b*e - a*f)*x^3)/(3*b^2) + (f*x^6)/(6*b) + (c*Log[x])/a - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a*b^3)

Rule 1821

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1620

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[Expon[Px, x], 2]

Rubi steps

$$\begin{aligned} \int \frac{c + dx^3 + ex^6 + fx^9}{x(a + bx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{c + dx + ex^2 + fx^3}{x(a + bx)} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{be - af}{b^2} + \frac{c}{ax} + \frac{fx}{b} + \frac{-b^3c + ab^2d - a^2be + a^3f}{ab^2(a + bx)} \right) dx, x, x^3 \right) \\ &= \frac{(be - af)x^3}{3b^2} + \frac{fx^6}{6b} + \frac{c \log(x)}{a} - \frac{(b^3c - ab^2d + a^2be - a^3f) \log(a + bx^3)}{3ab^3} \end{aligned}$$

Mathematica [A] time = 0.0306707, size = 75, normalized size = 0.94

$$\frac{-2 \log(a + bx^3) (a^2be + a^3(-f) - ab^2d + b^3c) + abx^3 (-2af + 2be + bfx^3) + 6b^3c \log(x)}{6ab^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)), x]

[Out] (a*b*x^3*(2*b*e - 2*a*f + b*f*x^3) + 6*b^3*c*Log[x] - 2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(6*a*b^3)

Maple [A] time = 0.005, size = 97, normalized size = 1.2

$$\frac{fx^6}{6b} - \frac{ax^3f}{3b^2} + \frac{ex^3}{3b} + \frac{a^2 \ln(bx^3 + a)f}{3b^3} - \frac{ae \ln(bx^3 + a)}{3b^2} + \frac{d \ln(bx^3 + a)}{3b} - \frac{c \ln(bx^3 + a)}{3a} + \frac{c \ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a), x)

[Out] 1/6*f*x^6/b-1/3/b^2*x^3*a*f+1/3*e*x^3/b+1/3*a^2/b^3*ln(b*x^3+a)*f-1/3*a*e*ln(b*x^3+a)/b^2+1/3*d*ln(b*x^3+a)/b-1/3*c*ln(b*x^3+a)/a+c*ln(x)/a

Maxima [A] time = 0.958248, size = 104, normalized size = 1.3

$$\frac{c \log(x^3)}{3a} + \frac{bfx^6 + 2(be - af)x^3}{6b^2} - \frac{(b^3c - ab^2d + a^2be - a^3f) \log(bx^3 + a)}{3ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a),x, algorithm="maxima")

[Out] $\frac{1}{3}c \log(x^3)/a + \frac{1}{6}(bfx^6 + 2(b^2e - a^2f)x^3)/b^2 - \frac{1}{3}(b^3c - a^2b^2d + a^2b^2e - a^3f) \log(bx^3 + a)/(ab^3)$

Fricas [A] time = 1.58809, size = 171, normalized size = 2.14

$$\frac{ab^2fx^6 + 6b^3c \log(x) + 2(ab^2e - a^2bf)x^3 - 2(b^3c - ab^2d + a^2be - a^3f) \log(bx^3 + a)}{6ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a),x, algorithm="fricas")

[Out] $\frac{1}{6}(ab^2fx^6 + 6b^3c \log(x) + 2(ab^2e - a^2bf)x^3 - 2(b^3c - ab^2d + a^2be - a^3f) \log(bx^3 + a))/(ab^3)$

Sympy [A] time = 4.91323, size = 68, normalized size = 0.85

$$\frac{fx^6}{6b} - \frac{x^3(af - be)}{3b^2} + \frac{c \log(x)}{a} + \frac{(a^3f - a^2be + ab^2d - b^3c) \log\left(\frac{a}{b} + x^3\right)}{3ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x/(b*x**3+a),x)

[Out] $\frac{fx^6}{6b} - \frac{x^3(af - b^2e)}{3b^2} + \frac{c \log(x)}{a} + \frac{(a^3f - a^2b^2e + ab^2d - b^3c) \log(a/b + x^3)}{3ab^3}$

Giac [A] time = 1.05752, size = 107, normalized size = 1.34

$$\frac{c \log(|x|)}{a} + \frac{bfx^6 - 2afx^3 + 2bx^3e}{6b^2} - \frac{(b^3c - ab^2d - a^3f + a^2be) \log(|bx^3 + a|)}{3ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a),x, algorithm="giac")
```

```
[Out] c*log(abs(x))/a + 1/6*(b*f*x^6 - 2*a*f*x^3 + 2*b*x^3*e)/b^2 - 1/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*log(abs(b*x^3 + a))/(a*b^3)
```

$$3.228 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)} dx$$

Optimal. Leaf size=81

$$\frac{\log(a+bx^3)(a^2be+a^3(-f)-ab^2d+b^3c)}{3a^2b^2} - \frac{\log(x)(bc-ad)}{a^2} - \frac{c}{3ax^3} + \frac{fx^3}{3b}$$

[Out] $-c/(3*a*x^3) + (f*x^3)/(3*b) - ((b*c - a*d)*\text{Log}[x])/a^2 + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[a + b*x^3])/(3*a^2*b^2)$

Rubi [A] time = 0.116136, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1821, 1620}

$$\frac{\log(a+bx^3)(a^2be+a^3(-f)-ab^2d+b^3c)}{3a^2b^2} - \frac{\log(x)(bc-ad)}{a^2} - \frac{c}{3ax^3} + \frac{fx^3}{3b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)), x]$

[Out] $-c/(3*a*x^3) + (f*x^3)/(3*b) - ((b*c - a*d)*\text{Log}[x])/a^2 + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[a + b*x^3])/(3*a^2*b^2)$

Rule 1821

$\text{Int}[(Pq_*)(x_)^{(m_*)}((a_) + (b_*)(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*\text{SubstFor}[x^n, Pq, x]*(a + b*x)^p, x], x, x^n], x] /; \text{FreeQ}\{a, b, m, n, p\}, x\} \&\& \text{PolyQ}[Pq, x^n] \&\& \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$

Rule 1620

$\text{Int}[(Px_*)((a_) + (b_*)(x_))^{(m_*)}((c_) + (d_*)(x_))^{(n_)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x\} \&\& \text{PolyQ}[Px, x] \&\& (\text{IntegersQ}[m, n] \parallel \text{IGtQ}[m, -2]) \&\& \text{GtQ}[\text{Expon}[Px, x], 2]$

Rubi steps

$$\begin{aligned}
\int \frac{c + dx^3 + ex^6 + fx^9}{x^4(a + bx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{c + dx + ex^2 + fx^3}{x^2(a + bx)} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{f}{b} + \frac{c}{ax^2} + \frac{-bc + ad}{a^2x} + \frac{b^3c - ab^2d + a^2be - a^3f}{a^2b(a + bx)} \right) dx, x, x^3 \right) \\
&= -\frac{c}{3ax^3} + \frac{fx^3}{3b} - \frac{(bc - ad) \log(x)}{a^2} + \frac{(b^3c - ab^2d + a^2be - a^3f) \log(a + bx^3)}{3a^2b^2}
\end{aligned}$$

Mathematica [A] time = 0.0434588, size = 77, normalized size = 0.95

$$\frac{1}{3} \left(\frac{\log(a + bx^3)(a^2be + a^3(-f) - ab^2d + b^3c)}{a^2b^2} + \frac{3 \log(x)(ad - bc)}{a^2} - \frac{c}{ax^3} + \frac{fx^3}{b} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)),x]

[Out] $-(c/(a*x^3)) + (f*x^3)/b + (3*(-(b*c) + a*d)*\text{Log}[x])/a^2 + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[a + b*x^3])/(a^2*b^2)/3$

Maple [A] time = 0.007, size = 94, normalized size = 1.2

$$\frac{fx^3}{3b} - \frac{a \ln(bx^3 + a)f}{3b^2} + \frac{e \ln(bx^3 + a)}{3b} - \frac{d \ln(bx^3 + a)}{3a} + \frac{b \ln(bx^3 + a)c}{3a^2} - \frac{c}{3ax^3} + \frac{d \ln(x)}{a} - \frac{\ln(x)bc}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a),x)

[Out] $1/3*f*x^3/b - 1/3*a/b^2*\ln(b*x^3+a)*f + 1/3*e*\ln(b*x^3+a)/b - 1/3*d*\ln(b*x^3+a)/a + 1/3/a^2*b*\ln(b*x^3+a)*c - 1/3*c/a/x^3 + d*\ln(x)/a - 1/a^2*\ln(x)*b*c$

Maxima [A] time = 0.966186, size = 104, normalized size = 1.28

$$\frac{fx^3}{3b} - \frac{(bc - ad) \log(x^3)}{3a^2} + \frac{(b^3c - ab^2d + a^2be - a^3f) \log(bx^3 + a)}{3a^2b^2} - \frac{c}{3ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a),x, algorithm="maxima")

[Out] $\frac{1}{3}fx^3/b - \frac{1}{3}(b^3c - a^3d)\log(x^3)/a^2 + \frac{1}{3}(b^3c - a^2b^2d + a^2b^2e - a^3f)\log(bx^3 + a)/(a^2b^2) - \frac{1}{3}c/(ax^3)$

Fricas [A] time = 1.50273, size = 180, normalized size = 2.22

$$\frac{a^2bfx^6 + (b^3c - ab^2d + a^2be - a^3f)x^3 \log(bx^3 + a) - 3(b^3c - ab^2d)x^3 \log(x) - ab^2c}{3a^2b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a),x, algorithm="fricas")

[Out] $\frac{1}{3}(a^2b^2fx^6 + (b^3c - a^2b^2d + a^2b^2e - a^3f)x^3\log(bx^3 + a) - 3(b^3c - a^2b^2d)x^3\log(x) - a^2b^2c)/(a^2b^2x^3)$

Sympy [A] time = 11.4391, size = 70, normalized size = 0.86

$$\frac{fx^3}{3b} - \frac{c}{3ax^3} + \frac{(ad - bc)\log(x)}{a^2} - \frac{(a^3f - a^2be + ab^2d - b^3c)\log\left(\frac{a}{b} + x^3\right)}{3a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**4/(b*x**3+a),x)

[Out] $fx^3/(3*b) - c/(3*a*x^3) + (a*d - b*c)*\log(x)/a^2 - (a^3*f - a^2*b^2*e + a*b^2*d - b^3*c)*\log(a/b + x^3)/(3*a^2*b^2)$

Giac [A] time = 1.05868, size = 128, normalized size = 1.58

$$\frac{fx^3}{3b} - \frac{(bc - ad)\log(|x|)}{a^2} + \frac{(b^3c - ab^2d - a^3f + a^2be)\log(|bx^3 + a|)}{3a^2b^2} + \frac{bcx^3 - adx^3 - ac}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*f*x^3/b - (b*c - a*d)*log(abs(x))/a^2 + 1/3*(b^3*c - a*b^2*d - a^3*f +  
a^2*b*e)*log(abs(b*x^3 + a))/(a^2*b^2) + 1/3*(b*c*x^3 - a*d*x^3 - a*c)/(a^2  
*x^3)
```

$$3.229 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)} dx$$

Optimal. Leaf size=95

$$-\frac{\log(a+bx^3)(a^2be+a^3(-f)-ab^2d+b^3c)}{3a^3b} + \frac{\log(x)(a^2e-abd+b^2c)}{a^3} + \frac{bc-ad}{3a^2x^3} - \frac{c}{6ax^6}$$

[Out] $-c/(6*a*x^6) + (b*c - a*d)/(3*a^2*x^3) + ((b^2*c - a*b*d + a^2*e)*\text{Log}[x])/a^3 - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[a + b*x^3])/(3*a^3*b)$

Rubi [A] time = 0.128976, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1821, 1620}

$$-\frac{\log(a+bx^3)(a^2be+a^3(-f)-ab^2d+b^3c)}{3a^3b} + \frac{\log(x)(a^2e-abd+b^2c)}{a^3} + \frac{bc-ad}{3a^2x^3} - \frac{c}{6ax^6}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)), x]$

[Out] $-c/(6*a*x^6) + (b*c - a*d)/(3*a^2*x^3) + ((b^2*c - a*b*d + a^2*e)*\text{Log}[x])/a^3 - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[a + b*x^3])/(3*a^3*b)$

Rule 1821

$\text{Int}[(\text{Pq}_x)(x)^{(m)}((a) + (b)_x)(x)^{(n)})^p, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*\text{SubstFor}[x^n, \text{Pq}, x]*(a + b*x)^p, x], x, x^n], x] /; \text{FreeQ}\{a, b, m, n, p, x\} \&\& \text{PolyQ}[\text{Pq}, x^n] \&\& \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$

Rule 1620

$\text{Int}[(\text{Px}_x)((a) + (b)_x)(x)^{(m)}((c) + (d)_x)(x)^{(n)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\text{Px}*(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x \&\& \text{PolyQ}[\text{Px}, x] \&\& (\text{IntegersQ}[m, n] \parallel \text{IGtQ}[m, -2]) \&\& \text{GtQ}[\text{E xpon}[\text{Px}, x], 2]$

Rubi steps

$$\begin{aligned}
\int \frac{c + dx^3 + ex^6 + fx^9}{x^7(a + bx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{c + dx + ex^2 + fx^3}{x^3(a + bx)} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{c}{ax^3} + \frac{-bc + ad}{a^2x^2} + \frac{b^2c - abd + a^2e}{a^3x} + \frac{-b^3c + ab^2d - a^2be + a^3f}{a^3(a + bx)} \right) dx, x, x^3 \right) \\
&= -\frac{c}{6ax^6} + \frac{bc - ad}{3a^2x^3} + \frac{(b^2c - abd + a^2e) \log(x)}{a^3} - \frac{(b^3c - ab^2d + a^2be - a^3f) \log(a + bx^3)}{3a^3b}
\end{aligned}$$

Mathematica [A] time = 0.0686564, size = 88, normalized size = 0.93

$$\frac{\log(a + bx^3) \left(\frac{2a^3f}{b} - 2a^2e + 2abd - 2b^2c \right) + 6 \log(x) (a^2e - abd + b^2c) - \frac{a(ac + 2adx^3 - 2bcx^3)}{x^6}}{6a^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)), x]

[Out] (-((a*(a*c - 2*b*c*x^3 + 2*a*d*x^3))/x^6) + 6*(b^2*c - a*b*d + a^2*e)*Log[x] + (-2*b^2*c + 2*a*b*d - 2*a^2*e + (2*a^3*f)/b)*Log[a + b*x^3])/(6*a^3)

Maple [A] time = 0.007, size = 116, normalized size = 1.2

$$\frac{\ln(bx^3 + a)f}{3b} - \frac{e \ln(bx^3 + a)}{3a} + \frac{b \ln(bx^3 + a)d}{3a^2} - \frac{b^2 \ln(bx^3 + a)c}{3a^3} - \frac{c}{6ax^6} - \frac{d}{3ax^3} + \frac{bc}{3x^3a^2} + \frac{e \ln(x)}{a} - \frac{\ln(x)bd}{a^2} + \ln$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a), x)

[Out] 1/3/b*ln(b*x^3+a)*f-1/3*e*ln(b*x^3+a)/a+1/3/a^2*b*ln(b*x^3+a)*d-1/3/a^3*b^2*ln(b*x^3+a)*c-1/6*c/a/x^6-1/3/a/x^3*d+1/3/a^2/x^3*b*c+e*ln(x)/a-1/a^2*ln(x)*b*d+1/a^3*ln(x)*b^2*c

Maxima [A] time = 0.948345, size = 126, normalized size = 1.33

$$\frac{(b^2c - abd + a^2e) \log(x^3)}{3a^3} - \frac{(b^3c - ab^2d + a^2be - a^3f) \log(bx^3 + a)}{3a^3b} + \frac{2(bc - ad)x^3 - ac}{6a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a),x, algorithm="maxima")

[Out] $\frac{1}{3}(b^2c - a^2bd + a^2e) \log(x^3)/a^3 - \frac{1}{3}(b^3c - a^2bd + a^2be - a^3f) \log(bx^3 + a)/(a^3b) + \frac{1}{6}(2(b^2c - a^2d)x^3 - a^3c)/(a^2x^6)$

Fricas [A] time = 1.52655, size = 213, normalized size = 2.24

$$\frac{2(b^3c - ab^2d + a^2be - a^3f)x^6 \log(bx^3 + a) - 6(b^3c - ab^2d + a^2be)x^6 \log(x) + a^2bc - 2(ab^2c - a^2bd)x^3}{6a^3bx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a),x, algorithm="fricas")

[Out] $-\frac{1}{6}(2(b^3c - a^2bd + a^2be - a^3f)x^6 \log(bx^3 + a) - 6(b^3c - a^2bd + a^2be)x^6 \log(x) + a^2b^2c - 2(a^2b^2c - a^2b^2d)x^3)/(a^3bx^6)$

Sympy [A] time = 51.4964, size = 85, normalized size = 0.89

$$-\frac{ac + x^3(2ad - 2bc)}{6a^2x^6} + \frac{(a^2e - abd + b^2c) \log(x)}{a^3} + \frac{(a^3f - a^2be + ab^2d - b^3c) \log\left(\frac{a}{b} + x^3\right)}{3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**7/(b*x**3+a),x)

[Out] $-\frac{(a^3c + x^3(2a^2d - 2b^2c))}{(6a^3x^6)} + \frac{(a^2e - a^2bd + b^2c) \log(x)/a^3 + (a^3f - a^2be + a^2b^2d - b^3c) \log(a/b + x^3)/(3a^3b)}$

Giac [A] time = 1.05506, size = 170, normalized size = 1.79

$$\frac{(b^2c - abd + a^2e) \log(|x|)}{a^3} - \frac{(b^3c - ab^2d - a^3f + a^2be) \log(|bx^3 + a|)}{3a^3b} - \frac{3b^2cx^6 - 3abdx^6 + 3a^2x^6e - 2abcx^3 + 2a^2dx^3}{6a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a),x, algorithm="giac")
```

```
[Out] (b^2*c - a*b*d + a^2*e)*log(abs(x))/a^3 - 1/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*log(abs(b*x^3 + a))/(a^3*b) - 1/6*(3*b^2*c*x^6 - 3*a*b*d*x^6 + 3*a^2*x^6*e - 2*a*b*c*x^3 + 2*a^2*d*x^3 + a^2*c)/(a^3*x^6)
```

$$3.230 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)} dx$$

Optimal. Leaf size=128

$$\frac{\log(a+bx^3)(a^2be+a^3(-f)-ab^2d+b^3c)}{3a^4} - \frac{\log(x)(a^2be+a^3(-f)-ab^2d+b^3c)}{a^4} - \frac{a^2e-abd+b^2c}{3a^3x^3} + \frac{bc-ad}{6a^2x^6} - \frac{c}{9ax^9}$$

[Out] $-c/(9*a*x^9) + (b*c - a*d)/(6*a^2*x^6) - (b^2*c - a*b*d + a^2*e)/(3*a^3*x^3) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[x])/a^4 + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[a + b*x^3])/(3*a^4)$

Rubi [A] time = 0.161649, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1821, 1620}

$$\frac{\log(a+bx^3)(a^2be+a^3(-f)-ab^2d+b^3c)}{3a^4} - \frac{\log(x)(a^2be+a^3(-f)-ab^2d+b^3c)}{a^4} - \frac{a^2e-abd+b^2c}{3a^3x^3} + \frac{bc-ad}{6a^2x^6} - \frac{c}{9ax^9}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x^3 + e*x^6 + f*x^9)/(x^{10}(a + b*x^3)), x]$

[Out] $-c/(9*a*x^9) + (b*c - a*d)/(6*a^2*x^6) - (b^2*c - a*b*d + a^2*e)/(3*a^3*x^3) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[x])/a^4 + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[a + b*x^3])/(3*a^4)$

Rule 1821

$\text{Int}[(Pq_)*(x_)^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*\text{SubstFor}[x^n, Pq, x]*(a + b*x)^p, x], x, x^n], x] /;$ FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Simplify[(m+1)/n]]

Rule 1620

$\text{Int}[(Px_)*((a_) + (b_)*(x_))^{(m_)}*((c_) + (d_)*(x_))^{(n_)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[Px*(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[Expon[Px, x], 2]

Rubi steps

$$\begin{aligned} \int \frac{c + dx^3 + ex^6 + fx^9}{x^{10}(a + bx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{c + dx + ex^2 + fx^3}{x^4(a + bx)} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{c}{ax^4} + \frac{-bc + ad}{a^2x^3} + \frac{b^2c - abd + a^2e}{a^3x^2} + \frac{-b^3c + ab^2d - a^2be + a^3f}{a^4x} - \frac{b(-b^3c + a^3f)}{a^4} \right) dx, x, x^3 \right) \\ &= -\frac{c}{9ax^9} + \frac{bc - ad}{6a^2x^6} - \frac{b^2c - abd + a^2e}{3a^3x^3} - \frac{(b^3c - ab^2d + a^2be - a^3f) \log(x)}{a^4} + \frac{(b^3c - ab^2d + a^2be - a^3f) \log(x)}{a^4} \end{aligned}$$

Mathematica [A] time = 0.0643549, size = 128, normalized size = 1.

$$\frac{\log(a + bx^3)(a^2be + a^3(-f) - ab^2d + b^3c)}{3a^4} + \frac{\log(x)(-a^2be + a^3f + ab^2d - b^3c)}{a^4} + \frac{a^2(-e) + abd - b^2c}{3a^3x^3} + \frac{bc - ad}{6a^2x^6} - \frac{c}{9ax^9}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)), x]

[Out] -c/(9*a*x^9) + (b*c - a*d)/(6*a^2*x^6) + ((-b^2*c) + a*b*d - a^2*e)/(3*a^3*x^3) + (((-b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[x])/a^4 + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^4)

Maple [A] time = 0.007, size = 161, normalized size = 1.3

$$-\frac{\ln(bx^3 + a)f}{3a} + \frac{\ln(bx^3 + a)be}{3a^2} - \frac{\ln(bx^3 + a)b^2d}{3a^3} + \frac{\ln(bx^3 + a)b^3c}{3a^4} - \frac{c}{9ax^9} - \frac{d}{6ax^6} + \frac{bc}{6a^2x^6} - \frac{e}{3ax^3} + \frac{bd}{3x^3a^2} - \frac{b^3c}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a), x)

[Out] -1/3/a*ln(b*x^3+a)*f+1/3/a^2*ln(b*x^3+a)*b*e-1/3/a^3*ln(b*x^3+a)*b^2*d+1/3/a^4*ln(b*x^3+a)*b^3*c-1/9*c/a/x^9-1/6/a/x^6*d+1/6/a^2/x^6*b*c-1/3/a/x^3*e+1/3/a^2/x^3*b*d-1/3/a^3/x^3*b^2*c+1/a*ln(x)*f-1/a^2*ln(x)*b*e+1/a^3*ln(x)*b^2*d-1/a^4*ln(x)*b^3*c

Maxima [A] time = 0.95327, size = 169, normalized size = 1.32

$$\frac{(b^3c - ab^2d + a^2be - a^3f) \log(bx^3 + a)}{3a^4} - \frac{(b^3c - ab^2d + a^2be - a^3f) \log(x^3)}{3a^4} - \frac{6(b^2c - abd + a^2e)x^6 - 3(abc - a^2d)x^3}{18a^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a),x, algorithm="maxima")

[Out] 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(b*x^3 + a)/a^4 - 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x^3)/a^4 - 1/18*(6*(b^2*c - a*b*d + a^2*e)*x^6 - 3*(a*b*c - a^2*d)*x^3 + 2*a^2*c)/(a^3*x^9)

Fricas [A] time = 1.47109, size = 269, normalized size = 2.1

$$\frac{6(b^3c - ab^2d + a^2be - a^3f)x^9 \log(bx^3 + a) - 18(b^3c - ab^2d + a^2be - a^3f)x^9 \log(x) - 6(ab^2c - a^2bd + a^3e)x^6 - 2a^3c + 3(a^2b*c - a^3*d)x^3}{18a^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a),x, algorithm="fricas")

[Out] 1/18*(6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9*log(b*x^3 + a) - 18*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9*log(x) - 6*(a*b^2*c - a^2*b*d + a^3*e)*x^6 - 2*a^3*c + 3*(a^2*b*c - a^3*d)*x^3)/(a^4*x^9)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**10/(b*x**3+a),x)

[Out] Timed out

Giac [A] time = 1.06691, size = 248, normalized size = 1.94

$$-\frac{(b^3c - ab^2d - a^3f + a^2be) \log(|x|)}{a^4} + \frac{(b^4c - ab^3d - a^3bf + a^2b^2e) \log(|bx^3 + a|)}{3a^4b} + \frac{11b^3cx^9 - 11ab^2dx^9 - 11a^3fx^9 + 11a^2b^2ex^9}{3a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a),x, algorithm="giac")

[Out] $-(b^3c - a*b^2*d - a^3*f + a^2*b^2*e)*\log(\text{abs}(x))/a^4 + 1/3*(b^4*c - a*b^3*d - a^3*b*f + a^2*b^2*e)*\log(\text{abs}(b*x^3 + a))/(a^4*b) + 1/18*(11*b^3*c*x^9 - 11*a*b^2*d*x^9 - 11*a^3*f*x^9 + 11*a^2*b*x^9*e - 6*a*b^2*c*x^6 + 6*a^2*b*d*x^6 - 6*a^3*x^6*e + 3*a^2*b*c*x^3 - 3*a^3*d*x^3 - 2*a^3*c)/(a^4*x^9)$

$$3.231 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)} dx$$

Optimal. Leaf size=164

$$\frac{a^2be + a^3(-f) - ab^2d + b^3c}{3a^4x^3} - \frac{b \log(a + bx^3)(a^2be + a^3(-f) - ab^2d + b^3c)}{3a^5} + \frac{b \log(x)(a^2be + a^3(-f) - ab^2d + b^3c)}{a^5} - a$$

[Out] $-c/(12*a*x^{12}) + (b*c - a*d)/(9*a^2*x^9) - (b^2*c - a*b*d + a^2*e)/(6*a^3*x^6) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*x^3) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[x])/a^5 - (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[a + b*x^3])/(3*a^5)$

Rubi [A] time = 0.181439, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1821, 1620}

$$\frac{a^2be + a^3(-f) - ab^2d + b^3c}{3a^4x^3} - \frac{b \log(a + bx^3)(a^2be + a^3(-f) - ab^2d + b^3c)}{3a^5} + \frac{b \log(x)(a^2be + a^3(-f) - ab^2d + b^3c)}{a^5} - a$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x^3 + e*x^6 + f*x^9)/(x^{13}(a + b*x^3)), x]$

[Out] $-c/(12*a*x^{12}) + (b*c - a*d)/(9*a^2*x^9) - (b^2*c - a*b*d + a^2*e)/(6*a^3*x^6) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*x^3) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[x])/a^5 - (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[a + b*x^3])/(3*a^5)$

Rule 1821

$\text{Int}[(Pq_)*(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*\text{SubstFor}[x^n, Pq, x]}*(a + b*x)^p, x], x, x^n], x] /;$ FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1620

$\text{Int}[(Px_)*((a_.) + (b_.)*(x_))^{(m_.)}*((c_.) + (d_.)*(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[Px*(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E

xpon[Px, x], 2]

Rubi steps

$$\begin{aligned} \int \frac{c + dx^3 + ex^6 + fx^9}{x^{13}(a + bx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{c + dx + ex^2 + fx^3}{x^5(a + bx)} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{c}{ax^5} + \frac{-bc + ad}{a^2x^4} + \frac{b^2c - abd + a^2e}{a^3x^3} + \frac{-b^3c + ab^2d - a^2be + a^3f}{a^4x^2} - \frac{b(-b^3c + a^3f)}{a^5} \right) dx, x, x^3 \right) \\ &= -\frac{c}{12ax^{12}} + \frac{bc - ad}{9a^2x^9} - \frac{b^2c - abd + a^2e}{6a^3x^6} + \frac{b^3c - ab^2d + a^2be - a^3f}{3a^4x^3} + \frac{b(b^3c - ab^2d + a^2be - a^3f)}{a^5} \end{aligned}$$

Mathematica [A] time = 0.0624507, size = 164, normalized size = 1.

$$\frac{36bx^{12} \log(x) (a^2be + a^3(-f) - ab^2d + b^3c) - 12bx^{12} \log(a + bx^3) (a^2be + a^3(-f) - ab^2d + b^3c) - 6a^2b^2x^6 (c + 2dx^3) + 2a^5}{36a^5x^{12}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)), x]

[Out] (12*a*b^3*c*x^9 - 6*a^2*b^2*x^6*(c + 2*d*x^3) + 2*a^3*b*x^3*(2*c + 3*d*x^3 + 6*e*x^6) - a^4*(3*c + 4*d*x^3 + 6*e*x^6 + 12*f*x^9) + 36*b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^12*Log[x] - 12*b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^12*Log[a + b*x^3])/(36*a^5*x^12)

Maple [A] time = 0.009, size = 210, normalized size = 1.3

$$\frac{b \ln(bx^3 + a) f}{3a^2} - \frac{b^2 \ln(bx^3 + a) e}{3a^3} + \frac{b^3 \ln(bx^3 + a) d}{3a^4} - \frac{b^4 \ln(bx^3 + a) c}{3a^5} - \frac{c}{12ax^{12}} - \frac{d}{9ax^9} + \frac{bc}{9a^2x^9} - \frac{e}{6ax^6} + \frac{bd}{6a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a), x)

[Out] 1/3*b/a^2*ln(b*x^3+a)*f-1/3*b^2/a^3*ln(b*x^3+a)*e+1/3*b^3/a^4*ln(b*x^3+a)*d-1/3*b^4/a^5*ln(b*x^3+a)*c-1/12*c/a/x^12-1/9/a/x^9*d+1/9/a^2/x^9*b*c-1/6/a/x^6*e+1/6/a^2/x^6*b*d-1/6/a^3/x^6*b^2*c-1/3/a/x^3*f+1/3/a^2/x^3*b*e-1/3/a^3

$/x^3b^2d+1/3/a^4/x^3b^3c-1/a^2*b*\ln(x)*f+1/a^3*b^2*\ln(x)*e-1/a^4*b^3*\ln(x)*d+1/a^5*b^4*\ln(x)*c$

Maxima [A] time = 0.955154, size = 224, normalized size = 1.37

$$\frac{(b^4c - ab^3d + a^2b^2e - a^3bf) \log(bx^3 + a)}{3a^5} + \frac{(b^4c - ab^3d + a^2b^2e - a^3bf) \log(x^3)}{3a^5} + \frac{12(b^3c - ab^2d + a^2be - a^3f)x^9 - 36(b^4c - ab^3d + a^2b^2e - a^3bf)x^{12} \log(x) - 12(ab^3c - a^2b^2d + a^3be - a^4f)x^9 + 6(a^2b^2c - a^3b*d + a^4e)x^6 + 3a^4c - 4(a^3b*c - a^4d)x^3}{36a^5x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a),x, algorithm="maxima")

[Out] $-1/3*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*\log(b*x^3 + a)/a^5 + 1/3*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*\log(x^3)/a^5 + 1/36*(12*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9 - 6*(a*b^2*c - a^2*b*d + a^3*e)*x^6 - 3*a^3*c + 4*(a^2*b*c - a^3*d)*x^3)/(a^4*x^{12})$

Fricas [A] time = 1.5128, size = 355, normalized size = 2.16

$$\frac{12(b^4c - ab^3d + a^2b^2e - a^3bf)x^{12} \log(bx^3 + a) - 36(b^4c - ab^3d + a^2b^2e - a^3bf)x^{12} \log(x) - 12(ab^3c - a^2b^2d + a^3be - a^4f)x^9 + 6(a^2b^2c - a^3b*d + a^4e)x^6 + 3a^4c - 4(a^3b*c - a^4d)x^3}{36a^5x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a),x, algorithm="fricas")

[Out] $-1/36*(12*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^{12}*\log(b*x^3 + a) - 36*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^{12}*\log(x) - 12*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^9 + 6*(a^2*b^2*c - a^3*b*d + a^4*e)*x^6 + 3*a^4*c - 4*(a^3*b*c - a^4*d)*x^3)/(a^5*x^{12})$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**13/(b*x**3+a),x)

[Out] Timed out

Giac [A] time = 1.06825, size = 317, normalized size = 1.93

$$\frac{(b^4c - ab^3d - a^3bf + a^2b^2e) \log(|x|)}{a^5} - \frac{(b^5c - ab^4d - a^3b^2f + a^2b^3e) \log(|bx^3 + a|)}{3a^5b} - \frac{25b^4cx^{12} - 25ab^3dx^{12} - 25a^3bfx^{12}}{3a^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a),x, algorithm="giac")

[Out] (b^4*c - a*b^3*d - a^3*b*f + a^2*b^2*e)*log(abs(x))/a^5 - 1/3*(b^5*c - a*b^4*d - a^3*b^2*f + a^2*b^3*e)*log(abs(b*x^3 + a))/(a^5*b) - 1/36*(25*b^4*c*x^12 - 25*a*b^3*d*x^12 - 25*a^3*b*f*x^12 + 25*a^2*b^2*x^12*e - 12*a*b^3*c*x^9 + 12*a^2*b^2*d*x^9 + 12*a^4*f*x^9 - 12*a^3*b*x^9*e + 6*a^2*b^2*c*x^6 - 6*a^3*b*d*x^6 + 6*a^4*x^6*e - 4*a^3*b*c*x^3 + 4*a^4*d*x^3 + 3*a^4*c)/(a^5*x^12)

$$3.232 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^{16}(a+bx^3)} dx$$

Optimal. Leaf size=205

$$-\frac{b(a^2be + a^3(-f) - ab^2d + b^3c)}{3a^5x^3} + \frac{a^2be + a^3(-f) - ab^2d + b^3c}{6a^4x^6} + \frac{b^2 \log(a + bx^3)(a^2be + a^3(-f) - ab^2d + b^3c)}{3a^6} - \frac{b^2 \log(a + bx^3)}{3a^6}$$

[Out] $-c/(15*a*x^{15}) + (b*c - a*d)/(12*a^2*x^{12}) - (b^2*c - a*b*d + a^2*e)/(9*a^3*x^9) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a^4*x^6) - (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*a^5*x^3) - (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[x])/a^6 + (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[a + b*x^3])/(3*a^6)$

Rubi [A] time = 0.208872, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1821, 1620}

$$-\frac{b(a^2be + a^3(-f) - ab^2d + b^3c)}{3a^5x^3} + \frac{a^2be + a^3(-f) - ab^2d + b^3c}{6a^4x^6} + \frac{b^2 \log(a + bx^3)(a^2be + a^3(-f) - ab^2d + b^3c)}{3a^6} - \frac{b^2 \log(a + bx^3)}{3a^6}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^16*(a + b*x^3)), x]

[Out] $-c/(15*a*x^{15}) + (b*c - a*d)/(12*a^2*x^{12}) - (b^2*c - a*b*d + a^2*e)/(9*a^3*x^9) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a^4*x^6) - (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*a^5*x^3) - (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[x])/a^6 + (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[a + b*x^3])/(3*a^6)$

Rule 1821

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1620

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]

, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[Expon[Px, x], 2]

Rubi steps

$$\begin{aligned} \int \frac{c + dx^3 + ex^6 + fx^9}{x^{16}(a + bx^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{c + dx + ex^2 + fx^3}{x^6(a + bx)} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{c}{ax^6} + \frac{-bc + ad}{a^2x^5} + \frac{b^2c - abd + a^2e}{a^3x^4} + \frac{-b^3c + ab^2d - a^2be + a^3f}{a^4x^3} - \frac{b(-b^3c + a^3f)}{a^4x^3} \right) dx, x, x^3 \right) \\ &= -\frac{c}{15ax^{15}} + \frac{bc - ad}{12a^2x^{12}} - \frac{b^2c - abd + a^2e}{9a^3x^9} + \frac{b^3c - ab^2d + a^2be - a^3f}{6a^4x^6} - \frac{b(b^3c - ab^2d + a^2be - a^3f)}{3a^5x^3} \end{aligned}$$

Mathematica [A] time = 0.143637, size = 194, normalized size = 0.95

$$\frac{a(10a^2b^2x^6(2c+3dx^3+6ex^6)-5a^3bx^3(3c+4dx^3+6ex^6+12fx^9)+a^4(12c+15dx^3+20ex^6+30fx^9)-30ab^3x^9(c+2dx^3)+60b^4cx^{12})}{x^{15}} - \frac{60b^2 \log(a + bx^3)(a^2be - a^3f)}{180a^6}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^16*(a + b*x^3)),x]

[Out] -((a*(60*b^4*c*x^12 - 30*a*b^3*x^9*(c + 2*d*x^3) + 10*a^2*b^2*x^6*(2*c + 3*d*x^3 + 6*e*x^6) - 5*a^3*b*x^3*(3*c + 4*d*x^3 + 6*e*x^6 + 12*f*x^9) + a^4*(12*c + 15*d*x^3 + 20*e*x^6 + 30*f*x^9)))/x^15 + 180*b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[x] - 60*b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(180*a^6)

Maple [A] time = 0.01, size = 260, normalized size = 1.3

$$-\frac{b^2 \ln(bx^3 + a)f}{3a^3} + \frac{b^3 \ln(bx^3 + a)e}{3a^4} - \frac{b^4 \ln(bx^3 + a)d}{3a^5} + \frac{b^5 \ln(bx^3 + a)c}{3a^6} - \frac{c}{15ax^{15}} - \frac{d}{12ax^{12}} + \frac{bc}{12a^2x^{12}} - \frac{e}{9ax^9} + \frac{f}{9a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/x^16/(b*x^3+a),x)

[Out] $-1/3*b^2/a^3*\ln(b*x^3+a)*f+1/3*b^3/a^4*\ln(b*x^3+a)*e-1/3*b^4/a^5*\ln(b*x^3+a)*d+1/3*b^5/a^6*\ln(b*x^3+a)*c-1/15*c/a/x^{15}-1/12/a/x^{12}*d+1/12/a^2/x^{12}*b*c-1/9/a/x^9*e+1/9/a^2/x^9*b*d-1/9/a^3/x^9*b^2*c-1/6/a/x^6*f+1/6/a^2/x^6*b*e-1/6/a^3/x^6*b^2*d+1/6/a^4/x^6*b^3*c+1/a^3*b^2*\ln(x)*f-1/a^4*b^3*\ln(x)*e+1/a^5*b^4*\ln(x)*d-1/a^6*b^5*\ln(x)*c+1/3/a^2*b/x^3*f-1/3/a^3*b^2/x^3*e+1/3/a^4*b^3/x^3*d-1/3/a^5*b^4/x^3*c$

Maxima [A] time = 0.958911, size = 281, normalized size = 1.37

$$\frac{(b^5c - ab^4d + a^2b^3e - a^3b^2f) \log(bx^3 + a)}{3a^6} - \frac{(b^5c - ab^4d + a^2b^3e - a^3b^2f) \log(x^3)}{3a^6} - \frac{60(b^4c - ab^3d + a^2b^2e - a^3bf)x^{15}}{180a^6x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^9+e*x^6+d*x^3+c)/x^16/(b*x^3+a),x, algorithm="maxima")`

[Out] $1/3*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*\log(b*x^3 + a)/a^6 - 1/3*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*\log(x^3)/a^6 - 1/180*(60*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^{12} - 30*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^9 + 20*(a^2*b^2*c - a^3*b*d + a^4*e)*x^6 + 12*a^4*c - 15*(a^3*b*c - a^4*d)*x^3)/(a^5*x^{15})$

Fricas [A] time = 1.6669, size = 441, normalized size = 2.15

$$\frac{60(b^5c - ab^4d + a^2b^3e - a^3b^2f)x^{15} \log(bx^3 + a) - 180(b^5c - ab^4d + a^2b^3e - a^3b^2f)x^{15} \log(x) - 60(ab^4c - a^2b^3d + a^3bf)x^{15}}{180a^6x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^9+e*x^6+d*x^3+c)/x^16/(b*x^3+a),x, algorithm="fricas")`

[Out] $1/180*(60*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^{15}*\log(b*x^3 + a) - 180*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^{15}*\log(x) - 60*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^{12} + 30*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x^9 - 20*(a^3*b^2*c - a^4*b*d + a^5*e)*x^6 - 12*a^5*c + 15*(a^4*b*c - a^5*d)*x^3)/(a^6*x^{15})$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**16/(b*x**3+a),x)

[Out] Timed out

Giac [A] time = 1.06632, size = 387, normalized size = 1.89

$$-\frac{(b^5c - ab^4d - a^3b^2f + a^2b^3e) \log(|x|)}{a^6} + \frac{(b^6c - ab^5d - a^3b^3f + a^2b^4e) \log(|bx^3 + a|)}{3a^6b} + \frac{137b^5cx^{15} - 137ab^4dx^{15} - 137c^2x^{15}}{180a^6b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^16/(b*x^3+a),x, algorithm="giac")

[Out] $-(b^5c - a*b^4*d - a^3*b^2*f + a^2*b^3*e)*\log(\text{abs}(x))/a^6 + 1/3*(b^6*c - a*b^5*d - a^3*b^3*f + a^2*b^4*e)*\log(\text{abs}(b*x^3 + a))/(a^6*b) + 1/180*(137*b^5*c*x^{15} - 137*a*b^4*d*x^{15} - 137*a^3*b^2*f*x^{15} + 137*a^2*b^3*x^{15}*e - 60*a*b^4*c*x^{12} + 60*a^2*b^3*d*x^{12} + 60*a^4*b*f*x^{12} - 60*a^3*b^2*x^{12}*e + 30*a^2*b^3*c*x^9 - 30*a^3*b^2*d*x^9 - 30*a^5*f*x^9 + 30*a^4*b*x^9*e - 20*a^3*b^2*c*x^6 + 20*a^4*b*d*x^6 - 20*a^5*x^6*e + 15*a^4*b*c*x^3 - 15*a^5*d*x^3 - 12*a^5*c)/(a^6*x^{15})$

$$3.233 \quad \int \frac{x^9(c+dx^3+ex^6+fx^9)}{a+bx^3} dx$$

Optimal. Leaf size=348

$$\frac{x^7(a^2be + a^3(-f) - ab^2d + b^3c)}{7b^4} - \frac{ax^4(a^2be + a^3(-f) - ab^2d + b^3c)}{4b^5} + \frac{a^{7/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^2be + a^3(-f))}{6b^{19/3}}$$

[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^6 - (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^4)/(4*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^7)/(7*b^4) + ((b^2*d - a*b*e + a^2*f)*x^10)/(10*b^3) + ((b*e - a*f)*x^13)/(13*b^2) + (f*x^16)/(16*b) + (a^(7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(19/3)) - (a^(7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(19/3)) + (a^(7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(19/3))

Rubi [A] time = 0.332909, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1836, 1488, 200, 31, 634, 617, 204, 628}

$$\frac{x^7(a^2be + a^3(-f) - ab^2d + b^3c)}{7b^4} - \frac{ax^4(a^2be + a^3(-f) - ab^2d + b^3c)}{4b^5} + \frac{a^{7/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^2be + a^3(-f))}{6b^{19/3}}$$

Antiderivative was successfully verified.

[In] Int[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]

[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^6 - (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^4)/(4*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^7)/(7*b^4) + ((b^2*d - a*b*e + a^2*f)*x^10)/(10*b^3) + ((b*e - a*f)*x^13)/(13*b^2) + (f*x^16)/(16*b) + (a^(7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(19/3)) - (a^(7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(19/3)) + (a^(7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(19/3))

Rule 1836

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m + q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1), x]] /; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1488

```
Int[((f_)*(x_))^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_))^(p_)*(d_ + (e_)*(x_)^(n_))^(q_), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 200

```
Int[((a_) + (b_)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(p_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_ + (e_)*(x_))/(a_ + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

a, 0] || LtQ[b, 0])

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
 imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^9(c + dx^3 + ex^6 + fx^9)}{a + bx^3} dx &= \frac{fx^{16}}{16b} + \frac{\int \frac{x^9(16bc + 16bdx^3 + 16(be - af)x^6)}{a + bx^3} dx}{16b} \\
 &= \frac{fx^{16}}{16b} + \frac{\int \left(\frac{16a^2(b^3c - ab^2d + a^2be - a^3f)}{b^5} - \frac{16a(b^3c - ab^2d + a^2be - a^3f)x^3}{b^4} + \frac{16(b^3c - ab^2d + a^2be - a^3f)x^6}{b^3} + \frac{16(b^3c - ab^2d + a^2be - a^3f)x^9}{b^2} \right) dx}{16b} \\
 &= \frac{a^2(b^3c - ab^2d + a^2be - a^3f)x}{b^6} - \frac{a(b^3c - ab^2d + a^2be - a^3f)x^4}{4b^5} + \frac{(b^3c - ab^2d + a^2be - a^3f)x^7}{7b^4} \\
 &= \frac{a^2(b^3c - ab^2d + a^2be - a^3f)x}{b^6} - \frac{a(b^3c - ab^2d + a^2be - a^3f)x^4}{4b^5} + \frac{(b^3c - ab^2d + a^2be - a^3f)x^7}{7b^4} \\
 &= \frac{a^2(b^3c - ab^2d + a^2be - a^3f)x}{b^6} - \frac{a(b^3c - ab^2d + a^2be - a^3f)x^4}{4b^5} + \frac{(b^3c - ab^2d + a^2be - a^3f)x^7}{7b^4} \\
 &= \frac{a^2(b^3c - ab^2d + a^2be - a^3f)x}{b^6} - \frac{a(b^3c - ab^2d + a^2be - a^3f)x^4}{4b^5} + \frac{(b^3c - ab^2d + a^2be - a^3f)x^7}{7b^4} \\
 &= \frac{a^2(b^3c - ab^2d + a^2be - a^3f)x}{b^6} - \frac{a(b^3c - ab^2d + a^2be - a^3f)x^4}{4b^5} + \frac{(b^3c - ab^2d + a^2be - a^3f)x^7}{7b^4}
 \end{aligned}$$

Mathematica [A] time = 0.0786584, size = 351, normalized size = 1.01

$$\frac{x^7(a^2be + a^3(-f) - ab^2d + b^3c)}{7b^4} + \frac{ax^4(-a^2be + a^3f + ab^2d - b^3c)}{4b^5} - \frac{a^{7/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(-a^2be + a^3f + ab^2d - b^3c)}{6b^{19/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3), x]

```
[Out] -((a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/b^6) + (a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^4)/(4*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^7)/(7*b^4) + ((b^2*d - a*b*e + a^2*f)*x^10)/(10*b^3) + ((b*e - a*f)*x^13)/(13*b^2) + (f*x^16)/(16*b) + (a^(7/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(19/3)) + (a^(7/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(19/3)) - (a^(7/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(19/3))
```

Maple [A] time = 0.003, size = 592, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] -1/b^4*a^3*d*x+1/b^3*a^2*c*x+1/10/b^3*x^10*a^2*f-1/10/b^2*x^10*a*e-1/7/b^4*x^7*a^3*f-1/13/b^2*x^13*a*f-1/b^6*a^5*f*x+1/b^5*a^4*e*x-1/6*a^6/b^7/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f+1/6*a^5/b^6/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e-1/3*a^3/b^4/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c+1/3*a^4/b^5/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d+1/7/b*x^7*c+1/13/b*x^13*e+1/10/b*x^10*d+1/3*a^6/b^7/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*f-1/3*a^5/b^6/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*e+1/3*a^4/b^5/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d-1/3*a^3/b^4/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c-1/6*a^4/b^5/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d+1/6*a^3/b^4/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+1/3*a^6/b^7/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f+1/7/b^3*x^7*a^2*e-1/7/b^2*x^7*a*d+1/4/b^5*x^4*a^4*f-1/4/b^4*x^4*a^3*e+1/4/b^3*x^4*a^2*d-1/4/b^2*x^4*a*c+1/16*f*x^16/b-1/3*a^5/b^6/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```


$f - a^{**3}b^*e + a^{**2}b^{**2}d - a*b^{**3}c)/(4*b^{**5}) - x*(a^{**5}f - a^{**4}b^*e + a^{**3}b^{**2}d - a^{**2}b^{**3}c)/b^{**6}$

Giac [A] time = 1.0624, size = 613, normalized size = 1.76

$$\frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}a^2b^3c - (-ab^2)^{\frac{1}{3}}a^3b^2d - (-ab^2)^{\frac{1}{3}}a^5f + (-ab^2)^{\frac{1}{3}}a^4be\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^7} - \frac{\left((-ab^2)^{\frac{1}{3}}a^2b^3c - (-ab^2)^{\frac{1}{3}}a^3b^2d - (-ab^2)^{\frac{1}{3}}a^5f + (-ab^2)^{\frac{1}{3}}a^4be\right)}{3b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")

[Out] $-1/3*\sqrt{3}*((-a*b^2)^{(1/3)}*a^2*b^3*c - (-a*b^2)^{(1/3)}*a^3*b^2*d - (-a*b^2)^{(1/3)}*a^5*f + (-a*b^2)^{(1/3)}*a^4*b*e)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/b^7 - 1/6*((-a*b^2)^{(1/3)}*a^2*b^3*c - (-a*b^2)^{(1/3)}*a^3*b^2*d - (-a*b^2)^{(1/3)}*a^5*f + (-a*b^2)^{(1/3)}*a^4*b*e)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/b^7 + 1/3*(a^3*b^13*c - a^4*b^12*d - a^6*b^10*f + a^5*b^11*e)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/a*b^16 + 1/7280*(455*b^15*f*x^16 - 560*a*b^14*f*x^13 + 560*b^15*x^13*e + 728*b^15*d*x^10 + 728*a^2*b^13*f*x^10 - 728*a*b^14*x^10*e + 1040*b^15*c*x^7 - 1040*a*b^14*d*x^7 - 1040*a^3*b^12*f*x^7 + 1040*a^2*b^13*x^7*e - 1820*a*b^14*c*x^4 + 1820*a^2*b^13*d*x^4 + 1820*a^4*b^11*f*x^4 - 1820*a^3*b^12*x^4*e + 7280*a^2*b^13*c*x - 7280*a^3*b^12*d*x - 7280*a^5*b^10*f*x + 7280*a^4*b^11*x*e)/b^16$

$$3.234 \quad \int \frac{x^7(c+dx^3+ex^6+fx^9)}{a+bx^3} dx$$

Optimal. Leaf size=316

$$\frac{x^5(a^2be + a^3(-f) - ab^2d + b^3c)}{5b^4} - \frac{ax^2(a^2be + a^3(-f) - ab^2d + b^3c)}{2b^5} + \frac{a^{5/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^2be + a^3(-f))}{6b^{17/3}}$$

[Out] $-(a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(2*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^5)/(5*b^4) + ((b^2*d - a*b*e + a^2*f)*x^8)/(8*b^3) + ((b*e - a*f)*x^{11})/(11*b^2) + (f*x^{14})/(14*b) - (a^{(5/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*b^{(17/3)}) - (a^{(5/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{(1/3)} + b^{(1/3)}*x])/(3*b^{(17/3)}) + (a^{(5/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(6*b^{(17/3)})$

Rubi [A] time = 0.306078, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1836, 1488, 292, 31, 634, 617, 204, 628}

$$\frac{x^5(a^2be + a^3(-f) - ab^2d + b^3c)}{5b^4} - \frac{ax^2(a^2be + a^3(-f) - ab^2d + b^3c)}{2b^5} + \frac{a^{5/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^2be + a^3(-f))}{6b^{17/3}}$$

Antiderivative was successfully verified.

[In] Int[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]

[Out] $-(a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(2*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^5)/(5*b^4) + ((b^2*d - a*b*e + a^2*f)*x^8)/(8*b^3) + ((b*e - a*f)*x^{11})/(11*b^2) + (f*x^{14})/(14*b) - (a^{(5/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*b^{(17/3)}) - (a^{(5/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{(1/3)} + b^{(1/3)}*x])/(3*b^{(17/3)}) + (a^{(5/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(6*b^{(17/3)})$

Rule 1836

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*]

```
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x] /;
NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
+ 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^( -1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^( -1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x^7 (c + dx^3 + ex^6 + fx^9)}{a + bx^3} dx &= \frac{fx^{14}}{14b} + \frac{\int \frac{x^7(14bc+14bdx^3+14(be-af)x^6)}{a+bx^3} dx}{14b} \\ &= \frac{fx^{14}}{14b} + \frac{\int \left(-\frac{14a(b^3c-ab^2d+a^2be-a^3f)x}{b^4} + \frac{14(b^3c-ab^2d+a^2be-a^3f)x^4}{b^3} + \frac{14(b^2d-abe+a^2f)x^7}{b^2} + \frac{14(be-af)x^{10}}{b} \right) dx}{14b} \\ &= -\frac{a(b^3c-ab^2d+a^2be-a^3f)x^2}{2b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)x^5}{5b^4} + \frac{(b^2d-abe+a^2f)x^8}{8b^3} \\ &= -\frac{a(b^3c-ab^2d+a^2be-a^3f)x^2}{2b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)x^5}{5b^4} + \frac{(b^2d-abe+a^2f)x^8}{8b^3} \\ &= -\frac{a(b^3c-ab^2d+a^2be-a^3f)x^2}{2b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)x^5}{5b^4} + \frac{(b^2d-abe+a^2f)x^8}{8b^3} \\ &= -\frac{a(b^3c-ab^2d+a^2be-a^3f)x^2}{2b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)x^5}{5b^4} + \frac{(b^2d-abe+a^2f)x^8}{8b^3} \\ &= -\frac{a(b^3c-ab^2d+a^2be-a^3f)x^2}{2b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)x^5}{5b^4} + \frac{(b^2d-abe+a^2f)x^8}{8b^3} \end{aligned}$$

Mathematica [A] time = 0.0975826, size = 311, normalized size = 0.98

$$\frac{x^5 (a^2be + a^3(-f) - ab^2d + b^3c)}{5b^4} + \frac{ax^2 (-a^2be + a^3f + ab^2d - b^3c)}{2b^5} - \frac{a^{5/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2) (-a^2be + a^3f + ab^2d - b^3c)}{6b^{17/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]

[Out] (a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(2*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^5)/(5*b^4) + ((b^2*d - a*b*e + a^2*f)*x^8)/(8*b^3) + (

$$\begin{aligned} & (b^2e - a^2f)x^{11}/(11b^2) + (fx^{14})/(14b) + (a^{5/3})(-(b^3c) + ab^2d \\ & - a^2be + a^3f) \operatorname{ArcTan}[(1 - (2b^{1/3})x)/a^{1/3}]/\sqrt{3}]/(\sqrt{3}b^{17/3}) \\ & + (a^{5/3})(-(b^3c) + ab^2d - a^2be + a^3f) \operatorname{Log}[a^{1/3} + b^{1/3}x] \\ & / (3b^{17/3}) - (a^{5/3})(-(b^3c) + ab^2d - a^2be + a^3f) \operatorname{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2] \\ & / (6b^{17/3}) \end{aligned}$$

Maple [B] time = 0.003, size = 554, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}(x^7(fx^9+ex^6+dx^3+c)/(bx^3+a), x)$

[Out] $\frac{1}{3}a^3/b^4/(1/b^3a)^{1/3} \ln(x+(1/b^3a)^{1/3})d - \frac{1}{3}a^2/b^3/(1/b^3a)^{1/3} \ln(x+(1/b^3a)^{1/3})c - \frac{1}{6}a^5/b^6/(1/b^3a)^{1/3} \ln(x^2-(1/b^3a)^{1/3}x+(1/b^3a)^{2/3})f + \frac{1}{6}a^4/b^5/(1/b^3a)^{1/3} \ln(x^2-(1/b^3a)^{1/3}x+(1/b^3a)^{2/3})e - \frac{1}{6}a^3/b^4/(1/b^3a)^{1/3} \ln(x^2-(1/b^3a)^{1/3}x+(1/b^3a)^{2/3})d + \frac{1}{6}a^2/b^3/(1/b^3a)^{1/3} \ln(x^2-(1/b^3a)^{1/3}x+(1/b^3a)^{2/3})c - \frac{1}{3}a^4/b^5/(1/b^3a)^{1/3} \ln(x+(1/b^3a)^{1/3})e + \frac{1}{11}bx^{11}e + \frac{1}{8}bx^8d + \frac{1}{5}bx^5c - \frac{1}{3}a^5/b^6 \cdot 3^{1/2}/(1/b^3a)^{1/3} \arctan(1/3 \cdot 3^{1/2} \cdot (2/(1/b^3a)^{1/3}x-1))f + \frac{1}{3}a^4/b^5 \cdot 3^{1/2}/(1/b^3a)^{1/3} \arctan(1/3 \cdot 3^{1/2} \cdot (2/(1/b^3a)^{1/3}x-1))e - \frac{1}{3}a^3/b^4 \cdot 3^{1/2}/(1/b^3a)^{1/3} \arctan(1/3 \cdot 3^{1/2} \cdot (2/(1/b^3a)^{1/3}x-1))d + \frac{1}{3}a^2/b^3 \cdot 3^{1/2}/(1/b^3a)^{1/3} \arctan(1/3 \cdot 3^{1/2} \cdot (2/(1/b^3a)^{1/3}x-1))c + \frac{1}{3}a^5/b^6/(1/b^3a)^{1/3} \ln(x+(1/b^3a)^{1/3})f + \frac{1}{2}b^5x^2a^4f - \frac{1}{2}b^4x^2a^3e - \frac{1}{11}b^2x^{11}af + \frac{1}{8}b^3x^8a^2f - \frac{1}{8}b^2x^8ae - \frac{1}{2}b^2x^2ac - \frac{1}{5}b^4x^5a^3f + \frac{1}{5}b^3x^5a^2e - \frac{1}{5}b^2x^5ad + \frac{1}{2}b^3x^2a^2d + \frac{1}{14}fx^{14}/b$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}(x^7(fx^9+ex^6+dx^3+c)/(bx^3+a), x, \operatorname{algorithm}="maxima")$

[Out] Exception raised: ValueError

Fricas [A] time = 1.44431, size = 726, normalized size = 2.3

$$660b^4fx^{14} + 840(b^4e - ab^3f)x^{11} + 1155(b^4d - ab^3e + a^2b^2f)x^8 + 1848(b^4c - ab^3d + a^2b^2e - a^3bf)x^5 - 4620(ab^3c - a^2b^2d + a^3b^2e - a^4bf)x^2 + 3080\sqrt{3}(ab^3c - a^2b^2d + a^3b^2e - a^4bf)(a^2/b^2)^{1/3} \arctan(1/3(2\sqrt{3}bx(a^2/b^2)^{1/3} - \sqrt{3}a)/a) + 1540(ab^3c - a^2b^2d + a^3b^2e - a^4bf)(a^2/b^2)^{1/3} \log(ax^2 - bx(a^2/b^2)^{2/3} + a(a^2/b^2)^{1/3}) - 3080(ab^3c - a^2b^2d + a^3b^2e - a^4bf)(a^2/b^2)^{1/3} \log(ax + b(a^2/b^2)^{2/3})/b^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")

[Out] 1/9240*(660*b^4*f*x^14 + 840*(b^4*e - a*b^3*f)*x^11 + 1155*(b^4*d - a*b^3*e + a^2*b^2*f)*x^8 + 1848*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^5 - 4620*(a*b^3*c - a^2*b^2*d + a^3*b^2*e - a^4*b*f)*x^2 + 3080*sqrt(3)*(a*b^3*c - a^2*b^2*d + a^3*b^2*e - a^4*b*f)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) + 1540*(a*b^3*c - a^2*b^2*d + a^3*b^2*e - a^4*b*f)*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a*(a^2/b^2)^(1/3)) - 3080*(a*b^3*c - a^2*b^2*d + a^3*b^2*e - a^4*b*f)*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3))/b^5

Sympy [A] time = 1.45072, size = 496, normalized size = 1.57

$$\text{RootSum}\left(27t^3b^{17} - a^{14}f^3 + 3a^{13}bef^2 - 3a^{12}b^2df^2 - 3a^{12}b^2e^2f + 3a^{11}b^3cf^2 + 6a^{11}b^3def + a^{11}b^3e^3 - 6a^{10}b^4cef - 3a^{10}b^4e^2f + 3a^{10}b^4e^2e^2f + 3a^{11}b^3c^2f + 6a^{11}b^3d^2ef + a^{11}b^3e^2e^2f - 6a^{10}b^4c^2ef - 3a^{10}b^4d^2e^2f - 3a^{10}b^4d^2e^2e^2 + 6a^{9}b^5c^2df + 3a^{9}b^5c^2e^2 + 3a^{9}b^5d^2e^2 - 3a^{8}b^6c^2f - 6a^{8}b^6c^2de - a^{8}b^6d^2e^3 + 3a^{7}b^7c^2e^2 + 3a^{7}b^7c^2d^2 - 3a^{6}b^8c^2d + a^{5}b^9c^3, \text{Lambda}(t, t \log(9t^{11}/(a^9f^2 - 2a^8b^2ef + 2a^7b^2d^2f + a^7b^2e^2 - 2a^6b^3c^2f - 2a^6b^3d^2e + 2a^5b^4c^2e + a^5b^4d^2 - 2a^4b^5c^2d + a^3b^6c^2) + x)) + f*x^{14}/(14*b) - x^{11}*(af - be)/(11*b^2) + x^8*(a^2*f - a*b*e + b^2*d)/(8*b^3) - x^5*(a^3*f - a^2*b*e +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)

[Out] RootSum(27*_t**3*b**17 - a**14*f**3 + 3*a**13*b*e*f**2 - 3*a**12*b**2*d*f**2 - 3*a**12*b**2*e**2*f + 3*a**11*b**3*c*f**2 + 6*a**11*b**3*d*e*f + a**11*b**3*e**3 - 6*a**10*b**4*c*e*f - 3*a**10*b**4*d**2*f - 3*a**10*b**4*d*e**2 + 6*a**9*b**5*c*d*f + 3*a**9*b**5*c*e**2 + 3*a**9*b**5*d**2*e - 3*a**8*b**6*c**2*f - 6*a**8*b**6*c*d*e - a**8*b**6*d**3 + 3*a**7*b**7*c**2*e + 3*a**7*b**7*c*d**2 - 3*a**6*b**8*c**2*d + a**5*b**9*c**3, Lambda(_t, _t*log(9*_t**2*b**11/(a**9*f**2 - 2*a**8*b**2*e*f + 2*a**7*b**2*d**2*f + a**7*b**2*e**2 - 2*a**6*b**3*c*f - 2*a**6*b**3*d**2*e + 2*a**5*b**4*c**2*e + a**5*b**4*d**2 - 2*a**4*b**5*c*d + a**3*b**6*c**2) + x))) + f*x**14/(14*b) - x**11*(a*f - b*e)/(11*b**2) + x**8*(a**2*f - a*b*e + b**2*d)/(8*b**3) - x**5*(a**3*f - a**2*b*e +

$a*b**2*d - b**3*c)/(5*b**4) + x**2*(a**4*f - a**3*b*e + a**2*b**2*d - a*b**3*c)/(2*b**5)$

Giac [A] time = 1.0825, size = 595, normalized size = 1.88

$$\frac{\sqrt{3}\left((-ab^2)^{\frac{2}{3}}ab^3c - (-ab^2)^{\frac{2}{3}}a^2b^2d - (-ab^2)^{\frac{2}{3}}a^4f + (-ab^2)^{\frac{2}{3}}a^3be\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^7} + \frac{\left((-ab^2)^{\frac{2}{3}}ab^3c - (-ab^2)^{\frac{2}{3}}a^2b^2d - (-ab^2)^{\frac{2}{3}}a^4f + (-ab^2)^{\frac{2}{3}}a^3be\right)\log\left(x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")

[Out] $-1/3*\sqrt{3}*((-a*b^2)^{(2/3)}*a*b^3*c - (-a*b^2)^{(2/3)}*a^2*b^2*d - (-a*b^2)^{(2/3)}*a^4*f + (-a*b^2)^{(2/3)}*a^3*b*e)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/b^7 + 1/6*((-a*b^2)^{(2/3)}*a*b^3*c - (-a*b^2)^{(2/3)}*a^2*b^2*d - (-a*b^2)^{(2/3)}*a^4*f + (-a*b^2)^{(2/3)}*a^3*b*e)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/b^7 - 1/3*(a^2*b^12*c*(-a/b)^{(1/3)} - a^3*b^11*d*(-a/b)^{(1/3)} - a^5*b^9*f*(-a/b)^{(1/3)} + a^4*b^10*(-a/b)^{(1/3)}*e)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/a*b^14 + 1/3080*(220*b^13*f*x^14 - 280*a*b^12*f*x^11 + 280*b^13*x^11*e + 385*b^13*d*x^8 + 385*a^2*b^11*f*x^8 - 385*a*b^12*x^8*e + 616*b^13*c*x^5 - 616*a*b^12*d*x^5 - 616*a^3*b^10*f*x^5 + 616*a^2*b^11*x^5*e - 1540*a*b^12*c*x^2 + 1540*a^2*b^11*d*x^2 + 1540*a^4*b^9*f*x^2 - 1540*a^3*b^10*x^2*e)/b^14$

$$3.235 \quad \int \frac{x^6(c+dx^3+ex^6+fx^9)}{a+bx^3} dx$$

Optimal. Leaf size=312

$$\frac{x^4(a^2be + a^3(-f) - ab^2d + b^3c)}{4b^4} - \frac{a^{4/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^{16/3}} - \frac{ax(a^2be + a^3(-f) - ab^2d + b^3c)}{b^5}$$

[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^4)/(4*b^4) + ((b^2*d - a*b*e + a^2*f)*x^7)/(7*b^3) + ((b*e - a*f)*x^10)/(10*b^2) + (f*x^13)/(13*b) - (a^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(16/3)) + (a^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(16/3)) - (a^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(16/3))

Rubi [A] time = 0.297768, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1836, 1488, 200, 31, 634, 617, 204, 628}

$$\frac{x^4(a^2be + a^3(-f) - ab^2d + b^3c)}{4b^4} - \frac{a^{4/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^{16/3}} - \frac{ax(a^2be + a^3(-f) - ab^2d + b^3c)}{b^5}$$

Antiderivative was successfully verified.

[In] Int[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]

[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^4)/(4*b^4) + ((b^2*d - a*b*e + a^2*f)*x^7)/(7*b^3) + ((b*e - a*f)*x^10)/(10*b^2) + (f*x^13)/(13*b) - (a^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(16/3)) + (a^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(16/3)) - (a^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(16/3))

Rule 1836

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq]

```
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
+ 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```


Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6 (c + dx^3 + ex^6 + fx^9)}{a + bx^3} dx &= \frac{fx^{13}}{13b} + \frac{\int \frac{x^6(13bc+13bdx^3+13(be-af)x^6)}{a+bx^3} dx}{13b} \\
&= \frac{fx^{13}}{13b} + \frac{\int \left(-\frac{13a(b^3c-ab^2d+a^2be-a^3f)}{b^4} + \frac{13(b^3c-ab^2d+a^2be-a^3f)x^3}{b^3} + \frac{13(b^2d-abe+a^2f)x^6}{b^2} + \frac{13(be-af)}{b} \right) dx}{13b} \\
&= -\frac{a(b^3c-ab^2d+a^2be-a^3f)x}{b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)x^4}{4b^4} + \frac{(b^2d-abe+a^2f)x^7}{7b^3} \\
&= -\frac{a(b^3c-ab^2d+a^2be-a^3f)x}{b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)x^4}{4b^4} + \frac{(b^2d-abe+a^2f)x^7}{7b^3} \\
&= -\frac{a(b^3c-ab^2d+a^2be-a^3f)x}{b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)x^4}{4b^4} + \frac{(b^2d-abe+a^2f)x^7}{7b^3} \\
&= -\frac{a(b^3c-ab^2d+a^2be-a^3f)x}{b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)x^4}{4b^4} + \frac{(b^2d-abe+a^2f)x^7}{7b^3} \\
&= -\frac{a(b^3c-ab^2d+a^2be-a^3f)x}{b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)x^4}{4b^4} + \frac{(b^2d-abe+a^2f)x^7}{7b^3}
\end{aligned}$$

Mathematica [A] time = 0.0934935, size = 306, normalized size = 0.98

$$\frac{x^4 (a^2be + a^3(-f) - ab^2d + b^3c)}{4b^4} + \frac{a^{4/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3}x^2) (-a^2be + a^3f + ab^2d - b^3c)}{6b^{16/3}} + \frac{ax (-a^2be + a^3f + ab^2d - b^3c)}{b^5}$$

Antiderivative was successfully verified.

[In] Integrate[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]

[Out] (a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/b^5 + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^4)/(4*b^4) + ((b^2*d - a*b*e + a^2*f)*x^7)/(7*b^3) + ((b*e -

$$\begin{aligned} & a^f x^{10} / (10 b^2) + (f x^{13}) / (13 b) + (a^{4/3} (-b^3 c) + a b^2 d - a^2 \\ & * b^e + a^3 f) \operatorname{ArcTan}[(1 - (2 b^{1/3} x) / a^{1/3}) / \sqrt{3}] / (\sqrt{3} b^{16/3}) \\ & - (a^{4/3} (-b^3 c) + a b^2 d - a^2 b^e + a^3 f) \operatorname{Log}[a^{1/3} + b^{1/3} x] / (3 b^{16/3}) \\ & + (a^{4/3} (-b^3 c) + a b^2 d - a^2 b^e + a^3 f) \operatorname{Log}[a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2] / (6 b^{16/3}) \end{aligned}$$

Maple [B] time = 0.004, size = 544, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)`

[Out]
$$\begin{aligned} & -1/7/b^2*x^7*a*e-1/4/b^4*x^4*a^3*f-1/10/b^2*x^10*a*f+1/b^3*a^2*d*x-1/b^2*a* \\ & c*x+1/4/b^3*x^4*a^2*e-1/4/b^2*x^4*a*d+1/b^5*a^4*f*x-1/b^4*a^3*e*x-1/6*a^2/b \\ & ^3/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*c+1/6*a^3/b^4/(1/b*a \\ &)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*d-1/3*a^3/b^4/(1/b*a)^{(2/3)}*3 \\ & ^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*d+1/3*a^2/b^3/(1/b*a)^{(2/3)} \\ &)*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*c+1/3*a^4/b^5/(1/b*a)^{(\\ & 2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*e-1/3*a^5/b^6/(1/b*a \\ &)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*f+1/3*a^4/b^5/(1/ \\ & b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*e-1/3*a^3/b^4/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/ \\ & 3)})*d+1/3*a^2/b^3/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*c+1/6*a^5/b^6/(1/b*a)^{(\\ & 2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*f-1/6*a^4/b^5/(1/b*a)^{(2/3)}*\ln(x \\ & ^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*e+1/10/b*x^10*e+1/7/b*x^7*d+1/4/b*x^4*c-1 \\ & /3*a^5/b^6/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*f+1/7/b^3*x^7*a^2*f+1/13*f*x^1 \\ & 3/b \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 1.37597, size = 679, normalized size = 2.18

$$420 b^4 f x^{13} + 546 (b^4 e - ab^3 f) x^{10} + 780 (b^4 d - ab^3 e + a^2 b^2 f) x^7 + 1365 (b^4 c - ab^3 d + a^2 b^2 e - a^3 b f) x^4 - 1820 \sqrt{3} (ab^3 c - a^2 b^2 d + a^3 b e - a^4 f) x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")

[Out] 1/5460*(420*b^4*f*x^13 + 546*(b^4*e - a*b^3*f)*x^10 + 780*(b^4*d - a*b^3*e + a^2*b^2*f)*x^7 + 1365*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^4 - 1820*sqrt(3)*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) + 910*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) - 1820*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*(-a/b)^(1/3)*log(x - (-a/b)^(1/3)) - 5460*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x/b^5

Sympy [A] time = 1.82208, size = 411, normalized size = 1.32

$$\text{RootSum}\left(27t^3b^{16} + a^{13}f^3 - 3a^{12}bef^2 + 3a^{11}b^2df^2 + 3a^{11}b^2e^2f - 3a^{10}b^3cf^2 - 6a^{10}b^3def - a^{10}b^3e^3 + 6a^9b^4cef + 3a^9b^4e^2f - 3a^8b^5c^2d - 3a^8b^5c^2e + 3a^7b^6c^2d + 3a^7b^6c^2e - 3a^6b^7c^2d - 3a^6b^7c^2e + 3a^5b^8c^2d - a^4b^9c^3, \text{Lambda}(t, t \cdot \log(-3 \cdot t \cdot b^5 / (a^4 \cdot f - a^3 \cdot b \cdot e + a^2 \cdot b^2 \cdot d - a \cdot b^3 \cdot c) + x))\right) + f \cdot x^{13} / (13 \cdot b) - x^{10} \cdot (a \cdot f - b \cdot e) / (10 \cdot b^2) + x^7 \cdot (a^2 \cdot f - a \cdot b \cdot e + b^2 \cdot d) / (7 \cdot b^3) - x^4 \cdot (a^3 \cdot f - a^2 \cdot b \cdot e + a \cdot b^2 \cdot d - b^3 \cdot c) / (4 \cdot b^4) + x \cdot (a^4 \cdot f - a^3 \cdot b \cdot e + a^2 \cdot b^2 \cdot d - a \cdot b^3 \cdot c) / b^5$$

Giac [A] time = 1.07153, size = 541, normalized size = 1.73

$$\frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}ab^3c - (-ab^2)^{\frac{1}{3}}a^2b^2d - (-ab^2)^{\frac{1}{3}}a^4f + (-ab^2)^{\frac{1}{3}}a^3be\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^6} + \frac{\left((-ab^2)^{\frac{1}{3}}ab^3c - (-ab^2)^{\frac{1}{3}}a^2b^2d - (-ab^2)^{\frac{1}{3}}a^4f + (-ab^2)^{\frac{1}{3}}a^3be\right)}{3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")

[Out] $\frac{1}{3}\sqrt{3}\left((-ab^2)^{\frac{1}{3}}ab^3c - (-ab^2)^{\frac{1}{3}}a^2b^2d - (-ab^2)^{\frac{1}{3}}a^4f + (-ab^2)^{\frac{1}{3}}a^3be\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right) + \frac{1}{6}\left((-ab^2)^{\frac{1}{3}}ab^3c - (-ab^2)^{\frac{1}{3}}a^2b^2d - (-ab^2)^{\frac{1}{3}}a^4f + (-ab^2)^{\frac{1}{3}}a^3be\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right) - \frac{1}{3}\left(a^2b^{11}c - a^3b^{10}d - a^5b^8f + a^4b^9e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\text{abs}\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\right) + \frac{1}{1820}\left(140b^{12}fx^{13} - 182a^2b^{11}fx^{10} + 182b^{12}x^{10}e + 260b^{12}dx^7 + 260a^2b^{10}fx^7 - 260ab^{11}x^7e + 455b^{12}cx^4 - 455ab^{11}dx^4 - 455a^3b^9fx^4 + 455a^2b^{10}x^4e - 1820ab^{11}cx + 1820a^2b^{10}dx + 1820a^4b^8fx - 1820a^3b^9xe\right)/b^{13}$

$$3.236 \quad \int \frac{x^4(c+dx^3+ex^6+fx^9)}{a+bx^3} dx$$

Optimal. Leaf size=279

$$\frac{x^2(a^2be + a^3(-f) - ab^2d + b^3c)}{2b^4} - \frac{a^{2/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^{14/3}} + \frac{a^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{bx})}{6b^{14/3}}$$

[Out] $((b^3c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(2*b^4) + ((b^2*d - a*b*e + a^2*f)*x^5)/(5*b^3) + ((b*e - a*f)*x^8)/(8*b^2) + (f*x^{11})/(11*b) + (a^{(2/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*b^{(14/3)}) + (a^{(2/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{(1/3)} + b^{(1/3)}*x])/(3*b^{(14/3)}) - (a^{(2/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(6*b^{(14/3)})$

Rubi [A] time = 0.273338, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1836, 1488, 292, 31, 634, 617, 204, 628}

$$\frac{x^2(a^2be + a^3(-f) - ab^2d + b^3c)}{2b^4} - \frac{a^{2/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^{14/3}} + \frac{a^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{bx})}{6b^{14/3}}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3), x]

[Out] $((b^3c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(2*b^4) + ((b^2*d - a*b*e + a^2*f)*x^5)/(5*b^3) + ((b*e - a*f)*x^8)/(8*b^2) + (f*x^{11})/(11*b) + (a^{(2/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*b^{(14/3)}) + (a^{(2/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{(1/3)} + b^{(1/3)}*x])/(3*b^{(14/3)}) - (a^{(2/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(6*b^{(14/3)})$

Rule 1836

Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m + q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;

NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*n)]) /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]

Rule 1488

Int[((f_)*(x_)^(m_))*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_)*(d_) + (e_)*(x_)^(n_)]^(q_), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 292

Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x^4(c + dx^3 + ex^6 + fx^9)}{a + bx^3} dx &= \frac{fx^{11}}{11b} + \frac{\int \frac{x^4(11bc + 11bdx^3 + 11(be - af)x^6)}{a + bx^3} dx}{11b} \\ &= \frac{fx^{11}}{11b} + \frac{\int \left(\frac{11(b^3c - ab^2d + a^2be - a^3f)x}{b^3} + \frac{11(b^2d - abe + a^2f)x^4}{b^2} + \frac{11(be - af)x^7}{b} + \frac{11(-ab^3c + a^2b^2d - a^3be + a^4f)}{b^3(a + bx^3)} \right) dx}{11b} \\ &= \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{2b^4} + \frac{(b^2d - abe + a^2f)x^5}{5b^3} + \frac{(be - af)x^8}{8b^2} + \frac{fx^{11}}{11b} - \frac{(a(b^3c - ab^2d + a^2be - a^3f))x^2}{11b} \\ &= \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{2b^4} + \frac{(b^2d - abe + a^2f)x^5}{5b^3} + \frac{(be - af)x^8}{8b^2} + \frac{fx^{11}}{11b} + \frac{(a^{2/3}(b^3c - ab^2d + a^2be - a^3f))x^2}{11b} \\ &= \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{2b^4} + \frac{(b^2d - abe + a^2f)x^5}{5b^3} + \frac{(be - af)x^8}{8b^2} + \frac{fx^{11}}{11b} + \frac{a^{2/3}(b^3c - ab^2d + a^2be - a^3f)x^2}{11b} \\ &= \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{2b^4} + \frac{(b^2d - abe + a^2f)x^5}{5b^3} + \frac{(be - af)x^8}{8b^2} + \frac{fx^{11}}{11b} + \frac{a^{2/3}(b^3c - ab^2d + a^2be - a^3f)x^2}{11b} \\ &= \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{2b^4} + \frac{(b^2d - abe + a^2f)x^5}{5b^3} + \frac{(be - af)x^8}{8b^2} + \frac{fx^{11}}{11b} + \frac{a^{2/3}(b^3c - ab^2d + a^2be - a^3f)x^2}{11b} \end{aligned}$$

Mathematica [A] time = 0.0915794, size = 266, normalized size = 0.95

$$660b^{2/3}x^2(a^2be + a^3(-f) - ab^2d + b^3c) + 220a^{2/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(-a^2be + a^3f + ab^2d - b^3c) - 440a^{2/3} \log$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3), x]

[Out] (660*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2 + 264*b^(5/3)*(b^2*d - a*b*e + a^2*f)*x^5 + 165*b^(8/3)*(b*e - a*f)*x^8 + 120*b^(11/3)*f*x^11 - 440*sqrt[3]*a^(2/3)*(-b^3*c + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^

$$\frac{(1/3)*x/a^{(1/3)}/\text{Sqrt}[3]}{3} - 440*a^{(2/3)}*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*\text{Log}[a^{(1/3)} + b^{(1/3)}*x] + 220*a^{(2/3)}*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2]/(1320*b^{(14/3)})$$

Maple [B] time = 0.004, size = 502, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)`

[Out] $\frac{1}{11}f*x^{11}/b - \frac{1}{8}b^2*x^8*a*f + \frac{1}{8}b*x^8*e + \frac{1}{5}b^3*x^5*a^2*f - \frac{1}{5}b^2*x^5*a*e + \frac{1}{5}b*x^5*d - \frac{1}{2}b^4*x^2*a^3*f + \frac{1}{2}b^3*x^2*a^2*e - \frac{1}{2}b^2*x^2*a*d + \frac{1}{2}b*x^2*c - \frac{1}{3}a^4/b^5/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})*f + \frac{1}{3}a^3/b^4/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})*e - \frac{1}{3}a^2/b^3/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})*d + \frac{1}{3}a/b^2/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})*c + \frac{1}{6}a^4/b^5/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*f - \frac{1}{6}a^3/b^4/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*e + \frac{1}{6}a^2/b^3/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*d - \frac{1}{6}a/b^2/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*c + \frac{1}{3}a^4/b^5*3^{(1/2)}/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*f - \frac{1}{3}a^3/b^4*3^{(1/2)}/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*e + \frac{1}{3}a^2/b^3*3^{(1/2)}/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*d - \frac{1}{3}a/b^2*3^{(1/2)}/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*c$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 1.40658, size = 635, normalized size = 2.28

$$120 b^3 f x^{11} + 165 (b^3 e - a b^2 f) x^8 + 264 (b^3 d - a b^2 e + a^2 b f) x^5 + 660 (b^3 c - a b^2 d + a^2 b e - a^3 f) x^2 - 440 \sqrt{3} (b^3 c - a b^2 d +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")

[Out] 1/1320*(120*b^3*f*x^11 + 165*(b^3*e - a*b^2*f)*x^8 + 264*(b^3*d - a*b^2*e + a^2*b*f)*x^5 + 660*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2 - 440*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a^2/b^2)^(1/3) + sqrt(3)*a)/a) + 220*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2)^(2/3) - a*(-a^2/b^2)^(1/3)) - 440*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2/b^2)^(1/3)*log(a*x + b*(-a^2/b^2)^(2/3))/b^4

Sympy [A] time = 1.38275, size = 459, normalized size = 1.65

$$\text{RootSum}\left(27t^3b^{14} + a^{11}f^3 - 3a^{10}bef^2 + 3a^9b^2df^2 + 3a^9b^2e^2f - 3a^8b^3cf^2 - 6a^8b^3def - a^8b^3e^3 + 6a^7b^4cef + 3a^7b^4d^2f\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)

[Out] RootSum(27*_t**3*b**14 + a**11*f**3 - 3*a**10*b*e*f**2 + 3*a**9*b**2*d*f**2 + 3*a**9*b**2*e**2*f - 3*a**8*b**3*c*f**2 - 6*a**8*b**3*d*e*f - a**8*b**3*e**3 + 6*a**7*b**4*c*e*f + 3*a**7*b**4*d**2*f + 3*a**7*b**4*d*e**2 - 6*a**6*b**5*c*d*f - 3*a**6*b**5*c*e**2 - 3*a**6*b**5*d**2*e + 3*a**5*b**6*c**2*f + 6*a**5*b**6*c*d*e + a**5*b**6*d**3 - 3*a**4*b**7*c**2*e - 3*a**4*b**7*c*d**2 + 3*a**3*b**8*c**2*d - a**2*b**9*c**3, Lambda(_t, _t*log(9*_t**2*b**9/(a**7*f**2 - 2*a**6*b*e*f + 2*a**5*b**2*d*f + a**5*b**2*e**2 - 2*a**4*b**3*c*f - 2*a**4*b**3*d*e + 2*a**3*b**4*c*e + a**3*b**4*d**2 - 2*a**2*b**5*c*d + a*b**6*c**2) + x))) + f*x**11/(11*b) - x**8*(a*f - b*e)/(8*b**2) + x**5*(a**2*f - a*b*e + b**2*d)/(5*b**3) - x**2*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(2*b**4)

Giac [A] time = 1.07492, size = 521, normalized size = 1.87

$$\frac{\sqrt{3} \left((-ab^2)^{\frac{2}{3}} b^3 c - (-ab^2)^{\frac{2}{3}} ab^2 d - (-ab^2)^{\frac{2}{3}} a^3 f + (-ab^2)^{\frac{2}{3}} a^2 b e \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{3 b^6} - \frac{\left((-ab^2)^{\frac{2}{3}} b^3 c - (-ab^2)^{\frac{2}{3}} ab^2 d - \right)}{3 b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")

[Out] $\frac{1}{3} \sqrt{3} \left((-ab^2)^{\frac{2}{3}} b^3 c - (-ab^2)^{\frac{2}{3}} ab^2 d - (-ab^2)^{\frac{2}{3}} a^3 f + (-ab^2)^{\frac{2}{3}} a^2 b e \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b} \right)^{\frac{1}{3}}} \right) - \frac{1}{6} \left((-ab^2)^{\frac{2}{3}} b^3 c - (-ab^2)^{\frac{2}{3}} ab^2 d - (-ab^2)^{\frac{2}{3}} a^3 f + (-ab^2)^{\frac{2}{3}} a^2 b e \right) \log \left(x^2 + x \left(-\frac{a}{b} \right)^{\frac{1}{3}} + \left(-\frac{a}{b} \right)^{\frac{2}{3}} \right) - \frac{1}{3} \left(a^2 b^9 d \left(-\frac{a}{b} \right)^{\frac{1}{3}} - a^4 b^7 f \left(-\frac{a}{b} \right)^{\frac{1}{3}} + a^3 b^8 e \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right) \log \left(\text{abs} \left(x - \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right) \right) - \frac{1}{440} \left(40 b^{10} f x^{11} - 55 a b^9 f x^8 + 55 b^{10} e x^8 + 88 b^{10} d x^5 + 88 a^2 b^8 f x^5 - 88 a b^9 e x^5 + 220 b^{10} c x^2 - 220 a b^9 d x^2 - 220 a^3 b^7 f x^2 + 220 a^2 b^8 e x^2 \right) / b^{11}$

$$3.237 \quad \int \frac{x^3(c+dx^3+ex^6+fx^9)}{a+bx^3} dx$$

Optimal. Leaf size=274

$$\frac{\sqrt[3]{a} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^{13/3}} + \frac{x(a^2be + a^3(-f) - ab^2d + b^3c)}{b^4} - \frac{\sqrt[3]{a} \log(\sqrt[3]{a} + \sqrt[3]{bx})}{b^4}$$

[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4 + ((b^2*d - a*b*e + a^2*f)*x^4)/(4*b^3) + ((b*e - a*f)*x^7)/(7*b^2) + (f*x^10)/(10*b) + (a^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(13/3)) - (a^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(13/3)) + (a^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(13/3))

Rubi [A] time = 0.26722, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1836, 1488, 200, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{a} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^{13/3}} + \frac{x(a^2be + a^3(-f) - ab^2d + b^3c)}{b^4} - \frac{\sqrt[3]{a} \log(\sqrt[3]{a} + \sqrt[3]{bx})}{b^4}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3), x]

[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4 + ((b^2*d - a*b*e + a^2*f)*x^4)/(4*b^3) + ((b*e - a*f)*x^7)/(7*b^2) + (f*x^10)/(10*b) + (a^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(13/3)) - (a^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(13/3)) + (a^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(13/3))

Rule 1836

Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m + q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;

NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]

Rule 1488

Int[((f_)*(x_)^(m_))*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_)*(d_) + (e_)*(x_)^(n_)]^(q_), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 200

Int[((a_) + (b_)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 31

Int[((a_) + (b_)*(x_))(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(c + dx^3 + ex^6 + fx^9)}{a + bx^3} dx &= \frac{fx^{10}}{10b} + \frac{\int \frac{x^3(10bc + 10bdx^3 + 10(be - af)x^6)}{a + bx^3} dx}{10b} \\
&= \frac{fx^{10}}{10b} + \frac{\int \left(\frac{10(b^3c - ab^2d + a^2be - a^3f)}{b^3} + \frac{10(b^2d - abe + a^2f)x^3}{b^2} + \frac{10(be - af)x^6}{b} + \frac{10(-ab^3c + a^2b^2d - a^3be + a^4f)}{b^3(a + bx^3)} \right) dx}{10b} \\
&= \frac{(b^3c - ab^2d + a^2be - a^3f)x}{b^4} + \frac{(b^2d - abe + a^2f)x^4}{4b^3} + \frac{(be - af)x^7}{7b^2} + \frac{fx^{10}}{10b} - \frac{(a(b^3c - ab^2d + a^2be - a^3f))}{10b} \\
&= \frac{(b^3c - ab^2d + a^2be - a^3f)x}{b^4} + \frac{(b^2d - abe + a^2f)x^4}{4b^3} + \frac{(be - af)x^7}{7b^2} + \frac{fx^{10}}{10b} - \frac{(\sqrt[3]{a}(b^3c - ab^2d + a^2be - a^3f))}{10b} \\
&= \frac{(b^3c - ab^2d + a^2be - a^3f)x}{b^4} + \frac{(b^2d - abe + a^2f)x^4}{4b^3} + \frac{(be - af)x^7}{7b^2} + \frac{fx^{10}}{10b} - \frac{\sqrt[3]{a}(b^3c - ab^2d + a^2be - a^3f)}{10b} \\
&= \frac{(b^3c - ab^2d + a^2be - a^3f)x}{b^4} + \frac{(b^2d - abe + a^2f)x^4}{4b^3} + \frac{(be - af)x^7}{7b^2} + \frac{fx^{10}}{10b} - \frac{\sqrt[3]{a}(b^3c - ab^2d + a^2be - a^3f)}{10b} \\
&= \frac{(b^3c - ab^2d + a^2be - a^3f)x}{b^4} + \frac{(b^2d - abe + a^2f)x^4}{4b^3} + \frac{(be - af)x^7}{7b^2} + \frac{fx^{10}}{10b} + \frac{\sqrt[3]{a}(b^3c - ab^2d + a^2be - a^3f)}{10b}
\end{aligned}$$

Mathematica [A] time = 0.0911862, size = 264, normalized size = 0.96

$$-70\sqrt[3]{a} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2) (-a^2be + a^3f + ab^2d - b^3c) + 420\sqrt[3]{bx} (a^2be + a^3(-f) - ab^2d + b^3c) + 140\sqrt[3]{a} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3), x]

[Out] (420*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x + 105*b^(4/3)*(b^2*d - a*b*e + a^2*f)*x^4 + 60*b^(7/3)*(b*e - a*f)*x^7 + 42*b^(10/3)*f*x^10 - 140*Sqrt[3]*a^(1/3)*(-b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)

```
)x)/a^(1/3))/Sqrt[3]] + 140*a^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)
*Log[a^(1/3) + b^(1/3)*x] - 70*a^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*
f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)]/(420*b^(13/3))
```

Maple [B] time = 0.003, size = 492, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] 1/10*f*x^10/b-1/7/b^2*x^7*a*f+1/7/b*x^7*e+1/4/b^3*x^4*a^2*f-1/4/b^2*x^4*a*e
+1/4/b*x^4*d-1/b^4*a^3*f*x+1/b^3*a^2*e*x-1/b^2*a*d*x+c*x/b+1/3*a^4/b^5/(1/b
*a)^(2/3)*ln(x+(1/b*a)^(1/3))*f-1/3*a^3/b^4/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3
))*e+1/3*a^2/b^3/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d-1/3*a/b^2/(1/b*a)^(2/3
)*ln(x+(1/b*a)^(1/3))*c-1/6*a^4/b^5/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1
/b*a)^(2/3))*f+1/6*a^3/b^4/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/
3))*e-1/6*a^2/b^3/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d+1/6
*a/b^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+1/3*a^4/b^5/(1
/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f-1/3*a^3/b^4
/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e+1/3*a^2/
b^3/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d-1/3*a
/b^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.29716, size = 566, normalized size = 2.07

$$42b^3fx^{10} + 60(b^3e - ab^2f)x^7 + 105(b^3d - ab^2e + a^2bf)x^4 - 140\sqrt{3}(b^3c - ab^2d + a^2be - a^3f)\left(\frac{a}{b}\right)^{\frac{1}{3}} \arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}} - \sqrt{3}a}{3a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")

[Out] 1/420*(42*b^3*f*x^10 + 60*(b^3*e - a*b^2*f)*x^7 + 105*(b^3*d - a*b^2*e + a^2*b*f)*x^4 - 140*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 70*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 140*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 420*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4

Sympy [A] time = 1.71881, size = 371, normalized size = 1.35

$$\text{RootSum}\left(27t^3b^{13} - a^{10}f^3 + 3a^9bef^2 - 3a^8b^2df^2 - 3a^8b^2e^2f + 3a^7b^3cf^2 + 6a^7b^3def + a^7b^3e^3 - 6a^6b^4cef - 3a^6b^4d^2f\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)

[Out] RootSum(27*_t**3*b**13 - a**10*f**3 + 3*a**9*b*e*f**2 - 3*a**8*b**2*d*f**2 - 3*a**8*b**2*e**2*f + 3*a**7*b**3*c*f**2 + 6*a**7*b**3*d*e*f + a**7*b**3*e**3 - 6*a**6*b**4*c*e*f - 3*a**6*b**4*d**2*f - 3*a**6*b**4*d*e**2 + 6*a**5*b**5*c*d*f + 3*a**5*b**5*c*e**2 + 3*a**5*b**5*d**2*e - 3*a**4*b**6*c**2*f - 6*a**4*b**6*c*d*e - a**4*b**6*d**3 + 3*a**3*b**7*c**2*e + 3*a**3*b**7*c*d**2 - 3*a**2*b**8*c**2*d + a*b**9*c**3, Lambda(_t, _t*log(3*_t*b**4/(a**3*f - a**2*b*e + a*b**2*d - b**3*c) + x))) + f*x**10/(10*b) - x**7*(a*f - b*e)/(7*b**2) + x**4*(a**2*f - a*b*e + b**2*d)/(4*b**3) - x*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/b**4

Giac [A] time = 1.08937, size = 467, normalized size = 1.7

$$\frac{\sqrt{3} \left((-ab^2)^{\frac{1}{3}} b^3 c - (-ab^2)^{\frac{1}{3}} ab^2 d - (-ab^2)^{\frac{1}{3}} a^3 f + (-ab^2)^{\frac{1}{3}} a^2 b e \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{3b^5} \left((-ab^2)^{\frac{1}{3}} b^3 c - (-ab^2)^{\frac{1}{3}} ab^2 d \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")

[Out] -1/3*sqrt(3)*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d - (-a*b^2)^(1/3)*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^5 - 1/6*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d - (-a*b^2)^(1/3)*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^5 + 1/3*(a*b^9*c - a^2*b^8*d - a^4*b^6*f + a^3*b^7*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^10) + 1/140*(14*b^9*f*x^10 - 20*a*b^8*f*x^7 + 20*b^9*x^7*e + 35*b^9*d*x^4 + 35*a^2*b^7*f*x^4 - 35*a*b^8*x^4*e + 140*b^9*c*x - 140*a*b^8*d*x - 140*a^3*b^6*f*x + 140*a^2*b^7*x*e)/b^10

$$3.238 \quad \int \frac{x(c+dx^3+ex^6+fx^9)}{a+bx^3} dx$$

Optimal. Leaf size=245

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{6\sqrt[3]{ab^{11/3}}} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{3\sqrt[3]{ab^{11/3}}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt[3]{ab^{11/3}}}\right)}{3\sqrt[3]{ab^{11/3}}}$$

[Out] $((b^2*d - a*b*e + a^2*f)*x^2)/(2*b^3) + ((b*e - a*f)*x^5)/(5*b^2) + (f*x^8)/(8*b) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{ArcTan}[(a^{1/3} - 2*b^{1/3}*x)/(\text{Sqrt}[3]*a^{1/3})]) / (\text{Sqrt}[3]*a^{1/3}*b^{11/3}) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[a^{1/3} + b^{1/3}*x]) / (3*a^{1/3}*b^{11/3}) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2]) / (6*a^{1/3}*b^{11/3})$

Rubi [A] time = 0.215567, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1836, 1488, 292, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{6\sqrt[3]{ab^{11/3}}} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{3\sqrt[3]{ab^{11/3}}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt[3]{ab^{11/3}}}\right)}{3\sqrt[3]{ab^{11/3}}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3), x]$

[Out] $((b^2*d - a*b*e + a^2*f)*x^2)/(2*b^3) + ((b*e - a*f)*x^5)/(5*b^2) + (f*x^8)/(8*b) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{ArcTan}[(a^{1/3} - 2*b^{1/3}*x)/(\text{Sqrt}[3]*a^{1/3})]) / (\text{Sqrt}[3]*a^{1/3}*b^{11/3}) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[a^{1/3} + b^{1/3}*x]) / (3*a^{1/3}*b^{11/3}) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2]) / (6*a^{1/3}*b^{11/3})$

Rule 1836

$\text{Int}[(Pq_*)*((c_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{With}[\{q = \text{Expon}[Pq, x]\}, \text{With}[\{Pqq = \text{Coeff}[Pq, x, q]\}, \text{Dist}[1/(b*(m + q + n*p + 1)), \text{Int}[(c*x)^m*\text{ExpandToSum}[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^{(q - n)}, x]*(a + b*x^n)^p, x], x] + \text{Simp}[(Pqq*(c*x)^{(m + q - n + 1)}*(a + b*x^n)^{(p + 1)})/(b*c^{(q - n + 1)}*(m + q + n*p + 1)), x]] /;$

NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*n)]) /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]

Rule 1488

Int[((f_)*(x_)^(m_))*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_)*(d_) + (e_)*(x_)^(n_)]^(q_), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 292

Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(c + dx^3 + ex^6 + fx^9)}{a + bx^3} dx &= \frac{fx^8}{8b} + \frac{\int \frac{x(8bc + 8bdx^3 + 8(be-af)x^6)}{a+bx^3} dx}{8b} \\
&= \frac{fx^8}{8b} + \frac{\int \left(\frac{8(b^2d - abe + a^2f)x}{b^2} + \frac{8(be-af)x^4}{b} + \frac{8(b^3c - ab^2d + a^2be - a^3f)x}{b^2(a+bx^3)} \right) dx}{8b} \\
&= \frac{(b^2d - abe + a^2f)x^2}{2b^3} + \frac{(be - af)x^5}{5b^2} + \frac{fx^8}{8b} + \frac{(b^3c - ab^2d + a^2be - a^3f) \int \frac{x}{a+bx^3} dx}{b^3} \\
&= \frac{(b^2d - abe + a^2f)x^2}{2b^3} + \frac{(be - af)x^5}{5b^2} + \frac{fx^8}{8b} - \frac{(b^3c - ab^2d + a^2be - a^3f) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{bx}} dx}{3\sqrt[3]{ab^{10/3}}} + \\
&= \frac{(b^2d - abe + a^2f)x^2}{2b^3} + \frac{(be - af)x^5}{5b^2} + \frac{fx^8}{8b} - \frac{(b^3c - ab^2d + a^2be - a^3f) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3\sqrt[3]{ab^{11/3}}} \\
&= \frac{(b^2d - abe + a^2f)x^2}{2b^3} + \frac{(be - af)x^5}{5b^2} + \frac{fx^8}{8b} - \frac{(b^3c - ab^2d + a^2be - a^3f) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3\sqrt[3]{ab^{11/3}}} \\
&= \frac{(b^2d - abe + a^2f)x^2}{2b^3} + \frac{(be - af)x^5}{5b^2} + \frac{fx^8}{8b} - \frac{(b^3c - ab^2d + a^2be - a^3f) \tan^{-1}\left(\frac{\sqrt[3]{a-2\sqrt[3]{b}}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{ab^{11/3}}}
\end{aligned}$$

Mathematica [A] time = 0.160058, size = 231, normalized size = 0.94

$$\frac{20 \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)(a^2be + a^3(-f) - ab^2d + b^3c)}{\sqrt[3]{a}} + \frac{40 \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)(-a^2be + a^3f + ab^2d - b^3c)}{\sqrt[3]{a}} + \frac{40\sqrt{3} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)(-a^2be + a^3f + ab^2d - b^3c)}{\sqrt[3]{a}} + 60$$

$$120b^{11/3}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]

[Out] (60*b^(2/3)*(b^2*d - a*b*e + a^2*f)*x^2 + 24*b^(5/3)*(b*e - a*f)*x^5 + 15*b^(8/3)*f*x^8 + (40*sqrt[3]*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1

$$- (2*b^{(1/3)*x}/a^{(1/3)})/\text{Sqrt}[3]]/a^{(1/3)} + (40*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*\text{Log}[a^{(1/3)} + b^{(1/3)*x}])/a^{(1/3)} + (20*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)*x} + b^{(2/3)*x^2}])/a^{(1/3)})/(120*b^{(11/3)})$$

Maple [B] time = 0.003, size = 450, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)`

[Out] $1/8*f*x^8/b - 1/5/b^2*x^5*a*f + 1/5/b*x^5*e + 1/2/b^3*x^2*a^2*f - 1/2/b^2*x^2*a*e + 1/2*d*x^2/b + 1/3/b^4/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})*a^3*f - 1/3/b^3/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})*a^2*e + 1/3/b^2/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})*a*d - 1/3/b/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})*c - 1/6/b^4/(1/b*a)^{(1/3)}*\ln(x^2 - (1/b*a)^{(1/3)}*x + (1/b*a)^{(2/3)})*a^3*f + 1/6/b^3/(1/b*a)^{(1/3)}*\ln(x^2 - (1/b*a)^{(1/3)}*x + (1/b*a)^{(2/3)})*a^2*e - 1/6/b^2/(1/b*a)^{(1/3)}*\ln(x^2 - (1/b*a)^{(1/3)}*x + (1/b*a)^{(2/3)})*a*d + 1/6/b/(1/b*a)^{(1/3)}*\ln(x^2 - (1/b*a)^{(1/3)}*x + (1/b*a)^{(2/3)})*c - 1/3/b^4*3^{(1/2)}/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*a^3*f + 1/3/b^3*3^{(1/2)}/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*a^2*e - 1/3/b^2*3^{(1/2)}/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*a*d + 1/3/b*3^{(1/2)}/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*c$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 1.51902, size = 1289, normalized size = 5.26

$$15 ab^4 f x^8 + 24 (ab^4 e - a^2 b^3 f) x^5 + 60 (ab^4 d - a^2 b^3 e + a^3 b^2 f) x^2 - 60 \sqrt{\frac{1}{3}} (ab^4 c - a^2 b^3 d + a^3 b^2 e - a^4 b f) \sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a} \log \left(\dots \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")

[Out] [1/120*(15*a*b^4*f*x^8 + 24*(a*b^4*e - a^2*b^3*f)*x^5 + 60*(a*b^4*d - a^2*b^3*e + a^3*b^2*f)*x^2 - 60*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) + 20*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 40*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a*b^5), 1/120*(15*a*b^4*f*x^8 + 24*(a*b^4*e - a^2*b^3*f)*x^5 + 60*(a*b^4*d - a^2*b^3*e + a^3*b^2*f)*x^2 - 120*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*sqrt((a*b^2)^(1/3)/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) + 20*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 40*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a*b^5)]

Sympy [A] time = 1.47288, size = 422, normalized size = 1.72

$$\text{RootSum}\left(27t^3 ab^{11} - a^9 f^3 + 3a^8 b e f^2 - 3a^7 b^2 d f^2 - 3a^7 b^2 e^2 f + 3a^6 b^3 c f^2 + 6a^6 b^3 d e f + a^6 b^3 e^3 - 6a^5 b^4 c e f - 3a^5 b^4 d^2 f\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)

[Out] RootSum(27*_t**3*a*b**11 - a**9*f**3 + 3*a**8*b*e*f**2 - 3*a**7*b**2*d*f**2 - 3*a**7*b**2*e**2*f + 3*a**6*b**3*c*f**2 + 6*a**6*b**3*d*e*f + a**6*b**3*

```

e**3 - 6*a**5*b**4*c*e*f - 3*a**5*b**4*d**2*f - 3*a**5*b**4*d*e**2 + 6*a**4
*b**5*c*d*f + 3*a**4*b**5*c*e**2 + 3*a**4*b**5*d**2*e - 3*a**3*b**6*c**2*f
- 6*a**3*b**6*c*d*e - a**3*b**6*d**3 + 3*a**2*b**7*c**2*e + 3*a**2*b**7*c*d
**2 - 3*a*b**8*c**2*d + b**9*c**3, Lambda(_t, _t*log(9*_t**2*a*b**7/(a**6*f
**2 - 2*a**5*b*e*f + 2*a**4*b**2*d*f + a**4*b**2*e**2 - 2*a**3*b**3*c*f - 2
*a**3*b**3*d*e + 2*a**2*b**4*c*e + a**2*b**4*d**2 - 2*a*b**5*c*d + b**6*c**
2) + x))) + f*x**8/(8*b) - x**5*(a*f - b*e)/(5*b**2) + x**2*(a**2*f - a*b*e
+ b**2*d)/(2*b**3)

```

Giac [A] time = 1.08358, size = 466, normalized size = 1.9

$$\frac{\sqrt{3} \left((-ab^2)^{\frac{2}{3}} b^3 c - (-ab^2)^{\frac{2}{3}} ab^2 d - (-ab^2)^{\frac{2}{3}} a^3 f + (-ab^2)^{\frac{2}{3}} a^2 b e \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{3 ab^5} + \frac{\left((-ab^2)^{\frac{2}{3}} b^3 c - (-ab^2)^{\frac{2}{3}} ab^2 d \right)}{3 ab^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")

[Out]
$$-1/3*\sqrt{3}*((-a*b^2)^{(2/3)}*b^3*c - (-a*b^2)^{(2/3)}*a*b^2*d - (-a*b^2)^{(2/3)}*a^3*f + (-a*b^2)^{(2/3)}*a^2*b*e)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(a*b^5) + 1/6*((-a*b^2)^{(2/3)}*b^3*c - (-a*b^2)^{(2/3)}*a*b^2*d - (-a*b^2)^{(2/3)}*a^3*f + (-a*b^2)^{(2/3)}*a^2*b*e)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a*b^5) - 1/3*(b^8*c*(-a/b)^{(1/3)} - a*b^7*d*(-a/b)^{(1/3)} - a^3*b^5*f*(-a/b)^{(1/3)} + a^2*b^6*(-a/b)^{(1/3)}*e)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/(a*b^8) + 1/40*(5*b^7*f*x^8 - 8*a*b^6*f*x^5 + 8*b^7*x^5*e + 20*b^7*d*x^2 + 20*a^2*b^5*f*x^2 - 20*a*b^6*x^2*e)/b^8$$

$$3.239 \quad \int \frac{c+dx^3+ex^6+fx^9}{a+bx^3} dx$$

Optimal. Leaf size=240

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{6a^{2/3}b^{10/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{3a^{2/3}b^{10/3}} - \frac{\tan^{-1}\left(\frac{(b^2d - a^2f)x + (b^3c - a^3f)}{\sqrt[3]{a} + \sqrt[3]{bx}}\right)}{\sqrt[3]{a} + \sqrt[3]{bx}}$$

[Out] $((b^2*d - a*b*e + a^2*f)*x)/b^3 + ((b*e - a*f)*x^4)/(4*b^2) + (f*x^7)/(7*b) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{ArcTan}[(a^{1/3} - 2*b^{1/3}*x)/(\text{Sqrt}[3]*a^{1/3})]) / (\text{Sqrt}[3]*a^{2/3}*b^{10/3}) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[a^{1/3} + b^{1/3}*x]) / (3*a^{2/3}*b^{10/3}) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2]) / (6*a^{2/3}*b^{10/3})$

Rubi [A] time = 0.152689, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {1887, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{6a^{2/3}b^{10/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{3a^{2/3}b^{10/3}} - \frac{\tan^{-1}\left(\frac{(b^2d - a^2f)x + (b^3c - a^3f)}{\sqrt[3]{a} + \sqrt[3]{bx}}\right)}{\sqrt[3]{a} + \sqrt[3]{bx}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3), x]

[Out] $((b^2*d - a*b*e + a^2*f)*x)/b^3 + ((b*e - a*f)*x^4)/(4*b^2) + (f*x^7)/(7*b) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{ArcTan}[(a^{1/3} - 2*b^{1/3}*x)/(\text{Sqrt}[3]*a^{1/3})]) / (\text{Sqrt}[3]*a^{2/3}*b^{10/3}) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[a^{1/3} + b^{1/3}*x]) / (3*a^{2/3}*b^{10/3}) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2]) / (6*a^{2/3}*b^{10/3})$

Rule 1887

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a + b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_)^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx^3 + ex^6 + fx^9}{a + bx^3} dx &= \int \left(\frac{b^2d - abe + a^2f}{b^3} + \frac{(be - af)x^3}{b^2} + \frac{fx^6}{b} + \frac{b^3c - ab^2d + a^2be - a^3f}{b^3(a + bx^3)} \right) dx \\
&= \frac{(b^2d - abe + a^2f)x}{b^3} + \frac{(be - af)x^4}{4b^2} + \frac{fx^7}{7b} + \frac{(b^3c - ab^2d + a^2be - a^3f) \int \frac{1}{a+bx^3} dx}{b^3} \\
&= \frac{(b^2d - abe + a^2f)x}{b^3} + \frac{(be - af)x^4}{4b^2} + \frac{fx^7}{7b} + \frac{(b^3c - ab^2d + a^2be - a^3f) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{bx}} dx}{3a^{2/3}b^3} + \dots \\
&= \frac{(b^2d - abe + a^2f)x}{b^3} + \frac{(be - af)x^4}{4b^2} + \frac{fx^7}{7b} + \frac{(b^3c - ab^2d + a^2be - a^3f) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{2/3}b^{10/3}} - \dots \\
&= \frac{(b^2d - abe + a^2f)x}{b^3} + \frac{(be - af)x^4}{4b^2} + \frac{fx^7}{7b} + \frac{(b^3c - ab^2d + a^2be - a^3f) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{2/3}b^{10/3}} - \dots \\
&= \frac{(b^2d - abe + a^2f)x}{b^3} + \frac{(be - af)x^4}{4b^2} + \frac{fx^7}{7b} - \frac{(b^3c - ab^2d + a^2be - a^3f) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{10/3}} + \dots
\end{aligned}$$

Mathematica [A] time = 0.147161, size = 229, normalized size = 0.95

$$\frac{14 \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)(-a^2be + a^3f + ab^2d - b^3c)}{a^{2/3}} + \frac{28 \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)(a^2be + a^3(-f) - ab^2d + b^3c)}{a^{2/3}} + \frac{28\sqrt{3} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)(-a^2be + a^3f + ab^2d - b^3c)}{a^{2/3}} + 84b^{10/3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3), x]

[Out] (84*b^(1/3)*(b^2*d - a*b*e + a^2*f)*x + 21*b^(4/3)*(b*e - a*f)*x^4 + 12*b^(7/3)*f*x^7 + (28*sqrt(3)*(-b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt(3)])/a^(2/3) + (28*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (14*(-b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3))/(84*b^(10/3))

Maple [B] time = 0.004, size = 442, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)`

[Out] $\frac{1}{7}f*x^7/b - \frac{1}{4}b^2*x^4*a*f + \frac{1}{4}b*x^4*e + \frac{1}{b^3}a^2*f*x - \frac{1}{b^2}a*e*x + d*x/b - \frac{1}{3}b^4/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*a^3*f + \frac{1}{3}b^3/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*a^2*e - \frac{1}{3}b^2/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*a*d + \frac{1}{3}c/b/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)}) + \frac{1}{6}b^4/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*a^3*f - \frac{1}{6}b^3/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*a^2*e + \frac{1}{6}b^2/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*a*d - \frac{1}{6}c/b/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)}) - \frac{1}{3}b^4/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*a^3*f + \frac{1}{3}b^3/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*a^2*e - \frac{1}{3}b^2/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*a*d + \frac{1}{3}c/b/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 1.36121, size = 1350, normalized size = 5.62

$$\left[\frac{12a^2b^3fx^7 + 21(a^2b^3e - a^3b^2f)x^4 - 42\sqrt{\frac{1}{3}}(ab^4c - a^2b^3d + a^3b^2e - a^4bf)\sqrt{\frac{(-a^2b)^{\frac{1}{3}}}{b}} \log\left(\frac{2abx^3 + 3(-a^2b)^{\frac{1}{3}}ax - a^2 - 3\sqrt{\frac{1}{3}}(2abx^2 + (-a^2b)^{\frac{1}{3}})}{bx^3 + a}\right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/84*(12*a^2*b^3*f*x^7 + 21*(a^2*b^3*e - a^3*b^2*f)*x^4 - 42*sqrt(1/3)*(a*
b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*
x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*
x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a) - 14*(b^3*c - a*
b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-
a^2*b)^(1/3)*a) + 28*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*log
(a*b*x + (-a^2*b)^(2/3)) + 84*(a^2*b^3*d - a^3*b^2*e + a^4*b*f)*x/(a^2*b^4
), 1/84*(12*a^2*b^3*f*x^7 + 21*(a^2*b^3*e - a^3*b^2*f)*x^4 + 84*sqrt(1/3)*(
a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(s
qrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^
2) - 14*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-
a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)
*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) + 84*(a^2*b^3*d - a^3*b^2*e + a
^4*b*f)*x)/(a^2*b^4)]
```

Sympy [A] time = 1.89, size = 340, normalized size = 1.42

$$\text{RootSum}\left(27t^3a^2b^{10} + a^9f^3 - 3a^8bef^2 + 3a^7b^2df^2 + 3a^7b^2e^2f - 3a^6b^3cf^2 - 6a^6b^3def - a^6b^3e^3 + 6a^5b^4cef + 3a^5b^4d^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**10 + a**9*f**3 - 3*a**8*b*e*f**2 + 3*a**7*b**2*d*f
**2 + 3*a**7*b**2*e**2*f - 3*a**6*b**3*c*f**2 - 6*a**6*b**3*d*e*f - a**6*b*
**3*e**3 + 6*a**5*b**4*c*e*f + 3*a**5*b**4*d**2*f + 3*a**5*b**4*d*e**2 - 6*a
**4*b**5*c*d*f - 3*a**4*b**5*c*e**2 - 3*a**4*b**5*d**2*e + 3*a**3*b**6*c**2
*f + 6*a**3*b**6*c*d*e + a**3*b**6*d**3 - 3*a**2*b**7*c**2*e - 3*a**2*b**7*
c*d**2 + 3*a*b**8*c**2*d - b**9*c**3, Lambda(_t, _t*log(-3*_t*a*b**3/(a**3*
f - a**2*b*e + a*b**2*d - b**3*c) + x))) + f*x**7/(7*b) - x**4*(a*f - b*e)/
(4*b**2) + x*(a**2*f - a*b*e + b**2*d)/b**3
```

Giac [A] time = 1.07513, size = 414, normalized size = 1.72

$$\frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}b^3c - (-ab^2)^{\frac{1}{3}}ab^2d - (-ab^2)^{\frac{1}{3}}a^3f + (-ab^2)^{\frac{1}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^4} + \frac{\left((-ab^2)^{\frac{1}{3}}b^3c - (-ab^2)^{\frac{1}{3}}ab^2d - \dots\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")

[Out] $\frac{1}{3}\sqrt{3}\left((-ab^2)^{\frac{1}{3}}b^3c - (-ab^2)^{\frac{1}{3}}ab^2d - (-ab^2)^{\frac{1}{3}}a^3f + (-ab^2)^{\frac{1}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right) + \frac{1}{6}\left((-ab^2)^{\frac{1}{3}}b^3c - (-ab^2)^{\frac{1}{3}}ab^2d - (-ab^2)^{\frac{1}{3}}a^3f + (-ab^2)^{\frac{1}{3}}a^2be\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right) + \frac{1}{3}\left(b^7c - ab^6d - a^3b^4f + a^2b^5e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right) + \frac{1}{28}\left(4b^6fx^7 - 7a^5b^5fx^4 + 7b^6x^4e + 28b^6dx + 28a^2b^4fx - 28a^5b^5xe\right)/b^7$

$$3.240 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)} dx$$

Optimal. Leaf size=227

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{6a^{4/3}b^{8/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{3a^{4/3}b^{8/3}} + \frac{\tan^{-1}\left(\frac{a^{1/3} - 2b^{1/3}x}{\sqrt[3]{a^2 + 3bx}}\right)}{3a^{4/3}b^{8/3}}$$

[Out] $-(c/(a*x)) + ((b*e - a*f)*x^2)/(2*b^2) + (f*x^5)/(5*b) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{ArcTan}[(a^{1/3} - 2*b^{1/3}*x)/(\text{Sqrt}[3]*a^{1/3})]) / (\text{Sqrt}[3]*a^{4/3}*b^{8/3}) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[a^{1/3} + b^{1/3}*x]) / (3*a^{4/3}*b^{8/3}) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2]) / (6*a^{4/3}*b^{8/3})$

Rubi [A] time = 0.192612, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1834, 292, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{6a^{4/3}b^{8/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{3a^{4/3}b^{8/3}} + \frac{\tan^{-1}\left(\frac{a^{1/3} - 2b^{1/3}x}{\sqrt[3]{a^2 + 3bx}}\right)}{3a^{4/3}b^{8/3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)), x]$

[Out] $-(c/(a*x)) + ((b*e - a*f)*x^2)/(2*b^2) + (f*x^5)/(5*b) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{ArcTan}[(a^{1/3} - 2*b^{1/3}*x)/(\text{Sqrt}[3]*a^{1/3})]) / (\text{Sqrt}[3]*a^{4/3}*b^{8/3}) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[a^{1/3} + b^{1/3}*x]) / (3*a^{4/3}*b^{8/3}) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*\text{Log}[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2]) / (6*a^{4/3}*b^{8/3})$

Rule 1834

$\text{Int}[(\text{Pq}_x)((c_x)(x_x)^{m_x})/((a_x) + (b_x)(x_x)^{n_x}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c_x)^m \text{Pq}_x / (a_x + b_x x_x^n), x], x] /;$ FreeQ[{a, b, c, m}, x] & & PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx^3 + ex^6 + fx^9}{x^2(a + bx^3)} dx &= \int \left(\frac{c}{ax^2} + \frac{(be - af)x}{b^2} + \frac{fx^4}{b} + \frac{(-b^3c + ab^2d - a^2be + a^3f)x}{ab^2(a + bx^3)} \right) dx \\
&= -\frac{c}{ax} + \frac{(be - af)x^2}{2b^2} + \frac{fx^5}{5b} + \frac{(-b^3c + ab^2d - a^2be + a^3f) \int \frac{x}{a+bx^3} dx}{ab^2} \\
&= -\frac{c}{ax} + \frac{(be - af)x^2}{2b^2} + \frac{fx^5}{5b} + \frac{(b^3c - ab^2d + a^2be - a^3f) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{bx}} dx}{3a^{4/3}b^{7/3}} - \frac{(b^3c - ab^2d + a^2be - a^3f)}{3a^{4/3}b^{8/3}} \\
&= -\frac{c}{ax} + \frac{(be - af)x^2}{2b^2} + \frac{fx^5}{5b} + \frac{(b^3c - ab^2d + a^2be - a^3f) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{4/3}b^{8/3}} - \frac{(b^3c - ab^2d + a^2be - a^3f)}{3a^{4/3}b^{8/3}} \\
&= -\frac{c}{ax} + \frac{(be - af)x^2}{2b^2} + \frac{fx^5}{5b} + \frac{(b^3c - ab^2d + a^2be - a^3f) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{4/3}b^{8/3}} - \frac{(b^3c - ab^2d + a^2be - a^3f)}{3a^{4/3}b^{8/3}} \\
&= -\frac{c}{ax} + \frac{(be - af)x^2}{2b^2} + \frac{fx^5}{5b} + \frac{(b^3c - ab^2d + a^2be - a^3f) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}b^{8/3}} + \frac{(b^3c - ab^2d + a^2be - a^3f)}{3a^{4/3}b^{8/3}}
\end{aligned}$$

Mathematica [A] time = 0.128406, size = 224, normalized size = 0.99

$$\begin{aligned}
&-5x \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^2be + a^3(-f) - ab^2d + b^3c) + 10x \log(\sqrt[3]{a} + \sqrt[3]{bx})(a^2be + a^3(-f) - ab^2d + b^3c) + 10 \\
&\frac{-5x \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^2be + a^3(-f) - ab^2d + b^3c) + 10x \log(\sqrt[3]{a} + \sqrt[3]{bx})(a^2be + a^3(-f) - ab^2d + b^3c) + 10}{30a^{4/3}b^{8/3}x}
\end{aligned}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)),x]

[Out] $(-30*a^{(1/3)}*b^{(8/3)}*c + 15*a^{(4/3)}*b^{(2/3)}*(b*e - a*f)*x^3 + 6*a^{(4/3)}*b^{(5/3)}*f*x^6 + 10*\text{Sqrt}[3]*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x*\text{ArcTan}[(1 - (2*b^{(1/3)}*x)/a^{(1/3)})/\text{Sqrt}[3]] + 10*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x*\text{Log}[a^{(1/3)} + b^{(1/3)}*x] - 5*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(30*a^{(4/3)}*b^{(8/3)}*x)$

Maple [B] time = 0.004, size = 419, normalized size = 1.9

$$\frac{fx^5}{5b} - \frac{ax^2f}{2b^2} + \frac{ex^2}{2b} - \frac{a^2f}{3b^3} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \frac{1}{\sqrt[3]{\frac{a}{b}}} + \frac{ae}{3b^2} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \frac{1}{\sqrt[3]{\frac{a}{b}}} - \frac{d}{3b} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \frac{1}{\sqrt[3]{\frac{a}{b}}} + \frac{c}{3a} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \frac{1}{\sqrt[3]{\frac{a}{b}}} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a),x)`

[Out] $\frac{1}{5}f x^5/b - \frac{1}{2}b^2 x^2 a f + \frac{1}{2}e x^2/b - \frac{1}{3}b^3 a^2/(1/b a)^{1/3} \ln(x + (1/b a)^{1/3}) * f + \frac{1}{3}b^2 a/(1/b a)^{1/3} \ln(x + (1/b a)^{1/3}) * e - \frac{1}{3}d/b/(1/b a)^{1/3} \ln(x + (1/b a)^{1/3}) + \frac{1}{3}a/(1/b a)^{1/3} \ln(x + (1/b a)^{1/3}) * c + \frac{1}{6}b^3 a^2/(1/b a)^{1/3} \ln(x^2 - (1/b a)^{1/3} * x + (1/b a)^{2/3}) * f - \frac{1}{6}b^2 a/(1/b a)^{1/3} \ln(x^2 - (1/b a)^{1/3} * x + (1/b a)^{2/3}) * e + \frac{1}{6}d/b/(1/b a)^{1/3} \ln(x^2 - (1/b a)^{1/3} * x + (1/b a)^{2/3}) - \frac{1}{6}a/(1/b a)^{1/3} \ln(x^2 - (1/b a)^{1/3} * x + (1/b a)^{2/3}) * c + \frac{1}{3}b^3 a^2 3^{1/2}/(1/b a)^{1/3} \arctan(1/3 * 3^{1/2} * (2/(1/b a)^{1/3} * x - 1)) * f - \frac{1}{3}b^2 a 3^{1/2}/(1/b a)^{1/3} \arctan(1/3 * 3^{1/2} * (2/(1/b a)^{1/3} * x - 1)) * e + \frac{1}{3}d 3^{1/2}/b/(1/b a)^{1/3} \arctan(1/3 * 3^{1/2} * (2/(1/b a)^{1/3} * x - 1)) - \frac{1}{3}a 3^{1/2}/(1/b a)^{1/3} \arctan(1/3 * 3^{1/2} * (2/(1/b a)^{1/3} * x - 1)) * c - c/a/x$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 1.44421, size = 1260, normalized size = 5.55

$$\left[\frac{6 a^2 b^3 f x^6 - 30 a b^4 c + 15 (a^2 b^3 e - a^3 b^2 f) x^3 - 15 \sqrt{\frac{1}{3}} (a b^4 c - a^2 b^3 d + a^3 b^2 e - a^4 b f) x \sqrt{\frac{(-a b^2)^{\frac{1}{3}}}{a}} \log \left(\frac{2 b^2 x^3 - a b + 3 \sqrt{\frac{1}{3}} (a b x + 2 (-a b^2)^{\frac{1}{3}})}{(-a b^2)^{\frac{1}{3}}} \right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a),x, algorithm="fricas")

[Out] [1/30*(6*a^2*b^3*f*x^6 - 30*a*b^4*c + 15*(a^2*b^3*e - a^3*b^2*f)*x^3 - 15*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - 5*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^(2/3)*x*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 10*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^(2/3)*x*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^4*x), 1/30*(6*a^2*b^3*f*x^6 - 30*a*b^4*c + 15*(a^2*b^3*e - a^3*b^2*f)*x^3 - 30*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - 5*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^(2/3)*x*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 10*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^(2/3)*x*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^4*x)]

Sympy [A] time = 2.51496, size = 406, normalized size = 1.79

RootSum($27t^3a^4b^8 + a^9f^3 - 3a^8bef^2 + 3a^7b^2df^2 + 3a^7b^2e^2f - 3a^6b^3cf^2 - 6a^6b^3def - a^6b^3e^3 + 6a^5b^4cef + 3a^5b^4d^2f$)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**2/(b*x**3+a),x)

[Out] RootSum(27*_t**3*a**4*b**8 + a**9*f**3 - 3*a**8*b*e*f**2 + 3*a**7*b**2*d*f**2 + 3*a**7*b**2*e**2*f - 3*a**6*b**3*c*f**2 - 6*a**6*b**3*d*e*f - a**6*b**3*e**3 + 6*a**5*b**4*c*e*f + 3*a**5*b**4*d**2*f + 3*a**5*b**4*d*e**2 - 6*a**4*b**5*c*d*f - 3*a**4*b**5*c*e**2 - 3*a**4*b**5*d**2*e + 3*a**3*b**6*c**2*f + 6*a**3*b**6*c*d*e + a**3*b**6*d**3 - 3*a**2*b**7*c**2*e - 3*a**2*b**7*c*d**2 + 3*a*b**8*c**2*d - b**9*c**3, Lambda(_t, _t*log(9*_t**2*a**3*b**5/(a**6*f**2 - 2*a**5*b*e*f + 2*a**4*b**2*d*f + a**4*b**2*e**2 - 2*a**3*b**3*c*f - 2*a**3*b**3*d*e + 2*a**2*b**4*c*e + a**2*b**4*d**2 - 2*a*b**5*c*d + b**6*c**2) + x))) + f*x**5/(5*b) - x**2*(a*f - b*e)/(2*b**2) - c/(a*x)

Giac [A] time = 1.09501, size = 428, normalized size = 1.89

$$-\frac{c}{ax} + \frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^2b^2} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c - \left(-ab^2\right)^{\frac{2}{3}}ab^2d\right)}{3a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a),x, algorithm="giac")

[Out] $-\frac{c}{ax} + \frac{1}{3} \cdot \frac{b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^2b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e}{a^2b^2} + \frac{1}{3} \cdot \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c - \left(-ab^2\right)^{\frac{2}{3}}ab^2d\right)}{a^2b^2} + \frac{1}{10} \cdot \frac{2b^4fx^5 - 5a^3b^3fx^2 + 5b^4x^2e}{b^5} - \frac{1}{6} \cdot \frac{\left(-ab^2\right)^{\frac{2}{3}}b^3c - \left(-ab^2\right)^{\frac{2}{3}}a^2b^2d - \left(-ab^2\right)^{\frac{2}{3}}a^3f + \left(-ab^2\right)^{\frac{2}{3}}a^2b^2e}{a^2b^4} \cdot \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)$

$$3.241 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)} dx$$

Optimal. Leaf size=224

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{6a^{5/3}b^{7/3}} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{3a^{5/3}b^{7/3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{bx}}{\sqrt[3]{a} + \sqrt[3]{bx}}\right)}{\sqrt[3]{a} + \sqrt[3]{bx}}$$

[Out] $-c/(2*a*x^2) + ((b*e - a*f)*x)/b^2 + (f*x^4)/(4*b) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)*b^(7/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(5/3)*b^(7/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)*b^(7/3))$

Rubi [A] time = 0.169822, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1834, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{6a^{5/3}b^{7/3}} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{3a^{5/3}b^{7/3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{bx}}{\sqrt[3]{a} + \sqrt[3]{bx}}\right)}{\sqrt[3]{a} + \sqrt[3]{bx}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)), x]$

[Out] $-c/(2*a*x^2) + ((b*e - a*f)*x)/b^2 + (f*x^4)/(4*b) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)*b^(7/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(5/3)*b^(7/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)*b^(7/3))$

Rule 1834

$\text{Int}[(\text{Pq}_x)((c_x)(x_x)^{m_x})/((a_x) + (b_x)(x_x)^{n_x}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c_x)^m \text{Pq}_x/(a_x + b_x x_x^n), x], x] /;$ FreeQ[{a, b, c, m}, x] & & PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_)^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx^3 + ex^6 + fx^9}{x^3(a + bx^3)} dx &= \int \left(\frac{be - af}{b^2} + \frac{c}{ax^3} + \frac{fx^3}{b} + \frac{-b^3c + ab^2d - a^2be + a^3f}{ab^2(a + bx^3)} \right) dx \\
&= -\frac{c}{2ax^2} + \frac{(be - af)x}{b^2} + \frac{fx^4}{4b} + \frac{(-b^3c + ab^2d - a^2be + a^3f) \int \frac{1}{a+bx^3} dx}{ab^2} \\
&= -\frac{c}{2ax^2} + \frac{(be - af)x}{b^2} + \frac{fx^4}{4b} - \frac{(b^3c - ab^2d + a^2be - a^3f) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{bx}} dx}{3a^{5/3}b^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)}{ab^2} \\
&= -\frac{c}{2ax^2} + \frac{(be - af)x}{b^2} + \frac{fx^4}{4b} - \frac{(b^3c - ab^2d + a^2be - a^3f) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{5/3}b^{7/3}} + \frac{(b^3c - ab^2d + a^2be - a^3f)}{ab^2} \\
&= -\frac{c}{2ax^2} + \frac{(be - af)x}{b^2} + \frac{fx^4}{4b} - \frac{(b^3c - ab^2d + a^2be - a^3f) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{5/3}b^{7/3}} + \frac{(b^3c - ab^2d + a^2be - a^3f)}{ab^2} \\
&= -\frac{c}{2ax^2} + \frac{(be - af)x}{b^2} + \frac{fx^4}{4b} + \frac{(b^3c - ab^2d + a^2be - a^3f) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}b^{7/3}} - \frac{(b^3c - ab^2d + a^2be - a^3f)}{ab^2}
\end{aligned}$$

Mathematica [A] time = 0.117203, size = 218, normalized size = 0.97

$$\frac{1}{12} \left(\frac{2 \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^2be + a^3(-f) - ab^2d + b^3c)}{a^{5/3}b^{7/3}} + \frac{4 \log(\sqrt[3]{a} + \sqrt[3]{bx})(-a^2be + a^3f + ab^2d - b^3c)}{a^{5/3}b^{7/3}} + \frac{4\sqrt{3} \arctan\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)(b^3c - ab^2d + a^2be - a^3f)}{\sqrt{3}a^{5/3}b^{7/3}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)),x]

[Out] ((-6*c)/(a*x^2) + (12*(b*e - a*f)*x)/b^2 + (3*f*x^4)/b + (4*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt(3)])/(a^(5/3)*b^(7/3)) + (4*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(a^(5/3)*b^(7/3)) + (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(a^(5/3)*b^(7/3))/12

Maple [B] time = 0.005, size = 414, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a),x)`

[Out] $\frac{1}{4}f x^4/b - \frac{1}{b^2} a f x + \frac{e x}{b} + \frac{1}{3} b^3 a^2 / (1/b a)^{2/3} \ln(x + (1/b a)^{1/3})$
 $-\frac{1}{3} b^2 a / (1/b a)^{2/3} \ln(x + (1/b a)^{1/3}) + \frac{e + 1/3 b / (1/b a)^{2/3}}{1/b a)^{2/3}} \ln(x + (1/b a)^{1/3})$
 $-\frac{1}{3} a / (1/b a)^{2/3} \ln(x + (1/b a)^{1/3}) + \frac{c - 1/6 b^3 a^2 / (1/b a)^{2/3}}{1/b a)^{2/3}} \ln(x^2 - (1/b a)^{1/3} x + (1/b a)^{2/3})$
 $-\frac{1}{6} b^2 a / (1/b a)^{2/3} \ln(x^2 - (1/b a)^{1/3} x + (1/b a)^{2/3})$
 $-\frac{1}{6} b / (1/b a)^{2/3} \ln(x^2 - (1/b a)^{1/3} x + (1/b a)^{2/3})$
 $+\frac{1}{6} a / (1/b a)^{2/3} \ln(x^2 - (1/b a)^{1/3} x + (1/b a)^{2/3})$
 $+\frac{1}{3} b^3 a^2 / (1/b a)^{2/3} 3^{1/2} \arctan(1/3 3^{1/2} (2/(1/b a)^{1/3} x - 1))$
 $-\frac{1}{3} b^2 a / (1/b a)^{2/3} 3^{1/2} \arctan(1/3 3^{1/2} (2/(1/b a)^{1/3} x - 1))$
 $+\frac{1}{3} b / (1/b a)^{2/3} 3^{1/2} \arctan(1/3 3^{1/2} (2/(1/b a)^{1/3} x - 1))$
 $-\frac{1}{3} a / (1/b a)^{2/3} 3^{1/2} \arctan(1/3 3^{1/2} (2/(1/b a)^{1/3} x - 1))$
 $-\frac{1}{2} c a / x^2$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 1.53733, size = 1280, normalized size = 5.71

$$\left[\frac{3 a^3 b^2 f x^6 - 6 a^2 b^3 c - 6 \sqrt{\frac{1}{3}} (a b^4 c - a^2 b^3 d + a^3 b^2 e - a^4 b f) x^2 \sqrt{-\frac{(a^2 b)^{\frac{1}{3}}}{b}} \log \left(\frac{2 a b x^3 - 3 (a^2 b)^{\frac{1}{3}} a x - a^2 + 3 \sqrt{\frac{1}{3}} (2 a b x^2 + (a^2 b)^{\frac{2}{3}} x - (a^2 b)^{\frac{1}{3}} a)}{b x^3 + a} \right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a),x, algorithm="fricas")

[Out] [1/12*(3*a^3*b^2*f*x^6 - 6*a^2*b^3*c - 6*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^2*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) + 2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a^2*b)^(2/3)*x^2*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 4*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a^2*b)^(2/3)*x^2*log(a*b*x + (a^2*b)^(2/3)) + 12*(a^3*b^2*e - a^4*b*f)*x^3)/(a^3*b^3*x^2), 1/12*(3*a^3*b^2*f*x^6 - 6*a^2*b^3*c - 12*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^2*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + 2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a^2*b)^(2/3)*x^2*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 4*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a^2*b)^(2/3)*x^2*log(a*b*x + (a^2*b)^(2/3)) + 12*(a^3*b^2*e - a^4*b*f)*x^3)/(a^3*b^3*x^2)]

Sympy [A] time = 3.05793, size = 326, normalized size = 1.46

RootSum($27t^3a^5b^7 - a^9f^3 + 3a^8bef^2 - 3a^7b^2df^2 - 3a^7b^2e^2f + 3a^6b^3cf^2 + 6a^6b^3def + a^6b^3e^3 - 6a^5b^4cef - 3a^5b^4d^2f$)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**3/(b*x**3+a),x)

[Out] RootSum(27*_t**3*a**5*b**7 - a**9*f**3 + 3*a**8*b*e*f**2 - 3*a**7*b**2*d*f**2 - 3*a**7*b**2*e**2*f + 3*a**6*b**3*c*f**2 + 6*a**6*b**3*d*e*f + a**6*b**3*e**3 - 6*a**5*b**4*c*e*f - 3*a**5*b**4*d**2*f - 3*a**5*b**4*d*e**2 + 6*a**4*b**5*c*d*f + 3*a**4*b**5*c*e**2 + 3*a**4*b**5*d**2*e - 3*a**3*b**6*c**2*f - 6*a**3*b**6*c*d*e - a**3*b**6*d**3 + 3*a**2*b**7*c**2*e + 3*a**2*b**7*c*d**2 - 3*a*b**8*c**2*d + b**9*c**3, Lambda(_t, _t*log(3*_t*a**2*b**2/(a**3*f - a**2*b*e + a*b**2*d - b**3*c) + x))) + f*x**4/(4*b) - x*(a*f - b*e)/b**2 - c/(2*a*x**2)

Giac [A] time = 1.07595, size = 378, normalized size = 1.69

$$\frac{(b^3c - ab^2d - a^3f + a^2be) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^2b^2} - \frac{c}{2ax^2} - \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c - \left(-ab^2\right)^{\frac{1}{3}}ab^2d - \left(-ab^2\right)^{\frac{1}{3}}a^3f + \left(-ab^2\right)^{\frac{1}{3}}\right)}{3a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a),x, algorithm="giac")

[Out] $\frac{1}{3}(b^3c - ab^2d - a^3f + a^2be)(-a/b)^{1/3} \log(\text{abs}(x - (-a/b)^{1/3})) / (a^2b^2) - \frac{1}{2}c/(ax^2) - \frac{1}{3}\sqrt{3} * ((-ab^2)^{1/3}b^3c - (-ab^2)^{1/3}ab^2d - (-ab^2)^{1/3}a^3f + (-ab^2)^{1/3}a^2be) \arctan(1/3\sqrt{3} * (2x + (-a/b)^{1/3}) / (-a/b)^{1/3}) / (a^2b^3) + \frac{1}{4}(b^3fx^4 - 4ab^2fx + 4b^3xe) / b^4 - \frac{1}{6} * ((-ab^2)^{1/3}b^3c - (-ab^2)^{1/3}ab^2d - (-ab^2)^{1/3}a^3f + (-ab^2)^{1/3}a^2be) \log(x^2 + x(-a/b)^{1/3} + (-a/b)^{2/3}) / (a^2b^3)$

$$3.242 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)} dx$$

Optimal. Leaf size=227

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{6a^{7/3}b^{5/3}} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{3a^{7/3}b^{5/3}} - \tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{bx}}{\sqrt[3]{a} + \sqrt[3]{bx}}\right)$$

[Out] $-c/(4*a*x^4) + (b*c - a*d)/(a^2*x) + (f*x^2)/(2*b) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)*b^(5/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(7/3)*b^(5/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(7/3)*b^(5/3))$

Rubi [A] time = 0.185703, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1834, 292, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{6a^{7/3}b^{5/3}} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{3a^{7/3}b^{5/3}} - \tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{bx}}{\sqrt[3]{a} + \sqrt[3]{bx}}\right)$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)),x]

[Out] $-c/(4*a*x^4) + (b*c - a*d)/(a^2*x) + (f*x^2)/(2*b) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)*b^(5/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(7/3)*b^(5/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(7/3)*b^(5/3))$

Rule 1834

Int[((Pq_)*((c_)*(x_)^(m_)))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^( -1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^( -1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx^3 + ex^6 + fx^9}{x^5(a + bx^3)} dx &= \int \left(\frac{c}{ax^5} + \frac{-bc + ad}{a^2x^2} + \frac{fx}{b} - \frac{(-b^3c + ab^2d - a^2be + a^3f)x}{a^2b(a + bx^3)} \right) dx \\
&= -\frac{c}{4ax^4} + \frac{bc - ad}{a^2x} + \frac{fx^2}{2b} - \frac{(-b^3c + ab^2d - a^2be + a^3f) \int \frac{x}{a+bx^3} dx}{a^2b} \\
&= -\frac{c}{4ax^4} + \frac{bc - ad}{a^2x} + \frac{fx^2}{2b} - \frac{(b^3c - ab^2d + a^2be - a^3f) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{bx}} dx}{3a^{7/3}b^{4/3}} + \frac{(b^3c - ab^2d + a^2be - a^3f)}{3a^{7/3}b^{5/3}} \\
&= -\frac{c}{4ax^4} + \frac{bc - ad}{a^2x} + \frac{fx^2}{2b} - \frac{(b^3c - ab^2d + a^2be - a^3f) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{7/3}b^{5/3}} + \frac{(b^3c - ab^2d + a^2be - a^3f)}{3a^{7/3}b^{5/3}} \\
&= -\frac{c}{4ax^4} + \frac{bc - ad}{a^2x} + \frac{fx^2}{2b} - \frac{(b^3c - ab^2d + a^2be - a^3f) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{7/3}b^{5/3}} + \frac{(b^3c - ab^2d + a^2be - a^3f)}{3a^{7/3}b^{5/3}} \\
&= -\frac{c}{4ax^4} + \frac{bc - ad}{a^2x} + \frac{fx^2}{2b} - \frac{(b^3c - ab^2d + a^2be - a^3f) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}b^{5/3}} - \frac{(b^3c - ab^2d + a^2be - a^3f)}{3a^{7/3}b^{5/3}}
\end{aligned}$$

Mathematica [A] time = 0.111304, size = 220, normalized size = 0.97

$$\frac{1}{12} \left(\frac{2 \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^2be + a^3(-f) - ab^2d + b^3c)}{a^{7/3}b^{5/3}} + \frac{4 \log(\sqrt[3]{a} + \sqrt[3]{bx})(-a^2be + a^3f + ab^2d - b^3c)}{a^{7/3}b^{5/3}} + \frac{4 \sqrt[3]{a} - 2 \sqrt[3]{bx}}{\sqrt{3} \sqrt[3]{a}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)),x]

[Out] ((-3*c)/(a*x^4) + (12*(b*c - a*d))/(a^2*x) + (6*f*x^2)/b + (4*sqrt[3]*(-b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]])/(a^(7/3)*b^(5/3)) + (4*(-b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x]/(a^(7/3)*b^(5/3)) + (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(a^(7/3)*b^(5/3)))/12

Maple [B] time = 0.007, size = 412, normalized size = 1.8

$$\frac{fx^2}{2b} + \frac{af}{3b^2} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \frac{1}{\sqrt[3]{\frac{a}{b}}} - \frac{e}{3b} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \frac{1}{\sqrt[3]{\frac{a}{b}}} + \frac{d}{3a} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \frac{1}{\sqrt[3]{\frac{a}{b}}} - \frac{bc}{3a^2} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \frac{1}{\sqrt[3]{\frac{a}{b}}} - \frac{af}{6b^2} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a),x)

[Out] $\frac{1}{2}fx^2/b + \frac{1}{3}b^2a/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})*f - \frac{1}{3}b/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})*e + \frac{1}{3}a/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})*d - \frac{1}{3}b/a^2/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})*c - \frac{1}{6}b^2a/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*f + \frac{1}{6}b/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*e - \frac{1}{6}a/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*d + \frac{1}{6}b/a^2/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*c - \frac{1}{3}b^2a^3^{(1/2)}/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*f + \frac{1}{3}b^3^{(1/2)}/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*e - \frac{1}{3}a^3^{(1/2)}/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*d + \frac{1}{3}b/a^2*3^{(1/2)}/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*c - \frac{1}{4}c/a/x^4 - d/a/x + 1/a^2/x*b*c$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.38581, size = 1251, normalized size = 5.51

$$\left[\frac{6a^3b^2fx^6 - 6\sqrt{\frac{1}{3}}(ab^4c - a^2b^3d + a^3b^2e - a^4bf)x^4\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}} \log\left(\frac{2b^2x^3 - ab - 3\sqrt{\frac{1}{3}}\left(abx + 2(ab^2)^{\frac{2}{3}}x^2 - (ab^2)^{\frac{1}{3}}a\right)\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}} - 3(ab^2)^{\frac{2}{3}}x}{bx^3 + a}}\right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a),x, algorithm="fricas")

[Out] [1/12*(6*a^3*b^2*f*x^6 - 6*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^4*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) + 2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^2)^(2/3)*x^4*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 4*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^2)^(2/3)*x^4*log(b*x + (a*b^2)^(1/3)) - 3*a^2*b^3*c + 12*(a*b^4*c - a^2*b^3*d)*x^3)/(a^3*b^3*x^4), 1/12*(6*a^3*b^2*f*x^6 - 12*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^4*sqrt((a*b^2)^(1/3)/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) + 2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^2)^(2/3)*x^4*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 4*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^2)^(2/3)*x^4*log(b*x + (a*b^2)^(1/3)) - 3*a^2*b^3*c + 12*(a*b^4*c - a^2*b^3*d)*x^3)/(a^3*b^3*x^4)]

Sympy [A] time = 8.04656, size = 411, normalized size = 1.81

$$\text{RootSum}\left(27t^3a^7b^5 - a^9f^3 + 3a^8bef^2 - 3a^7b^2df^2 - 3a^7b^2e^2f + 3a^6b^3cf^2 + 6a^6b^3def + a^6b^3e^3 - 6a^5b^4cef - 3a^5b^4d^2f\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**5/(b*x**3+a),x)

[Out] RootSum(27*_t**3*a**7*b**5 - a**9*f**3 + 3*a**8*b*e*f**2 - 3*a**7*b**2*d*f**2 - 3*a**7*b**2*e**2*f + 3*a**6*b**3*c*f**2 + 6*a**6*b**3*d*e*f + a**6*b**3

$3e^{**3} - 6a^{**5}b^{**4}c^*e^*f - 3a^{**5}b^{**4}d^{**2}f - 3a^{**5}b^{**4}d^*e^{**2} + 6a^{**4}b^{**5}c^*d^*f + 3a^{**4}b^{**5}c^*e^{**2} + 3a^{**4}b^{**5}d^{**2}e - 3a^{**3}b^{**6}c^{**2}f - 6a^{**3}b^{**6}c^*d^*e - a^{**3}b^{**6}d^{**3} + 3a^{**2}b^{**7}c^{**2}e + 3a^{**2}b^{**7}c^*d^{**2} - 3a^*b^{**8}c^{**2}d + b^{**9}c^{**3}, \text{Lambda}(_t, _t \cdot \log(9_t^{**2}a^{**5}b^{**3}/(a^{**6}f^{**2} - 2a^{**5}b^*e^*f + 2a^{**4}b^{**2}d^*f + a^{**4}b^{**2}e^{**2} - 2a^{**3}b^{**3}c^*f - 2a^{**3}b^{**3}d^*e + 2a^{**2}b^{**4}c^*e + a^{**2}b^{**4}d^{**2} - 2a^*b^{**5}c^*d + b^{**6}c^{**2}) + x))) + f*x^{**2}/(2*b) - (a*c + x^{**3}(4*a*d - 4*b*c))/(4*a^{**2}*x^{**4})$

Giac [A] time = 1.08082, size = 417, normalized size = 1.84

$$\frac{fx^2}{2b} - \frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^3b} - \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c - \left(-ab^2\right)^{\frac{2}{3}}ab^2d\right)}{3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a),x, algorithm="giac")

[Out] $\frac{1}{2}fx^2/b - \frac{1}{3}(b^3c(-a/b)^{1/3} - a^3f(-a/b)^{1/3} + a^2b(-a/b)^{1/3}e)(-a/b)^{1/3}\log(\text{abs}(x - (-a/b)^{1/3}))/a^3b - \frac{1}{3}\sqrt{3}((-ab^2)^{2/3}b^3c - (-ab^2)^{2/3}ab^2d)/a^3b + \frac{1}{6}((-ab^2)^{2/3}b^3c - (-ab^2)^{2/3}ab^2d - (-ab^2)^{2/3}a^3f + (-ab^2)^{2/3}a^2b^2e)\arctan(1/3\sqrt{3}(2x + (-a/b)^{1/3}))/(-a/b)^{1/3}/(a^3b^3) + 1/4*(4b^3cx^3 - 4a^2dx^3 - a^3c)/(a^2x^4)$

$$3.243 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)} dx$$

Optimal. Leaf size=225

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{6a^{8/3}b^{4/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{3a^{8/3}b^{4/3}} - \tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{bx}}{\sqrt[3]{a} + \sqrt[3]{bx}}\right)$$

[Out] $-c/(5*a*x^5) + (b*c - a*d)/(2*a^2*x^2) + (f*x)/b - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(8/3)*b^(4/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(8/3)*b^(4/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(8/3)*b^(4/3))$

Rubi [A] time = 0.169774, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1834, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{6a^{8/3}b^{4/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{3a^{8/3}b^{4/3}} - \tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{bx}}{\sqrt[3]{a} + \sqrt[3]{bx}}\right)$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)),x]

[Out] $-c/(5*a*x^5) + (b*c - a*d)/(2*a^2*x^2) + (f*x)/b - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(8/3)*b^(4/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(8/3)*b^(4/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(8/3)*b^(4/3))$

Rule 1834

Int[((Pq_)*((c_)*(x_)^(m_)))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] & & PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_)^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx^3 + ex^6 + fx^9}{x^6(a + bx^3)} dx &= \int \left(\frac{f}{b} + \frac{c}{ax^6} + \frac{-bc + ad}{a^2x^3} + \frac{b^3c - ab^2d + a^2be - a^3f}{a^2b(a + bx^3)} \right) dx \\
&= -\frac{c}{5ax^5} + \frac{bc - ad}{2a^2x^2} + \frac{fx}{b} + \frac{(b^3c - ab^2d + a^2be - a^3f) \int \frac{1}{a+bx^3} dx}{a^2b} \\
&= -\frac{c}{5ax^5} + \frac{bc - ad}{2a^2x^2} + \frac{fx}{b} + \frac{(b^3c - ab^2d + a^2be - a^3f) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{bx}} dx}{3a^{8/3}b} + \frac{(b^3c - ab^2d + a^2be - a^3f)}{3a^{8/3}b} \\
&= -\frac{c}{5ax^5} + \frac{bc - ad}{2a^2x^2} + \frac{fx}{b} + \frac{(b^3c - ab^2d + a^2be - a^3f) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{8/3}b^{4/3}} - \frac{(b^3c - ab^2d + a^2be - a^3f)}{3a^{8/3}b^{4/3}} \\
&= -\frac{c}{5ax^5} + \frac{bc - ad}{2a^2x^2} + \frac{fx}{b} + \frac{(b^3c - ab^2d + a^2be - a^3f) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{8/3}b^{4/3}} - \frac{(b^3c - ab^2d + a^2be - a^3f)}{3a^{8/3}b^{4/3}} \\
&= -\frac{c}{5ax^5} + \frac{bc - ad}{2a^2x^2} + \frac{fx}{b} - \frac{(b^3c - ab^2d + a^2be - a^3f) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}b^{4/3}} + \frac{(b^3c - ab^2d + a^2be - a^3f)}{3a^{8/3}b^{4/3}}
\end{aligned}$$

Mathematica [A] time = 0.08175, size = 220, normalized size = 0.98

$$\frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(-a^2be + a^3f + ab^2d - b^3c)}{6a^{8/3}b^{4/3}} + \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(a^2be + a^3(-f) - ab^2d + b^3c)}{3a^{8/3}b^{4/3}} + \frac{\tan^{-1}\left(\frac{1-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}b^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)),x]

[Out] $-\frac{c}{5a^5x^5} + \frac{bc - ad}{2a^2x^2} + \frac{fx}{b} + \frac{((-b^3c) + a^2b^2d - a^2b^2e + a^3f) \operatorname{ArcTan}\left[\frac{1 - (2b^{1/3}x)/a^{1/3}}{\sqrt{3}}\right]}{(\sqrt{3}a^{8/3}b^{4/3})} + \frac{((b^3c - a^2b^2d + a^2b^2e - a^3f) \operatorname{Log}[a^{1/3} + b^{1/3}x])}{(3a^{8/3}b^{4/3})} + \frac{((-b^3c) + a^2b^2d - a^2b^2e + a^3f) \operatorname{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2]}{(6a^{8/3}b^{4/3})}$

Maple [B] time = 0.005, size = 410, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a),x)`

[Out] $f*x/b-1/3/b^2*a/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*f+1/3/b/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*e-1/3/a/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*d+1/3*b/a^2/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*c+1/6/b^2*a/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*f-1/6/b/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*e+1/6/a/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*d-1/6*b/a^2/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*c-1/3/b^2*a/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*f+1/3/b/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*e-1/3/a/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*d+1/3*b/a^2/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*c-1/5*c/a/x^5-1/2*d/a/x^2+1/2/a^2/x^2*b*c$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 1.30809, size = 1312, normalized size = 5.83

$$\left[30a^4bfx^6 - 15\sqrt{\frac{1}{3}}(ab^4c - a^2b^3d + a^3b^2e - a^4bf)x^5\sqrt{\frac{(-a^2b)^{\frac{1}{3}}}{b}} \log\left(\frac{2abx^3 + 3(-a^2b)^{\frac{1}{3}}ax - a^2 - 3\sqrt{\frac{1}{3}}\left(2abx^2 + (-a^2b)^{\frac{2}{3}}x + (-a^2b)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a^2b)^{\frac{1}{3}}}{b}}}{bx^3 + a}\right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a),x, algorithm="fricas")

[Out] [1/30*(30*a^4*b*f*x^6 - 15*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^5*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a) - 5*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*x^5*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 10*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*x^5*log(a*b*x + (-a^2*b)^(2/3)) - 6*a^3*b^2*c + 15*(a^2*b^3*c - a^3*b^2*d)*x^3)/(a^4*b^2*x^5), 1/30*(30*a^4*b*f*x^6 + 30*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^5*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) - 5*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*x^5*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 10*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*x^5*log(a*b*x + (-a^2*b)^(2/3)) - 6*a^3*b^2*c + 15*(a^2*b^3*c - a^3*b^2*d)*x^3)/(a^4*b^2*x^5)]

Sympy [A] time = 9.84072, size = 328, normalized size = 1.46

RootSum($27t^3a^8b^4 + a^9f^3 - 3a^8bef^2 + 3a^7b^2df^2 + 3a^7b^2e^2f - 3a^6b^3cf^2 - 6a^6b^3def - a^6b^3e^3 + 6a^5b^4cef + 3a^5b^4d^2f$)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**6/(b*x**3+a),x)

[Out] RootSum($27*_t**3*a**8*b**4 + a**9*f**3 - 3*a**8*b*e*f**2 + 3*a**7*b**2*d*f**2 + 3*a**7*b**2*e**2*f - 3*a**6*b**3*c*f**2 - 6*a**6*b**3*d*e*f - a**6*b**3*e**3 + 6*a**5*b**4*c*e*f + 3*a**5*b**4*d**2*f + 3*a**5*b**4*d*e**2 - 6*a**4*b**5*c*d*f - 3*a**4*b**5*c*e**2 - 3*a**4*b**5*d**2*e + 3*a**3*b**6*c**2*f + 6*a**3*b**6*c*d*e + a**3*b**6*d**3 - 3*a**2*b**7*c**2*e - 3*a**2*b**7*c*d**2 + 3*a*b**8*c**2*d - b**9*c**3, Lambda(_t, _t*log(-3*_t*a**3*b/(a**3*f - a**2*b*e + a*b**2*d - b**3*c) + x))) + f*x/b - (2*a*c + x**3*(5*a*d - 5*b*c))/(10*a**2*x**5)$)

Giac [A] time = 1.07951, size = 370, normalized size = 1.64

$$\frac{fx}{b} - \frac{(b^3c - ab^2d - a^3f + a^2be)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^3b} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c - \left(-ab^2\right)^{\frac{1}{3}}ab^2d - \left(-ab^2\right)^{\frac{1}{3}}a^3f + \left(-ab^2\right)^{\frac{1}{3}}\right)}{3a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a),x, algorithm="giac")

[Out] $f*x/b - 1/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*(-a/b)^{1/3}*\log(\text{abs}(x - (-a/b)^{1/3}))/a^3*b + 1/3*\sqrt{3}*((-a*b^2)^{1/3}*b^3*c - (-a*b^2)^{1/3}*a*b^2*d - (-a*b^2)^{1/3}*a^3*f + (-a*b^2)^{1/3}*a^2*b*e)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{1/3})/(-a/b)^{1/3})/a^3*b^2 + 1/6*((-a*b^2)^{1/3}*b^3*c - (-a*b^2)^{1/3}*a*b^2*d - (-a*b^2)^{1/3}*a^3*f + (-a*b^2)^{1/3}*a^2*b*e)*\log(x^2 + x*(-a/b)^{1/3} + (-a/b)^{2/3})/a^3*b^2 + 1/10*(5*b*c*x^3 - 5*a*d*x^3 - 2*a*c)/a^2*x^5$

$$3.244 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)} dx$$

Optimal. Leaf size=242

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{6a^{10/3}b^{2/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{3a^{10/3}b^{2/3}} + \frac{\tan^{-1}\left(\frac{a^{1/3} - 2b^{1/3}x}{\sqrt{3a^{1/3} + b^{1/3}x}}\right)}{1}$$

[Out] $-c/(7*a*x^7) + (b*c - a*d)/(4*a^2*x^4) - (b^2*c - a*b*d + a^2*e)/(a^3*x) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^{1/3} - 2*b^{1/3}*x)/(Sqrt[3]*a^{1/3})])/(Sqrt[3]*a^{10/3}*b^{2/3}) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{1/3} + b^{1/3}*x])/(3*a^{10/3}*b^{2/3}) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2])/(6*a^{10/3}*b^{2/3})$

Rubi [A] time = 0.187964, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1834, 292, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{6a^{10/3}b^{2/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{3a^{10/3}b^{2/3}} + \frac{\tan^{-1}\left(\frac{a^{1/3} - 2b^{1/3}x}{\sqrt{3a^{1/3} + b^{1/3}x}}\right)}{1}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)),x]

[Out] $-c/(7*a*x^7) + (b*c - a*d)/(4*a^2*x^4) - (b^2*c - a*b*d + a^2*e)/(a^3*x) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^{1/3} - 2*b^{1/3}*x)/(Sqrt[3]*a^{1/3})])/(Sqrt[3]*a^{10/3}*b^{2/3}) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{1/3} + b^{1/3}*x])/(3*a^{10/3}*b^{2/3}) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2])/(6*a^{10/3}*b^{2/3})$

Rule 1834

Int[((Pq_)*((c_)*(x_))^(m_))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]

Rule 292

```
Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx^3 + ex^6 + fx^9}{x^8(a + bx^3)} dx &= \int \left(\frac{c}{ax^8} + \frac{-bc + ad}{a^2x^5} + \frac{b^2c - abd + a^2e}{a^3x^2} + \frac{(-b^3c + ab^2d - a^2be + a^3f)x}{a^3(a + bx^3)} \right) dx \\
&= -\frac{c}{7ax^7} + \frac{bc - ad}{4a^2x^4} - \frac{b^2c - abd + a^2e}{a^3x} + \frac{(-b^3c + ab^2d - a^2be + a^3f) \int \frac{x}{a+bx^3} dx}{a^3} \\
&= -\frac{c}{7ax^7} + \frac{bc - ad}{4a^2x^4} - \frac{b^2c - abd + a^2e}{a^3x} + \frac{(b^3c - ab^2d + a^2be - a^3f) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{bx}} dx}{3a^{10/3}\sqrt[3]{b}} - \frac{(b^3c - ab^2d + a^2be - a^3f)}{3a^{10/3}\sqrt[3]{b}} \\
&= -\frac{c}{7ax^7} + \frac{bc - ad}{4a^2x^4} - \frac{b^2c - abd + a^2e}{a^3x} + \frac{(b^3c - ab^2d + a^2be - a^3f) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{10/3}b^{2/3}} - \frac{(b^3c - ab^2d + a^2be - a^3f)}{3a^{10/3}b^{2/3}} \\
&= -\frac{c}{7ax^7} + \frac{bc - ad}{4a^2x^4} - \frac{b^2c - abd + a^2e}{a^3x} + \frac{(b^3c - ab^2d + a^2be - a^3f) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{10/3}b^{2/3}} - \frac{(b^3c - ab^2d + a^2be - a^3f)}{3a^{10/3}b^{2/3}} \\
&= -\frac{c}{7ax^7} + \frac{bc - ad}{4a^2x^4} - \frac{b^2c - abd + a^2e}{a^3x} + \frac{(b^3c - ab^2d + a^2be - a^3f) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{10/3}b^{2/3}} + \frac{(b^3c - ab^2d + a^2be - a^3f)}{\sqrt{3}a^{10/3}b^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.111245, size = 231, normalized size = 0.95

$$\frac{14 \log\left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3}x^2\right)(-a^2be + a^3f + ab^2d - b^3c)}{b^{2/3}} + \frac{28 \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)(a^2be + a^3(-f) - ab^2d + b^3c)}{b^{2/3}} + \frac{28\sqrt{3} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)(a^2be + a^3(-f) - ab^2d + b^3c)}{b^{2/3}} - \frac{84a^{10/3}}{b^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)),x]

[Out] ((-12*a^(7/3)*c)/x^7 + (21*a^(4/3)*(b*c - a*d))/x^4 - (84*a^(1/3)*(b^2*c - a*b*d + a^2*e))/x + (28*sqrt[3]*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]])/b^(2/3) + (28*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (14*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(84*a^(10/3))

Maple [B] time = 0.007, size = 440, normalized size = 1.8

$$-\frac{f}{3b} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \frac{1}{\sqrt[3]{\frac{a}{b}}} + \frac{e}{3a} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \frac{1}{\sqrt[3]{\frac{a}{b}}} - \frac{bd}{3a^2} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \frac{1}{\sqrt[3]{\frac{a}{b}}} + \frac{b^2c}{3a^3} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \frac{1}{\sqrt[3]{\frac{a}{b}}} + \frac{f}{6b} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \sqrt[3]{\frac{a}{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a),x)

[Out]
$$\begin{aligned} & -1/3/b/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})*f+1/3/a/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)}) \\ & *e-1/3/a^2*b/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})*d+1/3/a^3*b^2/(1/b*a)^{(1/3)} \\ & *\ln(x+(1/b*a)^{(1/3)})*c+1/6/b/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)}) \\ & *f-1/6/a/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*e+1/6/a^2*b/(1/b*a)^{(1/3)} \\ & *\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*d-1/6/a^3*b^2/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)}) \\ & *c+1/3*3^{(1/2)}/b/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*f-1/3/a*3^{(1/2)}/(1/b*a)^{(1/3)} \\ & *\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*e+1/3/a^2*3^{(1/2)}*b/(1/b*a)^{(1/3)} \\ & *\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*d-1/3/a^3*3^{(1/2)}*b^2/(1/b*a)^{(1/3)} \\ & *\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*c-1/7*c/a/x^7-1/4/a/x^4*d+1/4/a^2/x^4*b*c-e/a/x+1/a^2/x*b*d-1/a^3/x*b^2*c \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.43548, size = 1359, normalized size = 5.62

$$\left[42 \sqrt{\frac{1}{3}} (ab^4c - a^2b^3d + a^3b^2e - a^4bf) x^7 \sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}} \log \left(\frac{2b^2x^3 - ab + 3 \sqrt{\frac{1}{3}} \left(abx + 2(-ab^2)^{\frac{2}{3}}x^2 + (-ab^2)^{\frac{1}{3}}a \right) \sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}} - 3(-ab^2)^{\frac{2}{3}}x}{bx^3 + a}} \right) + 14(b \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a),x, algorithm="fricas")

[Out] [-1/84*(42*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^7*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + 14*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^(2/3)*x^7*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^(2/3)*x^7*log(b*x - (-a*b^2)^(1/3)) + 84*(a*b^4*c - a^2*b^3*d + a^3*b^2*e)*x^6 + 12*a^3*b^2*c - 21*(a^2*b^3*c - a^3*b^2*d)*x^3)/(a^4*b^2*x^7), -1/84*(84*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^7*sqrt((-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt((-a*b^2)^(1/3)/a)/b) + 14*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^(2/3)*x^7*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^(2/3)*x^7*log(b*x - (-a*b^2)^(1/3)) + 84*(a*b^4*c - a^2*b^3*d + a^3*b^2*e)*x^6 + 12*a^3*b^2*c - 21*(a^2*b^3*c - a^3*b^2*d)*x^3)/(a^4*b^2*x^7)]

Sympy [A] time = 30.4187, size = 432, normalized size = 1.79

RootSum(27t^3a^10b^2 + a^9f^3 - 3a^8bef^2 + 3a^7b^2df^2 + 3a^7b^2e^2f - 3a^6b^3cf^2 - 6a^6b^3def - a^6b^3e^3 + 6a^5b^4cef + 3a^5b^4d^2)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**8/(b*x**3+a),x)

[Out] RootSum(27*_t**3*a**10*b**2 + a**9*f**3 - 3*a**8*b*e*f**2 + 3*a**7*b**2*d*f**2 + 3*a**7*b**2*e**2*f - 3*a**6*b**3*c*f**2 - 6*a**6*b**3*d*e*f - a**6*b*

```

*3***3 + 6*a**5*b**4*c*e*f + 3*a**5*b**4*d**2*f + 3*a**5*b**4*d*e**2 - 6*a
**4*b**5*c*d*f - 3*a**4*b**5*c*e**2 - 3*a**4*b**5*d**2*e + 3*a**3*b**6*c**2
*f + 6*a**3*b**6*c*d*e + a**3*b**6*d**3 - 3*a**2*b**7*c**2*e - 3*a**2*b**7*
c*d**2 + 3*a*b**8*c**2*d - b**9*c**3, Lambda(_t, _t*log(9*_t**2*a**7*b/(a**
6*f**2 - 2*a**5*b*e*f + 2*a**4*b**2*d*f + a**4*b**2*e**2 - 2*a**3*b**3*c*f
- 2*a**3*b**3*d*e + 2*a**2*b**4*c*e + a**2*b**4*d**2 - 2*a*b**5*c*d + b**6*
c**2) + x))) - (4*a**2*c + x**6*(28*a**2*e - 28*a*b*d + 28*b**2*c) + x**3*(
7*a**2*d - 7*a*b*c))/(28*a**3*x**7)

```

Giac [A] time = 1.07466, size = 444, normalized size = 1.83

$$\frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^4} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c - \left(-ab^2\right)^{\frac{2}{3}}ab^2d - \left(-ab\right)^{\frac{2}{3}}a^3f + \left(-ab\right)^{\frac{2}{3}}a^2b^2e\right)\arctan\left(\frac{1}{3}\sqrt{3}\frac{2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}}{\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\left(a^4b^2 - \frac{1}{6}\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c - \left(-ab^2\right)^{\frac{2}{3}}ab^2d - \left(-ab^2\right)^{\frac{2}{3}}a^3f + \left(-ab^2\right)^{\frac{2}{3}}a^2b^2e\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right) - \frac{1}{28}\left(28b^2cx^6 - 28abdx^6 + 28a^2x^6e - 7abcx^3 + 7a^2dx^3 + 4a^2c\right)}{\left(a^3x^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a),x, algorithm="giac")
```

```

[Out] 1/3*(b^3*c*(-a/b)^(1/3) - a*b^2*d*(-a/b)^(1/3) - a^3*f*(-a/b)^(1/3) + a^2*b
*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 + 1/3*sqrt(3)*
((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*b^2)^(2/3)*a^3*f + (-a
*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/
(a^4*b^2) - 1/6*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*b^2)^(
2/3)*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3
))/ (a^4*b^2) - 1/28*(28*b^2*c*x^6 - 28*a*b*d*x^6 + 28*a^2*x^6*e - 7*a*b*c*x
^3 + 7*a^2*d*x^3 + 4*a^2*c)/(a^3*x^7)

```

$$3.245 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)} dx$$

Optimal. Leaf size=244

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{6a^{11/3}\sqrt[3]{b}} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{3a^{11/3}\sqrt[3]{b}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{bx}}{\sqrt[3]{a} + \sqrt[3]{bx}}\right)}{\sqrt[3]{a} + \sqrt[3]{bx}}$$

```
[Out] -c/(8*a*x^8) + (b*c - a*d)/(5*a^2*x^5) - (b^2*c - a*b*d + a^2*e)/(2*a^3*x^2)
) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(11/3)*b^(1/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(11/3)*b^(1/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(11/3)*b^(1/3))
```

Rubi [A] time = 0.174033, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1834, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{6a^{11/3}\sqrt[3]{b}} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{3a^{11/3}\sqrt[3]{b}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{bx}}{\sqrt[3]{a} + \sqrt[3]{bx}}\right)}{\sqrt[3]{a} + \sqrt[3]{bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)),x]
```

```
[Out] -c/(8*a*x^8) + (b*c - a*d)/(5*a^2*x^5) - (b^2*c - a*b*d + a^2*e)/(2*a^3*x^2)
) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(11/3)*b^(1/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(11/3)*b^(1/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(11/3)*b^(1/3))
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_)^(m_.)))/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Int[ExpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_)^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx^3 + ex^6 + fx^9}{x^9(a + bx^3)} dx &= \int \left(\frac{c}{ax^9} + \frac{-bc + ad}{a^2x^6} + \frac{b^2c - abd + a^2e}{a^3x^3} + \frac{-b^3c + ab^2d - a^2be + a^3f}{a^3(a + bx^3)} \right) dx \\
&= -\frac{c}{8ax^8} + \frac{bc - ad}{5a^2x^5} - \frac{b^2c - abd + a^2e}{2a^3x^2} + \frac{(-b^3c + ab^2d - a^2be + a^3f) \int \frac{1}{a+bx^3} dx}{a^3} \\
&= -\frac{c}{8ax^8} + \frac{bc - ad}{5a^2x^5} - \frac{b^2c - abd + a^2e}{2a^3x^2} - \frac{(b^3c - ab^2d + a^2be - a^3f) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{bx}} dx}{3a^{11/3}} - \frac{(b^3c - ab^2d + a^2be - a^3f) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{11/3} \sqrt[3]{b}} \\
&= -\frac{c}{8ax^8} + \frac{bc - ad}{5a^2x^5} - \frac{b^2c - abd + a^2e}{2a^3x^2} - \frac{(b^3c - ab^2d + a^2be - a^3f) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{11/3} \sqrt[3]{b}} + \frac{(b^3c - ab^2d + a^2be - a^3f) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{11/3} \sqrt[3]{b}} - \frac{(b^3c - ab^2d + a^2be - a^3f)}{120a^{11/3}}
\end{aligned}$$

Mathematica [A] time = 0.111757, size = 231, normalized size = 0.95

$$\frac{20 \log\left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2\right) (a^2be + a^3(-f) - ab^2d + b^3c)}{\sqrt[3]{b}} + \frac{40 \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right) (-a^2be + a^3f + ab^2d - b^3c)}{\sqrt[3]{b}} + \frac{40\sqrt{3} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right) (a^2be + a^3(-f) - ab^2d + b^3c)}{\sqrt[3]{b}} - \frac{(b^3c - ab^2d + a^2be - a^3f)}{120a^{11/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)),x]

[Out] ((-15*a^(8/3)*c)/x^8 + (24*a^(5/3)*(b*c - a*d))/x^5 - (60*a^(2/3)*(b^2*c - a*b*d + a^2*e))/x^2 + (40*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt(3)]/b^(1/3) + (40*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + (20*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3))/(120*a^(11/3))

Maple [B] time = 0.008, size = 441, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a),x)`

[Out] $\frac{1}{3} \frac{b}{(1/b*a)^{2/3}} \ln(x+(1/b*a)^{1/3}) * f - \frac{1}{3} \frac{a}{(1/b*a)^{2/3}} \ln(x+(1/b*a)^{1/3}) * e + \frac{1}{3} \frac{a^2*b}{(1/b*a)^{2/3}} \ln(x+(1/b*a)^{1/3}) * d - \frac{1}{3} \frac{a^3*b^2}{(1/b*a)^{2/3}} \ln(x+(1/b*a)^{1/3}) * c - \frac{1}{6} \frac{b}{(1/b*a)^{2/3}} \ln(x^2-(1/b*a)^{1/3}*x+(1/b*a)^{2/3}) * f + \frac{1}{6} \frac{a}{(1/b*a)^{2/3}} \ln(x^2-(1/b*a)^{1/3}*x+(1/b*a)^{2/3}) * e - \frac{1}{6} \frac{a^2*b}{(1/b*a)^{2/3}} \ln(x^2-(1/b*a)^{1/3}*x+(1/b*a)^{2/3}) * d + \frac{1}{6} \frac{a^3*b^2}{(1/b*a)^{2/3}} \ln(x^2-(1/b*a)^{1/3}*x+(1/b*a)^{2/3}) * c + \frac{1}{3} \frac{b}{(1/b*a)^{2/3}} * 3^{1/2} * \arctan(1/3 * 3^{1/2} * (2/(1/b*a)^{1/3} * x - 1)) * f - \frac{1}{3} \frac{a}{(1/b*a)^{2/3}} * 3^{1/2} * \arctan(1/3 * 3^{1/2} * (2/(1/b*a)^{1/3} * x - 1)) * e + \frac{1}{3} \frac{a^2*b}{(1/b*a)^{2/3}} * 3^{1/2} * \arctan(1/3 * 3^{1/2} * (2/(1/b*a)^{1/3} * x - 1)) * d - \frac{1}{3} \frac{a^3*b^2}{(1/b*a)^{2/3}} * 3^{1/2} * \arctan(1/3 * 3^{1/2} * (2/(1/b*a)^{1/3} * x - 1)) * c - \frac{1}{8} \frac{c}{a} * x^{-8} - \frac{1}{5} \frac{a}{x^5} * d + \frac{1}{5} \frac{a^2}{x^5} * b * c - \frac{1}{2} \frac{a}{x^2} * e + \frac{1}{2} \frac{a^2}{x^2} * b * d - \frac{1}{2} \frac{a^3}{x^2} * b^2 * c$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 1.37799, size = 1353, normalized size = 5.55

$$\left[60 \sqrt{\frac{1}{3}} (ab^4c - a^2b^3d + a^3b^2e - a^4bf) x^8 \sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}} \log \left(\frac{2abx^3 - 3(a^2b)^{\frac{1}{3}}ax - a^2 + 3\sqrt{\frac{1}{3}} \left(2abx^2 + (a^2b)^{\frac{2}{3}}x - (a^2b)^{\frac{1}{3}}a \right) \sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}}{bx^3 + a} \right) - 20(b^3c - \dots \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/120*(60*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^8*sqrt(-
(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*
(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*
x^3 + a) - 20*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a^2*b)^(2/3)*x^8*log(a*
b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 40*(b^3*c - a*b^2*d + a^2*b*e
- a^3*f)*(a^2*b)^(2/3)*x^8*log(a*b*x + (a^2*b)^(2/3)) + 60*(a^2*b^3*c - a^3
*b^2*d + a^4*b*e)*x^6 + 15*a^4*b*c - 24*(a^3*b^2*c - a^4*b*d)*x^3)/(a^5*b*x
^8), -1/120*(120*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^8*
sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a
)*sqrt((a^2*b)^(1/3)/b)/a^2) - 20*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a^2*
b)^(2/3)*x^8*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 40*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*(a^2*b)^(2/3)*x^8*log(a*b*x + (a^2*b)^(2/3)) +
60*(a^2*b^3*c - a^3*b^2*d + a^4*b*e)*x^6 + 15*a^4*b*c - 24*(a^3*b^2*c - a^4
*b*d)*x^3)/(a^5*b*x^8)]
```

Sympy [A] time = 52.7116, size = 348, normalized size = 1.43

$$\text{RootSum}\left(27t^3a^{11}b - a^9f^3 + 3a^8bef^2 - 3a^7b^2df^2 - 3a^7b^2e^2f + 3a^6b^3cf^2 + 6a^6b^3def + a^6b^3e^3 - 6a^5b^4cef - 3a^5b^4d^2f\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**9/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**11*b - a**9*f**3 + 3*a**8*b*e*f**2 - 3*a**7*b**2*d*f**2
- 3*a**7*b**2*e**2*f + 3*a**6*b**3*c*f**2 + 6*a**6*b**3*d*e*f + a**6*b**3*
e**3 - 6*a**5*b**4*c*e*f - 3*a**5*b**4*d**2*f - 3*a**5*b**4*d*e**2 + 6*a**4
*b**5*c*d*f + 3*a**4*b**5*c*e**2 + 3*a**4*b**5*d**2*e - 3*a**3*b**6*c**2*f
- 6*a**3*b**6*c*d*e - a**3*b**6*d**3 + 3*a**2*b**7*c**2*e + 3*a**2*b**7*c*d
**2 - 3*a*b**8*c**2*d + b**9*c**3, Lambda(_t, _t*log(3*_t*a**4/(a**3*f - a
**2*b*e + a*b**2*d - b**3*c) + x))) - (5*a**2*c + x**6*(20*a**2*e - 20*a*b*d
+ 20*b**2*c) + x**3*(8*a**2*d - 8*a*b*c))/(40*a**3*x**8)
```

Giac [A] time = 1.07419, size = 401, normalized size = 1.64

$$\frac{(b^3c - ab^2d - a^3f + a^2be) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^4} - \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c - \left(-ab^2\right)^{\frac{1}{3}}ab^2d - \left(-ab^2\right)^{\frac{1}{3}}a^3f + \left(-ab^2\right)^{\frac{1}{3}}a^2be\right) \arctan\left(\frac{2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}}{\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a),x, algorithm="giac")

[Out] $\frac{1}{3}*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/a^4 - \frac{1}{3}*sqrt(3)*((-a*b^2)^{(1/3)}*b^3*c - (-a*b^2)^{(1/3)}*a*b^2*d - (-a*b^2)^{(1/3)}*a^3*f + (-a*b^2)^{(1/3)}*a^2*b*e)*\arctan(1/3*sqrt(3)*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(a^4*b) - \frac{1}{6}*((-a*b^2)^{(1/3)}*b^3*c - (-a*b^2)^{(1/3)}*a*b^2*d - (-a*b^2)^{(1/3)}*a^3*f + (-a*b^2)^{(1/3)}*a^2*b*e)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a^4*b) - \frac{1}{40}*(20*b^2*c*x^6 - 20*a*b*d*x^6 + 20*a^2*x^6*e - 8*a*b*c*x^3 + 8*a^2*d*x^3 + 5*a^2*c)/(a^3*x^8)$

$$3.246 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)} dx$$

Optimal. Leaf size=277

$$\frac{\sqrt[3]{b} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2) (a^2 b e + a^3 (-f) - a b^2 d + b^3 c)}{6 a^{13/3}} + \frac{a^2 b e + a^3 (-f) - a b^2 d + b^3 c}{a^4 x} - \frac{\sqrt[3]{b} \log(\sqrt[3]{a} + \sqrt[3]{bx}) (a^2 b e + a^3 (-f) - a b^2 d + b^3 c)}{3 a^{13/3}}$$

[Out] $-c/(10*a*x^{10}) + (b*c - a*d)/(7*a^2*x^7) - (b^2*c - a*b*d + a^2*e)/(4*a^3*x^4) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a^4*x) - (b^{(1/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*a^{(13/3)}) - (b^{(1/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{(1/3)} + b^{(1/3)}*x])/(3*a^{(13/3)}) + (b^{(1/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(6*a^{(13/3)})$

Rubi [A] time = 0.222478, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1834, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{b} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2) (a^2 b e + a^3 (-f) - a b^2 d + b^3 c)}{6 a^{13/3}} + \frac{a^2 b e + a^3 (-f) - a b^2 d + b^3 c}{a^4 x} - \frac{\sqrt[3]{b} \log(\sqrt[3]{a} + \sqrt[3]{bx}) (a^2 b e + a^3 (-f) - a b^2 d + b^3 c)}{3 a^{13/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)),x]

[Out] $-c/(10*a*x^{10}) + (b*c - a*d)/(7*a^2*x^7) - (b^2*c - a*b*d + a^2*e)/(4*a^3*x^4) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a^4*x) - (b^{(1/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*a^{(13/3)}) - (b^{(1/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{(1/3)} + b^{(1/3)}*x])/(3*a^{(13/3)}) + (b^{(1/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(6*a^{(13/3)})$

Rule 1834

Int[((Pq_)*((c_)*(x_))^(m_))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]

Rule 292

```
Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^( -1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^( -1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx^3 + ex^6 + fx^9}{x^{11}(a + bx^3)} dx &= \int \left(\frac{c}{ax^{11}} + \frac{-bc + ad}{a^2x^8} + \frac{b^2c - abd + a^2e}{a^3x^5} + \frac{-b^3c + ab^2d - a^2be + a^3f}{a^4x^2} - \frac{b(-b^3c + ab^2d - a^2be + a^3f)}{a^4(a + bx^3)} \right) dx \\
&= -\frac{c}{10ax^{10}} + \frac{bc - ad}{7a^2x^7} - \frac{b^2c - abd + a^2e}{4a^3x^4} + \frac{b^3c - ab^2d + a^2be - a^3f}{a^4x} + \frac{(b(b^3c - ab^2d + a^2be - a^3f))}{a^4(a + bx^3)} \\
&= -\frac{c}{10ax^{10}} + \frac{bc - ad}{7a^2x^7} - \frac{b^2c - abd + a^2e}{4a^3x^4} + \frac{b^3c - ab^2d + a^2be - a^3f}{a^4x} - \frac{(b^{2/3}(b^3c - ab^2d + a^2be - a^3f))}{3a^{10/3}\sqrt{a + bx^3}} \\
&= -\frac{c}{10ax^{10}} + \frac{bc - ad}{7a^2x^7} - \frac{b^2c - abd + a^2e}{4a^3x^4} + \frac{b^3c - ab^2d + a^2be - a^3f}{a^4x} - \frac{\sqrt[3]{b}(b^3c - ab^2d + a^2be - a^3f)}{3a^{10/3}\sqrt{a + bx^3}} \\
&= -\frac{c}{10ax^{10}} + \frac{bc - ad}{7a^2x^7} - \frac{b^2c - abd + a^2e}{4a^3x^4} + \frac{b^3c - ab^2d + a^2be - a^3f}{a^4x} - \frac{\sqrt[3]{b}(b^3c - ab^2d + a^2be - a^3f)}{3a^{10/3}\sqrt{a + bx^3}} \\
&= -\frac{c}{10ax^{10}} + \frac{bc - ad}{7a^2x^7} - \frac{b^2c - abd + a^2e}{4a^3x^4} + \frac{b^3c - ab^2d + a^2be - a^3f}{a^4x} - \frac{\sqrt[3]{b}(b^3c - ab^2d + a^2be - a^3f)}{\sqrt{3}a^{10/3}\sqrt{a + bx^3}}
\end{aligned}$$

Mathematica [A] time = 0.112328, size = 266, normalized size = 0.96

$$70\sqrt[3]{b} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^2be + a^3(-f) - ab^2d + b^3c) + \frac{420\sqrt[3]{a}(a^2be + a^3(-f) - ab^2d + b^3c)}{x} + 140\sqrt[3]{b} \log(\sqrt[3]{a} + \sqrt[3]{bx})$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)),x]

[Out] ((-42*a^(10/3)*c)/x^10 + (60*a^(7/3)*(b*c - a*d))/x^7 - (105*a^(4/3)*(b^2*c - a*b*d + a^2*e))/x^4 + (420*a^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/x - 140*sqrt[3]*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]] + 140*b^(1/3)*(-b^3*c + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 70*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(420*a^(13/3))

Maple [B] time = 0.009, size = 491, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a),x)`

[Out] $\frac{1}{3} \frac{a}{(1/b*a)^{1/3}} \ln(x+(1/b*a)^{1/3}) * f - \frac{1}{3} \frac{b}{a^2} \frac{1}{(1/b*a)^{1/3}} \ln(x+(1/b*a)^{1/3}) * e + \frac{1}{3} \frac{b^2}{a^3} \frac{1}{(1/b*a)^{1/3}} \ln(x+(1/b*a)^{1/3}) * d - \frac{1}{3} \frac{b^3}{a^4} \frac{1}{(1/b*a)^{1/3}} \ln(x+(1/b*a)^{1/3}) * c - \frac{1}{6} \frac{a}{(1/b*a)^{1/3}} \ln(x^2-(1/b*a)^{1/3}) * x + \frac{1}{6} \frac{b}{a^2} \frac{1}{(1/b*a)^{1/3}} \ln(x^2-(1/b*a)^{1/3}) * x + \frac{1}{6} \frac{b^2}{a^3} \frac{1}{(1/b*a)^{1/3}} \ln(x^2-(1/b*a)^{1/3}) * x + \frac{1}{6} \frac{b^3}{a^4} \frac{1}{(1/b*a)^{1/3}} \ln(x^2-(1/b*a)^{1/3}) * x + \frac{1}{2} \frac{1}{(1/b*a)^{1/3}} \arctan(1/3 * 3^{1/2} * (2/(1/b*a)^{1/3}) * x - 1) * f + \frac{1}{3} \frac{b}{a^2} 3^{1/2} \frac{1}{(1/b*a)^{1/3}} \arctan(1/3 * 3^{1/2} * (2/(1/b*a)^{1/3}) * x - 1) * e - \frac{1}{3} \frac{b^2}{a^3} 3^{1/2} \frac{1}{(1/b*a)^{1/3}} \arctan(1/3 * 3^{1/2} * (2/(1/b*a)^{1/3}) * x - 1) * d + \frac{1}{3} \frac{b^3}{a^4} 3^{1/2} \frac{1}{(1/b*a)^{1/3}} \arctan(1/3 * 3^{1/2} * (2/(1/b*a)^{1/3}) * x - 1) * c - \frac{1}{10} \frac{c}{a} x^{10} - \frac{1}{7} \frac{d}{a} x^7 + \frac{1}{7} \frac{d}{a^2} x^7 * b * c - \frac{1}{4} \frac{e}{a} x^4 + \frac{1}{4} \frac{e}{a^2} x^4 * b * d - \frac{1}{4} \frac{e}{a^3} x^4 * b^2 * c - \frac{1}{a} x * f + \frac{1}{a^2} x * b * e - \frac{1}{a^3} x * b^2 * d + \frac{1}{a^4} x * b^3 * c$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 1.40998, size = 597, normalized size = 2.16

$140 \sqrt{3} (b^3 c - a b^2 d + a^2 b e - a^3 f) x^{10} \left(\frac{b}{a}\right)^{\frac{1}{3}} \arctan\left(\frac{2}{3} \sqrt{3} x \left(\frac{b}{a}\right)^{\frac{1}{3}} - \frac{1}{3} \sqrt{3}\right) + 70 (b^3 c - a b^2 d + a^2 b e - a^3 f) x^{10} \left(\frac{b}{a}\right)^{\frac{1}{3}} \log\left(bx^2 - \dots\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a),x, algorithm="fricas")`

[Out] $\frac{1}{420} * (140 * \text{sqrt}(3) * (b^3 * c - a * b^2 * d + a^2 * b * e - a^3 * f) * x^{10} * (b/a)^{1/3} * \text{arctan}(2/3 * \text{sqrt}(3) * x * (b/a)^{1/3} - 1/3 * \text{sqrt}(3)) + 70 * (b^3 * c - a * b^2 * d + a^2 * b * e - a^3 * f) * x^{10} * (b/a)^{1/3} * \log(bx^2 - \dots)$

$$e - a^3 f) x^{10} (b/a)^{1/3} \log(bx^2 - a x (b/a)^{2/3} + a (b/a)^{1/3}) - 140 (b^3 c - a b^2 d + a^2 b e - a^3 f) x^{10} (b/a)^{1/3} \log(bx + a (b/a)^{2/3}) + 420 (b^3 c - a b^2 d + a^2 b e - a^3 f) x^9 - 105 (a b^2 c - a^2 b d + a^3 e) x^6 - 42 a^3 c + 60 (a^2 b c - a^3 d) x^3 / (a^4 x^{10})$$

Sympy [A] time = 142.622, size = 473, normalized size = 1.71

$$\text{RootSum}\left(27t^3 a^{13} - a^9 b f^3 + 3a^8 b^2 e f^2 - 3a^7 b^3 d f^2 - 3a^7 b^3 e^2 f + 3a^6 b^4 c f^2 + 6a^6 b^4 d e f + a^6 b^4 e^3 - 6a^5 b^5 c e f - 3a^5 b^5 d^2 f\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**11/(b*x**3+a),x)

[Out] RootSum(27*_t**3*a**13 - a**9*b*f**3 + 3*a**8*b**2*e*f**2 - 3*a**7*b**3*d*f**2 - 3*a**7*b**3*e**2*f + 3*a**6*b**4*c*f**2 + 6*a**6*b**4*d*e*f + a**6*b**4*e**3 - 6*a**5*b**5*c*e*f - 3*a**5*b**5*d**2*f - 3*a**5*b**5*d*e**2 + 6*a**4*b**6*c*d*f + 3*a**4*b**6*c*e**2 + 3*a**4*b**6*d**2*e - 3*a**3*b**7*c**2*f - 6*a**3*b**7*c*d*e - a**3*b**7*d**3 + 3*a**2*b**8*c**2*e + 3*a**2*b**8*c*d**2 - 3*a*b**9*c**2*d + b**10*c**3, Lambda(_t, _t*log(9*_t**2*a**9/(a**6*b*f**2 - 2*a**5*b**2*e*f + 2*a**4*b**3*d*f + a**4*b**3*e**2 - 2*a**3*b**4*c*f - 2*a**3*b**4*d*e + 2*a**2*b**5*c*e + a**2*b**5*d**2 - 2*a*b**6*c*d + b**7*c**2) + x))) - (14*a**3*c + x**9*(140*a**3*f - 140*a**2*b*e + 140*a*b**2*d - 140*b**3*c) + x**6*(35*a**3*e - 35*a**2*b*d + 35*a*b**2*c) + x**3*(20*a**3*d - 20*a**2*b*c))/(140*a**4*x**10)

Giac [A] time = 1.08536, size = 508, normalized size = 1.83

$$\frac{\left(b^4 c \left(-\frac{a}{b}\right)^{\frac{1}{3}} - a b^3 d \left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^3 b f \left(-\frac{a}{b}\right)^{\frac{1}{3}} + a^2 b^2 \left(-\frac{a}{b}\right)^{\frac{1}{3}} e\right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right) + \sqrt{3} \left(\left(-ab^2\right)^{\frac{2}{3}} b^3 c - \left(-ab^2\right)^{\frac{2}{3}} ab^2 d - \dots\right)}{3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a),x, algorithm="giac")

[Out] -1/3*(b^4*c*(-a/b)^(1/3) - a*b^3*d*(-a/b)^(1/3) - a^3*b*f*(-a/b)^(1/3) + a^2*b^2*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^5 - 1/3*sqr

$$\begin{aligned}
& t(3) * ((-a*b^2)^{(2/3)} * b^3 * c - (-a*b^2)^{(2/3)} * a * b^2 * d - (-a*b^2)^{(2/3)} * a^3 * f \\
& + (-a*b^2)^{(2/3)} * a^2 * b * e) * \arctan(1/3 * \sqrt{3} * (2*x + (-a/b)^{(1/3)}) / (-a/b)^{(1/3)}) / (a^5 * b) \\
& + 1/6 * ((-a*b^2)^{(2/3)} * b^3 * c - (-a*b^2)^{(2/3)} * a * b^2 * d - (-a*b^2)^{(2/3)} * a^3 * f \\
& + (-a*b^2)^{(2/3)} * a^2 * b * e) * \log(x^2 + x * (-a/b)^{(1/3)} + (-a/b)^{(2/3)}) / (a^5 * b) \\
& + 1/140 * (140 * b^3 * c * x^9 - 140 * a * b^2 * d * x^9 - 140 * a^3 * f * x^9 + 140 * a^2 * b * x^9 * e \\
& - 35 * a * b^2 * c * x^6 + 35 * a^2 * b * d * x^6 - 35 * a^3 * x^6 * e + 20 * a^2 * b * c * x^3 \\
& - 20 * a^3 * d * x^3 - 14 * a^3 * c) / (a^4 * x^{10})
\end{aligned}$$

$$3.247 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)} dx$$

Optimal. Leaf size=280

$$\frac{a^2be + a^3(-f) - ab^2d + b^3c}{2a^4x^2} - \frac{b^{2/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^{14/3}} + \frac{b^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{bx})(a^2be + a^3(-f) - ab^2d + b^3c)}{3a^{14/3}}$$

[Out] $-c/(11*a*x^{11}) + (b*c - a*d)/(8*a^2*x^8) - (b^2*c - a*b*d + a^2*e)/(5*a^3*x^5) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(2*a^4*x^2) - (b^{(2/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*a^{(14/3)}) + (b^{(2/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{(1/3)} + b^{(1/3)}*x])/(3*a^{(14/3)}) - (b^{(2/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(6*a^{(14/3)})$

Rubi [A] time = 0.199354, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1834, 200, 31, 634, 617, 204, 628}

$$\frac{a^2be + a^3(-f) - ab^2d + b^3c}{2a^4x^2} - \frac{b^{2/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^{14/3}} + \frac{b^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{bx})(a^2be + a^3(-f) - ab^2d + b^3c)}{3a^{14/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)),x]

[Out] $-c/(11*a*x^{11}) + (b*c - a*d)/(8*a^2*x^8) - (b^2*c - a*b*d + a^2*e)/(5*a^3*x^5) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(2*a^4*x^2) - (b^{(2/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*a^{(14/3)}) + (b^{(2/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{(1/3)} + b^{(1/3)}*x])/(3*a^{(14/3)}) - (b^{(2/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(6*a^{(14/3)})$

Rule 1834

Int[((Pq_)*((c_)*(x_))^(m_))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_)^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx^3 + ex^6 + fx^9}{x^{12}(a + bx^3)} dx &= \int \left(\frac{c}{ax^{12}} + \frac{-bc + ad}{a^2x^9} + \frac{b^2c - abd + a^2e}{a^3x^6} + \frac{-b^3c + ab^2d - a^2be + a^3f}{a^4x^3} - \frac{b(-b^3c + ab^2d - a^2be + a^3f)}{a^4(a + bx^3)} \right) dx \\
&= -\frac{c}{11ax^{11}} + \frac{bc - ad}{8a^2x^8} - \frac{b^2c - abd + a^2e}{5a^3x^5} + \frac{b^3c - ab^2d + a^2be - a^3f}{2a^4x^2} + \frac{(b(b^3c - ab^2d + a^2be - a^3f))}{a^4} \\
&= -\frac{c}{11ax^{11}} + \frac{bc - ad}{8a^2x^8} - \frac{b^2c - abd + a^2e}{5a^3x^5} + \frac{b^3c - ab^2d + a^2be - a^3f}{2a^4x^2} + \frac{(b(b^3c - ab^2d + a^2be - a^3f))}{3a^{14}} \\
&= -\frac{c}{11ax^{11}} + \frac{bc - ad}{8a^2x^8} - \frac{b^2c - abd + a^2e}{5a^3x^5} + \frac{b^3c - ab^2d + a^2be - a^3f}{2a^4x^2} + \frac{b^{2/3}(b^3c - ab^2d + a^2be - a^3f)}{3a^{14}} \\
&= -\frac{c}{11ax^{11}} + \frac{bc - ad}{8a^2x^8} - \frac{b^2c - abd + a^2e}{5a^3x^5} + \frac{b^3c - ab^2d + a^2be - a^3f}{2a^4x^2} + \frac{b^{2/3}(b^3c - ab^2d + a^2be - a^3f)}{3a^{14}} \\
&= -\frac{c}{11ax^{11}} + \frac{bc - ad}{8a^2x^8} - \frac{b^2c - abd + a^2e}{5a^3x^5} + \frac{b^3c - ab^2d + a^2be - a^3f}{2a^4x^2} - \frac{b^{2/3}(b^3c - ab^2d + a^2be - a^3f)}{\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.114654, size = 266, normalized size = 0.95

$$\frac{660a^{2/3}(a^2be + a^3(-f) - ab^2d + b^3c)}{x^2} + 220b^{2/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(-a^2be + a^3f + ab^2d - b^3c) + 440b^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{bx})$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)),x]

[Out] ((-120*a^(11/3)*c)/x^11 + (165*a^(8/3)*(b*c - a*d))/x^8 - (264*a^(5/3)*(b^2*c - a*b*d + a^2*e))/x^5 + (660*a^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/x^2 - 440*sqrt(3)*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt(3)] + 440*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 220*b^(2/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(1320*a^(14/3))

Maple [B] time = 0.008, size = 493, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a),x)`

[Out]
$$-1/3/a/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*f+1/3*b/a^2/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*e-1/3*b^2/a^3/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*d+1/3*b^3/a^4/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*c+1/6/a/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*f-1/6*b/a^2/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*e+1/6*b^2/a^3/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*d-1/6*b^3/a^4/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*c-1/3/a/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*f+1/3*b/a^2/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*e-1/3*b^2/a^3/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*d+1/3*b^3/a^4/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*c-1/11*c/a/x^11-1/8/a/x^8*d+1/8/a^2/x^8*b*c-1/5/a/x^5*e+1/5/a^2/x^5*b*d-1/5/a^3/x^5*b^2*c-1/2/a/x^2*f+1/2/a^2/x^2*b*e-1/2/a^3/x^2*b^2*d+1/2/a^4/x^2*b^3*c$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 1.39613, size = 667, normalized size = 2.38

$$440\sqrt{3}(b^3c - ab^2d + a^2be - a^3f)x^{11}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}} - \sqrt{3}b}{3b}\right) - 220(b^3c - ab^2d + a^2be - a^3f)x^{11}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(\dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a),x, algorithm="fricas")`

```
[Out] -1/1320*(440*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^11*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 220*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^11*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 440*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^11*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)) - 660*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9 + 264*(a*b^2*c - a^2*b*d + a^3*e)*x^6 + 120*a^3*c - 165*(a^2*b*c - a^3*d)*x^3)/(a^4*x^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**12/(b*x**3+a), x)
```

```
[Out] Timed out
```

Giac [A] time = 1.07223, size = 456, normalized size = 1.63

$$\frac{\sqrt{3} \left((-ab^2)^{\frac{1}{3}} b^3 c - (-ab^2)^{\frac{1}{3}} ab^2 d - (-ab^2)^{\frac{1}{3}} a^3 f + (-ab^2)^{\frac{1}{3}} a^2 b e \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{3 a^5} - \frac{(b^4 c - ab^3 d - a^3 b f + a^2 b^2 e)}{3 a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a), x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d - (-a*b^2)^(1/3)*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^5 - 1/3*(b^4*c - a*b^3*d - a^3*b*f + a^2*b^2*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^5 + 1/6*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d - (-a*b^2)^(1/3)*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^5 + 1/440*(220*b^3*c*x^9 - 220*a*b^2*d*x^9 - 220*a^3*f*x^9 + 220*a^2*b*x^9*e - 88*a*b^2*c*x^6 + 88*a^2*b*d*x^6 - 88*a^3*x^6*e + 55*a^2*b*c*x^3 - 55*a^3*d*x^3 - 40*a^3*c)/(a^4*x^11)
```

$$3.248 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)} dx$$

Optimal. Leaf size=313

$$\frac{a^2be + a^3(-f) - ab^2d + b^3c}{4a^4x^4} - \frac{b^{4/3} \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^{16/3}} - \frac{b(a^2be + a^3(-f) - ab^2d + b^3c)}{a^5x}$$

[Out] $-c/(13*a*x^{13}) + (b*c - a*d)/(10*a^2*x^{10}) - (b^2*c - a*b*d + a^2*e)/(7*a^3*x^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(4*a^4*x^4) - (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(a^5*x) + (b^{(4/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*a^{(16/3)}) + (b^{(4/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{(1/3)} + b^{(1/3)}*x])/(3*a^{(16/3)}) - (b^{(4/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(6*a^{(16/3)})$

Rubi [A] time = 0.238236, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1834, 292, 31, 634, 617, 204, 628}

$$\frac{a^2be + a^3(-f) - ab^2d + b^3c}{4a^4x^4} - \frac{b^{4/3} \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^{16/3}} - \frac{b(a^2be + a^3(-f) - ab^2d + b^3c)}{a^5x}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)), x]

[Out] $-c/(13*a*x^{13}) + (b*c - a*d)/(10*a^2*x^{10}) - (b^2*c - a*b*d + a^2*e)/(7*a^3*x^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(4*a^4*x^4) - (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(a^5*x) + (b^{(4/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*a^{(16/3)}) + (b^{(4/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{(1/3)} + b^{(1/3)}*x])/(3*a^{(16/3)}) - (b^{(4/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(6*a^{(16/3)})$

Rule 1834

Int[((Pq_)*((c_)*(x_))^(m_))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &

& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^-1, x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
\int \frac{c + dx^3 + ex^6 + fx^9}{x^{14}(a + bx^3)} dx &= \int \left(\frac{c}{ax^{14}} + \frac{-bc + ad}{a^2x^{11}} + \frac{b^2c - abd + a^2e}{a^3x^8} + \frac{-b^3c + ab^2d - a^2be + a^3f}{a^4x^5} - \frac{b(-b^3c + ab^2d - a^2be + a^3f)}{a^5x^2} \right) dx \\
&= -\frac{c}{13ax^{13}} + \frac{bc - ad}{10a^2x^{10}} - \frac{b^2c - abd + a^2e}{7a^3x^7} + \frac{b^3c - ab^2d + a^2be - a^3f}{4a^4x^4} - \frac{b(b^3c - ab^2d + a^2be - a^3f)}{a^5x} \\
&= -\frac{c}{13ax^{13}} + \frac{bc - ad}{10a^2x^{10}} - \frac{b^2c - abd + a^2e}{7a^3x^7} + \frac{b^3c - ab^2d + a^2be - a^3f}{4a^4x^4} - \frac{b(b^3c - ab^2d + a^2be - a^3f)}{a^5x} \\
&= -\frac{c}{13ax^{13}} + \frac{bc - ad}{10a^2x^{10}} - \frac{b^2c - abd + a^2e}{7a^3x^7} + \frac{b^3c - ab^2d + a^2be - a^3f}{4a^4x^4} - \frac{b(b^3c - ab^2d + a^2be - a^3f)}{a^5x} \\
&= -\frac{c}{13ax^{13}} + \frac{bc - ad}{10a^2x^{10}} - \frac{b^2c - abd + a^2e}{7a^3x^7} + \frac{b^3c - ab^2d + a^2be - a^3f}{4a^4x^4} - \frac{b(b^3c - ab^2d + a^2be - a^3f)}{a^5x} \\
&= -\frac{c}{13ax^{13}} + \frac{bc - ad}{10a^2x^{10}} - \frac{b^2c - abd + a^2e}{7a^3x^7} + \frac{b^3c - ab^2d + a^2be - a^3f}{4a^4x^4} - \frac{b(b^3c - ab^2d + a^2be - a^3f)}{a^5x}
\end{aligned}$$

Mathematica [A] time = 0.0985146, size = 308, normalized size = 0.98

$$\frac{a^2be + a^3(-f) - ab^2d + b^3c}{4a^4x^4} + \frac{b^{4/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(-a^2be + a^3f + ab^2d - b^3c)}{6a^{16/3}} + \frac{b(-a^2be + a^3f + ab^2d - b^3c)}{a^5x}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)),x]

[Out] $-\frac{c}{13ax^{13}} + \frac{bc - ad}{10a^2x^{10}} - \frac{b^2c - abd + a^2e}{7a^3x^7} + \frac{b^3c - ab^2d + a^2be - a^3f}{4a^4x^4} + \frac{b(-b^3c + ab^2d - a^2be + a^3f)}{a^5x} + \frac{b^{4/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(-a^2be + a^3f + ab^2d - b^3c)}{6a^{16/3}}$

Maple [B] time = 0.009, size = 546, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((f*x^9+e*x^6+d*x^3+c)/x^{14}/(b*x^3+a), x)$

[Out]
$$\begin{aligned} & -1/10/a/x^{10}*d-1/7/a/x^7*e-1/4/a/x^4*f+1/4/a^2/x^4*b*e-1/4/a^3/x^4*b^2*d+1/ \\ & 4/a^4/x^4*b^3*c+1/a^2*b/x*f-1/a^3*b^2/x*e+1/a^4*b^3/x*d-1/a^5*b^4/x*c+1/7/a \\ & ^2/x^7*b*d-1/7/a^3/x^7*b^2*c+1/10/a^2/x^{10}*b*c-1/13*c/a/x^{13}-1/6*b^2/a^3/(1 \\ & /b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*e-1/6*b^4/a^5/(1/b*a)^{(1/ \\ & 3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*c+1/6*b^3/a^4/(1/b*a)^{(1/3)}*\ln(x^2 \\ & -(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*d-1/3*b^3/a^4/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1 \\ & /3)})*d+1/3*b^4/a^5/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})*c+1/6*b/a^2/(1/b*a)^{(1 \\ & /3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*f-1/3*b/a^2/(1/b*a)^{(1/3)}*\ln(x+(1 \\ & /b*a)^{(1/3)})*f+1/3*b^2/a^3/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})*e+1/3*b/a^2*3^ \\ & (1/2)/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*f-1/3*b^2/a^3 \\ & *3^{(1/2)/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*e+1/3*b^3/ \\ & a^4*3^{(1/2)/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*d-1/3*b \\ & ^4/a^5*3^{(1/2)/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*c \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((f*x^9+e*x^6+d*x^3+c)/x^{14}/(b*x^3+a), x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

Fricas [A] time = 1.34294, size = 711, normalized size = 2.27

$$1820 \sqrt{3}(b^4c - ab^3d + a^2b^2e - a^3bf)x^{13} \left(-\frac{b}{a}\right)^{\frac{1}{3}} \arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{3}} + \frac{1}{3}\sqrt{3}\right) - 910(b^4c - ab^3d + a^2b^2e - a^3bf)x^{13} \left(-\frac{b}{a}\right)^{\frac{1}{3}} \arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{3}} + \frac{1}{3}\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((f*x^9+e*x^6+d*x^3+c)/x^{14}/(b*x^3+a), x, \text{algorithm}="fricas")$

[Out]
$$\begin{aligned} & -1/5460*(1820*\text{sqrt}(3)*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^{13}*(-b/a)^{(\\ & 1/3)}*\arctan(2/3*\text{sqrt}(3)*x*(-b/a)^{(1/3)} + 1/3*\text{sqrt}(3)) - 910*(b^4*c - a*b^3* \end{aligned}$$

$$d + a^2 b^2 e - a^3 b f) x^{13} (-b/a)^{1/3} \log(b x^2 - a x (-b/a)^{2/3} - a (-b/a)^{1/3}) + 1820 (b^4 c - a b^3 d + a^2 b^2 e - a^3 b f) x^{13} (-b/a)^{1/3} \log(b x + a (-b/a)^{2/3}) + 5460 (b^4 c - a b^3 d + a^2 b^2 e - a^3 b f) x^{12} - 1365 (a b^3 c - a^2 b^2 d + a^3 b e - a^4 f) x^9 + 780 (a^2 b^2 c - a^3 b d + a^4 e) x^6 + 420 a^4 c - 546 (a^3 b c - a^4 d) x^3 / (a^5 x^{13})$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**14/(b*x**3+a), x)

[Out] Timed out

Giac [A] time = 1.07779, size = 566, normalized size = 1.81

$$\frac{\sqrt{3} \left((-ab^2)^{\frac{2}{3}} b^3 c - (-ab^2)^{\frac{2}{3}} ab^2 d - (-ab^2)^{\frac{2}{3}} a^3 f + (-ab^2)^{\frac{2}{3}} a^2 b e \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{3 a^6} + \frac{\left(b^5 c \left(-\frac{a}{b} \right)^{\frac{1}{3}} - ab^4 d \left(-\frac{a}{b} \right)^{\frac{1}{3}} - a^3 b^3 f \left(-\frac{a}{b} \right)^{\frac{1}{3}} + a^2 b^2 e \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a), x, algorithm="giac")

[Out] $\frac{1}{3} \sqrt{3} \left((-a b^2)^{2/3} b^3 c - (-a b^2)^{2/3} a b^2 d - (-a b^2)^{2/3} a^3 f + (-a b^2)^{2/3} a^2 b e \right) \arctan \left(\frac{\sqrt{3} \left(2x + (-a/b)^{1/3} \right)}{3 \left(-a/b \right)^{1/3}} \right) / (-a/b)^{1/3} + \frac{1}{3} (b^5 c (-a/b)^{1/3} - a b^4 d (-a/b)^{1/3} - a^3 b^3 f (-a/b)^{1/3} + a^2 b^2 e (-a/b)^{1/3}) / a^6 - \frac{1}{6} \left((-a b^2)^{2/3} b^3 c - (-a b^2)^{2/3} a b^2 d - (-a b^2)^{2/3} a^3 f + (-a b^2)^{2/3} a^2 b e \right) \log(x^2 + x (-a/b)^{1/3} + (-a/b)^{2/3}) / a^6 - \frac{1}{1820} (1820 b^4 c x^{12} - 1820 a b^3 d x^{12} - 1820 a^3 b f x^{12} + 1820 a^2 b^2 e x^{12} - 455 a b^3 c x^9 + 455 a^2 b^2 d x^9 + 455 a^4 f x^9 - 455 a^3 b e x^9 + 260 a^2 b^2 c x^6 - 260 a^3 b d x^6 + 260 a^4 e x^6 - 182 a^3 b c x^3 + 182 a^4 d x^3 + 140 a^4 c) / (a^5 x^{13})$

$$3.249 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^{15}(a+bx^3)} dx$$

Optimal. Leaf size=315

$$-\frac{b(a^2be + a^3(-f) - ab^2d + b^3c)}{2a^5x^2} + \frac{a^2be + a^3(-f) - ab^2d + b^3c}{5a^4x^5} + \frac{b^{5/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^{17/3}}$$

[Out] $-c/(14*a*x^{14}) + (b*c - a*d)/(11*a^2*x^{11}) - (b^2*c - a*b*d + a^2*e)/(8*a^3*x^8) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5) - (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(2*a^5*x^2) + (b^{(5/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})]/(Sqrt[3]*a^{(17/3)}) - (b^{(5/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{(1/3)} + b^{(1/3)}*x])/(3*a^{(17/3)}) + (b^{(5/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(6*a^{(17/3)})$

Rubi [A] time = 0.228995, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1834, 200, 31, 634, 617, 204, 628}

$$-\frac{b(a^2be + a^3(-f) - ab^2d + b^3c)}{2a^5x^2} + \frac{a^2be + a^3(-f) - ab^2d + b^3c}{5a^4x^5} + \frac{b^{5/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^{17/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^15*(a + b*x^3)),x]

[Out] $-c/(14*a*x^{14}) + (b*c - a*d)/(11*a^2*x^{11}) - (b^2*c - a*b*d + a^2*e)/(8*a^3*x^8) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5) - (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(2*a^5*x^2) + (b^{(5/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})]/(Sqrt[3]*a^{(17/3)}) - (b^{(5/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{(1/3)} + b^{(1/3)}*x])/(3*a^{(17/3)}) + (b^{(5/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(6*a^{(17/3)})$

Rule 1834

Int[((Pq_)*((c_)*(x_))^(m_))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &

& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 31

Int[((a_) + (b_.)*(x_)^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
\int \frac{c + dx^3 + ex^6 + fx^9}{x^{15}(a + bx^3)} dx &= \int \left(\frac{c}{ax^{15}} + \frac{-bc + ad}{a^2x^{12}} + \frac{b^2c - abd + a^2e}{a^3x^9} + \frac{-b^3c + ab^2d - a^2be + a^3f}{a^4x^6} - \frac{b(-b^3c + ab^2d - a^2be + a^3f)}{a^5x^3} \right) dx \\
&= -\frac{c}{14ax^{14}} + \frac{bc - ad}{11a^2x^{11}} - \frac{b^2c - abd + a^2e}{8a^3x^8} + \frac{b^3c - ab^2d + a^2be - a^3f}{5a^4x^5} - \frac{b(b^3c - ab^2d + a^2be - a^3f)}{2a^5x^2} \\
&= -\frac{c}{14ax^{14}} + \frac{bc - ad}{11a^2x^{11}} - \frac{b^2c - abd + a^2e}{8a^3x^8} + \frac{b^3c - ab^2d + a^2be - a^3f}{5a^4x^5} - \frac{b(b^3c - ab^2d + a^2be - a^3f)}{2a^5x^2} \\
&= -\frac{c}{14ax^{14}} + \frac{bc - ad}{11a^2x^{11}} - \frac{b^2c - abd + a^2e}{8a^3x^8} + \frac{b^3c - ab^2d + a^2be - a^3f}{5a^4x^5} - \frac{b(b^3c - ab^2d + a^2be - a^3f)}{2a^5x^2} \\
&= -\frac{c}{14ax^{14}} + \frac{bc - ad}{11a^2x^{11}} - \frac{b^2c - abd + a^2e}{8a^3x^8} + \frac{b^3c - ab^2d + a^2be - a^3f}{5a^4x^5} - \frac{b(b^3c - ab^2d + a^2be - a^3f)}{2a^5x^2}
\end{aligned}$$

Mathematica [A] time = 0.0974872, size = 311, normalized size = 0.99

$$\frac{b(-a^2be + a^3f + ab^2d - b^3c)}{2a^5x^2} + \frac{a^2be + a^3(-f) - ab^2d + b^3c}{5a^4x^5} + \frac{b^{5/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^{17/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^15*(a + b*x^3)),x]

[Out] $-\frac{c}{14ax^{14}} + \frac{bc - ad}{11a^2x^{11}} - \frac{b^2c - abd + a^2e}{8a^3x^8} + \frac{b^3c - ab^2d + a^2be - a^3f}{5a^4x^5} + \frac{b(-b^3c + ab^2d - a^2be + a^3f)}{2a^5x^2} + \frac{b^{5/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^{17/3}}$

Maple [B] time = 0.009, size = 548, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x^9+e*x^6+d*x^3+c)/x^15/(b*x^3+a),x)`

[Out]
$$-1/6*b/a^2/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*f+1/6*b^2/a^3/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*e-1/6*b^3/a^4/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*d+1/6*b^4/a^5/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*c+1/3*b/a^2/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*f-1/3*b^2/a^3/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*e-1/5/a/x^5*f-1/11/a/x^11*d-1/8/a/x^8*e+1/11/a^2/x^11*b*c+1/8/a^2/x^8*b*d-1/8/a^3/x^8*b^2*c+1/5/a^2/x^5*b*e-1/5/a^3/x^5*b^2*d+1/5/a^4/x^5*b^3*c+1/2/a^2*b/x^2*f-1/14*c/a/x^14+1/3*b^3/a^4/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*d-1/3*b^4/a^5/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*c-1/3*b^4/a^5/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*c-1/2/a^3*b^2/x^2*e+1/2/a^4*b^3/x^2*d-1/2/a^5*b^4/x^2*c+1/3*b/a^2/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*f-1/3*b^2/a^3/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*e+1/3*b^3/a^4/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*d$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^9+e*x^6+d*x^3+c)/x^15/(b*x^3+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 1.38373, size = 759, normalized size = 2.41

$$3080 \sqrt{3} (b^4 c - ab^3 d + a^2 b^2 e - a^3 b f) x^{14} \left(\frac{b^2}{a^2} \right)^{\frac{1}{3}} \arctan \left(\frac{2 \sqrt{3} a x \left(\frac{b^2}{a^2} \right)^{\frac{2}{3}} - \sqrt{3} b}{3 b} \right) - 1540 (b^4 c - ab^3 d + a^2 b^2 e - a^3 b f) x^{14} \left(\frac{b^2}{a^2} \right)^{\frac{1}{3}} \log$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^9+e*x^6+d*x^3+c)/x^15/(b*x^3+a),x, algorithm="fricas")`

```
[Out] -1/9240*(3080*sqrt(3)*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^14*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 1540*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^14*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) + 3080*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^14*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)) + 4620*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^12 - 1848*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^9 + 1155*(a^2*b^2*c - a^3*b*d + a^4*e)*x^6 + 660*a^4*c - 840*(a^3*b*c - a^4*d)*x^3)/(a^5*x^14)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**15/(b*x**3+a),x)
```

[Out] Timed out

Giac [A] time = 1.07733, size = 531, normalized size = 1.69

$$\frac{\sqrt{3} \left((-ab^2)^{\frac{1}{3}} b^4 c - (-ab^2)^{\frac{1}{3}} ab^3 d - (-ab^2)^{\frac{1}{3}} a^3 b f + (-ab^2)^{\frac{1}{3}} a^2 b^2 e \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{3 a^6} + \frac{(b^5 c - ab^4 d - a^3 b^2 f + a^2 b^2 e)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^15/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(1/3)*b^4*c - (-a*b^2)^(1/3)*a*b^3*d - (-a*b^2)^(1/3)*a^3*b*f + (-a*b^2)^(1/3)*a^2*b^2*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3)))/(-a/b)^(1/3))/a^6 + 1/3*(b^5*c - a*b^4*d - a^3*b^2*f + a^2*b^3*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^6 - 1/6*((-a*b^2)^(1/3)*b^4*c - (-a*b^2)^(1/3)*a*b^3*d - (-a*b^2)^(1/3)*a^3*b*f + (-a*b^2)^(1/3)*a^2*b^2*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^6 - 1/3080*(1540*b^4*c*x^12 - 1540*a*b^3*d*x^12 - 1540*a^3*b*f*x^12 + 1540*a^2*b^2*x^12*e - 616*a*b^3*c*x^9 + 616*a^2*b^2*d*x^9 + 616*a^4*f*x^9 - 616*a^3*b*x^9*e + 385*a^2*b^2*c*x^6 - 385*a^3*b*d*x^6 + 385*a^4*x^6*e - 280*a^3*b*c*x^3 + 280*a^4*d*x^3 + 220*a^4*c)/
```

$$(a^5 x^{14})$$

$$3.250 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^{17}(a+bx^3)} dx$$

Optimal. Leaf size=351

$$-\frac{b(a^2be + a^3(-f) - ab^2d + b^3c)}{4a^5x^4} + \frac{a^2be + a^3(-f) - ab^2d + b^3c}{7a^4x^7} + \frac{b^{7/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^{19/3}}$$

[Out] $-c/(16*a*x^{16}) + (b*c - a*d)/(13*a^2*x^{13}) - (b^2*c - a*b*d + a^2*e)/(10*a^3*x^{10}) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(7*a^4*x^7) - (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(4*a^5*x^4) + (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(a^6*x) - (b^{(7/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*a^{(19/3)}) - (b^{(7/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{(1/3)} + b^{(1/3)}*x])/(3*a^{(19/3)}) + (b^{(7/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(6*a^{(19/3)})$

Rubi [A] time = 0.257811, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1834, 292, 31, 634, 617, 204, 628}

$$-\frac{b(a^2be + a^3(-f) - ab^2d + b^3c)}{4a^5x^4} + \frac{a^2be + a^3(-f) - ab^2d + b^3c}{7a^4x^7} + \frac{b^{7/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^{19/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^17*(a + b*x^3)),x]

[Out] $-c/(16*a*x^{16}) + (b*c - a*d)/(13*a^2*x^{13}) - (b^2*c - a*b*d + a^2*e)/(10*a^3*x^{10}) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(7*a^4*x^7) - (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(4*a^5*x^4) + (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(a^6*x) - (b^{(7/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*a^{(19/3)}) - (b^{(7/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{(1/3)} + b^{(1/3)}*x])/(3*a^{(19/3)}) + (b^{(7/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(6*a^{(19/3)})$

Rule 1834

```
Int[((Pq_)*((c_)*(x_))^(m_))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] & & PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx^3 + ex^6 + fx^9}{x^{17}(a + bx^3)} dx &= \int \left(\frac{c}{ax^{17}} + \frac{-bc + ad}{a^2x^{14}} + \frac{b^2c - abd + a^2e}{a^3x^{11}} + \frac{-b^3c + ab^2d - a^2be + a^3f}{a^4x^8} - \frac{b(-b^3c + ab^2d - a^2be + a^3f)}{a^5x^5} \right) dx \\
&= -\frac{c}{16ax^{16}} + \frac{bc - ad}{13a^2x^{13}} - \frac{b^2c - abd + a^2e}{10a^3x^{10}} + \frac{b^3c - ab^2d + a^2be - a^3f}{7a^4x^7} - \frac{b(b^3c - ab^2d + a^2be - a^3f)}{4a^5x^4} \\
&= -\frac{c}{16ax^{16}} + \frac{bc - ad}{13a^2x^{13}} - \frac{b^2c - abd + a^2e}{10a^3x^{10}} + \frac{b^3c - ab^2d + a^2be - a^3f}{7a^4x^7} - \frac{b(b^3c - ab^2d + a^2be - a^3f)}{4a^5x^4} \\
&= -\frac{c}{16ax^{16}} + \frac{bc - ad}{13a^2x^{13}} - \frac{b^2c - abd + a^2e}{10a^3x^{10}} + \frac{b^3c - ab^2d + a^2be - a^3f}{7a^4x^7} - \frac{b(b^3c - ab^2d + a^2be - a^3f)}{4a^5x^4} \\
&= -\frac{c}{16ax^{16}} + \frac{bc - ad}{13a^2x^{13}} - \frac{b^2c - abd + a^2e}{10a^3x^{10}} + \frac{b^3c - ab^2d + a^2be - a^3f}{7a^4x^7} - \frac{b(b^3c - ab^2d + a^2be - a^3f)}{4a^5x^4}
\end{aligned}$$

Mathematica [A] time = 0.110059, size = 346, normalized size = 0.99

$$\frac{b(-a^2be + a^3f + ab^2d - b^3c)}{4a^5x^4} + \frac{a^2be + a^3(-f) - ab^2d + b^3c}{7a^4x^7} + \frac{b^{7/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^{19/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^17*(a + b*x^3)),x]

[Out] $-\frac{c}{16ax^{16}} + \frac{(bc - ad)}{13a^2x^{13}} - \frac{(b^2c - abd + a^2e)}{10a^3x^{10}} + \frac{(b^3c - ab^2d + a^2be - a^3f)}{7a^4x^7} + \frac{(b(-b^3c) + a^2b^2d - a^2b^2e + a^3f)}{4a^5x^4} + \frac{(b^2(b^3c - ab^2d + a^2be - a^3f))}{a^6x} + \frac{(b^{7/3}(-b^3c) + a^2b^2d - a^2b^2e + a^3f) \operatorname{ArcTan}\left[\frac{1 - (2b^{1/3}x)/a^{1/3}}{\sqrt{3}}\right]}{\sqrt{3}a^{19/3}} + \frac{(b^{7/3}(-b^3c) + a^2b^2d - a^2b^2e + a^3f) \operatorname{Log}[a^{1/3} + b^{1/3}x]}{3a^{19/3}} + \frac{(b^{7/3}(b^3c - ab^2d + a^2be - a^3f) \operatorname{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2])}{6a^{19/3}}$

Maple [A] time = 0.01, size = 600, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((f*x^9+e*x^6+d*x^3+c)/x^{17}/(b*x^3+a), x)$

[Out]
$$\begin{aligned} & -1/10/a/x^{10}*e-1/7/a/x^7*f-1/13/a/x^{13}*d-1/16*c/a/x^{16}+1/6*b^3/a^4/(1/b*a)^{1/3} \\ & * \ln(x^2-(1/b*a)^{1/3}*x+(1/b*a)^{2/3})*e-1/6*b^4/a^5/(1/b*a)^{1/3} * \ln(x^2-(1/b*a)^{1/3} \\ & *x+(1/b*a)^{2/3})*d+1/6*b^5/a^6/(1/b*a)^{1/3} * \ln(x^2-(1/b*a)^{1/3})*x+(1/b*a)^{2/3})*c \\ & +1/3*b^2/a^3/(1/b*a)^{1/3} * \ln(x+(1/b*a)^{1/3})*f-1/3*b^3/a^4/(1/b*a)^{1/3} * \ln(x+(1/b*a)^{1/3})*e \\ & +1/3*b^4/a^5/(1/b*a)^{1/3} * \ln(x+(1/b*a)^{1/3})*d-1/3*b^5/a^6/(1/b*a)^{1/3} * \ln(x+(1/b*a)^{1/3})*c \\ & -1/6*b^2/a^3/(1/b*a)^{1/3} * \ln(x^2-(1/b*a)^{1/3})*x+(1/b*a)^{2/3})*f-1/3*b^2/a^3*3^{1/2} \\ & / (1/b*a)^{1/3} * \arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1))*f+1/3*b^3/a^4*3^{1/2} \\ & / (1/b*a)^{1/3} * \arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1))*e-1/3*b^4/a^5*3^{1/2} \\ & / (1/b*a)^{1/3} * \arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1))*d+1/3*b^5/a^6*3^{1/2} \\ & / (1/b*a)^{1/3} * \arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1))*c+1/a^6*b^5/x*c+1/4/a^2*b/x^4*f \\ & -1/4/a^3*b^2/x^4*e+1/4/a^4*b^3/x^4*d-1/4/a^5*b^4/x^4*c+1/13/a^2/x^{13}*b*c+1/10/a^2/x^{10}*b*d \\ & -1/10/a^3/x^{10}*b^2*c+1/7/a^2/x^7*b*e-1/7/a^3/x^7*b^2*d+1/7/a^4/x^7*b^3*c-1/a^3*b^2/x*f+1/a^4*b^3/x*e \\ & -1/a^5*b^4/x*d \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((f*x^9+e*x^6+d*x^3+c)/x^{17}/(b*x^3+a), x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

Fricas [A] time = 1.2725, size = 798, normalized size = 2.27

$$7280 \sqrt{3} (b^5c - ab^4d + a^2b^3e - a^3b^2f) x^{16} \left(\frac{b}{a}\right)^{\frac{1}{3}} \arctan\left(\frac{2}{3} \sqrt{3} x \left(\frac{b}{a}\right)^{\frac{1}{3}} - \frac{1}{3} \sqrt{3}\right) + 3640 (b^5c - ab^4d + a^2b^3e - a^3b^2f) x^{16} \left(\frac{b}{a}\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^9+e*x^6+d*x^3+c)/x^17/(b*x^3+a),x, algorithm="fricas")`

[Out] $\frac{1}{21840} \cdot (7280 \sqrt{3}) \cdot (b^5 c - a b^4 d + a^2 b^3 e - a^3 b^2 f) x^{16} \left(\frac{b}{a}\right)^{\frac{1}{3}} \arctan\left(\frac{2}{3} \sqrt{3} x \left(\frac{b}{a}\right)^{\frac{1}{3}} - \frac{1}{3} \sqrt{3}\right) + 3640 (b^5 c - a b^4 d + a^2 b^3 e - a^3 b^2 f) x^{16} \left(\frac{b}{a}\right)^{\frac{1}{3}} \log(b x^2 - a x \left(\frac{b}{a}\right)^{\frac{2}{3}} + a \left(\frac{b}{a}\right)^{\frac{1}{3}}) - 7280 (b^5 c - a b^4 d + a^2 b^3 e - a^3 b^2 f) x^{16} \left(\frac{b}{a}\right)^{\frac{1}{3}} \log(b x + a \left(\frac{b}{a}\right)^{\frac{2}{3}}) + 21840 (b^5 c - a b^4 d + a^2 b^3 e - a^3 b^2 f) x^{15} - 5460 (a b^4 c - a^2 b^3 d + a^3 b^2 e - a^4 b f) x^{12} + 3120 (a^2 b^3 c - a^3 b^2 d + a^4 b e - a^5 f) x^9 - 2184 (a^3 b^2 c - a^4 b d + a^5 e) x^6 - 1365 a^5 c + 1680 (a^4 b c - a^5 d) x^3 / (a^6 x^{16})$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x**9+e*x**6+d*x**3+c)/x**17/(b*x**3+a),x)`

[Out] Timed out

Giac [A] time = 1.10368, size = 640, normalized size = 1.82

$$\frac{\sqrt{3} \left((-ab^2)^{\frac{2}{3}} b^4 c - (-ab^2)^{\frac{2}{3}} ab^3 d - (-ab^2)^{\frac{2}{3}} a^3 b f + (-ab^2)^{\frac{2}{3}} a^2 b^2 e \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}} \right)}{3 a^7} - \frac{\left(b^6 c \left(-\frac{a}{b}\right)^{\frac{1}{3}} - ab^5 d \left(-\frac{a}{b}\right)^{\frac{1}{3}} \right)}{3 a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^9+e*x^6+d*x^3+c)/x^17/(b*x^3+a),x, algorithm="giac")`

[Out] $-\frac{1}{3} \sqrt{3} \left((-a b^2)^{\frac{2}{3}} b^4 c - (-a b^2)^{\frac{2}{3}} a b^3 d - (-a b^2)^{\frac{2}{3}} a^2 b^2 e \right) \arctan\left(\frac{1}{3} \sqrt{3} (2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}})\right) / \left(-\frac{a}{b}\right)^{\frac{1}{3}} / a^7 - \frac{1}{3} (b^6 c \left(-\frac{a}{b}\right)^{\frac{1}{3}} - a b^5 d \left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^3 b^3 f \left(-\frac{a}{b}\right)^{\frac{1}{3}} + a^2 b^4 e \left(-\frac{a}{b}\right)^{\frac{1}{3}}) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left| x - \left(-\frac{a}{b}\right)^{\frac{1}{3}} \right|\right) / a^7 + \frac{1}{6} \left((-a b^2)^{\frac{2}{3}} b^4 c - (-a b^2)^{\frac{2}{3}} a b^3 d - (-a b^2)^{\frac{2}{3}} a^2 b^2 e \right) / a^7$

$$\begin{aligned}
& ^2)^{(2/3)}*a^3*b*f + (-a*b^2)^{(2/3)}*a^2*b^2*e)*\log(x^2 + x*(-a/b)^{(1/3)} + (- \\
& a/b)^{(2/3)))/a^7 + 1/7280*(7280*b^5*c*x^{15} - 7280*a*b^4*d*x^{15} - 7280*a^3*b^ \\
& 2*f*x^{15} + 7280*a^2*b^3*x^{15}*e - 1820*a*b^4*c*x^{12} + 1820*a^2*b^3*d*x^{12} + \\
& 1820*a^4*b*f*x^{12} - 1820*a^3*b^2*x^{12}*e + 1040*a^2*b^3*c*x^9 - 1040*a^3*b^2 \\
& *d*x^9 - 1040*a^5*f*x^9 + 1040*a^4*b*x^9*e - 728*a^3*b^2*c*x^6 + 728*a^4*b* \\
& d*x^6 - 728*a^5*x^6*e + 560*a^4*b*c*x^3 - 560*a^5*d*x^3 - 455*a^5*c)/(a^6*x \\
& ^{16})
\end{aligned}$$

$$3.251 \quad \int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=220

$$\frac{x^6(3a^2be - 4a^3f - 2ab^2d + b^3c)}{6b^5} - \frac{ax^3(4a^2be - 5a^3f - 3ab^2d + 2b^3c)}{3b^6} + \frac{a^3(a^2be + a^3(-f) - ab^2d + b^3c)}{3b^7(a + bx^3)} + \frac{a^2 \log(a + bx^3)}{3b^7}$$

[Out] $-(a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*x^3)/(3*b^6) + ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^6)/(6*b^5) + ((b^2*d - 2*a*b*e + 3*a^2*f)*x^9)/(9*b^4) + ((b*e - 2*a*f)*x^{12})/(12*b^3) + (f*x^{15})/(15*b^2) + (a^3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*b^7*(a + b*x^3)) + (a^2*(3*b^3*c - 4*a*b^2*d + 5*a^2*b*e - 6*a^3*f)*\text{Log}[a + b*x^3])/(3*b^7)$

Rubi [A] time = 0.341178, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1821, 1620}

$$\frac{x^6(3a^2be - 4a^3f - 2ab^2d + b^3c)}{6b^5} - \frac{ax^3(4a^2be - 5a^3f - 3ab^2d + 2b^3c)}{3b^6} + \frac{a^3(a^2be + a^3(-f) - ab^2d + b^3c)}{3b^7(a + bx^3)} + \frac{a^2 \log(a + bx^3)}{3b^7}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^{11}(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2, x]$

[Out] $-(a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*x^3)/(3*b^6) + ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^6)/(6*b^5) + ((b^2*d - 2*a*b*e + 3*a^2*f)*x^9)/(9*b^4) + ((b*e - 2*a*f)*x^{12})/(12*b^3) + (f*x^{15})/(15*b^2) + (a^3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*b^7*(a + b*x^3)) + (a^2*(3*b^3*c - 4*a*b^2*d + 5*a^2*b*e - 6*a^3*f)*\text{Log}[a + b*x^3])/(3*b^7)$

Rule 1821

$\text{Int}[(\text{Pq}_x)(x)^{m_x}((a_x) + (b_x)(x)^{n_x})^{p_x}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*\text{SubstFor}[x^n, \text{Pq}, x](a + b*x)^p, x], x, x^n], x] /; \text{FreeQ}\{a, b, m, n, p, x\} \ \&\& \ \text{PolyQ}[\text{Pq}, x^n] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol]
:= Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned} \int \frac{x^{11}(c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^3(c + dx + ex^2 + fx^3)}{(a + bx)^2} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{a(-2b^3c + 3ab^2d - 4a^2be + 5a^3f)}{b^6} + \frac{(b^3c - 2ab^2d + 3a^2be - 4a^3f)x}{b^5} + \right. \right. \\ &= -\frac{a(2b^3c - 3ab^2d + 4a^2be - 5a^3f)x^3}{3b^6} + \frac{(b^3c - 2ab^2d + 3a^2be - 4a^3f)x^6}{6b^5} + \frac{(b^2d - 2ab^2e + a^2c)x^9}{9b^4} \end{aligned}$$

Mathematica [A] time = 0.138523, size = 205, normalized size = 0.93

$$\frac{30b^2x^6(3a^2be - 4a^3f - 2ab^2d + b^3c) + 60abx^3(-4a^2be + 5a^3f + 3ab^2d - 2b^3c) - \frac{60a^3(-a^2be + a^3f + ab^2d - b^3c)}{a + bx^3} + 60a^2 \log(a + bx^3)}{180b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (60*a*b*(-2*b^3*c + 3*a*b^2*d - 4*a^2*b*e + 5*a^3*f)*x^3 + 30*b^2*(b^3*c -
2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^6 + 20*b^3*(b^2*d - 2*a*b*e + 3*a^2*f)*x
^9 + 15*b^4*(b*e - 2*a*f)*x^12 + 12*b^5*f*x^15 - (60*a^3*(-(b^3*c) + a*b^2*
d - a^2*b*e + a^3*f))/(a + b*x^3) + 60*a^2*(3*b^3*c - 4*a*b^2*d + 5*a^2*b*e
- 6*a^3*f)*Log[a + b*x^3])/(180*b^7)
```

Maple [A] time = 0.012, size = 288, normalized size = 1.3

$$\frac{fx^{15}}{15b^2} - \frac{x^{12}af}{6b^3} + \frac{x^{12}e}{12b^2} + \frac{x^9a^2f}{3b^4} - \frac{2x^9ae}{9b^3} + \frac{x^9d}{9b^2} - \frac{2a^3fx^6}{3b^5} + \frac{a^2ex^6}{2b^4} - \frac{adx^6}{3b^3} + \frac{x^6c}{6b^2} + \frac{5a^4fx^3}{3b^6} - \frac{4a^3ex^3}{3b^5} + \frac{a^2dx^3}{b^4} - \frac{2acx^3}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)`

[Out] $\frac{1}{15}f*x^{15}/b^2 - \frac{1}{6}/b^3*x^{12}*a*f + \frac{1}{12}/b^2*x^{12}*e + \frac{1}{3}/b^4*x^9*a^2*f - \frac{2}{9}/b^3*x^9*a*e + \frac{1}{9}/b^2*x^9*d - \frac{2}{3}/b^5*x^6*a^3*f + \frac{1}{2}/b^4*x^6*a^2*e - \frac{1}{3}/b^3*x^6*a*d + \frac{1}{6}/b^2*x^6*c + \frac{5}{3}/b^6*a^4*f*x^3 - \frac{4}{3}/b^5*a^3*e*x^3 + \frac{1}{b^4*a^2*d*x^3} - \frac{2}{3}/b^3*a*c*x^3 - \frac{2*a^5}{b^7}*\ln(b*x^3+a)*f + \frac{5}{3*a^4}/b^6*\ln(b*x^3+a)*e - \frac{4}{3*a^3}/b^5*\ln(b*x^3+a)*d + \frac{a^2}{b^4}*\ln(b*x^3+a)*c - \frac{1}{3*a^6}/b^7/(b*x^3+a)*f + \frac{1}{3*a^5}/b^6/(b*x^3+a)*e - \frac{1}{3*a^4}/b^5/(b*x^3+a)*d + \frac{1}{3*a^3}/b^4/(b*x^3+a)*c$

Maxima [A] time = 0.958348, size = 300, normalized size = 1.36

$$\frac{a^3b^3c - a^4b^2d + a^5be - a^6f}{3(b^8x^3 + ab^7)} + \frac{12b^4fx^{15} + 15(b^4e - 2ab^3f)x^{12} + 20(b^4d - 2ab^3e + 3a^2b^2f)x^9 + 30(b^4c - 2ab^3d + 3a^2b^2e - 4a^3b^2f)x^6 - 60(2a^2b^3c - 3a^2b^2d + 4a^3b^2e - 5a^4bf)x^3}{180b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")`

[Out] $\frac{1}{3}(a^3b^3c - a^4b^2d + a^5b^2e - a^6f)/(b^8x^3 + ab^7) + \frac{1}{180}(12b^4fx^{15} + 15(b^4e - 2a^2b^3f)x^{12} + 20(b^4d - 2a^2b^3e + 3a^2b^2f)x^9 + 30(b^4c - 2a^2b^3d + 3a^2b^2e - 4a^3b^2f)x^6 - 60(2a^2b^3c - 3a^2b^2d + 4a^3b^2e - 5a^4bf)x^3)/b^6 + \frac{1}{3}(3a^2b^3c - 4a^3b^2d + 5a^4b^2e - 6a^5bf)*\log(b*x^3 + a)/b^7$

Fricas [A] time = 1.244, size = 656, normalized size = 2.98

$$\frac{12b^6fx^{18} + 3(5b^6e - 6ab^5f)x^{15} + 5(4b^6d - 5ab^5e + 6a^2b^4f)x^{12} + 10(3b^6c - 4ab^5d + 5a^2b^4e - 6a^3b^3f)x^9 + 60a^3b^3c}{180b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")`

[Out] $\frac{1}{180}(12b^6fx^{18} + 3(5b^6e - 6a^2b^5f)x^{15} + 5(4b^6d - 5a^2b^5e + 6a^2b^4f)x^{12} + 10(3b^6c - 4a^2b^5d + 5a^2b^4e - 6a^3b^3f)x^9 + 60a^3b^3c - 60a^4b^2d + 60a^5b^2e - 60a^6f - 30(3a^2b^5c - 4a^2b^4d + 5a^3b^3e - 6a^4b^2f)x^6 - 60(2a^2b^4c - 3a^3b^3d + 4a^4b^2e - 5a^5b^2f)x^3 + 60(3a^3b^3c - 4a^4b^2d + 5a^5b^2e - 6a^6f)*\log(b*x^3 + a)/b^7$

$$\frac{b^2 e - 6a^6 f + (3a^2 b^4 c - 4a^3 b^3 d + 5a^4 b^2 e - 6a^5 b f) x^3}{b^7} \log(bx^3 + a) / (b^8 x^3 + a b^7)$$

Sympy [A] time = 11.5094, size = 224, normalized size = 1.02

$$\frac{a^2 (6a^3 f - 5a^2 b e + 4ab^2 d - 3b^3 c) \log(a + bx^3)}{3b^7} - \frac{a^6 f - a^5 b e + a^4 b^2 d - a^3 b^3 c}{3ab^7 + 3b^8 x^3} + \frac{f x^{15}}{15b^2} - \frac{x^{12} (2af - be)}{12b^3} + \frac{x^9 (3a^2 f - 2a^3 b e)}{9b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**11*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)

[Out]
$$-a^{**2}*(6*a^{**3}*f - 5*a^{**2}*b*e + 4*a*b^{**2}*d - 3*b^{**3}*c)*\log(a + b*x^{**3})/(3*b^{**7}) - (a^{**6}*f - a^{**5}*b*e + a^{**4}*b^{**2}*d - a^{**3}*b^{**3}*c)/(3*a*b^{**7} + 3*b^{**8}*x^{**3}) + f*x^{**15}/(15*b^{**2}) - x^{**12}*(2*a*f - b*e)/(12*b^{**3}) + x^{**9}*(3*a^{**2}*f - 2*a*b*e + b^{**2}*d)/(9*b^{**4}) - x^{**6}*(4*a^{**3}*f - 3*a^{**2}*b*e + 2*a*b^{**2}*d - b^{**3}*c)/(6*b^{**5}) + x^{**3}*(5*a^{**4}*f - 4*a^{**3}*b*e + 3*a^{**2}*b^{**2}*d - 2*a*b^{**3}*c)/(3*b^{**6})$$

Giac [A] time = 1.06618, size = 405, normalized size = 1.84

$$\frac{(3a^2 b^3 c - 4a^3 b^2 d - 6a^5 f + 5a^4 b e) \log(|bx^3 + a|)}{3b^7} - \frac{3a^2 b^4 c x^3 - 4a^3 b^3 d x^3 - 6a^5 b f x^3 + 5a^4 b^2 x^3 e + 2a^3 b^3 c - 3a^4 b^2 d - 3a^5 b f x^3 + 5a^4 b^2 x^3 e + 2a^3 b^3 c - 3a^4 b^2 d - 3a^5 b f x^3 + 5a^4 b^2 x^3 e}{3(bx^3 + a)b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")

[Out]
$$\frac{1}{3}*(3*a^2*b^3*c - 4*a^3*b^2*d - 6*a^5*f + 5*a^4*b*e)*\log(\text{abs}(b*x^3 + a))/b^7 - \frac{1}{3}*(3*a^2*b^4*c*x^3 - 4*a^3*b^3*d*x^3 - 6*a^5*b*f*x^3 + 5*a^4*b^2*x^3*e + 2*a^3*b^3*c - 3*a^4*b^2*d - 5*a^6*f + 4*a^5*b*e)/((b*x^3 + a)*b^7) + \frac{1}{180}*(12*b^8*f*x^{15} - 30*a*b^7*f*x^{12} + 15*b^8*x^{12}*e + 20*b^8*d*x^9 + 60*a^2*b^6*f*x^9 - 40*a*b^7*x^9*e + 30*b^8*c*x^6 - 60*a*b^7*d*x^6 - 120*a^3*b^5*f*x^6 + 90*a^2*b^6*x^6*e - 120*a*b^7*c*x^3 + 180*a^2*b^6*d*x^3 + 300*a^4*b^4*f*x^3 - 240*a^3*b^5*x^3*e)/b^{10}$$

$$3.252 \quad \int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=180

$$\frac{x^3(3a^2be - 4a^3f - 2ab^2d + b^3c)}{3b^5} - \frac{a^2(a^2be + a^3(-f) - ab^2d + b^3c)}{3b^6(a+bx^3)} - \frac{a \log(a+bx^3)(4a^2be - 5a^3f - 3ab^2d + 2b^3c)}{3b^6} +$$

[Out] $((b^3c - 2ab^2d + 3a^2be - 4a^3f)x^3)/(3b^5) + ((b^2d - 2abe + 3a^2f)x^6)/(6b^4) + ((be - 2af)x^9)/(9b^3) + (fx^{12})/(12b^2) - (a^2(b^3c - ab^2d + a^2be - a^3f))/(3b^6(a+bx^3)) - (a(2b^3c - 3ab^2d + 4a^2be - 5a^3f) \text{Log}[a+bx^3])/(3b^6)$

Rubi [A] time = 0.26477, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1821, 1620}

$$\frac{x^3(3a^2be - 4a^3f - 2ab^2d + b^3c)}{3b^5} - \frac{a^2(a^2be + a^3(-f) - ab^2d + b^3c)}{3b^6(a+bx^3)} - \frac{a \log(a+bx^3)(4a^2be - 5a^3f - 3ab^2d + 2b^3c)}{3b^6} +$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^8(c + dx^3 + ex^6 + fx^9))/(a + bx^3)^2, x]$

[Out] $((b^3c - 2ab^2d + 3a^2be - 4a^3f)x^3)/(3b^5) + ((b^2d - 2abe + 3a^2f)x^6)/(6b^4) + ((be - 2af)x^9)/(9b^3) + (fx^{12})/(12b^2) - (a^2(b^3c - ab^2d + a^2be - a^3f))/(3b^6(a+bx^3)) - (a(2b^3c - 3ab^2d + 4a^2be - 5a^3f) \text{Log}[a+bx^3])/(3b^6)$

Rule 1821

$\text{Int}[(Pq_)*(x_)^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*\text{SubstFor}[x^n, Pq, x]*(a+bx)^p, x], x, x^n], x] /;$ FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Simplify[(m+1)/n]]

Rule 1620

$\text{Int}[(Px_)*((a_) + (b_)*(x_))^{(m_)}*((c_) + (d_)*(x_))^{(n_)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[Px*(a+bx)^m*(c+dx)^n, x], x] /;$ FreeQ[{a, b, c

, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]

Rubi steps

$$\begin{aligned} \int \frac{x^8 (c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2 (c + dx + ex^2 + fx^3)}{(a + bx)^2} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{b^3c - 2ab^2d + 3a^2be - 4a^3f}{b^5} + \frac{(b^2d - 2abe + 3a^2f)x}{b^4} + \frac{(be - 2af)x^2}{b^3} + \frac{fx^3}{b^2} \right) dx, x, x^3 \right) \\ &= \frac{(b^3c - 2ab^2d + 3a^2be - 4a^3f)x^3}{3b^5} + \frac{(b^2d - 2abe + 3a^2f)x^6}{6b^4} + \frac{(be - 2af)x^9}{9b^3} + \frac{fx^{12}}{12b^2} - \frac{a^2}{b^2} \end{aligned}$$

Mathematica [A] time = 0.110585, size = 167, normalized size = 0.93

$$\frac{12bx^3(3a^2be - 4a^3f - 2ab^2d + b^3c) + \frac{12a^2(-a^2be + a^3f + ab^2d - b^3c)}{a+bx^3} + 12a \log(a + bx^3)(-4a^2be + 5a^3f + 3ab^2d - 2b^3c) + 6b^2x^6}{36b^6}$$

Antiderivative was successfully verified.

[In] Integrate[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]

[Out] (12*b*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^3 + 6*b^2*(b^2*d - 2*a*b*e + 3*a^2*f)*x^6 + 4*b^3*(b*e - 2*a*f)*x^9 + 3*b^4*f*x^12 + (12*a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3) + 12*a*(-2*b^3*c + 3*a*b^2*d - 4*a^2*b*e + 5*a^3*f)*Log[a + b*x^3])/(36*b^6)

Maple [A] time = 0.013, size = 240, normalized size = 1.3

$$\frac{fx^{12}}{12b^2} - \frac{2x^9af}{9b^3} + \frac{x^9e}{9b^2} + \frac{a^2fx^6}{2b^4} - \frac{aex^6}{3b^3} + \frac{dx^6}{6b^2} - \frac{4a^3fx^3}{3b^5} + \frac{a^2ex^3}{b^4} - \frac{2adx^3}{3b^3} + \frac{cx^3}{3b^2} + \frac{5a^4 \ln(bx^3 + a)f}{3b^6} - \frac{4a^3 \ln(bx^3 + a)}{3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)

[Out] $1/12*f*x^{12}/b^2-2/9/b^3*x^9*a*f+1/9/b^2*x^9*e+1/2/b^4*x^6*a^2*f-1/3/b^3*x^6*a*e+1/6/b^2*x^6*d-4/3/b^5*a^3*f*x^3+1/b^4*a^2*e*x^3-2/3/b^3*a*d*x^3+1/3/b^2*c*x^3+5/3*a^4/b^6*\ln(b*x^3+a)*f-4/3*a^3/b^5*\ln(b*x^3+a)*e+a^2/b^4*\ln(b*x^3+a)*d-2/3*a/b^3*\ln(b*x^3+a)*c+1/3*a^5/b^6/(b*x^3+a)*f-1/3*a^4/b^5/(b*x^3+a)*e+1/3*a^3/b^4/(b*x^3+a)*d-1/3*a^2/b^3/(b*x^3+a)*c$

Maxima [A] time = 0.964983, size = 243, normalized size = 1.35

$$\frac{a^2b^3c - a^3b^2d + a^4be - a^5f}{3(b^7x^3 + ab^6)} + \frac{3b^3fx^{12} + 4(b^3e - 2ab^2f)x^9 + 6(b^3d - 2ab^2e + 3a^2bf)x^6 + 12(b^3c - 2ab^2d + 3a^2be)}{36b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")`

[Out] $-1/3*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)/(b^7*x^3 + a*b^6) + 1/36*(3*b^3*f*x^{12} + 4*(b^3*e - 2*a*b^2*f)*x^9 + 6*(b^3*d - 2*a*b^2*e + 3*a^2*b*f)*x^6 + 12*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^3)/b^5 - 1/3*(2*a*b^3*c - 3*a^2*b^2*d + 4*a^3*b*e - 5*a^4*f)*\log(b*x^3 + a)/b^6$

Fricas [A] time = 1.2588, size = 555, normalized size = 3.08

$$\frac{3b^5fx^{15} + (4b^5e - 5ab^4f)x^{12} + 2(3b^5d - 4ab^4e + 5a^2b^3f)x^9 + 6(2b^5c - 3ab^4d + 4a^2b^3e - 5a^3b^2f)x^6 - 12a^2b^3c + 12a^3b^2d + 4a^4b^3e - 5a^5f}{36b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")`

[Out] $1/36*(3*b^5*f*x^{15} + (4*b^5*e - 5*a*b^4*f)*x^{12} + 2*(3*b^5*d - 4*a*b^4*e + 5*a^2*b^3*f)*x^9 + 6*(2*b^5*c - 3*a*b^4*d + 4*a^2*b^3*e - 5*a^3*b^2*f)*x^6 - 12*a^2*b^3*c + 12*a^3*b^2*d - 12*a^4*b^3*e + 12*a^5*f + 12*(a*b^4*c - 2*a^2*b^3*d + 3*a^3*b^2*e - 4*a^4*b*f)*x^3 - 12*(2*a^2*b^3*c - 3*a^3*b^2*d + 4*a^4*b*e - 5*a^5*f + (2*a*b^4*c - 3*a^2*b^3*d + 4*a^3*b^2*e - 5*a^4*b*f)*x^3)*\log(b*x^3 + a))/(b^7*x^3 + a*b^6)$

Sympy [A] time = 11.0927, size = 180, normalized size = 1.

$$\frac{a(5a^3f - 4a^2be + 3ab^2d - 2b^3c) \log(a + bx^3)}{3b^6} + \frac{a^5f - a^4be + a^3b^2d - a^2b^3c}{3ab^6 + 3b^7x^3} + \frac{fx^{12}}{12b^2} - \frac{x^9(2af - be)}{9b^3} + \frac{x^6(3a^2f - 2abe - b^2d)}{6b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)

[Out] a*(5*a**3*f - 4*a**2*b*e + 3*a*b**2*d - 2*b**3*c)*log(a + b*x**3)/(3*b**6) + (a**5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c)/(3*a*b**6 + 3*b**7*x**3) + f*x**12/(12*b**2) - x**9*(2*a*f - b*e)/(9*b**3) + x**6*(3*a**2*f - 2*a*b*e + b**2*d)/(6*b**4) - x**3*(4*a**3*f - 3*a**2*b*e + 2*a*b**2*d - b**3*c)/(3*b**5)

Giac [A] time = 1.05787, size = 335, normalized size = 1.86

$$\frac{(2ab^3c - 3a^2b^2d - 5a^4f + 4a^3be) \log(|bx^3 + a|)}{3b^6} + \frac{2ab^4cx^3 - 3a^2b^3dx^3 - 5a^4bfx^3 + 4a^3b^2x^3e + a^2b^3c - 2a^3b^2d - 4a^4f + 3a^3be}{3(bx^3 + a)b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")

[Out] -1/3*(2*a*b^3*c - 3*a^2*b^2*d - 5*a^4*f + 4*a^3*b*e)*log(abs(b*x^3 + a))/b^6 + 1/3*(2*a*b^4*c*x^3 - 3*a^2*b^3*d*x^3 - 5*a^4*b*f*x^3 + 4*a^3*b^2*x^3*e + a^2*b^3*c - 2*a^3*b^2*d - 4*a^5*f + 3*a^4*b*e)/((b*x^3 + a)*b^6) + 1/36*(3*b^6*f*x^12 - 8*a*b^5*f*x^9 + 4*b^6*x^9*e + 6*b^6*d*x^6 + 18*a^2*b^4*f*x^6 - 12*a*b^5*x^6*e + 12*b^6*c*x^3 - 24*a*b^5*d*x^3 - 48*a^3*b^3*f*x^3 + 36*a^2*b^4*x^3*e)/b^8

$$3.253 \quad \int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=140

$$\frac{a(a^2be + a^3(-f) - ab^2d + b^3c)}{3b^5(a + bx^3)} + \frac{\log(a + bx^3)(3a^2be - 4a^3f - 2ab^2d + b^3c)}{3b^5} + \frac{x^3(3a^2f - 2abe + b^2d)}{3b^4} + \frac{x^6(be - 2af)}{6b^3}$$

[Out] $((b^2*d - 2*a*b*e + 3*a^2*f)*x^3)/(3*b^4) + ((b*e - 2*a*f)*x^6)/(6*b^3) + (f*x^9)/(9*b^2) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*b^5*(a + b*x^3)) + ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*\text{Log}[a + b*x^3])/(3*b^5)$

Rubi [A] time = 0.199417, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1821, 1620}

$$\frac{a(a^2be + a^3(-f) - ab^2d + b^3c)}{3b^5(a + bx^3)} + \frac{\log(a + bx^3)(3a^2be - 4a^3f - 2ab^2d + b^3c)}{3b^5} + \frac{x^3(3a^2f - 2abe + b^2d)}{3b^4} + \frac{x^6(be - 2af)}{6b^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2, x]$

[Out] $((b^2*d - 2*a*b*e + 3*a^2*f)*x^3)/(3*b^4) + ((b*e - 2*a*f)*x^6)/(6*b^3) + (f*x^9)/(9*b^2) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*b^5*(a + b*x^3)) + ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*\text{Log}[a + b*x^3])/(3*b^5)$

Rule 1821

$\text{Int}[(Pq_)*(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*\text{SubstFor}[x^n, Pq, x]*(a + b*x)^p, x], x, x^n], x] /; \text{FreeQ}\{a, b, m, n, p\}, x] \&\& \text{PolyQ}[Pq, x^n] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 1620

$\text{Int}[(Px_)*((a_.) + (b_.)*(x_))^{(m_.)}*((c_.) + (d_.)*(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \text{PolyQ}[Px, x] \&\& (\text{IntegersQ}[m, n] \|\| \text{IGtQ}[m, -2]) \&\& \text{GtQ}[\text{Expon}[Px, x], 2]$

Rubi steps

$$\int \frac{x^5 (c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^2} dx = \frac{1}{3} \text{Subst} \left(\int \frac{x (c + dx + ex^2 + fx^3)}{(a + bx)^2} dx, x, x^3 \right)$$

$$= \frac{1}{3} \text{Subst} \left(\int \left(\frac{b^2 d - 2abe + 3a^2 f}{b^4} + \frac{(be - 2af)x}{b^3} + \frac{fx^2}{b^2} + \frac{a(-b^3 c + ab^2 d - a^2 be + a^3 f)}{b^4 (a + bx)^2} \right) dx, x, x^3 \right)$$

$$= \frac{(b^2 d - 2abe + 3a^2 f)x^3}{3b^4} + \frac{(be - 2af)x^6}{6b^3} + \frac{fx^9}{9b^2} + \frac{a(b^3 c - ab^2 d + a^2 be - a^3 f)}{3b^5 (a + bx^3)} + \frac{(b^3 c - 2ab^2 d + a^2 be - a^3 f)}{3b^5}$$

Mathematica [A] time = 0.0984255, size = 129, normalized size = 0.92

$$\frac{6a(a^2 be + a^3(-f) - ab^2 d + b^3 c)}{a + bx^3} + 6 \log(a + bx^3) (3a^2 be - 4a^3 f - 2ab^2 d + b^3 c) + 6bx^3 (3a^2 f - 2abe + b^2 d) + 3b^2 x^6 (be - 2af) + 2b^3 c - 2ab^2 d + a^2 be - a^3 f}{18b^5}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]

[Out] (6*b*(b^2*d - 2*a*b*e + 3*a^2*f)*x^3 + 3*b^2*(b*e - 2*a*f)*x^6 + 2*b^3*f*x^9 + (6*a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(a + b*x^3) + 6*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*Log[a + b*x^3])/(18*b^5)

Maple [A] time = 0.01, size = 192, normalized size = 1.4

$$\frac{fx^9}{9b^2} - \frac{x^6 af}{3b^3} + \frac{ex^6}{6b^2} + \frac{a^2 fx^3}{b^4} - \frac{2aex^3}{3b^3} + \frac{dx^3}{3b^2} - \frac{4 \ln(bx^3 + a) a^3 f}{3b^5} + \frac{\ln(bx^3 + a) a^2 e}{b^4} - \frac{2 \ln(bx^3 + a) ad}{3b^3} + \frac{\ln(bx^3 + a)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)

[Out] 1/9*f*x^9/b^2-1/3/b^3*x^6*a*f+1/6/b^2*x^6*e+1/b^4*a^2*f*x^3-2/3/b^3*a*e*x^3+1/3/b^2*d*x^3-4/3/b^5*ln(b*x^3+a)*a^3*f+1/b^4*ln(b*x^3+a)*a^2*e-2/3/b^3*ln(b*x^3+a)*a*d+1/3/b^2*ln(b*x^3+a)*c-1/3/b^5*a^4/(b*x^3+a)*f+1/3/b^4*a^3/(b*x^3+a)*e-1/3/b^3*a^2/(b*x^3+a)*d+1/3/b^2*a/(b*x^3+a)*c

Maxima [A] time = 0.958848, size = 186, normalized size = 1.33

$$\frac{ab^3c - a^2b^2d + a^3be - a^4f}{3(b^6x^3 + ab^5)} + \frac{2b^2fx^9 + 3(b^2e - 2abf)x^6 + 6(b^2d - 2abe + 3a^2f)x^3}{18b^4} + \frac{(b^3c - 2ab^2d + 3a^2be - 4a^3f)}{3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")

[Out] 1/3*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)/(b^6*x^3 + a*b^5) + 1/18*(2*b^2*f*x^9 + 3*(b^2*e - 2*a*b*f)*x^6 + 6*(b^2*d - 2*a*b*e + 3*a^2*f)*x^3)/b^4 + 1/3*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*log(b*x^3 + a)/b^5

Fricas [A] time = 1.30494, size = 432, normalized size = 3.09

$$\frac{2b^4fx^{12} + (3b^4e - 4ab^3f)x^9 + 3(2b^4d - 3ab^3e + 4a^2b^2f)x^6 + 6ab^3c - 6a^2b^2d + 6a^3be - 6a^4f + 6(ab^3d - 2a^2b^2e + ab^3c - 2ab^2d + 3a^2be - 4a^3f)}{18(b^6x^3 + ab^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] 1/18*(2*b^4*f*x^12 + (3*b^4*e - 4*a*b^3*f)*x^9 + 3*(2*b^4*d - 3*a*b^3*e + 4*a^2*b^2*f)*x^6 + 6*a*b^3*c - 6*a^2*b^2*d + 6*a^3*b*e - 6*a^4*f + 6*(a*b^3*d - 2*a^2*b^2*e + 3*a^3*b*f)*x^3 + 6*(a*b^3*c - 2*a^2*b^2*d + 3*a^3*b*e - 4*a^4*f + (b^4*c - 2*a*b^3*d + 3*a^2*b^2*e - 4*a^3*b*f)*x^3)*log(b*x^3 + a))/(b^6*x^3 + a*b^5)

Sympy [A] time = 10.6441, size = 138, normalized size = 0.99

$$-\frac{a^4f - a^3be + a^2b^2d - ab^3c}{3ab^5 + 3b^6x^3} + \frac{fx^9}{9b^2} - \frac{x^6(2af - be)}{6b^3} + \frac{x^3(3a^2f - 2abe + b^2d)}{3b^4} - \frac{(4a^3f - 3a^2be + 2ab^2d - b^3c) \log(a + b^3x^3)}{3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)

[Out] $-(a^{**4}f - a^{**3}b*e + a^{**2}b^{**2}d - a*b^{**3}c)/(3*a*b^{**5} + 3*b^{**6}*x^{**3}) + f*x^{**9}/(9*b^{**2}) - x^{**6}*(2*a*f - b*e)/(6*b^{**3}) + x^{**3}*(3*a^{**2}f - 2*a*b*e + b^{**2}d)/(3*b^{**4}) - (4*a^{**3}f - 3*a^{**2}b*e + 2*a*b^{**2}d - b^{**3}c)*\log(a + b*x^{**3})/(3*b^{**5})$

Giac [A] time = 1.06383, size = 293, normalized size = 2.09

$$\frac{(bx^3+a)^3 \left(2f - \frac{3(4abf-b^2e)}{(bx^3+a)b} + \frac{6(b^4d+6a^2b^2f-3ab^3e)}{(bx^3+a)^2b^2} \right) - \frac{6(b^3c-2ab^2d-4a^3f+3a^2be) \log\left(\frac{|bx^3+a|}{(bx^3+a)^2|b|}\right)}{b^4} + \frac{6\left(\frac{ab^6c}{bx^3+a} - \frac{a^2b^5d}{bx^3+a} - \frac{a^4b^3f}{bx^3+a} + \frac{a^3b^4e}{bx^3+a}\right)}{b^7}}{18b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")`

[Out] $\frac{1}{18} * ((b*x^3 + a)^3 * (2*f - 3*(4*a*b*f - b^2*e) / ((b*x^3 + a)*b) + 6*(b^4*d + 6*a^2*b^2*f - 3*a*b^3*e) / ((b*x^3 + a)^2*b^2)) / b^4 - 6*(b^3*c - 2*a*b^2*d - 4*a^3*f + 3*a^2*b*e) * \log(\text{abs}(b*x^3 + a) / ((b*x^3 + a)^2*\text{abs}(b))) / b^4 + 6*(a*b^6*c / (b*x^3 + a) - a^2*b^5*d / (b*x^3 + a) - a^4*b^3*f / (b*x^3 + a) + a^3*b^4*e / (b*x^3 + a)) / b^7 / b$

$$3.254 \quad \int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=103

$$-\frac{a^2be + a^3(-f) - ab^2d + b^3c}{3b^4(a+bx^3)} + \frac{\log(a+bx^3)(3a^2f - 2abe + b^2d)}{3b^4} + \frac{x^3(be - 2af)}{3b^3} + \frac{fx^6}{6b^2}$$

[Out] $((b*e - 2*a*f)*x^3)/(3*b^3) + (f*x^6)/(6*b^2) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*b^4*(a + b*x^3)) + ((b^2*d - 2*a*b*e + 3*a^2*f)*\text{Log}[a + b*x^3])/(3*b^4)$

Rubi [A] time = 0.14512, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1819, 1850}

$$-\frac{a^2be + a^3(-f) - ab^2d + b^3c}{3b^4(a+bx^3)} + \frac{\log(a+bx^3)(3a^2f - 2abe + b^2d)}{3b^4} + \frac{x^3(be - 2af)}{3b^3} + \frac{fx^6}{6b^2}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]

[Out] $((b*e - 2*a*f)*x^3)/(3*b^3) + (f*x^6)/(6*b^2) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*b^4*(a + b*x^3)) + ((b^2*d - 2*a*b*e + 3*a^2*f)*\text{Log}[a + b*x^3])/(3*b^4)$

Rule 1819

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]

Rule 1850

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int \frac{x^2 (c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{c + dx + ex^2 + fx^3}{(a + bx)^2} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{be - 2af}{b^3} + \frac{fx}{b^2} + \frac{b^3c - ab^2d + a^2be - a^3f}{b^3(a + bx)^2} + \frac{b^2d - 2abe + 3a^2f}{b^3(a + bx)} \right) dx, x, x^3 \right) \\ &= \frac{(be - 2af)x^3}{3b^3} + \frac{fx^6}{6b^2} - \frac{b^3c - ab^2d + a^2be - a^3f}{3b^4(a + bx^3)} + \frac{(b^2d - 2abe + 3a^2f) \log(a + bx^3)}{3b^4} \end{aligned}$$

Mathematica [A] time = 0.0584239, size = 93, normalized size = 0.9

$$\frac{\frac{2(-a^2be + a^3f + ab^2d - b^3c)}{a + bx^3} + 2 \log(a + bx^3)(3a^2f - 2abe + b^2d) + 2bx^3(be - 2af) + b^2fx^6}{6b^4}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]

[Out] (2*b*(b*e - 2*a*f)*x^3 + b^2*f*x^6 + (2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3) + 2*(b^2*d - 2*a*b*e + 3*a^2*f)*Log[a + b*x^3])/(6*b^4)

Maple [A] time = 0.01, size = 142, normalized size = 1.4

$$\frac{fx^6}{6b^2} - \frac{2ax^3f}{3b^3} + \frac{x^3e}{3b^2} + \frac{\ln(bx^3 + a)a^2f}{b^4} - \frac{2 \ln(bx^3 + a)ae}{3b^3} + \frac{\ln(bx^3 + a)d}{3b^2} + \frac{a^3f}{3b^4(bx^3 + a)} - \frac{a^2e}{3b^3(bx^3 + a)} + \frac{a}{3b^2(bx^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)

[Out] 1/6*f*x^6/b^2-2/3/b^3*x^3*a*f+1/3/b^2*x^3*e+1/b^4*ln(b*x^3+a)*a^2*f-2/3/b^3*ln(b*x^3+a)*a*e+1/3/b^2*ln(b*x^3+a)*d+1/3/b^4/(b*x^3+a)*a^3*f-1/3/b^3/(b*x^3+a)*a^2*e+1/3/b^2/(b*x^3+a)*a*d-1/3/b/(b*x^3+a)*c

Maxima [A] time = 0.959441, size = 132, normalized size = 1.28

$$\frac{b^3c - ab^2d + a^2be - a^3f}{3(b^5x^3 + ab^4)} + \frac{bfx^6 + 2(be - 2af)x^3}{6b^3} + \frac{(b^2d - 2abe + 3a^2f)\log(bx^3 + a)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")

[Out] -1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(b^5*x^3 + a*b^4) + 1/6*(b*f*x^6 + 2*(b*e - 2*a*f)*x^3)/b^3 + 1/3*(b^2*d - 2*a*b*e + 3*a^2*f)*log(b*x^3 + a)/b^4

Fricas [A] time = 1.13548, size = 305, normalized size = 2.96

$$\frac{b^3fx^9 + (2b^3e - 3ab^2f)x^6 - 2b^3c + 2ab^2d - 2a^2be + 2a^3f + 2(ab^2e - 2a^2bf)x^3 + 2(ab^2d - 2a^2be + 3a^3f + (b^3d - 2a^2b^2e + 3a^3f)\log(bx^3 + a))}{6(b^5x^3 + ab^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] 1/6*(b^3*f*x^9 + (2*b^3*e - 3*a*b^2*f)*x^6 - 2*b^3*c + 2*a*b^2*d - 2*a^2*b*e + 2*a^3*f + 2*(a*b^2*e - 2*a^2*b*f)*x^3 + 2*(a*b^2*d - 2*a^2*b*e + 3*a^3*f + (b^3*d - 2*a*b^2*e + 3*a^2*b*f)*x^3)*log(b*x^3 + a))/(b^5*x^3 + a*b^4)

Sympy [A] time = 9.22015, size = 97, normalized size = 0.94

$$\frac{a^3f - a^2be + ab^2d - b^3c}{3ab^4 + 3b^5x^3} + \frac{fx^6}{6b^2} - \frac{x^3(2af - be)}{3b^3} + \frac{(3a^2f - 2abe + b^2d)\log(a + bx^3)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)

[Out] (a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*a*b**4 + 3*b**5*x**3) + f*x**6/(6*b**2) - x**3*(2*a*f - b*e)/(3*b**3) + (3*a**2*f - 2*a*b*e + b**2*d)*log(a

$$+ b*x**3)/(3*b**4)$$

Giac [B] time = 1.05823, size = 278, normalized size = 2.7

$$-\frac{1}{6} f \left(\frac{(bx^3 + a)^2 \left(\frac{6a}{bx^3 + a} - 1 \right)}{b^4} + \frac{6a^2 \log\left(\frac{|bx^3 + a|}{(bx^3 + a)^2 |b|}\right)}{b^4} - \frac{2a^3}{(bx^3 + a)b^4} \right) + \frac{1}{3} \left(\frac{2a \log\left(\frac{|bx^3 + a|}{(bx^3 + a)^2 |b|}\right)}{b^3} + \frac{bx^3 + a}{b^3} - \frac{a^2}{(bx^3 + a)b^3} \right) e^{-}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")

[Out] -1/6*f*((b*x^3 + a)^2*(6*a/(b*x^3 + a) - 1)/b^4 + 6*a^2*log(abs(b*x^3 + a)/((b*x^3 + a)^2*abs(b)))/b^4 - 2*a^3/((b*x^3 + a)*b^4)) + 1/3*(2*a*log(abs(b*x^3 + a)/((b*x^3 + a)^2*abs(b)))/b^3 + (b*x^3 + a)/b^3 - a^2/((b*x^3 + a)*b^3))*e - 1/3*d*(log(abs(b*x^3 + a)/((b*x^3 + a)^2*abs(b)))/b - a/((b*x^3 + a)*b))/b - 1/3*c/((b*x^3 + a)*b)

$$3.255 \quad \int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)^2} dx$$

Optimal. Leaf size=100

$$\frac{a^2be + a^3(-f) - ab^2d + b^3c}{3ab^3(a + bx^3)} - \frac{\log(a + bx^3)(-a^2be + 2a^3f + b^3c)}{3a^2b^3} + \frac{c \log(x)}{a^2} + \frac{fx^3}{3b^2}$$

[Out] (f*x^3)/(3*b^2) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a*b^3*(a + b*x^3)) + (c*Log[x])/a^2 - ((b^3*c - a^2*b*e + 2*a^3*f)*Log[a + b*x^3])/(3*a^2*b^3)

Rubi [A] time = 0.125302, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1821, 1620}

$$\frac{a^2be + a^3(-f) - ab^2d + b^3c}{3ab^3(a + bx^3)} - \frac{\log(a + bx^3)(-a^2be + 2a^3f + b^3c)}{3a^2b^3} + \frac{c \log(x)}{a^2} + \frac{fx^3}{3b^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^2),x]

[Out] (f*x^3)/(3*b^2) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a*b^3*(a + b*x^3)) + (c*Log[x])/a^2 - ((b^3*c - a^2*b*e + 2*a^3*f)*Log[a + b*x^3])/(3*a^2*b^3)

Rule 1821

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1620

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[Expon[Px, x], 2]

Rubi steps

$$\begin{aligned}
\int \frac{c + dx^3 + ex^6 + fx^9}{x(a + bx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{c + dx + ex^2 + fx^3}{x(a + bx)^2} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{f}{b^2} + \frac{c}{a^2x} + \frac{-b^3c + ab^2d - a^2be + a^3f}{ab^2(a + bx)^2} + \frac{-b^3c + a^2be - 2a^3f}{a^2b^2(a + bx)} \right) dx, x, x^3 \right) \\
&= \frac{fx^3}{3b^2} + \frac{b^3c - ab^2d + a^2be - a^3f}{3ab^3(a + bx^3)} + \frac{c \log(x)}{a^2} - \frac{(b^3c - a^2be + 2a^3f) \log(a + bx^3)}{3a^2b^3}
\end{aligned}$$

Mathematica [A] time = 0.122146, size = 95, normalized size = 0.95

$$\frac{\frac{a(a^2b(e+fx^3)+a^3(-f)+ab^2(fx^6-d)+b^3c)}{a+bx^3} + \log(a+bx^3)(a^2be-2a^3f-b^3c)}{b^3} + 3c \log(x)}{3a^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^2), x]

[Out] (3*c*Log[x] + ((a*(b^3*c - a^3*f + a^2*b*(e + f*x^3) + a*b^2*(-d + f*x^6)))/(a + b*x^3) + (-b^3*c) + a^2*b*e - 2*a^3*f)*Log[a + b*x^3])/b^3/(3*a^2)

Maple [A] time = 0.011, size = 125, normalized size = 1.3

$$\frac{fx^3}{3b^2} - \frac{2a \ln(bx^3 + a)f}{3b^3} + \frac{\ln(bx^3 + a)e}{3b^2} - \frac{c \ln(bx^3 + a)}{3a^2} - \frac{a^2f}{3b^3(bx^3 + a)} + \frac{ae}{3b^2(bx^3 + a)} - \frac{d}{3b(bx^3 + a)} + \frac{c}{3a(bx^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^2,x)

[Out] 1/3*f*x^3/b^2-2/3/b^3*a*ln(b*x^3+a)*f+1/3/b^2*ln(b*x^3+a)*e-1/3*c*ln(b*x^3+a)/a^2-1/3/b^3*a^2/(b*x^3+a)*f+1/3/b^2*a/(b*x^3+a)*e-1/3/b/(b*x^3+a)*d+1/3/a/(b*x^3+a)*c+c*ln(x)/a^2

Maxima [A] time = 0.95645, size = 135, normalized size = 1.35

$$\frac{fx^3}{3b^2} + \frac{b^3c - ab^2d + a^2be - a^3f}{3(ab^4x^3 + a^2b^3)} + \frac{c \log(x^3)}{3a^2} - \frac{(b^3c - a^2be + 2a^3f) \log(bx^3 + a)}{3a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^2,x, algorithm="maxima")

[Out] 1/3*f*x^3/b^2 + 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a*b^4*x^3 + a^2*b^3) + 1/3*c*log(x^3)/a^2 - 1/3*(b^3*c - a^2*b*e + 2*a^3*f)*log(b*x^3 + a)/(a^2*b^3)

Fricas [A] time = 1.43537, size = 293, normalized size = 2.93

$$\frac{a^2b^2fx^6 + a^3bfx^3 + ab^3c - a^2b^2d + a^3be - a^4f - (ab^3c - a^3be + 2a^4f + (b^4c - a^2b^2e + 2a^3bf)x^3) \log(bx^3 + a) + 3(b^4c - a^2b^2e + 2a^3bf)x^3}{3(a^2b^4x^3 + a^3b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^2,x, algorithm="fricas")

[Out] 1/3*(a^2*b^2*f*x^6 + a^3*b*f*x^3 + a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f - (a*b^3*c - a^3*b*e + 2*a^4*f + (b^4*c - a^2*b^2*e + 2*a^3*b*f)*x^3)*log(b*x^3 + a) + 3*(b^4*c*x^3 + a*b^3*c)*log(x))/(a^2*b^4*x^3 + a^3*b^3)

Sympy [A] time = 33.8253, size = 95, normalized size = 0.95

$$-\frac{a^3f - a^2be + ab^2d - b^3c}{3a^2b^3 + 3ab^4x^3} + \frac{fx^3}{3b^2} + \frac{c \log(x)}{a^2} - \frac{(2a^3f - a^2be + b^3c) \log\left(\frac{a}{b} + x^3\right)}{3a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x/(b*x**3+a)**2,x)

[Out] -(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*a**2*b**3 + 3*a*b**4*x**3) + f*x**3/(3*b**2) + c*log(x)/a**2 - (2*a**3*f - a**2*b*e + b**3*c)*log(a/b + x**3)

*3)/(3*a**2*b**3)

Giac [A] time = 1.07303, size = 169, normalized size = 1.69

$$\frac{fx^3}{3b^2} + \frac{c \log(|x|)}{a^2} - \frac{(b^3c + 2a^3f - a^2be) \log(|bx^3 + a|)}{3a^2b^3} + \frac{b^4cx^3 + 2a^3bfx^3 - a^2b^2x^3e + 2ab^3c - a^2b^2d + a^4f}{3(bx^3 + a)a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^2,x, algorithm="giac")

[Out] 1/3*f*x^3/b^2 + c*log(abs(x))/a^2 - 1/3*(b^3*c + 2*a^3*f - a^2*b*e)*log(abs(b*x^3 + a))/(a^2*b^3) + 1/3*(b^4*c*x^3 + 2*a^3*b*f*x^3 - a^2*b^2*x^3*e + 2*a*b^3*c - a^2*b^2*d + a^4*f)/((b*x^3 + a)*a^2*b^3)

$$3.256 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)^2} dx$$

Optimal. Leaf size=109

$$-\frac{a^2be + a^3(-f) - ab^2d + b^3c}{3a^2b^2(a+bx^3)} + \frac{\log(a+bx^3)(a^3f - ab^2d + 2b^3c)}{3a^3b^2} - \frac{\log(x)(2bc - ad)}{a^3} - \frac{c}{3a^2x^3}$$

[Out] $-c/(3*a^2*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^2*b^2*(a + b*x^3)) - ((2*b*c - a*d)*\text{Log}[x])/a^3 + ((2*b^3*c - a*b^2*d + a^3*f)*\text{Log}[a + b*x^3])/ (3*a^3*b^2)$

Rubi [A] time = 0.14216, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1821, 1620}

$$-\frac{a^2be + a^3(-f) - ab^2d + b^3c}{3a^2b^2(a+bx^3)} + \frac{\log(a+bx^3)(a^3f - ab^2d + 2b^3c)}{3a^3b^2} - \frac{\log(x)(2bc - ad)}{a^3} - \frac{c}{3a^2x^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^2),x]

[Out] $-c/(3*a^2*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^2*b^2*(a + b*x^3)) - ((2*b*c - a*d)*\text{Log}[x])/a^3 + ((2*b^3*c - a*b^2*d + a^3*f)*\text{Log}[a + b*x^3])/ (3*a^3*b^2)$

Rule 1821

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1620

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[Expon[Px, x], 2]

Rubi steps

$$\begin{aligned}
\int \frac{c + dx^3 + ex^6 + fx^9}{x^4 (a + bx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{c + dx + ex^2 + fx^3}{x^2 (a + bx)^2} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{c}{a^2 x^2} + \frac{-2bc + ad}{a^3 x} + \frac{b^3 c - ab^2 d + a^2 b e - a^3 f}{a^2 b (a + bx)^2} + \frac{2b^3 c - ab^2 d + a^3 f}{a^3 b (a + bx)} \right) dx, x, x^3 \right) \\
&= -\frac{c}{3a^2 x^3} - \frac{b^3 c - ab^2 d + a^2 b e - a^3 f}{3a^2 b^2 (a + bx^3)} - \frac{(2bc - ad) \log(x)}{a^3} + \frac{(2b^3 c - ab^2 d + a^3 f) \log(a + bx^3)}{3a^3 b^2}
\end{aligned}$$

Mathematica [A] time = 0.0767274, size = 97, normalized size = 0.89

$$\frac{\frac{a(-a^2be+a^3f+ab^2d-b^3c)}{b^2(a+bx^3)} + \frac{\log(a+bx^3)(a^3f-ab^2d+2b^3c)}{b^2} + 3 \log(x)(ad-2bc) - \frac{ac}{x^3}}{3a^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^2), x]

[Out] (-((a*c)/x^3) + (a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(b^2*(a + b*x^3)) + 3*(-2*b*c + a*d)*Log[x] + ((2*b^3*c - a*b^2*d + a^3*f)*Log[a + b*x^3]))/b^2)/(3*a^3)

Maple [A] time = 0.014, size = 132, normalized size = 1.2

$$\frac{f \ln(bx^3 + a)}{3b^2} - \frac{d \ln(bx^3 + a)}{3a^2} + \frac{2bc \ln(bx^3 + a)}{3a^3} + \frac{af}{3b^2(bx^3 + a)} - \frac{e}{3b(bx^3 + a)} + \frac{d}{3a(bx^3 + a)} - \frac{bc}{3a^2(bx^3 + a)} - \frac{c}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^2,x)

[Out] 1/3*f*ln(b*x^3+a)/b^2-1/3*d*ln(b*x^3+a)/a^2+2/3*b*c*ln(b*x^3+a)/a^3+1/3*a/b^2/(b*x^3+a)*f-1/3/b/(b*x^3+a)*e+1/3/a/(b*x^3+a)*d-1/3/a^2*b/(b*x^3+a)*c-1/3*c/x^3/a^2+d*ln(x)/a^2-2*b*c*ln(x)/a^3

Maxima [A] time = 0.954806, size = 157, normalized size = 1.44

$$-\frac{ab^2c + (2b^3c - ab^2d + a^2be - a^3f)x^3}{3(a^2b^3x^6 + a^3b^2x^3)} - \frac{(2bc - ad)\log(x^3)}{3a^3} + \frac{(2b^3c - ab^2d + a^3f)\log(bx^3 + a)}{3a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^2,x, algorithm="maxima")

[Out] -1/3*(a*b^2*c + (2*b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(a^2*b^3*x^6 + a^3*b^2*x^3) - 1/3*(2*b*c - a*d)*log(x^3)/a^3 + 1/3*(2*b^3*c - a*b^2*d + a^3*f)*log(b*x^3 + a)/(a^3*b^2)

Fricas [A] time = 1.49415, size = 340, normalized size = 3.12

$$\frac{a^2b^2c + (2ab^3c - a^2b^2d + a^3be - a^4f)x^3 - ((2b^4c - ab^3d + a^3bf)x^6 + (2ab^3c - a^2b^2d + a^4f)x^3)\log(bx^3 + a) + 3((2b^4c - ab^3d + a^3bf)x^6 + (2ab^3c - a^2b^2d + a^4f)x^3)\log(x)}{3(a^3b^3x^6 + a^4b^2x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^2,x, algorithm="fricas")

[Out] -1/3*(a^2*b^2*c + (2*a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^3 - ((2*b^4*c - a*b^3*d + a^3*b*f)*x^6 + (2*a*b^3*c - a^2*b^2*d + a^4*f)*x^3)*log(b*x^3 + a) + 3*((2*b^4*c - a*b^3*d)*x^6 + (2*a*b^3*c - a^2*b^2*d)*x^3)*log(x)/(a^3*b^3*x^6 + a^4*b^2*x^3)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**4/(b*x**3+a)**2,x)

[Out] Timed out

Giac [A] time = 1.05914, size = 177, normalized size = 1.62

$$-\frac{(2bc - ad) \log(|x|)}{a^3} + \frac{(2b^3c - ab^2d + a^3f) \log(|bx^3 + a|)}{3a^3b^2} - \frac{a^2bfx^6 + 4b^3cx^3 - 2ab^2dx^3 - a^3fx^3 + 2a^2bx^3e + 2ab^2c}{6(bx^6 + ax^3)a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^2,x, algorithm="giac")

[Out] -(2*b*c - a*d)*log(abs(x))/a^3 + 1/3*(2*b^3*c - a*b^2*d + a^3*f)*log(abs(b*x^3 + a))/(a^3*b^2) - 1/6*(a^2*b*f*x^6 + 4*b^3*c*x^3 - 2*a*b^2*d*x^3 - a^3*f*x^3 + 2*a^2*b*x^3*e + 2*a*b^2*c)/((b*x^6 + a*x^3)*a^2*b^2)

$$3.257 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)^2} dx$$

Optimal. Leaf size=130

$$\frac{a^2be + a^3(-f) - ab^2d + b^3c}{3a^3b(a+bx^3)} - \frac{\log(a+bx^3)(a^2e - 2abd + 3b^2c)}{3a^4} + \frac{\log(x)(a^2e - 2abd + 3b^2c)}{a^4} + \frac{2bc - ad}{3a^3x^3} - \frac{c}{6a^2x^6}$$

[Out] $-c/(6*a^2*x^6) + (2*b*c - a*d)/(3*a^3*x^3) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^3*b*(a + b*x^3)) + ((3*b^2*c - 2*a*b*d + a^2*e)*\text{Log}[x])/a^4 - ((3*b^2*c - 2*a*b*d + a^2*e)*\text{Log}[a + b*x^3])/(3*a^4)$

Rubi [A] time = 0.153838, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1821, 1620}

$$\frac{a^2be + a^3(-f) - ab^2d + b^3c}{3a^3b(a+bx^3)} - \frac{\log(a+bx^3)(a^2e - 2abd + 3b^2c)}{3a^4} + \frac{\log(x)(a^2e - 2abd + 3b^2c)}{a^4} + \frac{2bc - ad}{3a^3x^3} - \frac{c}{6a^2x^6}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^2), x]$

[Out] $-c/(6*a^2*x^6) + (2*b*c - a*d)/(3*a^3*x^3) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^3*b*(a + b*x^3)) + ((3*b^2*c - 2*a*b*d + a^2*e)*\text{Log}[x])/a^4 - ((3*b^2*c - 2*a*b*d + a^2*e)*\text{Log}[a + b*x^3])/(3*a^4)$

Rule 1821

$\text{Int}[(Pq_*)(x_)^{(m_*)}*((a_) + (b_*)(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*\text{SubstFor}[x^n, Pq, x]*(a + b*x)^p, x], x, x^n], x] /; \text{FreeQ}\{a, b, m, n, p\}, x] \&\& \text{PolyQ}[Pq, x^n] \&\& \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$

Rule 1620

$\text{Int}[(Px_)*((a_) + (b_*)(x_))^{(m_*)}*((c_) + (d_*)(x_))^{(n_)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \text{PolyQ}[Px, x] \&\& (\text{IntegersQ}[m, n] \parallel \text{IGtQ}[m, -2]) \&\& \text{GtQ}[\text{Expon}[Px, x], 2]$

Rubi steps

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^7 (a + bx^3)^2} dx = \frac{1}{3} \text{Subst} \left(\int \frac{c + dx + ex^2 + fx^3}{x^3 (a + bx)^2} dx, x, x^3 \right)$$

$$= \frac{1}{3} \text{Subst} \left(\int \left(\frac{c}{a^2 x^3} + \frac{-2bc + ad}{a^3 x^2} + \frac{3b^2c - 2abd + a^2e}{a^4 x} + \frac{-b^3c + ab^2d - a^2be + a^3f}{a^3 (a + bx)^2} - \frac{b(3b^2c - 2abd + a^2e)}{a^4 (a + bx)} \right) dx, x, x^3 \right)$$

$$= -\frac{c}{6a^2 x^6} + \frac{2bc - ad}{3a^3 x^3} + \frac{b^3c - ab^2d + a^2be - a^3f}{3a^3 b (a + bx^3)} + \frac{(3b^2c - 2abd + a^2e) \log(x)}{a^4} - \frac{b(3b^2c - 2abd + a^2e)}{a^4 (a + bx^3)}$$

Mathematica [A] time = 0.114931, size = 118, normalized size = 0.91

$$\frac{2a(-a^2be + a^3f + ab^2d - b^3c)}{b(a + bx^3)} + 2 \log(a + bx^3) (a^2e - 2abd + 3b^2c) - 6 \log(x) (a^2e - 2abd + 3b^2c) + \frac{a^2c}{x^6} + \frac{2a(ad - 2bc)}{x^3}$$

$$6a^4$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^2), x]

[Out] -((a^2*c)/x^6 + (2*a*(-2*b*c + a*d))/x^3 + (2*a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(b*(a + b*x^3)) - 6*(3*b^2*c - 2*a*b*d + a^2*e)*Log[x] + 2*(3*b^2*c - 2*a*b*d + a^2*e)*Log[a + b*x^3])/(6*a^4)

Maple [A] time = 0.015, size = 167, normalized size = 1.3

$$-\frac{e \ln(bx^3 + a)}{3a^2} + \frac{2 \ln(bx^3 + a)bd}{3a^3} - \frac{\ln(bx^3 + a)b^2c}{a^4} - \frac{f}{3b(bx^3 + a)} + \frac{e}{3a(bx^3 + a)} - \frac{bd}{3a^2(bx^3 + a)} + \frac{b^2c}{3a^3(bx^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^2,x)

[Out] -1/3*e*ln(b*x^3+a)/a^2+2/3/a^3*ln(b*x^3+a)*b*d-1/a^4*ln(b*x^3+a)*b^2*c-1/3/b/(b*x^3+a)*f+1/3/a/(b*x^3+a)*e-1/3/a^2*b/(b*x^3+a)*d+1/3/a^3*b^2/(b*x^3+a)*c-1/6*c/a^2/x^6-1/3/a^2/x^3*d+2/3/a^3/x^3*b*c+e*ln(x)/a^2-2/a^3*ln(x)*b*d+3/a^4*ln(x)*b^2*c

Maxima [A] time = 0.979299, size = 186, normalized size = 1.43

$$\frac{2(3b^3c - 2ab^2d + a^2be - a^3f)x^6 - a^2bc + (3ab^2c - 2a^2bd)x^3}{6(a^3b^2x^9 + a^4bx^6)} - \frac{(3b^2c - 2abd + a^2e)\log(bx^3 + a)}{3a^4} + \frac{(3b^2c - 2abd + a^2e)\log(bx^3 + a)}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^2,x, algorithm="maxima")

[Out] 1/6*(2*(3*b^3*c - 2*a*b^2*d + a^2*b*e - a^3*f)*x^6 - a^2*b*c + (3*a*b^2*c - 2*a^2*b*d)*x^3)/(a^3*b^2*x^9 + a^4*b*x^6) - 1/3*(3*b^2*c - 2*a*b*d + a^2*e)*log(b*x^3 + a)/a^4 + 1/3*(3*b^2*c - 2*a*b*d + a^2*e)*log(x^3)/a^4

Fricas [A] time = 1.42823, size = 431, normalized size = 3.32

$$\frac{2(3ab^3c - 2a^2b^2d + a^3be - a^4f)x^6 - a^3bc + (3a^2b^2c - 2a^3bd)x^3 - 2((3b^4c - 2ab^3d + a^2b^2e)x^9 + (3ab^3c - 2a^2b^2d + a^3be)x^6)\log(bx^3 + a) + 6((3b^4c - 2ab^3d + a^2b^2e)x^9 + (3ab^3c - 2a^2b^2d + a^3be)x^6)\log(x)}{6(a^4b^2x^9 + a^5bx^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^2,x, algorithm="fricas")

[Out] 1/6*(2*(3*a*b^3*c - 2*a^2*b^2*d + a^3*b*e - a^4*f)*x^6 - a^3*b*c + (3*a^2*b^2*c - 2*a^3*b*d)*x^3 - 2*((3*b^4*c - 2*a*b^3*d + a^2*b^2*e)*x^9 + (3*a*b^3*c - 2*a^2*b^2*d + a^3*b*e)*x^6)*log(b*x^3 + a) + 6*((3*b^4*c - 2*a*b^3*d + a^2*b^2*e)*x^9 + (3*a*b^3*c - 2*a^2*b^2*d + a^3*b*e)*x^6)*log(x))/(a^4*b^2*x^9 + a^5*b*x^6)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**7/(b*x**3+a)**2,x)

[Out] Timed out

Giac [A] time = 1.07027, size = 271, normalized size = 2.08

$$\frac{(3b^2c - 2abd + a^2e) \log(|x|)}{a^4} - \frac{(3b^3c - 2ab^2d + a^2be) \log(|bx^3 + a|)}{3a^4b} + \frac{3b^4cx^3 - 2ab^3dx^3 + a^2b^2x^3e + 4ab^3c - 3a^2b^2d}{3(bx^3 + a)a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^2,x, algorithm="giac")

[Out] $(3b^2c - 2a*b*d + a^2*e)*\log(\text{abs}(x))/a^4 - 1/3*(3b^3c - 2a*b^2*d + a^2*b*e)*\log(\text{abs}(b*x^3 + a))/(a^4*b) + 1/3*(3b^4*c*x^3 - 2a*b^3*d*x^3 + a^2*b^2*x^3*e + 4a*b^3*c - 3a^2*b^2*d - a^4*f + 2a^3*b*e)/((b*x^3 + a)*a^4*b) - 1/6*(9b^2*c*x^6 - 6a*b*d*x^6 + 3a^2*x^6*e - 4a*b*c*x^3 + 2a^2*d*x^3 + a^2*c)/(a^4*x^6)$

$$3.258 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)^2} dx$$

Optimal. Leaf size=175

$$-\frac{a^2be + a^3(-f) - ab^2d + b^3c}{3a^4(a+bx^3)} + \frac{\log(a+bx^3)(2a^2be + a^3(-f) - 3ab^2d + 4b^3c)}{3a^5} - \frac{\log(x)(2a^2be + a^3(-f) - 3ab^2d + 4b^3c)}{a^5}$$

[Out] $-c/(9*a^2*x^9) + (2*b*c - a*d)/(6*a^3*x^6) - (3*b^2*c - 2*a*b*d + a^2*e)/(3*a^4*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*(a + b*x^3)) - ((4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*\text{Log}[x])/a^5 + ((4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*\text{Log}[a + b*x^3])/(3*a^5)$

Rubi [A] time = 0.202541, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1821, 1620}

$$-\frac{a^2be + a^3(-f) - ab^2d + b^3c}{3a^4(a+bx^3)} + \frac{\log(a+bx^3)(2a^2be + a^3(-f) - 3ab^2d + 4b^3c)}{3a^5} - \frac{\log(x)(2a^2be + a^3(-f) - 3ab^2d + 4b^3c)}{a^5}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x^3 + e*x^6 + f*x^9)/(x^{10}*(a + b*x^3)^2), x]$

[Out] $-c/(9*a^2*x^9) + (2*b*c - a*d)/(6*a^3*x^6) - (3*b^2*c - 2*a*b*d + a^2*e)/(3*a^4*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*(a + b*x^3)) - ((4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*\text{Log}[x])/a^5 + ((4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*\text{Log}[a + b*x^3])/(3*a^5)$

Rule 1821

$\text{Int}[(Pq_*)(x_)^{(m_*)}*((a_) + (b_*)(x_)^{(n_)})^{(p_*)}, x_Symbol] := \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*\text{SubstFor}[x^n, Pq, x]*(a + b*x)^p, x], x, x^n], x] /;$ FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Simplify[(m+1)/n]]

Rule 1620

$\text{Int}[(Px_)*((a_) + (b_*)(x_))^{(m_*)}*((c_) + (d_*)(x_))^{(n_*)}, x_Symbol] := \text{Int}[\text{ExpandIntegrand}[Px*(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c

, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]

Rubi steps

$$\begin{aligned} \int \frac{c + dx^3 + ex^6 + fx^9}{x^{10}(a + bx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{c + dx + ex^2 + fx^3}{x^4(a + bx)^2} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{c}{a^2x^4} + \frac{-2bc + ad}{a^3x^3} + \frac{3b^2c - 2abd + a^2e}{a^4x^2} + \frac{-4b^3c + 3ab^2d - 2a^2be + a^3f}{a^5x} - \frac{b^4}{a^5} \right) dx, x, x^3 \right) \\ &= -\frac{c}{9a^2x^9} + \frac{2bc - ad}{6a^3x^6} - \frac{3b^2c - 2abd + a^2e}{3a^4x^3} - \frac{b^3c - ab^2d + a^2be - a^3f}{3a^4(a + bx^3)} - \frac{(4b^3c - 3ab^2d + 2a^2be - a^3f)}{a^5} \end{aligned}$$

Mathematica [A] time = 0.10454, size = 160, normalized size = 0.91

$$\frac{6a(-a^2be + a^3f + ab^2d - b^3c)}{a + bx^3} + 6 \log(a + bx^3)(2a^2be + a^3(-f) - 3ab^2d + 4b^3c) + 18 \log(x)(-2a^2be + a^3f + 3ab^2d - 4b^3c) - \frac{6a^4}{18a^5}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^2), x]

[Out] ((-2*a^3*c)/x^9 - (3*a^2*(-2*b*c + a*d))/x^6 - (6*a*(3*b^2*c - 2*a*b*d + a^2*e))/x^3 + (6*a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3) + 18*(-4*b^3*c + 3*a*b^2*d - 2*a^2*b*e + a^3*f)*Log[x] + 6*(4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*Log[a + b*x^3])/(18*a^5)

Maple [A] time = 0.018, size = 229, normalized size = 1.3

$$-\frac{\ln(bx^3 + a)f}{3a^2} + \frac{2b \ln(bx^3 + a)e}{3a^3} - \frac{b^2 \ln(bx^3 + a)d}{a^4} + \frac{4b^3 \ln(bx^3 + a)c}{3a^5} + \frac{f}{3a(bx^3 + a)} - \frac{be}{3a^2(bx^3 + a)} + \frac{b^2d}{3a^3(bx^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^2,x)

[Out]
$$-1/3/a^2*\ln(b*x^3+a)*f+2/3*b/a^3*\ln(b*x^3+a)*e-b^2/a^4*\ln(b*x^3+a)*d+4/3*b^3/a^5*\ln(b*x^3+a)*c+1/3/a/(b*x^3+a)*f-1/3*b/a^2/(b*x^3+a)*e+1/3*b^2/a^3/(b*x^3+a)*d-1/3*b^3/a^4/(b*x^3+a)*c-1/9*c/a^2/x^9-1/6/a^2/x^6*d+1/3/a^3/x^6*b*c-1/3/a^2/x^3*e+2/3/a^3/x^3*b*d-1/a^4/x^3*b^2*c+1/a^2*\ln(x)*f-2/a^3*\ln(x)*b*e+3/a^4*\ln(x)*b^2*d-4/a^5*\ln(x)*b^3*c$$

Maxima [A] time = 1.04895, size = 244, normalized size = 1.39

$$\frac{6(4b^3c - 3ab^2d + 2a^2be - a^3f)x^9 + 3(4ab^2c - 3a^2bd + 2a^3e)x^6 + 2a^3c - (4a^2bc - 3a^3d)x^3}{18(a^4bx^{12} + a^5x^9)} + \frac{(4b^3c - 3ab^2d + 2a^2be - a^3f)x^9 + 3(4ab^2c - 3a^2bd + 2a^3e)x^6 + 2a^3c - (4a^2bc - 3a^3d)x^3}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^2,x, algorithm="maxima")`

[Out]
$$-1/18*(6*(4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*x^9 + 3*(4*a*b^2*c - 3*a^2*b*d + 2*a^3*e)*x^6 + 2*a^3*c - (4*a^2*b*c - 3*a^3*d)*x^3)/(a^4*b*x^{12} + a^5*x^9) + 1/3*(4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*\log(b*x^3 + a)/a^5 - 1/3*(4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*\log(x^3)/a^5$$

Fricas [A] time = 1.6199, size = 549, normalized size = 3.14

$$\frac{6(4ab^3c - 3a^2b^2d + 2a^3be - a^4f)x^9 + 3(4a^2b^2c - 3a^3bd + 2a^4e)x^6 + 2a^4c - (4a^3bc - 3a^4d)x^3 - 6((4b^4c - 3ab^3d + 2a^2b^2e - a^3bf)x^{12} + (4ab^3c - 3a^2b^2d + 2a^3be - a^4f)x^9)*\log(b*x^3 + a) + 18((4b^4c - 3ab^3d + 2a^2b^2e - a^3bf)x^{12} + (4ab^3c - 3a^2b^2d + 2a^3be - a^4f)x^9)*\log(x)}{a^5*b*x^{12} + a^6*x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^2,x, algorithm="fricas")`

[Out]
$$-1/18*(6*(4*a*b^3*c - 3*a^2*b^2*d + 2*a^3*b*e - a^4*f)*x^9 + 3*(4*a^2*b^2*c - 3*a^3*b*d + 2*a^4*e)*x^6 + 2*a^4*c - (4*a^3*b*c - 3*a^4*d)*x^3 - 6*((4*b^4*c - 3*a*b^3*d + 2*a^2*b^2*e - a^3*b*f)*x^{12} + (4*a*b^3*c - 3*a^2*b^2*d + 2*a^3*b*e - a^4*f)*x^9)*\log(b*x^3 + a) + 18*((4*b^4*c - 3*a*b^3*d + 2*a^2*b^2*e - a^3*b*f)*x^{12} + (4*a*b^3*c - 3*a^2*b^2*d + 2*a^3*b*e - a^4*f)*x^9)*\log(x)/(a^5*b*x^{12} + a^6*x^9)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**10/(b*x**3+a)**2,x)

[Out] Timed out

Giac [A] time = 1.06707, size = 371, normalized size = 2.12

$$-\frac{(4b^3c - 3ab^2d - a^3f + 2a^2be) \log(|x|)}{a^5} + \frac{(4b^4c - 3ab^3d - a^3bf + 2a^2b^2e) \log(|bx^3 + a|)}{3a^5b} - \frac{4b^4cx^3 - 3ab^3dx^3 - a^3bf}{3a^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^2,x, algorithm="giac")

[Out] $-(4*b^3*c - 3*a*b^2*d - a^3*f + 2*a^2*b*e)*\log(\text{abs}(x))/a^5 + 1/3*(4*b^4*c - 3*a*b^3*d - a^3*b*f + 2*a^2*b^2*e)*\log(\text{abs}(b*x^3 + a))/(a^5*b) - 1/3*(4*b^4*c*x^3 - 3*a*b^3*d*x^3 - a^3*b*f*x^3 + 2*a^2*b^2*x^3*e + 5*a*b^3*c - 4*a^2*b^2*d - 2*a^4*f + 3*a^3*b*e)/((b*x^3 + a)*a^5) + 1/18*(44*b^3*c*x^9 - 33*a*b^2*d*x^9 - 11*a^3*f*x^9 + 22*a^2*b*x^9*e - 18*a*b^2*c*x^6 + 12*a^2*b*d*x^6 - 6*a^3*x^6*e + 6*a^2*b*c*x^3 - 3*a^3*d*x^3 - 2*a^3*c)/(a^5*x^9)$

$$3.259 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)^2} dx$$

Optimal. Leaf size=214

$$\frac{b(a^2be + a^3(-f) - ab^2d + b^3c)}{3a^5(a + bx^3)} + \frac{2a^2be + a^3(-f) - 3ab^2d + 4b^3c}{3a^5x^3} - \frac{b \log(a + bx^3)(3a^2be - 2a^3f - 4ab^2d + 5b^3c)}{3a^6} + \dots$$

[Out] $-c/(12*a^2*x^{12}) + (2*b*c - a*d)/(9*a^3*x^9) - (3*b^2*c - 2*a*b*d + a^2*e)/(6*a^4*x^6) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(3*a^5*x^3) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*a^5*(a + b*x^3)) + (b*(5*b^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f)*Log[x])/a^6 - (b*(5*b^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f)*Log[a + b*x^3])/(3*a^6)$

Rubi [A] time = 0.233997, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1821, 1620}

$$\frac{b(a^2be + a^3(-f) - ab^2d + b^3c)}{3a^5(a + bx^3)} + \frac{2a^2be + a^3(-f) - 3ab^2d + 4b^3c}{3a^5x^3} - \frac{b \log(a + bx^3)(3a^2be - 2a^3f - 4ab^2d + 5b^3c)}{3a^6} + \dots$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^2), x]

[Out] $-c/(12*a^2*x^{12}) + (2*b*c - a*d)/(9*a^3*x^9) - (3*b^2*c - 2*a*b*d + a^2*e)/(6*a^4*x^6) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(3*a^5*x^3) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*a^5*(a + b*x^3)) + (b*(5*b^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f)*Log[x])/a^6 - (b*(5*b^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f)*Log[a + b*x^3])/(3*a^6)$

Rule 1821

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol]
:= Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned} \int \frac{c + dx^3 + ex^6 + fx^9}{x^{13}(a + bx^3)^2} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{c + dx + ex^2 + fx^3}{x^5(a + bx)^2} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{c}{a^2x^5} + \frac{-2bc + ad}{a^3x^4} + \frac{3b^2c - 2abd + a^2e}{a^4x^3} + \frac{-4b^3c + 3ab^2d - 2a^2be + a^3f}{a^5x^2} - \frac{b(b^3c - ab^2d + a^2e - a^3f)}{a^5x} \right) dx, x, x^3 \right) \\ &= -\frac{c}{12a^2x^{12}} + \frac{2bc - ad}{9a^3x^9} - \frac{3b^2c - 2abd + a^2e}{6a^4x^6} + \frac{4b^3c - 3ab^2d + 2a^2be - a^3f}{3a^5x^3} + \frac{b(b^3c - ab^2d + a^2e - a^3f)}{3a^5(a + bx^3)} \end{aligned}$$

Mathematica [A] time = 0.202961, size = 198, normalized size = 0.93

$$\frac{\frac{12ab(-a^2be+a^3f+ab^2d-b^3c)}{a+bx^3} + \frac{12a(-2a^2be+a^3f+3ab^2d-4b^3c)}{x^3} + 12b \log(a + bx^3)(3a^2be - 2a^3f - 4ab^2d + 5b^3c) - 36b \log(x)(3a^2be - 2a^3f - 4ab^2d + 5b^3c)}{36a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^2), x]
```

```
[Out] -((3*a^4*c)/x^12 + (4*a^3*(-2*b*c + a*d))/x^9 + (6*a^2*(3*b^2*c - 2*a*b*d + a^2*e))/x^6 + (12*a*(-4*b^3*c + 3*a*b^2*d - 2*a^2*b*e + a^3*f))/x^3 + (12*a*b*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3) - 36*b*(5*b^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f)*Log[x] + 12*b*(5*b^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f)*Log[a + b*x^3])/(36*a^6)
```

Maple [A] time = 0.02, size = 282, normalized size = 1.3

$$\frac{2b \ln(bx^3 + a)f}{3a^3} - \frac{b^2 \ln(bx^3 + a)e}{a^4} + \frac{4b^3 \ln(bx^3 + a)d}{3a^5} - \frac{5b^4 \ln(bx^3 + a)c}{3a^6} - \frac{fb}{3a^2(bx^3 + a)} + \frac{eb^2}{3a^3(bx^3 + a)} - \frac{b(b^3c - ab^2d + a^2e - a^3f)}{3a^5(a + bx^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^2,x)

[Out] $\frac{2}{3} \frac{b}{a^3} \ln(bx^3+a) * f - \frac{b^2}{a^4} \ln(bx^3+a) * e + \frac{4}{3} \frac{b^3}{a^5} \ln(bx^3+a) * d - \frac{5}{3} \frac{b^4}{a^6} \ln(bx^3+a) * c - \frac{1}{3} \frac{b}{a^2} \frac{f}{(bx^3+a)} + \frac{1}{3} \frac{b^2}{a^3} \frac{e}{(bx^3+a)} - \frac{1}{3} \frac{b^3}{a^4} \frac{d}{(bx^3+a)} + \frac{1}{3} \frac{b^4}{a^5} \frac{c}{(bx^3+a)} - \frac{1}{12} \frac{c}{a^2} \frac{1}{x^{12}} - \frac{1}{9} \frac{d}{a^2} \frac{1}{x^9} + \frac{2}{9} \frac{1}{a^3} \frac{1}{x^9} * b * c - \frac{1}{6} \frac{1}{a^2} \frac{1}{x^6} * e + \frac{1}{3} \frac{1}{a^3} \frac{1}{x^6} * b * d - \frac{1}{2} \frac{1}{a^4} \frac{1}{x^6} * b^2 * c - \frac{1}{3} \frac{1}{a^2} \frac{1}{x^3} * f + \frac{2}{3} \frac{1}{a^3} \frac{1}{x^3} * b * e - \frac{1}{a^4} \frac{1}{x^3} * b^2 * d + \frac{4}{3} \frac{1}{a^5} \frac{1}{x^3} * b^3 * c - 2 * \frac{b}{a^3} \ln(x) * f + 3 * \frac{b^2}{a^4} \ln(x) * e - 4 * \frac{b^3}{a^5} \ln(x) * d + 5 * \frac{b^4}{a^6} \ln(x) * c$

Maxima [A] time = 1.01741, size = 305, normalized size = 1.43

$$\frac{12(5b^4c - 4ab^3d + 3a^2b^2e - 2a^3bf)x^{12} + 6(5ab^3c - 4a^2b^2d + 3a^3be - 2a^4f)x^9 - 2(5a^2b^2c - 4a^3bd + 3a^4e)x^6 - 3a^5}{36(a^5bx^{15} + a^6x^{12})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^2,x, algorithm="maxima")

[Out] $\frac{1}{36} * (12 * (5 * b^4 * c - 4 * a * b^3 * d + 3 * a^2 * b^2 * e - 2 * a^3 * b * f) * x^{12} + 6 * (5 * a * b^3 * c - 4 * a^2 * b^2 * d + 3 * a^3 * b * e - 2 * a^4 * f) * x^9 - 2 * (5 * a^2 * b^2 * c - 4 * a^3 * b * d + 3 * a^4 * e) * x^6 - 3 * a^5 * c + (5 * a^3 * b * c - 4 * a^4 * d) * x^3) / (a^5 * b * x^{15} + a^6 * x^{12}) - \frac{1}{3} * (5 * b^4 * c - 4 * a * b^3 * d + 3 * a^2 * b^2 * e - 2 * a^3 * b * f) * \log(bx^3 + a) / a^6 + \frac{1}{3} * (5 * b^4 * c - 4 * a * b^3 * d + 3 * a^2 * b^2 * e - 2 * a^3 * b * f) * \log(x^3) / a^6$

Fricas [A] time = 1.60778, size = 670, normalized size = 3.13

$$\frac{12(5ab^4c - 4a^2b^3d + 3a^3b^2e - 2a^4bf)x^{12} + 6(5a^2b^3c - 4a^3b^2d + 3a^4be - 2a^5f)x^9 - 2(5a^3b^2c - 4a^4bd + 3a^5e)x^6 - 3a^5}{36(a^5bx^{15} + a^6x^{12})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^2,x, algorithm="fricas")

[Out] $\frac{1}{36} * (12 * (5 * a * b^4 * c - 4 * a^2 * b^3 * d + 3 * a^3 * b^2 * e - 2 * a^4 * b * f) * x^{12} + 6 * (5 * a^2 * b^3 * c - 4 * a^3 * b^2 * d + 3 * a^4 * b * e - 2 * a^5 * f) * x^9 - 2 * (5 * a^3 * b^2 * c - 4 * a^4 * b * d + 3 * a^5 * e) * x^6 - 3 * a^5 * c + (5 * a^4 * b * c - 4 * a^5 * d) * x^3 - 12 * ((5 * b^5 * c - 4 * a * b^4 * d + 3 * a^2 * b^3 * e - 2 * a^3 * b^2 * f) * x^{15} + (5 * a * b^4 * c - 4 * a^2 * b^3 * d + 3 * a^3 * b^2 * e - 2 * a^4 * b * f) * x^{12}) * \log(bx^3 + a) + 36 * ((5 * b^5 * c - 4 * a * b^4 * d + 3 * a^2 * b^3 * e - 2 * a^3 * b^2 * f) * x^{15} + (5 * a * b^4 * c - 4 * a^2 * b^3 * d + 3 * a^3 * b^2 * e - 2 * a^4 * b * f) * x^{12}) * \log(x^3) / a^6$

$$\frac{2b^3e - 2a^3b^2f)x^{15} + (5ab^4c - 4a^2b^3d + 3a^3b^2e - 2a^4b^2f)x^{12} \log(x)}{(a^6bx^{15} + a^7x^{12})}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**13/(b*x**3+a)**2,x)

[Out] Timed out

Giac [A] time = 1.07398, size = 447, normalized size = 2.09

$$\frac{(5b^4c - 4ab^3d - 2a^3bf + 3a^2b^2e) \log(|x|)}{a^6} - \frac{(5b^5c - 4ab^4d - 2a^3b^2f + 3a^2b^3e) \log(|bx^3 + a|)}{3a^6b} + \frac{5b^5cx^3 - 4ab^4dx^3 - \dots}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^2,x, algorithm="giac")

[Out] $(5b^4c - 4a^3b^3d - 2a^3b^2f + 3a^2b^3e) \log(\text{abs}(x)) / a^6 - 1/3 * (5b^5c - 4a^3b^4d - 2a^3b^2f + 3a^2b^3e) \log(\text{abs}(bx^3 + a)) / (a^6b) + 1/3 * (5b^5c * x^3 - 4a^3b^4d * x^3 - 2a^3b^2f * x^3 + 3a^2b^3e * x^3) / ((bx^3 + a) * a^6) - 1/36 * (12 * 5b^4c * x^{12} - 100 * a^3b^3d * x^{12} - 50 * a^3b^2f * x^{12} + 75 * a^2b^3e * x^{12} - 48 * a^4b^3c * x^9 + 36 * a^2b^2d * x^9 + 12 * a^4f * x^9 - 24 * a^3b * x^9 * e + 18 * a^2b^2 * c * x^6 - 12 * a^3b * d * x^6 + 6 * a^4 * x^6 * e - 8 * a^3b * c * x^3 + 4 * a^4 * d * x^3 + 3 * a^4 * c) / (a^6 * x^{12})$

$$3.260 \quad \int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=369

$$\frac{x^4(3a^2be - 4a^3f - 2ab^2d + b^3c)}{4b^5} - \frac{a^2x(a^2be + a^3(-f) - ab^2d + b^3c)}{3b^6(a + bx^3)} - \frac{a^{4/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(13a^2be - 16a^3)}{18b^{19/3}}$$

[Out] $-\left(\frac{a(2b^3c - 3ab^2d + 4a^2b^2e - 5a^3f)x}{b^6}\right) + \left(\frac{(b^3c - 2ab^2d + 3a^2b^2e - 4a^3f)x^4}{4b^5}\right) + \left(\frac{(b^2d - 2ab^2e + 3a^2f)x^7}{(7b^4)}\right) + \left(\frac{(be - 2af)x^{10}}{(10b^3)}\right) + \left(\frac{fx^{13}}{(13b^2)}\right) - \left(\frac{a^2(b^3c - ab^2d + a^2b^2e - a^3f)x}{(3b^6(a + bx^3))}\right) - \left(\frac{a^{4/3}(7b^3c - 10ab^2d + 13a^2b^2e - 16a^3f)\text{ArcTan}\left[\frac{a^{1/3} - 2b^{1/3}x}{\sqrt{3}a^{1/3}}\right]}{(3\sqrt{3}b^{19/3})}\right) + \left(\frac{a^{4/3}(7b^3c - 10ab^2d + 13a^2b^2e - 16a^3f)\text{Log}\left[a^{1/3} + b^{1/3}x\right]}{(9b^{19/3})}\right) - \left(\frac{a^{4/3}(7b^3c - 10ab^2d + 13a^2b^2e - 16a^3f)\text{Log}\left[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2\right]}{(18b^{19/3})}\right)$

Rubi [A] time = 0.467051, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1828, 1887, 200, 31, 634, 617, 204, 628}

$$\frac{x^4(3a^2be - 4a^3f - 2ab^2d + b^3c)}{4b^5} - \frac{a^2x(a^2be + a^3(-f) - ab^2d + b^3c)}{3b^6(a + bx^3)} - \frac{a^{4/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(13a^2be - 16a^3)}{18b^{19/3}}$$

Antiderivative was successfully verified.

[In] Int[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]

[Out] $-\left(\frac{a(2b^3c - 3ab^2d + 4a^2b^2e - 5a^3f)x}{b^6}\right) + \left(\frac{(b^3c - 2ab^2d + 3a^2b^2e - 4a^3f)x^4}{4b^5}\right) + \left(\frac{(b^2d - 2ab^2e + 3a^2f)x^7}{(7b^4)}\right) + \left(\frac{(be - 2af)x^{10}}{(10b^3)}\right) + \left(\frac{fx^{13}}{(13b^2)}\right) - \left(\frac{a^2(b^3c - ab^2d + a^2b^2e - a^3f)x}{(3b^6(a + bx^3))}\right) - \left(\frac{a^{4/3}(7b^3c - 10ab^2d + 13a^2b^2e - 16a^3f)\text{ArcTan}\left[\frac{a^{1/3} - 2b^{1/3}x}{\sqrt{3}a^{1/3}}\right]}{(3\sqrt{3}b^{19/3})}\right) + \left(\frac{a^{4/3}(7b^3c - 10ab^2d + 13a^2b^2e - 16a^3f)\text{Log}\left[a^{1/3} + b^{1/3}x\right]}{(9b^{19/3})}\right) - \left(\frac{a^{4/3}(7b^3c - 10ab^2d + 13a^2b^2e - 16a^3f)\text{Log}\left[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2\right]}{(18b^{19/3})}\right)$

Rule 1828

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q =
m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
+ b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 200

```
Int[((a_) + (b_)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(p_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
```

`-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

Rule 628

`Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`

Rubi steps

$$\begin{aligned} \int \frac{x^9 (c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^2} dx &= -\frac{a^2 (b^3c - ab^2d + a^2be - a^3f)x}{3b^6 (a + bx^3)} - \frac{\int \frac{-a^3(b^3c - ab^2d + a^2be - a^3f) + 3a^2b(b^3c - ab^2d + a^2be - a^3f)x^3 - 3ab^2(b^3c - ab^2d + a^2be - a^3f)x^6 + a^3(b^3c - ab^2d + a^2be - a^3f)x^9}{(a + bx^3)^2} dx}{3b^6 (a + bx^3)} \\ &= -\frac{a^2 (b^3c - ab^2d + a^2be - a^3f)x}{3b^6 (a + bx^3)} - \frac{\int (3a^2 (2b^3c - 3ab^2d + 4a^2be - 5a^3f) - 3ab (b^3c - ab^2d + a^2be - a^3f)x^3 + a^3 (b^3c - ab^2d + a^2be - a^3f)x^6 - a^3 (b^3c - ab^2d + a^2be - a^3f)x^9)}{(a + bx^3)^2} dx}{3b^6 (a + bx^3)} \\ &= -\frac{a (2b^3c - 3ab^2d + 4a^2be - 5a^3f)x}{b^6} + \frac{(b^3c - 2ab^2d + 3a^2be - 4a^3f)x^4}{4b^5} + \frac{(b^2d - 2ab^2e + a^2f)x^7}{7b^6} \\ &= -\frac{a (2b^3c - 3ab^2d + 4a^2be - 5a^3f)x}{b^6} + \frac{(b^3c - 2ab^2d + 3a^2be - 4a^3f)x^4}{4b^5} + \frac{(b^2d - 2ab^2e + a^2f)x^7}{7b^6} \\ &= -\frac{a (2b^3c - 3ab^2d + 4a^2be - 5a^3f)x}{b^6} + \frac{(b^3c - 2ab^2d + 3a^2be - 4a^3f)x^4}{4b^5} + \frac{(b^2d - 2ab^2e + a^2f)x^7}{7b^6} \\ &= -\frac{a (2b^3c - 3ab^2d + 4a^2be - 5a^3f)x}{b^6} + \frac{(b^3c - 2ab^2d + 3a^2be - 4a^3f)x^4}{4b^5} + \frac{(b^2d - 2ab^2e + a^2f)x^7}{7b^6} \\ &= -\frac{a (2b^3c - 3ab^2d + 4a^2be - 5a^3f)x}{b^6} + \frac{(b^3c - 2ab^2d + 3a^2be - 4a^3f)x^4}{4b^5} + \frac{(b^2d - 2ab^2e + a^2f)x^7}{7b^6} \end{aligned}$$

Mathematica [A] time = 0.352521, size = 364, normalized size = 0.99

$$\frac{x^4 (3a^2be - 4a^3f - 2ab^2d + b^3c)}{4b^5} + \frac{a^2x (-a^2be + a^3f + ab^2d - b^3c)}{3b^6 (a + bx^3)} + \frac{a^{4/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2) (-13a^2be + 16a^3f - 12ab^2d + b^3c)}{18b^{19/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]

[Out] (a*(-2*b^3*c + 3*a*b^2*d - 4*a^2*b*e + 5*a^3*f)*x)/b^6 + ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^4)/(4*b^5) + ((b^2*d - 2*a*b*e + 3*a^2*f)*x^7)/(7*b^4) + ((b*e - 2*a*f)*x^10)/(10*b^3) + (f*x^13)/(13*b^2) + (a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(3*b^6*(a + b*x^3)) + (a^(4/3)*(-7*b^3*c + 10*a*b^2*d - 13*a^2*b*e + 16*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(19/3)) - (a^(4/3)*(-7*b^3*c + 10*a*b^2*d - 13*a^2*b*e + 16*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(19/3)) + (a^(4/3)*(-7*b^3*c + 10*a*b^2*d - 13*a^2*b*e + 16*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*b^(19/3))

Maple [A] time = 0.01, size = 622, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)

[Out] 7/9*a^2/b^4*c/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-16/9*a^5/b^7*f/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+13/9*a^4/b^6*e/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/10/b^2*x^10*e+1/7/b^2*x^7*d+1/4/b^2*x^4*c-10/9*a^3/b^5*d/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-7/18*a^2/b^4*c/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-1/3*a^4/b^5*x/(b*x^3+a)*e+1/3*a^3/b^4*x/(b*x^3+a)*d-1/3*a^2/b^3*x/(b*x^3+a)*c+1/3*a^5/b^6*x/(b*x^3+a)*f+7/9*a^2/b^4*c/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+13/9*a^4/b^6*e/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-13/18*a^4/b^6*e/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-10/9*a^3/b^5*d/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+5/9*a^3/b^5*d/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+8/9*a^5/b^7*f/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-16/9*a^5/b^7*f/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+1/13*f*x^13/b^2-1/5/b^3*x^10*a*f+3/7/b^4*x^7*a^2*f-2/7/b^3*x^7*a*e-1/b^5*x^4*a^3*f+3/4/b^4*x^4*a^2*e-1/2/b^3*x^4*a*d+5/b^6*a^4*f*x-4/b^5*a^3*e*x+3/b^4*a^2*d*x-2/b^3*a*c*x

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.41347, size = 1145, normalized size = 3.1

$1260 b^5 f x^{16} + 126 (13 b^5 e - 16 a b^4 f) x^{13} + 234 (10 b^5 d - 13 a b^4 e + 16 a^2 b^3 f) x^{10} + 585 (7 b^5 c - 10 a b^4 d + 13 a^2 b^3 e - 16 a^3 b^2 f) x^7 - 4095 (7 a b^4 c - 10 a^2 b^3 d + 13 a^3 b^2 e - 16 a^4 b f) x^4 - 1820 \sqrt{3} (7 a^2 b^3 c - 10 a^3 b^2 d + 13 a^4 b e - 16 a^5 f + (7 a b^4 c - 10 a^2 b^3 d + 13 a^3 b^2 e - 16 a^4 b f) x^3) (-a/b)^{1/3} \arctan(1/3 (2 \sqrt{3} b x (-a/b)^{2/3} - \sqrt{3} a) / a) + 910 (7 a^2 b^3 c - 10 a^3 b^2 d + 13 a^4 b e - 16 a^5 f + (7 a b^4 c - 10 a^2 b^3 d + 13 a^3 b^2 e - 16 a^4 b f) x^3) (-a/b)^{1/3} \log(x^2 + x (-a/b)^{1/3} + (-a/b)^{2/3}) - 1820 (7 a^2 b^3 c - 10 a^3 b^2 d + 13 a^4 b e - 16 a^5 f + (7 a b^4 c - 10 a^2 b^3 d + 13 a^3 b^2 e - 16 a^4 b f) x^3) (-a/b)^{1/3} \log(x - (-a/b)^{1/3}) - 5460 (7 a^2 b^3 c - 10 a^3 b^2 d + 13 a^4 b e - 16 a^5 f) x / (b^7 x^3 + a b^6)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] $1/16380 * (1260 * b^5 * f * x^{16} + 126 * (13 * b^5 * e - 16 * a * b^4 * f) * x^{13} + 234 * (10 * b^5 * d - 13 * a * b^4 * e + 16 * a^2 * b^3 * f) * x^{10} + 585 * (7 * b^5 * c - 10 * a * b^4 * d + 13 * a^2 * b^3 * e - 16 * a^3 * b^2 * f) * x^7 - 4095 * (7 * a * b^4 * c - 10 * a^2 * b^3 * d + 13 * a^3 * b^2 * e - 16 * a^4 * b * f) * x^4 - 1820 * \sqrt{3} * (7 * a^2 * b^3 * c - 10 * a^3 * b^2 * d + 13 * a^4 * b * e - 16 * a^5 * f + (7 * a * b^4 * c - 10 * a^2 * b^3 * d + 13 * a^3 * b^2 * e - 16 * a^4 * b * f) * x^3) * (-a/b)^{1/3} * \arctan(1/3 * (2 * \sqrt{3} * b * x * (-a/b)^{2/3} - \sqrt{3} * a) / a) + 910 * (7 * a^2 * b^3 * c - 10 * a^3 * b^2 * d + 13 * a^4 * b * e - 16 * a^5 * f + (7 * a * b^4 * c - 10 * a^2 * b^3 * d + 13 * a^3 * b^2 * e - 16 * a^4 * b * f) * x^3) * (-a/b)^{1/3} * \log(x^2 + x * (-a/b)^{1/3} + (-a/b)^{2/3}) - 1820 * (7 * a^2 * b^3 * c - 10 * a^3 * b^2 * d + 13 * a^4 * b * e - 16 * a^5 * f + (7 * a * b^4 * c - 10 * a^2 * b^3 * d + 13 * a^3 * b^2 * e - 16 * a^4 * b * f) * x^3) * (-a/b)^{1/3} * \log(x - (-a/b)^{1/3}) - 5460 * (7 * a^2 * b^3 * c - 10 * a^3 * b^2 * d + 13 * a^4 * b * e - 16 * a^5 * f) * x / (b^7 * x^3 + a * b^6)$

Sympy [A] time = 10.5078, size = 490, normalized size = 1.33

$\frac{x(a^5 f - a^4 b e + a^3 b^2 d - a^2 b^3 c)}{3 a b^6 + 3 b^7 x^3} + \text{RootSum}\left(729 t^3 b^{19} + 4096 a^{13} f^3 - 9984 a^{12} b e f^2 + 7680 a^{11} b^2 d f^2 + 8112 a^{11} b^2 e^2 f - 5460 a^{10} b^3 c f^2 - 1820 a^9 b^4 c e f + 1820 a^8 b^5 c^2\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**9*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)

[Out] x*(a**5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c)/(3*a*b**6 + 3*b**7*x**3) + RootSum(729*_t**3*b**19 + 4096*a**13*f**3 - 9984*a**12*b*e*f**2 + 7680*a**11*b**2*d*f**2 + 8112*a**11*b**2*e**2*f - 5376*a**10*b**3*c*f**2 - 12480*a**10*b**3*d*e*f - 2197*a**10*b**3*e**3 + 8736*a**9*b**4*c*e*f + 4800*a**9*b**4*d**2*f + 5070*a**9*b**4*d*e**2 - 6720*a**8*b**5*c*d*f - 3549*a**8*b**5*c*e**2 - 3900*a**8*b**5*d**2*e + 2352*a**7*b**6*c**2*f + 5460*a**7*b**6*c*d*e + 1000*a**7*b**6*d**3 - 1911*a**6*b**7*c**2*e - 2100*a**6*b**7*c*d**2 + 1470*a**5*b**8*c**2*d - 343*a**4*b**9*c**3, Lambda(_t, _t*log(-9*_t*b**6/(16*a**4*f - 13*a**3*b*e + 10*a**2*b**2*d - 7*a*b**3*c) + x))) + f*x**13/(13*b**2) - x**10*(2*a*f - b*e)/(10*b**3) + x**7*(3*a**2*f - 2*a*b*e + b**2*d)/(7*b**4) - x**4*(4*a**3*f - 3*a**2*b*e + 2*a*b**2*d - b**3*c)/(4*b**5) + x*(5*a**4*f - 4*a**3*b*e + 3*a**2*b**2*d - 2*a*b**3*c)/b**6

Giac [A] time = 1.07843, size = 609, normalized size = 1.65

$$\frac{\sqrt{3} \left(7 (-ab^2)^{\frac{1}{3}} ab^3c - 10 (-ab^2)^{\frac{1}{3}} a^2b^2d - 16 (-ab^2)^{\frac{1}{3}} a^4f + 13 (-ab^2)^{\frac{1}{3}} a^3be \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{9b^7} (7a^2b^3c - 10a^3b^2d)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")

[Out] 1/9*sqrt(3)*(7*(-a*b^2)^(1/3)*a*b^3*c - 10*(-a*b^2)^(1/3)*a^2*b^2*d - 16*(-a*b^2)^(1/3)*a^4*f + 13*(-a*b^2)^(1/3)*a^3*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^7 - 1/9*(7*a^2*b^3*c - 10*a^3*b^2*d - 16*a^5*f + 13*a^4*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^6) + 1/18*(7*(-a*b^2)^(1/3)*a*b^3*c - 10*(-a*b^2)^(1/3)*a^2*b^2*d - 16*(-a*b^2)^(1/3)*a^4*f + 13*(-a*b^2)^(1/3)*a^3*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^7 - 1/3*(a^2*b^3*c*x - a^3*b^2*d*x - a^5*f*x + a^4*b*x*e)/((b*x^3 + a)*b^6) + 1/1820*(140*b^24*f*x^13 - 364*a*b^23*f*x^10 + 182*b^24*x^10*e + 260*b^24*d*x^7 + 780*a^2*b^22*f*x^7 - 520*a*b^23*x^7*e + 455*b^24*c*x^4 - 910*a*b^23*d*x^4 - 1820*a^3*b^21*f*x^4 + 1365*a^2*b^22*x^4*e - 3640*a*b^23*c*x + 5460*a^2*b^22*d*x + 9100*a^4*b^20*f*x - 7280*a^3*b^21*x*e)/b^26

$$3.261 \quad \int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=335

$$\frac{x^2(3a^2be - 4a^3f - 2ab^2d + b^3c)}{2b^5} + \frac{ax^2(a^2be + a^3(-f) - ab^2d + b^3c)}{3b^5(a + bx^3)} - \frac{a^{2/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(11a^2be - 14a^3)}{18b^{17/3}}$$

[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^2)/(2*b^5) + ((b^2*d - 2*a*b*e + 3*a^2*f)*x^5)/(5*b^4) + ((b*e - 2*a*f)*x^8)/(8*b^3) + (f*x^11)/(11*b^2) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*b^5*(a + b*x^3)) + (a^(2/3)*(5*b^3*c - 8*a*b^2*d + 11*a^2*b*e - 14*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*b^(17/3)) + (a^(2/3)*(5*b^3*c - 8*a*b^2*d + 11*a^2*b*e - 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(17/3)) - (a^(2/3)*(5*b^3*c - 8*a*b^2*d + 11*a^2*b*e - 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*b^(17/3))

Rubi [A] time = 0.705022, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1828, 1851, 1836, 1488, 292, 31, 634, 617, 204, 628}

$$\frac{x^2(3a^2be - 4a^3f - 2ab^2d + b^3c)}{2b^5} + \frac{ax^2(a^2be + a^3(-f) - ab^2d + b^3c)}{3b^5(a + bx^3)} - \frac{a^{2/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(11a^2be - 14a^3)}{18b^{17/3}}$$

Antiderivative was successfully verified.

[In] Int[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]

[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^2)/(2*b^5) + ((b^2*d - 2*a*b*e + 3*a^2*f)*x^5)/(5*b^4) + ((b*e - 2*a*f)*x^8)/(8*b^3) + (f*x^11)/(11*b^2) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*b^5*(a + b*x^3)) + (a^(2/3)*(5*b^3*c - 8*a*b^2*d + 11*a^2*b*e - 14*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*b^(17/3)) + (a^(2/3)*(5*b^3*c - 8*a*b^2*d + 11*a^2*b*e - 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(17/3)) - (a^(2/3)*(5*b^3*c - 8*a*b^2*d + 11*a^2*b*e - 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*b^(17/3))

Rule 1828

```

Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q =
m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
+ b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
LtQ[p, -1] && IGtQ[m, 0]

```

Rule 1851

```

Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[x*PolynomialQuot
ient[Pq, x, x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x]
&& EqQ[Coeff[Pq, x, 0], 0] && !MatchQ[Pq, x^(m_)*(u_)] /; IntegerQ[m]

```

Rule 1836

```

Int[(Pq_)*((c_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
+ 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
+ 1)/(2*n)]) /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]

```

Rule 1488

```

Int[((f_)*(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_))^(p_)*
(d_) + (e_)*(x_)^(n_))^(q_), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]

```

Rule 292

```

Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]

```

Rule 31

```

Int[((a_) + (b_)*(x_))^(p_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]

```


Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7 (c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^2} dx &= \frac{a(b^3c - ab^2d + a^2be - a^3f)x^2}{3b^5(a + bx^3)} - \frac{\int \frac{2a^2b(b^3c - ab^2d + a^2be - a^3f)x - 3ab^2(b^3c - ab^2d + a^2be - a^3f)x^4 - 3ab^3(b^2c - ab^2d + a^2be - a^3f)x^7}{a + bx^3} dx}{3ab^6} \\
&= \frac{a(b^3c - ab^2d + a^2be - a^3f)x^2}{3b^5(a + bx^3)} - \frac{\int \frac{x(2a^2b(b^3c - ab^2d + a^2be - a^3f) - 3ab^2(b^3c - ab^2d + a^2be - a^3f)x^3 - 3ab^3(b^2c - ab^2d + a^2be - a^3f)x^6)}{a + bx^3} dx}{3ab^6} \\
&= \frac{fx^{11}}{11b^2} + \frac{a(b^3c - ab^2d + a^2be - a^3f)x^2}{3b^5(a + bx^3)} - \frac{\int \frac{x(22a^2b^2(b^3c - ab^2d + a^2be - a^3f) - 33ab^3(b^3c - ab^2d + a^2be - a^3f)x^3 - 33ab^4(b^2c - ab^2d + a^2be - a^3f)x^6)}{a + bx^3} dx}{33ab^7} \\
&= \frac{(be - 2af)x^8}{8b^3} + \frac{fx^{11}}{11b^2} + \frac{a(b^3c - ab^2d + a^2be - a^3f)x^2}{3b^5(a + bx^3)} - \frac{\int \frac{x(176a^2b^3(b^3c - ab^2d + a^2be - a^3f) - 264ab^4(b^2c - ab^2d + a^2be - a^3f)x^3 - 264ab^5(b^2c - ab^2d + a^2be - a^3f)x^6)}{a + bx^3} dx}{264ab^8} \\
&= \frac{(be - 2af)x^8}{8b^3} + \frac{fx^{11}}{11b^2} + \frac{a(b^3c - ab^2d + a^2be - a^3f)x^2}{3b^5(a + bx^3)} - \frac{\int \left(-264ab^3(b^3c - 2ab^2d + 3a^2be - 4a^3f)x^2 + \dots \right)}{264ab^8} \\
&= \frac{(b^3c - 2ab^2d + 3a^2be - 4a^3f)x^2}{2b^5} + \frac{(b^2d - 2abe + 3a^2f)x^5}{5b^4} + \frac{(be - 2af)x^8}{8b^3} + \frac{fx^{11}}{11b^2} + \frac{a}{11b^2} \\
&= \frac{(b^3c - 2ab^2d + 3a^2be - 4a^3f)x^2}{2b^5} + \frac{(b^2d - 2abe + 3a^2f)x^5}{5b^4} + \frac{(be - 2af)x^8}{8b^3} + \frac{fx^{11}}{11b^2} + \frac{a}{11b^2} \\
&= \frac{(b^3c - 2ab^2d + 3a^2be - 4a^3f)x^2}{2b^5} + \frac{(b^2d - 2abe + 3a^2f)x^5}{5b^4} + \frac{(be - 2af)x^8}{8b^3} + \frac{fx^{11}}{11b^2} + \frac{a}{11b^2} \\
&= \frac{(b^3c - 2ab^2d + 3a^2be - 4a^3f)x^2}{2b^5} + \frac{(b^2d - 2abe + 3a^2f)x^5}{5b^4} + \frac{(be - 2af)x^8}{8b^3} + \frac{fx^{11}}{11b^2} + \frac{a}{11b^2} \\
&= \frac{(b^3c - 2ab^2d + 3a^2be - 4a^3f)x^2}{2b^5} + \frac{(b^2d - 2abe + 3a^2f)x^5}{5b^4} + \frac{(be - 2af)x^8}{8b^3} + \frac{fx^{11}}{11b^2} + \frac{a}{11b^2}
\end{aligned}$$

Mathematica [A] time = 0.165613, size = 319, normalized size = 0.95

$$1980b^{2/3}x^2(3a^2be - 4a^3f - 2ab^2d + b^3c) + \frac{1320ab^{2/3}x^2(a^2be + a^3(-f) - ab^2d + b^3c)}{a + bx^3} + 220a^{2/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(-11a^2be$$

Antiderivative was successfully verified.

[In] Integrate[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]

[Out] (1980*b^(2/3)*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^2 + 792*b^(5/3)*(b^2*d - 2*a*b*e + 3*a^2*f)*x^5 + 495*b^(8/3)*(b*e - 2*a*f)*x^8 + 360*b^(11/3)*f*x^11 + (1320*a*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(a + b*x^3) - 440*sqrt[3]*a^(2/3)*(-5*b^3*c + 8*a*b^2*d - 11*a^2*b*e + 14*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]] - 440*a^(2/3)*(-5*b^3*c + 8*a*b^2*d - 11*a^2*b*e + 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 220*a^(2/3)*(-5*b^3*c + 8*a*b^2*d - 11*a^2*b*e + 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(3960*b^(17/3))

Maple [B] time = 0.01, size = 584, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)

[Out] -11/9*a^3/b^5*e*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+8/9*a^2/b^4*d*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/8/b^2*x^8*e+1/5/b^2*x^5*d+1/2/b^2*x^2*c-1/4/b^3*x^8*a*f+3/5/b^4*x^5*a^2*f-5/9*a/b^3*c*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+14/9*a^4/b^6*f*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+5/9*a/b^3*c/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-5/18*a/b^3*c/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-1/3*a^2/b^3*x^2/(b*x^3+a)*d+1/3*a/b^2*x^2/(b*x^3+a)*c-14/9*a^4/b^6*f/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+7/9*a^4/b^6*f/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+11/9*a^3/b^5*e/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-11/18*a^3/b^5*e/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-8/9*a^2/b^4*d/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+4/9*a^2/b^4*d/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-1/3*a^4/b^5*x^2/(b*x^3+a)*f+1/3*a^3/b^4*x^2/(b*x^3+a)*e+1/11*f*x^11/b^2-2/5/b^3*x^5*a*e-2/b^5*x^2*a^3*f+3/2/b^4*x^2*a^2*e-1/b^3*x^2*a*d

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.29785, size = 1057, normalized size = 3.16

$$360b^4fx^{14} + 45(11b^4e - 14ab^3f)x^{11} + 99(8b^4d - 11ab^3e + 14a^2b^2f)x^8 + 396(5b^4c - 8ab^3d + 11a^2b^2e - 14a^3bf)x^5 + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] $\frac{1}{3960} \cdot (360b^4fx^{14} + 45(11b^4e - 14a^2b^3f)x^{11} + 99(8b^4d - 11a^3b^3e + 14a^2b^2f)x^8 + 396(5b^4c - 8a^2b^3d + 11a^2b^2e - 14a^3b^3f)x^5 + 660(5a^2b^3c - 8a^2b^2d + 11a^3b^3e - 14a^4f)x^2 - 440\sqrt{3}(5a^2b^3c - 8a^2b^2d + 11a^3b^3e - 14a^4f + (5b^4c - 8a^2b^3d + 11a^2b^2e - 14a^3b^3f)x^3) \cdot (-a^2/b^2)^{1/3} \cdot \arctan(1/3 \cdot (2\sqrt{3}bx \cdot (-a^2/b^2)^{1/3} + \sqrt{3}a)/a) + 220(5a^2b^3c - 8a^2b^2d + 11a^3b^3e - 14a^4f + (5b^4c - 8a^2b^3d + 11a^2b^2e - 14a^3b^3f)x^3) \cdot (-a^2/b^2)^{1/3} \cdot \log(ax^2 - bx \cdot (-a^2/b^2)^{2/3} - a \cdot (-a^2/b^2)^{1/3}) - 440(5a^2b^3c - 8a^2b^2d + 11a^3b^3e - 14a^4f + (5b^4c - 8a^2b^3d + 11a^2b^2e - 14a^3b^3f)x^3) \cdot (-a^2/b^2)^{1/3} \cdot \log(ax + b \cdot (-a^2/b^2)^{2/3})) / (b^6x^3 + ab^5)$

Sympy [A] time = 37.4541, size = 530, normalized size = 1.58

$$-\frac{x^2(a^4f - a^3be + a^2b^2d - ab^3c)}{3ab^5 + 3b^6x^3} + \text{RootSum}\left(729t^3b^{17} + 2744a^{11}f^3 - 6468a^{10}bef^2 + 4704a^9b^2df^2 + 5082a^9b^2e^2f - 29\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)

[Out] $-x^{**2} \cdot (a^{**4}f - a^{**3}b^*e + a^{**2}b^{**2}d - a*b^{**3}c) / (3a*b^{**5} + 3b^{**6}x^{**3}) + \text{RootSum}(729*_t^{**3}b^{**17} + 2744*a^{**11}f^{**3} - 6468*a^{**10}b^*e*f^{**2} + 4704*a$

```

**9*b**2*d*f**2 + 5082*a**9*b**2*e**2*f - 2940*a**8*b**3*c*f**2 - 7392*a**8
*b**3*d*e*f - 1331*a**8*b**3*e**3 + 4620*a**7*b**4*c*e*f + 2688*a**7*b**4*d
**2*f + 2904*a**7*b**4*d*e**2 - 3360*a**6*b**5*c*d*f - 1815*a**6*b**5*c*e**
2 - 2112*a**6*b**5*d**2*e + 1050*a**5*b**6*c**2*f + 2640*a**5*b**6*c*d*e +
512*a**5*b**6*d**3 - 825*a**4*b**7*c**2*e - 960*a**4*b**7*c*d**2 + 600*a**3
*b**8*c**2*d - 125*a**2*b**9*c**3, Lambda(_t, _t*log(81*_t**2*b**11/(196*a
**7*f**2 - 308*a**6*b*e*f + 224*a**5*b**2*d*f + 121*a**5*b**2*e**2 - 140*a**
4*b**3*c*f - 176*a**4*b**3*d*e + 110*a**3*b**4*c*e + 64*a**3*b**4*d**2 - 80
*a**2*b**5*c*d + 25*a*b**6*c**2) + x))) + f*x**11/(11*b**2) - x**8*(2*a*f -
b*e)/(8*b**3) + x**5*(3*a**2*f - 2*a*b*e + b**2*d)/(5*b**4) - x**2*(4*a**3
*f - 3*a**2*b*e + 2*a*b**2*d - b**3*c)/(2*b**5)

```

Giac [A] time = 1.08749, size = 597, normalized size = 1.78

$$\frac{\left(5ab^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 8a^2b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 14a^4f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 11a^3b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right) + \sqrt{3}\left(5(-ab^2)^{\frac{2}{3}}b^3c - 8(-ab^2)^{\frac{2}{3}}b^3d\right)}{9ab^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")

```

[Out] 1/9*(5*a*b^3*c*(-a/b)^(1/3) - 8*a^2*b^2*d*(-a/b)^(1/3) - 14*a^4*f*(-a/b)^(1
/3) + 11*a^3*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b
^5) + 1/9*sqrt(3)*(5*(-a*b^2)^(2/3)*b^3*c - 8*(-a*b^2)^(2/3)*a*b^2*d - 14*(
-a*b^2)^(2/3)*a^3*f + 11*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x +
(-a/b)^(1/3))/(-a/b)^(1/3))/b^7 + 1/3*(a*b^3*c*x^2 - a^2*b^2*d*x^2 - a^4*f*x
^2 + a^3*b*x^2*e)/((b*x^3 + a)*b^5) - 1/18*(5*(-a*b^2)^(2/3)*b^3*c - 8*(-a
*b^2)^(2/3)*a*b^2*d - 14*(-a*b^2)^(2/3)*a^3*f + 11*(-a*b^2)^(2/3)*a^2*b*e)*
log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^7 + 1/440*(40*b^20*f*x^11 - 110*
a*b^19*f*x^8 + 55*b^20*x^8*e + 88*b^20*d*x^5 + 264*a^2*b^18*f*x^5 - 176*a*b
^19*x^5*e + 220*b^20*c*x^2 - 440*a*b^19*d*x^2 - 880*a^3*b^17*f*x^2 + 660*a^
2*b^18*x^2*e)/b^22

```

$$3.262 \quad \int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=328

$$\frac{ax(a^2be + a^3(-f) - ab^2d + b^3c)}{3b^5(a + bx^3)} + \frac{\sqrt[3]{a} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(10a^2be - 13a^3f - 7ab^2d + 4b^3c)}{18b^{16/3}} + \frac{x(3a^2be - 4a^3f - b^3c)}{b^5}$$

[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x)/b^5 + ((b^2*d - 2*a*b*e + 3*a^2*f)*x^4)/(4*b^4) + ((b*e - 2*a*f)*x^7)/(7*b^3) + (f*x^10)/(10*b^2) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*b^5*(a + b*x^3)) + (a^(1/3)*(4*b^3*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*b^(16/3)) - (a^(1/3)*(4*b^3*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(16/3)) + (a^(1/3)*(4*b^3*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*b^(16/3))

Rubi [A] time = 0.368865, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1828, 1887, 200, 31, 634, 617, 204, 628}

$$\frac{ax(a^2be + a^3(-f) - ab^2d + b^3c)}{3b^5(a + bx^3)} + \frac{\sqrt[3]{a} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(10a^2be - 13a^3f - 7ab^2d + 4b^3c)}{18b^{16/3}} + \frac{x(3a^2be - 4a^3f - b^3c)}{b^5}$$

Antiderivative was successfully verified.

[In] Int[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]

[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x)/b^5 + ((b^2*d - 2*a*b*e + 3*a^2*f)*x^4)/(4*b^4) + ((b*e - 2*a*f)*x^7)/(7*b^3) + (f*x^10)/(10*b^2) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*b^5*(a + b*x^3)) + (a^(1/3)*(4*b^3*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*b^(16/3)) - (a^(1/3)*(4*b^3*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(16/3)) + (a^(1/3)*(4*b^3*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*b^(16/3))

Rule 1828

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q =
m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
+ b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 200

```
Int[((a_) + (b_)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(p_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

a, 0] || LtQ[b, 0])

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x^6 (c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^2} dx &= \frac{a(b^3c - ab^2d + a^2be - a^3f)x}{3b^5(a + bx^3)} - \int \frac{a^2(b^3c - ab^2d + a^2be - a^3f) - 3ab(b^3c - ab^2d + a^2be - a^3f)x^3 - 3ab^2(b^2d - abe - a^2f)x^6}{(a + bx^3)^3} dx \\ &= \frac{a(b^3c - ab^2d + a^2be - a^3f)x}{3b^5(a + bx^3)} - \int \frac{(-3a(b^3c - 2ab^2d + 3a^2be - 4a^3f) - 3ab(b^2d - 2abe + 3a^2f))x^3 + (-3a^2be + 3a^3f - 3ab^2d + 3a^2be - 4a^3f)x^6}{(a + bx^3)^3} dx \\ &= \frac{(b^3c - 2ab^2d + 3a^2be - 4a^3f)x}{b^5} + \frac{(b^2d - 2abe + 3a^2f)x^4}{4b^4} + \frac{(be - 2af)x^7}{7b^3} + \frac{fx^{10}}{10b^2} + \frac{a}{10b^2} \\ &= \frac{(b^3c - 2ab^2d + 3a^2be - 4a^3f)x}{b^5} + \frac{(b^2d - 2abe + 3a^2f)x^4}{4b^4} + \frac{(be - 2af)x^7}{7b^3} + \frac{fx^{10}}{10b^2} + \frac{a}{10b^2} \\ &= \frac{(b^3c - 2ab^2d + 3a^2be - 4a^3f)x}{b^5} + \frac{(b^2d - 2abe + 3a^2f)x^4}{4b^4} + \frac{(be - 2af)x^7}{7b^3} + \frac{fx^{10}}{10b^2} + \frac{a}{10b^2} \\ &= \frac{(b^3c - 2ab^2d + 3a^2be - 4a^3f)x}{b^5} + \frac{(b^2d - 2abe + 3a^2f)x^4}{4b^4} + \frac{(be - 2af)x^7}{7b^3} + \frac{fx^{10}}{10b^2} + \frac{a}{10b^2} \\ &= \frac{(b^3c - 2ab^2d + 3a^2be - 4a^3f)x}{b^5} + \frac{(b^2d - 2abe + 3a^2f)x^4}{4b^4} + \frac{(be - 2af)x^7}{7b^3} + \frac{fx^{10}}{10b^2} + \frac{a}{10b^2} \end{aligned}$$

Mathematica [A] time = 0.248407, size = 315, normalized size = 0.96

$$\frac{420a\sqrt[3]{bx}(a^2be + a^3(-f) - ab^2d + b^3c)}{a + bx^3} - 70\sqrt[3]{a} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2) (-10a^2be + 13a^3f + 7ab^2d - 4b^3c) + 1260\sqrt[3]{bx}(3a^2be - 4a^3f)$$

Antiderivative was successfully verified.

[In] Integrate[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]

[Out] (1260*b^(1/3)*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x + 315*b^(4/3)*(b^2*d - 2*a*b*e + 3*a^2*f)*x^4 + 180*b^(7/3)*(b*e - 2*a*f)*x^7 + 126*b^(10/3)*f*x^10 + (420*a*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(a + b*x^3) - 140*sqrt[3]*a^(1/3)*(-4*b^3*c + 7*a*b^2*d - 10*a^2*b*e + 13*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]] + 140*a^(1/3)*(-4*b^3*c + 7*a*b^2*d - 10*a^2*b*e + 13*a^3*f)*Log[a^(1/3) + b^(1/3)*x] - 70*a^(1/3)*(-4*b^3*c + 7*a*b^2*d - 10*a^2*b*e + 13*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(1260*b^(16/3))

Maple [B] time = 0.01, size = 567, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)

[Out] 5/9*a^3/b^5*e/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+7/9*a^2/b^4*d/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+1/7/b^2*x^7*e+1/4/b^2*x^4*d+1/b^2*c*x+7/9*a^2/b^4*d/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/10*f*x^10/b^2+1/3*a/b^2*x/(b*x^3+a)*c+13/9*a^4/b^6*f/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-13/18*a^4/b^6*f/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-10/9*a^3/b^5*e/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-7/18*a^2/b^4*d/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-4/9*a/b^3*c/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+2/9*a/b^3*c/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-1/3*a^4/b^5*x/(b*x^3+a)*f+1/3*a^3/b^4*x/(b*x^3+a)*e-1/3*a^2/b^3*x/(b*x^3+a)*d-2/b^3*a*d*x-2/7/b^3*x^7*a*f+3/4/b^4*x^4*a^2*f-1/2/b^3*x^4*a*e-4/b^5*a^3*f*x+3/b^4*a^2*e*x-4/9*a/b^3*c/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+13/9*a^4/b^6*f/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-10/9*a^3/b^5*e/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36585, size = 992, normalized size = 3.02

$$126b^4fx^{13} + 18(10b^4e - 13ab^3f)x^{10} + 45(7b^4d - 10ab^3e + 13a^2b^2f)x^7 + 315(4b^4c - 7ab^3d + 10a^2b^2e - 13a^3bf)x^4 -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/1260*(126*b^4*f*x^13 + 18*(10*b^4*e - 13*a*b^3*f)*x^10 + 45*(7*b^4*d - 10*
*a*b^3*e + 13*a^2*b^2*f)*x^7 + 315*(4*b^4*c - 7*a*b^3*d + 10*a^2*b^2*e - 13*
*a^3*b*f)*x^4 - 140*sqrt(3)*(4*a*b^3*c - 7*a^2*b^2*d + 10*a^3*b*e - 13*a^4*
f + (4*b^4*c - 7*a*b^3*d + 10*a^2*b^2*e - 13*a^3*b*f)*x^3)*(a/b)^(1/3)*arct
an(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 70*(4*a*b^3*c - 7*a^2*b
^2*d + 10*a^3*b*e - 13*a^4*f + (4*b^4*c - 7*a*b^3*d + 10*a^2*b^2*e - 13*a^3
*b*f)*x^3)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 140*(4*a*b^
3*c - 7*a^2*b^2*d + 10*a^3*b*e - 13*a^4*f + (4*b^4*c - 7*a*b^3*d + 10*a^2*b
^2*e - 13*a^3*b*f)*x^3)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 420*(4*a*b^3*c -
7*a^2*b^2*d + 10*a^3*b*e - 13*a^4*f)*x)/(b^6*x^3 + a*b^5)
```

Sympy [A] time = 10.9149, size = 440, normalized size = 1.34

$$-\frac{x(a^4f - a^3be + a^2b^2d - ab^3c)}{3ab^5 + 3b^6x^3} + \text{RootSum}\left(729t^3b^{16} - 2197a^{10}f^3 + 5070a^9bef^2 - 3549a^8b^2df^2 - 3900a^8b^2e^2f + 2028a^7b^3c^2f^2 - 5460a^7b^3cde^2f + 1000a^7b^3e^3 - 3120a^6b^4c^2ef - 1911a^6b^4d^2e^2f + 1000a^6b^4de^3 - 3120a^6b^4c^2ef - 1911a^6b^4d^2e^2f + 1000a^6b^4de^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] -x*(a**4*f - a**3*b*e + a**2*b**2*d - a*b**3*c)/(3*a*b**5 + 3*b**6*x**3) +
RootSum(729*_t**3*b**16 - 2197*a**10*f**3 + 5070*a**9*b*e*f**2 - 3549*a**8*
b**2*d*f**2 - 3900*a**8*b**2*e**2*f + 2028*a**7*b**3*c*f**2 + 5460*a**7*b**
3*d*e*f + 1000*a**7*b**3*e**3 - 3120*a**6*b**4*c*e*f - 1911*a**6*b**4*d**2*
```

$f - 2100a^{**6}b^{**4}d^{**e**2} + 2184a^{**5}b^{**5}c^{**d}f + 1200a^{**5}b^{**5}c^{**e**2} +$
 $1470a^{**5}b^{**5}d^{**2}e - 624a^{**4}b^{**6}c^{**2}f - 1680a^{**4}b^{**6}c^{**d}e - 343a^{**4}b^{**6}d^{**3}$
 $+ 480a^{**3}b^{**7}c^{**2}e + 588a^{**3}b^{**7}c^{**d**2} - 336a^{**2}b^{**8}$
 $c^{**2}d + 64a*b^{**9}c^{**3}, \text{Lambda}(_t, _t*\log(9*_t*b^{**5}/(13*a^{**3}f - 10*a^{**2}$
 $b*e + 7*a*b^{**2}d - 4*b^{**3}c) + x)) + f*x^{**10}/(10*b^{**2}) - x^{**7}*(2*a*f - b*e$
 $)/(7*b^{**3}) + x^{**4}*(3*a^{**2}f - 2*a*b*e + b^{**2}d)/(4*b^{**4}) - x*(4*a^{**3}f - 3*$
 $a^{**2}b*e + 2*a*b^{**2}d - b^{**3}c)/b^{**5}$

Giac [A] time = 1.0802, size = 532, normalized size = 1.62

$$\frac{\sqrt{3}\left(4(-ab^2)^{\frac{1}{3}}b^3c - 7(-ab^2)^{\frac{1}{3}}ab^2d - 13(-ab^2)^{\frac{1}{3}}a^3f + 10(-ab^2)^{\frac{1}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right) + (4ab^3c - 7a^2b^2d)}{9b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")

[Out] $-1/9*\sqrt{3}*(4*(-a*b^2)^{(1/3)}*b^3*c - 7*(-a*b^2)^{(1/3)}*a*b^2*d - 13*(-a*b^2)^{(1/3)}*a^3*f + 10*(-a*b^2)^{(1/3)}*a^2*b*e)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/b^6 + 1/9*(4*a*b^3*c - 7*a^2*b^2*d - 13*a^4*f + 10*a^3*b*e)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/(a*b^5) - 1/18*(4*(-a*b^2)^{(1/3)}*b^3*c - 7*(-a*b^2)^{(1/3)}*a*b^2*d - 13*(-a*b^2)^{(1/3)}*a^3*f + 10*(-a*b^2)^{(1/3)}*a^2*b*e)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/b^6 + 1/3*(a*b^3*c*x - a^2*b^2*d*x - a^4*f*x + a^3*b*x*e)/((b*x^3 + a)*b^5) + 1/140*(14*b^18*f*x^10 - 40*a*b^17*f*x^7 + 20*b^18*d*x^7*e + 35*b^18*d*x^4 + 105*a^2*b^16*f*x^4 - 70*a*b^17*x^4*e + 140*b^18*c*x - 280*a*b^17*d*x - 560*a^3*b^15*f*x + 420*a^2*b^16*x*e)/b^20$

$$3.263 \quad \int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=298

$$-\frac{x^2(a^2be + a^3(-f) - ab^2d + b^3c)}{3b^4(a + bx^3)} + \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(8a^2be - 11a^3f - 5ab^2d + 2b^3c)}{18\sqrt[3]{ab^{14/3}}} - \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(8a^2be - 11a^3f - 5ab^2d + 2b^3c)}{18\sqrt[3]{ab^{14/3}}}$$

[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x^2)/(2*b^4) + ((b*e - 2*a*f)*x^5)/(5*b^3) + (f*x^8)/(8*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*b^4*(a + b*x^3)) - ((2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*b^(14/3)) - ((2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(1/3)*b^(14/3)) + ((2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(1/3)*b^(14/3))

Rubi [A] time = 0.462677, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1828, 1851, 1836, 1488, 292, 31, 634, 617, 204, 628}

$$-\frac{x^2(a^2be + a^3(-f) - ab^2d + b^3c)}{3b^4(a + bx^3)} + \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(8a^2be - 11a^3f - 5ab^2d + 2b^3c)}{18\sqrt[3]{ab^{14/3}}} - \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(8a^2be - 11a^3f - 5ab^2d + 2b^3c)}{18\sqrt[3]{ab^{14/3}}}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]

[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x^2)/(2*b^4) + ((b*e - 2*a*f)*x^5)/(5*b^3) + (f*x^8)/(8*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*b^4*(a + b*x^3)) - ((2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*b^(14/3)) - ((2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(1/3)*b^(14/3)) + ((2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(1/3)*b^(14/3))

Rule 1828

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)

```
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
+ b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1851

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[x*PolynomialQuot
ient[Pq, x, x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x]
&& EqQ[Coeff[Pq, x, 0], 0] && !MatchQ[Pq, x^(m_)*(u_)] /; IntegerQ[m]
```

Rule 1836

```
Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
+ 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
+ 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1488

```
Int[((f_)*(x_)^(m_))*((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_))^(p_)*(
(d_) + (e_)*(x_)^(n_))^(q_), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4(c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^2} dx &= -\frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3b^4(a + bx^3)} - \frac{\int \frac{-2ab(b^3c - ab^2d + a^2be - a^3f)x - 3ab^2(b^2d - abe + a^2f)x^4 - 3ab^3(be - af)}{a + bx^3}}{3ab^5} \\
&= -\frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3b^4(a + bx^3)} - \frac{\int \frac{x(-2ab(b^3c - ab^2d + a^2be - a^3f) - 3ab^2(b^2d - abe + a^2f)x^3 - 3ab^3(be - af))}{a + bx^3}}{3ab^5} \\
&= \frac{fx^8}{8b^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3b^4(a + bx^3)} - \frac{\int \frac{x(-16ab^2(b^3c - ab^2d + a^2be - a^3f) - 24ab^3(b^2d - abe + a^2f)x^3 - 24ab^3(be - af))}{a + bx^3}}{24ab^6} \\
&= \frac{fx^8}{8b^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3b^4(a + bx^3)} - \frac{\int \left(-24ab^2(b^2d - 2abe + 3a^2f)x - 24ab^3(be - af) \right)}{24ab^6} \\
&= \frac{(b^2d - 2abe + 3a^2f)x^2}{2b^4} + \frac{(be - 2af)x^5}{5b^3} + \frac{fx^8}{8b^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3b^4(a + bx^3)} + \frac{(2b^3c - ab^2d + a^2be - a^3f)x^2}{2b^4} \\
&= \frac{(b^2d - 2abe + 3a^2f)x^2}{2b^4} + \frac{(be - 2af)x^5}{5b^3} + \frac{fx^8}{8b^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3b^4(a + bx^3)} - \frac{(2b^3c - ab^2d + a^2be - a^3f)x^2}{2b^4} \\
&= \frac{(b^2d - 2abe + 3a^2f)x^2}{2b^4} + \frac{(be - 2af)x^5}{5b^3} + \frac{fx^8}{8b^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3b^4(a + bx^3)} - \frac{(2b^3c - ab^2d + a^2be - a^3f)x^2}{2b^4} \\
&= \frac{(b^2d - 2abe + 3a^2f)x^2}{2b^4} + \frac{(be - 2af)x^5}{5b^3} + \frac{fx^8}{8b^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3b^4(a + bx^3)} - \frac{(2b^3c - ab^2d + a^2be - a^3f)x^2}{2b^4} \\
&= \frac{(b^2d - 2abe + 3a^2f)x^2}{2b^4} + \frac{(be - 2af)x^5}{5b^3} + \frac{fx^8}{8b^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3b^4(a + bx^3)} - \frac{(2b^3c - ab^2d + a^2be - a^3f)x^2}{2b^4}
\end{aligned}$$

Mathematica [A] time = 0.156751, size = 282, normalized size = 0.95

$$\frac{-\frac{120b^{2/3}x^2(a^2be + a^3(-f) - ab^2d + b^3c)}{a + bx^3} + \frac{20 \log\left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx + b^{2/3}x^2}\right)(8a^2be - 11a^3f - 5ab^2d + 2b^3c)}{\sqrt[3]{a}} + \frac{40 \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)(-8a^2be + 11a^3f + 5ab^2d - 2b^3c)}{\sqrt[3]{a}}}{360b^{14/3}} + \dots$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]

```
[Out] (180*b^(2/3)*(b^2*d - 2*a*b*e + 3*a^2*f)*x^2 + 72*b^(5/3)*(b*e - 2*a*f)*x^5
+ 45*b^(8/3)*f*x^8 - (120*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)
/(a + b*x^3) + (40*sqrt(3)*(-2*b^3*c + 5*a*b^2*d - 8*a^2*b*e + 11*a^3*f)*Ar
cTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt(3)])/a^(1/3) + (40*(-2*b^3*c + 5*a*b^
2*d - 8*a^2*b*e + 11*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + (20*(2*b^3*
c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/a^(1/3)/(360*b^(14/3))
```

Maple [B] time = 0.009, size = 529, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/8*f*x^8/b^2-2/5/b^3*x^5*a*f+1/5/b^2*x^5*e+3/2/b^4*x^2*a^2*f-1/b^3*x^2*a*e
+1/2/b^2*x^2*d+1/3/b^4*x^2/(b*x^3+a)*a^3*f-1/3/b^3*x^2/(b*x^3+a)*a^2*e+1/3/
b^2*x^2/(b*x^3+a)*a*d-1/3/b*x^2/(b*x^3+a)*c+11/9/b^5*a^3*f/(1/b*a)^(1/3)*ln
(x+(1/b*a)^(1/3))-11/18/b^5*a^3*f/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b
*a)^(2/3))-11/9/b^5*a^3*f*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*
a)^(1/3)*x-1))-8/9/b^4*a^2*e/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+4/9/b^4*a^2*
e/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+8/9/b^4*a^2*e*3^(1/2)
/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+5/9/b^3*a*d/(1/b*a
)^(1/3)*ln(x+(1/b*a)^(1/3))-5/18/b^3*a*d/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)
*x+(1/b*a)^(2/3))-5/9/b^3*a*d*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(
1/b*a)^(1/3)*x-1))-2/9/b^2*c/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/9/b^2*c/(1
/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2/9/b^2*c*3^(1/2)/(1/b*a
)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7054, size = 2094, normalized size = 7.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] [1/360*(45*a*b^5*f*x^11 + 9*(8*a*b^5*e - 11*a^2*b^4*f)*x^8 + 36*(5*a*b^5*d - 8*a^2*b^4*e + 11*a^3*b^3*f)*x^5 - 60*(2*a*b^5*c - 5*a^2*b^4*d + 8*a^3*b^3*e - 11*a^4*b^2*f)*x^2 - 60*sqrt(1/3)*(2*a^2*b^4*c - 5*a^3*b^3*d + 8*a^4*b^2*e - 11*a^5*b*f + (2*a*b^5*c - 5*a^2*b^4*d + 8*a^3*b^3*e - 11*a^4*b^2*f)*x^3)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) + 20*(2*a*b^3*c - 5*a^2*b^2*d + 8*a^3*b*e - 11*a^4*f + (2*b^4*c - 5*a*b^3*d + 8*a^2*b^2*e - 11*a^3*b*f)*x^3)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 40*(2*a*b^3*c - 5*a^2*b^2*d + 8*a^3*b*e - 11*a^4*f + (2*b^4*c - 5*a*b^3*d + 8*a^2*b^2*e - 11*a^3*b*f)*x^3)*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a*b^7*x^3 + a^2*b^6), 1/360*(45*a*b^5*f*x^11 + 9*(8*a*b^5*e - 11*a^2*b^4*f)*x^8 + 36*(5*a*b^5*d - 8*a^2*b^4*e + 11*a^3*b^3*f)*x^5 - 60*(2*a*b^5*c - 5*a^2*b^4*d + 8*a^3*b^3*e - 11*a^4*b^2*f)*x^2 - 120*sqrt(1/3)*(2*a^2*b^4*c - 5*a^3*b^3*d + 8*a^4*b^2*e - 11*a^5*b*f + (2*a*b^5*c - 5*a^2*b^4*d + 8*a^3*b^3*e - 11*a^4*b^2*f)*x^3)*sqrt((a*b^2)^(1/3)/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) + 20*(2*a*b^3*c - 5*a^2*b^2*d + 8*a^3*b*e - 11*a^4*f + (2*b^4*c - 5*a*b^3*d + 8*a^2*b^2*e - 11*a^3*b*f)*x^3)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 40*(2*a*b^3*c - 5*a^2*b^2*d + 8*a^3*b*e - 11*a^4*f + (2*b^4*c - 5*a*b^3*d + 8*a^2*b^2*e - 11*a^3*b*f)*x^3)*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a*b^7*x^3 + a^2*b^6)]

Sympy [A] time = 35.7815, size = 484, normalized size = 1.62

$$\frac{x^2(a^3f - a^2be + ab^2d - b^3c)}{3ab^4 + 3b^5x^3} + \text{RootSum}\left(729t^3ab^{14} - 1331a^9f^3 + 2904a^8bef^2 - 1815a^7b^2df^2 - 2112a^7b^2e^2f + 726a^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)

[Out] x**2*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*a*b**4 + 3*b**5*x**3) + RootSum(729*_t**3*a*b**14 - 1331*a**9*f**3 + 2904*a**8*b*e*f**2 - 1815*a**7*b*

```

*2*d*f**2 - 2112*a**7*b**2*e**2*f + 726*a**6*b**3*c*f**2 + 2640*a**6*b**3*d
*e*f + 512*a**6*b**3*e**3 - 1056*a**5*b**4*c*e*f - 825*a**5*b**4*d**2*f - 9
60*a**5*b**4*d*e**2 + 660*a**4*b**5*c*d*f + 384*a**4*b**5*c*e**2 + 600*a**4
*b**5*d**2*e - 132*a**3*b**6*c**2*f - 480*a**3*b**6*c*d*e - 125*a**3*b**6*d
**3 + 96*a**2*b**7*c**2*e + 150*a**2*b**7*c*d**2 - 60*a*b**8*c**2*d + 8*b**
9*c**3, Lambda(_t, _t*log(81*_t**2*a*b**9/(121*a**6*f**2 - 176*a**5*b*e*f +
110*a**4*b**2*d*f + 64*a**4*b**2*e**2 - 44*a**3*b**3*c*f - 80*a**3*b**3*d*
e + 32*a**2*b**4*c*e + 25*a**2*b**4*d**2 - 20*a*b**5*c*d + 4*b**6*c**2) + x
))) + f*x**8/(8*b**2) - x**5*(2*a*f - b*e)/(5*b**3) + x**2*(3*a**2*f - 2*a*
b*e + b**2*d)/(2*b**4)

```

Giac [A] time = 1.08374, size = 537, normalized size = 1.8

$$\frac{\left(2b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 5ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 11a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 8a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9ab^4} - \frac{b^3cx^2 - ab^2dx^2 - a^3fx^2 + a^2bx}{3(bx^3 + a)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```

[Out] -1/9*(2*b^3*c*(-a/b)^(1/3) - 5*a*b^2*d*(-a/b)^(1/3) - 11*a^3*f*(-a/b)^(1/3)
+ 8*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4)
- 1/3*(b^3*c*x^2 - a*b^2*d*x^2 - a^3*f*x^2 + a^2*b*x^2*e)/((b*x^3 + a)*b^4)
- 1/9*sqrt(3)*(2*(-a*b^2)^(2/3)*b^3*c - 5*(-a*b^2)^(2/3)*a*b^2*d - 11*(-a*
b^2)^(2/3)*a^3*f + 8*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/
b)^(1/3))/(-a/b)^(1/3))/(a*b^6) + 1/18*(2*(-a*b^2)^(2/3)*b^3*c - 5*(-a*b^2)
^(2/3)*a*b^2*d - 11*(-a*b^2)^(2/3)*a^3*f + 8*(-a*b^2)^(2/3)*a^2*b*e)*log(x^
2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^6) + 1/40*(5*b^14*f*x^8 - 16*a*b^13
*f*x^5 + 8*b^14*x^5*e + 20*b^14*d*x^2 + 60*a^2*b^12*f*x^2 - 40*a*b^13*x^2*e
)/b^16

```

$$3.264 \quad \int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=288

$$\frac{x(a^2be + a^3(-f) - ab^2d + b^3c)}{3b^4(a + bx^3)} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(7a^2be - 10a^3f - 4ab^2d + b^3c)}{18a^{2/3}b^{13/3}} + \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(7a^2be - 10a^3f - 4ab^2d + b^3c)}{9a^{2/3}b^{13/3}}$$

```
[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x)/b^4 + ((b*e - 2*a*f)*x^4)/(4*b^3) + (f*x^7)/(7*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*b^4*(a + b*x^3)) - ((b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*b^(13/3)) + ((b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(2/3)*b^(13/3)) - ((b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(2/3)*b^(13/3))
```

Rubi [A] time = 0.32561, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1828, 1887, 200, 31, 634, 617, 204, 628}

$$\frac{x(a^2be + a^3(-f) - ab^2d + b^3c)}{3b^4(a + bx^3)} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(7a^2be - 10a^3f - 4ab^2d + b^3c)}{18a^{2/3}b^{13/3}} + \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(7a^2be - 10a^3f - 4ab^2d + b^3c)}{9a^{2/3}b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x)/b^4 + ((b*e - 2*a*f)*x^4)/(4*b^3) + (f*x^7)/(7*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*b^4*(a + b*x^3)) - ((b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*b^(13/3)) + ((b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(2/3)*b^(13/3)) - ((b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(2/3)*b^(13/3))
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
```

```
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
+ b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 200

```
Int[((a_) + (b_)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x^3(c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^2} dx &= -\frac{(b^3c - ab^2d + a^2be - a^3f)x}{3b^4(a + bx^3)} - \int \frac{-a(b^3c - ab^2d + a^2be - a^3f) - 3ab(b^2d - abe + a^2f)x^3 - 3ab^2(be - af)x^6 - 3ab^3fx^9}{a + bx^3} dx \\ &= -\frac{(b^3c - ab^2d + a^2be - a^3f)x}{3b^4(a + bx^3)} - \frac{\int (-3a(b^2d - 2abe + 3a^2f) - 3ab(be - 2af)x^3 - 3ab^2fx^6 - 3ab^3fx^9) dx}{3ab^4} \\ &= \frac{(b^2d - 2abe + 3a^2f)x}{b^4} + \frac{(be - 2af)x^4}{4b^3} + \frac{fx^7}{7b^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3b^4(a + bx^3)} + \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3b^4(a + bx^3)} \\ &= \frac{(b^2d - 2abe + 3a^2f)x}{b^4} + \frac{(be - 2af)x^4}{4b^3} + \frac{fx^7}{7b^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3b^4(a + bx^3)} + \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3b^4(a + bx^3)} \\ &= \frac{(b^2d - 2abe + 3a^2f)x}{b^4} + \frac{(be - 2af)x^4}{4b^3} + \frac{fx^7}{7b^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3b^4(a + bx^3)} + \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3b^4(a + bx^3)} \\ &= \frac{(b^2d - 2abe + 3a^2f)x}{b^4} + \frac{(be - 2af)x^4}{4b^3} + \frac{fx^7}{7b^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3b^4(a + bx^3)} + \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3b^4(a + bx^3)} \\ &= \frac{(b^2d - 2abe + 3a^2f)x}{b^4} + \frac{(be - 2af)x^4}{4b^3} + \frac{fx^7}{7b^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3b^4(a + bx^3)} + \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3b^4(a + bx^3)} \end{aligned}$$

Mathematica [A] time = 0.157155, size = 277, normalized size = 0.96

$$\frac{84 \sqrt[3]{bx}(a^2be + a^3(-f) - ab^2d + b^3c)}{a + bx^3} + \frac{14 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx + b^{2/3}x^2})(-7a^2be + 10a^3f + 4ab^2d - b^3c)}{a^{2/3}} + \frac{28 \log(\sqrt[3]{a} + \sqrt[3]{bx})(7a^2be - 10a^3f - 4ab^2d + b^3c)}{a^{2/3}} + \frac{28 \sqrt[3]{ta}}{252b^{13/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]

[Out] $(252*b^{(1/3)}*(b^2*d - 2*a*b*e + 3*a^2*f)*x + 63*b^{(4/3)}*(b*e - 2*a*f)*x^4 + 36*b^{(7/3)}*f*x^7 - (84*b^{(1/3)}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(a + b*x^3) + (28*\sqrt{3}*(-(b^3*c) + 4*a*b^2*d - 7*a^2*b*e + 10*a^3*f)*\text{ArcTan}[(1 - (2*b^{(1/3)}*x)/a^{(1/3)})/\sqrt{3}])/a^{(2/3)} + (28*(b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*\text{Log}[a^{(1/3)} + b^{(1/3)}*x])/a^{(2/3)} + (14*(-(b^3*c) + 4*a*b^2*d - 7*a^2*b*e + 10*a^3*f)*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/a^{(2/3)})/(252*b^{(13/3)})$

Maple [B] time = 0.01, size = 514, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)

[Out] $1/7*f*x^7/b^2 - 1/2/b^3*x^4*a*f + 1/4/b^2*x^4*e + 3/b^4*a^2*f*x - 2/b^3*a*e*x + 1/b^2*d*x + 1/3/b^4*x/(b*x^3+a)*a^3*f - 1/3/b^3*x/(b*x^3+a)*a^2*e + 1/3/b^2*x/(b*x^3+a)*a*d - 1/3/b*x/(b*x^3+a)*c - 10/9/b^5*a^3*f/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)}) + 5/9/b^5*a^3*f/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)}) - 10/9/b^5*a^3*f/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1)) + 7/9/b^4*a^2*e/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)}) - 7/18/b^4*a^2*e/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)}) + 7/9/b^4*a^2*e/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1)) - 4/9/b^3*a*d/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)}) + 2/9/b^3*a*d/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)}) - 4/9/b^3*a*d/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1)) + 1/9/b^2*c/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)}) - 1/18/b^2*c/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)}) + 1/9/b^2*c/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.44706, size = 2125, normalized size = 7.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/252*(36*a^2*b^4*f*x^{10} + 9*(7*a^2*b^4*e - 10*a^3*b^3*f)*x^7 + 63*(4*a^2*b^4*d - 7*a^3*b^3*e + 10*a^4*b^2*f)*x^4 - 42*\sqrt{1/3}*(a^2*b^4*c - 4*a^3*b^3*d + 7*a^4*b^2*e - 10*a^5*b*f + (a*b^5*c - 4*a^2*b^4*d + 7*a^3*b^3*e - 10*a^4*b^2*f)*x^3) * \sqrt{(-a^2*b)^{(1/3)}/b} * \log((2*a*b*x^3 + 3*(-a^2*b)^{(1/3)}*a*x - a^2 - 3*\sqrt{1/3}*(2*a*b*x^2 + (-a^2*b)^{(2/3)}*x + (-a^2*b)^{(1/3)}*a)*\sqrt{(-a^2*b)^{(1/3)}/b})) / (b*x^3 + a)) - 14*(a*b^3*c - 4*a^2*b^2*d + 7*a^3*b*e - 10*a^4*f + (b^4*c - 4*a*b^3*d + 7*a^2*b^2*e - 10*a^3*b*f)*x^3) * (-a^2*b)^{(2/3)} * \log(a*b*x^2 - (-a^2*b)^{(2/3)}*x - (-a^2*b)^{(1/3)}*a) + 28*(a*b^3*c - 4*a^2*b^2*d + 7*a^3*b*e - 10*a^4*f + (b^4*c - 4*a*b^3*d + 7*a^2*b^2*e - 10*a^3*b*f)*x^3) * (-a^2*b)^{(2/3)} * \log(a*b*x + (-a^2*b)^{(2/3)}) - 84*(a^2*b^4*c - 4*a^3*b^3*d + 7*a^4*b^2*e - 10*a^5*b*f)*x / (a^2*b^6*x^3 + a^3*b^5), 1/252*(36*a^2*b^4*f*x^{10} + 9*(7*a^2*b^4*e - 10*a^3*b^3*f)*x^7 + 63*(4*a^2*b^4*d - 7*a^3*b^3*e + 10*a^4*b^2*f)*x^4 + 84*\sqrt{1/3}*(a^2*b^4*c - 4*a^3*b^3*d + 7*a^4*b^2*e - 10*a^5*b*f + (a*b^5*c - 4*a^2*b^4*d + 7*a^3*b^3*e - 10*a^4*b^2*f)*x^3) * \sqrt{-(-a^2*b)^{(1/3)}/b} * \arctan(\sqrt{1/3}*(2*(-a^2*b)^{(2/3)}*x + (-a^2*b)^{(1/3)}*a) * \sqrt{-(-a^2*b)^{(1/3)}/b} / a^2) - 14*(a*b^3*c - 4*a^2*b^2*d + 7*a^3*b*e - 10*a^4*f + (b^4*c - 4*a*b^3*d + 7*a^2*b^2*e - 10*a^3*b*f)*x^3) * (-a^2*b)^{(2/3)} * \log(a*b*x^2 - (-a^2*b)^{(2/3)}*x - (-a^2*b)^{(1/3)}*a) + 28*(a*b^3*c - 4*a^2*b^2*d + 7*a^3*b*e - 10*a^4*f + (b^4*c - 4*a*b^3*d + 7*a^2*b^2*e - 10*a^3*b*f)*x^3) * (-a^2*b)^{(2/3)} * \log(a*b*x + (-a^2*b)^{(2/3)}) - 84*(a^2*b^4*c - 4*a^3*b^3*d + 7*a^4*b^2*e - 10*a^5*b*f)*x / (a^2*b^6*x^3 + a^3*b^5)] \end{aligned}$$

Sympy [A] time = 10.1333, size = 398, normalized size = 1.38

$$\frac{x(a^3f - a^2be + ab^2d - b^3c)}{3ab^4 + 3b^5x^3} + \text{RootSum}\left(729t^3a^2b^{13} + 1000a^9f^3 - 2100a^8bef^2 + 1200a^7b^2df^2 + 1470a^7b^2e^2f - 300a^6b^3e^2f^2 - 300a^6b^3e^2f^2 - 300a^6b^3e^2f^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)

```
[Out] x*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*a*b**4 + 3*b**5*x**3) + RootSu
m(729*_t**3*a**2*b**13 + 1000*a**9*f**3 - 2100*a**8*b*e*f**2 + 1200*a**7*b*
*2*d*f**2 + 1470*a**7*b**2*e**2*f - 300*a**6*b**3*c*f**2 - 1680*a**6*b**3*d
*e*f - 343*a**6*b**3*e**3 + 420*a**5*b**4*c*e*f + 480*a**5*b**4*d**2*f + 58
8*a**5*b**4*d*e**2 - 240*a**4*b**5*c*d*f - 147*a**4*b**5*c*e**2 - 336*a**4*
b**5*d**2*e + 30*a**3*b**6*c**2*f + 168*a**3*b**6*c*d*e + 64*a**3*b**6*d**3
- 21*a**2*b**7*c**2*e - 48*a**2*b**7*c*d**2 + 12*a*b**8*c**2*d - b**9*c**3
, Lambda(_t, _t*log(-9*_t*a*b**4/(10*a**3*f - 7*a**2*b*e + 4*a*b**2*d - b**
3*c) + x))) + f*x**7/(7*b**2) - x**4*(2*a*f - b*e)/(4*b**3) + x*(3*a**2*f -
2*a*b*e + b**2*d)/b**4
```

Giac [A] time = 1.10032, size = 471, normalized size = 1.64

$$\frac{(b^3c - 4ab^2d - 10a^3f + 7a^2be)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9ab^4} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c - 4\left(-ab^2\right)^{\frac{1}{3}}ab^2d - 10\left(-ab^2\right)^{\frac{1}{3}}a^3f + 7\left(-ab^2\right)^{\frac{1}{3}}a^2be\right)}{9ab^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(b^3*c - 4*a*b^2*d - 10*a^3*f + 7*a^2*b*e)*(-a/b)^(1/3)*log(abs(x - (-
a/b)^(1/3)))/(a*b^4) + 1/9*sqrt(3)*((-a*b^2)^(1/3)*b^3*c - 4*(-a*b^2)^(1/3)
*a*b^2*d - 10*(-a*b^2)^(1/3)*a^3*f + 7*(-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*s
qrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^5) - 1/3*(b^3*c*x - a*b^2*d*
x - a^3*f*x + a^2*b*x*e)/((b*x^3 + a)*b^4) + 1/18*((-a*b^2)^(1/3)*b^3*c - 4
*(-a*b^2)^(1/3)*a*b^2*d - 10*(-a*b^2)^(1/3)*a^3*f + 7*(-a*b^2)^(1/3)*a^2*b*
e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^5) + 1/28*(4*b^12*f*x^7 -
14*a*b^11*f*x^4 + 7*b^12*x^4*e + 28*b^12*d*x + 84*a^2*b^10*f*x - 56*a*b^11*
x*e)/b^14
```


$$3.265 \quad \int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=271

$$\frac{x^2(a^2be + a^3(-f) - ab^2d + b^3c)}{3ab^3(a + bx^3)} + \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(-5a^2be + 8a^3f + 2ab^2d + b^3c)}{18a^{4/3}b^{11/3}} - \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(-5a^2be + a^3(-f) - ab^2d + b^3c)}{9a^{4/3}b^{11/3}}$$

[Out] $((b*e - 2*a*f)*x^2)/(2*b^3) + (f*x^5)/(5*b^2) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*a*b^3*(a + b*x^3)) - ((b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(4/3)*b^(11/3)) - ((b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*Log[a^(1/3) + b^(1/3)*x]/(9*a^(4/3)*b^(11/3)) + ((b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(18*a^(4/3)*b^(11/3))$

Rubi [A] time = 0.289389, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1828, 1594, 1488, 292, 31, 634, 617, 204, 628}

$$\frac{x^2(a^2be + a^3(-f) - ab^2d + b^3c)}{3ab^3(a + bx^3)} + \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(-5a^2be + 8a^3f + 2ab^2d + b^3c)}{18a^{4/3}b^{11/3}} - \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(-5a^2be + a^3(-f) - ab^2d + b^3c)}{9a^{4/3}b^{11/3}}$$

Antiderivative was successfully verified.

[In] Int[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]

[Out] $((b*e - 2*a*f)*x^2)/(2*b^3) + (f*x^5)/(5*b^2) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*a*b^3*(a + b*x^3)) - ((b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(4/3)*b^(11/3)) - ((b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*Log[a^(1/3) + b^(1/3)*x]/(9*a^(4/3)*b^(11/3)) + ((b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(18*a^(4/3)*b^(11/3))$

Rule 1828

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^m*Pq, a + b*x^n, x]

```
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
+ b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1594

```
Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x
_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1488

```
Int[((f_)*(x_)^(m_))*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_))^(p_)*(
(d_) + (e_)*(x_)^(n_))^(q_), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^( -1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x(c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^2} dx &= \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3ab^3(a + bx^3)} - \frac{\int \frac{-b(b^3c + 2ab^2d - 2a^2be + 2a^3f)x - 3ab^2(be - af)x^4 - 3ab^3fx^7}{a + bx^3} dx}{3ab^4} \\
 &= \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3ab^3(a + bx^3)} - \frac{\int \frac{x(-b(b^3c + 2ab^2d - 2a^2be + 2a^3f) - 3ab^2(be - af)x^3 - 3ab^3fx^6)}{a + bx^3} dx}{3ab^4} \\
 &= \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3ab^3(a + bx^3)} - \frac{\int \left(-3ab(be - 2af)x - 3ab^2fx^4 + \frac{(-b^4c - 2ab^3d + 5a^2b^2e - 8a^3b^2f)}{a + bx^3} \right) dx}{3ab^4} \\
 &= \frac{(be - 2af)x^2}{2b^3} + \frac{fx^5}{5b^2} + \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3ab^3(a + bx^3)} + \frac{(b^3c + 2ab^2d - 5a^2be + 8a^3f) \int}{3ab^3} \\
 &= \frac{(be - 2af)x^2}{2b^3} + \frac{fx^5}{5b^2} + \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3ab^3(a + bx^3)} - \frac{(b^3c + 2ab^2d - 5a^2be + 8a^3f) \int}{9a^{4/3}b^{10/3}} \\
 &= \frac{(be - 2af)x^2}{2b^3} + \frac{fx^5}{5b^2} + \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3ab^3(a + bx^3)} - \frac{(b^3c + 2ab^2d - 5a^2be + 8a^3f) \int}{9a^{4/3}b^{11/3}} \\
 &= \frac{(be - 2af)x^2}{2b^3} + \frac{fx^5}{5b^2} + \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3ab^3(a + bx^3)} - \frac{(b^3c + 2ab^2d - 5a^2be + 8a^3f) \log}{9a^{4/3}b^{11/3}} \\
 &= \frac{(be - 2af)x^2}{2b^3} + \frac{fx^5}{5b^2} + \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3ab^3(a + bx^3)} - \frac{(b^3c + 2ab^2d - 5a^2be + 8a^3f) \operatorname{atanh}}{3\sqrt{3}a^{4/3}b^{11/3}}
 \end{aligned}$$

Mathematica [A] time = 0.152679, size = 255, normalized size = 0.94

$$\frac{30b^{2/3}x^2(a^2be+a^3(-f)-ab^2d+b^3c)}{a(a+bx^3)} + \frac{5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx+b^{2/3}x^2}\right)(-5a^2be+8a^3f+2ab^2d+b^3c)}{a^{4/3}} - \frac{10\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)(-5a^2be+8a^3f+2ab^2d+b^3c)}{a^{4/3}} - \frac{10\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}bx+b^{2/3}x^2}{a}\right)}{90b^{11/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]

[Out] $(45b^{2/3}(b^3e - 2a^2f)x^2 + 18b^{5/3}f^2x^5 + (30b^{2/3}(b^3c - a^2d + a^2b^2e - a^3f)x^2)/(a(a + bx^3)) - (10\sqrt{3}(b^3c + 2a^2b^2d - 5a^2b^2e + 8a^3f)\text{ArcTan}[(1 - (2b^{1/3}x)/a^{1/3})/\sqrt{3}])/a^{4/3} - (10(b^3c + 2a^2b^2d - 5a^2b^2e + 8a^3f)\text{Log}[a^{1/3} + b^{1/3}x])/a^{4/3} + (5(b^3c + 2a^2b^2d - 5a^2b^2e + 8a^3f)\text{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2])/a^{4/3})/(90b^{11/3})$

Maple [B] time = 0.009, size = 495, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)

[Out] $\frac{1}{5}f^2x^5/b^2 - 1/b^3x^2af + 1/2/b^2x^2e - 1/3/b^3x^2a^2/(b^3x^3+a) + f + 1/3/b^2x^2a/(b^3x^3+a) + e - 1/3x^2d/(b^3x^3+a) + b + 1/3x^2/a/(b^3x^3+a) + c - 8/9/b^4a^2/(1/b^3a)^{1/3} \ln(x + (1/b^3a)^{1/3}) + 5/9/b^3a/(1/b^3a)^{1/3} \ln(x + (1/b^3a)^{1/3}) + e - 2/9/b^2/(1/b^3a)^{1/3} \ln(x + (1/b^3a)^{1/3}) + d - 1/9/b/a/(1/b^3a)^{1/3} \ln(x + (1/b^3a)^{1/3}) + c + 4/9/b^4a^2/(1/b^3a)^{1/3} \ln(x^2 - (1/b^3a)^{1/3}x + (1/b^3a)^{2/3}) + f - 5/18/b^3a/(1/b^3a)^{1/3} \ln(x^2 - (1/b^3a)^{1/3}x + (1/b^3a)^{2/3}) + e + 1/9/b^2/(1/b^3a)^{1/3} \ln(x^2 - (1/b^3a)^{1/3}x + (1/b^3a)^{2/3}) + d + 1/18/b/a/(1/b^3a)^{1/3} \ln(x^2 - (1/b^3a)^{1/3}x + (1/b^3a)^{2/3}) + c + 8/9/b^4a^23^{1/2}/(1/b^3a)^{1/3} \arctan(1/33^{1/2}*(2/(1/b^3a)^{1/3}x - 1)) + f - 5/9/b^3a3^{1/2}/(1/b^3a)^{1/3} \arctan(1/33^{1/2}*(2/(1/b^3a)^{1/3}x - 1)) + e + 2/9/b^23^{1/2}/(1/b^3a)^{1/3} \arctan(1/33^{1/2}*(2/(1/b^3a)^{1/3}x - 1)) + d + 1/9/b/a3^{1/2}/(1/b^3a)^{1/3} \arctan(1/33^{1/2}*(2/(1/b^3a)^{1/3}x - 1)) + c$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.48168, size = 1935, normalized size = 7.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] [1/90*(18*a^2*b^4*f*x^8 + 9*(5*a^2*b^4*e - 8*a^3*b^3*f)*x^5 + 15*(2*a*b^5*c - 2*a^2*b^4*d + 5*a^3*b^3*e - 8*a^4*b^2*f)*x^2 + 15*sqrt(1/3)*(a^2*b^4*c + 2*a^3*b^3*d - 5*a^4*b^2*e + 8*a^5*b*f + (a*b^5*c + 2*a^2*b^4*d - 5*a^3*b^3*e + 8*a^4*b^2*f)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + 5*(a*b^3*c + 2*a^2*b^2*d - 5*a^3*b*e + 8*a^4*f + (b^4*c + 2*a*b^3*d - 5*a^2*b^2*e + 8*a^3*b*f)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 10*(a*b^3*c + 2*a^2*b^2*d - 5*a^3*b*e + 8*a^4*f + (b^4*c + 2*a*b^3*d - 5*a^2*b^2*e + 8*a^3*b*f)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3))/(a^2*b^6*x^3 + a^3*b^5), 1/90*(18*a^2*b^4*f*x^8 + 9*(5*a^2*b^4*e - 8*a^3*b^3*f)*x^5 + 15*(2*a*b^5*c - 2*a^2*b^4*d + 5*a^3*b^3*e - 8*a^4*b^2*f)*x^2 + 30*sqrt(1/3)*(a^2*b^4*c + 2*a^3*b^3*d - 5*a^4*b^2*e + 8*a^5*b*f + (a*b^5*c + 2*a^2*b^4*d - 5*a^3*b^3*e + 8*a^4*b^2*f)*x^3)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 5*(a*b^3*c + 2*a^2*b^2*d - 5*a^3*b*e + 8*a^4*f + (b^4*c + 2*a*b^3*d - 5*a^2*b^2*e + 8*a^3*b*f)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 10*(a*b^3*c + 2*a^2*b^2*d - 5*a^3*b*e + 8*a^4*f + (b^4*c + 2*a*b^3*d - 5*a^2*b^2*e + 8*a^3*b*f)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3))/(a^2*b^6*x^3 + a^3*b^5)]

Sympy [A] time = 16.8514, size = 461, normalized size = 1.7

$$-\frac{x^2(a^3f - a^2be + ab^2d - b^3c)}{3a^2b^3 + 3ab^4x^3} + \text{RootSum}\left(729t^3a^4b^{11} + 512a^9f^3 - 960a^8bef^2 + 384a^7b^2df^2 + 600a^7b^2e^2f + 192a^6b^3c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)

[Out] -x**2*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*a**2*b**3 + 3*a*b**4*x**3) + RootSum(729*_t**3*a**4*b**11 + 512*a**9*f**3 - 960*a**8*b*e*f**2 + 384*a**7*b**2*d*f**2 + 600*a**7*b**2*e**2*f + 192*a**6*b**3*c*f**2 - 480*a**6*b**3*d*e*f - 125*a**6*b**3*e**3 - 240*a**5*b**4*c*e*f + 96*a**5*b**4*d**2*f + 150*a**5*b**4*d*e**2 + 96*a**4*b**5*c*d*f + 75*a**4*b**5*c*e**2 - 60*a**4*b**5*d**2*e + 24*a**3*b**6*c**2*f - 60*a**3*b**6*c*d*e + 8*a**3*b**6*d**3 - 15*a**2*b**7*c**2*e + 12*a**2*b**7*c*d**2 + 6*a*b**8*c**2*d + b**9*c**3, Lambda(_t, _t*log(81*_t**2*a**3*b**7/(64*a**6*f**2 - 80*a**5*b*e*f + 32*a**4*b**2*d*f + 25*a**4*b**2*e**2 + 16*a**3*b**3*c*f - 20*a**3*b**3*d*e - 10*a**2*b**4*c*e + 4*a**2*b**4*d**2 + 4*a*b**5*c*d + b**6*c**2) + x))) + f*x**5/(5*b**2) - x**2*(2*a*f - b*e)/(2*b**3)

Giac [A] time = 1.07596, size = 494, normalized size = 1.82

$$\frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 2ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 8a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 5a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^2b^3} + \frac{b^3cx^2 - ab^2dx^2 - a^3fx^2 + a^2bx^2e}{3(bx^3 + a)ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")

[Out] -1/9*(b^3*c*(-a/b)^(1/3) + 2*a*b^2*d*(-a/b)^(1/3) + 8*a^3*f*(-a/b)^(1/3) - 5*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^3) + 1/3*(b^3*c*x^2 - a*b^2*d*x^2 - a^3*f*x^2 + a^2*b*x^2*e)/((b*x^3 + a)*a*b^3) - 1/9*sqrt(3)*((-a*b^2)^(2/3)*b^3*c + 2*(-a*b^2)^(2/3)*a*b^2*d + 8*(-a*b^2)^(2/3)*a^3*f - 5*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3)))/(-a/b)^(1/3)/(a^2*b^5) + 1/18*((-a*b^2)^(2/3)*b^3*c + 2*(-a*b^2)^(2/3)*a*b^2*d + 8*(-a*b^2)^(2/3)*a^3*f - 5*(-a*b^2)^(2/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^5) + 1/10*(2*b^8*f*x^5 - 10*a*b^7*f*x^2 + 5*b^8*x^2*e)/b^10

$$3.266 \quad \int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^2} dx$$

Optimal. Leaf size=264

$$\frac{x(a^2be + a^3(-f) - ab^2d + b^3c)}{3ab^3(a + bx^3)} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(-4a^2be + 7a^3f + ab^2d + 2b^3c)}{18a^{5/3}b^{10/3}} + \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(-4a^2be + a^3(-f) - ab^2d + b^3c)}{9a^{5/3}b^{10/3}}$$

[Out] ((b*e - 2*a*f)*x)/b^3 + (f*x^4)/(4*b^2) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*a*b^3*(a + b*x^3)) - ((2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(10/3)) + ((2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(10/3)) - ((2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(10/3))

Rubi [A] time = 0.264159, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1858, 1411, 388, 200, 31, 634, 617, 204, 628}

$$\frac{x(a^2be + a^3(-f) - ab^2d + b^3c)}{3ab^3(a + bx^3)} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(-4a^2be + 7a^3f + ab^2d + 2b^3c)}{18a^{5/3}b^{10/3}} + \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(-4a^2be + a^3(-f) - ab^2d + b^3c)}{9a^{5/3}b^{10/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^2,x]

[Out] ((b*e - 2*a*f)*x)/b^3 + (f*x^4)/(4*b^2) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*a*b^3*(a + b*x^3)) - ((2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(10/3)) + ((2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(10/3)) - ((2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(10/3))

Rule 1858

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan

```
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a +
  b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1411

```
Int[((d_) + (e_)*(x_)^(n_))^(q_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_
)), x_Symbol] := Simp[(c*x^(n + 1)*(d + e*x^n)^(q + 1))/(e*(n*(q + 2) + 1))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
  (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
  n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rule 388

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
  p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
  c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 200

```
Int[((a_) + (b_)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(
  Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
  t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_)*(x_))(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
  x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]))] /; Free
```


$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 204

$\text{Int}[\{(a_)+(b_)*(x_)^2\}^{-1}, x_Symbol] \ :> \ -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2]*x]/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 628

$\text{Int}[\{(d_)+(e_)*(x_)\}/\{(a_)+(b_)*(x_)+(c_)*(x_)^2\}, x_Symbol] \ :> \ \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rubi steps

$$\begin{aligned} \int \frac{c + dx^3 + ex^6 + fx^9}{(a + bx^3)^2} dx &= \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3ab^3(a + bx^3)} - \frac{\int \frac{-2b^3c - ab^2d + a^2be - a^3f - 3ab(be - af)x^3 - 3ab^2fx^6}{a + bx^3} dx}{3ab^3} \\ &= \frac{fx^4}{4b^2} + \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3ab^3(a + bx^3)} - \frac{\int \frac{4b(-2b^3c - ab^2d + a^2be - a^3f) - (-12a^2b^2f + 12ab^2(be - af))x^3}{a + bx^3} dx}{12ab^4} \\ &= \frac{(be - 2af)x}{b^3} + \frac{fx^4}{4b^2} + \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3ab^3(a + bx^3)} + \frac{(2b^3c + ab^2d - 4a^2be + 7a^3f) \int \frac{1}{a + bx^3}}{3ab^3} \\ &= \frac{(be - 2af)x}{b^3} + \frac{fx^4}{4b^2} + \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3ab^3(a + bx^3)} + \frac{(2b^3c + ab^2d - 4a^2be + 7a^3f) \int \frac{1}{\sqrt[3]{a + bx^3}}}}{9a^{5/3}b^3} \\ &= \frac{(be - 2af)x}{b^3} + \frac{fx^4}{4b^2} + \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3ab^3(a + bx^3)} + \frac{(2b^3c + ab^2d - 4a^2be + 7a^3f) \log(\sqrt[3]{a + bx^3})}{9a^{5/3}b^{10/3}} \\ &= \frac{(be - 2af)x}{b^3} + \frac{fx^4}{4b^2} + \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3ab^3(a + bx^3)} + \frac{(2b^3c + ab^2d - 4a^2be + 7a^3f) \log(\sqrt[3]{a + bx^3})}{9a^{5/3}b^{10/3}} \\ &= \frac{(be - 2af)x}{b^3} + \frac{fx^4}{4b^2} + \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3ab^3(a + bx^3)} - \frac{(2b^3c + ab^2d - 4a^2be + 7a^3f) \tan^{-1}\left(\frac{\sqrt[3]{a + bx^3}}{\sqrt[3]{a}}\right)}{3\sqrt[3]{3}a^{5/3}b^{10/3}} \end{aligned}$$

Mathematica [A] time = 0.144892, size = 251, normalized size = 0.95

$$\frac{\frac{12\sqrt[3]{bx}(a^2be+a^3(-f)-ab^2d+b^3c)}{a(a+bx^3)} - \frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx+b^{2/3}x^2}\right)(-4a^2be+7a^3f+ab^2d+2b^3c)}{a^{5/3}} + \frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)(-4a^2be+7a^3f+ab^2d+2b^3c)}{a^{5/3}} - \frac{4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt{3}\sqrt[3]{bx+b^{2/3}x^2}}{1-\sqrt{3}\sqrt[3]{bx+b^{2/3}x^2}}\right)}{36b^{10/3}}}{36b^{10/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^2,x]

[Out] (36*b^(1/3)*(b*e - 2*a*f)*x + 9*b^(4/3)*f*x^4 + (12*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(a*(a + b*x^3)) - (4*sqrt[3]*(2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]])/a^(5/3) + (4*(2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) - (2*(2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(36*b^(10/3))

Maple [B] time = 0.009, size = 482, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)

[Out] 1/4*f*x^4/b^2-2/b^3*a*f*x+1/b^2*x*e-1/3/b^3*x*a^2/(b*x^3+a)*f+1/3/b^2*x*a/(b*x^3+a)*e-1/3/b*x/(b*x^3+a)*d+1/3*c*x/a/(b*x^3+a)+7/9/b^4*a^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*f-4/9/b^3*a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*e+1/9/b^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d+2/9*c/a/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-7/18/b^4*a^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f+2/9/b^3*a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e-1/18/b^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d-1/9*c/a/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+7/9/b^4*a^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f-4/9/b^3*a/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e+1/9/b^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d+2/9*c/a/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.52419, size = 1917, normalized size = 7.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/36*(9*a^3*b^3*f*x^7 + 9*(4*a^3*b^3*e - 7*a^4*b^2*f)*x^4 + 6*\sqrt{1/3}*(2 \\ & *a^2*b^4*c + a^3*b^3*d - 4*a^4*b^2*e + 7*a^5*b*f + (2*a*b^5*c + a^2*b^4*d - \\ & 4*a^3*b^3*e + 7*a^4*b^2*f)*x^3)*\sqrt{-(a^2*b)^{(1/3)}/b}*\log((2*a*b*x^3 - 3* \\ & (a^2*b)^{(1/3)}*a*x - a^2 + 3*\sqrt{1/3}*(2*a*b*x^2 + (a^2*b)^{(2/3)}*x - (a^2*b \\ &)^{(1/3)}*a)*\sqrt{-(a^2*b)^{(1/3)}/b}))/ (b*x^3 + a)) - 2*(2*a*b^3*c + a^2*b^2*d \\ & - 4*a^3*b*e + 7*a^4*f + (2*b^4*c + a*b^3*d - 4*a^2*b^2*e + 7*a^3*b*f)*x^3)* \\ & (a^2*b)^{(2/3)}*\log(a*b*x^2 - (a^2*b)^{(2/3)}*x + (a^2*b)^{(1/3)}*a) + 4*(2*a*b^3 \\ & *c + a^2*b^2*d - 4*a^3*b*e + 7*a^4*f + (2*b^4*c + a*b^3*d - 4*a^2*b^2*e + 7 \\ & *a^3*b*f)*x^3)*(a^2*b)^{(2/3)}*\log(a*b*x + (a^2*b)^{(2/3)}) + 12*(a^2*b^4*c - a \\ & ^3*b^3*d + 4*a^4*b^2*e - 7*a^5*b*f)*x)/(a^3*b^5*x^3 + a^4*b^4), 1/36*(9*a^3 \\ & *b^3*f*x^7 + 9*(4*a^3*b^3*e - 7*a^4*b^2*f)*x^4 + 12*\sqrt{1/3}*(2*a^2*b^4*c \\ & + a^3*b^3*d - 4*a^4*b^2*e + 7*a^5*b*f + (2*a*b^5*c + a^2*b^4*d - 4*a^3*b^3* \\ & e + 7*a^4*b^2*f)*x^3)*\sqrt{(a^2*b)^{(1/3)}/b}*\arctan(\sqrt{1/3}*(2*(a^2*b)^{(2/ \\ & 3)}*x - (a^2*b)^{(1/3)}*a)*\sqrt{(a^2*b)^{(1/3)}/b}/a^2) - 2*(2*a*b^3*c + a^2*b^2 \\ & *d - 4*a^3*b*e + 7*a^4*f + (2*b^4*c + a*b^3*d - 4*a^2*b^2*e + 7*a^3*b*f)*x^ \\ & 3)*(a^2*b)^{(2/3)}*\log(a*b*x^2 - (a^2*b)^{(2/3)}*x + (a^2*b)^{(1/3)}*a) + 4*(2*a* \\ & b^3*c + a^2*b^2*d - 4*a^3*b*e + 7*a^4*f + (2*b^4*c + a*b^3*d - 4*a^2*b^2*e \\ & + 7*a^3*b*f)*x^3)*(a^2*b)^{(2/3)}*\log(a*b*x + (a^2*b)^{(2/3)}) + 12*(a^2*b^4*c \\ & - a^3*b^3*d + 4*a^4*b^2*e - 7*a^5*b*f)*x)/(a^3*b^5*x^3 + a^4*b^4)] \end{aligned}$$

Sympy [A] time = 6.35866, size = 376, normalized size = 1.42

$$-\frac{x(a^3f - a^2be + ab^2d - b^3c)}{3a^2b^3 + 3ab^4x^3} + \text{RootSum}\left(729t^3a^5b^{10} - 343a^9f^3 + 588a^8bef^2 - 147a^7b^2df^2 - 336a^7b^2e^2f - 294a^6b^3cf\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)

[Out] -x*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*a**2*b**3 + 3*a*b**4*x**3) + RootSum(729*_t**3*a**5*b**10 - 343*a**9*f**3 + 588*a**8*b*e*f**2 - 147*a**7*b**2*d*f**2 - 336*a**7*b**2*e**2*f - 294*a**6*b**3*c*f**2 + 168*a**6*b**3*d*e*f + 64*a**6*b**3*e**3 + 336*a**5*b**4*c*e*f - 21*a**5*b**4*d**2*f - 48*a**5*b**4*d*e**2 - 84*a**4*b**5*c*d*f - 96*a**4*b**5*c*e**2 + 12*a**4*b**5*d**2*e - 84*a**3*b**6*c**2*f + 48*a**3*b**6*c*d*e - a**3*b**6*d**3 + 48*a**2*b**7*c**2*e - 6*a**2*b**7*c*d**2 - 12*a*b**8*c**2*d - 8*b**9*c**3, Lambda(_t, _t*log(9*_t*a**2*b**3/(7*a**3*f - 4*a**2*b*e + a*b**2*d + 2*b**3*c) + x))) + f*x**4/(4*b**2) - x*(2*a*f - b*e)/b**3

Giac [A] time = 1.07211, size = 433, normalized size = 1.64

$$\frac{(2b^3c + ab^2d + 7a^3f - 4a^2be)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^2b^3} + \frac{\sqrt{3}\left(2(-ab^2)^{\frac{1}{3}}b^3c + (-ab^2)^{\frac{1}{3}}ab^2d + 7(-ab^2)^{\frac{1}{3}}a^3f - 4(-ab^2)^{\frac{1}{3}}a^2be\right)}{9a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")

[Out] -1/9*(2*b^3*c + a*b^2*d + 7*a^3*f - 4*a^2*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^3) + 1/9*sqrt(3)*(2*(-a*b^2)^(1/3)*b^3*c + (-a*b^2)^(1/3)*a*b^2*d + 7*(-a*b^2)^(1/3)*a^3*f - 4*(-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^4) + 1/3*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*x*e)/((b*x^3 + a)*a*b^3) + 1/18*(2*(-a*b^2)^(1/3)*b^3*c + (-a*b^2)^(1/3)*a*b^2*d + 7*(-a*b^2)^(1/3)*a^3*f - 4*(-a*b^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^4) + 1/4*(b^6*f*x^4 - 8*a*b^5*f*x + 4*b^6*x*e)/b^8

$$3.267 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)^2} dx$$

Optimal. Leaf size=265

$$\frac{x^2(a^2be + a^3(-f) - ab^2d + b^3c)}{3a^2b^2(a + bx^3)} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(-2a^2be + 5a^3f - ab^2d + 4b^3c)}{18a^{7/3}b^{8/3}} + \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(-2a^2be + a^3(-f) - ab^2d + b^3c)}{18a^{7/3}b^{8/3}}$$

[Out] $-(c/(a^2*x)) + (f*x^2)/(2*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*a^2*b^2*(a + b*x^3)) + ((4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*ArcTan[(a^{1/3} - 2*b^{1/3}*x)/(Sqrt[3]*a^{1/3})])/(3*Sqrt[3]*a^{7/3}*b^{8/3}) + ((4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*Log[a^{1/3} + b^{1/3}*x])/(9*a^{7/3}*b^{8/3}) - ((4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*Log[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2])/(18*a^{7/3}*b^{8/3})$

Rubi [A] time = 0.253483, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1829, 1488, 292, 31, 634, 617, 204, 628}

$$\frac{x^2(a^2be + a^3(-f) - ab^2d + b^3c)}{3a^2b^2(a + bx^3)} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(-2a^2be + 5a^3f - ab^2d + 4b^3c)}{18a^{7/3}b^{8/3}} + \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(-2a^2be + a^3(-f) - ab^2d + b^3c)}{18a^{7/3}b^{8/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^2), x]

[Out] $-(c/(a^2*x)) + (f*x^2)/(2*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*a^2*b^2*(a + b*x^3)) + ((4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*ArcTan[(a^{1/3} - 2*b^{1/3}*x)/(Sqrt[3]*a^{1/3})])/(3*Sqrt[3]*a^{7/3}*b^{8/3}) + ((4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*Log[a^{1/3} + b^{1/3}*x])/(9*a^{7/3}*b^{8/3}) - ((4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*Log[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2])/(18*a^{7/3}*b^{8/3})$

Rule 1829

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[

```
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^( -1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^( -1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
 imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{c + dx^3 + ex^6 + fx^9}{x^2(a + bx^3)^2} dx &= -\frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3a^2b^2(a + bx^3)} - \frac{\int \frac{-3b^3c + b\left(\frac{b^3c}{a} - b^2d - 2abe + 2a^2f\right)x^3 - 3ab^2fx^6}{x^2(a + bx^3)} dx}{3ab^3} \\
 &= -\frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3a^2b^2(a + bx^3)} - \frac{\int \left(-\frac{3b^3c}{ax^2} - 3abfx + \frac{b(4b^3c - ab^2d - 2a^2be + 5a^3f)x}{a(a + bx^3)}\right) dx}{3ab^3} \\
 &= -\frac{c}{a^2x} + \frac{fx^2}{2b^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3a^2b^2(a + bx^3)} - \frac{(4b^3c - ab^2d - 2a^2be + 5a^3f) \int \frac{x}{a + bx^3} dx}{3a^2b^2} \\
 &= -\frac{c}{a^2x} + \frac{fx^2}{2b^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3a^2b^2(a + bx^3)} + \frac{(4b^3c - ab^2d - 2a^2be + 5a^3f) \int \frac{1}{\sqrt[3]{a + \sqrt[3]{bx}}} dx}{9a^{7/3}b^{7/3}} \\
 &= -\frac{c}{a^2x} + \frac{fx^2}{2b^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3a^2b^2(a + bx^3)} + \frac{(4b^3c - ab^2d - 2a^2be + 5a^3f) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{7/3}b^{8/3}} \\
 &= -\frac{c}{a^2x} + \frac{fx^2}{2b^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3a^2b^2(a + bx^3)} + \frac{(4b^3c - ab^2d - 2a^2be + 5a^3f) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{7/3}b^{8/3}} \\
 &= -\frac{c}{a^2x} + \frac{fx^2}{2b^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3a^2b^2(a + bx^3)} + \frac{(4b^3c - ab^2d - 2a^2be + 5a^3f) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a + \sqrt[3]{bx}}}\right)}{3\sqrt{3}a^{7/3}b^{8/3}}
 \end{aligned}$$

Mathematica [A] time = 0.152978, size = 255, normalized size = 0.96

$$\frac{1}{18} \left(\frac{6x^2(-a^2be + a^3f + ab^2d - b^3c)}{a^2b^2(a + bx^3)} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(-2a^2be + 5a^3f - ab^2d + 4b^3c)}{a^{7/3}b^{8/3}} + \frac{2 \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3\sqrt{3}a^{7/3}b^{8/3}} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] ((-18*c)/(a^2*x) + (9*f*x^2)/b^2 + (6*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)
)*x^2)/(a^2*b^2*(a + b*x^3)) + (2*Sqrt[3]*(4*b^3*c - a*b^2*d - 2*a^2*b*e +
5*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(a^(7/3)*b^(8/3)) + (
2*(4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(a^(7
/3)*b^(8/3)) - ((4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(a^(7/3)*b^(8/3)))/18
```

Maple [B] time = 0.012, size = 474, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^2,x)
```

```
[Out] 1/2*f*x^2/b^2+1/3/b^2*a*x^2/(b*x^3+a)*f-1/3/b*x^2/(b*x^3+a)*e+1/3*d*x^2/a/(
b*x^3+a)-1/3*b/a^2*x^2/(b*x^3+a)*c+5/9/b^3*a*f/(1/b*a)^(1/3)*ln(x+(1/b*a)^(
1/3))-5/18/b^3*a*f/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-5/9/
b^3*a*f*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/9
*d/a/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/18*d/a/b/(1/b*a)^(1/3)*ln(x^2-(1
/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/9*d/a*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(
1/2)*(2/(1/b*a)^(1/3)*x-1))+4/9/a^2*c/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-2/9
/a^2*c/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-4/9/a^2*c*3^(1/2
)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-2/9/b^2*e/(1/b*a)
^(1/3)*ln(x+(1/b*a)^(1/3))+1/9/b^2*e/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(
1/b*a)^(2/3))+2/9/b^2*e*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)
^(1/3)*x-1))-1/a^2*c/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```


[Out] Exception raised: ValueError

Fricas [A] time = 1.48022, size = 1877, normalized size = 7.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^2,x, algorithm="fricas")

[Out] [1/18*(9*a^3*b^3*f*x^6 - 18*a^2*b^4*c - 3*(8*a*b^5*c - 2*a^2*b^4*d + 2*a^3*b^3*e - 5*a^4*b^2*f)*x^3 + 3*sqrt(1/3)*((4*a*b^5*c - a^2*b^4*d - 2*a^3*b^3*e + 5*a^4*b^2*f)*x^4 + (4*a^2*b^4*c - a^3*b^3*d - 2*a^4*b^2*e + 5*a^5*b*f)*x)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) - ((4*b^4*c - a*b^3*d - 2*a^2*b^2*e + 5*a^3*b*f)*x^4 + (4*a*b^3*c - a^2*b^2*d - 2*a^3*b*e + 5*a^4*f)*x)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) + 2*((4*b^4*c - a*b^3*d - 2*a^2*b^2*e + 5*a^3*b*f)*x^4 + (4*a*b^3*c - a^2*b^2*d - 2*a^3*b*e + 5*a^4*f)*x)*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^5*x^4 + a^4*b^4*x), 1/18*(9*a^3*b^3*f*x^6 - 18*a^2*b^4*c - 3*(8*a*b^5*c - 2*a^2*b^4*d + 2*a^3*b^3*e - 5*a^4*b^2*f)*x^3 + 6*sqrt(1/3)*((4*a*b^5*c - a^2*b^4*d - 2*a^3*b^3*e + 5*a^4*b^2*f)*x^4 + (4*a^2*b^4*c - a^3*b^3*d - 2*a^4*b^2*e + 5*a^5*b*f)*x)*sqrt((a*b^2)^(1/3)/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) - ((4*b^4*c - a*b^3*d - 2*a^2*b^2*e + 5*a^3*b*f)*x^4 + (4*a*b^3*c - a^2*b^2*d - 2*a^3*b*e + 5*a^4*f)*x)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) + 2*((4*b^4*c - a*b^3*d - 2*a^2*b^2*e + 5*a^3*b*f)*x^4 + (4*a*b^3*c - a^2*b^2*d - 2*a^3*b*e + 5*a^4*f)*x)*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^5*x^4 + a^4*b^4*x)]

Sympy [A] time = 26.6082, size = 457, normalized size = 1.72

$$\frac{-3ab^2c + x^3(a^3f - a^2be + ab^2d - 4b^3c)}{3a^3b^2x + 3a^2b^3x^4} + \text{RootSum}\left(729t^3a^7b^8 - 125a^9f^3 + 150a^8bef^2 + 75a^7b^2df^2 - 60a^7b^2e^2f - 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**2/(b*x**3+a)**2,x)

```
[Out] (-3*a*b**2*c + x**3*(a**3*f - a**2*b*e + a*b**2*d - 4*b**3*c))/(3*a**3*b**2
*x + 3*a**2*b**3*x**4) + RootSum(729*_t**3*a**7*b**8 - 125*a**9*f**3 + 150*
a**8*b*e*f**2 + 75*a**7*b**2*d*f**2 - 60*a**7*b**2*e**2*f - 300*a**6*b**3*c
*f**2 - 60*a**6*b**3*d*e*f + 8*a**6*b**3*e**3 + 240*a**5*b**4*c*e*f - 15*a*
*5*b**4*d**2*f + 12*a**5*b**4*d*e**2 + 120*a**4*b**5*c*d*f - 48*a**4*b**5*c
*e**2 + 6*a**4*b**5*d**2*e - 240*a**3*b**6*c**2*f - 48*a**3*b**6*c*d*e + a*
*3*b**6*d**3 + 96*a**2*b**7*c**2*e - 12*a**2*b**7*c*d**2 + 48*a*b**8*c**2*d
- 64*b**9*c**3, Lambda(_t, _t*log(81*_t**2*a**5*b**5/(25*a**6*f**2 - 20*a*
*5*b*e*f - 10*a**4*b**2*d*f + 4*a**4*b**2*e**2 + 40*a**3*b**3*c*f + 4*a**3*
b**3*d*e - 16*a**2*b**4*c*e + a**2*b**4*d**2 - 8*a*b**5*c*d + 16*b**6*c**2)
+ x))) + f*x**2/(2*b**2)
```

Giac [A] time = 1.07628, size = 477, normalized size = 1.8

$$\frac{fx^2}{2b^2} + \frac{\left(4b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 5a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 2a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^3b^2} - \frac{4b^3cx^3 - ab^2dx^3 - a^3fx^3 + 3(bx^4 + ax)a^2}{3(bx^4 + ax)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*f*x^2/b^2 + 1/9*(4*b^3*c*(-a/b)^(1/3) - a*b^2*d*(-a/b)^(1/3) + 5*a^3*f*
(-a/b)^(1/3) - 2*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)
)))/(a^3*b^2) - 1/3*(4*b^3*c*x^3 - a*b^2*d*x^3 - a^3*f*x^3 + a^2*b*x^3*e +
3*a*b^2*c)/((b*x^4 + a*x)*a^2*b^2) + 1/9*sqrt(3)*(4*(-a*b^2)^(2/3)*b^3*c -
(-a*b^2)^(2/3)*a*b^2*d + 5*(-a*b^2)^(2/3)*a^3*f - 2*(-a*b^2)^(2/3)*a^2*b*e)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^4) - 1/18*(4*
(-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d + 5*(-a*b^2)^(2/3)*a^3*f - 2*
(-a*b^2)^(2/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^4)
```

$$3.268 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)^2} dx$$

Optimal. Leaf size=260

$$\frac{x(a^2be + a^3(-f) - ab^2d + b^3c)}{3a^2b^2(a + bx^3)} + \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(-a^2be + 4a^3f - 2ab^2d + 5b^3c)}{18a^{8/3}b^{7/3}} - \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(-a^2be + 4a^3f - 2ab^2d + 5b^3c)}{18a^{8/3}b^{7/3}}$$

[Out] $-c/(2*a^2*x^2) + (f*x)/b^2 - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*a^2*b^2*(a + b*x^3)) + ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*ArcTan[(a^{1/3} - 2*b^{1/3}*x)/(Sqrt[3]*a^{1/3})])/(3*Sqrt[3]*a^{8/3}*b^{7/3}) - ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*Log[a^{1/3} + b^{1/3}*x])/(9*a^{8/3}*b^{7/3}) + ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*Log[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2])/(18*a^{8/3}*b^{7/3})$

Rubi [A] time = 0.246217, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1829, 1488, 200, 31, 634, 617, 204, 628}

$$\frac{x(a^2be + a^3(-f) - ab^2d + b^3c)}{3a^2b^2(a + bx^3)} + \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(-a^2be + 4a^3f - 2ab^2d + 5b^3c)}{18a^{8/3}b^{7/3}} - \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(-a^2be + 4a^3f - 2ab^2d + 5b^3c)}{18a^{8/3}b^{7/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^2), x]

[Out] $-c/(2*a^2*x^2) + (f*x)/b^2 - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*a^2*b^2*(a + b*x^3)) + ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*ArcTan[(a^{1/3} - 2*b^{1/3}*x)/(Sqrt[3]*a^{1/3})])/(3*Sqrt[3]*a^{8/3}*b^{7/3}) - ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*Log[a^{1/3} + b^{1/3}*x])/(9*a^{8/3}*b^{7/3}) + ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*Log[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2])/(18*a^{8/3}*b^{7/3})$

Rule 1829

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[

```
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1488

```
Int[((f_)*(x_))^(m_)*((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_))^(p_)*(d_) + (e_)*(x_)^(n_))^(q_), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 200

```
Int[((a_) + (b_)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(p_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> S
 imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{c + dx^3 + ex^6 + fx^9}{x^3(a + bx^3)^2} dx &= -\frac{(b^3c - ab^2d + a^2be - a^3f)x}{3a^2b^2(a + bx^3)} - \frac{\int \frac{-3b^3c + b\left(\frac{2b^3c}{a} - 2b^2d - abe + a^2f\right)x^3 - 3ab^2fx^6}{x^3(a + bx^3)} dx}{3ab^3} \\
 &= -\frac{(b^3c - ab^2d + a^2be - a^3f)x}{3a^2b^2(a + bx^3)} - \frac{\int \left(-3abf - \frac{3b^3c}{ax^3} + \frac{b(5b^3c - 2ab^2d - a^2be + 4a^3f)}{a(a + bx^3)}\right) dx}{3ab^3} \\
 &= -\frac{c}{2a^2x^2} + \frac{fx}{b^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3a^2b^2(a + bx^3)} - \frac{(5b^3c - 2ab^2d - a^2be + 4a^3f) \int \frac{1}{a + bx^3} dx}{3a^2b^2} \\
 &= -\frac{c}{2a^2x^2} + \frac{fx}{b^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3a^2b^2(a + bx^3)} - \frac{(5b^3c - 2ab^2d - a^2be + 4a^3f) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{bx}} dx}{9a^{8/3}b^2} \\
 &= -\frac{c}{2a^2x^2} + \frac{fx}{b^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3a^2b^2(a + bx^3)} - \frac{(5b^3c - 2ab^2d - a^2be + 4a^3f) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{8/3}b^{7/3}} \\
 &= -\frac{c}{2a^2x^2} + \frac{fx}{b^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3a^2b^2(a + bx^3)} - \frac{(5b^3c - 2ab^2d - a^2be + 4a^3f) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{8/3}b^{7/3}} \\
 &= -\frac{c}{2a^2x^2} + \frac{fx}{b^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3a^2b^2(a + bx^3)} + \frac{(5b^3c - 2ab^2d - a^2be + 4a^3f) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a + bx^3}}\right)}{3\sqrt{3}a^{8/3}b^{7/3}}
 \end{aligned}$$

Mathematica [A] time = 0.152077, size = 250, normalized size = 0.96

$$\frac{1}{18} \left(\frac{6x(-a^2be + a^3f + ab^2d - b^3c)}{a^2b^2(a + bx^3)} + \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(-a^2be + 4a^3f - 2ab^2d + 5b^3c)}{a^{8/3}b^{7/3}} - \frac{2 \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3\sqrt{3}a^{8/3}b^{7/3}} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] ((-9*c)/(a^2*x^2) + (18*f*x)/b^2 + (6*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(a^2*b^2*(a + b*x^3)) + (2*Sqrt[3]*(5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(a^(8/3)*b^(7/3)) - (2*(5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(a^(8/3)*b^(7/3)) + ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(a^(8/3)*b^(7/3)))/18
```

Maple [B] time = 0.011, size = 463, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^2,x)
```

```
[Out] f*x/b^2+1/3/b^2*a*x/(b*x^3+a)*f-1/3/b*x/(b*x^3+a)*e+1/3/a*x/(b*x^3+a)*d-1/3*b/a^2*x/(b*x^3+a)*c-4/9/b^3*a*f/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+2/9/b^3*a*f/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-4/9/b^3*a*f/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+2/9/b/a*d/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/9/b/a*d/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2/9/b/a*d/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-5/9/a^2*c/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+5/18/a^2*c/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-5/9/a^2*c/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/9/b^2*e/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/18/b^2*e/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/9/b^2*e/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/2*c/a^2/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [A] time = 1.4632, size = 1952, normalized size = 7.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^2,x, algorithm="fricas")

[Out] [1/18*(18*a^4*b^2*f*x^6 - 9*a^3*b^3*c - 3*(5*a^2*b^4*c - 2*a^3*b^3*d + 2*a^4*b^2*e - 8*a^5*b*f)*x^3 + 3*sqrt(1/3)*((5*a*b^5*c - 2*a^2*b^4*d - a^3*b^3*e + 4*a^4*b^2*f)*x^5 + (5*a^2*b^4*c - 2*a^3*b^3*d - a^4*b^2*e + 4*a^5*b*f)*x^2)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a) + ((5*b^4*c - 2*a*b^3*d - a^2*b^2*e + 4*a^3*b*f)*x^5 + (5*a*b^3*c - 2*a^2*b^2*d - a^3*b*e + 4*a^4*f)*x^2)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) - 2*((5*b^4*c - 2*a*b^3*d - a^2*b^2*e + 4*a^3*b*f)*x^5 + (5*a*b^3*c - 2*a^2*b^2*d - a^3*b*e + 4*a^4*f)*x^2)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))/(a^4*b^4*x^5 + a^5*b^3*x^2), 1/18*(18*a^4*b^2*f*x^6 - 9*a^3*b^3*c - 3*(5*a^2*b^4*c - 2*a^3*b^3*d + 2*a^4*b^2*e - 8*a^5*b*f)*x^3 - 6*sqrt(1/3)*((5*a*b^5*c - 2*a^2*b^4*d - a^3*b^3*e + 4*a^4*b^2*f)*x^5 + (5*a^2*b^4*c - 2*a^3*b^3*d - a^4*b^2*e + 4*a^5*b*f)*x^2)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) + ((5*b^4*c - 2*a*b^3*d - a^2*b^2*e + 4*a^3*b*f)*x^5 + (5*a*b^3*c - 2*a^2*b^2*d - a^3*b*e + 4*a^4*f)*x^2)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) - 2*((5*b^4*c - 2*a*b^3*d - a^2*b^2*e + 4*a^3*b*f)*x^5 + (5*a*b^3*c - 2*a^2*b^2*d - a^3*b*e + 4*a^4*f)*x^2)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))/(a^4*b^4*x^5 + a^5*b^3*x^2)]

Sympy [A] time = 58.6862, size = 381, normalized size = 1.47

$$\frac{-3ab^2c + x^3(2a^3f - 2a^2be + 2ab^2d - 5b^3c)}{6a^3b^2x^2 + 6a^2b^3x^5} + \text{RootSum}\left(729t^3a^8b^7 + 64a^9f^3 - 48a^8bef^2 - 96a^7b^2df^2 + 12a^7b^2e^2f + \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**3/(b*x**3+a)**2,x)

```
[Out] (-3*a*b**2*c + x**3*(2*a**3*f - 2*a**2*b*e + 2*a*b**2*d - 5*b**3*c))/(6*a**
3*b**2*x**2 + 6*a**2*b**3*x**5) + RootSum(729*_t**3*a**8*b**7 + 64*a**9*f**
3 - 48*a**8*b*e*f**2 - 96*a**7*b**2*d*f**2 + 12*a**7*b**2*e**2*f + 240*a**6
*b**3*c*f**2 + 48*a**6*b**3*d*e*f - a**6*b**3*e**3 - 120*a**5*b**4*c*e*f +
48*a**5*b**4*d**2*f - 6*a**5*b**4*d*e**2 - 240*a**4*b**5*c*d*f + 15*a**4*b*
*5*c*e**2 - 12*a**4*b**5*d**2*e + 300*a**3*b**6*c**2*f + 60*a**3*b**6*c*d*e
- 8*a**3*b**6*d**3 - 75*a**2*b**7*c**2*e + 60*a**2*b**7*c*d**2 - 150*a*b**
8*c**2*d + 125*b**9*c**3, Lambda(_t, _t*log(-9*_t*a**3*b**2/(4*a**3*f - a**
2*b*e - 2*a*b**2*d + 5*b**3*c) + x))) + f*x/b**2
```

Giac [A] time = 1.08146, size = 417, normalized size = 1.6

$$\frac{fx}{b^2} + \frac{(5b^3c - 2ab^2d + 4a^3f - a^2be) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^3b^2} - \frac{c}{2a^2x^2} - \frac{\sqrt{3}\left(5(-ab^2)^{\frac{1}{3}}b^3c - 2(-ab^2)^{\frac{1}{3}}ab^2d + 4(-ab^2)^{\frac{1}{3}}a^3f - (-ab^2)^{\frac{1}{3}}a^2be\right)}{9a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] f*x/b^2 + 1/9*(5*b^3*c - 2*a*b^2*d + 4*a^3*f - a^2*b*e)*(-a/b)^(1/3)*log(ab
s(x - (-a/b)^(1/3)))/(a^3*b^2) - 1/2*c/(a^2*x^2) - 1/9*sqrt(3)*(5*(-a*b^2)^(
1/3)*b^3*c - 2*(-a*b^2)^(1/3)*a*b^2*d + 4*(-a*b^2)^(1/3)*a^3*f - (-a*b^2)^(
1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b
^3) - 1/3*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*x*e)/((b*x^3 + a)*a^2*b^2)
- 1/18*(5*(-a*b^2)^(1/3)*b^3*c - 2*(-a*b^2)^(1/3)*a*b^2*d + 4*(-a*b^2)^(1/
3)*a^3*f - (-a*b^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))
/(a^3*b^3)
```


$$3.269 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)^2} dx$$

Optimal. Leaf size=269

$$\frac{x^2(a^2be + a^3(-f) - ab^2d + b^3c)}{3a^3b(a + bx^3)} + \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^2be + 2a^3f - 4ab^2d + 7b^3c)}{18a^{10/3}b^{5/3}} - \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(a^2be + 2a^3f - 4ab^2d + 7b^3c)}{9a^{10}}$$

[Out] $-c/(4*a^2*x^4) + (2*b*c - a*d)/(a^3*x) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*a^3*b*(a + b*x^3)) - ((7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*\text{ArcTan}[(a^{1/3} - 2*b^{1/3}*x)/(\text{Sqrt}[3]*a^{1/3})])/(3*\text{Sqrt}[3]*a^{10/3}*b^{5/3}) - ((7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*\text{Log}[a^{1/3} + b^{1/3}*x])/(9*a^{10/3}*b^{5/3}) + ((7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*\text{Log}[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2])/(18*a^{10/3}*b^{5/3})$

Rubi [A] time = 0.288055, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1829, 1488, 292, 31, 634, 617, 204, 628}

$$\frac{x^2(a^2be + a^3(-f) - ab^2d + b^3c)}{3a^3b(a + bx^3)} + \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^2be + 2a^3f - 4ab^2d + 7b^3c)}{18a^{10/3}b^{5/3}} - \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(a^2be + 2a^3f - 4ab^2d + 7b^3c)}{9a^{10}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^2), x]

[Out] $-c/(4*a^2*x^4) + (2*b*c - a*d)/(a^3*x) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*a^3*b*(a + b*x^3)) - ((7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*\text{ArcTan}[(a^{1/3} - 2*b^{1/3}*x)/(\text{Sqrt}[3]*a^{1/3})])/(3*\text{Sqrt}[3]*a^{10/3}*b^{5/3}) - ((7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*\text{Log}[a^{1/3} + b^{1/3}*x])/(9*a^{10/3}*b^{5/3}) + ((7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*\text{Log}[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2])/(18*a^{10/3}*b^{5/3})$

Rule 1829

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[

```
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^( -1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^( -1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
 imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{c + dx^3 + ex^6 + fx^9}{x^5(a + bx^3)^2} dx &= \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3a^3b(a + bx^3)} - \frac{\int \frac{-3b^3c + 3b^3\left(\frac{bc}{a} - d\right)x^3 - b^2\left(\frac{b^2c}{a^2} - \frac{b^2d}{a} + be + 2af\right)x^6}{x^5(a + bx^3)} dx}{3ab^3} \\
 &= \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3a^3b(a + bx^3)} - \frac{\int \left(-\frac{3b^3c}{ax^5} - \frac{3b^3(-2bc + ad)}{a^2x^2} - \frac{b^2(7b^3c - 4ab^2d + a^2be + 2a^3f)x}{a^2(a + bx^3)} \right) dx}{3ab^3} \\
 &= -\frac{c}{4a^2x^4} + \frac{2bc - ad}{a^3x} + \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3a^3b(a + bx^3)} + \frac{(7b^3c - 4ab^2d + a^2be + 2a^3f) \int \frac{x}{a + bx^3}}{3a^3b} \\
 &= -\frac{c}{4a^2x^4} + \frac{2bc - ad}{a^3x} + \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3a^3b(a + bx^3)} - \frac{(7b^3c - 4ab^2d + a^2be + 2a^3f) \int \frac{1}{\sqrt[3]{a + bx^3}}}{9a^{10/3}b^{4/3}} \\
 &= -\frac{c}{4a^2x^4} + \frac{2bc - ad}{a^3x} + \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3a^3b(a + bx^3)} - \frac{(7b^3c - 4ab^2d + a^2be + 2a^3f) \log(\sqrt[3]{a + bx^3})}{9a^{10/3}b^{5/3}} \\
 &= -\frac{c}{4a^2x^4} + \frac{2bc - ad}{a^3x} + \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3a^3b(a + bx^3)} - \frac{(7b^3c - 4ab^2d + a^2be + 2a^3f) \log(\sqrt[3]{a + bx^3})}{9a^{10/3}b^{5/3}} \\
 &= -\frac{c}{4a^2x^4} + \frac{2bc - ad}{a^3x} + \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3a^3b(a + bx^3)} - \frac{(7b^3c - 4ab^2d + a^2be + 2a^3f) \tan^{-1}\left(\frac{\sqrt[3]{a + bx^3}}{\sqrt[3]{a}}\right)}{3\sqrt[3]{a}^{10/3}b^{5/3}}
 \end{aligned}$$

Mathematica [A] time = 0.168302, size = 255, normalized size = 0.95

$$\frac{-\frac{12\sqrt[3]{ax^2}(-a^2be + a^3f + ab^2d - b^3c)}{b(a + bx^3)} + \frac{2\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx + b^{2/3}x^2}\right)(a^2be + 2a^3f - 4ab^2d + 7b^3c)}{b^{5/3}} - \frac{4\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)(a^2be + 2a^3f - 4ab^2d + 7b^3c)}{b^{5/3}} - \frac{4\sqrt[3]{3}\tan^{-1}\left(\frac{\sqrt[3]{a + bx^3}}{\sqrt[3]{a}}\right)}{36a^{10/3}}}{36a^{10/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^2), x]

[Out]
$$\begin{aligned} &((-9*a^{(4/3)}*c)/x^4 - (36*a^{(1/3)}*(-2*b*c + a*d))/x - (12*a^{(1/3)}*(-(b^3*c) \\ &+ a*b^2*d - a^2*b*e + a^3*f)*x^2)/(b*(a + b*x^3)) - (4*\text{Sqrt}[3]*(7*b^3*c - \\ &4*a*b^2*d + a^2*b*e + 2*a^3*f)*\text{ArcTan}[(1 - (2*b^{(1/3)}*x)/a^{(1/3)})/\text{Sqrt}[3]]) \\ &/b^{(5/3)} - (4*(7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*\text{Log}[a^{(1/3)} + b^{(1/3)} \\ &3*x])/b^{(5/3)} + (2*(7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*\text{Log}[a^{(2/3)} - \\ &a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/b^{(5/3)})/(36*a^{(10/3)}) \end{aligned}$$

Maple [B] time = 0.013, size = 486, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^2,x)

[Out]
$$\begin{aligned} &-1/3/b*x^2/(b*x^3+a)*f+1/3/a*x^2/(b*x^3+a)*e-1/3/a^2*b*x^2/(b*x^3+a)*d+1/3/ \\ &a^3*b^2*x^2/(b*x^3+a)*c-2/9/b^2/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})*f-1/9/a/b \\ &/ (1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})*e+4/9/a^2/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)}) \\ &3)*d-7/9/a^3*b/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})*c+1/9/b^2/(1/b*a)^{(1/3)}* \\ &\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*f+1/18/a/b/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a) \\ &)^{(1/3)}*x+(1/b*a)^{(2/3)})*e-2/9/a^2/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/ \\ &b*a)^{(2/3)})*d+7/18/a^3*b/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)} \\ &)*c+2/9/b^2*3^{(1/2)}/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1)) \\ & *f+1/9/a/b*3^{(1/2)}/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))* \\ & e-4/9/a^2*3^{(1/2)}/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))* \\ & +7/9/a^3*b*3^{(1/2)}/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))* \\ & c-1/4*c/a^2/x^4-d/a^2/x+2/a^3/x*b*c \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.52471, size = 1968, normalized size = 7.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^2,x, algorithm="fricas")

[Out] [-1/36*(9*a^3*b^3*c - 12*(7*a*b^5*c - 4*a^2*b^4*d + a^3*b^3*e - a^4*b^2*f)*x^6 - 9*(7*a^2*b^4*c - 4*a^3*b^3*d)*x^3 - 6*sqrt(1/3)*((7*a*b^5*c - 4*a^2*b^4*d + a^3*b^3*e + 2*a^4*b^2*f)*x^7 + (7*a^2*b^4*c - 4*a^3*b^3*d + a^4*b^2*e + 2*a^5*b*f)*x^4)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - 2*((7*b^4*c - 4*a*b^3*d + a^2*b^2*e + 2*a^3*b*f)*x^7 + (7*a*b^3*c - 4*a^2*b^2*d + a^3*b*e + 2*a^4*f)*x^4)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 4*((7*b^4*c - 4*a*b^3*d + a^2*b^2*e + 2*a^3*b*f)*x^7 + (7*a*b^3*c - 4*a^2*b^2*d + a^3*b*e + 2*a^4*f)*x^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^4*b^4*x^7 + a^5*b^3*x^4), -1/36*(9*a^3*b^3*c - 12*(7*a*b^5*c - 4*a^2*b^4*d + a^3*b^3*e - a^4*b^2*f)*x^6 - 9*(7*a^2*b^4*c - 4*a^3*b^3*d)*x^3 - 12*sqrt(1/3)*((7*a*b^5*c - 4*a^2*b^4*d + a^3*b^3*e + 2*a^4*b^2*f)*x^7 + (7*a^2*b^4*c - 4*a^3*b^3*d + a^4*b^2*e + 2*a^5*b*f)*x^4)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - 2*((7*b^4*c - 4*a*b^3*d + a^2*b^2*e + 2*a^3*b*f)*x^7 + (7*a*b^3*c - 4*a^2*b^2*d + a^3*b*e + 2*a^4*f)*x^4)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 4*((7*b^4*c - 4*a*b^3*d + a^2*b^2*e + 2*a^3*b*f)*x^7 + (7*a*b^3*c - 4*a^2*b^2*d + a^3*b*e + 2*a^4*f)*x^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^4*b^4*x^7 + a^5*b^3*x^4)]

Sympy [A] time = 138.391, size = 473, normalized size = 1.76

RootSum($729t^3a^{10}b^5 + 8a^9f^3 + 12a^8bef^2 - 48a^7b^2df^2 + 6a^7b^2e^2f + 84a^6b^3cf^2 - 48a^6b^3def + a^6b^3e^3 + 84a^5b^4cef +$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**5/(b*x**3+a)**2,x)

[Out] RootSum(729*_t**3*a**10*b**5 + 8*a**9*f**3 + 12*a**8*b*e*f**2 - 48*a**7*b**2*d*f**2 + 6*a**7*b**2*e**2*f + 84*a**6*b**3*c*f**2 - 48*a**6*b**3*d*e*f +

$a^{**6}b^{**3}e^{**3} + 84a^{**5}b^{**4}c*e*f + 96a^{**5}b^{**4}d^{**2}*f - 12a^{**5}b^{**4}d*
e^{**2} - 336a^{**4}b^{**5}c*d*f + 21a^{**4}b^{**5}c*e^{**2} + 48a^{**4}b^{**5}d^{**2}*e + 29
4a^{**3}b^{**6}c^{**2}*f - 168a^{**3}b^{**6}c*d*e - 64a^{**3}b^{**6}d^{**3} + 147a^{**2}b^{**
7c^{**2}*e + 336a^{**2}b^{**7}c*d^{**2} - 588a*b^{**8}c^{**2}*d + 343b^{**9}c^{**3}, \text{Lambda}$
 $(_t, _t*\log(81*_t^{**2}*a^{**7}b^{**3}/(4*a^{**6}f^{**2} + 4*a^{**5}b*e*f - 16*a^{**4}b^{**2}d
*f + a^{**4}b^{**2}e^{**2} + 28*a^{**3}b^{**3}c*f - 8*a^{**3}b^{**3}d*e + 14*a^{**2}b^{**4}c*e
+ 16*a^{**2}b^{**4}d^{**2} - 56*a*b^{**5}c*d + 49*b^{**6}c^{**2}) + x))) - (3*a^{**2}b*c +
x^{**6}*(4*a^{**3}f - 4*a^{**2}b*e + 16*a*b^{**2}d - 28*b^{**3}c) + x^{**3}*(12*a^{**2}b*d
- 21*a*b^{**2}c))/(12*a^{**4}b*x^{**4} + 12*a^{**3}b^{**2}*x^{**7})$

Giac [A] time = 1.08189, size = 483, normalized size = 1.8

$$\frac{\left(7b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 4ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 2a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^4b} + \frac{b^3cx^2 - ab^2dx^2 - a^3fx^2 + a^2bx^2e}{3(bx^3 + a)a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^2,x, algorithm="giac")

[Out] $-1/9*(7*b^3*c*(-a/b)^{(1/3)} - 4*a*b^2*d*(-a/b)^{(1/3)} + 2*a^3*f*(-a/b)^{(1/3)}$
 $+ a^2*b*(-a/b)^{(1/3)}*e)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/a^4*b + 1$
 $/3*(b^3*c*x^2 - a*b^2*d*x^2 - a^3*f*x^2 + a^2*b*x^2*e)/((b*x^3 + a)*a^3*b)$
 $- 1/9*\text{sqrt}(3)*(7*(-a*b^2)^{(2/3)}*b^3*c - 4*(-a*b^2)^{(2/3)}*a*b^2*d + 2*(-a*b^2)^{(2/3)}*a^3*f$
 $+ (-a*b^2)^{(2/3)}*a^2*b*e)*\text{arctan}(1/3*\text{sqrt}(3)*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/a^4*b^3$
 $+ 1/18*(7*(-a*b^2)^{(2/3)}*b^3*c - 4*(-a*b^2)^{(2/3)}*a*b^2*d + 2*(-a*b^2)^{(2/3)}*a^3*f$
 $+ (-a*b^2)^{(2/3)}*a^2*b*e)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/a^4*b^3 + 1/4*(8*b*c*x^3 - 4*a*d*x^3 - a*c)/a^3*x^4$

$$3.270 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)^2} dx$$

Optimal. Leaf size=270

$$\frac{x(a^2be + a^3(-f) - ab^2d + b^3c)}{3a^3b(a + bx^3)} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(2a^2be + a^3f - 5ab^2d + 8b^3c)}{18a^{11/3}b^{4/3}} + \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(2a^2be + a^3f - 5ab^2d + 8b^3c)}{9a^{11/3}b^{4/3}}$$

[Out] $-c/(5*a^2*x^5) + (2*b*c - a*d)/(2*a^3*x^2) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*a^3*b*(a + b*x^3)) - ((8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f) * \text{ArcTan}[(a^{1/3} - 2*b^{1/3}*x)/(\text{Sqrt}[3]*a^{1/3})])/(3*\text{Sqrt}[3]*a^{11/3}*b^{4/3}) + ((8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*\text{Log}[a^{1/3} + b^{1/3}*x])/(9*a^{11/3}*b^{4/3}) - ((8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*\text{Log}[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2])/(18*a^{11/3}*b^{4/3})$

Rubi [A] time = 0.27243, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1829, 1488, 200, 31, 634, 617, 204, 628}

$$\frac{x(a^2be + a^3(-f) - ab^2d + b^3c)}{3a^3b(a + bx^3)} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(2a^2be + a^3f - 5ab^2d + 8b^3c)}{18a^{11/3}b^{4/3}} + \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(2a^2be + a^3f - 5ab^2d + 8b^3c)}{9a^{11/3}b^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^2), x]

[Out] $-c/(5*a^2*x^5) + (2*b*c - a*d)/(2*a^3*x^2) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*a^3*b*(a + b*x^3)) - ((8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f) * \text{ArcTan}[(a^{1/3} - 2*b^{1/3}*x)/(\text{Sqrt}[3]*a^{1/3})])/(3*\text{Sqrt}[3]*a^{11/3}*b^{4/3}) + ((8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*\text{Log}[a^{1/3} + b^{1/3}*x])/(9*a^{11/3}*b^{4/3}) - ((8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*\text{Log}[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2])/(18*a^{11/3}*b^{4/3})$

Rule 1829

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[

```
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1488

```
Int[((f_)*(x_))^(m_)*((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_))^(p_)*(d_) + (e_)*(x_)^(n_))^(q_), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 200

```
Int[((a_) + (b_)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(p_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```


Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
 imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{c + dx^3 + ex^6 + fx^9}{x^6(a + bx^3)^2} dx &= \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3a^3b(a + bx^3)} - \frac{\int \frac{-3b^3c + 3b^3\left(\frac{bc}{a} - d\right)x^3 - b^2\left(\frac{2b^3c}{a^2} - \frac{2b^2d}{a} + 2be + af\right)x^6}{x^6(a + bx^3)} dx}{3ab^3} \\
 &= \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3a^3b(a + bx^3)} - \frac{\int \left(-\frac{3b^3c}{ax^6} - \frac{3b^3(-2bc + ad)}{a^2x^3} - \frac{b^2(8b^3c - 5ab^2d + 2a^2be + a^3f)}{a^2(a + bx^3)} \right) dx}{3ab^3} \\
 &= -\frac{c}{5a^2x^5} + \frac{2bc - ad}{2a^3x^2} + \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3a^3b(a + bx^3)} + \frac{(8b^3c - 5ab^2d + 2a^2be + a^3f) \int \frac{1}{a + bx^3}}{3a^3b} \\
 &= -\frac{c}{5a^2x^5} + \frac{2bc - ad}{2a^3x^2} + \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3a^3b(a + bx^3)} + \frac{(8b^3c - 5ab^2d + 2a^2be + a^3f) \int \frac{1}{\sqrt[3]{a + bx^3}}}{9a^{11/3}b} \\
 &= -\frac{c}{5a^2x^5} + \frac{2bc - ad}{2a^3x^2} + \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3a^3b(a + bx^3)} + \frac{(8b^3c - 5ab^2d + 2a^2be + a^3f) \log(\sqrt[3]{a + bx^3})}{9a^{11/3}b^{4/3}} \\
 &= -\frac{c}{5a^2x^5} + \frac{2bc - ad}{2a^3x^2} + \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3a^3b(a + bx^3)} + \frac{(8b^3c - 5ab^2d + 2a^2be + a^3f) \log(\sqrt[3]{a + bx^3})}{9a^{11/3}b^{4/3}} \\
 &= -\frac{c}{5a^2x^5} + \frac{2bc - ad}{2a^3x^2} + \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3a^3b(a + bx^3)} - \frac{(8b^3c - 5ab^2d + 2a^2be + a^3f) \tan^{-1}\left(\frac{\sqrt[3]{a + bx^3}}{\sqrt[3]{a}}\right)}{3\sqrt[3]{3}a^{11/3}b^{4/3}}
 \end{aligned}$$

Mathematica [A] time = 0.162183, size = 253, normalized size = 0.94

$$\frac{30a^{2/3}x(-a^2be + a^3f + ab^2d - b^3c)}{b(a + bx^3)} - \frac{5 \log\left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx + b^{2/3}x^2}\right)(2a^2be + a^3f - 5ab^2d + 8b^3c)}{b^{4/3}} + \frac{10 \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)(2a^2be + a^3f - 5ab^2d + 8b^3c)}{b^{4/3}} - \frac{10\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{a + bx^3}}{\sqrt[3]{a}}\right)}{90a^{11/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^2),x]

[Out]
$$\begin{aligned} &((-18*a^{(5/3)}*c)/x^5 - (45*a^{(2/3)}*(-2*b*c + a*d))/x^2 - (30*a^{(2/3)}*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(b*(a + b*x^3)) - (10*\sqrt{3}*(8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*\text{ArcTan}[(1 - (2*b^{(1/3)}*x)/a^{(1/3)})/\sqrt{3}])/b^{(4/3)} + (10*(8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*\text{Log}[a^{(1/3)} + b^{(1/3)*x}])/b^{(4/3)} - (5*(8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)*x} + b^{(2/3)*x^2}])/b^{(4/3)})/(90*a^{(11/3)}) \end{aligned}$$

Maple [B] time = 0.011, size = 477, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^2,x)

[Out]
$$\begin{aligned} &-1/3/b*x/(b*x^3+a)*f+1/3/a*x/(b*x^3+a)*e-1/3/a^2*b*x/(b*x^3+a)*d+1/3/a^3*b^2*x/(b*x^3+a)*c+1/9/b^2/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*f+2/9/a/b/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*e-5/9/a^2/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*d+8/9/a^3*b/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*c-1/18/b^2/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)*x}+(1/b*a)^{(2/3)})*f-1/9/a/b/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)*x}+(1/b*a)^{(2/3)})*e+5/18/a^2/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)*x}+(1/b*a)^{(2/3)})*d-4/9/a^3*b/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)*x}+(1/b*a)^{(2/3)})*c+1/9/b^2/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)*x}-1))*f+2/9/a/b/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)*x}-1))*e-5/9/a^2/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)*x}-1))*d+8/9/a^3*b/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)*x}-1))*c-1/5*c/a^2/x^5-1/2*d/a^2/x^2+1/a^3/x^2*b*c \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.55885, size = 1991, normalized size = 7.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/90*(18*a^4*b^2*c - 15*(8*a^2*b^4*c - 5*a^3*b^3*d + 2*a^4*b^2*e - 2*a^5*b*f)*x^6 - 9*(8*a^3*b^3*c - 5*a^4*b^2*d)*x^3 - 15*\sqrt{1/3}*((8*a*b^5*c - 5*a^2*b^4*d + 2*a^3*b^3*e + a^4*b^2*f)*x^8 + (8*a^2*b^4*c - 5*a^3*b^3*d + 2*a^4*b^2*e + a^5*b*f)*x^5)*\sqrt{-(a^2*b)^{(1/3)}/b}*\log((2*a*b*x^3 - 3*(a^2*b)^{(1/3)}*a*x - a^2 + 3*\sqrt{1/3}*(2*a*b*x^2 + (a^2*b)^{(2/3)}*x - (a^2*b)^{(1/3)}*a)*\sqrt{-(a^2*b)^{(1/3)}/b})/(b*x^3 + a)) + 5*((8*b^4*c - 5*a*b^3*d + 2*a^2*b^2*e + a^3*b*f)*x^8 + (8*a*b^3*c - 5*a^2*b^2*d + 2*a^3*b*e + a^4*f)*x^5)*(a^2*b)^{(2/3)}*\log(a*b*x^2 - (a^2*b)^{(2/3)}*x + (a^2*b)^{(1/3)}*a) - 10*((8*b^4*c - 5*a*b^3*d + 2*a^2*b^2*e + a^3*b*f)*x^8 + (8*a*b^3*c - 5*a^2*b^2*d + 2*a^3*b*e + a^4*f)*x^5)*(a^2*b)^{(2/3)}*\log(a*b*x + (a^2*b)^{(2/3)})]/(a^5*b^3*x^8 + a^6*b^2*x^5), \\ & -1/90*(18*a^4*b^2*c - 15*(8*a^2*b^4*c - 5*a^3*b^3*d + 2*a^4*b^2*e - 2*a^5*b*f)*x^6 - 9*(8*a^3*b^3*c - 5*a^4*b^2*d)*x^3 - 30*\sqrt{1/3}*((8*a*b^5*c - 5*a^2*b^4*d + 2*a^3*b^3*e + a^4*b^2*f)*x^8 + (8*a^2*b^4*c - 5*a^3*b^3*d + 2*a^4*b^2*e + a^5*b*f)*x^5)*\sqrt{(a^2*b)^{(1/3)}/b}*\arctan(\sqrt{1/3}*(2*(a^2*b)^{(2/3)}*x - (a^2*b)^{(1/3)}*a)*\sqrt{(a^2*b)^{(1/3)}/b})/a^2) + 5*((8*b^4*c - 5*a*b^3*d + 2*a^2*b^2*e + a^3*b*f)*x^8 + (8*a*b^3*c - 5*a^2*b^2*d + 2*a^3*b*e + a^4*f)*x^5)*(a^2*b)^{(2/3)}*\log(a*b*x^2 - (a^2*b)^{(2/3)}*x + (a^2*b)^{(1/3)}*a) - 10*((8*b^4*c - 5*a*b^3*d + 2*a^2*b^2*e + a^3*b*f)*x^8 + (8*a*b^3*c - 5*a^2*b^2*d + 2*a^3*b*e + a^4*f)*x^5)*(a^2*b)^{(2/3)}*\log(a*b*x + (a^2*b)^{(2/3)})]/(a^5*b^3*x^8 + a^6*b^2*x^5)] \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**6/(b*x**3+a)**2,x)

[Out] Timed out

Giac [A] time = 1.08144, size = 429, normalized size = 1.59

$$\frac{(8b^3c - 5ab^2d + a^3f + 2a^2be) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^4b} + \frac{\sqrt{3} \left(8(-ab^2)^{\frac{1}{3}}b^3c - 5(-ab^2)^{\frac{1}{3}}ab^2d + (-ab^2)^{\frac{1}{3}}a^3f + 2(-ab^2)^{\frac{1}{3}}a^2be\right)}{9a^4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^2,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/9*(8*b^3*c - 5*a*b^2*d + a^3*f + 2*a^2*b*e)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)})) / (a^4*b) \\ & + 1/9*\text{sqrt}(3)*(8*(-a*b^2)^{(1/3)}*b^3*c - 5*(-a*b^2)^{(1/3)}*a*b^2*d + (-a*b^2)^{(1/3)}*a^3*f \\ & + 2*(-a*b^2)^{(1/3)}*a^2*b*e)*\text{arctan}(1/3*\text{sqrt}(3)*(2*x + (-a/b)^{(1/3)}) / (-a/b)^{(1/3)}) / (a^4*b^2) \\ & + 1/3*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*x*e) / ((b*x^3 + a)*a^3*b) \\ & + 1/18*(8*(-a*b^2)^{(1/3)}*b^3*c - 5*(-a*b^2)^{(1/3)}*a*b^2*d + (-a*b^2)^{(1/3)}*a^3*f + 2*(-a*b^2)^{(1/3)}*a^2*b*e) \\ & *\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)}) / (a^4*b^2) \\ & + 1/10*(10*b*c*x^3 - 5*a*d*x^3 - 2*a*c) / (a^3*x^5) \end{aligned}$$

$$3.271 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)^2} dx$$

Optimal. Leaf size=297

$$\frac{x^2(a^2be + a^3(-f) - ab^2d + b^3c)}{3a^4(a + bx^3)} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(4a^2be + a^3(-f) - 7ab^2d + 10b^3c)}{18a^{13/3}b^{2/3}} + \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})}{18a^{13/3}b^{2/3}}$$

[Out] $-c/(7*a^2*x^7) + (2*b*c - a*d)/(4*a^3*x^4) - (3*b^2*c - 2*a*b*d + a^2*e)/(a^4*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*a^4*(a + b*x^3)) + ((10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*ArcTan[(a^{1/3} - 2*b^{1/3}*x)/(Sqrt[3]*a^{1/3})])/(3*Sqrt[3]*a^{13/3}*b^{2/3}) + ((10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*Log[a^{1/3} + b^{1/3}*x])/(9*a^{13/3}*b^{2/3}) - ((10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*Log[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}]*x^2))/(18*a^{13/3}*b^{2/3})$

Rubi [A] time = 0.383572, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1829, 1834, 292, 31, 634, 617, 204, 628}

$$\frac{x^2(a^2be + a^3(-f) - ab^2d + b^3c)}{3a^4(a + bx^3)} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(4a^2be + a^3(-f) - 7ab^2d + 10b^3c)}{18a^{13/3}b^{2/3}} + \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})}{18a^{13/3}b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^2), x]

[Out] $-c/(7*a^2*x^7) + (2*b*c - a*d)/(4*a^3*x^4) - (3*b^2*c - 2*a*b*d + a^2*e)/(a^4*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*a^4*(a + b*x^3)) + ((10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*ArcTan[(a^{1/3} - 2*b^{1/3}*x)/(Sqrt[3]*a^{1/3})])/(3*Sqrt[3]*a^{13/3}*b^{2/3}) + ((10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*Log[a^{1/3} + b^{1/3}*x])/(9*a^{13/3}*b^{2/3}) - ((10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*Log[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}]*x^2))/(18*a^{13/3}*b^{2/3})$

Rule 1829

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^

```
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_)*(x_))^(m_))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^( -1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^( -1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{c + dx^3 + ex^6 + fx^9}{x^8(a + bx^3)^2} dx &= -\frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3a^4(a + bx^3)} - \frac{\int \frac{-3b^3c + 3b^3\left(\frac{bc}{a} - d\right)x^3 - \frac{3b^3(b^2c - abd + a^2e)x^6}{a^2} + \frac{b^3(b^3c - ab^2d + a^2be - a^3f)x^9}{a^3}}{x^8(a + bx^3)} dx}{3ab^3} \\ &= -\frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3a^4(a + bx^3)} - \frac{\int \left(-\frac{3b^3c}{ax^8} - \frac{3b^3(-2bc + ad)}{a^2x^5} - \frac{3b^3(3b^2c - 2abd + a^2e)}{a^3x^2} - \frac{b^3(-10b^3c + 7ab^2d + a^2be - a^3f)}{a^3(a + bx^3)} \right) dx}{3ab^3} \\ &= -\frac{c}{7a^2x^7} + \frac{2bc - ad}{4a^3x^4} - \frac{3b^2c - 2abd + a^2e}{a^4x} - \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3a^4(a + bx^3)} - \frac{(10b^3c - 7ab^2d + a^2be - a^3f)}{a^3(a + bx^3)} \\ &= -\frac{c}{7a^2x^7} + \frac{2bc - ad}{4a^3x^4} - \frac{3b^2c - 2abd + a^2e}{a^4x} - \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3a^4(a + bx^3)} + \frac{(10b^3c - 7ab^2d + a^2be - a^3f)}{a^3(a + bx^3)} \\ &= -\frac{c}{7a^2x^7} + \frac{2bc - ad}{4a^3x^4} - \frac{3b^2c - 2abd + a^2e}{a^4x} - \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3a^4(a + bx^3)} + \frac{(10b^3c - 7ab^2d + a^2be - a^3f)}{a^3(a + bx^3)} \\ &= -\frac{c}{7a^2x^7} + \frac{2bc - ad}{4a^3x^4} - \frac{3b^2c - 2abd + a^2e}{a^4x} - \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{3a^4(a + bx^3)} + \frac{(10b^3c - 7ab^2d + a^2be - a^3f)}{a^3(a + bx^3)} \end{aligned}$$

Mathematica [A] time = 0.179237, size = 281, normalized size = 0.95

$$\frac{84 \sqrt[3]{ax^2(-a^2be + a^3f + ab^2d - b^3c)}}{a + bx^3} + \frac{14 \log\left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx + b^{2/3}x^2}\right)(-4a^2be + a^3f + 7ab^2d - 10b^3c)}{b^{2/3}} + \frac{28 \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)(4a^2be + a^3(-f) - 7ab^2d + 10b^3c)}{b^{2/3}} + \frac{28\sqrt{3}t}{52a^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^2),x]
```

```
[Out] ((-36*a^(7/3)*c)/x^7 - (63*a^(4/3)*(-2*b*c + a*d))/x^4 - (252*a^(1/3)*(3*b^2*c - 2*a*b*d + a^2*e))/x + (84*a^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(a + b*x^3) + (28*sqrt(3)*(10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt(3)])/b^(2/3) + (28*(10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (14*(-10*b^3*c + 7*a*b^2*d - 4*a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(252*a^(13/3))
```

Maple [B] time = 0.015, size = 529, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^2,x)
```

```
[Out] 1/3/a*x^2/(b*x^3+a)*f-1/3/a^2*x^2/(b*x^3+a)*b*e+1/3/a^3*x^2/(b*x^3+a)*b^2*d-1/3/a^4*x^2/(b*x^3+a)*b^3*c+4/9/a^2*e/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-2/9/a^2*e/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-4/9/a^2*e*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-7/9/a^3*b*d/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+7/18/a^3*b*d/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+7/9/a^3*b*d*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+10/9/a^4*b^2*c/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-5/9/a^4*b^2*c/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-10/9/a^4*b^2*c*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/9/a*f/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/18/a*f/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/9/a*f*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/7*c/a^2/x^7-1/4/a^2/x^4*d+1/2/a^3/x^4*b*c-e/a^2/x+2/a^3/x*b*d-3/a^4/x*b^2*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^2,x, algorithm="maxima")
```


[Out] Exception raised: ValueError

Fricas [A] time = 1.46461, size = 2159, normalized size = 7.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/252*(84*(10*a*b^5*c - 7*a^2*b^4*d + 4*a^3*b^3*e - a^4*b^2*f)*x^9 + 36*a^4*b^2*c \\ & + 63*(10*a^2*b^4*c - 7*a^3*b^3*d + 4*a^4*b^2*e)*x^6 - 9*(10*a^3*b^3*c - 7*a^4*b^2*d)*x^3 \\ & + 42*\sqrt{1/3}*((10*a*b^5*c - 7*a^2*b^4*d + 4*a^3*b^3*e - a^4*b^2*f)*x^{10} + (10*a^2*b^4*c - 7*a^3*b^3*d \\ & + 4*a^4*b^2*e - a^5*b*f)*x^7)*\sqrt{(-a*b^2)^{(1/3)}/a}*\log((2*b^2*x^3 - a*b + 3*\sqrt{1/3}*(a*b*x + 2 \\ & *(-a*b^2)^{(2/3)}*x^2 + (-a*b^2)^{(1/3)}*a)*\sqrt{(-a*b^2)^{(1/3)}/a} - 3*(-a*b^2)^{(2/3)}*x)/(b*x^3 + a)) \\ & + 14*((10*b^4*c - 7*a*b^3*d + 4*a^2*b^2*e - a^3*b*f)*x^{10} + (10*a*b^3*c - 7*a^2*b^2*d + 4*a^3*b*e - a^4*f)*x^7) \\ & *(-a*b^2)^{(2/3)}*\log(b^2*x^2 + (-a*b^2)^{(1/3)}*b*x + (-a*b^2)^{(2/3)}) - 28*((10*b^4*c - 7*a*b^3*d \\ & + 4*a^2*b^2*e - a^3*b*f)*x^{10} + (10*a*b^3*c - 7*a^2*b^2*d + 4*a^3*b*e - a^4*f)*x^7) \\ & *(-a*b^2)^{(2/3)}*\log(b*x - (-a*b^2)^{(1/3)})/(a^5*b^3*x^{10} + a^6*b^2*x^7), \\ & -1/252*(84*(10*a*b^5*c - 7*a^2*b^4*d + 4*a^3*b^3*e - a^4*b^2*f)*x^9 + 36*a^4*b^2*c \\ & + 63*(10*a^2*b^4*c - 7*a^3*b^3*d + 4*a^4*b^2*e)*x^6 - 9*(10*a^3*b^3*c - 7*a^4*b^2*d)*x^3 \\ & + 84*\sqrt{1/3}*((10*a*b^5*c - 7*a^2*b^4*d + 4*a^3*b^3*e - a^4*b^2*f)*x^{10} + (10*a^2*b^4*c - 7*a^3*b^3*d \\ & + 4*a^4*b^2*e - a^5*b*f)*x^7)*\sqrt{-(-a*b^2)^{(1/3)}/a}*\arctan(\sqrt{1/3}*(2*b*x + (-a*b^2)^{(1/3)}) \\ & *\sqrt{-(-a*b^2)^{(1/3)}/a}/b) + 14*((10*b^4*c - 7*a*b^3*d + 4*a^2*b^2*e - a^3*b*f)*x^{10} + (10*a*b^3*c - 7*a^2*b^2*d \\ & + 4*a^3*b*e - a^4*f)*x^7) \\ & *(-a*b^2)^{(2/3)}*\log(b^2*x^2 + (-a*b^2)^{(1/3)}*b*x + (-a*b^2)^{(2/3)}) - 28*((10*b^4*c - 7*a*b^3*d + 4*a^2*b^2*e - a^3*b*f) \\ & *x^{10} + (10*a*b^3*c - 7*a^2*b^2*d + 4*a^3*b*e - a^4*f)*x^7) \\ & *(-a*b^2)^{(2/3)}*\log(b*x - (-a*b^2)^{(1/3)})/(a^5*b^3*x^{10} + a^6*b^2*x^7)] \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**8/(b*x**3+a)**2,x)

[Out] Timed out

Giac [A] time = 1.11736, size = 522, normalized size = 1.76

$$\frac{\left(10b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 7ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 4a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^5} - \frac{b^3cx^2 - ab^2dx^2 - a^3fx^2 + a^2bx^2e}{3(bx^3 + a)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^2,x, algorithm="giac")

[Out] $\frac{1}{9} \cdot \frac{10b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 7a^2b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 4a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e}{a^5} - \frac{1}{3} \cdot \frac{b^3cx^2 - a^2b^2dx^2 - a^3fx^2 + a^2bx^2e}{(bx^3 + a)a^4} + \frac{1}{9} \cdot \frac{\sqrt{3} \cdot (10(-ab^2)^{\frac{2}{3}}b^3c - 7(-ab^2)^{\frac{2}{3}}a^2b^2d - (-ab^2)^{\frac{2}{3}}a^3f + 4(-ab^2)^{\frac{2}{3}}a^2b^2e) \cdot \arctan\left(\frac{1}{3}\sqrt{3} \cdot (2x + (-\frac{a}{b})^{\frac{1}{3}})\right)}{(-\frac{a}{b})^{\frac{1}{3}}a^5b^2} - \frac{1}{18} \cdot \frac{10(-ab^2)^{\frac{2}{3}}b^3c - 7(-ab^2)^{\frac{2}{3}}a^2b^2d - (-ab^2)^{\frac{2}{3}}a^3f + 4(-ab^2)^{\frac{2}{3}}a^2b^2e}{a^5b^2} \cdot \log(x^2 + x(-\frac{a}{b})^{\frac{1}{3}} + (-\frac{a}{b})^{\frac{2}{3}})}{a^5b^2} - \frac{1}{28} \cdot \frac{84b^2cx^6 - 56a^2bdx^6 + 28a^2x^6e - 14a^2bcx^3 + 7a^2dx^3 + 4a^2c}{a^4x^7}$

$$3.272 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)^2} dx$$

Optimal. Leaf size=297

$$\frac{x(a^2be + a^3(-f) - ab^2d + b^3c)}{3a^4(a + bx^3)} + \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(5a^2be - 2a^3f - 8ab^2d + 11b^3c)}{18a^{14/3}\sqrt[3]{b}} - \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(5a^2be + a^3(-f) - ab^2d + b^3c)}{18a^{14/3}\sqrt[3]{b}}$$

[Out] $-c/(8*a^2*x^8) + (2*b*c - a*d)/(5*a^3*x^5) - (3*b^2*c - 2*a*b*d + a^2*e)/(2*a^4*x^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*a^4*(a + b*x^3)) + ((11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(14/3)*b^(1/3)) - ((11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(14/3)*b^(1/3)) + ((11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(14/3)*b^(1/3))$

Rubi [A] time = 0.369952, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1829, 1834, 200, 31, 634, 617, 204, 628}

$$\frac{x(a^2be + a^3(-f) - ab^2d + b^3c)}{3a^4(a + bx^3)} + \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(5a^2be - 2a^3f - 8ab^2d + 11b^3c)}{18a^{14/3}\sqrt[3]{b}} - \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(5a^2be + a^3(-f) - ab^2d + b^3c)}{18a^{14/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^2), x]

[Out] $-c/(8*a^2*x^8) + (2*b*c - a*d)/(5*a^3*x^5) - (3*b^2*c - 2*a*b*d + a^2*e)/(2*a^4*x^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*a^4*(a + b*x^3)) + ((11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(14/3)*b^(1/3)) - ((11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(14/3)*b^(1/3)) + ((11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(14/3)*b^(1/3))$

Rule 1829

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^

```
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_)*(x_))^(m_))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[Ex
pandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 200

```
Int[((a_) + (b_)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
 imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^9(a + bx^3)^2} dx = -\frac{(b^3c - ab^2d + a^2be - a^3f)x}{3a^4(a + bx^3)} - \frac{\int \frac{-3b^3c + 3b^3\left(\frac{bc}{a} - d\right)x^3 - \frac{3b^3(b^2c - abd + a^2e)x^6}{a^2} + \frac{2b^3(b^3c - ab^2d + a^2be - a^3f)x^9}{a^3}}{x^9(a + bx^3)} dx}{3ab^3}$$

$$= -\frac{(b^3c - ab^2d + a^2be - a^3f)x}{3a^4(a + bx^3)} - \frac{\int \left(-\frac{3b^3c}{ax^9} - \frac{3b^3(-2bc + ad)}{a^2x^6} - \frac{3b^3(3b^2c - 2abd + a^2e)}{a^3x^3} - \frac{b^3(-11b^3c + 8ab^2d - 11b^3c + 8ab^2d)}{a^3(a + bx^3)} \right)}{3ab^3}$$

$$= -\frac{c}{8a^2x^8} + \frac{2bc - ad}{5a^3x^5} - \frac{3b^2c - 2abd + a^2e}{2a^4x^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3a^4(a + bx^3)} - \frac{(11b^3c - 8ab^2d + 11b^3c - 8ab^2d)}{360a^{14/3}}$$

$$= -\frac{c}{8a^2x^8} + \frac{2bc - ad}{5a^3x^5} - \frac{3b^2c - 2abd + a^2e}{2a^4x^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3a^4(a + bx^3)} - \frac{(11b^3c - 8ab^2d + 11b^3c - 8ab^2d)}{360a^{14/3}}$$

$$= -\frac{c}{8a^2x^8} + \frac{2bc - ad}{5a^3x^5} - \frac{3b^2c - 2abd + a^2e}{2a^4x^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3a^4(a + bx^3)} - \frac{(11b^3c - 8ab^2d + 11b^3c - 8ab^2d)}{360a^{14/3}}$$

$$= -\frac{c}{8a^2x^8} + \frac{2bc - ad}{5a^3x^5} - \frac{3b^2c - 2abd + a^2e}{2a^4x^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)x}{3a^4(a + bx^3)} - \frac{(11b^3c - 8ab^2d + 11b^3c - 8ab^2d)}{360a^{14/3}}$$

Mathematica [A] time = 0.178107, size = 280, normalized size = 0.94

$$\frac{120a^{2/3}x(-a^2be + a^3f + ab^2d - b^3c)}{a + bx^3} + \frac{20 \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx + b^{2/3}x^2}\right)(5a^2be - 2a^3f - 8ab^2d + 11b^3c)}{\sqrt[3]{b}} + \frac{40 \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)(-5a^2be + 2a^3f + 8ab^2d - 11b^3c)}{\sqrt[3]{b}} + \frac{40\sqrt{3}t}{360a^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^2),x]
```

```
[Out] ((-45*a^(8/3)*c)/x^8 - (72*a^(5/3)*(-2*b*c + a*d))/x^5 - (180*a^(2/3)*(3*b^2*c - 2*a*b*d + a^2*e))/x^2 + (120*a^(2/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(a + b*x^3) + (40*sqrt(3)*(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt(3)])/b^(1/3) + (40*(-11*b^3*c + 8*a*b^2*d - 5*a^2*b*e + 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + (20*(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3))/(360*a^(14/3))
```

Maple [B] time = 0.014, size = 520, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^2,x)
```

```
[Out] 1/3/a*x/(b*x^3+a)*f-1/3/a^2*x/(b*x^3+a)*b*e+1/3/a^3*x/(b*x^3+a)*b^2*d-1/3/a^4*x/(b*x^3+a)*b^3*c-5/9/a^2*e/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+5/18/a^2*e/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-5/9/a^2*e/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+8/9/a^3*b*d/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-4/9/a^3*b*d/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+8/9/a^3*b*d/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-11/9/a^4*b^2*c/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+11/18/a^4*b^2*c/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-11/9/a^4*b^2*c/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+2/9/a*f/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/9/a*f/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2/9/a*f/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/8*c/a^2/x^8-1/5/a^2/x^5*d+2/5/a^3/x^5*b*c-1/2/a^2/x^2*e+1/a^3/x^2*b*d-3/2/a^4/x^2*b^2*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^2,x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [A] time = 1.55454, size = 2182, normalized size = 7.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/360*(60*(11*a^2*b^4*c - 8*a^3*b^3*d + 5*a^4*b^2*e - 2*a^5*b*f)*x^9 + 45 \\ & *a^5*b*c + 36*(11*a^3*b^3*c - 8*a^4*b^2*d + 5*a^5*b*e)*x^6 - 9*(11*a^4*b^2*c \\ & - 8*a^5*b*d)*x^3 + 60*\sqrt{1/3}*((11*a*b^5*c - 8*a^2*b^4*d + 5*a^3*b^3*e \\ & - 2*a^4*b^2*f)*x^{11} + (11*a^2*b^4*c - 8*a^3*b^3*d + 5*a^4*b^2*e - 2*a^5*b*f) \\ &)*x^8)*\sqrt{-(a^2*b)^{(1/3)}/b}*\log((2*a*b*x^3 - 3*(a^2*b)^{(1/3)}*a*x - a^2 + \\ & 3*\sqrt{1/3}*(2*a*b*x^2 + (a^2*b)^{(2/3)}*x - (a^2*b)^{(1/3)}*a)*\sqrt{-(a^2*b)^{(1/3)}/b}) \\ &)/(b*x^3 + a)) - 20*((11*b^4*c - 8*a*b^3*d + 5*a^2*b^2*e - 2*a^3*b*f) \\ &)*x^{11} + (11*a*b^3*c - 8*a^2*b^2*d + 5*a^3*b*e - 2*a^4*f)*x^8)*(a^2*b)^{(2/3)} \\ &)*\log(a*b*x^2 - (a^2*b)^{(2/3)}*x + (a^2*b)^{(1/3)}*a) + 40*((11*b^4*c - 8*a*b^3*d \\ & + 5*a^2*b^2*e - 2*a^3*b*f)*x^{11} + (11*a*b^3*c - 8*a^2*b^2*d + 5*a^3*b*e \\ & - 2*a^4*f)*x^8)*(a^2*b)^{(2/3)}*\log(a*b*x + (a^2*b)^{(2/3)}))/((a^6*b^2*x^{11} + \\ & a^7*b*x^8), -1/360*(60*(11*a^2*b^4*c - 8*a^3*b^3*d + 5*a^4*b^2*e - 2*a^5*b*f) \\ &)*x^9 + 45*a^5*b*c + 36*(11*a^3*b^3*c - 8*a^4*b^2*d + 5*a^5*b*e)*x^6 - 9*(\\ & 11*a^4*b^2*c - 8*a^5*b*d)*x^3 + 120*\sqrt{1/3}*((11*a*b^5*c - 8*a^2*b^4*d + \\ & 5*a^3*b^3*e - 2*a^4*b^2*f)*x^{11} + (11*a^2*b^4*c - 8*a^3*b^3*d + 5*a^4*b^2*e \\ & - 2*a^5*b*f)*x^8)*\sqrt{(a^2*b)^{(1/3)}/b}*\arctan(\sqrt{1/3}*(2*(a^2*b)^{(2/3)}* \\ & x - (a^2*b)^{(1/3)}*a)*\sqrt{(a^2*b)^{(1/3)}/b}/a^2) - 20*((11*b^4*c - 8*a*b^3*d \\ & + 5*a^2*b^2*e - 2*a^3*b*f)*x^{11} + (11*a*b^3*c - 8*a^2*b^2*d + 5*a^3*b*e - \\ & 2*a^4*f)*x^8)*(a^2*b)^{(2/3)}*\log(a*b*x^2 - (a^2*b)^{(2/3)}*x + (a^2*b)^{(1/3)}*a) \\ &) + 40*((11*b^4*c - 8*a*b^3*d + 5*a^2*b^2*e - 2*a^3*b*f)*x^{11} + (11*a*b^3*c \\ & - 8*a^2*b^2*d + 5*a^3*b*e - 2*a^4*f)*x^8)*(a^2*b)^{(2/3)}*\log(a*b*x + (a^2*b)^{(2/3)}))/ \\ & ((a^6*b^2*x^{11} + a^7*b*x^8)] \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**9/(b*x**3+a)**2,x)

[Out] Timed out

Giac [A] time = 1.0871, size = 468, normalized size = 1.58

$$\frac{(11b^3c - 8ab^2d - 2a^3f + 5a^2be)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^5} - \frac{\sqrt{3}\left(11(-ab^2)^{\frac{1}{3}}b^3c - 8(-ab^2)^{\frac{1}{3}}ab^2d - 2(-ab^2)^{\frac{1}{3}}a^3f + 5(-ab^2)^{\frac{1}{3}}a^2be\right)}{9a^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^2,x, algorithm="giac")

[Out] $\frac{1}{9} \frac{(11b^3c - 8ab^2d - 2a^3f + 5a^2be) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^5} - \frac{1}{9} \frac{\sqrt{3} \left(11(-ab^2)^{\frac{1}{3}}b^3c - 8(-ab^2)^{\frac{1}{3}}ab^2d - 2(-ab^2)^{\frac{1}{3}}a^3f + 5(-ab^2)^{\frac{1}{3}}a^2be\right)}{a^5b} - \frac{1}{3} \frac{\arctan\left(\frac{\sqrt{3}(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}})}{\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^5b} - \frac{1}{18} \frac{(11(-ab^2)^{\frac{1}{3}}b^3c - 8(-ab^2)^{\frac{1}{3}}ab^2d - 2(-ab^2)^{\frac{1}{3}}a^3f + 5(-ab^2)^{\frac{1}{3}}a^2be) \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^5b} - \frac{1}{40} \frac{(60b^2cx^6 - 40abd^2x^6 + 20a^2x^6e - 16abcx^3 + 8a^2dx^3 + 5a^2c)}{a^4x^8}$

$$3.273 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)^2} dx$$

Optimal. Leaf size=334

$$\frac{bx^2(a^2be + a^3(-f) - ab^2d + b^3c)}{3a^5(a + bx^3)} + \frac{\sqrt[3]{b} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(7a^2be - 4a^3f - 10ab^2d + 13b^3c)}{18a^{16/3}} + \frac{2a^2be + a^3(-f)}{a^5}$$

[Out] $-c/(10*a^2*x^{10}) + (2*b*c - a*d)/(7*a^3*x^7) - (3*b^2*c - 2*a*b*d + a^2*e)/(4*a^4*x^4) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(a^5*x) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*a^5*(a + b*x^3)) - (b^{(1/3)}*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(3*Sqrt[3]*a^{(16/3)}) - (b^{(1/3)}*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*Log[a^{(1/3)} + b^{(1/3)}*x])/(9*a^{(16/3)}) + (b^{(1/3)}*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(18*a^{(16/3)})$

Rubi [A] time = 0.456739, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1829, 1834, 292, 31, 634, 617, 204, 628}

$$\frac{bx^2(a^2be + a^3(-f) - ab^2d + b^3c)}{3a^5(a + bx^3)} + \frac{\sqrt[3]{b} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(7a^2be - 4a^3f - 10ab^2d + 13b^3c)}{18a^{16/3}} + \frac{2a^2be + a^3(-f)}{a^5}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^2), x]

[Out] $-c/(10*a^2*x^{10}) + (2*b*c - a*d)/(7*a^3*x^7) - (3*b^2*c - 2*a*b*d + a^2*e)/(4*a^4*x^4) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(a^5*x) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*a^5*(a + b*x^3)) - (b^{(1/3)}*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(3*Sqrt[3]*a^{(16/3)}) - (b^{(1/3)}*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*Log[a^{(1/3)} + b^{(1/3)}*x])/(9*a^{(16/3)}) + (b^{(1/3)}*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(18*a^{(16/3)})$

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Int[Ex
pandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^( -1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^( -1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

a, 0] || LtQ[b, 0])

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
 imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{c + dx^3 + ex^6 + fx^9}{x^{11} (a + bx^3)^2} dx &= \frac{b(b^3c - ab^2d + a^2be - a^3f)x^2}{3a^5(a + bx^3)} - \frac{\int \frac{-3b^3c + 3b^3\left(\frac{bc}{a} - d\right)x^3 - \frac{3b^3(b^2c - abd + a^2e)x^6}{a^2} + \frac{3b^3(b^3c - ab^2d + a^2be - a^3f)x^9}{a^3} - b^4(b^3c - ab^2d + a^2be - a^3f)}{x^{11}(a + bx^3)} dx}{3ab^3} \\ &= \frac{b(b^3c - ab^2d + a^2be - a^3f)x^2}{3a^5(a + bx^3)} - \frac{\int \left(-\frac{3b^3c}{ax^{11}} - \frac{3b^3(-2bc + ad)}{a^2x^8} - \frac{3b^3(3b^2c - 2abd + a^2e)}{a^3x^5} - \frac{3b^3(-4b^3c + 3ab^2d + a^2be - a^3f)}{a^4x^2} \right) dx}{3ab^3} \\ &= -\frac{c}{10a^2x^{10}} + \frac{2bc - ad}{7a^3x^7} - \frac{3b^2c - 2abd + a^2e}{4a^4x^4} + \frac{4b^3c - 3ab^2d + 2a^2be - a^3f}{a^5x} + \frac{b(b^3c - ab^2d + a^2be - a^3f)}{3a^5(a + bx^3)} \\ &= -\frac{c}{10a^2x^{10}} + \frac{2bc - ad}{7a^3x^7} - \frac{3b^2c - 2abd + a^2e}{4a^4x^4} + \frac{4b^3c - 3ab^2d + 2a^2be - a^3f}{a^5x} + \frac{b(b^3c - ab^2d + a^2be - a^3f)}{3a^5(a + bx^3)} \\ &= -\frac{c}{10a^2x^{10}} + \frac{2bc - ad}{7a^3x^7} - \frac{3b^2c - 2abd + a^2e}{4a^4x^4} + \frac{4b^3c - 3ab^2d + 2a^2be - a^3f}{a^5x} + \frac{b(b^3c - ab^2d + a^2be - a^3f)}{3a^5(a + bx^3)} \\ &= -\frac{c}{10a^2x^{10}} + \frac{2bc - ad}{7a^3x^7} - \frac{3b^2c - 2abd + a^2e}{4a^4x^4} + \frac{4b^3c - 3ab^2d + 2a^2be - a^3f}{a^5x} + \frac{b(b^3c - ab^2d + a^2be - a^3f)}{3a^5(a + bx^3)} \\ &= -\frac{c}{10a^2x^{10}} + \frac{2bc - ad}{7a^3x^7} - \frac{3b^2c - 2abd + a^2e}{4a^4x^4} + \frac{4b^3c - 3ab^2d + 2a^2be - a^3f}{a^5x} + \frac{b(b^3c - ab^2d + a^2be - a^3f)}{3a^5(a + bx^3)} \end{aligned}$$

Mathematica [A] time = 0.181351, size = 319, normalized size = 0.96

$$-\frac{420 \sqrt[3]{abx^2(-a^2be + a^3f + ab^2d - b^3c)}}{a + bx^3} + 70 \sqrt[3]{b} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3}x^2) (7a^2be - 4a^3f - 10ab^2d + 13b^3c) - \frac{1260 \sqrt[3]{a}(-2a^2be + a^3f + 3b^3c)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^2),x]

[Out]
$$\begin{aligned} &((-126*a^{(10/3)*c}/x^{10} - (180*a^{(7/3)*(-2*b*c + a*d)}/x^7 - (315*a^{(4/3)*} \\ &3*b^2*c - 2*a*b*d + a^2*e))/x^4 - (1260*a^{(1/3)*(-4*b^3*c + 3*a*b^2*d - 2*a \\ &^2*b*e + a^3*f))/x - (420*a^{(1/3)*b*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*} \\ &x^2)/(a + b*x^3) - 140*\text{Sqrt}[3]*b^{(1/3)*}(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e - \\ &4*a^3*f)*\text{ArcTan}[(1 - (2*b^{(1/3)*x}/a^{(1/3)})/\text{Sqrt}[3]] + 140*b^{(1/3)*}(-13*b^3 \\ &3*c + 10*a*b^2*d - 7*a^2*b*e + 4*a^3*f)*\text{Log}[a^{(1/3)} + b^{(1/3)*x}] + 70*b^{(1/3)*} \\ &(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*\text{Log}[a^{(2/3)} - a^{(1/3)*b^{(1/3)*}x} \\ &+ b^{(2/3)*x^2}]/(1260*a^{(16/3)}) \end{aligned}$$

Maple [A] time = 0.016, size = 575, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^2,x)

[Out]
$$\begin{aligned} &1/3*b^2/a^3*x^2/(b*x^3+a)*e-1/10*c/a^2/x^{10}+10/9*b^2/a^4*d/(1/b*a)^{(1/3)*}\ln \\ &(x+(1/b*a)^{(1/3)})-13/9*b^3/a^5*c/(1/b*a)^{(1/3)*}\ln(x+(1/b*a)^{(1/3)})+13/18*b^ \\ &3/a^5*c/(1/b*a)^{(1/3)*}\ln(x^2-(1/b*a)^{(1/3)*x}+(1/b*a)^{(2/3)})-1/3*b/a^2*x^2/(\\ &b*x^3+a)*f-7/9*b/a^3*e/(1/b*a)^{(1/3)*}\ln(x+(1/b*a)^{(1/3)})+4/9/a^2*f/(1/b*a)^{(\\ &1/3)*}\ln(x+(1/b*a)^{(1/3)})-2/9/a^2*f/(1/b*a)^{(1/3)*}\ln(x^2-(1/b*a)^{(1/3)*x}+(1 \\ &/b*a)^{(2/3)})+2/7/a^3/x^7*b*c+1/2/a^3/x^4*b*d-3/4/a^4/x^4*b^2*c+2/a^3/x*b*e- \\ &3/a^4/x*b^2*d+4/a^5/x*b^3*c-1/a^2/x*f-1/7/a^2/x^7*d-1/4/a^2/x^4*e+7/18*b/a^ \\ &3*e/(1/b*a)^{(1/3)*}\ln(x^2-(1/b*a)^{(1/3)*x}+(1/b*a)^{(2/3)})+1/3*b^4/a^5*x^2/(b* \\ &x^3+a)*c-5/9*b^2/a^4*d/(1/b*a)^{(1/3)*}\ln(x^2-(1/b*a)^{(1/3)*x}+(1/b*a)^{(2/3)})- \\ &4/9/a^2*f*3^{(1/2)}/(1/b*a)^{(1/3)*}\arctan(1/3*3^{(1/2)*(2/(1/b*a)^{(1/3)*x}-1)})-1 \\ &/3*b^3/a^4*x^2/(b*x^3+a)*d+7/9*b/a^3*e*3^{(1/2)}/(1/b*a)^{(1/3)*}\arctan(1/3*3^{(\\ &1/2)*(2/(1/b*a)^{(1/3)*x}-1)})-10/9*b^2/a^4*d*3^{(1/2)}/(1/b*a)^{(1/3)*}\arctan(1/3 \\ &*3^{(1/2)*(2/(1/b*a)^{(1/3)*x}-1)})+13/9*b^3/a^5*c*3^{(1/2)}/(1/b*a)^{(1/3)*}\arctan \\ &(1/3*3^{(1/2)*(2/(1/b*a)^{(1/3)*x}-1)}) \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 1.37247, size = 1034, normalized size = 3.1

$$420(13b^4c - 10ab^3d + 7a^2b^2e - 4a^3bf)x^{12} + 315(13ab^3c - 10a^2b^2d + 7a^3be - 4a^4f)x^9 - 45(13a^2b^2c - 10a^3bd + 7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^2,x, algorithm="fricas")`

[Out]
$$\frac{1}{1260} \cdot (420 \cdot (13b^4c - 10ab^3d + 7a^2b^2e - 4a^3bf) \cdot x^{12} + 315 \cdot (13ab^3c - 10a^2b^2d + 7a^3be - 4a^4f) \cdot x^9 - 45 \cdot (13a^2b^2c - 10a^3bd + 7a^4e) \cdot x^6 - 126a^4c + 18 \cdot (13a^3b^3c - 10a^4d) \cdot x^3 + 140 \cdot \sqrt{3} \cdot ((13b^4c - 10ab^3d + 7a^2b^2e - 4a^3bf) \cdot x^{13} + (13ab^3c - 10a^2b^2d + 7a^3be - 4a^4f) \cdot x^{10}) \cdot (b/a)^{1/3} \cdot \arctan(2/3 \cdot \sqrt{3} \cdot x \cdot (b/a)^{1/3} - 1/3 \cdot \sqrt{3}) + 70 \cdot ((13b^4c - 10ab^3d + 7a^2b^2e - 4a^3bf) \cdot x^{13} + (13ab^3c - 10a^2b^2d + 7a^3be - 4a^4f) \cdot x^{10}) \cdot (b/a)^{1/3} \cdot \log(bx^2 - ax \cdot (b/a)^{2/3} + a \cdot (b/a)^{1/3}) - 140 \cdot ((13b^4c - 10ab^3d + 7a^2b^2e - 4a^3bf) \cdot x^{13} + (13ab^3c - 10a^2b^2d + 7a^3be - 4a^4f) \cdot x^{10}) \cdot (b/a)^{1/3} \cdot \log(bx + a \cdot (b/a)^{2/3})) / (a^5bx^{13} + a^6x^{10})$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x**9+e*x**6+d*x**3+c)/x**11/(b*x**3+a)**2,x)`

[Out] Timed out

Giac [A] time = 1.1012, size = 590, normalized size = 1.77

$$\frac{\left(13b^4c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 10ab^3d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 4a^3bf\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 7a^2b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right) + \sqrt{3}\left(13\left(-ab^2\right)^{\frac{2}{3}}b^3c - 10\left(-ab^2\right)^{\frac{2}{3}}b^2d - 4\left(-ab^2\right)^{\frac{2}{3}}a^3f + 7\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\arctan\left(\frac{1}{3}\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\right)}{9a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^2,x, algorithm="giac")

[Out] -1/9*(13*b^4*c*(-a/b)^(1/3) - 10*a*b^3*d*(-a/b)^(1/3) - 4*a^3*b*f*(-a/b)^(1/3) + 7*a^2*b^2*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^6 - 1/9*sqrt(3)*(13*(-a*b^2)^(2/3)*b^3*c - 10*(-a*b^2)^(2/3)*a*b^2*d - 4*(-a*b^2)^(2/3)*a^3*f + 7*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3)))/(-a/b)^(1/3)/(a^6*b) + 1/3*(b^4*c*x^2 - a*b^3*d*x^2 - a^3*b*f*x^2 + a^2*b^2*x^2*e)/((b*x^3 + a)*a^5) + 1/18*(13*(-a*b^2)^(2/3)*b^3*c - 10*(-a*b^2)^(2/3)*a*b^2*d - 4*(-a*b^2)^(2/3)*a^3*f + 7*(-a*b^2)^(2/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^6*b) + 1/140*(560*b^3*c*x^9 - 420*a*b^2*d*x^9 - 140*a^3*f*x^9 + 280*a^2*b*x^9*e - 105*a*b^2*c*x^6 + 70*a^2*b*d*x^6 - 35*a^3*x^6*e + 40*a^2*b*c*x^3 - 20*a^3*d*x^3 - 14*a^3*c)/(a^5*x^10)

$$3.274 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)^2} dx$$

Optimal. Leaf size=335

$$\frac{bx(a^2be + a^3(-f) - ab^2d + b^3c)}{3a^5(a + bx^3)} + \frac{2a^2be + a^3(-f) - 3ab^2d + 4b^3c}{2a^5x^2} - \frac{b^{2/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(8a^2be - 5a^3f - 1)}{18a^{17/3}}$$

[Out] $-c/(11*a^2*x^{11}) + (2*b*c - a*d)/(8*a^3*x^8) - (3*b^2*c - 2*a*b*d + a^2*e)/(5*a^4*x^5) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(2*a^5*x^2) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*a^5*(a + b*x^3)) - (b^{(2/3)}*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(3*Sqrt[3]*a^{(17/3)}) + (b^{(2/3)}*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*Log[a^{(1/3)} + b^{(1/3)}*x])/(9*a^{(17/3)}) - (b^{(2/3)}*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(18*a^{(17/3)})$

Rubi [A] time = 0.434195, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1829, 1834, 200, 31, 634, 617, 204, 628}

$$\frac{bx(a^2be + a^3(-f) - ab^2d + b^3c)}{3a^5(a + bx^3)} + \frac{2a^2be + a^3(-f) - 3ab^2d + 4b^3c}{2a^5x^2} - \frac{b^{2/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(8a^2be - 5a^3f - 1)}{18a^{17/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^2), x]

[Out] $-c/(11*a^2*x^{11}) + (2*b*c - a*d)/(8*a^3*x^8) - (3*b^2*c - 2*a*b*d + a^2*e)/(5*a^4*x^5) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(2*a^5*x^2) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*a^5*(a + b*x^3)) - (b^{(2/3)}*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(3*Sqrt[3]*a^{(17/3)}) + (b^{(2/3)}*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*Log[a^{(1/3)} + b^{(1/3)}*x])/(9*a^{(17/3)}) - (b^{(2/3)}*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(18*a^{(17/3)})$

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Int[Ex
pandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(p_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```


a, 0] || LtQ[b, 0])

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
 imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{c + dx^3 + ex^6 + fx^9}{x^{12}(a + bx^3)^2} dx &= \frac{b(b^3c - ab^2d + a^2be - a^3f)x}{3a^5(a + bx^3)} - \frac{\int \frac{-3b^3c + 3b^3\left(\frac{bc}{a} - d\right)x^3 - \frac{3b^3(b^2c - abd + a^2e)x^6}{a^2} + \frac{3b^3(b^3c - ab^2d + a^2be - a^3f)x^9}{a^3} - \frac{2b^4(b^3c - ab^2d + a^2be - a^3f)x^{12}}{a^4}}{x^{12}(a + bx^3)} dx}{3ab^3} \\ &= \frac{b(b^3c - ab^2d + a^2be - a^3f)x}{3a^5(a + bx^3)} - \frac{\int \left(-\frac{3b^3c}{ax^{12}} - \frac{3b^3(-2bc + ad)}{a^2x^9} - \frac{3b^3(3b^2c - 2abd + a^2e)}{a^3x^6} - \frac{3b^3(-4b^3c + 3ab^2d - 2a^2be + a^3f)}{a^4x^3} \right) dx}{3ab^3} \\ &= -\frac{c}{11a^2x^{11}} + \frac{2bc - ad}{8a^3x^8} - \frac{3b^2c - 2abd + a^2e}{5a^4x^5} + \frac{4b^3c - 3ab^2d + 2a^2be - a^3f}{2a^5x^2} + \frac{b(b^3c - ab^2d + a^2be - a^3f)}{3a^5(a + bx^3)} \\ &= -\frac{c}{11a^2x^{11}} + \frac{2bc - ad}{8a^3x^8} - \frac{3b^2c - 2abd + a^2e}{5a^4x^5} + \frac{4b^3c - 3ab^2d + 2a^2be - a^3f}{2a^5x^2} + \frac{b(b^3c - ab^2d + a^2be - a^3f)}{3a^5(a + bx^3)} \\ &= -\frac{c}{11a^2x^{11}} + \frac{2bc - ad}{8a^3x^8} - \frac{3b^2c - 2abd + a^2e}{5a^4x^5} + \frac{4b^3c - 3ab^2d + 2a^2be - a^3f}{2a^5x^2} + \frac{b(b^3c - ab^2d + a^2be - a^3f)}{3a^5(a + bx^3)} \\ &= -\frac{c}{11a^2x^{11}} + \frac{2bc - ad}{8a^3x^8} - \frac{3b^2c - 2abd + a^2e}{5a^4x^5} + \frac{4b^3c - 3ab^2d + 2a^2be - a^3f}{2a^5x^2} + \frac{b(b^3c - ab^2d + a^2be - a^3f)}{3a^5(a + bx^3)} \\ &= -\frac{c}{11a^2x^{11}} + \frac{2bc - ad}{8a^3x^8} - \frac{3b^2c - 2abd + a^2e}{5a^4x^5} + \frac{4b^3c - 3ab^2d + 2a^2be - a^3f}{2a^5x^2} + \frac{b(b^3c - ab^2d + a^2be - a^3f)}{3a^5(a + bx^3)} \end{aligned}$$

Mathematica [A] time = 0.187121, size = 317, normalized size = 0.95

$$-\frac{1320a^{2/3}bx(-a^2be + a^3f + ab^2d - b^3c)}{a + bx^3} - \frac{1980a^{2/3}(-2a^2be + a^3f + 3ab^2d - 4b^3c)}{x^2} + 220b^{2/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2) (-8a^2be + 5a^3f + 11ab^2d - b^3c)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^2),x]

[Out]
$$\begin{aligned} &((-360*a^{(11/3)*c})/x^{11} - (495*a^{(8/3)*(-2*b*c + a*d)})/x^8 - (792*a^{(5/3)*} \\ &3*b^2*c - 2*a*b*d + a^2*e))/x^5 - (1980*a^{(2/3)*(-4*b^3*c + 3*a*b^2*d - 2*a \\ &^2*b*e + a^3*f))/x^2 - (1320*a^{(2/3)*b*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3* \\ &f)*x)/(a + b*x^3) - 440*sqrt[3]*b^{(2/3)*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e \\ &- 5*a^3*f)*ArcTan[(1 - (2*b^{(1/3)*x})/a^{(1/3)})/sqrt[3]] + 440*b^{(2/3)*(14*b^3*c \\ &- 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*Log[a^{(1/3)} + b^{(1/3)*x}] + 220*b^{(2 \\ &/3)*(-14*b^3*c + 11*a*b^2*d - 8*a^2*b*e + 5*a^3*f)*Log[a^{(2/3)} - a^{(1/3)*b^ \\ &(1/3)*x + b^{(2/3)*x^2}]/(3960*a^{(17/3)}) \end{aligned}$$

Maple [A] time = 0.017, size = 566, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^2,x)

[Out]
$$\begin{aligned} &-1/11*c/a^2/x^{11}+14/9*b^3/a^5*c/(1/b*a)^{(2/3)*ln(x+(1/b*a)^{(1/3)})}-7/9*b^3/a \\ &^5*c/(1/b*a)^{(2/3)*ln(x^2-(1/b*a)^{(1/3)*x+(1/b*a)^{(2/3)})}-1/3*b^3/a^4*x/(b*x \\ &^3+a)*d+1/3*b^4/a^5*x/(b*x^3+a)*c-5/9/a^2*f/(1/b*a)^{(2/3)*3^{(1/2)*arctan(1/ \\ &3*3^{(1/2)*(2/(1/b*a)^{(1/3)*x-1})+8/9*b/a^3*e/(1/b*a)^{(2/3)*ln(x+(1/b*a)^{(1/ \\ &3)})}-4/9*b/a^3*e/(1/b*a)^{(2/3)*ln(x^2-(1/b*a)^{(1/3)*x+(1/b*a)^{(2/3)})}-11/9*b^ \\ &2/a^4*d/(1/b*a)^{(2/3)*ln(x+(1/b*a)^{(1/3)})+11/18*b^2/a^4*d/(1/b*a)^{(2/3)*ln(\\ &x^2-(1/b*a)^{(1/3)*x+(1/b*a)^{(2/3)})}-1/2/a^2/x^2*f-1/8/a^2/x^8*d-1/5/a^2/x^5* \\ &e-5/9/a^2*f/(1/b*a)^{(2/3)*ln(x+(1/b*a)^{(1/3)})+5/18/a^2*f/(1/b*a)^{(2/3)*ln(x \\ &^2-(1/b*a)^{(1/3)*x+(1/b*a)^{(2/3)})+1/4/a^3/x^8*b*c+2/5/a^3/x^5*b*d-3/5/a^4/x \\ &^5*b^2*c+1/a^3/x^2*b*e-3/2/a^4/x^2*b^2*d+2/a^5/x^2*b^3*c+1/3*b^2/a^3*x/(b*x \\ &^3+a)*e-1/3*b/a^2*x/(b*x^3+a)*f+14/9*b^3/a^5*c/(1/b*a)^{(2/3)*3^{(1/2)*arctan \\ &(1/3*3^{(1/2)*(2/(1/b*a)^{(1/3)*x-1})+8/9*b/a^3*e/(1/b*a)^{(2/3)*3^{(1/2)*arcta \\ &n(1/3*3^{(1/2)*(2/(1/b*a)^{(1/3)*x-1})}-11/9*b^2/a^4*d/(1/b*a)^{(2/3)*3^{(1/2)*a \\ &rctan(1/3*3^{(1/2)*(2/(1/b*a)^{(1/3)*x-1})} \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.38418, size = 1099, normalized size = 3.28

$$660(14b^4c - 11ab^3d + 8a^2b^2e - 5a^3bf)x^{12} + 396(14ab^3c - 11a^2b^2d + 8a^3be - 5a^4f)x^9 - 99(14a^2b^2c - 11a^3bd + 8a^4e)x^6 - 360a^4c + 45(14a^3b^3c - 11a^4d)x^3 - 440\sqrt{3}((14b^4c - 11ab^3d + 8a^2b^2e - 5a^3bf)x^{14} + (14ab^3c - 11a^2b^2d + 8a^3be - 5a^4f)x^{11})\arctan\left(\frac{1}{3}\sqrt{3}\frac{ax(-b^2/a^2)^{2/3} - b}{b}\right) + 220((14b^4c - 11ab^3d + 8a^2b^2e - 5a^3bf)x^{14} + (14ab^3c - 11a^2b^2d + 8a^3be - 5a^4f)x^{11})\log(b^2x^2 + ax(-b^2/a^2)^{1/3} + a^2(-b^2/a^2)^{2/3}) - 440((14b^4c - 11ab^3d + 8a^2b^2e - 5a^3bf)x^{14} + (14ab^3c - 11a^2b^2d + 8a^3be - 5a^4f)x^{11})\log(bx - a(-b^2/a^2)^{1/3})/(a^5bx^{14} + a^6x^{11})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3960*(660*(14*b^4*c - 11*a*b^3*d + 8*a^2*b^2*e - 5*a^3*b*f)*x^12 + 396*(14*a*b^3*c - 11*a^2*b^2*d + 8*a^3*b*e - 5*a^4*f)*x^9 - 99*(14*a^2*b^2*c - 11*a^3*b*d + 8*a^4*e)*x^6 - 360*a^4*c + 45*(14*a^3*b*c - 11*a^4*d)*x^3 - 440*sqrt(3)*((14*b^4*c - 11*a*b^3*d + 8*a^2*b^2*e - 5*a^3*b*f)*x^14 + (14*a*b^3*c - 11*a^2*b^2*d + 8*a^3*b*e - 5*a^4*f)*x^11)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) + 220*((14*b^4*c - 11*a*b^3*d + 8*a^2*b^2*e - 5*a^3*b*f)*x^14 + (14*a*b^3*c - 11*a^2*b^2*d + 8*a^3*b*e - 5*a^4*f)*x^11)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - 440*((14*b^4*c - 11*a*b^3*d + 8*a^2*b^2*e - 5*a^3*b*f)*x^14 + (14*a*b^3*c - 11*a^2*b^2*d + 8*a^3*b*e - 5*a^4*f)*x^11)*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)))/(a^5*b*x^14 + a^6*x^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**12/(b*x**3+a)**2,x)
```

[Out] Timed out

Giac [A] time = 1.08584, size = 528, normalized size = 1.58

$$\frac{\sqrt{3} \left(14 (-ab^2)^{\frac{1}{3}} b^3 c - 11 (-ab^2)^{\frac{1}{3}} ab^2 d - 5 (-ab^2)^{\frac{1}{3}} a^3 f + 8 (-ab^2)^{\frac{1}{3}} a^2 b e \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{9 a^6} \quad (14 b^4 c - 11 ab^3 d - 5 a^3 f + 8 a^2 b e)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^2,x, algorithm="giac")

[Out] $\frac{1}{9} \sqrt{3} (14 (-a b^2)^{\frac{1}{3}} b^3 c - 11 (-a b^2)^{\frac{1}{3}} a b^2 d - 5 (-a b^2)^{\frac{1}{3}} a^3 f + 8 (-a b^2)^{\frac{1}{3}} a^2 b e) \arctan \left(\frac{\sqrt{3} (2x + (-a/b)^{\frac{1}{3}})}{3 (-a/b)^{\frac{1}{3}}} \right) / (-a/b)^{\frac{1}{3}} / a^6 - \frac{1}{9} (14 b^4 c - 11 a b^3 d - 5 a^3 b f + 8 a^2 b^2 e) (-a/b)^{\frac{1}{3}} \log(\text{abs}(x - (-a/b)^{\frac{1}{3}})) / a^6 + \frac{1}{18} (14 (-a b^2)^{\frac{1}{3}} b^3 c - 11 (-a b^2)^{\frac{1}{3}} a b^2 d - 5 (-a b^2)^{\frac{1}{3}} a^3 f + 8 (-a b^2)^{\frac{1}{3}} a^2 b e) \log(x^2 + x (-a/b)^{\frac{1}{3}} + (-a/b)^{\frac{2}{3}}) / a^6 + \frac{1}{3} (b^4 c x - a b^3 d x - a^3 b f x + a^2 b^2 x e) / ((b x^3 + a) a^5) + \frac{1}{440} (880 b^3 c x^9 - 660 a b^2 d x^9 - 220 a^3 f x^9 + 440 a^2 b x^9 e - 264 a b^2 c x^6 + 176 a^2 b d x^6 - 88 a^3 x^6 e + 110 a^2 b c x^3 - 55 a^3 d x^3 - 40 a^3 c) / (a^5 x^{11})$

$$3.275 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)^2} dx$$

Optimal. Leaf size=375

$$-\frac{b^2x^2(a^2be+a^3(-f)-ab^2d+b^3c)}{3a^6(a+bx^3)} + \frac{2a^2be+a^3(-f)-3ab^2d+4b^3c}{4a^5x^4} - \frac{b^{4/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(10a^2be-7a^3)}{18a^{19/3}}$$

[Out] $-c/(13a^2x^{13}) + (2bc - ad)/(10a^3x^{10}) - (3b^2c - 2abd + a^2e)/(7a^4x^7) + (4b^3c - 3ab^2d + 2a^2be - a^3f)/(4a^5x^4) - (b(5b^3c - 4ab^2d + 3a^2be - 2a^3f))/(a^6x) - (b^2(b^3c - ab^2d + a^2be - a^3f)x^2)/(3a^6(a + bx^3)) + (b^{4/3}(16b^3c - 13ab^2d + 10a^2be - 7a^3f) \operatorname{ArcTan}[(a^{1/3} - 2b^{1/3}x)/(\sqrt[3]{3}a^{1/3})])/(3\sqrt[3]{3}a^{19/3}) + (b^{4/3}(16b^3c - 13ab^2d + 10a^2be - 7a^3f) \operatorname{Log}[a^{1/3} + b^{1/3}x])/(9a^{19/3}) - (b^{4/3}(16b^3c - 13ab^2d + 10a^2be - 7a^3f) \operatorname{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2])/(18a^{19/3})$

Rubi [A] time = 0.533546, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1829, 1834, 292, 31, 634, 617, 204, 628}

$$-\frac{b^2x^2(a^2be+a^3(-f)-ab^2d+b^3c)}{3a^6(a+bx^3)} + \frac{2a^2be+a^3(-f)-3ab^2d+4b^3c}{4a^5x^4} - \frac{b^{4/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(10a^2be-7a^3)}{18a^{19/3}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c + dx^3 + ex^6 + fx^9)/(x^{14}(a + bx^3)^2), x]$

[Out] $-c/(13a^2x^{13}) + (2bc - ad)/(10a^3x^{10}) - (3b^2c - 2abd + a^2e)/(7a^4x^7) + (4b^3c - 3ab^2d + 2a^2be - a^3f)/(4a^5x^4) - (b(5b^3c - 4ab^2d + 3a^2be - 2a^3f))/(a^6x) - (b^2(b^3c - ab^2d + a^2be - a^3f)x^2)/(3a^6(a + bx^3)) + (b^{4/3}(16b^3c - 13ab^2d + 10a^2be - 7a^3f) \operatorname{ArcTan}[(a^{1/3} - 2b^{1/3}x)/(\sqrt[3]{3}a^{1/3})])/(3\sqrt[3]{3}a^{19/3}) + (b^{4/3}(16b^3c - 13ab^2d + 10a^2be - 7a^3f) \operatorname{Log}[a^{1/3} + b^{1/3}x])/(9a^{19/3}) - (b^{4/3}(16b^3c - 13ab^2d + 10a^2be - 7a^3f) \operatorname{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2])/(18a^{19/3})$

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_)^(m_.)))/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^( -1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{c + dx^3 + ex^6 + fx^9}{x^{14}(a + bx^3)^2} dx &= -\frac{b^2(b^3c - ab^2d + a^2be - a^3f)x^2}{3a^6(a + bx^3)} - \int \frac{-3b^3c + 3b^3\left(\frac{bc}{a} - d\right)x^3 - \frac{3b^3(b^2c - abd + a^2e)x^6}{a^2} + \frac{3b^3(b^3c - ab^2d + a^2be - a^3f)x^9}{a^3}}{x^{14}(a + bx^3)} dx \\
 &= -\frac{b^2(b^3c - ab^2d + a^2be - a^3f)x^2}{3a^6(a + bx^3)} - \int \left(-\frac{3b^3c}{ax^{14}} - \frac{3b^3(-2bc + ad)}{a^2x^{11}} - \frac{3b^3(3b^2c - 2abd + a^2e)}{a^3x^8} - \frac{3b^3(-4b^3c + 3ab^2d + 2a^2be - a^3f)}{a^4x^5} \right) dx \\
 &= -\frac{c}{13a^2x^{13}} + \frac{2bc - ad}{10a^3x^{10}} - \frac{3b^2c - 2abd + a^2e}{7a^4x^7} + \frac{4b^3c - 3ab^2d + 2a^2be - a^3f}{4a^5x^4} - \frac{b(5b^3c - 4ab^2d + 3a^2be - a^3f)}{3ab^3} \\
 &= -\frac{c}{13a^2x^{13}} + \frac{2bc - ad}{10a^3x^{10}} - \frac{3b^2c - 2abd + a^2e}{7a^4x^7} + \frac{4b^3c - 3ab^2d + 2a^2be - a^3f}{4a^5x^4} - \frac{b(5b^3c - 4ab^2d + 3a^2be - a^3f)}{3ab^3} \\
 &= -\frac{c}{13a^2x^{13}} + \frac{2bc - ad}{10a^3x^{10}} - \frac{3b^2c - 2abd + a^2e}{7a^4x^7} + \frac{4b^3c - 3ab^2d + 2a^2be - a^3f}{4a^5x^4} - \frac{b(5b^3c - 4ab^2d + 3a^2be - a^3f)}{3ab^3} \\
 &= -\frac{c}{13a^2x^{13}} + \frac{2bc - ad}{10a^3x^{10}} - \frac{3b^2c - 2abd + a^2e}{7a^4x^7} + \frac{4b^3c - 3ab^2d + 2a^2be - a^3f}{4a^5x^4} - \frac{b(5b^3c - 4ab^2d + 3a^2be - a^3f)}{3ab^3}
 \end{aligned}$$

Mathematica [A] time = 0.360017, size = 370, normalized size = 0.99

$$\frac{b^2 x^2 (-a^2 b e + a^3 f + a b^2 d - b^3 c)}{3 a^6 (a + b x^3)} + \frac{2 a^2 b e + a^3 (-f) - 3 a b^2 d + 4 b^3 c}{4 a^5 x^4} + \frac{b^{4/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b x} + b^{2/3} x^2) (-10 a^2 b e + 7 a^3 f + \dots)}{18 a^{19/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^2),x]

[Out]
$$-c/(13*a^2*x^{13}) + (2*b*c - a*d)/(10*a^3*x^{10}) - (3*b^2*c - 2*a*b*d + a^2*e)/(7*a^4*x^7) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(4*a^5*x^4) + (b*(-5*b^3*c + 4*a*b^2*d - 3*a^2*b*e + 2*a^3*f))/(a^6*x) + (b^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(3*a^6*(a + b*x^3)) + (b^{(4/3)}*(16*b^3*c - 13*a*b^2*d + 10*a^2*b*e - 7*a^3*f)*ArcTan[(1 - (2*b^{(1/3)}*x)/a^{(1/3)})/Sqrt[3]])/(3*Sqrt[3]*a^{(19/3)}) + (b^{(4/3)}*(16*b^3*c - 13*a*b^2*d + 10*a^2*b*e - 7*a^3*f)*Log[a^{(1/3)} + b^{(1/3)}*x])/(9*a^{(19/3)}) + (b^{(4/3)}*(-16*b^3*c + 13*a*b^2*d - 10*a^2*b*e + 7*a^3*f)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(18*a^{(19/3)})$$

Maple [A] time = 0.016, size = 631, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^2,x)

[Out]
$$13/18*b^3/a^5*d/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})+16/9*b^4/a^6*c/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})-8/9*b^4/a^6*c/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})+7/18*b/a^3*f/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})+10/9*b^2/a^4*e/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})-5/9*b^2/a^4*e/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})-13/9*b^3/a^5*d/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})-7/9*b/a^3*f/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})+7/9*b/a^3*f*3^{(1/2)}/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))-10/9*b^2/a^4*e*3^{(1/2)}/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))+13/9*b^3/a^5*d*3^{(1/2)}/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))-16/9*b^4/a^6*c*3^{(1/2)}/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))-1/13*c/a^2/x^{13}+1/2/a^3/x^4*b*e-3/4/a^4/x^4*b^2*d+1/a^5/x^4*b^3*c+2*b/a^3/x*f-3*b^2/a^4/x*e+4*b^3/a^5/x*d-5*b^4/a^6/x*c+1/5/a^3/x$$

$$\frac{10bc + 2/7a^3/x^7bd - 3/7a^4/x^7b^2c + 1/3b^2/a^3x^2/(bx^3+a)f - 1/3b^3/a^4x^2/(bx^3+a)e - 1/10a^2/x^10d - 1/7a^2/x^7e - 1/4a^2/x^4f + 1/3b^4/a^5x^2/(bx^3+a)d - 1/3b^5/a^6x^2/(bx^3+a)c}{}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.36125, size = 1188, normalized size = 3.17

$$5460(16b^5c - 13ab^4d + 10a^2b^3e - 7a^3b^2f)x^{15} + 4095(16ab^4c - 13a^2b^3d + 10a^3b^2e - 7a^4bf)x^{12} - 585(16a^2b^3c -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^2,x, algorithm="fricas")

[Out]
$$-1/16380(5460(16b^5c - 13ab^4d + 10a^2b^3e - 7a^3b^2f)x^{15} + 4095(16ab^4c - 13a^2b^3d + 10a^3b^2e - 7a^4bf)x^{12} - 585(16a^2b^3c - 13a^3b^2d + 10a^4be - 7a^5f)x^9 + 234(16a^3b^2c - 13a^4bd + 10a^5e)x^6 + 1260a^5c - 126(16a^4b^2c - 13a^5d)x^3 + 1820\sqrt{3}((16b^5c - 13ab^4d + 10a^2b^3e - 7a^3b^2f)x^{16} + (16ab^4c - 13a^2b^3d + 10a^3b^2e - 7a^4bf)x^{13})*(-b/a)^{(1/3)}a \operatorname{rctan}(2/3\sqrt{3}x*(-b/a)^{(1/3)} + 1/3\sqrt{3}) - 910((16b^5c - 13ab^4d + 10a^2b^3e - 7a^3b^2f)x^{16} + (16ab^4c - 13a^2b^3d + 10a^3b^2e - 7a^4bf)x^{13})*(-b/a)^{(1/3)}\log(bx^2 - ax*(-b/a)^{(2/3)} - a*(-b/a)^{(1/3)}) + 1820((16b^5c - 13ab^4d + 10a^2b^3e - 7a^3b^2f)x^{16} + (16ab^4c - 13a^2b^3d + 10a^3b^2e - 7a^4bf)x^{13})*(-b/a)^{(1/3)}\log(bx + a*(-b/a)^{(2/3)})/(a^6bx^{16} + a^7x^{13})$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**14/(b*x**3+a)**2,x)

[Out] Timed out

Giac [A] time = 1.08059, size = 651, normalized size = 1.74

$$\frac{\sqrt{3} \left(16 (-ab^2)^{\frac{2}{3}} b^3 c - 13 (-ab^2)^{\frac{2}{3}} ab^2 d - 7 (-ab^2)^{\frac{2}{3}} a^3 f + 10 (-ab^2)^{\frac{2}{3}} a^2 b e \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{9 a^7} + \frac{\left(16 b^5 c \left(-\frac{a}{b} \right)^{\frac{1}{3}} - 13 \right)}{9 a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^2,x, algorithm="giac")

[Out] 1/9*sqrt(3)*(16*(-a*b^2)^(2/3)*b^3*c - 13*(-a*b^2)^(2/3)*a*b^2*d - 7*(-a*b^2)^(2/3)*a^3*f + 10*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^7 + 1/9*(16*b^5*c*(-a/b)^(1/3) - 13*a*b^4*d*(-a/b)^(1/3) - 7*a^3*b^2*f*(-a/b)^(1/3) + 10*a^2*b^3*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^7 - 1/18*(16*(-a*b^2)^(2/3)*b^3*c - 13*(-a*b^2)^(2/3)*a*b^2*d - 7*(-a*b^2)^(2/3)*a^3*f + 10*(-a*b^2)^(2/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^7 - 1/3*(b^5*c*x^2 - a*b^4*d*x^2 - a^3*b^2*f*x^2 + a^2*b^3*x^2*e)/((b*x^3 + a)*a^6) - 1/1820*(9100*b^4*c*x^12 - 7280*a*b^3*d*x^12 - 3640*a^3*b*f*x^12 + 5460*a^2*b^2*x^12*e - 1820*a*b^3*c*x^9 + 1365*a^2*b^2*d*x^9 + 455*a^4*f*x^9 - 910*a^3*b*x^9*e + 780*a^2*b^2*c*x^6 - 520*a^3*b*d*x^6 + 260*a^4*x^6*e - 364*a^3*b*c*x^3 + 182*a^4*d*x^3 + 140*a^4*c)/(a^6*x^13)

$$3.276 \quad \int \frac{x^{14}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=266

$$\frac{x^6(6a^2be - 10a^3f - 3ab^2d + b^3c)}{6b^6} - \frac{ax^3(10a^2be - 15a^3f - 6ab^2d + 3b^3c)}{3b^7} + \frac{a^3(6a^2be - 7a^3f - 5ab^2d + 4b^3c)}{3b^8(a + bx^3)} - \frac{a^4(a^2 - 3ab^2d + 6a^2b^2e - 10a^3f)}{6b^6} + \frac{(b^2d - 3ab^2e + 6a^2f)x^9}{9b^5} + \frac{(b^3c - ab^2d + a^2b^2e - a^3f)x^{12}}{12b^4} + \frac{fx^{15}}{15b^3} - \frac{a^4(b^3c - ab^2d + a^2b^2e - a^3f)}{6b^8(a + bx^3)^2} + \frac{a^3(4b^3c - 5ab^2d + 6a^2b^2e - 7a^3f)}{3b^8(a + bx^3)} + \frac{a^2(6b^3c - 10ab^2d + 15a^2b^2e - 21a^3f) \operatorname{Log}[a + bx^3]}{3b^8}$$

[Out] $-(a(3b^3c - 6ab^2d + 10a^2b^2e - 15a^3f)x^3)/(3b^7) + ((b^3c - 3ab^2d + 6a^2b^2e - 10a^3f)x^6)/(6b^6) + ((b^2d - 3ab^2e + 6a^2f)x^9)/(9b^5) + ((b^3c - ab^2d + a^2b^2e - a^3f)x^{12})/(12b^4) + (fx^{15})/(15b^3) - (a^4(b^3c - ab^2d + a^2b^2e - a^3f))/(6b^8(a + bx^3)^2) + (a^3(4b^3c - 5ab^2d + 6a^2b^2e - 7a^3f))/(3b^8(a + bx^3)) + (a^2(6b^3c - 10ab^2d + 15a^2b^2e - 21a^3f) \operatorname{Log}[a + bx^3])/(3b^8)$

Rubi [A] time = 0.436041, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1821, 1620}

$$\frac{x^6(6a^2be - 10a^3f - 3ab^2d + b^3c)}{6b^6} - \frac{ax^3(10a^2be - 15a^3f - 6ab^2d + 3b^3c)}{3b^7} + \frac{a^3(6a^2be - 7a^3f - 5ab^2d + 4b^3c)}{3b^8(a + bx^3)} - \frac{a^4(a^2 - 3ab^2d + 6a^2b^2e - 10a^3f)}{6b^6} + \frac{(b^2d - 3ab^2e + 6a^2f)x^9}{9b^5} + \frac{(b^3c - ab^2d + a^2b^2e - a^3f)x^{12}}{12b^4} + \frac{fx^{15}}{15b^3} - \frac{a^4(b^3c - ab^2d + a^2b^2e - a^3f)}{6b^8(a + bx^3)^2} + \frac{a^3(4b^3c - 5ab^2d + 6a^2b^2e - 7a^3f)}{3b^8(a + bx^3)} + \frac{a^2(6b^3c - 10ab^2d + 15a^2b^2e - 21a^3f) \operatorname{Log}[a + bx^3]}{3b^8}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x^{14}(c + dx^3 + ex^6 + fx^9))/(a + bx^3)^3, x]$

[Out] $-(a(3b^3c - 6ab^2d + 10a^2b^2e - 15a^3f)x^3)/(3b^7) + ((b^3c - 3ab^2d + 6a^2b^2e - 10a^3f)x^6)/(6b^6) + ((b^2d - 3ab^2e + 6a^2f)x^9)/(9b^5) + ((b^3c - ab^2d + a^2b^2e - a^3f)x^{12})/(12b^4) + (fx^{15})/(15b^3) - (a^4(b^3c - ab^2d + a^2b^2e - a^3f))/(6b^8(a + bx^3)^2) + (a^3(4b^3c - 5ab^2d + 6a^2b^2e - 7a^3f))/(3b^8(a + bx^3)) + (a^2(6b^3c - 10ab^2d + 15a^2b^2e - 21a^3f) \operatorname{Log}[a + bx^3])/(3b^8)$

Rule 1821

$\operatorname{Int}[(Pq_*)(x_*)^{(m_*)}((a_*) + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Simplify}[(m + 1)/n] - 1) \operatorname{SubstFor}[x^n, Pq, x] (a + bx)^p, x], x, x^n], x] /;$ $\operatorname{FreeQ}\{a, b, m, n, p\}, x \ \&\& \operatorname{PolyQ}[Pq, x^n] \ \&\& \operatorname{IntegerQ}[\operatorname{Simplify}[(m + 1)/n]]$

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c,
d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned} \int \frac{x^{14}(c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^4(c + dx + ex^2 + fx^3)}{(a + bx)^3} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{a(-3b^3c + 6ab^2d - 10a^2be + 15a^3f)}{b^7} + \frac{(b^3c - 3ab^2d + 6a^2be - 10a^3f)x}{b^6} \right. \right. \\ &= -\frac{a(3b^3c - 6ab^2d + 10a^2be - 15a^3f)x^3}{3b^7} + \frac{(b^3c - 3ab^2d + 6a^2be - 10a^3f)x^6}{6b^6} + \frac{(b^2d - 3ab^2e + 3a^2f)x^9}{9b^5} \end{aligned}$$

Mathematica [A] time = 0.142904, size = 246, normalized size = 0.92

$$30b^2x^6(6a^2be - 10a^3f - 3ab^2d + b^3c) + 60abx^3(-10a^2be + 15a^3f + 6ab^2d - 3b^3c) - \frac{60a^3(-6a^2be + 7a^3f + 5ab^2d - 4b^3c)}{a+bx^3} + \frac{30a^4(-a^2be + 2a^3f + ab^2d - b^3c)}{a+bx^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^14*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]

[Out] (60*a*b*(-3*b^3*c + 6*a*b^2*d - 10*a^2*b*e + 15*a^3*f)*x^3 + 30*b^2*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^6 + 20*b^3*(b^2*d - 3*a*b*e + 6*a^2*f)*x^9 + 15*b^4*(b*e - 3*a*f)*x^12 + 12*b^5*f*x^15 + (30*a^4*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3)^2 - (60*a^3*(-4*b^3*c + 5*a*b^2*d - 6*a^2*b*e + 7*a^3*f))/(a + b*x^3) + 60*a^2*(6*b^3*c - 10*a*b^2*d + 15*a^2*b*e - 21*a^3*f)*Log[a + b*x^3]/(180*b^8)

Maple [A] time = 0.015, size = 361, normalized size = 1.4

$$\frac{fx^{15}}{15b^3} + 5\frac{a^4fx^3}{b^7} - \frac{10a^3ex^3}{3b^6} + 2\frac{a^2dx^3}{b^5} - \frac{acx^3}{b^4} + \frac{a^7f}{6b^8(bx^3+a)^2} - \frac{a^6e}{6b^7(bx^3+a)^2} + \frac{a^5d}{6b^6(bx^3+a)^2} - \frac{a^4c}{6b^5(bx^3+a)^2} - \frac{30a^4(-a^2be + 2a^3f + ab^2d - b^3c)}{a+bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^14*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)`

[Out]
$$\frac{1}{15}f*x^{15}/b^3 + 5/b^7*a^4*f*x^3 - 10/3/b^6*a^3*e*x^3 + 2/b^5*a^2*d*x^3 - 1/b^4*a*c*x^3 + 1/6*a^7/b^8/(b*x^3+a)^2*f - 1/6*a^6/b^7/(b*x^3+a)^2*e + 1/6*a^5/b^6/(b*x^3+a)^2*d - 1/6*a^4/b^5/(b*x^3+a)^2*c - 7/3*a^6/b^8/(b*x^3+a)*f + 2*a^5/b^7/(b*x^3+a)*e - 5/3*a^4/b^6/(b*x^3+a)*d + 4/3*a^3/b^5/(b*x^3+a)*c - 7*a^5/b^8*\ln(b*x^3+a)*f + 5*a^4/b^7*\ln(b*x^3+a)*e - 10/3*a^3/b^6*\ln(b*x^3+a)*d + 2*a^2/b^5*\ln(b*x^3+a)*c + 1/12/b^3*x^{12}*e + 1/9/b^3*x^9*d + 1/6/b^3*x^6*c - 1/4/b^4*x^{12}*a*f + 2/3/b^5*x^9*a^2*f - 1/3/b^4*x^9*a*e - 5/3/b^6*x^6*a^3*f + 1/b^5*x^6*a^2*e - 1/2/b^4*x^6*a*d$$

Maxima [A] time = 0.954004, size = 371, normalized size = 1.39

$$\frac{7a^4b^3c - 9a^5b^2d + 11a^6be - 13a^7f + 2(4a^3b^4c - 5a^4b^3d + 6a^5b^2e - 7a^6bf)x^3}{6(b^{10}x^6 + 2ab^9x^3 + a^2b^8)} + \frac{12b^4fx^{15} + 15(b^4e - 3ab^3f)x^{12} + \dots}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^14*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")`

[Out]
$$\frac{1}{6}(7a^4b^3c - 9a^5b^2d + 11a^6be - 13a^7f + 2(4a^3b^4c - 5a^4b^3d + 6a^5b^2e - 7a^6bf)x^3)/(b^{10}x^6 + 2a^2b^8 + 2ab^9x^3) + \frac{1}{180}(12b^4fx^{15} + 15(b^4e - 3ab^3f)x^{12} + 20(b^4d - 3a^2b^3e + 6a^2b^2f)x^9 + 30(b^4c - 3a^2b^3d + 6a^2b^2e - 10a^3bf)x^6 - 60(3a^2b^3c - 6a^2b^2d + 10a^3be - 15a^4f)x^3)/b^7 + \frac{1}{3}(6a^2b^3c - 10a^3b^2d + 15a^4be - 21a^5f)*\log(b*x^3 + a)/b^8$$

Fricas [A] time = 1.23426, size = 883, normalized size = 3.32

$$\frac{12b^7fx^{21} + 3(5b^7e - 7ab^6f)x^{18} + 2(10b^7d - 15ab^6e + 21a^2b^5f)x^{15} + 5(6b^7c - 10ab^6d + 15a^2b^5e - 21a^3b^4f)x^{12} + \dots}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^14*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")`

[Out]
$$\frac{1}{180}(12b^7fx^{21} + 3(5b^7e - 7a^2b^6f)x^{18} + 2(10b^7d - 15a^2b^6e + 21a^2b^5f)x^{15} + 5(6b^7c - 10a^2b^6d + 15a^2b^5e - 21a^3c$$

$$b^4 f) x^{12} - 20(6 a b^6 c - 10 a^2 b^5 d + 15 a^3 b^4 e - 21 a^4 b^3 f) x^9 + 210 a^4 b^3 c - 270 a^5 b^2 d + 330 a^6 b e - 390 a^7 f - 30(11 a^2 b^5 c - 21 a^3 b^4 d + 34 a^4 b^3 e - 50 a^5 b^2 f) x^6 + 60(a^3 b^4 c + a^4 b^3 d - 4 a^5 b^2 e + 8 a^6 b f) x^3 + 60(6 a^4 b^3 c - 10 a^5 b^2 d + 15 a^6 b e - 21 a^7 f + (6 a^2 b^5 c - 10 a^3 b^4 d + 15 a^4 b^3 e - 21 a^5 b^2 f) x^6 + 2(6 a^3 b^4 c - 10 a^4 b^3 d + 15 a^5 b^2 e - 21 a^6 b f) x^3) \log(b x^3 + a) / (b^{10} x^6 + 2 a b^9 x^3 + a^2 b^8)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**14*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.07827, size = 471, normalized size = 1.77

$$\frac{(6 a^2 b^3 c - 10 a^3 b^2 d - 21 a^5 f + 15 a^4 b e) \log(|b x^3 + a|)}{3 b^8} - \frac{18 a^2 b^5 c x^6 - 30 a^3 b^4 d x^6 - 63 a^5 b^2 f x^6 + 45 a^4 b^3 x^6 e + 28 a^3 b^4 c x^3}{3 b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^14*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")

[Out] $\frac{1}{3}(6 a^2 b^3 c - 10 a^3 b^2 d - 21 a^5 f + 15 a^4 b e) \log(\text{abs}(b x^3 + a)) / b^8 - \frac{1}{6}(18 a^2 b^5 c x^6 - 30 a^3 b^4 d x^6 - 63 a^5 b^2 f x^6 + 45 a^4 b^3 x^6 e + 28 a^3 b^4 c x^3 - 50 a^4 b^3 d x^3 - 112 a^6 b f x^3 + 78 a^5 b^2 x^3 e + 11 a^4 b^3 c - 21 a^5 b^2 d - 50 a^7 f + 34 a^6 b e) / ((b x^3 + a)^2 b^8) + \frac{1}{180}(12 b^{12} f x^{15} - 45 a b^{11} f x^{12} + 15 b^{12} x^{12} e + 20 b^{12} d x^9 + 120 a^2 b^{10} f x^9 - 60 a b^{11} x^9 e + 30 b^{12} c x^6 - 90 a b^{11} d x^6 - 300 a^3 b^9 f x^6 + 180 a^2 b^{10} x^6 e - 180 a b^{11} c x^3 + 360 a^2 b^{10} d x^3 + 900 a^4 b^8 f x^3 - 600 a^3 b^9 x^3 e) / b^{15}$

$$3.277 \quad \int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=226

$$\frac{x^3(6a^2be - 10a^3f - 3ab^2d + b^3c)}{3b^6} - \frac{a^2(5a^2be - 6a^3f - 4ab^2d + 3b^3c)}{3b^7(a+bx^3)} + \frac{a^3(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^7(a+bx^3)^2} - \frac{a \log(a+bx^3)}{3b^7}$$

[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^3)/(3*b^6) + ((b^2*d - 3*a*b*e + 6*a^2*f)*x^6)/(6*b^5) + ((b*e - 3*a*f)*x^9)/(9*b^4) + (f*x^12)/(12*b^3) + (a^3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(6*b^7*(a + b*x^3)^2) - (a^2*(3*b^3*c - 4*a*b^2*d + 5*a^2*b*e - 6*a^3*f))/(3*b^7*(a + b*x^3)) - (a*(3*b^3*c - 6*a*b^2*d + 10*a^2*b*e - 15*a^3*f)*Log[a + b*x^3])/(3*b^7)

Rubi [A] time = 0.330679, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1821, 1620}

$$\frac{x^3(6a^2be - 10a^3f - 3ab^2d + b^3c)}{3b^6} - \frac{a^2(5a^2be - 6a^3f - 4ab^2d + 3b^3c)}{3b^7(a+bx^3)} + \frac{a^3(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^7(a+bx^3)^2} - \frac{a \log(a+bx^3)}{3b^7}$$

Antiderivative was successfully verified.

[In] Int[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]

[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^3)/(3*b^6) + ((b^2*d - 3*a*b*e + 6*a^2*f)*x^6)/(6*b^5) + ((b*e - 3*a*f)*x^9)/(9*b^4) + (f*x^12)/(12*b^3) + (a^3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(6*b^7*(a + b*x^3)^2) - (a^2*(3*b^3*c - 4*a*b^2*d + 5*a^2*b*e - 6*a^3*f))/(3*b^7*(a + b*x^3)) - (a*(3*b^3*c - 6*a*b^2*d + 10*a^2*b*e - 15*a^3*f)*Log[a + b*x^3])/(3*b^7)

Rule 1821

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol]
:= Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned} \int \frac{x^{11}(c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^3(c + dx + ex^2 + fx^3)}{(a + bx)^3} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{b^3c - 3ab^2d + 6a^2be - 10a^3f}{b^6} + \frac{(b^2d - 3abe + 6a^2f)x}{b^5} + \frac{(be - 3af)x^2}{b^4} + \right. \right. \\ &= \frac{(b^3c - 3ab^2d + 6a^2be - 10a^3f)x^3}{3b^6} + \frac{(b^2d - 3abe + 6a^2f)x^6}{6b^5} + \frac{(be - 3af)x^9}{9b^4} + \frac{fx^{12}}{12b^3} + \end{aligned}$$

Mathematica [A] time = 0.120015, size = 208, normalized size = 0.92

$$\frac{12bx^3(6a^2be - 10a^3f - 3ab^2d + b^3c) + \frac{12a^2(-5a^2be + 6a^3f + 4ab^2d - 3b^3c)}{a+bx^3} + \frac{6a^3(a^2be + a^3(-f) - ab^2d + b^3c)}{(a+bx^3)^2} + 12a \log(a + bx^3)(-10a^2be - 12ab^2d + 6a^2be - 10a^3f)}{36b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (12*b*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^3 + 6*b^2*(b^2*d - 3*a*b
*e + 6*a^2*f)*x^6 + 4*b^3*(b*e - 3*a*f)*x^9 + 3*b^4*f*x^12 + (6*a^3*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f))/(a + b*x^3)^2 + (12*a^2*(-3*b^3*c + 4*a*b^2*d
- 5*a^2*b*e + 6*a^3*f))/(a + b*x^3) + 12*a*(-3*b^3*c + 6*a*b^2*d - 10*a^2*
b*e + 15*a^3*f)*Log[a + b*x^3])/(36*b^7)
```

Maple [A] time = 0.013, size = 313, normalized size = 1.4

$$\frac{fx^{12}}{12b^3} - \frac{x^9af}{3b^4} + \frac{x^9e}{9b^3} + \frac{a^2fx^6}{b^5} - \frac{aex^6}{2b^4} + \frac{dx^6}{6b^3} - \frac{10a^3fx^3}{3b^6} + 2\frac{a^2ex^3}{b^5} - \frac{adx^3}{b^4} + \frac{cx^3}{3b^3} - \frac{a^6f}{6b^7(bx^3 + a)^2} + \frac{a^5e}{6b^6(bx^3 + a)^2} - \frac{a^4f}{6b^5(bx^3 + a)^2} + \frac{a^3e}{6b^4(bx^3 + a)^2} - \frac{a^2f}{6b^3(bx^3 + a)^2} + \frac{a^2e}{6b^2(bx^3 + a)^2} - \frac{af}{6b(bx^3 + a)^2} + \frac{e}{6(bx^3 + a)^2} + \frac{a^3f}{6b^3} - \frac{a^2e}{6b^2} + \frac{af}{6b} - \frac{e}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^{11}(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)$

[Out] $\frac{1}{12}f*x^{12}/b^3 - \frac{1}{3}b^4*x^9*a*f + \frac{1}{9}b^3*x^9*e + \frac{1}{b^5}x^6*a^2*f - \frac{1}{2}b^4*x^6*a*e + \frac{1}{6}b^3*x^6*d - \frac{10}{3}b^6*a^3*f*x^3 + \frac{2}{b^5}a^2*e*x^3 - \frac{1}{b^4}a*d*x^3 + \frac{1}{3}b^3*c*x^3 - \frac{1}{6}a^6/b^7/(b*x^3+a)^2*f + \frac{1}{6}a^5/b^6/(b*x^3+a)^2*e - \frac{1}{6}a^4/b^5/(b*x^3+a)^2*d + \frac{1}{6}a^3/b^4/(b*x^3+a)^2*c + 5*a^4/b^7*\ln(b*x^3+a)*f - \frac{10}{3}a^3/b^6*\ln(b*x^3+a)*e + 2*a^2/b^5*\ln(b*x^3+a)*d - a/b^4*\ln(b*x^3+a)*c + 2*a^5/b^7/(b*x^3+a)*f - \frac{5}{3}a^4/b^6/(b*x^3+a)*e + \frac{4}{3}a^3/b^5/(b*x^3+a)*d - a^2/b^4/(b*x^3+a)*c$

Maxima [A] time = 0.963488, size = 315, normalized size = 1.39

$$\frac{5a^3b^3c - 7a^4b^2d + 9a^5be - 11a^6f + 2(3a^2b^4c - 4a^3b^3d + 5a^4b^2e - 6a^5bf)x^3}{6(b^9x^6 + 2ab^8x^3 + a^2b^7)} + \frac{3b^3fx^{12} + 4(b^3e - 3ab^2f)x^9 + 6(b^3c - 3ab^2d + 10a^3be - 15a^4f)x^6 - 12(b^3c - 3ab^2d + 6a^2be - 10a^3f)x^3}{b^6} - \frac{1}{3} \frac{(3ab^3c - 6a^2b^2d + 10a^3be - 15a^4f) \log(bx^3 + a)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^{11}(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, \text{algorithm}="maxima")$

[Out] $-\frac{1}{6}*(5*a^3*b^3*c - 7*a^4*b^2*d + 9*a^5*b*e - 11*a^6*f + 2*(3*a^2*b^4*c - 4*a^3*b^3*d + 5*a^4*b^2*e - 6*a^5*b*f)*x^3)/(b^9*x^6 + 2*a*b^8*x^3 + a^2*b^7) + \frac{1}{36}*(3*b^3*f*x^{12} + 4*(b^3*e - 3*a*b^2*f)*x^9 + 6*(b^3*d - 3*a*b^2*e + 6*a^2*b*f)*x^6 + 12*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^3)/b^6 - \frac{1}{3}*(3*a*b^3*c - 6*a^2*b^2*d + 10*a^3*b*e - 15*a^4*f)*\log(b*x^3 + a)/b^7$

Fricas [A] time = 1.34569, size = 765, normalized size = 3.38

$$\frac{3b^6fx^{18} + 2(2b^6e - 3ab^5f)x^{15} + (6b^6d - 10ab^5e + 15a^2b^4f)x^{12} + 4(3b^6c - 6ab^5d + 10a^2b^4e - 15a^3b^3f)x^9 - 30a^3b^3c}{b^6} - \frac{12(b^3c - 3ab^2d + 6a^2be - 10a^3f)x^3}{b^6} - \frac{1}{3} \frac{(3ab^3c - 6a^2b^2d + 10a^3be - 15a^4f) \log(bx^3 + a)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^{11}(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, \text{algorithm}="fricas")$

[Out] $\frac{1}{36}*(3*b^6*f*x^{18} + 2*(2*b^6*e - 3*a*b^5*f)*x^{15} + (6*b^6*d - 10*a*b^5*e + 15*a^2*b^4*f)*x^{12} + 4*(3*b^6*c - 6*a*b^5*d + 10*a^2*b^4*e - 15*a^3*b^3*f)*x^9 - 30*a^3*b^3*c + 42*a^4*b^2*d - 54*a^5*b*e + 66*a^6*f + 6*(4*a*b^5*c - 11*a^2*b^4*d + 21*a^3*b^3*e - 34*a^4*b^2*f)*x^6 - 12*(2*a^2*b^4*c - a^3*b^3*d - a^4*b^2*e + 4*a^5*b*f)*x^3 - 12*(3*a^3*b^3*c - 6*a^4*b^2*d + 10*a^5*b$

$$\frac{e^{-15a^6f} + (3ab^5c - 6a^2b^4d + 10a^3b^3e - 15a^4b^2f)x^6 + 2(3a^2b^4c - 6a^3b^3d + 10a^4b^2e - 15a^5b^1f)x^3}{(b^9x^6 + 2ab^8x^3 + a^2b^7)} \log(bx^3 + a)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**11*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.09688, size = 402, normalized size = 1.78

$$\frac{(3ab^3c - 6a^2b^2d - 15a^4f + 10a^3be) \log(|bx^3 + a|)}{3b^7} + \frac{9ab^5cx^6 - 18a^2b^4dx^6 - 45a^4b^2fx^6 + 30a^3b^3x^6e + 12a^2b^4cx^3 - 6a^3b^3d}{6b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")

[Out]
$$\frac{-1/3(3ab^3c - 6a^2b^2d - 15a^4f + 10a^3b^3e) \log(\text{abs}(bx^3 + a))}{b^7} + \frac{1/6(9ab^5cx^6 - 18a^2b^4d^2x^6 - 45a^4b^2f^2x^6 + 30a^3b^3e^2x^6 + 12a^2b^4c^2x^3 - 28a^3b^3d^2x^3 - 78a^5b^1f^2x^3 + 50a^4b^2e^2x^3 + 4a^3b^3c^2 - 11a^4b^2d^2 - 34a^6f^2 + 21a^5b^1e^2)}{(b^9x^6 + 2ab^8x^3 + a^2b^7)} + \frac{1/36(3b^9f^2x^{12} - 12ab^8fx^9 + 4b^9x^9e^2 + 6b^9d^2x^6 + 36a^2b^7f^2x^6 - 18ab^8fx^6e + 12b^9c^2x^3 - 36ab^8d^2x^3 - 120a^3b^6fx^3 + 72a^2b^7x^3e)}{b^{12}}$$

$$3.278 \quad \int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=186

$$\frac{a(4a^2be - 5a^3f - 3ab^2d + 2b^3c)}{3b^6(a+bx^3)} - \frac{a^2(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^6(a+bx^3)^2} + \frac{\log(a+bx^3)(6a^2be - 10a^3f - 3ab^2d + b^3c)}{3b^6} + \dots$$

[Out] $((b^2*d - 3*a*b*e + 6*a^2*f)*x^3)/(3*b^5) + ((b*e - 3*a*f)*x^6)/(6*b^4) + (f*x^9)/(9*b^3) - (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(6*b^6*(a + b*x^3)^2) + (a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f))/(3*b^6*(a + b*x^3)) + ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*\text{Log}[a + b*x^3])/(3*b^6)$

Rubi [A] time = 0.268936, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1821, 1620}

$$\frac{a(4a^2be - 5a^3f - 3ab^2d + 2b^3c)}{3b^6(a+bx^3)} - \frac{a^2(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^6(a+bx^3)^2} + \frac{\log(a+bx^3)(6a^2be - 10a^3f - 3ab^2d + b^3c)}{3b^6} + \dots$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3, x]$

[Out] $((b^2*d - 3*a*b*e + 6*a^2*f)*x^3)/(3*b^5) + ((b*e - 3*a*f)*x^6)/(6*b^4) + (f*x^9)/(9*b^3) - (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(6*b^6*(a + b*x^3)^2) + (a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f))/(3*b^6*(a + b*x^3)) + ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*\text{Log}[a + b*x^3])/(3*b^6)$

Rule 1821

$\text{Int}[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{\text{Simplify}[(m+1)/n] - 1}*\text{SubstFor}[x^n, Pq, x]*(a + b*x)^p, x], x, x^n], x] /;$ FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Simplify[(m+1)/n]]

Rule 1620

$\text{Int}[(Px_)*((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[Px*(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c

, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]

Rubi steps

$$\begin{aligned} \int \frac{x^8 (c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2 (c + dx + ex^2 + fx^3)}{(a + bx)^3} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{b^2 d - 3abe + 6a^2 f}{b^5} + \frac{(be - 3af)x}{b^4} + \frac{fx^2}{b^3} - \frac{a^2 (-b^3 c + ab^2 d - a^2 be + a^3 f)}{b^5 (a + bx)^3} \right) dx, x, x^3 \right) \\ &= \frac{(b^2 d - 3abe + 6a^2 f)x^3}{3b^5} + \frac{(be - 3af)x^6}{6b^4} + \frac{fx^9}{9b^3} - \frac{a^2 (b^3 c - ab^2 d + a^2 be - a^3 f)}{6b^6 (a + bx^3)^2} + \frac{a (2b^3 c - 3ab^2 d + a^2 be - a^3 f)}{18b^6} \end{aligned}$$

Mathematica [A] time = 0.0987472, size = 170, normalized size = 0.91

$$\frac{-\frac{6a(-4a^2be+5a^3f+3ab^2d-2b^3c)}{a+bx^3} + \frac{3a^2(-a^2be+a^3f+ab^2d-b^3c)}{(a+bx^3)^2} + 6 \log(a + bx^3)(6a^2be - 10a^3f - 3ab^2d + b^3c) + 6bx^3(6a^2f - 3abe + a^3f)}{18b^6}$$

Antiderivative was successfully verified.

[In] Integrate[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]

[Out] (6*b*(b^2*d - 3*a*b*e + 6*a^2*f)*x^3 + 3*b^2*(b*e - 3*a*f)*x^6 + 2*b^3*f*x^9 + (3*a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3)^2 - (6*a*(-2*b^3*c + 3*a*b^2*d - 4*a^2*b*e + 5*a^3*f))/(a + b*x^3) + 6*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*Log[a + b*x^3])/(18*b^6)

Maple [A] time = 0.012, size = 266, normalized size = 1.4

$$\frac{fx^9}{9b^3} - \frac{x^6af}{2b^4} + \frac{ex^6}{6b^3} + 2 \frac{a^2fx^3}{b^5} - \frac{aex^3}{b^4} + \frac{dx^3}{3b^3} + \frac{a^5f}{6b^6(bx^3+a)^2} - \frac{a^4e}{6b^5(bx^3+a)^2} + \frac{a^3d}{6b^4(bx^3+a)^2} - \frac{a^2c}{6b^3(bx^3+a)^2} - \frac{10a^2(b^3c - 3ab^2d + a^2be - a^3f)}{18b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)

[Out] $\frac{1}{9}f*x^9/b^3 - 1/2/b^4*x^6*a*f + 1/6/b^3*x^6*e + 2/b^5*a^2*f*x^3 - 1/b^4*a*e*x^3 + 1/3/b^3*d*x^3 + 1/6/b^6*a^5/(b*x^3+a)^2*f - 1/6/b^5*a^4/(b*x^3+a)^2*e + 1/6/b^4*a^3/(b*x^3+a)^2*d - 1/6/b^3*a^2/(b*x^3+a)^2*c - 10/3/b^6*\ln(b*x^3+a)*a^3*f + 2/b^5*\ln(b*x^3+a)*a^2*e - 1/b^4*\ln(b*x^3+a)*a*d + 1/3/b^3*\ln(b*x^3+a)*c - 5/3/b^6*a^4/(b*x^3+a)*f + 4/3/b^5*a^3/(b*x^3+a)*e - 1/b^4*a^2/(b*x^3+a)*d + 2/3/b^3*a/(b*x^3+a)*c$

Maxima [A] time = 0.95037, size = 258, normalized size = 1.39

$$\frac{3a^2b^3c - 5a^3b^2d + 7a^4be - 9a^5f + 2(2ab^4c - 3a^2b^3d + 4a^3b^2e - 5a^4bf)x^3}{6(b^8x^6 + 2ab^7x^3 + a^2b^6)} + \frac{2b^2fx^9 + 3(b^2e - 3abf)x^6 + 6(b^2d - 3a^2b^3c)}{18b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")`

[Out] $\frac{1}{6}*(3*a^2*b^3*c - 5*a^3*b^2*d + 7*a^4*b*e - 9*a^5*f + 2*(2*a*b^4*c - 3*a^2*b^3*d + 4*a^3*b^2*e - 5*a^4*b*f)*x^3)/(b^8*x^6 + 2*a*b^7*x^3 + a^2*b^6) + \frac{1}{18}*(2*b^2*f*x^9 + 3*(b^2*e - 3*a*b*f)*x^6 + 6*(b^2*d - 3*a*b*e + 6*a^2*f)*x^3)/b^5 + \frac{1}{3}*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*\log(b*x^3 + a)/b^6$

Fricas [A] time = 1.29363, size = 635, normalized size = 3.41

$$\frac{2b^5fx^{15} + (3b^5e - 5ab^4f)x^{12} + 2(3b^5d - 6ab^4e + 10a^2b^3f)x^9 + 3(4ab^4d - 11a^2b^3e + 21a^3b^2f)x^6 + 9a^2b^3c - 15a^3b^2d + 21a^4b^2e - 27a^5f + 6(2a^2b^3d + a^3b^2e + a^4bf)x^3 + 6((b^5c - 3a^2b^3e - 10a^3b^2f)x^6 + a^2b^3c - 3a^3b^2d + 6a^4be - 10a^5f + 2(a^2b^3d + 6a^3b^2e - 10a^4bf)x^3)*\log(b*x^3 + a)}{(b^8*x^6 + 2*a*b^7*x^3 + a^2*b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")`

[Out] $\frac{1}{18}*(2*b^5*f*x^{15} + (3*b^5*e - 5*a*b^4*f)*x^{12} + 2*(3*b^5*d - 6*a*b^4*e + 10*a^2*b^3*f)*x^9 + 3*(4*a*b^4*d - 11*a^2*b^3*e + 21*a^3*b^2*f)*x^6 + 9*a^2*b^3*c - 15*a^3*b^2*d + 21*a^4*b^2*e - 27*a^5*f + 6*(2*a^2*b^3*d + a^3*b^2*e + a^4*b*f)*x^3 + 6*((b^5*c - 3*a^2*b^3*e - 10*a^3*b^2*f)*x^6 + a^2*b^3*c - 3*a^3*b^2*d + 6*a^4*b*e - 10*a^5*f + 2*(a^2*b^3*d + 6*a^3*b^2*e - 10*a^4*b*f)*x^3)*\log(b*x^3 + a))/(b^8*x^6 + 2*a*b^7*x^3 + a^2*b^6)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.07923, size = 319, normalized size = 1.72

$$\frac{(b^3c - 3ab^2d - 10a^3f + 6a^2be) \log(|bx^3 + a|)}{3b^6} - \frac{3b^5cx^6 - 9ab^4dx^6 - 30a^3b^2fx^6 + 18a^2b^3x^6e + 2ab^4cx^3 - 12a^2b^3dx^3 - 6(bx^3 + a)^2b^6}{6(bx^3 + a)^2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")

[Out] $\frac{1}{3} \cdot \frac{(b^3c - 3ab^2d - 10a^3f + 6a^2be) \log(|bx^3 + a|)}{b^6} - \frac{1}{6} \cdot \frac{(3b^5cx^6 - 9ab^4dx^6 - 30a^3b^2fx^6 + 18a^2b^3x^6e + 2ab^4cx^3 - 12a^2b^3dx^3 - 50a^4b^2fx^3 + 28a^3b^2x^3e - 4a^3b^2d - 21a^5f + 11a^4be)}{(bx^3 + a)^2b^6} + \frac{1}{18} \cdot \frac{(2b^6fx^9 - 9ab^5fx^6 + 3b^6x^6e + 6b^6dx^3 + 36a^2b^4fx^3 - 18ab^5x^3e)}{b^9}$

$$3.279 \quad \int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=146

$$\frac{3a^2be - 4a^3f - 2ab^2d + b^3c}{3b^5(a+bx^3)} + \frac{a(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^5(a+bx^3)^2} + \frac{\log(a+bx^3)(6a^2f - 3abe + b^2d)}{3b^5} + \frac{x^3(be - 3af)}{3b^4} +$$

[Out] ((b*e - 3*a*f)*x^3)/(3*b^4) + (f*x^6)/(6*b^3) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(6*b^5*(a + b*x^3)^2) - (b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)/(3*b^5*(a + b*x^3)) + ((b^2*d - 3*a*b*e + 6*a^2*f)*Log[a + b*x^3])/(3*b^5)

Rubi [A] time = 0.200995, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1821, 1620}

$$\frac{3a^2be - 4a^3f - 2ab^2d + b^3c}{3b^5(a+bx^3)} + \frac{a(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^5(a+bx^3)^2} + \frac{\log(a+bx^3)(6a^2f - 3abe + b^2d)}{3b^5} + \frac{x^3(be - 3af)}{3b^4} +$$

Antiderivative was successfully verified.

[In] Int[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]

[Out] ((b*e - 3*a*f)*x^3)/(3*b^4) + (f*x^6)/(6*b^3) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(6*b^5*(a + b*x^3)^2) - (b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)/(3*b^5*(a + b*x^3)) + ((b^2*d - 3*a*b*e + 6*a^2*f)*Log[a + b*x^3])/(3*b^5)

Rule 1821

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p_.], x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1620

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c

, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]

Rubi steps

$$\begin{aligned} \int \frac{x^5 (c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x (c + dx + ex^2 + fx^3)}{(a + bx)^3} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{be - 3af}{b^4} + \frac{fx}{b^3} + \frac{a(-b^3c + ab^2d - a^2be + a^3f)}{b^4(a + bx)^3} + \frac{b^3c - 2ab^2d + 3a^2be -}{b^4(a + bx)^2} \right. \right. \\ &= \frac{(be - 3af)x^3}{3b^4} + \frac{fx^6}{6b^3} + \frac{a(b^3c - ab^2d + a^2be - a^3f)}{6b^5(a + bx^3)^2} - \frac{b^3c - 2ab^2d + 3a^2be - 4a^3f}{3b^5(a + bx^3)} + \frac{(b^2}{ \end{aligned}$$

Mathematica [A] time = 0.0737187, size = 145, normalized size = 0.99

$$\frac{a^2b^2(3d - 4ex^3 - 11fx^6) + 2(a + bx^3)^2 \log(a + bx^3)(6a^2f - 3abe + b^2d) + a^3b(2fx^3 - 5e) + 7a^4f - ab^3(c - 4x^3(d + e))}{6b^5(a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]

[Out] (7*a^4*f + a^3*b*(-5*e + 2*f*x^3) + a^2*b^2*(3*d - 4*e*x^3 - 11*f*x^6) + b^4*x^3*(-2*c + 2*e*x^6 + f*x^9) - a*b^3*(c - 4*x^3*(d + e*x^3 - f*x^6)) + 2*(b^2*d - 3*a*b*e + 6*a^2*f)*(a + b*x^3)^2*Log[a + b*x^3])/(6*b^5*(a + b*x^3)^2)

Maple [A] time = 0.013, size = 213, normalized size = 1.5

$$\frac{fx^6}{6b^3} - \frac{ax^3f}{b^4} + \frac{x^3e}{3b^3} - \frac{a^4f}{6b^5(bx^3 + a)^2} + \frac{a^3e}{6b^4(bx^3 + a)^2} - \frac{a^2d}{6b^3(bx^3 + a)^2} + \frac{ac}{6b^2(bx^3 + a)^2} + 2 \frac{\ln(bx^3 + a) a^2 f}{b^5} - \frac{\ln(bx^3 + a)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)

[Out] $1/6*f*x^6/b^3-1/b^4*x^3*a*f+1/3/b^3*x^3*e-1/6/b^5*a^4/(b*x^3+a)^2*f+1/6/b^4*a^3/(b*x^3+a)^2*e-1/6/b^3*a^2/(b*x^3+a)^2*d+1/6/b^2*a/(b*x^3+a)^2*c+2/b^5*\ln(b*x^3+a)*a^2*f-1/b^4*\ln(b*x^3+a)*a*e+1/3/b^3*\ln(b*x^3+a)*d+4/3/b^5/(b*x^3+a)*a^3*f-1/b^4/(b*x^3+a)*a^2*e+2/3/b^3/(b*x^3+a)*a*d-1/3/b^2/(b*x^3+a)*c$

Maxima [A] time = 0.952873, size = 198, normalized size = 1.36

$$\frac{ab^3c - 3a^2b^2d + 5a^3be - 7a^4f + 2(b^4c - 2ab^3d + 3a^2b^2e - 4a^3bf)x^3}{6(b^7x^6 + 2ab^6x^3 + a^2b^5)} + \frac{bfx^6 + 2(be - 3af)x^3}{6b^4} + \frac{(b^2d - 3abe + 6a^2f)\log(bx^3 + a)}{3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")`

[Out] $-1/6*(a*b^3*c - 3*a^2*b^2*d + 5*a^3*b*e - 7*a^4*f + 2*(b^4*c - 2*a*b^3*d + 3*a^2*b^2*e - 4*a^3*b*f)*x^3)/(b^7*x^6 + 2*a*b^6*x^3 + a^2*b^5) + 1/6*(b*f*x^6 + 2*(b*e - 3*a*f)*x^3)/b^4 + 1/3*(b^2*d - 3*a*b*e + 6*a^2*f)*\log(b*x^3 + a)/b^5$

Fricas [A] time = 1.25994, size = 470, normalized size = 3.22

$$\frac{b^4fx^{12} + 2(b^4e - 2ab^3f)x^9 + (4ab^3e - 11a^2b^2f)x^6 - ab^3c + 3a^2b^2d - 5a^3be + 7a^4f - 2(b^4c - 2ab^3d + 2a^2b^2e - a^3bf)}{6(b^7x^6 + 2ab^6x^3 + a^2b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")`

[Out] $1/6*(b^4*f*x^{12} + 2*(b^4*e - 2*a*b^3*f)*x^9 + (4*a*b^3*e - 11*a^2*b^2*f)*x^6 - a*b^3*c + 3*a^2*b^2*d - 5*a^3*b*e + 7*a^4*f - 2*(b^4*c - 2*a*b^3*d + 2*a^2*b^2*e - a^3*b*f)*x^3 + 2*((b^4*d - 3*a*b^3*e + 6*a^2*b^2*f)*x^6 + a^2*b^2*d - 3*a^3*b*e + 6*a^4*f + 2*(a*b^3*d - 3*a^2*b^2*e + 6*a^3*b*f)*x^3)*\log(b*x^3 + a)/(b^7*x^6 + 2*a*b^6*x^3 + a^2*b^5)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.08806, size = 197, normalized size = 1.35

$$\frac{(b^2d + 6a^2f - 3abe) \log(|bx^3 + a|)}{3b^5} + \frac{b^3fx^6 - 6ab^2fx^3 + 2b^3x^3e}{6b^6} - \frac{ab^3c - 3a^2b^2d - 7a^4f + 5a^3be + 2(b^4c - 2ab^3d - 4a^3b^2e)x^3}{6(bx^3 + a)^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")

[Out] 1/3*(b^2*d + 6*a^2*f - 3*a*b*e)*log(abs(b*x^3 + a))/b^5 + 1/6*(b^3*f*x^6 - 6*a*b^2*f*x^3 + 2*b^3*x^3*e)/b^6 - 1/6*(a*b^3*c - 3*a^2*b^2*d - 7*a^4*f + 5*a^3*b*e + 2*(b^4*c - 2*a*b^3*d - 4*a^3*b*f + 3*a^2*b^2*e)*x^3)/((b*x^3 + a)^2*b^5)

$$3.280 \quad \int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=109

$$-\frac{a^2be + a^3(-f) - ab^2d + b^3c}{6b^4(a+bx^3)^2} - \frac{3a^2f - 2abe + b^2d}{3b^4(a+bx^3)} + \frac{(be - 3af)\log(a+bx^3)}{3b^4} + \frac{fx^3}{3b^3}$$

[Out] (f*x^3)/(3*b^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*b^4*(a + b*x^3)^2) - (b^2*d - 2*a*b*e + 3*a^2*f)/(3*b^4*(a + b*x^3)) + ((b*e - 3*a*f)*Log[a + b*x^3])/(3*b^4)

Rubi [A] time = 0.151546, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1819, 1850}

$$-\frac{a^2be + a^3(-f) - ab^2d + b^3c}{6b^4(a+bx^3)^2} - \frac{3a^2f - 2abe + b^2d}{3b^4(a+bx^3)} + \frac{(be - 3af)\log(a+bx^3)}{3b^4} + \frac{fx^3}{3b^3}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]

[Out] (f*x^3)/(3*b^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*b^4*(a + b*x^3)^2) - (b^2*d - 2*a*b*e + 3*a^2*f)/(3*b^4*(a + b*x^3)) + ((b*e - 3*a*f)*Log[a + b*x^3])/(3*b^4)

Rule 1819

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]

Rule 1850

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\int \frac{x^2 (c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^3} dx = \frac{1}{3} \text{Subst} \left(\int \frac{c + dx + ex^2 + fx^3}{(a + bx)^3} dx, x, x^3 \right)$$

$$= \frac{1}{3} \text{Subst} \left(\int \left(\frac{f}{b^3} + \frac{b^3c - ab^2d + a^2be - a^3f}{b^3(a + bx)^3} + \frac{b^2d - 2abe + 3a^2f}{b^3(a + bx)^2} + \frac{be - 3af}{b^3(a + bx)} \right) dx, x, x^3 \right)$$

$$= \frac{fx^3}{3b^3} - \frac{b^3c - ab^2d + a^2be - a^3f}{6b^4 (a + bx^3)^2} - \frac{b^2d - 2abe + 3a^2f}{3b^4 (a + bx^3)} + \frac{(be - 3af) \log(a + bx^3)}{3b^4}$$

Mathematica [A] time = 0.0541032, size = 105, normalized size = 0.96

$$\frac{a^2b(3e - 4fx^3) - 5a^3f + ab^2(-d + 4ex^3 + 4fx^6) + 2(a + bx^3)^2(be - 3af) \log(a + bx^3) - b^3(c + 2dx^3 - 2fx^9)}{6b^4(a + bx^3)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]

[Out] (-5*a^3*f + a^2*b*(3*e - 4*f*x^3) + a*b^2*(-d + 4*e*x^3 + 4*f*x^6) - b^3*(c + 2*d*x^3 - 2*f*x^9) + 2*(b*e - 3*a*f)*(a + b*x^3)^2*Log[a + b*x^3])/(6*b^4*(a + b*x^3)^2)

Maple [A] time = 0.01, size = 156, normalized size = 1.4

$$\frac{fx^3}{3b^3} + \frac{a^3f}{6b^4(bx^3 + a)^2} - \frac{a^2e}{6b^3(bx^3 + a)^2} + \frac{ad}{6b^2(bx^3 + a)^2} - \frac{c}{6b(bx^3 + a)^2} - \frac{\ln(bx^3 + a)af}{b^4} + \frac{\ln(bx^3 + a)e}{3b^3} - \frac{a^2f}{b^4(bx^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)

[Out] 1/3*f*x^3/b^3+1/6/b^4/(b*x^3+a)^2*a^3*f-1/6/b^3/(b*x^3+a)^2*a^2*e+1/6/b^2/(b*x^3+a)^2*a*d-1/6/b/(b*x^3+a)^2*c-1/b^4*ln(b*x^3+a)*a*f+1/3/b^3*ln(b*x^3+a)*e-1/b^4/(b*x^3+a)*a^2*f+2/3/b^3/(b*x^3+a)*a*e-1/3/b^2/(b*x^3+a)*d

Maxima [A] time = 0.951215, size = 147, normalized size = 1.35

$$\frac{fx^3}{3b^3} - \frac{b^3c + ab^2d - 3a^2be + 5a^3f + 2(b^3d - 2ab^2e + 3a^2bf)x^3}{6(b^6x^6 + 2ab^5x^3 + a^2b^4)} + \frac{(be - 3af)\log(bx^3 + a)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")

[Out] 1/3*f*x^3/b^3 - 1/6*(b^3*c + a*b^2*d - 3*a^2*b*e + 5*a^3*f + 2*(b^3*d - 2*a*b^2*e + 3*a^2*b*f)*x^3)/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4) + 1/3*(b*e - 3*a*f)*log(b*x^3 + a)/b^4

Fricas [A] time = 1.165, size = 329, normalized size = 3.02

$$\frac{2b^3fx^9 + 4ab^2fx^6 - b^3c - ab^2d + 3a^2be - 5a^3f - 2(b^3d - 2ab^2e + 2a^2bf)x^3 + 2((b^3e - 3ab^2f)x^6 + a^2be - 3a^3f + 2ab^2e - 3a^2bf)x^3}{6(b^6x^6 + 2ab^5x^3 + a^2b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] 1/6*(2*b^3*f*x^9 + 4*a*b^2*f*x^6 - b^3*c - a*b^2*d + 3*a^2*b*e - 5*a^3*f - 2*(b^3*d - 2*a*b^2*e + 2*a^2*b*f)*x^3 + 2*((b^3*e - 3*a*b^2*f)*x^6 + a^2*b*e - 3*a^3*f + 2*(a*b^2*e - 3*a^2*b*f)*x^3)*log(b*x^3 + a)/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.07817, size = 135, normalized size = 1.24

$$\frac{fx^3}{3b^3} - \frac{(3af - be) \log(|bx^3 + a|)}{3b^4} - \frac{b^3c + ab^2d + 5a^3f + 2(b^3d + 3a^2bf - 2ab^2e)x^3 - 3a^2be}{6(bx^3 + a)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")

[Out] 1/3*f*x^3/b^3 - 1/3*(3*a*f - b*e)*log(abs(b*x^3 + a))/b^4 - 1/6*(b^3*c + a*b^2*d + 5*a^3*f + 2*(b^3*d + 3*a^2*b*f - 2*a*b^2*e)*x^3 - 3*a^2*b*e)/((b*x^3 + a)^2*b^4)

$$3.281 \quad \int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)^3} dx$$

Optimal. Leaf size=114

$$\frac{a^2be + a^3(-f) - ab^2d + b^3c}{6ab^3(a+bx^3)^2} + \frac{-a^2be + 2a^3f + b^3c}{3a^2b^3(a+bx^3)} - \frac{1}{3} \left(\frac{c}{a^3} - \frac{f}{b^3} \right) \log(a+bx^3) + \frac{c \log(x)}{a^3}$$

[Out] (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a*b^3*(a + b*x^3)^2) + (b^3*c - a^2*b*e + 2*a^3*f)/(3*a^2*b^3*(a + b*x^3)) + (c*Log[x])/a^3 - ((c/a^3 - f/b^3)*Log[a + b*x^3])/3

Rubi [A] time = 0.15477, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1821, 1620}

$$\frac{a^2be + a^3(-f) - ab^2d + b^3c}{6ab^3(a+bx^3)^2} + \frac{-a^2be + 2a^3f + b^3c}{3a^2b^3(a+bx^3)} - \frac{1}{3} \left(\frac{c}{a^3} - \frac{f}{b^3} \right) \log(a+bx^3) + \frac{c \log(x)}{a^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^3),x]

[Out] (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a*b^3*(a + b*x^3)^2) + (b^3*c - a^2*b*e + 2*a^3*f)/(3*a^2*b^3*(a + b*x^3)) + (c*Log[x])/a^3 - ((c/a^3 - f/b^3)*Log[a + b*x^3])/3

Rule 1821

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1620

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[Expon[Px, x], 2]

Rubi steps

$$\begin{aligned}
\int \frac{c + dx^3 + ex^6 + fx^9}{x(a + bx^3)^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{c + dx + ex^2 + fx^3}{x(a + bx)^3} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{c}{a^3 x} + \frac{-b^3 c + ab^2 d - a^2 b e + a^3 f}{ab^2 (a + bx)^3} + \frac{-b^3 c + a^2 b e - 2a^3 f}{a^2 b^2 (a + bx)^2} + \frac{-b^3 c + a^3 f}{a^3 b^2 (a + bx)} \right) dx, x \right) \\
&= \frac{b^3 c - ab^2 d + a^2 b e - a^3 f}{6ab^3 (a + bx^3)^2} + \frac{b^3 c - a^2 b e + 2a^3 f}{3a^2 b^3 (a + bx^3)} + \frac{c \log(x)}{a^3} - \frac{1}{3} \left(\frac{c}{a^3} - \frac{f}{b^3} \right) \log(a + bx^3)
\end{aligned}$$

Mathematica [A] time = 0.105827, size = 104, normalized size = 0.91

$$\frac{\frac{a(-a^2 b^2 (d + 2ex^3) - a^3 b(e - 4fx^3) + 3a^4 f + 3ab^3 c + 2b^4 cx^3)}{(a + bx^3)^2} + 2(a^3 f - b^3 c) \log(a + bx^3)}{b^3} + 6c \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^3), x]

[Out] (6*c*Log[x] + ((a*(3*a*b^3*c + 3*a^4*f + 2*b^4*c*x^3 - a^2*b^2*(d + 2*e*x^3) - a^3*b*(e - 4*f*x^3)))/(a + b*x^3)^2 + 2*(-(b^3*c) + a^3*f)*Log[a + b*x^3])/b^3)/(6*a^3)

Maple [A] time = 0.015, size = 147, normalized size = 1.3

$$-\frac{a^2 f}{6 b^3 (b x^3 + a)^2} + \frac{a e}{6 b^2 (b x^3 + a)^2} - \frac{d}{6 b (b x^3 + a)^2} + \frac{c}{6 a (b x^3 + a)^2} + \frac{\ln(b x^3 + a) f}{3 b^3} - \frac{c \ln(b x^3 + a)}{3 a^3} + \frac{2 a f}{3 b^3 (b x^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^3,x)

[Out] -1/6*a^2/b^3/(b*x^3+a)^2*f+1/6*a/b^2/(b*x^3+a)^2*e-1/6/b/(b*x^3+a)^2*d+1/6/a/(b*x^3+a)^2*c+1/3/b^3*ln(b*x^3+a)*f-1/3*c*ln(b*x^3+a)/a^3+2/3*a/b^3/(b*x^3+a)*f-1/3/b^2/(b*x^3+a)*e+1/3/a^2/(b*x^3+a)*c+c*ln(x)/a^3

Maxima [A] time = 0.953155, size = 174, normalized size = 1.53

$$\frac{3ab^3c - a^2b^2d - a^3be + 3a^4f + 2(b^4c - a^2b^2e + 2a^3bf)x^3}{6(a^2b^5x^6 + 2a^3b^4x^3 + a^4b^3)} + \frac{c \log(x^3)}{3a^3} - \frac{(b^3c - a^3f) \log(bx^3 + a)}{3a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^3,x, algorithm="maxima")

[Out] 1/6*(3*a*b^3*c - a^2*b^2*d - a^3*b*e + 3*a^4*f + 2*(b^4*c - a^2*b^2*e + 2*a^3*b*f)*x^3)/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3) + 1/3*c*log(x^3)/a^3 - 1/3*(b^3*c - a^3*f)*log(b*x^3 + a)/(a^3*b^3)

Fricas [A] time = 1.36771, size = 374, normalized size = 3.28

$$\frac{3a^2b^3c - a^3b^2d - a^4be + 3a^5f + 2(ab^4c - a^3b^2e + 2a^4bf)x^3 - 2((b^5c - a^3b^2f)x^6 + a^2b^3c - a^5f + 2(ab^4c - a^4bf)x^3)}{6(a^3b^5x^6 + 2a^4b^4x^3 + a^5b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^3,x, algorithm="fricas")

[Out] 1/6*(3*a^2*b^3*c - a^3*b^2*d - a^4*b*e + 3*a^5*f + 2*(a*b^4*c - a^3*b^2*e + 2*a^4*b*f)*x^3 - 2*((b^5*c - a^3*b^2*f)*x^6 + a^2*b^3*c - a^5*f + 2*(a*b^4*c - a^4*b*f)*x^3)*log(b*x^3 + a) + 6*(b^5*c*x^6 + 2*a*b^4*c*x^3 + a^2*b^3*c)*log(x))/(a^3*b^5*x^6 + 2*a^4*b^4*x^3 + a^5*b^3)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.07024, size = 173, normalized size = 1.52

$$\frac{c \log(|x|)}{a^3} - \frac{(b^3c - a^3f) \log(|bx^3 + a|)}{3a^3b^3} + \frac{3b^4cx^6 - 3a^3bfx^6 + 8ab^3cx^3 - 2a^4fx^3 - 2a^3bx^3e + 6a^2b^2c - a^3bd - a^4e}{6(bx^3 + a)^2 a^3 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^3,x, algorithm="giac")

[Out] c*log(abs(x))/a^3 - 1/3*(b^3*c - a^3*f)*log(abs(b*x^3 + a))/(a^3*b^3) + 1/6
 *(3*b^4*c*x^6 - 3*a^3*b*f*x^6 + 8*a*b^3*c*x^3 - 2*a^4*f*x^3 - 2*a^3*b*x^3*e
 + 6*a^2*b^2*c - a^3*b*d - a^4*e)/((b*x^3 + a)^2*a^3*b^2)

$$3.282 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)^3} dx$$

Optimal. Leaf size=134

$$-\frac{a^2be + a^3(-f) - ab^2d + b^3c}{6a^2b^2(a+bx^3)^2} - \frac{a^3f - ab^2d + 2b^3c}{3a^3b^2(a+bx^3)} + \frac{(3bc - ad) \log(a+bx^3)}{3a^4} - \frac{\log(x)(3bc - ad)}{a^4} - \frac{c}{3a^3x^3}$$

[Out] $-c/(3*a^3*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a^2*b^2*(a + b*x^3)^2) - (2*b^3*c - a*b^2*d + a^3*f)/(3*a^3*b^2*(a + b*x^3)) - ((3*b*c - a*d)*\text{Log}[x])/a^4 + ((3*b*c - a*d)*\text{Log}[a + b*x^3])/(3*a^4)$

Rubi [A] time = 0.171251, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1821, 1620}

$$-\frac{a^2be + a^3(-f) - ab^2d + b^3c}{6a^2b^2(a+bx^3)^2} - \frac{a^3f - ab^2d + 2b^3c}{3a^3b^2(a+bx^3)} + \frac{(3bc - ad) \log(a+bx^3)}{3a^4} - \frac{\log(x)(3bc - ad)}{a^4} - \frac{c}{3a^3x^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^3), x]$

[Out] $-c/(3*a^3*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a^2*b^2*(a + b*x^3)^2) - (2*b^3*c - a*b^2*d + a^3*f)/(3*a^3*b^2*(a + b*x^3)) - ((3*b*c - a*d)*\text{Log}[x])/a^4 + ((3*b*c - a*d)*\text{Log}[a + b*x^3])/(3*a^4)$

Rule 1821

$\text{Int}[(\text{Pq}_x)^{m_x}((a_x) + (b_x)(x_x)^{n_x})^{p_x}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{\text{Simplify}[(m+1)/n] - 1} * \text{SubstFor}[x^n, \text{Pq}, x] * (a + b*x)^p, x], x, x^n], x] /;$ FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Simplify[(m+1)/n]]

Rule 1620

$\text{Int}[(\text{Px}_x)^{m_x}((a_x) + (b_x)(x_x)^{n_x})^{p_x}((c_x) + (d_x)(x_x)^{q_x}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\text{Px}*(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E

xpon[Px, x], 2]

Rubi steps

$$\begin{aligned} \int \frac{c + dx^3 + ex^6 + fx^9}{x^4 (a + bx^3)^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{c + dx + ex^2 + fx^3}{x^2 (a + bx)^3} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{c}{a^3 x^2} + \frac{-3bc + ad}{a^4 x} + \frac{b^3 c - ab^2 d + a^2 b e - a^3 f}{a^2 b (a + bx)^3} + \frac{2b^3 c - ab^2 d + a^3 f}{a^3 b (a + bx)^2} - \frac{b(-3bc + ad)}{a^4 (a + bx)} \right) dx, x, x^3 \right) \\ &= -\frac{c}{3a^3 x^3} - \frac{b^3 c - ab^2 d + a^2 b e - a^3 f}{6a^2 b^2 (a + bx^3)^2} - \frac{2b^3 c - ab^2 d + a^3 f}{3a^3 b^2 (a + bx^3)} - \frac{(3bc - ad) \log(x)}{a^4} + \frac{(3bc - ad) \log(a + bx^3)}{3a^4} \end{aligned}$$

Mathematica [A] time = 0.0914598, size = 121, normalized size = 0.9

$$\frac{\frac{a^2(-a^2be+a^3f+ab^2d-b^3c)}{b^2(a+bx^3)^2} - \frac{2a(a^3f-ab^2d+2b^3c)}{b^2(a+bx^3)} + 2(3bc-ad)\log(a+bx^3) + 6\log(x)(ad-3bc) - \frac{2ac}{x^3}}{6a^4}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^3), x]

[Out] ((-2*a*c)/x^3 + (a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(b^2*(a + b*x^3)^2) - (2*a*(2*b^3*c - a*b^2*d + a^3*f))/(b^2*(a + b*x^3)) + 6*(-3*b*c + a*d)*Log[x] + 2*(3*b*c - a*d)*Log[a + b*x^3])/(6*a^4)

Maple [A] time = 0.014, size = 163, normalized size = 1.2

$$\frac{af}{6b^2(bx^3+a)^2} - \frac{e}{6b(bx^3+a)^2} + \frac{d}{6a(bx^3+a)^2} - \frac{bc}{6a^2(bx^3+a)^2} - \frac{d \ln(bx^3+a)}{3a^3} + \frac{bc \ln(bx^3+a)}{a^4} - \frac{f}{3b^2(bx^3+a)} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^3,x)

[Out] 1/6*a/b^2/(b*x^3+a)^2*f-1/6/b/(b*x^3+a)^2*e+1/6/a/(b*x^3+a)^2*d-1/6/a^2*b/(b*x^3+a)^2*c-1/3*d*ln(b*x^3+a)/a^3+b*c*ln(b*x^3+a)/a^4-1/3/b^2/(b*x^3+a)*f+

$\frac{1}{3}a^2/(b*x^3+a)*d-2/3/a^3*b/(b*x^3+a)*c-1/3*c/a^3/x^3+d*\ln(x)/a^3-3*b*c*\ln(x)/a^4$

Maxima [A] time = 0.961867, size = 194, normalized size = 1.45

$$\frac{2(3b^4c - ab^3d + a^3bf)x^6 + 2a^2b^2c + (9ab^3c - 3a^2b^2d + a^3be + a^4f)x^3}{6(a^3b^4x^9 + 2a^4b^3x^6 + a^5b^2x^3)} + \frac{(3bc - ad)\log(bx^3 + a)}{3a^4} - \frac{(3bc - ad)\log(x)}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^3,x, algorithm="maxima")

[Out] $-1/6*(2*(3*b^4*c - a*b^3*d + a^3*b*f)*x^6 + 2*a^2*b^2*c + (9*a*b^3*c - 3*a^2*b^2*d + a^3*b*e + a^4*f)*x^3)/(a^3*b^4*x^9 + 2*a^4*b^3*x^6 + a^5*b^2*x^3) + 1/3*(3*b*c - a*d)*\log(b*x^3 + a)/a^4 - 1/3*(3*b*c - a*d)*\log(x^3)/a^4$

Fricas [A] time = 1.39977, size = 497, normalized size = 3.71

$$\frac{2(3ab^4c - a^2b^3d + a^4bf)x^6 + 2a^3b^2c + (9a^2b^3c - 3a^3b^2d + a^4be + a^5f)x^3 - 2((3b^5c - ab^4d)x^9 + 2(3ab^4c - a^2b^3d))}{6(a^4b^4x^9 + 2a^5b^3x^6 + a^6b^2x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^3,x, algorithm="fricas")

[Out] $-1/6*(2*(3*a*b^4*c - a^2*b^3*d + a^4*b*f)*x^6 + 2*a^3*b^2*c + (9*a^2*b^3*c - 3*a^3*b^2*d + a^4*b*e + a^5*f)*x^3 - 2*((3*b^5*c - a*b^4*d)*x^9 + 2*(3*a*b^4*c - a^2*b^3*d)*x^6 + (3*a^2*b^3*c - a^3*b^2*d)*x^3)*\log(b*x^3 + a) + 6*((3*b^5*c - a*b^4*d)*x^9 + 2*(3*a*b^4*c - a^2*b^3*d)*x^6 + (3*a^2*b^3*c - a^3*b^2*d)*x^3)*\log(x)/(a^4*b^4*x^9 + 2*a^5*b^3*x^6 + a^6*b^2*x^3)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**4/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.09671, size = 234, normalized size = 1.75

$$-\frac{(3bc - ad) \log(|x|)}{a^4} + \frac{(3b^2c - abd) \log(|bx^3 + a|)}{3a^4b} + \frac{3bcx^3 - adx^3 - ac}{3a^4x^3} - \frac{9b^5cx^6 - 3ab^4dx^6 + 22ab^4cx^3 - 8a^2b^3dx^3}{6(bx^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^3,x, algorithm="giac")

[Out] $-(3*b*c - a*d)*\log(\text{abs}(x))/a^4 + 1/3*(3*b^2*c - a*b*d)*\log(\text{abs}(b*x^3 + a))/(a^4*b) + 1/3*(3*b*c*x^3 - a*d*x^3 - a*c)/(a^4*x^3) - 1/6*(9*b^5*c*x^6 - 3*a*b^4*d*x^6 + 22*a*b^4*c*x^3 - 8*a^2*b^3*d*x^3 + 2*a^4*b*f*x^3 + 14*a^2*b^3*c - 6*a^3*b^2*d + a^5*f + a^4*b*e)/((b*x^3 + a)^2*a^4*b^2)$

$$3.283 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)^3} dx$$

Optimal. Leaf size=163

$$\frac{a^2be + a^3(-f) - ab^2d + b^3c}{6a^3b(a+bx^3)^2} + \frac{a^2e - 2abd + 3b^2c}{3a^4(a+bx^3)} - \frac{\log(a+bx^3)(a^2e - 3abd + 6b^2c)}{3a^5} + \frac{\log(x)(a^2e - 3abd + 6b^2c)}{a^5} + \dots$$

[Out] $-c/(6*a^3*x^6) + (3*b*c - a*d)/(3*a^4*x^3) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a^3*b*(a + b*x^3)^2) + (3*b^2*c - 2*a*b*d + a^2*e)/(3*a^4*(a + b*x^3)) + ((6*b^2*c - 3*a*b*d + a^2*e)*\text{Log}[x])/a^5 - ((6*b^2*c - 3*a*b*d + a^2*e)*\text{Log}[a + b*x^3])/(3*a^5)$

Rubi [A] time = 0.199532, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1821, 1620}

$$\frac{a^2be + a^3(-f) - ab^2d + b^3c}{6a^3b(a+bx^3)^2} + \frac{a^2e - 2abd + 3b^2c}{3a^4(a+bx^3)} - \frac{\log(a+bx^3)(a^2e - 3abd + 6b^2c)}{3a^5} + \frac{\log(x)(a^2e - 3abd + 6b^2c)}{a^5} + \dots$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^3), x]$

[Out] $-c/(6*a^3*x^6) + (3*b*c - a*d)/(3*a^4*x^3) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a^3*b*(a + b*x^3)^2) + (3*b^2*c - 2*a*b*d + a^2*e)/(3*a^4*(a + b*x^3)) + ((6*b^2*c - 3*a*b*d + a^2*e)*\text{Log}[x])/a^5 - ((6*b^2*c - 3*a*b*d + a^2*e)*\text{Log}[a + b*x^3])/(3*a^5)$

Rule 1821

$\text{Int}[(Pq_)*(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*\text{SubstFor}[x^n, Pq, x]*(a + b*x)^p, x], x, x^n], x] /;$ FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1620

$\text{Int}[(Px_)*((a_.) + (b_.)*(x_))^{(m_.)}*((c_.) + (d_.)*(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[Px*(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c}

, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]

Rubi steps

$$\begin{aligned} \int \frac{c + dx^3 + ex^6 + fx^9}{x^7 (a + bx^3)^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{c + dx + ex^2 + fx^3}{x^3 (a + bx)^3} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{c}{a^3 x^3} + \frac{-3bc + ad}{a^4 x^2} + \frac{6b^2c - 3abd + a^2e}{a^5 x} + \frac{-b^3c + ab^2d - a^2be + a^3f}{a^3 (a + bx)^3} - \frac{b(3b^2c - 3abd + a^2e)}{a^5 (a + bx)^2} \right) dx, x, x^3 \right) \\ &= -\frac{c}{6a^3 x^6} + \frac{3bc - ad}{3a^4 x^3} + \frac{b^3c - ab^2d + a^2be - a^3f}{6a^3 b (a + bx^3)^2} + \frac{3b^2c - 2abd + a^2e}{3a^4 (a + bx^3)} + \frac{(6b^2c - 3abd + a^2e) \ln(a + bx^3)}{a^5} \end{aligned}$$

Mathematica [A] time = 0.106458, size = 149, normalized size = 0.91

$$\frac{a^2(a^2be + a^3(-f) - ab^2d + b^3c)}{b(a + bx^3)^2} + \frac{2a(a^2e - 2abd + 3b^2c)}{a + bx^3} - 2 \log(a + bx^3) (a^2e - 3abd + 6b^2c) + 6 \log(x) (a^2e - 3abd + 6b^2c) - \frac{a^2c}{x^6} - \frac{2a(ad - b^2c)}{x^3} - \frac{b^3c - ab^2d + a^2be - a^3f}{6a^3 b (a + bx^3)^2} + \frac{3b^2c - 2abd + a^2e}{3a^4 (a + bx^3)} + \frac{(6b^2c - 3abd + a^2e) \ln(a + bx^3)}{a^5}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^3), x]

[Out] (-((a^2*c)/x^6) - (2*a*(-3*b*c + a*d))/x^3 + (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(b*(a + b*x^3)^2) + (2*a*(3*b^2*c - 2*a*b*d + a^2*e))/(a + b*x^3) + 6*(6*b^2*c - 3*a*b*d + a^2*e)*Log[x] - 2*(6*b^2*c - 3*a*b*d + a^2*e)*Log[a + b*x^3])/(6*a^5)

Maple [A] time = 0.017, size = 213, normalized size = 1.3

$$-\frac{f}{6b(bx^3 + a)^2} + \frac{e}{6a(bx^3 + a)^2} - \frac{bd}{6a^2(bx^3 + a)^2} + \frac{b^2c}{6a^3(bx^3 + a)^2} - \frac{e \ln(bx^3 + a)}{3a^3} + \frac{\ln(bx^3 + a)bd}{a^4} - 2 \frac{\ln(bx^3 + a)b^3c - ab^2d + a^2be - a^3f}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^3,x)

[Out]
$$-1/6/b/(b*x^3+a)^2*f+1/6/a/(b*x^3+a)^2*e-1/6/a^2*b/(b*x^3+a)^2*d+1/6/a^3*b^2/(b*x^3+a)^2*c-1/3*e*\ln(b*x^3+a)/a^3+1/a^4*\ln(b*x^3+a)*b*d-2/a^5*\ln(b*x^3+a)*b^2*c+1/3/a^2/(b*x^3+a)*e-2/3/a^3/(b*x^3+a)*b*d+1/a^4/(b*x^3+a)*b^2*c-1/6*c/a^3/x^6-1/3/a^3/x^3*d+1/a^4/x^3*b*c+e*\ln(x)/a^3-3/a^4*\ln(x)*b*d+6/a^5*\ln(x)*b^2*c$$

Maxima [A] time = 0.980084, size = 246, normalized size = 1.51

$$\frac{2(6b^4c - 3ab^3d + a^2b^2e)x^9 + (18ab^3c - 9a^2b^2d + 3a^3be - a^4f)x^6 - a^3bc + 2(2a^2b^2c - a^3bd)x^3 - (6b^2c - 3abd + a^2e)}{6(a^4b^3x^{12} + 2a^5b^2x^9 + a^6bx^6)} - \frac{(6b^2c - 3abd + a^2e)}{3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^3,x, algorithm="maxima")`

[Out]
$$1/6*(2*(6*b^4*c - 3*a*b^3*d + a^2*b^2*e)*x^9 + (18*a*b^3*c - 9*a^2*b^2*d + 3*a^3*b*e - a^4*f)*x^6 - a^3*b*c + 2*(2*a^2*b^2*c - a^3*b*d)*x^3)/(a^4*b^3*x^{12} + 2*a^5*b^2*x^9 + a^6*b*x^6) - 1/3*(6*b^2*c - 3*a*b*d + a^2*e)*\log(b*x^3 + a)/a^5 + 1/3*(6*b^2*c - 3*a*b*d + a^2*e)*\log(x^3)/a^5$$

Fricas [B] time = 1.31011, size = 652, normalized size = 4.

$$\frac{2(6ab^4c - 3a^2b^3d + a^3b^2e)x^9 + (18a^2b^3c - 9a^3b^2d + 3a^4be - a^5f)x^6 - a^4bc + 2(2a^3b^2c - a^4bd)x^3 - 2((6b^5c - 3ab^4d + a^2b^3e)x^{12} + 2(6a*b^4*c - 3*a^2*b^3*d + a^3*b^2*e)*x^9 + (6*a^2*b^3*c - 3*a^3*b^2*d + a^4*b*e)*x^6)*\log(b*x^3 + a) + 6*((6*b^5*c - 3*a*b^4*d + a^2*b^3*e)*x^{12} + 2*(6*a*b^4*c - 3*a^2*b^3*d + a^3*b^2*e)*x^9 + (6*a^2*b^3*c - 3*a^3*b^2*d + a^4*b*e)*x^6)*\log(x)}{(a^5*b^3*x^{12} + 2*a^6*b^2*x^9 + a^7*b*x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^3,x, algorithm="fricas")`

[Out]
$$1/6*(2*(6*a*b^4*c - 3*a^2*b^3*d + a^3*b^2*e)*x^9 + (18*a^2*b^3*c - 9*a^3*b^2*d + 3*a^4*b*e - a^5*f)*x^6 - a^4*b*c + 2*(2*a^3*b^2*c - a^4*b*d)*x^3 - 2*((6*b^5*c - 3*a*b^4*d + a^2*b^3*e)*x^{12} + 2*(6*a*b^4*c - 3*a^2*b^3*d + a^3*b^2*e)*x^9 + (6*a^2*b^3*c - 3*a^3*b^2*d + a^4*b*e)*x^6)*\log(b*x^3 + a) + 6*((6*b^5*c - 3*a*b^4*d + a^2*b^3*e)*x^{12} + 2*(6*a*b^4*c - 3*a^2*b^3*d + a^3*b^2*e)*x^9 + (6*a^2*b^3*c - 3*a^3*b^2*d + a^4*b*e)*x^6)*\log(x))/(a^5*b^3*x^{12} + 2*a^6*b^2*x^9 + a^7*b*x^6)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**7/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.07307, size = 255, normalized size = 1.56

$$\frac{(6b^2c - 3abd + a^2e) \log(|x|)}{a^5} - \frac{(6b^3c - 3ab^2d + a^2be) \log(|bx^3 + a|)}{3a^5b} + \frac{12b^4cx^9 - 6ab^3dx^9 + 2a^2b^2x^9e + 18ab^3cx^6 - 9a^2b^2cx^6 - 9a^2b^2ex^6 + 18a^2b^2cx^3 - 2a^3b^2d^2x^3 - a^3b^2c^2}{6(b^2x^6 + ax^3)^2a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^3,x, algorithm="giac")

[Out] (6*b^2*c - 3*a*b*d + a^2*e)*log(abs(x))/a^5 - 1/3*(6*b^3*c - 3*a*b^2*d + a^2*b*e)*log(abs(b*x^3 + a))/(a^5*b) + 1/6*(12*b^4*c*x^9 - 6*a*b^3*d*x^9 + 2*a^2*b^2*x^9*e + 18*a*b^3*c*x^6 - 9*a^2*b^2*d*x^6 - a^4*f*x^6 + 3*a^3*b*x^6*e + 4*a^2*b^2*c*x^3 - 2*a^3*b*d*x^3 - a^3*b*c)/((b*x^6 + a*x^3)^2*a^4*b)

$$3.284 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)^3} dx$$

Optimal. Leaf size=218

$$\frac{2a^2be + a^3(-f) - 3ab^2d + 4b^3c}{3a^5(a+bx^3)} - \frac{a^2be + a^3(-f) - ab^2d + b^3c}{6a^4(a+bx^3)^2} + \frac{\log(a+bx^3)(3a^2be + a^3(-f) - 6ab^2d + 10b^3c)}{3a^6} - \log$$

[Out] $-c/(9*a^3*x^9) + (3*b*c - a*d)/(6*a^4*x^6) - (6*b^2*c - 3*a*b*d + a^2*e)/(3*a^5*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a^4*(a + b*x^3)^2) - (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(3*a^5*(a + b*x^3)) - ((10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)*\text{Log}[x])/a^6 + ((10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)*\text{Log}[a + b*x^3])/(3*a^6)$

Rubi [A] time = 0.263711, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1821, 1620}

$$\frac{2a^2be + a^3(-f) - 3ab^2d + 4b^3c}{3a^5(a+bx^3)} - \frac{a^2be + a^3(-f) - ab^2d + b^3c}{6a^4(a+bx^3)^2} + \frac{\log(a+bx^3)(3a^2be + a^3(-f) - 6ab^2d + 10b^3c)}{3a^6} - \log$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x^3 + e*x^6 + f*x^9)/(x^{10}*(a + b*x^3)^3), x]$

[Out] $-c/(9*a^3*x^9) + (3*b*c - a*d)/(6*a^4*x^6) - (6*b^2*c - 3*a*b*d + a^2*e)/(3*a^5*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a^4*(a + b*x^3)^2) - (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(3*a^5*(a + b*x^3)) - ((10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)*\text{Log}[x])/a^6 + ((10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)*\text{Log}[a + b*x^3])/(3*a^6)$

Rule 1821

$\text{Int}[(Pq_)*(x_)^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] := \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*\text{SubstFor}[x^n, Pq, x]*(a + b*x)^p, x], x, x^n], x] /;$ FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned} \int \frac{c + dx^3 + ex^6 + fx^9}{x^{10}(a + bx^3)^3} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{c + dx + ex^2 + fx^3}{x^4(a + bx)^3} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{c}{a^3x^4} + \frac{-3bc + ad}{a^4x^3} + \frac{6b^2c - 3abd + a^2e}{a^5x^2} + \frac{-10b^3c + 6ab^2d - 3a^2be + a^3f - b^3}{a^6x} \right) dx, x, x^3 \right) \\ &= -\frac{c}{9a^3x^9} + \frac{3bc - ad}{6a^4x^6} - \frac{6b^2c - 3abd + a^2e}{3a^5x^3} - \frac{b^3c - ab^2d + a^2be - a^3f}{6a^4(a + bx^3)^2} - \frac{4b^3c - 3ab^2d + 2a^2be - b^3}{3a^5(a + bx^3)} \end{aligned}$$

Mathematica [A] time = 0.137317, size = 200, normalized size = 0.92

$$\frac{3a^2(-a^2be + a^3f + ab^2d - b^3c)}{(a + bx^3)^2} + \frac{6a(-2a^2be + a^3f + 3ab^2d - 4b^3c)}{a + bx^3} + 6 \log(a + bx^3) (3a^2be + a^3(-f) - 6ab^2d + 10b^3c) + 18 \log(x) (-3a^2be + b^3) \over 18a^6$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^3), x]
```

```
[Out] ((-2*a^3*c)/x^9 - (3*a^2*(-3*b*c + a*d))/x^6 - (6*a*(6*b^2*c - 3*a*b*d + a^2*e))/x^3 + (3*a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3)^2 + (6*a*(-4*b^3*c + 3*a*b^2*d - 2*a^2*b*e + a^3*f))/(a + b*x^3) + 18*(-10*b^3*c + 6*a*b^2*d - 3*a^2*b*e + a^3*f)*Log[x] + 6*(10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)*Log[a + b*x^3])/(18*a^6)
```

Maple [A] time = 0.02, size = 293, normalized size = 1.3

$$\frac{f}{6a(bx^3 + a)^2} - \frac{be}{6a^2(bx^3 + a)^2} + \frac{b^2d}{6a^3(bx^3 + a)^2} - \frac{b^3c}{6a^4(bx^3 + a)^2} - \frac{\ln(bx^3 + a)f}{3a^3} + \frac{b \ln(bx^3 + a)e}{a^4} - 2 \frac{b^2 \ln(bx^3 + a)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^3,x)`

[Out] $\frac{1}{6} \frac{1}{a} \frac{1}{(b^3x+a)^2} f - \frac{1}{6} \frac{b}{a^2} \frac{1}{(b^3x+a)^2} e + \frac{1}{6} \frac{b^2}{a^3} \frac{1}{(b^3x+a)^2} d - \frac{1}{6} \frac{b^3}{a^4} \frac{1}{(b^3x+a)^2} c - \frac{1}{3} \frac{1}{a^3} \ln(b^3x+a) f + \frac{b}{a^4} \ln(b^3x+a) e - 2 \frac{b^2}{a^5} \ln(b^3x+a) d + \frac{10}{3} \frac{b^3}{a^6} \ln(b^3x+a) c + \frac{1}{3} \frac{1}{a^2} \frac{1}{(b^3x+a)} f - \frac{2}{3} \frac{b}{a^3} \frac{1}{(b^3x+a)} e + \frac{b^2}{a^4} \frac{1}{(b^3x+a)} d - \frac{4}{3} \frac{b^3}{a^5} \frac{1}{(b^3x+a)} c - \frac{1}{9} \frac{c}{a^3} x^{-9} - \frac{1}{6} \frac{1}{a^3} x^{-6} d + \frac{1}{2} \frac{1}{a^4} x^{-6} b c - \frac{1}{3} \frac{1}{a^3} x^{-3} e + \frac{1}{a^4} x^{-3} b d - \frac{2}{a^5} x^{-3} b^2 c + \frac{1}{a^3} \ln(x) f - \frac{3}{a^4} \ln(x) b e + \frac{6}{a^5} \ln(x) b^2 d - \frac{10}{a^6} \ln(x) b^3 c$

Maxima [A] time = 0.980614, size = 313, normalized size = 1.44

$$\frac{6(10b^4c - 6ab^3d + 3a^2b^2e - a^3bf)x^{12} + 9(10ab^3c - 6a^2b^2d + 3a^3be - a^4f)x^9 + 2(10a^2b^2c - 6a^3bd + 3a^4e)x^6 + 2}{18(a^5b^2x^{15} + 2a^6bx^{12} + a^7x^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^3,x, algorithm="maxima")`

[Out] $-\frac{1}{18} (6(10b^4c - 6a^2b^3d + 3a^3b^2e - a^3bf)x^{12} + 9(10a^2b^3c - 6a^2b^2d + 3a^3be - a^4f)x^9 + 2(10a^2b^2c - 6a^3bd + 3a^4e)x^6 + 2a^4c - (5a^3b^3c - 3a^4d)x^3) / (a^5b^2x^{15} + 2a^6bx^{12} + a^7x^9) + \frac{1}{3} (10b^3c - 6a^2b^2d + 3a^2b^2e - a^3f) \log(b^3x + a) / a^6 - \frac{1}{3} (10b^3c - 6a^2b^2d + 3a^2b^2e - a^3f) \log(x^3) / a^6$

Fricas [A] time = 1.44256, size = 837, normalized size = 3.84

$$\frac{6(10ab^4c - 6a^2b^3d + 3a^3b^2e - a^4bf)x^{12} + 9(10a^2b^3c - 6a^3b^2d + 3a^4be - a^5f)x^9 + 2(10a^3b^2c - 6a^4bd + 3a^5e)x^6 + 2}{18(a^5b^2x^{15} + 2a^6bx^{12} + a^7x^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^3,x, algorithm="fricas")`

[Out] $-\frac{1}{18} (6(10a^2b^4c - 6a^2b^3d + 3a^3b^2e - a^4bf)x^{12} + 9(10a^2b^3c - 6a^3b^2d + 3a^4be - a^5f)x^9 + 2(10a^3b^2c - 6a^4bd + 3a^5e)x^6 + 2a^5c - (5a^4b^3c - 3a^5d)x^3 - 6((10b^5c - 6a^2b^4d + 3a^2b^3e - a^3b^2f)x^{15} + 2(10a^2b^4c - 6a^2b^3d + 3a^3b^2e - a^4bf)x^{12} + (10a^2b^3c - 6a^3b^2d + 3a^4be - a^5f)x^9 + 2(10a^3b^2c - 6a^4bd + 3a^5e)x^6 + 2a^5c)) / (a^5b^2x^{15} + 2a^6bx^{12} + a^7x^9) + \frac{1}{3} (10b^3c - 6a^2b^2d + 3a^2b^2e - a^3f) \log(b^3x + a) / a^6 - \frac{1}{3} (10b^3c - 6a^2b^2d + 3a^2b^2e - a^3f) \log(x^3) / a^6$

$$x^9) \cdot \log(bx^3 + a) + 18 \cdot ((10b^5c - 6a^2b^4d + 3a^3b^3e - a^4b^2f) \cdot x^{15} + 2 \cdot (10ab^4c - 6a^2b^3d + 3a^3b^2e - a^4bf) \cdot x^{12} + (10a^2b^3c - 6a^3b^2d + 3a^4be - a^5f) \cdot x^9) \cdot \log(x) / (a^6b^2x^{15} + 2a^7bx^{12} + a^8x^9)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**10/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.1005, size = 437, normalized size = 2.

$$-\frac{(10b^3c - 6ab^2d - a^3f + 3a^2be) \log(|x|)}{a^6} + \frac{(10b^4c - 6ab^3d - a^3bf + 3a^2b^2e) \log(|bx^3 + a|)}{3a^6b} - \frac{30b^5cx^6 - 18ab^4dx^6 - 30b^4cx^5 - 18ab^3dx^5 - 30b^3cx^4 - 18ab^2dx^4 - 30b^2cx^3 - 18abdx^3 - 30bcx^2 - 18bdx^2 - 30cdx - 18cd}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^3,x, algorithm="giac")

[Out] $-(10b^3c - 6a^2b^4d - a^3bf + 3a^2b^3e) \cdot \log(\text{abs}(x)) / a^6 + 1/3 \cdot (10b^4c - 6a^2b^3d - a^3bf + 3a^2b^2e) \cdot \log(\text{abs}(bx^3 + a)) / (a^6b) - 1/6 \cdot (30b^5cx^6 - 18a^2b^4dx^6 - 3a^3b^3cx^5 + 9a^2b^3cx^4e + 68a^2b^4cx^3 - 42a^2b^3d^2x^3 - 8a^4b^2fx^3 + 22a^3b^2x^3e + 39a^2b^3c - 25a^3b^2d - 6a^5f + 14a^4b^2e) / ((bx^3 + a)^2 a^6) + 1/18 \cdot (110b^3cx^9 - 66a^2b^2dx^9 - 11a^3f^2x^9 + 33a^2b^2cx^9e - 36a^2b^2c^2x^6 + 18a^2b^2bd^2x^6 - 6a^3x^6e + 9a^2b^2c^2x^3 - 3a^3d^2x^3 - 2a^3c) / (a^6x^9)$

$$3.285 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)^3} dx$$

Optimal. Leaf size=258

$$\frac{b(3a^2be - 2a^3f - 4ab^2d + 5b^3c)}{3a^6(a + bx^3)} + \frac{b(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^5(a + bx^3)^2} + \frac{3a^2be + a^3(-f) - 6ab^2d + 10b^3c}{3a^6x^3} - \frac{b \log(a + bx^3)}{3a^6x^3}$$

[Out] $-c/(12*a^3*x^{12}) + (3*b*c - a*d)/(9*a^4*x^9) - (6*b^2*c - 3*a*b*d + a^2*e)/(6*a^5*x^6) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(3*a^6*x^3) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(6*a^5*(a + b*x^3)^2) + (b*(5*b^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f))/(3*a^6*(a + b*x^3)) + (b*(15*b^3*c - 10*a*b^2*d + 6*a^2*b*e - 3*a^3*f)*Log[x])/a^7 - (b*(15*b^3*c - 10*a*b^2*d + 6*a^2*b*e - 3*a^3*f)*Log[a + b*x^3])/(3*a^7)$

Rubi [A] time = 0.304493, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1821, 1620}

$$\frac{b(3a^2be - 2a^3f - 4ab^2d + 5b^3c)}{3a^6(a + bx^3)} + \frac{b(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^5(a + bx^3)^2} + \frac{3a^2be + a^3(-f) - 6ab^2d + 10b^3c}{3a^6x^3} - \frac{b \log(a + bx^3)}{3a^6x^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^3),x]

[Out] $-c/(12*a^3*x^{12}) + (3*b*c - a*d)/(9*a^4*x^9) - (6*b^2*c - 3*a*b*d + a^2*e)/(6*a^5*x^6) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(3*a^6*x^3) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(6*a^5*(a + b*x^3)^2) + (b*(5*b^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f))/(3*a^6*(a + b*x^3)) + (b*(15*b^3*c - 10*a*b^2*d + 6*a^2*b*e - 3*a^3*f)*Log[x])/a^7 - (b*(15*b^3*c - 10*a*b^2*d + 6*a^2*b*e - 3*a^3*f)*Log[a + b*x^3])/(3*a^7)$

Rule 1821

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c,
d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^{13}(a + bx^3)^3} dx = \frac{1}{3} \text{Subst} \left(\int \frac{c + dx + ex^2 + fx^3}{x^5(a + bx)^3} dx, x, x^3 \right)$$

$$= \frac{1}{3} \text{Subst} \left(\int \left(\frac{c}{a^3x^5} + \frac{-3bc + ad}{a^4x^4} + \frac{6b^2c - 3abd + a^2e}{a^5x^3} + \frac{-10b^3c + 6ab^2d - 3a^2be + a^3f - b^3}{a^6x^2} - \frac{b^4}{a^7x} \right) dx, x, x^3 \right)$$

$$= -\frac{c}{12a^3x^{12}} + \frac{3bc - ad}{9a^4x^9} - \frac{6b^2c - 3abd + a^2e}{6a^5x^6} + \frac{10b^3c - 6ab^2d + 3a^2be - a^3f}{3a^6x^3} + \frac{b(b^3c - ab^2d - a^3f - b^4)}{6a^5(a + bx^3)}$$

Mathematica [A] time = 0.179807, size = 238, normalized size = 0.92

$$\frac{a^3b^2x^6(15c+40dx^3-108ex^6+36fx^9)-12a^2b^3x^9(5c-15dx^3+6ex^6)-2a^4bx^3(3c+5dx^3+12ex^6-27fx^9)+a^5(3c+4dx^3+6ex^6+12fx^9)+30ab^4x^{12}(4dx^3-9c)-180b^5cx^{15}}{x^{12}(a+bx^3)^2} \quad 36a^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^3), x]
```

```
[Out] (-((a*(-180*b^5*c*x^15 + 30*a*b^4*x^12*(-9*c + 4*d*x^3) - 12*a^2*b^3*x^9*(5*c - 15*d*x^3 + 6*e*x^6) - 2*a^4*b*x^3*(3*c + 5*d*x^3 + 12*e*x^6 - 27*f*x^9) + a^5*(3*c + 4*d*x^3 + 6*e*x^6 + 12*f*x^9) + a^3*b^2*x^6*(15*c + 40*d*x^3 - 108*e*x^6 + 36*f*x^9))))/(x^12*(a + b*x^3)^2)) + 36*b*(15*b^3*c - 10*a*b^2*d + 6*a^2*b*e - 3*a^3*f)*Log[x] + 12*b*(-15*b^3*c + 10*a*b^2*d - 6*a^2*b*e + 3*a^3*f)*Log[a + b*x^3])/(36*a^7)
```

Maple [A] time = 0.019, size = 349, normalized size = 1.4

$$-\frac{d}{9a^3x^9} - \frac{e}{6a^3x^6} - \frac{f}{3a^3x^3} - \frac{c}{12a^3x^{12}} - 2 \frac{b^2 \ln(bx^3 + a)e}{a^5} + \frac{10b^3 \ln(bx^3 + a)d}{3a^6} + \frac{bc}{3a^4x^9} + \frac{bd}{2a^4x^6} - \frac{b^2c}{a^5x^6} + \frac{be}{a^4x^3} - 2 \frac{b^4}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((f*x^9+e*x^6+d*x^3+c)/x^{13}/(b*x^3+a)^3, x)$

[Out] $-1/9/a^3/x^9*d-1/6/a^3/x^6*e-1/3/a^3/x^3*f-1/12*c/a^3/x^{12}-2*b^2/a^5*\ln(b*x^3+a)*e+10/3*b^3/a^6*\ln(b*x^3+a)*d+1/3/a^4/x^9*b*c+1/2/a^4/x^6*b*d-1/a^5/x^6*b^2*c+1/a^4/x^3*b*e-2/a^5/x^3*b^2*d+10/3/a^6/x^3*b^3*c-3*b/a^4*\ln(x)*f+6*b^2/a^5*\ln(x)*e-10*b^3/a^6*\ln(x)*d+15*b^4/a^7*\ln(x)*c-1/6*b/a^2/(b*x^3+a)^2*f+1/6*b^2/a^3/(b*x^3+a)^2*e-1/6*b^3/a^4/(b*x^3+a)^2*d+1/6*b^4/a^5/(b*x^3+a)^2*c+b/a^4*\ln(b*x^3+a)*f+5/3*b^4/a^6/(b*x^3+a)*c-2/3*b/a^3/(b*x^3+a)*f-5*b^4/a^7*\ln(b*x^3+a)*c+b^2/a^4/(b*x^3+a)*e-4/3*b^3/a^5/(b*x^3+a)*d$

Maxima [A] time = 0.988422, size = 378, normalized size = 1.47

$$\frac{12(15b^5c - 10ab^4d + 6a^2b^3e - 3a^3b^2f)x^{15} + 18(15ab^4c - 10a^2b^3d + 6a^3b^2e - 3a^4bf)x^{12} + 4(15a^2b^3c - 10a^3b^2d + 6a^4b^2e - 3a^5bf)x^9 - (15a^3b^2c - 10a^4b^2d + 6a^5b^2e - 3a^6bf)x^6 - 3a^5c + 2(3a^4b^2c - 2a^5b^2d + 6a^6b^2e - 3a^7bf)x^3}{36(a^6b^2x^{18} + 2a^7bx^{15} + a^8x^{12})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((f*x^9+e*x^6+d*x^3+c)/x^{13}/(b*x^3+a)^3, x, \text{algorithm}="maxima")$

[Out] $1/36*(12*(15*b^5*c - 10*a*b^4*d + 6*a^2*b^3*e - 3*a^3*b^2*f)*x^{15} + 18*(15*a*b^4*c - 10*a^2*b^3*d + 6*a^3*b^2*e - 3*a^4*b*f)*x^{12} + 4*(15*a^2*b^3*c - 10*a^3*b^2*d + 6*a^4*b^2*e - 3*a^5*b*f)*x^9 - (15*a^3*b^2*c - 10*a^4*b^2*d + 6*a^5*b^2*e - 3*a^6*b*f)*x^6 - 3*a^5*c + 2*(3*a^4*b^2*c - 2*a^5*b^2*d + 6*a^6*b^2*e - 3*a^7*b*f)*x^3)/(a^6*b^2*x^{18} + 2*a^7*b*x^{15} + a^8*x^{12}) - 1/3*(15*b^4*c - 10*a*b^3*d + 6*a^2*b^2*e - 3*a^3*b*f)*\log(b*x^3 + a)/a^7 + 1/3*(15*b^4*c - 10*a*b^3*d + 6*a^2*b^2*e - 3*a^3*b*f)*\log(x^3)/a^7$

Fricas [A] time = 1.63972, size = 984, normalized size = 3.81

$$\frac{12(15ab^5c - 10a^2b^4d + 6a^3b^3e - 3a^4b^2f)x^{15} + 18(15a^2b^4c - 10a^3b^3d + 6a^4b^2e - 3a^5bf)x^{12} + 4(15a^3b^3c - 10a^4b^2d + 6a^5b^2e - 3a^6bf)x^9 - (15a^4b^2c - 10a^5b^2d + 6a^6b^2e - 3a^7bf)x^6 - 3a^6c + 2(3a^5b^2c - 2a^6b^2d + 6a^7b^2e - 3a^8bf)x^3}{36(a^6b^2x^{18} + 2a^7bx^{15} + a^8x^{12})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((f*x^9+e*x^6+d*x^3+c)/x^{13}/(b*x^3+a)^3, x, \text{algorithm}="fricas")$

```
[Out] 1/36*(12*(15*a*b^5*c - 10*a^2*b^4*d + 6*a^3*b^3*e - 3*a^4*b^2*f)*x^15 + 18*
(15*a^2*b^4*c - 10*a^3*b^3*d + 6*a^4*b^2*e - 3*a^5*b*f)*x^12 + 4*(15*a^3*b^
3*c - 10*a^4*b^2*d + 6*a^5*b*e - 3*a^6*f)*x^9 - 3*a^6*c - (15*a^4*b^2*c - 1
0*a^5*b*d + 6*a^6*e)*x^6 + 2*(3*a^5*b*c - 2*a^6*d)*x^3 - 12*((15*b^6*c - 10
*a*b^5*d + 6*a^2*b^4*e - 3*a^3*b^3*f)*x^18 + 2*(15*a*b^5*c - 10*a^2*b^4*d +
6*a^3*b^3*e - 3*a^4*b^2*f)*x^15 + (15*a^2*b^4*c - 10*a^3*b^3*d + 6*a^4*b^2
*e - 3*a^5*b*f)*x^12)*log(b*x^3 + a) + 36*((15*b^6*c - 10*a*b^5*d + 6*a^2*b
^4*e - 3*a^3*b^3*f)*x^18 + 2*(15*a*b^5*c - 10*a^2*b^4*d + 6*a^3*b^3*e - 3*a
^4*b^2*f)*x^15 + (15*a^2*b^4*c - 10*a^3*b^3*d + 6*a^4*b^2*e - 3*a^5*b*f)*x^
12)*log(x))/(a^7*b^2*x^18 + 2*a^8*b*x^15 + a^9*x^12)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**13/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.07742, size = 513, normalized size = 1.99

$$\frac{(15b^4c - 10ab^3d - 3a^3bf + 6a^2b^2e) \log(|x|)}{a^7} - \frac{(15b^5c - 10ab^4d - 3a^3b^2f + 6a^2b^3e) \log(|bx^3 + a|)}{3a^7b} + \frac{45b^6cx^6 - 30ab^5c^2x^5 + 15a^2b^4c^3x^4 - 5a^3b^3c^4x^3 + 5a^4b^2c^5x^2 - 5a^5b^3c^6x + 5a^6c^7}{3a^7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] (15*b^4*c - 10*a*b^3*d - 3*a^3*b*f + 6*a^2*b^2*e)*log(abs(x))/a^7 - 1/3*(15
*b^5*c - 10*a*b^4*d - 3*a^3*b^2*f + 6*a^2*b^3*e)*log(abs(b*x^3 + a))/(a^7*b
) + 1/6*(45*b^6*c*x^6 - 30*a*b^5*d*x^6 - 9*a^3*b^3*f*x^6 + 18*a^2*b^4*x^6*e
+ 100*a*b^5*c*x^3 - 68*a^2*b^4*d*x^3 - 22*a^4*b^2*f*x^3 + 42*a^3*b^3*x^3*e
+ 56*a^2*b^4*c - 39*a^3*b^3*d - 14*a^5*b*f + 25*a^4*b^2*e)/((b*x^3 + a)^2*
a^7) - 1/36*(375*b^4*c*x^12 - 250*a*b^3*d*x^12 - 75*a^3*b*f*x^12 + 150*a^2
*b^2*x^12*e - 120*a*b^3*c*x^9 + 72*a^2*b^2*d*x^9 + 12*a^4*f*x^9 - 36*a^3*b*x
^9*e + 36*a^2*b^2*c*x^6 - 18*a^3*b*d*x^6 + 6*a^4*x^6*e - 12*a^3*b*c*x^3 + 4
*a^4*d*x^3 + 3*a^4*c)/(a^7*x^12)
```

$$3.286 \quad \int \frac{x^{12}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=416

$$\frac{x^4(6a^2be - 10a^3f - 3ab^2d + b^3c)}{4b^6} - \frac{a^2x(31a^2be - 37a^3f - 25ab^2d + 19b^3c)}{18b^7(a + bx^3)} + \frac{a^3x(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^7(a + bx^3)^2} - \frac{a^{4/3}}{b^{2/3}}$$

[Out] $-\left(\frac{a(3b^3c - 6ab^2d + 10a^2b^2e - 15a^3f)x}{b^7} + \frac{(b^3c - 3ab^2d + 6a^2b^2e - 10a^3f)x^4}{(4b^6)} + \frac{(b^2d - 3ab^2e + 6a^2f)x^7}{(7b^5)} + \frac{(be - 3af)x^{10}}{(10b^4)} + \frac{fx^{13}}{(13b^3)} + \frac{a^3(b^3c - ab^2d + a^2b^2e - a^3f)x}{(6b^7(a + bx^3)^2)} - \frac{a^2(19b^3c - 25ab^2d + 31a^2b^2e - 37a^3f)x}{(18b^7(a + bx^3))} - \frac{a^{4/3}(35b^3c - 65ab^2d + 104a^2b^2e - 152a^3f)\text{ArcTan}\left[\frac{a^{1/3} - 2b^{1/3}x}{\sqrt{3}a^{1/3}}\right]}{(9\sqrt{3}b^{22/3})} + \frac{a^{4/3}(35b^3c - 65ab^2d + 104a^2b^2e - 152a^3f)\text{Log}\left[\frac{a^{1/3} + b^{1/3}x}{27b^{22/3}}\right]}{(27b^{22/3})} - \frac{a^{4/3}(35b^3c - 65ab^2d + 104a^2b^2e - 152a^3f)\text{Log}\left[\frac{a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2}{54b^{22/3}}\right]}{(54b^{22/3})}\right)$

Rubi [A] time = 0.742162, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {1828, 1858, 1887, 200, 31, 634, 617, 204, 628}

$$\frac{x^4(6a^2be - 10a^3f - 3ab^2d + b^3c)}{4b^6} - \frac{a^2x(31a^2be - 37a^3f - 25ab^2d + 19b^3c)}{18b^7(a + bx^3)} + \frac{a^3x(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^7(a + bx^3)^2} - \frac{a^{4/3}}{b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(x^12*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]

[Out] $-\left(\frac{a(3b^3c - 6ab^2d + 10a^2b^2e - 15a^3f)x}{b^7} + \frac{(b^3c - 3ab^2d + 6a^2b^2e - 10a^3f)x^4}{(4b^6)} + \frac{(b^2d - 3ab^2e + 6a^2f)x^7}{(7b^5)} + \frac{(be - 3af)x^{10}}{(10b^4)} + \frac{fx^{13}}{(13b^3)} + \frac{a^3(b^3c - ab^2d + a^2b^2e - a^3f)x}{(6b^7(a + bx^3)^2)} - \frac{a^2(19b^3c - 25ab^2d + 31a^2b^2e - 37a^3f)x}{(18b^7(a + bx^3))} - \frac{a^{4/3}(35b^3c - 65ab^2d + 104a^2b^2e - 152a^3f)\text{ArcTan}\left[\frac{a^{1/3} - 2b^{1/3}x}{\sqrt{3}a^{1/3}}\right]}{(9\sqrt{3}b^{22/3})} + \frac{a^{4/3}(35b^3c - 65ab^2d + 104a^2b^2e - 152a^3f)\text{Log}\left[\frac{a^{1/3} + b^{1/3}x}{27b^{22/3}}\right]}{(27b^{22/3})} - \frac{a^{4/3}(35b^3c - 65ab^2d + 104a^2b^2e - 152a^3f)\text{Log}\left[\frac{a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2}{54b^{22/3}}\right]}{(54b^{22/3})}\right)$

$(4/3)*(35*b^3*c - 65*a*b^2*d + 104*a^2*b*e - 152*a^3*f)*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2]/(54*b^{(22/3)})$

Rule 1828

$\text{Int}[(Pq_)*(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_.)})^{(p_)}, x_Symbol] \rightarrow \text{With}[\{q = m + \text{Expon}[Pq, x]\}, \text{Module}[\{Q = \text{PolynomialQuotient}[b^{(\text{Floor}[(q - 1)/n] + 1)}*x^m*Pq, a + b*x^n, x], R = \text{PolynomialRemainder}[b^{(\text{Floor}[(q - 1)/n] + 1)}*x^m*Pq, a + b*x^n, x]\}, \text{Dist}[1/(a*n*(p + 1)*b^{(\text{Floor}[(q - 1)/n] + 1)}), \text{Int}[(a + b*x^n)^{(p + 1)}*\text{ExpandToSum}[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - \text{Simp}[(x*R*(a + b*x^n)^{(p + 1)})/(a*n*(p + 1)*b^{(\text{Floor}[(q - 1)/n] + 1)}), x]] /; \text{GeQ}[q, n]] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[p, -1] \&\& \text{IGtQ}[m, 0]$

Rule 1858

$\text{Int}[(Pq_)*((a_) + (b_.)*(x_)^{(n_.)})^{(p_)}, x_Symbol] \rightarrow \text{With}[\{q = \text{Expon}[Pq, x]\}, \text{Module}[\{Q = \text{PolynomialQuotient}[b^{(\text{Floor}[(q - 1)/n] + 1)}*Pq, a + b*x^n, x], R = \text{PolynomialRemainder}[b^{(\text{Floor}[(q - 1)/n] + 1)}*Pq, a + b*x^n, x]\}, \text{Dist}[1/(a*n*(p + 1)*b^{(\text{Floor}[(q - 1)/n] + 1)}), \text{Int}[(a + b*x^n)^{(p + 1)}*\text{ExpandToSum}[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - \text{Simp}[(x*R*(a + b*x^n)^{(p + 1)})/(a*n*(p + 1)*b^{(\text{Floor}[(q - 1)/n] + 1)}), x]] /; \text{GeQ}[q, n]] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[p, -1]$

Rule 1887

$\text{Int}[(Pq_)/((a_) + (b_.)*(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[Pq/(a + b*x^n), x], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IntegerQ}[n]$

Rule 200

$\text{Int}[((a_) + (b_.)*(x_)^3)^{(-1)}, x_Symbol] \rightarrow \text{Dist}[1/(3*\text{Rt}[a, 3]^2), \text{Int}[1/(\text{Rt}[a, 3] + \text{Rt}[b, 3]*x), x], x] + \text{Dist}[1/(3*\text{Rt}[a, 3]^2), \text{Int}[(2*\text{Rt}[a, 3] - \text{Rt}[b, 3]*x)/(\text{Rt}[a, 3]^2 - \text{Rt}[a, 3]*\text{Rt}[b, 3]*x + \text{Rt}[b, 3]^2*x^2), x], x] /; \text{FreeQ}[\{a, b\}, x]$

Rule 31

$\text{Int}(((a_) + (b_.)*(x_))^{(-1)}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}[\{a, b\}, x]$

Rule 634

$\text{Int}(((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] \rightarrow \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{In}$

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{12}(c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^3} dx &= \frac{a^3(b^3c - ab^2d + a^2be - a^3f)x}{6b^7(a + bx^3)^2} - \int \frac{a^4(b^3c - ab^2d + a^2be - a^3f) - 6a^3b(b^3c - ab^2d + a^2be - a^3f)x^3 + 6a^2b^2(b^3c - ab^2d + a^2be - a^3f)x^6 - 6ab^3(b^3c - ab^2d + a^2be - a^3f)x^9 + 6a^4(b^3c - ab^2d + a^2be - a^3f)x^{12}}{(a + bx^3)^3} dx \\
&= \frac{a^3(b^3c - ab^2d + a^2be - a^3f)x}{6b^7(a + bx^3)^2} - \frac{a^2(19b^3c - 25ab^2d + 31a^2be - 37a^3f)x}{18b^7(a + bx^3)} + \int \frac{2a^4b^6(8b^3c - 12ab^2d + 10a^2be - 10a^3f)}{(a + bx^3)^3} dx \\
&= \frac{a^3(b^3c - ab^2d + a^2be - a^3f)x}{6b^7(a + bx^3)^2} - \frac{a^2(19b^3c - 25ab^2d + 31a^2be - 37a^3f)x}{18b^7(a + bx^3)} + \int (-18a^3b^6) \frac{1}{(a + bx^3)^3} dx \\
&= -\frac{a(3b^3c - 6ab^2d + 10a^2be - 15a^3f)x}{b^7} + \frac{(b^3c - 3ab^2d + 6a^2be - 10a^3f)x^4}{4b^6} + \frac{(b^2d - 3b^3c + 3a^2be - 3a^3f)}{4b^6} \\
&= -\frac{a(3b^3c - 6ab^2d + 10a^2be - 15a^3f)x}{b^7} + \frac{(b^3c - 3ab^2d + 6a^2be - 10a^3f)x^4}{4b^6} + \frac{(b^2d - 3b^3c + 3a^2be - 3a^3f)}{4b^6} \\
&= -\frac{a(3b^3c - 6ab^2d + 10a^2be - 15a^3f)x}{b^7} + \frac{(b^3c - 3ab^2d + 6a^2be - 10a^3f)x^4}{4b^6} + \frac{(b^2d - 3b^3c + 3a^2be - 3a^3f)}{4b^6} \\
&= -\frac{a(3b^3c - 6ab^2d + 10a^2be - 15a^3f)x}{b^7} + \frac{(b^3c - 3ab^2d + 6a^2be - 10a^3f)x^4}{4b^6} + \frac{(b^2d - 3b^3c + 3a^2be - 3a^3f)}{4b^6} \\
&= -\frac{a(3b^3c - 6ab^2d + 10a^2be - 15a^3f)x}{b^7} + \frac{(b^3c - 3ab^2d + 6a^2be - 10a^3f)x^4}{4b^6} + \frac{(b^2d - 3b^3c + 3a^2be - 3a^3f)}{4b^6}
\end{aligned}$$

Mathematica [A] time = 0.482441, size = 411, normalized size = 0.99

$$\frac{x^4(6a^2be - 10a^3f - 3ab^2d + b^3c)}{4b^6} + \frac{a^2x(-31a^2be + 37a^3f + 25ab^2d - 19b^3c)}{18b^7(a + bx^3)} + \frac{a^3x(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^7(a + bx^3)^2} + \frac{a^4(3b^3c - 6ab^2d + 10a^2be - 10a^3f)}{4b^6}$$

Antiderivative was successfully verified.

[In] Integrate[(x^12*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]

```
[Out] (a*(-3*b^3*c + 6*a*b^2*d - 10*a^2*b*e + 15*a^3*f)*x)/b^7 + ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^4)/(4*b^6) + ((b^2*d - 3*a*b*e + 6*a^2*f)*x^7)/(7*b^5) + ((b*e - 3*a*f)*x^10)/(10*b^4) + (f*x^13)/(13*b^3) + (a^3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b^7*(a + b*x^3)^2) + (a^2*(-19*b^3*c + 25*a*b^2*d - 31*a^2*b*e + 37*a^3*f)*x)/(18*b^7*(a + b*x^3)) + (a^(4/3)*(-35*b^3*c + 65*a*b^2*d - 104*a^2*b*e + 152*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(22/3)) - (a^(4/3)*(-35*b^3*c + 65*a*b^2*d - 104*a^2*b*e + 152*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(22/3)) + (a^(4/3)*(-35*b^3*c + 65*a*b^2*d - 104*a^2*b*e + 152*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(22/3))
```

Maple [A] time = 0.013, size = 706, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] -19/18*a^2/b^3/(b*x^3+a)^2*x^4*c+17/9*a^6/b^7/(b*x^3+a)^2*f*x-14/9*a^5/b^6/(b*x^3+a)^2*e*x+11/9*a^4/b^5/(b*x^3+a)^2*d*x-8/9*a^3/b^4/(b*x^3+a)^2*c*x+1/13*f*x^13/b^3+37/18*a^5/b^6/(b*x^3+a)^2*x^4*f-31/18*a^4/b^5/(b*x^3+a)^2*x^4*e+25/18*a^3/b^4/(b*x^3+a)^2*x^4*d+104/27*a^4/b^7*e/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-35/54*a^2/b^5*c/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-65/27*a^3/b^6*d/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+65/54*a^3/b^6*d/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-152/27*a^5/b^8*f/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+76/27*a^5/b^8*f/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+35/27*a^2/b^5*c/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-52/27*a^4/b^7*e/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-3/10/b^4*x^10*a*f+6/7/b^5*x^7*a^2*f-3/7/b^4*x^7*a*e-5/2/b^6*x^4*a^3*f+3/2/b^5*x^4*a^2*e-3/4/b^4*x^4*a*d+15/b^7*a^4*f*x-10/b^6*a^3*e*x+6/b^5*a^2*d*x-3/b^4*a*c*x+1/10/b^3*x^10*e+1/7/b^3*x^7*d+1/4/b^3*x^4*c-152/27*a^5/b^8*f/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+104/27*a^4/b^7*e/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-65/27*a^3/b^6*d/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+35/27*a^2/b^5*c/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32608, size = 1589, normalized size = 3.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/49140*(3780*b^6*f*x^19 + 378*(13*b^6*e - 19*a*b^5*f)*x^16 + 108*(65*b^6*d
- 104*a*b^5*e + 152*a^2*b^4*f)*x^13 + 351*(35*b^6*c - 65*a*b^5*d + 104*a^2
*b^4*e - 152*a^3*b^3*f)*x^10 - 3510*(35*a*b^5*c - 65*a^2*b^4*d + 104*a^3*b^
3*e - 152*a^4*b^2*f)*x^7 - 9555*(35*a^2*b^4*c - 65*a^3*b^3*d + 104*a^4*b^2*
e - 152*a^5*b*f)*x^4 - 1820*sqrt(3)*(35*a^3*b^3*c - 65*a^4*b^2*d + 104*a^5*
b*e - 152*a^6*f + (35*a*b^5*c - 65*a^2*b^4*d + 104*a^3*b^3*e - 152*a^4*b^2*
f)*x^6 + 2*(35*a^2*b^4*c - 65*a^3*b^3*d + 104*a^4*b^2*e - 152*a^5*b*f)*x^3)
*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) + 910*
(35*a^3*b^3*c - 65*a^4*b^2*d + 104*a^5*b*e - 152*a^6*f + (35*a*b^5*c - 65*a
^2*b^4*d + 104*a^3*b^3*e - 152*a^4*b^2*f)*x^6 + 2*(35*a^2*b^4*c - 65*a^3*b^
3*d + 104*a^4*b^2*e - 152*a^5*b*f)*x^3)*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/
3) + (-a/b)^(2/3)) - 1820*(35*a^3*b^3*c - 65*a^4*b^2*d + 104*a^5*b*e - 152*
a^6*f + (35*a*b^5*c - 65*a^2*b^4*d + 104*a^3*b^3*e - 152*a^4*b^2*f)*x^6 + 2
*(35*a^2*b^4*c - 65*a^3*b^3*d + 104*a^4*b^2*e - 152*a^5*b*f)*x^3)*(-a/b)^(1
/3)*log(x - (-a/b)^(1/3)) - 5460*(35*a^3*b^3*c - 65*a^4*b^2*d + 104*a^5*b*e
- 152*a^6*f)*x)/(b^9*x^6 + 2*a*b^8*x^3 + a^2*b^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```


[Out] Timed out

Giac [A] time = 1.09094, size = 675, normalized size = 1.62

$$\frac{\sqrt{3} \left(35 (-ab^2)^{\frac{1}{3}} ab^3c - 65 (-ab^2)^{\frac{1}{3}} a^2b^2d - 152 (-ab^2)^{\frac{1}{3}} a^4f + 104 (-ab^2)^{\frac{1}{3}} a^3be \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{27 b^8} \quad (35 a^2 b^3 c -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^12*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")`

[Out]
$$\frac{1}{27} \sqrt{3} (35 (-a b^2)^{1/3} a b^3 c - 65 (-a b^2)^{1/3} a^2 b^2 d - 152 (-a b^2)^{1/3} a^4 f + 104 (-a b^2)^{1/3} a^3 b e) \arctan \left(\frac{1}{3} \sqrt{3} \left(2 x + \left(-\frac{a}{b} \right)^{1/3} \right) / \left(-\frac{a}{b} \right)^{1/3} \right) / b^8 - \frac{1}{27} (35 a^2 b^3 c - 65 a^3 b^2 d - 152 a^5 f + 104 a^4 b e) \left(-\frac{a}{b} \right)^{1/3} \log \left(\text{abs} \left(x - \left(-\frac{a}{b} \right)^{1/3} \right) \right) / (a b^7) + \frac{1}{54} (35 (-a b^2)^{1/3} a b^3 c - 65 (-a b^2)^{1/3} a^2 b^2 d - 152 (-a b^2)^{1/3} a^4 f + 104 (-a b^2)^{1/3} a^3 b e) \log \left(x^2 + x \left(-\frac{a}{b} \right)^{1/3} + \left(-\frac{a}{b} \right)^{2/3} \right) / b^8 - \frac{1}{18} (19 a^2 b^4 c x^4 - 25 a^3 b^3 d x^4 - 37 a^5 b f x^4 + 31 a^4 b^2 x^4 e + 16 a^3 b^3 c x - 22 a^4 b^2 d x - 34 a^6 f x + 28 a^5 b x e) / ((b x^3 + a)^2 b^7) + \frac{1}{1820} (140 b^36 f x^13 - 546 a b^35 f x^10 + 182 b^36 x^10 e + 260 b^36 d x^7 + 1560 a^2 b^34 f x^7 - 780 a b^35 x^7 e + 455 b^36 c x^4 - 1365 a b^35 d x^4 - 4550 a^3 b^33 f x^4 + 2730 a^2 b^34 x^4 e - 5460 a b^35 c x + 10920 a^2 b^34 d x + 27300 a^4 b^32 f x - 18200 a^3 b^33 x e) / b^39$$

$$3.287 \quad \int \frac{x^{10}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=384

$$\frac{x^2(6a^2be - 10a^3f - 3ab^2d + b^3c)}{2b^6} + \frac{ax^2(13a^2be - 16a^3f - 10ab^2d + 7b^3c)}{9b^6(a + bx^3)} - \frac{a^2x^2(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^6(a + bx^3)^2} - \frac{a^{2/3} \log\left(\frac{a^{1/3} + b^{1/3}x}{a^{1/3} + b^{1/3}x}\right)}{54b^{20/3}}$$

[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^2)/(2*b^6) + ((b^2*d - 3*a*b*e + 6*a^2*f)*x^5)/(5*b^5) + ((b*e - 3*a*f)*x^8)/(8*b^4) + (f*x^11)/(11*b^3) - (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*b^6*(a + b*x^3)^2) + (a*(7*b^3*c - 10*a*b^2*d + 13*a^2*b*e - 16*a^3*f)*x^2)/(9*b^6*(a + b*x^3)) + (a^(2/3)*(20*b^3*c - 44*a*b^2*d + 77*a^2*b*e - 119*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*b^(20/3)) + (a^(2/3)*(20*b^3*c - 44*a*b^2*d + 77*a^2*b*e - 119*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(20/3)) - (a^(2/3)*(20*b^3*c - 44*a*b^2*d + 77*a^2*b*e - 119*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(20/3))

Rubi [A] time = 1.05046, antiderivative size = 384, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1828, 1851, 1836, 1488, 292, 31, 634, 617, 204, 628}

$$\frac{x^2(6a^2be - 10a^3f - 3ab^2d + b^3c)}{2b^6} + \frac{ax^2(13a^2be - 16a^3f - 10ab^2d + 7b^3c)}{9b^6(a + bx^3)} - \frac{a^2x^2(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^6(a + bx^3)^2} - \frac{a^{2/3} \log\left(\frac{a^{1/3} + b^{1/3}x}{a^{1/3} + b^{1/3}x}\right)}{54b^{20/3}}$$

Antiderivative was successfully verified.

[In] Int[(x^10*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]

[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^2)/(2*b^6) + ((b^2*d - 3*a*b*e + 6*a^2*f)*x^5)/(5*b^5) + ((b*e - 3*a*f)*x^8)/(8*b^4) + (f*x^11)/(11*b^3) - (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*b^6*(a + b*x^3)^2) + (a*(7*b^3*c - 10*a*b^2*d + 13*a^2*b*e - 16*a^3*f)*x^2)/(9*b^6*(a + b*x^3)) + (a^(2/3)*(20*b^3*c - 44*a*b^2*d + 77*a^2*b*e - 119*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*b^(20/3)) + (a^(2/3)*(20*b^3*c - 44*a*b^2*d + 77*a^2*b*e - 119*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(20/3)) - (a^(2/3)*(20*b^3*c - 44*a*b^2*d + 77*a^2*b*e - 119*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(20/3))

Rule 1828

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q =
m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
+ b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1851

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[x*PolynomialQuot
ient[Pq, x, x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x]
&& EqQ[Coeff[Pq, x, 0], 0] && !MatchQ[Pq, x^(m_)*(u_)] /; IntegerQ[m]]
```

Rule 1836

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
+ 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
+ 1)/(2*n)]) /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1488

```
Int[((f_)*(x_))^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_))^(p_)*((
d_) + (e_)*(x_)^(n_))^(q_), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(p_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{10}(c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^3} dx &= -\frac{a^2(b^3c - ab^2d + a^2be - a^3f)x^2}{6b^6(a + bx^3)^2} - \frac{\int \frac{-2a^3b(b^3c - ab^2d + a^2be - a^3f)x + 6a^2b^2(b^3c - ab^2d + a^2be - a^3f)x^4}{(a + bx^3)^3} dx}{6b^6(a + bx^3)^2} \\
&= -\frac{a^2(b^3c - ab^2d + a^2be - a^3f)x^2}{6b^6(a + bx^3)^2} - \frac{\int \frac{x(-2a^3b(b^3c - ab^2d + a^2be - a^3f) + 6a^2b^2(b^3c - ab^2d + a^2be - a^3f)x^4)}{(a + bx^3)^3} dx}{6b^6(a + bx^3)^2} \\
&= -\frac{a^2(b^3c - ab^2d + a^2be - a^3f)x^2}{6b^6(a + bx^3)^2} + \frac{a(7b^3c - 10ab^2d + 13a^2be - 16a^3f)x^2}{9b^6(a + bx^3)} + \frac{\int \frac{-2a^3b}{(a + bx^3)^3} dx}{9b^6(a + bx^3)} \\
&= -\frac{a^2(b^3c - ab^2d + a^2be - a^3f)x^2}{6b^6(a + bx^3)^2} + \frac{a(7b^3c - 10ab^2d + 13a^2be - 16a^3f)x^2}{9b^6(a + bx^3)} + \frac{\int \frac{x(-2a^3b)}{(a + bx^3)^3} dx}{9b^6(a + bx^3)} \\
&= \frac{fx^{11}}{11b^3} - \frac{a^2(b^3c - ab^2d + a^2be - a^3f)x^2}{6b^6(a + bx^3)^2} + \frac{a(7b^3c - 10ab^2d + 13a^2be - 16a^3f)x^2}{9b^6(a + bx^3)} + \frac{\int \frac{x(-2a^3b)}{(a + bx^3)^3} dx}{9b^6(a + bx^3)} \\
&= \frac{(be - 3af)x^8}{8b^4} + \frac{fx^{11}}{11b^3} - \frac{a^2(b^3c - ab^2d + a^2be - a^3f)x^2}{6b^6(a + bx^3)^2} + \frac{a(7b^3c - 10ab^2d + 13a^2be - 16a^3f)x^2}{9b^6(a + bx^3)} + \frac{\int \frac{x(-2a^3b)}{(a + bx^3)^3} dx}{9b^6(a + bx^3)} \\
&= \frac{(be - 3af)x^8}{8b^4} + \frac{fx^{11}}{11b^3} - \frac{a^2(b^3c - ab^2d + a^2be - a^3f)x^2}{6b^6(a + bx^3)^2} + \frac{a(7b^3c - 10ab^2d + 13a^2be - 16a^3f)x^2}{9b^6(a + bx^3)} + \frac{\int \frac{x(-2a^3b)}{(a + bx^3)^3} dx}{9b^6(a + bx^3)} \\
&= \frac{(b^3c - 3ab^2d + 6a^2be - 10a^3f)x^2}{2b^6} + \frac{(b^2d - 3abe + 6a^2f)x^5}{5b^5} + \frac{(be - 3af)x^8}{8b^4} + \frac{fx^{11}}{11b^3} \\
&= \frac{(b^3c - 3ab^2d + 6a^2be - 10a^3f)x^2}{2b^6} + \frac{(b^2d - 3abe + 6a^2f)x^5}{5b^5} + \frac{(be - 3af)x^8}{8b^4} + \frac{fx^{11}}{11b^3} \\
&= \frac{(b^3c - 3ab^2d + 6a^2be - 10a^3f)x^2}{2b^6} + \frac{(b^2d - 3abe + 6a^2f)x^5}{5b^5} + \frac{(be - 3af)x^8}{8b^4} + \frac{fx^{11}}{11b^3} \\
&= \frac{(b^3c - 3ab^2d + 6a^2be - 10a^3f)x^2}{2b^6} + \frac{(b^2d - 3abe + 6a^2f)x^5}{5b^5} + \frac{(be - 3af)x^8}{8b^4} + \frac{fx^{11}}{11b^3} \\
&= \frac{(b^3c - 3ab^2d + 6a^2be - 10a^3f)x^2}{2b^6} + \frac{(b^2d - 3abe + 6a^2f)x^5}{5b^5} + \frac{(be - 3af)x^8}{8b^4} + \frac{fx^{11}}{11b^3}
\end{aligned}$$

Mathematica [A] time = 0.402336, size = 380, normalized size = 0.99

$$\frac{x^2(6a^2be - 10a^3f - 3ab^2d + b^3c)}{2b^6} + \frac{ax^2(13a^2be - 16a^3f - 10ab^2d + 7b^3c)}{9b^6(a + bx^3)} + \frac{a^2x^2(-a^2be + a^3f + ab^2d - b^3c)}{6b^6(a + bx^3)^2} + \frac{a^{2/3} \log}{a^{2/3} \log}$$

Antiderivative was successfully verified.

[In] Integrate[(x^10*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]

[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^2)/(2*b^6) + ((b^2*d - 3*a*b*e + 6*a^2*f)*x^5)/(5*b^5) + ((b*e - 3*a*f)*x^8)/(8*b^4) + (f*x^11)/(11*b^3) + (a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(6*b^6*(a + b*x^3)^2) + (a*(7*b^3*c - 10*a*b^2*d + 13*a^2*b*e - 16*a^3*f)*x^2)/(9*b^6*(a + b*x^3)) - (a^(2/3)*(-20*b^3*c + 44*a*b^2*d - 77*a^2*b*e + 119*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(20/3)) - (a^(2/3)*(-20*b^3*c + 44*a*b^2*d - 77*a^2*b*e + 119*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(20/3)) + (a^(2/3)*(-20*b^3*c + 44*a*b^2*d - 77*a^2*b*e + 119*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(20/3))

Maple [B] time = 0.013, size = 668, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^10*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)

[Out] -17/18*a^3/b^4/(b*x^3+a)^2*x^2*d+11/18*a^2/b^3/(b*x^3+a)^2*x^2*c+7/9*a/b^2/(b*x^3+a)^2*x^5*c-29/18*a^5/b^6/(b*x^3+a)^2*x^2*f+23/18*a^4/b^5/(b*x^3+a)^2*x^2*e-10/27*a/b^4*c/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-119/27*a^4/b^7*f/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+119/54*a^4/b^7*f/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+77/27*a^3/b^6*e/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-77/54*a^3/b^6*e/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-44/27*a^2/b^5*d/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-16/9*a^4/b^5/(b*x^3+a)^2*x^5*f+13/9*a^3/b^4/(b*x^3+a)^2*x^5*e-10/9*a^2/b^3/(b*x^3+a)^2*x^5*d-77/27*a^3/b^6*e*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/11*f*x^11/b^3+3/b^5*x^2*a^2*e-3/2/b^4*x^2*a*d-3/8/b^4*x^8*a*f+6/5/b^5*x^5*a^2*f-3/5/b^4*x^5*a*e-5/b^6*x^2*a^3*f+22/27*a^2/b^5*d/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+20/27*a/b^4*c/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))

$$\frac{1}{b^2 a^{1/3}} + \frac{1}{8 b^3 x^8 e} + \frac{1}{5 b^3 x^5 d} + \frac{1}{2 b^3 x^2 c} + \frac{44}{27} \frac{a^2}{b^5 d} 3^{\left(\frac{1}{2}\right)} / \left(\frac{1}{b^2 a^{1/3}}\right) \arctan\left(\frac{1}{3} 3^{\left(\frac{1}{2}\right)} \left(\frac{2}{(1/b^2 a^{1/3}) x - 1}\right)\right) - \frac{20}{27} \frac{a}{b^4 c} 3^{\left(\frac{1}{2}\right)} / \left(\frac{1}{b^2 a^{1/3}}\right) \arctan\left(\frac{1}{3} 3^{\left(\frac{1}{2}\right)} \left(\frac{2}{(1/b^2 a^{1/3}) x - 1}\right)\right) + \frac{119}{27} \frac{a^4}{b^7 f} 3^{\left(\frac{1}{2}\right)} / \left(\frac{1}{b^2 a^{1/3}}\right) \arctan\left(\frac{1}{3} 3^{\left(\frac{1}{2}\right)} \left(\frac{2}{(1/b^2 a^{1/3}) x - 1}\right)\right)$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹⁰*(f*x⁹+e*x⁶+d*x³+c)/(b*x³+a)³,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.35236, size = 1496, normalized size = 3.9

$$1080 b^5 f x^{17} + 135 (11 b^5 e - 17 a b^4 f) x^{14} + 54 (44 b^5 d - 77 a b^4 e + 119 a^2 b^3 f) x^{11} + 297 (20 b^5 c - 44 a b^4 d + 77 a^2 b^3 e - 119 a^3 b^2 f) x^8 + 1056 (20 a b^4 c - 44 a^2 b^3 d + 77 a^3 b^2 e - 119 a^4 b f) x^5 + 660 (20 a^2 b^3 c - 44 a^3 b^2 d + 77 a^4 b e - 119 a^5 f) x^2 - 440 \sqrt{3} ((20 b^5 c - 44 a b^4 d + 77 a^2 b^3 e - 119 a^3 b^2 f) x^6 + 20 a^2 b^3 c - 44 a^3 b^2 d + 77 a^4 b e - 119 a^5 f + 2 (20 a b^4 c - 44 a^2 b^3 d + 77 a^3 b^2 e - 119 a^4 b f) x^3) (-a^2/b^2)^{1/3} \arctan\left(\frac{1}{3} (2 \sqrt{3} b x (-a^2/b^2)^{1/3} + \sqrt{3} a) / a\right) + 220 ((20 b^5 c - 44 a b^4 d + 77 a^2 b^3 e - 119 a^3 b^2 f) x^6 + 20 a^2 b^3 c - 44 a^3 b^2 d + 77 a^4 b e - 119 a^5 f + 2 (20 a b^4 c - 44 a^2 b^3 d + 77 a^3 b^2 e - 119 a^4 b f) x^3) (-a^2/b^2)^{1/3} \log(a x^2 - b x (-a^2/b^2)^{1/3} - a (-a^2/b^2)^{1/3}) - 440 ((20 b^5 c - 44 a b^4 d + 77 a^2 b^3 e - 119 a^3 b^2 f) x^6 + 20 a^2 b^3 c - 44 a^3 b^2 d + 77 a^4 b e - 119 a^5 f + 2 (20 a b^4 c - 44 a^2 b^3 d + 77 a^3 b^2 e - 119 a^4 b f) x^3) (-a^2/b^2)^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹⁰*(f*x⁹+e*x⁶+d*x³+c)/(b*x³+a)³,x, algorithm="fricas")

[Out] $\frac{1}{11880} (1080 b^5 f x^{17} + 135 (11 b^5 e - 17 a b^4 f) x^{14} + 54 (44 b^5 d - 77 a b^4 e + 119 a^2 b^3 f) x^{11} + 297 (20 b^5 c - 44 a b^4 d + 77 a^2 b^3 e - 119 a^3 b^2 f) x^8 + 1056 (20 a b^4 c - 44 a^2 b^3 d + 77 a^3 b^2 e - 119 a^4 b f) x^5 + 660 (20 a^2 b^3 c - 44 a^3 b^2 d + 77 a^4 b e - 119 a^5 f) x^2 - 440 \sqrt{3} ((20 b^5 c - 44 a b^4 d + 77 a^2 b^3 e - 119 a^3 b^2 f) x^6 + 20 a^2 b^3 c - 44 a^3 b^2 d + 77 a^4 b e - 119 a^5 f + 2 (20 a b^4 c - 44 a^2 b^3 d + 77 a^3 b^2 e - 119 a^4 b f) x^3) (-a^2/b^2)^{1/3} \arctan\left(\frac{1}{3} (2 \sqrt{3} b x (-a^2/b^2)^{1/3} + \sqrt{3} a) / a\right) + 220 ((20 b^5 c - 44 a b^4 d + 77 a^2 b^3 e - 119 a^3 b^2 f) x^6 + 20 a^2 b^3 c - 44 a^3 b^2 d + 77 a^4 b e - 119 a^5 f + 2 (20 a b^4 c - 44 a^2 b^3 d + 77 a^3 b^2 e - 119 a^4 b f) x^3) (-a^2/b^2)^{1/3} \log(a x^2 - b x (-a^2/b^2)^{1/3} - a (-a^2/b^2)^{1/3}) - 440 ((20 b^5 c - 44 a b^4 d + 77 a^2 b^3 e - 119 a^3 b^2 f) x^6 + 20 a^2 b^3 c - 44 a^3 b^2 d + 77 a^4 b e - 119 a^5 f + 2 (20 a b^4 c - 44 a^2 b^3 d + 77 a^3 b^2 e - 119 a^4 b f) x^3) (-a^2/b^2)^{1/3}$

$$x^6 + 20a^2b^3c - 44a^3b^2d + 77a^4b^2e - 119a^5f + 2(20ab^4c - 44a^2b^3d + 77a^3b^2e - 119a^4b^2f)x^3) \cdot (-a^2/b^2)^{(1/3)} \cdot \log(ax + b(-a^2/b^2)^{(2/3)}) / (b^8x^6 + 2ab^7x^3 + a^2b^6)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**10*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.09179, size = 663, normalized size = 1.73

$$\frac{\left(20ab^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 44a^2b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 119a^4f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 77a^3b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27ab^6} + \frac{\sqrt{3}\left(20(-ab^2)^{\frac{2}{3}}b^3c - 44a^2b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 119a^4f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 77a^3b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)}{27ab^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")

[Out] $\frac{1}{27} \cdot (20ab^3c \cdot (-a/b)^{(1/3)} - 44a^2b^2d \cdot (-a/b)^{(1/3)} - 119a^4f \cdot (-a/b)^{(1/3)} + 77a^3b \cdot (-a/b)^{(1/3)} e) \cdot (-a/b)^{(1/3)} \cdot \log(\text{abs}(x - (-a/b)^{(1/3)})) / (ab^6) + \frac{1}{27} \cdot \sqrt{3} \cdot (20 \cdot (-ab^2)^{(2/3)} \cdot b^3c - 44 \cdot (-ab^2)^{(2/3)} \cdot a^2b^2d - 119 \cdot (-ab^2)^{(2/3)} \cdot a^4f + 77 \cdot (-ab^2)^{(2/3)} \cdot a^2b^2e) \cdot \arctan(1/3 \cdot \sqrt{3} \cdot (2x + (-a/b)^{(1/3)}) / (-a/b)^{(1/3)}) / b^8 - \frac{1}{54} \cdot (20 \cdot (-ab^2)^{(2/3)} \cdot b^3c - 4 \cdot (-ab^2)^{(2/3)} \cdot a^2b^2d - 119 \cdot (-ab^2)^{(2/3)} \cdot a^4f + 77 \cdot (-ab^2)^{(2/3)} \cdot a^2b^2e) \cdot \log(x^2 + x \cdot (-a/b)^{(1/3)} + (-a/b)^{(2/3)}) / b^8 + \frac{1}{18} \cdot (14 \cdot a^2b^4c \cdot x^5 - 20 \cdot a^2b^3d \cdot x^5 - 32 \cdot a^4b^2f \cdot x^5 + 26 \cdot a^3b^2e \cdot x^5 + 11 \cdot a^2b^3c \cdot x^2 - 17 \cdot a^3b^2d \cdot x^2 - 29 \cdot a^5f \cdot x^2 + 23 \cdot a^4b^2e \cdot x^2) / ((b \cdot x^3 + a)^2 \cdot b^6) + \frac{1}{4} \cdot (40 \cdot b^30 \cdot f \cdot x^{11} - 165 \cdot a \cdot b^{29} \cdot f \cdot x^8 + 55 \cdot b^{30} \cdot x^8 \cdot e + 88 \cdot b^{30} \cdot d \cdot x^5 + 528 \cdot a^2 \cdot b^{28} \cdot f \cdot x^5 - 264 \cdot a \cdot b^{29} \cdot x^5 \cdot e + 220 \cdot b^{30} \cdot c \cdot x^2 - 660 \cdot a \cdot b^{29} \cdot d \cdot x^2 - 2200 \cdot a^3 \cdot b^{27} \cdot f \cdot x^2 + 1320 \cdot a^2 \cdot b^{28} \cdot x^2 \cdot e) / b^{33}$

$$3.288 \quad \int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=375

$$\frac{ax(25a^2be - 31a^3f - 19ab^2d + 13b^3c)}{18b^6(a+bx^3)} - \frac{a^2x(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^6(a+bx^3)^2} + \frac{\sqrt[3]{a} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(65a^2be - 54b^{19/3})}{54b^{19/3}}$$

[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x)/b^6 + ((b^2*d - 3*a*b*e + 6*a^2*f)*x^4)/(4*b^5) + ((b*e - 3*a*f)*x^7)/(7*b^4) + (f*x^10)/(10*b^3) - (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b^6*(a + b*x^3)^2) + (a*(13*b^3*c - 19*a*b^2*d + 25*a^2*b*e - 31*a^3*f)*x)/(18*b^6*(a + b*x^3)) + (a^(1/3)*(14*b^3*c - 35*a*b^2*d + 65*a^2*b*e - 104*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*b^(19/3)) - (a^(1/3)*(14*b^3*c - 35*a*b^2*d + 65*a^2*b*e - 104*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(19/3)) + (a^(1/3)*(14*b^3*c - 35*a*b^2*d + 65*a^2*b*e - 104*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(19/3))

Rubi [A] time = 0.606352, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {1828, 1858, 1887, 200, 31, 634, 617, 204, 628}

$$\frac{ax(25a^2be - 31a^3f - 19ab^2d + 13b^3c)}{18b^6(a+bx^3)} - \frac{a^2x(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^6(a+bx^3)^2} + \frac{\sqrt[3]{a} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(65a^2be - 54b^{19/3})}{54b^{19/3}}$$

Antiderivative was successfully verified.

[In] Int[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]

[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x)/b^6 + ((b^2*d - 3*a*b*e + 6*a^2*f)*x^4)/(4*b^5) + ((b*e - 3*a*f)*x^7)/(7*b^4) + (f*x^10)/(10*b^3) - (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b^6*(a + b*x^3)^2) + (a*(13*b^3*c - 19*a*b^2*d + 25*a^2*b*e - 31*a^3*f)*x)/(18*b^6*(a + b*x^3)) + (a^(1/3)*(14*b^3*c - 35*a*b^2*d + 65*a^2*b*e - 104*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*b^(19/3)) - (a^(1/3)*(14*b^3*c - 35*a*b^2*d + 65*a^2*b*e - 104*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(19/3)) + (a^(1/3)*(14*b^3*c - 35*a*b^2*d + 65*a^2*b*e - 104*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(19/3))

Rule 1828

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q =
m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
+ b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a +
b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 200

```
Int[((a_) + (b_)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(p_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9 (c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^3} dx &= -\frac{a^2 (b^3c - ab^2d + a^2be - a^3f) x}{6b^6 (a + bx^3)^2} - \int \frac{-a^3(b^3c - ab^2d + a^2be - a^3f) + 6a^2b(b^3c - ab^2d + a^2be - a^3f)x^3 - 6ab^2(b^3c - ab^2d + a^2be - a^3f)x^6 + a^3(b^3c - ab^2d + a^2be - a^3f)x^9}{(a + bx^3)^3} dx \\
&= -\frac{a^2 (b^3c - ab^2d + a^2be - a^3f) x}{6b^6 (a + bx^3)^2} + \frac{a (13b^3c - 19ab^2d + 25a^2be - 31a^3f) x}{18b^6 (a + bx^3)} + \int \frac{-2a^3b^5(5b^3c - 7ab^2d + 3a^2be - 3a^3f)}{(a + bx^3)^3} dx \\
&= -\frac{a^2 (b^3c - ab^2d + a^2be - a^3f) x}{6b^6 (a + bx^3)^2} + \frac{a (13b^3c - 19ab^2d + 25a^2be - 31a^3f) x}{18b^6 (a + bx^3)} + \int (18a^2b^5) \frac{1}{(a + bx^3)^3} dx \\
&= \frac{(b^3c - 3ab^2d + 6a^2be - 10a^3f) x}{b^6} + \frac{(b^2d - 3abe + 6a^2f) x^4}{4b^5} + \frac{(be - 3af)x^7}{7b^4} + \frac{fx^{10}}{10b^3} - \frac{a^3}{10b^3} \\
&= \frac{(b^3c - 3ab^2d + 6a^2be - 10a^3f) x}{b^6} + \frac{(b^2d - 3abe + 6a^2f) x^4}{4b^5} + \frac{(be - 3af)x^7}{7b^4} + \frac{fx^{10}}{10b^3} - \frac{a^3}{10b^3} \\
&= \frac{(b^3c - 3ab^2d + 6a^2be - 10a^3f) x}{b^6} + \frac{(b^2d - 3abe + 6a^2f) x^4}{4b^5} + \frac{(be - 3af)x^7}{7b^4} + \frac{fx^{10}}{10b^3} - \frac{a^3}{10b^3} \\
&= \frac{(b^3c - 3ab^2d + 6a^2be - 10a^3f) x}{b^6} + \frac{(b^2d - 3abe + 6a^2f) x^4}{4b^5} + \frac{(be - 3af)x^7}{7b^4} + \frac{fx^{10}}{10b^3} - \frac{a^3}{10b^3} \\
&= \frac{(b^3c - 3ab^2d + 6a^2be - 10a^3f) x}{b^6} + \frac{(b^2d - 3abe + 6a^2f) x^4}{4b^5} + \frac{(be - 3af)x^7}{7b^4} + \frac{fx^{10}}{10b^3} - \frac{a^3}{10b^3}
\end{aligned}$$

Mathematica [A] time = 0.37398, size = 362, normalized size = 0.97

$$\frac{210a \sqrt[3]{bx}(25a^2be - 31a^3f - 19ab^2d + 13b^3c)}{a + bx^3} + \frac{630a^2 \sqrt[3]{bx}(-a^2be + a^3f + ab^2d - b^3c)}{(a + bx^3)^2} - 70 \sqrt[3]{a} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3}x^2) (-65a^2be + 104a^3f + 3)$$

Antiderivative was successfully verified.

[In] Integrate[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]

```
[Out] (3780*b^(1/3)*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x + 945*b^(4/3)*(b
^2*d - 3*a*b*e + 6*a^2*f)*x^4 + 540*b^(7/3)*(b*e - 3*a*f)*x^7 + 378*b^(10/3
)*f*x^10 + (630*a^2*b^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(a +
b*x^3)^2 + (210*a*b^(1/3)*(13*b^3*c - 19*a*b^2*d + 25*a^2*b*e - 31*a^3*f)*x
)/(a + b*x^3) - 140*sqrt(3)*a^(1/3)*(-14*b^3*c + 35*a*b^2*d - 65*a^2*b*e +
104*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt(3)] + 140*a^(1/3)*(-14*b
^3*c + 35*a*b^2*d - 65*a^2*b*e + 104*a^3*f)*Log[a^(1/3) + b^(1/3)*x] - 70*a
^(1/3)*(-14*b^3*c + 35*a*b^2*d - 65*a^2*b*e + 104*a^3*f)*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2]/(3780*b^(19/3))
```

Maple [A] time = 0.012, size = 651, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] -3/7/b^4*x^7*a*f+3/2/b^5*x^4*a^2*f-3/4/b^4*x^4*a*e-10/b^6*a^3*f*x+6/b^5*a^2
*e*x-3/b^4*a*d*x-19/18*a^2/b^3/(b*x^3+a)^2*x^4*d+1/10*f*x^10/b^3-14/9*a^5/b
^6/(b*x^3+a)^2*f*x+11/9*a^4/b^5/(b*x^3+a)^2*e*x-8/9*a^3/b^4/(b*x^3+a)^2*d*x
+5/9*a^2/b^3/(b*x^3+a)^2*c*x+104/27*a^4/b^7*f/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1
/3))-65/27*a^3/b^6*e/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+65/54*a^3/b^6*e/(1/b
*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+35/27*a^2/b^5*d/(1/b*a)^(2/
3)*ln(x+(1/b*a)^(1/3))-35/54*a^2/b^5*d/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x
+(1/b*a)^(2/3))-14/27*a/b^4*c/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-31/18*a^4/b
^5/(b*x^3+a)^2*x^4*f+25/18*a^3/b^4/(b*x^3+a)^2*x^4*e-52/27*a^4/b^7*f/(1/b*a
)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/7/b^3*x^7*e+1/4/b^3*x^4*d+1
/b^3*c*x+7/27*a/b^4*c/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1
3/18*a/b^2/(b*x^3+a)^2*x^4*c+104/27*a^4/b^7*f/(1/b*a)^(2/3)*3^(1/2)*arctan(
1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-65/27*a^3/b^6*e/(1/b*a)^(2/3)*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+35/27*a^2/b^5*d/(1/b*a)^(2/3)*3^(1/
2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-14/27*a/b^4*c/(1/b*a)^(2/3)*3^(
1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.44258, size = 1424, normalized size = 3.8

$$378b^5fx^{16} + 108(5b^5e - 8ab^4f)x^{13} + 27(35b^5d - 65ab^4e + 104a^2b^3f)x^{10} + 270(14b^5c - 35ab^4d + 65a^2b^3e - 104a^3b^2f)x^7 + 735(14ab^4c - 35a^2b^3d + 65a^3b^2e - 104a^4b^1f)x^4 - 140\sqrt{3}((14b^5c - 35ab^4d + 65a^2b^3e - 104a^3b^2f)x^6 + 14a^2b^3c - 35a^3b^2d + 65a^4b^1e - 104a^5f + 2(14ab^4c - 35a^2b^3d + 65a^3b^2e - 104a^4b^1f)x^3)(a/b)^{1/3}\arctan(1/3(2\sqrt{3}bx(a/b)^{2/3} - \sqrt{3}a)/a) + 70((14b^5c - 35ab^4d + 65a^2b^3e - 104a^3b^2f)x^6 + 14a^2b^3c - 35a^3b^2d + 65a^4b^1e - 104a^5f + 2(14ab^4c - 35a^2b^3d + 65a^3b^2e - 104a^4b^1f)x^3)(a/b)^{1/3}\log(x^2 - x(a/b)^{1/3} + (a/b)^{2/3}) - 140((14b^5c - 35ab^4d + 65a^2b^3e - 104a^3b^2f)x^6 + 14a^2b^3c - 35a^3b^2d + 65a^4b^1e - 104a^5f + 2(14ab^4c - 35a^2b^3d + 65a^3b^2e - 104a^4b^1f)x^3)(a/b)^{1/3}\log(x + (a/b)^{1/3}) + 420(14a^2b^3c - 35a^3b^2d + 65a^4b^1e - 104a^5f)x/(b^8x^6 + 2ab^7x^3 + a^2b^6)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3780*(378*b^5*f*x^16 + 108*(5*b^5*e - 8*a*b^4*f)*x^13 + 27*(35*b^5*d - 65*a*b^4*e + 104*a^2*b^3*f)*x^10 + 270*(14*b^5*c - 35*a*b^4*d + 65*a^2*b^3*e - 104*a^3*b^2*f)*x^7 + 735*(14*a*b^4*c - 35*a^2*b^3*d + 65*a^3*b^2*e - 104*a^4*b^1*f)*x^4 - 140*sqrt(3)*((14*b^5*c - 35*a*b^4*d + 65*a^2*b^3*e - 104*a^3*b^2*f)*x^6 + 14*a^2*b^3*c - 35*a^3*b^2*d + 65*a^4*b^1*e - 104*a^5*f + 2*(14*a*b^4*c - 35*a^2*b^3*d + 65*a^3*b^2*e - 104*a^4*b^1*f)*x^3)*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 70*((14*b^5*c - 35*a*b^4*d + 65*a^2*b^3*e - 104*a^3*b^2*f)*x^6 + 14*a^2*b^3*c - 35*a^3*b^2*d + 65*a^4*b^1*e - 104*a^5*f + 2*(14*a*b^4*c - 35*a^2*b^3*d + 65*a^3*b^2*e - 104*a^4*b^1*f)*x^3)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 140*((14*b^5*c - 35*a*b^4*d + 65*a^2*b^3*e - 104*a^3*b^2*f)*x^6 + 14*a^2*b^3*c - 35*a^3*b^2*d + 65*a^4*b^1*e - 104*a^5*f + 2*(14*a*b^4*c - 35*a^2*b^3*d + 65*a^3*b^2*e - 104*a^4*b^1*f)*x^3)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 420*(14*a^2*b^3*c - 35*a^3*b^2*d + 65*a^4*b^1*e - 104*a^5*f)*x/(b^8*x^6 + 2*a*b^7*x^3 + a^2*b^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

[Out] Timed out

Giac [A] time = 1.08754, size = 598, normalized size = 1.59

$$\frac{\sqrt{3} \left(14 (-ab^2)^{\frac{1}{3}} b^3 c - 35 (-ab^2)^{\frac{1}{3}} ab^2 d - 104 (-ab^2)^{\frac{1}{3}} a^3 f + 65 (-ab^2)^{\frac{1}{3}} a^2 b e \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}} \right)}{27 b^7} + \frac{(14 ab^3 c - 35 a^2 b^2 d - 104 a^3 f + 65 a^2 b e) \log \left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}} \right)}{b^7} + \frac{1}{18} \frac{(13 a^4 b^4 c x^4 - 19 a^4 b^3 d x^4 - 31 a^4 b^2 f x^4 + 25 a^4 b x^4 e + 10 a^4 x^4 e + 10 a^3 b^3 c x - 16 a^3 b^2 d x - 28 a^3 b f x + 22 a^3 b x e)}{(b x^3 + a)^2 b^6} + \frac{1}{140} \frac{(14 b^27 f x^{10} - 60 a b^{26} f x^7 + 20 b^{27} x^7 e + 35 b^{27} d x^4 + 210 a^2 b^{25} f x^4 - 105 a b^{26} x^4 e + 140 b^{27} c x - 420 a b^{26} d x - 1400 a^3 b^{24} f x + 840 a^2 b^{25} x e)}{b^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")

[Out]
$$-1/27*\sqrt{3}*(14*(-a*b^2)^{(1/3)}*b^3*c - 35*(-a*b^2)^{(1/3)}*a*b^2*d - 104*(-a*b^2)^{(1/3)}*a^3*f + 65*(-a*b^2)^{(1/3)}*a^2*b*e)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/b^7 + 1/27*(14*a*b^3*c - 35*a^2*b^2*d - 104*a^4*f + 65*a^3*b*e)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/ (a*b^6) - 1/54*(14*(-a*b^2)^{(1/3)}*b^3*c - 35*(-a*b^2)^{(1/3)}*a*b^2*d - 104*(-a*b^2)^{(1/3)}*a^3*f + 65*(-a*b^2)^{(1/3)}*a^2*b*e)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/b^7 + 1/18*(13*a*b^4*c*x^4 - 19*a^2*b^3*d*x^4 - 31*a^4*b*f*x^4 + 25*a^3*b^2*x^4*e + 10*a^2*b^3*c*x - 16*a^3*b^2*d*x - 28*a^5*f*x + 22*a^4*b*x*e)/((b*x^3 + a)^2*b^6) + 1/140*(14*b^27*f*x^{10} - 60*a*b^{26}*f*x^7 + 20*b^{27}*x^7*e + 35*b^{27}*d*x^4 + 210*a^2*b^{25}*f*x^4 - 105*a*b^{26}*x^4*e + 140*b^{27}*c*x - 420*a*b^{26}*d*x - 1400*a^3*b^{24}*f*x + 840*a^2*b^{25}*x*e)/b^{30}$$

$$3.289 \quad \int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=345

$$-\frac{x^2(10a^2be - 13a^3f - 7ab^2d + 4b^3c)}{9b^5(a+bx^3)} + \frac{ax^2(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^5(a+bx^3)^2} + \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(44a^2be - 77a^3f)}{54\sqrt[3]{ab^{17/3}}}$$

[Out] $((b^2d - 3a*b*e + 6*a^2*f)*x^2)/(2*b^5) + ((b*e - 3*a*f)*x^5)/(5*b^4) + (f*x^8)/(8*b^3) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*b^5*(a + b*x^3)^2) - ((4*b^3*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^3*f)*x^2)/(9*b^5*(a + b*x^3)) - ((5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*ArcTan[a^(1/3) - 2*b^(1/3)*x]/(Sqrt[3]*a^(1/3)))/(9*Sqrt[3]*a^(1/3)*b^(17/3)) - ((5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*Log[a^(1/3) + b^(1/3)*x]/(27*a^(1/3)*b^(17/3)) + ((5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(54*a^(1/3)*b^(17/3)))$

Rubi [A] time = 0.763222, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1828, 1851, 1836, 1488, 292, 31, 634, 617, 204, 628}

$$-\frac{x^2(10a^2be - 13a^3f - 7ab^2d + 4b^3c)}{9b^5(a+bx^3)} + \frac{ax^2(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^5(a+bx^3)^2} + \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(44a^2be - 77a^3f)}{54\sqrt[3]{ab^{17/3}}}$$

Antiderivative was successfully verified.

[In] Int[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]

[Out] $((b^2d - 3a*b*e + 6*a^2*f)*x^2)/(2*b^5) + ((b*e - 3*a*f)*x^5)/(5*b^4) + (f*x^8)/(8*b^3) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*b^5*(a + b*x^3)^2) - ((4*b^3*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^3*f)*x^2)/(9*b^5*(a + b*x^3)) - ((5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*ArcTan[a^(1/3) - 2*b^(1/3)*x]/(Sqrt[3]*a^(1/3)))/(9*Sqrt[3]*a^(1/3)*b^(17/3)) - ((5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*Log[a^(1/3) + b^(1/3)*x]/(27*a^(1/3)*b^(17/3)) + ((5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(54*a^(1/3)*b^(17/3)))$

Rule 1828


```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q =
m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
+ b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1851

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[x*PolynomialQuot
ient[Pq, x, x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x]
&& EqQ[Coeff[Pq, x, 0], 0] && !MatchQ[Pq, x^(m_)*(u_)] /; IntegerQ[m]]
```

Rule 1836

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
+ 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
+ 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1488

```
Int[((f_)*(x_))^(m_)*((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_))^(p_)*((
d_) + (e_)*(x_)^(n_))^(q_), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(p_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7(c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^3} dx &= \frac{a(b^3c - ab^2d + a^2be - a^3f)x^2}{6b^5(a + bx^3)^2} - \frac{\int \frac{2a^2b(b^3c - ab^2d + a^2be - a^3f)x - 6ab^2(b^3c - ab^2d + a^2be - a^3f)x^4 - 6ab^3(b^3c - ab^2d + a^2be - a^3f)x^7}{(a + bx^3)^2} dx}{6ab^6} \\
&= \frac{a(b^3c - ab^2d + a^2be - a^3f)x^2}{6b^5(a + bx^3)^2} - \frac{\int \frac{x(2a^2b(b^3c - ab^2d + a^2be - a^3f) - 6ab^2(b^3c - ab^2d + a^2be - a^3f)x^3 - 6ab^3(b^3c - ab^2d + a^2be - a^3f)x^6)}{(a + bx^3)^2} dx}{6ab^6} \\
&= \frac{a(b^3c - ab^2d + a^2be - a^3f)x^2}{6b^5(a + bx^3)^2} - \frac{(4b^3c - 7ab^2d + 10a^2be - 13a^3f)x^2}{9b^5(a + bx^3)} + \frac{\int \frac{2a^2b^6(5b^3c - 13b^2d + 10abe - 13a^3f)}{(a + bx^3)^2} dx}{6ab^6} \\
&= \frac{a(b^3c - ab^2d + a^2be - a^3f)x^2}{6b^5(a + bx^3)^2} - \frac{(4b^3c - 7ab^2d + 10a^2be - 13a^3f)x^2}{9b^5(a + bx^3)} + \frac{\int \frac{x(2a^2b^6(5b^3c - 13b^2d + 10abe - 13a^3f))}{(a + bx^3)^2} dx}{6ab^6} \\
&= \frac{fx^8}{8b^3} + \frac{a(b^3c - ab^2d + a^2be - a^3f)x^2}{6b^5(a + bx^3)^2} - \frac{(4b^3c - 7ab^2d + 10a^2be - 13a^3f)x^2}{9b^5(a + bx^3)} + \frac{\int \frac{x(16a^2b^6(5b^3c - 13b^2d + 10abe - 13a^3f))}{(a + bx^3)^2} dx}{6ab^6} \\
&= \frac{fx^8}{8b^3} + \frac{a(b^3c - ab^2d + a^2be - a^3f)x^2}{6b^5(a + bx^3)^2} - \frac{(4b^3c - 7ab^2d + 10a^2be - 13a^3f)x^2}{9b^5(a + bx^3)} + \frac{\int (14a^2b^6(5b^3c - 13b^2d + 10abe - 13a^3f))}{6ab^6} \\
&= \frac{(b^2d - 3abe + 6a^2f)x^2}{2b^5} + \frac{(be - 3af)x^5}{5b^4} + \frac{fx^8}{8b^3} + \frac{a(b^3c - ab^2d + a^2be - a^3f)x^2}{6b^5(a + bx^3)^2} - \frac{(4b^3c - 7ab^2d + 10a^2be - 13a^3f)x^2}{9b^5(a + bx^3)} \\
&= \frac{(b^2d - 3abe + 6a^2f)x^2}{2b^5} + \frac{(be - 3af)x^5}{5b^4} + \frac{fx^8}{8b^3} + \frac{a(b^3c - ab^2d + a^2be - a^3f)x^2}{6b^5(a + bx^3)^2} - \frac{(4b^3c - 7ab^2d + 10a^2be - 13a^3f)x^2}{9b^5(a + bx^3)} \\
&= \frac{(b^2d - 3abe + 6a^2f)x^2}{2b^5} + \frac{(be - 3af)x^5}{5b^4} + \frac{fx^8}{8b^3} + \frac{a(b^3c - ab^2d + a^2be - a^3f)x^2}{6b^5(a + bx^3)^2} - \frac{(4b^3c - 7ab^2d + 10a^2be - 13a^3f)x^2}{9b^5(a + bx^3)} \\
&= \frac{(b^2d - 3abe + 6a^2f)x^2}{2b^5} + \frac{(be - 3af)x^5}{5b^4} + \frac{fx^8}{8b^3} + \frac{a(b^3c - ab^2d + a^2be - a^3f)x^2}{6b^5(a + bx^3)^2} - \frac{(4b^3c - 7ab^2d + 10a^2be - 13a^3f)x^2}{9b^5(a + bx^3)} \\
&= \frac{(b^2d - 3abe + 6a^2f)x^2}{2b^5} + \frac{(be - 3af)x^5}{5b^4} + \frac{fx^8}{8b^3} + \frac{a(b^3c - ab^2d + a^2be - a^3f)x^2}{6b^5(a + bx^3)^2} - \frac{(4b^3c - 7ab^2d + 10a^2be - 13a^3f)x^2}{9b^5(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.223697, size = 329, normalized size = 0.95

$$\frac{-\frac{120b^{2/3}x^2(10a^2be-13a^3f-7ab^2d+4b^3c)}{a+bx^3} + \frac{180ab^{2/3}x^2(a^2be+a^3(-f)-ab^2d+b^3c)}{(a+bx^3)^2} + \frac{20\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx+b^{2/3}x^2}\right)(44a^2be-77a^3f-20ab^2d+5b^3c)}{\sqrt[3]{a}} + \frac{40\log\left(\sqrt[3]{\frac{bx+b^{2/3}x^2}{a}}\right)}{\sqrt[3]{a}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]

[Out] (540*b^(2/3)*(b^2*d - 3*a*b*e + 6*a^2*f)*x^2 + 216*b^(5/3)*(b*e - 3*a*f)*x^5 + 135*b^(8/3)*f*x^8 + (180*a*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(a + b*x^3)^2 - (120*b^(2/3)*(4*b^3*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^3*f)*x^2)/(a + b*x^3) + (40*sqrt(3)*(-5*b^3*c + 20*a*b^2*d - 44*a^2*b*e + 77*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt(3)])/a^(1/3) + (40*(-5*b^3*c + 20*a*b^2*d - 44*a^2*b*e + 77*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + (20*(5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(1/3))/(1080*b^(17/3))

Maple [B] time = 0.013, size = 611, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)

[Out] -3/2/b^4*x^2*a*e-4/9/b/(b*x^3+a)^2*x^5*c-5/27/b^3*c/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+5/54/b^3*c/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-3/5/b^4*x^5*a*f+3/b^5*x^2*a^2*f+1/8*f*x^8/b^3-5/18/b^2/(b*x^3+a)^2*x^2*a*c+1/18/b^3/(b*x^3+a)^2*x^2*a^2*d+1/5/b^3*x^5*e+1/2/b^3*x^2*d-17/18/b^4/(b*x^3+a)^2*x^2*a^3*e+23/18/b^5/(b*x^3+a)^2*x^2*a^4*f+13/9/b^4/(b*x^3+a)^2*x^5*f*a^3-10/9/b^3/(b*x^3+a)^2*x^5*a^2*e+7/9/b^2/(b*x^3+a)^2*x^5*a*d+5/27/b^3*c^3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-10/27/b^4*a*d/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+77/27/b^6*a^3*f/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-77/54/b^6*a^3*f/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-44/27/b^5*a^2*e/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+22/27/b^5*a^2*e/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+20/27/b^4*a*d/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-77/27/b^6*a^3*f*3^(1/2)/(1/b*a)^(1/3)

```
*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+44/27/b^5*a^2*e*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-20/27/b^4*a*d*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.48138, size = 2915, normalized size = 8.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/1080*(135*a*b^6*f*x^14 + 54*(4*a*b^6*e - 7*a^2*b^5*f)*x^11 + 27*(20*a*b^6*d - 44*a^2*b^5*e + 77*a^3*b^4*f)*x^8 - 96*(5*a*b^6*c - 20*a^2*b^5*d + 44*a^3*b^4*e - 77*a^4*b^3*f)*x^5 - 60*(5*a^2*b^5*c - 20*a^3*b^4*d + 44*a^4*b^3*e - 77*a^5*b^2*f)*x^2 - 60*sqrt(1/3)*(5*a^3*b^4*c - 20*a^4*b^3*d + 44*a^5*b^2*e - 77*a^6*b*f + (5*a*b^6*c - 20*a^2*b^5*d + 44*a^3*b^4*e - 77*a^4*b^3*f)*x^6 + 2*(5*a^2*b^5*c - 20*a^3*b^4*d + 44*a^4*b^3*e - 77*a^5*b^2*f)*x^3)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) + 20*((5*b^5*c - 20*a*b^4*d + 44*a^2*b^3*e - 77*a^3*b^2*f)*x^6 + 5*a^2*b^3*c - 20*a^3*b^2*d + 44*a^4*b*e - 77*a^5*f + 2*(5*a*b^4*c - 20*a^2*b^3*d + 44*a^3*b^2*e - 77*a^4*b*f)*x^3)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 40*((5*b^5*c - 20*a*b^4*d + 44*a^2*b^3*e - 77*a^3*b^2*f)*x^6 + 5*a^2*b^3*c - 20*a^3*b^2*d + 44*a^4*b*e - 77*a^5*f + 2*(5*a*b^4*c - 20*a^2*b^3*d + 44*a^3*b^2*e - 77*a^4*b*f)*x^3)*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a*b^9*x^6 + 2*a^2*b^8*x^3 + a^3*b^7), 1/1080*(135*a*b^6*f*x^14 + 54*(4*a*b^6*e - 7*a^2*b^5*f)*x^11 + 27*(20*a*b^6*d - 44*a^2*b^5*e + 77*a^3*b^4*f)*x^8 - 96*(5*a*b^6*c - 20*a^2*b^5*d + 44*a^3*b^4*e - 77*a^4*b^3*f)*x^5 - 60*(5*a^2*b^5*c - 20*a^3*b^4*d + 44*a^4*b^3*e - 77*a^5
```

$$\begin{aligned}
& *b^2*f)*x^2 - 120*\sqrt{1/3}*(5*a^3*b^4*c - 20*a^4*b^3*d + 44*a^5*b^2*e - 77 \\
& *a^6*b*f + (5*a*b^6*c - 20*a^2*b^5*d + 44*a^3*b^4*e - 77*a^4*b^3*f)*x^6 + 2 \\
& *(5*a^2*b^5*c - 20*a^3*b^4*d + 44*a^4*b^3*e - 77*a^5*b^2*f)*x^3)*\sqrt{(a*b^2)^{1/3}/a} \\
& *\arctan(-\sqrt{1/3}*(2*b*x - (a*b^2)^{1/3}))*\sqrt{(a*b^2)^{1/3}/a} \\
& /b) + 20*((5*b^5*c - 20*a*b^4*d + 44*a^2*b^3*e - 77*a^3*b^2*f)*x^6 + 5*a^2*b^3*c \\
& - 20*a^3*b^2*d + 44*a^4*b*e - 77*a^5*f + 2*(5*a*b^4*c - 20*a^2*b^3*d \\
& + 44*a^3*b^2*e - 77*a^4*b*f)*x^3)*(a*b^2)^{2/3}*\log(b^2*x^2 - (a*b^2)^{1/3} \\
& *b*x + (a*b^2)^{2/3}) - 40*((5*b^5*c - 20*a*b^4*d + 44*a^2*b^3*e - 77*a^3*b^2*f)*x^6 \\
& + 5*a^2*b^3*c - 20*a^3*b^2*d + 44*a^4*b*e - 77*a^5*f + 2*(5*a*b^4*c \\
& *c - 20*a^2*b^3*d + 44*a^3*b^2*e - 77*a^4*b*f)*x^3)*(a*b^2)^{2/3}*\log(b*x + \\
& (a*b^2)^{1/3}))/((a*b^9*x^6 + 2*a^2*b^8*x^3 + a^3*b^7)]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.09243, size = 601, normalized size = 1.74

$$\frac{\left(5b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 20ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 77a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 44a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right) + \sqrt{3}\left(5(-ab^2)^{\frac{2}{3}}b^3c - 20(-a\right)}{27ab^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")

[Out]
$$\begin{aligned}
& -1/27*(5*b^3*c*(-a/b)^{1/3} - 20*a*b^2*d*(-a/b)^{1/3} - 77*a^3*f*(-a/b)^{1/3} \\
& + 44*a^2*b*(-a/b)^{1/3}*e)*(-a/b)^{1/3}*\log(\text{abs}(x - (-a/b)^{1/3}))/((a*b^5 \\
& - 1/27*\sqrt{3}*(5*(-a*b^2)^{2/3}*b^3*c - 20*(-a*b^2)^{2/3}*a*b^2*d - 77* \\
& (-a*b^2)^{2/3}*a^3*f + 44*(-a*b^2)^{2/3}*a^2*b*e))*\arctan(1/3*\sqrt{3}*(2*x + \\
& (-a/b)^{1/3}))/(-a/b)^{1/3}))/((a*b^7) - 1/18*(8*b^4*c*x^5 - 14*a*b^3*d*x^5 -
\end{aligned}$$

$$\frac{26a^3bf^5x^5 + 20a^2b^2x^5e + 5a^3c^2x^2 - 11a^2b^2dx^2 - 23a^4f^2x^2 + 17a^3bx^2e}{(bx^3 + a)^2b^5} + \frac{1}{54} \frac{5(-ab^2)^{2/3}b^3c - 20(-ab^2)^{2/3}ab^2d - 77(-ab^2)^{2/3}a^3f + 44(-ab^2)^{2/3}a^2be}{(a^2b^7)} \log(x^2 + x(-a/b)^{1/3} + (-a/b)^{2/3}) + \frac{1}{40} \frac{5b^{21}f^8 - 24ab^{20}f^5x^5 + 8b^{21}x^5e + 20b^{21}dx^2 + 120a^2b^{19}f^2x^2 - 60ab^{20}x^2e}{b^{24}}$$

$$3.290 \quad \int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=336

$$-\frac{x(19a^2be - 25a^3f - 13ab^2d + 7b^3c)}{18b^5(a+bx^3)} + \frac{ax(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^5(a+bx^3)^2} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(35a^2be - 65a^3f)}{54a^{2/3}b^{16/3}}$$

[Out] ((b^2*d - 3*a*b*e + 6*a^2*f)*x)/b^5 + ((b*e - 3*a*f)*x^4)/(4*b^4) + (f*x^7)/(7*b^3) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b^5*(a + b*x^3)^2) - ((7*b^3*c - 13*a*b^2*d + 19*a^2*b*e - 25*a^3*f)*x)/(18*b^5*(a + b*x^3)) - ((2*b^3*c - 14*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(2/3)*b^(16/3)) + ((2*b^3*c - 14*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(2/3)*b^(16/3)) - ((2*b^3*c - 14*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(2/3)*b^(16/3))

Rubi [A] time = 0.508736, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {1828, 1858, 1887, 200, 31, 634, 617, 204, 628}

$$-\frac{x(19a^2be - 25a^3f - 13ab^2d + 7b^3c)}{18b^5(a+bx^3)} + \frac{ax(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^5(a+bx^3)^2} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(35a^2be - 65a^3f)}{54a^{2/3}b^{16/3}}$$

Antiderivative was successfully verified.

[In] Int[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]

[Out] ((b^2*d - 3*a*b*e + 6*a^2*f)*x)/b^5 + ((b*e - 3*a*f)*x^4)/(4*b^4) + (f*x^7)/(7*b^3) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b^5*(a + b*x^3)^2) - ((7*b^3*c - 13*a*b^2*d + 19*a^2*b*e - 25*a^3*f)*x)/(18*b^5*(a + b*x^3)) - ((2*b^3*c - 14*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(2/3)*b^(16/3)) + ((2*b^3*c - 14*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(2/3)*b^(16/3)) - ((2*b^3*c - 14*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(2/3)*b^(16/3))

Rule 1828


```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q =
m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
+ b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a +
b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 200

```
Int[((a_) + (b_)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(p_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
 implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6(c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^3} dx &= \frac{a(b^3c - ab^2d + a^2be - a^3f)x}{6b^5(a + bx^3)^2} - \frac{\int \frac{a^2(b^3c - ab^2d + a^2be - a^3f) - 6ab(b^3c - ab^2d + a^2be - a^3f)x^3 - 6a^2(b^2d - ab^2c + ab^2e - a^3f)x^6}{(a + bx^3)^2} dx}{6ab^5} \\
&= \frac{a(b^3c - ab^2d + a^2be - a^3f)x}{6b^5(a + bx^3)^2} - \frac{(7b^3c - 13ab^2d + 19a^2be - 25a^3f)x}{18b^5(a + bx^3)} + \frac{\int \frac{2a^2b^4(2b^3c - 5ab^2d + 5a^2be - a^3f)}{(a + bx^3)^2} dx}{6ab^5} \\
&= \frac{a(b^3c - ab^2d + a^2be - a^3f)x}{6b^5(a + bx^3)^2} - \frac{(7b^3c - 13ab^2d + 19a^2be - 25a^3f)x}{18b^5(a + bx^3)} + \frac{\int (18a^2b^4(b^2d - ab^2c + ab^2e - a^3f)) dx}{6ab^5} \\
&= \frac{(b^2d - 3abe + 6a^2f)x}{b^5} + \frac{(be - 3af)x^4}{4b^4} + \frac{fx^7}{7b^3} + \frac{a(b^3c - ab^2d + a^2be - a^3f)x}{6b^5(a + bx^3)^2} - \frac{(7b^3c - 13ab^2d + 19a^2be - 25a^3f)x}{18b^5(a + bx^3)} \\
&= \frac{(b^2d - 3abe + 6a^2f)x}{b^5} + \frac{(be - 3af)x^4}{4b^4} + \frac{fx^7}{7b^3} + \frac{a(b^3c - ab^2d + a^2be - a^3f)x}{6b^5(a + bx^3)^2} - \frac{(7b^3c - 13ab^2d + 19a^2be - 25a^3f)x}{18b^5(a + bx^3)} \\
&= \frac{(b^2d - 3abe + 6a^2f)x}{b^5} + \frac{(be - 3af)x^4}{4b^4} + \frac{fx^7}{7b^3} + \frac{a(b^3c - ab^2d + a^2be - a^3f)x}{6b^5(a + bx^3)^2} - \frac{(7b^3c - 13ab^2d + 19a^2be - 25a^3f)x}{18b^5(a + bx^3)} \\
&= \frac{(b^2d - 3abe + 6a^2f)x}{b^5} + \frac{(be - 3af)x^4}{4b^4} + \frac{fx^7}{7b^3} + \frac{a(b^3c - ab^2d + a^2be - a^3f)x}{6b^5(a + bx^3)^2} - \frac{(7b^3c - 13ab^2d + 19a^2be - 25a^3f)x}{18b^5(a + bx^3)} \\
&= \frac{(b^2d - 3abe + 6a^2f)x}{b^5} + \frac{(be - 3af)x^4}{4b^4} + \frac{fx^7}{7b^3} + \frac{a(b^3c - ab^2d + a^2be - a^3f)x}{6b^5(a + bx^3)^2} - \frac{(7b^3c - 13ab^2d + 19a^2be - 25a^3f)x}{18b^5(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.313184, size = 323, normalized size = 0.96

$$-\frac{42\sqrt[3]{bx}(19a^2be - 25a^3f - 13ab^2d + 7b^3c)}{a + bx^3} + \frac{126a\sqrt[3]{bx}(a^2be + a^3(-f) - ab^2d + b^3c)}{(a + bx^3)^2} + \frac{14\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx + b^{2/3}x^2})(-35a^2be + 65a^3f + 14ab^2d - 2b^3c)}{a^{2/3}} + \frac{28\log(\sqrt[3]{a})}{a}$$

Antiderivative was successfully verified.

[In] Integrate[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]

```
[Out] (756*b^(1/3)*(b^2*d - 3*a*b*e + 6*a^2*f)*x + 189*b^(4/3)*(b*e - 3*a*f)*x^4
+ 108*b^(7/3)*f*x^7 + (126*a*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)
/(a + b*x^3)^2 - (42*b^(1/3)*(7*b^3*c - 13*a*b^2*d + 19*a^2*b*e - 25*a^3*f)
*x)/(a + b*x^3) + (28*sqrt(3)*(-2*b^3*c + 14*a*b^2*d - 35*a^2*b*e + 65*a^3*
f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt(3)])/a^(2/3) + (28*(2*b^3*c - 14
*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (14*(
-2*b^3*c + 14*a*b^2*d - 35*a^2*b*e + 65*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/a^(2/3))/(756*b^(16/3))
```

Maple [B] time = 0.012, size = 596, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] -14/27/b^4*a*d/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+7/27/b^4*a*d/(1/b*a)^(2/3)
*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2/27/b^3*c/(1/b*a)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+13/18/b^2/(b*x^3+a)^2*x^4*a*d-1/27/b
^3*c/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-3/4/b^4*x^4*a*f+6/
b^5*a^2*f*x-3/b^4*a*e*x-7/18/b/(b*x^3+a)^2*x^4*c+2/27/b^3*c/(1/b*a)^(2/3)*l
n(x+(1/b*a)^(1/3))+1/7*f*x^7/b^3+1/4/b^3*x^4*e+1/b^3*d*x+35/27/b^5*a^2*e/(1
/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-35/54/b^5*a^2*e/(1/b*a)^(2/3)*ln(x^2-(1/b*a
)^(1/3)*x+(1/b*a)^(2/3))-2/9/b^2/(b*x^3+a)^2*a*c*x+25/18/b^4/(b*x^3+a)^2*x^
4*a^3*f-19/18/b^3/(b*x^3+a)^2*x^4*a^2*e-65/27/b^6*a^3*f/(1/b*a)^(2/3)*ln(x+
(1/b*a)^(1/3))+65/54/b^6*a^3*f/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)
^(2/3))+5/9/b^3/(b*x^3+a)^2*a^2*d*x+11/9/b^5/(b*x^3+a)^2*a^4*f*x-8/9/b^4/(b
*x^3+a)^2*a^3*e*x-65/27/b^6*a^3*f/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*
(2/(1/b*a)^(1/3)*x-1))+35/27/b^5*a^2*e/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(
1/2)*(2/(1/b*a)^(1/3)*x-1))-14/27/b^4*a*d/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*
3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [B] time = 1.46143, size = 2986, normalized size = 8.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] [1/756*(108*a^2*b^5*f*x^13 + 27*(7*a^2*b^5*e - 13*a^3*b^4*f)*x^10 + 54*(14*a^2*b^5*d - 35*a^3*b^4*e + 65*a^4*b^3*f)*x^7 - 147*(2*a^2*b^5*c - 14*a^3*b^4*d + 35*a^4*b^3*e - 65*a^5*b^2*f)*x^4 - 42*sqrt(1/3)*(2*a^3*b^4*c - 14*a^4*b^3*d + 35*a^5*b^2*e - 65*a^6*b*f + (2*a*b^6*c - 14*a^2*b^5*d + 35*a^3*b^4*e - 65*a^4*b^3*f)*x^6 + 2*(2*a^2*b^5*c - 14*a^3*b^4*d + 35*a^4*b^3*e - 65*a^5*b^2*f)*x^3)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 14*((2*b^5*c - 14*a*b^4*d + 35*a^2*b^3*e - 65*a^3*b^2*f)*x^6 + 2*a^2*b^3*c - 14*a^3*b^2*d + 35*a^4*b*e - 65*a^5*f + 2*(2*a*b^4*c - 14*a^2*b^3*d + 35*a^3*b^2*e - 65*a^4*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*((2*b^5*c - 14*a*b^4*d + 35*a^2*b^3*e - 65*a^3*b^2*f)*x^6 + 2*a^2*b^3*c - 14*a^3*b^2*d + 35*a^4*b*e - 65*a^5*f + 2*(2*a*b^4*c - 14*a^2*b^3*d + 35*a^3*b^2*e - 65*a^4*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) - 84*(2*a^3*b^4*c - 14*a^4*b^3*d + 35*a^5*b^2*e - 65*a^6*b*f)*x)/(a^2*b^8*x^6 + 2*a^3*b^7*x^3 + a^4*b^6), 1/756*(108*a^2*b^5*f*x^13 + 27*(7*a^2*b^5*e - 13*a^3*b^4*f)*x^10 + 54*(14*a^2*b^5*d - 35*a^3*b^4*e + 65*a^4*b^3*f)*x^7 - 147*(2*a^2*b^5*c - 14*a^3*b^4*d + 35*a^4*b^3*e - 65*a^5*b^2*f)*x^4 + 84*sqrt(1/3)*(2*a^3*b^4*c - 14*a^4*b^3*d + 35*a^5*b^2*e - 65*a^6*b*f + (2*a*b^6*c - 14*a^2*b^5*d + 35*a^3*b^4*e - 65*a^4*b^3*f)*x^6 + 2*(2*a^2*b^5*c - 14*a^3*b^4*d + 35*a^4*b^3*e - 65*a^5*b^2*f)*x^3)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) - 14*((2*b^5*c - 14*a*b^4*d + 35*a^2*b^3*e - 65*a^3*b^2*f)*x^6 + 2*a^2*b^3*c - 14*a^3*b^2*d + 35*a^4*b*e - 65*a^5*f + 2*(2*a*b^4*c - 14*a^2*b^3*d + 35*a^3*b^2*e - 65*a^4*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*((2*b^5*c - 14*a*b^4*d + 35*a^2*b^3*e - 65*a^3*b^2*f)*x^6 + 2*a^2*b^3*c - 14*a^3*b^2*d + 35*a^4*b*e - 65*a^5*f + 2*(2*a*b^4*c - 14*a^2*b^3*d + 35*a^3*b^2*e - 65*a^4*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) - 84*(2*a^3*b^4*c - 14*a^4*b^3*d + 35*a^5*b^2*e - 65*a^6*b*f)*x)/(a^2*b^8*x^6 + 2*a^3*b^7*x^3 + a^4*b^6)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.10782, size = 539, normalized size = 1.6

$$\frac{(2b^3c - 14ab^2d - 65a^3f + 35a^2be)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27ab^5} + \frac{\sqrt{3}\left(2\left(-ab^2\right)^{\frac{1}{3}}b^3c - 14\left(-ab^2\right)^{\frac{1}{3}}ab^2d - 65\left(-ab^2\right)^{\frac{1}{3}}a^3f\right)}{27ab^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/27*(2*b^3*c - 14*a*b^2*d - 65*a^3*f + 35*a^2*b*e)*(-a/b)^(1/3)*\log(\text{abs}(x \\ & - (-a/b)^(1/3)))/(a*b^5) + 1/27*\text{sqrt}(3)*(2*(-a*b^2)^(1/3)*b^3*c - 14*(-a*b \\ & ^2)^(1/3)*a*b^2*d - 65*(-a*b^2)^(1/3)*a^3*f + 35*(-a*b^2)^(1/3)*a^2*b*e)*\text{ar} \\ & \text{ctan}(1/3*\text{sqrt}(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^6) + 1/54*(2*(-a*b \\ & ^2)^(1/3)*b^3*c - 14*(-a*b^2)^(1/3)*a*b^2*d - 65*(-a*b^2)^(1/3)*a^3*f + 35* \\ & (-a*b^2)^(1/3)*a^2*b*e)*\log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^6) - \\ & 1/18*(7*b^4*c*x^4 - 13*a*b^3*d*x^4 - 25*a^3*b*f*x^4 + 19*a^2*b^2*x^4*e + 4* \\ & a*b^3*c*x - 10*a^2*b^2*d*x - 22*a^4*f*x + 16*a^3*b*x*e)/((b*x^3 + a)^2*b^5) \\ & + 1/28*(4*b^18*f*x^7 - 21*a*b^17*f*x^4 + 7*b^18*x^4*e + 28*b^18*d*x + 168* \\ & a^2*b^16*f*x - 84*a*b^17*x*e)/b^21 \end{aligned}$$

$$3.291 \quad \int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=316

$$\frac{x^2(7a^2be - 10a^3f - 4ab^2d + b^3c)}{9ab^4(a + bx^3)} - \frac{x^2(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^4(a + bx^3)^2} + \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx + b^{2/3}x^2})(-20a^2be + 44a^3f)}{54a^{4/3}b^{14/3}}$$

[Out] ((b*e - 3*a*f)*x^2)/(2*b^4) + (f*x^5)/(5*b^3) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*b^4*(a + b*x^3)^2) + ((b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*x^2)/(9*a*b^4*(a + b*x^3)) - ((b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(4/3)*b^(14/3)) - ((b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(4/3)*b^(14/3)) + ((b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(4/3)*b^(14/3))

Rubi [A] time = 0.504646, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1828, 1851, 1594, 1488, 292, 31, 634, 617, 204, 628}

$$\frac{x^2(7a^2be - 10a^3f - 4ab^2d + b^3c)}{9ab^4(a + bx^3)} - \frac{x^2(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^4(a + bx^3)^2} + \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx + b^{2/3}x^2})(-20a^2be + 44a^3f)}{54a^{4/3}b^{14/3}}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]

[Out] ((b*e - 3*a*f)*x^2)/(2*b^4) + (f*x^5)/(5*b^3) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*b^4*(a + b*x^3)^2) + ((b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*x^2)/(9*a*b^4*(a + b*x^3)) - ((b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(4/3)*b^(14/3)) - ((b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(4/3)*b^(14/3)) + ((b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(4/3)*b^(14/3))

Rule 1828

Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q = m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)

```
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1851

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[x*PolynomialQuot
ient[Pq, x, x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x]
&& EqQ[Coeff[Pq, x, 0], 0] && !MatchQ[Pq, x^(m_)*(u_)] /; IntegerQ[m]
```

Rule 1594

```
Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x
_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1488

```
Int[((f_)*(x_)^(m_))*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_))^(p_)*(
(d_) + (e_)*(x_)^(n_))^(q_), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d
 + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```


Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4(c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^3} dx &= -\frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6b^4(a + bx^3)^2} - \frac{\int \frac{-2ab(b^3c - ab^2d + a^2be - a^3f)x - 6ab^2(b^2d - abe + a^2f)x^4 - 6ab^3(be - af)x^7}{(a + bx^3)^2}}{6ab^5} \\
&= -\frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6b^4(a + bx^3)^2} - \frac{\int \frac{x(-2ab(b^3c - ab^2d + a^2be - a^3f) - 6ab^2(b^2d - abe + a^2f)x^3 - 6ab^3(be - af)x^6)}{(a + bx^3)^2}}{6ab^5} \\
&= -\frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6b^4(a + bx^3)^2} + \frac{(b^3c - 4ab^2d + 7a^2be - 10a^3f)x^2}{9ab^4(a + bx^3)} + \frac{\int \frac{2ab^5(b^3c + 5ab^2d - 11a^3f)x^9}{(a + bx^3)^2}}{6ab^5} \\
&= -\frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6b^4(a + bx^3)^2} + \frac{(b^3c - 4ab^2d + 7a^2be - 10a^3f)x^2}{9ab^4(a + bx^3)} + \frac{\int \frac{x(2ab^5(b^3c + 5ab^2d - 11a^3f) - 6ab^6(b^2d - abe + a^2f)x^3 - 6ab^7(be - af)x^6)}{(a + bx^3)^2}}{6ab^5} \\
&= -\frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6b^4(a + bx^3)^2} + \frac{(b^3c - 4ab^2d + 7a^2be - 10a^3f)x^2}{9ab^4(a + bx^3)} + \frac{\int (18a^2b^5(be - 3af)x^9)}{6ab^5} \\
&= \frac{(be - 3af)x^2}{2b^4} + \frac{fx^5}{5b^3} - \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6b^4(a + bx^3)^2} + \frac{(b^3c - 4ab^2d + 7a^2be - 10a^3f)x^2}{9ab^4(a + bx^3)} \\
&= \frac{(be - 3af)x^2}{2b^4} + \frac{fx^5}{5b^3} - \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6b^4(a + bx^3)^2} + \frac{(b^3c - 4ab^2d + 7a^2be - 10a^3f)x^2}{9ab^4(a + bx^3)} \\
&= \frac{(be - 3af)x^2}{2b^4} + \frac{fx^5}{5b^3} - \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6b^4(a + bx^3)^2} + \frac{(b^3c - 4ab^2d + 7a^2be - 10a^3f)x^2}{9ab^4(a + bx^3)} \\
&= \frac{(be - 3af)x^2}{2b^4} + \frac{fx^5}{5b^3} - \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6b^4(a + bx^3)^2} + \frac{(b^3c - 4ab^2d + 7a^2be - 10a^3f)x^2}{9ab^4(a + bx^3)} \\
&= \frac{(be - 3af)x^2}{2b^4} + \frac{fx^5}{5b^3} - \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6b^4(a + bx^3)^2} + \frac{(b^3c - 4ab^2d + 7a^2be - 10a^3f)x^2}{9ab^4(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.205988, size = 300, normalized size = 0.95

$$\frac{30b^{2/3}x^2(7a^2be-10a^3f-4ab^2d+b^3c)}{a(a+bx^3)} - \frac{45b^{2/3}x^2(a^2be+a^3(-f)-ab^2d+b^3c)}{(a+bx^3)^2} + \frac{5 \log\left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx+b^{2/3}x^2}\right)(-20a^2be+44a^3f+5ab^2d+b^3c)}{a^{4/3}} - \frac{10 \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{270b^{14/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]

[Out] (135*b^(2/3)*(b*e - 3*a*f)*x^2 + 54*b^(5/3)*f*x^5 - (45*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(a + b*x^3)^2 + (30*b^(2/3)*(b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*x^2)/(a*(a + b*x^3)) - (10*sqrt(3)*(b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt(3)])/a^(4/3) - (10*(b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(4/3) + (5*(b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3))/(270*b^(14/3))

Maple [B] time = 0.011, size = 574, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)

[Out] 1/5*f*x^5/b^3-3/2/b^4*x^2*a*f+1/2/b^3*x^2*e-10/9/b^3/(b*x^3+a)^2*a^2*x^5*f+7/9/b^2/(b*x^3+a)^2*a*x^5*e-4/9/b/(b*x^3+a)^2*x^5*d+1/9/(b*x^3+a)^2/a*x^5*c-17/18/b^4/(b*x^3+a)^2*x^2*a^3*f+11/18/b^3/(b*x^3+a)^2*x^2*a^2*e-5/18/b^2/(b*x^3+a)^2*x^2*a*d-1/18/b/(b*x^3+a)^2*x^2*c-44/27/b^5*a^2/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*f+20/27/b^4*a/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*e-5/27/b^3/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*d-1/27/b^2/a/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*c+22/27/b^5*a^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f-10/27/b^4*a/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e+5/54/b^3/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d+1/54/b^2/a/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+44/27/b^5*a^2*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f-20/27/b^4*a*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e+5/27/b^3*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d+1/27/b^2/a*3^(1/2)/(1

$$/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*c$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 1.5351, size = 2723, normalized size = 8.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/270*(54*a^2*b^5*f*x^{11} + 27*(5*a^2*b^5*e - 11*a^3*b^4*f)*x^8 + 6*(5*a*b^6*c - 20*a^2*b^5*d + 80*a^3*b^4*e - 176*a^4*b^3*f)*x^5 - 15*(a^2*b^5*c + 5*a^3*b^4*d - 20*a^4*b^3*e + 44*a^5*b^2*f)*x^2 + 15*\sqrt{1/3}*(a^3*b^4*c + 5*a^4*b^3*d - 20*a^5*b^2*e + 44*a^6*b*f + (a*b^6*c + 5*a^2*b^5*d - 20*a^3*b^4*e + 44*a^4*b^3*f)*x^6 + 2*(a^2*b^5*c + 5*a^3*b^4*d - 20*a^4*b^3*e + 44*a^5*b^2*f)*x^3)*\sqrt{(-a*b^2)^{(1/3)}/a}*\log((2*b^2*x^3 - a*b + 3*\sqrt{1/3}*(a*b*x + 2*(-a*b^2)^{(2/3)}*x^2 + (-a*b^2)^{(1/3)}*a)*\sqrt{(-a*b^2)^{(1/3)}/a} - 3*(-a*b^2)^{(2/3)}*x)/(b*x^3 + a)) + 5*((b^5*c + 5*a*b^4*d - 20*a^2*b^3*e + 44*a^3*b^2*f)*x^6 + a^2*b^3*c + 5*a^3*b^2*d - 20*a^4*b*e + 44*a^5*f + 2*(a*b^4*c + 5*a^2*b^3*d - 20*a^3*b^2*e + 44*a^4*b*f)*x^3)*(-a*b^2)^{(2/3)}*\log(b^2*x^2 + (-a*b^2)^{(1/3)}*b*x + (-a*b^2)^{(2/3)}) - 10*((b^5*c + 5*a*b^4*d - 20*a^2*b^3*e + 44*a^3*b^2*f)*x^6 + a^2*b^3*c + 5*a^3*b^2*d - 20*a^4*b*e + 44*a^5*f + 2*(a*b^4*c + 5*a^2*b^3*d - 20*a^3*b^2*e + 44*a^4*b*f)*x^3)*(-a*b^2)^{(2/3)}*\log(b*x - (-a*b^2)^{(1/3)})]/(a^2*b^8*x^6 + 2*a^3*b^7*x^3 + a^4*b^6), 1/270*(54*a^2*b^5*f*x^{11} + 27*(5*a^2*b^5*e - 11*a^3*b^4*f)*x^8 + 6*(5*a*b^6*c - 20*a^2*b^5*d + 80*a^3*b^4*e - 176*a^4*b^3*f)*x^5 - 15*(a^2*b^5*c + 5*a^3*b^4*d - 20*a^4*b^3*e + 44*a^5*b^2*f)*x^2 + 30*\sqrt{1/3}*(a^3*b^4*c + 5*a^4*b^3*d - 20*a^5*b^2*e + 44*a^6*b*f + (a*b^6*c + 5*a^2*b^5*d - 20*a^3*b^4*e + 44*a^4*b^3*f)*x^6 + 2*(a^2*b^5*c + 5*a^3*b^4*d - 20*a^4*b^3*e + 44*a^5*b^2*f)*x^3)*\sqrt{(-a*b^2)^{(1/3)}/a}*\arctan(\sqrt{1/3}*(2*b*x + (-a*b^2)^{(1/3)})*\sqrt{(-a*b^2)^{(1/3)}/a}) \end{aligned}$$

$$t(-(-a*b^2)^{(1/3)}/a)/b) + 5*((b^5*c + 5*a*b^4*d - 20*a^2*b^3*e + 44*a^3*b^2*f)*x^6 + a^2*b^3*c + 5*a^3*b^2*d - 20*a^4*b*e + 44*a^5*f + 2*(a*b^4*c + 5*a^2*b^3*d - 20*a^3*b^2*e + 44*a^4*b*f)*x^3)*(-a*b^2)^{(2/3)}*\log(b^2*x^2 + (-a*b^2)^{(1/3)}*b*x + (-a*b^2)^{(2/3)}) - 10*((b^5*c + 5*a*b^4*d - 20*a^2*b^3*e + 44*a^3*b^2*f)*x^6 + a^2*b^3*c + 5*a^3*b^2*d - 20*a^4*b*e + 44*a^5*f + 2*(a*b^4*c + 5*a^2*b^3*d - 20*a^3*b^2*e + 44*a^4*b*f)*x^3)*(-a*b^2)^{(2/3)}*\log(b*x - (-a*b^2)^{(1/3)})/(a^2*b^8*x^6 + 2*a^3*b^7*x^3 + a^4*b^6)]$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.09331, size = 558, normalized size = 1.77

$$\frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 5ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 44a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 20a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^2b^4} - \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c + 5\left(-ab^2\right)^{\frac{2}{3}}\right)}{27a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/27*(b^3*c*(-a/b)^{(1/3)} + 5*a*b^2*d*(-a/b)^{(1/3)} + 44*a^3*f*(-a/b)^{(1/3)} \\ & - 20*a^2*b*(-a/b)^{(1/3)}*e)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)})))/(a^2*b^4) \\ & - 1/27*\text{sqrt}(3)*((-a*b^2)^{(2/3)}*b^3*c + 5*(-a*b^2)^{(2/3)}*a*b^2*d + 44*(-a*b^2)^{(2/3)}*a^3*f \\ & - 20*(-a*b^2)^{(2/3)}*a^2*b*e)*\text{arctan}(1/3*\text{sqrt}(3)*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(a^2*b^6) + 1/18*(2*b^4*c*x^5 - 8*a*b^3*d*x^5 - 20*a^3*b*f*x^5 \\ & + 14*a^2*b^2*x^5*e - a*b^3*c*x^2 - 5*a^2*b^2*d*x^2 - 17*a^4*f*x^2 + 11*a^3*b*x^2*e)/((b*x^3 + a)^2*a*b^4) + 1/54*((-a*b^2)^{(2/3)}*b^3*c + 5*(-a*b^2)^{(2/3)}*a*b^2*d \\ & + 44*(-a*b^2)^{(2/3)}*a^3*f - 20*(-a*b^2)^{(2/3)}*a^2*b*e)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a^2*b^6) + 1/10*(2*b^12*f*x^5 - 15*a*b^11*f*x^2 + 5*b^12*x^2*e)/b^15 \end{aligned}$$

$$3.292 \quad \int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=307

$$\frac{x(13a^2be - 19a^3f - 7ab^2d + b^3c)}{18ab^4(a + bx^3)} - \frac{x(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^4(a + bx^3)^2} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx + b^{2/3}x^2})(-14a^2be + 35a^3f + 27a^2b^2d - 19a^3f)}{54a^{5/3}b^{13/3}}$$

[Out] ((b*e - 3*a*f)*x)/b^4 + (f*x^4)/(4*b^3) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b^4*(a + b*x^3)^2) + ((b^3*c - 7*a*b^2*d + 13*a^2*b*e - 19*a^3*f)*x)/(18*a*b^4*(a + b*x^3)) - ((b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(13/3)) + ((b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(5/3)*b^(13/3)) - ((b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(13/3))

Rubi [A] time = 0.411702, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1828, 1858, 1411, 388, 200, 31, 634, 617, 204, 628}

$$\frac{x(13a^2be - 19a^3f - 7ab^2d + b^3c)}{18ab^4(a + bx^3)} - \frac{x(a^2be + a^3(-f) - ab^2d + b^3c)}{6b^4(a + bx^3)^2} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx + b^{2/3}x^2})(-14a^2be + 35a^3f + 27a^2b^2d - 19a^3f)}{54a^{5/3}b^{13/3}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]

[Out] ((b*e - 3*a*f)*x)/b^4 + (f*x^4)/(4*b^3) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b^4*(a + b*x^3)^2) + ((b^3*c - 7*a*b^2*d + 13*a^2*b*e - 19*a^3*f)*x)/(18*a*b^4*(a + b*x^3)) - ((b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(13/3)) + ((b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(5/3)*b^(13/3)) - ((b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(13/3))

Rule 1828

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)

```
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
+ b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a +
b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1411

```
Int[((d_) + (e_)*(x_)^(n_))^(q_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_
)), x_Symbol] := Simp[(c*x^(n + 1)*(d + e*x^n)^(q + 1))/(e*(n*(q + 2) + 1))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
(c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rule 388

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 200

```
Int[((a_) + (b_)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^3} dx &= -\frac{(b^3c - ab^2d + a^2be - a^3f)x}{6b^4(a + bx^3)^2} - \frac{\int \frac{-a(b^3c - ab^2d + a^2be - a^3f) - 6ab(b^2d - abe + a^2f)x^3 - 6ab^2(be - af)x^6 - 6a^2f}{(a + bx^3)^2}}{6ab^4} \\
&= -\frac{(b^3c - ab^2d + a^2be - a^3f)x}{6b^4(a + bx^3)^2} + \frac{(b^3c - 7ab^2d + 13a^2be - 19a^3f)x}{18ab^4(a + bx^3)} + \frac{\int \frac{2ab^3(b^3c + 2ab^2d - 5a^2f)}{(a + bx^3)^2}}{18ab^4} \\
&= \frac{fx^4}{4b^3} - \frac{(b^3c - ab^2d + a^2be - a^3f)x}{6b^4(a + bx^3)^2} + \frac{(b^3c - 7ab^2d + 13a^2be - 19a^3f)x}{18ab^4(a + bx^3)} + \frac{\int \frac{8ab^4(b^3c + 2ab^2d - 5a^2f)}{(a + bx^3)^2}}{18ab^4} \\
&= \frac{(be - 3af)x}{b^4} + \frac{fx^4}{4b^3} - \frac{(b^3c - ab^2d + a^2be - a^3f)x}{6b^4(a + bx^3)^2} + \frac{(b^3c - 7ab^2d + 13a^2be - 19a^3f)x}{18ab^4(a + bx^3)} \\
&= \frac{(be - 3af)x}{b^4} + \frac{fx^4}{4b^3} - \frac{(b^3c - ab^2d + a^2be - a^3f)x}{6b^4(a + bx^3)^2} + \frac{(b^3c - 7ab^2d + 13a^2be - 19a^3f)x}{18ab^4(a + bx^3)} \\
&= \frac{(be - 3af)x}{b^4} + \frac{fx^4}{4b^3} - \frac{(b^3c - ab^2d + a^2be - a^3f)x}{6b^4(a + bx^3)^2} + \frac{(b^3c - 7ab^2d + 13a^2be - 19a^3f)x}{18ab^4(a + bx^3)} \\
&= \frac{(be - 3af)x}{b^4} + \frac{fx^4}{4b^3} - \frac{(b^3c - ab^2d + a^2be - a^3f)x}{6b^4(a + bx^3)^2} + \frac{(b^3c - 7ab^2d + 13a^2be - 19a^3f)x}{18ab^4(a + bx^3)} \\
&= \frac{(be - 3af)x}{b^4} + \frac{fx^4}{4b^3} - \frac{(b^3c - ab^2d + a^2be - a^3f)x}{6b^4(a + bx^3)^2} + \frac{(b^3c - 7ab^2d + 13a^2be - 19a^3f)x}{18ab^4(a + bx^3)} \\
&= \frac{(be - 3af)x}{b^4} + \frac{fx^4}{4b^3} - \frac{(b^3c - ab^2d + a^2be - a^3f)x}{6b^4(a + bx^3)^2} + \frac{(b^3c - 7ab^2d + 13a^2be - 19a^3f)x}{18ab^4(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.201663, size = 294, normalized size = 0.96

$$\frac{6\sqrt[3]{bx}(13a^2be - 19a^3f - 7ab^2d + b^3c)}{a(a + bx^3)} - \frac{18\sqrt[3]{bx}(a^2be + a^3(-f) - ab^2d + b^3c)}{(a + bx^3)^2} - \frac{2\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx + b^{2/3}x^2}\right)(-14a^2be + 35a^3f + 2ab^2d + b^3c)}{a^{5/3}} + \frac{4\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)(-14a^2be + 35a^3f + 2ab^2d + b^3c)}{108b^{13/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]

```
[Out] (108*b^(1/3)*(b*e - 3*a*f)*x + 27*b^(4/3)*f*x^4 - (18*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(a + b*x^3)^2 + (6*b^(1/3)*(b^3*c - 7*a*b^2*d + 13*a^2*b*e - 19*a^3*f)*x)/(a*(a + b*x^3)) - (4*Sqrt[3]*(b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(5/3) + (4*(b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) - (2*(b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(108*b^(13/3))
```

Maple [B] time = 0.011, size = 561, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/4*f*x^4/b^3-3/b^4*a*f*x+1/b^3*x*e-19/18/b^3/(b*x^3+a)^2*x^4*a^2*f+13/18/b^2/(b*x^3+a)^2*x^4*a*e-7/18/b/(b*x^3+a)^2*x^4*d+1/18/(b*x^3+a)^2/a*x^4*c-8/9/b^4/(b*x^3+a)^2*a^3*f*x+5/9/b^3/(b*x^3+a)^2*a^2*e*x-2/9/b^2/(b*x^3+a)^2*a*d*x-1/9/b/(b*x^3+a)^2*c*x+35/27/b^5*a^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*f-14/27/b^4*a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*e+2/27/b^3/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d+1/27/b^2/a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c-35/54/b^5*a^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f+7/27/b^4*a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e-1/27/b^3/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d-1/54/b^2/a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+35/27/b^5*a^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f-14/27/b^4*a/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e+2/27/b^3/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d+1/27/b^2/a/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [B] time = 1.60752, size = 2716, normalized size = 8.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x³*(f*x⁹+e*x⁶+d*x³+c)/(b*x³+a)³,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/108*(27*a^3*b^4*f*x^{10} + 54*(2*a^3*b^4*e - 5*a^4*b^3*f)*x^7 + 3*(2*a^2*b^5*c - 14*a^3*b^4*d + 98*a^4*b^3*e - 245*a^5*b^2*f)*x^4 + 6*\sqrt{1/3}*(a^3*b^4*c + 2*a^4*b^3*d - 14*a^5*b^2*e + 35*a^6*b*f + (a*b^6*c + 2*a^2*b^5*d - 14*a^3*b^4*e + 35*a^4*b^3*f)*x^6 + 2*(a^2*b^5*c + 2*a^3*b^4*d - 14*a^4*b^3*e + 35*a^5*b^2*f)*x^3)*\sqrt{-(a^2*b)^{(1/3)}/b}*\log((2*a*b*x^3 - 3*(a^2*b)^{(1/3)}*a*x - a^2 + 3*\sqrt{1/3}*(2*a*b*x^2 + (a^2*b)^{(2/3)}*x - (a^2*b)^{(1/3)}*a)*\sqrt{-(a^2*b)^{(1/3)}/b})/(b*x^3 + a)) - 2*((b^5*c + 2*a*b^4*d - 14*a^2*b^3*e + 35*a^3*b^2*f)*x^6 + a^2*b^3*c + 2*a^3*b^2*d - 14*a^4*b*e + 35*a^5*f + 2*(a*b^4*c + 2*a^2*b^3*d - 14*a^3*b^2*e + 35*a^4*b*f)*x^3)*(a^2*b)^{(2/3)}*\log(a*b*x^2 - (a^2*b)^{(2/3)}*x + (a^2*b)^{(1/3)}*a) + 4*((b^5*c + 2*a*b^4*d - 14*a^2*b^3*e + 35*a^3*b^2*f)*x^6 + a^2*b^3*c + 2*a^3*b^2*d - 14*a^4*b*e + 35*a^5*f + 2*(a*b^4*c + 2*a^2*b^3*d - 14*a^3*b^2*e + 35*a^4*b*f)*x^3)*(a^2*b)^{(2/3)}*\log(a*b*x + (a^2*b)^{(2/3)}) - 12*(a^3*b^4*c + 2*a^4*b^3*d - 14*a^5*b^2*e + 35*a^6*b*f)*x)/(a^3*b^7*x^6 + 2*a^4*b^6*x^3 + a^5*b^5), 1/108*(27*a^3*b^4*f*x^{10} + 54*(2*a^3*b^4*e - 5*a^4*b^3*f)*x^7 + 3*(2*a^2*b^5*c - 14*a^3*b^4*d + 98*a^4*b^3*e - 245*a^5*b^2*f)*x^4 + 12*\sqrt{1/3}*(a^3*b^4*c + 2*a^4*b^3*d - 14*a^5*b^2*e + 35*a^6*b*f + (a*b^6*c + 2*a^2*b^5*d - 14*a^3*b^4*e + 35*a^4*b^3*f)*x^6 + 2*(a^2*b^5*c + 2*a^3*b^4*d - 14*a^4*b^3*e + 35*a^5*b^2*f)*x^3)*\sqrt{(a^2*b)^{(1/3)}/b}*\arctan(\sqrt{1/3}*(2*(a^2*b)^{(2/3)}*x - (a^2*b)^{(1/3)}*a)*\sqrt{(a^2*b)^{(1/3)}/b}/a^2) - 2*((b^5*c + 2*a*b^4*d - 14*a^2*b^3*e + 35*a^3*b^2*f)*x^6 + a^2*b^3*c + 2*a^3*b^2*d - 14*a^4*b*e + 35*a^5*f + 2*(a*b^4*c + 2*a^2*b^3*d - 14*a^3*b^2*e + 35*a^4*b*f)*x^3)*(a^2*b)^{(2/3)}*\log(a*b*x^2 - (a^2*b)^{(2/3)}*x + (a^2*b)^{(1/3)}*a) + 4*((b^5*c + 2*a*b^4*d - 14*a^2*b^3*e + 35*a^3*b^2*f)*x^6 + a^2*b^3*c + 2*a^3*b^2*d - 14*a^4*b*e + 35*a^5*f + 2*(a*b^4*c + 2*a^2*b^3*d - 14*a^3*b^2*e + 35*a^4*b*f)*x^3)*(a^2*b)^{(2/3)}*\log(a*b*x + (a^2*b)^{(2/3)}) - 12*(a^3*b^4*c + 2*a^4*b^3*d - 14*a^5*b^2*e + 35*a^6*b*f)*x)/(a^3*b^7*x^6 + 2*a^4*b^6*x^3 + a^5*b^5)] \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.09215, size = 495, normalized size = 1.61

$$\frac{(b^3c + 2ab^2d + 35a^3f - 14a^2be) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^2b^4} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c + 2\left(-ab^2\right)^{\frac{1}{3}}ab^2d + 35\left(-ab^2\right)^{\frac{1}{3}}a^3f - 14a^2be\right)}{27a^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/27*(b^3*c + 2*a*b^2*d + 35*a^3*f - 14*a^2*b*e)*(-a/b)^{(1/3)}*\log(\text{abs}(x - \\ & (-a/b)^{(1/3)}))/ (a^2*b^4) + 1/27*\text{sqrt}(3)*((-a*b^2)^{(1/3)}*b^3*c + 2*(-a*b^2)^{(1/3)} \\ & *a*b^2*d + 35*(-a*b^2)^{(1/3)}*a^3*f - 14*(-a*b^2)^{(1/3)}*a^2*b*e)*\text{arctan} \\ & (1/3*\text{sqrt}(3)*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(a^2*b^5) + 1/54*((-a*b^2)^{(1/3)} \\ & *b^3*c + 2*(-a*b^2)^{(1/3)}*a*b^2*d + 35*(-a*b^2)^{(1/3)}*a^3*f - 14*(-a*b^2)^{(1/3)} \\ & *a^2*b*e)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a^2*b^5) + 1/1 \\ & 8*(b^4*c*x^4 - 7*a*b^3*d*x^4 - 19*a^3*b*f*x^4 + 13*a^2*b^2*x^4*e - 2*a*b^3*c*x \\ & - 4*a^2*b^2*d*x - 16*a^4*f*x + 10*a^3*b*x*e)/((b*x^3 + a)^2*a*b^4) + 1/ \\ & 4*(b^9*f*x^4 - 12*a*b^8*f*x + 4*b^9*x*e)/b^{12} \end{aligned}$$

$$3.293 \quad \int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=301

$$\frac{x^2(-4a^2be + 7a^3f + ab^2d + 2b^3c)}{9a^2b^3(a+bx^3)} + \frac{x^2(a^2be + a^3(-f) - ab^2d + b^3c)}{6ab^3(a+bx^3)^2} + \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(5a^2be - 20a^3f + 5a^2b^2d + 2b^3c)}{54a^{7/3}b^{11/3}}$$

[Out] (f*x^2)/(2*b^3) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a*b^3*(a + b*x^3)^2) + ((2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*x^2)/(9*a^2*b^3*(a + b*x^3)) - ((2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(7/3)*b^(11/3)) - ((2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*Log[a^(1/3) + b^(1/3)*x]/(27*a^(7/3)*b^(11/3))) + ((2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(54*a^(7/3)*b^(11/3)))

Rubi [A] time = 0.368046, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1828, 1594, 1482, 459, 292, 31, 634, 617, 204, 628}

$$\frac{x^2(-4a^2be + 7a^3f + ab^2d + 2b^3c)}{9a^2b^3(a+bx^3)} + \frac{x^2(a^2be + a^3(-f) - ab^2d + b^3c)}{6ab^3(a+bx^3)^2} + \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(5a^2be - 20a^3f + 5a^2b^2d + 2b^3c)}{54a^{7/3}b^{11/3}}$$

Antiderivative was successfully verified.

[In] Int[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]

[Out] (f*x^2)/(2*b^3) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a*b^3*(a + b*x^3)^2) + ((2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*x^2)/(9*a^2*b^3*(a + b*x^3)) - ((2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(7/3)*b^(11/3)) - ((2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*Log[a^(1/3) + b^(1/3)*x]/(27*a^(7/3)*b^(11/3))) + ((2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(54*a^(7/3)*b^(11/3)))

Rule 1828

Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q = m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)

```
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
+ b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1594

```
Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1482

```
Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_))^(p_)*((d_) + (
e_)*(x_)^(n_))^(q_), x_Symbol] :> Simp[((-d)^(m - Mod[m, n])/n - 1)*(c*d^
2 - b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*(d + e*x^n)^(q + 1))/(n*e^(2*p + (m
- Mod[m, n])/n)*(q + 1)), x] + Dist[1/(n*e^(2*p + (m - Mod[m, n])/n)*(q + 1
)), Int[x^Mod[m, n]*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1*(n*e^(2*p +
(m - Mod[m, n])/n)*(q + 1)*x^(m - Mod[m, n]))*(a + b*x^n + c*x^(2*n))^p - (
-d)^(m - Mod[m, n])/n - 1)*(c*d^2 - b*d*e + a*e^2)^p*(d*(Mod[m, n] + 1) +
e*(Mod[m, n] + n*(q + 1) + 1)*x^n)]]/(d + e*x^n)], x], x] /; FreeQ[{a,
b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IGt
Q[p, 0] && ILtQ[q, -1] && IGtQ[m, 0]
```

Rule 459

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
+ 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
+ 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^( -1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^3} dx &= \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6ab^3(a + bx^3)^2} - \frac{\int \frac{-2b(2b^3c + ab^2d - a^2be + a^3f)x - 6ab^2(be - af)x^4 - 6ab^3fx^7}{(a + bx^3)^2} dx}{6ab^4} \\
&= \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6ab^3(a + bx^3)^2} - \frac{\int \frac{x(-2b(2b^3c + ab^2d - a^2be + a^3f) - 6ab^2(be - af)x^3 - 6ab^3fx^6)}{(a + bx^3)^2} dx}{6ab^4} \\
&= \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6ab^3(a + bx^3)^2} + \frac{(2b^3c + ab^2d - 4a^2be + 7a^3f)x^2}{9a^2b^3(a + bx^3)} + \frac{\int \frac{x(2b^3(\frac{2b^3c}{a} + b^2d + 5abe - \dots)}{a + bx^3} dx}{18ab^3} \\
&= \frac{fx^2}{2b^3} + \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6ab^3(a + bx^3)^2} + \frac{(2b^3c + ab^2d - 4a^2be + 7a^3f)x^2}{9a^2b^3(a + bx^3)} + \frac{(2b^3c + ab^2d - \dots)}{18ab^3} \\
&= \frac{fx^2}{2b^3} + \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6ab^3(a + bx^3)^2} + \frac{(2b^3c + ab^2d - 4a^2be + 7a^3f)x^2}{9a^2b^3(a + bx^3)} - \frac{(2b^3c + ab^2d - \dots)}{18ab^3} \\
&= \frac{fx^2}{2b^3} + \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6ab^3(a + bx^3)^2} + \frac{(2b^3c + ab^2d - 4a^2be + 7a^3f)x^2}{9a^2b^3(a + bx^3)} - \frac{(2b^3c + ab^2d - \dots)}{18ab^3} \\
&= \frac{fx^2}{2b^3} + \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6ab^3(a + bx^3)^2} + \frac{(2b^3c + ab^2d - 4a^2be + 7a^3f)x^2}{9a^2b^3(a + bx^3)} - \frac{(2b^3c + ab^2d - \dots)}{18ab^3} \\
&= \frac{fx^2}{2b^3} + \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6ab^3(a + bx^3)^2} + \frac{(2b^3c + ab^2d - 4a^2be + 7a^3f)x^2}{9a^2b^3(a + bx^3)} - \frac{(2b^3c + ab^2d - \dots)}{18ab^3}
\end{aligned}$$

Mathematica [A] time = 0.200122, size = 284, normalized size = 0.94

$$\frac{6b^{2/3}x^2(-4a^2be + 7a^3f + ab^2d + 2b^3c)}{a^2(a + bx^3)} + \frac{9b^{2/3}x^2(a^2be + a^3(-f) - ab^2d + b^3c)}{a(a + bx^3)^2} + \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx + b^{2/3}x^2})(5a^2be - 20a^3f + ab^2d + 2b^3c)}{a^{7/3}} - \frac{2\log(\sqrt[3]{a} + \sqrt[3]{bx})(5a^2be - 20a^3f + ab^2d + 2b^3c)}{a^{7/3}}$$

$54b^{11/3}$

Antiderivative was successfully verified.

[In] Integrate[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]


```
[Out] (27*b^(2/3)*f*x^2 + (9*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(a*(a + b*x^3)^2) + (6*b^(2/3)*(2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*x^2)/(a^2*(a + b*x^3)) - (2*sqrt(3)*(2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt(3)])/a^(7/3) - (2*(2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(7/3) + ((2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(7/3))/(54*b^(11/3))
```

Maple [B] time = 0.011, size = 550, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/2*f*x^2/b^3+7/9/b^2/(b*x^3+a)^2*a*x^5*f-4/9/b/(b*x^3+a)^2*x^5*e+1/9/(b*x^3+a)^2/a*x^5*d+2/9*b/(b*x^3+a)^2/a^2*x^5*c+11/18/b^3/(b*x^3+a)^2*a^2*x^2*f-5/18/b^2/(b*x^3+a)^2*a*x^2*e-1/18/b/(b*x^3+a)^2*x^2*d+7/18/(b*x^3+a)^2/a*x^2*c+20/27/b^4*a/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*f-5/27/b^3/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*e-1/27/b^2/a/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*d-2/27/b/a^2/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*c-10/27/b^4*a/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f+5/54/b^3/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e+1/54/b^2/a/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d+1/27/b/a^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c-20/27/b^4*a*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f+5/27/b^3*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e+1/27/b^2/a*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d+2/27/b/a^2*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [B] time = 1.49314, size = 2538, normalized size = 8.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] [1/54*(27*a^3*b^4*f*x^8 + 6*(2*a*b^6*c + a^2*b^5*d - 4*a^3*b^4*e + 16*a^4*b^3*f)*x^5 + 3*(7*a^2*b^5*c - a^3*b^4*d - 5*a^4*b^3*e + 20*a^5*b^2*f)*x^2 - 3*sqrt(1/3)*(2*a^3*b^4*c + a^4*b^3*d + 5*a^5*b^2*e - 20*a^6*b*f + (2*a*b^6*c + a^2*b^5*d + 5*a^3*b^4*e - 20*a^4*b^3*f)*x^6 + 2*(2*a^2*b^5*c + a^3*b^4*d + 5*a^4*b^3*e - 20*a^5*b^2*f)*x^3)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) + ((2*b^5*c + a*b^4*d + 5*a^2*b^3*e - 20*a^3*b^2*f)*x^6 + 2*a^2*b^3*c + a^3*b^2*d + 5*a^4*b*e - 20*a^5*f + 2*(2*a*b^4*c + a^2*b^3*d + 5*a^3*b^2*e - 20*a^4*b*f)*x^3)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 2*((2*b^5*c + a*b^4*d + 5*a^2*b^3*e - 20*a^3*b^2*f)*x^6 + 2*a^2*b^3*c + a^3*b^2*d + 5*a^4*b*e - 20*a^5*f + 2*(2*a*b^4*c + a^2*b^3*d + 5*a^3*b^2*e - 20*a^4*b*f)*x^3)*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^7*x^6 + 2*a^4*b^6*x^3 + a^5*b^5), 1/54*(27*a^3*b^4*f*x^8 + 6*(2*a*b^6*c + a^2*b^5*d - 4*a^3*b^4*e + 16*a^4*b^3*f)*x^5 + 3*(7*a^2*b^5*c - a^3*b^4*d - 5*a^4*b^3*e + 20*a^5*b^2*f)*x^2 - 6*sqrt(1/3)*(2*a^3*b^4*c + a^4*b^3*d + 5*a^5*b^2*e - 20*a^6*b*f + (2*a*b^6*c + a^2*b^5*d + 5*a^3*b^4*e - 20*a^4*b^3*f)*x^6 + 2*(2*a^2*b^5*c + a^3*b^4*d + 5*a^4*b^3*e - 20*a^5*b^2*f)*x^3)*sqrt((a*b^2)^(1/3)/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) + ((2*b^5*c + a*b^4*d + 5*a^2*b^3*e - 20*a^3*b^2*f)*x^6 + 2*a^2*b^3*c + a^3*b^2*d + 5*a^4*b*e - 20*a^5*f + 2*(2*a*b^4*c + a^2*b^3*d + 5*a^3*b^2*e - 20*a^4*b*f)*x^3)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 2*((2*b^5*c + a*b^4*d + 5*a^2*b^3*e - 20*a^3*b^2*f)*x^6 + 2*a^2*b^3*c + a^3*b^2*d + 5*a^4*b*e - 20*a^5*f + 2*(2*a*b^4*c + a^2*b^3*d + 5*a^3*b^2*e - 20*a^4*b*f)*x^3)*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^7*x^6 + 2*a^4*b^6*x^3 + a^5*b^5)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.09405, size = 522, normalized size = 1.73

$$\frac{fx^2}{2b^3} - \frac{\left(2b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} + ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 20a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 5a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^3b^3} - \frac{\sqrt{3}\left(2\left(-ab^2\right)^{\frac{2}{3}}b^3c + (-a\right)}{27a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")

[Out] $\frac{1}{2}fx^2/b^3 - \frac{1}{27}(2b^3c(-a/b)^{1/3} + a^2b^2d(-a/b)^{1/3} - 20a^3f(-a/b)^{1/3} + 5a^2b(-a/b)^{1/3}e)(-a/b)^{1/3} \log(\text{abs}(x - (-a/b)^{1/3}))/a^3b^3 - \frac{1}{27}\sqrt{3}(2(-ab^2)^{2/3}b^3c + (-ab^2)^{2/3}a^2b^2d - 20(-ab^2)^{2/3}a^3f + 5(-ab^2)^{2/3}a^2b^2e) \arctan(1/3\sqrt{3}(2x + (-a/b)^{1/3})/(-a/b)^{1/3})/a^3b^5 + \frac{1}{18}(4b^4cx^5 + 2a^3b^3dx^5 + 14a^3b^2fx^5 - 8a^2b^2x^5e + 7a^2b^3cx^2 - a^2b^2dx^2 + 11a^4fx^2 - 5a^3b^2x^2e)/(b^3x^3 + a)^2a^2b^3 + \frac{1}{54}(2(-ab^2)^{2/3}b^3c + (-ab^2)^{2/3}a^2b^2d - 20(-ab^2)^{2/3}a^3f + 5(-ab^2)^{2/3}a^2b^2e) \log(x^2 + x(-a/b)^{1/3} + (-a/b)^{2/3})/a^3b^5$

$$3.294 \quad \int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^3} dx$$

Optimal. Leaf size=292

$$\frac{x(-7a^2be + 13a^3f + ab^2d + 5b^3c)}{18a^2b^3(a + bx^3)} + \frac{x(a^2be + a^3(-f) - ab^2d + b^3c)}{6ab^3(a + bx^3)^2} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(2a^2be - 14a^3f + ab^2d + 5b^3c)}{54a^{8/3}b^{10/3}}$$

[Out] (f*x)/b^3 + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a*b^3*(a + b*x^3)^2) + ((5*b^3*c + a*b^2*d - 7*a^2*b*e + 13*a^3*f)*x)/(18*a^2*b^3*(a + b*x^3)) - ((5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(10/3)) + ((5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(10/3)) - ((5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(10/3))

Rubi [A] time = 0.307193, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1858, 1409, 388, 200, 31, 634, 617, 204, 628}

$$\frac{x(-7a^2be + 13a^3f + ab^2d + 5b^3c)}{18a^2b^3(a + bx^3)} + \frac{x(a^2be + a^3(-f) - ab^2d + b^3c)}{6ab^3(a + bx^3)^2} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(2a^2be - 14a^3f + ab^2d + 5b^3c)}{54a^{8/3}b^{10/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^3, x]

[Out] (f*x)/b^3 + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a*b^3*(a + b*x^3)^2) + ((5*b^3*c + a*b^2*d - 7*a^2*b*e + 13*a^3*f)*x)/(18*a^2*b^3*(a + b*x^3)) - ((5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(10/3)) + ((5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(10/3)) - ((5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(10/3))

Rule 1858

Int[(Pq)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,

```
x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x], Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x] /; GeQ[q, n] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1409

```
Int[((d_) + (e_)*(x_)^(n_))^(q_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x_Symbol] := -Simp[((c*d^2 - b*d*e + a*e^2)*x*(d + e*x^n)^(q + 1))/(d*e^2*n*(q + 1)), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Simp[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 200

```
Int[((a_) + (b_)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_)*(x_))(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b]
```

], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{c + dx^3 + ex^6 + fx^9}{(a + bx^3)^3} dx &= \frac{(b^3c - ab^2d + a^2be - a^3f)x}{6ab^3(a + bx^3)^2} - \frac{\int \frac{-5b^3c - ab^2d + a^2be - a^3f - 6ab(be - af)x^3 - 6ab^2fx^6}{(a + bx^3)^2} dx}{6ab^3} \\
 &= \frac{(b^3c - ab^2d + a^2be - a^3f)x}{6ab^3(a + bx^3)^2} + \frac{(5b^3c + ab^2d - 7a^2be + 13a^3f)x}{18a^2b^3(a + bx^3)} + \frac{\int \frac{2b^2(5b^3c + ab^2d + 2a^2be - 5a^3f) + a^2e}{a + bx^3}}{18a^2b^5} \\
 &= \frac{fx}{b^3} + \frac{(b^3c - ab^2d + a^2be - a^3f)x}{6ab^3(a + bx^3)^2} + \frac{(5b^3c + ab^2d - 7a^2be + 13a^3f)x}{18a^2b^3(a + bx^3)} + \frac{(5b^3c + ab^2d + 2a^2e)}{9} \\
 &= \frac{fx}{b^3} + \frac{(b^3c - ab^2d + a^2be - a^3f)x}{6ab^3(a + bx^3)^2} + \frac{(5b^3c + ab^2d - 7a^2be + 13a^3f)x}{18a^2b^3(a + bx^3)} + \frac{(5b^3c + ab^2d + 2a^2e)}{2} \\
 &= \frac{fx}{b^3} + \frac{(b^3c - ab^2d + a^2be - a^3f)x}{6ab^3(a + bx^3)^2} + \frac{(5b^3c + ab^2d - 7a^2be + 13a^3f)x}{18a^2b^3(a + bx^3)} + \frac{(5b^3c + ab^2d + 2a^2e)}{2} \\
 &= \frac{fx}{b^3} + \frac{(b^3c - ab^2d + a^2be - a^3f)x}{6ab^3(a + bx^3)^2} + \frac{(5b^3c + ab^2d - 7a^2be + 13a^3f)x}{18a^2b^3(a + bx^3)} + \frac{(5b^3c + ab^2d + 2a^2e)}{2} \\
 &= \frac{fx}{b^3} + \frac{(b^3c - ab^2d + a^2be - a^3f)x}{6ab^3(a + bx^3)^2} + \frac{(5b^3c + ab^2d - 7a^2be + 13a^3f)x}{18a^2b^3(a + bx^3)} - \frac{(5b^3c + ab^2d + 2a^2e)}{9}
 \end{aligned}$$

Mathematica [A] time = 0.189997, size = 279, normalized size = 0.96

$$\frac{3\sqrt[3]{bx}(-7a^2be+13a^3f+ab^2d+5b^3c)}{a^2(a+bx^3)} + \frac{9\sqrt[3]{bx}(a^2be+a^3(-f)-ab^2d+b^3c)}{a(a+bx^3)^2} - \frac{\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2)(2a^2be-14a^3f+ab^2d+5b^3c)}{a^{8/3}} + \frac{2\log(\sqrt[3]{a}+\sqrt[3]{bx})(2a^2be-14a^3f+ab^2d+5b^3c)}{a^{8/3}}$$

$$54b^{10/3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^3,x]

[Out] $(54*b^{1/3}*f*x + (9*b^{1/3}*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(a*(a + b*x^3)^2) + (3*b^{1/3}*(5*b^3*c + a*b^2*d - 7*a^2*b*e + 13*a^3*f)*x)/(a^2*(a + b*x^3)) - (2*sqrt(3)*(5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*ArcTan[(1 - (2*b^{1/3}*x)/a^{1/3})/sqrt(3)])/a^{8/3} + (2*(5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*Log[a^{1/3} + b^{1/3}*x])/a^{8/3} - ((5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*Log[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2])/a^{8/3})/(54*b^{10/3})$

Maple [B] time = 0.011, size = 539, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)

[Out] $f*x/b^3+13/18/b^2/(b*x^3+a)^2*x^4*a*f-7/18/b/(b*x^3+a)^2*x^4*e+1/18/(b*x^3+a)^2/a*x^4*d+5/18*b/(b*x^3+a)^2/a^2*x^4*c+5/9/b^3/(b*x^3+a)^2*a^2*f*x-2/9/b^2/(b*x^3+a)^2*a*e*x-1/9/b/(b*x^3+a)^2*d*x+4/9*c/a*x/(b*x^3+a)^2-14/27/b^4*a/(1/b*a)^{2/3}*ln(x+(1/b*a)^{1/3})*f+2/27/b^3/(1/b*a)^{2/3}*ln(x+(1/b*a)^{1/3})*e+1/27/b^2/a/(1/b*a)^{2/3}*ln(x+(1/b*a)^{1/3})*d+5/27*c/a^2/b/(1/b*a)^{2/3}*ln(x+(1/b*a)^{1/3}))+7/27/b^4*a/(1/b*a)^{2/3}*ln(x^2-(1/b*a)^{1/3}*x+(1/b*a)^{2/3})*f-1/27/b^3/(1/b*a)^{2/3}*ln(x^2-(1/b*a)^{1/3}*x+(1/b*a)^{2/3})*e-1/54/b^2/a/(1/b*a)^{2/3}*ln(x^2-(1/b*a)^{1/3}*x+(1/b*a)^{2/3})*d-5/54*c/a^2/b/(1/b*a)^{2/3}*ln(x^2-(1/b*a)^{1/3}*x+(1/b*a)^{2/3}))-14/27/b^4*a/(1/b*a)^{2/3}*3^{1/2}*arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1))*f+2/27/b^3/(1/b*a)^{2/3}*3^{1/2}*arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1))*e+1/27/b^2/a/(1/b*a)^{2/3}*3^{1/2}*arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1))*d+5/27*c/a^2/b/(1/b*a)^{2/3}*3^{1/2}*arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1))$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 1.49873, size = 2587, normalized size = 8.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/54*(54*a^4*b^3*f*x^7 + 3*(5*a^2*b^5*c + a^3*b^4*d - 7*a^4*b^3*e + 49*a^5*b^2*f)*x^4 - 3*\sqrt{1/3}*(5*a^3*b^4*c + a^4*b^3*d + 2*a^5*b^2*e - 14*a^6*b*f + (5*a*b^6*c + a^2*b^5*d + 2*a^3*b^4*e - 14*a^4*b^3*f)*x^6 + 2*(5*a^2*b^5*c + a^3*b^4*d + 2*a^4*b^3*e - 14*a^5*b^2*f)*x^3)*\sqrt{(-a^2*b)^{(1/3)}/b}*1 \\ & \log((2*a*b*x^3 + 3*(-a^2*b)^{(1/3)}*a*x - a^2 - 3*\sqrt{1/3}*(2*a*b*x^2 + (-a^2*b)^{(2/3)}*x + (-a^2*b)^{(1/3)}*a)*\sqrt{(-a^2*b)^{(1/3)}/b}))/ (b*x^3 + a)) - ((5*b^5*c + a*b^4*d + 2*a^2*b^3*e - 14*a^3*b^2*f)*x^6 + 5*a^2*b^3*c + a^3*b^2*d \\ & + 2*a^4*b*e - 14*a^5*f + 2*(5*a*b^4*c + a^2*b^3*d + 2*a^3*b^2*e - 14*a^4*b*f)*x^3)*(-a^2*b)^{(2/3)}*\log(a*b*x^2 - (-a^2*b)^{(2/3)}*x - (-a^2*b)^{(1/3)}*a) \\ & + 2*((5*b^5*c + a*b^4*d + 2*a^2*b^3*e - 14*a^3*b^2*f)*x^6 + 5*a^2*b^3*c + a^3*b^2*d + 2*a^4*b*e - 14*a^5*f + 2*(5*a*b^4*c + a^2*b^3*d + 2*a^3*b^2*e - 14*a^4*b*f)*x^3)*(-a^2*b)^{(2/3)}*\log(a*b*x + (-a^2*b)^{(2/3)}) + 6*(4*a^3*b^4*c - a^4*b^3*d - 2*a^5*b^2*e + 14*a^6*b*f)*x)/(a^4*b^6*x^6 + 2*a^5*b^5*x^3 + a^6*b^4), \\ & 1/54*(54*a^4*b^3*f*x^7 + 3*(5*a^2*b^5*c + a^3*b^4*d - 7*a^4*b^3*e + 49*a^5*b^2*f)*x^4 + 6*\sqrt{1/3}*(5*a^3*b^4*c + a^4*b^3*d + 2*a^5*b^2*e - 14*a^6*b*f + (5*a*b^6*c + a^2*b^5*d + 2*a^3*b^4*e - 14*a^4*b^3*f)*x^6 + 2*(5*a^2*b^5*c + a^3*b^4*d + 2*a^4*b^3*e - 14*a^5*b^2*f)*x^3)*\sqrt{-(-a^2*b)^{(1/3)}/b}* \\ & \arctan(\sqrt{1/3}*(2*(-a^2*b)^{(2/3)}*x + (-a^2*b)^{(1/3)}*a)*\sqrt{-(-a^2*b)^{(1/3)}/b}/a^2) - ((5*b^5*c + a*b^4*d + 2*a^2*b^3*e - 14*a^3*b^2*f)*x^6 + 5*a^2*b^3*c + a^3*b^2*d + 2*a^4*b*e - 14*a^5*f + 2*(5*a*b^4*c + a^2*b^3*d + 2*a^3*b^2*e - 14*a^4*b*f)*x^3)*(-a^2*b)^{(2/3)}*\log(a*b*x^2 - (-a^2*b)^{(2/3)}*x - (-a^2*b)^{(1/3)}*a) + 2*((5*b^5*c + a*b^4*d + 2*a^2*b^3*e - 14*a^3*b^2*f)*x^6 + 5*a^2*b^3*c + a^3*b^2*d + 2*a^4*b*e - 14*a^5*f + 2*(5*a*b^4*c + a^2*b^3*d + 2*a^3*b^2*e - 14*a^4*b*f)*x^3)*(-a^2*b)^{(2/3)}*\log(a*b*x + (-a^2*b)^{(2/3)}) + 6*(4*a^3*b^4*c - a^4*b^3*d - 2*a^5*b^2*e + 14*a^6*b*f)*x)/(a^4*b^6*x^6 + 2*a^5*b^5*x^3 + a^6*b^4) \end{aligned}$$

$$\begin{aligned} &^2*f)*x^6 + 5*a^2*b^3*c + a^3*b^2*d + 2*a^4*b*e - 14*a^5*f + 2*(5*a*b^4*c + \\ &a^2*b^3*d + 2*a^3*b^2*e - 14*a^4*b*f)*x^3)*(-a^2*b)^{(2/3)}*\log(a*b*x + (-a^2*b)^{(2/3})) + 6*(4*a^3*b^4*c - a^4*b^3*d - 2*a^5*b^2*e + 14*a^6*b*f)*x)/(a^4*b^6*x^6 + 2*a^5*b^5*x^3 + a^6*b^4)] \end{aligned}$$

Sympy [A] time = 170.751, size = 418, normalized size = 1.43

$$\frac{x^4(13a^3bf - 7a^2b^2e + ab^3d + 5b^4c) + x(10a^4f - 4a^3be - 2a^2b^2d + 8ab^3c)}{18a^4b^3 + 36a^3b^4x^3 + 18a^2b^5x^6} + \text{RootSum}\left(19683t^3a^8b^{10} + 2744a^9f^3 - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)

[Out] (x**4*(13*a**3*b*f - 7*a**2*b**2*e + a*b**3*d + 5*b**4*c) + x*(10*a**4*f - 4*a**3*b*e - 2*a**2*b**2*d + 8*a*b**3*c))/(18*a**4*b**3 + 36*a**3*b**4*x**3 + 18*a**2*b**5*x**6) + RootSum(19683*_t**3*a**8*b**10 + 2744*a**9*f**3 - 176*a**8*b*e*f**2 - 588*a**7*b**2*d*f**2 + 168*a**7*b**2*e**2*f - 2940*a**6*b**3*c*f**2 + 168*a**6*b**3*d*e*f - 8*a**6*b**3*e**3 + 840*a**5*b**4*c*e*f + 42*a**5*b**4*d**2*f - 12*a**5*b**4*d*e**2 + 420*a**4*b**5*c*d*f - 60*a**4*b**5*c*e**2 - 6*a**4*b**5*d**2*e + 1050*a**3*b**6*c**2*f - 60*a**3*b**6*c*d*e - a**3*b**6*d**3 - 150*a**2*b**7*c**2*e - 15*a**2*b**7*c*d**2 - 75*a*b**8*c**2*d - 125*b**9*c**3, Lambda(_t, _t*log(-27*_t*a**3*b**3/(14*a**3*f - 2*a**2*b*e - a*b**2*d - 5*b**3*c) + x))) + f*x/b**3

Giac [A] time = 1.09674, size = 463, normalized size = 1.59

$$\frac{fx}{b^3} - \frac{(5b^3c + ab^2d - 14a^3f + 2a^2be)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^3b^3} + \frac{\sqrt{3}\left(5(-ab^2)^{\frac{1}{3}}b^3c + (-ab^2)^{\frac{1}{3}}ab^2d - 14(-ab^2)^{\frac{1}{3}}a^3f\right)}{27a^3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")

[Out] f*x/b^3 - 1/27*(5*b^3*c + a*b^2*d - 14*a^3*f + 2*a^2*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^3) + 1/27*sqrt(3)*(5*(-a*b^2)^(1/3)*b^3*c + (

$$\begin{aligned}
& -a*b^2)^{(1/3)}*a*b^2*d - 14*(-a*b^2)^{(1/3)}*a^3*f + 2*(-a*b^2)^{(1/3)}*a^2*b*e) \\
& *arctan(1/3*sqrt(3)*(2*x + (-a/b)^{(1/3)))/(-a/b)^{(1/3)))/(a^3*b^4) + 1/54*(5* \\
& (-a*b^2)^{(1/3)}*b^3*c + (-a*b^2)^{(1/3)}*a*b^2*d - 14*(-a*b^2)^{(1/3)}*a^3*f + 2 \\
& *(-a*b^2)^{(1/3)}*a^2*b*e)*log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)))/(a^3*b^4) \\
& + 1/18*(5*b^4*c*x^4 + a*b^3*d*x^4 + 13*a^3*b*f*x^4 - 7*a^2*b^2*x^4*e + 8*a \\
& *b^3*c*x - 2*a^2*b^2*d*x + 10*a^4*f*x - 4*a^3*b*x*e)/((b*x^3 + a)^2*a^2*b^3 \\
&)
\end{aligned}$$

$$3.295 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)^3} dx$$

Optimal. Leaf size=303

$$\frac{x^2(-a^2be + 4a^3f - 2ab^2d + 5b^3c)}{9a^3b^2(a + bx^3)} - \frac{x^2(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^2b^2(a + bx^3)^2} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(-a^2be - 5a^3f - \dots)}{54a^{10/3}b^{8/3}}$$

[Out] $-(c/(a^3*x)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^2*b^2*(a + b*x^3)^2) - ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*x^2)/(9*a^3*b^2*(a + b*x^3)) + ((14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)*b^(8/3)) + ((14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*b^(8/3)) - ((14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(10/3)*b^(8/3))$

Rubi [A] time = 0.339039, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {1829, 1484, 453, 292, 31, 634, 617, 204, 628}

$$\frac{x^2(-a^2be + 4a^3f - 2ab^2d + 5b^3c)}{9a^3b^2(a + bx^3)} - \frac{x^2(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^2b^2(a + bx^3)^2} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(-a^2be - 5a^3f - \dots)}{54a^{10/3}b^{8/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^3), x]

[Out] $-(c/(a^3*x)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^2*b^2*(a + b*x^3)^2) - ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*x^2)/(9*a^3*b^2*(a + b*x^3)) + ((14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)*b^(8/3)) + ((14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*b^(8/3)) - ((14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(10/3)*b^(8/3))$

Rule 1829

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^

```
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1484

```
Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_)*((d_) + (e
_)*(x_)^(n_)]^(q_), x_Symbol] := Simp[((-d)^((m - Mod[m, n])/n - 1)*(c*d^2
- b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*(d + e*x^n)^(q + 1))/(n*e^(2*p + (m -
Mod[m, n])/n)*(q + 1)), x] + Dist[(-d)^((m - Mod[m, n])/n - 1)/(n*e^(2*p)*
(q + 1)), Int[x^m*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1*(n*(-d)^(-(m
- Mod[m, n])/n) + 1)*e^(2*p)*(q + 1)*(a + b*x^n + c*x^(2*n))^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^((m - Mod[m, n])/n)*x^(m - Mod[m, n])))*(d*(Mod[m, n]
+ 1) + e*(Mod[m, n] + n*(q + 1) + 1)*x^n))]/(d + e*x^n)], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m, 0]
```

Rule 453

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_)]^(p_)*((c_) + (d_)*(x_)^(n
_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 292

```
Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx^3 + ex^6 + fx^9}{x^2(a + bx^3)^3} dx &= -\frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6a^2b^2(a + bx^3)^2} - \frac{\int \frac{-6b^3c + 2b\left(\frac{2b^3c}{a} - 2b^2d - abe + a^2f\right)x^3 - 6ab^2fx^6}{x^2(a + bx^3)^2} dx}{6ab^3} \\
&= -\frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6a^2b^2(a + bx^3)^2} - \frac{(5b^3c - 2ab^2d - a^2be + 4a^3f)x^2}{9a^3b^2(a + bx^3)} + \frac{\int \frac{18ab^5c - 2b^3(5b^3c - 2ab^2d - a^2be + 4a^3f)x^3 - 6ab^2fx^6}{x^2(a + bx^3)^2} dx}{18a^3b^5} \\
&= -\frac{c}{a^3x} - \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6a^2b^2(a + bx^3)^2} - \frac{(5b^3c - 2ab^2d - a^2be + 4a^3f)x^2}{9a^3b^2(a + bx^3)} - \frac{(14b^3c - 2ab^2d - a^2be + 4a^3f)x^2}{18a^3b^5} \\
&= -\frac{c}{a^3x} - \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6a^2b^2(a + bx^3)^2} - \frac{(5b^3c - 2ab^2d - a^2be + 4a^3f)x^2}{9a^3b^2(a + bx^3)} + \frac{(14b^3c - 2ab^2d - a^2be + 4a^3f)x^2}{18a^3b^5} \\
&= -\frac{c}{a^3x} - \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6a^2b^2(a + bx^3)^2} - \frac{(5b^3c - 2ab^2d - a^2be + 4a^3f)x^2}{9a^3b^2(a + bx^3)} + \frac{(14b^3c - 2ab^2d - a^2be + 4a^3f)x^2}{18a^3b^5} \\
&= -\frac{c}{a^3x} - \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6a^2b^2(a + bx^3)^2} - \frac{(5b^3c - 2ab^2d - a^2be + 4a^3f)x^2}{9a^3b^2(a + bx^3)} + \frac{(14b^3c - 2ab^2d - a^2be + 4a^3f)x^2}{18a^3b^5} \\
&= -\frac{c}{a^3x} - \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6a^2b^2(a + bx^3)^2} - \frac{(5b^3c - 2ab^2d - a^2be + 4a^3f)x^2}{9a^3b^2(a + bx^3)} + \frac{(14b^3c - 2ab^2d - a^2be + 4a^3f)x^2}{18a^3b^5}
\end{aligned}$$

Mathematica [A] time = 0.222532, size = 286, normalized size = 0.94

$$\frac{-6\sqrt[3]{ax^2(-a^2be+4a^3f-2ab^2d+5b^3c)}}{b^2(a+bx^3)} + \frac{9a^{4/3}x^2(-a^2be+a^3f+ab^2d-b^3c)}{b^2(a+bx^3)^2} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx+b^{2/3}x^2}\right)(a^2be+5a^3f+2ab^2d-14b^3c)}{b^{8/3}} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)(a^2be+5a^3f+2ab^2d-14b^3c)}{b^{8/3}}$$

$$54a^{10/3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^3), x]

[Out] ((-54*a^(1/3)*c)/x + (9*a^(4/3)*(-b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(b^2*(a + b*x^3)^2) - (6*a^(1/3)*(5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)

$$\begin{aligned} & *x^2)/(b^2*(a + b*x^3)) + (2*\text{Sqrt}[3]*(14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^3*f)*\text{ArcTan}[(1 - (2*b^{1/3}*x)/a^{1/3})/\text{Sqrt}[3]])/b^{8/3} - (2*(-14*b^3*c + \\ & 2*a*b^2*d + a^2*b*e + 5*a^3*f)*\text{Log}[a^{1/3} + b^{1/3}*x])/b^{8/3} + ((-14*b^3*c + 2*a*b^2*d + a^2*b*e + 5*a^3*f)*\text{Log}[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2])/b^{8/3})/(54*a^{10/3}) \end{aligned}$$

Maple [B] time = 0.014, size = 547, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^3,x)`

[Out]
$$\begin{aligned} & -4/9/(b*x^3+a)^2/b*x^5*f+1/9/a/(b*x^3+a)^2*x^5*e+2/9/a^2/(b*x^3+a)^2*b*x^5* \\ & d-5/9/a^3/(b*x^3+a)^2*b^2*x^5*c-5/18*a/(b*x^3+a)^2/b^2*x^2*f-1/18/(b*x^3+a) \\ & ^2/b*x^2*e+7/18*d/a*x^2/(b*x^3+a)^2-13/18/a^2/(b*x^3+a)^2*b*x^2*c-5/27/b^3/ \\ & (1/b*a)^{1/3}*ln(x+(1/b*a)^{1/3})*f-1/27/a/b^2/(1/b*a)^{1/3}*ln(x+(1/b*a)^{1/3})* \\ & e-2/27*d/a^2/b/(1/b*a)^{1/3}*ln(x+(1/b*a)^{1/3}))+14/27/a^3/(1/b*a)^{1/3} \\ & *ln(x+(1/b*a)^{1/3})*c+5/54/b^3/(1/b*a)^{1/3}*ln(x^2-(1/b*a)^{1/3}*x+(1/ \\ & b*a)^{2/3})*f+1/54/a/b^2/(1/b*a)^{1/3}*ln(x^2-(1/b*a)^{1/3}*x+(1/b*a)^{2/3})* \\ & e+1/27*d/a^2/b/(1/b*a)^{1/3}*ln(x^2-(1/b*a)^{1/3}*x+(1/b*a)^{2/3}))-7/27/a \\ & ^3/(1/b*a)^{1/3}*ln(x^2-(1/b*a)^{1/3}*x+(1/b*a)^{2/3})*c+5/27/b^3*3^{1/2}/(\\ & 1/b*a)^{1/3}*arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1))*f+1/27/a/b^2*3^{1/2} \\ & /(1/b*a)^{1/3}*arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1))*e+2/27*d/a^2*3^{1/2} \\ & /b/(1/b*a)^{1/3}*arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1))-14/27/a^3*3^{1/2} \\ & /(1/b*a)^{1/3}*arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1))*c-c/a^3/x \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [B] time = 1.50544, size = 2591, normalized size = 8.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/54*(54*a^3*b^4*c + 6*(14*a*b^6*c - 2*a^2*b^5*d - a^3*b^4*e + 4*a^4*b^3*f) \\ & *x^6 + 3*(49*a^2*b^5*c - 7*a^3*b^4*d + a^4*b^3*e + 5*a^5*b^2*f)*x^3 + 3*s \\ & \text{qrt}(1/3)*((14*a*b^6*c - 2*a^2*b^5*d - a^3*b^4*e - 5*a^4*b^3*f)*x^7 + 2*(14* \\ & a^2*b^5*c - 2*a^3*b^4*d - a^4*b^3*e - 5*a^5*b^2*f)*x^4 + (14*a^3*b^4*c - 2* \\ & a^4*b^3*d - a^5*b^2*e - 5*a^6*b*f)*x)*\text{sqrt}((-a*b^2)^{(1/3)}/a)*\log((2*b^2*x^3 \\ & - a*b + 3*\text{sqrt}(1/3)*(a*b*x + 2*(-a*b^2)^{(2/3)}*x^2 + (-a*b^2)^{(1/3)}*a)*\text{sqrt} \\ & ((-a*b^2)^{(1/3)}/a) - 3*(-a*b^2)^{(2/3)}*x)/(b*x^3 + a)) + ((14*b^5*c - 2*a*b^4*d \\ & - a^2*b^3*e - 5*a^3*b^2*f)*x^7 + 2*(14*a*b^4*c - 2*a^2*b^3*d - a^3*b^2*e \\ & - 5*a^4*b*f)*x^4 + (14*a^2*b^3*c - 2*a^3*b^2*d - a^4*b*e - 5*a^5*f)*x)*(- \\ & a*b^2)^{(2/3)}*\log(b^2*x^2 + (-a*b^2)^{(1/3)}*b*x + (-a*b^2)^{(2/3)}) - 2*((14*b^5*c \\ & - 2*a*b^4*d - a^2*b^3*e - 5*a^3*b^2*f)*x^7 + 2*(14*a*b^4*c - 2*a^2*b^3*d \\ & - a^3*b^2*e - 5*a^4*b*f)*x^4 + (14*a^2*b^3*c - 2*a^3*b^2*d - a^4*b*e - 5* \\ & a^5*f)*x)*(-a*b^2)^{(2/3)}*\log(b*x - (-a*b^2)^{(1/3)})]/(a^4*b^6*x^7 + 2*a^5*b^5*x^4 \\ & + a^6*b^4*x), -1/54*(54*a^3*b^4*c + 6*(14*a*b^6*c - 2*a^2*b^5*d - a^3 \\ & *b^4*e + 4*a^4*b^3*f)*x^6 + 3*(49*a^2*b^5*c - 7*a^3*b^4*d + a^4*b^3*e + 5*a \\ & ^5*b^2*f)*x^3 + 6*\text{sqrt}(1/3)*((14*a*b^6*c - 2*a^2*b^5*d - a^3*b^4*e - 5*a^4* \\ & b^3*f)*x^7 + 2*(14*a^2*b^5*c - 2*a^3*b^4*d - a^4*b^3*e - 5*a^5*b^2*f)*x^4 + \\ & (14*a^3*b^4*c - 2*a^4*b^3*d - a^5*b^2*e - 5*a^6*b*f)*x)*\text{sqrt}((-a*b^2)^{(1/3)}/a)*\text{arctan}(\text{sqrt}(1/3)*(2*b*x + (-a*b^2)^{(1/3)})*\text{sqrt}((-a*b^2)^{(1/3)}/a)/b) \\ & + ((14*b^5*c - 2*a*b^4*d - a^2*b^3*e - 5*a^3*b^2*f)*x^7 + 2*(14*a*b^4*c - 2 \\ & *a^2*b^3*d - a^3*b^2*e - 5*a^4*b*f)*x^4 + (14*a^2*b^3*c - 2*a^3*b^2*d - a^4 \\ & *b*e - 5*a^5*f)*x)*(-a*b^2)^{(2/3)}*\log(b^2*x^2 + (-a*b^2)^{(1/3)}*b*x + (-a*b^2)^{(2/3)}) - 2*((14*b^5*c - 2*a*b^4*d - a^2*b^3*e - 5*a^3*b^2*f)*x^7 + 2*(14 \\ & *a*b^4*c - 2*a^2*b^3*d - a^3*b^2*e - 5*a^4*b*f)*x^4 + (14*a^2*b^3*c - 2*a^3 \\ & *b^2*d - a^4*b*e - 5*a^5*f)*x)*(-a*b^2)^{(2/3)}*\log(b*x - (-a*b^2)^{(1/3)})]/(a^4*b^6*x^7 + 2*a^5*b^5*x^4 + a^6*b^4*x)] \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**2/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.11206, size = 525, normalized size = 1.73

$$-\frac{c}{a^3x} + \frac{\left(14b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 2ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 5a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^4b^2} + \frac{\sqrt{3}\left(14(-ab^2)^{\frac{2}{3}}b^3c - 2\right)}{27a^4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^3,x, algorithm="giac")

[Out]
$$-c/(a^3x) + 1/27*(14*b^3*c*(-a/b)^{(1/3)} - 2*a*b^2*d*(-a/b)^{(1/3)} - 5*a^3*f*(-a/b)^{(1/3)} - a^2*b*(-a/b)^{(1/3)}*e)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)})))/(a^4*b^2) + 1/27*\text{sqrt}(3)*(14*(-a*b^2)^{(2/3)}*b^3*c - 2*(-a*b^2)^{(2/3)}*a*b^2*d - 5*(-a*b^2)^{(2/3)}*a^3*f - (-a*b^2)^{(2/3)}*a^2*b*e)*\arctan(1/3*\text{sqrt}(3)*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(a^4*b^4) - 1/18*(10*b^4*c*x^5 - 4*a*b^3*d*x^5 + 8*a^3*b*f*x^5 - 2*a^2*b^2*x^5*e + 13*a*b^3*c*x^2 - 7*a^2*b^2*d*x^2 + 5*a^4*f*x^2 + a^3*b*x^2*e)/((b*x^3 + a)^2*a^3*b^2) - 1/54*(14*(-a*b^2)^{(2/3)}*b^3*c - 2*(-a*b^2)^{(2/3)}*a*b^2*d - 5*(-a*b^2)^{(2/3)}*a^3*f - (-a*b^2)^{(2/3)}*a^2*b*e)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a^4*b^4)$$

$$3.296 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)^3} dx$$

Optimal. Leaf size=301

$$\frac{x(-a^2be + 7a^3f - 5ab^2d + 11b^3c)}{18a^3b^2(a+bx^3)} - \frac{x(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^2b^2(a+bx^3)^2} + \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(-a^2be - 2a^3f - 5ab^2d + 11b^3c)}{54a^{11/3}b^{7/3}}$$

[Out] $-c/(2*a^3*x^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^2*b^2*(a + b*x^3)^2) - ((11*b^3*c - 5*a*b^2*d - a^2*b*e + 7*a^3*f)*x)/(18*a^3*b^2*(a + b*x^3)) + ((20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)*b^(7/3)) - ((20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*b^(7/3)) + ((20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(11/3)*b^(7/3))$

Rubi [A] time = 0.329154, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {1829, 1484, 453, 200, 31, 634, 617, 204, 628}

$$\frac{x(-a^2be + 7a^3f - 5ab^2d + 11b^3c)}{18a^3b^2(a+bx^3)} - \frac{x(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^2b^2(a+bx^3)^2} + \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(-a^2be - 2a^3f - 5ab^2d + 11b^3c)}{54a^{11/3}b^{7/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^3), x]

[Out] $-c/(2*a^3*x^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^2*b^2*(a + b*x^3)^2) - ((11*b^3*c - 5*a*b^2*d - a^2*b*e + 7*a^3*f)*x)/(18*a^3*b^2*(a + b*x^3)) + ((20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)*b^(7/3)) - ((20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*b^(7/3)) + ((20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(11/3)*b^(7/3))$

Rule 1829

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^

```
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i], Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1484

```
Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_)*((d_) + (e
_)*(x_)^(n_)]^(q_), x_Symbol] := Simp[(-d)^(m - Mod[m, n])/n - 1*(c*d^2
- b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*(d + e*x^n)^(q + 1))/(n*e^(2*p + (m -
Mod[m, n])/n)*(q + 1)), x] + Dist[(-d)^(m - Mod[m, n])/n - 1/(n*e^(2*p)*
(q + 1)), Int[x^m*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1*(n*(-d)^(-(m
- Mod[m, n])/n) + 1)*e^(2*p)*(q + 1)*(a + b*x^n + c*x^(2*n))^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^((m - Mod[m, n])/n)*x^(m - Mod[m, n]))*(d*(Mod[m, n]
+ 1) + e*(Mod[m, n] + n*(q + 1) + 1)*x^n))]/(d + e*x^n)], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m, 0]
```

Rule 453

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_)]^(p_)*((c_) + (d_)*(x_)^(n
_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 200

```
Int[((a_) + (b_)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(p_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx^3 + ex^6 + fx^9}{x^3(a + bx^3)^3} dx &= -\frac{(b^3c - ab^2d + a^2be - a^3f)x}{6a^2b^2(a + bx^3)^2} - \frac{\int \frac{-6b^3c + b\left(\frac{5b^3c}{a} - 5b^2d - abe + a^2f\right)x^3 - 6ab^2fx^6}{x^3(a + bx^3)^2} dx}{6ab^3} \\
&= -\frac{(b^3c - ab^2d + a^2be - a^3f)x}{6a^2b^2(a + bx^3)^2} - \frac{(11b^3c - 5ab^2d - a^2be + 7a^3f)x}{18a^3b^2(a + bx^3)} + \frac{\int \frac{18ab^5c - 2b^3(11b^3c - 5ab^2d - a^2be + 7a^3f)x}{x^3(a + bx^3)}}{18a^3b^5} \\
&= -\frac{c}{2a^3x^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)x}{6a^2b^2(a + bx^3)^2} - \frac{(11b^3c - 5ab^2d - a^2be + 7a^3f)x}{18a^3b^2(a + bx^3)} - \frac{(20b^3c - 5ab^2d - a^2be + 7a^3f)x}{18a^3b^2(a + bx^3)} \\
&= -\frac{c}{2a^3x^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)x}{6a^2b^2(a + bx^3)^2} - \frac{(11b^3c - 5ab^2d - a^2be + 7a^3f)x}{18a^3b^2(a + bx^3)} - \frac{(20b^3c - 5ab^2d - a^2be + 7a^3f)x}{18a^3b^2(a + bx^3)} \\
&= -\frac{c}{2a^3x^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)x}{6a^2b^2(a + bx^3)^2} - \frac{(11b^3c - 5ab^2d - a^2be + 7a^3f)x}{18a^3b^2(a + bx^3)} - \frac{(20b^3c - 5ab^2d - a^2be + 7a^3f)x}{18a^3b^2(a + bx^3)} \\
&= -\frac{c}{2a^3x^2} - \frac{(b^3c - ab^2d + a^2be - a^3f)x}{6a^2b^2(a + bx^3)^2} - \frac{(11b^3c - 5ab^2d - a^2be + 7a^3f)x}{18a^3b^2(a + bx^3)} - \frac{(20b^3c - 5ab^2d - a^2be + 7a^3f)x}{18a^3b^2(a + bx^3)} + \frac{(20b^3c - 5ab^2d - a^2be + 7a^3f)x}{18a^3b^2(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.213489, size = 283, normalized size = 0.94

$$\frac{9a^{5/3}x(-a^2be + a^3f + ab^2d - b^3c)}{b^2(a + bx^3)^2} - \frac{3a^{2/3}x(-a^2be + 7a^3f - 5ab^2d + 11b^3c)}{b^2(a + bx^3)} - \frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx + b^{2/3}x^2}\right)(a^2be + 2a^3f + 5ab^2d - 20b^3c)}{b^{7/3}} + \frac{2\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)(a^2be + 2a^3f + 5ab^2d - 20b^3c)}{b^{7/3}}$$

$$54a^{11/3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^3), x]

[Out] ((-27*a^(2/3)*c)/x^2 + (9*a^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(b^2*(a + b*x^3)^2) - (3*a^(2/3)*(11*b^3*c - 5*a*b^2*d - a^2*b*e + 7*a^3*f)

$$\begin{aligned} &) * x) / (b^2 * (a + b * x^3)) + (2 * \text{Sqrt}[3] * (20 * b^3 * c - 5 * a * b^2 * d - a^2 * b * e - 2 * a^3 * \\ & * f) * \text{ArcTan}[(1 - (2 * b^{(1/3)} * x) / a^{(1/3)}) / \text{Sqrt}[3]]) / b^{(7/3)} + (2 * (-20 * b^3 * c + \\ & 5 * a * b^2 * d + a^2 * b * e + 2 * a^3 * f) * \text{Log}[a^{(1/3)} + b^{(1/3)} * x]) / b^{(7/3)} - ((-20 * b^3 * c + \\ & 5 * a * b^2 * d + a^2 * b * e + 2 * a^3 * f) * \text{Log}[a^{(2/3)} - a^{(1/3)} * b^{(1/3)} * x + b^{(2/3)} * x^2]) / b^{(7/3)}) / (54 * a^{(11/3)}) \end{aligned}$$

Maple [B] time = 0.013, size = 539, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^3,x)`

[Out]
$$\begin{aligned} & -7/18/(b*x^3+a)^2/b*x^4*f+1/18/a/(b*x^3+a)^2*x^4*e+5/18/a^2/(b*x^3+a)^2*b*x \\ & ^4*d-11/18/a^3/(b*x^3+a)^2*b^2*x^4*c-2/9*a/(b*x^3+a)^2/b^2*x*f-1/9/(b*x^3+a \\ &)^2/b*x*e+4/9/a/(b*x^3+a)^2*x*d-7/9/a^2/(b*x^3+a)^2*b*x*c+2/27/b^3/(1/b*a)^ \\ & (2/3)*\ln(x+(1/b*a)^{(1/3)})*f+1/27/a/b^2/(1/b*a)^{(2/3)*\ln(x+(1/b*a)^{(1/3)})*e+ \\ & 5/27/a^2/b/(1/b*a)^{(2/3)*\ln(x+(1/b*a)^{(1/3)})*d-20/27/a^3/(1/b*a)^{(2/3)*\ln(x \\ & +(1/b*a)^{(1/3)})*c-1/27/b^3/(1/b*a)^{(2/3)*\ln(x^2-(1/b*a)^{(1/3)*x+(1/b*a)^{(2/ \\ & 3))*f-1/54/a/b^2/(1/b*a)^{(2/3)*\ln(x^2-(1/b*a)^{(1/3)*x+(1/b*a)^{(2/3))*e-5/54 \\ & /a^2/b/(1/b*a)^{(2/3)*\ln(x^2-(1/b*a)^{(1/3)*x+(1/b*a)^{(2/3))*d+10/27/a^3/(1/b \\ & *a)^{(2/3)*\ln(x^2-(1/b*a)^{(1/3)*x+(1/b*a)^{(2/3))*c+2/27/b^3/(1/b*a)^{(2/3)*3^ \\ & (1/2)*\arctan(1/3*3^{(1/2)*(2/(1/b*a)^{(1/3)*x-1}))*f+1/27/a/b^2/(1/b*a)^{(2/3)* \\ & 3^{(1/2)*\arctan(1/3*3^{(1/2)*(2/(1/b*a)^{(1/3)*x-1}))*e+5/27/a^2/b/(1/b*a)^{(2/3) \\ &)*3^{(1/2)*\arctan(1/3*3^{(1/2)*(2/(1/b*a)^{(1/3)*x-1}))*d-20/27/a^3/(1/b*a)^{(2/ \\ & 3)*3^{(1/2)*\arctan(1/3*3^{(1/2)*(2/(1/b*a)^{(1/3)*x-1}))*c-1/2*c/a^3/x^2} \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [B] time = 1.84402, size = 2618, normalized size = 8.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/54*(27*a^4*b^3*c + 3*(20*a^2*b^5*c - 5*a^3*b^4*d - a^4*b^3*e + 7*a^5*b^2*f)*x^6 + 6*(16*a^3*b^4*c - 4*a^4*b^3*d + a^5*b^2*e + 2*a^6*b*f)*x^3 + 3*\sqrt[3]{1/3}*((20*a*b^6*c - 5*a^2*b^5*d - a^3*b^4*e - 2*a^4*b^3*f)*x^8 + 2*(20*a^2*b^5*c - 5*a^3*b^4*d - a^4*b^3*e - 2*a^5*b^2*f)*x^5 + (20*a^3*b^4*c - 5*a^4*b^3*d - a^5*b^2*e - 2*a^6*b*f)*x^2)*\sqrt{-(a^2*b)^{1/3}/b}*\log((2*a*b*x^3 - 3*(a^2*b)^{1/3}*a*x - a^2 + 3*\sqrt[3]{1/3}*(2*a*b*x^2 + (a^2*b)^{2/3})*x - (a^2*b)^{1/3}*a)*\sqrt{-(a^2*b)^{1/3}/b})/(b*x^3 + a) - ((20*b^5*c - 5*a*b^4*d - a^2*b^3*e - 2*a^3*b^2*f)*x^8 + 2*(20*a*b^4*c - 5*a^2*b^3*d - a^3*b^2*e - 2*a^4*b*f)*x^5 + (20*a^2*b^3*c - 5*a^3*b^2*d - a^4*b*e - 2*a^5*f)*x^2)*(a^2*b)^{2/3}*\log(a*b*x^2 - (a^2*b)^{2/3}*x + (a^2*b)^{1/3}*a) + 2*((20*b^5*c - 5*a*b^4*d - a^2*b^3*e - 2*a^3*b^2*f)*x^8 + 2*(20*a*b^4*c - 5*a^2*b^3*d - a^3*b^2*e - 2*a^4*b*f)*x^5 + (20*a^2*b^3*c - 5*a^3*b^2*d - a^4*b*e - 2*a^5*f)*x^2)*(a^2*b)^{2/3}*\log(a*b*x + (a^2*b)^{2/3})/(a^5*b^5*x^8 + 2*a^6*b^4*x^5 + a^7*b^3*x^2), -1/54*(27*a^4*b^3*c + 3*(20*a^2*b^5*c - 5*a^3*b^4*d - a^4*b^3*e + 7*a^5*b^2*f)*x^6 + 6*(16*a^3*b^4*c - 4*a^4*b^3*d + a^5*b^2*e + 2*a^6*b*f)*x^3 + 6*\sqrt[3]{1/3}*((20*a*b^6*c - 5*a^2*b^5*d - a^3*b^4*e - 2*a^4*b^3*f)*x^8 + 2*(20*a^2*b^5*c - 5*a^3*b^4*d - a^4*b^3*e - 2*a^5*b^2*f)*x^5 + (20*a^3*b^4*c - 5*a^4*b^3*d - a^5*b^2*e - 2*a^6*b*f)*x^2)*\sqrt{(a^2*b)^{1/3}/b}*\arctan(\sqrt[3]{1/3}*(2*(a^2*b)^{2/3}*x - (a^2*b)^{1/3}*a)*\sqrt{(a^2*b)^{1/3}/b})/a^2) - ((20*b^5*c - 5*a*b^4*d - a^2*b^3*e - 2*a^3*b^2*f)*x^8 + 2*(20*a*b^4*c - 5*a^2*b^3*d - a^3*b^2*e - 2*a^4*b*f)*x^5 + (20*a^2*b^3*c - 5*a^3*b^2*d - a^4*b*e - 2*a^5*f)*x^2)*(a^2*b)^{2/3}*\log(a*b*x^2 - (a^2*b)^{2/3}*x + (a^2*b)^{1/3}*a) + 2*((20*b^5*c - 5*a*b^4*d - a^2*b^3*e - 2*a^3*b^2*f)*x^8 + 2*(20*a*b^4*c - 5*a^2*b^3*d - a^3*b^2*e - 2*a^4*b*f)*x^5 + (20*a^2*b^3*c - 5*a^3*b^2*d - a^4*b*e - 2*a^5*f)*x^2)*(a^2*b)^{2/3}*\log(a*b*x + (a^2*b)^{2/3})/(a^5*b^5*x^8 + 2*a^6*b^4*x^5 + a^7*b^3*x^2)] \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**3/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.084, size = 486, normalized size = 1.61

$$\frac{(20b^3c - 5ab^2d - 2a^3f - a^2be)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^4b^2} - \frac{\sqrt{3}\left(20(-ab^2)^{\frac{1}{3}}b^3c - 5(-ab^2)^{\frac{1}{3}}ab^2d - 2(-ab^2)^{\frac{1}{3}}a^3f - (-ab^2)^{\frac{1}{3}}a^2be\right)}{27a^4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^3,x, algorithm="giac")

[Out] $\frac{1}{27} \cdot (20b^3c - 5a^2b^2d - 2a^3f - a^2be) \cdot \left(-\frac{a}{b}\right)^{\frac{1}{3}} \cdot \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right) / (a^4b^2) - \frac{1}{27} \cdot \sqrt{3} \cdot (20(-ab^2)^{\frac{1}{3}}b^3c - 5(-ab^2)^{\frac{1}{3}}ab^2d - 2(-ab^2)^{\frac{1}{3}}a^3f - (-ab^2)^{\frac{1}{3}}a^2be) / (27a^4b^3)$

$$3.297 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)^3} dx$$

Optimal. Leaf size=317

$$\frac{x^2(2a^2be + a^3f - 5ab^2d + 8b^3c)}{9a^4b(a + bx^3)} + \frac{x^2(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^3b(a + bx^3)^2} + \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(2a^2be + a^3f - 14ab^2d + 8b^3c)}{54a^{13/3}b^{5/3}}$$

[Out] $-c/(4*a^3*x^4) + (3*b*c - a*d)/(a^4*x) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^3*b*(a + b*x^3)^2) + ((8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*x^2)/(9*a^4*b*(a + b*x^3)) - ((35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(13/3)*b^(5/3)) - ((35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(13/3)*b^(5/3)) + ((35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(13/3)*b^(5/3))$

Rubi [A] time = 0.369523, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {1829, 1484, 1488, 292, 31, 634, 617, 204, 628}

$$\frac{x^2(2a^2be + a^3f - 5ab^2d + 8b^3c)}{9a^4b(a + bx^3)} + \frac{x^2(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^3b(a + bx^3)^2} + \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(2a^2be + a^3f - 14ab^2d + 8b^3c)}{54a^{13/3}b^{5/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^3), x]

[Out] $-c/(4*a^3*x^4) + (3*b*c - a*d)/(a^4*x) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^3*b*(a + b*x^3)^2) + ((8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*x^2)/(9*a^4*b*(a + b*x^3)) - ((35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(13/3)*b^(5/3)) - ((35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(13/3)*b^(5/3)) + ((35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(13/3)*b^(5/3))$

Rule 1829

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^

```
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1484

```
Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_)*((d_) + (e
_)*(x_)^(n_)]^(q_), x_Symbol] := Simp[((-d)^((m - Mod[m, n])/n - 1)*(c*d^2
- b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*(d + e*x^n)^(q + 1))/(n*e^(2*p + (m -
Mod[m, n])/n)*(q + 1)), x] + Dist[(-d)^((m - Mod[m, n])/n - 1)/(n*e^(2*p)*
(q + 1)), Int[x^m*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1*(n*(-d)^(-(m
- Mod[m, n])/n) + 1)*e^(2*p)*(q + 1)*(a + b*x^n + c*x^(2*n))^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^((m - Mod[m, n])/n)*x^(m - Mod[m, n])))*(d*(Mod[m, n]
+ 1) + e*(Mod[m, n] + n*(q + 1) + 1)*x^n))]/(d + e*x^n)], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m, 0]
```

Rule 1488

```
Int[((f_)*(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_)*
(d_) + (e_)*(x_)^(n_)]^(q_), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx^3 + ex^6 + fx^9}{x^5 (a + bx^3)^3} dx &= \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6a^3b(a + bx^3)^2} - \frac{\int \frac{-6b^3c + 6b^3\left(\frac{bc}{a} - d\right)x^3 - 2b^2\left(\frac{2b^3c}{a^2} - \frac{2b^2d}{a} + 2be + af\right)x^6}{x^5(a + bx^3)^2} dx}{6ab^3} \\
&= \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6a^3b(a + bx^3)^2} + \frac{(8b^3c - 5ab^2d + 2a^2be + a^3f)x^2}{9a^4b(a + bx^3)} - \frac{\int \frac{-18a^2b^5c + 18ab^5(2bc - ad)x^3 - 18b^5(3bc - ad)}{x^5(a + bx^3)^2} dx}{18ab^3} \\
&= \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6a^3b(a + bx^3)^2} + \frac{(8b^3c - 5ab^2d + 2a^2be + a^3f)x^2}{9a^4b(a + bx^3)} - \frac{\int \left(-\frac{18ab^5c}{x^5} + \frac{18b^5(3bc - ad)}{x^2} \right) dx}{18ab^3} \\
&= -\frac{c}{4a^3x^4} + \frac{3bc - ad}{a^4x} + \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6a^3b(a + bx^3)^2} + \frac{(8b^3c - 5ab^2d + 2a^2be + a^3f)x^2}{9a^4b(a + bx^3)} + \frac{3b^5(3bc - ad)}{18ab^3} \\
&= -\frac{c}{4a^3x^4} + \frac{3bc - ad}{a^4x} + \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6a^3b(a + bx^3)^2} + \frac{(8b^3c - 5ab^2d + 2a^2be + a^3f)x^2}{9a^4b(a + bx^3)} - \frac{3b^5(3bc - ad)}{18ab^3} \\
&= -\frac{c}{4a^3x^4} + \frac{3bc - ad}{a^4x} + \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6a^3b(a + bx^3)^2} + \frac{(8b^3c - 5ab^2d + 2a^2be + a^3f)x^2}{9a^4b(a + bx^3)} - \frac{3b^5(3bc - ad)}{18ab^3} \\
&= -\frac{c}{4a^3x^4} + \frac{3bc - ad}{a^4x} + \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6a^3b(a + bx^3)^2} + \frac{(8b^3c - 5ab^2d + 2a^2be + a^3f)x^2}{9a^4b(a + bx^3)} - \frac{3b^5(3bc - ad)}{18ab^3} \\
&= -\frac{c}{4a^3x^4} + \frac{3bc - ad}{a^4x} + \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6a^3b(a + bx^3)^2} + \frac{(8b^3c - 5ab^2d + 2a^2be + a^3f)x^2}{9a^4b(a + bx^3)} - \frac{3b^5(3bc - ad)}{18ab^3}
\end{aligned}$$

Mathematica [A] time = 0.232386, size = 303, normalized size = 0.96

$$\frac{12\sqrt[3]{a}x^2(2a^2be + a^3f - 5ab^2d + 8b^3c)}{b(a + bx^3)} - \frac{18a^4/3x^2(-a^2be + a^3f + ab^2d - b^3c)}{b(a + bx^3)^2} + \frac{2\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx + b^{2/3}x^2}\right)(2a^2be + a^3f - 14ab^2d + 35b^3c)}{b^{5/3}} - \frac{4\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)(2a^2be + a^3f - 14ab^2d + 35b^3c)}{b^{5/3}}$$

$108a^{13/3}$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^3), x]

```
[Out] ((-27*a^(4/3)*c)/x^4 - (108*a^(1/3)*(-3*b*c + a*d))/x - (18*a^(4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(b*(a + b*x^3)^2) + (12*a^(1/3)*(8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*x^2)/(b*(a + b*x^3)) - (4*sqrt(3)*(35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt(3)])/b^(5/3) - (4*(35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(5/3) + (2*(35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3))/(108*a^(13/3))
```

Maple [B] time = 0.013, size = 574, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^3,x)
```

```
[Out] 1/9/a/(b*x^3+a)^2*x^5*f+2/9/a^2/(b*x^3+a)^2*x^5*b*e-5/9/a^3/(b*x^3+a)^2*x^5*b^2*d+8/9/a^4/(b*x^3+a)^2*x^5*b^3*c-1/18/(b*x^3+a)^2/b*x^2*f+7/18/a/(b*x^3+a)^2*x^2*e-13/18/a^2/(b*x^3+a)^2*b*x^2*d+19/18/a^3/(b*x^3+a)^2*b^2*x^2*c-1/27/a/b^2/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*f-2/27/a^2/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*e+14/27/a^3/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*d-35/27/a^4*b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*c+1/54/a/b^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f+1/27/a^2/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e-7/27/a^3/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d+35/54/a^4*b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+1/27/a/b^2*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f+2/27/a^2/b*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e-14/27/a^3*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d+35/27/a^4*b*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c-1/4*c/a^3/x^4-d/a^3/x+3/a^4/x*b*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^3,x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [B] time = 2.07189, size = 2773, normalized size = 8.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^3,x, algorithm="fricas")

[Out] [1/108*(12*(35*a*b^6*c - 14*a^2*b^5*d + 2*a^3*b^4*e + a^4*b^3*f)*x^9 - 27*a^4*b^3*c + 3*(245*a^2*b^5*c - 98*a^3*b^4*d + 14*a^4*b^3*e - 2*a^5*b^2*f)*x^6 + 54*(5*a^3*b^4*c - 2*a^4*b^3*d)*x^3 + 6*sqrt(1/3)*((35*a*b^6*c - 14*a^2*b^5*d + 2*a^3*b^4*e + a^4*b^3*f)*x^10 + 2*(35*a^2*b^5*c - 14*a^3*b^4*d + 2*a^4*b^3*e + a^5*b^2*f)*x^7 + (35*a^3*b^4*c - 14*a^4*b^3*d + 2*a^5*b^2*e + a^6*b*f)*x^4)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + 2*((35*b^5*c - 14*a*b^4*d + 2*a^2*b^3*e + a^3*b^2*f)*x^10 + 2*(35*a*b^4*c - 14*a^2*b^3*d + 2*a^3*b^2*e + a^4*b*f)*x^7 + (35*a^2*b^3*c - 14*a^3*b^2*d + 2*a^4*b*e + a^5*f)*x^4)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 4*((35*b^5*c - 14*a*b^4*d + 2*a^2*b^3*e + a^3*b^2*f)*x^10 + 2*(35*a*b^4*c - 14*a^2*b^3*d + 2*a^3*b^2*e + a^4*b*f)*x^7 + (35*a^2*b^3*c - 14*a^3*b^2*d + 2*a^4*b*e + a^5*f)*x^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^5*x^10 + 2*a^6*b^4*x^7 + a^7*b^3*x^4), 1/108*(12*(35*a*b^6*c - 14*a^2*b^5*d + 2*a^3*b^4*e + a^4*b^3*f)*x^9 - 27*a^4*b^3*c + 3*(245*a^2*b^5*c - 98*a^3*b^4*d + 14*a^4*b^3*e - 2*a^5*b^2*f)*x^6 + 54*(5*a^3*b^4*c - 2*a^4*b^3*d)*x^3 + 12*sqrt(1/3)*((35*a*b^6*c - 14*a^2*b^5*d + 2*a^3*b^4*e + a^4*b^3*f)*x^10 + 2*(35*a^2*b^5*c - 14*a^3*b^4*d + 2*a^4*b^3*e + a^5*b^2*f)*x^7 + (35*a^3*b^4*c - 14*a^4*b^3*d + 2*a^5*b^2*e + a^6*b*f)*x^4)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 2*((35*b^5*c - 14*a*b^4*d + 2*a^2*b^3*e + a^3*b^2*f)*x^10 + 2*(35*a*b^4*c - 14*a^2*b^3*d + 2*a^3*b^2*e + a^4*b*f)*x^7 + (35*a^2*b^3*c - 14*a^3*b^2*d + 2*a^4*b*e + a^5*f)*x^4)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 4*((35*b^5*c - 14*a*b^4*d + 2*a^2*b^3*e + a^3*b^2*f)*x^10 + 2*(35*a*b^4*c - 14*a^2*b^3*d + 2*a^3*b^2*e + a^4*b*f)*x^7 + (35*a^2*b^3*c - 14*a^3*b^2*d + 2*a^4*b*e + a^5*f)*x^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^5*x^10 + 2*a^6*b^4*x^7 + a^7*b^3*x^4)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**5/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.10135, size = 547, normalized size = 1.73

$$\frac{\left(35b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 14ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 2a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right) + \sqrt{3}\left(35(-ab^2)^{\frac{2}{3}}b^3c - 14(-ab^2)^{\frac{2}{3}}b^2d + (-ab^2)^{\frac{2}{3}}a^3f + 2(-ab^2)^{\frac{2}{3}}a^2be\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x + (-a/b)^{\frac{1}{3}})\right)}{27a^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^3,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/27*(35*b^3*c*(-a/b)^{(1/3)} - 14*a*b^2*d*(-a/b)^{(1/3)} + a^3*f*(-a/b)^{(1/3)} \\ & + 2*a^2*b*(-a/b)^{(1/3)}*e)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/ (a^5*b) \\ & - 1/27*\text{sqrt}(3)*(35*(-a*b^2)^{(2/3)}*b^3*c - 14*(-a*b^2)^{(2/3)}*a*b^2*d + (-a*b \\ & ^2)^{(2/3)}*a^3*f + 2*(-a*b^2)^{(2/3)}*a^2*b*e)*\arctan(1/3*\text{sqrt}(3)*(2*x + (-a/b \\ &)^{(1/3)})/(-a/b)^{(1/3)})/ (a^5*b^3) + 1/18*(16*b^4*c*x^5 - 10*a*b^3*d*x^5 + 2* \\ & a^3*b*f*x^5 + 4*a^2*b^2*x^5*e + 19*a*b^3*c*x^2 - 13*a^2*b^2*d*x^2 - a^4*f*x \\ & ^2 + 7*a^3*b*x^2*e)/((b*x^3 + a)^2*a^4*b) + 1/54*(35*(-a*b^2)^{(2/3)}*b^3*c - \\ & 14*(-a*b^2)^{(2/3)}*a*b^2*d + (-a*b^2)^{(2/3)}*a^3*f + 2*(-a*b^2)^{(2/3)}*a^2*b* \\ & e)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/ (a^5*b^3) + 1/4*(12*b*c*x^3 - 4 \\ & *a*d*x^3 - a*c)/ (a^4*x^4) \end{aligned}$$

$$3.298 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)^3} dx$$

Optimal. Leaf size=316

$$\frac{x(5a^2be + a^3f - 11ab^2d + 17b^3c)}{18a^4b(a + bx^3)} + \frac{x(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^3b(a + bx^3)^2} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(5a^2be + a^3f - 20ab^2)}{54a^{14/3}b^{4/3}}$$

[Out] $-c/(5*a^3*x^5) + (3*b*c - a*d)/(2*a^4*x^2) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^3*b*(a + b*x^3)^2) + ((17*b^3*c - 11*a*b^2*d + 5*a^2*b*e + a^3*f)*x)/(18*a^4*b*(a + b*x^3)) - ((44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(14/3)*b^(4/3)) + ((44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x]/(27*a^(14/3)*b^(4/3)) - ((44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(54*a^(14/3)*b^(4/3))$

Rubi [A] time = 0.368028, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {1829, 1484, 1488, 200, 31, 634, 617, 204, 628}

$$\frac{x(5a^2be + a^3f - 11ab^2d + 17b^3c)}{18a^4b(a + bx^3)} + \frac{x(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^3b(a + bx^3)^2} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(5a^2be + a^3f - 20ab^2)}{54a^{14/3}b^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^3), x]

[Out] $-c/(5*a^3*x^5) + (3*b*c - a*d)/(2*a^4*x^2) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^3*b*(a + b*x^3)^2) + ((17*b^3*c - 11*a*b^2*d + 5*a^2*b*e + a^3*f)*x)/(18*a^4*b*(a + b*x^3)) - ((44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(14/3)*b^(4/3)) + ((44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x]/(27*a^(14/3)*b^(4/3)) - ((44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(54*a^(14/3)*b^(4/3))$

Rule 1829

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^


```
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i], Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1484

```
Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_)*((d_) + (e
_)*(x_)^(n_)]^(q_), x_Symbol] := Simp[(-d)^(m - Mod[m, n])/n - 1*(c*d^2
- b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*(d + e*x^n)^(q + 1))/(n*e^(2*p + (m -
Mod[m, n])/n)*(q + 1)), x] + Dist[(-d)^(m - Mod[m, n])/n - 1/(n*e^(2*p)*
(q + 1)), Int[x^m*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1*(n*(-d)^(-(m
- Mod[m, n])/n) + 1)*e^(2*p)*(q + 1)*(a + b*x^n + c*x^(2*n))^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^((m - Mod[m, n])/n)*x^(m - Mod[m, n])))*(d*(Mod[m, n]
+ 1) + e*(Mod[m, n] + n*(q + 1) + 1)*x^n))]/(d + e*x^n)], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m, 0]
```

Rule 1488

```
Int[((f_)*(x_))^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_)*
(d_) + (e_)*(x_)^(n_)]^(q_), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 200

```
Int[((a_) + (b_)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_)*(x_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx^3 + ex^6 + fx^9}{x^6(a + bx^3)^3} dx &= \frac{(b^3c - ab^2d + a^2be - a^3f)x}{6a^3b(a + bx^3)^2} - \frac{\int \frac{-6b^3c + 6b^3\left(\frac{bc}{a} - d\right)x^3 - b^2\left(\frac{5b^3c}{a^2} - \frac{5b^2d}{a} + 5be + af\right)x^6}{x^6(a + bx^3)^2} dx}{6ab^3} \\
&= \frac{(b^3c - ab^2d + a^2be - a^3f)x}{6a^3b(a + bx^3)^2} + \frac{(17b^3c - 11ab^2d + 5a^2be + a^3f)x}{18a^4b(a + bx^3)} - \frac{\int \frac{-18a^2b^5c + 18ab^5(2bc - ad)x^3}{x^6(a + bx^3)^2} dx}{18a^4b} \\
&= \frac{(b^3c - ab^2d + a^2be - a^3f)x}{6a^3b(a + bx^3)^2} + \frac{(17b^3c - 11ab^2d + 5a^2be + a^3f)x}{18a^4b(a + bx^3)} - \frac{\int \left(-\frac{18ab^5c}{x^6} + \frac{18b^5(3bc - ad)}{x^3} \right) dx}{18a^4b} \\
&= -\frac{c}{5a^3x^5} + \frac{3bc - ad}{2a^4x^2} + \frac{(b^3c - ab^2d + a^2be - a^3f)x}{6a^3b(a + bx^3)^2} + \frac{(17b^3c - 11ab^2d + 5a^2be + a^3f)x}{18a^4b(a + bx^3)} + \dots \\
&= -\frac{c}{5a^3x^5} + \frac{3bc - ad}{2a^4x^2} + \frac{(b^3c - ab^2d + a^2be - a^3f)x}{6a^3b(a + bx^3)^2} + \frac{(17b^3c - 11ab^2d + 5a^2be + a^3f)x}{18a^4b(a + bx^3)} + \dots \\
&= -\frac{c}{5a^3x^5} + \frac{3bc - ad}{2a^4x^2} + \frac{(b^3c - ab^2d + a^2be - a^3f)x}{6a^3b(a + bx^3)^2} + \frac{(17b^3c - 11ab^2d + 5a^2be + a^3f)x}{18a^4b(a + bx^3)} + \dots \\
&= -\frac{c}{5a^3x^5} + \frac{3bc - ad}{2a^4x^2} + \frac{(b^3c - ab^2d + a^2be - a^3f)x}{6a^3b(a + bx^3)^2} + \frac{(17b^3c - 11ab^2d + 5a^2be + a^3f)x}{18a^4b(a + bx^3)} + \dots \\
&= -\frac{c}{5a^3x^5} + \frac{3bc - ad}{2a^4x^2} + \frac{(b^3c - ab^2d + a^2be - a^3f)x}{6a^3b(a + bx^3)^2} + \frac{(17b^3c - 11ab^2d + 5a^2be + a^3f)x}{18a^4b(a + bx^3)} + \dots
\end{aligned}$$

Mathematica [A] time = 0.233369, size = 299, normalized size = 0.95

$$\frac{-\frac{45a^{5/3}x(-a^2be + a^3f + ab^2d - b^3c)}{b(a + bx^3)^2} + \frac{15a^{2/3}x(5a^2be + a^3f - 11ab^2d + 17b^3c)}{b(a + bx^3)} - \frac{5 \log\left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx + b^{2/3}x^2}\right)(5a^2be + a^3f - 20ab^2d + 44b^3c)}{b^{4/3}} + \frac{10 \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)(5a^2be + a^3f - 20ab^2d + 44b^3c)}{b^{4/3}}}{270a^{14/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^3), x]

```
[Out] ((-54*a^(5/3)*c)/x^5 - (135*a^(2/3)*(-3*b*c + a*d))/x^2 - (45*a^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(b*(a + b*x^3)^2) + (15*a^(2/3)*(17*b^3*c - 11*a*b^2*d + 5*a^2*b*e + a^3*f)*x)/(b*(a + b*x^3)) - (10*Sqrt[3]*(44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(4/3) + (10*(44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(4/3) - (5*(44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(4/3))/(270*a^(14/3))
```

Maple [B] time = 0.017, size = 566, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^3,x)
```

```
[Out] 1/18/a/(b*x^3+a)^2*x^4*f+5/18/a^2/(b*x^3+a)^2*x^4*b*e-11/18/a^3/(b*x^3+a)^2*x^4*b^2*d+17/18/a^4/(b*x^3+a)^2*x^4*b^3*c-1/9/(b*x^3+a)^2/b*x*f+4/9/a/(b*x^3+a)^2*x*e-7/9/a^2/(b*x^3+a)^2*b*x*d+10/9/a^3/(b*x^3+a)^2*b^2*x*c+1/27/a/b^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*f+5/27/a^2/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*e-20/27/a^3/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d+44/27/a^4*b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c-1/54/a/b^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f-5/54/a^2/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e+10/27/a^3/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d-2/27/a^4*b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+1/27/a/b^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f+5/27/a^2/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e-20/27/a^3/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d+44/27/a^4*b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c-1/5*c/a^3/x^5-1/2*d/a^3/x^2+3/2/a^4/x^2*b*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^3,x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [B] time = 1.83024, size = 2785, normalized size = 8.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/270*(15*(44*a^2*b^5*c - 20*a^3*b^4*d + 5*a^4*b^3*e + a^5*b^2*f)*x^9 - 54 \\ & *a^5*b^2*c + 6*(176*a^3*b^4*c - 80*a^4*b^3*d + 20*a^5*b^2*e - 5*a^6*b*f)*x^6 \\ & + 27*(11*a^4*b^3*c - 5*a^5*b^2*d)*x^3 + 15*\sqrt{1/3}*((44*a*b^6*c - 20*a^2 \\ & *b^5*d + 5*a^3*b^4*e + a^4*b^3*f)*x^{11} + 2*(44*a^2*b^5*c - 20*a^3*b^4*d + \\ & 5*a^4*b^3*e + a^5*b^2*f)*x^8 + (44*a^3*b^4*c - 20*a^4*b^3*d + 5*a^5*b^2*e + \\ & a^6*b*f)*x^5)*\sqrt{-(a^2*b)^{(1/3)}/b}*\log((2*a*b*x^3 - 3*(a^2*b)^{(1/3)}*a*x \\ & - a^2 + 3*\sqrt{1/3}*(2*a*b*x^2 + (a^2*b)^{(2/3)}*x - (a^2*b)^{(1/3)}*a)*\sqrt{-(a^2*b)^{(1/3)}/b}) \\ & / (b*x^3 + a)) - 5*((44*b^5*c - 20*a*b^4*d + 5*a^2*b^3*e + a^3*b^2*f)*x^{11} + 2*(44*a*b^4*c \\ & - 20*a^2*b^3*d + 5*a^3*b^2*e + a^4*b*f)*x^8 + (44*a^2*b^3*c - 20*a^3*b^2*d + 5*a^4*b*e + a^5*f)*x^5) \\ & *(a^2*b)^{(2/3)}*\log(a*b*x^2 - (a^2*b)^{(2/3)}*x + (a^2*b)^{(1/3)}*a) + 10*((44*b^5*c - 20*a*b^4*d + \\ & 5*a^2*b^3*e + a^3*b^2*f)*x^{11} + 2*(44*a*b^4*c - 20*a^2*b^3*d + 5*a^3*b^2*e \\ & + a^4*b*f)*x^8 + (44*a^2*b^3*c - 20*a^3*b^2*d + 5*a^4*b*e + a^5*f)*x^5)*(a^2*b)^{(2/3)} \\ & *\log(a*b*x + (a^2*b)^{(2/3)}))/ (a^6*b^4*x^{11} + 2*a^7*b^3*x^8 + a^8*b^2*x^5), \\ & 1/270*(15*(44*a^2*b^5*c - 20*a^3*b^4*d + 5*a^4*b^3*e + a^5*b^2*f)*x^9 - 54*a^5*b^2*c \\ & + 6*(176*a^3*b^4*c - 80*a^4*b^3*d + 20*a^5*b^2*e - 5*a^6*b*f)*x^6 + 27*(11*a^4*b^3*c \\ & - 5*a^5*b^2*d)*x^3 + 30*\sqrt{1/3}*((44*a*b^6*c - 20*a^2*b^5*d + 5*a^3*b^4*e + a^4*b^3*f)*x^{11} \\ & + 2*(44*a^2*b^5*c - 20*a^3*b^4*d + 5*a^4*b^3*e + a^5*b^2*f)*x^8 + (44*a^3*b^4*c - 20*a^4*b^3*d \\ & + 5*a^5*b^2*e + a^6*b*f)*x^5)*\sqrt{(a^2*b)^{(1/3)}/b}*\arctan(\sqrt{1/3}*(2*(a^2*b)^{(2/3)}*x \\ & - (a^2*b)^{(1/3)}*a)*\sqrt{(a^2*b)^{(1/3)}/b}/a^2) - 5*((44*b^5*c - 20*a*b^4*d + 5*a^2*b^3*e \\ & + a^3*b^2*f)*x^{11} + 2*(44*a*b^4*c - 20*a^2*b^3*d + 5*a^3*b^2*e + a^4*b*f)*x^8 + (44*a^2*b^3*c \\ & - 20*a^3*b^2*d + 5*a^4*b*e + a^5*f)*x^5)*(a^2*b)^{(2/3)}*\log(a*b*x^2 - (a^2*b)^{(2/3)}*x \\ & + (a^2*b)^{(1/3)}*a) + 10*((44*b^5*c - 20*a*b^4*d + 5*a^2*b^3*e + a^3*b^2*f)*x^{11} + 2*(44*a*b^4*c \\ & - 20*a^2*b^3*d + 5*a^3*b^2*e + a^4*b*f)*x^8 + (44*a^2*b^3*c - 20*a^3*b^2*d + 5*a^4*b*e + a^5*f)*x^5) \\ & *(a^2*b)^{(2/3)}*\log(a*b*x + (a^2*b)^{(2/3)}))/ (a^6*b^4*x^{11} + 2*a^7*b^3*x^8 + a^8*b^2*x^5)] \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**6/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.09964, size = 491, normalized size = 1.55

$$\frac{(44b^3c - 20ab^2d + a^3f + 5a^2be) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^5b} + \frac{\sqrt{3}\left(44(-ab^2)^{\frac{1}{3}}b^3c - 20(-ab^2)^{\frac{1}{3}}ab^2d + (-ab^2)^{\frac{1}{3}}a^3f + 5a^2be\right)}{27a^5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^3,x, algorithm="giac")

[Out]
$$-1/27*(44*b^3*c - 20*a*b^2*d + a^3*f + 5*a^2*b*e)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/(a^5*b) + 1/27*\text{sqrt}(3)*(44*(-a*b^2)^{(1/3)}*b^3*c - 20*(-a*b^2)^{(1/3)}*a*b^2*d + (-a*b^2)^{(1/3)}*a^3*f + 5*(-a*b^2)^{(1/3)}*a^2*b*e)*\arctan(1/3*\text{sqrt}(3)*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(a^5*b^2) + 1/54*(44*(-a*b^2)^{(1/3)}*b^3*c - 20*(-a*b^2)^{(1/3)}*a*b^2*d + (-a*b^2)^{(1/3)}*a^3*f + 5*(-a*b^2)^{(1/3)}*a^2*b*e)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a^5*b^2) + 1/18*(17*b^4*c*x^4 - 11*a*b^3*d*x^4 + a^3*b*f*x^4 + 5*a^2*b^2*x^4*e + 20*a*b^3*c*x - 14*a^2*b^2*d*x - 2*a^4*f*x + 8*a^3*b*x*e)/((b*x^3 + a)^2*a^4*b) + 1/10*(15*b*c*x^3 - 5*a*d*x^3 - 2*a*c)/(a^4*x^5)$$

$$3.299 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)^3} dx$$

Optimal. Leaf size=343

$$\frac{x^2(5a^2be - 2a^3f - 8ab^2d + 11b^3c)}{9a^5(a + bx^3)} - \frac{x^2(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^4(a + bx^3)^2} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(14a^2be - 2a^3f - 54a^{16/3}b^{2/3})}{54a^{16/3}b^{2/3}}$$

[Out] $-c/(7*a^3*x^7) + (3*b*c - a*d)/(4*a^4*x^4) - (6*b^2*c - 3*a*b*d + a^2*e)/(a^5*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^4*(a + b*x^3)^2) - ((11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*x^2)/(9*a^5*(a + b*x^3)) + ((65*b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(16/3)*b^(2/3)) + ((65*b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(16/3)*b^(2/3)) - ((65*b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(16/3)*b^(2/3))$

Rubi [A] time = 0.56931, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1829, 1834, 292, 31, 634, 617, 204, 628}

$$\frac{x^2(5a^2be - 2a^3f - 8ab^2d + 11b^3c)}{9a^5(a + bx^3)} - \frac{x^2(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^4(a + bx^3)^2} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(14a^2be - 2a^3f - 54a^{16/3}b^{2/3})}{54a^{16/3}b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^3), x]

[Out] $-c/(7*a^3*x^7) + (3*b*c - a*d)/(4*a^4*x^4) - (6*b^2*c - 3*a*b*d + a^2*e)/(a^5*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^4*(a + b*x^3)^2) - ((11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*x^2)/(9*a^5*(a + b*x^3)) + ((65*b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(16/3)*b^(2/3)) + ((65*b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(16/3)*b^(2/3)) - ((65*b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(16/3)*b^(2/3))$

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^( -1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^( -1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```


a, 0] || LtQ[b, 0])

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
 imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{c + dx^3 + ex^6 + fx^9}{x^8(a + bx^3)^3} dx &= -\frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6a^4(a + bx^3)^2} - \frac{\int \frac{-6b^3c + 6b^3\left(\frac{bc}{a} - d\right)x^3 - \frac{6b^3(b^2c - abd + a^2e)x^6}{a^2} + \frac{4b^3(b^3c - ab^2d + a^2be - a^3f)x^9}{a^3}}{x^8(a + bx^3)^2} dx}{6ab^3} \\
 &= -\frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6a^4(a + bx^3)^2} - \frac{(11b^3c - 8ab^2d + 5a^2be - 2a^3f)x^2}{9a^5(a + bx^3)} + \frac{\int \frac{18b^6c - 18b^6\left(\frac{2bc}{a} - d\right)x^3}{x^8(a + bx^3)^2} dx}{9a^5} \\
 &= -\frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6a^4(a + bx^3)^2} - \frac{(11b^3c - 8ab^2d + 5a^2be - 2a^3f)x^2}{9a^5(a + bx^3)} + \frac{\int \left(\frac{18b^6c}{ax^8} + \frac{18b^6(-3bc - d)}{a^2x^5}\right) dx}{9a^5} \\
 &= -\frac{c}{7a^3x^7} + \frac{3bc - ad}{4a^4x^4} - \frac{6b^2c - 3abd + a^2e}{a^5x} - \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6a^4(a + bx^3)^2} - \frac{(11b^3c - 8ab^2d + 5a^2be - 2a^3f)x^2}{9a^5(a + bx^3)} \\
 &= -\frac{c}{7a^3x^7} + \frac{3bc - ad}{4a^4x^4} - \frac{6b^2c - 3abd + a^2e}{a^5x} - \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6a^4(a + bx^3)^2} - \frac{(11b^3c - 8ab^2d + 5a^2be - 2a^3f)x^2}{9a^5(a + bx^3)} \\
 &= -\frac{c}{7a^3x^7} + \frac{3bc - ad}{4a^4x^4} - \frac{6b^2c - 3abd + a^2e}{a^5x} - \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6a^4(a + bx^3)^2} - \frac{(11b^3c - 8ab^2d + 5a^2be - 2a^3f)x^2}{9a^5(a + bx^3)} \\
 &= -\frac{c}{7a^3x^7} + \frac{3bc - ad}{4a^4x^4} - \frac{6b^2c - 3abd + a^2e}{a^5x} - \frac{(b^3c - ab^2d + a^2be - a^3f)x^2}{6a^4(a + bx^3)^2} - \frac{(11b^3c - 8ab^2d + 5a^2be - 2a^3f)x^2}{9a^5(a + bx^3)}
 \end{aligned}$$

Mathematica [A] time = 0.23793, size = 328, normalized size = 0.96

$$\frac{126a^{4/3}x^2(-a^2be+a^3f+ab^2d-b^3c)}{(a+bx^3)^2} + \frac{84\sqrt[3]{ax^2}(-5a^2be+2a^3f+8ab^2d-11b^3c)}{a+bx^3} + \frac{14\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx+b^{2/3}x^2}\right)(-14a^2be+2a^3f+35ab^2d-65b^3c)}{b^{2/3}} + \frac{28\log\left(\sqrt[3]{a}+\sqrt[3]{bx+b^{2/3}x^2}\right)}{b^{2/3}}$$

756a¹⁶

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^3),x]

[Out] ((-108*a^(7/3)*c)/x^7 - (189*a^(4/3)*(-3*b*c + a*d))/x^4 - (756*a^(1/3)*(6*b^2*c - 3*a*b*d + a^2*e))/x + (126*a^(4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(a + b*x^3)^2 + (84*a^(1/3)*(-11*b^3*c + 8*a*b^2*d - 5*a^2*b*e + 2*a^3*f)*x^2)/(a + b*x^3) + (28*sqrt[3]*(65*b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]])/b^(2/3) + (28*(65*b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (14*(-65*b^3*c + 35*a*b^2*d - 14*a^2*b*e + 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(756*a^(16/3))

Maple [B] time = 0.02, size = 611, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^3,x)

[Out] 2/9/a^2/(b*x^3+a)^2*x^5*b*f-5/9/a^3/(b*x^3+a)^2*x^5*e*b^2-e/a^3/x-1/7*c/a^3/x^7+8/9/a^4/(b*x^3+a)^2*x^5*d*b^3-11/9/a^5/(b*x^3+a)^2*x^5*c*b^4-13/18/a^2/(b*x^3+a)^2*x^2*b*e+19/18/a^3/(b*x^3+a)^2*x^2*b^2*d-25/18/a^4/(b*x^3+a)^2*x^2*b^3*c-14/27/a^3*e*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-35/27/a^4*b*d/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+35/54/a^4*b*d/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+65/27/a^5*b^2*c/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-65/54/a^5*b^2*c/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-2/27/a^2*f/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/27/a^2*f/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+7/18/a/(b*x^3+a)^2*x^2*f+14/27/a^3*e/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-7/27/a^3*e/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+3/4/a^4/x^4*b*c+3/a^4/x*b*d-6/a^5/x*b^2*c-1/4/a^3/x^4*d+35/27/a^4*b*d*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*

$$\left(\frac{2}{(1/b*a)^{(1/3)*x-1}}\right) - 65/27/a^5*b^2*c*3^{(1/2)}/(1/b*a)^{(1/3)*\arctan(1/3*3^{(1/2)*(2/(1/b*a)^{(1/3)*x-1})} + 2/27/a^2*f*3^{(1/2)}/b/(1/b*a)^{(1/3)*\arctan(1/3*3^{(1/2)*(2/(1/b*a)^{(1/3)*x-1})})}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 1.76082, size = 3031, normalized size = 8.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/756*(84*(65*a*b^6*c - 35*a^2*b^5*d + 14*a^3*b^4*e - 2*a^4*b^3*f)*x^{12} + \\ & 147*(65*a^2*b^5*c - 35*a^3*b^4*d + 14*a^4*b^3*e - 2*a^5*b^2*f)*x^9 + 108*a^5*b^2*c + \\ & 54*(65*a^3*b^4*c - 35*a^4*b^3*d + 14*a^5*b^2*e)*x^6 - 27*(13*a^4*b^3*c - 7*a^5*b^2*d)*x^3 + \\ & 42*\sqrt{1/3}*((65*a*b^6*c - 35*a^2*b^5*d + 14*a^3*b^4*e - 2*a^4*b^3*f)*x^{13} + \\ & 2*(65*a^2*b^5*c - 35*a^3*b^4*d + 14*a^4*b^3*e - 2*a^5*b^2*f)*x^{10} + (65*a^3*b^4*c - \\ & 35*a^4*b^3*d + 14*a^5*b^2*e - 2*a^6*b*f)*x^7)*\sqrt{(-a*b^2)^{(1/3)}/a}*\log((2*b^2*x^3 - a*b + 3*\sqrt{1/3}*(a*b*x \\ & + 2*(-a*b^2)^{(2/3)*x^2} + (-a*b^2)^{(1/3)*a})*\sqrt{(-a*b^2)^{(1/3)}/a} - 3*(-a*b^2)^{(2/3)*x} \\ & / (b*x^3 + a)) + 14*((65*b^5*c - 35*a*b^4*d + 14*a^2*b^3*e - 2*a^3*b^2*f)*x^{13} + \\ & 2*(65*a*b^4*c - 35*a^2*b^3*d + 14*a^3*b^2*e - 2*a^4*b*f)*x^{10} + (65*a^2*b^3*c - \\ & 35*a^3*b^2*d + 14*a^4*b*e - 2*a^5*f)*x^7)*(-a*b^2)^{(2/3)*\log(b^2*x^2 + (-a*b^2)^{(1/3)*b*x} + (-a*b^2)^{(2/3)})} \\ & - 28*((65*b^5*c - 35*a*b^4*d + 14*a^2*b^3*e - 2*a^3*b^2*f)*x^{13} + 2*(65*a*b^4*c - \\ & 35*a^2*b^3*d + 14*a^3*b^2*e - 2*a^4*b*f)*x^{10} + (65*a^2*b^3*c - 35*a^3*b^2*d + \\ & 14*a^4*b*e - 2*a^5*f)*x^7)*(-a*b^2)^{(2/3)*\log(b*x - (-a*b^2)^{(1/3)})} / (a^6*b^4*x^{13} \\ & + 2*a^7*b^3*x^{10} + a^8*b^2*x^7), -1/756*(84*(65*a*b^6*c - 35*a^2*b^5*d + 14 \\ & *a^3*b^4*e - 2*a^4*b^3*f)*x^{12} + 147*(65*a^2*b^5*c - 35*a^3*b^4*d + 14*a^4*b^3*e - \\ & 2*a^5*b^2*f)*x^9 + 108*a^5*b^2*c + 54*(65*a^3*b^4*c - 35*a^4*b^3*d \end{aligned}$$

$$+ 14a^5b^2e)x^6 - 27(13a^4b^3c - 7a^5b^2d)x^3 + 84\sqrt{1/3}((65ab^6c - 35a^2b^5d + 14a^3b^4e - 2a^4b^3f)x^{13} + 2(65a^2b^5c - 35a^3b^4d + 14a^4b^3e - 2a^5b^2f)x^{10} + (65a^3b^4c - 35a^4b^3d + 14a^5b^2e - 2a^6b^1f)x^7)\sqrt{-(-ab^2)^{1/3}/a} \arctan(\sqrt{1/3}(2bx + (-ab^2)^{1/3})\sqrt{-(-ab^2)^{1/3}/a}/b) + 14((65b^5c - 35ab^4d + 14a^2b^3e - 2a^3b^2f)x^{13} + 2(65ab^4c - 35a^2b^3d + 14a^3b^2e - 2a^4b^1f)x^{10} + (65a^2b^3c - 35a^3b^2d + 14a^4b^1e - 2a^5f)x^7)(-ab^2)^{2/3} \log(b^2x^2 + (-ab^2)^{1/3}bx + (-ab^2)^{2/3}) - 28((65b^5c - 35ab^4d + 14a^2b^3e - 2a^3b^2f)x^{13} + 2(65ab^4c - 35a^2b^3d + 14a^3b^2e - 2a^4b^1f)x^{10} + (65a^2b^3c - 35a^3b^2d + 14a^4b^1e - 2a^5f)x^7)(-ab^2)^{2/3} \log(bx - (-ab^2)^{1/3})/(a^6b^4x^{13} + 2a^7b^3x^{10} + a^8b^2x^7)]$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**8/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.09876, size = 586, normalized size = 1.71

$$\frac{\left(65b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 35ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 2a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 14a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^6} + \frac{\sqrt{3}\left(65(-ab^2)^{\frac{2}{3}}b^3c - 35(-a\right)}{27a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^3,x, algorithm="giac")

[Out] 1/27*(65*b^3*c*(-a/b)^(1/3) - 35*a*b^2*d*(-a/b)^(1/3) - 2*a^3*f*(-a/b)^(1/3) + 14*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^6 + 1/27*sqrt(3)*(65*(-a*b^2)^(2/3)*b^3*c - 35*(-a*b^2)^(2/3)*a*b^2*d - 2*(-a*b^2)^(2/3)*a^3*f + 14*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/

$$\begin{aligned}
& b)^{(1/3)} / (-a/b)^{(1/3)} / (a^6 b^2) - 1/18 * (22 * b^4 * c * x^5 - 16 * a * b^3 * d * x^5 - 4 \\
& * a^3 * b * f * x^5 + 10 * a^2 * b^2 * x^5 * e + 25 * a * b^3 * c * x^2 - 19 * a^2 * b^2 * d * x^2 - 7 * a^4 \\
& * f * x^2 + 13 * a^3 * b * x^2 * e) / ((b * x^3 + a)^2 * a^5) - 1/54 * (65 * (-a * b^2)^{(2/3)} * b^3 * \\
& c - 35 * (-a * b^2)^{(2/3)} * a * b^2 * d - 2 * (-a * b^2)^{(2/3)} * a^3 * f + 14 * (-a * b^2)^{(2/3)} * \\
& a^2 * b * e) * \log(x^2 + x * (-a/b)^{(1/3)} + (-a/b)^{(2/3)}) / (a^6 * b^2) - 1/28 * (168 * b^2 \\
& * c * x^6 - 84 * a * b * d * x^6 + 28 * a^2 * x^6 * e - 21 * a * b * c * x^3 + 7 * a^2 * d * x^3 + 4 * a^2 * c \\
&) / (a^5 * x^7)
\end{aligned}$$

$$3.300 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)^3} dx$$

Optimal. Leaf size=341

$$\frac{x(11a^2be - 5a^3f - 17ab^2d + 23b^3c)}{18a^5(a+bx^3)} - \frac{x(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^4(a+bx^3)^2} + \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(20a^2be - 5a^3f - 54a^{17/3}\sqrt[3]{b})}{54a^{17/3}\sqrt[3]{b}}$$

[Out] $-c/(8*a^3*x^8) + (3*b*c - a*d)/(5*a^4*x^5) - (6*b^2*c - 3*a*b*d + a^2*e)/(2*a^5*x^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^4*(a + b*x^3)^2) - ((23*b^3*c - 17*a*b^2*d + 11*a^2*b*e - 5*a^3*f)*x)/(18*a^5*(a + b*x^3)) + ((77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(17/3)*b^(1/3)) - ((77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(17/3)*b^(1/3)) + ((77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(17/3)*b^(1/3))$

Rubi [A] time = 0.54676, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1829, 1834, 200, 31, 634, 617, 204, 628}

$$\frac{x(11a^2be - 5a^3f - 17ab^2d + 23b^3c)}{18a^5(a+bx^3)} - \frac{x(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^4(a+bx^3)^2} + \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(20a^2be - 5a^3f - 54a^{17/3}\sqrt[3]{b})}{54a^{17/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^3), x]

[Out] $-c/(8*a^3*x^8) + (3*b*c - a*d)/(5*a^4*x^5) - (6*b^2*c - 3*a*b*d + a^2*e)/(2*a^5*x^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^4*(a + b*x^3)^2) - ((23*b^3*c - 17*a*b^2*d + 11*a^2*b*e - 5*a^3*f)*x)/(18*a^5*(a + b*x^3)) + ((77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(17/3)*b^(1/3)) - ((77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(17/3)*b^(1/3)) + ((77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(17/3)*b^(1/3))$

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```


Mathematica [A] time = 0.235911, size = 324, normalized size = 0.95

$$\frac{180a^{5/3}x(-a^2be+a^3f+ab^2d-b^3c)}{(a+bx^3)^2} + \frac{60a^{2/3}x(-11a^2be+5a^3f+17ab^2d-23b^3c)}{a+bx^3} + \frac{20 \log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx+b^{2/3}x^2}\right)(20a^2be-5a^3f-44ab^2d+77b^3c)}{\sqrt[3]{b}} + \frac{40 \log\left(\sqrt[3]{a}+\sqrt[3]{bx+b^{2/3}x^2}\right)(20a^2be-5a^3f-44ab^2d+77b^3c)}{\sqrt[3]{b}}$$

1080

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^3), x]

[Out] $\left(\frac{-135a^{8/3}c}{x^8} - \frac{(216a^{5/3}(-3b^3c + a^2d))}{x^5} - \frac{(540a^{2/3}(6b^2c - 3ab^2d + a^2e))}{x^2} + (180a^{5/3}(-b^3c) + a^2b^2d - a^2b^2e + a^3f)x/(a + b^2x^3)^2 + (60a^{2/3}(-23b^3c + 17ab^2d - 11a^2b^2e + 5a^3f)x)/(a + b^2x^3) + (40\sqrt[3]{3}(77b^3c - 44ab^2d + 20a^2b^2e - 5a^3f)\text{ArcTan}\left[\frac{1 - (2b^{1/3}x)/a^{1/3}}{\sqrt[3]{3}}\right])/b^{1/3} + (40(-77b^3c + 44ab^2d - 20a^2b^2e + 5a^3f)\text{Log}[a^{1/3} + b^{1/3}x])/b^{1/3} + (20(77b^3c - 44ab^2d + 20a^2b^2e - 5a^3f)\text{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2])/b^{1/3}\right)/(1080a^{17/3})$

Maple [B] time = 0.017, size = 603, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^3, x)

[Out] $\frac{-1}{8} \frac{c}{a^3} \frac{1}{x^8} - \frac{23}{18} \frac{d}{a^5} \frac{1}{(b^2x^3+a)^2} - \frac{7}{9} \frac{e}{a^2} \frac{1}{(b^2x^3+a)^2} - \frac{4}{9} \frac{f}{a} \frac{1}{(b^2x^3+a)^2} - \frac{20}{27} \frac{e}{a^3} \frac{1}{(b^2x^3+a)^2} \ln\left(x + \frac{1}{b^2x^3+a}\right) + \frac{10}{27} \frac{e}{a^3} \frac{1}{(b^2x^3+a)^2} \ln\left(x^2 - \frac{1}{b^2x^3+a} + \frac{1}{b^2x^3+a}\right) + \frac{3}{5} \frac{d}{a^4} \frac{1}{x^5} + \frac{3}{2} \frac{d}{a^4} \frac{1}{x^2} - \frac{3}{a^5} \frac{d}{x^2} - \frac{1}{5} \frac{d}{a^3} \frac{1}{x^5} - \frac{1}{2} \frac{d}{a^3} \frac{1}{x^2} + \frac{10}{9} \frac{e}{a^3} \frac{1}{(b^2x^3+a)^2} - \frac{13}{9} \frac{e}{a^4} \frac{1}{(b^2x^3+a)^2} - \frac{77}{54} \frac{e}{a^5} \frac{1}{b^2x^3+a} \ln\left(x^2 - \frac{1}{b^2x^3+a} + \frac{1}{b^2x^3+a}\right) + \frac{5}{18} \frac{e}{a^2} \frac{1}{(b^2x^3+a)^2} - \frac{11}{18} \frac{e}{a^3} \frac{1}{(b^2x^3+a)^2} - \frac{17}{18} \frac{e}{a^4} \frac{1}{(b^2x^3+a)^2} + \frac{5}{27} \frac{f}{a^2} \frac{1}{b^2x^3+a} \ln\left(x + \frac{1}{b^2x^3+a}\right) - \frac{5}{54} \frac{f}{a^2} \frac{1}{b^2x^3+a} \ln\left(x^2 - \frac{1}{b^2x^3+a} + \frac{1}{b^2x^3+a}\right) - \frac{20}{27} \frac{e}{a^3} \frac{1}{(b^2x^3+a)^2} \arctan\left(\frac{1}{3} \sqrt[3]{\frac{2}{b^2x^3+a}}\right) + \frac{44}{27} \frac{d}{a^4} \frac{1}{b^2x^3+a} \ln\left(x + \frac{1}{b^2x^3+a}\right) - \frac{22}{27} \frac{d}{a^4} \frac{1}{b^2x^3+a} \ln\left(x^2 - \frac{1}{b^2x^3+a} + \frac{1}{b^2x^3+a}\right) - \frac{77}{27} \frac{e}{a^5} \frac{1}{b^2x^3+a} \ln\left(x + \frac{1}{b^2x^3+a}\right) + \frac{44}{27} \frac{d}{a^4} \frac{1}{b^2x^3+a} \ln\left(x + \frac{1}{b^2x^3+a}\right) \arctan\left(\frac{1}{3} \sqrt[3]{\frac{2}{b^2x^3+a}}\right)$

$$\begin{aligned} & \sqrt{\frac{2}{1/b*a}} \cdot \sqrt[3]{x-1} - \frac{77}{27} \frac{a^5 b^2 c}{(1/b*a)^{2/3}} \sqrt[3]{\frac{1}{2} \arctan\left(\frac{1}{3} \sqrt[3]{\frac{2}{1/b*a}} \sqrt[3]{x-1}\right) + \frac{5}{27} \frac{a^2 f/b}{(1/b*a)^{2/3}} \sqrt[3]{\frac{1}{2} \arctan\left(\frac{1}{3} \sqrt[3]{\frac{2}{1/b*a}} \sqrt[3]{x-1}\right)}} \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 1.77692, size = 3013, normalized size = 8.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & \left[-\frac{1}{1080} (60(77a^2b^5c - 44a^3b^4d + 20a^4b^3e - 5a^5b^2f)x^{12} + 96(77a^3b^4c - 44a^4b^3d + 20a^5b^2e - 5a^6b^1f)x^9 + 135a^6b^1c + 27(77a^4b^3c - 44a^5b^2d + 20a^6b^1e)x^6 - 54(7a^5b^2c - 4a^6b^1d)x^3 + 60\sqrt{1/3}((77ab^6c - 44a^2b^5d + 20a^3b^4e - 5a^4b^3f)x^{14} + 2(77a^2b^5c - 44a^3b^4d + 20a^4b^3e - 5a^5b^2f)x^{11} + (77a^3b^4c - 44a^4b^3d + 20a^5b^2e - 5a^6b^1f)x^8) \sqrt{-(a^2b)^{1/3}/b} \log((2abx^3 - 3(a^2b)^{1/3}ax - a^2 + 3\sqrt{1/3}(2abx^2 + (a^2b)^{2/3}x - (a^2b)^{1/3}a) \sqrt{-(a^2b)^{1/3}/b})/(b^3 + a)) - 20((77b^5c - 44ab^4d + 20a^2b^3e - 5a^3b^2f)x^{14} + 2(77ab^4c - 44a^2b^3d + 20a^3b^2e - 5a^4b^1f)x^{11} + (77a^2b^3c - 44a^3b^2d + 20a^4b^1e - 5a^5f)x^8) (a^2b)^{2/3} \log(ax^2 - (a^2b)^{2/3}x + (a^2b)^{1/3}a) + 40((77b^5c - 44ab^4d + 20a^2b^3e - 5a^3b^2f)x^{14} + 2(77ab^4c - 44a^2b^3d + 20a^3b^2e - 5a^4b^1f)x^{11} + (77a^2b^3c - 44a^3b^2d + 20a^4b^1e - 5a^5f)x^8) (a^2b)^{2/3} \log(ax + (a^2b)^{2/3}) \right] / (a^7b^3x^{14} + 2a^8b^2x^{11} + a^9b^1x^8), -\frac{1}{1080} (60(77a^2b^5c - 44a^3b^4d + 20a^4b^3e - 5a^5b^2f)x^{12} + 96(77a^3b^4c - 44a^4b^3d + 20a^5b^2e - 5a^6b^1f)x^9 + 135a^6b^1c + 27(77a^4b^3c - 44a^5b^2d + 20a^6b^1e)x^6 - 54(7a^5b^2c - 4a^6b^1d)x^3 + 60\sqrt{1/3}((77ab^6c - 44a^2b^5d + 20a^3b^4e - 5a^4b^3f)x^{14} + 2(77a^2b^5c - 44a^3b^4d + 20a^4b^3e - 5a^5b^2f)x^{11} + (77a^3b^4c - 44a^4b^3d + 20a^5b^2e - 5a^6b^1f)x^8) \sqrt{-(a^2b)^{1/3}/b} \log((2abx^3 - 3(a^2b)^{1/3}ax - a^2 + 3\sqrt{1/3}(2abx^2 + (a^2b)^{2/3}x - (a^2b)^{1/3}a) \sqrt{-(a^2b)^{1/3}/b})/(b^3 + a)) - 20((77b^5c - 44ab^4d + 20a^2b^3e - 5a^3b^2f)x^{14} + 2(77ab^4c - 44a^2b^3d + 20a^3b^2e - 5a^4b^1f)x^{11} + (77a^2b^3c - 44a^3b^2d + 20a^4b^1e - 5a^5f)x^8) (a^2b)^{2/3} \log(ax^2 - (a^2b)^{2/3}x + (a^2b)^{1/3}a) + 40((77b^5c - 44ab^4d + 20a^2b^3e - 5a^3b^2f)x^{14} + 2(77ab^4c - 44a^2b^3d + 20a^3b^2e - 5a^4b^1f)x^{11} + (77a^2b^3c - 44a^3b^2d + 20a^4b^1e - 5a^5f)x^8) (a^2b)^{2/3} \log(ax + (a^2b)^{2/3}) \right] / (a^7b^3x^{14} + 2a^8b^2x^{11} + a^9b^1x^8) \end{aligned}$$

$$\begin{aligned} &^6 - 54*(7*a^5*b^2*c - 4*a^6*b*d)*x^3 + 120*\sqrt{1/3}*((77*a*b^6*c - 44*a^2 \\ &*b^5*d + 20*a^3*b^4*e - 5*a^4*b^3*f)*x^{14} + 2*(77*a^2*b^5*c - 44*a^3*b^4*d \\ &+ 20*a^4*b^3*e - 5*a^5*b^2*f)*x^{11} + (77*a^3*b^4*c - 44*a^4*b^3*d + 20*a^5* \\ &b^2*e - 5*a^6*b*f)*x^8)*\sqrt{(a^2*b)^{(1/3)}/b}*\arctan(\sqrt{1/3}*(2*(a^2*b)^{(\\ &2/3)*x - (a^2*b)^{(1/3)*a}*\sqrt{(a^2*b)^{(1/3)}/b}/a^2) - 20*((77*b^5*c - 44*a \\ &*b^4*d + 20*a^2*b^3*e - 5*a^3*b^2*f)*x^{14} + 2*(77*a*b^4*c - 44*a^2*b^3*d + \\ &20*a^3*b^2*e - 5*a^4*b*f)*x^{11} + (77*a^2*b^3*c - 44*a^3*b^2*d + 20*a^4*b*e \\ &- 5*a^5*f)*x^8)*(a^2*b)^{(2/3)*\log(a*b*x^2 - (a^2*b)^{(2/3)*x + (a^2*b)^{(1/3) \\ &*a) + 40*((77*b^5*c - 44*a*b^4*d + 20*a^2*b^3*e - 5*a^3*b^2*f)*x^{14} + 2*(77 \\ &*a*b^4*c - 44*a^2*b^3*d + 20*a^3*b^2*e - 5*a^4*b*f)*x^{11} + (77*a^2*b^3*c - \\ &44*a^3*b^2*d + 20*a^4*b*e - 5*a^5*f)*x^8)*(a^2*b)^{(2/3)*\log(a*b*x + (a^2*b) \\ &^{(2/3))})/(a^7*b^3*x^{14} + 2*a^8*b^2*x^{11} + a^9*b*x^8)] \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**9/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.09785, size = 532, normalized size = 1.56

$$\frac{(77b^3c - 44ab^2d - 5a^3f + 20a^2be) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^6} - \frac{\sqrt{3}\left(77(-ab^2)^{\frac{1}{3}}b^3c - 44(-ab^2)^{\frac{1}{3}}ab^2d - 5(-ab^2)^{\frac{1}{3}}a^3f\right)}{27a^6b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^3,x, algorithm="giac")

[Out] $\frac{1}{27}*(77*b^3*c - 44*a*b^2*d - 5*a^3*f + 20*a^2*b*e)*(-a/b)^{(1/3)*\log(\text{abs}(x - (-a/b)^{(1/3)})))/a^6 - 1/27*\sqrt{3}*(77*(-a*b^2)^{(1/3)*b^3*c - 44*(-a*b^2)^{(1/3)*a*b^2*d - 5*(-a*b^2)^{(1/3)*a^3*f + 20*(-a*b^2)^{(1/3)*a^2*b*e})*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(a^6*b) - 1/54*(77*(-a*b^2)^{($

$$\begin{aligned}
& \left(\frac{1}{3}\right)b^3c - 44(-ab^2)^{(1/3)}ab^2d - 5(-ab^2)^{(1/3)}a^3f + 20(-ab^2)^{(1/3)}a^2b^2e) \log(x^2 + x(-a/b)^{(1/3)} + (-a/b)^{(2/3)}) / (a^6b) - 1/18 * \\
& (23b^4c^2x^4 - 17ab^3d^2x^4 - 5a^3b^2f^2x^4 + 11a^2b^2x^4e + 26ab^3c^2x - 20a^2b^2d^2x - 8a^4f^2x + 14a^3b^2x^2e) / ((bx^3 + a)^2a^5) - 1/ \\
& 40(120b^2c^2x^6 - 60ab^2d^2x^6 + 20a^2x^6e - 24ab^2c^2x^3 + 8a^2d^2x^3 + 5a^2c^2) / (a^5x^8)
\end{aligned}$$

$$3.301 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)^3} dx$$

Optimal. Leaf size=381

$$\frac{bx^2(8a^2be - 5a^3f - 11ab^2d + 14b^3c)}{9a^6(a+bx^3)} + \frac{bx^2(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^5(a+bx^3)^2} + \frac{\sqrt[3]{b} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(35a^2be - 14ab^2d + 11a^3f)}{54a^{19/3}}$$

[Out] $-c/(10*a^3*x^{10}) + (3*b*c - a*d)/(7*a^4*x^7) - (6*b^2*c - 3*a*b*d + a^2*e)/(4*a^5*x^4) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(a^6*x) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^5*(a + b*x^3)^2) + (b*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*x^2)/(9*a^6*(a + b*x^3)) - (b^{(1/3)}*(104*b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(9*Sqrt[3]*a^{(19/3)}) - (b^{(1/3)}*(104*b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f)*Log[a^{(1/3)} + b^{(1/3)}*x])/(27*a^{(19/3)}) + (b^{(1/3)}*(104*b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(54*a^{(19/3)})$

Rubi [A] time = 0.713572, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1829, 1834, 292, 31, 634, 617, 204, 628}

$$\frac{bx^2(8a^2be - 5a^3f - 11ab^2d + 14b^3c)}{9a^6(a+bx^3)} + \frac{bx^2(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^5(a+bx^3)^2} + \frac{\sqrt[3]{b} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(35a^2be - 14ab^2d + 11a^3f)}{54a^{19/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^3), x]

[Out] $-c/(10*a^3*x^{10}) + (3*b*c - a*d)/(7*a^4*x^7) - (6*b^2*c - 3*a*b*d + a^2*e)/(4*a^5*x^4) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(a^6*x) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^5*(a + b*x^3)^2) + (b*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*x^2)/(9*a^6*(a + b*x^3)) - (b^{(1/3)}*(104*b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(9*Sqrt[3]*a^{(19/3)}) - (b^{(1/3)}*(104*b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f)*Log[a^{(1/3)} + b^{(1/3)}*x])/(27*a^{(19/3)}) + (b^{(1/3)}*(104*b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(54*a^{(19/3)})$

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_)^(m_.)))/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^( -1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^{11}(a + bx^3)^3} dx = \frac{b(b^3c - ab^2d + a^2be - a^3f)x^2}{6a^5(a + bx^3)^2} - \frac{\int \frac{-6b^3c + 6b^3\left(\frac{bc}{a} - d\right)x^3 - \frac{6b^3(b^2c - abd + a^2e)x^6}{a^2} + \frac{6b^3(b^3c - ab^2d + a^2be - a^3f)x^9}{a^3} - 4b^4}{x^{11}(a + bx^3)^2} dx}{6ab^3}$$

$$= \frac{b(b^3c - ab^2d + a^2be - a^3f)x^2}{6a^5(a + bx^3)^2} + \frac{b(14b^3c - 11ab^2d + 8a^2be - 5a^3f)x^2}{9a^6(a + bx^3)} + \frac{\int \frac{18b^7c - 18b^7\left(\frac{2bc}{a} - d\right)}{x^{11}(a + bx^3)^2} dx}{6ab^3}$$

$$= \frac{b(b^3c - ab^2d + a^2be - a^3f)x^2}{6a^5(a + bx^3)^2} + \frac{b(14b^3c - 11ab^2d + 8a^2be - 5a^3f)x^2}{9a^6(a + bx^3)} + \frac{\int \left(\frac{18b^7c}{ax^{11}} + \frac{18b^7\left(\frac{2bc}{a} - d\right)}{a^2x^8}\right) dx}{6ab^3}$$

$$= -\frac{c}{10a^3x^{10}} + \frac{3bc - ad}{7a^4x^7} - \frac{6b^2c - 3abd + a^2e}{4a^5x^4} + \frac{10b^3c - 6ab^2d + 3a^2be - a^3f}{a^6x} + \frac{b(b^3c - ab^2d + a^2be - a^3f)}{6a^5(a + bx^3)}$$

$$= -\frac{c}{10a^3x^{10}} + \frac{3bc - ad}{7a^4x^7} - \frac{6b^2c - 3abd + a^2e}{4a^5x^4} + \frac{10b^3c - 6ab^2d + 3a^2be - a^3f}{a^6x} + \frac{b(b^3c - ab^2d + a^2be - a^3f)}{6a^5(a + bx^3)}$$

$$= -\frac{c}{10a^3x^{10}} + \frac{3bc - ad}{7a^4x^7} - \frac{6b^2c - 3abd + a^2e}{4a^5x^4} + \frac{10b^3c - 6ab^2d + 3a^2be - a^3f}{a^6x} + \frac{b(b^3c - ab^2d + a^2be - a^3f)}{6a^5(a + bx^3)}$$

$$= -\frac{c}{10a^3x^{10}} + \frac{3bc - ad}{7a^4x^7} - \frac{6b^2c - 3abd + a^2e}{4a^5x^4} + \frac{10b^3c - 6ab^2d + 3a^2be - a^3f}{a^6x} + \frac{b(b^3c - ab^2d + a^2be - a^3f)}{6a^5(a + bx^3)}$$

$$= -\frac{c}{10a^3x^{10}} + \frac{3bc - ad}{7a^4x^7} - \frac{6b^2c - 3abd + a^2e}{4a^5x^4} + \frac{10b^3c - 6ab^2d + 3a^2be - a^3f}{a^6x} + \frac{b(b^3c - ab^2d + a^2be - a^3f)}{6a^5(a + bx^3)}$$

Mathematica [A] time = 0.392011, size = 366, normalized size = 0.96

$$\frac{630a^{4/3}bx^2(-a^2be+a^3f+ab^2d-b^3c)}{(a+bx^3)^2} - \frac{420\sqrt[3]{abx^2}(-8a^2be+5a^3f+11ab^2d-14b^3c)}{a+bx^3} + 70\sqrt[3]{b} \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right) (35a^2be - 14a^3f - 65a^2b^2c + 3a^3bd - a^4e)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^3),x]

[Out] ((-378*a^(10/3)*c)/x^10 - (540*a^(7/3)*(-3*b*c + a*d))/x^7 - (945*a^(4/3)*(6*b^2*c - 3*a*b*d + a^2*e))/x^4 - (3780*a^(1/3)*(-10*b^3*c + 6*a*b^2*d - 3*a^2*b*e + a^3*f))/x - (630*a^(4/3)*b*(-b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2/(a + b*x^3)^2 - (420*a^(1/3)*b*(-14*b^3*c + 11*a*b^2*d - 8*a^2*b*e + 5*a^3*f)*x^2)/(a + b*x^3) - 140*sqrt[3]*b^(1/3)*(104*b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]] + 140*b^(1/3)*(-104*b^3*c + 65*a*b^2*d - 35*a^2*b*e + 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 70*b^(1/3)*(104*b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(3780*a^(19/3))

Maple [A] time = 0.019, size = 659, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^3,x)

[Out] -104/27*b^3/a^6*c/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-6/a^5/x*b^2*d+10/a^6/x*b^3*c-25/18*b^3/a^4/(b*x^3+a)^2*x^2*d+65/27*b^2/a^5*d/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-65/54*b^2/a^5*d/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-1/10*c/a^3/x^10-11/9*b^4/a^5/(b*x^3+a)^2*x^5*d+14/9*b^5/a^6/(b*x^3+a)^2*x^5*c+14/27/a^3*f/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-7/27/a^3*f/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+3/7/a^4/x^7*b*c+3/4/a^4/x^4*b*d-3/2/a^5/x^4*b^2*c+3/a^4/x*b*e-1/7/a^3/x^7*d-1/4/a^3/x^4*e-1/a^3/x*f+35/27*b/a^4*e*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-65/27*b^2/a^5*d*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+104/27*b^3/a^6*c*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-5/9*b^2/a^3/(b*x^3+a)^2*x^5*f+8/9*b^3/a^4/(b*x^3+a)^2*x^5*e+52/27*b^3/a^6*c/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-14/27/a^3*f*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))

$$\frac{1/2}{(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))-35/27*b/a^4*e/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})+35/54*b/a^4*e/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})-13/18*b/a^2/(b*x^3+a)^2*x^2*f+19/18*b^2/a^3/(b*x^3+a)^2*x^2*e+31/18*b^4/a^5/(b*x^3+a)^2*x^2*c}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.61915, size = 1472, normalized size = 3.86

$$420(104b^5c - 65ab^4d + 35a^2b^3e - 14a^3b^2f)x^{15} + 735(104ab^4c - 65a^2b^3d + 35a^3b^2e - 14a^4bf)x^{12} + 270(104a^2b^3c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^3,x, algorithm="fricas")

[Out] $\frac{1}{3780} \cdot (420 \cdot (104 \cdot b^5 \cdot c - 65 \cdot a \cdot b^4 \cdot d + 35 \cdot a^2 \cdot b^3 \cdot e - 14 \cdot a^3 \cdot b^2 \cdot f) \cdot x^{15} + 735 \cdot (104 \cdot a \cdot b^4 \cdot c - 65 \cdot a^2 \cdot b^3 \cdot d + 35 \cdot a^3 \cdot b^2 \cdot e - 14 \cdot a^4 \cdot b \cdot f) \cdot x^{12} + 270 \cdot (104 \cdot a^2 \cdot b^3 \cdot c - 65 \cdot a^3 \cdot b^2 \cdot d + 35 \cdot a^4 \cdot b \cdot e - 14 \cdot a^5 \cdot f) \cdot x^9 - 27 \cdot (104 \cdot a^3 \cdot b^2 \cdot c - 65 \cdot a^4 \cdot b \cdot d + 35 \cdot a^5 \cdot e) \cdot x^6 - 378 \cdot a^5 \cdot c + 108 \cdot (8 \cdot a^4 \cdot b \cdot c - 5 \cdot a^5 \cdot d) \cdot x^3 + 140 \cdot \sqrt{3} \cdot ((104 \cdot b^5 \cdot c - 65 \cdot a \cdot b^4 \cdot d + 35 \cdot a^2 \cdot b^3 \cdot e - 14 \cdot a^3 \cdot b^2 \cdot f) \cdot x^{16} + 2 \cdot (104 \cdot a \cdot b^4 \cdot c - 65 \cdot a^2 \cdot b^3 \cdot d + 35 \cdot a^3 \cdot b^2 \cdot e - 14 \cdot a^4 \cdot b \cdot f) \cdot x^{13} + (104 \cdot a^2 \cdot b^3 \cdot c - 65 \cdot a^3 \cdot b^2 \cdot d + 35 \cdot a^4 \cdot b \cdot e - 14 \cdot a^5 \cdot f) \cdot x^{10}) \cdot (b/a)^{(1/3)} \cdot \arctan(2/3 \cdot \sqrt{3} \cdot x \cdot (b/a)^{(1/3)} - 1/3 \cdot \sqrt{3}) + 70 \cdot ((104 \cdot b^5 \cdot c - 65 \cdot a \cdot b^4 \cdot d + 35 \cdot a^2 \cdot b^3 \cdot e - 14 \cdot a^3 \cdot b^2 \cdot f) \cdot x^{16} + 2 \cdot (104 \cdot a \cdot b^4 \cdot c - 65 \cdot a^2 \cdot b^3 \cdot d + 35 \cdot a^3 \cdot b^2 \cdot e - 14 \cdot a^4 \cdot b \cdot f) \cdot x^{13} + (104 \cdot a^2 \cdot b^3 \cdot c - 65 \cdot a^3 \cdot b^2 \cdot d + 35 \cdot a^4 \cdot b \cdot e - 14 \cdot a^5 \cdot f) \cdot x^{10}) \cdot (b/a)^{(1/3)} \cdot \log(b \cdot x^2 - a \cdot x \cdot (b/a)^{(2/3)} + a \cdot (b/a)^{(1/3)}) - 140 \cdot ((104 \cdot b^5 \cdot c - 65 \cdot a \cdot b^4 \cdot d + 35 \cdot a^2 \cdot b^3 \cdot e - 14 \cdot a^3 \cdot b^2 \cdot f) \cdot x^{16} + 2 \cdot (104 \cdot a \cdot b^4 \cdot c - 65 \cdot a^2 \cdot b^3 \cdot d + 35 \cdot a^3 \cdot b^2 \cdot e - 14 \cdot a^4 \cdot b \cdot f) \cdot x^{13} + (104 \cdot a^2 \cdot b^3 \cdot c - 65 \cdot a^3 \cdot b^2 \cdot d + 35 \cdot a^4 \cdot b \cdot e - 14 \cdot a^5 \cdot f) \cdot x^{10}) \cdot (b/a)^{(1/3)} \cdot \log(b \cdot x + a \cdot (b/a)^{(2/3)})) / (a$

$$^6*b^2*x^{16} + 2*a^7*b*x^{13} + a^8*x^{10})$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**11/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.09807, size = 656, normalized size = 1.72

$$\frac{\left(104 b^4 c \left(-\frac{a}{b}\right)^{\frac{1}{3}} - 65 a b^3 d \left(-\frac{a}{b}\right)^{\frac{1}{3}} - 14 a^3 b f \left(-\frac{a}{b}\right)^{\frac{1}{3}} + 35 a^2 b^2 \left(-\frac{a}{b}\right)^{\frac{1}{3}} e\right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right) + \sqrt{3} \left(104 (-ab^2)^{\frac{2}{3}} b^3 c - \dots\right)}{27 a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^3,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/27*(104*b^4*c*(-a/b)^{(1/3)} - 65*a*b^3*d*(-a/b)^{(1/3)} - 14*a^3*b*f*(-a/b)^{(1/3)} \\ & + 35*a^2*b^2*(-a/b)^{(1/3)}*e)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)})) \\ & /a^7 - 1/27*\text{sqrt}(3)*(104*(-a*b^2)^{(2/3)}*b^3*c - 65*(-a*b^2)^{(2/3)}*a*b^2*d - \\ & 14*(-a*b^2)^{(2/3)}*a^3*f + 35*(-a*b^2)^{(2/3)}*a^2*b*e)*\text{arctan}(1/3*\text{sqrt}(3)*(2 \\ & *x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(a^7*b) + 1/54*(104*(-a*b^2)^{(2/3)}*b^3*c - \\ & 65*(-a*b^2)^{(2/3)}*a*b^2*d - 14*(-a*b^2)^{(2/3)}*a^3*f + 35*(-a*b^2)^{(2/3)}*a^2 \\ & *b*e)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a^7*b) + 1/18*(28*b^5*c*x^5 \\ & - 22*a*b^4*d*x^5 - 10*a^3*b^2*f*x^5 + 16*a^2*b^3*x^5*e + 31*a*b^4*c*x^2 - \\ & 25*a^2*b^3*d*x^2 - 13*a^4*b*f*x^2 + 19*a^3*b^2*x^2*e)/((b*x^3 + a)^2*a^6) \\ & + 1/140*(1400*b^3*c*x^9 - 840*a*b^2*d*x^9 - 140*a^3*f*x^9 + 420*a^2*b*x^9*e \\ & - 210*a*b^2*c*x^6 + 105*a^2*b*d*x^6 - 35*a^3*x^6*e + 60*a^2*b*c*x^3 - 20*a^3 \\ & *d*x^3 - 14*a^3*c)/(a^6*x^{10}) \end{aligned}$$

$$3.302 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)^3} dx$$

Optimal. Leaf size=380

$$\frac{bx(17a^2be - 11a^3f - 23ab^2d + 29b^3c)}{18a^6(a+bx^3)} + \frac{3a^2be + a^3(-f) - 6ab^2d + 10b^3c}{2a^6x^2} + \frac{bx(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^5(a+bx^3)^2} - \frac{b^{2/3} \log}{b^{2/3} \log}$$

[Out] $-c/(11*a^3*x^{11}) + (3*b*c - a*d)/(8*a^4*x^8) - (6*b^2*c - 3*a*b*d + a^2*e)/(5*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(2*a^6*x^2) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^5*(a + b*x^3)^2) + (b*(29*b^3*c - 23*a*b^2*d + 17*a^2*b*e - 11*a^3*f)*x)/(18*a^6*(a + b*x^3)) - (b^{(2/3)}*(119*b^3*c - 77*a*b^2*d + 44*a^2*b*e - 20*a^3*f)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(9*Sqrt[3]*a^{(20/3)}) + (b^{(2/3)}*(119*b^3*c - 77*a*b^2*d + 44*a^2*b*e - 20*a^3*f)*Log[a^{(1/3)} + b^{(1/3)}*x])/(27*a^{(20/3)}) - (b^{(2/3)}*(119*b^3*c - 77*a*b^2*d + 44*a^2*b*e - 20*a^3*f)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(54*a^{(20/3)})$

Rubi [A] time = 0.668966, antiderivative size = 380, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1829, 1834, 200, 31, 634, 617, 204, 628}

$$\frac{bx(17a^2be - 11a^3f - 23ab^2d + 29b^3c)}{18a^6(a+bx^3)} + \frac{3a^2be + a^3(-f) - 6ab^2d + 10b^3c}{2a^6x^2} + \frac{bx(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^5(a+bx^3)^2} - \frac{b^{2/3} \log}{b^{2/3} \log}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^3), x]

[Out] $-c/(11*a^3*x^{11}) + (3*b*c - a*d)/(8*a^4*x^8) - (6*b^2*c - 3*a*b*d + a^2*e)/(5*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(2*a^6*x^2) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^5*(a + b*x^3)^2) + (b*(29*b^3*c - 23*a*b^2*d + 17*a^2*b*e - 11*a^3*f)*x)/(18*a^6*(a + b*x^3)) - (b^{(2/3)}*(119*b^3*c - 77*a*b^2*d + 44*a^2*b*e - 20*a^3*f)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(9*Sqrt[3]*a^{(20/3)}) + (b^{(2/3)}*(119*b^3*c - 77*a*b^2*d + 44*a^2*b*e - 20*a^3*f)*Log[a^{(1/3)} + b^{(1/3)}*x])/(27*a^{(20/3)}) - (b^{(2/3)}*(119*b^3*c - 77*a*b^2*d + 44*a^2*b*e - 20*a^3*f)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(54*a^{(20/3)})$

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_)^(m_.)))/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^{12}(a + bx^3)^3} dx = \frac{b(b^3c - ab^2d + a^2be - a^3f)x}{6a^5(a + bx^3)^2} - \frac{\int \frac{-6b^3c + 6b^3\left(\frac{bc}{a} - d\right)x^3 - \frac{6b^3(b^2c - abd + a^2e)x^6}{a^2} + \frac{6b^3(b^3c - ab^2d + a^2be - a^3f)x^9}{a^3} - \frac{5b^4(b^3c - ab^2d + a^2be - a^3f)x^{12}}{a^4}}{x^{12}(a + bx^3)^2} dx}{6ab^3}$$

$$= \frac{b(b^3c - ab^2d + a^2be - a^3f)x}{6a^5(a + bx^3)^2} + \frac{b(29b^3c - 23ab^2d + 17a^2be - 11a^3f)x}{18a^6(a + bx^3)} + \frac{\int \frac{18b^7c - 18b^7\left(\frac{2bc}{a} - d\right)x^3 - \frac{18b^7(b^2c - abd + a^2e)x^6}{a^2} + \frac{18b^7(b^3c - ab^2d + a^2be - a^3f)x^9}{a^3} - \frac{5b^8(b^3c - ab^2d + a^2be - a^3f)x^{12}}{a^4}}{x^{12}(a + bx^3)^2} dx}{18a^6}$$

$$= \frac{b(b^3c - ab^2d + a^2be - a^3f)x}{6a^5(a + bx^3)^2} + \frac{b(29b^3c - 23ab^2d + 17a^2be - 11a^3f)x}{18a^6(a + bx^3)} + \frac{\int \left(\frac{18b^7c}{ax^{12}} + \frac{18b^7\left(\frac{2bc}{a} - d\right)x^3}{a^2} - \frac{18b^7(b^2c - abd + a^2e)x^6}{a^3} + \frac{18b^7(b^3c - ab^2d + a^2be - a^3f)x^9}{a^4} - \frac{5b^8(b^3c - ab^2d + a^2be - a^3f)x^{12}}{a^5}\right) dx}{18a^6}$$

$$= -\frac{c}{11a^3x^{11}} + \frac{3bc - ad}{8a^4x^8} - \frac{6b^2c - 3abd + a^2e}{5a^5x^5} + \frac{10b^3c - 6ab^2d + 3a^2be - a^3f}{2a^6x^2} + \frac{b(b^3c - ab^2d + a^2be - a^3f)}{6a^5(a + bx^3)}$$

$$= -\frac{c}{11a^3x^{11}} + \frac{3bc - ad}{8a^4x^8} - \frac{6b^2c - 3abd + a^2e}{5a^5x^5} + \frac{10b^3c - 6ab^2d + 3a^2be - a^3f}{2a^6x^2} + \frac{b(b^3c - ab^2d + a^2be - a^3f)}{6a^5(a + bx^3)}$$

$$= -\frac{c}{11a^3x^{11}} + \frac{3bc - ad}{8a^4x^8} - \frac{6b^2c - 3abd + a^2e}{5a^5x^5} + \frac{10b^3c - 6ab^2d + 3a^2be - a^3f}{2a^6x^2} + \frac{b(b^3c - ab^2d + a^2be - a^3f)}{6a^5(a + bx^3)}$$

$$= -\frac{c}{11a^3x^{11}} + \frac{3bc - ad}{8a^4x^8} - \frac{6b^2c - 3abd + a^2e}{5a^5x^5} + \frac{10b^3c - 6ab^2d + 3a^2be - a^3f}{2a^6x^2} + \frac{b(b^3c - ab^2d + a^2be - a^3f)}{6a^5(a + bx^3)}$$

$$= -\frac{c}{11a^3x^{11}} + \frac{3bc - ad}{8a^4x^8} - \frac{6b^2c - 3abd + a^2e}{5a^5x^5} + \frac{10b^3c - 6ab^2d + 3a^2be - a^3f}{2a^6x^2} + \frac{b(b^3c - ab^2d + a^2be - a^3f)}{6a^5(a + bx^3)}$$

Mathematica [A] time = 0.428823, size = 376, normalized size = 0.99

$$\frac{bx(17a^2be - 11a^3f - 23ab^2d + 29b^3c)}{18a^6(a + bx^3)} + \frac{3a^2be + a^3(-f) - 6ab^2d + 10b^3c}{2a^6x^2} + \frac{bx(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^5(a + bx^3)^2} + \frac{b^{2/3} \log(\dots)}{\dots}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^3), x]

[Out]
$$-c/(11*a^3*x^{11}) + (3*b*c - a*d)/(8*a^4*x^8) - (6*b^2*c - 3*a*b*d + a^2*e)/(5*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(2*a^6*x^2) + (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^5*(a + b*x^3)^2) + (b*(29*b^3*c - 23*a*b^2*d + 17*a^2*b*e - 11*a^3*f)*x)/(18*a^6*(a + b*x^3)) + (b^{2/3}*(-119*b^3*c + 77*a*b^2*d - 44*a^2*b*e + 20*a^3*f)*ArcTan[(1 - (2*b^{1/3}*x)/a^{1/3})/Sqrt[3]])/(9*Sqrt[3]*a^{20/3}) + (b^{2/3}*(119*b^3*c - 77*a*b^2*d + 44*a^2*b*e - 20*a^3*f)*Log[a^{1/3} + b^{1/3}*x])/(27*a^{20/3}) + (b^{2/3}*(-119*b^3*c + 77*a*b^2*d - 44*a^2*b*e + 20*a^3*f)*Log[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2])/(54*a^{20/3})$$

Maple [A] time = 0.018, size = 651, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^3,x)

[Out]
$$\frac{3}{5}a^4/x^5*b*d - \frac{6}{5}a^5/x^5*b^2*c + \frac{3}{2}a^4/x^2*b*e - \frac{3}{a^5/x^2*b^2*d} + \frac{5}{a^6/x^2} * b^3*c - \frac{20}{27}a^3*f/(1/b*a)^{2/3} * \ln(x + (1/b*a)^{1/3}) + \frac{10}{27}a^3*f/(1/b*a)^{2/3} * \ln(x^2 - (1/b*a)^{1/3}*x + (1/b*a)^{2/3}) + \frac{3}{8}a^4/x^8*b*c - \frac{11}{18}b^2/a^3/(b*x^3+a)^2*x^4*f + \frac{17}{18}b^3/a^4/(b*x^3+a)^2*x^4*e - \frac{1}{11}c/a^3/x^{11} - \frac{23}{18}b^4/a^5/(b*x^3+a)^2*x^4*d + \frac{29}{18}b^5/a^6/(b*x^3+a)^2*x^4*c - \frac{7}{9}b/a^2/(b*x^3+a)^2*f*x - \frac{20}{27}a^3*f/(1/b*a)^{2/3} * 3^{1/2} * \arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x - 1)) + \frac{44}{27}b/a^4*e/(1/b*a)^{2/3} * \ln(x + (1/b*a)^{1/3}) - \frac{22}{27}b/a^4*e/(1/b*a)^{2/3} * \ln(x^2 - (1/b*a)^{1/3}*x + (1/b*a)^{2/3}) - \frac{77}{27}b^2/a^5*d/(1/b*a)^{2/3} * \ln(x + (1/b*a)^{1/3}) + \frac{77}{54}b^2/a^5*d/(1/b*a)^{2/3} * \ln(x^2 - (1/b*a)^{1/3}*x + (1/b*a)^{2/3}) + \frac{119}{27}b^3/a^6*c/(1/b*a)^{2/3} * \ln(x + (1/b*a)^{1/3}) - \frac{119}{54}b^3/a^6*c/(1/b*a)^{2/3} * \ln(x^2 - (1/b*a)^{1/3}*x + (1/b*a)^{2/3}) + \frac{10}{9}b^2/a^3/(b*x^3+a)^2*e*x - \frac{13}{9}b^3/a^4/(b*x^3+a)^2*d*x + \frac{16}{9}b^4/a^5/(b*x^3+a)^2*c*x - \frac{1}{8}a^3$$

$$\frac{1}{x^8 d - 1/5 a^3/x^5 e - 1/2 a^3/x^2 f - 77/27 b^2/a^5 d / (1/b a)^{2/3} * 3^{1/2} * \arctan(1/3 * 3^{1/2} * (2/(1/b a)^{1/3} * x - 1)) + 119/27 * b^3/a^6 * c / (1/b a)^{2/3} * 3^{1/2} * \arctan(1/3 * 3^{1/2} * (2/(1/b a)^{1/3} * x - 1)) + 44/27 * b/a^4 * e / (1/b a)^{2/3} * 3^{1/2} * \arctan(1/3 * 3^{1/2} * (2/(1/b a)^{1/3} * x - 1))}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.62499, size = 1543, normalized size = 4.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^3,x, algorithm="fricas")

[Out]
$$\frac{1}{11880} * (660 * (119 * b^5 * c - 77 * a * b^4 * d + 44 * a^2 * b^3 * e - 20 * a^3 * b^2 * f) * x^{15} + 1056 * (119 * a * b^4 * c - 77 * a^2 * b^3 * d + 44 * a^3 * b^2 * e - 20 * a^4 * b * f) * x^{12} + 297 * (119 * a^2 * b^3 * c - 77 * a^3 * b^2 * d + 44 * a^4 * b * e - 20 * a^5 * f) * x^9 - 54 * (119 * a^3 * b^2 * c - 77 * a^4 * b * d + 44 * a^5 * e) * x^6 - 1080 * a^5 * c + 135 * (17 * a^4 * b * c - 11 * a^5 * d) * x^3 - 440 * \sqrt{3} * ((119 * b^5 * c - 77 * a * b^4 * d + 44 * a^2 * b^3 * e - 20 * a^3 * b^2 * f) * x^{17} + 2 * (119 * a * b^4 * c - 77 * a^2 * b^3 * d + 44 * a^3 * b^2 * e - 20 * a^4 * b * f) * x^{14} + (119 * a^2 * b^3 * c - 77 * a^3 * b^2 * d + 44 * a^4 * b * e - 20 * a^5 * f) * x^{11}) * (-b^2/a^2)^{1/3} * \arctan(1/3 * (2 * \sqrt{3} * a * x * (-b^2/a^2)^{2/3} - \sqrt{3} * b) / b) + 220 * ((119 * b^5 * c - 77 * a * b^4 * d + 44 * a^2 * b^3 * e - 20 * a^3 * b^2 * f) * x^{17} + 2 * (119 * a * b^4 * c - 77 * a^2 * b^3 * d + 44 * a^3 * b^2 * e - 20 * a^4 * b * f) * x^{14} + (119 * a^2 * b^3 * c - 77 * a^3 * b^2 * d + 44 * a^4 * b * e - 20 * a^5 * f) * x^{11}) * (-b^2/a^2)^{1/3} * \log(b^2 * x^2 + a * b * x * (-b^2/a^2)^{1/3} + a^2 * (-b^2/a^2)^{2/3}) - 440 * ((119 * b^5 * c - 77 * a * b^4 * d + 44 * a^2 * b^3 * e - 20 * a^3 * b^2 * f) * x^{17} + 2 * (119 * a * b^4 * c - 77 * a^2 * b^3 * d + 44 * a^3 * b^2 * e - 20 * a^4 * b * f) * x^{14} + (119 * a^2 * b^3 * c - 77 * a^3 * b^2 * d + 44 * a^4 * b * e - 20 * a^5 * f) * x^{11}) * (-b^2/a^2)^{1/3} * \log(b * x - a * (-b^2/a^2)^{1/3})) / (a^6 * b^2 * x^{17} + 2 * a^7 * b * x^{14} + a^8 * x^{11})$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**12/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.07959, size = 594, normalized size = 1.56

$$\frac{\sqrt{3} \left(119 (-ab^2)^{\frac{1}{3}} b^3 c - 77 (-ab^2)^{\frac{1}{3}} ab^2 d - 20 (-ab^2)^{\frac{1}{3}} a^3 f + 44 (-ab^2)^{\frac{1}{3}} a^2 b e \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{27 a^7} \quad (119 b^4 c - 77 ab^3 d - 20 a^3 b^2 f + 44 a^2 b^3 e)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^3,x, algorithm="giac")

[Out] 1/27*sqrt(3)*(119*(-a*b^2)^(1/3)*b^3*c - 77*(-a*b^2)^(1/3)*a*b^2*d - 20*(-a*b^2)^(1/3)*a^3*f + 44*(-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^7 - 1/27*(119*b^4*c - 77*a*b^3*d - 20*a^3*b^2*f + 44*a^2*b^3*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^7 + 1/54*(119*(-a*b^2)^(1/3)*b^3*c - 77*(-a*b^2)^(1/3)*a*b^2*d - 20*(-a*b^2)^(1/3)*a^3*f + 44*(-a*b^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^7 + 1/18*(29*b^5*c*x^4 - 23*a*b^4*d*x^4 - 11*a^3*b^2*f*x^4 + 17*a^2*b^3*x^4*e + 32*a*b^4*c*x - 26*a^2*b^3*d*x - 14*a^4*b*f*x + 20*a^3*b^2*x*e)/((b*x^3 + a)^2*a^6) + 1/440*(2200*b^3*c*x^9 - 1320*a*b^2*d*x^9 - 220*a^3*f*x^9 + 660*a^2*b*x^9*e - 528*a*b^2*c*x^6 + 264*a^2*b*d*x^6 - 88*a^3*x^6*e + 165*a^2*b*c*x^3 - 55*a^3*d*x^3 - 40*a^3*c)/(a^6*x^11)

$$3.303 \quad \int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)^3} dx$$

Optimal. Leaf size=424

$$\frac{b^2x^2(11a^2be - 8a^3f - 14ab^2d + 17b^3c)}{9a^7(a+bx^3)} - \frac{b^2x^2(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^6(a+bx^3)^2} + \frac{3a^2be + a^3(-f) - 6ab^2d + 10b^3c}{4a^6x^4} - \frac{b^4}{3}$$

[Out] $-c/(13*a^3*x^{13}) + (3*b*c - a*d)/(10*a^4*x^{10}) - (6*b^2*c - 3*a*b*d + a^2*e)/(7*a^5*x^7) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(4*a^6*x^4) - (b*(15*b^3*c - 10*a*b^2*d + 6*a^2*b*e - 3*a^3*f))/(a^7*x) - (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^6*(a + b*x^3)^2) - (b^2*(17*b^3*c - 14*a*b^2*d + 11*a^2*b*e - 8*a^3*f)*x^2)/(9*a^7*(a + b*x^3)) + (b^{(4/3)}*(152*b^3*c - 104*a*b^2*d + 65*a^2*b*e - 35*a^3*f))*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})]/(9*Sqrt[3]*a^{(22/3)}) + (b^{(4/3)}*(152*b^3*c - 104*a*b^2*d + 65*a^2*b*e - 35*a^3*f))*Log[a^{(1/3)} + b^{(1/3)}*x]/(27*a^{(22/3)}) - (b^{(4/3)}*(152*b^3*c - 104*a*b^2*d + 65*a^2*b*e - 35*a^3*f))*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2]/(54*a^{(22/3)})$

Rubi [A] time = 0.853138, antiderivative size = 424, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1829, 1834, 292, 31, 634, 617, 204, 628}

$$\frac{b^2x^2(11a^2be - 8a^3f - 14ab^2d + 17b^3c)}{9a^7(a+bx^3)} - \frac{b^2x^2(a^2be + a^3(-f) - ab^2d + b^3c)}{6a^6(a+bx^3)^2} + \frac{3a^2be + a^3(-f) - 6ab^2d + 10b^3c}{4a^6x^4} - \frac{b^4}{3}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^3), x]

[Out] $-c/(13*a^3*x^{13}) + (3*b*c - a*d)/(10*a^4*x^{10}) - (6*b^2*c - 3*a*b*d + a^2*e)/(7*a^5*x^7) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(4*a^6*x^4) - (b*(15*b^3*c - 10*a*b^2*d + 6*a^2*b*e - 3*a^3*f))/(a^7*x) - (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^6*(a + b*x^3)^2) - (b^2*(17*b^3*c - 14*a*b^2*d + 11*a^2*b*e - 8*a^3*f)*x^2)/(9*a^7*(a + b*x^3)) + (b^{(4/3)}*(152*b^3*c - 104*a*b^2*d + 65*a^2*b*e - 35*a^3*f))*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})]/(9*Sqrt[3]*a^{(22/3)}) + (b^{(4/3)}*(152*b^3*c - 104*a*b^2*d + 65*a^2*b*e - 35*a^3*f))*Log[a^{(1/3)} + b^{(1/3)}*x]/(27*a^{(22/3)}) - (b^{(4/3)}*(152*b^3*c - 104*a*b^2*d + 65*a^2*b*e - 35*a^3*f))*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2]/(54*a^{(22/3)})$

$$(152*b^3*c - 104*a*b^2*d + 65*a^2*b*e - 35*a^3*f)*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2]/(54*a^{(22/3)})$$

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_)*(x_)^(m_)))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[Ex
pandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^( -1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
```

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 204

$\text{Int}[\frac{(a_.) + (b_.)x^{-2}}{x}, x_Symbol] \rightarrow -\text{Simp}[\frac{\text{ArcTan}[\frac{\sqrt{-b, 2}x}{\sqrt{-a, 2}}]}{\sqrt{-a, 2}\sqrt{-b, 2}}, x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 628

$\text{Int}[\frac{(d_.) + (e_.)x}{(a_.) + (b_.)x + (c_.)x^2}, x_Symbol] \rightarrow \text{Simp}[\frac{d \cdot \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]}{b, x}] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2cd - be, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{c + dx^3 + ex^6 + fx^9}{x^{14}(a + bx^3)^3} dx &= -\frac{b^2(b^3c - ab^2d + a^2be - a^3f)x^2}{6a^6(a + bx^3)^2} - \int \frac{-6b^3c + 6b^3\left(\frac{bc}{a} - d\right)x^3 - \frac{6b^3(b^2c - abd + a^2e)x^6}{a^2} + \frac{6b^3(b^3c - ab^2d + a^2be - a^3f)x^9}{a^3} - \frac{6b^4}{x^{14}(a + bx^3)^2}}{6ab^3} \\
&= -\frac{b^2(b^3c - ab^2d + a^2be - a^3f)x^2}{6a^6(a + bx^3)^2} - \frac{b^2(17b^3c - 14ab^2d + 11a^2be - 8a^3f)x^2}{9a^7(a + bx^3)} + \int \frac{18b^8c - 18b^8\left(\frac{2b}{a}\right)}{x^{14}(a + bx^3)^2} \\
&= -\frac{b^2(b^3c - ab^2d + a^2be - a^3f)x^2}{6a^6(a + bx^3)^2} - \frac{b^2(17b^3c - 14ab^2d + 11a^2be - 8a^3f)x^2}{9a^7(a + bx^3)} + \int \left(\frac{18b^8c}{ax^{14}} + \frac{18b^8}{a^2x^{14}}\right) \\
&= -\frac{c}{13a^3x^{13}} + \frac{3bc - ad}{10a^4x^{10}} - \frac{6b^2c - 3abd + a^2e}{7a^5x^7} + \frac{10b^3c - 6ab^2d + 3a^2be - a^3f}{4a^6x^4} - \frac{b(15b^3c - 10a^2b^2d + 5a^3be - 3a^4f)}{9a^7(a + bx^3)} \\
&= -\frac{c}{13a^3x^{13}} + \frac{3bc - ad}{10a^4x^{10}} - \frac{6b^2c - 3abd + a^2e}{7a^5x^7} + \frac{10b^3c - 6ab^2d + 3a^2be - a^3f}{4a^6x^4} - \frac{b(15b^3c - 10a^2b^2d + 5a^3be - 3a^4f)}{9a^7(a + bx^3)} \\
&= -\frac{c}{13a^3x^{13}} + \frac{3bc - ad}{10a^4x^{10}} - \frac{6b^2c - 3abd + a^2e}{7a^5x^7} + \frac{10b^3c - 6ab^2d + 3a^2be - a^3f}{4a^6x^4} - \frac{b(15b^3c - 10a^2b^2d + 5a^3be - 3a^4f)}{9a^7(a + bx^3)} \\
&= -\frac{c}{13a^3x^{13}} + \frac{3bc - ad}{10a^4x^{10}} - \frac{6b^2c - 3abd + a^2e}{7a^5x^7} + \frac{10b^3c - 6ab^2d + 3a^2be - a^3f}{4a^6x^4} - \frac{b(15b^3c - 10a^2b^2d + 5a^3be - 3a^4f)}{9a^7(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.465972, size = 419, normalized size = 0.99

$$\frac{b^2x^2(-11a^2be + 8a^3f + 14ab^2d - 17b^3c)}{9a^7(a + bx^3)} + \frac{b^2x^2(-a^2be + a^3f + ab^2d - b^3c)}{6a^6(a + bx^3)^2} + \frac{3a^2be + a^3(-f) - 6ab^2d + 10b^3c}{4a^6x^4} + \frac{b^{4/3} \log(a + bx^3)}{3a^{10/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^3), x]

```
[Out] -c/(13*a^3*x^13) + (3*b*c - a*d)/(10*a^4*x^10) - (6*b^2*c - 3*a*b*d + a^2*e)
)/(7*a^5*x^7) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(4*a^6*x^4) + (b
*(-15*b^3*c + 10*a*b^2*d - 6*a^2*b*e + 3*a^3*f))/(a^7*x) + (b^2*(-(b^3*c) +
a*b^2*d - a^2*b*e + a^3*f)*x^2)/(6*a^6*(a + b*x^3)^2) + (b^2*(-17*b^3*c +
14*a*b^2*d - 11*a^2*b*e + 8*a^3*f)*x^2)/(9*a^7*(a + b*x^3)) + (b^(4/3)*(152
*b^3*c - 104*a*b^2*d + 65*a^2*b*e - 35*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(
1/3))/Sqrt[3]])/(9*Sqrt[3]*a^(22/3)) + (b^(4/3)*(152*b^3*c - 104*a*b^2*d +
65*a^2*b*e - 35*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(22/3)) + (b^(4/3)*(-
152*b^3*c + 104*a*b^2*d - 65*a^2*b*e + 35*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2])/(54*a^(22/3))
```

Maple [A] time = 0.022, size = 716, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^3,x)
```

```
[Out] 8/9*b^3/a^4/(b*x^3+a)^2*x^5*f-11/9*b^4/a^5/(b*x^3+a)^2*x^5*e+14/9*b^5/a^6/(
b*x^3+a)^2*x^5*d+3/10/a^4/x^10*b*c+3/7/a^4/x^7*b*d-6/7/a^5/x^7*b^2*c+3/4/a^
4/x^4*b*e-3/2/a^5/x^4*b^2*d+5/2/a^6/x^4*b^3*c+3*b/a^4/x*f-6*b^2/a^5/x*e+10*
b^3/a^6/x*d-15*b^4/a^7/x*c+104/27*b^3/a^6*d*3^(1/2)/(1/b*a)^(1/3)*arctan(1/
3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-152/27*b^4/a^7*c*3^(1/2)/(1/b*a)^(1/3)*arc
tan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/13*c/a^3/x^13-1/10/a^3/x^10*d-1/7/
a^3/x^7*e-1/4/a^3/x^4*f-104/27*b^3/a^6*d/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+
52/27*b^3/a^6*d/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+152/27*
b^4/a^7*c/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-76/27*b^4/a^7*c/(1/b*a)^(1/3)*l
n(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-35/27*b/a^4*f/(1/b*a)^(1/3)*ln(x+(1/b*
a)^(1/3))+35/54*b/a^4*f/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))
+65/27*b^2/a^5*e/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-65/54*b^2/a^5*e/(1/b*a)^(
1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-37/18*b^5/a^6/(b*x^3+a)^2*x^2*c
-25/18*b^3/a^4/(b*x^3+a)^2*x^2*e+31/18*b^4/a^5/(b*x^3+a)^2*x^2*d-17/9*b^6/a
^7/(b*x^3+a)^2*x^5*c+19/18*b^2/a^3/(b*x^3+a)^2*x^2*f+35/27*b/a^4*f*3^(1/2)/
(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-65/27*b^2/a^5*e*3^(
1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68566, size = 1638, normalized size = 3.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/49140*(5460*(152*b^6*c - 104*a*b^5*d + 65*a^2*b^4*e - 35*a^3*b^3*f)*x^18
+ 9555*(152*a*b^5*c - 104*a^2*b^4*d + 65*a^3*b^3*e - 35*a^4*b^2*f)*x^15 +
3510*(152*a^2*b^4*c - 104*a^3*b^3*d + 65*a^4*b^2*e - 35*a^5*b*f)*x^12 - 351
*(152*a^3*b^3*c - 104*a^4*b^2*d + 65*a^5*b*e - 35*a^6*f)*x^9 + 3780*a^6*c +
108*(152*a^4*b^2*c - 104*a^5*b*d + 65*a^6*e)*x^6 - 378*(19*a^5*b*c - 13*a^
6*d)*x^3 + 1820*sqrt(3)*((152*b^6*c - 104*a*b^5*d + 65*a^2*b^4*e - 35*a^3*b
^3*f)*x^19 + 2*(152*a*b^5*c - 104*a^2*b^4*d + 65*a^3*b^3*e - 35*a^4*b^2*f)*
x^16 + (152*a^2*b^4*c - 104*a^3*b^3*d + 65*a^4*b^2*e - 35*a^5*b*f)*x^13)*(-
b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 910*((152*b^6
*c - 104*a*b^5*d + 65*a^2*b^4*e - 35*a^3*b^3*f)*x^19 + 2*(152*a*b^5*c - 104
*a^2*b^4*d + 65*a^3*b^3*e - 35*a^4*b^2*f)*x^16 + (152*a^2*b^4*c - 104*a^3*b
^3*d + 65*a^4*b^2*e - 35*a^5*b*f)*x^13)*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)
^(2/3) - a*(-b/a)^(1/3)) + 1820*((152*b^6*c - 104*a*b^5*d + 65*a^2*b^4*e -
35*a^3*b^3*f)*x^19 + 2*(152*a*b^5*c - 104*a^2*b^4*d + 65*a^3*b^3*e - 35*a^4
*b^2*f)*x^16 + (152*a^2*b^4*c - 104*a^3*b^3*d + 65*a^4*b^2*e - 35*a^5*b*f)*
x^13)*(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)))/(a^7*b^2*x^19 + 2*a^8*b*x^16
+ a^9*x^13)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**14/(b*x**3+a)**3,x)
```

[Out] Timed out

Giac [A] time = 1.1053, size = 717, normalized size = 1.69

$$\frac{\sqrt{3} \left(152 (-ab^2)^{\frac{2}{3}} b^3 c - 104 (-ab^2)^{\frac{2}{3}} ab^2 d - 35 (-ab^2)^{\frac{2}{3}} a^3 f + 65 (-ab^2)^{\frac{2}{3}} a^2 b e \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{27 a^8} + \frac{\left(152 b^5 c \left(-\frac{a}{b} \right) \right)}{27 a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^3,x, algorithm="giac")`

[Out]
$$\frac{1}{27} \sqrt{3} (152 (-a b^2)^{2/3} b^3 c - 104 (-a b^2)^{2/3} a b^2 d - 35 (-a b^2)^{2/3} a^3 f + 65 (-a b^2)^{2/3} a^2 b e) \arctan\left(\frac{1}{3} \sqrt{3} (2x + (-a/b)^{1/3}) / (-a/b)^{1/3}\right) / a^8 + \frac{1}{27} (152 b^5 c (-a/b)^{1/3} - 104 a b^4 d (-a/b)^{1/3} - 35 a^3 b^2 f (-a/b)^{1/3} + 65 a^2 b^3 e (-a/b)^{1/3}) \log(\text{abs}(x - (-a/b)^{1/3})) / a^8 - \frac{1}{54} (152 (-a b^2)^{2/3} b^3 c - 104 (-a b^2)^{2/3} a b^2 d - 35 (-a b^2)^{2/3} a^3 f + 65 (-a b^2)^{2/3} a^2 b e) \log(x^2 + x (-a/b)^{1/3} + (-a/b)^{2/3}) / a^8 - \frac{1}{18} (34 b^6 c x^5 - 28 a b^5 d x^5 - 16 a^3 b^3 f x^5 + 22 a^2 b^4 e x^5 + 37 a b^5 c x^2 - 31 a^2 b^4 d x^2 - 19 a^4 b^2 f x^2 + 25 a^3 b^3 e x^2) / ((b x^3 + a)^2 a^7) - \frac{1}{1820} (27300 b^4 c x^{12} - 18200 a b^3 d x^{12} - 5460 a^3 b f x^{12} + 10920 a^2 b^2 e x^{12} - 4550 a b^3 c x^9 + 2730 a^2 b^2 d x^9 + 455 a^4 f x^9 - 1365 a^3 b e x^9 + 1560 a^2 b^2 c x^6 - 780 a^3 b d x^6 + 260 a^4 e x^6 - 546 a^3 b c x^3 + 182 a^4 d x^3 + 140 a^4 e) / (a^7 x^{13})$$

$$3.304 \quad \int \frac{(1-x)x^4}{1+x^3} dx$$

Optimal. Leaf size=54

$$-\frac{x^3}{3} + \frac{x^2}{2} + \frac{1}{6} \log(x^2 - x + 1) + \frac{2}{3} \log(x + 1) + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] $x^2/2 - x^3/3 + \text{ArcTan}[(1 - 2*x)/\text{Sqrt}[3]]/\text{Sqrt}[3] + (2*\text{Log}[1 + x])/3 + \text{Log}[1 - x + x^2]/6$

Rubi [A] time = 0.0719371, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1887, 1874, 31, 634, 618, 204, 628}

$$-\frac{x^3}{3} + \frac{x^2}{2} + \frac{1}{6} \log(x^2 - x + 1) + \frac{2}{3} \log(x + 1) + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 - x)*x^4/(1 + x^3), x]$

[Out] $x^2/2 - x^3/3 + \text{ArcTan}[(1 - 2*x)/\text{Sqrt}[3]]/\text{Sqrt}[3] + (2*\text{Log}[1 + x])/3 + \text{Log}[1 - x + x^2]/6$

Rule 1887

$\text{Int}[(Pq_)/((a_)+(b_)*(x_)^{n_}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[Pq/(a + b*x^n), x], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{IntegerQ}[n]$

Rule 1874

$\text{Int}[(P2_)/((a_)+(b_)*(x_)^3), x_Symbol] \rightarrow \text{With}\{A = \text{Coeff}[P2, x, 0], B = \text{Coeff}[P2, x, 1], C = \text{Coeff}[P2, x, 2], q = (a/b)^{(1/3)}\}, \text{Dist}[(q*(A - B*q + C*q^2))/(3*a), \text{Int}[1/(q + x), x], x] + \text{Dist}[q/(3*a), \text{Int}[(q*(2*A + B*q - C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; \text{NeQ}[a*B^3 - b*A^3, 0] \ \&\& \ \text{NeQ}[A - B*q + C*q^2, 0] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PolyQ}[P2, x, 2] \ \&\& \ \text{GtQ}[a/b, 0]$

Rule 31


```
Int[((a_) + (b_)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^-1, x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-x)x^4}{1+x^3} dx &= \int \left(x - x^2 + \frac{(-1+x)x}{1+x^3} \right) dx \\
&= \frac{x^2}{2} - \frac{x^3}{3} + \int \frac{(-1+x)x}{1+x^3} dx \\
&= \frac{x^2}{2} - \frac{x^3}{3} + \frac{1}{3} \int \frac{-2+x}{1-x+x^2} dx + \frac{2}{3} \int \frac{1}{1+x} dx \\
&= \frac{x^2}{2} - \frac{x^3}{3} + \frac{2}{3} \log(1+x) + \frac{1}{6} \int \frac{-1+2x}{1-x+x^2} dx - \frac{1}{2} \int \frac{1}{1-x+x^2} dx \\
&= \frac{x^2}{2} - \frac{x^3}{3} + \frac{2}{3} \log(1+x) + \frac{1}{6} \log(1-x+x^2) + \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x \right) \\
&= \frac{x^2}{2} - \frac{x^3}{3} - \frac{\tan^{-1} \left(\frac{-1+2x}{\sqrt{3}} \right)}{\sqrt{3}} + \frac{2}{3} \log(1+x) + \frac{1}{6} \log(1-x+x^2)
\end{aligned}$$

Mathematica [A] time = 0.0139, size = 59, normalized size = 1.09

$$\frac{1}{6} \left(-2x^3 + 3x^2 - \log(x^2 - x + 1) + 2 \log(x^3 + 1) + 2 \log(x + 1) - 2\sqrt{3} \tan^{-1} \left(\frac{2x - 1}{\sqrt{3}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((1 - x)*x^4)/(1 + x^3), x]

[Out] (3*x^2 - 2*x^3 - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Log[1 + x] - Log[1 - x + x^2] + 2*Log[1 + x^3])/6

Maple [A] time = 0.005, size = 45, normalized size = 0.8

$$-\frac{x^3}{3} + \frac{x^2}{2} + \frac{\ln(x^2 - x + 1)}{6} - \frac{\sqrt{3}}{3} \arctan\left(\frac{(2x - 1)\sqrt{3}}{3}\right) + \frac{2 \ln(1 + x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-x)*x^4/(x^3+1), x)

[Out] -1/3*x^3+1/2*x^2+1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+2/3*ln(1+x)

Maxima [A] time = 1.4146, size = 59, normalized size = 1.09

$$-\frac{1}{3}x^3 + \frac{1}{2}x^2 - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{6}\log(x^2-x+1) + \frac{2}{3}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)*x^4/(x^3+1),x, algorithm="maxima")

[Out] -1/3*x^3 + 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) + 2/3*log(x + 1)

Fricas [A] time = 1.55536, size = 140, normalized size = 2.59

$$-\frac{1}{3}x^3 + \frac{1}{2}x^2 - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{6}\log(x^2-x+1) + \frac{2}{3}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)*x^4/(x^3+1),x, algorithm="fricas")

[Out] -1/3*x^3 + 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) + 2/3*log(x + 1)

Sympy [A] time = 0.1289, size = 53, normalized size = 0.98

$$-\frac{x^3}{3} + \frac{x^2}{2} + \frac{2\log(x+1)}{3} + \frac{\log(x^2-x+1)}{6} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)*x**4/(x**3+1),x)

[Out] -x**3/3 + x**2/2 + 2*log(x + 1)/3 + log(x**2 - x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3

Giac [A] time = 1.05003, size = 61, normalized size = 1.13

$$-\frac{1}{3}x^3 + \frac{1}{2}x^2 - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{6}\log(x^2-x+1) + \frac{2}{3}\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x^4/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/3*x^3 + 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) + 2/3*log(abs(x + 1))
```

$$3.305 \quad \int \frac{(1-x)x^3}{1+x^3} dx$$

Optimal. Leaf size=30

$$-\frac{x^2}{2} + \frac{1}{3} \log(x^2 - x + 1) + x - \frac{2}{3} \log(x + 1)$$

[Out] $x - x^2/2 - (2*\text{Log}[1 + x])/3 + \text{Log}[1 - x + x^2]/3$

Rubi [A] time = 0.0400098, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1887, 1860, 31, 628}

$$-\frac{x^2}{2} + \frac{1}{3} \log(x^2 - x + 1) + x - \frac{2}{3} \log(x + 1)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\frac{(1-x)*x^3}{(1+x^3)}, x]$

[Out] $x - x^2/2 - (2*\text{Log}[1 + x])/3 + \text{Log}[1 - x + x^2]/3$

Rule 1887

$\text{Int}[\frac{\text{Pq}_x}{(\text{a}_x + (\text{b}_x)(x_x)^{n_x})}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\frac{\text{Pq}_x}{\text{a}_x + \text{b}_x x_x^n}, x], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PolyQ}[\text{Pq}_x, x] \ \&\& \ \text{IntegerQ}[n]$

Rule 1860

$\text{Int}[\frac{(A_x + (B_x)(x_x))}{(\text{a}_x + (\text{b}_x)(x_x)^3)}, x_Symbol] \rightarrow \text{With}[\{r = \text{Numerator}[\text{Rt}[a/b, 3]], s = \text{Denominator}[\text{Rt}[a/b, 3]]\}, -\text{Dist}[(r*(B*r - A*s))/(3*a*s), \text{Int}[1/(r + s*x), x], x] + \text{Dist}[r/(3*a*s), \text{Int}[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; \text{FreeQ}[\{a, b, A, B\}, x] \ \&\& \ \text{NeQ}[a*B^3 - b*A^3, 0] \ \&\& \ \text{PosQ}[a/b]$

Rule 31

$\text{Int}[\frac{(\text{a}_x + (\text{b}_x)(x_x))^{-1}}{x_Symbol}] \rightarrow \text{Simp}[\frac{\text{Log}[\text{RemoveContent}[a + b*x, x]]}{b}, x] /; \text{FreeQ}[\{a, b\}, x]$

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(1-x)x^3}{1+x^3} dx &= \int \left(1-x - \frac{1-x}{1+x^3} \right) dx \\ &= x - \frac{x^2}{2} - \int \frac{1-x}{1+x^3} dx \\ &= x - \frac{x^2}{2} - \frac{1}{3} \int \frac{1-2x}{1-x+x^2} dx - \frac{2}{3} \int \frac{1}{1+x} dx \\ &= x - \frac{x^2}{2} - \frac{2}{3} \log(1+x) + \frac{1}{3} \log(1-x+x^2) \end{aligned}$$

Mathematica [A] time = 0.0043203, size = 30, normalized size = 1.

$$-\frac{x^2}{2} + \frac{1}{3} \log(x^2 - x + 1) + x - \frac{2}{3} \log(x + 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - x)*x^3)/(1 + x^3),x]
```

```
[Out] x - x^2/2 - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3
```

Maple [A] time = 0.004, size = 25, normalized size = 0.8

$$x - \frac{x^2}{2} - \frac{2 \ln(1+x)}{3} + \frac{\ln(x^2 - x + 1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)*x^3/(x^3+1),x)
```

```
[Out] x-1/2*x^2-2/3*ln(1+x)+1/3*ln(x^2-x+1)
```

Maxima [A] time = 1.42281, size = 32, normalized size = 1.07

$$-\frac{1}{2}x^2 + x + \frac{1}{3}\log(x^2 - x + 1) - \frac{2}{3}\log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)*x^3/(x^3+1),x, algorithm="maxima")

[Out] -1/2*x^2 + x + 1/3*log(x^2 - x + 1) - 2/3*log(x + 1)

Fricas [A] time = 1.51494, size = 73, normalized size = 2.43

$$-\frac{1}{2}x^2 + x + \frac{1}{3}\log(x^2 - x + 1) - \frac{2}{3}\log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)*x^3/(x^3+1),x, algorithm="fricas")

[Out] -1/2*x^2 + x + 1/3*log(x^2 - x + 1) - 2/3*log(x + 1)

Sympy [A] time = 0.089936, size = 24, normalized size = 0.8

$$-\frac{x^2}{2} + x - \frac{2\log(x + 1)}{3} + \frac{\log(x^2 - x + 1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)*x**3/(x**3+1),x)

[Out] -x**2/2 + x - 2*log(x + 1)/3 + log(x**2 - x + 1)/3

Giac [A] time = 1.07455, size = 34, normalized size = 1.13

$$-\frac{1}{2}x^2 + x + \frac{1}{3}\log(x^2 - x + 1) - \frac{2}{3}\log(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x^3/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/2*x^2 + x + 1/3*log(x^2 - x + 1) - 2/3*log(abs(x + 1))
```


$$3.306 \quad \int \frac{(1-x)x^2}{1+x^3} dx$$

Optimal. Leaf size=44

$$\frac{1}{6} \log(x^2 - x + 1) - x + \frac{2}{3} \log(x + 1) - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] $-x - \text{ArcTan}[(1 - 2*x)/\text{Sqrt}[3]]/\text{Sqrt}[3] + (2*\text{Log}[1 + x])/3 + \text{Log}[1 - x + x^2]/6$

Rubi [A] time = 0.0598778, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1887, 1874, 31, 634, 618, 204, 628}

$$\frac{1}{6} \log(x^2 - x + 1) - x + \frac{2}{3} \log(x + 1) - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 - x)*x^2/(1 + x^3), x]$

[Out] $-x - \text{ArcTan}[(1 - 2*x)/\text{Sqrt}[3]]/\text{Sqrt}[3] + (2*\text{Log}[1 + x])/3 + \text{Log}[1 - x + x^2]/6$

Rule 1887

$\text{Int}[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[Pq/(a + b*x^n), x], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{IntegerQ}[n]$

Rule 1874

$\text{Int}[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] \rightarrow \text{With}\{A = \text{Coeff}[P2, x, 0], B = \text{Coeff}[P2, x, 1], C = \text{Coeff}[P2, x, 2], q = (a/b)^{(1/3)}\}, \text{Dist}[(q*(A - B*q + C*q^2))/(3*a), \text{Int}[1/(q + x), x], x] + \text{Dist}[q/(3*a), \text{Int}[(q*(2*A + B*q - C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; \text{NeQ}[a*B^3 - b*A^3, 0] \ \&\& \ \text{NeQ}[A - B*q + C*q^2, 0] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PolyQ}[P2, x, 2] \ \&\& \ \text{GtQ}[a/b, 0]$

Rule 31

```
Int[((a_) + (b_.)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^( -1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^( -1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-x)x^2}{1+x^3} dx &= \int \left(-1 + \frac{1+x^2}{1+x^3} \right) dx \\
&= -x + \int \frac{1+x^2}{1+x^3} dx \\
&= -x + \frac{1}{3} \int \frac{1+x}{1-x+x^2} dx + \frac{2}{3} \int \frac{1}{1+x} dx \\
&= -x + \frac{2}{3} \log(1+x) + \frac{1}{6} \int \frac{-1+2x}{1-x+x^2} dx + \frac{1}{2} \int \frac{1}{1-x+x^2} dx \\
&= -x + \frac{2}{3} \log(1+x) + \frac{1}{6} \log(1-x+x^2) - \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x \right) \\
&= -x + \frac{\tan^{-1} \left(\frac{-1+2x}{\sqrt{3}} \right)}{\sqrt{3}} + \frac{2}{3} \log(1+x) + \frac{1}{6} \log(1-x+x^2)
\end{aligned}$$

Mathematica [A] time = 0.0086687, size = 53, normalized size = 1.2

$$-\frac{1}{6} \log(x^2 - x + 1) + \frac{1}{3} \log(x^3 + 1) - x + \frac{1}{3} \log(x + 1) + \frac{\tan^{-1} \left(\frac{2x-1}{\sqrt{3}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[((1 - x)*x^2)/(1 + x^3), x]

[Out] -x + ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 + x]/3 - Log[1 - x + x^2]/6 + Log[1 + x^3]/3

Maple [A] time = 0.003, size = 38, normalized size = 0.9

$$-x + \frac{\ln(x^2 - x + 1)}{6} + \frac{\sqrt{3}}{3} \arctan \left(\frac{(2x-1)\sqrt{3}}{3} \right) + \frac{2 \ln(1+x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-x)*x^2/(x^3+1), x)

[Out] -x+1/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+2/3*ln(1+x)

Maxima [A] time = 1.41157, size = 50, normalized size = 1.14

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x-1)\right) - x + \frac{1}{6} \log(x^2 - x + 1) + \frac{2}{3} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)*x^2/(x^3+1),x, algorithm="maxima")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - x + 1/6*log(x^2 - x + 1) + 2/3*log(x + 1)

Fricas [A] time = 1.49866, size = 117, normalized size = 2.66

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x-1)\right) - x + \frac{1}{6} \log(x^2 - x + 1) + \frac{2}{3} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)*x^2/(x^3+1),x, algorithm="fricas")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - x + 1/6*log(x^2 - x + 1) + 2/3*log(x + 1)

Sympy [A] time = 0.123329, size = 44, normalized size = 1.

$$-x + \frac{2 \log(x + 1)}{3} + \frac{\log(x^2 - x + 1)}{6} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)*x**2/(x**3+1),x)

[Out] -x + 2*log(x + 1)/3 + log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3

Giac [A] time = 1.06137, size = 51, normalized size = 1.16

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x-1)\right) - x + \frac{1}{6} \log(x^2 - x + 1) + \frac{2}{3} \log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x^2/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - x + 1/6*log(x^2 - x + 1) + 2/3*log(abs(x + 1))
```

3.307

$$\int \frac{(1-x)x}{1+x^3} dx$$

Optimal. Leaf size=41

$$-\frac{1}{6} \log(x^2 - x + 1) - \frac{2}{3} \log(x + 1) - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] $-(\text{ArcTan}[(1 - 2*x)/\text{Sqrt}[3]]/\text{Sqrt}[3]) - (2*\text{Log}[1 + x])/3 - \text{Log}[1 - x + x^2]/6$

Rubi [A] time = 0.0418063, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1874, 31, 634, 618, 204, 628}

$$-\frac{1}{6} \log(x^2 - x + 1) - \frac{2}{3} \log(x + 1) - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[((1 - x)*x)/(1 + x^3),x]

[Out] $-(\text{ArcTan}[(1 - 2*x)/\text{Sqrt}[3]]/\text{Sqrt}[3]) - (2*\text{Log}[1 + x])/3 - \text{Log}[1 - x + x^2]/6$

Rule 1874

Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[(q*(A - B*q + C*q^2))/(3*a), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q - C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*A^3, 0] && NeQ[A - B*q + C*q^2, 0] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2] && GtQ[a/b, 0]

Rule 31

Int[((a_) + (b_.)*(x_))^-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{(1-x)x}{1+x^3} dx &= \frac{1}{3} \int \frac{2-x}{1-x+x^2} dx - \frac{2}{3} \int \frac{1}{1+x} dx \\
 &= -\frac{2}{3} \log(1+x) - \frac{1}{6} \int \frac{-1+2x}{1-x+x^2} dx + \frac{1}{2} \int \frac{1}{1-x+x^2} dx \\
 &= -\frac{2}{3} \log(1+x) - \frac{1}{6} \log(1-x+x^2) - \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x \right) \\
 &= \frac{\tan^{-1} \left(\frac{-1+2x}{\sqrt{3}} \right)}{\sqrt{3}} - \frac{2}{3} \log(1+x) - \frac{1}{6} \log(1-x+x^2)
 \end{aligned}$$

Mathematica [A] time = 0.0072435, size = 50, normalized size = 1.22

$$\frac{1}{6} \log(x^2 - x + 1) - \frac{1}{3} \log(x^3 + 1) - \frac{1}{3} \log(x + 1) + \frac{\tan^{-1} \left(\frac{2x-1}{\sqrt{3}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[((1 - x)*x)/(1 + x^3),x]

[Out] ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x + x^2]/6 - Log[1 + x^3]/3

Maple [A] time = 0.004, size = 35, normalized size = 0.9

$$-\frac{\ln(x^2 - x + 1)}{6} + \frac{\sqrt{3}}{3} \arctan\left(\frac{(2x - 1)\sqrt{3}}{3}\right) - \frac{2 \ln(1 + x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-x)*x/(x^3+1),x)

[Out] -1/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-2/3*ln(1+x)

Maxima [A] time = 1.41225, size = 46, normalized size = 1.12

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x - 1)\right) - \frac{1}{6} \log(x^2 - x + 1) - \frac{2}{3} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)*x/(x^3+1),x, algorithm="maxima")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*log(x + 1)

Fricas [A] time = 1.54614, size = 112, normalized size = 2.73

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x - 1)\right) - \frac{1}{6} \log(x^2 - x + 1) - \frac{2}{3} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)*x/(x^3+1),x, algorithm="fricas")

[Out] $\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{6}\log(x^2-x+1) - \frac{2}{3}\log(x+1)$

Sympy [A] time = 0.131201, size = 42, normalized size = 1.02

$$-\frac{2\log(x+1)}{3} - \frac{\log(x^2-x+1)}{6} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-x)*x/(x**3+1),x)`

[Out] $-2*\log(x+1)/3 - \log(x**2-x+1)/6 + \sqrt{3}*\operatorname{atan}(2*\sqrt{3}*x/3 - \sqrt{3})/3$

Giac [A] time = 1.05767, size = 47, normalized size = 1.15

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{6}\log(x^2-x+1) - \frac{2}{3}\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-x)*x/(x^3+1),x, algorithm="giac")`

[Out] $\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{6}\log(x^2-x+1) - \frac{2}{3}\log(\operatorname{abs}(x+1))$

3.308

$$\int \frac{1-x}{x(1+x^3)} dx$$

Optimal. Leaf size=42

$$-\frac{1}{6} \log(x^2 - x + 1) + \log(x) - \frac{2}{3} \log(x + 1) + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[x] - (2*Log[1 + x])/3 - Log[1 - x + x^2]/6

Rubi [A] time = 0.0493821, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1834, 634, 618, 204, 628}

$$-\frac{1}{6} \log(x^2 - x + 1) + \log(x) - \frac{2}{3} \log(x + 1) + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x)/(x*(1 + x^3)), x]

[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[x] - (2*Log[1 + x])/3 - Log[1 - x + x^2]/6

Rule 1834

Int[((Pq_)*((c_)*(x_)^(m_)))/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] & & PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1-x}{x(1+x^3)} dx &= \int \left(\frac{1}{x} - \frac{2}{3(1+x)} + \frac{-1-x}{3(1-x+x^2)} \right) dx \\
 &= \log(x) - \frac{2}{3} \log(1+x) + \frac{1}{3} \int \frac{-1-x}{1-x+x^2} dx \\
 &= \log(x) - \frac{2}{3} \log(1+x) - \frac{1}{6} \int \frac{-1+2x}{1-x+x^2} dx - \frac{1}{2} \int \frac{1}{1-x+x^2} dx \\
 &= \log(x) - \frac{2}{3} \log(1+x) - \frac{1}{6} \log(1-x+x^2) + \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x \right) \\
 &= -\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \log(x) - \frac{2}{3} \log(1+x) - \frac{1}{6} \log(1-x+x^2)
 \end{aligned}$$

Mathematica [A] time = 0.0076564, size = 53, normalized size = 1.26

$$\frac{1}{6} \log(x^2 - x + 1) - \frac{1}{3} \log(x^3 + 1) + \log(x) - \frac{1}{3} \log(x + 1) - \frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/(x*(1 + x^3)), x]
```

[Out] $-(\text{ArcTan}[(-1 + 2x)/\text{Sqrt}[3]]/\text{Sqrt}[3]) + \text{Log}[x] - \text{Log}[1 + x]/3 + \text{Log}[1 - x + x^2]/6 - \text{Log}[1 + x^3]/3$

Maple [A] time = 0.006, size = 37, normalized size = 0.9

$$-\frac{\ln(x^2 - x + 1)}{6} - \frac{\sqrt{3}}{3} \arctan\left(\frac{(2x - 1)\sqrt{3}}{3}\right) + \ln(x) - \frac{2 \ln(1 + x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1-x)/x/(x^3+1),x)`

[Out] $-1/6*\ln(x^2-x+1)-1/3*3^{(1/2)}*\arctan(1/3*(2*x-1)*3^{(1/2)})+\ln(x)-2/3*\ln(1+x)$

Maxima [A] time = 1.42337, size = 49, normalized size = 1.17

$$-\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x - 1)\right) - \frac{1}{6} \log(x^2 - x + 1) - \frac{2}{3} \log(x + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-x)/x/(x^3+1),x, algorithm="maxima")`

[Out] $-1/3*\text{sqrt}(3)*\arctan(1/3*\text{sqrt}(3)*(2*x - 1)) - 1/6*\log(x^2 - x + 1) - 2/3*\log(x + 1) + \log(x)$

Fricas [A] time = 1.4933, size = 126, normalized size = 3.

$$-\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x - 1)\right) - \frac{1}{6} \log(x^2 - x + 1) - \frac{2}{3} \log(x + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-x)/x/(x^3+1),x, algorithm="fricas")`

[Out] $-1/3*\text{sqrt}(3)*\arctan(1/3*\text{sqrt}(3)*(2*x - 1)) - 1/6*\log(x^2 - x + 1) - 2/3*\log(x + 1) + \log(x)$

Sympy [A] time = 0.175426, size = 46, normalized size = 1.1

$$\log(x) - \frac{2\log(x+1)}{3} - \frac{\log(x^2 - x + 1)}{6} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)/x/(x**3+1),x)

[Out] log(x) - 2*log(x + 1)/3 - log(x**2 - x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3

Giac [A] time = 1.08784, size = 51, normalized size = 1.21

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{6}\log(x^2 - x + 1) - \frac{2}{3}\log(|x+1|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)/x/(x^3+1),x, algorithm="giac")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*log(abs(x + 1)) + log(abs(x))

$$3.309 \quad \int \frac{1-x}{x^2(1+x^3)} dx$$

Optimal. Leaf size=49

$$\frac{1}{6} \log(x^2 - x + 1) - \frac{1}{x} - \log(x) + \frac{2}{3} \log(x + 1) + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] $-x^{-1} + \text{ArcTan}[(1 - 2*x)/\text{Sqrt}[3]]/\text{Sqrt}[3] - \text{Log}[x] + (2*\text{Log}[1 + x])/3 + \text{Log}[1 - x + x^2]/6$

Rubi [A] time = 0.049683, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1834, 634, 618, 204, 628}

$$\frac{1}{6} \log(x^2 - x + 1) - \frac{1}{x} - \log(x) + \frac{2}{3} \log(x + 1) + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 - x)/(x^2*(1 + x^3)), x]$

[Out] $-x^{-1} + \text{ArcTan}[(1 - 2*x)/\text{Sqrt}[3]]/\text{Sqrt}[3] - \text{Log}[x] + (2*\text{Log}[1 + x])/3 + \text{Log}[1 - x + x^2]/6$

Rule 1834

$\text{Int}[(\text{Pq}_*) * ((\text{c}_*) * (\text{x}_*))^{(\text{m}_*)}) / ((\text{a}_*) + (\text{b}_*) * (\text{x}_*)^{(\text{n}_*)}), \text{x_Symbol}] :> \text{Int}[\text{ExpandIntegrand}[(\text{c}*x)^{\text{m}}*\text{Pq}]/(\text{a} + \text{b}*x^{\text{n}}), \text{x}], \text{x}] /; \text{FreeQ}[\{\text{a}, \text{b}, \text{c}, \text{m}\}, \text{x}] \& \& \text{PolyQ}[\text{Pq}, \text{x}] \& \& \text{IntegerQ}[\text{n}] \& \& !\text{IGtQ}[\text{m}, 0]$

Rule 634

$\text{Int}[(\text{d}_*) + (\text{e}_*) * (\text{x}_*)] / ((\text{a}_*) + (\text{b}_*) * (\text{x}_*) + (\text{c}_*) * (\text{x}_*)^2), \text{x_Symbol}] :> \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(\text{a} + \text{b}*x + \text{c}*x^2), \text{x}], \text{x}] + \text{Dist}[e/(2*c), \text{Int}[(\text{b} + 2*c*x)/(\text{a} + \text{b}*x + \text{c}*x^2), \text{x}], \text{x}] /; \text{FreeQ}[\{\text{a}, \text{b}, \text{c}, \text{d}, \text{e}\}, \text{x}] \& \& \text{NeQ}[\text{b}^2 - 4*a*c, 0] \& \& \text{NeQ}[\text{b}^2 - 4*a*c, 0] \& \& !\text{NiceSqrtQ}[\text{b}^2 - 4*a*c]$

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1-x}{x^2(1+x^3)} dx &= \int \left(\frac{1}{x^2} - \frac{1}{x} + \frac{2}{3(1+x)} + \frac{-2+x}{3(1-x+x^2)} \right) dx \\ &= -\frac{1}{x} - \log(x) + \frac{2}{3} \log(1+x) + \frac{1}{3} \int \frac{-2+x}{1-x+x^2} dx \\ &= -\frac{1}{x} - \log(x) + \frac{2}{3} \log(1+x) + \frac{1}{6} \int \frac{-1+2x}{1-x+x^2} dx - \frac{1}{2} \int \frac{1}{1-x+x^2} dx \\ &= -\frac{1}{x} - \log(x) + \frac{2}{3} \log(1+x) + \frac{1}{6} \log(1-x+x^2) + \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x \right) \\ &= -\frac{1}{x} - \frac{\tan^{-1} \left(\frac{-1+2x}{\sqrt{3}} \right)}{\sqrt{3}} - \log(x) + \frac{2}{3} \log(1+x) + \frac{1}{6} \log(1-x+x^2) \end{aligned}$$

Mathematica [A] time = 0.0153023, size = 60, normalized size = 1.22

$$-\frac{1}{6} \log(x^2 - x + 1) + \frac{1}{3} \log(x^3 + 1) - \frac{1}{x} - \log(x) + \frac{1}{3} \log(x + 1) - \frac{\tan^{-1} \left(\frac{2x-1}{\sqrt{3}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/(x^2*(1 + x^3)), x]
```

[Out] $-x^{-1} - \text{ArcTan}[-1 + 2x]/\text{Sqrt}[3]/\text{Sqrt}[3] - \text{Log}[x] + \text{Log}[1 + x]/3 - \text{Log}[1 - x + x^2]/6 + \text{Log}[1 + x^3]/3$

Maple [A] time = 0.008, size = 44, normalized size = 0.9

$$\frac{\ln(x^2 - x + 1)}{6} - \frac{\sqrt{3}}{3} \arctan\left(\frac{(2x - 1)\sqrt{3}}{3}\right) - x^{-1} - \ln(x) + \frac{2 \ln(1 + x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1-x)/x^2/(x^3+1),x)`

[Out] $1/6*\ln(x^2-x+1)-1/3*3^{(1/2)}*\arctan(1/3*(2*x-1)*3^{(1/2)})-1/x-\ln(x)+2/3*\ln(1+x)$

Maxima [A] time = 1.47203, size = 58, normalized size = 1.18

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{x} + \frac{1}{6}\log(x^2-x+1) + \frac{2}{3}\log(x+1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-x)/x^2/(x^3+1),x, algorithm="maxima")`

[Out] $-1/3*\text{sqrt}(3)*\arctan(1/3*\text{sqrt}(3)*(2*x - 1)) - 1/x + 1/6*\log(x^2 - x + 1) + 2/3*\log(x + 1) - \log(x)$

Fricas [A] time = 1.53385, size = 144, normalized size = 2.94

$$\frac{2\sqrt{3}x\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - x\log(x^2-x+1) - 4x\log(x+1) + 6x\log(x) + 6}{6x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-x)/x^2/(x^3+1),x, algorithm="fricas")`

[Out] $-1/6*(2*\sqrt{3}*x*\arctan(1/3*\sqrt{3}*(2*x - 1)) - x*\log(x^2 - x + 1) - 4*x*\log(x + 1) + 6*x*\log(x) + 6)/x$

Sympy [A] time = 0.194682, size = 49, normalized size = 1.

$$-\log(x) + \frac{2\log(x+1)}{3} + \frac{\log(x^2-x+1)}{6} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-x)/x**2/(x**3+1),x)`

[Out] $-\log(x) + 2*\log(x + 1)/3 + \log(x**2 - x + 1)/6 - \sqrt{3}*\operatorname{atan}(2*\sqrt{3}*x/3 - \sqrt{3}/3)/3 - 1/x$

Giac [A] time = 1.06451, size = 61, normalized size = 1.24

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{x} + \frac{1}{6}\log(x^2-x+1) + \frac{2}{3}\log(|x+1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-x)/x^2/(x^3+1),x, algorithm="giac")`

[Out] $-1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - 1)) - 1/x + 1/6*\log(x^2 - x + 1) + 2/3*\log(\operatorname{abs}(x + 1)) - \log(\operatorname{abs}(x))$

$$3.310 \quad \int \frac{1-x}{x^3(1+x^3)} dx$$

Optimal. Leaf size=32

$$-\frac{1}{2x^2} + \frac{1}{3} \log(x^2 - x + 1) + \frac{1}{x} - \frac{2}{3} \log(x + 1)$$

[Out] $-1/(2*x^2) + x^{(-1)} - (2*\text{Log}[1 + x])/3 + \text{Log}[1 - x + x^2]/3$

Rubi [A] time = 0.0326286, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1834, 628}

$$-\frac{1}{2x^2} + \frac{1}{3} \log(x^2 - x + 1) + \frac{1}{x} - \frac{2}{3} \log(x + 1)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 - x)/(x^3*(1 + x^3)), x]$

[Out] $-1/(2*x^2) + x^{(-1)} - (2*\text{Log}[1 + x])/3 + \text{Log}[1 - x + x^2]/3$

Rule 1834

$\text{Int}[(Pq_)*((c_)*(x_))^{(m_)}]/((a_) + (b_)*(x_)^{(n_)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*Pq]/(a + b*x^n), x] /;$ FreeQ[{a, b, c, m}, x] & PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]

Rule 628

$\text{Int}(((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /;$ FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{1-x}{x^3(1+x^3)} dx &= \int \left(\frac{1}{x^3} - \frac{1}{x^2} - \frac{2}{3(1+x)} + \frac{-1+2x}{3(1-x+x^2)} \right) dx \\ &= -\frac{1}{2x^2} + \frac{1}{x} - \frac{2}{3} \log(1+x) + \frac{1}{3} \int \frac{-1+2x}{1-x+x^2} dx \\ &= -\frac{1}{2x^2} + \frac{1}{x} - \frac{2}{3} \log(1+x) + \frac{1}{3} \log(1-x+x^2) \end{aligned}$$

Mathematica [A] time = 0.003987, size = 32, normalized size = 1.

$$-\frac{1}{2x^2} + \frac{1}{3} \log(x^2 - x + 1) + \frac{1}{x} - \frac{2}{3} \log(x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x)/(x^3*(1 + x^3)),x]

[Out] -1/(2*x^2) + x^(-1) - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3

Maple [A] time = 0.007, size = 27, normalized size = 0.8

$$-\frac{1}{2x^2} + x^{-1} - \frac{2 \ln(1+x)}{3} + \frac{\ln(x^2 - x + 1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-x)/x^3/(x^3+1),x)

[Out] -1/2/x^2+1/x-2/3*ln(1+x)+1/3*ln(x^2-x+1)

Maxima [A] time = 1.40062, size = 38, normalized size = 1.19

$$\frac{2x-1}{2x^2} + \frac{1}{3} \log(x^2 - x + 1) - \frac{2}{3} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)/x^3/(x^3+1),x, algorithm="maxima")

[Out] $1/2*(2*x - 1)/x^2 + 1/3*\log(x^2 - x + 1) - 2/3*\log(x + 1)$

Fricas [A] time = 1.42257, size = 85, normalized size = 2.66

$$\frac{2x^2 \log(x^2 - x + 1) - 4x^2 \log(x + 1) + 6x - 3}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-x)/x^3/(x^3+1),x, algorithm="fricas")`

[Out] $1/6*(2*x^2*\log(x^2 - x + 1) - 4*x^2*\log(x + 1) + 6*x - 3)/x^2$

Sympy [A] time = 0.109497, size = 27, normalized size = 0.84

$$-\frac{2 \log(x + 1)}{3} + \frac{\log(x^2 - x + 1)}{3} + \frac{2x - 1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-x)/x**3/(x**3+1),x)`

[Out] $-2*\log(x + 1)/3 + \log(x**2 - x + 1)/3 + (2*x - 1)/(2*x**2)$

Giac [A] time = 1.04714, size = 39, normalized size = 1.22

$$\frac{2x - 1}{2x^2} + \frac{1}{3} \log(x^2 - x + 1) - \frac{2}{3} \log(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-x)/x^3/(x^3+1),x, algorithm="giac")`

[Out] $1/2*(2*x - 1)/x^2 + 1/3*\log(x^2 - x + 1) - 2/3*\log(\text{abs}(x + 1))$

$$3.311 \quad \int \frac{x(1+2x)}{1+x^3} dx$$

Optimal. Leaf size=41

$$\frac{5}{6} \log(x^2 - x + 1) + \frac{1}{3} \log(x + 1) - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] $-(\text{ArcTan}[(1 - 2*x)/\text{Sqrt}[3]]/\text{Sqrt}[3]) + \text{Log}[1 + x]/3 + (5*\text{Log}[1 - x + x^2])/6$

Rubi [A] time = 0.0410974, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1874, 31, 634, 618, 204, 628}

$$\frac{5}{6} \log(x^2 - x + 1) + \frac{1}{3} \log(x + 1) - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x*(1 + 2*x))/(1 + x^3), x]$

[Out] $-(\text{ArcTan}[(1 - 2*x)/\text{Sqrt}[3]]/\text{Sqrt}[3]) + \text{Log}[1 + x]/3 + (5*\text{Log}[1 - x + x^2])/6$

Rule 1874

$\text{Int}[(P2_)/((a_)+(b_)*(x_)^3), x_Symbol] \rightarrow \text{With}\{A = \text{Coeff}[P2, x, 0], B = \text{Coeff}[P2, x, 1], C = \text{Coeff}[P2, x, 2], q = (a/b)^{(1/3)}\}, \text{Dist}[(q*(A - B*q + C*q^2))/(3*a), \text{Int}[1/(q + x), x], x] + \text{Dist}[q/(3*a), \text{Int}[(q*(2*A + B*q - C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; \text{NeQ}[a*B^3 - b*A^3, 0] \&\& \text{NeQ}[A - B*q + C*q^2, 0] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PolyQ}[P2, x, 2] \&\& \text{GtQ}[a/b, 0]$

Rule 31

$\text{Int}[(a_)+(b_)*(x_)^{-1}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}\{a, b\}, x]$

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x(1+2x)}{1+x^3} dx &= \frac{1}{3} \int \frac{1}{1+x} dx + \frac{1}{3} \int \frac{-1+5x}{1-x+x^2} dx \\ &= \frac{1}{3} \log(1+x) + \frac{1}{2} \int \frac{1}{1-x+x^2} dx + \frac{5}{6} \int \frac{-1+2x}{1-x+x^2} dx \\ &= \frac{1}{3} \log(1+x) + \frac{5}{6} \log(1-x+x^2) - \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x \right) \\ &= \frac{\tan^{-1} \left(\frac{-1+2x}{\sqrt{3}} \right)}{\sqrt{3}} + \frac{1}{3} \log(1+x) + \frac{5}{6} \log(1-x+x^2) \end{aligned}$$

Mathematica [A] time = 0.0073541, size = 47, normalized size = 1.15

$$\frac{1}{6} \left(\log(x^2 - x + 1) + 4 \log(x^3 + 1) - 2 \log(x + 1) + 2\sqrt{3} \tan^{-1} \left(\frac{2x - 1}{\sqrt{3}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x*(1 + 2*x))/(1 + x^3),x]

[Out] (2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Log[1 + x] + Log[1 - x + x^2] + 4*Log[1 + x^3])/6

Maple [A] time = 0.004, size = 35, normalized size = 0.9

$$\frac{5 \ln(x^2 - x + 1)}{6} + \frac{\sqrt{3}}{3} \arctan\left(\frac{(2x - 1)\sqrt{3}}{3}\right) + \frac{\ln(1 + x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(1+2*x)/(x^3+1),x)

[Out] 5/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/3*ln(1+x)

Maxima [A] time = 1.40721, size = 46, normalized size = 1.12

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x - 1)\right) + \frac{5}{6} \log(x^2 - x + 1) + \frac{1}{3} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(1+2*x)/(x^3+1),x, algorithm="maxima")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 5/6*log(x^2 - x + 1) + 1/3*log(x + 1)

Fricas [A] time = 1.54763, size = 112, normalized size = 2.73

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x - 1)\right) + \frac{5}{6} \log(x^2 - x + 1) + \frac{1}{3} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(1+2*x)/(x^3+1),x, algorithm="fricas")

[Out] $\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{5}{6}\log(x^2-x+1) + \frac{1}{3}\log(x+1)$

Sympy [A] time = 0.138427, size = 42, normalized size = 1.02

$$\frac{\log(x+1)}{3} + \frac{5\log(x^2-x+1)}{6} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(1+2*x)/(x**3+1),x)`

[Out] $\log(x+1)/3 + 5*\log(x**2-x+1)/6 + \sqrt{3}*\operatorname{atan}(2*\sqrt{3}*x/3 - \sqrt{3}/3)/3$

Giac [A] time = 1.06795, size = 47, normalized size = 1.15

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{5}{6}\log(x^2-x+1) + \frac{1}{3}\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(1+2*x)/(x^3+1),x, algorithm="giac")`

[Out] $\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{5}{6}\log(x^2-x+1) + \frac{1}{3}\log(\operatorname{abs}(x+1))$

$$3.312 \quad \int \frac{x(1+2x)}{1-x^3} dx$$

Optimal. Leaf size=39

$$-\frac{1}{2} \log(x^2 + x + 1) - \log(1 - x) - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] $-(\text{ArcTan}[(1 + 2*x)/\text{Sqrt}[3]]/\text{Sqrt}[3]) - \text{Log}[1 - x] - \text{Log}[1 + x + x^2]/2$

Rubi [A] time = 0.0395694, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1875, 31, 634, 618, 204, 628}

$$-\frac{1}{2} \log(x^2 + x + 1) - \log(1 - x) - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x*(1 + 2*x))/(1 - x^3), x]$

[Out] $-(\text{ArcTan}[(1 + 2*x)/\text{Sqrt}[3]]/\text{Sqrt}[3]) - \text{Log}[1 - x] - \text{Log}[1 + x + x^2]/2$

Rule 1875

$\text{Int}[(P2_)/((a_ + (b_)*(x_)^3), x_Symbol] := \text{With}[\{A = \text{Coeff}[P2, x, 0], B = \text{Coeff}[P2, x, 1], C = \text{Coeff}[P2, x, 2], q = (-a/b)^{(1/3)}\}, \text{Dist}[(q*(A + B*q + C*q^2))/(3*a), \text{Int}[1/(q - x), x], x] + \text{Dist}[q/(3*a), \text{Int}[(q*(2*A - B*q - C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; \text{NeQ}[a*B^3 - b*A^3, 0] \&\& \text{NeQ}[A + B*q + C*q^2, 0] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[P2, x, 2] \&\& \text{LtQ}[a/b, 0]$

Rule 31

$\text{Int}[((a_ + (b_)*(x_))^{(-1)}, x_Symbol] := \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}[\{a, b\}, x]$

Rule 634

$\text{Int}[((d_ + (e_)*(x_))/((a_ + (b_)*(x_ + (c_)*(x_)^2), x_Symbol] := \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{In}$

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{x(1+2x)}{1-x^3} dx &= \frac{1}{3} \int \frac{-3-3x}{1+x+x^2} dx + \int \frac{1}{1-x} dx \\
 &= -\log(1-x) - \frac{1}{2} \int \frac{1}{1+x+x^2} dx - \frac{1}{2} \int \frac{1+2x}{1+x+x^2} dx \\
 &= -\log(1-x) - \frac{1}{2} \log(1+x+x^2) + \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1+2x\right) \\
 &= -\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \log(1-x) - \frac{1}{2} \log(1+x+x^2)
 \end{aligned}$$

Mathematica [A] time = 0.010297, size = 53, normalized size = 1.36

$$\frac{1}{6} \log(x^2 + x + 1) - \frac{2}{3} \log(1 - x^3) - \frac{1}{3} \log(1 - x) - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + 2*x))/(1 - x^3), x]
```

[Out] $-(\text{ArcTan}[(1 + 2x)/\text{Sqrt}[3]]/\text{Sqrt}[3]) - \text{Log}[1 - x]/3 + \text{Log}[1 + x + x^2]/6 - (2*\text{Log}[1 - x^3])/3$

Maple [A] time = 0.005, size = 33, normalized size = 0.9

$$-\ln(-1+x) - \frac{\ln(x^2+x+1)}{2} - \frac{\sqrt{3}}{3} \arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(1+2*x)/(-x^3+1),x)`

[Out] $-\ln(-1+x) - 1/2*\ln(x^2+x+1) - 1/3*\arctan(1/3*(1+2*x)*3^{(1/2)})*3^{(1/2)}$

Maxima [A] time = 1.40839, size = 43, normalized size = 1.1

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{2}\log(x^2+x+1) - \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(1+2*x)/(-x^3+1),x, algorithm="maxima")`

[Out] $-1/3*\text{sqrt}(3)*\arctan(1/3*\text{sqrt}(3)*(2*x + 1)) - 1/2*\log(x^2 + x + 1) - \log(x - 1)$

Fricas [A] time = 1.50109, size = 108, normalized size = 2.77

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{2}\log(x^2+x+1) - \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(1+2*x)/(-x^3+1),x, algorithm="fricas")`

[Out] $-1/3*\text{sqrt}(3)*\arctan(1/3*\text{sqrt}(3)*(2*x + 1)) - 1/2*\log(x^2 + x + 1) - \log(x - 1)$

Sympy [A] time = 0.12768, size = 41, normalized size = 1.05

$$-\log(x-1) - \frac{\log(x^2+x+1)}{2} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(1+2*x)/(-x**3+1),x)

[Out] -log(x - 1) - log(x**2 + x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3

Giac [A] time = 1.05645, size = 45, normalized size = 1.15

$$-\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x+1)\right) - \frac{1}{2} \log(x^2+x+1) - \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(1+2*x)/(-x^3+1),x, algorithm="giac")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1) - log(abs(x - 1))

3.313 $\int x^2 (c + dx + ex^2) (a + bx^3) dx$

Optimal. Leaf size=55

$$\frac{1}{3}acx^3 + \frac{1}{4}adx^4 + \frac{1}{5}aex^5 + \frac{1}{6}bcx^6 + \frac{1}{7}bdx^7 + \frac{1}{8}bex^8$$

[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + (b*c*x^6)/6 + (b*d*x^7)/7 + (b*e*x^8)/8

Rubi [A] time = 0.0554037, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1628}

$$\frac{1}{3}acx^3 + \frac{1}{4}adx^4 + \frac{1}{5}aex^5 + \frac{1}{6}bcx^6 + \frac{1}{7}bdx^7 + \frac{1}{8}bex^8$$

Antiderivative was successfully verified.

[In] Int[x^2*(c + d*x + e*x^2)*(a + b*x^3),x]

[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + (b*c*x^6)/6 + (b*d*x^7)/7 + (b*e*x^8)/8

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int x^2 (c + dx + ex^2) (a + bx^3) dx &= \int (acx^2 + adx^3 + aex^4 + bcx^5 + bdx^6 + bex^7) dx \\ &= \frac{1}{3}acx^3 + \frac{1}{4}adx^4 + \frac{1}{5}aex^5 + \frac{1}{6}bcx^6 + \frac{1}{7}bdx^7 + \frac{1}{8}bex^8 \end{aligned}$$

Mathematica [A] time = 0.0032462, size = 55, normalized size = 1.

$$\frac{1}{3}acx^3 + \frac{1}{4}adx^4 + \frac{1}{5}aex^5 + \frac{1}{6}bcx^6 + \frac{1}{7}bdx^7 + \frac{1}{8}bex^8$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3),x]

[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + (b*c*x^6)/6 + (b*d*x^7)/7 + (b*e*x^8)/8

Maple [A] time = 0.002, size = 44, normalized size = 0.8

$$\frac{acx^3}{3} + \frac{adx^4}{4} + \frac{aex^5}{5} + \frac{bcx^6}{6} + \frac{bdx^7}{7} + \frac{bex^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(e*x^2+d*x+c)*(b*x^3+a),x)

[Out] 1/3*a*c*x^3+1/4*a*d*x^4+1/5*a*e*x^5+1/6*b*c*x^6+1/7*b*d*x^7+1/8*b*e*x^8

Maxima [A] time = 0.928077, size = 58, normalized size = 1.05

$$\frac{1}{8}bex^8 + \frac{1}{7}bdx^7 + \frac{1}{6}bcx^6 + \frac{1}{5}aex^5 + \frac{1}{4}adx^4 + \frac{1}{3}acx^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="maxima")

[Out] 1/8*b*e*x^8 + 1/7*b*d*x^7 + 1/6*b*c*x^6 + 1/5*a*e*x^5 + 1/4*a*d*x^4 + 1/3*a*c*x^3

Fricas [A] time = 1.29515, size = 112, normalized size = 2.04

$$\frac{1}{8}x^8eb + \frac{1}{7}x^7db + \frac{1}{6}x^6cb + \frac{1}{5}x^5ea + \frac{1}{4}x^4da + \frac{1}{3}x^3ca$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="fricas")

[Out] $1/8*x^8*e*b + 1/7*x^7*d*b + 1/6*x^6*c*b + 1/5*x^5*e*a + 1/4*x^4*d*a + 1/3*x^3*c*a$

Sympy [A] time = 0.063593, size = 49, normalized size = 0.89

$$\frac{acx^3}{3} + \frac{adx^4}{4} + \frac{aex^5}{5} + \frac{bcx^6}{6} + \frac{bdx^7}{7} + \frac{bex^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a),x)`

[Out] $a*c*x**3/3 + a*d*x**4/4 + a*e*x**5/5 + b*c*x**6/6 + b*d*x**7/7 + b*e*x**8/8$

Giac [A] time = 1.06384, size = 61, normalized size = 1.11

$$\frac{1}{8}bx^8e + \frac{1}{7}bdx^7 + \frac{1}{6}bcx^6 + \frac{1}{5}ax^5e + \frac{1}{4}adx^4 + \frac{1}{3}acx^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="giac")`

[Out] $1/8*b*x^8*e + 1/7*b*d*x^7 + 1/6*b*c*x^6 + 1/5*a*x^5*e + 1/4*a*d*x^4 + 1/3*a*c*x^3$

3.314 $\int x(c + dx + ex^2)(a + bx^3) dx$

Optimal. Leaf size=55

$$\frac{1}{2}acx^2 + \frac{1}{3}adx^3 + \frac{1}{4}aex^4 + \frac{1}{5}bcx^5 + \frac{1}{6}bdx^6 + \frac{1}{7}bex^7$$

[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)/6 + (b*e*x^7)/7

Rubi [A] time = 0.0376349, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1628}

$$\frac{1}{2}acx^2 + \frac{1}{3}adx^3 + \frac{1}{4}aex^4 + \frac{1}{5}bcx^5 + \frac{1}{6}bdx^6 + \frac{1}{7}bex^7$$

Antiderivative was successfully verified.

[In] Int[x*(c + d*x + e*x^2)*(a + b*x^3),x]

[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)/6 + (b*e*x^7)/7

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int x(c + dx + ex^2)(a + bx^3) dx &= \int (acx + adx^2 + aex^3 + bcx^4 + bdx^5 + bex^6) dx \\ &= \frac{1}{2}acx^2 + \frac{1}{3}adx^3 + \frac{1}{4}aex^4 + \frac{1}{5}bcx^5 + \frac{1}{6}bdx^6 + \frac{1}{7}bex^7 \end{aligned}$$

Mathematica [A] time = 0.0022814, size = 55, normalized size = 1.

$$\frac{1}{2}acx^2 + \frac{1}{3}adx^3 + \frac{1}{4}aex^4 + \frac{1}{5}bcx^5 + \frac{1}{6}bdx^6 + \frac{1}{7}bex^7$$

Antiderivative was successfully verified.

[In] Integrate[x*(c + d*x + e*x^2)*(a + b*x^3),x]

[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)/6 + (b*e*x^7)/7

Maple [A] time = 0.001, size = 44, normalized size = 0.8

$$\frac{acx^2}{2} + \frac{adx^3}{3} + \frac{aex^4}{4} + \frac{bcx^5}{5} + \frac{bdx^6}{6} + \frac{bex^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(e*x^2+d*x+c)*(b*x^3+a),x)

[Out] 1/2*a*c*x^2+1/3*a*d*x^3+1/4*a*e*x^4+1/5*b*c*x^5+1/6*b*d*x^6+1/7*b*e*x^7

Maxima [A] time = 0.945712, size = 58, normalized size = 1.05

$$\frac{1}{7}bex^7 + \frac{1}{6}bdx^6 + \frac{1}{5}bcx^5 + \frac{1}{4}aex^4 + \frac{1}{3}adx^3 + \frac{1}{2}acx^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="maxima")

[Out] 1/7*b*e*x^7 + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*e*x^4 + 1/3*a*d*x^3 + 1/2*a*c*x^2

Fricas [A] time = 1.28433, size = 112, normalized size = 2.04

$$\frac{1}{7}x^7eb + \frac{1}{6}x^6db + \frac{1}{5}x^5cb + \frac{1}{4}x^4ea + \frac{1}{3}x^3da + \frac{1}{2}x^2ca$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="fricas")

[Out] $1/7*x^7*e*b + 1/6*x^6*d*b + 1/5*x^5*c*b + 1/4*x^4*e*a + 1/3*x^3*d*a + 1/2*x^2*c*a$

Sympy [A] time = 0.063504, size = 49, normalized size = 0.89

$$\frac{acx^2}{2} + \frac{adx^3}{3} + \frac{aex^4}{4} + \frac{bcx^5}{5} + \frac{bdx^6}{6} + \frac{bex^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x**2+d*x+c)*(b*x**3+a),x)`

[Out] $a*c*x**2/2 + a*d*x**3/3 + a*e*x**4/4 + b*c*x**5/5 + b*d*x**6/6 + b*e*x**7/7$

Giac [A] time = 1.06715, size = 61, normalized size = 1.11

$$\frac{1}{7}bx^7e + \frac{1}{6}bdx^6 + \frac{1}{5}bcx^5 + \frac{1}{4}ax^4e + \frac{1}{3}adx^3 + \frac{1}{2}acx^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="giac")`

[Out] $1/7*b*x^7*e + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*x^4*e + 1/3*a*d*x^3 + 1/2*a*c*x^2$

3.315 $\int (c + dx + ex^2)(a + bx^3) dx$

Optimal. Leaf size=50

$$acx + \frac{1}{2}adx^2 + \frac{1}{3}aex^3 + \frac{1}{4}bcx^4 + \frac{1}{5}bdx^5 + \frac{1}{6}bex^6$$

[Out] $a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (b*c*x^4)/4 + (b*d*x^5)/5 + (b*e*x^6)/6$

Rubi [A] time = 0.0245263, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1657}

$$acx + \frac{1}{2}adx^2 + \frac{1}{3}aex^3 + \frac{1}{4}bcx^4 + \frac{1}{5}bdx^5 + \frac{1}{6}bex^6$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)*(a + b*x^3),x]

[Out] $a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (b*c*x^4)/4 + (b*d*x^5)/5 + (b*e*x^6)/6$

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (c + dx + ex^2)(a + bx^3) dx &= \int (ac + adx + aex^2 + bcx^3 + bdx^4 + bex^5) dx \\ &= acx + \frac{1}{2}adx^2 + \frac{1}{3}aex^3 + \frac{1}{4}bcx^4 + \frac{1}{5}bdx^5 + \frac{1}{6}bex^6 \end{aligned}$$

Mathematica [A] time = 0.0018268, size = 50, normalized size = 1.

$$acx + \frac{1}{2}adx^2 + \frac{1}{3}aex^3 + \frac{1}{4}bcx^4 + \frac{1}{5}bdx^5 + \frac{1}{6}bex^6$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)*(a + b*x^3),x]

[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (b*c*x^4)/4 + (b*d*x^5)/5 + (b*e*x^6)/6

Maple [A] time = 0.002, size = 41, normalized size = 0.8

$$acx + \frac{adx^2}{2} + \frac{aex^3}{3} + \frac{bcx^4}{4} + \frac{bdx^5}{5} + \frac{bex^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)*(b*x^3+a),x)

[Out] a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*b*c*x^4+1/5*b*d*x^5+1/6*b*e*x^6

Maxima [A] time = 0.950306, size = 54, normalized size = 1.08

$$\frac{1}{6}bex^6 + \frac{1}{5}bdx^5 + \frac{1}{4}bcx^4 + \frac{1}{3}aex^3 + \frac{1}{2}adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a),x, algorithm="maxima")

[Out] 1/6*b*e*x^6 + 1/5*b*d*x^5 + 1/4*b*c*x^4 + 1/3*a*e*x^3 + 1/2*a*d*x^2 + a*c*x

Fricas [A] time = 1.28266, size = 104, normalized size = 2.08

$$\frac{1}{6}x^6eb + \frac{1}{5}x^5db + \frac{1}{4}x^4cb + \frac{1}{3}x^3ea + \frac{1}{2}x^2da + xca$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a),x, algorithm="fricas")

[Out] $1/6*x^6*e*b + 1/5*x^5*d*b + 1/4*x^4*c*b + 1/3*x^3*e*a + 1/2*x^2*d*a + x*c*a$

Sympy [A] time = 0.064803, size = 46, normalized size = 0.92

$$acx + \frac{adx^2}{2} + \frac{aex^3}{3} + \frac{bcx^4}{4} + \frac{bdx^5}{5} + \frac{bex^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d*x+c)*(b*x**3+a),x)`

[Out] $a*c*x + a*d*x**2/2 + a*e*x**3/3 + b*c*x**4/4 + b*d*x**5/5 + b*e*x**6/6$

Giac [A] time = 1.04892, size = 57, normalized size = 1.14

$$\frac{1}{6}bx^6e + \frac{1}{5}bdx^5 + \frac{1}{4}bcx^4 + \frac{1}{3}ax^3e + \frac{1}{2}adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d*x+c)*(b*x^3+a),x, algorithm="giac")`

[Out] $1/6*b*x^6*e + 1/5*b*d*x^5 + 1/4*b*c*x^4 + 1/3*a*x^3*e + 1/2*a*d*x^2 + a*c*x$

$$3.316 \quad \int \frac{(c+dx+ex^2)(a+bx^3)}{x} dx$$

Optimal. Leaf size=46

$$ac \log(x) + adx + \frac{1}{2}aex^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4 + \frac{1}{5}bex^5$$

[Out] a*d*x + (a*e*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4 + (b*e*x^5)/5 + a*c*Log[x]

Rubi [A] time = 0.0245177, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1628}

$$ac \log(x) + adx + \frac{1}{2}aex^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4 + \frac{1}{5}bex^5$$

Antiderivative was successfully verified.

[In] Int[((c + d*x + e*x^2)*(a + b*x^3))/x,x]

[Out] a*d*x + (a*e*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4 + (b*e*x^5)/5 + a*c*Log[x]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{(c+dx+ex^2)(a+bx^3)}{x} dx &= \int \left(ad + \frac{ac}{x} + aex + bcx^2 + bdx^3 + bex^4 \right) dx \\ &= adx + \frac{1}{2}aex^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4 + \frac{1}{5}bex^5 + ac \log(x) \end{aligned}$$

Mathematica [A] time = 0.0041212, size = 46, normalized size = 1.

$$ac \log(x) + adx + \frac{1}{2}aex^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4 + \frac{1}{5}bex^5$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3))/x,x]

[Out] a*d*x + (a*e*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4 + (b*e*x^5)/5 + a*c*Log[x]

Maple [A] time = 0.003, size = 39, normalized size = 0.9

$$adx + \frac{aex^2}{2} + \frac{bcx^3}{3} + \frac{bdx^4}{4} + \frac{bex^5}{5} + ac \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)*(b*x^3+a)/x,x)

[Out] a*d*x+1/2*a*e*x^2+1/3*b*c*x^3+1/4*b*d*x^4+1/5*b*e*x^5+a*c*ln(x)

Maxima [A] time = 0.949336, size = 51, normalized size = 1.11

$$\frac{1}{5}bex^5 + \frac{1}{4}bdx^4 + \frac{1}{3}bcx^3 + \frac{1}{2}aex^2 + adx + ac \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x,x, algorithm="maxima")

[Out] 1/5*b*e*x^5 + 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*e*x^2 + a*d*x + a*c*log(x)

Fricas [A] time = 1.49063, size = 103, normalized size = 2.24

$$\frac{1}{5}bex^5 + \frac{1}{4}bdx^4 + \frac{1}{3}bcx^3 + \frac{1}{2}aex^2 + adx + ac \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x,x, algorithm="fricas")

[Out] $1/5*b*e*x^5 + 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*e*x^2 + a*d*x + a*c*\log(x)$

Sympy [A] time = 0.272387, size = 44, normalized size = 0.96

$$ac \log(x) + adx + \frac{aex^2}{2} + \frac{bcx^3}{3} + \frac{bdx^4}{4} + \frac{bex^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d*x+c)*(b*x**3+a)/x,x)`

[Out] $a*c*\log(x) + a*d*x + a*e*x**2/2 + b*c*x**3/3 + b*d*x**4/4 + b*e*x**5/5$

Giac [A] time = 1.08311, size = 55, normalized size = 1.2

$$\frac{1}{5}bx^5e + \frac{1}{4}bdx^4 + \frac{1}{3}bcx^3 + \frac{1}{2}ax^2e + adx + ac \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d*x+c)*(b*x^3+a)/x,x, algorithm="giac")`

[Out] $1/5*b*x^5*e + 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*x^2*e + a*d*x + a*c*\log(\text{abs}(x))$

$$3.317 \quad \int \frac{(c+dx+ex^2)(a+bx^3)}{x^2} dx$$

Optimal. Leaf size=44

$$-\frac{ac}{x} + ad \log(x) + aex + \frac{1}{2}bcx^2 + \frac{1}{3}bdx^3 + \frac{1}{4}bex^4$$

[Out] $-\frac{(a*c)}{x} + a*e*x + (b*c*x^2)/2 + (b*d*x^3)/3 + (b*e*x^4)/4 + a*d*\text{Log}[x]$

Rubi [A] time = 0.0350813, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1628}

$$-\frac{ac}{x} + ad \log(x) + aex + \frac{1}{2}bcx^2 + \frac{1}{3}bdx^3 + \frac{1}{4}bex^4$$

Antiderivative was successfully verified.

[In] $\text{Int}[\frac{(c + d*x + e*x^2)*(a + b*x^3)}{x^2}, x]$

[Out] $-\frac{(a*c)}{x} + a*e*x + (b*c*x^2)/2 + (b*d*x^3)/3 + (b*e*x^4)/4 + a*d*\text{Log}[x]$

Rule 1628

$\text{Int}[(Pq_)*((d_.) + (e_.)*(x_))^{(m_.)}*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^{(p_.)}, x_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, m\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[p, -2]$

Rubi steps

$$\begin{aligned} \int \frac{(c+dx+ex^2)(a+bx^3)}{x^2} dx &= \int \left(ae + \frac{ac}{x^2} + \frac{ad}{x} + bcx + bdx^2 + bex^3 \right) dx \\ &= -\frac{ac}{x} + aex + \frac{1}{2}bcx^2 + \frac{1}{3}bdx^3 + \frac{1}{4}bex^4 + ad \log(x) \end{aligned}$$

Mathematica [A] time = 0.0048538, size = 44, normalized size = 1.

$$-\frac{ac}{x} + ad \log(x) + aex + \frac{1}{2}bcx^2 + \frac{1}{3}bdx^3 + \frac{1}{4}bex^4$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3))/x^2,x]

[Out] -((a*c)/x) + a*e*x + (b*c*x^2)/2 + (b*d*x^3)/3 + (b*e*x^4)/4 + a*d*Log[x]

Maple [A] time = 0.004, size = 39, normalized size = 0.9

$$-\frac{ac}{x} + aex + \frac{bcx^2}{2} + \frac{bdx^3}{3} + \frac{bex^4}{4} + ad \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)*(b*x^3+a)/x^2,x)

[Out] -a*c/x+a*e*x+1/2*b*c*x^2+1/3*b*d*x^3+1/4*b*e*x^4+a*d*ln(x)

Maxima [A] time = 0.943359, size = 51, normalized size = 1.16

$$\frac{1}{4} bex^4 + \frac{1}{3} bdx^3 + \frac{1}{2} bcx^2 + aex + ad \log(x) - \frac{ac}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x^2,x, algorithm="maxima")

[Out] 1/4*b*e*x^4 + 1/3*b*d*x^3 + 1/2*b*c*x^2 + a*e*x + a*d*log(x) - a*c/x

Fricas [A] time = 1.43856, size = 113, normalized size = 2.57

$$\frac{3 bex^5 + 4 bdx^4 + 6 bcx^3 + 12 aex^2 + 12 adx \log(x) - 12 ac}{12x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x^2,x, algorithm="fricas")

[Out] $1/12*(3*b*e*x^5 + 4*b*d*x^4 + 6*b*c*x^3 + 12*a*e*x^2 + 12*a*d*x*\log(x) - 12*a*c)/x$

Sympy [A] time = 0.296316, size = 41, normalized size = 0.93

$$-\frac{ac}{x} + ad \log(x) + aex + \frac{bcx^2}{2} + \frac{bdx^3}{3} + \frac{bex^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d*x+c)*(b*x**3+a)/x**2,x)`

[Out] `-a*c/x + a*d*log(x) + a*e*x + b*c*x**2/2 + b*d*x**3/3 + b*e*x**4/4`

Giac [A] time = 1.06735, size = 55, normalized size = 1.25

$$\frac{1}{4}bx^4e + \frac{1}{3}bdx^3 + \frac{1}{2}bcx^2 + axe + ad \log(|x|) - \frac{ac}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d*x+c)*(b*x^3+a)/x^2,x, algorithm="giac")`

[Out] `1/4*b*x^4*e + 1/3*b*d*x^3 + 1/2*b*c*x^2 + a*x*e + a*d*log(abs(x)) - a*c/x`

$$3.318 \quad \int \frac{(c+dx+ex^2)(a+bx^3)}{x^3} dx$$

Optimal. Leaf size=44

$$-\frac{ac}{2x^2} - \frac{ad}{x} + ae \log(x) + bcx + \frac{1}{2}bdx^2 + \frac{1}{3}bex^3$$

[Out] $-(a*c)/(2*x^2) - (a*d)/x + b*c*x + (b*d*x^2)/2 + (b*e*x^3)/3 + a*e*\text{Log}[x]$

Rubi [A] time = 0.0341978, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1628}

$$-\frac{ac}{2x^2} - \frac{ad}{x} + ae \log(x) + bcx + \frac{1}{2}bdx^2 + \frac{1}{3}bex^3$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x + e*x^2)*(a + b*x^3)/x^3, x]$

[Out] $-(a*c)/(2*x^2) - (a*d)/x + b*c*x + (b*d*x^2)/2 + (b*e*x^3)/3 + a*e*\text{Log}[x]$

Rule 1628

$\text{Int}[(Pq_)*((d_.) + (e_.)*(x_))^{(m_.)}*((a_.) + (b_.)*(x_)) + (c_.)*(x_)^2]^{(p_.)}, x_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /;$ FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{(c+dx+ex^2)(a+bx^3)}{x^3} dx &= \int \left(bc + \frac{ac}{x^3} + \frac{ad}{x^2} + \frac{ae}{x} + bdx + bex^2 \right) dx \\ &= -\frac{ac}{2x^2} - \frac{ad}{x} + bcx + \frac{1}{2}bdx^2 + \frac{1}{3}bex^3 + ae \log(x) \end{aligned}$$

Mathematica [A] time = 0.0040648, size = 44, normalized size = 1.

$$-\frac{ac}{2x^2} - \frac{ad}{x} + ae \log(x) + bcx + \frac{1}{2}bdx^2 + \frac{1}{3}bex^3$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3))/x^3,x]

[Out] $-(a*c)/(2*x^2) - (a*d)/x + b*c*x + (b*d*x^2)/2 + (b*e*x^3)/3 + a*e*\text{Log}[x]$

Maple [A] time = 0.005, size = 39, normalized size = 0.9

$$-\frac{ac}{2x^2} - \frac{ad}{x} + bcx + \frac{bdx^2}{2} + \frac{bex^3}{3} + ae \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)*(b*x^3+a)/x^3,x)

[Out] $-1/2*a*c/x^2 - a*d/x + b*c*x + 1/2*b*d*x^2 + 1/3*b*e*x^3 + a*e*\ln(x)$

Maxima [A] time = 0.937409, size = 51, normalized size = 1.16

$$\frac{1}{3}bex^3 + \frac{1}{2}bdx^2 + bcx + ae \log(x) - \frac{2adx + ac}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x^3,x, algorithm="maxima")

[Out] $1/3*b*e*x^3 + 1/2*b*d*x^2 + b*c*x + a*e*\log(x) - 1/2*(2*a*d*x + a*c)/x^2$

Fricas [A] time = 1.50428, size = 111, normalized size = 2.52

$$\frac{2bex^5 + 3bdx^4 + 6bcx^3 + 6aex^2 \log(x) - 6adx - 3ac}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x^3,x, algorithm="fricas")

[Out] $\frac{1}{6}(2be^5x + 3bd^4x + 6b^3cx^3 + 6ae^2x \log(x) - 6ad^2x - 3ac^2)/x^2$

Sympy [A] time = 0.365596, size = 42, normalized size = 0.95

$$ae \log(x) + bcx + \frac{bdx^2}{2} + \frac{bex^3}{3} - \frac{ac + 2adx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d*x+c)*(b*x**3+a)/x**3,x)`

[Out] $a \cdot e \cdot \log(x) + b \cdot c \cdot x + b \cdot d \cdot x^2 / 2 + b \cdot e \cdot x^3 / 3 - (a \cdot c + 2 \cdot a \cdot d \cdot x) / (2 \cdot x^2)$

Giac [A] time = 1.06201, size = 55, normalized size = 1.25

$$\frac{1}{3}bx^3e + \frac{1}{2}bdx^2 + bcx + ae \log(|x|) - \frac{2adx + ac}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d*x+c)*(b*x^3+a)/x^3,x, algorithm="giac")`

[Out] $\frac{1}{3}bx^3e + \frac{1}{2}bdx^2 + bcx + ae \log(\text{abs}(x)) - \frac{1}{2}(2ad^2x + ac^2)/x^2$

$$3.319 \quad \int x^2 (c + dx + ex^2) (a + bx^3)^2 dx$$

Optimal. Leaf size=82

$$\frac{1}{4}a^2dx^4 + \frac{1}{5}a^2ex^5 + \frac{c(a + bx^3)^3}{9b} + \frac{2}{7}abdx^7 + \frac{1}{4}abex^8 + \frac{1}{10}b^2dx^{10} + \frac{1}{11}b^2ex^{11}$$

[Out] (a^2*d*x^4)/4 + (a^2*e*x^5)/5 + (2*a*b*d*x^7)/7 + (a*b*e*x^8)/4 + (b^2*d*x^10)/10 + (b^2*e*x^11)/11 + (c*(a + b*x^3)^3)/(9*b)

Rubi [A] time = 0.0589412, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1582, 1850}

$$\frac{1}{4}a^2dx^4 + \frac{1}{5}a^2ex^5 + \frac{c(a + bx^3)^3}{9b} + \frac{2}{7}abdx^7 + \frac{1}{4}abex^8 + \frac{1}{10}b^2dx^{10} + \frac{1}{11}b^2ex^{11}$$

Antiderivative was successfully verified.

[In] Int[x^2*(c + d*x + e*x^2)*(a + b*x^3)^2,x]

[Out] (a^2*d*x^4)/4 + (a^2*e*x^5)/5 + (2*a*b*d*x^7)/7 + (a*b*e*x^8)/4 + (b^2*d*x^10)/10 + (b^2*e*x^11)/11 + (c*(a + b*x^3)^3)/(9*b)

Rule 1582

Int[(Px_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(Coeff[Px, x, n - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_.))^(q_) /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a + b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]

Rule 1850

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned}
\int x^2 (c + dx + ex^2) (a + bx^3)^2 dx &= \frac{c(a + bx^3)^3}{9b} + \int (a + bx^3)^2 (-cx^2 + x^2(c + dx + ex^2)) dx \\
&= \frac{c(a + bx^3)^3}{9b} + \int (a^2 dx^3 + a^2 ex^4 + 2abdx^6 + 2abex^7 + b^2 dx^9 + b^2 ex^{10}) dx \\
&= \frac{1}{4} a^2 dx^4 + \frac{1}{5} a^2 ex^5 + \frac{2}{7} abdx^7 + \frac{1}{4} abex^8 + \frac{1}{10} b^2 dx^{10} + \frac{1}{11} b^2 ex^{11} + \frac{c(a + bx^3)^3}{9b}
\end{aligned}$$

Mathematica [A] time = 0.0035515, size = 97, normalized size = 1.18

$$\frac{1}{3} a^2 cx^3 + \frac{1}{4} a^2 dx^4 + \frac{1}{5} a^2 ex^5 + \frac{1}{3} abcx^6 + \frac{2}{7} abdx^7 + \frac{1}{4} abex^8 + \frac{1}{9} b^2 cx^9 + \frac{1}{10} b^2 dx^{10} + \frac{1}{11} b^2 ex^{11}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3)^2,x]

[Out] (a^2*c*x^3)/3 + (a^2*d*x^4)/4 + (a^2*e*x^5)/5 + (a*b*c*x^6)/3 + (2*a*b*d*x^7)/7 + (a*b*e*x^8)/4 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10 + (b^2*e*x^11)/11

Maple [A] time = 0.002, size = 80, normalized size = 1.

$$\frac{b^2 ex^{11}}{11} + \frac{b^2 dx^{10}}{10} + \frac{b^2 cx^9}{9} + \frac{abex^8}{4} + \frac{2 abdx^7}{7} + \frac{abcx^6}{3} + \frac{a^2 ex^5}{5} + \frac{a^2 dx^4}{4} + \frac{a^2 cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^2,x)

[Out] 1/11*b^2*e*x^11+1/10*b^2*d*x^10+1/9*b^2*c*x^9+1/4*a*b*e*x^8+2/7*a*b*d*x^7+1/3*a*b*c*x^6+1/5*a^2*e*x^5+1/4*a^2*d*x^4+1/3*a^2*c*x^3

Maxima [A] time = 0.938983, size = 107, normalized size = 1.3

$$\frac{1}{11} b^2 ex^{11} + \frac{1}{10} b^2 dx^{10} + \frac{1}{9} b^2 cx^9 + \frac{1}{4} abex^8 + \frac{2}{7} abdx^7 + \frac{1}{3} abcx^6 + \frac{1}{5} a^2 ex^5 + \frac{1}{4} a^2 dx^4 + \frac{1}{3} a^2 cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="maxima")`

[Out] $\frac{1}{11}b^2e*x^{11} + \frac{1}{10}b^2d*x^{10} + \frac{1}{9}b^2c*x^9 + \frac{1}{4}a*b*e*x^8 + \frac{2}{7}a*b*d*x^7 + \frac{1}{3}a*b*c*x^6 + \frac{1}{5}a^2*e*x^5 + \frac{1}{4}a^2*d*x^4 + \frac{1}{3}a^2*c*x^3$

Fricas [A] time = 1.22065, size = 198, normalized size = 2.41

$$\frac{1}{11}x^{11}eb^2 + \frac{1}{10}x^{10}db^2 + \frac{1}{9}x^9cb^2 + \frac{1}{4}x^8eba + \frac{2}{7}x^7dba + \frac{1}{3}x^6cba + \frac{1}{5}x^5ea^2 + \frac{1}{4}x^4da^2 + \frac{1}{3}x^3ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="fricas")`

[Out] $\frac{1}{11}x^{11}e*b^2 + \frac{1}{10}x^{10}d*b^2 + \frac{1}{9}x^9*c*b^2 + \frac{1}{4}x^8*e*b*a + \frac{2}{7}x^7*d*b*a + \frac{1}{3}x^6*c*b*a + \frac{1}{5}x^5*e*a^2 + \frac{1}{4}x^4*d*a^2 + \frac{1}{3}x^3*c*a^2$

Sympy [A] time = 0.07362, size = 92, normalized size = 1.12

$$\frac{a^2cx^3}{3} + \frac{a^2dx^4}{4} + \frac{a^2ex^5}{5} + \frac{abcx^6}{3} + \frac{2abdx^7}{7} + \frac{abex^8}{4} + \frac{b^2cx^9}{9} + \frac{b^2dx^{10}}{10} + \frac{b^2ex^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**2,x)`

[Out] $a**2*c*x**3/3 + a**2*d*x**4/4 + a**2*e*x**5/5 + a*b*c*x**6/3 + 2*a*b*d*x**7/7 + a*b*e*x**8/4 + b**2*c*x**9/9 + b**2*d*x**10/10 + b**2*e*x**11/11$

Giac [A] time = 1.08003, size = 111, normalized size = 1.35

$$\frac{1}{11}b^2x^{11}e + \frac{1}{10}b^2dx^{10} + \frac{1}{9}b^2cx^9 + \frac{1}{4}abx^8e + \frac{2}{7}abdx^7 + \frac{1}{3}abcx^6 + \frac{1}{5}a^2x^5e + \frac{1}{4}a^2dx^4 + \frac{1}{3}a^2cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/11*b^2*x^11*e + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a*b*x^8*e + 2/7*a*b  
*d*x^7 + 1/3*a*b*c*x^6 + 1/5*a^2*x^5*e + 1/4*a^2*d*x^4 + 1/3*a^2*c*x^3
```

3.320 $\int x(c + dx + ex^2)(a + bx^3)^2 dx$

Optimal. Leaf size=82

$$\frac{1}{2}a^2cx^2 + \frac{1}{4}a^2ex^4 + \frac{2}{5}abcx^5 + \frac{d(a + bx^3)^3}{9b} + \frac{2}{7}abex^7 + \frac{1}{8}b^2cx^8 + \frac{1}{10}b^2ex^{10}$$

[Out] $(a^2cx^2)/2 + (a^2ex^4)/4 + (2abcx^5)/5 + (2abex^7)/7 + (b^2cx^8)/8 + (b^2ex^{10})/10 + (d(a + bx^3)^3)/(9b)$

Rubi [A] time = 0.0510239, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1582, 1850}

$$\frac{1}{2}a^2cx^2 + \frac{1}{4}a^2ex^4 + \frac{2}{5}abcx^5 + \frac{d(a + bx^3)^3}{9b} + \frac{2}{7}abex^7 + \frac{1}{8}b^2cx^8 + \frac{1}{10}b^2ex^{10}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*(c + d*x + e*x^2)*(a + b*x^3)^2, x]$

[Out] $(a^2cx^2)/2 + (a^2ex^4)/4 + (2abcx^5)/5 + (2abex^7)/7 + (b^2cx^8)/8 + (b^2ex^{10})/10 + (d(a + bx^3)^3)/(9b)$

Rule 1582

$\text{Int}[(Px_*)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] \rightarrow \text{Simp}[(\text{Coeff}[Px, x, n - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + \text{Int}[(Px - \text{Coeff}[Px, x, n - 1]*x^(n - 1))*(a + b*x^n)^p, x] /;$ FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_)*((c_) + (d_)*x^(m_))^(q_)] /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a + b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]

Rule 1850

$\text{Int}[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[Pq*(a + b*x^n)^p, x], x] /;$ FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned}
\int x(c + dx + ex^2)(a + bx^3)^2 dx &= \frac{d(a + bx^3)^3}{9b} + \int (a + bx^3)^2 (-dx^2 + x(c + dx + ex^2)) dx \\
&= \frac{d(a + bx^3)^3}{9b} + \int (a^2cx + a^2ex^3 + 2abcx^4 + 2abex^6 + b^2cx^7 + b^2ex^9) dx \\
&= \frac{1}{2}a^2cx^2 + \frac{1}{4}a^2ex^4 + \frac{2}{5}abcx^5 + \frac{2}{7}abex^7 + \frac{1}{8}b^2cx^8 + \frac{1}{10}b^2ex^{10} + \frac{d(a + bx^3)^3}{9b}
\end{aligned}$$

Mathematica [A] time = 0.0030289, size = 97, normalized size = 1.18

$$\frac{1}{2}a^2cx^2 + \frac{1}{3}a^2dx^3 + \frac{1}{4}a^2ex^4 + \frac{2}{5}abcx^5 + \frac{1}{3}abdx^6 + \frac{2}{7}abex^7 + \frac{1}{8}b^2cx^8 + \frac{1}{9}b^2dx^9 + \frac{1}{10}b^2ex^{10}$$

Antiderivative was successfully verified.

[In] Integrate[x*(c + d*x + e*x^2)*(a + b*x^3)^2,x]

[Out] (a^2*c*x^2)/2 + (a^2*d*x^3)/3 + (a^2*e*x^4)/4 + (2*a*b*c*x^5)/5 + (a*b*d*x^6)/3 + (2*a*b*e*x^7)/7 + (b^2*c*x^8)/8 + (b^2*d*x^9)/9 + (b^2*e*x^10)/10

Maple [A] time = 0., size = 80, normalized size = 1.

$$\frac{b^2ex^{10}}{10} + \frac{b^2dx^9}{9} + \frac{b^2cx^8}{8} + \frac{2abex^7}{7} + \frac{abdx^6}{3} + \frac{2abcx^5}{5} + \frac{a^2ex^4}{4} + \frac{a^2dx^3}{3} + \frac{a^2cx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(e*x^2+d*x+c)*(b*x^3+a)^2,x)

[Out] 1/10*b^2*e*x^10+1/9*b^2*d*x^9+1/8*b^2*c*x^8+2/7*a*b*e*x^7+1/3*a*b*d*x^6+2/5*a*b*c*x^5+1/4*a^2*e*x^4+1/3*a^2*d*x^3+1/2*a^2*c*x^2

Maxima [A] time = 0.932844, size = 107, normalized size = 1.3

$$\frac{1}{10}b^2ex^{10} + \frac{1}{9}b^2dx^9 + \frac{1}{8}b^2cx^8 + \frac{2}{7}abex^7 + \frac{1}{3}abdx^6 + \frac{2}{5}abcx^5 + \frac{1}{4}a^2ex^4 + \frac{1}{3}a^2dx^3 + \frac{1}{2}a^2cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="maxima")`

[Out] $\frac{1}{10}b^2e*x^{10} + \frac{1}{9}b^2d*x^9 + \frac{1}{8}b^2c*x^8 + \frac{2}{7}a*b*e*x^7 + \frac{1}{3}a*b*d*x^6 + \frac{2}{5}a*b*c*x^5 + \frac{1}{4}a^2*e*x^4 + \frac{1}{3}a^2*d*x^3 + \frac{1}{2}a^2*c*x^2$

Fricas [A] time = 1.31605, size = 196, normalized size = 2.39

$$\frac{1}{10}x^{10}eb^2 + \frac{1}{9}x^9db^2 + \frac{1}{8}x^8cb^2 + \frac{2}{7}x^7eba + \frac{1}{3}x^6dba + \frac{2}{5}x^5cba + \frac{1}{4}x^4ea^2 + \frac{1}{3}x^3da^2 + \frac{1}{2}x^2ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="fricas")`

[Out] $\frac{1}{10}x^{10}e*b^2 + \frac{1}{9}x^9*d*b^2 + \frac{1}{8}x^8*c*b^2 + \frac{2}{7}x^7*e*b*a + \frac{1}{3}x^6*d*b*a + \frac{2}{5}x^5*c*b*a + \frac{1}{4}x^4*e*a^2 + \frac{1}{3}x^3*d*a^2 + \frac{1}{2}x^2*c*a^2$

Sympy [A] time = 0.071493, size = 94, normalized size = 1.15

$$\frac{a^2cx^2}{2} + \frac{a^2dx^3}{3} + \frac{a^2ex^4}{4} + \frac{2abcx^5}{5} + \frac{abdx^6}{3} + \frac{2abex^7}{7} + \frac{b^2cx^8}{8} + \frac{b^2dx^9}{9} + \frac{b^2ex^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**2,x)`

[Out] $a**2*c*x**2/2 + a**2*d*x**3/3 + a**2*e*x**4/4 + 2*a*b*c*x**5/5 + a*b*d*x**6/3 + 2*a*b*e*x**7/7 + b**2*c*x**8/8 + b**2*d*x**9/9 + b**2*e*x**10/10$

Giac [A] time = 1.0604, size = 111, normalized size = 1.35

$$\frac{1}{10}b^2x^{10}e + \frac{1}{9}b^2dx^9 + \frac{1}{8}b^2cx^8 + \frac{2}{7}abx^7e + \frac{1}{3}abdx^6 + \frac{2}{5}abcx^5 + \frac{1}{4}a^2x^4e + \frac{1}{3}a^2dx^3 + \frac{1}{2}a^2cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/10*b^2*x^10*e + 1/9*b^2*d*x^9 + 1/8*b^2*c*x^8 + 2/7*a*b*x^7*e + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*x^4*e + 1/3*a^2*d*x^3 + 1/2*a^2*c*x^2
```

3.321 $\int (c + dx + ex^2)(a + bx^3)^2 dx$

Optimal. Leaf size=77

$$a^2cx + \frac{1}{2}a^2dx^2 + \frac{1}{2}abcx^4 + \frac{2}{5}abdx^5 + \frac{e(a + bx^3)^3}{9b} + \frac{1}{7}b^2cx^7 + \frac{1}{8}b^2dx^8$$

[Out] $a^2c*x + (a^2*d*x^2)/2 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (b^2*c*x^7)/7 + (b^2*d*x^8)/8 + (e*(a + b*x^3)^3)/(9*b)$

Rubi [A] time = 0.0610721, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {1582, 1850}

$$a^2cx + \frac{1}{2}a^2dx^2 + \frac{1}{2}abcx^4 + \frac{2}{5}abdx^5 + \frac{e(a + bx^3)^3}{9b} + \frac{1}{7}b^2cx^7 + \frac{1}{8}b^2dx^8$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x + e*x^2)*(a + b*x^3)^2, x]$

[Out] $a^2c*x + (a^2*d*x^2)/2 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (b^2*c*x^7)/7 + (b^2*d*x^8)/8 + (e*(a + b*x^3)^3)/(9*b)$

Rule 1582

$\text{Int}[(P_x) * ((a_) + (b_) * (x_)^{(n_)})^{(p_)}, x_Symbol] := \text{Simp}[(\text{Coeff}[P_x, x, n - 1] * (a + b * x^n)^{(p + 1)}) / (b * n * (p + 1)), x] + \text{Int}[(P_x - \text{Coeff}[P_x, x, n - 1] * x^{(n - 1)}) * (a + b * x^n)^p, x] /;$ FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1] * x^{(n - 1)}] && !MatchQ[Px, (Qx_) * ((c_) + (d_) * x^{(m_)})^{(q_) /}; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx * (a + b * x^n)^p, x, m - 1], 0] && GtQ[m * q, n * p]

Rule 1850

$\text{Int}[(P_q) * ((a_) + (b_) * (x_)^{(n_)})^{(p_)}, x_Symbol] := \text{Int}[\text{ExpandIntegrand}[P_q * (a + b * x^n)^p, x], x] /;$ FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned}
\int (c + dx + ex^2)(a + bx^3)^2 dx &= \frac{e(a + bx^3)^3}{9b} + \int (c + dx)(a + bx^3)^2 dx \\
&= \frac{e(a + bx^3)^3}{9b} + \int (a^2c + a^2dx + 2abcx^3 + 2abdx^4 + b^2cx^6 + b^2dx^7) dx \\
&= a^2cx + \frac{1}{2}a^2dx^2 + \frac{1}{2}abcx^4 + \frac{2}{5}abdx^5 + \frac{1}{7}b^2cx^7 + \frac{1}{8}b^2dx^8 + \frac{e(a + bx^3)^3}{9b}
\end{aligned}$$

Mathematica [A] time = 0.0027036, size = 92, normalized size = 1.19

$$a^2cx + \frac{1}{2}a^2dx^2 + \frac{1}{3}a^2ex^3 + \frac{1}{2}abcx^4 + \frac{2}{5}abdx^5 + \frac{1}{3}abex^6 + \frac{1}{7}b^2cx^7 + \frac{1}{8}b^2dx^8 + \frac{1}{9}b^2ex^9$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)*(a + b*x^3)^2,x]

[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (a*b*e*x^6)/3 + (b^2*c*x^7)/7 + (b^2*d*x^8)/8 + (b^2*e*x^9)/9

Maple [A] time = 0.001, size = 77, normalized size = 1.

$$\frac{b^2ex^9}{9} + \frac{b^2dx^8}{8} + \frac{b^2cx^7}{7} + \frac{abex^6}{3} + \frac{2abdx^5}{5} + \frac{abcx^4}{2} + \frac{a^2ex^3}{3} + \frac{a^2dx^2}{2} + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)*(b*x^3+a)^2,x)

[Out] 1/9*b^2*e*x^9+1/8*b^2*d*x^8+1/7*b^2*c*x^7+1/3*a*b*e*x^6+2/5*a*b*d*x^5+1/2*a*b*c*x^4+1/3*a^2*e*x^3+1/2*a^2*d*x^2+a^2*c*x

Maxima [A] time = 0.939354, size = 103, normalized size = 1.34

$$\frac{1}{9}b^2ex^9 + \frac{1}{8}b^2dx^8 + \frac{1}{7}b^2cx^7 + \frac{1}{3}abex^6 + \frac{2}{5}abdx^5 + \frac{1}{2}abcx^4 + \frac{1}{3}a^2ex^3 + \frac{1}{2}a^2dx^2 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="maxima")

[Out] $\frac{1}{9}b^2e*x^9 + \frac{1}{8}b^2d*x^8 + \frac{1}{7}b^2c*x^7 + \frac{1}{3}a*b*e*x^6 + \frac{2}{5}a*b*d*x^5 + \frac{1}{2}a*b*c*x^4 + \frac{1}{3}a^2*e*x^3 + \frac{1}{2}a^2*d*x^2 + a^2*c*x$

Fricas [A] time = 1.36238, size = 185, normalized size = 2.4

$$\frac{1}{9}x^9eb^2 + \frac{1}{8}x^8db^2 + \frac{1}{7}x^7cb^2 + \frac{1}{3}x^6eba + \frac{2}{5}x^5dba + \frac{1}{2}x^4cba + \frac{1}{3}x^3ea^2 + \frac{1}{2}x^2da^2 + xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="fricas")

[Out] $\frac{1}{9}x^9e*b^2 + \frac{1}{8}x^8d*b^2 + \frac{1}{7}x^7c*b^2 + \frac{1}{3}x^6e*b*a + \frac{2}{5}x^5*d*b*a + \frac{1}{2}x^4*c*b*a + \frac{1}{3}x^3*e*a^2 + \frac{1}{2}x^2*d*a^2 + x*c*a^2$

Sympy [A] time = 0.097755, size = 88, normalized size = 1.14

$$a^2cx + \frac{a^2dx^2}{2} + \frac{a^2ex^3}{3} + \frac{abcx^4}{2} + \frac{2abdx^5}{5} + \frac{abex^6}{3} + \frac{b^2cx^7}{7} + \frac{b^2dx^8}{8} + \frac{b^2ex^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**2,x)

[Out] $a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + a*b*c*x**4/2 + 2*a*b*d*x**5/5 + a*b*e*x**6/3 + b**2*c*x**7/7 + b**2*d*x**8/8 + b**2*e*x**9/9$

Giac [A] time = 1.06257, size = 107, normalized size = 1.39

$$\frac{1}{9}b^2x^9e + \frac{1}{8}b^2dx^8 + \frac{1}{7}b^2cx^7 + \frac{1}{3}abx^6e + \frac{2}{5}abdx^5 + \frac{1}{2}abcx^4 + \frac{1}{3}a^2x^3e + \frac{1}{2}a^2dx^2 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*b^2*x^9*e + 1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 1/3*a*b*x^6*e + 2/5*a*b*d*x^5 + 1/2*a*b*c*x^4 + 1/3*a^2*x^3*e + 1/2*a^2*d*x^2 + a^2*c*x
```

$$3.322 \quad \int \frac{(c+dx+ex^2)(a+bx^3)^2}{x} dx$$

Optimal. Leaf size=88

$$a^2c \log(x) + a^2dx + \frac{1}{2}a^2ex^2 + \frac{2}{3}abcx^3 + \frac{1}{2}abdx^4 + \frac{2}{5}abex^5 + \frac{1}{6}b^2cx^6 + \frac{1}{7}b^2dx^7 + \frac{1}{8}b^2ex^8$$

[Out] $a^2d*x + (a^2*e*x^2)/2 + (2*a*b*c*x^3)/3 + (a*b*d*x^4)/2 + (2*a*b*e*x^5)/5 + (b^2*c*x^6)/6 + (b^2*d*x^7)/7 + (b^2*e*x^8)/8 + a^2*c*Log[x]$

Rubi [A] time = 0.0518618, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1628}

$$a^2c \log(x) + a^2dx + \frac{1}{2}a^2ex^2 + \frac{2}{3}abcx^3 + \frac{1}{2}abdx^4 + \frac{2}{5}abex^5 + \frac{1}{6}b^2cx^6 + \frac{1}{7}b^2dx^7 + \frac{1}{8}b^2ex^8$$

Antiderivative was successfully verified.

[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^2)/x,x]

[Out] $a^2d*x + (a^2*e*x^2)/2 + (2*a*b*c*x^3)/3 + (a*b*d*x^4)/2 + (2*a*b*e*x^5)/5 + (b^2*c*x^6)/6 + (b^2*d*x^7)/7 + (b^2*e*x^8)/8 + a^2*c*Log[x]$

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{(c+dx+ex^2)(a+bx^3)^2}{x} dx &= \int \left(a^2d + \frac{a^2c}{x} + a^2ex + 2abcx^2 + 2abdx^3 + 2abex^4 + b^2cx^5 + b^2dx^6 + b^2ex^7 \right) dx \\ &= a^2dx + \frac{1}{2}a^2ex^2 + \frac{2}{3}abcx^3 + \frac{1}{2}abdx^4 + \frac{2}{5}abex^5 + \frac{1}{6}b^2cx^6 + \frac{1}{7}b^2dx^7 + \frac{1}{8}b^2ex^8 + a^2c \log \end{aligned}$$

Mathematica [A] time = 0.0076852, size = 88, normalized size = 1.

$$a^2c \log(x) + a^2dx + \frac{1}{2}a^2ex^2 + \frac{2}{3}abcx^3 + \frac{1}{2}abdx^4 + \frac{2}{5}abex^5 + \frac{1}{6}b^2cx^6 + \frac{1}{7}b^2dx^7 + \frac{1}{8}b^2ex^8$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^2)/x,x]

[Out] a^2*d*x + (a^2*e*x^2)/2 + (2*a*b*c*x^3)/3 + (a*b*d*x^4)/2 + (2*a*b*e*x^5)/5 + (b^2*c*x^6)/6 + (b^2*d*x^7)/7 + (b^2*e*x^8)/8 + a^2*c*Log[x]

Maple [A] time = 0.001, size = 75, normalized size = 0.9

$$a^2dx + \frac{a^2ex^2}{2} + \frac{2abcx^3}{3} + \frac{abdx^4}{2} + \frac{2abex^5}{5} + \frac{b^2cx^6}{6} + \frac{b^2dx^7}{7} + \frac{b^2ex^8}{8} + a^2c \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)*(b*x^3+a)^2/x,x)

[Out] a^2*d*x+1/2*a^2*e*x^2+2/3*a*b*c*x^3+1/2*a*b*d*x^4+2/5*a*b*e*x^5+1/6*b^2*c*x^6+1/7*b^2*d*x^7+1/8*b^2*e*x^8+a^2*c*ln(x)

Maxima [A] time = 0.948685, size = 100, normalized size = 1.14

$$\frac{1}{8}b^2ex^8 + \frac{1}{7}b^2dx^7 + \frac{1}{6}b^2cx^6 + \frac{2}{5}abex^5 + \frac{1}{2}abdx^4 + \frac{2}{3}abcx^3 + \frac{1}{2}a^2ex^2 + a^2dx + a^2c \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x,x, algorithm="maxima")

[Out] 1/8*b^2*e*x^8 + 1/7*b^2*d*x^7 + 1/6*b^2*c*x^6 + 2/5*a*b*e*x^5 + 1/2*a*b*d*x^4 + 2/3*a*b*c*x^3 + 1/2*a^2*e*x^2 + a^2*d*x + a^2*c*log(x)

Fricas [A] time = 1.49492, size = 184, normalized size = 2.09

$$\frac{1}{8}b^2ex^8 + \frac{1}{7}b^2dx^7 + \frac{1}{6}b^2cx^6 + \frac{2}{5}abex^5 + \frac{1}{2}abdx^4 + \frac{2}{3}abcx^3 + \frac{1}{2}a^2ex^2 + a^2dx + a^2c \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x,x, algorithm="fricas")

[Out] $\frac{1}{8}b^2e*x^8 + \frac{1}{7}b^2d*x^7 + \frac{1}{6}b^2c*x^6 + \frac{2}{5}a*b*e*x^5 + \frac{1}{2}a*b*d*x^4 + \frac{2}{3}a*b*c*x^3 + \frac{1}{2}a^2e*x^2 + a^2d*x + a^2c*\log(x)$

Sympy [A] time = 0.352949, size = 88, normalized size = 1.

$$a^2c \log(x) + a^2dx + \frac{a^2ex^2}{2} + \frac{2abcx^3}{3} + \frac{abdx^4}{2} + \frac{2abex^5}{5} + \frac{b^2cx^6}{6} + \frac{b^2dx^7}{7} + \frac{b^2ex^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**2/x,x)

[Out] $a**2*c*\log(x) + a**2*d*x + a**2*e*x**2/2 + 2*a*b*c*x**3/3 + a*b*d*x**4/2 + 2*a*b*e*x**5/5 + b**2*c*x**6/6 + b**2*d*x**7/7 + b**2*e*x**8/8$

Giac [A] time = 1.05081, size = 105, normalized size = 1.19

$$\frac{1}{8}b^2x^8e + \frac{1}{7}b^2dx^7 + \frac{1}{6}b^2cx^6 + \frac{2}{5}abx^5e + \frac{1}{2}abdx^4 + \frac{2}{3}abcx^3 + \frac{1}{2}a^2x^2e + a^2dx + a^2c \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x,x, algorithm="giac")

[Out] $\frac{1}{8}b^2*x^8*e + \frac{1}{7}b^2*d*x^7 + \frac{1}{6}b^2*c*x^6 + \frac{2}{5}a*b*x^5*e + \frac{1}{2}a*b*d*x^4 + \frac{2}{3}a*b*c*x^3 + \frac{1}{2}a^2*x^2*e + a^2*d*x + a^2*c*\log(\text{abs}(x))$

$$3.323 \quad \int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^2} dx$$

Optimal. Leaf size=83

$$-\frac{a^2c}{x} + a^2d \log(x) + a^2ex + abcx^2 + \frac{2}{3}abdx^3 + \frac{1}{2}abex^4 + \frac{1}{5}b^2cx^5 + \frac{1}{6}b^2dx^6 + \frac{1}{7}b^2ex^7$$

[Out] $-\frac{a^2c}{x} + a^2ex + abcx^2 + \frac{2a^2bdx^3}{3} + \frac{a^2bex^4}{2} + \frac{b^2cx^5}{5} + \frac{b^2dx^6}{6} + \frac{b^2ex^7}{7} + a^2d \log(x)$

Rubi [A] time = 0.0629493, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1628}

$$-\frac{a^2c}{x} + a^2d \log(x) + a^2ex + abcx^2 + \frac{2}{3}abdx^3 + \frac{1}{2}abex^4 + \frac{1}{5}b^2cx^5 + \frac{1}{6}b^2dx^6 + \frac{1}{7}b^2ex^7$$

Antiderivative was successfully verified.

[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^2)/x^2,x]

[Out] $-\frac{a^2c}{x} + a^2ex + abcx^2 + \frac{2a^2bdx^3}{3} + \frac{a^2bex^4}{2} + \frac{b^2cx^5}{5} + \frac{b^2dx^6}{6} + \frac{b^2ex^7}{7} + a^2d \log(x)$

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^2} dx &= \int \left(a^2e + \frac{a^2c}{x^2} + \frac{a^2d}{x} + 2abcx + 2abdx^2 + 2abex^3 + b^2cx^4 + b^2dx^5 + b^2ex^6 \right) dx \\ &= -\frac{a^2c}{x} + a^2ex + abcx^2 + \frac{2}{3}abdx^3 + \frac{1}{2}abex^4 + \frac{1}{5}b^2cx^5 + \frac{1}{6}b^2dx^6 + \frac{1}{7}b^2ex^7 + a^2d \log(x) \end{aligned}$$

Mathematica [A] time = 0.0077615, size = 83, normalized size = 1.

$$-\frac{a^2c}{x} + a^2d \log(x) + a^2ex + abcx^2 + \frac{2}{3}abdx^3 + \frac{1}{2}abex^4 + \frac{1}{5}b^2cx^5 + \frac{1}{6}b^2dx^6 + \frac{1}{7}b^2ex^7$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^2)/x^2,x]

[Out] -((a^2*c)/x) + a^2*e*x + a*b*c*x^2 + (2*a*b*d*x^3)/3 + (a*b*e*x^4)/2 + (b^2*c*x^5)/5 + (b^2*d*x^6)/6 + (b^2*e*x^7)/7 + a^2*d*Log[x]

Maple [A] time = 0.005, size = 74, normalized size = 0.9

$$-\frac{a^2c}{x} + a^2ex + abcx^2 + \frac{2}{3}abdx^3 + \frac{abex^4}{2} + \frac{b^2cx^5}{5} + \frac{b^2dx^6}{6} + \frac{b^2ex^7}{7} + a^2d \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)*(b*x^3+a)^2/x^2,x)

[Out] -a^2*c/x+a^2*e*x+a*b*c*x^2+2/3*a*b*d*x^3+1/2*a*b*e*x^4+1/5*b^2*c*x^5+1/6*b^2*d*x^6+1/7*b^2*e*x^7+a^2*d*ln(x)

Maxima [A] time = 0.948298, size = 99, normalized size = 1.19

$$\frac{1}{7}b^2ex^7 + \frac{1}{6}b^2dx^6 + \frac{1}{5}b^2cx^5 + \frac{1}{2}abex^4 + \frac{2}{3}abdx^3 + abcx^2 + a^2ex + a^2d \log(x) - \frac{a^2c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^2,x, algorithm="maxima")

[Out] 1/7*b^2*e*x^7 + 1/6*b^2*d*x^6 + 1/5*b^2*c*x^5 + 1/2*a*b*e*x^4 + 2/3*a*b*d*x^3 + a*b*c*x^2 + a^2*e*x + a^2*d*log(x) - a^2*c/x

Fricas [A] time = 1.53432, size = 204, normalized size = 2.46

$$\frac{30 b^2 e x^8 + 35 b^2 d x^7 + 42 b^2 c x^6 + 105 a b e x^5 + 140 a b d x^4 + 210 a b c x^3 + 210 a^2 e x^2 + 210 a^2 d x \log(x) - 210 a^2 c}{210 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^2,x, algorithm="fricas")

[Out] 1/210*(30*b^2*e*x^8 + 35*b^2*d*x^7 + 42*b^2*c*x^6 + 105*a*b*e*x^5 + 140*a*b*d*x^4 + 210*a*b*c*x^3 + 210*a^2*e*x^2 + 210*a^2*d*x*log(x) - 210*a^2*c)/x

Sympy [A] time = 0.339739, size = 82, normalized size = 0.99

$$-\frac{a^2c}{x} + a^2d \log(x) + a^2ex + abcx^2 + \frac{2abdx^3}{3} + \frac{abex^4}{2} + \frac{b^2cx^5}{5} + \frac{b^2dx^6}{6} + \frac{b^2ex^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**2/x**2,x)

[Out] -a**2*c/x + a**2*d*log(x) + a**2*e*x + a*b*c*x**2 + 2*a*b*d*x**3/3 + a*b*e*x**4/2 + b**2*c*x**5/5 + b**2*d*x**6/6 + b**2*e*x**7/7

Giac [A] time = 1.05151, size = 104, normalized size = 1.25

$$\frac{1}{7}b^2x^7e + \frac{1}{6}b^2dx^6 + \frac{1}{5}b^2cx^5 + \frac{1}{2}abx^4e + \frac{2}{3}abdx^3 + abcx^2 + a^2xe + a^2d \log(|x|) - \frac{a^2c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^2,x, algorithm="giac")

[Out] 1/7*b^2*x^7*e + 1/6*b^2*d*x^6 + 1/5*b^2*c*x^5 + 1/2*a*b*x^4*e + 2/3*a*b*d*x^3 + a*b*c*x^2 + a^2*x*e + a^2*d*log(abs(x)) - a^2*c/x

$$3.324 \quad \int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^3} dx$$

Optimal. Leaf size=84

$$-\frac{a^2c}{2x^2} - \frac{a^2d}{x} + a^2e \log(x) + 2abcx + abdx^2 + \frac{2}{3}abex^3 + \frac{1}{4}b^2cx^4 + \frac{1}{5}b^2dx^5 + \frac{1}{6}b^2ex^6$$

[Out] $-(a^2*c)/(2*x^2) - (a^2*d)/x + 2*a*b*c*x + a*b*d*x^2 + (2*a*b*e*x^3)/3 + (b^2*c*x^4)/4 + (b^2*d*x^5)/5 + (b^2*e*x^6)/6 + a^2*e*\text{Log}[x]$

Rubi [A] time = 0.0641961, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1628}

$$-\frac{a^2c}{2x^2} - \frac{a^2d}{x} + a^2e \log(x) + 2abcx + abdx^2 + \frac{2}{3}abex^3 + \frac{1}{4}b^2cx^4 + \frac{1}{5}b^2dx^5 + \frac{1}{6}b^2ex^6$$

Antiderivative was successfully verified.

[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^2)/x^3,x]

[Out] $-(a^2*c)/(2*x^2) - (a^2*d)/x + 2*a*b*c*x + a*b*d*x^2 + (2*a*b*e*x^3)/3 + (b^2*c*x^4)/4 + (b^2*d*x^5)/5 + (b^2*e*x^6)/6 + a^2*e*\text{Log}[x]$

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^3} dx &= \int \left(2abc + \frac{a^2c}{x^3} + \frac{a^2d}{x^2} + \frac{a^2e}{x} + 2abdx + 2abex^2 + b^2cx^3 + b^2dx^4 + b^2ex^5 \right) dx \\ &= -\frac{a^2c}{2x^2} - \frac{a^2d}{x} + 2abcx + abdx^2 + \frac{2}{3}abex^3 + \frac{1}{4}b^2cx^4 + \frac{1}{5}b^2dx^5 + \frac{1}{6}b^2ex^6 + a^2e \log(x) \end{aligned}$$

Mathematica [A] time = 0.0084296, size = 84, normalized size = 1.

$$-\frac{a^2c}{2x^2} - \frac{a^2d}{x} + a^2e \log(x) + 2abcx + abdx^2 + \frac{2}{3}abex^3 + \frac{1}{4}b^2cx^4 + \frac{1}{5}b^2dx^5 + \frac{1}{6}b^2ex^6$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^2)/x^3,x]

[Out] $-(a^2*c)/(2*x^2) - (a^2*d)/x + 2*a*b*c*x + a*b*d*x^2 + (2*a*b*e*x^3)/3 + (b^2*c*x^4)/4 + (b^2*d*x^5)/5 + (b^2*e*x^6)/6 + a^2*e*\text{Log}[x]$

Maple [A] time = 0.006, size = 75, normalized size = 0.9

$$-\frac{a^2c}{2x^2} - \frac{a^2d}{x} + 2abcx + abdx^2 + \frac{2abex^3}{3} + \frac{b^2cx^4}{4} + \frac{b^2dx^5}{5} + \frac{b^2ex^6}{6} + a^2e \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)*(b*x^3+a)^2/x^3,x)

[Out] $-1/2*a^2*c/x^2 - a^2*d/x + 2*a*b*c*x + a*b*d*x^2 + 2/3*a*b*e*x^3 + 1/4*b^2*c*x^4 + 1/5*b^2*d*x^5 + 1/6*b^2*e*x^6 + a^2*e*\ln(x)$

Maxima [A] time = 0.943366, size = 100, normalized size = 1.19

$$\frac{1}{6}b^2ex^6 + \frac{1}{5}b^2dx^5 + \frac{1}{4}b^2cx^4 + \frac{2}{3}abex^3 + abdx^2 + 2abcx + a^2e \log(x) - \frac{2a^2dx + a^2c}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^3,x, algorithm="maxima")

[Out] $1/6*b^2*e*x^6 + 1/5*b^2*d*x^5 + 1/4*b^2*c*x^4 + 2/3*a*b*e*x^3 + a*b*d*x^2 + 2*a*b*c*x + a^2*e*\log(x) - 1/2*(2*a^2*d*x + a^2*c)/x^2$

Fricas [A] time = 1.46572, size = 198, normalized size = 2.36

$$\frac{10b^2ex^8 + 12b^2dx^7 + 15b^2cx^6 + 40abex^5 + 60abdx^4 + 120abcx^3 + 60a^2ex^2 \log(x) - 60a^2dx - 30a^2c}{60x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^3,x, algorithm="fricas")`

[Out] $\frac{1}{60}(10*b^2*e*x^8 + 12*b^2*d*x^7 + 15*b^2*c*x^6 + 40*a*b*e*x^5 + 60*a*b*d*x^4 + 120*a*b*c*x^3 + 60*a^2*e*x^2*\log(x) - 60*a^2*d*x - 30*a^2*c)/x^2$

Sympy [A] time = 0.394683, size = 85, normalized size = 1.01

$$a^2e \log(x) + 2abcx + abdx^2 + \frac{2abex^3}{3} + \frac{b^2cx^4}{4} + \frac{b^2dx^5}{5} + \frac{b^2ex^6}{6} - \frac{a^2c + 2a^2dx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d*x+c)*(b*x**3+a)**2/x**3,x)`

[Out] $a**2*e*\log(x) + 2*a*b*c*x + a*b*d*x**2 + 2*a*b*e*x**3/3 + b**2*c*x**4/4 + b**2*d*x**5/5 + b**2*e*x**6/6 - (a**2*c + 2*a**2*d*x)/(2*x**2)$

Giac [A] time = 1.05293, size = 105, normalized size = 1.25

$$\frac{1}{6}b^2x^6e + \frac{1}{5}b^2dx^5 + \frac{1}{4}b^2cx^4 + \frac{2}{3}abx^3e + abdx^2 + 2abcx + a^2e \log(|x|) - \frac{2a^2dx + a^2c}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^3,x, algorithm="giac")`

[Out] $\frac{1}{6}*b^2*x^6*e + \frac{1}{5}*b^2*d*x^5 + \frac{1}{4}*b^2*c*x^4 + \frac{2}{3}*a*b*x^3*e + a*b*d*x^2 + 2*a*b*c*x + a^2*e*\log(\text{abs}(x)) - \frac{1}{2}*(2*a^2*d*x + a^2*c)/x^2$

3.325 $\int x^2 (c + dx + ex^2) (a + bx^3)^3 dx$

Optimal. Leaf size=110

$$\frac{3}{7}a^2bdx^7 + \frac{3}{8}a^2bex^8 + \frac{1}{4}a^3dx^4 + \frac{1}{5}a^3ex^5 + \frac{3}{10}ab^2dx^{10} + \frac{3}{11}ab^2ex^{11} + \frac{c(a+bx^3)^4}{12b} + \frac{1}{13}b^3dx^{13} + \frac{1}{14}b^3ex^{14}$$

[Out] $(a^3d*x^4)/4 + (a^3e*x^5)/5 + (3*a^2*b*d*x^7)/7 + (3*a^2*b*e*x^8)/8 + (3*a*b^2*d*x^{10})/10 + (3*a*b^2*e*x^{11})/11 + (b^3*d*x^{13})/13 + (b^3*e*x^{14})/14 + (c*(a + b*x^3)^4)/(12*b)$

Rubi [A] time = 0.0787603, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1582, 1850}

$$\frac{3}{7}a^2bdx^7 + \frac{3}{8}a^2bex^8 + \frac{1}{4}a^3dx^4 + \frac{1}{5}a^3ex^5 + \frac{3}{10}ab^2dx^{10} + \frac{3}{11}ab^2ex^{11} + \frac{c(a+bx^3)^4}{12b} + \frac{1}{13}b^3dx^{13} + \frac{1}{14}b^3ex^{14}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^2*(c + d*x + e*x^2)*(a + b*x^3)^3, x]$

[Out] $(a^3d*x^4)/4 + (a^3e*x^5)/5 + (3*a^2*b*d*x^7)/7 + (3*a^2*b*e*x^8)/8 + (3*a*b^2*d*x^{10})/10 + (3*a*b^2*e*x^{11})/11 + (b^3*d*x^{13})/13 + (b^3*e*x^{14})/14 + (c*(a + b*x^3)^4)/(12*b)$

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(Coeff[Px, x, n - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a + b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
```

, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int x^2 (c + dx + ex^2) (a + bx^3)^3 dx &= \frac{c(a + bx^3)^4}{12b} + \int (a + bx^3)^3 (-cx^2 + x^2(c + dx + ex^2)) dx \\ &= \frac{c(a + bx^3)^4}{12b} + \int (a^3 dx^3 + a^3 ex^4 + 3a^2 b dx^6 + 3a^2 b ex^7 + 3ab^2 dx^9 + 3ab^2 ex^{10} + b^3 dx^{13} + b^3 ex^{14}) dx \\ &= \frac{1}{4} a^3 dx^4 + \frac{1}{5} a^3 ex^5 + \frac{3}{7} a^2 b dx^7 + \frac{3}{8} a^2 b ex^8 + \frac{3}{10} ab^2 dx^{10} + \frac{3}{11} ab^2 ex^{11} + \frac{1}{13} b^3 dx^{13} + \frac{1}{14} b^3 ex^{14} \end{aligned}$$

Mathematica [A] time = 0.0038531, size = 139, normalized size = 1.26

$$\frac{1}{2} a^2 b c x^6 + \frac{3}{7} a^2 b d x^7 + \frac{3}{8} a^2 b e x^8 + \frac{1}{3} a^3 c x^3 + \frac{1}{4} a^3 d x^4 + \frac{1}{5} a^3 e x^5 + \frac{1}{3} a b^2 c x^9 + \frac{3}{10} a b^2 d x^{10} + \frac{3}{11} a b^2 e x^{11} + \frac{1}{12} b^3 c x^{12} + \frac{1}{13} b^3 d x^{13} + \frac{1}{14} b^3 e x^{14}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3)^3,x]

[Out] (a^3*c*x^3)/3 + (a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (a^2*b*c*x^6)/2 + (3*a^2*b*d*x^7)/7 + (3*a^2*b*e*x^8)/8 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2*e*x^11)/11 + (b^3*c*x^12)/12 + (b^3*d*x^13)/13 + (b^3*e*x^14)/14

Maple [A] time = 0.001, size = 116, normalized size = 1.1

$$\frac{b^3 e x^{14}}{14} + \frac{b^3 d x^{13}}{13} + \frac{b^3 c x^{12}}{12} + \frac{3 a b^2 e x^{11}}{11} + \frac{3 a b^2 d x^{10}}{10} + \frac{a b^2 c x^9}{3} + \frac{3 a^2 b e x^8}{8} + \frac{3 a^2 b d x^7}{7} + \frac{a^2 b c x^6}{2} + \frac{a^3 e x^5}{5} + \frac{a^3 d x^4}{4} + \frac{a^3 c x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^3,x)

[Out] 1/14*b^3*e*x^14+1/13*b^3*d*x^13+1/12*b^3*c*x^12+3/11*a*b^2*e*x^11+3/10*a*b^2*d*x^10+1/3*a*b^2*c*x^9+3/8*a^2*b*e*x^8+3/7*a^2*b*d*x^7+1/2*a^2*b*c*x^6+1/5*a^3*e*x^5+1/4*a^3*d*x^4+1/3*a^3*c*x^3

Maxima [A] time = 0.941417, size = 155, normalized size = 1.41

$$\frac{1}{14}b^3ex^{14} + \frac{1}{13}b^3dx^{13} + \frac{1}{12}b^3cx^{12} + \frac{3}{11}ab^2ex^{11} + \frac{3}{10}ab^2dx^{10} + \frac{1}{3}ab^2cx^9 + \frac{3}{8}a^2bex^8 + \frac{3}{7}a^2bdx^7 + \frac{1}{2}a^2bcx^6 + \frac{1}{5}a^3ex^5 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="maxima")

[Out] 1/14*b^3*e*x^14 + 1/13*b^3*d*x^13 + 1/12*b^3*c*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3/8*a^2*b*e*x^8 + 3/7*a^2*b*d*x^7 + 1/2*a^2*b*c*x^6 + 1/5*a^3*e*x^5 + 1/4*a^3*d*x^4 + 1/3*a^3*c*x^3

Fricas [A] time = 1.30698, size = 288, normalized size = 2.62

$$\frac{1}{14}x^{14}eb^3 + \frac{1}{13}x^{13}db^3 + \frac{1}{12}x^{12}cb^3 + \frac{3}{11}x^{11}eb^2a + \frac{3}{10}x^{10}db^2a + \frac{1}{3}x^9cb^2a + \frac{3}{8}x^8eba^2 + \frac{3}{7}x^7dba^2 + \frac{1}{2}x^6cba^2 + \frac{1}{5}x^5ea^3 + \frac{1}{4}x^4da^3 + \frac{1}{3}x^3ca^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="fricas")

[Out] 1/14*x^14*e*b^3 + 1/13*x^13*d*b^3 + 1/12*x^12*c*b^3 + 3/11*x^11*e*b^2*a + 3/10*x^10*d*b^2*a + 1/3*x^9*c*b^2*a + 3/8*x^8*e*b*a^2 + 3/7*x^7*d*b*a^2 + 1/2*x^6*c*b*a^2 + 1/5*x^5*e*a^3 + 1/4*x^4*d*a^3 + 1/3*x^3*c*a^3

Sympy [A] time = 0.075398, size = 138, normalized size = 1.25

$$\frac{a^3cx^3}{3} + \frac{a^3dx^4}{4} + \frac{a^3ex^5}{5} + \frac{a^2bcx^6}{2} + \frac{3a^2bdx^7}{7} + \frac{3a^2bex^8}{8} + \frac{ab^2cx^9}{3} + \frac{3ab^2dx^{10}}{10} + \frac{3ab^2ex^{11}}{11} + \frac{b^3cx^{12}}{12} + \frac{b^3dx^{13}}{13} + \frac{b^3ex^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**3,x)

[Out] a**3*c*x**3/3 + a**3*d*x**4/4 + a**3*e*x**5/5 + a**2*b*c*x**6/2 + 3*a**2*b*d*x**7/7 + 3*a**2*b*e*x**8/8 + a*b**2*c*x**9/3 + 3*a*b**2*d*x**10/10 + 3*a*b**2*e*x**11/11 + b**3*c*x**12/12 + b**3*d*x**13/13 + b**3*e*x**14/14

Giac [A] time = 1.05876, size = 161, normalized size = 1.46

$$\frac{1}{14} b^3 x^{14} e + \frac{1}{13} b^3 d x^{13} + \frac{1}{12} b^3 c x^{12} + \frac{3}{11} a b^2 x^{11} e + \frac{3}{10} a b^2 d x^{10} + \frac{1}{3} a b^2 c x^9 + \frac{3}{8} a^2 b x^8 e + \frac{3}{7} a^2 b d x^7 + \frac{1}{2} a^2 b c x^6 + \frac{1}{5} a^3 x^5 e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="giac")

[Out] 1/14*b^3*x^14*e + 1/13*b^3*d*x^13 + 1/12*b^3*c*x^12 + 3/11*a*b^2*x^11*e + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3/8*a^2*b*x^8*e + 3/7*a^2*b*d*x^7 + 1/2*a^2*b*c*x^6 + 1/5*a^3*x^5*e + 1/4*a^3*d*x^4 + 1/3*a^3*c*x^3

3.326 $\int x(c + dx + ex^2)(a + bx^3)^3 dx$

Optimal. Leaf size=110

$$\frac{3}{5}a^2bcx^5 + \frac{3}{7}a^2bex^7 + \frac{1}{2}a^3cx^2 + \frac{1}{4}a^3ex^4 + \frac{3}{8}ab^2cx^8 + \frac{3}{10}ab^2ex^{10} + \frac{d(a + bx^3)^4}{12b} + \frac{1}{11}b^3cx^{11} + \frac{1}{13}b^3ex^{13}$$

[Out] $(a^3c*x^2)/2 + (a^3e*x^4)/4 + (3*a^2*b*c*x^5)/5 + (3*a^2*b*e*x^7)/7 + (3*a*b^2*c*x^8)/8 + (3*a*b^2*e*x^{10})/10 + (b^3*c*x^{11})/11 + (b^3*e*x^{13})/13 + (d*(a + b*x^3)^4)/(12*b)$

Rubi [A] time = 0.0693723, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1582, 1850}

$$\frac{3}{5}a^2bcx^5 + \frac{3}{7}a^2bex^7 + \frac{1}{2}a^3cx^2 + \frac{1}{4}a^3ex^4 + \frac{3}{8}ab^2cx^8 + \frac{3}{10}ab^2ex^{10} + \frac{d(a + bx^3)^4}{12b} + \frac{1}{11}b^3cx^{11} + \frac{1}{13}b^3ex^{13}$$

Antiderivative was successfully verified.

[In] Int[x*(c + d*x + e*x^2)*(a + b*x^3)^3,x]

[Out] $(a^3c*x^2)/2 + (a^3e*x^4)/4 + (3*a^2*b*c*x^5)/5 + (3*a^2*b*e*x^7)/7 + (3*a*b^2*c*x^8)/8 + (3*a*b^2*e*x^{10})/10 + (b^3*c*x^{11})/11 + (b^3*e*x^{13})/13 + (d*(a + b*x^3)^4)/(12*b)$

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(Coeff[Px, x, n - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a + b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
```


, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int x(c + dx + ex^2)(a + bx^3)^3 dx &= \frac{d(a + bx^3)^4}{12b} + \int (a + bx^3)^3 (-dx^2 + x(c + dx + ex^2)) dx \\ &= \frac{d(a + bx^3)^4}{12b} + \int (a^3cx + a^3ex^3 + 3a^2bcx^4 + 3a^2bex^6 + 3ab^2cx^7 + 3ab^2ex^9 + b^3cx^{10}) dx \\ &= \frac{1}{2}a^3cx^2 + \frac{1}{4}a^3ex^4 + \frac{3}{5}a^2bcx^5 + \frac{3}{7}a^2bex^7 + \frac{3}{8}ab^2cx^8 + \frac{3}{10}ab^2ex^{10} + \frac{1}{11}b^3cx^{11} + \frac{1}{13}b^3cx^{13} \end{aligned}$$

Mathematica [A] time = 0.0036847, size = 139, normalized size = 1.26

$$\frac{3}{5}a^2bcx^5 + \frac{1}{2}a^2bdx^6 + \frac{3}{7}a^2bex^7 + \frac{1}{2}a^3cx^2 + \frac{1}{3}a^3dx^3 + \frac{1}{4}a^3ex^4 + \frac{3}{8}ab^2cx^8 + \frac{1}{3}ab^2dx^9 + \frac{3}{10}ab^2ex^{10} + \frac{1}{11}b^3cx^{11} + \frac{1}{12}b^3dx^{12}$$

Antiderivative was successfully verified.

[In] Integrate[x*(c + d*x + e*x^2)*(a + b*x^3)^3,x]

[Out] (a^3*c*x^2)/2 + (a^3*d*x^3)/3 + (a^3*e*x^4)/4 + (3*a^2*b*c*x^5)/5 + (a^2*b*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (3*a*b^2*c*x^8)/8 + (a*b^2*d*x^9)/3 + (3*a*b^2*e*x^10)/10 + (b^3*c*x^11)/11 + (b^3*d*x^12)/12 + (b^3*e*x^13)/13

Maple [A] time = 0.001, size = 116, normalized size = 1.1

$$\frac{b^3ex^{13}}{13} + \frac{b^3dx^{12}}{12} + \frac{b^3cx^{11}}{11} + \frac{3ab^2ex^{10}}{10} + \frac{ab^2dx^9}{3} + \frac{3ab^2cx^8}{8} + \frac{3a^2bex^7}{7} + \frac{a^2bdx^6}{2} + \frac{3a^2bcx^5}{5} + \frac{a^3ex^4}{4} + \frac{a^3dx^3}{3} + \frac{a^3cx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(e*x^2+d*x+c)*(b*x^3+a)^3,x)

[Out] 1/13*b^3*e*x^13+1/12*b^3*d*x^12+1/11*b^3*c*x^11+3/10*a*b^2*e*x^10+1/3*a*b^2*d*x^9+3/8*a*b^2*c*x^8+3/7*a^2*b*e*x^7+1/2*a^2*b*d*x^6+3/5*a^2*b*c*x^5+1/4*a^3*e*x^4+1/3*a^3*d*x^3+1/2*a^3*c*x^2

Maxima [A] time = 0.919483, size = 155, normalized size = 1.41

$$\frac{1}{13} b^3 e x^{13} + \frac{1}{12} b^3 d x^{12} + \frac{1}{11} b^3 c x^{11} + \frac{3}{10} a b^2 e x^{10} + \frac{1}{3} a b^2 d x^9 + \frac{3}{8} a b^2 c x^8 + \frac{3}{7} a^2 b e x^7 + \frac{1}{2} a^2 b d x^6 + \frac{3}{5} a^2 b c x^5 + \frac{1}{4} a^3 e x^4 + \frac{1}{3} a^3 d x^3 + \frac{1}{2} a^3 c x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="maxima")

[Out] 1/13*b^3*e*x^13 + 1/12*b^3*d*x^12 + 1/11*b^3*c*x^11 + 3/10*a*b^2*e*x^10 + 1/3*a*b^2*d*x^9 + 3/8*a*b^2*c*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*a^3*e*x^4 + 1/3*a^3*d*x^3 + 1/2*a^3*c*x^2

Fricas [A] time = 1.26776, size = 285, normalized size = 2.59

$$\frac{1}{13} x^{13} e b^3 + \frac{1}{12} x^{12} d b^3 + \frac{1}{11} x^{11} c b^3 + \frac{3}{10} x^{10} e b^2 a + \frac{1}{3} x^9 d b^2 a + \frac{3}{8} x^8 c b^2 a + \frac{3}{7} x^7 e b a^2 + \frac{1}{2} x^6 d b a^2 + \frac{3}{5} x^5 c b a^2 + \frac{1}{4} x^4 e a^3 + \frac{1}{3} x^3 d a^3 + \frac{1}{2} x^2 c a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="fricas")

[Out] 1/13*x^13*e*b^3 + 1/12*x^12*d*b^3 + 1/11*x^11*c*b^3 + 3/10*x^10*e*b^2*a + 1/3*x^9*d*b^2*a + 3/8*x^8*c*b^2*a + 3/7*x^7*e*b*a^2 + 1/2*x^6*d*b*a^2 + 3/5*x^5*c*b*a^2 + 1/4*x^4*e*a^3 + 1/3*x^3*d*a^3 + 1/2*x^2*c*a^3

Sympy [A] time = 0.076247, size = 138, normalized size = 1.25

$$\frac{a^3 c x^2}{2} + \frac{a^3 d x^3}{3} + \frac{a^3 e x^4}{4} + \frac{3 a^2 b c x^5}{5} + \frac{a^2 b d x^6}{2} + \frac{3 a^2 b e x^7}{7} + \frac{3 a b^2 c x^8}{8} + \frac{a b^2 d x^9}{3} + \frac{3 a b^2 e x^{10}}{10} + \frac{b^3 c x^{11}}{11} + \frac{b^3 d x^{12}}{12} + \frac{b^3 e x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**3,x)

[Out] a**3*c*x**2/2 + a**3*d*x**3/3 + a**3*e*x**4/4 + 3*a**2*b*c*x**5/5 + a**2*b*d*x**6/2 + 3*a**2*b*e*x**7/7 + 3*a*b**2*c*x**8/8 + a*b**2*d*x**9/3 + 3*a*b**2*e*x**10/10 + b**3*c*x**11/11 + b**3*d*x**12/12 + b**3*e*x**13/13

Giac [A] time = 1.08196, size = 161, normalized size = 1.46

$$\frac{1}{13} b^3 x^{13} e + \frac{1}{12} b^3 d x^{12} + \frac{1}{11} b^3 c x^{11} + \frac{3}{10} a b^2 x^{10} e + \frac{1}{3} a b^2 d x^9 + \frac{3}{8} a b^2 c x^8 + \frac{3}{7} a^2 b x^7 e + \frac{1}{2} a^2 b d x^6 + \frac{3}{5} a^2 b c x^5 + \frac{1}{4} a^3 x^4 e +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="giac")`

[Out] `1/13*b^3*x^13*e + 1/12*b^3*d*x^12 + 1/11*b^3*c*x^11 + 3/10*a*b^2*x^10*e + 1/3*a*b^2*d*x^9 + 3/8*a*b^2*c*x^8 + 3/7*a^2*b*x^7*e + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*a^3*x^4*e + 1/3*a^3*d*x^3 + 1/2*a^3*c*x^2`

3.327 $\int (c + dx + ex^2)(a + bx^3)^3 dx$

Optimal. Leaf size=105

$$\frac{3}{4}a^2bcx^4 + \frac{3}{5}a^2bdx^5 + a^3cx + \frac{1}{2}a^3dx^2 + \frac{3}{7}ab^2cx^7 + \frac{3}{8}ab^2dx^8 + \frac{e(a + bx^3)^4}{12b} + \frac{1}{10}b^3cx^{10} + \frac{1}{11}b^3dx^{11}$$

[Out] $a^3c*x + (a^3*d*x^2)/2 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^5)/5 + (3*a*b^2*c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (b^3*c*x^{10})/10 + (b^3*d*x^{11})/11 + (e*(a + b*x^3)^4)/(12*b)$

Rubi [A] time = 0.0969183, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {1582, 1850}

$$\frac{3}{4}a^2bcx^4 + \frac{3}{5}a^2bdx^5 + a^3cx + \frac{1}{2}a^3dx^2 + \frac{3}{7}ab^2cx^7 + \frac{3}{8}ab^2dx^8 + \frac{e(a + bx^3)^4}{12b} + \frac{1}{10}b^3cx^{10} + \frac{1}{11}b^3dx^{11}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)*(a + b*x^3)^3,x]

[Out] $a^3c*x + (a^3*d*x^2)/2 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^5)/5 + (3*a*b^2*c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (b^3*c*x^{10})/10 + (b^3*d*x^{11})/11 + (e*(a + b*x^3)^4)/(12*b)$

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(Coeff[Px, x, n - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a + b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
```

, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int (c + dx + ex^2)(a + bx^3)^3 dx &= \frac{e(a + bx^3)^4}{12b} + \int (c + dx)(a + bx^3)^3 dx \\ &= \frac{e(a + bx^3)^4}{12b} + \int (a^3c + a^3dx + 3a^2bcx^3 + 3a^2bdx^4 + 3ab^2cx^6 + 3ab^2dx^7 + b^3cx^9 + b^3dx^{11}) dx \\ &= a^3cx + \frac{1}{2}a^3dx^2 + \frac{3}{4}a^2bcx^4 + \frac{3}{5}a^2bdx^5 + \frac{3}{7}ab^2cx^7 + \frac{3}{8}ab^2dx^8 + \frac{1}{10}b^3cx^{10} + \frac{1}{11}b^3dx^{11} \end{aligned}$$

Mathematica [A] time = 0.0036741, size = 134, normalized size = 1.28

$$\frac{3}{4}a^2bcx^4 + \frac{3}{5}a^2bdx^5 + \frac{1}{2}a^2bex^6 + a^3cx + \frac{1}{2}a^3dx^2 + \frac{1}{3}a^3ex^3 + \frac{3}{7}ab^2cx^7 + \frac{3}{8}ab^2dx^8 + \frac{1}{3}ab^2ex^9 + \frac{1}{10}b^3cx^{10} + \frac{1}{11}b^3dx^{11} +$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)*(a + b*x^3)^3,x]

[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^5)/5 + (a^2*b*e*x^6)/2 + (3*a*b^2*c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (a*b^2*e*x^9)/3 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11 + (b^3*e*x^12)/12

Maple [A] time = 0.001, size = 113, normalized size = 1.1

$$\frac{b^3ex^{12}}{12} + \frac{b^3dx^{11}}{11} + \frac{b^3cx^{10}}{10} + \frac{ab^2ex^9}{3} + \frac{3ab^2dx^8}{8} + \frac{3ab^2cx^7}{7} + \frac{a^2bex^6}{2} + \frac{3a^2bdx^5}{5} + \frac{3a^2bcx^4}{4} + \frac{a^3ex^3}{3} + \frac{a^3dx^2}{2} + a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)*(b*x^3+a)^3,x)

[Out] 1/12*b^3*e*x^12+1/11*b^3*d*x^11+1/10*b^3*c*x^10+1/3*a*b^2*e*x^9+3/8*a*b^2*d*x^8+3/7*a*b^2*c*x^7+1/2*a^2*b*e*x^6+3/5*a^2*b*d*x^5+3/4*a^2*b*c*x^4+1/3*a^3*e*x^3+1/2*a^3*d*x^2+a^3*c*x

Maxima [A] time = 0.93441, size = 151, normalized size = 1.44

$$\frac{1}{12}b^3ex^{12} + \frac{1}{11}b^3dx^{11} + \frac{1}{10}b^3cx^{10} + \frac{1}{3}ab^2ex^9 + \frac{3}{8}ab^2dx^8 + \frac{3}{7}ab^2cx^7 + \frac{1}{2}a^2bex^6 + \frac{3}{5}a^2bdx^5 + \frac{3}{4}a^2bcx^4 + \frac{1}{3}a^3ex^3 + \frac{1}{2}a^3dx^2 + a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="maxima")

[Out] 1/12*b^3*e*x^12 + 1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 1/3*a*b^2*e*x^9 + 3/8*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 1/2*a^2*b*e*x^6 + 3/5*a^2*b*d*x^5 + 3/4*a^2*b*c*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x

Fricas [A] time = 1.28377, size = 274, normalized size = 2.61

$$\frac{1}{12}x^{12}eb^3 + \frac{1}{11}x^{11}db^3 + \frac{1}{10}x^{10}cb^3 + \frac{1}{3}x^9eb^2a + \frac{3}{8}x^8db^2a + \frac{3}{7}x^7cb^2a + \frac{1}{2}x^6eba^2 + \frac{3}{5}x^5dba^2 + \frac{3}{4}x^4cba^2 + \frac{1}{3}x^3ea^3 + \frac{1}{2}x^2da^3 + xa^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="fricas")

[Out] 1/12*x^12*e*b^3 + 1/11*x^11*d*b^3 + 1/10*x^10*c*b^3 + 1/3*x^9*e*b^2*a + 3/8*x^8*d*b^2*a + 3/7*x^7*c*b^2*a + 1/2*x^6*e*b*a^2 + 3/5*x^5*d*b*a^2 + 3/4*x^4*c*b*a^2 + 1/3*x^3*e*a^3 + 1/2*x^2*d*a^3 + x*c*a^3

Sympy [A] time = 0.075057, size = 134, normalized size = 1.28

$$a^3cx + \frac{a^3dx^2}{2} + \frac{a^3ex^3}{3} + \frac{3a^2bcx^4}{4} + \frac{3a^2bdx^5}{5} + \frac{a^2bex^6}{2} + \frac{3ab^2cx^7}{7} + \frac{3ab^2dx^8}{8} + \frac{ab^2ex^9}{3} + \frac{b^3cx^{10}}{10} + \frac{b^3dx^{11}}{11} + \frac{b^3ex^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**3,x)

[Out] a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + 3*a**2*b*c*x**4/4 + 3*a**2*b*d*x**5/5 + a**2*b*e*x**6/2 + 3*a*b**2*c*x**7/7 + 3*a*b**2*d*x**8/8 + a*b**2*e*x**9/3 + b**3*c*x**10/10 + b**3*d*x**11/11 + b**3*e*x**12/12

Giac [A] time = 1.08264, size = 157, normalized size = 1.5

$$\frac{1}{12} b^3 x^{12} e + \frac{1}{11} b^3 d x^{11} + \frac{1}{10} b^3 c x^{10} + \frac{1}{3} a b^2 x^9 e + \frac{3}{8} a b^2 d x^8 + \frac{3}{7} a b^2 c x^7 + \frac{1}{2} a^2 b x^6 e + \frac{3}{5} a^2 b d x^5 + \frac{3}{4} a^2 b c x^4 + \frac{1}{3} a^3 x^3 e + \frac{1}{2} a^3 d x^2 + a^3 c x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="giac")

[Out] 1/12*b^3*x^12*e + 1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 1/3*a*b^2*x^9*e + 3/8*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 1/2*a^2*b*x^6*e + 3/5*a^2*b*d*x^5 + 3/4*a^2*b*c*x^4 + 1/3*a^3*x^3*e + 1/2*a^3*d*x^2 + a^3*c*x

$$3.328 \quad \int \frac{(c+dx+ex^2)(a+bx^3)^3}{x} dx$$

Optimal. Leaf size=127

$$a^2bcx^3 + \frac{3}{4}a^2bdx^4 + \frac{3}{5}a^2bex^5 + a^3c \log(x) + a^3dx + \frac{1}{2}a^3ex^2 + \frac{1}{2}ab^2cx^6 + \frac{3}{7}ab^2dx^7 + \frac{3}{8}ab^2ex^8 + \frac{1}{9}b^3cx^9 + \frac{1}{10}b^3dx^{10} + \frac{1}{11}$$

[Out] $a^3d*x + (a^3*e*x^2)/2 + a^2*b*c*x^3 + (3*a^2*b*d*x^4)/4 + (3*a^2*b*e*x^5)/5 + (a*b^2*c*x^6)/2 + (3*a*b^2*d*x^7)/7 + (3*a*b^2*e*x^8)/8 + (b^3*c*x^9)/9 + (b^3*d*x^{10})/10 + (b^3*e*x^{11})/11 + a^3*c*Log[x]$

Rubi [A] time = 0.0736885, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1628}

$$a^2bcx^3 + \frac{3}{4}a^2bdx^4 + \frac{3}{5}a^2bex^5 + a^3c \log(x) + a^3dx + \frac{1}{2}a^3ex^2 + \frac{1}{2}ab^2cx^6 + \frac{3}{7}ab^2dx^7 + \frac{3}{8}ab^2ex^8 + \frac{1}{9}b^3cx^9 + \frac{1}{10}b^3dx^{10} + \frac{1}{11}$$

Antiderivative was successfully verified.

[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^3)/x,x]

[Out] $a^3d*x + (a^3*e*x^2)/2 + a^2*b*c*x^3 + (3*a^2*b*d*x^4)/4 + (3*a^2*b*e*x^5)/5 + (a*b^2*c*x^6)/2 + (3*a*b^2*d*x^7)/7 + (3*a*b^2*e*x^8)/8 + (b^3*c*x^9)/9 + (b^3*d*x^{10})/10 + (b^3*e*x^{11})/11 + a^3*c*Log[x]$

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{(c+dx+ex^2)(a+bx^3)^3}{x} dx &= \int \left(a^3d + \frac{a^3c}{x} + a^3ex + 3a^2bcx^2 + 3a^2bdx^3 + 3a^2bex^4 + 3ab^2cx^5 + 3ab^2dx^6 + 3ab^2ex^7 \right. \\ &\quad \left. + a^3dx + \frac{1}{2}a^3ex^2 + a^2bcx^3 + \frac{3}{4}a^2bdx^4 + \frac{3}{5}a^2bex^5 + \frac{1}{2}ab^2cx^6 + \frac{3}{7}ab^2dx^7 + \frac{3}{8}ab^2ex^8 + \frac{1}{9}b^3cx^9 \right) dx \end{aligned}$$

Mathematica [A] time = 0.0087516, size = 127, normalized size = 1.

$$a^2bcx^3 + \frac{3}{4}a^2bdx^4 + \frac{3}{5}a^2bex^5 + a^3c \log(x) + a^3dx + \frac{1}{2}a^3ex^2 + \frac{1}{2}ab^2cx^6 + \frac{3}{7}ab^2dx^7 + \frac{3}{8}ab^2ex^8 + \frac{1}{9}b^3cx^9 + \frac{1}{10}b^3dx^{10} + \frac{1}{11}b^3ex^{11} + a^3c \ln(x)$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^3)/x,x]

[Out] a^3*d*x + (a^3*e*x^2)/2 + a^2*b*c*x^3 + (3*a^2*b*d*x^4)/4 + (3*a^2*b*e*x^5)/5 + (a*b^2*c*x^6)/2 + (3*a*b^2*d*x^7)/7 + (3*a*b^2*e*x^8)/8 + (b^3*c*x^9)/9 + (b^3*d*x^10)/10 + (b^3*e*x^11)/11 + a^3*c*Log[x]

Maple [A] time = 0.001, size = 110, normalized size = 0.9

$$a^3dx + \frac{a^3ex^2}{2} + a^2bcx^3 + \frac{3a^2bdx^4}{4} + \frac{3a^2bex^5}{5} + \frac{ab^2cx^6}{2} + \frac{3ab^2dx^7}{7} + \frac{3ab^2ex^8}{8} + \frac{b^3cx^9}{9} + \frac{b^3dx^{10}}{10} + \frac{b^3ex^{11}}{11} + a^3c \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)*(b*x^3+a)^3/x,x)

[Out] a^3*d*x+1/2*a^3*e*x^2+a^2*b*c*x^3+3/4*a^2*b*d*x^4+3/5*a^2*b*e*x^5+1/2*a*b^2*c*x^6+3/7*a*b^2*d*x^7+3/8*a*b^2*e*x^8+1/9*b^3*c*x^9+1/10*b^3*d*x^10+1/11*b^3*e*x^11+a^3*c*ln(x)

Maxima [A] time = 0.952839, size = 147, normalized size = 1.16

$$\frac{1}{11}b^3ex^{11} + \frac{1}{10}b^3dx^{10} + \frac{1}{9}b^3cx^9 + \frac{3}{8}ab^2ex^8 + \frac{3}{7}ab^2dx^7 + \frac{1}{2}ab^2cx^6 + \frac{3}{5}a^2bex^5 + \frac{3}{4}a^2bdx^4 + a^2bcx^3 + \frac{1}{2}a^3ex^2 + a^3dx + a^3c \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x,x, algorithm="maxima")

[Out] 1/11*b^3*e*x^11 + 1/10*b^3*d*x^10 + 1/9*b^3*c*x^9 + 3/8*a*b^2*e*x^8 + 3/7*a*b^2*d*x^7 + 1/2*a*b^2*c*x^6 + 3/5*a^2*b*e*x^5 + 3/4*a^2*b*d*x^4 + a^2*b*c*x^3 + 1/2*a^3*e*x^2 + a^3*d*x + a^3*c*log(x)

Fricas [A] time = 1.49194, size = 265, normalized size = 2.09

$$\frac{1}{11} b^3 e x^{11} + \frac{1}{10} b^3 d x^{10} + \frac{1}{9} b^3 c x^9 + \frac{3}{8} a b^2 e x^8 + \frac{3}{7} a b^2 d x^7 + \frac{1}{2} a b^2 c x^6 + \frac{3}{5} a^2 b e x^5 + \frac{3}{4} a^2 b d x^4 + a^2 b c x^3 + \frac{1}{2} a^3 e x^2 + a^3 d x +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x,x, algorithm="fricas")

[Out] 1/11*b^3*e*x^11 + 1/10*b^3*d*x^10 + 1/9*b^3*c*x^9 + 3/8*a*b^2*e*x^8 + 3/7*a*b^2*d*x^7 + 1/2*a*b^2*c*x^6 + 3/5*a^2*b*e*x^5 + 3/4*a^2*b*d*x^4 + a^2*b*c*x^3 + 1/2*a^3*e*x^2 + a^3*d*x + a^3*c*log(x)

Sympy [A] time = 0.375604, size = 131, normalized size = 1.03

$$a^3 c \log(x) + a^3 d x + \frac{a^3 e x^2}{2} + a^2 b c x^3 + \frac{3 a^2 b d x^4}{4} + \frac{3 a^2 b e x^5}{5} + \frac{a b^2 c x^6}{2} + \frac{3 a b^2 d x^7}{7} + \frac{3 a b^2 e x^8}{8} + \frac{b^3 c x^9}{9} + \frac{b^3 d x^{10}}{10} + \frac{b^3 e x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**3/x,x)

[Out] a**3*c*log(x) + a**3*d*x + a**3*e*x**2/2 + a**2*b*c*x**3 + 3*a**2*b*d*x**4/4 + 3*a**2*b*e*x**5/5 + a*b**2*c*x**6/2 + 3*a*b**2*d*x**7/7 + 3*a*b**2*e*x**8/8 + b**3*c*x**9/9 + b**3*d*x**10/10 + b**3*e*x**11/11

Giac [A] time = 1.07317, size = 154, normalized size = 1.21

$$\frac{1}{11} b^3 x^{11} e + \frac{1}{10} b^3 d x^{10} + \frac{1}{9} b^3 c x^9 + \frac{3}{8} a b^2 x^8 e + \frac{3}{7} a b^2 d x^7 + \frac{1}{2} a b^2 c x^6 + \frac{3}{5} a^2 b x^5 e + \frac{3}{4} a^2 b d x^4 + a^2 b c x^3 + \frac{1}{2} a^3 x^2 e + a^3 d x +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x,x, algorithm="giac")

[Out] 1/11*b^3*x^11*e + 1/10*b^3*d*x^10 + 1/9*b^3*c*x^9 + 3/8*a*b^2*x^8*e + 3/7*a*b^2*d*x^7 + 1/2*a*b^2*c*x^6 + 3/5*a^2*b*x^5*e + 3/4*a^2*b*d*x^4 + a^2*b*c*x^3 + 1/2*a^3*x^2*e + a^3*d*x + a^3*c*log(abs(x))

$$3.329 \quad \int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^2} dx$$

Optimal. Leaf size=125

$$\frac{3}{2}a^2bcx^2 + a^2bdx^3 + \frac{3}{4}a^2bex^4 - \frac{a^3c}{x} + a^3d \log(x) + a^3ex + \frac{3}{5}ab^2cx^5 + \frac{1}{2}ab^2dx^6 + \frac{3}{7}ab^2ex^7 + \frac{1}{8}b^3cx^8 + \frac{1}{9}b^3dx^9 + \frac{1}{10}b^3ex^{10}$$

[Out] $-\left(\frac{a^3c}{x}\right) + a^3e*x + \frac{3a^2b*c*x^2}{2} + a^2*b*d*x^3 + \frac{3a^2*b*e*x^4}{4} + \frac{3a*b^2*c*x^5}{5} + \frac{a*b^2*d*x^6}{2} + \frac{3a*b^2*e*x^7}{7} + \frac{b^3*c*x^8}{8} + \frac{b^3*d*x^9}{9} + \frac{b^3*e*x^{10}}{10} + a^3*d*\text{Log}[x]$

Rubi [A] time = 0.0922203, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1628}

$$\frac{3}{2}a^2bcx^2 + a^2bdx^3 + \frac{3}{4}a^2bex^4 - \frac{a^3c}{x} + a^3d \log(x) + a^3ex + \frac{3}{5}ab^2cx^5 + \frac{1}{2}ab^2dx^6 + \frac{3}{7}ab^2ex^7 + \frac{1}{8}b^3cx^8 + \frac{1}{9}b^3dx^9 + \frac{1}{10}b^3ex^{10}$$

Antiderivative was successfully verified.

[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^3)/x^2,x]

[Out] $-\left(\frac{a^3c}{x}\right) + a^3e*x + \frac{3a^2b*c*x^2}{2} + a^2*b*d*x^3 + \frac{3a^2*b*e*x^4}{4} + \frac{3a*b^2*c*x^5}{5} + \frac{a*b^2*d*x^6}{2} + \frac{3a*b^2*e*x^7}{7} + \frac{b^3*c*x^8}{8} + \frac{b^3*d*x^9}{9} + \frac{b^3*e*x^{10}}{10} + a^3*d*\text{Log}[x]$

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^2} dx &= \int \left(a^3e + \frac{a^3c}{x^2} + \frac{a^3d}{x} + 3a^2bcx + 3a^2bdx^2 + 3a^2bex^3 + 3ab^2cx^4 + 3ab^2dx^5 + 3ab^2ex^6 \right. \\ &\quad \left. - \frac{a^3c}{x} + a^3ex + \frac{3}{2}a^2bcx^2 + a^2bdx^3 + \frac{3}{4}a^2bex^4 + \frac{3}{5}ab^2cx^5 + \frac{1}{2}ab^2dx^6 + \frac{3}{7}ab^2ex^7 + \frac{1}{8}b^3cx^8 \right. \\ &\quad \left. + \frac{1}{9}b^3dx^9 + \frac{1}{10}b^3ex^{10} \right) dx \end{aligned}$$

Mathematica [A] time = 0.0083931, size = 125, normalized size = 1.

$$\frac{3}{2}a^2bcx^2 + a^2bdx^3 + \frac{3}{4}a^2bex^4 - \frac{a^3c}{x} + a^3d \log(x) + a^3ex + \frac{3}{5}ab^2cx^5 + \frac{1}{2}ab^2dx^6 + \frac{3}{7}ab^2ex^7 + \frac{1}{8}b^3cx^8 + \frac{1}{9}b^3dx^9 + \frac{1}{10}b^3ex^{10}$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^3)/x^2,x]

[Out] -((a^3*c)/x) + a^3*e*x + (3*a^2*b*c*x^2)/2 + a^2*b*d*x^3 + (3*a^2*b*e*x^4)/4 + (3*a*b^2*c*x^5)/5 + (a*b^2*d*x^6)/2 + (3*a*b^2*e*x^7)/7 + (b^3*c*x^8)/8 + (b^3*d*x^9)/9 + (b^3*e*x^10)/10 + a^3*d*Log[x]

Maple [A] time = 0.006, size = 110, normalized size = 0.9

$$-\frac{a^3c}{x} + a^3ex + \frac{3a^2bcx^2}{2} + a^2bdx^3 + \frac{3a^2bex^4}{4} + \frac{3ab^2cx^5}{5} + \frac{ab^2dx^6}{2} + \frac{3ab^2ex^7}{7} + \frac{b^3cx^8}{8} + \frac{b^3dx^9}{9} + \frac{b^3ex^{10}}{10} + a^3d \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)*(b*x^3+a)^3/x^2,x)

[Out] -a^3*c/x+a^3*e*x+3/2*a^2*b*c*x^2+a^2*b*d*x^3+3/4*a^2*b*e*x^4+3/5*a*b^2*c*x^5+1/2*a*b^2*d*x^6+3/7*a*b^2*e*x^7+1/8*b^3*c*x^8+1/9*b^3*d*x^9+1/10*b^3*e*x^10+a^3*d*ln(x)

Maxima [A] time = 0.940401, size = 147, normalized size = 1.18

$$\frac{1}{10}b^3ex^{10} + \frac{1}{9}b^3dx^9 + \frac{1}{8}b^3cx^8 + \frac{3}{7}ab^2ex^7 + \frac{1}{2}ab^2dx^6 + \frac{3}{5}ab^2cx^5 + \frac{3}{4}a^2bex^4 + a^2bdx^3 + \frac{3}{2}a^2bcx^2 + a^3ex + a^3d \log(x) -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^2,x, algorithm="maxima")

[Out] 1/10*b^3*e*x^10 + 1/9*b^3*d*x^9 + 1/8*b^3*c*x^8 + 3/7*a*b^2*e*x^7 + 1/2*a*b^2*d*x^6 + 3/5*a*b^2*c*x^5 + 3/4*a^2*b*e*x^4 + a^2*b*d*x^3 + 3/2*a^2*b*c*x^2 + a^3*e*x + a^3*d*log(x) - a^3*c/x

Fricas [A] time = 1.45317, size = 305, normalized size = 2.44

$$\frac{252b^3ex^{11} + 280b^3dx^{10} + 315b^3cx^9 + 1080ab^2ex^8 + 1260ab^2dx^7 + 1512ab^2cx^6 + 1890a^2bex^5 + 2520a^2bdx^4 + 3780a^2b^2cx^3 + 2520a^3ex^2 + 2520a^3d^2x + 2520a^3c}{2520x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^2,x, algorithm="fricas")

[Out] 1/2520*(252*b^3*e*x^11 + 280*b^3*d*x^10 + 315*b^3*c*x^9 + 1080*a*b^2*e*x^8 + 1260*a*b^2*d*x^7 + 1512*a*b^2*c*x^6 + 1890*a^2*b*e*x^5 + 2520*a^2*b*d*x^4 + 3780*a^2*b*c*x^3 + 2520*a^3*e*x^2 + 2520*a^3*d*x*log(x) - 2520*a^3*c)/x

Sympy [A] time = 0.39922, size = 128, normalized size = 1.02

$$-\frac{a^3c}{x} + a^3d \log(x) + a^3ex + \frac{3a^2bcx^2}{2} + a^2bdx^3 + \frac{3a^2bex^4}{4} + \frac{3ab^2cx^5}{5} + \frac{ab^2dx^6}{2} + \frac{3ab^2ex^7}{7} + \frac{b^3cx^8}{8} + \frac{b^3dx^9}{9} + \frac{b^3ex^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**3/x**2,x)

[Out] -a**3*c/x + a**3*d*log(x) + a**3*e*x + 3*a**2*b*c*x**2/2 + a**2*b*d*x**3 + 3*a**2*b*e*x**4/4 + 3*a*b**2*c*x**5/5 + a*b**2*d*x**6/2 + 3*a*b**2*e*x**7/7 + b**3*c*x**8/8 + b**3*d*x**9/9 + b**3*e*x**10/10

Giac [A] time = 1.05804, size = 154, normalized size = 1.23

$$\frac{1}{10}b^3x^{10}e + \frac{1}{9}b^3dx^9 + \frac{1}{8}b^3cx^8 + \frac{3}{7}ab^2x^7e + \frac{1}{2}ab^2dx^6 + \frac{3}{5}ab^2cx^5 + \frac{3}{4}a^2bx^4e + a^2bdx^3 + \frac{3}{2}a^2bcx^2 + a^3xe + a^3d \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^2,x, algorithm="giac")

[Out] 1/10*b^3*x^10*e + 1/9*b^3*d*x^9 + 1/8*b^3*c*x^8 + 3/7*a*b^2*x^7*e + 1/2*a*b^2*d*x^6 + 3/5*a*b^2*c*x^5 + 3/4*a^2*b*x^4*e + a^2*b*d*x^3 + 3/2*a^2*b*c*x^2 + a^3*x*e + a^3*d*log(abs(x)) - a^3*c/x

$$3.330 \quad \int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^3} dx$$

Optimal. Leaf size=126

$$3a^2bcx + \frac{3}{2}a^2bdx^2 + a^2bex^3 - \frac{a^3c}{2x^2} - \frac{a^3d}{x} + a^3e \log(x) + \frac{3}{4}ab^2cx^4 + \frac{3}{5}ab^2dx^5 + \frac{1}{2}ab^2ex^6 + \frac{1}{7}b^3cx^7 + \frac{1}{8}b^3dx^8 + \frac{1}{9}b^3ex^9$$

$$\begin{aligned} [\text{Out}] & -(a^3c)/(2*x^2) - (a^3d)/x + 3*a^2*b*c*x + (3*a^2*b*d*x^2)/2 + a^2*b*e*x^3 \\ & + (3*a*b^2*c*x^4)/4 + (3*a*b^2*d*x^5)/5 + (a*b^2*e*x^6)/2 + (b^3*c*x^7)/7 \\ & + (b^3*d*x^8)/8 + (b^3*e*x^9)/9 + a^3*e*\text{Log}[x] \end{aligned}$$

Rubi [A] time = 0.0864589, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1628}

$$3a^2bcx + \frac{3}{2}a^2bdx^2 + a^2bex^3 - \frac{a^3c}{2x^2} - \frac{a^3d}{x} + a^3e \log(x) + \frac{3}{4}ab^2cx^4 + \frac{3}{5}ab^2dx^5 + \frac{1}{2}ab^2ex^6 + \frac{1}{7}b^3cx^7 + \frac{1}{8}b^3dx^8 + \frac{1}{9}b^3ex^9$$

Antiderivative was successfully verified.

$$[\text{In}] \text{Int}[\frac{(c + d*x + e*x^2)*(a + b*x^3)^3}{x^3}, x]$$

$$\begin{aligned} [\text{Out}] & -(a^3c)/(2*x^2) - (a^3d)/x + 3*a^2*b*c*x + (3*a^2*b*d*x^2)/2 + a^2*b*e*x^3 \\ & + (3*a*b^2*c*x^4)/4 + (3*a*b^2*d*x^5)/5 + (a*b^2*e*x^6)/2 + (b^3*c*x^7)/7 \\ & + (b^3*d*x^8)/8 + (b^3*e*x^9)/9 + a^3*e*\text{Log}[x] \end{aligned}$$

Rule 1628

$\text{Int}[(\text{Pq}_.) * ((\text{d}_.) + (\text{e}_.) * (\text{x}_.)^m) * ((\text{a}_.) + (\text{b}_.) * (\text{x}_.) + (\text{c}_.) * (\text{x}_.)^2)^p, \text{x_Symbol}] \rightarrow \text{Int}[\text{ExpandIntegrand}[(\text{d} + \text{e} * \text{x})^m * \text{Pq} * (\text{a} + \text{b} * \text{x} + \text{c} * \text{x}^2)^p, \text{x}], \text{x}] /;$ FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{(c + dx + ex^2)(a + bx^3)^3}{x^3} dx &= \int \left(3a^2bc + \frac{a^3c}{x^3} + \frac{a^3d}{x^2} + \frac{a^3e}{x} + 3a^2bdx + 3a^2bex^2 + 3ab^2cx^3 + 3ab^2dx^4 + 3ab^2ex^5 + b^3cx^6 \right. \\ &\quad \left. - \frac{a^3c}{2x^2} - \frac{a^3d}{x} + 3a^2bcx + \frac{3}{2}a^2bdx^2 + a^2bex^3 + \frac{3}{4}ab^2cx^4 + \frac{3}{5}ab^2dx^5 + \frac{1}{2}ab^2ex^6 + \frac{1}{7}b^3cx^7 \right) dx \end{aligned}$$

Mathematica [A] time = 0.0090364, size = 126, normalized size = 1.

$$3a^2bcx + \frac{3}{2}a^2bdx^2 + a^2bex^3 - \frac{a^3c}{2x^2} - \frac{a^3d}{x} + a^3e \log(x) + \frac{3}{4}ab^2cx^4 + \frac{3}{5}ab^2dx^5 + \frac{1}{2}ab^2ex^6 + \frac{1}{7}b^3cx^7 + \frac{1}{8}b^3dx^8 + \frac{1}{9}b^3ex^9$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^3)/x^3, x]

[Out] $-(a^3c)/(2x^2) - (a^3d)/x + 3a^2b^2cx + (3a^2b^2d*x^2)/2 + a^2b^2e*x^3 + (3a^2b^2c*x^4)/4 + (3a^2b^2d*x^5)/5 + (a^2b^2e*x^6)/2 + (b^3c*x^7)/7 + (b^3d*x^8)/8 + (b^3e*x^9)/9 + a^3e*\text{Log}[x]$

Maple [A] time = 0.006, size = 111, normalized size = 0.9

$$-\frac{a^3c}{2x^2} - \frac{a^3d}{x} + 3a^2bcx + \frac{3a^2bdx^2}{2} + a^2bex^3 + \frac{3ab^2cx^4}{4} + \frac{3ab^2dx^5}{5} + \frac{ab^2ex^6}{2} + \frac{b^3cx^7}{7} + \frac{b^3dx^8}{8} + \frac{b^3ex^9}{9} + a^3e \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)*(b*x^3+a)^3/x^3, x)

[Out] $-1/2*a^3*c/x^2 - a^3*d/x + 3*a^2*b^2*c*x + 3/2*a^2*b^2*d*x^2 + a^2*b^2*e*x^3 + 3/4*a^2*b^2*c*x^4 + 3/5*a^2*b^2*d*x^5 + 1/2*a^2*b^2*e*x^6 + 1/7*b^3*c*x^7 + 1/8*b^3*d*x^8 + 1/9*b^3*e*x^9 + a^3*e*\ln(x)$

Maxima [A] time = 0.935902, size = 149, normalized size = 1.18

$$\frac{1}{9}b^3ex^9 + \frac{1}{8}b^3dx^8 + \frac{1}{7}b^3cx^7 + \frac{1}{2}ab^2ex^6 + \frac{3}{5}ab^2dx^5 + \frac{3}{4}ab^2cx^4 + a^2bex^3 + \frac{3}{2}a^2bdx^2 + 3a^2bcx + a^3e \log(x) - \frac{2a^3dx + a^3c}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^3, x, algorithm="maxima")

[Out] $1/9*b^3*e*x^9 + 1/8*b^3*d*x^8 + 1/7*b^3*c*x^7 + 1/2*a^2*b^2*e*x^6 + 3/5*a^2*b^2*d*x^5 + 3/4*a^2*b^2*c*x^4 + a^2*b^2*e*x^3 + 3/2*a^2*b^2*d*x^2 + 3*a^2*b^2*c*x + a^3*e*\log(x) - 1/2*(2*a^3*d*x + a^3*c)/x^2$

Fricas [A] time = 1.46691, size = 308, normalized size = 2.44

$$\frac{280 b^3 e x^{11} + 315 b^3 d x^{10} + 360 b^3 c x^9 + 1260 a b^2 e x^8 + 1512 a b^2 d x^7 + 1890 a b^2 c x^6 + 2520 a^2 b e x^5 + 3780 a^2 b d x^4 + 7560 a^2 b c x^3 + 2520 a^3 e x^2 \log(x) - 2520 a^3 d x - 1260 a^3 c}{2520 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^3,x, algorithm="fricas")

[Out] 1/2520*(280*b^3*e*x^11 + 315*b^3*d*x^10 + 360*b^3*c*x^9 + 1260*a*b^2*e*x^8 + 1512*a*b^2*d*x^7 + 1890*a*b^2*c*x^6 + 2520*a^2*b*e*x^5 + 3780*a^2*b*d*x^4 + 7560*a^2*b*c*x^3 + 2520*a^3*e*x^2*log(x) - 2520*a^3*d*x - 1260*a^3*c)/x^2

Sympy [A] time = 0.460926, size = 129, normalized size = 1.02

$$a^3 e \log(x) + 3a^2 b c x + \frac{3a^2 b d x^2}{2} + a^2 b e x^3 + \frac{3a b^2 c x^4}{4} + \frac{3a b^2 d x^5}{5} + \frac{a b^2 e x^6}{2} + \frac{b^3 c x^7}{7} + \frac{b^3 d x^8}{8} + \frac{b^3 e x^9}{9} - \frac{a^3 c + 2a^3 d x}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**3/x**3,x)

[Out] a**3*e*log(x) + 3*a**2*b*c*x + 3*a**2*b*d*x**2/2 + a**2*b*e*x**3 + 3*a*b**2*c*x**4/4 + 3*a*b**2*d*x**5/5 + a*b**2*e*x**6/2 + b**3*c*x**7/7 + b**3*d*x**8/8 + b**3*e*x**9/9 - (a**3*c + 2*a**3*d*x)/(2*x**2)

Giac [A] time = 1.0553, size = 155, normalized size = 1.23

$$\frac{1}{9} b^3 x^9 e + \frac{1}{8} b^3 d x^8 + \frac{1}{7} b^3 c x^7 + \frac{1}{2} a b^2 x^6 e + \frac{3}{5} a b^2 d x^5 + \frac{3}{4} a b^2 c x^4 + a^2 b x^3 e + \frac{3}{2} a^2 b d x^2 + 3 a^2 b c x + a^3 e \log(|x|) - \frac{2 a^3 d x + a^3 c}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^3,x, algorithm="giac")

[Out] 1/9*b^3*x^9*e + 1/8*b^3*d*x^8 + 1/7*b^3*c*x^7 + 1/2*a*b^2*x^6*e + 3/5*a*b^2*d*x^5 + 3/4*a*b^2*c*x^4 + a^2*b*x^3*e + 3/2*a^2*b*d*x^2 + 3*a^2*b*c*x + a^3*e*log(abs(x)) - 1/2*(2*a^3*d*x + a^3*c)/x^2

3.331 $\int x^2 (c + dx + ex^2) (a + bx^3)^4 dx$

Optimal. Leaf size=138

$$\frac{3}{5}a^2b^2dx^{10} + \frac{6}{11}a^2b^2ex^{11} + \frac{4}{7}a^3bdx^7 + \frac{1}{2}a^3bex^8 + \frac{1}{4}a^4dx^4 + \frac{1}{5}a^4ex^5 + \frac{4}{13}ab^3dx^{13} + \frac{2}{7}ab^3ex^{14} + \frac{c(a+bx^3)^5}{15b} + \frac{1}{16}b^4dx^{16}$$

[Out] $(a^4d*x^4)/4 + (a^4e*x^5)/5 + (4*a^3*b*d*x^7)/7 + (a^3*b*e*x^8)/2 + (3*a^2*b^2*d*x^{10})/5 + (6*a^2*b^2*e*x^{11})/11 + (4*a*b^3*d*x^{13})/13 + (2*a*b^3*e*x^{14})/7 + (b^4*d*x^{16})/16 + (b^4*e*x^{17})/17 + (c*(a + b*x^3)^5)/(15*b)$

Rubi [A] time = 0.0985253, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1582, 1850}

$$\frac{3}{5}a^2b^2dx^{10} + \frac{6}{11}a^2b^2ex^{11} + \frac{4}{7}a^3bdx^7 + \frac{1}{2}a^3bex^8 + \frac{1}{4}a^4dx^4 + \frac{1}{5}a^4ex^5 + \frac{4}{13}ab^3dx^{13} + \frac{2}{7}ab^3ex^{14} + \frac{c(a+bx^3)^5}{15b} + \frac{1}{16}b^4dx^{16}$$

Antiderivative was successfully verified.

[In] Int[x^2*(c + d*x + e*x^2)*(a + b*x^3)^4,x]

[Out] $(a^4*d*x^4)/4 + (a^4*e*x^5)/5 + (4*a^3*b*d*x^7)/7 + (a^3*b*e*x^8)/2 + (3*a^2*b^2*d*x^{10})/5 + (6*a^2*b^2*e*x^{11})/11 + (4*a*b^3*d*x^{13})/13 + (2*a*b^3*e*x^{14})/7 + (b^4*d*x^{16})/16 + (b^4*e*x^{17})/17 + (c*(a + b*x^3)^5)/(15*b)$

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(Coeff[Px, x, n - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a + b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
```

, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int x^2 (c + dx + ex^2) (a + bx^3)^4 dx &= \frac{c(a + bx^3)^5}{15b} + \int (a + bx^3)^4 (-cx^2 + x^2(c + dx + ex^2)) dx \\ &= \frac{c(a + bx^3)^5}{15b} + \int (a^4 dx^3 + a^4 ex^4 + 4a^3 b dx^6 + 4a^3 b ex^7 + 6a^2 b^2 dx^9 + 6a^2 b^2 ex^{10} + 4a^2 b^2 cx^9 + 4a^2 b^2 dx^{10} + 4a^2 b^2 ex^{11} + 4a^2 b^2 cx^{12} + 4a^2 b^2 dx^{13} + 4a^2 b^2 ex^{14} + 4a^2 b^2 cx^{15} + 4a^2 b^2 dx^{16} + 4a^2 b^2 ex^{17} + 4a^2 b^2 cx^{18}) dx \\ &= \frac{1}{4} a^4 dx^4 + \frac{1}{5} a^4 ex^5 + \frac{4}{7} a^3 b dx^7 + \frac{1}{2} a^3 b ex^8 + \frac{3}{5} a^2 b^2 dx^{10} + \frac{6}{11} a^2 b^2 ex^{11} + \frac{4}{13} ab^3 dx^{13} + \frac{4}{13} ab^3 ex^{14} + \frac{4}{15} a^2 b^2 cx^{15} + \frac{4}{15} a^2 b^2 dx^{16} + \frac{4}{17} a^2 b^2 ex^{17} + \frac{4}{17} a^2 b^2 cx^{18} \end{aligned}$$

Mathematica [A] time = 0.0048061, size = 181, normalized size = 1.31

$$\frac{2}{3} a^2 b^2 cx^9 + \frac{3}{5} a^2 b^2 dx^{10} + \frac{6}{11} a^2 b^2 ex^{11} + \frac{2}{3} a^3 bcx^6 + \frac{4}{7} a^3 b dx^7 + \frac{1}{2} a^3 b ex^8 + \frac{1}{3} a^4 cx^3 + \frac{1}{4} a^4 dx^4 + \frac{1}{5} a^4 ex^5 + \frac{1}{3} ab^3 cx^{12} + \frac{4}{13} ab^3 dx^{13} + \frac{4}{13} ab^3 ex^{14} + \frac{4}{15} a^2 b^2 cx^{15} + \frac{4}{15} a^2 b^2 dx^{16} + \frac{4}{17} a^2 b^2 ex^{17} + \frac{4}{17} a^2 b^2 cx^{18}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3)^4,x]

[Out] (a^4*c*x^3)/3 + (a^4*d*x^4)/4 + (a^4*e*x^5)/5 + (2*a^3*b*c*x^6)/3 + (4*a^3*b*d*x^7)/7 + (a^3*b*e*x^8)/2 + (2*a^2*b^2*c*x^9)/3 + (3*a^2*b^2*d*x^10)/5 + (6*a^2*b^2*e*x^11)/11 + (a*b^3*c*x^12)/3 + (4*a*b^3*d*x^13)/13 + (2*a*b^3*e*x^14)/7 + (b^4*c*x^15)/15 + (b^4*d*x^16)/16 + (b^4*e*x^17)/17

Maple [A] time = 0.001, size = 152, normalized size = 1.1

$$\frac{b^4 ex^{17}}{17} + \frac{b^4 dx^{16}}{16} + \frac{b^4 cx^{15}}{15} + \frac{2 ab^3 ex^{14}}{7} + \frac{4 ab^3 dx^{13}}{13} + \frac{ab^3 cx^{12}}{3} + \frac{6 a^2 b^2 ex^{11}}{11} + \frac{3 a^2 b^2 dx^{10}}{5} + \frac{2 a^2 b^2 cx^9}{3} + \frac{a^3 b ex^8}{2} + \frac{4 a^3 b cx^6}{3} + \frac{a^4 dx^4}{4} + \frac{a^4 ex^5}{5} + \frac{a^4 cx^3}{3} + \frac{ab^3 dx^{13}}{13} + \frac{ab^3 ex^{14}}{13} + \frac{ab^3 cx^{12}}{3} + \frac{a^2 b^2 dx^{10}}{5} + \frac{a^2 b^2 ex^{11}}{11} + \frac{a^2 b^2 cx^9}{3} + \frac{a^3 b dx^7}{7} + \frac{a^3 b ex^8}{2} + \frac{a^4 dx^4}{4} + \frac{a^4 ex^5}{5} + \frac{a^4 cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^4,x)

[Out] 1/17*b^4*e*x^17+1/16*b^4*d*x^16+1/15*b^4*c*x^15+2/7*a*b^3*e*x^14+4/13*a*b^3*d*x^13+1/3*a*b^3*c*x^12+6/11*a^2*b^2*e*x^11+3/5*a^2*b^2*d*x^10+2/3*a^2*b^2*c*x^9+1/2*a^3*b*e*x^8+4/7*a^3*b*d*x^7+2/3*a^3*b*c*x^6+1/5*a^4*e*x^5+1/4*a^4*d*x^4+1/3*a^4*c*x^3

$$4*d*x^4+1/3*a^4*c*x^3$$

Maxima [A] time = 0.949499, size = 204, normalized size = 1.48

$$\frac{1}{17} b^4 e x^{17} + \frac{1}{16} b^4 d x^{16} + \frac{1}{15} b^4 c x^{15} + \frac{2}{7} a b^3 e x^{14} + \frac{4}{13} a b^3 d x^{13} + \frac{1}{3} a b^3 c x^{12} + \frac{6}{11} a^2 b^2 e x^{11} + \frac{3}{5} a^2 b^2 d x^{10} + \frac{2}{3} a^2 b^2 c x^9 + \frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="maxima")

[Out] 1/17*b^4*e*x^17 + 1/16*b^4*d*x^16 + 1/15*b^4*c*x^15 + 2/7*a*b^3*e*x^14 + 4/13*a*b^3*d*x^13 + 1/3*a*b^3*c*x^12 + 6/11*a^2*b^2*e*x^11 + 3/5*a^2*b^2*d*x^10 + 2/3*a^2*b^2*c*x^9 + 1/2*a^3*b*e*x^8 + 4/7*a^3*b*d*x^7 + 2/3*a^3*b*c*x^6 + 1/5*a^4*e*x^5 + 1/4*a^4*d*x^4 + 1/3*a^4*c*x^3

Fricas [A] time = 1.30364, size = 373, normalized size = 2.7

$$\frac{1}{17} x^{17} e b^4 + \frac{1}{16} x^{16} d b^4 + \frac{1}{15} x^{15} c b^4 + \frac{2}{7} x^{14} e b^3 a + \frac{4}{13} x^{13} d b^3 a + \frac{1}{3} x^{12} c b^3 a + \frac{6}{11} x^{11} e b^2 a^2 + \frac{3}{5} x^{10} d b^2 a^2 + \frac{2}{3} x^9 c b^2 a^2 + \frac{1}{2} x^8 e b a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="fricas")

[Out] 1/17*x^17*e*b^4 + 1/16*x^16*d*b^4 + 1/15*x^15*c*b^4 + 2/7*x^14*e*b^3*a + 4/13*x^13*d*b^3*a + 1/3*x^12*c*b^3*a + 6/11*x^11*e*b^2*a^2 + 3/5*x^10*d*b^2*a^2 + 2/3*x^9*c*b^2*a^2 + 1/2*x^8*e*b*a^3 + 4/7*x^7*d*b*a^3 + 2/3*x^6*c*b*a^3 + 1/5*x^5*e*a^4 + 1/4*x^4*d*a^4 + 1/3*x^3*c*a^4

Sympy [A] time = 0.086843, size = 184, normalized size = 1.33

$$\frac{a^4 c x^3}{3} + \frac{a^4 d x^4}{4} + \frac{a^4 e x^5}{5} + \frac{2 a^3 b c x^6}{3} + \frac{4 a^3 b d x^7}{7} + \frac{a^3 b e x^8}{2} + \frac{2 a^2 b^2 c x^9}{3} + \frac{3 a^2 b^2 d x^{10}}{5} + \frac{6 a^2 b^2 e x^{11}}{11} + \frac{a b^3 c x^{12}}{3} + \frac{4 a b^3 d x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**4,x)

[Out] $a^{**4}c*x^{**3}/3 + a^{**4}d*x^{**4}/4 + a^{**4}e*x^{**5}/5 + 2*a^{**3}b*c*x^{**6}/3 + 4*a^{**3}b*d*x^{**7}/7 + a^{**3}b*e*x^{**8}/2 + 2*a^{**2}b^{**2}c*x^{**9}/3 + 3*a^{**2}b^{**2}d*x^{**10}/5 + 6*a^{**2}b^{**2}e*x^{**11}/11 + a*b^{**3}c*x^{**12}/3 + 4*a*b^{**3}d*x^{**13}/13 + 2*a*b^{**3}e*x^{**14}/7 + b^{**4}c*x^{**15}/15 + b^{**4}d*x^{**16}/16 + b^{**4}e*x^{**17}/17$

Giac [A] time = 1.06244, size = 211, normalized size = 1.53

$$\frac{1}{17}b^4x^{17}e + \frac{1}{16}b^4dx^{16} + \frac{1}{15}b^4cx^{15} + \frac{2}{7}ab^3x^{14}e + \frac{4}{13}ab^3dx^{13} + \frac{1}{3}ab^3cx^{12} + \frac{6}{11}a^2b^2x^{11}e + \frac{3}{5}a^2b^2dx^{10} + \frac{2}{3}a^2b^2cx^9 + \frac{1}{2}a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="giac")`

[Out] $1/17*b^4*x^{17}*e + 1/16*b^4*d*x^{16} + 1/15*b^4*c*x^{15} + 2/7*a*b^3*x^{14}*e + 4/13*a*b^3*d*x^{13} + 1/3*a*b^3*c*x^{12} + 6/11*a^2*b^2*x^{11}*e + 3/5*a^2*b^2*d*x^{10} + 2/3*a^2*b^2*c*x^9 + 1/2*a^3*b*x^8*e + 4/7*a^3*b*d*x^7 + 2/3*a^3*b*c*x^6 + 1/5*a^4*x^5*e + 1/4*a^4*d*x^4 + 1/3*a^4*c*x^3$

3.332 $\int x(c + dx + ex^2)(a + bx^3)^4 dx$

Optimal. Leaf size=138

$$\frac{3}{4}a^2b^2cx^8 + \frac{3}{5}a^2b^2ex^{10} + \frac{4}{5}a^3bcx^5 + \frac{4}{7}a^3bex^7 + \frac{1}{2}a^4cx^2 + \frac{1}{4}a^4ex^4 + \frac{4}{11}ab^3cx^{11} + \frac{4}{13}ab^3ex^{13} + \frac{d(a + bx^3)^5}{15b} + \frac{1}{14}b^4cx^{14}$$

[Out] $(a^4cx^2)/2 + (a^4ex^4)/4 + (4a^3bcx^5)/5 + (4a^3bex^7)/7 + (3a^2b^2cx^8)/4 + (3a^2b^2ex^{10})/5 + (4a^2b^3cx^{11})/11 + (4a^2b^3ex^{13})/13 + (b^4cx^{14})/14 + (d(a + bx^3)^5)/(15b)$

Rubi [A] time = 0.0934948, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1582, 1850}

$$\frac{3}{4}a^2b^2cx^8 + \frac{3}{5}a^2b^2ex^{10} + \frac{4}{5}a^3bcx^5 + \frac{4}{7}a^3bex^7 + \frac{1}{2}a^4cx^2 + \frac{1}{4}a^4ex^4 + \frac{4}{11}ab^3cx^{11} + \frac{4}{13}ab^3ex^{13} + \frac{d(a + bx^3)^5}{15b} + \frac{1}{14}b^4cx^{14}$$

Antiderivative was successfully verified.

[In] Int[x*(c + d*x + e*x^2)*(a + b*x^3)^4,x]

[Out] $(a^4cx^2)/2 + (a^4ex^4)/4 + (4a^3bcx^5)/5 + (4a^3bex^7)/7 + (3a^2b^2cx^8)/4 + (3a^2b^2ex^{10})/5 + (4a^2b^3cx^{11})/11 + (4a^2b^3ex^{13})/13 + (b^4cx^{14})/14 + (d(a + bx^3)^5)/(15b)$

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(Coeff[Px, x, n - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a + b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
```

, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int x(c + dx + ex^2)(a + bx^3)^4 dx &= \frac{d(a + bx^3)^5}{15b} + \int (a + bx^3)^4 (-dx^2 + x(c + dx + ex^2)) dx \\ &= \frac{d(a + bx^3)^5}{15b} + \int (a^4cx + a^4ex^3 + 4a^3bcx^4 + 4a^3bex^6 + 6a^2b^2cx^7 + 6a^2b^2ex^9 + 4ab^3cx^{10} + 4ab^3ex^{12}) dx \\ &= \frac{1}{2}a^4cx^2 + \frac{1}{4}a^4ex^4 + \frac{4}{5}a^3bcx^5 + \frac{4}{7}a^3bex^7 + \frac{3}{4}a^2b^2cx^8 + \frac{3}{5}a^2b^2ex^{10} + \frac{4}{11}ab^3cx^{11} + \frac{4}{13}ab^3ex^{13} \end{aligned}$$

Mathematica [A] time = 0.0042877, size = 181, normalized size = 1.31

$$\frac{3}{4}a^2b^2cx^8 + \frac{2}{3}a^2b^2dx^9 + \frac{3}{5}a^2b^2ex^{10} + \frac{4}{5}a^3bcx^5 + \frac{2}{3}a^3bdx^6 + \frac{4}{7}a^3bex^7 + \frac{1}{2}a^4cx^2 + \frac{1}{3}a^4dx^3 + \frac{1}{4}a^4ex^4 + \frac{4}{11}ab^3cx^{11} + \frac{1}{3}ab^3ex^{13}$$

Antiderivative was successfully verified.

[In] Integrate[x*(c + d*x + e*x^2)*(a + b*x^3)^4,x]

[Out] (a^4*c*x^2)/2 + (a^4*d*x^3)/3 + (a^4*e*x^4)/4 + (4*a^3*b*c*x^5)/5 + (2*a^3*b*d*x^6)/3 + (4*a^3*b*e*x^7)/7 + (3*a^2*b^2*c*x^8)/4 + (2*a^2*b^2*d*x^9)/3 + (3*a^2*b^2*e*x^10)/5 + (4*a*b^3*c*x^11)/11 + (a*b^3*d*x^12)/3 + (4*a*b^3*e*x^13)/13 + (b^4*c*x^14)/14 + (b^4*d*x^15)/15 + (b^4*e*x^16)/16

Maple [A] time = 0.002, size = 152, normalized size = 1.1

$$\frac{b^4ex^{16}}{16} + \frac{b^4dx^{15}}{15} + \frac{b^4cx^{14}}{14} + \frac{4ab^3ex^{13}}{13} + \frac{ab^3dx^{12}}{3} + \frac{4ab^3cx^{11}}{11} + \frac{3a^2b^2ex^{10}}{5} + \frac{2a^2b^2dx^9}{3} + \frac{3a^2b^2cx^8}{4} + \frac{4a^3bex^7}{7} + \frac{2a^3b^2cx^6}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(e*x^2+d*x+c)*(b*x^3+a)^4,x)

[Out] 1/16*b^4*e*x^16+1/15*b^4*d*x^15+1/14*b^4*c*x^14+4/13*a*b^3*e*x^13+1/3*a*b^3*d*x^12+4/11*a*b^3*c*x^11+3/5*a^2*b^2*e*x^10+2/3*a^2*b^2*d*x^9+3/4*a^2*b^2*c*x^8+4/7*a^3*b*e*x^7+2/3*a^3*b*d*x^6+4/5*a^3*b*c*x^5+1/4*a^4*e*x^4+1/3*a^4*d*x^3+1/4*a^4*c*x^2

$$*d*x^3+1/2*a^4*c*x^2$$

Maxima [A] time = 0.939755, size = 204, normalized size = 1.48

$$\frac{1}{16} b^4 e x^{16} + \frac{1}{15} b^4 d x^{15} + \frac{1}{14} b^4 c x^{14} + \frac{4}{13} a b^3 e x^{13} + \frac{1}{3} a b^3 d x^{12} + \frac{4}{11} a b^3 c x^{11} + \frac{3}{5} a^2 b^2 e x^{10} + \frac{2}{3} a^2 b^2 d x^9 + \frac{3}{4} a^2 b^2 c x^8 + \frac{4}{7} a^2 b^2 e x^7 + \frac{2}{3} a^2 b^2 d x^6 + \frac{4}{5} a^2 b^2 c x^5 + \frac{1}{4} a^2 b^2 e x^4 + \frac{1}{3} a^2 b^2 d x^3 + \frac{1}{2} a^2 b^2 c x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="maxima")

[Out] 1/16*b^4*e*x^16 + 1/15*b^4*d*x^15 + 1/14*b^4*c*x^14 + 4/13*a*b^3*e*x^13 + 1/3*a*b^3*d*x^12 + 4/11*a*b^3*c*x^11 + 3/5*a^2*b^2*e*x^10 + 2/3*a^2*b^2*d*x^9 + 3/4*a^2*b^2*c*x^8 + 4/7*a^2*b^2*e*x^7 + 2/3*a^2*b^2*d*x^6 + 4/5*a^2*b^2*c*x^5 + 1/4*a^2*b^2*e*x^4 + 1/3*a^2*b^2*d*x^3 + 1/2*a^2*b^2*c*x^2

Fricas [A] time = 1.30116, size = 371, normalized size = 2.69

$$\frac{1}{16} x^{16} e b^4 + \frac{1}{15} x^{15} d b^4 + \frac{1}{14} x^{14} c b^4 + \frac{4}{13} x^{13} e b^3 a + \frac{1}{3} x^{12} d b^3 a + \frac{4}{11} x^{11} c b^3 a + \frac{3}{5} x^{10} e b^2 a^2 + \frac{2}{3} x^9 d b^2 a^2 + \frac{3}{4} x^8 c b^2 a^2 + \frac{4}{7} x^7 e b a^3 + \frac{2}{3} x^6 d b a^3 + \frac{4}{5} x^5 c b a^3 + \frac{1}{4} x^4 e a^4 + \frac{1}{3} x^3 d a^4 + \frac{1}{2} x^2 c a^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="fricas")

[Out] 1/16*x^16*e*b^4 + 1/15*x^15*d*b^4 + 1/14*x^14*c*b^4 + 4/13*x^13*e*b^3*a + 1/3*x^12*d*b^3*a + 4/11*x^11*c*b^3*a + 3/5*x^10*e*b^2*a^2 + 2/3*x^9*d*b^2*a^2 + 3/4*x^8*c*b^2*a^2 + 4/7*x^7*e*b*a^3 + 2/3*x^6*d*b*a^3 + 4/5*x^5*c*b*a^3 + 1/4*x^4*e*a^4 + 1/3*x^3*d*a^4 + 1/2*x^2*c*a^4

Sympy [A] time = 0.087054, size = 185, normalized size = 1.34

$$\frac{a^4 c x^2}{2} + \frac{a^4 d x^3}{3} + \frac{a^4 e x^4}{4} + \frac{4 a^3 b c x^5}{5} + \frac{2 a^3 b d x^6}{3} + \frac{4 a^3 b e x^7}{7} + \frac{3 a^2 b^2 c x^8}{4} + \frac{2 a^2 b^2 d x^9}{3} + \frac{3 a^2 b^2 e x^{10}}{5} + \frac{4 a b^3 c x^{11}}{11} + \frac{a b^3 d x^{12}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**4,x)

[Out] $a^{**4}c*x^{**2}/2 + a^{**4}d*x^{**3}/3 + a^{**4}e*x^{**4}/4 + 4*a^{**3}b*c*x^{**5}/5 + 2*a^{**3}b*d*x^{**6}/3 + 4*a^{**3}b*e*x^{**7}/7 + 3*a^{**2}b^{**2}c*x^{**8}/4 + 2*a^{**2}b^{**2}d*x^{**9}/3 + 3*a^{**2}b^{**2}e*x^{**10}/5 + 4*a*b^{**3}c*x^{**11}/11 + a*b^{**3}d*x^{**12}/3 + 4*a*b^{**3}e*x^{**13}/13 + b^{**4}c*x^{**14}/14 + b^{**4}d*x^{**15}/15 + b^{**4}e*x^{**16}/16$

Giac [A] time = 1.05148, size = 211, normalized size = 1.53

$$\frac{1}{16} b^4 x^{16} e + \frac{1}{15} b^4 d x^{15} + \frac{1}{14} b^4 c x^{14} + \frac{4}{13} a b^3 x^{13} e + \frac{1}{3} a b^3 d x^{12} + \frac{4}{11} a b^3 c x^{11} + \frac{3}{5} a^2 b^2 x^{10} e + \frac{2}{3} a^2 b^2 d x^9 + \frac{3}{4} a^2 b^2 c x^8 + \frac{4}{7} a^3 b c x^7 e + \frac{2}{3} a^3 b c d x^6 + \frac{4}{5} a^3 b c^2 x^5 + \frac{1}{4} a^4 c x^4 e + \frac{1}{3} a^4 d x^3 + \frac{1}{2} a^4 c x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="giac")`

[Out] $1/16*b^4*x^16*e + 1/15*b^4*d*x^15 + 1/14*b^4*c*x^14 + 4/13*a*b^3*x^13*e + 1/3*a*b^3*d*x^12 + 4/11*a*b^3*c*x^11 + 3/5*a^2*b^2*x^10*e + 2/3*a^2*b^2*d*x^9 + 3/4*a^2*b^2*c*x^8 + 4/7*a^3*b*x^7*e + 2/3*a^3*b*d*x^6 + 4/5*a^3*b*c*x^5 + 1/4*a^4*x^4*e + 1/3*a^4*d*x^3 + 1/2*a^4*c*x^2$

3.333 $\int (c + dx + ex^2)(a + bx^3)^4 dx$

Optimal. Leaf size=130

$$\frac{6}{7}a^2b^2cx^7 + \frac{3}{4}a^2b^2dx^8 + a^3bcx^4 + \frac{4}{5}a^3bdx^5 + a^4cx + \frac{1}{2}a^4dx^2 + \frac{2}{5}ab^3cx^{10} + \frac{4}{11}ab^3dx^{11} + \frac{e(a + bx^3)^5}{15b} + \frac{1}{13}b^4cx^{13} + \frac{1}{14}b^4dx^{14}$$

[Out] $a^4c*x + (a^4*d*x^2)/2 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5 + (6*a^2*b^2*c*x^7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a*b^3*c*x^{10})/5 + (4*a*b^3*d*x^{11})/11 + (b^4*c*x^{13})/13 + (b^4*d*x^{14})/14 + (e*(a + b*x^3)^5)/(15*b)$

Rubi [A] time = 0.144909, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {1582, 1850}

$$\frac{6}{7}a^2b^2cx^7 + \frac{3}{4}a^2b^2dx^8 + a^3bcx^4 + \frac{4}{5}a^3bdx^5 + a^4cx + \frac{1}{2}a^4dx^2 + \frac{2}{5}ab^3cx^{10} + \frac{4}{11}ab^3dx^{11} + \frac{e(a + bx^3)^5}{15b} + \frac{1}{13}b^4cx^{13} + \frac{1}{14}b^4dx^{14}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)*(a + b*x^3)^4,x]

[Out] $a^4c*x + (a^4*d*x^2)/2 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5 + (6*a^2*b^2*c*x^7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a*b^3*c*x^{10})/5 + (4*a*b^3*d*x^{11})/11 + (b^4*c*x^{13})/13 + (b^4*d*x^{14})/14 + (e*(a + b*x^3)^5)/(15*b)$

Rule 1582

Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(Coeff[Px, x, n - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a + b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]

Rule 1850

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p

, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int (c + dx + ex^2)(a + bx^3)^4 dx &= \frac{e(a + bx^3)^5}{15b} + \int (c + dx)(a + bx^3)^4 dx \\ &= \frac{e(a + bx^3)^5}{15b} + \int (a^4c + a^4dx + 4a^3bcx^3 + 4a^3bdx^4 + 6a^2b^2cx^6 + 6a^2b^2dx^7 + 4ab^3cx^9 + 4ab^3dx^{10}) dx \\ &= a^4cx + \frac{1}{2}a^4dx^2 + a^3bcx^4 + \frac{4}{5}a^3bdx^5 + \frac{6}{7}a^2b^2cx^7 + \frac{3}{4}a^2b^2dx^8 + \frac{2}{5}ab^3cx^{10} + \frac{4}{11}ab^3dx^{11} \end{aligned}$$

Mathematica [A] time = 0.0038972, size = 173, normalized size = 1.33

$$\frac{6}{7}a^2b^2cx^7 + \frac{3}{4}a^2b^2dx^8 + \frac{2}{3}a^2b^2ex^9 + a^3bcx^4 + \frac{4}{5}a^3bdx^5 + \frac{2}{3}a^3bex^6 + a^4cx + \frac{1}{2}a^4dx^2 + \frac{1}{3}a^4ex^3 + \frac{2}{5}ab^3cx^{10} + \frac{4}{11}ab^3dx^{11} + \dots$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)*(a + b*x^3)^4,x]

[Out] a^4*c*x + (a^4*d*x^2)/2 + (a^4*e*x^3)/3 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5 + (2*a^3*b*e*x^6)/3 + (6*a^2*b^2*c*x^7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a^2*b^2*e*x^9)/3 + (2*a*b^3*c*x^10)/5 + (4*a*b^3*d*x^11)/11 + (a*b^3*e*x^12)/3 + (b^4*c*x^13)/13 + (b^4*d*x^14)/14 + (b^4*e*x^15)/15

Maple [A] time = 0.001, size = 148, normalized size = 1.1

$$\frac{b^4ex^{15}}{15} + \frac{b^4dx^{14}}{14} + \frac{b^4cx^{13}}{13} + \frac{ab^3ex^{12}}{3} + \frac{4ab^3dx^{11}}{11} + \frac{2ab^3cx^{10}}{5} + \frac{2a^2b^2ex^9}{3} + \frac{3a^2b^2dx^8}{4} + \frac{6a^2b^2cx^7}{7} + \frac{2a^3bex^6}{3} + \frac{4a^3ba^4cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)*(b*x^3+a)^4,x)

[Out] 1/15*b^4*e*x^15+1/14*b^4*d*x^14+1/13*b^4*c*x^13+1/3*a*b^3*e*x^12+4/11*a*b^3*d*x^11+2/5*a*b^3*c*x^10+2/3*a^2*b^2*e*x^9+3/4*a^2*b^2*d*x^8+6/7*a^2*b^2*c*x^7+2/3*a^3*b*e*x^6+4/5*a^3*b*d*x^5+a^3*b*c*x^4+1/3*a^4*e*x^3+1/2*a^4*d*x^2

$+a^4*c*x$

Maxima [A] time = 0.935401, size = 198, normalized size = 1.52

$$\frac{1}{15} b^4 e x^{15} + \frac{1}{14} b^4 d x^{14} + \frac{1}{13} b^4 c x^{13} + \frac{1}{3} a b^3 e x^{12} + \frac{4}{11} a b^3 d x^{11} + \frac{2}{5} a b^3 c x^{10} + \frac{2}{3} a^2 b^2 e x^9 + \frac{3}{4} a^2 b^2 d x^8 + \frac{6}{7} a^2 b^2 c x^7 + \frac{2}{3} a^3 b e x^6 + \frac{4}{5} a^3 b d x^5 + a^3 b c x^4 + \frac{1}{3} a^4 e x^3 + \frac{1}{2} a^4 d x^2 + a^4 c x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="maxima")

[Out] 1/15*b^4*e*x^15 + 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 1/3*a*b^3*e*x^12 + 4/11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 + 2/3*a^2*b^2*e*x^9 + 3/4*a^2*b^2*d*x^8 + 6/7*a^2*b^2*c*x^7 + 2/3*a^3*b*e*x^6 + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 + 1/3*a^4*e*x^3 + 1/2*a^4*d*x^2 + a^4*c*x

Fricas [A] time = 1.28289, size = 355, normalized size = 2.73

$$\frac{1}{15} x^{15} e b^4 + \frac{1}{14} x^{14} d b^4 + \frac{1}{13} x^{13} c b^4 + \frac{1}{3} x^{12} e b^3 a + \frac{4}{11} x^{11} d b^3 a + \frac{2}{5} x^{10} c b^3 a + \frac{2}{3} x^9 e b^2 a^2 + \frac{3}{4} x^8 d b^2 a^2 + \frac{6}{7} x^7 c b^2 a^2 + \frac{2}{3} x^6 e b a^3 + \frac{4}{5} x^5 d b a^3 + x^4 c b a^3 + \frac{1}{3} x^3 e a^4 + \frac{1}{2} x^2 d a^4 + x c a^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="fricas")

[Out] 1/15*x^15*e*b^4 + 1/14*x^14*d*b^4 + 1/13*x^13*c*b^4 + 1/3*x^12*e*b^3*a + 4/11*x^11*d*b^3*a + 2/5*x^10*c*b^3*a + 2/3*x^9*e*b^2*a^2 + 3/4*x^8*d*b^2*a^2 + 6/7*x^7*c*b^2*a^2 + 2/3*x^6*e*b*a^3 + 4/5*x^5*d*b*a^3 + x^4*c*b*a^3 + 1/3*x^3*e*a^4 + 1/2*x^2*d*a^4 + x*c*a^4

Sympy [A] time = 0.083989, size = 178, normalized size = 1.37

$$a^4 c x + \frac{a^4 d x^2}{2} + \frac{a^4 e x^3}{3} + a^3 b c x^4 + \frac{4 a^3 b d x^5}{5} + \frac{2 a^3 b e x^6}{3} + \frac{6 a^2 b^2 c x^7}{7} + \frac{3 a^2 b^2 d x^8}{4} + \frac{2 a^2 b^2 e x^9}{3} + \frac{2 a b^3 c x^{10}}{5} + \frac{4 a b^3 d x^{11}}{11} + \frac{2 a^3 b e x^6}{3} + \frac{4 a^3 b d x^5}{5} + a^3 b c x^4 + \frac{1}{3} a^4 e x^3 + \frac{1}{2} a^4 d x^2 + a^4 c x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**4,x)

```
[Out] a**4*c*x + a**4*d*x**2/2 + a**4*e*x**3/3 + a**3*b*c*x**4 + 4*a**3*b*d*x**5/
5 + 2*a**3*b*e*x**6/3 + 6*a**2*b**2*c*x**7/7 + 3*a**2*b**2*d*x**8/4 + 2*a**
2*b**2*e*x**9/3 + 2*a*b**3*c*x**10/5 + 4*a*b**3*d*x**11/11 + a*b**3*e*x**12
/3 + b**4*c*x**13/13 + b**4*d*x**14/14 + b**4*e*x**15/15
```

Giac [A] time = 1.06435, size = 205, normalized size = 1.58

$$\frac{1}{15} b^4 x^{15} e + \frac{1}{14} b^4 dx^{14} + \frac{1}{13} b^4 cx^{13} + \frac{1}{3} ab^3 x^{12} e + \frac{4}{11} ab^3 dx^{11} + \frac{2}{5} ab^3 cx^{10} + \frac{2}{3} a^2 b^2 x^9 e + \frac{3}{4} a^2 b^2 dx^8 + \frac{6}{7} a^2 b^2 cx^7 + \frac{2}{3} a^3 bx^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] 1/15*b^4*x^15*e + 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 1/3*a*b^3*x^12*e + 4/
11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 + 2/3*a^2*b^2*x^9*e + 3/4*a^2*b^2*d*x^8
+ 6/7*a^2*b^2*c*x^7 + 2/3*a^3*b*x^6*e + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 + 1/3
*a^4*x^3*e + 1/2*a^4*d*x^2 + a^4*c*x
```

$$3.334 \quad \int \frac{(c+dx+ex^2)(a+bx^3)^4}{x} dx$$

Optimal. Leaf size=166

$$a^2b^2cx^6 + \frac{6}{7}a^2b^2dx^7 + \frac{3}{4}a^2b^2ex^8 + \frac{4}{3}a^3bcx^3 + a^3bdx^4 + \frac{4}{5}a^3bex^5 + a^4c \log(x) + a^4dx + \frac{1}{2}a^4ex^2 + \frac{4}{9}ab^3cx^9 + \frac{2}{5}ab^3dx^{10} + \dots$$

[Out] $a^4d*x + (a^4*e*x^2)/2 + (4*a^3*b*c*x^3)/3 + a^3*b*d*x^4 + (4*a^3*b*e*x^5)/5 + a^2*b^2*c*x^6 + (6*a^2*b^2*d*x^7)/7 + (3*a^2*b^2*e*x^8)/4 + (4*a*b^3*c*x^9)/9 + (2*a*b^3*d*x^{10})/5 + (4*a*b^3*e*x^{11})/11 + (b^4*c*x^{12})/12 + (b^4*d*x^{13})/13 + (b^4*e*x^{14})/14 + a^4*c*Log[x]$

Rubi [A] time = 0.108534, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1628}

$$a^2b^2cx^6 + \frac{6}{7}a^2b^2dx^7 + \frac{3}{4}a^2b^2ex^8 + \frac{4}{3}a^3bcx^3 + a^3bdx^4 + \frac{4}{5}a^3bex^5 + a^4c \log(x) + a^4dx + \frac{1}{2}a^4ex^2 + \frac{4}{9}ab^3cx^9 + \frac{2}{5}ab^3dx^{10} + \dots$$

Antiderivative was successfully verified.

[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^4)/x,x]

[Out] $a^4d*x + (a^4*e*x^2)/2 + (4*a^3*b*c*x^3)/3 + a^3*b*d*x^4 + (4*a^3*b*e*x^5)/5 + a^2*b^2*c*x^6 + (6*a^2*b^2*d*x^7)/7 + (3*a^2*b^2*e*x^8)/4 + (4*a*b^3*c*x^9)/9 + (2*a*b^3*d*x^{10})/5 + (4*a*b^3*e*x^{11})/11 + (b^4*c*x^{12})/12 + (b^4*d*x^{13})/13 + (b^4*e*x^{14})/14 + a^4*c*Log[x]$

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x} dx = \int \left(a^4d + \frac{a^4c}{x} + a^4ex + 4a^3bcx^2 + 4a^3bdx^3 + 4a^3bex^4 + 6a^2b^2cx^5 + 6a^2b^2dx^6 + 6a^2b^2ex^7 + \dots \right) dx$$

$$= a^4dx + \frac{1}{2}a^4ex^2 + \frac{4}{3}a^3bcx^3 + a^3bdx^4 + \frac{4}{5}a^3bex^5 + a^2b^2cx^6 + \frac{6}{7}a^2b^2dx^7 + \frac{3}{4}a^2b^2ex^8 + \dots$$

Mathematica [A] time = 0.0088974, size = 166, normalized size = 1.

$$a^2b^2cx^6 + \frac{6}{7}a^2b^2dx^7 + \frac{3}{4}a^2b^2ex^8 + \frac{4}{3}a^3bcx^3 + a^3bdx^4 + \frac{4}{5}a^3bex^5 + a^4c \log(x) + a^4dx + \frac{1}{2}a^4ex^2 + \frac{4}{9}ab^3cx^9 + \frac{2}{5}ab^3dx^{10} +$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^4)/x,x]

[Out] a^4*d*x + (a^4*e*x^2)/2 + (4*a^3*b*c*x^3)/3 + a^3*b*d*x^4 + (4*a^3*b*e*x^5)/5 + a^2*b^2*c*x^6 + (6*a^2*b^2*d*x^7)/7 + (3*a^2*b^2*e*x^8)/4 + (4*a*b^3*c*x^9)/9 + (2*a*b^3*d*x^10)/5 + (4*a*b^3*e*x^11)/11 + (b^4*c*x^12)/12 + (b^4*d*x^13)/13 + (b^4*e*x^14)/14 + a^4*c*Log[x]

Maple [A] time = 0.003, size = 145, normalized size = 0.9

$$a^4dx + \frac{a^4ex^2}{2} + \frac{4a^3bcx^3}{3} + a^3bdx^4 + \frac{4a^3bex^5}{5} + a^2b^2cx^6 + \frac{6a^2b^2dx^7}{7} + \frac{3a^2b^2ex^8}{4} + \frac{4ab^3cx^9}{9} + \frac{2ab^3dx^{10}}{5} + \frac{4ab^3ex^{11}}{11} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)*(b*x^3+a)^4/x,x)

[Out] a^4*d*x+1/2*a^4*e*x^2+4/3*a^3*b*c*x^3+a^3*b*d*x^4+4/5*a^3*b*e*x^5+a^2*b^2*c*x^6+6/7*a^2*b^2*d*x^7+3/4*a^2*b^2*e*x^8+4/9*a*b^3*c*x^9+2/5*a*b^3*d*x^10+4/11*a*b^3*e*x^11+1/12*b^4*c*x^12+1/13*b^4*d*x^13+1/14*b^4*e*x^14+a^4*c*ln(x)

Maxima [A] time = 0.936197, size = 194, normalized size = 1.17

$$\frac{1}{14}b^4ex^{14} + \frac{1}{13}b^4dx^{13} + \frac{1}{12}b^4cx^{12} + \frac{4}{11}ab^3ex^{11} + \frac{2}{5}ab^3dx^{10} + \frac{4}{9}ab^3cx^9 + \frac{3}{4}a^2b^2ex^8 + \frac{6}{7}a^2b^2dx^7 + a^2b^2cx^6 + \frac{4}{5}a^3bex^5 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x,x, algorithm="maxima")

[Out] 1/14*b^4*e*x^14 + 1/13*b^4*d*x^13 + 1/12*b^4*c*x^12 + 4/11*a*b^3*e*x^11 + 2/5*a*b^3*d*x^10 + 4/9*a*b^3*c*x^9 + 3/4*a^2*b^2*e*x^8 + 6/7*a^2*b^2*d*x^7 + a^2*b^2*c*x^6 + 4/5*a^3*b*e*x^5 + a^3*b*d*x^4 + 4/3*a^3*b*c*x^3 + 1/2*a^4*

$$e*x^2 + a^4*d*x + a^4*c*log(x)$$

Fricas [A] time = 1.46817, size = 347, normalized size = 2.09

$$\frac{1}{14} b^4 e x^{14} + \frac{1}{13} b^4 d x^{13} + \frac{1}{12} b^4 c x^{12} + \frac{4}{11} a b^3 e x^{11} + \frac{2}{5} a b^3 d x^{10} + \frac{4}{9} a b^3 c x^9 + \frac{3}{4} a^2 b^2 e x^8 + \frac{6}{7} a^2 b^2 d x^7 + a^2 b^2 c x^6 + \frac{4}{5} a^3 b e x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x,x, algorithm="fricas")

[Out] 1/14*b^4*e*x^14 + 1/13*b^4*d*x^13 + 1/12*b^4*c*x^12 + 4/11*a*b^3*e*x^11 + 2/5*a*b^3*d*x^10 + 4/9*a*b^3*c*x^9 + 3/4*a^2*b^2*e*x^8 + 6/7*a^2*b^2*d*x^7 + a^2*b^2*c*x^6 + 4/5*a^3*b*e*x^5 + a^3*b*d*x^4 + 4/3*a^3*b*c*x^3 + 1/2*a^4*e*x^2 + a^4*d*x + a^4*c*log(x)

Sympy [A] time = 0.456358, size = 175, normalized size = 1.05

$$a^4 c \log(x) + a^4 d x + \frac{a^4 e x^2}{2} + \frac{4 a^3 b c x^3}{3} + a^3 b d x^4 + \frac{4 a^3 b e x^5}{5} + a^2 b^2 c x^6 + \frac{6 a^2 b^2 d x^7}{7} + \frac{3 a^2 b^2 e x^8}{4} + \frac{4 a b^3 c x^9}{9} + \frac{2 a b^3 d x^{10}}{5} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**4/x,x)

[Out] a**4*c*log(x) + a**4*d*x + a**4*e*x**2/2 + 4*a**3*b*c*x**3/3 + a**3*b*d*x**4 + 4*a**3*b*e*x**5/5 + a**2*b**2*c*x**6 + 6*a**2*b**2*d*x**7/7 + 3*a**2*b**2*e*x**8/4 + 4*a*b**3*c*x**9/9 + 2*a*b**3*d*x**10/5 + 4*a*b**3*e*x**11/11 + b**4*c*x**12/12 + b**4*d*x**13/13 + b**4*e*x**14/14

Giac [A] time = 1.06076, size = 203, normalized size = 1.22

$$\frac{1}{14} b^4 x^{14} e + \frac{1}{13} b^4 d x^{13} + \frac{1}{12} b^4 c x^{12} + \frac{4}{11} a b^3 x^{11} e + \frac{2}{5} a b^3 d x^{10} + \frac{4}{9} a b^3 c x^9 + \frac{3}{4} a^2 b^2 x^8 e + \frac{6}{7} a^2 b^2 d x^7 + a^2 b^2 c x^6 + \frac{4}{5} a^3 b x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x,x, algorithm="giac")

```
[Out] 1/14*b^4*x^14*e + 1/13*b^4*d*x^13 + 1/12*b^4*c*x^12 + 4/11*a*b^3*x^11*e + 2/5*a*b^3*d*x^10 + 4/9*a*b^3*c*x^9 + 3/4*a^2*b^2*x^8*e + 6/7*a^2*b^2*d*x^7 + a^2*b^2*c*x^6 + 4/5*a^3*b*x^5*e + a^3*b*d*x^4 + 4/3*a^3*b*c*x^3 + 1/2*a^4*x^2*e + a^4*d*x + a^4*c*log(abs(x))
```


$$3.335 \quad \int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^2} dx$$

Optimal. Leaf size=162

$$\frac{6}{5}a^2b^2cx^5 + a^2b^2dx^6 + \frac{6}{7}a^2b^2ex^7 + 2a^3bcx^2 + \frac{4}{3}a^3bdx^3 + a^3bex^4 - \frac{a^4c}{x} + a^4d \log(x) + a^4ex + \frac{1}{2}ab^3cx^8 + \frac{4}{9}ab^3dx^9 + \frac{2}{5}ab$$

[Out] $-\left(\frac{a^4c}{x}\right) + a^4e*x + 2*a^3*b*c*x^2 + (4*a^3*b*d*x^3)/3 + a^3*b*e*x^4 + (6*a^2*b^2*c*x^5)/5 + a^2*b^2*d*x^6 + (6*a^2*b^2*e*x^7)/7 + (a*b^3*c*x^8)/2 + (4*a*b^3*d*x^9)/9 + (2*a*b^3*e*x^10)/5 + (b^4*c*x^11)/11 + (b^4*d*x^12)/12 + (b^4*e*x^13)/13 + a^4*d*\text{Log}[x]$

Rubi [A] time = 0.13332, antiderivative size = 162, normalized size of antiderivative = 1, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1628}

$$\frac{6}{5}a^2b^2cx^5 + a^2b^2dx^6 + \frac{6}{7}a^2b^2ex^7 + 2a^3bcx^2 + \frac{4}{3}a^3bdx^3 + a^3bex^4 - \frac{a^4c}{x} + a^4d \log(x) + a^4ex + \frac{1}{2}ab^3cx^8 + \frac{4}{9}ab^3dx^9 + \frac{2}{5}ab$$

Antiderivative was successfully verified.

[In] $\text{Int}[\left((c + d*x + e*x^2)*(a + b*x^3)^4\right)/x^2, x]$

[Out] $-\left(\frac{a^4c}{x}\right) + a^4e*x + 2*a^3*b*c*x^2 + (4*a^3*b*d*x^3)/3 + a^3*b*e*x^4 + (6*a^2*b^2*c*x^5)/5 + a^2*b^2*d*x^6 + (6*a^2*b^2*e*x^7)/7 + (a*b^3*c*x^8)/2 + (4*a*b^3*d*x^9)/9 + (2*a*b^3*e*x^10)/5 + (b^4*c*x^11)/11 + (b^4*d*x^12)/12 + (b^4*e*x^13)/13 + a^4*d*\text{Log}[x]$

Rule 1628

$\text{Int}[(Pq_)*((d_.) + (e_.)*(x_))^m*(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2]^p, x_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, m\}, x \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{IGtQ}[p, -2]$

Rubi steps

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^2} dx = \int \left(a^4e + \frac{a^4c}{x^2} + \frac{a^4d}{x} + 4a^3bcx + 4a^3bdx^2 + 4a^3bex^3 + 6a^2b^2cx^4 + 6a^2b^2dx^5 + 6a^2b^2ex^6 \right. \\ \left. - \frac{a^4c}{x} + a^4ex + 2a^3bcx^2 + \frac{4}{3}a^3bdx^3 + a^3bex^4 + \frac{6}{5}a^2b^2cx^5 + a^2b^2dx^6 + \frac{6}{7}a^2b^2ex^7 + \frac{1}{2}a^2b^2ex^8 \right) dx$$

Mathematica [A] time = 0.0087761, size = 162, normalized size = 1.

$$\frac{6}{5}a^2b^2cx^5 + a^2b^2dx^6 + \frac{6}{7}a^2b^2ex^7 + 2a^3bcx^2 + \frac{4}{3}a^3bdx^3 + a^3bex^4 - \frac{a^4c}{x} + a^4d \log(x) + a^4ex + \frac{1}{2}ab^3cx^8 + \frac{4}{9}ab^3dx^9 + \frac{2}{5}ab^3$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^4)/x^2,x]

[Out] -((a^4*c)/x) + a^4*e*x + 2*a^3*b*c*x^2 + (4*a^3*b*d*x^3)/3 + a^3*b*e*x^4 + (6*a^2*b^2*c*x^5)/5 + a^2*b^2*d*x^6 + (6*a^2*b^2*e*x^7)/7 + (a*b^3*c*x^8)/2 + (4*a*b^3*d*x^9)/9 + (2*a*b^3*e*x^10)/5 + (b^4*c*x^11)/11 + (b^4*d*x^12)/12 + (b^4*e*x^13)/13 + a^4*d*Log[x]

Maple [A] time = 0.006, size = 145, normalized size = 0.9

$$-\frac{a^4c}{x} + a^4ex + 2a^3bcx^2 + \frac{4a^3bdx^3}{3} + a^3bex^4 + \frac{6a^2b^2cx^5}{5} + a^2b^2dx^6 + \frac{6a^2b^2ex^7}{7} + \frac{ab^3cx^8}{2} + \frac{4ab^3dx^9}{9} + \frac{2ab^3ex^{10}}{5} + \frac{b^4c}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)*(b*x^3+a)^4/x^2,x)

[Out] -a^4*c/x+a^4*e*x+2*a^3*b*c*x^2+4/3*a^3*b*d*x^3+a^3*b*e*x^4+6/5*a^2*b^2*c*x^5+a^2*b^2*d*x^6+6/7*a^2*b^2*e*x^7+1/2*a*b^3*c*x^8+4/9*a*b^3*d*x^9+2/5*a*b^3*e*x^10+1/11*b^4*c*x^11+1/12*b^4*d*x^12+1/13*b^4*e*x^13+a^4*d*ln(x)

Maxima [A] time = 0.941723, size = 194, normalized size = 1.2

$$\frac{1}{13}b^4ex^{13} + \frac{1}{12}b^4dx^{12} + \frac{1}{11}b^4cx^{11} + \frac{2}{5}ab^3ex^{10} + \frac{4}{9}ab^3dx^9 + \frac{1}{2}ab^3cx^8 + \frac{6}{7}a^2b^2ex^7 + a^2b^2dx^6 + \frac{6}{5}a^2b^2cx^5 + a^3bex^4 + \frac{4}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^2,x, algorithm="maxima")

[Out] 1/13*b^4*e*x^13 + 1/12*b^4*d*x^12 + 1/11*b^4*c*x^11 + 2/5*a*b^3*e*x^10 + 4/9*a*b^3*d*x^9 + 1/2*a*b^3*c*x^8 + 6/7*a^2*b^2*e*x^7 + a^2*b^2*d*x^6 + 6/5*a^2*b^2*c*x^5 + a^3*b*e*x^4 + 4/3*a^3*b*d*x^3 + 2*a^3*b*c*x^2 + a^4*e*x + a^4

$$4*d*\log(x) - a^4*c/x$$

Fricas [A] time = 1.4897, size = 433, normalized size = 2.67

$$13860 b^4 e x^{14} + 15015 b^4 d x^{13} + 16380 b^4 c x^{12} + 72072 a b^3 e x^{11} + 80080 a b^3 d x^{10} + 90090 a b^3 c x^9 + 154440 a^2 b^2 e x^8 + 180180 a^2 b^2 d x^7 + 216216 a^2 b^2 c x^6 + 180180 a^3 b e x^5 + 240240 a^3 b d x^4 + 360360 a^3 b c x^3 + 180180 a^4 e x^2 + 180180 a^4 d x \log(x) - 180180 a^4 c / x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^2,x, algorithm="fricas")

[Out] 1/180180*(13860*b^4*e*x^14 + 15015*b^4*d*x^13 + 16380*b^4*c*x^12 + 72072*a*b^3*e*x^11 + 80080*a*b^3*d*x^10 + 90090*a*b^3*c*x^9 + 154440*a^2*b^2*e*x^8 + 180180*a^2*b^2*d*x^7 + 216216*a^2*b^2*c*x^6 + 180180*a^3*b*e*x^5 + 240240*a^3*b*d*x^4 + 360360*a^3*b*c*x^3 + 180180*a^4*e*x^2 + 180180*a^4*d*x*log(x) - 180180*a^4*c)/x

Sympy [A] time = 0.47288, size = 168, normalized size = 1.04

$$-\frac{a^4 c}{x} + a^4 d \log(x) + a^4 e x + 2a^3 b c x^2 + \frac{4a^3 b d x^3}{3} + a^3 b e x^4 + \frac{6a^2 b^2 c x^5}{5} + a^2 b^2 d x^6 + \frac{6a^2 b^2 e x^7}{7} + \frac{a b^3 c x^8}{2} + \frac{4a b^3 d x^9}{9} + \frac{2a b^3 e x^{10}}{11} + \frac{b^4 c x^{11}}{12} + \frac{b^4 d x^{12}}{13} + \frac{b^4 e x^{13}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**4/x**2,x)

[Out] -a**4*c/x + a**4*d*log(x) + a**4*e*x + 2*a**3*b*c*x**2 + 4*a**3*b*d*x**3/3 + a**3*b*e*x**4 + 6*a**2*b**2*c*x**5/5 + a**2*b**2*d*x**6 + 6*a**2*b**2*e*x**7/7 + a*b**3*c*x**8/2 + 4*a*b**3*d*x**9/9 + 2*a*b**3*e*x**10/5 + b**4*c*x**11/11 + b**4*d*x**12/12 + b**4*e*x**13/13

Giac [A] time = 1.0459, size = 203, normalized size = 1.25

$$\frac{1}{13} b^4 x^{13} e + \frac{1}{12} b^4 d x^{12} + \frac{1}{11} b^4 c x^{11} + \frac{2}{5} a b^3 x^{10} e + \frac{4}{9} a b^3 d x^9 + \frac{1}{2} a b^3 c x^8 + \frac{6}{7} a^2 b^2 x^7 e + a^2 b^2 d x^6 + \frac{6}{5} a^2 b^2 c x^5 + a^3 b x^4 e + \frac{4}{3} a^3 b d x^3 + a^3 b c x^2 + a^4 e x + a^4 d \log(x) - \frac{a^4 c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^2,x, algorithm="giac")
```

```
[Out] 1/13*b^4*x^13*e + 1/12*b^4*d*x^12 + 1/11*b^4*c*x^11 + 2/5*a*b^3*x^10*e + 4/9*a*b^3*d*x^9 + 1/2*a*b^3*c*x^8 + 6/7*a^2*b^2*x^7*e + a^2*b^2*d*x^6 + 6/5*a^2*b^2*c*x^5 + a^3*b*x^4*e + 4/3*a^3*b*d*x^3 + 2*a^3*b*c*x^2 + a^4*x*e + a^4*d*log(abs(x)) - a^4*c/x
```

$$3.336 \quad \int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^3} dx$$

Optimal. Leaf size=166

$$\frac{3}{2}a^2b^2cx^4 + \frac{6}{5}a^2b^2dx^5 + a^2b^2ex^6 + 4a^3bcx + 2a^3bdx^2 + \frac{4}{3}a^3bex^3 - \frac{a^4c}{2x^2} - \frac{a^4d}{x} + a^4e \log(x) + \frac{4}{7}ab^3cx^7 + \frac{1}{2}ab^3dx^8 + \frac{4}{9}ab^3ex^9$$

[Out] $-(a^4c)/(2x^2) - (a^4d)/x + 4a^3b^2cx + 2a^3b^2dx^2 + (4a^3b^2ex^3)/3 + (3a^2b^2c^2x^4)/2 + (6a^2b^2d^2x^5)/5 + a^2b^2e^2x^6 + (4a^2b^3cx^7)/7 + (a^2b^3d^2x^8)/2 + (4a^2b^3ex^9)/9 + (b^4c^2x^{10})/10 + (b^4d^2x^{11})/11 + (b^4e^2x^{12})/12 + a^4e \text{Log}[x]$

Rubi [A] time = 0.124775, antiderivative size = 166, normalized size of antiderivative = 1, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1628}

$$\frac{3}{2}a^2b^2cx^4 + \frac{6}{5}a^2b^2dx^5 + a^2b^2ex^6 + 4a^3bcx + 2a^3bdx^2 + \frac{4}{3}a^3bex^3 - \frac{a^4c}{2x^2} - \frac{a^4d}{x} + a^4e \log(x) + \frac{4}{7}ab^3cx^7 + \frac{1}{2}ab^3dx^8 + \frac{4}{9}ab^3ex^9$$

Antiderivative was successfully verified.

[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^4)/x^3, x]

[Out] $-(a^4c)/(2x^2) - (a^4d)/x + 4a^3b^2cx + 2a^3b^2dx^2 + (4a^3b^2ex^3)/3 + (3a^2b^2c^2x^4)/2 + (6a^2b^2d^2x^5)/5 + a^2b^2e^2x^6 + (4a^2b^3cx^7)/7 + (a^2b^3d^2x^8)/2 + (4a^2b^3ex^9)/9 + (b^4c^2x^{10})/10 + (b^4d^2x^{11})/11 + (b^4e^2x^{12})/12 + a^4e \text{Log}[x]$

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^3} dx = \int \left(4a^3bc + \frac{a^4c}{x^3} + \frac{a^4d}{x^2} + \frac{a^4e}{x} + 4a^3bdx + 4a^3bex^2 + 6a^2b^2cx^3 + 6a^2b^2dx^4 + 6a^2b^2ex^5 - \frac{a^4c}{2x^2} - \frac{a^4d}{x} + 4a^3bcx + 2a^3bdx^2 + \frac{4}{3}a^3bex^3 + \frac{3}{2}a^2b^2cx^4 + \frac{6}{5}a^2b^2dx^5 + a^2b^2ex^6 + \frac{4}{9}ab^3cx^7 + \frac{1}{2}ab^3dx^8 + \frac{4}{9}ab^3ex^9 \right) dx$$

Mathematica [A] time = 0.0089017, size = 166, normalized size = 1.

$$\frac{3}{2}a^2b^2cx^4 + \frac{6}{5}a^2b^2dx^5 + a^2b^2ex^6 + 4a^3bcx + 2a^3bdx^2 + \frac{4}{3}a^3bex^3 - \frac{a^4c}{2x^2} - \frac{a^4d}{x} + a^4e \log(x) + \frac{4}{7}ab^3cx^7 + \frac{1}{2}ab^3dx^8 + \frac{4}{9}ab^3ex^9$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^4)/x^3,x]

[Out] $-(a^4*c)/(2*x^2) - (a^4*d)/x + 4*a^3*b*c*x + 2*a^3*b*d*x^2 + (4*a^3*b*e*x^3)/3 + (3*a^2*b^2*c*x^4)/2 + (6*a^2*b^2*d*x^5)/5 + a^2*b^2*e*x^6 + (4*a*b^3*c*x^7)/7 + (a*b^3*d*x^8)/2 + (4*a*b^3*e*x^9)/9 + (b^4*c*x^{10})/10 + (b^4*d*x^{11})/11 + (b^4*e*x^{12})/12 + a^4*e*\text{Log}[x]$

Maple [A] time = 0.006, size = 147, normalized size = 0.9

$$-\frac{a^4c}{2x^2} - \frac{a^4d}{x} + 4a^3bcx + 2a^3bdx^2 + \frac{4a^3bex^3}{3} + \frac{3a^2b^2cx^4}{2} + \frac{6a^2b^2dx^5}{5} + a^2b^2ex^6 + \frac{4ab^3cx^7}{7} + \frac{ab^3dx^8}{2} + \frac{4ab^3ex^9}{9} + \frac{b^4c}{10} + \frac{b^4d}{11} + \frac{b^4e}{12} + a^4e \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)*(b*x^3+a)^4/x^3,x)

[Out] $-1/2*a^4*c/x^2 - a^4*d/x + 4*a^3*b*c*x + 2*a^3*b*d*x^2 + 4/3*a^3*b*e*x^3 + 3/2*a^2*b^2*c*x^4 + 6/5*a^2*b^2*d*x^5 + a^2*b^2*e*x^6 + 4/7*a*b^3*c*x^7 + 1/2*a*b^3*d*x^8 + 4/9*a*b^3*e*x^9 + 1/10*b^4*c*x^{10} + 1/11*b^4*d*x^{11} + 1/12*b^4*e*x^{12} + a^4*e*\ln(x)$

Maxima [A] time = 0.949211, size = 197, normalized size = 1.19

$$\frac{1}{12}b^4ex^{12} + \frac{1}{11}b^4dx^{11} + \frac{1}{10}b^4cx^{10} + \frac{4}{9}ab^3ex^9 + \frac{1}{2}ab^3dx^8 + \frac{4}{7}ab^3cx^7 + a^2b^2ex^6 + \frac{6}{5}a^2b^2dx^5 + \frac{3}{2}a^2b^2cx^4 + \frac{4}{3}a^3bex^3 + 2a^3bdx^2 + 4a^3bcx + a^4e \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^3,x, algorithm="maxima")

[Out] $1/12*b^4*e*x^{12} + 1/11*b^4*d*x^{11} + 1/10*b^4*c*x^{10} + 4/9*a*b^3*e*x^9 + 1/2*a*b^3*d*x^8 + 4/7*a*b^3*c*x^7 + a^2*b^2*e*x^6 + 6/5*a^2*b^2*d*x^5 + 3/2*a^2*b^2*c*x^4 + 4/3*a^3*b*e*x^3 + 2*a^3*b*d*x^2 + 4*a^3*b*c*x + a^4*e*\log(x)$

$$- \frac{1}{2}(2a^4dx + a^4c)/x^2$$

Fricas [A] time = 1.52702, size = 413, normalized size = 2.49

$$\frac{1155 b^4 e x^{14} + 1260 b^4 d x^{13} + 1386 b^4 c x^{12} + 6160 a b^3 e x^{11} + 6930 a b^3 d x^{10} + 7920 a b^3 c x^9 + 13860 a^2 b^2 e x^8 + 16632 a^2 b^2 d x^7 + 20790 a^2 b^2 c x^6 + 18480 a^3 b e x^5 + 27720 a^3 b d x^4 + 55440 a^3 b c x^3 + 13860 a^4 e x^2 \log(x) - 13860 a^4 d x - 6930 a^4 c}{13860}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^3,x, algorithm="fricas")

[Out] 1/13860*(1155*b^4*e*x^14 + 1260*b^4*d*x^13 + 1386*b^4*c*x^12 + 6160*a*b^3*e*x^11 + 6930*a*b^3*d*x^10 + 7920*a*b^3*c*x^9 + 13860*a^2*b^2*e*x^8 + 16632*a^2*b^2*d*x^7 + 20790*a^2*b^2*c*x^6 + 18480*a^3*b*e*x^5 + 27720*a^3*b*d*x^4 + 55440*a^3*b*c*x^3 + 13860*a^4*e*x^2*log(x) - 13860*a^4*d*x - 6930*a^4*c)/x^2

Sympy [A] time = 0.558303, size = 173, normalized size = 1.04

$$a^4 e \log(x) + 4a^3 b c x + 2a^3 b d x^2 + \frac{4a^3 b e x^3}{3} + \frac{3a^2 b^2 c x^4}{2} + \frac{6a^2 b^2 d x^5}{5} + a^2 b^2 e x^6 + \frac{4a b^3 c x^7}{7} + \frac{a b^3 d x^8}{2} + \frac{4a b^3 e x^9}{9} + \frac{b^4 c x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**4/x**3,x)

[Out] a**4*e*log(x) + 4*a**3*b*c*x + 2*a**3*b*d*x**2 + 4*a**3*b*e*x**3/3 + 3*a**2*b**2*c*x**4/2 + 6*a**2*b**2*d*x**5/5 + a**2*b**2*e*x**6 + 4*a*b**3*c*x**7/7 + a*b**3*d*x**8/2 + 4*a*b**3*e*x**9/9 + b**4*c*x**10/10 + b**4*d*x**11/11 + b**4*e*x**12/12 - (a**4*c + 2*a**4*d*x)/(2*x**2)

Giac [A] time = 1.05512, size = 205, normalized size = 1.23

$$\frac{1}{12} b^4 x^{12} e + \frac{1}{11} b^4 d x^{11} + \frac{1}{10} b^4 c x^{10} + \frac{4}{9} a b^3 x^9 e + \frac{1}{2} a b^3 d x^8 + \frac{4}{7} a b^3 c x^7 + a^2 b^2 x^6 e + \frac{6}{5} a^2 b^2 d x^5 + \frac{3}{2} a^2 b^2 c x^4 + \frac{4}{3} a^3 b x^3 e +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^3,x, algorithm="giac")
```

```
[Out] 1/12*b^4*x^12*e + 1/11*b^4*d*x^11 + 1/10*b^4*c*x^10 + 4/9*a*b^3*x^9*e + 1/2
*a*b^3*d*x^8 + 4/7*a*b^3*c*x^7 + a^2*b^2*x^6*e + 6/5*a^2*b^2*d*x^5 + 3/2*a^
2*b^2*c*x^4 + 4/3*a^3*b*x^3*e + 2*a^3*b*d*x^2 + 4*a^3*b*c*x + a^4*e*log(abs
(x)) - 1/2*(2*a^4*d*x + a^4*c)/x^2
```


$$3.337 \quad \int \frac{x^3(c+dx+ex^2)}{a+bx^3} dx$$

Optimal. Leaf size=205

$$\frac{\sqrt[3]{a} \left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}} \right) \log \left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2 \right)}{6b^{4/3}} - \frac{\sqrt[3]{a} \left(\sqrt[3]{bc} - \sqrt[3]{ad} \right) \log \left(\sqrt[3]{a} + \sqrt[3]{bx} \right)}{3b^{5/3}} + \frac{\sqrt[3]{a} \left(\sqrt[3]{ad} + \sqrt[3]{bc} \right) \tan^{-1} \left(\frac{\sqrt[3]{a-2\sqrt[3]{bx}}}{\sqrt{3} \sqrt[3]{a}} \right)}{\sqrt{3} b^{5/3}}$$

[Out] (c*x)/b + (d*x^2)/(2*b) + (e*x^3)/(3*b) + (a^(1/3)*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*b^(5/3)) - (a^(1/3)*(b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x]/(3*b^(5/3)) + (a^(1/3)*(c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*b^(4/3)) - (a*e*Log[a + b*x^3])/(3*b^2)

Rubi [A] time = 0.261464, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\sqrt[3]{a} \left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}} \right) \log \left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2 \right)}{6b^{4/3}} - \frac{\sqrt[3]{a} \left(\sqrt[3]{bc} - \sqrt[3]{ad} \right) \log \left(\sqrt[3]{a} + \sqrt[3]{bx} \right)}{3b^{5/3}} + \frac{\sqrt[3]{a} \left(\sqrt[3]{ad} + \sqrt[3]{bc} \right) \tan^{-1} \left(\frac{\sqrt[3]{a-2\sqrt[3]{bx}}}{\sqrt{3} \sqrt[3]{a}} \right)}{\sqrt{3} b^{5/3}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(c + d*x + e*x^2))/(a + b*x^3), x]

[Out] (c*x)/b + (d*x^2)/(2*b) + (e*x^3)/(3*b) + (a^(1/3)*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*b^(5/3)) - (a^(1/3)*(b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x]/(3*b^(5/3)) + (a^(1/3)*(c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*b^(4/3)) - (a*e*Log[a + b*x^3])/(3*b^2)

Rule 1887

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Pq/(a + b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]

Rule 1871

Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di

```
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^( -1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^( -1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned}
 \int \frac{x^3(c + dx + ex^2)}{a + bx^3} dx &= \int \left(\frac{c}{b} + \frac{dx}{b} + \frac{ex^2}{b} - \frac{ac + adx + aex^2}{b(a + bx^3)} \right) dx \\
 &= \frac{cx}{b} + \frac{dx^2}{2b} + \frac{ex^3}{3b} - \frac{\int \frac{ac + adx + aex^2}{a + bx^3} dx}{b} \\
 &= \frac{cx}{b} + \frac{dx^2}{2b} + \frac{ex^3}{3b} - \frac{\int \frac{ac + adx}{a + bx^3} dx}{b} - \frac{(ae) \int \frac{x^2}{a + bx^3} dx}{b} \\
 &= \frac{cx}{b} + \frac{dx^2}{2b} + \frac{ex^3}{3b} - \frac{ae \log(a + bx^3)}{3b^2} - \frac{\int \frac{\sqrt[3]{a}(2a\sqrt[3]{bc} + a^{4/3}d) + \sqrt[3]{b}(-a\sqrt[3]{bc} + a^{4/3}d)x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} dx}{3a^{2/3}b^{4/3}} - \frac{\left(\sqrt[3]{a}\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)\right)}{3b} \\
 &= \frac{cx}{b} + \frac{dx^2}{2b} + \frac{ex^3}{3b} - \frac{\sqrt[3]{a}\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3b^{4/3}} - \frac{ae \log(a + bx^3)}{3b^2} - \frac{(a^{2/3}(\sqrt[3]{bc} + \sqrt[3]{ad})) \int}{2b^{4/3}} \\
 &= \frac{cx}{b} + \frac{dx^2}{2b} + \frac{ex^3}{3b} - \frac{\sqrt[3]{a}\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3b^{4/3}} + \frac{\sqrt[3]{a}\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{6b^{4/3}} \\
 &= \frac{cx}{b} + \frac{dx^2}{2b} + \frac{ex^3}{3b} + \frac{\sqrt[3]{a}(\sqrt[3]{bc} + \sqrt[3]{ad}) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}} - \frac{\sqrt[3]{a}\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3b^{4/3}} + \frac{\sqrt[3]{a}}{3b}
 \end{aligned}$$

Mathematica [A] time = 0.104917, size = 191, normalized size = 0.93

$$\frac{\sqrt[3]{b}(\sqrt[3]{a}\sqrt[3]{bc} - a^{2/3}d) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2) + 2\sqrt[3]{b}(a^{2/3}d - \sqrt[3]{a}\sqrt[3]{bc}) \log(\sqrt[3]{a} + \sqrt[3]{bx}) + 2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}(\sqrt[3]{ad} + \sqrt[3]{bc}) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{6b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(c + d*x + e*x^2))/(a + b*x^3), x]

[Out] (6*b*c*x + 3*b*d*x^2 + 2*b*e*x^3 + 2*Sqrt[3]*a^(1/3)*b^(1/3)*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*b^(1/3)*(-(a^(1/3)*b^(1/3)*c) + a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*(a^(1/3)*b^(1/3)*c - a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*a*e*Lo

$g[a + b*x^3]/(6*b^2)$

Maple [A] time = 0.003, size = 231, normalized size = 1.1

$$\frac{ex^3}{3b} + \frac{dx^2}{2b} + \frac{cx}{b} - \frac{ac}{3b^2} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{ac}{6b^2} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{a\sqrt{3}c}{3b^2} \arctan\left(\frac{\sqrt{3}}{3} \left(2x \frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) \left(\frac{a}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(e*x^2+d*x+c)/(b*x^3+a),x)`

[Out] $\frac{1}{3}e*x^3/b + \frac{1}{2}d*x^2/b + c*x/b - \frac{1}{3}a/b^2/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*c + \frac{1}{6}a/b^2/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*c - \frac{1}{3}a/b^2/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*c + \frac{1}{3}b^2/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})*a*d - \frac{1}{6}b^2/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*a*d - \frac{1}{3}b^2*3^{(1/2)}/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*a*d - \frac{1}{3}a*e*\ln(b*x^3+a)/b^2$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [C] time = 9.92844, size = 10954, normalized size = 53.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")`

```
[Out] 1/36*(12*b*e*x^3 + 18*b*d*x^2 - 2*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d
+ a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*
b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*
b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)
*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^
3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)*b^2*log(1/36*((-I*sqrt(3) + 1)*
(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 +
a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3
- (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6
+ 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*
b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)^2*b^4*d
+ 2*a*b*c*d^2 - a*b*c^2*e + a^2*d*e^2 + 1/6*(b^3*c^2 - 2*a*b^2*d*e)*((-I*sq
rt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/5
4*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^
3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*
a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6
- 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^
2) + (b^2*c^3 + a*b*d^3)*x) + 36*b*c*x + (((-I*sqrt(3) + 1)*(a^2*e^2/b^4 -
(a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 +
1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d
*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3
+ a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e
^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)*b^2 + 3*sqrt(1/3)*b^2*s
qrt(-(((I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*
e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1
/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3)
+ 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2
*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/
3) + 6*a*e/b^2)^2*b^4 - 12*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*
e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d +
a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)
^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5
+ 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c
*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)*a*b^2*e + 144*a*b*c*d + 36*a^2*e^2)/b^
4) - 18*a*e)*log(-1/36*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)
/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2
*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/
3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/
18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e
)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)^2*b^4*d - 2*a*b*c*d^2 + a*b*c^2*e - a^2*d*
e^2 - 1/6*(b^3*c^2 - 2*a*b^2*d*e)*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d
+ a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*
b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*
b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)
*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^
```

$$\begin{aligned}
& 3 - 3*c*d*e)*a^2*b)/b^6)^{(1/3)} + 6*a*e/b^2) + 2*(b^2*c^3 + a*b*d^3)*x + 1/1 \\
& 2*sqrt(1/3)*(((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/ \\
& 27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/ \\
& b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^{(1/3)} + 9*(I* \\
& sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c* \\
& d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b \\
& ^6)^{(1/3)} + 6*a*e/b^2)*b^4*d - 6*b^3*c^2 - 6*a*b^2*d*e)*sqrt(-(((-I*sqrt(3) \\
& + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b* \\
& c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a \\
& ^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^{(1/3)} + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e \\
& ^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/ \\
& 54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^{(1/3)} + 6*a*e/b^2)^2* \\
& b^4 - 12*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a \\
& ^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 \\
& - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^{(1/3)} + 9*(I*sqrt \\
& (3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + \\
& a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^ \\
& (1/3) + 6*a*e/b^2)*a*b^2*e + 144*a*b*c*d + 36*a^2*e^2)/b^4)) + (((-I*sqrt(3) \\
&) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b \\
& *c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + \\
& a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^{(1/3)} + 9*(I*sqrt(3) + 1)*(-1/27*a^3* \\
& e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1 \\
& /54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^{(1/3)} + 6*a*e/b^2)*b \\
& ^2 - 3*sqrt(1/3)*b^2*sqrt(-(((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2 \\
& *e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d \\
& + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6 \\
&)^{(1/3)} + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 \\
& + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3* \\
& c*d*e)*a^2*b)/b^6)^{(1/3)} + 6*a*e/b^2)^2*b^4 - 12*((-I*sqrt(3) + 1)*(a^2*e^2 \\
& /b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a \\
& /b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 \\
& - 3*c*d*e)*a^2*b)/b^6)^{(1/3)} + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54* \\
& (b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 \\
& + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^{(1/3)} + 6*a*e/b^2)*a*b^2*e + 144*a* \\
& b*c*d + 36*a^2*e^2)/b^4) - 18*a*e)*log(-1/36*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 \\
& - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 \\
& + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3* \\
& c*d*e)*a^2*b)/b^6)^{(1/3)} + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c \\
& ^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^ \\
& 3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^{(1/3)} + 6*a*e/b^2)^2*b^4*d - 2*a*b*c*d^ \\
& 2 + a*b*c^2*e - a^2*d*e^2 - 1/6*(b^3*c^2 - 2*a*b^2*d*e)*((-I*sqrt(3) + 1)* \\
& (a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a \\
& *d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 \\
& - (d^3 - 3*c*d*e)*a^2*b)/b^6)^{(1/3)} + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 \\
& + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b
\end{aligned}$$

$$\begin{aligned} &^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^{(1/3)} + 6*a*e/b^2) + 2*(b^2* \\ &c^3 + a*b*d^3)*x - 1/12*sqrt(1/3)*(((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c* \\ &d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a \\ &*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2 \\ &*b)/b^6)^{(1/3)} + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3 \\ &)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d \\ &^3 - 3*c*d*e)*a^2*b)/b^6)^{(1/3)} + 6*a*e/b^2)*b^4*d - 6*b^3*c^2 - 6*a*b^2*d* \\ &e)*sqrt(-(((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27* \\ &a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 \\ &- 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^{(1/3)} + 9*(I*sqrt \\ &t(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + \\ &a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6) \\ &^{(1/3)} + 6*a*e/b^2)^2*b^4 - 12*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + \\ &a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c \\ &*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/ \\ &b^6)^{(1/3)} + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/ \\ &b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - \\ &3*c*d*e)*a^2*b)/b^6)^{(1/3)} + 6*a*e/b^2)*a*b^2*e + 144*a*b*c*d + 36*a^2*e^2 \\ &)/b^4)))/b^2 \end{aligned}$$

Sympy [A] time = 1.59667, size = 178, normalized size = 0.87

$$\text{RootSum}\left(27t^3b^6 + 27t^2ab^4e + t(9a^2b^2e^2 + 9ab^3cd) + a^3e^3 + 3a^2bcde - a^2bd^3 + ab^2c^3, \left(t \mapsto t \log\left(x + \frac{9t^2b^4d + 6tab^2}{\dots}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(e*x**2+d*x+c)/(b*x**3+a), x)

[Out] RootSum(27*_t**3*b**6 + 27*_t**2*a*b**4*e + _t*(9*a**2*b**2*e**2 + 9*a*b**3*c*d) + a**3*e**3 + 3*a**2*b*c*d*e - a**2*b*d**3 + a*b**2*c**3, Lambda(_t, _t*log(x + (9*_t**2*b**4*d + 6*_t*a*b**2*d*e - 3*_t*b**3*c**2 + a**2*d*e**2 - a*b*c**2*e + 2*a*b*c*d**2)/(a*b*d**3 + b**2*c**3)))) + c*x/b + d*x**2/(2*b) + e*x**3/(3*b)

Giac [A] time = 1.08081, size = 302, normalized size = 1.47

$$\frac{ae \log(|bx^3 + a|)}{3b^2} + \frac{2b^2x^3e + 3b^2dx^2 + 6b^2cx}{6b^3} - \frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}ab^2c - (-ab^2)^{\frac{2}{3}}abd\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^4} - \left((-ab^2)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")`

[Out]
$$-1/3*a*e*\log(\text{abs}(b*x^3 + a))/b^2 + 1/6*(2*b^2*x^3*e + 3*b^2*d*x^2 + 6*b^2*c*x)/b^3 - 1/3*\sqrt{3}*((-a*b^2)^{1/3}*a*b^2*c - (-a*b^2)^{2/3}*a*b*d)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{1/3})/(-a/b)^{1/3})/(a*b^4) - 1/6*((-a*b^2)^{1/3}*a*b^2*c + (-a*b^2)^{2/3}*a*b*d)*\log(x^2 + x*(-a/b)^{1/3} + (-a/b)^{2/3})/(a*b^4) + 1/3*(a*b^6*d*(-a/b)^{1/3} + a*b^6*c)*(-a/b)^{1/3}*\log(\text{abs}(x - (-a/b)^{1/3}))/a*b^7$$

$$3.338 \quad \int \frac{x^2(c+dx+ex^2)}{a+bx^3} dx$$

Optimal. Leaf size=193

$$\frac{\sqrt[3]{a} \left(d - \frac{\sqrt[3]{ae}}{\sqrt[3]{b}} \right) \log \left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2 \right)}{6b^{4/3}} - \frac{\sqrt[3]{a} \left(\sqrt[3]{bd} - \sqrt[3]{ae} \right) \log \left(\sqrt[3]{a} + \sqrt[3]{bx} \right)}{3b^{5/3}} + \frac{\sqrt[3]{a} \left(\sqrt[3]{ae} + \sqrt[3]{bd} \right) \tan^{-1} \left(\frac{\sqrt[3]{a-2\sqrt[3]{bx}}}{\sqrt{3} \sqrt[3]{a}} \right)}{\sqrt{3} b^{5/3}} +$$

[Out] (d*x)/b + (e*x^2)/(2*b) + (a^(1/3)*(b^(1/3)*d + a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*b^(5/3)) - (a^(1/3)*(b^(1/3)*d - a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x]/(3*b^(5/3)) + (a^(1/3)*(d - (a^(1/3)*e)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*b^(4/3)) + (c*Log[a + b*x^3])/(3*b)

Rubi [A] time = 0.247859, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\sqrt[3]{a} \left(d - \frac{\sqrt[3]{ae}}{\sqrt[3]{b}} \right) \log \left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2 \right)}{6b^{4/3}} - \frac{\sqrt[3]{a} \left(\sqrt[3]{bd} - \sqrt[3]{ae} \right) \log \left(\sqrt[3]{a} + \sqrt[3]{bx} \right)}{3b^{5/3}} + \frac{\sqrt[3]{a} \left(\sqrt[3]{ae} + \sqrt[3]{bd} \right) \tan^{-1} \left(\frac{\sqrt[3]{a-2\sqrt[3]{bx}}}{\sqrt{3} \sqrt[3]{a}} \right)}{\sqrt{3} b^{5/3}} +$$

Antiderivative was successfully verified.

[In] Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3), x]

[Out] (d*x)/b + (e*x^2)/(2*b) + (a^(1/3)*(b^(1/3)*d + a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*b^(5/3)) - (a^(1/3)*(b^(1/3)*d - a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x]/(3*b^(5/3)) + (a^(1/3)*(d - (a^(1/3)*e)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*b^(4/3)) + (c*Log[a + b*x^3])/(3*b)

Rule 1887

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Pq/(a + b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]

Rule 1871

Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di

```
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2(c + dx + ex^2)}{a + bx^3} dx &= \int \left(\frac{d}{b} + \frac{ex}{b} - \frac{ad + aex - bcx^2}{b(a + bx^3)} \right) dx \\
 &= \frac{dx}{b} + \frac{ex^2}{2b} - \frac{\int \frac{ad + aex - bcx^2}{a + bx^3} dx}{b} \\
 &= \frac{dx}{b} + \frac{ex^2}{2b} - \frac{\int \frac{ad + aex}{a + bx^3} dx}{b} + c \int \frac{x^2}{a + bx^3} dx \\
 &= \frac{dx}{b} + \frac{ex^2}{2b} + \frac{c \log(a + bx^3)}{3b} - \frac{\int \frac{\sqrt[3]{a}(2a\sqrt[3]{bd + a^{4/3}e}) + \sqrt[3]{b}(-a\sqrt[3]{bd + a^{4/3}e})x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} dx}{3a^{2/3}b^{4/3}} - \frac{\left(\sqrt[3]{a}\left(d - \frac{\sqrt[3]{ae}}{\sqrt[3]{b}}\right)\right) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{bx}} dx}{3b} \\
 &= \frac{dx}{b} + \frac{ex^2}{2b} - \frac{\sqrt[3]{a}\left(d - \frac{\sqrt[3]{ae}}{\sqrt[3]{b}}\right) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3b^{4/3}} + \frac{c \log(a + bx^3)}{3b} - \frac{(a^{2/3}(\sqrt[3]{bd} + \sqrt[3]{ae})) \int \frac{1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} dx}{2b^{4/3}} \\
 &= \frac{dx}{b} + \frac{ex^2}{2b} - \frac{\sqrt[3]{a}\left(d - \frac{\sqrt[3]{ae}}{\sqrt[3]{b}}\right) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3b^{4/3}} + \frac{\sqrt[3]{a}\left(d - \frac{\sqrt[3]{ae}}{\sqrt[3]{b}}\right) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{6b^{4/3}} + \frac{c \log(a + bx^3)}{3b} \\
 &= \frac{dx}{b} + \frac{ex^2}{2b} + \frac{\sqrt[3]{a}(\sqrt[3]{bd} + \sqrt[3]{ae}) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}} - \frac{\sqrt[3]{a}\left(d - \frac{\sqrt[3]{ae}}{\sqrt[3]{b}}\right) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3b^{4/3}} + \frac{\sqrt[3]{a}\left(d - \frac{\sqrt[3]{ae}}{\sqrt[3]{b}}\right) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{6b^{4/3}}
 \end{aligned}$$

Mathematica [A] time = 0.0885497, size = 184, normalized size = 0.95

$$\frac{-\left(a^{2/3}e - \sqrt[3]{a}\sqrt[3]{bd}\right) \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right) + 2\left(a^{2/3}e - \sqrt[3]{a}\sqrt[3]{bd}\right) \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right) + 2b^{2/3}c \log\left(a + bx^3\right) + 2\sqrt{3}\sqrt[3]{a}\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{6b^{5/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3), x]

[Out] (6*b^(2/3)*d*x + 3*b^(2/3)*e*x^2 + 2*Sqrt[3]*a^(1/3)*(b^(1/3)*d + a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(-(a^(1/3)*b^(1/3)*d) + a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x] - (-(a^(1/3)*b^(1/3)*d) + a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*b^(2/3)*c*Log[a + b*x^3]/(

$6*b^{(5/3)}$

Maple [A] time = 0.003, size = 221, normalized size = 1.2

$$\frac{ex^2}{2b} + \frac{dx}{b} - \frac{ad}{3b^2} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{ad}{6b^2} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{a\sqrt{3}d}{3b^2} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(e*x^2+d*x+c)/(b*x^3+a),x)`

[Out] $\frac{1}{2}e*x^2/b + d*x/b - 1/3/b^2/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*a*d + 1/6/b^2/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*a*d - 1/3/b^2/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*a*d + 1/3/b^2*a/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})*e - 1/6/b^2*a/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*e - 1/3/b^2*a*3^{(1/2)}/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*e + 1/3*c*\ln(b*x^3+a)/b$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [C] time = 9.57806, size = 9513, normalized size = 49.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")`

```
[Out] 1/12*(6*e*x^2 - 2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b*log(1/4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)^2*b^3*e + b*c*d^2 + b*c^2*e + 2*a*d*e^2 + 1/2*(b^2*d^2 + 2*b^2*c*e)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b) + (b*d^3 + a*e^3)*x) + 1
2*d*x + ((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b + 3*sqrt(1/3)*b*sqrt(-(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)^2*b^3 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b
^2*c + 4*b*c^2 + 16*a*d*e)/b^3) + 6*c)*log(-1/4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)^2*b^3*e - b*c*d^2 - b*c^2*e - 2*a*d*e^2 - 1/2*(b^2*d^2 + 2*b^2*c*e)
*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b) + 2*(b*d^3 + a*e^3)*x + 3/4*sqrt(1/3)*((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b^3*e - 2*b^2*d^2 + 2*b^2*c*e)*sqrt(-(2
```


$$\frac{3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^{(1/3)} - 2*c/b)*b^2*c + 4*b*c^2 + 16*a*d*e)/b^3)))/b$$

Sympy [A] time = 1.50061, size = 150, normalized size = 0.78

$$\text{RootSum}\left(27t^3b^5 - 27t^2b^4c + t(9ab^2de + 9b^3c^2) - a^2e^3 - 3abcde + abd^3 - b^2c^3, \left(t \mapsto t \log\left(x + \frac{9t^2b^3e - 6tb^2ce - 3tb^2}{ae^3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a), x)

[Out] RootSum(27*_t**3*b**5 - 27*_t**2*b**4*c + _t*(9*a*b**2*d*e + 9*b**3*c**2) - a**2*e**3 - 3*a*b*c*d*e + a*b*d**3 - b**2*c**3, Lambda(_t, _t*log(x + (9*_t**2*b**3*e - 6*_t*b**2*c*e - 3*_t*b**2*d**2 + 2*a*d*e**2 + b*c**2*e + b*c*d**2)/(a*e**3 + b*d**3)))) + d*x/b + e*x**2/(2*b)

Giac [A] time = 1.07844, size = 285, normalized size = 1.48

$$\frac{c \log(|bx^3 + a|)}{3b} + \frac{bx^2e + 2bdx}{2b^2} - \frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}} ab^2d - (-ab^2)^{\frac{2}{3}} abe\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^4} - \frac{\left((-ab^2)^{\frac{1}{3}} ab^2d + (-ab^2)^{\frac{2}{3}} abe\right)}{3ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a), x, algorithm="giac")

[Out] 1/3*c*log(abs(b*x^3 + a))/b + 1/2*(b*x^2*e + 2*b*d*x)/b^2 - 1/3*sqrt(3)*((-a*b^2)^(1/3)*a*b^2*d - (-a*b^2)^(2/3)*a*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^4) - 1/6*((-a*b^2)^(1/3)*a*b^2*d + (-a*b^2)^(2/3)*a*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^4) + 1/3*(a*b^4*(-a/b)^(1/3)*e + a*b^4*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5)

$$3.339 \quad \int \frac{x(c+dx+ex^2)}{a+bx^3} dx$$

Optimal. Leaf size=183

$$\frac{(a^{2/3}e + b^{2/3}c) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{6\sqrt[3]{ab^{4/3}}} - \frac{(a^{2/3}e + b^{2/3}c) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3\sqrt[3]{ab^{4/3}}} - \frac{(b^{2/3}c - a^{2/3}e) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{ab^{4/3}}} + \frac{d \log(a + bx^3)}{3b}$$

[Out] (e*x)/b - ((b^(2/3)*c - a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3)*b^(4/3)) - ((b^(2/3)*c + a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x]/(3*a^(1/3)*b^(4/3)) + ((b^(2/3)*c + a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*a^(1/3)*b^(4/3)) + (d*Log[a + b*x^3])/(3*b)

Rubi [A] time = 0.225588, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{(a^{2/3}e + b^{2/3}c) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{6\sqrt[3]{ab^{4/3}}} - \frac{(a^{2/3}e + b^{2/3}c) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3\sqrt[3]{ab^{4/3}}} - \frac{(b^{2/3}c - a^{2/3}e) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{ab^{4/3}}} + \frac{d \log(a + bx^3)}{3b}$$

Antiderivative was successfully verified.

[In] Int[(x*(c + d*x + e*x^2))/(a + b*x^3), x]

[Out] (e*x)/b - ((b^(2/3)*c - a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3)*b^(4/3)) - ((b^(2/3)*c + a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x]/(3*a^(1/3)*b^(4/3)) + ((b^(2/3)*c + a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*a^(1/3)*b^(4/3)) + (d*Log[a + b*x^3])/(3*b)

Rule 1887

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Pq/(a + b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]

Rule 1871

Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di

st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

Rule 1860

Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]

Rule 31

Int[((a_) + (b_)*(x_))^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned}
 \int \frac{x(c + dx + ex^2)}{a + bx^3} dx &= \int \left(\frac{e}{b} - \frac{ae - bcx - bdx^2}{b(a + bx^3)} \right) dx \\
 &= \frac{ex}{b} - \frac{\int \frac{ae - bcx - bdx^2}{a + bx^3} dx}{b} \\
 &= \frac{ex}{b} - \frac{\int \frac{ae - bcx}{a + bx^3} dx}{b} + d \int \frac{x^2}{a + bx^3} dx \\
 &= \frac{ex}{b} + \frac{d \log(a + bx^3)}{3b} - \frac{\int \frac{\sqrt[3]{a}(-\sqrt[3]{a}bc + 2a\sqrt[3]{be}) + \sqrt[3]{b}(-\sqrt[3]{abc} - a\sqrt[3]{be})x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} dx}{3a^{2/3}b^{4/3}} - \frac{(b^{2/3}c + a^{2/3}e) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{bx}} dx}{3\sqrt[3]{ab}} \\
 &= \frac{ex}{b} - \frac{(b^{2/3}c + a^{2/3}e) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3\sqrt[3]{ab}^{4/3}} + \frac{d \log(a + bx^3)}{3b} + \frac{(b^{2/3}c - a^{2/3}e) \int \frac{1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} dx}{2b} + \\
 &= \frac{ex}{b} - \frac{(b^{2/3}c + a^{2/3}e) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3\sqrt[3]{ab}^{4/3}} + \frac{(b^{2/3}c + a^{2/3}e) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{6\sqrt[3]{ab}^{4/3}} + \frac{d \log(a + bx^3)}{3b} \\
 &= \frac{ex}{b} - \frac{(b^{2/3}c - a^{2/3}e) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{ab}^{4/3}} - \frac{(b^{2/3}c + a^{2/3}e) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3\sqrt[3]{ab}^{4/3}} + \frac{(b^{2/3}c + a^{2/3}e) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{6\sqrt[3]{ab}^{4/3}}
 \end{aligned}$$

Mathematica [A] time = 0.0558075, size = 200, normalized size = 1.09

$$-\frac{(a^{4/3}(-\sqrt[3]{b})e - a^{2/3}bc) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{6ab^{5/3}} + \frac{(a^{4/3}(-\sqrt[3]{b})e - a^{2/3}bc) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3ab^{5/3}} + \frac{(a^{2/3}bc - a^{4/3}\sqrt[3]{be}) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}ab^{5/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3), x]

[Out] (e*x)/b + ((a^(2/3)*b*c - a^(4/3)*b^(1/3)*e)*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a*b^(5/3)) + ((-a^(2/3)*b*c) - a^(4/3)*b^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x]/(3*a*b^(5/3)) - ((-a^(2/3)*b*c) - a^(4/3)*b^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*a*b^(5/3)) +

$$(d \cdot \text{Log}[a + b \cdot x^3]) / (3 \cdot b)$$

Maple [A] time = 0.003, size = 209, normalized size = 1.1

$$\frac{ex}{b} - \frac{ae}{3b^2} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{ae}{6b^2} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{a\sqrt{3}e}{3b^2} \arctan\left(\frac{\sqrt{3}}{3} \left(2x \frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{c}{3b} \ln\left(\dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(e*x^2+d*x+c)/(b*x^3+a),x)`

[Out] `e*x/b-1/3/b^2*a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*e+1/6/b^2*a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e-1/3/b^2*a/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e-1/3/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*c+1/6/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+1/3/b*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c+1/3*d*ln(b*x^3+a)/b`

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [C] time = 9.56091, size = 9754, normalized size = 53.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")`

```
[Out] -1/12*(2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)*b*log(-1/4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)^2*a*b^3*c - a*b*c*d^2 + 2*a*b*c^2*e + a^2*d*e^2 - 1/2*(2*a*b^2*c*d - a^2*b*e^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b) - (b^2*c^3 - a^2*e^3)*x) - 12*e*x - ((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)*b - 3*sqrt(1/3)*b*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)^2*b^2 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)*b*d + 4*d^2 - 16*c*e)/b^2) + 6*d)*log(1/4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)^2*a*b^3*c + a*b*c*d^2 - 2*a*b*c^2*e - a^2*d*e^2 + 1/2*(2*a*b^2*c*d - a^2*b*e^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b) - 2*(b^2*c^3 - a^2*e^3)*x + 3/4*sqrt(1/3)*((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)
```

$$\begin{aligned}
& 4) - (b^2c^3 - a^2e^3)/(ab^4)^{1/3} - 2d/b * ab^3c + 2ab^2cd + 2a^2b^2e^2 * \sqrt{-((2(1/2)^{2/3} * (-\sqrt{3}) + 1) * (d^2/b^2 - (d^2 - ce)/b^2)) / (2d^3/b^3 - 3(d^2 - ce)d/b^3 - (b^2c^3 + a^2e^3 - (d^3 - 3cde)) * ab)} / (ab^4) - (b^2c^3 - a^2e^3)/(ab^4)^{1/3} + (1/2)^{1/3} * (\sqrt{3} + 1) * (2d^3/b^3 - 3(d^2 - ce)d/b^3 - (b^2c^3 + a^2e^3 - (d^3 - 3cde)) * ab) / (ab^4) - (b^2c^3 - a^2e^3)/(ab^4)^{1/3} - 2d/b^2b^2 + 4 * (2 * (1/2)^{2/3} * (-\sqrt{3}) + 1) * (d^2/b^2 - (d^2 - ce)/b^2) / (2d^3/b^3 - 3(d^2 - ce)d/b^3 - (b^2c^3 + a^2e^3 - (d^3 - 3cde)) * ab) / (ab^4) - (b^2c^3 - a^2e^3)/(ab^4)^{1/3} + (1/2)^{1/3} * (\sqrt{3} + 1) * (2d^3/b^3 - 3(d^2 - ce)d/b^3 - (b^2c^3 + a^2e^3 - (d^3 - 3cde)) * ab) / (ab^4) - (b^2c^3 - a^2e^3)/(ab^4)^{1/3} - 2d/b * bd + 4d^2 - 16ce/b^2) - ((2 * (1/2)^{2/3} * (-\sqrt{3}) + 1) * (d^2/b^2 - (d^2 - ce)/b^2) / (2d^3/b^3 - 3(d^2 - ce)d/b^3 - (b^2c^3 + a^2e^3 - (d^3 - 3cde)) * ab) / (ab^4) - (b^2c^3 - a^2e^3)/(ab^4))^{1/3} + (1/2)^{1/3} * (\sqrt{3} + 1) * (2d^3/b^3 - 3(d^2 - ce)d/b^3 - (b^2c^3 + a^2e^3 - (d^3 - 3cde)) * ab) / (ab^4) - (b^2c^3 - a^2e^3)/(ab^4)^{1/3} - 2d/b * b + 3\sqrt{1/3} * b * \sqrt{-((2(1/2)^{2/3} * (-\sqrt{3}) + 1) * (d^2/b^2 - (d^2 - ce)/b^2)) / (2d^3/b^3 - 3(d^2 - ce)d/b^3 - (b^2c^3 + a^2e^3 - (d^3 - 3cde)) * ab)} / (ab^4) - (b^2c^3 - a^2e^3)/(ab^4)^{1/3} + (1/2)^{1/3} * (\sqrt{3} + 1) * (2d^3/b^3 - 3(d^2 - ce)d/b^3 - (b^2c^3 + a^2e^3 - (d^3 - 3cde)) * ab) / (ab^4) - (b^2c^3 - a^2e^3)/(ab^4)^{1/3} - 2d/b^2b^2 + 4 * (2 * (1/2)^{2/3} * (-\sqrt{3}) + 1) * (d^2/b^2 - (d^2 - ce)/b^2) / (2d^3/b^3 - 3(d^2 - ce)d/b^3 - (b^2c^3 + a^2e^3 - (d^3 - 3cde)) * ab) / (ab^4) - (b^2c^3 - a^2e^3)/(ab^4)^{1/3} + (1/2)^{1/3} * (\sqrt{3} + 1) * (2d^3/b^3 - 3(d^2 - ce)d/b^3 - (b^2c^3 + a^2e^3 - (d^3 - 3cde)) * ab) / (ab^4) - (b^2c^3 - a^2e^3)/(ab^4)^{1/3} - 2d/b * bd + 4d^2 - 16ce/b^2) + 6d * \log(1/4 * (2 * (1/2)^{2/3} * (-\sqrt{3}) + 1) * (d^2/b^2 - (d^2 - ce)/b^2) / (2d^3/b^3 - 3(d^2 - ce)d/b^3 - (b^2c^3 + a^2e^3 - (d^3 - 3cde)) * ab) / (ab^4) - (b^2c^3 - a^2e^3)/(ab^4))^{1/3} + (1/2)^{1/3} * (\sqrt{3} + 1) * (2d^3/b^3 - 3(d^2 - ce)d/b^3 - (b^2c^3 + a^2e^3 - (d^3 - 3cde)) * ab) / (ab^4) - (b^2c^3 - a^2e^3)/(ab^4)^{1/3} - 2d/b^2 * ab^3c + ab * cd^2 - 2ab * c^2e - a^2 * de^2 + 1/2 * (2 * ab^2 * cd - a^2 * b^2 * e^2) * (2 * (1/2)^{2/3} * (-\sqrt{3}) + 1) * (d^2/b^2 - (d^2 - ce)/b^2) / (2d^3/b^3 - 3(d^2 - ce)d/b^3 - (b^2c^3 + a^2e^3 - (d^3 - 3cde)) * ab) / (ab^4) - (b^2c^3 - a^2e^3)/(ab^4)^{1/3} + (1/2)^{1/3} * (\sqrt{3} + 1) * (2d^3/b^3 - 3(d^2 - ce)d/b^3 - (b^2c^3 + a^2e^3 - (d^3 - 3cde)) * ab) / (ab^4) - (b^2c^3 - a^2e^3)/(ab^4)^{1/3} - 2d/b - 2 * (b^2c^3 - a^2e^3) * x - 3/4 * \sqrt{1/3} * ((2 * (1/2)^{2/3} * (-\sqrt{3}) + 1) * (d^2/b^2 - (d^2 - ce)/b^2) / (2d^3/b^3 - 3(d^2 - ce)d/b^3 - (b^2c^3 + a^2e^3 - (d^3 - 3cde)) * ab) / (ab^4) - (b^2c^3 - a^2e^3)/(ab^4))^{1/3} + (1/2)^{1/3} * (\sqrt{3} + 1) * (2d^3/b^3 - 3(d^2 - ce)d/b^3 - (b^2c^3 + a^2e^3 - (d^3 - 3cde)) * ab) / (ab^4) - (b^2c^3 - a^2e^3)/(ab^4)^{1/3} - 2d/b * ab^3c + 2ab^2cd + 2a^2b^2e^2 * \sqrt{-((2(1/2)^{2/3} * (-\sqrt{3}) + 1) * (d^2/b^2 - (d^2 - ce)/b^2)) / (2d^3/b^3 - 3(d^2 - ce)d/b^3 - (b^2c^3 + a^2e^3 - (d^3 - 3cde)) * ab)} / (ab^4) - (b^2c^3 - a^2e^3)/(ab^4)^{1/3} + (1/2)^{1/3} * (\sqrt{3} + 1) * (2d^3/b^3 - 3(d^2 - ce)d/b^3 - (b^2c^3 + a^2e^3 - (d^3 - 3cde)) * ab) / (ab^4) - (b^2c^3 - a^2e^3)/(ab^4)^{1/3}
\end{aligned}$$

$$\begin{aligned} & \sqrt[3]{d^3 + a^2 e^3 - (d^3 - 3cd^2e)ab} / (ab^4) - (b^2 c^3 - a^2 e^3) / (ab^4)^{1/3} \\ & - 2d/b \sqrt[3]{b^2} + 4 \cdot (2 \cdot (1/2)^{2/3}) \cdot (-I \sqrt[3]{3} + 1) \cdot (d^2/b^2 - (d^2 - c^2 e)/b^2) / (2d^3/b^3 - 3(d^2 - c^2 e)d/b^3 - (b^2 c^3 + a^2 e^3 - (d^3 - 3cd^2e)ab) / (ab^4) - (b^2 c^3 - a^2 e^3) / (ab^4)^{1/3} + (1/2)^{1/3} \cdot (I \sqrt[3]{3} + 1) \cdot (2d^3/b^3 - 3(d^2 - c^2 e)d/b^3 - (b^2 c^3 + a^2 e^3 - (d^3 - 3cd^2e)ab) / (ab^4) - (b^2 c^3 - a^2 e^3) / (ab^4)^{1/3} - 2d/b) \cdot b \cdot d + 4 \cdot d^2 - 16c^2 e) / b^2) \end{aligned}$$

Sympy [A] time = 1.65329, size = 160, normalized size = 0.87

$$\text{RootSum}\left(27t^3 ab^4 - 27t^2 ab^3 d + t(-9ab^2 ce + 9ab^2 d^2) + a^2 e^3 + 3abcde - abd^3 + b^2 c^3, \left(t \mapsto t \log\left(x + \frac{-9t^2 ab^3 c - 3ta^2 b^3 d}{\dots}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x**2+d*x+c)/(b*x**3+a),x)

[Out] RootSum(27*_t**3*a*b**4 - 27*_t**2*a*b**3*d + _t*(-9*a*b**2*c*e + 9*a*b**2*d**2) + a**2*e**3 + 3*a*b*c*d*e - a*b*d**3 + b**2*c**3, Lambda(_t, _t*log(x + (-9*_t**2*a*b**3*c - 3*_t*a**2*b*e**2 + 6*_t*a*b**2*c*d + a**2*d*e**2 + 2*a*b*c**2*e - a*b*c*d**2)/(a**2*e**3 - b**2*c**3)))) + e*x/b

Giac [A] time = 1.07542, size = 257, normalized size = 1.4

$$\frac{xe}{b} + \frac{d \log(|bx^3 + a|)}{3b} - \frac{\sqrt{3} \left((-ab^2)^{\frac{1}{3}} ae + (-ab^2)^{\frac{2}{3}} c \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{3ab^2} - \frac{\left((-ab^2)^{\frac{1}{3}} ae - (-ab^2)^{\frac{2}{3}} c \right) \log \left(x^2 + x \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{6ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")

[Out] x*e/b + 1/3*d*log(abs(b*x^3 + a))/b - 1/3*sqrt(3)*((-a*b^2)^(1/3)*a*e + (-a*b^2)^(2/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) - 1/6*((-a*b^2)^(1/3)*a*e - (-a*b^2)^(2/3)*c)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2) - 1/3*(b^3*c*(-a/b)^(1/3) - a*b^2*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3)

$$3.340 \quad \int \frac{c+dx+ex^2}{a+bx^3} dx$$

Optimal. Leaf size=177

$$-\frac{\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}} + \frac{\left(\sqrt[3]{bc} - \sqrt[3]{ad}\right) \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{3a^{2/3}b^{2/3}} - \frac{\left(\sqrt[3]{ad} + \sqrt[3]{bc}\right) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}} + \frac{e \log(a + bx^3)}{3b}$$

[Out] -(((b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3)) + ((b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(2/3)) - ((c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3)) + (e*Log[a + b*x^3])/(3*b)

Rubi [A] time = 0.132435, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}} + \frac{\left(\sqrt[3]{bc} - \sqrt[3]{ad}\right) \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{3a^{2/3}b^{2/3}} - \frac{\left(\sqrt[3]{ad} + \sqrt[3]{bc}\right) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}} + \frac{e \log(a + bx^3)}{3b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/(a + b*x^3), x]

[Out] -(((b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3)) + ((b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(2/3)) - ((c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3)) + (e*Log[a + b*x^3])/(3*b)

Rule 1871

Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Dist[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^n), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2}{a + bx^3} dx &= e \int \frac{x^2}{a + bx^3} dx + \int \frac{c + dx}{a + bx^3} dx \\
&= \frac{e \log(a + bx^3)}{3b} + \frac{\int \frac{\sqrt[3]{a}(2\sqrt[3]{bc} + \sqrt[3]{ad}) + \sqrt[3]{b}(-\sqrt[3]{bc} + \sqrt[3]{ad})x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} dx}{3a^{2/3}\sqrt[3]{b}} + \frac{\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{bx}} dx}{3a^{2/3}} \\
&= \frac{\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{2/3}\sqrt[3]{b}} + \frac{e \log(a + bx^3)}{3b} - \frac{(\sqrt[3]{bc} - \sqrt[3]{ad}) \int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} dx}{6a^{2/3}b^{2/3}} + \frac{1}{2} \left(\frac{c}{\sqrt[3]{a}} + \frac{d}{\sqrt[3]{b}}\right) \\
&= \frac{\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{2/3}\sqrt[3]{b}} - \frac{(\sqrt[3]{bc} - \sqrt[3]{ad}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{6a^{2/3}b^{2/3}} + \frac{e \log(a + bx^3)}{3b} + \frac{(\sqrt[3]{bc} - \sqrt[3]{ad}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{\sqrt{3}a^{2/3}b^{2/3}} \\
&= -\frac{(\sqrt[3]{bc} - \sqrt[3]{ad}) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}} + \frac{\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{2/3}\sqrt[3]{b}} - \frac{(\sqrt[3]{bc} - \sqrt[3]{ad}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{6a^{2/3}b^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.0916627, size = 176, normalized size = 0.99

$$-\sqrt[3]{b}(\sqrt[3]{a}\sqrt[3]{bc} - a^{2/3}d) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2) + 2\sqrt[3]{b}(\sqrt[3]{a}\sqrt[3]{bc} - a^{2/3}d) \log(\sqrt[3]{a} + \sqrt[3]{bx}) - 2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}(\sqrt[3]{ad} + \sqrt[3]{bc}) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)$$

$6ab$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(a + b*x^3), x]

[Out] $(-2*\text{Sqrt}[3]*a^{(1/3)}*b^{(1/3)}*(b^{(1/3)}*c + a^{(1/3)}*d)*\text{ArcTan}[(1 - (2*b^{(1/3)}*x)/a^{(1/3)})/\text{Sqrt}[3]] + 2*b^{(1/3)}*(a^{(1/3)}*b^{(1/3)}*c - a^{(2/3)}*d)*\text{Log}[a^{(1/3)} + b^{(1/3)}*x] - b^{(1/3)}*(a^{(1/3)}*b^{(1/3)}*c - a^{(2/3)}*d)*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2] + 2*a*e*\text{Log}[a + b*x^3])/(6*a*b)$

Maple [A] time = 0.003, size = 200, normalized size = 1.1

$$\frac{c}{3b} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{c}{6b} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{c\sqrt{3}}{3b} \arctan\left(\frac{\sqrt{3}}{3} \left(2x \frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{d}{3b} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d*x+c)/(b*x^3+a),x)`

[Out] $\frac{1}{3} \frac{c}{b} \frac{1}{(b*a)^{2/3}} \ln(x + (1/b*a)^{1/3}) - \frac{1}{6} \frac{c}{b} \frac{1}{(b*a)^{2/3}} \ln(x^2 - (1/b*a)^{1/3} * x + (1/b*a)^{2/3}) + \frac{1}{3} \frac{c}{b} \frac{1}{(b*a)^{2/3}} * 3^{1/2} * \arctan(1/3 * 3^{1/2} * (2/(1/b*a)^{1/3} * x - 1)) - \frac{1}{3} \frac{d}{b} \frac{1}{(b*a)^{1/3}} \ln(x + (1/b*a)^{1/3}) + \frac{1}{6} \frac{d}{b} \frac{1}{(b*a)^{1/3}} \ln(x^2 - (1/b*a)^{1/3} * x + (1/b*a)^{2/3}) + \frac{1}{3} \frac{d}{b} \frac{1}{(b*a)^{1/3}} * 3^{1/2} / b \frac{1}{(b*a)^{1/3}} * \arctan(1/3 * 3^{1/2} * (2/(1/b*a)^{1/3} * x - 1)) + \frac{1}{3} * e * \ln(b*x^3+a) / b$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [C] time = 9.51533, size = 10437, normalized size = 58.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")`

[Out]
$$\begin{aligned} & -\frac{1}{12} * (2 * (2 * (1/2)^{(2/3)} * (-I * \sqrt{3}) + 1) * (e^2/b^2 - (b*c*d + a*e^2)/(a*b^2)) / (2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^{1/3} + (1/2)^{(1/3)} * (I * \sqrt{3} + 1) * (2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^{1/3} - 2*e/b) * b * \log(1/4 * (2 * (1/2)^{(2/3)} * (-I * \sqrt{3}) + 1) * (e^2/b^2 - (b*c*d + a*e^2)/(a*b^2)) / (2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^{1/3} + (1/2)^{(1/3)} * (I * \sqrt{3} + 1) * (2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^{1/3} - 2*e/b)^2 * a^2 * b^2 * d + 2*a*b*c*d^2 - a*b*c^2*e + a^2*d*e^2 - 1/2*(a*b^2*c^2 - 2*a^2*b*d*e) * (2 * (1/2)^{(2/3)} * (-I * \sqrt{3}) + 1) * (e^2/b^2 - (b*c*d + a*e^2)/(a*b^2)) / (2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^{1/3} + (1/2)^{(1/3)} * (I * \sqrt{3} + 1) * (2 * (1/2)^{(2/3)} * (-I * \sqrt{3}) + 1) * (e^2/b^2 - (b*c*d + a*e^2)/(a*b^2)) / (2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^{1/3} \end{aligned}$$

$$\begin{aligned}
& b^2c^3 + a^2e^3 - (d^3 - 3c*d*e)*a*b)/(a^2*b^3))^{1/3} + (1/2)^{1/3}*(I* \\
& \text{sqrt}(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^ \\
& 2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3c*d*e)*a*b)/(a^2*b^3))^{1/3} - 2*e/b \\
&) + (b^2*c^3 + a*b*d^3)*x - ((2*(1/2)^{2/3})*(-I*\text{sqrt}(3) + 1)*(e^2/b^2 - (b \\
& *c*d + a*e^2)/(a*b^2)))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + \\
& a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3c*d*e)*a*b)/(a^2*b^3))^{1/3} \\
& + (1/2)^{1/3}*(I*\text{sqrt}(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + \\
& (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3c*d*e)*a*b)/(a^2 \\
& *b^3))^{1/3} - 2*e/b)*b + 3*\text{sqrt}(1/3)*b*\text{sqrt}(-((2*(1/2)^{2/3})*(-I*\text{sqrt}(3) + \\
& 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2)))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a \\
& *b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3c*d*e)*a* \\
& b)/(a^2*b^3))^{1/3} + (1/2)^{1/3}*(I*\text{sqrt}(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a \\
& *e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3 \\
& *c*d*e)*a*b)/(a^2*b^3))^{1/3} - 2*e/b)^2*a*b^2 + 4*(2*(1/2)^{2/3})*(-I*\text{sqrt}(\\
& 3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2)))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)* \\
& e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3c*d*e \\
&)*a*b)/(a^2*b^3))^{1/3} + (1/2)^{1/3}*(I*\text{sqrt}(3) + 1)*(2*e^3/b^3 - 3*(b*c*d \\
& + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 \\
& - 3c*d*e)*a*b)/(a^2*b^3))^{1/3} - 2*e/b)*a*b*e + 16*b*c*d + 4*a*e^2)/(a*b \\
& ^2) + 6*e)*\log(-1/4*(2*(1/2)^{2/3})*(-I*\text{sqrt}(3) + 1)*(e^2/b^2 - (b*c*d + a* \\
& e^2)/(a*b^2)))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a \\
& ^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3c*d*e)*a*b)/(a^2*b^3))^{1/3} + (1/2 \\
&)^{1/3}*(I*\text{sqrt}(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + \\
& a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3c*d*e)*a*b)/(a^2*b^3))^{1/3} \\
& - 2*e/b)^2*a^2*b^2*d - 2*a*b*c*d^2 + a*b*c^2*e - a^2*d*e^2 + 1/2*(a*b^2 \\
& *c^2 - 2*a^2*b*d*e)*(2*(1/2)^{2/3})*(-I*\text{sqrt}(3) + 1)*(e^2/b^2 - (b*c*d + a*e \\
& ^2)/(a*b^2)))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^ \\
& 2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3c*d*e)*a*b)/(a^2*b^3))^{1/3} + (1/2) \\
& ^{1/3}*(I*\text{sqrt}(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + \\
& a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3c*d*e)*a*b)/(a^2*b^3))^{1/3} \\
& - 2*e/b) + 2*(b^2*c^3 + a*b*d^3)*x + 3/4*\text{sqrt}(1/3)*((2*(1/2)^{2/3})*(-I*s \\
& \text{qrt}(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2)))/(2*e^3/b^3 - 3*(b*c*d + a*e \\
& ^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3c \\
& *d*e)*a*b)/(a^2*b^3))^{1/3} + (1/2)^{1/3}*(I*\text{sqrt}(3) + 1)*(2*e^3/b^3 - 3*(b \\
& *c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - \\
& (d^3 - 3c*d*e)*a*b)/(a^2*b^3))^{1/3} - 2*e/b)*a^2*b^2*d + 2*a*b^2*c^2 + 2* \\
& a^2*b*d*e)*\text{sqrt}(-((2*(1/2)^{2/3})*(-I*\text{sqrt}(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2 \\
&)/(a*b^2)))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2* \\
& b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3c*d*e)*a*b)/(a^2*b^3))^{1/3} + (1/2)^{(\\
& 1/3}*(I*\text{sqrt}(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a* \\
& d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3c*d*e)*a*b)/(a^2*b^3))^{1/3} \\
& - 2*e/b)^2*a*b^2 + 4*(2*(1/2)^{2/3})*(-I*\text{sqrt}(3) + 1)*(e^2/b^2 - (b*c*d + a \\
& *e^2)/(a*b^2)))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(\\
& a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3c*d*e)*a*b)/(a^2*b^3))^{1/3} + (1/ \\
& 2)^{1/3}*(I*\text{sqrt}(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3
\end{aligned}$$

$$\begin{aligned}
& + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^{(1/3)} - 2*e/b)*a*b*e + 16*b*c*d + 4*a*e^2)/(a*b^2))) - ((2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2)))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^{(1/3)} - 2*e/b)*b - 3*\sqrt{1/3}*b*\sqrt{-(2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2)))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^{(1/3)} - 2*e/b)^2*a*b^2 + 4*(2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2)))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^{(1/3)} - 2*e/b)*a*b*e + 16*b*c*d + 4*a*e^2)/(a*b^2)) + 6*e)*\log(-1/4*(2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2)))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^{(1/3)} - 2*e/b)^2*a^2*b^2*d - 2*a*b*c*d^2 + a*b*c^2*e - a^2*d*e^2 + 1/2*(a*b^2*c^2 - 2*a^2*b*d*e)*(2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2)))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^{(1/3)} - 2*e/b) + 2*(b^2*c^3 + a*b*d^3)*x - 3/4*\sqrt{1/3)*((2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2)))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^{(1/3)} - 2*e/b)*a^2*b^2*d + 2*a*b^2*c^2 + 2*a^2*b*d*e)*\sqrt{-(2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2)))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^{(1/3)} - 2*e/b)^2*a*b^2 + 4*(2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2)))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^{(1/3)} - 2*e/b)*a*b*e + 16*b*c*d + 4*a*e^2)/(a*b^2))
\end{aligned}$$

))/b

Sympy [A] time = 1.27179, size = 160, normalized size = 0.9

RootSum($27t^3a^2b^3 - 27t^2a^2b^2e + t(9a^2be^2 + 9ab^2cd) - a^2e^3 - 3abcde + abd^3 - b^2c^3$, $t \mapsto t \log\left(x + \frac{9t^2a^2b^2d - 6ta^2t}{\dots}\right)$)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/(b*x**3+a),x)

[Out] RootSum($27*_t**3*a**2*b**3 - 27*_t**2*a**2*b**2*e + _t*(9*a**2*b*e**2 + 9*a*b**2*c*d) - a**2*e**3 - 3*a*b*c*d*e + a*b*d**3 - b**2*c**3$, Lambda(_t, _t*log(x + (9*_t**2*a**2*b**2*d - 6*_t*a**2*b*d*e + 3*_t*a*b**2*c**2 + a**2*d*e**2 - a*b*c**2*e + 2*a*b*c*d**2)/(a*b*d**3 + b**2*c**3))))

Giac [A] time = 1.07264, size = 246, normalized size = 1.39

$$\frac{e \log(|bx^3 + a|)}{3b} - \frac{\left(bd\left(-\frac{a}{b}\right)^{\frac{1}{3}} + bc\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}bc - \left(-ab^2\right)^{\frac{2}{3}}d\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")

[Out] $\frac{1}{3}e \log(\text{abs}(b*x^3 + a))/b - \frac{1}{3}*(b*d*(-a/b)^{(1/3)} + b*c)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/(a*b) + \frac{1}{3}*sqrt(3)*((-a*b^2)^{(1/3)}*b*c - (-a*b^2)^{(2/3)}*d)*\arctan(1/3*sqrt(3)*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(a*b^2) + \frac{1}{6}*((-a*b^2)^{(1/3)}*a*b^3*c + (-a*b^2)^{(2/3)}*a*b^2*d)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a^2*b^4)$

$$3.341 \quad \int \frac{c+dx+ex^2}{x(a+bx^3)} dx$$

Optimal. Leaf size=184

$$\frac{\left(d - \frac{\sqrt[3]{ae}}{\sqrt[3]{b}}\right) \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}} + \frac{\left(\sqrt[3]{bd} - \sqrt[3]{ae}\right) \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{3a^{2/3}b^{2/3}} - \frac{\left(\sqrt[3]{ae} + \sqrt[3]{bd}\right) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}} - \frac{c \log(a+bx)}{3a}$$

[Out] -(((b^(1/3)*d + a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3)) + (c*Log[x])/a + ((b^(1/3)*d - a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(2/3)) - ((d - (a^(1/3)*e)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3)) - (c*Log[a + b*x^3])/(3*a)

Rubi [A] time = 0.206411, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\left(d - \frac{\sqrt[3]{ae}}{\sqrt[3]{b}}\right) \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}} + \frac{\left(\sqrt[3]{bd} - \sqrt[3]{ae}\right) \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{3a^{2/3}b^{2/3}} - \frac{\left(\sqrt[3]{ae} + \sqrt[3]{bd}\right) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}} - \frac{c \log(a+bx)}{3a}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/(x*(a + b*x^3)), x]

[Out] -(((b^(1/3)*d + a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3)) + (c*Log[x])/a + ((b^(1/3)*d - a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(2/3)) - ((d - (a^(1/3)*e)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3)) - (c*Log[a + b*x^3])/(3*a)

Rule 1834

Int[((Pq_)*((c_)*(x_)^(m_)))/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] & & PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]

Rule 1871

```
Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Dist[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

`Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]`

Rubi steps

$$\begin{aligned}
 \int \frac{c + dx + ex^2}{x(a + bx^3)} dx &= \int \left(\frac{c}{ax} + \frac{ad + aex - bcx^2}{a(a + bx^3)} \right) dx \\
 &= \frac{c \log(x)}{a} + \frac{\int \frac{ad + aex - bcx^2}{a + bx^3} dx}{a} \\
 &= \frac{c \log(x)}{a} + \frac{\int \frac{ad + aex}{a + bx^3} dx}{a} - \frac{(bc) \int \frac{x^2}{a + bx^3} dx}{a} \\
 &= \frac{c \log(x)}{a} - \frac{c \log(a + bx^3)}{3a} + \frac{\int \frac{\sqrt[3]{a}(2a\sqrt[3]{bd} + a^{4/3}e) + \sqrt[3]{b}(-a\sqrt[3]{bd} + a^{4/3}e)x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} dx}{3a^{5/3}\sqrt[3]{b}} + \frac{\left(d - \frac{\sqrt[3]{ae}}{\sqrt[3]{b}}\right) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{bx}} dx}{3a^{2/3}} \\
 &= \frac{c \log(x)}{a} + \frac{\left(d - \frac{\sqrt[3]{ae}}{\sqrt[3]{b}}\right) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{2/3}\sqrt[3]{b}} - \frac{c \log(a + bx^3)}{3a} + \frac{1}{2} \left(\frac{d}{\sqrt[3]{a}} + \frac{e}{\sqrt[3]{b}} \right) \int \frac{1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} \\
 &= \frac{c \log(x)}{a} + \frac{\left(d - \frac{\sqrt[3]{ae}}{\sqrt[3]{b}}\right) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{2/3}\sqrt[3]{b}} - \frac{\left(d - \frac{\sqrt[3]{ae}}{\sqrt[3]{b}}\right) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{6a^{2/3}\sqrt[3]{b}} - \frac{c \log(a + bx^3)}{3a} + \\
 &= -\frac{(\sqrt[3]{bd} + \sqrt[3]{ae}) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}} + \frac{c \log(x)}{a} + \frac{\left(d - \frac{\sqrt[3]{ae}}{\sqrt[3]{b}}\right) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{2/3}\sqrt[3]{b}} - \frac{\left(d - \frac{\sqrt[3]{ae}}{\sqrt[3]{b}}\right) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{6a^{2/3}\sqrt[3]{b}}
 \end{aligned}$$

Mathematica [A] time = 0.088848, size = 176, normalized size = 0.96

$$\frac{(a^{2/3}e - \sqrt[3]{a}\sqrt[3]{bd}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2) + 2(\sqrt[3]{a}\sqrt[3]{bd} - a^{2/3}e) \log(\sqrt[3]{a} + \sqrt[3]{bx}) - 2b^{2/3}c \log(a + bx^3) - 2\sqrt{3}\sqrt[3]{a}(\sqrt[3]{ae})}{6ab^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)), x]

[Out] (-2*Sqrt[3]*a^(1/3)*(b^(1/3)*d + a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 6*b^(2/3)*c*Log[x] + 2*(a^(1/3)*b^(1/3)*d - a^(2/3)*e)*Log[a

$$\frac{1}{3} \left[a^{1/3} + b^{1/3} x \right] + \frac{(-a^{1/3} b^{1/3} d + a^{2/3} e) \operatorname{Log}[a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2] - 2 b^{2/3} c \operatorname{Log}[a + b x^3]}{(6 a b^{2/3})}$$

Maple [A] time = 0.005, size = 207, normalized size = 1.1

$$\frac{d}{3b} \ln \left(x + \sqrt[3]{\frac{a}{b}} \right) \left(\frac{a}{b} \right)^{-\frac{2}{3}} - \frac{d}{6b} \ln \left(x^2 - \sqrt[3]{\frac{a}{b}} x + \left(\frac{a}{b} \right)^{\frac{2}{3}} \right) \left(\frac{a}{b} \right)^{-\frac{2}{3}} + \frac{d\sqrt{3}}{3b} \arctan \left(\frac{\sqrt{3}}{3} \left(2x \frac{1}{\sqrt[3]{\frac{a}{b}}} - 1 \right) \right) \left(\frac{a}{b} \right)^{-\frac{2}{3}} - \frac{e}{3b} \ln \left(x + \sqrt[3]{\frac{a}{b}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d*x+c)/x/(b*x^3+a),x)`

[Out] $\frac{1}{3} \frac{d}{b} \frac{1}{(1/b*a)^{2/3}} \ln(x + (1/b*a)^{1/3}) - \frac{1}{6} \frac{d}{b} \frac{1}{(1/b*a)^{2/3}} \ln(x^2 - (1/b*a)^{1/3} x + (1/b*a)^{2/3}) + \frac{1}{3} \frac{d}{b} \frac{1}{(1/b*a)^{2/3}} \arctan\left(\frac{1}{3} \sqrt{3} \left(2x \frac{1}{\sqrt[3]{a/b}} - 1 \right)\right) - \frac{e}{3b} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) + \frac{1}{6} \frac{e}{b} \frac{1}{(1/b*a)^{1/3}} \ln(x^2 - (1/b*a)^{1/3} x + (1/b*a)^{2/3}) + \frac{1}{3} \frac{e}{b} \frac{1}{(1/b*a)^{1/3}} \arctan\left(\frac{1}{3} \sqrt{3} \left(2x \frac{1}{\sqrt[3]{a/b}} - 1 \right)\right) - \frac{1}{3} \frac{c}{a} \ln(b*x^3+a) + c \ln(x)/a$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [C] time = 10.6854, size = 10657, normalized size = 57.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="fricas")`

[Out]
$$\begin{aligned}
& -1/36*(2*((-I*\sqrt{3}) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b)))/(-1/27*c^3/a^3 \\
& + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54 \\
& *(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^{(1/3)} + 9*(I*\sqrt{3}) \\
& + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3) \\
& /(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^{(1/3)} \\
&) + 6*c/a)*a*\log(1/36*((-I*\sqrt{3}) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b)) \\
& /(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) \\
& - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^{(1/3)} + \\
& 9*(I*\sqrt{3}) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b \\
& *d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a \\
& ^3*b^2))^{(1/3)} + 6*c/a)^2*a^2*b*e + b*c*d^2 + b*c^2*e + 2*a*d*e^2 - 1/6*(a* \\
& b*d^2 + 2*a*b*c*e)*((-I*\sqrt{3}) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(- \\
& 1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b \\
& ^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^{(1/3)} + 9*(\\
& I*\sqrt{3}) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^ \\
& 3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3* \\
& b^2))^{(1/3)} + 6*c/a) + (b*d^3 + a*e^3)*x - (((-I*\sqrt{3}) + 1)*(c^2/a^2 - (\\
& b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1 \\
& /54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a \\
& *b)/(a^3*b^2))^{(1/3)} + 9*(I*\sqrt{3}) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d \\
& *e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - \\
& (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^{(1/3)} + 6*c/a)*a + 3*\sqrt{1/3}*a*\sqrt{-(((\\
& -I*\sqrt{3}) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b \\
& *c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + \\
& a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^{(1/3)} + 9*(I*\sqrt{3}) + 1)*(-1/27* \\
& c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - \\
& 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^{(1/3)} + 6*c/a)^2 \\
& *a^2*b - 12*((-I*\sqrt{3}) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^ \\
& 3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1 \\
& /54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^{(1/3)} + 9*(I*\sqrt{3} \\
& (3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e \\
& ^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^{(\\
& 1/3)} + 6*c/a)*a*b*c + 36*b*c^2 + 144*a*d*e)/(a^2*b)) - 18*c)*\log(-1/36*((-I \\
& *\sqrt{3}) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b* \\
& c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a \\
& ^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^{(1/3)} + 9*(I*\sqrt{3}) + 1)*(-1/27*c \\
& ^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - \\
& 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^{(1/3)} + 6*c/a)^2* \\
& a^2*b*e - b*c*d^2 - b*c^2*e - 2*a*d*e^2 + 1/6*(a*b*d^2 + 2*a*b*c*e)*((-I*\sqrt{3} \\
& + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2 \\
& + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2* \\
& e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^{(1/3)} + 9*(I*\sqrt{3}) + 1)*(-1/27*c^3/ \\
& a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/5 \\
& 4*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^{(1/3)} + 6*c/a) + 2*(\\
& b*d^3 + a*e^3)*x + 1/12*\sqrt{1/3}*(((-I*\sqrt{3}) + 1)*(c^2/a^2 - (b*c^2 + a
\end{aligned}$$

$$\begin{aligned}
& d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 \\
& + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b \\
& ^2))^{\frac{1}{3}} + 9*(I*\sqrt{3} + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3 \\
& *b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c \\
& *d*e)*a*b)/(a^3*b^2))^{\frac{1}{3}} + 6*c/a)*a^2*b*e + 6*a*b*d^2 - 6*a*b*c*e)*\sqrt{ \\
& -(((-I*\sqrt{3} + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b)))/(-1/27*c^3/a^3 + 1/ \\
& 18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c \\
& ^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^{\frac{1}{3}} + 9*(I*\sqrt{3} + 1)*(- \\
& 1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b \\
& ^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^{\frac{1}{3}} + 6*c \\
& /a)^2*a^2*b - 12*((-I*\sqrt{3} + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b)))/(-1/ \\
& 27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2 \\
&) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^{\frac{1}{3}} + 9*(I* \\
& \sqrt{3} + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 \\
& + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^ \\
& 2))^{\frac{1}{3}} + 6*c/a)*a*b*c + 36*b*c^2 + 144*a*d*e)/(a^2*b))) - (((-I*\sqrt{3} \\
& + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b)))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d \\
& *e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - \\
& (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^{\frac{1}{3}} + 9*(I*\sqrt{3} + 1)*(-1/27*c^3/a^3 + \\
& 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2 \\
& *c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^{\frac{1}{3}} + 6*c/a)*a - 3*\sqrt{ \\
& \frac{1}{3}})*a*\sqrt{-(((-I*\sqrt{3} + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b)))/(-1/27* \\
& c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - \\
& 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^{\frac{1}{3}} + 9*(I*\sqrt{ \\
& 3} + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a \\
& *e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2)) \\
& ^{\frac{1}{3}} + 6*c/a)^2*a^2*b - 12*((-I*\sqrt{3} + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(\\
& a^2*b)))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e \\
& ^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^{\frac{1}{3}} \\
& + 9*(I*\sqrt{3} + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + \\
& 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)* \\
& a*b)/(a^3*b^2))^{\frac{1}{3}} + 6*c/a)*a*b*c + 36*b*c^2 + 144*a*d*e)/(a^2*b)) - 18* \\
& c)*\log(-1/36*((-I*\sqrt{3} + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b)))/(-1/27*c \\
& ^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - \\
& 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^{\frac{1}{3}} + 9*(I*\sqrt{ \\
& 3} + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a* \\
& e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^{\frac{1}{3}} \\
& + 6*c/a)^2*a^2*b*e - b*c*d^2 - b*c^2*e - 2*a*d*e^2 + 1/6*(a*b*d^2 + 2 \\
& *a*b*c*e))*((-I*\sqrt{3} + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b)))/(-1/27*c^3/ \\
& a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/5 \\
& 4*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^{\frac{1}{3}} + 9*(I*\sqrt{3} \\
& + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3 \\
&)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^{\frac{1}{3}} \\
& + 6*c/a) + 2*(b*d^3 + a*e^3)*x - 1/12*\sqrt{\frac{1}{3}})*(((-I*\sqrt{3} + 1)*(c^2/ \\
& a^2 - (b*c^2 + a*d*e)/(a^2*b)))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3
\end{aligned}$$

$$\begin{aligned}
& *b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^{(1/3)} + 9*(I*\sqrt{3} + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^{(1/3)} + 6*c/a)*a^2*b*e + 6*a*b*d^2 - 6*a*b*c*e)*\sqrt{-(((-I*\sqrt{3} + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b)))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^{(1/3)} + 9*(I*\sqrt{3} + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^{(1/3)} + 6*c/a)^2*a^2*b - 12*((-I*\sqrt{3} + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b)))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^{(1/3)} + 9*(I*\sqrt{3} + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^{(1/3)} + 6*c/a)*a*b*c + 36*b*c^2 + 144*a*d*e)/(a^2*b)) - 36*c*log(x))/a
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/x/(b*x**3+a),x)

[Out] Timed out

Giac [A] time = 1.08962, size = 267, normalized size = 1.45

$$-\frac{c \log(|bx^3 + a|)}{3a} + \frac{c \log(|x|)}{a} + \frac{\sqrt{3} \left((-ab^2)^{\frac{1}{3}} bd - (-ab^2)^{\frac{2}{3}} e \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{3ab^2} - \frac{\left(a^2b \left(-\frac{a}{b} \right)^{\frac{1}{3}} e + a^2bd \right) \left(-\frac{a}{b} \right)^{\frac{1}{3}} \log \left(\dots \right)}{3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="giac")

[Out] -1/3*c*log(abs(b*x^3 + a))/a + c*log(abs(x))/a + 1/3*sqrt(3)*((-a*b^2)^(1/3))*b*d - (-a*b^2)^(2/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1

$$\begin{aligned} & /3)/(a*b^2) - 1/3*(a^2*b*(-a/b)^{(1/3)}*e + a^2*b*d)*(-a/b)^{(1/3)}*\log(\text{abs}(x \\ & - (-a/b)^{(1/3)}))/(a^3*b) + 1/6*((-a*b^2)^{(1/3)}*a*b^3*d + (-a*b^2)^{(2/3)}*a*b \\ & ^2*e)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a^2*b^4) \end{aligned}$$

$$3.342 \quad \int \frac{c+dx+ex^2}{x^2(a+bx^3)} dx$$

Optimal. Leaf size=192

$$-\frac{(a^{2/3}e + b^{2/3}c) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{6a^{4/3}\sqrt[3]{b}} + \frac{(a^{2/3}e + b^{2/3}c) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{4/3}\sqrt[3]{b}} + \frac{(b^{2/3}c - a^{2/3}e) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}\sqrt[3]{b}} - \frac{d \log}{d \log}$$

[Out] $-(c/(a*x)) + ((b^{(2/3)*c} - a^{(2/3)*e})*\text{ArcTan}[(a^{(1/3)} - 2*b^{(1/3)*x})/(\text{Sqrt}[3]*a^{(1/3)})]) / (\text{Sqrt}[3]*a^{(4/3)*b^{(1/3)}}) + (d*\text{Log}[x])/a + ((b^{(2/3)*c} + a^{(2/3)*e})*\text{Log}[a^{(1/3)} + b^{(1/3)*x}] / (3*a^{(4/3)*b^{(1/3)}}) - ((b^{(2/3)*c} + a^{(2/3)*e})*\text{Log}[a^{(2/3)} - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2}] / (6*a^{(4/3)*b^{(1/3)}}) - (d*\text{Log}[a + b*x^3]) / (3*a)$

Rubi [A] time = 0.213757, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{(a^{2/3}e + b^{2/3}c) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{6a^{4/3}\sqrt[3]{b}} + \frac{(a^{2/3}e + b^{2/3}c) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{4/3}\sqrt[3]{b}} + \frac{(b^{2/3}c - a^{2/3}e) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}\sqrt[3]{b}} - \frac{d \log}{d \log}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x + e*x^2)/(x^2*(a + b*x^3)), x]$

[Out] $-(c/(a*x)) + ((b^{(2/3)*c} - a^{(2/3)*e})*\text{ArcTan}[(a^{(1/3)} - 2*b^{(1/3)*x})/(\text{Sqrt}[3]*a^{(1/3)})]) / (\text{Sqrt}[3]*a^{(4/3)*b^{(1/3)}}) + (d*\text{Log}[x])/a + ((b^{(2/3)*c} + a^{(2/3)*e})*\text{Log}[a^{(1/3)} + b^{(1/3)*x}] / (3*a^{(4/3)*b^{(1/3)}}) - ((b^{(2/3)*c} + a^{(2/3)*e})*\text{Log}[a^{(2/3)} - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2}] / (6*a^{(4/3)*b^{(1/3)}}) - (d*\text{Log}[a + b*x^3]) / (3*a)$

Rule 1834

$\text{Int}[(\text{Pq}_*) * ((c_*) * (x_*)^{(m_*)}) / ((a_*) + (b_*) * (x_*)^{(n_*)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m * \text{Pq}]/(a + b*x^n), x], x] /; \text{FreeQ}[\{a, b, c, m\}, x] \& \& \text{PolyQ}[\text{Pq}, x] \& \& \text{IntegerQ}[n] \& \& !\text{IGtQ}[m, 0]$

Rule 1871

```
Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Dist[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned}
 \int \frac{c + dx + ex^2}{x^2(a + bx^3)} dx &= \int \left(\frac{c}{ax^2} + \frac{d}{ax} + \frac{ae - bcx - bdx^2}{a(a + bx^3)} \right) dx \\
 &= -\frac{c}{ax} + \frac{d \log(x)}{a} + \frac{\int \frac{ae - bcx - bdx^2}{a + bx^3} dx}{a} \\
 &= -\frac{c}{ax} + \frac{d \log(x)}{a} + \frac{\int \frac{ae - bcx}{a + bx^3} dx}{a} - \frac{(bd) \int \frac{x^2}{a + bx^3} dx}{a} \\
 &= -\frac{c}{ax} + \frac{d \log(x)}{a} - \frac{d \log(a + bx^3)}{3a} + \frac{\int \frac{\sqrt[3]{a}(-\sqrt[3]{abc} + 2a\sqrt[3]{be}) + \sqrt[3]{b}(-\sqrt[3]{abc} - a\sqrt[3]{be})x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} dx}{3a^{5/3}\sqrt[3]{b}} + \frac{(b^{2/3}c + a^{2/3}e) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{bx}} dx}{3a^{4/3}} \\
 &= -\frac{c}{ax} + \frac{d \log(x)}{a} + \frac{(b^{2/3}c + a^{2/3}e) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{4/3}\sqrt[3]{b}} - \frac{d \log(a + bx^3)}{3a} - \frac{(b^{2/3}c - a^{2/3}e) \int \frac{1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} dx}{2a} \\
 &= -\frac{c}{ax} + \frac{d \log(x)}{a} + \frac{(b^{2/3}c + a^{2/3}e) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{4/3}\sqrt[3]{b}} - \frac{(b^{2/3}c + a^{2/3}e) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{6a^{4/3}\sqrt[3]{b}} - \frac{d \log(a + bx^3)}{3a} \\
 &= -\frac{c}{ax} + \frac{(b^{2/3}c - a^{2/3}e) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}\sqrt[3]{b}} + \frac{d \log(x)}{a} + \frac{(b^{2/3}c + a^{2/3}e) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{4/3}\sqrt[3]{b}} - \frac{(b^{2/3}c + a^{2/3}e) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{6a^{4/3}\sqrt[3]{b}}
 \end{aligned}$$

Mathematica [A] time = 0.238511, size = 184, normalized size = 0.96

$$\frac{(a^{2/3}b^{2/3}c + a^{4/3}e) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{\sqrt[3]{b}} - \frac{2(a^{2/3}b^{2/3}c + a^{4/3}e) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{\sqrt[3]{b}} + \frac{2\sqrt{3}a^{2/3}(a^{2/3}e - b^{2/3}c) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}} + 2ad \log(a + bx^3) + \frac{6ac}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)), x]

[Out] $-\left(\frac{6ac}{x} + (2\sqrt[3]{a^{2/3}}(-b^{2/3}c) + a^{2/3}e)\operatorname{ArcTan}\left[\frac{1 - (2b^{1/3}x)/a^{1/3}}{\sqrt[3]{3}}\right]\right)/b^{1/3} - 6ad\operatorname{Log}[x] - (2(a^{2/3}b^{2/3}c + a^{4/3}e)\operatorname{Log}[a^{1/3} + b^{1/3}x])/b^{1/3} + ((a^{2/3}b^{2/3}c + a^{4/3}e)\operatorname{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2])/b^{1/3} + 2ad\operatorname{Log}[a + b^3x^3]/(6a^2)$

Maple [A] time = 0.007, size = 216, normalized size = 1.1

$$\frac{e}{3b} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{e}{6b} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{\sqrt{3}e}{3b} \arctan\left(\frac{\sqrt{3}}{3} \left(2x \frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{c}{3a} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d*x+c)/x^2/(b*x^3+a), x)`

[Out] $\frac{1}{3} \frac{b}{(1/ba)^{2/3}} \ln(x + (1/ba)^{1/3}) e - \frac{1}{6} \frac{b}{(1/ba)^{2/3}} \ln(x^2 - (1/ba)^{1/3}x + (1/ba)^{2/3}) e + \frac{1}{3} \frac{b}{(1/ba)^{2/3}} 3^{1/2} \arctan(1/3 3^{1/2} (2/(1/ba)^{1/3}x - 1)) e + \frac{1}{3} \frac{a}{(1/ba)^{1/3}} \ln(x + (1/ba)^{1/3}) c - \frac{1}{6} \frac{a}{(1/ba)^{1/3}} \ln(x^2 - (1/ba)^{1/3}x + (1/ba)^{2/3}) c - \frac{1}{3} \frac{a}{3^{1/2}} \frac{3^{1/2}}{(1/ba)^{1/3}} \arctan(1/3 3^{1/2} (2/(1/ba)^{1/3}x - 1)) c - \frac{1}{3} d \ln(bx^3+a) / a + d \ln(x) / a - c/a/x$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d*x+c)/x^2/(b*x^3+a), x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [C] time = 9.23121, size = 10066, normalized size = 52.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/36*(2*((-I*\sqrt{3}) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) \\ & - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^{1/3} + 9*(I*\sqrt{3}) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) \\ & - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^{1/3} + 6*d/a)*a*x*\log(-1/36*((-I*\sqrt{3}) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) \\ & - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^{1/3} + 9*(I*\sqrt{3}) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) \\ & - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^{1/3} + 6*d/a)^2*a^3*b*c - a*b*c*d^2 + 2*a*b*c^2*e + a^2*d*e^2 + 1/6*(2*a^2*b*c*d - a^3*e^2)*((-I*\sqrt{3}) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) \\ & - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^{1/3} + 9*(I*\sqrt{3}) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) \\ & - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^{1/3} + 6*d/a - (b^2*c^3 - a^2*e^3)*x) - 36*d*x*\log(x) - (((-I*\sqrt{3}) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) \\ & - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^{1/3} + 9*(I*\sqrt{3}) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) \\ & - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^{1/3} + 6*d/a)*a*x - 3*\sqrt{1/3}*a*x*\sqrt{-(((-I*\sqrt{3}) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) \\ & - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^{1/3} + 9*(I*\sqrt{3}) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) \\ & - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^{1/3} + 6*d/a)^2*a^2 - 12*((-I*\sqrt{3}) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) \\ & - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^{1/3} + 9*(I*\sqrt{3}) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) \\ & - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^{1/3} + 6*d/a)*a*d + 36*d^2 - 144*c*e)/a^2 - 18*d*x)*\log(1/36*((-I*\sqrt{3}) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) \\ & - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^{1/3} + 9*(I*\sqrt{3}) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) \\ & - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^{1/3} + 6*d/a)^2*a^3*b*c + a*b*c*d^2 - 2*a*b*c^2*e - a^2*d*e^2 - 1/6*(2*a^2*b*c*d - a^3*e^2)*((-I*\sqrt{3}) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) \\ & - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^{1/3} + 9*(I*\sqrt{3}) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) \\ & - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^{1/3} + 6*d/a) - 2*(b^2*c^3 - \end{aligned}$$

$$\begin{aligned}
& a^2 e^3 x + 1/12 \sqrt{1/3} * (((-I \sqrt{3}) + 1) * (d^2/a^2 - (d^2 - c e)/a^2) / \\
& (-1/27 d^3/a^3 + 1/18 (d^2 - c e) d/a^3 + 1/54 (b^2 c^3 + a^2 e^3 - (d^3 - \\
& 3 c d e) a b) / (a^4 b) - 1/54 (b^2 c^3 - a^2 e^3) / (a^4 b))^{1/3} + 9 * (I \sqrt{3} \\
& (3) + 1) * (-1/27 d^3/a^3 + 1/18 (d^2 - c e) d/a^3 + 1/54 (b^2 c^3 + a^2 e^3 \\
& - (d^3 - 3 c d e) a b) / (a^4 b) - 1/54 (b^2 c^3 - a^2 e^3) / (a^4 b))^{1/3} + \\
& 6 d/a) a^3 b c - 6 a^2 b c d - 6 a^3 e^2) \sqrt{-(((-I \sqrt{3}) + 1) * (d^2/a^2 \\
& - (d^2 - c e)/a^2) / (-1/27 d^3/a^3 + 1/18 (d^2 - c e) d/a^3 + 1/54 (b^2 c^3 \\
& + a^2 e^3 - (d^3 - 3 c d e) a b) / (a^4 b) - 1/54 (b^2 c^3 - a^2 e^3) / (a^4 b \\
&))^{1/3} + 9 * (I \sqrt{3} + 1) * (-1/27 d^3/a^3 + 1/18 (d^2 - c e) d/a^3 + 1/54 \\
& * (b^2 c^3 + a^2 e^3 - (d^3 - 3 c d e) a b) / (a^4 b) - 1/54 (b^2 c^3 - a^2 e^3) / (a^4 b))^{1/3} \\
& + 6 d/a)^2 a^2 - 12 * ((-I \sqrt{3}) + 1) * (d^2/a^2 - (d^2 - c \\
& e)/a^2) / (-1/27 d^3/a^3 + 1/18 (d^2 - c e) d/a^3 + 1/54 (b^2 c^3 + a^2 e^3 \\
& - (d^3 - 3 c d e) a b) / (a^4 b) - 1/54 (b^2 c^3 - a^2 e^3) / (a^4 b))^{1/3} + \\
& 9 * (I \sqrt{3} + 1) * (-1/27 d^3/a^3 + 1/18 (d^2 - c e) d/a^3 + 1/54 (b^2 c^3 + \\
& a^2 e^3 - (d^3 - 3 c d e) a b) / (a^4 b) - 1/54 (b^2 c^3 - a^2 e^3) / (a^4 b)) \\
& ^{1/3} + 6 d/a) a d + 36 d^2 - 144 c e) / a^2) - (((-I \sqrt{3}) + 1) * (d^2/a^2 \\
& - (d^2 - c e)/a^2) / (-1/27 d^3/a^3 + 1/18 (d^2 - c e) d/a^3 + 1/54 (b^2 c^3 \\
& + a^2 e^3 - (d^3 - 3 c d e) a b) / (a^4 b) - 1/54 (b^2 c^3 - a^2 e^3) / (a^4 b \\
&))^{1/3} + 9 * (I \sqrt{3} + 1) * (-1/27 d^3/a^3 + 1/18 (d^2 - c e) d/a^3 + 1/54 \\
& * (b^2 c^3 + a^2 e^3 - (d^3 - 3 c d e) a b) / (a^4 b) - 1/54 (b^2 c^3 - a^2 e^3) / (a^4 b))^{1/3} \\
& + 6 d/a) a x + 3 \sqrt{1/3} a x \sqrt{-(((-I \sqrt{3}) + 1) * (d^2/a^2 - (d^2 - c e)/a^2) / (-1/27 d^3/a^3 + 1/18 (d^2 - c e) d/a^3 + 1/54 (b^2 c^3 + a^2 e^3 - (d^3 - 3 c d e) a b) / (a^4 b) - 1/54 (b^2 c^3 - a^2 e^3) / (a^4 b))^{1/3} + 9 * (I \sqrt{3} + 1) * (-1/27 d^3/a^3 + 1/18 (d^2 - c e) d/a^3 + 1/54 (b^2 c^3 + a^2 e^3 - (d^3 - 3 c d e) a b) / (a^4 b) - 1/54 (b^2 c^3 - a^2 e^3) / (a^4 b))^{1/3} + 6 d/a)^2 a^2 - 12 * ((-I \sqrt{3}) + 1) * (d^2/a^2 - (d^2 - c e)/a^2) / (-1/27 d^3/a^3 + 1/18 (d^2 - c e) d/a^3 + 1/54 (b^2 c^3 + a^2 e^3 - (d^3 - 3 c d e) a b) / (a^4 b) - 1/54 (b^2 c^3 - a^2 e^3) / (a^4 b))^{1/3} + 9 * (I \sqrt{3} + 1) * (-1/27 d^3/a^3 + 1/18 (d^2 - c e) d/a^3 + 1/54 (b^2 c^3 + a^2 e^3 - (d^3 - 3 c d e) a b) / (a^4 b) - 1/54 (b^2 c^3 - a^2 e^3) / (a^4 b))^{1/3} + 6 d/a) a d + 36 d^2 - 144 c e) / a^2) - 18 d x) \log(1/36 * ((-I \sqrt{3}) + 1) * (d^2/a^2 - (d^2 - c e)/a^2) / (-1/27 d^3/a^3 + 1/18 (d^2 - c e) d/a^3 + 1/54 (b^2 c^3 + a^2 e^3 - (d^3 - 3 c d e) a b) / (a^4 b) - 1/54 (b^2 c^3 - a^2 e^3) / (a^4 b))^{1/3} + 9 * (I \sqrt{3} + 1) * (-1/27 d^3/a^3 + 1/18 (d^2 - c e) d/a^3 + 1/54 (b^2 c^3 + a^2 e^3 - (d^3 - 3 c d e) a b) / (a^4 b) - 1/54 (b^2 c^3 - a^2 e^3) / (a^4 b))^{1/3} + 6 d/a)^2 a^3 b c + a b c d^2 - 2 a b c^2 e - a^2 d e^2 - 1/6 * (2 a^2 b c d - a^3 e^2) * (((-I \sqrt{3}) + 1) * (d^2/a^2 - (d^2 - c e)/a^2) / (-1/27 d^3/a^3 + 1/18 (d^2 - c e) d/a^3 + 1/54 (b^2 c^3 + a^2 e^3 - (d^3 - 3 c d e) a b) / (a^4 b) - 1/54 (b^2 c^3 - a^2 e^3) / (a^4 b))^{1/3} + 9 * (I \sqrt{3} + 1) * (-1/27 d^3/a^3 + 1/18 (d^2 - c e) d/a^3 + 1/54 (b^2 c^3 + a^2 e^3 - (d^3 - 3 c d e) a b) / (a^4 b) - 1/54 (b^2 c^3 - a^2 e^3) / (a^4 b))^{1/3} + 6 d/a) - 2 * (b^2 c^3 - a^2 e^3) x - 1/12 \sqrt{1/3} * (((-I \sqrt{3}) + 1) * (d^2/a^2 - (d^2 - c e)/a^2) / (-1/27 d^3/a^3 + 1/18 (d^2 - c e) d/a^3 + 1/54 (b^2 c^3 + a^2 e^3 - (d^3 - 3 c d e) a b) / (a^4 b) - 1/54 (b^2 c^3 - a^2 e^3) / (a^4 b))^{1/3} + 9 * (I \sqrt{3} + 1) * (-1/27 d^3/a^3 + 1/18
\end{aligned}$$

$$\begin{aligned} &*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) \\ &- 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^{(1/3)} + 6*d/a)*a^3*b*c - 6*a^2*b*c*d - \\ &6*a^3*e^2)*\sqrt{-(((-I*\sqrt{3}) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3 \\ &/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a \\ &*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^{(1/3)} + 9*(I*\sqrt{3}) + 1)*(\\ &-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3 \\ &*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^{(1/3)} + 6*d/a)^2*a \\ &^2 - 12*((-I*\sqrt{3}) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18 \\ &*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) \\ &- 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^{(1/3)} + 9*(I*\sqrt{3}) + 1)*(-1/27*d^3/a \\ &^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b \\ &)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^{(1/3)} + 6*d/a)*a*d + 36*d^2 - \\ &144*c*e)/a^2)) + 36*c)/(a*x) \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a),x)

[Out] Timed out

Giac [A] time = 1.08301, size = 279, normalized size = 1.45

$$-\frac{d \log(|bx^3 + a|)}{3a} + \frac{d \log(|x|)}{a} - \frac{c}{ax} + \frac{\left((-ab^2)^{\frac{1}{3}} ae - (-ab^2)^{\frac{2}{3}} c \right) \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}} \right)}{6a^2b} + \frac{\left(ab^2c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^2be \right) \left(-\frac{a}{b}\right)^{\frac{1}{3}}}{3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="giac")

[Out] $-\frac{1}{3}d*\log(\text{abs}(b*x^3 + a))/a + d*\log(\text{abs}(x))/a - c/(a*x) + \frac{1}{6}*((-a*b^2)^{(1/3)}*a*e - (-a*b^2)^{(2/3)}*c)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a^2*b) + \frac{1}{3}*(a*b^2*c*(-a/b)^{(1/3)} - a^2*b*e)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/(a^3*b) + \frac{1}{3}*\sqrt{3}*((-a*b^2)^{(1/3)}*a*b^2*e + (-a*b^2)^{(2/3)}*b^2*c)$

$$\text{*arctan}(1/3*\text{sqrt}(3)*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(a^2*b^3)$$

$$3.343 \quad \int \frac{c+dx+ex^2}{x^3(a+bx^3)} dx$$

Optimal. Leaf size=203

$$\frac{b^{2/3} \left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}} \right) \log \left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2 \right)}{6a^{5/3}} - \frac{\sqrt[3]{b} \left(\sqrt[3]{bc} - \sqrt[3]{ad} \right) \log \left(\sqrt[3]{a} + \sqrt[3]{bx} \right)}{3a^{5/3}} + \frac{\sqrt[3]{b} \left(\sqrt[3]{ad} + \sqrt[3]{bc} \right) \tan^{-1} \left(\frac{\sqrt[3]{a-2\sqrt[3]{bx}}}{\sqrt[3]{3}\sqrt[3]{a}} \right)}{\sqrt[3]{3}a^{5/3}} - e$$

[Out] $-c/(2*a*x^2) - d/(a*x) + (b^{(1/3)}*(b^{(1/3)}*c + a^{(1/3)}*d)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})]/(Sqrt[3]*a^{(5/3)}) + (e*Log[x])/a - (b^{(1/3)}*(b^{(1/3)}*c - a^{(1/3)}*d)*Log[a^{(1/3)} + b^{(1/3)}*x]/(3*a^{(5/3)}) + (b^{(2/3)}*(c - (a^{(1/3)}*d)/b^{(1/3)})*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(6*a^{(5/3)}) - (e*Log[a + b*x^3])/(3*a)$

Rubi [A] time = 0.193687, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{b^{2/3} \left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}} \right) \log \left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2 \right)}{6a^{5/3}} - \frac{\sqrt[3]{b} \left(\sqrt[3]{bc} - \sqrt[3]{ad} \right) \log \left(\sqrt[3]{a} + \sqrt[3]{bx} \right)}{3a^{5/3}} + \frac{\sqrt[3]{b} \left(\sqrt[3]{ad} + \sqrt[3]{bc} \right) \tan^{-1} \left(\frac{\sqrt[3]{a-2\sqrt[3]{bx}}}{\sqrt[3]{3}\sqrt[3]{a}} \right)}{\sqrt[3]{3}a^{5/3}} - e$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/(x^3*(a + b*x^3)),x]

[Out] $-c/(2*a*x^2) - d/(a*x) + (b^{(1/3)}*(b^{(1/3)}*c + a^{(1/3)}*d)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})]/(Sqrt[3]*a^{(5/3)}) + (e*Log[x])/a - (b^{(1/3)}*(b^{(1/3)}*c - a^{(1/3)}*d)*Log[a^{(1/3)} + b^{(1/3)}*x]/(3*a^{(5/3)}) + (b^{(2/3)}*(c - (a^{(1/3)}*d)/b^{(1/3)})*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(6*a^{(5/3)}) - (e*Log[a + b*x^3])/(3*a)$

Rule 1834

Int[((Pq_)*((c_)*(x_))^(m_))/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*Pq]/(a + b*x^n), x] /; FreeQ[{a, b, c, m}, x] & & PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]

Rule 1871

```
Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Dist[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned}
 \int \frac{c + dx + ex^2}{x^3(a + bx^3)} dx &= \int \left(\frac{c}{ax^3} + \frac{d}{ax^2} + \frac{e}{ax} - \frac{b(c + dx + ex^2)}{a(a + bx^3)} \right) dx \\
 &= -\frac{c}{2ax^2} - \frac{d}{ax} + \frac{e \log(x)}{a} - \frac{b \int \frac{c+dx+ex^2}{a+bx^3} dx}{a} \\
 &= -\frac{c}{2ax^2} - \frac{d}{ax} + \frac{e \log(x)}{a} - \frac{b \int \frac{c+dx}{a+bx^3} dx}{a} - \frac{(be) \int \frac{x^2}{a+bx^3} dx}{a} \\
 &= -\frac{c}{2ax^2} - \frac{d}{ax} + \frac{e \log(x)}{a} - \frac{e \log(a + bx^3)}{3a} - \frac{b^{2/3} \int \frac{\sqrt[3]{a}(2\sqrt[3]{bc} + \sqrt[3]{ad}) + \sqrt[3]{b}(-\sqrt[3]{bc} + \sqrt[3]{ad})x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} dx}{3a^{5/3}} - \frac{b \left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}} \right)}{3a^{5/3}} \\
 &= -\frac{c}{2ax^2} - \frac{d}{ax} + \frac{e \log(x)}{a} - \frac{b^{2/3} \left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}} \right) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{5/3}} - \frac{e \log(a + bx^3)}{3a} + \frac{(\sqrt[3]{b}(\sqrt[3]{bc} - \sqrt[3]{ad})) \int \frac{x^2}{a+bx^3} dx}{6a^{5/3}} \\
 &= -\frac{c}{2ax^2} - \frac{d}{ax} + \frac{e \log(x)}{a} - \frac{b^{2/3} \left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}} \right) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{5/3}} + \frac{\sqrt[3]{b}(\sqrt[3]{bc} - \sqrt[3]{ad}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{6a^{5/3}} \\
 &= -\frac{c}{2ax^2} - \frac{d}{ax} + \frac{\sqrt[3]{b}(\sqrt[3]{bc} + \sqrt[3]{ad}) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}} + \frac{e \log(x)}{a} - \frac{b^{2/3} \left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}} \right) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{5/3}} + \frac{\sqrt[3]{b}(\sqrt[3]{bc} - \sqrt[3]{ad}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{6a^{5/3}}
 \end{aligned}$$

Mathematica [A] time = 0.177999, size = 192, normalized size = 0.95

$$\frac{\sqrt[3]{b}(\sqrt[3]{a}\sqrt[3]{bc} - a^{2/3}d) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2) + 2\sqrt[3]{b}(a^{2/3}d - \sqrt[3]{a}\sqrt[3]{bc}) \log(\sqrt[3]{a} + \sqrt[3]{bx}) + 2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}(\sqrt[3]{ad} + \sqrt[3]{bc}) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{6a^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(x^3*(a + b*x^3)), x]

[Out] ((-3*a*c)/x^2 - (6*a*d)/x + 2*Sqrt[3]*a^(1/3)*b^(1/3)*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 6*a*e*Log[x] + 2*b^(1/3)*

$$-(a^{1/3}b^{1/3}c + a^{2/3}d)\text{Log}[a^{1/3} + b^{1/3}x] + b^{1/3}(a^{1/3}b^{1/3}c - a^{2/3}d)\text{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2] - 2ae\text{Log}[a + bx^3]/(6a^2)$$

Maple [A] time = 0.005, size = 225, normalized size = 1.1

$$-\frac{c}{3a} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{c}{6a} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{c\sqrt{3}}{3a} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{d}{3a} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)/x^3/(b*x^3+a),x)

[Out] $-\frac{1}{3} \frac{c}{a} \frac{1}{(b \cdot a)^{2/3}} \ln(x + (b \cdot a)^{1/3}) + \frac{1}{6} \frac{c}{a} \frac{1}{(b \cdot a)^{2/3}} \ln(x^2 - (b \cdot a)^{1/3}x + (b \cdot a)^{2/3}) - \frac{1}{3} \frac{c}{a} \frac{1}{(b \cdot a)^{2/3}} \arctan\left(\frac{1}{3} \frac{3^{1/2}}{(b \cdot a)^{1/3}} (2x - (b \cdot a)^{1/3})\right) + \frac{d}{3} \frac{1}{a} \ln(x + (b \cdot a)^{1/3}) - \frac{1}{6} \frac{d}{a} \frac{1}{(b \cdot a)^{1/3}} \ln(x^2 - (b \cdot a)^{1/3}x + (b \cdot a)^{2/3}) - \frac{1}{3} \frac{d}{a} \frac{1}{(b \cdot a)^{1/3}} \arctan\left(\frac{1}{3} \frac{3^{1/2}}{(b \cdot a)^{1/3}} (2x - (b \cdot a)^{1/3})\right) - \frac{1}{3} \frac{e}{a} \ln(b \cdot x^3 + a) + \frac{d}{a} \frac{1}{x} + \frac{e}{a} \ln(x) - \frac{1}{2} \frac{c}{a} \frac{1}{x^2}$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [C] time = 8.21964, size = 9917, normalized size = 48.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="fricas")

[Out]
$$-1/36*(2*((-I*\sqrt{3}) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 9*(I*\sqrt{3}) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 6*e/a)*a*x^2*\log(1/36*((-I*\sqrt{3}) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 9*(I*\sqrt{3}) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 6*e/a)^2*a^4*d + 2*a*b*c*d^2 - a*b*c^2*e + a^2*d*e^2 + 1/6*(a^2*b*c^2 - 2*a^3*d*e)*((-I*\sqrt{3}) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 9*(I*\sqrt{3}) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 6*e/a) + (b^2*c^3 + a*b*d^3)*x) - 36*e*x^2*\log(x) + 36*d*x - (((-I*\sqrt{3}) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 9*(I*\sqrt{3}) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 6*e/a)*a*x^2 + 3*\sqrt{1/3}*a*x^2*\sqrt{(-(((-I*\sqrt{3}) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 9*(I*\sqrt{3}) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 6*e/a)^2*a^3 - 12*((-I*\sqrt{3}) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 9*(I*\sqrt{3}) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 6*e/a)*a^2*e + 144*b*c*d + 36*a*e^2)/a^3) - 18*e*x^2)*\log(-1/36*((-I*\sqrt{3}) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 9*(I*\sqrt{3}) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 6*e/a)^2*a^4*d - 2*a*b*c*d^2 + a*b*c^2*e - a^2*d*e^2 - 1/6*(a^2*b*c^2 - 2*a^3*d*e)*((-I*\sqrt{3}) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 9*(I*\sqrt{3}) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 6*e/a) + 2*(b^2*c^3 + a*b*d^3)*x + 1/12*\sqrt{1/3}*(((-I*\sqrt{3}) + 1)$$

$$\begin{aligned}
& 1) * (e^2/a^2 - (b*c*d + a*e^2)/a^3) / (-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 9*(I*\sqrt{3} + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 6*e/a)*a^4*d - 6*a^2*b*c^2 - 6*a^3*d*e)*\sqrt{-(((-I*\sqrt{3} + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3) / (-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 9*(I*\sqrt{3} + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 6*e/a)^2*a^3 - 12*((-I*\sqrt{3} + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3) / (-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 9*(I*\sqrt{3} + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 6*e/a)*a^2*e + 144*b*c*d + 36*a*e^2)/a^3)) - (((-I*\sqrt{3} + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3) / (-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 9*(I*\sqrt{3} + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 6*e/a)*a*x^2 - 3*\sqrt{1/3}*a*x^2*\sqrt{-(((-I*\sqrt{3} + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3) / (-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 9*(I*\sqrt{3} + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 6*e/a)^2*a^3 - 12*((-I*\sqrt{3} + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3) / (-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 9*(I*\sqrt{3} + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 6*e/a)*a^2*e + 144*b*c*d + 36*a*e^2)/a^3) - 18*e*x^2)*\log(-1/36*((-I*\sqrt{3} + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3) / (-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 9*(I*\sqrt{3} + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 6*e/a)^2*a^4*d - 2*a*b*c*d^2 + a*b*c^2*e - a^2*d*e^2 - 1/6*(a^2*b*c^2 - 2*a^3*d*e)*((-I*\sqrt{3} + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3) / (-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 9*(I*\sqrt{3} + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 6*e/a) + 2*(b^2*c^3 + a*b*d^3)*x - 1/12*\sqrt{1/3}*(((-I*\sqrt{3} + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3) / (-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 9*(I*\sqrt{3} + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^{(1/3)} + 6*e/a)^2)
\end{aligned}$$

$$\begin{aligned} &) * a * b) / a^5)^{(1/3)} + 6 * e / a) * a^4 * d - 6 * a^2 * b * c^2 - 6 * a^3 * d * e) * \text{sqrt}(-(((-I * \text{sqrt} \\ & \text{t}(3) + 1) * (e^2 / a^2 - (b * c * d + a * e^2) / a^3) / (-1 / 27 * e^3 / a^3 + 1 / 18 * (b * c * d + a * \\ & e^2) * e / a^4 + 1 / 54 * (b * c^3 + a * d^3) * b / a^5 - 1 / 54 * (b^2 * c^3 + a^2 * e^3 - (d^3 - \\ & 3 * c * d * e) * a * b) / a^5)^{(1/3)} + 9 * (I * \text{sqrt}(3) + 1) * (-1 / 27 * e^3 / a^3 + 1 / 18 * (b * c * d + \\ & a * e^2) * e / a^4 + 1 / 54 * (b * c^3 + a * d^3) * b / a^5 - 1 / 54 * (b^2 * c^3 + a^2 * e^3 - (d^3 - \\ & 3 * c * d * e) * a * b) / a^5)^{(1/3)} + 6 * e / a)^2 * a^3 - 12 * ((-I * \text{sqrt}(3) + 1) * (e^2 / a^2 \\ & - (b * c * d + a * e^2) / a^3) / (-1 / 27 * e^3 / a^3 + 1 / 18 * (b * c * d + a * e^2) * e / a^4 + 1 / 54 * (\\ & b * c^3 + a * d^3) * b / a^5 - 1 / 54 * (b^2 * c^3 + a^2 * e^3 - (d^3 - 3 * c * d * e) * a * b) / a^5)^{(1/3)} \\ & + 9 * (I * \text{sqrt}(3) + 1) * (-1 / 27 * e^3 / a^3 + 1 / 18 * (b * c * d + a * e^2) * e / a^4 + 1 / 5 \\ & 4 * (b * c^3 + a * d^3) * b / a^5 - 1 / 54 * (b^2 * c^3 + a^2 * e^3 - (d^3 - 3 * c * d * e) * a * b) / a^ \\ & 5)^{(1/3)} + 6 * e / a) * a^2 * e + 144 * b * c * d + 36 * a * e^2) / a^3)) + 18 * c) / (a * x^2) \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/x**3/(b*x**3+a),x)

[Out] Timed out

Giac [A] time = 1.07828, size = 285, normalized size = 1.4

$$-\frac{e \log(|bx^3 + a|)}{3a} + \frac{e \log(|x|)}{a} - \frac{\left((-ab^2)^{\frac{1}{3}} bc + (-ab^2)^{\frac{2}{3}} d \right) \log\left(x^2 + x \left(-\frac{a}{b} \right)^{\frac{1}{3}} + \left(-\frac{a}{b} \right)^{\frac{2}{3}} \right)}{6a^2b} + \frac{\left(ab^2 d \left(-\frac{a}{b} \right)^{\frac{1}{3}} + ab^2 c \right) \left(-\frac{a}{b} \right)^{\frac{1}{3}} \log\left(-\frac{a}{b} \right)}{3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="giac")

$$\begin{aligned} \text{[Out]} & -1/3 * e * \log(\text{abs}(b * x^3 + a)) / a + e * \log(\text{abs}(x)) / a - 1/6 * ((-a * b^2)^{(1/3)} * b * c + \\ & (-a * b^2)^{(2/3)} * d) * \log(x^2 + x * (-a/b)^{(1/3)} + (-a/b)^{(2/3)}) / (a^2 * b) + 1/3 * (a \\ & * b^2 * d * (-a/b)^{(1/3)} + a * b^2 * c) * (-a/b)^{(1/3)} * \log(\text{abs}(x - (-a/b)^{(1/3)})) / (a^3 \\ & * b) - 1/2 * (2 * d * x + c) / (a * x^2) - 1/3 * \text{sqrt}(3) * ((-a * b^2)^{(1/3)} * a * b^3 * c - (-a * b \\ & ^2)^{(2/3)} * a * b^2 * d) * \arctan(1/3 * \text{sqrt}(3) * (2 * x + (-a/b)^{(1/3)}) / (-a/b)^{(1/3)}) / (a \\ & ^3 * b^3) \end{aligned}$$

$$3.344 \quad \int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=190

$$\frac{\left(d - \frac{2\sqrt[3]{ae}}{\sqrt[3]{b}}\right) \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{18a^{2/3}b^{4/3}} + \frac{\left(\sqrt[3]{bd} - 2\sqrt[3]{ae}\right) \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{9a^{2/3}b^{5/3}} - \frac{\left(2\sqrt[3]{ae} + \sqrt[3]{bd}\right) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{5/3}} - \frac{c+dx}{3b(a+bx^3)}$$

[Out] $-(c + d*x + e*x^2)/(3*b*(a + b*x^3)) - ((b^{(1/3)}*d + 2*a^{(1/3)}*e)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(3*Sqrt[3]*a^{(2/3)}*b^{(5/3)}) + ((b^{(1/3)}*d - 2*a^{(1/3)}*e)*Log[a^{(1/3)} + b^{(1/3)}*x])/(9*a^{(2/3)}*b^{(5/3)}) - ((d - (2*a^{(1/3)}*e)/b^{(1/3)})*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(18*a^{(2/3)}*b^{(4/3)})$

Rubi [A] time = 0.16616, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1823, 1860, 31, 634, 617, 204, 628}

$$\frac{\left(d - \frac{2\sqrt[3]{ae}}{\sqrt[3]{b}}\right) \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{18a^{2/3}b^{4/3}} + \frac{\left(\sqrt[3]{bd} - 2\sqrt[3]{ae}\right) \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{9a^{2/3}b^{5/3}} - \frac{\left(2\sqrt[3]{ae} + \sqrt[3]{bd}\right) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{5/3}} - \frac{c+dx}{3b(a+bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^2,x]

[Out] $-(c + d*x + e*x^2)/(3*b*(a + b*x^3)) - ((b^{(1/3)}*d + 2*a^{(1/3)}*e)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(3*Sqrt[3]*a^{(2/3)}*b^{(5/3)}) + ((b^{(1/3)}*d - 2*a^{(1/3)}*e)*Log[a^{(1/3)} + b^{(1/3)}*x])/(9*a^{(2/3)}*b^{(5/3)}) - ((d - (2*a^{(1/3)}*e)/b^{(1/3)})*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(18*a^{(2/3)}*b^{(4/3)})$

Rule 1823

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(Pq*(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && EqQ[m - n + 1, 0] && LtQ[p, -1]

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^( -1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^( -1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^2} dx &= -\frac{c+dx+ex^2}{3b(a+bx^3)} + \frac{\int \frac{d+2ex}{a+bx^3} dx}{3b} \\
&= -\frac{c+dx+ex^2}{3b(a+bx^3)} + \frac{\int \frac{\sqrt[3]{a}(2\sqrt[3]{bd}+2\sqrt[3]{ae})+\sqrt[3]{b}(-\sqrt[3]{bd}+2\sqrt[3]{ae})x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2} dx}{9a^{2/3}b^{4/3}} + \frac{\left(d-\frac{2\sqrt[3]{ae}}{\sqrt[3]{b}}\right) \int \frac{1}{\sqrt[3]{a}+\sqrt[3]{bx}} dx}{9a^{2/3}b} \\
&= -\frac{c+dx+ex^2}{3b(a+bx^3)} + \frac{\left(d-\frac{2\sqrt[3]{ae}}{\sqrt[3]{b}}\right) \log(\sqrt[3]{a}+\sqrt[3]{bx})}{9a^{2/3}b^{4/3}} + \frac{\left(\frac{\sqrt[3]{bd}}{\sqrt[3]{a}}+2e\right) \int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2} dx}{6b^{4/3}} - \frac{(\sqrt[3]{bd}-2\sqrt[3]{ae}) \log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2)}{18a^{2/3}b^{5/3}} + \\
&= -\frac{c+dx+ex^2}{3b(a+bx^3)} + \frac{\left(d-\frac{2\sqrt[3]{ae}}{\sqrt[3]{b}}\right) \log(\sqrt[3]{a}+\sqrt[3]{bx})}{9a^{2/3}b^{4/3}} - \frac{(\sqrt[3]{bd}-2\sqrt[3]{ae}) \log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2)}{18a^{2/3}b^{5/3}} + \\
&= -\frac{c+dx+ex^2}{3b(a+bx^3)} - \frac{(\sqrt[3]{bd}+2\sqrt[3]{ae}) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{5/3}} + \frac{\left(d-\frac{2\sqrt[3]{ae}}{\sqrt[3]{b}}\right) \log(\sqrt[3]{a}+\sqrt[3]{bx})}{9a^{2/3}b^{4/3}} - \frac{(\sqrt[3]{bd}-2\sqrt[3]{ae}) \log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2)}{18a^{2/3}b^{5/3}}
\end{aligned}$$

Mathematica [A] time = 0.128857, size = 174, normalized size = 0.92

$$\frac{\frac{(2\sqrt[3]{ae}-\sqrt[3]{bd}) \log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2)}{a^{2/3}} + \frac{2(\sqrt[3]{bd}-2\sqrt[3]{ae}) \log(\sqrt[3]{a}+\sqrt[3]{bx})}{a^{2/3}} - \frac{2\sqrt{3}(2\sqrt[3]{ae}+\sqrt[3]{bd}) \tan^{-1}\left(\frac{1-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}} - \frac{6b^{2/3}(c+x(d+ex))}{a+bx^3}}{18b^{5/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^2, x]

[Out] ((-6*b^(2/3)*(c + x*(d + e*x)))/(a + b*x^3) - (2*sqrt(3)*(b^(1/3)*d + 2*a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt(3)])/a^(2/3) + (2*(b^(1/3)*d - 2*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + ((-(b^(1/3)*d) + 2*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3))/(18*b^(5/3))

Maple [A] time = 0.009, size = 219, normalized size = 1.2

$$\frac{1}{bx^3+a} \left(-\frac{ex^2}{3b} - \frac{dx}{3b} - \frac{c}{3b} \right) + \frac{d}{9b^2} \ln \left(x + \sqrt[3]{\frac{a}{b}} \right) \left(\frac{a}{b} \right)^{-\frac{2}{3}} - \frac{d}{18b^2} \ln \left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b} \right)^{\frac{2}{3}} \right) \left(\frac{a}{b} \right)^{-\frac{2}{3}} + \frac{d\sqrt{3}}{9b^2} \arctan \left(\frac{\sqrt{3}}{3} \left(2x - \sqrt[3]{\frac{a}{b}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] (-1/3*e*x^2/b-1/3*d*x/b-1/3*c/b)/(b*x^3+a)+1/9/b^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d-1/18/b^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d+1/9/b^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d-2/9/b^2*e/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/9/b^2*e/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2/9/b^2*e*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 6.23643, size = 4988, normalized size = 26.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(12*e*x^2 + 2*(b^2*x^3 + a*b)*((1/2)^(1/3)*(I*sqrt(3) + 1))*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3))*log(1/2*((1/2)^(1/3)*(I*sqrt(3) + 1))*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))^2*a^2*b^3*e - 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1))*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3))*a*b^2*d^2 + 8*a*d*e^2 + (b*d^3 + 8*a*e^3)*x + 12*d*x - ((b^2*x^3 + a
```


Sympy [A] time = 2.42221, size = 109, normalized size = 0.57

$$\text{RootSum}\left(729t^3a^2b^5 + 54tab^2de + 8ae^3 - bd^3, \left(t \mapsto t \log\left(x + \frac{162t^2a^2b^3e + 9tab^2d^2 + 8ade^2}{8ae^3 + bd^3}\right)\right)\right) - \frac{c + dx + ex^2}{3ab + 3b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**2,x)

[Out] RootSum(729*_t**3*a**2*b**5 + 54*_t*a*b**2*d*e + 8*a*e**3 - b*d**3, Lambda(_t, _t*log(x + (162*_t**2*a**2*b**3*e + 9*_t*a*b**2*d**2 + 8*a*d*e**2)/(8*a*e**3 + b*d**3)))) - (c + d*x + e*x**2)/(3*a*b + 3*b**2*x**3)

Giac [A] time = 1.0744, size = 258, normalized size = 1.36

$$\frac{\left(2\left(-\frac{a}{b}\right)^{\frac{1}{3}}e + d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9ab} - \frac{x^2e + dx + c}{3(bx^3 + a)b} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}bd - 2\left(-ab^2\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9ab^3} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac")

[Out] -1/9*(2*(-a/b)^(1/3)*e + d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b) - 1/3*(x^2*e + d*x + c)/((b*x^3 + a)*b) + 1/9*sqrt(3)*((-a*b^2)^(1/3)*b*d - 2*(-a*b^2)^(2/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^3) + 1/18*((-a*b^2)^(1/3)*a*b^2*d + 2*(-a*b^2)^(2/3)*a*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^4)

$$3.345 \quad \int \frac{x(c+dx+ex^2)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=200

$$\frac{(b^{2/3}c - a^{2/3}e) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{18a^{4/3}b^{4/3}} - \frac{(b^{2/3}c - a^{2/3}e) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{4/3}b^{4/3}} - \frac{(a^{2/3}e + b^{2/3}c) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{4/3}} - \frac{x(ae - b^2c)}{3a^2}$$

[Out] $-(x*(a*e - b*c*x - b*d*x^2))/(3*a*b*(a + b*x^3)) - ((b^{(2/3)*c} + a^{(2/3)*e}) * \text{ArcTan}[(a^{(1/3)} - 2*b^{(1/3)*x})/(\text{Sqrt}[3]*a^{(1/3)})])/(3*\text{Sqrt}[3]*a^{(4/3)}*b^{(4/3)}) - ((b^{(2/3)*c} - a^{(2/3)*e}) * \text{Log}[a^{(1/3)} + b^{(1/3)*x}])/(9*a^{(4/3)}*b^{(4/3)}) + ((b^{(2/3)*c} - a^{(2/3)*e}) * \text{Log}[a^{(2/3)} - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2}])/(18*a^{(4/3)}*b^{(4/3)})$

Rubi [A] time = 0.15269, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1828, 1860, 31, 634, 617, 204, 628}

$$\frac{(b^{2/3}c - a^{2/3}e) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{18a^{4/3}b^{4/3}} - \frac{(b^{2/3}c - a^{2/3}e) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{4/3}b^{4/3}} - \frac{(a^{2/3}e + b^{2/3}c) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{4/3}} - \frac{x(ae - b^2c)}{3a^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x*(c + d*x + e*x^2))/(a + b*x^3)^2, x]$

[Out] $-(x*(a*e - b*c*x - b*d*x^2))/(3*a*b*(a + b*x^3)) - ((b^{(2/3)*c} + a^{(2/3)*e}) * \text{ArcTan}[(a^{(1/3)} - 2*b^{(1/3)*x})/(\text{Sqrt}[3]*a^{(1/3)})])/(3*\text{Sqrt}[3]*a^{(4/3)}*b^{(4/3)}) - ((b^{(2/3)*c} - a^{(2/3)*e}) * \text{Log}[a^{(1/3)} + b^{(1/3)*x}])/(9*a^{(4/3)}*b^{(4/3)}) + ((b^{(2/3)*c} - a^{(2/3)*e}) * \text{Log}[a^{(2/3)} - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2}])/(18*a^{(4/3)}*b^{(4/3)})$

Rule 1828

$\text{Int}[(Pq_*)(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x_Symbol] := \text{With}[\{q = m + \text{Expon}[Pq, x]\}, \text{Module}[\{Q = \text{PolynomialQuotient}[b^{(\text{Floor}[(q - 1)/n] + 1)*x^m*Pq, a + b*x^n, x], R = \text{PolynomialRemainder}[b^{(\text{Floor}[(q - 1)/n] + 1)*x^m*Pq, a + b*x^n, x]\}, \text{Dist}[1/(a*n*(p + 1)*b^{(\text{Floor}[(q - 1)/n] + 1)}), \text{Int}[(a + b*x^n)^{(p + 1)}*\text{ExpandToSum}[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],$

$x], x] - \text{Simp}[(x^R*(a + b*x^n)^{(p + 1)})/(a*n*(p + 1)*b^{(\text{Floor}[(q - 1)/n] + 1)}), x]] /; \text{GeQ}[q, n]] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[p, -1] \&\& \text{IGtQ}[m, 0]$

Rule 1860

$\text{Int}[(A + B*x)/(a + b*x^3), x_Symbol] := \text{With}\{r = \text{Numerator}[\text{Rt}[a/b, 3]], s = \text{Denominator}[\text{Rt}[a/b, 3]]\}, -\text{Dist}[(r*(B*r - A*s))/(3*a*s), \text{Int}[1/(r + s*x), x], x] + \text{Dist}[r/(3*a*s), \text{Int}[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; \text{FreeQ}\{a, b, A, B\}, x] \&\& \text{NeQ}[a*B^3 - b*A^3, 0] \&\& \text{PosQ}[a/b]$

Rule 31

$\text{Int}[(a + b*x)^{-1}, x_Symbol] := \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}\{a, b\}, x]$

Rule 634

$\text{Int}[(d + e*x)/(a + b*x + c*x^2), x_Symbol] := \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rule 617

$\text{Int}[(a + b*x + c*x^2)^{-1}, x_Symbol] := \text{With}\{q = 1 - 4*S\text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \text{ || } \text{!RationalQ}[b^2 - 4*a*c]) /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 204

$\text{Int}[(a + b*x^2)^{-1}, x_Symbol] := -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{LtQ}[a, 0] \text{ || } \text{LtQ}[b, 0])$

Rule 628

$\text{Int}[(d + e*x)/(a + b*x + c*x^2), x_Symbol] := \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{x(c + dx + ex^2)}{(a + bx^3)^2} dx &= -\frac{x(ae - bcx - bdx^2)}{3ab(a + bx^3)} - \frac{\int \frac{-ae - bcx}{a + bx^3} dx}{3ab} \\
&= -\frac{x(ae - bcx - bdx^2)}{3ab(a + bx^3)} - \frac{\int \frac{\sqrt[3]{a}(-\sqrt[3]{abc} - 2a\sqrt[3]{be}) + \sqrt[3]{b}(-\sqrt[3]{abc} + a\sqrt[3]{be})x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} dx}{9a^{5/3}b^{4/3}} - \frac{(b^{2/3}c - a^{2/3}e) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{bx}} dx}{9a^{4/3}b} \\
&= -\frac{x(ae - bcx - bdx^2)}{3ab(a + bx^3)} - \frac{(b^{2/3}c - a^{2/3}e) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{4/3}b^{4/3}} + \frac{(b^{2/3}c - a^{2/3}e) \int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} dx}{18a^{4/3}b^{4/3}} \\
&= -\frac{x(ae - bcx - bdx^2)}{3ab(a + bx^3)} - \frac{(b^{2/3}c - a^{2/3}e) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{4/3}b^{4/3}} + \frac{(b^{2/3}c - a^{2/3}e) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{18a^{4/3}b^{4/3}} \\
&= -\frac{x(ae - bcx - bdx^2)}{3ab(a + bx^3)} - \frac{(b^{2/3}c + a^{2/3}e) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{4/3}} - \frac{(b^{2/3}c - a^{2/3}e) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{4/3}b^{4/3}} + \frac{(b^{2/3}c - a^{2/3}e) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{18a^{4/3}b^{4/3}}
\end{aligned}$$

Mathematica [A] time = 0.173585, size = 186, normalized size = 0.93

$$\frac{-(a^{4/3}\sqrt[3]{be} - a^{2/3}bc) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2) + 2(a^{4/3}\sqrt[3]{be} - a^{2/3}bc) \log(\sqrt[3]{a} + \sqrt[3]{bx}) - 2\sqrt{3}(a^{2/3}bc + a^{4/3}\sqrt[3]{be}) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{18a^2b^{5/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3)^2,x]

[Out] ((-6*a*b^(2/3)*(-b*c*x^2 + a*(d + e*x)))/(a + b*x^3) - 2*sqrt(3)*(a^(2/3)*b*c + a^(4/3)*b^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt(3)] + 2*(-(a^(2/3)*b*c) + a^(4/3)*b^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x] - ((a^(2/3)*b*c) + a^(4/3)*b^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^2*b^(5/3))

Maple [A] time = 0.007, size = 228, normalized size = 1.1

$$\frac{1}{bx^3 + a} \left(\frac{cx^2}{3a} - \frac{ex}{3b} - \frac{d}{3b} \right) + \frac{e}{9b^2} \ln \left(x + \sqrt[3]{\frac{a}{b}} \right) \left(\frac{a}{b} \right)^{-\frac{2}{3}} - \frac{e}{18b^2} \ln \left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b} \right)^{\frac{2}{3}} \right) \left(\frac{a}{b} \right)^{-\frac{2}{3}} + \frac{e\sqrt{3}}{9b^2} \arctan \left(\frac{\sqrt{3}}{3} \left(2x \sqrt[3]{\frac{a}{b}} - 1 \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] (1/3*c/a*x^2-1/3*e*x/b-1/3/b*d)/(b*x^3+a)+1/9/b^2*e/(1/b*a)^(2/3)*ln(x+(1/b
*a)^(1/3))-1/18/b^2*e/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1
/9/b^2*e/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/
9/b/a/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*c+1/18/b/a/(1/b*a)^(1/3)*ln(x^2-(1/
b*a)^(1/3)*x+(1/b*a)^(2/3))*c+1/9/b/a*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1
/2)*(2/(1/b*a)^(1/3)*x-1))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 6.77424, size = 5293, normalized size = 26.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*b*c*x^2 - 12*a*e*x - 2*(a*b^2*x^3 + a^2*b)*((1/2)^(1/3)*(I*sqrt(3)
+ 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)
- 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^
4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))*log(1/4*((1/2)^(1/3)*(I*sqrt(3)
+ 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)
- 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^
4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))^2*a^3*b^3*c - 1/2*((1/2)^(1/3)*
(I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b
^4))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^
3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))*a^3*b*e^2 + 2*a*b*c^2
```

$$\begin{aligned}
& *e + (b^2*c^3 + a^2*e^3)*x) - 12*a*d + ((a*b^2*x^3 + a^2*b)*((1/2)^{(1/3)}*(I \\
& *sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4 \\
&))^{(1/3)} - 2*(1/2)^{(2/3)}*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3) \\
& / (a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^{(1/3)})) + 3*sqrt(1/3)*(a*b^2*x^ \\
& 3 + a^2*b)*sqrt(-(((1/2)^{(1/3)}*(I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^ \\
& 4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^{(1/3)} - 2*(1/2)^{(2/3)}*c*e*(-I*sqrt(3) + \\
& 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4) \\
&))^{(1/3)}))^2*a^2*b^2 + 16*c*e)/(a^2*b^2))) * log(-1/4*((1/2)^{(1/3)}*(I*sqrt(3) \\
& + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^{(1/3)} \\
& - 2*(1/2)^{(2/3)}*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) \\
&) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^{(1/3)}))^2*a^3*b^3*c + 1/2*((1/2)^{(1/3)}*(\\
& I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^ \\
& 4))^{(1/3)} - 2*(1/2)^{(2/3)}*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3) \\
&)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^{(1/3)})) * a^3*b*e^2 - 2*a*b*c^2* \\
& e + 2*(b^2*c^3 + a^2*e^3)*x + 3/4*sqrt(1/3)*(((1/2)^{(1/3)}*(I*sqrt(3) + 1)*(\\
& (b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^{(1/3)} - 2*(1 \\
& /2)^{(2/3)}*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b \\
& ^2*c^3 - a^2*e^3)/(a^4*b^4))^{(1/3)})) * a^3*b^3*c + 2*a^3*b*e^2)*sqrt(-(((1/2) \\
& ^{(1/3)}*(I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3) \\
& / (a^4*b^4))^{(1/3)} - 2*(1/2)^{(2/3)}*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + \\
& a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^{(1/3)}))^2*a^2*b^2 + 16 \\
& *c*e)/(a^2*b^2))) + ((a*b^2*x^3 + a^2*b)*((1/2)^{(1/3)}*(I*sqrt(3) + 1)*((b^2 \\
& *c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^{(1/3)} - 2*(1/2)^ \\
& (2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c \\
& ^3 - a^2*e^3)/(a^4*b^4))^{(1/3)})) - 3*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt(-((\\
& (1/2)^{(1/3)}*(I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2 \\
& *e^3)/(a^4*b^4))^{(1/3)} - 2*(1/2)^{(2/3)}*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2* \\
& c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^{(1/3)}))^2*a^2*b^2 \\
& + 16*c*e)/(a^2*b^2))) * log(-1/4*((1/2)^{(1/3)}*(I*sqrt(3) + 1)*((b^2*c^3 + a^ \\
& 2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^{(1/3)} - 2*(1/2)^{(2/3)}*c*e \\
& *(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2* \\
& e^3)/(a^4*b^4))^{(1/3)}))^2*a^3*b^3*c + 1/2*((1/2)^{(1/3)}*(I*sqrt(3) + 1)*((b^ \\
& 2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^{(1/3)} - 2*(1/2) \\
& ^{(2/3)}*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2* \\
& c^3 - a^2*e^3)/(a^4*b^4))^{(1/3)})) * a^3*b*e^2 - 2*a*b*c^2*e + 2*(b^2*c^3 + a^ \\
& 2*e^3)*x - 3/4*sqrt(1/3)*(((1/2)^{(1/3)}*(I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3) \\
& / (a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^{(1/3)} - 2*(1/2)^{(2/3)}*c*e*(-I*s \\
& qrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(\\
& a^4*b^4))^{(1/3)})) * a^3*b^3*c + 2*a^3*b*e^2)*sqrt(-(((1/2)^{(1/3)}*(I*sqrt(3) + \\
& 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^{(1/3)} - \\
& 2*(1/2)^{(2/3)}*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) \\
& - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^{(1/3)}))^2*a^2*b^2 + 16*c*e)/(a^2*b^2))))/ \\
& (a*b^2*x^3 + a^2*b)
\end{aligned}$$

Sympy [A] time = 1.66737, size = 124, normalized size = 0.62

$$\text{RootSum}\left(729t^3a^4b^4 + 27ta^2b^2ce - a^2e^3 + b^2c^3, \left(t \mapsto t \log\left(x + \frac{81t^2a^3b^3c + 9ta^3be^2 + 2abc^2e}{a^2e^3 + b^2c^3}\right)\right)\right) + \frac{-ad - aex + bcx^2}{3a^2b + 3ab^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**2,x)

[Out] RootSum(729*_t**3*a**4*b**4 + 27*_t*a**2*b**2*c*e - a**2*e**3 + b**2*c**3, Lambda(_t, _t*log(x + (81*_t**2*a**3*b**3*c + 9*_t*a**3*b*e**2 + 2*a*b*c**2*e)/(a**2*e**3 + b**2*c**3)))) + (-a*d - a*e*x + b*c*x**2)/(3*a**2*b + 3*a*b**2*x**3)

Giac [A] time = 1.10256, size = 273, normalized size = 1.36

$$\frac{\left(bc\left(-\frac{a}{b}\right)^{\frac{1}{3}} + ae\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^2b} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}ae - \left(-ab^2\right)^{\frac{2}{3}}c\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^2b^2} + \frac{bcx^2 - axe - ad}{3(bx^3 + a)ab} + \left(\dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac")

[Out] -1/9*(b*c*(-a/b)^(1/3) + a*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b) + 1/9*sqrt(3)*((-a*b^2)^(1/3)*a*e - (-a*b^2)^(2/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^2) + 1/3*(b*c*x^2 - a*x*e - a*d)/((b*x^3 + a)*a*b) + 1/18*((-a*b^2)^(1/3)*a*b^2*e + (-a*b^2)^(2/3)*b^2*c)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^4)

$$3.346 \quad \int \frac{c+dx+ex^2}{(a+bx^3)^2} dx$$

Optimal. Leaf size=199

$$-\frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{18a^{5/3}b^{2/3}} + \frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{5/3}b^{2/3}} - \frac{(\sqrt[3]{ad} + 2\sqrt[3]{bc}) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} - \frac{ae}{3}$$

[Out] $-(a*e - b*x*(c + d*x))/(3*a*b*(a + b*x^3)) - ((2*b^{(1/3)}*c + a^{(1/3)}*d)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(3*Sqrt[3]*a^{(5/3)}*b^{(2/3)}) + ((2*b^{(1/3)}*c - a^{(1/3)}*d)*Log[a^{(1/3)} + b^{(1/3)}*x])/(9*a^{(5/3)}*b^{(2/3)}) - ((2*b^{(1/3)}*c - a^{(1/3)}*d)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(18*a^{(5/3)}*b^{(2/3)})$

Rubi [A] time = 0.132274, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$, Rules used = {1854, 1860, 31, 634, 617, 204, 628}

$$-\frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{18a^{5/3}b^{2/3}} + \frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{5/3}b^{2/3}} - \frac{(\sqrt[3]{ad} + 2\sqrt[3]{bc}) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} - \frac{ae}{3}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/(a + b*x^3)^2, x]

[Out] $-(a*e - b*x*(c + d*x))/(3*a*b*(a + b*x^3)) - ((2*b^{(1/3)}*c + a^{(1/3)}*d)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(3*Sqrt[3]*a^{(5/3)}*b^{(2/3)}) + ((2*b^{(1/3)}*c - a^{(1/3)}*d)*Log[a^{(1/3)} + b^{(1/3)}*x])/(9*a^{(5/3)}*b^{(2/3)}) - ((2*b^{(1/3)}*c - a^{(1/3)}*d)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(18*a^{(5/3)}*b^{(2/3)})$

Rule 1854

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq, x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]]*(a + b*x^n)^(p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,

0] && LtQ[p, -1]

Rule 1860

Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]

Rule 31

Int[((a_) + (b_)*(x_))^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2}{(a + bx^3)^2} dx &= \frac{ae - bx(c + dx)}{3ab(a + bx^3)} - \frac{\int \frac{-2c - dx}{a + bx^3} dx}{3a} \\
&= \frac{ae - bx(c + dx)}{3ab(a + bx^3)} - \frac{\int \frac{\sqrt[3]{a}(-4\sqrt[3]{bc} - \sqrt[3]{ad}) + \sqrt[3]{b}(2\sqrt[3]{bc} - \sqrt[3]{ad})x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} dx}{9a^{5/3}\sqrt[3]{b}} + \frac{\left(2c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{bx}} dx}{9a^{5/3}} \\
&= \frac{ae - bx(c + dx)}{3ab(a + bx^3)} + \frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{5/3}b^{2/3}} - \frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} dx}{18a^{5/3}b^{2/3}} + \frac{(2c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{bx}} dx}{9a^{5/3}} \\
&= \frac{ae - bx(c + dx)}{3ab(a + bx^3)} + \frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{5/3}b^{2/3}} - \frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{18a^{5/3}b^{2/3}} \\
&= \frac{ae - bx(c + dx)}{3ab(a + bx^3)} - \frac{(2\sqrt[3]{bc} + \sqrt[3]{ad}) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} + \frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{5/3}b^{2/3}} - \frac{(2\sqrt[3]{bc} - \sqrt[3]{ad}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{18a^{5/3}b^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.178538, size = 189, normalized size = 0.95

$$\frac{\sqrt[3]{a}\sqrt[3]{b}(\sqrt[3]{ad} - 2\sqrt[3]{bc}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2) + (4\sqrt[3]{ab}b^{2/3}c - 2a^{2/3}\sqrt[3]{bd}) \log(\sqrt[3]{a} + \sqrt[3]{bx}) + \frac{6a(bx(c+dx) - ae)}{a+bx^3} - 2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}{18a^2b}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(a + b*x^3)^2, x]

[Out] ((6*a*(-(a*e) + b*x*(c + d*x)))/(a + b*x^3) - 2*Sqrt[3]*a^(1/3)*b^(1/3)*(2*b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + (4*a^(1/3)*b^(2/3)*c - 2*a^(2/3)*b^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x] + a^(1/3)*b^(1/3)*(-2*b^(1/3)*c + a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^2*b)

Maple [A] time = 0.002, size = 253, normalized size = 1.3

$$\frac{cx}{3a(bx^3 + a)} + \frac{2c}{9ab} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{c}{9ab} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{2c\sqrt{3}}{9ab} \arctan\left(\frac{\sqrt{3}}{3} \left(2x\sqrt[3]{\frac{1}{a}} - 1\right)\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/3*c*x/a/(b*x^3+a)+2/9*c/a/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/9*c/a/b/(
1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2/9*c/a/b/(1/b*a)^(2/3)*
3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/3*d*x^2/a/(b*x^3+a)-1/9
*d/a/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/18*d/a/b/(1/b*a)^(1/3)*ln(x^2-(1
/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/9*d/a*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(
1/2)*(2/(1/b*a)^(1/3)*x-1))-1/3/b/(b*x^3+a)*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 6.24559, size = 5011, normalized size = 25.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*b*d*x^2 + 12*b*c*x - 2*(a*b^2*x^3 + a^2*b)*((1/2)^(1/3)*(I*sqrt(3)
+ 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4
*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b
*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b
*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3
)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^
3)/(a^5*b^2))^(1/3)))^2*a^4*b*d - 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3
+ a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d
*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a
^5*b^2))^(1/3)))*a^2*b*c^2 + 4*a*c*d^2 + (8*b*c^3 + a*d^3)*x) - 12*a*e + ((
```


Sympy [A] time = 1.28355, size = 116, normalized size = 0.58

$$\text{RootSum}\left(729t^3a^5b^2 + 54ta^2bcd + ad^3 - 8bc^3, \left(t \mapsto t \log\left(x + \frac{81t^2a^4bd + 36ta^2bc^2 + 4acd^2}{ad^3 + 8bc^3}\right)\right)\right) + \frac{-ae + bcx + bdx^2}{3a^2b + 3ab^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/(b*x**3+a)**2,x)

[Out] RootSum(729*_t**3*a**5*b**2 + 54*_t*a**2*b*c*d + a*d**3 - 8*b*c**3, Lambda(_t, _t*log(x + (81*_t**2*a**4*b*d + 36*_t*a**2*b*c**2 + 4*a*c*d**2)/(a*d**3 + 8*b*c**3)))) + (-a*e + b*c*x + b*d*x**2)/(3*a**2*b + 3*a*b**2*x**3)

Giac [A] time = 1.07851, size = 266, normalized size = 1.34

$$\frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 2c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^2} + \frac{\sqrt{3}\left(2\left(-ab^2\right)^{\frac{1}{3}}bc - \left(-ab^2\right)^{\frac{2}{3}}d\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^2b^2} + \frac{bdx^2 + bcx - ae}{3(bx^3 + a)ab} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac")

[Out] -1/9*(d*(-a/b)^(1/3) + 2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1/9*sqrt(3)*(2*(-a*b^2)^(1/3)*b*c - (-a*b^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^2) + 1/3*(b*d*x^2 + b*c*x - a*e)/((b*x^3 + a)*a*b) + 1/18*(2*(-a*b^2)^(1/3)*a*b^3*c + (-a*b^2)^(2/3)*a*b^2*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^4)

$$3.347 \quad \int \frac{c+dx+ex^2}{x(a+bx^3)^2} dx$$

Optimal. Leaf size=222

$$-\frac{(2\sqrt[3]{bd} - \sqrt[3]{ae}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{18a^{5/3}b^{2/3}} + \frac{(2\sqrt[3]{bd} - \sqrt[3]{ae}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{5/3}b^{2/3}} - \frac{(\sqrt[3]{ae} + 2\sqrt[3]{bd}) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} + x$$

[Out] (x*(a*d + a*e*x - b*c*x^2))/(3*a^2*(a + b*x^3)) - ((2*b^(1/3)*d + a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(5/3)*b^(2/3)) + (c*Log[x])/a^2 + ((2*b^(1/3)*d - a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/ (9*a^(5/3)*b^(2/3)) - ((2*b^(1/3)*d - a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/ (18*a^(5/3)*b^(2/3)) - (c*Log[a + b*x^3])/ (3*a^2)

Rubi [A] time = 0.312688, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{(2\sqrt[3]{bd} - \sqrt[3]{ae}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{18a^{5/3}b^{2/3}} + \frac{(2\sqrt[3]{bd} - \sqrt[3]{ae}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{5/3}b^{2/3}} - \frac{(\sqrt[3]{ae} + 2\sqrt[3]{bd}) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} + x$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/(x*(a + b*x^3)^2), x]

[Out] (x*(a*d + a*e*x - b*c*x^2))/(3*a^2*(a + b*x^3)) - ((2*b^(1/3)*d + a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(5/3)*b^(2/3)) + (c*Log[x])/a^2 + ((2*b^(1/3)*d - a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/ (9*a^(5/3)*b^(2/3)) - ((2*b^(1/3)*d - a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/ (18*a^(5/3)*b^(2/3)) - (c*Log[a + b*x^3])/ (3*a^2)

Rule 1829

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(

$(a + b*x^n)^{(p + 1)}/(a^{2*n*(p + 1)}*b^{(Floor[(q - 1)/n] + 1)}, x]] /; FreeQ$
 $[{a, b}, x] \&\& PolyQ[Pq, x] \&\& IGtQ[n, 0] \&\& LtQ[p, -1] \&\& ILtQ[m, 0]$

Rule 1834

$Int[((Pq_)*((c_)*(x_))^{(m_)})/((a_) + (b_)*(x_)^{(n_)}), x_Symbol] := Int[Ex$
 $pendIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] \&$
 $\& PolyQ[Pq, x] \&\& IntegerQ[n] \&\& !IGtQ[m, 0]$

Rule 1871

$Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B$
 $= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di$
 $st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a$
 $/b]] /; FreeQ[{a, b}, x] \&\& PolyQ[P2, x, 2]$

Rule 1860

$Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numer$
 $ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*$
 $s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r$
 $- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] \&\& Ne$
 $Q[a*B^3 - b*A^3, 0] \&\& PosQ[a/b]$

Rule 31

$Int[((a_) + (b_)*(x_))^{(-1)}, x_Symbol] := Simp[Log[RemoveContent[a + b*x,$
 $x]]/b, x] /; FreeQ[{a, b}, x]$

Rule 634

$Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D$
 $ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In$
 $t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] \&\& NeQ$
 $[2*c*d - b*e, 0] \&\& NeQ[b^2 - 4*a*c, 0] \&\& !NiceSqrtQ[b^2 - 4*a*c]$

Rule 617

$Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^{(-1)}, x_Symbol] := With[{q = 1 - 4*S$
 $implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b$
 $], x] /; RationalQ[q] \&\& (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free$
 $Q[{a, b, c}, x] \&\& NeQ[b^2 - 4*a*c, 0]$

Rule 204


```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2}{x(a + bx^3)^2} dx &= \frac{x(ad + aex - bcx^2)}{3a^2(a + bx^3)} - \frac{\int \frac{-3bc - 2bdx - bex^2}{x(a + bx^3)} dx}{3ab} \\
&= \frac{x(ad + aex - bcx^2)}{3a^2(a + bx^3)} - \frac{\int \left(-\frac{3bc}{ax} - \frac{b(2ad + aex - 3bcx^2)}{a(a + bx^3)} \right) dx}{3ab} \\
&= \frac{x(ad + aex - bcx^2)}{3a^2(a + bx^3)} + \frac{c \log(x)}{a^2} + \frac{\int \frac{2ad + aex - 3bcx^2}{a + bx^3} dx}{3a^2} \\
&= \frac{x(ad + aex - bcx^2)}{3a^2(a + bx^3)} + \frac{c \log(x)}{a^2} + \frac{\int \frac{2ad + aex}{a + bx^3} dx}{3a^2} - \frac{(bc) \int \frac{x^2}{a + bx^3} dx}{a^2} \\
&= \frac{x(ad + aex - bcx^2)}{3a^2(a + bx^3)} + \frac{c \log(x)}{a^2} - \frac{c \log(a + bx^3)}{3a^2} + \frac{\int \frac{\sqrt[3]{a}(4a\sqrt[3]{bd} + a^{4/3}e) + \sqrt[3]{b}(-2a\sqrt[3]{bd} + a^{4/3}e)x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} dx}{9a^{8/3}\sqrt[3]{b}} + \frac{(2d - \frac{3}{2}e)}{3a^2} \\
&= \frac{x(ad + aex - bcx^2)}{3a^2(a + bx^3)} + \frac{c \log(x)}{a^2} + \frac{(2\sqrt[3]{bd} - \sqrt[3]{ae}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{5/3}b^{2/3}} - \frac{c \log(a + bx^3)}{3a^2} - \frac{(2\sqrt[3]{bd} - \sqrt[3]{ae})}{18a^{5/3}b^{2/3}} \\
&= \frac{x(ad + aex - bcx^2)}{3a^2(a + bx^3)} + \frac{c \log(x)}{a^2} + \frac{(2\sqrt[3]{bd} - \sqrt[3]{ae}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{5/3}b^{2/3}} - \frac{(2\sqrt[3]{bd} - \sqrt[3]{ae}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx})}{18a^{5/3}b^{2/3}} \\
&= \frac{x(ad + aex - bcx^2)}{3a^2(a + bx^3)} - \frac{(2\sqrt[3]{bd} + \sqrt[3]{ae}) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} + \frac{c \log(x)}{a^2} + \frac{(2\sqrt[3]{bd} - \sqrt[3]{ae}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{5/3}b^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.156545, size = 199, normalized size = 0.9

$$\frac{\frac{(a^{2/3}e - 2\sqrt[3]{a}\sqrt[3]{bd}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{b^{2/3}} + \frac{2(2\sqrt[3]{a}\sqrt[3]{bd} - a^{2/3}e) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{b^{2/3}} - \frac{2\sqrt{3}\sqrt[3]{a}(\sqrt[3]{ae} + 2\sqrt[3]{bd}) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{2/3}} + \frac{6a(c + x(d + ex))}{a + bx^3} - 6c \log(a + bx^3)}{18a^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)^2), x]

[Out] ((6*a*(c + x*(d + e*x)))/(a + b*x^3) - (2*Sqrt[3]*a^(1/3)*(2*b^(1/3)*d + a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + 18*c*Log[x]

$$+ (2*(2*a^{(1/3)}*b^{(1/3)}*d - a^{(2/3)}*e)*\text{Log}[a^{(1/3)} + b^{(1/3)}*x])/b^{(2/3)} + ((-2*a^{(1/3)}*b^{(1/3)}*d + a^{(2/3)}*e)*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/b^{(2/3)} - 6*c*\text{Log}[a + b*x^3]/(18*a^2)$$

Maple [A] time = 0.01, size = 274, normalized size = 1.2

$$\frac{ex^2}{3a(bx^3+a)} + \frac{dx}{3a(bx^3+a)} + \frac{c}{3a(bx^3+a)} + \frac{2d}{9ab} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{d}{9ab} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{2d\sqrt{3}}{9ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)/x/(b*x^3+a)^2,x)

[Out] $\frac{1}{3} \frac{a x^2}{(b x^3+a) e} + \frac{1}{3} \frac{a x}{(b x^3+a) d} + \frac{1}{3} \frac{a}{(b x^3+a) c} + \frac{2}{9} \frac{b}{a} \frac{d}{(1/b * a)^{(2/3)} * \ln(x + (1/b * a)^{(1/3)})} - \frac{1}{9} \frac{b}{a} \frac{d}{(1/b * a)^{(2/3)} * \ln(x^2 - (1/b * a)^{(1/3)} * x + (1/b * a)^{(2/3)})} + \frac{2}{9} \frac{b}{a} \frac{d}{(1/b * a)^{(2/3)} * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (2/(1/b * a)^{(1/3)} * x - 1))} - \frac{1}{9} \frac{a}{b} \frac{d}{(1/b * a)^{(1/3)} * \ln(x + (1/b * a)^{(1/3)})} * e + \frac{1}{18} \frac{a}{b} \frac{d}{(1/b * a)^{(1/3)} * \ln(x^2 - (1/b * a)^{(1/3)} * x + (1/b * a)^{(2/3)})} * e + \frac{1}{9} \frac{a}{b} * 3^{(1/2)} / (1/b * a)^{(1/3)} * \arctan(1/3 * 3^{(1/2)} * (2/(1/b * a)^{(1/3)} * x - 1)) * e - \frac{1}{3} * c * \ln(b * x^3 + a) / a^2 + c * \ln(x) / a^2$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [C] time = 7.16404, size = 12189, normalized size = 54.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

$$\begin{aligned}
& *a*d*e)/c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 \\
& + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^{(1/3)} + 54*c/a^2)^2*a^4*b* \\
& e - 12*b*c*d^2 - 9*b*c^2*e - 4*a*d*e^2 + 1/9*(2*a^2*b*d^2 + 3*a^2*b*c*e)*((\\
& -I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b)))/(-1/27*c^3/a^6 + \\
& 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - \\
& 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^{(1/3)} + \\
& 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1 \\
& /1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 \\
& - 9*c*d*e)*a*b)/(a^6*b^2))^{(1/3)} + 54*c/a^2) + 2*(8*b*d^3 + a*e^3)*x + 1/1 \\
& 08*sqrt(1/3)*(((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b)))/(- \\
& 1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e \\
& ^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^ \\
& 6*b^2))^{(1/3)} + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d* \\
& e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^ \\
& 2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^{(1/3)} + 54*c/a^2)*a^4*b*e + 72* \\
& a^2*b*d^2 - 54*a^2*b*c*e)*sqrt(-(((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + \\
& 2*a*d*e)/(a^4*b)))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/ \\
& 1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 \\
& - 9*c*d*e)*a*b)/(a^6*b^2))^{(1/3)} + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/16 \\
& 2*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/14 \\
& 58*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^{(1/3)} + 54*c \\
& /a^2)^2*a^4*b - 108*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4 \\
& *b)))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 \\
& + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a* \\
& b)/(a^6*b^2))^{(1/3)} + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + \\
& 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^ \\
& 3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^{(1/3)} + 54*c/a^2)*a^2*b*c \\
& + 2916*b*c^2 + 2592*a*d*e)/(a^4*b))) - (162*b*c*x^3 - (a^2*b*x^3 + a^3))*((\\
& -I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b)))/(-1/27*c^3/a^6 + \\
& 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - \\
& 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^{(1/3)} + \\
& 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1 \\
& /1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 \\
& - 9*c*d*e)*a*b)/(a^6*b^2))^{(1/3)} + 54*c/a^2) + 162*a*c + 3*sqrt(1/3)*(a^2* \\
& b*x^3 + a^3)*sqrt(-(((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4 \\
& *b)))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 \\
& + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a* \\
& b)/(a^6*b^2))^{(1/3)} + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + \\
& 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^ \\
& 3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^{(1/3)} + 54*c/a^2)^2*a^4*b \\
& - 108*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b)))/(-1/27*c \\
& ^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^ \\
& 5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2)) \\
& ^{(1/3)} + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a \\
& ^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 -
\end{aligned}$$

$$\begin{aligned}
& 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^{(1/3)} + 54*c/a^2)*a^2*b*c + 2916*b*c^2 \\
& + 2592*a*d*e)/(a^4*b))) * \log(-1/324*((-I*\sqrt{3}) + 1)*(9*c^2/a^4 - (9*b*c^2 \\
& + 2*a*d*e)/(a^4*b)))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + \\
& 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 \\
& ^3 - 9*c*d*e)*a*b)/(a^6*b^2))^{(1/3)} + 81*(I*\sqrt{3}) + 1)*(-1/27*c^3/a^6 + 1 \\
& /162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1 \\
& /1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^{(1/3)} + 5 \\
& 4*c/a^2)^2*a^4*b*e - 12*b*c*d^2 - 9*b*c^2*e - 4*a*d*e^2 + 1/9*(2*a^2*b*d^2 \\
& + 3*a^2*b*c*e)*((-I*\sqrt{3}) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b)))/ \\
& (-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a* \\
& e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a \\
& ^6*b^2))^{(1/3)} + 81*(I*\sqrt{3}) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d \\
& *e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a \\
& ^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^{(1/3)} + 54*c/a^2) + 2*(8*b*d^3 \\
& + a*e^3)*x - 1/108*\sqrt{1/3}*(((-I*\sqrt{3}) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2* \\
& a*d*e)/(a^4*b)))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/14 \\
& 58*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - \\
& 9*c*d*e)*a*b)/(a^6*b^2))^{(1/3)} + 81*(I*\sqrt{3}) + 1)*(-1/27*c^3/a^6 + 1/162* \\
& (9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458 \\
& *(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^{(1/3)} + 54*c/a \\
& ^2)*a^4*b*e + 72*a^2*b*d^2 - 54*a^2*b*c*e)*\sqrt{-(((-I*\sqrt{3}) + 1)*(9*c^2/ \\
& a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b)))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d* \\
& e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^ \\
& 2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^{(1/3)} + 81*(I*\sqrt{3}) + 1)*(-1/ \\
& 27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3) \\
& / (a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b \\
& ^2))^{(1/3)} + 54*c/a^2)^2*a^4*b - 108*((-I*\sqrt{3}) + 1)*(9*c^2/a^4 - (9*b*c^ \\
& 2 + 2*a*d*e)/(a^4*b)))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) \\
& + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4* \\
& d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^{(1/3)} + 81*(I*\sqrt{3}) + 1)*(-1/27*c^3/a^6 + \\
& 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - \\
& 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^{(1/3)} + \\
& 54*c/a^2)*a^2*b*c + 2916*b*c^2 + 2592*a*d*e)/(a^4*b))) + 324*(b*c*x^3 + a*c \\
&)*\log(x))/(a^2*b*x^3 + a^3)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/x/(b*x**3+a)**2,x)

[Out] Timed out

Giac [A] time = 1.09506, size = 311, normalized size = 1.4

$$-\frac{c \log(|bx^3 + a|)}{3a^2} + \frac{c \log(|x|)}{a^2} + \frac{\sqrt{3} \left(2 \left(-ab^2 \right)^{\frac{1}{3}} bd - \left(-ab^2 \right)^{\frac{2}{3}} e \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{9a^2b^2} + \frac{ax^2e + adx + ac}{3(bx^3 + a)a^2} - \frac{\left(a^3b \left(-\frac{a}{b} \right) \right)}{3(bx^3 + a)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="giac")

[Out]
$$-1/3*c*\log(\text{abs}(b*x^3 + a))/a^2 + c*\log(\text{abs}(x))/a^2 + 1/9*\text{sqrt}(3)*(2*(-a*b^2)^{\frac{1}{3}}*b*d - (-a*b^2)^{\frac{2}{3}}*e)*\arctan(1/3*\text{sqrt}(3)*(2*x + (-a/b)^{\frac{1}{3}})/(-a/b)^{\frac{1}{3}})/(a^2*b^2) + 1/3*(a*x^2*e + a*d*x + a*c)/((b*x^3 + a)*a^2) - 1/9*(a^3*b*(-a/b)^{\frac{1}{3}}*e + 2*a^3*b*d)*(-a/b)^{\frac{1}{3}}*\log(\text{abs}(x - (-a/b)^{\frac{1}{3}}))/(a^5*b) + 1/18*(2*(-a*b^2)^{\frac{1}{3}}*a*b^3*d + (-a*b^2)^{\frac{2}{3}}*a*b^2*e)*\log(x^2 + x*(-a/b)^{\frac{1}{3}} + (-a/b)^{\frac{2}{3}})/(a^3*b^4)$$

$$3.348 \quad \int \frac{c+dx+ex^2}{x^2(a+bx^3)^2} dx$$

Optimal. Leaf size=231

$$\frac{(a^{2/3}e + 2b^{2/3}c) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{9a^{7/3}\sqrt[3]{b}} + \frac{2(a^{2/3}e + 2b^{2/3}c) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{7/3}\sqrt[3]{b}} + \frac{2(2b^{2/3}c - a^{2/3}e) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}\sqrt[3]{b}}$$

[Out] $-(c/(a^2*x)) + (x*(a*e - b*c*x - b*d*x^2))/(3*a^2*(a + b*x^3)) + (2*(2*b^(2/3)*c - a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(7/3)*b^(1/3)) + (d*Log[x])/a^2 + (2*(2*b^(2/3)*c + a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x]/(9*a^(7/3)*b^(1/3)) - ((2*b^(2/3)*c + a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(9*a^(7/3)*b^(1/3)) - (d*Log[a + b*x^3])/(3*a^2)$

Rubi [A] time = 0.343126, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{(a^{2/3}e + 2b^{2/3}c) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{9a^{7/3}\sqrt[3]{b}} + \frac{2(a^{2/3}e + 2b^{2/3}c) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{7/3}\sqrt[3]{b}} + \frac{2(2b^{2/3}c - a^{2/3}e) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^2), x]

[Out] $-(c/(a^2*x)) + (x*(a*e - b*c*x - b*d*x^2))/(3*a^2*(a + b*x^3)) + (2*(2*b^(2/3)*c - a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(7/3)*b^(1/3)) + (d*Log[x])/a^2 + (2*(2*b^(2/3)*c + a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x]/(9*a^(7/3)*b^(1/3)) - ((2*b^(2/3)*c + a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(9*a^(7/3)*b^(1/3)) - (d*Log[a + b*x^3])/(3*a^2)$

Rule 1829

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[

$x^m(a + b x^n)^{(p+1)}$ ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]

Rule 1834

Int[((Pq_)*((c_)*(x_))^(m_))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((c*x)^(m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]

Rule 1871

Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Dist[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

Rule 1860

Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]

Rule 31

Int[((a_) + (b_)*(x_))^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2}{x^2(a + bx^3)^2} dx &= \frac{x(ae - bcx - bdx^2)}{3a^2(a + bx^3)} - \frac{\int \frac{-3bc - 3bdx - 2bex^2 + \frac{b^2cx^3}{a}}{x^2(a + bx^3)} dx}{3ab} \\
&= \frac{x(ae - bcx - bdx^2)}{3a^2(a + bx^3)} - \frac{\int \left(-\frac{3bc}{ax^2} - \frac{3bd}{ax} - \frac{b(2ae - 4bcx - 3bdx^2)}{a(a + bx^3)} \right) dx}{3ab} \\
&= -\frac{c}{a^2x} + \frac{x(ae - bcx - bdx^2)}{3a^2(a + bx^3)} + \frac{d \log(x)}{a^2} + \frac{\int \frac{2ae - 4bcx - 3bdx^2}{a + bx^3} dx}{3a^2} \\
&= -\frac{c}{a^2x} + \frac{x(ae - bcx - bdx^2)}{3a^2(a + bx^3)} + \frac{d \log(x)}{a^2} + \frac{\int \frac{2ae - 4bcx}{a + bx^3} dx}{3a^2} - \frac{(bd) \int \frac{x^2}{a + bx^3} dx}{a^2} \\
&= -\frac{c}{a^2x} + \frac{x(ae - bcx - bdx^2)}{3a^2(a + bx^3)} + \frac{d \log(x)}{a^2} - \frac{d \log(a + bx^3)}{3a^2} + \frac{\int \frac{\sqrt[3]{a}(-4\sqrt[3]{a}bc + 4a\sqrt[3]{be}) + \sqrt[3]{b}(-4\sqrt[3]{a}bc - 2a\sqrt[3]{be})}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}}{9a^{8/3}\sqrt[3]{b}} \\
&= -\frac{c}{a^2x} + \frac{x(ae - bcx - bdx^2)}{3a^2(a + bx^3)} + \frac{d \log(x)}{a^2} + \frac{2(2b^{2/3}c + a^{2/3}e) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{7/3}\sqrt[3]{b}} - \frac{d \log(a + bx^3)}{3a^2} \\
&= -\frac{c}{a^2x} + \frac{x(ae - bcx - bdx^2)}{3a^2(a + bx^3)} + \frac{d \log(x)}{a^2} + \frac{2(2b^{2/3}c + a^{2/3}e) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{7/3}\sqrt[3]{b}} - \frac{(2b^{2/3}c + a^{2/3}e) \log(a + bx^3)}{3a^2} \\
&= -\frac{c}{a^2x} + \frac{x(ae - bcx - bdx^2)}{3a^2(a + bx^3)} + \frac{2(2b^{2/3}c - a^{2/3}e) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}\sqrt[3]{b}} + \frac{d \log(x)}{a^2} + \frac{2(2b^{2/3}c + a^{2/3}e)}{9a^{7/3}\sqrt[3]{b}}
\end{aligned}$$

Mathematica [A] time = 0.261757, size = 213, normalized size = 0.92

$$\frac{\frac{(2a^{2/3}b^{2/3}c + a^{4/3}e) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{\sqrt[3]{b}} - \frac{2(2a^{2/3}b^{2/3}c + a^{4/3}e) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{\sqrt[3]{b}}}{9a^3} + \frac{2\sqrt{3}a^{2/3}(a^{2/3}e - 2b^{2/3}c) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}} - \frac{3a(a(dx + ex^2) - bcx^2)}{a + bx^3} + 3\frac{d \log(x)}{a^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^2), x]

```
[Out] -((9*a*c)/x - (3*a*(-(b*c*x^2) + a*(d + e*x)))/(a + b*x^3) + (2*sqrt[3]*a^(2/3)*(-2*b^(2/3)*c + a^(2/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]])/b^(1/3) - 9*a*d*Log[x] - (2*(2*a^(2/3)*b^(2/3)*c + a^(4/3)*e)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + ((2*a^(2/3)*b^(2/3)*c + a^(4/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3) + 3*a*d*Log[a + b*x^3])/(9*a^3)
```

Maple [A] time = 0.012, size = 275, normalized size = 1.2

$$-\frac{bcx^2}{3a^2(bx^3+a)} + \frac{ex}{3a(bx^3+a)} + \frac{d}{3a(bx^3+a)} + \frac{2e}{9ab} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{e}{9ab} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{2\sqrt{3}}{9ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x)
```

```
[Out] -1/3*b/a^2*x^2/(b*x^3+a)*c+1/3*a*x/(b*x^3+a)*e+1/3/a/(b*x^3+a)*d+2/9/a/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*e-1/9/a/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e+2/9/a/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e+4/9/a^2*c/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-2/9/a^2*c/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-4/9/a^2*c*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/3*d*ln(b*x^3+a)/a^2+d*ln(x)/a^2-1/a^2*c/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 7.34174, size = 11634, normalized size = 50.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="fricas")

[Out]
$$-1/324*(432*b*c*x^3 - 108*a*e*x^2 - 108*a*d*x + 2*(a^2*b*x^4 + a^3*x))*((-I*\sqrt{3} + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{1/3} + 81*(I*\sqrt{3} + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{1/3} + 54*d/a^2)*\log(-1/324*((-I*\sqrt{3} + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{1/3} + 81*(I*\sqrt{3} + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{1/3} + 54*d/a^2)^2*a^5*b*c - 9*a*b*c*d^2 + 16*a*b*c^2*e + 3*a^2*d*e^2 + 1/18*(6*a^3*b*c*d - a^4*e^2))*((-I*\sqrt{3} + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{1/3} + 81*(I*\sqrt{3} + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{1/3} + 54*d/a^2) - 2*(8*b^2*c^3 - a^2*e^3)*x) + 324*a*c + (162*b*d*x^4 + 162*a*d*x - (a^2*b*x^4 + a^3*x))*((-I*\sqrt{3} + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{1/3} + 81*(I*\sqrt{3} + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{1/3} + 54*d/a^2) + 3*\sqrt{1/3)*(a^2*b*x^4 + a^3*x)*\sqrt{-(((-I*\sqrt{3} + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{1/3} + 81*(I*\sqrt{3} + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{1/3} + 54*d/a^2)^2*a^4 - 108*((-I*\sqrt{3} + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{1/3} + 81*(I*\sqrt{3} + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{1/3} + 54*d/a^2)*a^2*d + 2916*d^2 - 10368*c*e)/a^4))*\log(1/324*((-I*\sqrt{3} + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{1/3} + 81*(I*\sqrt{3} + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{1/3} + 54*d/a^2)$$

$$\begin{aligned}
& a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{(1/3)} + 5 \\
& 4*d/a^2)^2*a^5*b*c + 9*a*b*c*d^2 - 16*a*b*c^2*e - 3*a^2*d*e^2 - 1/18*(6*a^3*b*c*d - a^4*e^2)*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/2 \\
& 7*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{(1/3)} \\
& + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{(1/3)} \\
& + 54*d/a^2) - 4*(8*b^2*c^3 - a^2*e^3)*x + 1/108*sqrt(1/3)*(((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{(1/3)} \\
& + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{(1/3)} + 54*d/a^2)*a^5*b*c - 54*a^3*b*c*d - 18*a^4*e^2)*sqrt(-(((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{(1/3)} + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{(1/3)} + 54*d/a^2)^2*a^4 - 108*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{(1/3)} + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{(1/3)} + 54*d/a^2)*a^2*d + 2916*d^2 - 10368*c*e)/a^4)) + (162*b*d*x^4 + 162*a*d*x - (a^2*b*x^4 + a^3*x)*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{(1/3)} + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{(1/3)} + 54*d/a^2) - 3*sqrt(1/3)*(a^2*b*x^4 + a^3*x)*sqrt(-(((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{(1/3)} + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{(1/3)} + 54*d/a^2)^2*a^4 - 108*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{(1/3)} + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{(1/3)} + 54*d/a^2)*a^2*d + 2916*d^2 - 10368*c*e)/a^4))*log(1/324*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{(1/3)} + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{(1/3)} + 54*d/a^2)
\end{aligned}$$

$$\begin{aligned}
& - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2 \\
& *c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^ \\
& 2*e^3)/(a^7*b))^{(1/3)} + 81*(I*\sqrt{3} + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - \\
& 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a \\
& ^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{(1/3)} + 54*d/a^2)^2*a^5*b*c + \\
& 9*a*b*c*d^2 - 16*a*b*c^2*e - 3*a^2*d*e^2 - 1/18*(6*a^3*b*c*d - a^4*e^2)*((- \\
& I*\sqrt{3} + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^ \\
& 2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a \\
& *b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{(1/3)} + 81*(I*\sqrt{3} + \\
& 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^ \\
& 2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7 \\
& *b))^{(1/3)} + 54*d/a^2) - 4*(8*b^2*c^3 - a^2*e^3)*x - 1/108*\sqrt{1/3}*(((-I* \\
& \sqrt{3} + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^ \\
& 2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b \\
&)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{(1/3)} + 81*(I*\sqrt{3} + 1) \\
& *(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2* \\
& e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b \\
&))^{(1/3)} + 54*d/a^2)*a^5*b*c - 54*a^3*b*c*d - 18*a^4*e^2)*\sqrt{-(((-I*\sqrt{3} \\
& (3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8 \\
& *c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^ \\
& 7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{(1/3)} + 81*(I*\sqrt{3} + 1)*(-1/ \\
& 27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - \\
& 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{(1 \\
& /3)} + 54*d/a^2)^2*a^4 - 108*((-I*\sqrt{3} + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/ \\
& a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8* \\
& a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a \\
& ^7*b))^{(1/3)} + 81*(I*\sqrt{3} + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/ \\
& a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4 \\
& /729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^{(1/3)} + 54*d/a^2)*a^2*d + 2916*d^2 - 10 \\
& 368*c*e)/a^4)) - 324*(b*d*x^4 + a*d*x)*\log(x))/(a^2*b*x^4 + a^3*x)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**2,x)

[Out] Timed out

Giac [A] time = 1.10002, size = 328, normalized size = 1.42

$$-\frac{d \log(|bx^3 + a|)}{3a^2} + \frac{d \log(|x|)}{a^2} - \frac{4bcx^3 - ax^2e - adx + 3ac}{3(bx^4 + ax)a^2} + \frac{\left((-ab^2)^{\frac{1}{3}}ae - 2(-ab^2)^{\frac{2}{3}}c\right) \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9a^3b} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="giac")

[Out] $-\frac{1}{3}d \log(\text{abs}(b*x^3 + a))/a^2 + d \log(\text{abs}(x))/a^2 - \frac{1}{3} \frac{(4*b*c*x^3 - a*x^2*e - a*d*x + 3*a*c)}{(b*x^4 + a*x)*a^2} + \frac{1}{9} \frac{((-a*b^2)^{\frac{1}{3}}*a*e - 2*(-a*b^2)^{\frac{2}{3}}*c) \log(x^2 + x*(-a/b)^{\frac{1}{3}} + (-a/b)^{\frac{2}{3}})}{(a^3*b) + \frac{2}{9} \sqrt{3} * ((-a*b^2)^{\frac{1}{3}}*a*b^2*e + 2*(-a*b^2)^{\frac{2}{3}}*b^2*c) \arctan(\frac{1}{3} \sqrt{3} * (2*x + (-a/b)^{\frac{1}{3}}) / (-a/b)^{\frac{1}{3}})}{(a^3*b^3) + \frac{2}{9} * (2*a^2*b^2*c*(-a/b)^{\frac{1}{3}} - a^3*b*e) * (-a/b)^{\frac{1}{3}} \log(\text{abs}(x - (-a/b)^{\frac{1}{3}}))} / (a^5*b)}$

$$3.349 \quad \int \frac{c+dx+ex^2}{x^3(a+bx^3)^2} dx$$

Optimal. Leaf size=242

$$\frac{\sqrt[3]{b}(5\sqrt[3]{bc}-4\sqrt[3]{ad})\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2)}{18a^{8/3}} - \frac{x(bc+bdx+bex^2)}{3a^2(a+bx^3)} - \frac{\sqrt[3]{b}(5\sqrt[3]{bc}-4\sqrt[3]{ad})\log(\sqrt[3]{a}+\sqrt[3]{bx})}{9a^{8/3}} + \frac{\sqrt[3]{b}(4\sqrt[3]{a^2b}-3\sqrt[3]{ad})\log(\sqrt[3]{a}+\sqrt[3]{bx})}{9a^{8/3}}$$

[Out] $-c/(2*a^2*x^2) - d/(a^2*x) - (x*(b*c + b*d*x + b*e*x^2))/(3*a^2*(a + b*x^3)) + (b^(1/3)*(5*b^(1/3)*c + 4*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(8/3)) + (e*Log[x])/a^2 - (b^(1/3)*(5*b^(1/3)*c - 4*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x]/(9*a^(8/3)) + (b^(1/3)*(5*b^(1/3)*c - 4*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(18*a^(8/3)) - (e*Log[a + b*x^3])/ (3*a^2)$

Rubi [A] time = 0.345395, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\sqrt[3]{b}(5\sqrt[3]{bc}-4\sqrt[3]{ad})\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2)}{18a^{8/3}} - \frac{x(bc+bdx+bex^2)}{3a^2(a+bx^3)} - \frac{\sqrt[3]{b}(5\sqrt[3]{bc}-4\sqrt[3]{ad})\log(\sqrt[3]{a}+\sqrt[3]{bx})}{9a^{8/3}} + \frac{\sqrt[3]{b}(4\sqrt[3]{a^2b}-3\sqrt[3]{ad})\log(\sqrt[3]{a}+\sqrt[3]{bx})}{9a^{8/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^2), x]

[Out] $-c/(2*a^2*x^2) - d/(a^2*x) - (x*(b*c + b*d*x + b*e*x^2))/(3*a^2*(a + b*x^3)) + (b^(1/3)*(5*b^(1/3)*c + 4*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(8/3)) + (e*Log[x])/a^2 - (b^(1/3)*(5*b^(1/3)*c - 4*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x]/(9*a^(8/3)) + (b^(1/3)*(5*b^(1/3)*c - 4*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(18*a^(8/3)) - (e*Log[a + b*x^3])/ (3*a^2)$

Rule 1829

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[

```
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_)*(x_))^(m_))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Dist[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^( -1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2}{x^3(a + bx^3)^2} dx &= -\frac{x(bc + bdx + bex^2)}{3a^2(a + bx^3)} - \frac{\int \frac{-3bc - 3bdx - 3bex^2 + \frac{2b^2cx^3}{a} + \frac{b^2dx^4}{a}}{x^3(a + bx^3)} dx}{3ab} \\
&= -\frac{x(bc + bdx + bex^2)}{3a^2(a + bx^3)} - \frac{\int \left(-\frac{3bc}{ax^3} - \frac{3bd}{ax^2} - \frac{3be}{ax} + \frac{b^2(5c + 4dx + 3ex^2)}{a(a + bx^3)} \right) dx}{3ab} \\
&= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc + bdx + bex^2)}{3a^2(a + bx^3)} + \frac{e \log(x)}{a^2} - \frac{b \int \frac{5c + 4dx + 3ex^2}{a + bx^3} dx}{3a^2} \\
&= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc + bdx + bex^2)}{3a^2(a + bx^3)} + \frac{e \log(x)}{a^2} - \frac{b \int \frac{5c + 4dx}{a + bx^3} dx}{3a^2} - \frac{(be) \int \frac{x^2}{a + bx^3} dx}{a^2} \\
&= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc + bdx + bex^2)}{3a^2(a + bx^3)} + \frac{e \log(x)}{a^2} - \frac{e \log(a + bx^3)}{3a^2} - \frac{b^{2/3} \int \frac{\sqrt[3]{a}(10\sqrt[3]{bc} + 4\sqrt[3]{ad}) + \sqrt[3]{b}(-5\sqrt[3]{b})}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} dx}{9a^{8/3}} \\
&= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc + bdx + bex^2)}{3a^2(a + bx^3)} + \frac{e \log(x)}{a^2} - \frac{\sqrt[3]{b}(5\sqrt[3]{bc} - 4\sqrt[3]{ad}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{8/3}} - \frac{e \log(a + bx^3)}{3a^2} \\
&= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc + bdx + bex^2)}{3a^2(a + bx^3)} + \frac{e \log(x)}{a^2} - \frac{\sqrt[3]{b}(5\sqrt[3]{bc} - 4\sqrt[3]{ad}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{8/3}} + \frac{\sqrt[3]{b}(5\sqrt[3]{bc} - 4\sqrt[3]{ad}) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}} + \frac{e \log(x)}{a^2} - \frac{\sqrt[3]{b}(5\sqrt[3]{bc} - 4\sqrt[3]{ad})}{9a^{8/3}}
\end{aligned}$$

Mathematica [A] time = 0.169457, size = 221, normalized size = 0.91

$$\sqrt[3]{b}(5\sqrt[3]{a}\sqrt[3]{bc} - 4a^{2/3}d) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2) + 2\sqrt[3]{b}(4a^{2/3}d - 5\sqrt[3]{a}\sqrt[3]{bc}) \log(\sqrt[3]{a} + \sqrt[3]{bx}) + \frac{6a(ae - bx(c + dx))}{a + bx^3} + 2\sqrt{3}\sqrt[3]{a}$$

18a³

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^2), x]

[Out] ((-9*a*c)/x^2 - (18*a*d)/x + (6*a*(a*e - b*x*(c + d*x)))/(a + b*x^3) + 2*sqrt[3]*a^(1/3)*b^(1/3)*(5*b^(1/3)*c + 4*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)

$$\frac{1}{a^{1/3}} \sqrt[3]{3} + 18ae \operatorname{Log}[x] + 2b^{1/3}(-5a^{1/3}b^{1/3}c + 4a^{2/3}d) \operatorname{Log}[a^{1/3} + b^{1/3}x] + b^{1/3}(5a^{1/3}b^{1/3}c - 4a^{2/3}d) \operatorname{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2] - 6ae \operatorname{Log}[a + bx^3] / (18a^3)$$

Maple [A] time = 0.013, size = 276, normalized size = 1.1

$$-\frac{bdx^2}{3a^2(bx^3+a)} - \frac{bcx}{3a^2(bx^3+a)} + \frac{e}{3a(bx^3+a)} - \frac{5c}{9a^2} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{5c}{18a^2} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{5c}{9a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x)

[Out] $-\frac{1}{3} \frac{a^2 b x^2}{(b x^3 + a)^2} - \frac{1}{3} \frac{b a^2 x}{(b x^3 + a)^2} + \frac{1}{3} \frac{a}{(b x^3 + a)^2} e - \frac{5}{9} \frac{a^2 c}{(1/b a)^{2/3}} \ln(x + (1/b a)^{1/3}) + \frac{5}{18} \frac{a^2 c}{(1/b a)^{2/3}} \ln(x^2 - (1/b a)^{1/3} x + (1/b a)^{2/3}) - \frac{5}{9} \frac{a^2 c}{(1/b a)^{2/3}} 3^{1/2} \arctan(1/3 3^{1/2} (2/(1/b a)^{1/3} x - 1)) + \frac{4}{9} \frac{a^2 d}{(1/b a)^{2/3}} \ln(x + (1/b a)^{1/3}) - \frac{2}{9} \frac{a^2 d}{(1/b a)^{2/3}} \ln(x^2 - (1/b a)^{1/3} x + (1/b a)^{2/3}) - \frac{4}{9} \frac{a^2 d}{(1/b a)^{2/3}} 3^{1/2} \arctan(1/3 3^{1/2} (2/(1/b a)^{1/3} x - 1)) - \frac{1}{3} \frac{e \ln(b x^3 + a)}{a^2} - \frac{d}{a^2} \frac{1}{x} - \frac{1}{2} \frac{c}{a^2} \frac{1}{x^2} + e \ln(x) / a^2$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [C] time = 9.80153, size = 12122, normalized size = 50.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="fricas")

[Out]
$$-1/324*(432*b*d*x^4 + 270*b*c*x^3 - 108*a*e*x^2 + 324*a*d*x + 2*(a^2*b*x^5 + a^3*x^2))*((-I*\sqrt{3} + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{1/3} + 81*(I*\sqrt{3} + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{1/3} + 54*e/a^2*\log(1/81*((-I*\sqrt{3} + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{1/3} + 81*(I*\sqrt{3} + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{1/3} + 54*e/a^2)^2*a^6*d + 160*a*b*c*d^2 - 75*a*b*c^2*e + 36*a^2*d*e^2 + 1/18*(25*a^3*b*c^2 - 24*a^4*d*e)*((-I*\sqrt{3} + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{1/3} + 81*(I*\sqrt{3} + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{1/3} + 54*e/a^2) + (125*b^2*c^3 + 64*a*b*d^3)*x) + 162*a*c + (162*b*e*x^5 + 162*a*e*x^2 - (a^2*b*x^5 + a^3*x^2))*((-I*\sqrt{3} + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{1/3} + 81*(I*\sqrt{3} + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{1/3} + 54*e/a^2) - 3*\sqrt{1/3}*(a^2*b*x^5 + a^3*x^2)*\sqrt{-(((-I*\sqrt{3} + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{1/3} + 81*(I*\sqrt{3} + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{1/3} + 54*e/a^2)^2*a^5 - 108*((-I*\sqrt{3} + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{1/3} + 81*(I*\sqrt{3} + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{1/3} + 54*e/a^2)*a^3*e + 25920*b*c*d + 2916*a*e^2)/a^5)*\log(-1/81*((-I*\sqrt{3} + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{1/3} + 81*(I*\sqrt{3} + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{1/3} + 54*e/a^2)$$

$$\begin{aligned}
& c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{(1/3)} + 54*e/a^2)^2*a^6*d - 160*a*b*c*d^2 + 75*a*b*c^2*e - 36*a^2*d*e^2 - 1/18*(25*a^3*b*c^2 - 24*a^4*d*e)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{(1/3)} + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{(1/3)} + 54*e/a^2) + 2*(125*b^2*c^3 + 64*a*b*d^3)*x + 1/54*sqrt(1/3)*(2*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{(1/3)} + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{(1/3)} + 54*e/a^2)*a^6*d - 225*a^3*b*c^2 - 108*a^4*d*e)*sqrt(-(((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{(1/3)} + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{(1/3)} + 54*e/a^2)^2*a^5 - 108*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{(1/3)} + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{(1/3)} + 54*e/a^2)*a^3*e + 25920*b*c*d + 2916*a*e^2)/a^5)) + (162*b*e*x^5 + 162*a*e*x^2 - (a^2*b*x^5 + a^3*x^2))*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{(1/3)} + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{(1/3)} + 54*e/a^2) + 3*sqrt(1/3)*(a^2*b*x^5 + a^3*x^2)*sqrt(-(((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{(1/3)} + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{(1/3)} + 54*e/a^2)^2*a^5 - 108*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{(1/3)} + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{(1/3)} + 54*e/a^2)
\end{aligned}$$

$$\begin{aligned}
& *a*b)/a^8)^{(1/3)} + 54*e/a^2)*a^3*e + 25920*b*c*d + 2916*a*e^2)/a^5)) * \log(-1 \\
& /81*((-I*\sqrt{3}) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 \\
& + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - \\
& 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{(1/3)} + \\
& 81*(I*\sqrt{3}) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1 \\
& 458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16 \\
& *d^3 - 45*c*d*e)*a*b)/a^8)^{(1/3)} + 54*e/a^2)^2*a^6*d - 160*a*b*c*d^2 + 75*a \\
& *b*c^2*e - 36*a^2*d*e^2 - 1/18*(25*a^3*b*c^2 - 24*a^4*d*e)*((-I*\sqrt{3}) + 1 \\
&)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + \\
& 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 \\
& + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{(1/3)} + 81*(I*\sqrt{3}) + 1)* \\
& (-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64* \\
& a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b \\
&)/a^8)^{(1/3)} + 54*e/a^2) + 2*(125*b^2*c^3 + 64*a*b*d^3)*x - 1/54*\sqrt{1/3)* \\
& (2*((-I*\sqrt{3}) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 \\
& + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - \\
& 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{(1/3)} + \\
& 81*(I*\sqrt{3}) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/14 \\
& 58*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16* \\
& d^3 - 45*c*d*e)*a*b)/a^8)^{(1/3)} + 54*e/a^2)*a^6*d - 225*a^3*b*c^2 - 108*a^4 \\
& *d*e)*\sqrt{-(((-I*\sqrt{3}) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/2 \\
& 7*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3 \\
&)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8 \\
&)^{(1/3)} + 81*(I*\sqrt{3}) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/ \\
& a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^ \\
& 3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{(1/3)} + 54*e/a^2)^2*a^5 - 108*((-I*\sqrt{ \\
& 3}) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20* \\
& b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125* \\
& b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^{(1/3)} + 81*(I*\sqrt{3} \\
&) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^ \\
& 3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d \\
& *e)*a*b)/a^8)^{(1/3)} + 54*e/a^2)*a^3*e + 25920*b*c*d + 2916*a*e^2)/a^5)) - 3 \\
& 24*(b*e*x^5 + a*e*x^2)*\log(x))/(a^2*b*x^5 + a^3*x^2)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**2,x)

[Out] Timed out

Giac [A] time = 1.09414, size = 344, normalized size = 1.42

$$-\frac{e \log(|bx^3 + a|)}{3a^2} + \frac{e \log(|x|)}{a^2} - \frac{\left(5(-ab^2)^{\frac{1}{3}}bc + 4(-ab^2)^{\frac{2}{3}}d\right) \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^3b} + \frac{\left(4a^2b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 5a^2b^2c\right)}{9a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="giac")

[Out]
$$-1/3*e*\log(\text{abs}(b*x^3 + a))/a^2 + e*\log(\text{abs}(x))/a^2 - 1/18*(5*(-a*b^2)^{(1/3)}*b*c + 4*(-a*b^2)^{(2/3)}*d)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a^3*b) + 1/9*(4*a^2*b^2*d*(-a/b)^{(1/3)} + 5*a^2*b^2*c)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/(a^5*b) - 1/6*(8*b*d*x^4 + 5*b*c*x^3 - 2*a*x^2*e + 6*a*d*x + 3*a*c)/((b*x^3 + a)*a^2*x^2) - 1/9*\text{sqrt}(3)*(5*(-a*b^2)^{(1/3)}*a*b^3*c - 4*(-a*b^2)^{(2/3)}*a*b^2*d)*\text{arctan}(1/3*\text{sqrt}(3)*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(a^4*b^3)$$

$$3.350 \quad \int \frac{c+dx+ex^2}{x^4(a+bx^3)^2} dx$$

Optimal. Leaf size=262

$$\frac{x \left(-\frac{b^2cx^2}{a} + bd + bex \right)}{3a^2(a+bx^3)} + \frac{\sqrt[3]{b} (5\sqrt[3]{bd} - 4\sqrt[3]{ae}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{18a^{8/3}} + \frac{2bc \log(a+bx^3)}{3a^3} - \frac{2bc \log(x)}{a^3} - \frac{\sqrt[3]{b} (5\sqrt[3]{bd})}{a^3}$$

[Out] $-c/(3*a^2*x^3) - d/(2*a^2*x^2) - e/(a^2*x) - (x*(b*d + b*e*x - (b^2*c*x^2)/a))/(3*a^2*(a + b*x^3)) + (b^{(1/3)}*(5*b^{(1/3)}*d + 4*a^{(1/3)}*e)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})]/(3*Sqrt[3]*a^{(8/3)}) - (2*b*c*Log[x])/a^3 - (b^{(1/3)}*(5*b^{(1/3)}*d - 4*a^{(1/3)}*e)*Log[a^{(1/3)} + b^{(1/3)}*x]/(9*a^{(8/3)}) + (b^{(1/3)}*(5*b^{(1/3)}*d - 4*a^{(1/3)}*e)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2]/(18*a^{(8/3)}) + (2*b*c*Log[a + b*x^3])/ (3*a^3)$

Rubi [A] time = 0.403572, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{x \left(-\frac{b^2cx^2}{a} + bd + bex \right)}{3a^2(a+bx^3)} + \frac{\sqrt[3]{b} (5\sqrt[3]{bd} - 4\sqrt[3]{ae}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{18a^{8/3}} + \frac{2bc \log(a+bx^3)}{3a^3} - \frac{2bc \log(x)}{a^3} - \frac{\sqrt[3]{b} (5\sqrt[3]{bd})}{a^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^2), x]

[Out] $-c/(3*a^2*x^3) - d/(2*a^2*x^2) - e/(a^2*x) - (x*(b*d + b*e*x - (b^2*c*x^2)/a))/(3*a^2*(a + b*x^3)) + (b^{(1/3)}*(5*b^{(1/3)}*d + 4*a^{(1/3)}*e)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})]/(3*Sqrt[3]*a^{(8/3)}) - (2*b*c*Log[x])/a^3 - (b^{(1/3)}*(5*b^{(1/3)}*d - 4*a^{(1/3)}*e)*Log[a^{(1/3)} + b^{(1/3)}*x]/(9*a^{(8/3)}) + (b^{(1/3)}*(5*b^{(1/3)}*d - 4*a^{(1/3)}*e)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2]/(18*a^{(8/3)}) + (2*b*c*Log[a + b*x^3])/ (3*a^3)$

Rule 1829

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[

```
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_)*(x_))^(m_))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Dist[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^( -1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2}{x^4(a + bx^3)^2} dx &= -\frac{x\left(bd + bex - \frac{b^2cx^2}{a}\right)}{3a^2(a + bx^3)} - \frac{\int \frac{-3bc - 3bdx - 3bex^2 + \frac{3b^2cx^3}{a} + \frac{2b^2dx^4}{a} + \frac{b^2ex^5}{a}}{x^4(a + bx^3)} dx}{3ab} \\
&= -\frac{x\left(bd + bex - \frac{b^2cx^2}{a}\right)}{3a^2(a + bx^3)} - \frac{\int \left(-\frac{3bc}{ax^4} - \frac{3bd}{ax^3} - \frac{3be}{ax^2} + \frac{6b^2c}{a^2x} + \frac{b^2(5ad + 4aex - 6bcx^2)}{a^2(a + bx^3)}\right) dx}{3ab} \\
&= -\frac{c}{3a^2x^3} - \frac{d}{2a^2x^2} - \frac{e}{a^2x} - \frac{x\left(bd + bex - \frac{b^2cx^2}{a}\right)}{3a^2(a + bx^3)} - \frac{2bc \log(x)}{a^3} - \frac{b \int \frac{5ad + 4aex - 6bcx^2}{a + bx^3} dx}{3a^3} \\
&= -\frac{c}{3a^2x^3} - \frac{d}{2a^2x^2} - \frac{e}{a^2x} - \frac{x\left(bd + bex - \frac{b^2cx^2}{a}\right)}{3a^2(a + bx^3)} - \frac{2bc \log(x)}{a^3} - \frac{b \int \frac{5ad + 4aex}{a + bx^3} dx}{3a^3} + \frac{(2b^2c) \int \frac{x^2}{a + bx^3} dx}{a^3} \\
&= -\frac{c}{3a^2x^3} - \frac{d}{2a^2x^2} - \frac{e}{a^2x} - \frac{x\left(bd + bex - \frac{b^2cx^2}{a}\right)}{3a^2(a + bx^3)} - \frac{2bc \log(x)}{a^3} + \frac{2bc \log(a + bx^3)}{3a^3} - \frac{b^{2/3} \int \frac{\sqrt[3]{a}(10a\sqrt[3]{b}}{a + bx^3} dx}{9a^{8/3}} \\
&= -\frac{c}{3a^2x^3} - \frac{d}{2a^2x^2} - \frac{e}{a^2x} - \frac{x\left(bd + bex - \frac{b^2cx^2}{a}\right)}{3a^2(a + bx^3)} - \frac{2bc \log(x)}{a^3} - \frac{\sqrt[3]{b}(5\sqrt[3]{bd} - 4\sqrt[3]{ae}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{8/3}} \\
&= -\frac{c}{3a^2x^3} - \frac{d}{2a^2x^2} - \frac{e}{a^2x} - \frac{x\left(bd + bex - \frac{b^2cx^2}{a}\right)}{3a^2(a + bx^3)} - \frac{2bc \log(x)}{a^3} - \frac{\sqrt[3]{b}(5\sqrt[3]{bd} - 4\sqrt[3]{ae}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{8/3}} \\
&= -\frac{c}{3a^2x^3} - \frac{d}{2a^2x^2} - \frac{e}{a^2x} - \frac{x\left(bd + bex - \frac{b^2cx^2}{a}\right)}{3a^2(a + bx^3)} + \frac{\sqrt[3]{b}(5\sqrt[3]{bd} + 4\sqrt[3]{ae}) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}} - \frac{2bc \log(x)}{a^3}
\end{aligned}$$

Mathematica [A] time = 0.174743, size = 225, normalized size = 0.86

$$\sqrt[3]{b}(5\sqrt[3]{a}\sqrt[3]{bd} - 4a^{2/3}e) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2) + 2\sqrt[3]{b}(4a^{2/3}e - 5\sqrt[3]{a}\sqrt[3]{bd}) \log(\sqrt[3]{a} + \sqrt[3]{bx}) - \frac{6ab(c + x(d + ex))}{a + bx^3} + 12bc \log$$

18a³

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^2), x]

[Out] ((-6*a*c)/x^3 - (9*a*d)/x^2 - (18*a*e)/x - (6*a*b*(c + x*(d + e*x)))/(a + b*x^3) + 2*Sqrt[3]*a^(1/3)*b^(1/3)*(5*b^(1/3)*d + 4*a^(1/3)*e)*ArcTan[(1 - (

$$2*b^{(1/3)*x}/a^{(1/3)}/\text{Sqrt}[3] - 36*b*c*\text{Log}[x] + 2*b^{(1/3)}*(-5*a^{(1/3)}*b^{(1/3)}*d + 4*a^{(2/3)}*e)*\text{Log}[a^{(1/3)} + b^{(1/3)}*x] + b^{(1/3)}*(5*a^{(1/3)}*b^{(1/3)}*d - 4*a^{(2/3)}*e)*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2] + 12*b*c*\text{Log}[a + b*x^3]/(18*a^3)$$

Maple [A] time = 0.013, size = 289, normalized size = 1.1

$$-\frac{bex^2}{3a^2(bx^3+a)} - \frac{bdx}{3a^2(bx^3+a)} - \frac{bc}{3a^2(bx^3+a)} - \frac{5d}{9a^2} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{5d}{18a^2} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{5d}{9a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x)

[Out] $-\frac{1}{3}a^{-2}x^2/(b*x^3+a)*b*e - \frac{1}{3}a^{-2}b*x/(b*x^3+a)*d - \frac{1}{3}a^{-2}b/(b*x^3+a)*c - \frac{5}{9}a^{-2}/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*d + \frac{5}{18}a^{-2}/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*d - \frac{5}{9}a^{-2}/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*d + \frac{4}{9}a^{-2}*e/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)}) - \frac{2}{9}a^{-2}*e/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)}) - \frac{4}{9}a^{-2}*e*3^{(1/2)}/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1)) + \frac{2}{3}b*c*\ln(b*x^3+a)/a^3 - \frac{1}{2}d/a^2/x^2 - e/a^2/x - 1/3*c/x^3/a^2 - 2*b*c*\ln(x)/a^3$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [C] time = 10.1793, size = 12670, normalized size = 48.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="fricas")

[Out]
$$-1/36*(48*a*b*e*x^5 + 30*a*b*d*x^4 + 24*a*b*c*x^3 + 36*a^2*e*x^2 + 18*a^2*d*x + 12*a^2*c + 2*(a^3*b*x^6 + a^4*x^3)*(8*(1/2)^{(2/3)}*(-I*\sqrt{3}) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3}) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^{(1/3)} - 12*b*c/a^3)*\log((8*(1/2)^{(2/3)}*(-I*\sqrt{3}) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3}) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^{(1/3)} - 12*b*c/a^3)^2*a^6*e + 150*b^2*c*d^2 + 144*b^2*c^2*e + 160*a*b*d*e^2 + 1/2*(25*a^3*b*d^2 + 48*a^3*b*c*e)*(8*(1/2)^{(2/3)}*(-I*\sqrt{3}) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3}) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^{(1/3)} - 12*b*c/a^3) + (125*b^2*d^3 + 64*a*b*e^3)*x) - (36*b^2*c*x^6 + 36*a*b*c*x^3 + (a^3*b*x^6 + a^4*x^3)*(8*(1/2)^{(2/3)}*(-I*\sqrt{3}) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3}) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^{(1/3)} - 12*b*c/a^3) + 3*\sqrt{1/3)*(a^3*b*x^6 + a^4*x^3)*\sqrt{-((8*(1/2)^{(2/3)}*(-I*\sqrt{3}) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3}) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^{(1/3)} - 12*b*c/a^3)^2*a^6 + 24*(8*(1/2)^{(2/3)}*(-I*\sqrt{3}) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3}) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^{(1/3)} - 12*b*c/a^3)*a^3*b*c + 144*b^2*c^2 + 320*a*b*d*e)/a^6)*\log(-8*(1/2)^{(2/3)}*(-I*\sqrt{3}) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*$$

$$\begin{aligned}
& b^2c^2 + 5a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72 \\
& *c*d*e)*a*b^2)/a^9)^{(1/3)} + (1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*(432*b^3*c^3/a^9 + \\
& (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^ \\
& 3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^{(1/3)} - 12*b*c/a^3 \\
&)^2*a^6*e - 150*b^2*c*d^2 - 144*b^2*c^2*e - 160*a*b*d*e^2 - 1/2*(25*a^3*b*d \\
& ^2 + 48*a^3*b*c*e)*(8*(1/2)^{(2/3)}*(-I*\text{sqrt}(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c \\
& ^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72* \\
& (9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - \\
& 72*c*d*e)*a*b^2)/a^9)^{(1/3)} + (1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*(432*b^3*c^3/a^9 \\
& + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216 \\
& *b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^{(1/3)} - 12*b*c/a \\
& ^3) + 2*(125*b^2*d^3 + 64*a*b*e^3)*x + 3/2*\text{sqrt}(1/3)*(2*(8*(1/2)^{(2/3)}*(-I \\
& *\text{sqrt}(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^ \\
& 9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (21 \\
& 6*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^{(1/3)} + (1/2)^ \\
& (1/3)*(I*\text{sqrt}(3) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72* \\
& (9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - \\
& 72*c*d*e)*a*b^2)/a^9)^{(1/3)} - 12*b*c/a^3)*a^6*e - 25*a^3*b*d^2 + 24*a^3*b* \\
& c*e)*\text{sqrt}(-((8*(1/2)^{(2/3)}*(-I*\text{sqrt}(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5 \\
& *a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c \\
& ^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d \\
& *e)*a*b^2)/a^9)^{(1/3)} + (1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*(432*b^3*c^3/a^9 + (125 \\
& *b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^ \\
& 3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^{(1/3)} - 12*b*c/a^3)^2* \\
& a^6 + 24*(8*(1/2)^{(2/3)}*(-I*\text{sqrt}(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a* \\
& b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 \\
& + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e) \\
& *a*b^2)/a^9)^{(1/3)} + (1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*(432*b^3*c^3/a^9 + (125*b* \\
& d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + \\
& 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^{(1/3)} - 12*b*c/a^3)*a^3*b \\
& *c + 144*b^2*c^2 + 320*a*b*d*e)/a^6)) - (36*b^2*c*x^6 + 36*a*b*c*x^3 + (a^3 \\
& *b*x^6 + a^4*x^3)*(8*(1/2)^{(2/3)}*(-I*\text{sqrt}(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c \\
& ^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(\\
& 9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - \\
& 72*c*d*e)*a*b^2)/a^9)^{(1/3)} + (1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*(432*b^3*c^3/a^9 \\
& + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216* \\
& b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^{(1/3)} - 12*b*c/a \\
& ^3) - 3*\text{sqrt}(1/3)*(a^3*b*x^6 + a^4*x^3)*\text{sqrt}(-((8*(1/2)^{(2/3)}*(-I*\text{sqrt}(3) + \\
& 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b \\
& *d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 \\
& + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^{(1/3)} + (1/2)^{(1/3)}*(I*s \\
& \text{qrt}(3) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 \\
& + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e) \\
& *a*b^2)/a^9)^{(1/3)} - 12*b*c/a^3)^2*a^6 + 24*(8*(1/2)^{(2/3)}*(-I*\text{sqrt}(3) + 1) \\
& *(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^
\end{aligned}$$

$$\begin{aligned}
& 3 + 64*a*e^3*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^{(1/3)} - 12*b*c/a^3)*a^3*b*c + 144*b^2*c^2 + 320*a*b*d*e)/a^6))*\log(- (8*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^{(1/3)} - 12*b*c/a^3)^2*a^6*e - 150*b^2*c*d^2 - 144*b^2*c^2*e - 160*a*b*d*e^2 - 1/2*(25*a^3*b*d^2 + 48*a^3*b*c*e)*(8*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^{(1/3)} - 12*b*c/a^3) + 2*(125*b^2*d^3 + 64*a*b*e^3)*x - 3/2*\sqrt{1/3}*(2*(8*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^{(1/3)} - 12*b*c/a^3)^2*a^6 + 24*(8*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^{(1/3)} - 12*b*c/a^3)*a^3*b*c + 144*b^2*c^2 + 320*a*b*d*e)/a^6)) + 72*(b^2*c*x^6 + a*b*c*x^3)*\log(x))/(a^3*b*x^6 + a^4*x^3)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/x**4/(b*x**3+a)**2,x)

[Out] Timed out

Giac [A] time = 1.07759, size = 373, normalized size = 1.42

$$\frac{2bc \log(|bx^3 + a|)}{3a^3} - \frac{2bc \log(|x|)}{a^3} - \frac{\left(5(-ab^2)^{\frac{1}{3}}bd + 4(-ab^2)^{\frac{2}{3}}e\right) \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^3b} - \frac{\sqrt{3}\left(5(-ab^2)^{\frac{1}{3}}ab^3d - \dots\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="giac")

[Out] $\frac{2}{3}bc \log(\text{abs}(bx^3 + a))/a^3 - 2bc \log(\text{abs}(x))/a^3 - \frac{1}{18}(5(-ab^2)^{\frac{1}{3}}bd + 4(-ab^2)^{\frac{2}{3}}e) \log(x^2 + x(-a/b)^{\frac{1}{3}} + (-a/b)^{\frac{2}{3}})/(a^3b) - \frac{1}{9}\sqrt{3}(5(-ab^2)^{\frac{1}{3}}ab^3d - 4(-ab^2)^{\frac{2}{3}}ab^2e) \text{arctan}\left(\frac{1}{3}\sqrt{3}(2x + (-a/b)^{\frac{1}{3}})/(-a/b)^{\frac{1}{3}}\right)/(a^4b^3) + \frac{1}{9}(4a^4b^2(-a/b)^{\frac{1}{3}}e + 5a^4b^2d)(-a/b)^{\frac{1}{3}} \log(\text{abs}(x - (-a/b)^{\frac{1}{3}}))/(a^7b) - \frac{1}{6}(8abx^5e + 5abdx^4 + 4abcx^3 + 6a^2x^2e + 3a^2dx + 2a^2c)/((bx^3 + a)a^3x^3)$

$$3.351 \quad \int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=215

$$\frac{\left(d - \frac{\sqrt[3]{ae}}{\sqrt[3]{b}}\right) \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{54a^{5/3}b^{4/3}} + \frac{\left(\sqrt[3]{bd} - \sqrt[3]{ae}\right) \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{27a^{5/3}b^{5/3}} - \frac{\left(\sqrt[3]{ae} + \sqrt[3]{bd}\right) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{5/3}} - \frac{c+dx}{6b(a+bx^3)}$$

[Out] $-(c + d*x + e*x^2)/(6*b*(a + b*x^3)^2) + (x*(d + 2*e*x))/(18*a*b*(a + b*x^3)) - ((b^{(1/3)*d} + a^{(1/3)*e})*ArcTan[(a^{(1/3)} - 2*b^{(1/3)*x})/(Sqrt[3]*a^{(1/3)})])/(9*Sqrt[3]*a^{(5/3)*b^{(5/3)}}) + ((b^{(1/3)*d} - a^{(1/3)*e})*Log[a^{(1/3)} + b^{(1/3)*x}])/(27*a^{(5/3)*b^{(5/3)}}) - ((d - (a^{(1/3)*e})/b^{(1/3)})*Log[a^{(2/3)} - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2}])/(54*a^{(5/3)*b^{(4/3)}})$

Rubi [A] time = 0.196885, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {1823, 1855, 1860, 31, 634, 617, 204, 628}

$$\frac{\left(d - \frac{\sqrt[3]{ae}}{\sqrt[3]{b}}\right) \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{54a^{5/3}b^{4/3}} + \frac{\left(\sqrt[3]{bd} - \sqrt[3]{ae}\right) \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{27a^{5/3}b^{5/3}} - \frac{\left(\sqrt[3]{ae} + \sqrt[3]{bd}\right) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{5/3}} - \frac{c+dx}{6b(a+bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^3,x]

[Out] $-(c + d*x + e*x^2)/(6*b*(a + b*x^3)^2) + (x*(d + 2*e*x))/(18*a*b*(a + b*x^3)) - ((b^{(1/3)*d} + a^{(1/3)*e})*ArcTan[(a^{(1/3)} - 2*b^{(1/3)*x})/(Sqrt[3]*a^{(1/3)})])/(9*Sqrt[3]*a^{(5/3)*b^{(5/3)}}) + ((b^{(1/3)*d} - a^{(1/3)*e})*Log[a^{(1/3)} + b^{(1/3)*x}])/(27*a^{(5/3)*b^{(5/3)}}) - ((d - (a^{(1/3)*e})/b^{(1/3)})*Log[a^{(2/3)} - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2}])/(54*a^{(5/3)*b^{(4/3)}})$

Rule 1823

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Pq*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && EqQ[m - n + 1, 0] && LtQ[p, -1]

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
+ 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d},
```

e}], x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^3} dx &= -\frac{c+dx+ex^2}{6b(a+bx^3)^2} + \frac{\int \frac{d+2ex}{(a+bx^3)^2} dx}{6b} \\
 &= -\frac{c+dx+ex^2}{6b(a+bx^3)^2} + \frac{x(d+2ex)}{18ab(a+bx^3)} - \frac{\int \frac{-2d-2ex}{a+bx^3} dx}{18ab} \\
 &= -\frac{c+dx+ex^2}{6b(a+bx^3)^2} + \frac{x(d+2ex)}{18ab(a+bx^3)} - \frac{\int \frac{\sqrt[3]{a}(-4\sqrt[3]{bd}-2\sqrt[3]{ae}) + \sqrt[3]{b}(2\sqrt[3]{bd}-2\sqrt[3]{ae})x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2} dx}{54a^{5/3}b^{4/3}} + \frac{\left(d-\frac{\sqrt[3]{ae}}{\sqrt[3]{b}}\right) \int \frac{1}{\sqrt[3]{a}+\sqrt[3]{bx}}}{27a^{5/3}b} \\
 &= -\frac{c+dx+ex^2}{6b(a+bx^3)^2} + \frac{x(d+2ex)}{18ab(a+bx^3)} + \frac{\left(d-\frac{\sqrt[3]{ae}}{\sqrt[3]{b}}\right) \log(\sqrt[3]{a}+\sqrt[3]{bx})}{27a^{5/3}b^{4/3}} + \frac{(\sqrt[3]{bd}+\sqrt[3]{ae}) \int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}}{18a^{4/3}b^{4/3}} \\
 &= -\frac{c+dx+ex^2}{6b(a+bx^3)^2} + \frac{x(d+2ex)}{18ab(a+bx^3)} + \frac{\left(d-\frac{\sqrt[3]{ae}}{\sqrt[3]{b}}\right) \log(\sqrt[3]{a}+\sqrt[3]{bx})}{27a^{5/3}b^{4/3}} - \frac{\left(d-\frac{\sqrt[3]{ae}}{\sqrt[3]{b}}\right) \log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx})}{54a^{5/3}b^{4/3}} \\
 &= -\frac{c+dx+ex^2}{6b(a+bx^3)^2} + \frac{x(d+2ex)}{18ab(a+bx^3)} - \frac{(\sqrt[3]{bd}+\sqrt[3]{ae}) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{5/3}} + \frac{\left(d-\frac{\sqrt[3]{ae}}{\sqrt[3]{b}}\right) \log(\sqrt[3]{a}+\sqrt[3]{bx})}{27a^{5/3}b^{4/3}}
 \end{aligned}$$

Mathematica [A] time = 0.179103, size = 198, normalized size = 0.92

$$\frac{\frac{(\sqrt[3]{ae}-\sqrt[3]{bd}) \log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2)}{a^{5/3}} + \frac{2(\sqrt[3]{bd}-\sqrt[3]{ae}) \log(\sqrt[3]{a}+\sqrt[3]{bx})}{a^{5/3}} - \frac{2\sqrt{3}(\sqrt[3]{ae}+\sqrt[3]{bd}) \tan^{-1}\left(\frac{1-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}} - \frac{9b^{2/3}(c+x(d+ex))}{(a+bx^3)^2} + \frac{3b^{2/3}x(d+2ex)}{a(a+bx^3)}}{54b^{5/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^3,x]

[Out] ((3*b^(2/3)*x*(d + 2*e*x))/(a*(a + b*x^3)) - (9*b^(2/3)*(c + x*(d + e*x)))/(a + b*x^3)^2 - (2*sqrt(3)*(b^(1/3)*d + a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x

$$\left. \frac{1}{(bx^3 + a)^2} \left(\frac{ex^5}{9a} + \frac{dx^4}{18a} - \frac{ex^2}{18b} - \frac{dx}{9b} - \frac{c}{6b} \right) + \frac{d}{27b^2a} \ln \left(x + \sqrt[3]{\frac{a}{b}} \right) \left(\frac{a}{b} \right)^{-\frac{2}{3}} - \frac{d}{54b^2a} \ln \left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b} \right)^{\frac{2}{3}} \right) \left(\frac{a}{b} \right)^{-\frac{2}{3}} + \frac{d\sqrt{3}}{27b^2} \right) / a^{(1/3)} / \text{Sqrt}[3]] / a^{(5/3)} + (2*(b^{(1/3)}*d - a^{(1/3)}*e)*\text{Log}[a^{(1/3)} + b^{(1/3)}*x]) / a^{(5/3)} + ((-(b^{(1/3)}*d) + a^{(1/3)}*e)*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2]) / a^{(5/3)} / (54*b^{(5/3)})$$

Maple [A] time = 0.01, size = 255, normalized size = 1.2

$$\frac{1}{(bx^3 + a)^2} \left(\frac{ex^5}{9a} + \frac{dx^4}{18a} - \frac{ex^2}{18b} - \frac{dx}{9b} - \frac{c}{6b} \right) + \frac{d}{27b^2a} \ln \left(x + \sqrt[3]{\frac{a}{b}} \right) \left(\frac{a}{b} \right)^{-\frac{2}{3}} - \frac{d}{54b^2a} \ln \left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b} \right)^{\frac{2}{3}} \right) \left(\frac{a}{b} \right)^{-\frac{2}{3}} + \frac{d\sqrt{3}}{27b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^3,x)`

[Out] $(1/9/a*e*x^5+1/18*d/a*x^4-1/18*e*x^2/b-1/9*d*x/b-1/6*c/b)/(b*x^3+a)^2+1/27/b^2/a/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*d-1/54/b^2/a/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*d+1/27/b^2/a/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*d-1/27/a/b^2/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})*e+1/54/a/b^2/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*e+1/27/a/b^2*3^{(1/2)}/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*e$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [C] time = 9.74419, size = 5122, normalized size = 23.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")

[Out]
$$\frac{1}{108} \cdot (12 b^2 e x^5 + 6 b^2 d x^4 - 6 a^2 e x^2 - 12 a d x - 2 (a^3 b^3 x^6 + 2 a^2 b^2 x^3 + a^3 b) \cdot ((1/2)^{1/3} (I \sqrt{3} + 1) \cdot ((b d^3 + a e^3)/(a^5 b^5) + (b d^3 - a e^3)/(a^5 b^5))^{1/3} - 2 \cdot (1/2)^{2/3} d e \cdot (-I \sqrt{3} + 1)/(a^3 b^3 \cdot ((b d^3 + a e^3)/(a^5 b^5) + (b d^3 - a e^3)/(a^5 b^5))^{1/3})) \cdot \log(1/4 \cdot ((1/2)^{1/3} (I \sqrt{3} + 1) \cdot ((b d^3 + a e^3)/(a^5 b^5) + (b d^3 - a e^3)/(a^5 b^5))^{1/3} - 2 \cdot (1/2)^{2/3} d e \cdot (-I \sqrt{3} + 1)/(a^3 b^3 \cdot ((b d^3 + a e^3)/(a^5 b^5) + (b d^3 - a e^3)/(a^5 b^5))^{1/3}))^2 \cdot a^4 b^3 e - 1/2 \cdot ((1/2)^{1/3} (I \sqrt{3} + 1) \cdot ((b d^3 + a e^3)/(a^5 b^5) + (b d^3 - a e^3)/(a^5 b^5))^{1/3} - 2 \cdot (1/2)^{2/3} d e \cdot (-I \sqrt{3} + 1)/(a^3 b^3 \cdot ((b d^3 + a e^3)/(a^5 b^5) + (b d^3 - a e^3)/(a^5 b^5))^{1/3})) \cdot a^2 b^2 d^2 + 2 a d e^2 + (b d^3 + a e^3) x) - 18 a c + ((a b^3 x^6 + 2 a^2 b^2 x^3 + a^3 b) \cdot ((1/2)^{1/3} (I \sqrt{3} + 1) \cdot ((b d^3 + a e^3)/(a^5 b^5) + (b d^3 - a e^3)/(a^5 b^5))^{1/3} - 2 \cdot (1/2)^{2/3} d e \cdot (-I \sqrt{3} + 1)/(a^3 b^3 \cdot ((b d^3 + a e^3)/(a^5 b^5) + (b d^3 - a e^3)/(a^5 b^5))^{1/3})) + 3 \sqrt{1/3} \cdot (a b^3 x^6 + 2 a^2 b^2 x^3 + a^3 b) \cdot \sqrt{-(((1/2)^{1/3} (I \sqrt{3} + 1) \cdot ((b d^3 + a e^3)/(a^5 b^5) + (b d^3 - a e^3)/(a^5 b^5))^{1/3} - 2 \cdot (1/2)^{2/3} d e \cdot (-I \sqrt{3} + 1)/(a^3 b^3 \cdot ((b d^3 + a e^3)/(a^5 b^5) + (b d^3 - a e^3)/(a^5 b^5))^{1/3}))^2 \cdot a^3 b^3 + 16 d e)/(a^3 b^3))} \cdot \log(-1/4 \cdot ((1/2)^{1/3} (I \sqrt{3} + 1) \cdot ((b d^3 + a e^3)/(a^5 b^5) + (b d^3 - a e^3)/(a^5 b^5))^{1/3} - 2 \cdot (1/2)^{2/3} d e \cdot (-I \sqrt{3} + 1)/(a^3 b^3 \cdot ((b d^3 + a e^3)/(a^5 b^5) + (b d^3 - a e^3)/(a^5 b^5))^{1/3}))^2 \cdot a^4 b^3 e + 1/2 \cdot ((1/2)^{1/3} (I \sqrt{3} + 1) \cdot ((b d^3 + a e^3)/(a^5 b^5) + (b d^3 - a e^3)/(a^5 b^5))^{1/3} - 2 \cdot (1/2)^{2/3} d e \cdot (-I \sqrt{3} + 1)/(a^3 b^3 \cdot ((b d^3 + a e^3)/(a^5 b^5) + (b d^3 - a e^3)/(a^5 b^5))^{1/3}))^2 \cdot a^2 b^2 d^2 - 2 a d e^2 + 2 (b d^3 + a e^3) x + 3/4 \sqrt{1/3} \cdot (((1/2)^{1/3} (I \sqrt{3} + 1) \cdot ((b d^3 + a e^3)/(a^5 b^5) + (b d^3 - a e^3)/(a^5 b^5))^{1/3} - 2 \cdot (1/2)^{2/3} d e \cdot (-I \sqrt{3} + 1)/(a^3 b^3 \cdot ((b d^3 + a e^3)/(a^5 b^5) + (b d^3 - a e^3)/(a^5 b^5))^{1/3})) \cdot a^4 b^3 e + 2 a^2 b^2 d^2) \cdot \sqrt{-(((1/2)^{1/3} (I \sqrt{3} + 1) \cdot ((b d^3 + a e^3)/(a^5 b^5) + (b d^3 - a e^3)/(a^5 b^5))^{1/3} - 2 \cdot (1/2)^{2/3} d e \cdot (-I \sqrt{3} + 1)/(a^3 b^3 \cdot ((b d^3 + a e^3)/(a^5 b^5) + (b d^3 - a e^3)/(a^5 b^5))^{1/3}))^2 \cdot a^3 b^3 + 16 d e)/(a^3 b^3))} + ((a b^3 x^6 + 2 a^2 b^2 x^3 + a^3 b) \cdot ((1/2)^{1/3} (I \sqrt{3} + 1) \cdot ((b d^3 + a e^3)/(a^5 b^5) + (b d^3 - a e^3)/(a^5 b^5))^{1/3} - 2 \cdot (1/2)^{2/3} d e \cdot (-I \sqrt{3} + 1)/(a^3 b^3 \cdot ((b d^3 + a e^3)/(a^5 b^5) + (b d^3 - a e^3)/(a^5 b^5))^{1/3})) - 3 \sqrt{1/3} \cdot (a b^3 x^6 + 2 a^2 b^2 x^3 + a^3 b) \cdot \sqrt{-(((1/2)^{1/3} (I \sqrt{3} + 1) \cdot ((b d^3 + a e^3)/(a^5 b^5) + (b d^3 - a e^3)/(a^5 b^5))^{1/3} - 2 \cdot (1/2)^{2/3} d e \cdot (-I \sqrt{3} + 1)/(a^3 b^3 \cdot ((b d^3 + a e^3)/(a^5 b^5) + (b d^3 - a e^3)/(a^5 b^5))^{1/3}))^2 \cdot a^3 b^3 + 16 d e)/(a^3 b^3))} \cdot \log(-1/4 \cdot ((1/2)^{1/3} (I \sqrt{3} + 1) \cdot ((b d^3 + a e^3)/(a^5 b^5) + (b d^3 - a e^3)/(a^5 b^5))^{1/3} - 2 \cdot (1/2)^{2/3} d e \cdot (-I \sqrt{3} + 1)/(a^3 b^3 \cdot ((b d^3 + a e^3)/(a^5 b^5) + (b d^3 - a e^3)/(a^5 b^5))^{1/3}))^2 \cdot a^4 b^3 e + 1/2 \cdot ((1/2)^{1/3} (I \sqrt{3} + 1) \cdot ((b d^3 + a e^3)/(a^5 b^5) + (b d^3 - a e^3)/(a^5 b^5))^{1/3} - 2 \cdot (1/2)^{2/3} d e \cdot (-I \sqrt{3} + 1)$$

$$\begin{aligned} &)/(a^3 b^3 ((b d^3 + a e^3)/(a^5 b^5) + (b d^3 - a e^3)/(a^5 b^5))^{1/3}) * \\ & a^2 b^2 d^2 - 2 a d e^2 + 2 (b d^3 + a e^3) x - 3/4 \sqrt{1/3} * (((1/2)^{1/3} \\ & * (I \sqrt{3} + 1) * ((b d^3 + a e^3)/(a^5 b^5) + (b d^3 - a e^3)/(a^5 b^5))^{1/3} \\ & - 2 (1/2)^{2/3} d e * (-I \sqrt{3} + 1) / (a^3 b^3 ((b d^3 + a e^3)/(a^5 b^5) \\ & + (b d^3 - a e^3)/(a^5 b^5))^{1/3})) * a^4 b^3 e + 2 a^2 b^2 d^2) * \sqrt{-(((\\ & 1/2)^{1/3} * (I \sqrt{3} + 1) * ((b d^3 + a e^3)/(a^5 b^5) + (b d^3 - a e^3)/(a^5 \\ & b^5))^{1/3} - 2 (1/2)^{2/3} d e * (-I \sqrt{3} + 1) / (a^3 b^3 ((b d^3 + a e^3) \\ &) / (a^5 b^5) + (b d^3 - a e^3)/(a^5 b^5))^{1/3}))^2 a^3 b^3 + 16 d e) / (a^3 b \\ & ^3)) / (a b^3 x^6 + 2 a^2 b^2 x^3 + a^3 b) \end{aligned}$$

Sympy [A] time = 5.12363, size = 148, normalized size = 0.69

$$\text{RootSum}\left(19683 t^3 a^5 b^5 + 81 t a^2 b^2 d e + a e^3 - b d^3, \left(t \mapsto t \log\left(x + \frac{729 t^2 a^4 b^3 e + 27 t a^2 b^2 d^2 + 2 a d e^2}{a e^3 + b d^3}\right)\right)\right) + \frac{-3 a c - 2 a d x - a}{18 a^3 b + 36 a^2 b^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**3,x)

[Out] RootSum(19683*_t**3*a**5*b**5 + 81*_t*a**2*b**2*d*e + a*e**3 - b*d**3, Lamb
da(_t, _t*log(x + (729*_t**2*a**4*b**3*e + 27*_t*a**2*b**2*d**2 + 2*a*d*e**
2)/(a*e**3 + b*d**3)))) + (-3*a*c - 2*a*d*x - a*e*x**2 + b*d*x**4 + 2*b*e*x
5)/(18*a3*b + 36*a**2*b**2*x**3 + 18*a*b**3*x**6)

Giac [A] time = 1.11349, size = 288, normalized size = 1.34

$$\begin{aligned} & \frac{\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}} e + d\right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27 a^2 b} + \frac{\sqrt{3} \left(\left(-a b^2\right)^{\frac{1}{3}} b d - \left(-a b^2\right)^{\frac{2}{3}} e\right) \arctan\left(\frac{\sqrt{3} \left(2 x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27 a^2 b^3} + \frac{2 b x^5 e + b d x^4 - a x^2 e}{18 (b x^3 + a)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac")

[Out] -1/27*((-a/b)^(1/3)*e + d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b)
+ 1/27*sqrt(3)*((-a*b^2)^(1/3)*b*d - (-a*b^2)^(2/3)*e)*arctan(1/3*sqrt(3)*(
2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^3) + 1/18*(2*b*x^5*e + b*d*x^4 - a

$$\frac{x^2 e - 2 a d x - 3 a c}{(b x^3 + a)^2 a b} + \frac{1}{54} \frac{(-a b^2)^{1/3} a b^2 d + (-a b^2)^{2/3} a b e \log(x^2 + x(-a/b)^{1/3} + (-a/b)^{2/3})}{a^3 b^4}$$

$$3.352 \quad \int \frac{x(c+dx+ex^2)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=239

$$\frac{(2b^{2/3}c - a^{2/3}e) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{54a^{7/3}b^{4/3}} - \frac{(2b^{2/3}c - a^{2/3}e) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{27a^{7/3}b^{4/3}} - \frac{(a^{2/3}e + 2b^{2/3}c) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{4/3}} - \frac{3ad}{18}$$

[Out] $-(x*(a*e - b*c*x - b*d*x^2))/(6*a*b*(a + b*x^3)^2) - (3*a*d - x*(a*e + 4*b*c*x))/(18*a^2*b*(a + b*x^3)) - ((2*b^(2/3)*c + a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(7/3)*b^(4/3)) - ((2*b^(2/3)*c - a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x]/(27*a^(7/3)*b^(4/3)) + ((2*b^(2/3)*c - a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(54*a^(7/3)*b^(4/3))$

Rubi [A] time = 0.204025, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1828, 1854, 1860, 31, 634, 617, 204, 628}

$$\frac{(2b^{2/3}c - a^{2/3}e) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{54a^{7/3}b^{4/3}} - \frac{(2b^{2/3}c - a^{2/3}e) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{27a^{7/3}b^{4/3}} - \frac{(a^{2/3}e + 2b^{2/3}c) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{4/3}} - \frac{3ad}{18}$$

Antiderivative was successfully verified.

[In] Int[(x*(c + d*x + e*x^2))/(a + b*x^3)^3, x]

[Out] $-(x*(a*e - b*c*x - b*d*x^2))/(6*a*b*(a + b*x^3)^2) - (3*a*d - x*(a*e + 4*b*c*x))/(18*a^2*b*(a + b*x^3)) - ((2*b^(2/3)*c + a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(7/3)*b^(4/3)) - ((2*b^(2/3)*c - a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x]/(27*a^(7/3)*b^(4/3)) + ((2*b^(2/3)*c - a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(54*a^(7/3)*b^(4/3))$

Rule 1828

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^

```
m*Pq, a + b*x^n, x]], Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]]*(a + b*x^n)^(p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(p_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(c + dx + ex^2)}{(a + bx^3)^3} dx &= -\frac{x(ae - bcx - bdx^2)}{6ab(a + bx^3)^2} - \frac{\int \frac{-ae - 4bcx - 3bdx^2}{(a + bx^3)^2} dx}{6ab} \\
&= -\frac{x(ae - bcx - bdx^2)}{6ab(a + bx^3)^2} - \frac{3ad - x(ae + 4bcx)}{18a^2b(a + bx^3)} + \frac{\int \frac{2ae + 4bcx}{a + bx^3} dx}{18a^2b} \\
&= -\frac{x(ae - bcx - bdx^2)}{6ab(a + bx^3)^2} - \frac{3ad - x(ae + 4bcx)}{18a^2b(a + bx^3)} + \frac{\int \frac{\sqrt[3]{a}(4\sqrt[3]{abc} + 4a\sqrt[3]{be}) + \sqrt[3]{b}(4\sqrt[3]{abc} - 2a\sqrt[3]{be})x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} dx}{54a^{8/3}b^{4/3}} - \frac{(2b^{2/3}c - a^{2/3}e)}{54a^{8/3}b^{4/3}} \\
&= -\frac{x(ae - bcx - bdx^2)}{6ab(a + bx^3)^2} - \frac{3ad - x(ae + 4bcx)}{18a^2b(a + bx^3)} - \frac{(2b^{2/3}c - a^{2/3}e) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{27a^{7/3}b^{4/3}} + \frac{(2b^{2/3}c - a^{2/3}e)}{54a^{8/3}b^{4/3}} \\
&= -\frac{x(ae - bcx - bdx^2)}{6ab(a + bx^3)^2} - \frac{3ad - x(ae + 4bcx)}{18a^2b(a + bx^3)} - \frac{(2b^{2/3}c - a^{2/3}e) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{27a^{7/3}b^{4/3}} + \frac{(2b^{2/3}c - a^{2/3}e)}{54a^{8/3}b^{4/3}} \\
&= -\frac{x(ae - bcx - bdx^2)}{6ab(a + bx^3)^2} - \frac{3ad - x(ae + 4bcx)}{18a^2b(a + bx^3)} - \frac{(2b^{2/3}c + a^{2/3}e) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{4/3}} - \frac{(2b^{2/3}c - a^{2/3}e)}{27a^{8/3}b^{4/3}}
\end{aligned}$$

Mathematica [A] time = 0.28205, size = 214, normalized size = 0.9

$$\frac{3ab^{2/3}(-a^2(3d+2ex)+abx^2(7c+ex^2)+4b^2cx^5)}{(a+bx^3)^2} + (2a^{2/3}bc - a^{4/3}\sqrt[3]{be}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2) - 2\sqrt{3}a^{2/3}\sqrt[3]{b}(a^{2/3}e + 2b^{2/3}c) \tan^{-1}$$

$$54a^3b^{5/3}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3)^3,x]

[Out]
$$\frac{((3ab^{2/3})(4b^2cx^5 - a^2(3d + 2ex) + abx^2(7c + ex^2)))/(a + b^2x^3)^2 - 2\sqrt{3}a^{2/3}b^{1/3}(2b^{2/3}c + a^{2/3}e)\text{ArcTan}\left[\frac{1 - (2b^{1/3}x)/a^{1/3}}{\sqrt{3}}\right] + 2(-2a^{2/3}bc + a^{4/3}b^{1/3}e)\text{Log}[a^{1/3} + b^{1/3}x] + (2a^{2/3}bc - a^{4/3}b^{1/3}e)\text{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2]}{(54a^3b^{5/3})}$$

Maple [A] time = 0.01, size = 256, normalized size = 1.1

$$\frac{1}{(bx^3 + a)^2} \left(\frac{2bcx^5}{9a^2} + \frac{ex^4}{18a} + \frac{7cx^2}{18a} - \frac{ex}{9b} - \frac{d}{6b} \right) + \frac{e}{27b^2a} \ln \left(x + \sqrt[3]{\frac{a}{b}} \right) \left(\frac{a}{b} \right)^{-\frac{2}{3}} - \frac{e}{54b^2a} \ln \left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b} \right)^{\frac{2}{3}} \right) \left(\frac{a}{b} \right)^{-\frac{2}{3}} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(e*x^2+d*x+c)/(b*x^3+a)^3,x)

[Out]
$$\frac{2/9/a^2*c*b*x^5 + 1/18/a*e*x^4 + 7/18*c/a*x^2 - 1/9*e*x/b - 1/6/b*d}{(b*x^3+a)^2} + \frac{1}{27/a/b^2/(1/b*a)^{2/3}*ln(x+(1/b*a)^{1/3})} * e - \frac{1}{54/a/b^2/(1/b*a)^{2/3}*ln(x^2 - (1/b*a)^{1/3}*x + (1/b*a)^{2/3})} * e + \frac{1}{27/a/b^2/(1/b*a)^{2/3}*3^{1/2}*arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1))} * e - \frac{2}{27/b/a^2/(1/b*a)^{1/3}*ln(x+(1/b*a)^{1/3})} * c + \frac{1}{27/b/a^2/(1/b*a)^{1/3}*ln(x^2 - (1/b*a)^{1/3}*x + (1/b*a)^{2/3})} * c + \frac{2}{27/b/a^2*3^{1/2}/(1/b*a)^{1/3}*arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1))} * c$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

$$\begin{aligned}
& a^2e^3/(a^7b^4) - (8b^2c^3 - a^2e^3)/(a^7b^4)^{(1/3)}) - 3\sqrt[3]{1/3} \\
&)*(a^2b^3x^6 + 2a^3b^2x^3 + a^4b)*\sqrt{-(((1/2)^{(1/3)}*(I*\sqrt{3}) + 1) \\
& *((8b^2c^3 + a^2e^3)/(a^7b^4) - (8b^2c^3 - a^2e^3)/(a^7b^4)^{(1/3)} \\
& + 4*(1/2)^{(2/3)}*c*e*(I*\sqrt{3}) - 1)/(a^4b^2*((8b^2c^3 + a^2e^3)/(a^7b^4) \\
& - (8b^2c^3 - a^2e^3)/(a^7b^4)^{(1/3)}))^2*a^4b^2 + 32*c*e)/(a^4b^2) \\
&))*\log(-1/2*((1/2)^{(1/3)}*(I*\sqrt{3}) + 1)*((8b^2c^3 + a^2e^3)/(a^7b^4) - \\
& (8b^2c^3 - a^2e^3)/(a^7b^4)^{(1/3)} + 4*(1/2)^{(2/3)}*c*e*(I*\sqrt{3}) - 1) \\
& / (a^4b^2*((8b^2c^3 + a^2e^3)/(a^7b^4) - (8b^2c^3 - a^2e^3)/(a^7b^4) \\
&))^{(1/3)})^2*a^5b^3*c + 1/2*((1/2)^{(1/3)}*(I*\sqrt{3}) + 1)*((8b^2c^3 + a^2 \\
& *e^3)/(a^7b^4) - (8b^2c^3 - a^2e^3)/(a^7b^4)^{(1/3)} + 4*(1/2)^{(2/3)}*c* \\
& e*(I*\sqrt{3}) - 1)/(a^4b^2*((8b^2c^3 + a^2e^3)/(a^7b^4) - (8b^2c^3 - \\
& a^2e^3)/(a^7b^4)^{(1/3)}))*a^4*b*e^2 - 8*a*b*c^2*e + 2*(8b^2c^3 + a^2e^ \\
& 3)*x - 3/2*\sqrt[3]{1/3}*(((1/2)^{(1/3)}*(I*\sqrt{3}) + 1)*((8b^2c^3 + a^2e^3)/(\\
& a^7b^4) - (8b^2c^3 - a^2e^3)/(a^7b^4)^{(1/3)} + 4*(1/2)^{(2/3)}*c*e*(I*sq \\
& rt(3) - 1)/(a^4b^2*((8b^2c^3 + a^2e^3)/(a^7b^4) - (8b^2c^3 - a^2e^3) \\
&)/(a^7b^4))^{(1/3)}))*a^5b^3*c + a^4*b*e^2)*\sqrt{-(((1/2)^{(1/3)}*(I*\sqrt{3}) \\
& + 1)*((8b^2c^3 + a^2e^3)/(a^7b^4) - (8b^2c^3 - a^2e^3)/(a^7b^4)^{(1 \\
& /3) + 4*(1/2)^{(2/3)}*c*e*(I*\sqrt{3}) - 1)/(a^4b^2*((8b^2c^3 + a^2e^3)/(a^ \\
& 7b^4) - (8b^2c^3 - a^2e^3)/(a^7b^4)^{(1/3)}))^2*a^4b^2 + 32*c*e)/(a^4* \\
& b^2)))/ (a^2b^3x^6 + 2a^3b^2x^3 + a^4b)
\end{aligned}$$

Sympy [A] time = 3.38543, size = 170, normalized size = 0.71

$$\text{RootSum}\left(19683t^3a^7b^4 + 162ta^3b^2ce - a^2e^3 + 8b^2c^3, \left(t \mapsto t \log\left(x + \frac{1458t^2a^5b^3c + 27ta^4be^2 + 8abc^2e}{a^2e^3 + 8b^2c^3}\right)\right)\right) + \frac{-3a^2d - 2}{18a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e**x**2+d*x+c)/(b*x**3+a)**3,x)

[Out] RootSum(19683*_t**3*a**7*b**4 + 162*_t*a**3*b**2*c*e - a**2*e**3 + 8*b**2*c**3, Lambda(_t, _t*log(x + (1458*_t**2*a**5*b**3*c + 27*_t*a**4*b*e**2 + 8*a*b*c**2*e)/(a**2*e**3 + 8*b**2*c**3)))) + (-3*a**2*d - 2*a**2*e*x + 7*a*b*c*x**2 + a*b*e*x**4 + 4*b**2*c*x**5)/(18*a**4*b + 36*a**3*b**2*x**3 + 18*a**2*b**3*x**6)

Giac [A] time = 1.10307, size = 306, normalized size = 1.28

$$\frac{\left(2bc\left(-\frac{a}{b}\right)^{\frac{1}{3}} + ae\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^3b} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}ae - 2\left(-ab^2\right)^{\frac{2}{3}}c\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^3b^2} + \frac{4b^2cx^5 + abx^4e}{18\left(\dots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac")

[Out] $-1/27*(2*b*c*(-a/b)^{(1/3)} + a*e)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/(a^3*b) + 1/27*\text{sqrt}(3)*((-a*b^2)^{(1/3)}*a*e - 2*(-a*b^2)^{(2/3)}*c)*\arctan(1/3*\text{sqrt}(3)*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(a^3*b^2) + 1/18*(4*b^2*c*x^5 + a*b*x^4*e + 7*a*b*c*x^2 - 2*a^2*x*e - 3*a^2*d)/((b*x^3 + a)^2*a^2*b) + 1/54*((-a*b^2)^{(1/3)}*a*b^2*e + 2*(-a*b^2)^{(2/3)}*b^2*c)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a^3*b^4)$

$$3.353 \quad \int \frac{c+dx+ex^2}{(a+bx^3)^3} dx$$

Optimal. Leaf size=225

$$-\frac{(5\sqrt[3]{bc}-2\sqrt[3]{ad})\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2)}{54a^{8/3}b^{2/3}} + \frac{(5\sqrt[3]{bc}-2\sqrt[3]{ad})\log(\sqrt[3]{a}+\sqrt[3]{bx})}{27a^{8/3}b^{2/3}} - \frac{(2\sqrt[3]{ad}+5\sqrt[3]{bc})\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}}$$

[Out] (x*(5*c + 4*d*x))/(18*a^2*(a + b*x^3)) - (a*e - b*x*(c + d*x))/(6*a*b*(a + b*x^3)^2) - ((5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(8/3)*b^(2/3)) + ((5*b^(1/3)*c - 2*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x]/(27*a^(8/3)*b^(2/3)) - ((5*b^(1/3)*c - 2*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(54*a^(8/3)*b^(2/3))

Rubi [A] time = 0.188495, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {1854, 1855, 1860, 31, 634, 617, 204, 628}

$$-\frac{(5\sqrt[3]{bc}-2\sqrt[3]{ad})\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2)}{54a^{8/3}b^{2/3}} + \frac{(5\sqrt[3]{bc}-2\sqrt[3]{ad})\log(\sqrt[3]{a}+\sqrt[3]{bx})}{27a^{8/3}b^{2/3}} - \frac{(2\sqrt[3]{ad}+5\sqrt[3]{bc})\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/(a + b*x^3)^3, x]

[Out] (x*(5*c + 4*d*x))/(18*a^2*(a + b*x^3)) - (a*e - b*x*(c + d*x))/(6*a*b*(a + b*x^3)^2) - ((5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(8/3)*b^(2/3)) + ((5*b^(1/3)*c - 2*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x]/(27*a^(8/3)*b^(2/3)) - ((5*b^(1/3)*c - 2*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(54*a^(8/3)*b^(2/3))

Rule 1854

Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q, x])*(a + b*x^n)^(p + 1)/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1})*(a + b*x^n)^(p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,

0] && LtQ[p, -1]

Rule 1855

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]

Rule 1860

Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]

Rule 31

Int[((a_) + (b_.)*(x_))^(p_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2}{(a + bx^3)^3} dx &= \frac{ae - bx(c + dx)}{6ab(a + bx^3)^2} - \frac{\int \frac{-5c-4dx}{(a+bx^3)^2} dx}{6a} \\
&= \frac{x(5c + 4dx)}{18a^2(a + bx^3)} - \frac{ae - bx(c + dx)}{6ab(a + bx^3)^2} + \frac{\int \frac{10c+4dx}{a+bx^3} dx}{18a^2} \\
&= \frac{x(5c + 4dx)}{18a^2(a + bx^3)} - \frac{ae - bx(c + dx)}{6ab(a + bx^3)^2} + \frac{\int \frac{\sqrt[3]{a}(20\sqrt[3]{bc}+4\sqrt[3]{ad})+\sqrt[3]{b}(-10\sqrt[3]{bc}+4\sqrt[3]{ad})x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2} dx}{54a^{8/3}\sqrt[3]{b}} + \frac{(5c - \frac{2\sqrt[3]{ad}}{\sqrt[3]{b}}) \int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}}}{27a^{8/3}} \\
&= \frac{x(5c + 4dx)}{18a^2(a + bx^3)} - \frac{ae - bx(c + dx)}{6ab(a + bx^3)^2} + \frac{(5\sqrt[3]{bc} - 2\sqrt[3]{ad}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{27a^{8/3}b^{2/3}} - \frac{(5\sqrt[3]{bc} - 2\sqrt[3]{ad}) \int \frac{-\sqrt[3]{a}\sqrt[3]{b}}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}}}{54a^{8/3}b^{2/3}} \\
&= \frac{x(5c + 4dx)}{18a^2(a + bx^3)} - \frac{ae - bx(c + dx)}{6ab(a + bx^3)^2} + \frac{(5\sqrt[3]{bc} - 2\sqrt[3]{ad}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{27a^{8/3}b^{2/3}} - \frac{(5\sqrt[3]{bc} - 2\sqrt[3]{ad}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx})}{54a^{8/3}b^{2/3}} \\
&= \frac{x(5c + 4dx)}{18a^2(a + bx^3)} - \frac{ae - bx(c + dx)}{6ab(a + bx^3)^2} - \frac{(5\sqrt[3]{bc} + 2\sqrt[3]{ad}) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}} + \frac{(5\sqrt[3]{bc} - 2\sqrt[3]{ad}) \log(\sqrt[3]{a} - \sqrt[3]{bx})}{27a^{8/3}b^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.286109, size = 213, normalized size = 0.95

$$\frac{3a(-3a^2e+abx(8c+7dx)+b^2x^4(5c+4dx))}{(a+bx^3)^2} + \sqrt[3]{a}\sqrt[3]{b}(2\sqrt[3]{ad}-5\sqrt[3]{bc})\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2) + 2\sqrt[3]{b}(5\sqrt[3]{a}\sqrt[3]{bc}-2a^{2/3}d)\log(\sqrt[3]{a}-\sqrt[3]{bx})$$

54a³b

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(a + b*x^3)^3,x]

[Out] ((3*a*(-3*a^2*e + b^2*x^4*(5*c + 4*d*x) + a*b*x*(8*c + 7*d*x)))/(a + b*x^3)^2 - 2*Sqrt[3]*a^(1/3)*b^(1/3)*(5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(1 - (2*b

$$\frac{(1/3)*x/a^{(1/3)}/\text{Sqrt}[3]] + 2*b^{(1/3)}*(5*a^{(1/3)}*b^{(1/3)}*c - 2*a^{(2/3)}*d) * \text{Log}[a^{(1/3)} + b^{(1/3)}*x] + a^{(1/3)}*b^{(1/3)}*(-5*b^{(1/3)}*c + 2*a^{(1/3)}*d) * \text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2]/(54*a^3*b)$$

Maple [A] time = 0.003, size = 308, normalized size = 1.4

$$\frac{cx}{6a(bx^3+a)^2} + \frac{5cx}{18a^2(bx^3+a)} + \frac{5c}{27ba^2} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{5c}{54ba^2} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{5c\sqrt{3}}{27ba^2} \arctan\left(\frac{x + \sqrt[3]{\frac{a}{b}}}{\sqrt[3]{\frac{a}{b}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)/(b*x^3+a)^3,x)

[Out] $\frac{1}{6} * c / a * x / (b * x^3 + a)^2 + \frac{5}{18} * c / a^2 * x / (b * x^3 + a) + \frac{5}{27} * c / a^2 / b / (1/b * a)^{(2/3)} * \ln(x + (1/b * a)^{(1/3)}) - \frac{5}{54} * c / a^2 / b / (1/b * a)^{(2/3)} * \ln(x^2 - (1/b * a)^{(1/3)} * x + (1/b * a)^{(2/3)}) + \frac{5}{27} * c / a^2 / b / (1/b * a)^{(2/3)} * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (2 / (1/b * a)^{(1/3)} * x - 1)) + \frac{1}{6} * d / a * x^2 / (b * x^3 + a)^2 + \frac{2}{9} * d / a^2 * x^2 / (b * x^3 + a) - \frac{2}{27} * d / a^2 / b / (1/b * a)^{(1/3)} * \ln(x + (1/b * a)^{(1/3)}) + \frac{1}{27} * d / a^2 / b / (1/b * a)^{(1/3)} * \ln(x^2 - (1/b * a)^{(1/3)} * x + (1/b * a)^{(2/3)}) + \frac{2}{27} * d / a^2 * 3^{(1/2)} / b / (1/b * a)^{(1/3)} * \arctan(1/3 * 3^{(1/2)} * (2 / (1/b * a)^{(1/3)} * x - 1)) + \frac{1}{6} * e / a * x^3 / (b * x^3 + a)^2 - \frac{1}{6} * e / a / b / (b * x^3 + a)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [C] time = 8.38672, size = 5612, normalized size = 24.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

$$\begin{aligned}
& - 8*a*d^3/(a^8*b^2))^{(1/3)} - 20*(1/2)^{(2/3)}*c*d*(-I*\sqrt{3} + 1)/(a^5*b*(\\
& (125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^{(1/3)})^2 \\
& *a^6*b*d + 25/2*((1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b \\
& ^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^{(1/3)} - 20*(1/2)^{(2/3)}*c*d*(-I*\sqrt{3} \\
& (3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^ \\
& 8*b^2))^{(1/3)})) *a^3*b*c^2 - 40*a*c*d^2 + 2*(125*b*c^3 + 8*a*d^3)*x - 3/2*sq \\
& rt(1/3)*(((1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (1 \\
& 25*b*c^3 - 8*a*d^3)/(a^8*b^2))^{(1/3)} - 20*(1/2)^{(2/3)}*c*d*(-I*\sqrt{3} + 1)/ \\
& (a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^{(\\
& 1/3)})) *a^6*b*d + 25*a^3*b*c^2)*\sqrt{-(((1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((125*b \\
& *c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^{(1/3)} - 20*(1/ \\
& 2)^{(2/3)}*c*d*(-I*\sqrt{3} + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (12 \\
& 5*b*c^3 - 8*a*d^3)/(a^8*b^2))^{(1/3)}))^{2*a^5*b + 160*c*d)/(a^5*b)))/(a^2*b^ \\
& 3*x^6 + 2*a^3*b^2*x^3 + a^4*b)
\end{aligned}$$

Sympy [A] time = 2.11682, size = 163, normalized size = 0.72

$$\text{RootSum}\left(19683t^3a^8b^2 + 810ta^3bcd + 8ad^3 - 125bc^3, \left(t \mapsto t \log\left(x + \frac{1458t^2a^6bd + 675ta^3bc^2 + 40acd^2}{8ad^3 + 125bc^3}\right)\right)\right) + \frac{-3a^2e + 8}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/(b*x**3+a)**3,x)

[Out] RootSum(19683*_t**3*a**8*b**2 + 810*_t*a**3*b*c*d + 8*a*d**3 - 125*b*c**3, Lambda(_t, _t*log(x + (1458*_t**2*a**6*b*d + 675*_t*a**3*b*c**2 + 40*a*c*d**2)/(8*a*d**3 + 125*b*c**3)))) + (-3*a**2*e + 8*a*b*c*x + 7*a*b*d*x**2 + 5*b**2*c*x**4 + 4*b**2*d*x**5)/(18*a**4*b + 36*a**3*b**2*x**3 + 18*a**2*b**3*x**6)

Giac [A] time = 1.12697, size = 301, normalized size = 1.34

$$\frac{\left(2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 5c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^3} + \frac{\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}bc - 2\left(-ab^2\right)^{\frac{2}{3}}d\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^3b^2} + \frac{4b^2dx^5 + 5b^2c}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/27*(2*d*(-a/b)^{(1/3)} + 5*c)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/a^3 \\ & + 1/27*\sqrt{3}*(5*(-a*b^2)^{(1/3)}*b*c - 2*(-a*b^2)^{(2/3)}*d)*\arctan(1/3*\sqrt{3} \\ & *(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(a^3*b^2) + 1/18*(4*b^2*d*x^5 + 5*b^2 \\ & *c*x^4 + 7*a*b*d*x^2 + 8*a*b*c*x - 3*a^2*e)/((b*x^3 + a)^2*a^2*b) + 1/54*(5 \\ & *(-a*b^2)^{(1/3)}*a*b^3*c + 2*(-a*b^2)^{(2/3)}*a*b^2*d)*\log(x^2 + x*(-a/b)^{(1/3)} \\ &) + (-a/b)^{(2/3)}/(a^4*b^4) \end{aligned}$$

$$3.354 \quad \int \frac{c+dx+ex^2}{x(a+bx^3)^3} dx$$

Optimal. Leaf size=257

$$-\frac{(5\sqrt[3]{bd}-2\sqrt[3]{ae})\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2)}{54a^{8/3}b^{2/3}} + \frac{(5\sqrt[3]{bd}-2\sqrt[3]{ae})\log(\sqrt[3]{a}+\sqrt[3]{bx})}{27a^{8/3}b^{2/3}} - \frac{(2\sqrt[3]{ae}+5\sqrt[3]{bd})\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}} +$$

[Out] (x*(a*d + a*e*x - b*c*x^2))/(6*a^2*(a + b*x^3)^2) + (x*(5*a*d + 4*a*e*x - 9*b*c*x^2))/(18*a^3*(a + b*x^3)) - ((5*b^(1/3)*d + 2*a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(8/3)*b^(2/3)) + (c*Log[x])/a^3 + ((5*b^(1/3)*d - 2*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x]/(27*a^(8/3)*b^(2/3)) - ((5*b^(1/3)*d - 2*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(54*a^(8/3)*b^(2/3)) - (c*Log[a + b*x^3])/(3*a^3)

Rubi [A] time = 0.41382, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{(5\sqrt[3]{bd}-2\sqrt[3]{ae})\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2)}{54a^{8/3}b^{2/3}} + \frac{(5\sqrt[3]{bd}-2\sqrt[3]{ae})\log(\sqrt[3]{a}+\sqrt[3]{bx})}{27a^{8/3}b^{2/3}} - \frac{(2\sqrt[3]{ae}+5\sqrt[3]{bd})\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}} +$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/(x*(a + b*x^3)^3), x]

[Out] (x*(a*d + a*e*x - b*c*x^2))/(6*a^2*(a + b*x^3)^2) + (x*(5*a*d + 4*a*e*x - 9*b*c*x^2))/(18*a^3*(a + b*x^3)) - ((5*b^(1/3)*d + 2*a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(8/3)*b^(2/3)) + (c*Log[x])/a^3 + ((5*b^(1/3)*d - 2*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x]/(27*a^(8/3)*b^(2/3)) - ((5*b^(1/3)*d - 2*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(54*a^(8/3)*b^(2/3)) - (c*Log[a + b*x^3])/(3*a^3)

Rule 1829

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m


```
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_)*(x_))^(m_))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
```

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 204

$\text{Int}[\frac{(a_1 + (b_1)x^2)^{-1}}{x}, x_Symbol] \ :> \ -\text{Simp}[\frac{\text{ArcTan}[\frac{\sqrt{-b_1}x}{\sqrt{-a_1}}]}{\sqrt{-a_1}}], x] \ ; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 628

$\text{Int}[\frac{(d_1 + (e_1)x)}{(a_1 + (b_1)x + (c_1)x^2)}, x_Symbol] \ :> \ \text{Simp}[\frac{d_1 \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]}{b}], x] \ ; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2cd - be, 0]$

Rule 260

$\text{Int}[x^{(m_1)} / (a_1 + (b_1)x^{(n_1)}), x_Symbol] \ :> \ \text{Simp}[\frac{\text{Log}[\text{RemoveContent}[a + bx^n, x]]}{bn}], x] \ ; \ \text{FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2}{x(a + bx^3)^3} dx &= \frac{x(ad + aex - bcx^2)}{6a^2(a + bx^3)^2} - \frac{\int \frac{-6bc - 5bdx - 4bex^2 + \frac{3b^2cx^3}{a}}{x(a + bx^3)^2} dx}{6ab} \\
&= \frac{x(ad + aex - bcx^2)}{6a^2(a + bx^3)^2} + \frac{x(5ad + 4aex - 9bcx^2)}{18a^3(a + bx^3)} + \frac{\int \frac{18b^2c + 10b^2dx + 4b^2ex^2}{x(a + bx^3)} dx}{18a^2b^2} \\
&= \frac{x(ad + aex - bcx^2)}{6a^2(a + bx^3)^2} + \frac{x(5ad + 4aex - 9bcx^2)}{18a^3(a + bx^3)} + \frac{\int \left(\frac{18b^2c}{ax} + \frac{2b^2(5ad + 2aex - 9bcx^2)}{a(a + bx^3)} \right) dx}{18a^2b^2} \\
&= \frac{x(ad + aex - bcx^2)}{6a^2(a + bx^3)^2} + \frac{x(5ad + 4aex - 9bcx^2)}{18a^3(a + bx^3)} + \frac{c \log(x)}{a^3} + \frac{\int \frac{5ad + 2aex - 9bcx^2}{a + bx^3} dx}{9a^3} \\
&= \frac{x(ad + aex - bcx^2)}{6a^2(a + bx^3)^2} + \frac{x(5ad + 4aex - 9bcx^2)}{18a^3(a + bx^3)} + \frac{c \log(x)}{a^3} + \frac{\int \frac{5ad + 2aex}{a + bx^3} dx}{9a^3} - \frac{(bc) \int \frac{x^2}{a + bx^3} dx}{a^3} \\
&= \frac{x(ad + aex - bcx^2)}{6a^2(a + bx^3)^2} + \frac{x(5ad + 4aex - 9bcx^2)}{18a^3(a + bx^3)} + \frac{c \log(x)}{a^3} - \frac{c \log(a + bx^3)}{3a^3} + \frac{\int \frac{\sqrt[3]{a}(10a\sqrt[3]{bd} + 2a^{4/3}e)}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx^3}}}{27a^{1/3}} \\
&= \frac{x(ad + aex - bcx^2)}{6a^2(a + bx^3)^2} + \frac{x(5ad + 4aex - 9bcx^2)}{18a^3(a + bx^3)} + \frac{c \log(x)}{a^3} + \frac{(5\sqrt[3]{bd} - 2\sqrt[3]{ae}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{27a^{8/3}b^{2/3}} - \frac{c \log(a + bx^3)}{3a^3} \\
&= \frac{x(ad + aex - bcx^2)}{6a^2(a + bx^3)^2} + \frac{x(5ad + 4aex - 9bcx^2)}{18a^3(a + bx^3)} + \frac{c \log(x)}{a^3} + \frac{(5\sqrt[3]{bd} - 2\sqrt[3]{ae}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{27a^{8/3}b^{2/3}} - \frac{c \log(a + bx^3)}{3a^3} \\
&= \frac{x(ad + aex - bcx^2)}{6a^2(a + bx^3)^2} + \frac{x(5ad + 4aex - 9bcx^2)}{18a^3(a + bx^3)} - \frac{(5\sqrt[3]{bd} + 2\sqrt[3]{ae}) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}} + \frac{c \log(x)}{a^3} + \frac{c \log(a + bx^3)}{3a^3}
\end{aligned}$$

Mathematica [A] time = 0.177574, size = 229, normalized size = 0.89

$$\frac{(2a^{2/3}e - 5\sqrt[3]{a}\sqrt[3]{bd}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{b^{2/3}} + \frac{2(5\sqrt[3]{a}\sqrt[3]{bd} - 2a^{2/3}e) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{b^{2/3}} + \frac{9a^2(c + x(d + ex))}{(a + bx^3)^2} - \frac{2\sqrt{3}\sqrt[3]{a}(2\sqrt[3]{ae} + 5\sqrt[3]{bd}) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{2/3}} + \frac{3a(6c + x(d + ex))}{a^3}$$

$54a^3$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)^3),x]
```

```
[Out] ((9*a^2*(c + x*(d + e*x)))/(a + b*x^3)^2 + (3*a*(6*c + x*(5*d + 4*e*x)))/(a + b*x^3) - (2*sqrt[3]*a^(1/3)*(5*b^(1/3)*d + 2*a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]])/b^(2/3) + 54*c*Log[x] + (2*(5*a^(1/3)*b^(1/3)*d - 2*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((-5*a^(1/3)*b^(1/3)*d + 2*a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) - 18*c*Log[a + b*x^3])/(54*a^3)
```

Maple [A] time = 0.013, size = 331, normalized size = 1.3

$$\frac{2 b e x^5}{9 a^2 (b x^3 + a)^2} + \frac{5 b d x^4}{18 a^2 (b x^3 + a)^2} + \frac{b c x^3}{3 a^2 (b x^3 + a)^2} + \frac{7 e x^2}{18 a (b x^3 + a)^2} + \frac{4 d x}{9 a (b x^3 + a)^2} + \frac{c}{2 a (b x^3 + a)^2} + \frac{5 d}{27 b a^2} \ln \left(x + \frac{a}{b x^3 + a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x/(b*x^3+a)^3,x)
```

```
[Out] 2/9/a^2/(b*x^3+a)^2*x^5*b*e+5/18/a^2/(b*x^3+a)^2*b*x^4*d+1/3/a^2/(b*x^3+a)^2*b*c*x^3+7/18/a/(b*x^3+a)^2*x^2*e+4/9/a/(b*x^3+a)^2*x*d+1/2/a/(b*x^3+a)^2*c+5/27/a^2/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d-5/54/a^2/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d+5/27/a^2/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d-2/27/a^2/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*e+1/27/a^2/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e+2/27/a^2/b*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e-1/3*c*ln(b*x^3+a)/a^3+c*ln(x)/a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 10.6925, size = 13407, normalized size = 52.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="fricas")

[Out]
$$\frac{1}{2916} (648 a b e x^5 + 810 a b d x^4 + 972 a b c x^3 + 1134 a^2 e x^2 + 1296 a^2 d x + 1458 a^2 c - 2(a^3 b^2 x^6 + 2 a^4 b x^3 + a^5) ((-I \sqrt{3} + 1) (81 c^2/a^6 - (81 b c^2 + 10 a d e)/(a^6 b)) / (-1/27 c^3/a^9 + 1/1458 (81 b c^2 + 10 a d e) c / (a^9 b) + 1/39366 (125 b d^3 + 8 a e^3) / (a^8 b^2) - 1/39366 (729 b^2 c^3 + 8 a^2 e^3 - 5(25 d^3 - 54 c d e) a b) / (a^9 b^2))^{1/3} + 729 (I \sqrt{3} + 1) (-1/27 c^3/a^9 + 1/1458 (81 b c^2 + 10 a d e) c / (a^9 b) + 1/39366 (125 b d^3 + 8 a e^3) / (a^8 b^2) - 1/39366 (729 b^2 c^3 + 8 a^2 e^3 - 5(25 d^3 - 54 c d e) a b) / (a^9 b^2))^{1/3} + 486 c / a^3) \log(1/1458 ((-I \sqrt{3} + 1) (81 c^2/a^6 - (81 b c^2 + 10 a d e)/(a^6 b)) / (-1/27 c^3/a^9 + 1/1458 (81 b c^2 + 10 a d e) c / (a^9 b) + 1/39366 (125 b d^3 + 8 a e^3) / (a^8 b^2) - 1/39366 (729 b^2 c^3 + 8 a^2 e^3 - 5(25 d^3 - 54 c d e) a b) / (a^9 b^2))^{1/3} + 729 (I \sqrt{3} + 1) (-1/27 c^3/a^9 + 1/1458 (81 b c^2 + 10 a d e) c / (a^9 b) + 1/39366 (125 b d^3 + 8 a e^3) / (a^8 b^2) - 1/39366 (729 b^2 c^3 + 8 a^2 e^3 - 5(25 d^3 - 54 c d e) a b) / (a^9 b^2))^{1/3} + 486 c / a^3)^2 a^6 b e + 225 b c d^2 + 162 b c^2 e + 40 a d e^2 - 1/54 (25 a^3 b d^2 + 36 a^3 b c e) ((-I \sqrt{3} + 1) (81 c^2/a^6 - (81 b c^2 + 10 a d e)/(a^6 b)) / (-1/27 c^3/a^9 + 1/1458 (81 b c^2 + 10 a d e) c / (a^9 b) + 1/39366 (125 b d^3 + 8 a e^3) / (a^8 b^2) - 1/39366 (729 b^2 c^3 + 8 a^2 e^3 - 5(25 d^3 - 54 c d e) a b) / (a^9 b^2))^{1/3} + 729 (I \sqrt{3} + 1) (-1/27 c^3/a^9 + 1/1458 (81 b c^2 + 10 a d e) c / (a^9 b) + 1/39366 (125 b d^3 + 8 a e^3) / (a^8 b^2) - 1/39366 (729 b^2 c^3 + 8 a^2 e^3 - 5(25 d^3 - 54 c d e) a b) / (a^9 b^2))^{1/3} + 486 c / a^3) + (125 b d^3 + 8 a e^3) x - (1458 b^2 c x^6 + 2916 a b c x^3 + 1458 a^2 c - (a^3 b^2 x^6 + 2 a^4 b x^3 + a^5) ((-I \sqrt{3} + 1) (81 c^2/a^6 - (81 b c^2 + 10 a d e)/(a^6 b)) / (-1/27 c^3/a^9 + 1/1458 (81 b c^2 + 10 a d e) c / (a^9 b) + 1/39366 (125 b d^3 + 8 a e^3) / (a^8 b^2) - 1/39366 (729 b^2 c^3 + 8 a^2 e^3 - 5(25 d^3 - 54 c d e) a b) / (a^9 b^2))^{1/3} + 729 (I \sqrt{3} + 1) (-1/27 c^3/a^9 + 1/1458 (81 b c^2 + 10 a d e) c / (a^9 b) + 1/39366 (125 b d^3 + 8 a e^3) / (a^8 b^2) - 1/39366 (729 b^2 c^3 + 8 a^2 e^3 - 5(25 d^3 - 54 c d e) a b) / (a^9 b^2))^{1/3} + 486 c / a^3) - 3 \sqrt{1/3} (a^3 b^2 x^6 + 2 a^4 b x^3 + a^5) \sqrt{-(((-I \sqrt{3} + 1) (81 c^2/a^6 - (81 b c^2 + 10 a d e)/(a^6 b)) / (-1/27 c^3/a^9 + 1/1458 (81 b c^2 + 10 a d e) c / (a^9 b) + 1/39366 (125 b d^3 + 8 a e^3) / (a^8 b^2) - 1/39366 (729 b^2 c^3 + 8 a^2 e^3 - 5(25 d^3 - 54 c d e) a b) / (a^9 b^2))^{1/3} + 729 (I \sqrt{3} + 1) (-1/27 c^3/a^9 + 1/1458 (81 b c^2 + 10 a d e) c / (a^9 b) + 1/39366 (125 b d^3 + 8 a e^3) / (a^8 b^2) - 1/39366 (729 b^2 c^3 + 8 a^2 e^3 - 5(25 d^3 - 54 c d e) a b) / (a^9 b^2))^{1/3} + 486 c / a^3) -$$

$$\begin{aligned}
& 5*(25*d^3 - 54*c*d*e)*a*b/(a^9*b^2))^{(1/3)} + 486*c/a^3)^2*a^6*b - 972*((-I \\
& *sqrt(3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b)))/(-1/27*c^3/a^9 + \\
& 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^ \\
& 8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9 \\
& *b^2))^{(1/3)} + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a \\
& *d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^ \\
& 2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^{(1/3)} + 486*c/a^3 \\
&)*a^3*b*c + 236196*b*c^2 + 116640*a*d*e)/(a^6*b)))*log(-1/1458*((-I*sqrt(3) \\
& + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b)))/(-1/27*c^3/a^9 + 1/1458* \\
& (81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - \\
& 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^{(\\
& 1/3)} + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/ \\
& (a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + \\
& 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^{(1/3)} + 486*c/a^3)^2*a^6* \\
& b*e - 225*b*c*d^2 - 162*b*c^2*e - 40*a*d*e^2 + 1/54*(25*a^3*b*d^2 + 36*a^3* \\
& b*c*e))*((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b)))/(-1/2 \\
& 7*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8 \\
& *a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e \\
&)*a*b)/(a^9*b^2))^{(1/3)} + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b \\
& *c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39 \\
& 366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^{(1/3)} \\
& + 486*c/a^3) + 2*(125*b*d^3 + 8*a*e^3)*x + 1/486*sqrt(1/3)*(((-I*sqrt(3) + \\
& 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b)))/(-1/27*c^3/a^9 + 1/1458*(81 \\
& *b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/ \\
& 39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^{(1/3)} \\
&) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^ \\
& 9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a \\
& ^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^{(1/3)} + 486*c/a^3)*a^6*b*e + \\
& 675*a^3*b*d^2 - 486*a^3*b*c*e)*sqrt(-(((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81* \\
& b*c^2 + 10*a*d*e)/(a^6*b)))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/ \\
& (a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + \\
& 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^{(1/3)} + 729*(I*sqrt(3) + \\
& 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b \\
& *d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - \\
& 54*c*d*e)*a*b)/(a^9*b^2))^{(1/3)} + 486*c/a^3)^2*a^6*b - 972*((-I*sqrt(3) + 1 \\
&)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b)))/(-1/27*c^3/a^9 + 1/1458*(81* \\
& b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/3 \\
& 9366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^{(1/3)} \\
& + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9 \\
& *b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^ \\
& 2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^{(1/3)} + 486*c/a^3)*a^3*b*c + \\
& 236196*b*c^2 + 116640*a*d*e)/(a^6*b)) - (1458*b^2*c*x^6 + 2916*a*b*c*x^3 + \\
& 1458*a^2*c - (a^3*b^2*x^6 + 2*a^4*b*x^3 + a^5))*((-I*sqrt(3) + 1)*(81*c^2/a \\
& ^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b)))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10* \\
& a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b
\end{aligned}$$

$$\begin{aligned}
& ^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^{(1/3)} + 729*(I*\text{sqrt}(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^{(1/3)} + 486*c/a^3) + 3*\text{sqrt}(1/3)*(a^3*b^2*x^6 + 2*a^4*b*x^3 + a^5)*\text{sqrt}(-(((-I*\text{sqrt}(3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b)))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^{(1/3)} + 729*(I*\text{sqrt}(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^{(1/3)} + 486*c/a^3)^2*a^6*b - 972*((-I*\text{sqrt}(3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b)))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^{(1/3)} + 729*(I*\text{sqrt}(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^{(1/3)} + 486*c/a^3)^2*a^6*b*c + 236196*b*c^2 + 116640*a*d*e)/(a^6*b)))*\log(-1/1458*((-I*\text{sqrt}(3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b)))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^{(1/3)} + 729*(I*\text{sqrt}(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^{(1/3)} + 486*c/a^3)^2*a^6*b*e - 225*b*c*d^2 - 162*b*c^2*e - 40*a*d*e^2 + 1/54*(25*a^3*b*d^2 + 36*a^3*b*c*e))*((-I*\text{sqrt}(3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b)))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^{(1/3)} + 729*(I*\text{sqrt}(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^{(1/3)} + 486*c/a^3) + 2*(125*b*d^3 + 8*a*e^3)*x - 1/486*\text{sqrt}(1/3)*(((-I*\text{sqrt}(3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b)))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^{(1/3)} + 729*(I*\text{sqrt}(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^{(1/3)} + 486*c/a^3)^2*a^6*b*e + 675*a^3*b*d^2 - 486*a^3*b*c*e)*\text{sqrt}(-(((-I*\text{sqrt}(3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b)))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^{(1/3)} + 729*(I*\text{sqrt}(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^{(1/3)} + 486*c/a^3)^2*a^6*b - 972*((-I*\text{sqrt}(3) + 1)*(81*c^2/a^6 - (
\end{aligned}$$

$$\begin{aligned} & (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e) \\ & *c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 \\ & + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^{(1/3)} + 729*(I*sqrt(3) \\ & + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(12 \\ & 5*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 \\ & - 54*c*d*e)*a*b)/(a^9*b^2))^{(1/3)} + 486*c/a^3)*a^3*b*c + 236196*b*c^2 + 11 \\ & 6640*a*d*e)/(a^6*b))) + 2916*(b^2*c*x^6 + 2*a*b*c*x^3 + a^2*c)*log(x))/(a^3 \\ & *b^2*x^6 + 2*a^4*b*x^3 + a^5) \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/x/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.11628, size = 359, normalized size = 1.4

$$-\frac{c \log(|bx^3 + a|)}{3a^3} + \frac{c \log(|x|)}{a^3} + \frac{\sqrt{3} \left(5 (-ab^2)^{\frac{1}{3}} bd - 2 (-ab^2)^{\frac{2}{3}} e \right) \arctan \left(\frac{\sqrt{3} \left(2x + (-\frac{a}{b})^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{27 a^3 b^2} + \frac{4 abx^5 e + 5 abdx^4 + 6 abcx^3}{18 (bx^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="giac")

[Out]
$$-\frac{1}{3}c*\log(\text{abs}(b*x^3 + a))/a^3 + c*\log(\text{abs}(x))/a^3 + \frac{1}{27}*sqrt(3)*(5*(-a*b^2)^{(1/3)}*b*d - 2*(-a*b^2)^{(2/3)}*e)*\arctan(1/3*sqrt(3)*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(a^3*b^2) + \frac{1}{18}*(4*a*b*x^5*e + 5*a*b*d*x^4 + 6*a*b*c*x^3 + 7*a^2*x^2*e + 8*a^2*d*x + 9*a^2*c)/((b*x^3 + a)^2*a^3) - \frac{1}{27}*(2*a^4*b*(-a/b)^{(1/3)}*e + 5*a^4*b*d)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)})/(a^7*b) + \frac{1}{54}*(5*(-a*b^2)^{(1/3)}*a*b^3*d + 2*(-a*b^2)^{(2/3)}*a*b^2*e)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a^4*b^4)$$

$$3.355 \quad \int \frac{c+dx+ex^2}{x^2(a+bx^3)^3} dx$$

Optimal. Leaf size=267

$$-\frac{(5a^{2/3}e + 14b^{2/3}c) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{54a^{10/3}\sqrt[3]{b}} + \frac{(5a^{2/3}e + 14b^{2/3}c) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{27a^{10/3}\sqrt[3]{b}} + \frac{(14b^{2/3}c - 5a^{2/3}e) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{b}}\right)}{9\sqrt{3}a^{10/3}\sqrt[3]{b}}$$

[Out] $-(c/(a^3x)) + (x*(a*e - b*c*x - b*d*x^2))/(6*a^2*(a + b*x^3)^2) + (x*(5*a*e - 10*b*c*x - 9*b*d*x^2))/(18*a^3*(a + b*x^3)) + ((14*b^(2/3)*c - 5*a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(10/3)*b^(1/3)) + (d*Log[x])/a^3 + ((14*b^(2/3)*c + 5*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x]/(27*a^(10/3)*b^(1/3)) - ((14*b^(2/3)*c + 5*a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(54*a^(10/3)*b^(1/3)) - (d*Log[a + b*x^3])/(3*a^3)$

Rubi [A] time = 0.461799, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{(5a^{2/3}e + 14b^{2/3}c) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{54a^{10/3}\sqrt[3]{b}} + \frac{(5a^{2/3}e + 14b^{2/3}c) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{27a^{10/3}\sqrt[3]{b}} + \frac{(14b^{2/3}c - 5a^{2/3}e) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{b}}\right)}{9\sqrt{3}a^{10/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^3), x]

[Out] $-(c/(a^3x)) + (x*(a*e - b*c*x - b*d*x^2))/(6*a^2*(a + b*x^3)^2) + (x*(5*a*e - 10*b*c*x - 9*b*d*x^2))/(18*a^3*(a + b*x^3)) + ((14*b^(2/3)*c - 5*a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(10/3)*b^(1/3)) + (d*Log[x])/a^3 + ((14*b^(2/3)*c + 5*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x]/(27*a^(10/3)*b^(1/3)) - ((14*b^(2/3)*c + 5*a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(54*a^(10/3)*b^(1/3)) - (d*Log[a + b*x^3])/(3*a^3)$

Rule 1829

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^

```
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_)*(x_))^(m_))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[Ex
pandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^( -1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
```

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 204

$\text{Int}[\frac{(a_1 + (b_1)x^2)^{-1}}{x}, x_Symbol] \rightarrow -\text{Simp}[\frac{\text{ArcTan}[\frac{\text{Rt}[-b, 2]x}{\text{Rt}[-a, 2] - \text{Rt}[-a, 2]\text{Rt}[-b, 2]}}]{x}, x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 628

$\text{Int}[\frac{(d_1 + (e_1)x)}{(a_1 + (b_1)x + (c_1)x^2)}, x_Symbol] \rightarrow \text{Simp}[\frac{d \cdot \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]}{b}, x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2cd - be, 0]$

Rule 260

$\text{Int}[x^{(m_1)} / (a_1 + (b_1)x^n), x_Symbol] \rightarrow \text{Simp}[\frac{\text{Log}[\text{RemoveContent}[a + bx^n, x]]}{bn}, x] \ /; \ \text{FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2}{x^2(a + bx^3)^3} dx &= \frac{x(ae - bcx - bdx^2)}{6a^2(a + bx^3)^2} - \frac{\int \frac{-6bc - 6bdx - 5bex^2 + \frac{4b^2cx^3}{a} + \frac{3b^2dx^4}{a}}{x^2(a + bx^3)^2} dx}{6ab} \\
&= \frac{x(ae - bcx - bdx^2)}{6a^2(a + bx^3)^2} + \frac{x(5ae - 10bcx - 9bdx^2)}{18a^3(a + bx^3)} + \frac{\int \frac{18b^3c + 18b^3dx + 10b^3ex^2 - \frac{10b^4cx^3}{a}}{x^2(a + bx^3)} dx}{18a^2b^3} \\
&= \frac{x(ae - bcx - bdx^2)}{6a^2(a + bx^3)^2} + \frac{x(5ae - 10bcx - 9bdx^2)}{18a^3(a + bx^3)} + \frac{\int \left(\frac{18b^3c}{ax^2} + \frac{18b^3d}{ax} + \frac{2b^3(5ae - 14bcx - 9bdx^2)}{a(a + bx^3)} \right) dx}{18a^2b^3} \\
&= -\frac{c}{a^3x} + \frac{x(ae - bcx - bdx^2)}{6a^2(a + bx^3)^2} + \frac{x(5ae - 10bcx - 9bdx^2)}{18a^3(a + bx^3)} + \frac{d \log(x)}{a^3} + \frac{\int \frac{5ae - 14bcx - 9bdx^2}{a + bx^3} dx}{9a^3} \\
&= -\frac{c}{a^3x} + \frac{x(ae - bcx - bdx^2)}{6a^2(a + bx^3)^2} + \frac{x(5ae - 10bcx - 9bdx^2)}{18a^3(a + bx^3)} + \frac{d \log(x)}{a^3} + \frac{\int \frac{5ae - 14bcx}{a + bx^3} dx}{9a^3} - \frac{(bd) \int \frac{x^2}{a + bx^3} dx}{a^3} \\
&= -\frac{c}{a^3x} + \frac{x(ae - bcx - bdx^2)}{6a^2(a + bx^3)^2} + \frac{x(5ae - 10bcx - 9bdx^2)}{18a^3(a + bx^3)} + \frac{d \log(x)}{a^3} - \frac{d \log(a + bx^3)}{3a^3} + \frac{\int \frac{\sqrt[3]{a}(-14\sqrt[3]{a}}{a + bx^3} dx}{9a^3} \\
&= -\frac{c}{a^3x} + \frac{x(ae - bcx - bdx^2)}{6a^2(a + bx^3)^2} + \frac{x(5ae - 10bcx - 9bdx^2)}{18a^3(a + bx^3)} + \frac{d \log(x)}{a^3} + \frac{(14b^{2/3}c + 5a^{2/3}e) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{27a^{10/3}\sqrt[3]{b}} \\
&= -\frac{c}{a^3x} + \frac{x(ae - bcx - bdx^2)}{6a^2(a + bx^3)^2} + \frac{x(5ae - 10bcx - 9bdx^2)}{18a^3(a + bx^3)} + \frac{d \log(x)}{a^3} + \frac{(14b^{2/3}c + 5a^{2/3}e) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{27a^{10/3}\sqrt[3]{b}} \\
&= -\frac{c}{a^3x} + \frac{x(ae - bcx - bdx^2)}{6a^2(a + bx^3)^2} + \frac{x(5ae - 10bcx - 9bdx^2)}{18a^3(a + bx^3)} + \frac{(14b^{2/3}c - 5a^{2/3}e) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}\sqrt[3]{b}} + \frac{d \log(x)}{a^3}
\end{aligned}$$

Mathematica [A] time = 0.221198, size = 248, normalized size = 0.93

$$\frac{(14a^{2/3}b^{2/3}c + 5a^{4/3}e) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{\sqrt[3]{b}} + \frac{2(14a^{2/3}b^{2/3}c + 5a^{4/3}e) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{\sqrt[3]{b}} - \frac{2\sqrt{3}a^{2/3}(5a^{2/3}e - 14b^{2/3}c) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}} + \frac{9a^2(a(dx + ex^2) - bcx^2)}{(a + bx^3)^2}$$

54a⁴

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^3),x]

[Out]
$$\begin{aligned} &((-54*a*c)/x + (3*a*(6*a*d + 5*a*e*x - 10*b*c*x^2))/(a + b*x^3) + (9*a^2*(- \\ &(b*c*x^2) + a*(d + e*x)))/(a + b*x^3)^2 - (2*\text{Sqrt}[3]*a^{(2/3)}*(-14*b^{(2/3)}*c \\ &+ 5*a^{(2/3)}*e)*\text{ArcTan}[(1 - (2*b^{(1/3)}*x)/a^{(1/3)})/\text{Sqrt}[3]])/b^{(1/3)} + 54*a \\ &*d*\text{Log}[x] + (2*(14*a^{(2/3)}*b^{(2/3)}*c + 5*a^{(4/3)}*e)*\text{Log}[a^{(1/3)} + b^{(1/3)}*x \\ &])/b^{(1/3)} - ((14*a^{(2/3)}*b^{(2/3)}*c + 5*a^{(4/3)}*e)*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)} \\ &x + b^{(2/3)}*x^2])/b^{(1/3)} - 18*a*d*\text{Log}[a + b*x^3])/(54*a^4) \end{aligned}$$

Maple [A] time = 0.017, size = 334, normalized size = 1.3

$$-\frac{5b^2cx^5}{9a^3(bx^3+a)^2} + \frac{5bex^4}{18a^2(bx^3+a)^2} + \frac{bdx^3}{3a^2(bx^3+a)^2} - \frac{13bcx^2}{18a^2(bx^3+a)^2} + \frac{4ex}{9a(bx^3+a)^2} + \frac{d}{2a(bx^3+a)^2} + \frac{5e}{27ba^2} \ln$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x)

[Out]
$$\begin{aligned} &-5/9/a^3/(b*x^3+a)^2*b^2*x^5*c+5/18/a^2/(b*x^3+a)^2*x^4*b*e+1/3/a^2/(b*x^3+ \\ &a)^2*b*d*x^3-13/18/a^2/(b*x^3+a)^2*b*x^2*c+4/9/a/(b*x^3+a)^2*x*e+1/2/a/(b*x \\ &^3+a)^2*d+5/27/a^2/b/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*e-5/54/a^2/b/(1/b*a) \\ &^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*e+5/27/a^2/b/(1/b*a)^{(2/3)}*3^{(\\ &1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*e+14/27/a^3/(1/b*a)^{(1/3)}*\ln \\ &(x+(1/b*a)^{(1/3)})*c-7/27/a^3/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(\\ &2/3)})*c-14/27/a^3*3^{(1/2)}/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)} \\ &*x-1))*c-1/3*d*\ln(b*x^3+a)/a^3+d*\ln(x)/a^3-c/a^3/x \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [C] time = 10.0854, size = 13300, normalized size = 49.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/2916*(4536*b^2*c*x^6 - 810*a*b*e*x^5 - 972*a*b*d*x^4 + 7938*a*b*c*x^3 - \\ & 1296*a^2*e*x^2 - 1458*a^2*d*x + 2916*a^2*c + 2*(a^3*b^2*x^7 + 2*a^4*b*x^4 + \\ & a^5*x)*((-I*\sqrt{3}) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a \\ & ^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - \\ & 27*(27*d^3 - 70*c*d*e)*a*b)/(a^{10}*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3 \\ &)/(a^{10}*b))^{1/3} + 729*(I*\sqrt{3}) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 7 \\ & 0*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e) \\ & *a*b)/(a^{10}*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^{10}*b))^{1/3} + 486 \\ & *d/a^3)*\log(-7/1458*((-I*\sqrt{3}) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/ \\ & (-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 1 \\ & 25*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^{10}*b) - 1/39366*(2744*b^2*c^3 - \\ & 125*a^2*e^3)/(a^{10}*b))^{1/3} + 729*(I*\sqrt{3}) + 1)*(-1/27*d^3/a^9 + 1/1458 \\ & *(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 \\ & - 70*c*d*e)*a*b)/(a^{10}*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^{10}*b)) \\ & ^{1/3} + 486*d/a^3)^2*a^7*b*c - 1134*a*b*c*d^2 + 1960*a*b*c^2*e + 225*a^2*d \\ & *e^2 + 1/54*(252*a^4*b*c*d - 25*a^5*e^2)*((-I*\sqrt{3}) + 1)*(81*d^2/a^6 - (8 \\ & 1*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39 \\ & 366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^{10}*b) - 1/ \\ & 39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^{10}*b))^{1/3} + 729*(I*\sqrt{3}) + 1)*(- \\ & 1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125 \\ & *a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^{10}*b) - 1/39366*(2744*b^2*c^3 - 1 \\ & 25*a^2*e^3)/(a^{10}*b))^{1/3} + 486*d/a^3) - (2744*b^2*c^3 - 125*a^2*e^3)*x \\ & + (1458*b^2*d*x^7 + 2916*a*b*d*x^4 + 1458*a^2*d*x - (a^3*b^2*x^7 + 2*a^4*b* \\ & x^4 + a^5*x)*((-I*\sqrt{3}) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27* \\ & d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2* \\ & e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^{10}*b) - 1/39366*(2744*b^2*c^3 - 125*a^ \\ & 2*e^3)/(a^{10}*b))^{1/3} + 729*(I*\sqrt{3}) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^ \\ & 2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c \\ & *d*e)*a*b)/(a^{10}*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^{10}*b))^{1/3} \\ & + 486*d/a^3) + 3*\sqrt{1/3}*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*\sqrt{-(((-I* \\ & \sqrt{3}) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(\\ & 81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - \\ & 70*c*d*e)*a*b)/(a^{10}*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^{10}*b))^{ \\ & (1/3) + 729*(I*\sqrt{3}) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 \\ & + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^{10}*b} \end{aligned}$$

$$\begin{aligned}
&) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^{10}*b))^{(1/3)} + 486*d/a^3)^2*a^6 \\
& - 972*((-I*\sqrt{3}) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 \\
& + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - \\
& 27*(27*d^3 - 70*c*d*e)*a*b)/(a^{10}*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3) \\
& / (a^{10}*b))^{(1/3)} + 729*(I*\sqrt{3}) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70 \\
& *c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)* \\
& a*b)/(a^{10}*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^{10}*b))^{(1/3)} + 486* \\
& d/a^3)*a^3*d + 236196*d^2 - 816480*c*e)/a^6))*\log(7/1458*((-I*\sqrt{3}) + 1)* \\
& (81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c \\
& *e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a \\
& b)/(a^{10}*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^{10}*b))^{(1/3)} + 729*(I \\
& *\sqrt{3}) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(27 \\
& 44*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^{10}*b) - 1/39366*(\\
& 2744*b^2*c^3 - 125*a^2*e^3)/(a^{10}*b))^{(1/3)} + 486*d/a^3)^2*a^7*b*c + 1134*a \\
& *b*c*d^2 - 1960*a*b*c^2*e - 225*a^2*d*e^2 - 1/54*(252*a^4*b*c*d - 25*a^5*e^ \\
& 2))*((-I*\sqrt{3}) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + \\
& 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(\\
& 27*d^3 - 70*c*d*e)*a*b)/(a^{10}*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^ \\
& 10*b))^{(1/3)} + 729*(I*\sqrt{3}) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e \\
&)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b) \\
& / (a^{10}*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^{10}*b))^{(1/3)} + 486*d/a^ \\
& 3) - 2*(2744*b^2*c^3 - 125*a^2*e^3)*x + 1/486*\sqrt{1/3}*(7*((-I*\sqrt{3}) + 1 \\
&)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70 \\
& *c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)* \\
& a*b)/(a^{10}*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^{10}*b))^{(1/3)} + 729* \\
& (I*\sqrt{3}) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(\\
& 2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^{10}*b) - 1/39366 \\
& *(2744*b^2*c^3 - 125*a^2*e^3)/(a^{10}*b))^{(1/3)} + 486*d/a^3)*a^7*b*c - 3402*a \\
& ^4*b*c*d - 675*a^5*e^2)*\sqrt{-(((-I*\sqrt{3}) + 1)*(81*d^2/a^6 - (81*d^2 - 70 \\
& *c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744* \\
& b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^{10}*b) - 1/39366*(274 \\
& 4*b^2*c^3 - 125*a^2*e^3)/(a^{10}*b))^{(1/3)} + 729*(I*\sqrt{3}) + 1)*(-1/27*d^3/a \\
& ^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - \\
& 27*(27*d^3 - 70*c*d*e)*a*b)/(a^{10}*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3 \\
&)/(a^{10}*b))^{(1/3)} + 486*d/a^3)^2*a^6 - 972*((-I*\sqrt{3}) + 1)*(81*d^2/a^6 - \\
& (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/ \\
& 39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^{10}*b) - \\
& 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^{10}*b))^{(1/3)} + 729*(I*\sqrt{3}) + 1)* \\
& (-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 1 \\
& 25*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^{10}*b) - 1/39366*(2744*b^2*c^3 - \\
& 125*a^2*e^3)/(a^{10}*b))^{(1/3)} + 486*d/a^3)*a^3*d + 236196*d^2 - 816480*c*e) \\
& /a^6)) + (1458*b^2*d*x^7 + 2916*a*b*d*x^4 + 1458*a^2*d*x - (a^3*b^2*x^7 + 2 \\
& *a^4*b*x^4 + a^5*x))*((-I*\sqrt{3}) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/ \\
& (-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 1 \\
& 25*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^{10}*b) - 1/39366*(2744*b^2*c^3 -
\end{aligned}$$

$$\begin{aligned}
& 125*a^2*e^3)/(a^{10}*b))^{(1/3)} + 729*(I*\text{sqrt}(3) + 1)*(-1/27*d^3/a^9 + 1/1458 \\
& *(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 \\
& - 70*c*d*e)*a*b)/(a^{10}*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^{10}*b)) \\
& ^{(1/3)} + 486*d/a^3) - 3*\text{sqrt}(1/3)*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*\text{sqrt}(\\
& -(((-I*\text{sqrt}(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1 \\
& /1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(2 \\
& 7*d^3 - 70*c*d*e)*a*b)/(a^{10}*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^{1 \\
& 0*b}))^{(1/3)} + 729*(I*\text{sqrt}(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e) \\
& *d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/ \\
& (a^{10}*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^{10}*b))^{(1/3)} + 486*d/a^3 \\
&)^2*a^6 - 972*((-I*\text{sqrt}(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27 \\
& *d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2 \\
& *e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^{10}*b) - 1/39366*(2744*b^2*c^3 - 125*a \\
& ^2*e^3)/(a^{10}*b))^{(1/3)} + 729*(I*\text{sqrt}(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d \\
& ^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70* \\
& c*d*e)*a*b)/(a^{10}*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^{10}*b))^{(1/3)} \\
& + 486*d/a^3)*a^3*d + 236196*d^2 - 816480*c*e)/a^6))*\text{log}(7/1458*((-I*\text{sqrt}(3) \\
&) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d^2 \\
& - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c* \\
& d*e)*a*b)/(a^{10}*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^{10}*b))^{(1/3)} + \\
& 729*(I*\text{sqrt}(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39 \\
& 366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^{10}*b) - 1/ \\
& 39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^{10}*b))^{(1/3)} + 486*d/a^3)^2*a^7*b*c + \\
& 1134*a*b*c*d^2 - 1960*a*b*c^2*e - 225*a^2*d*e^2 - 1/54*(252*a^4*b*c*d - 25 \\
& *a^5*e^2))*((-I*\text{sqrt}(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3 \\
& /a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 \\
& - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^{10}*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e \\
& ^3)/(a^{10}*b))^{(1/3)} + 729*(I*\text{sqrt}(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - \\
& 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d* \\
& e)*a*b)/(a^{10}*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^{10}*b))^{(1/3)} + 4 \\
& 86*d/a^3) - 2*(2744*b^2*c^3 - 125*a^2*e^3)*x - 1/486*\text{sqrt}(1/3)*(7*((-I*\text{sqrt} \\
& (3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d \\
& ^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70* \\
& c*d*e)*a*b)/(a^{10}*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^{10}*b))^{(1/3)} \\
& + 729*(I*\text{sqrt}(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/ \\
& 39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^{10}*b) - \\
& 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^{10}*b))^{(1/3)} + 486*d/a^3)*a^7*b*c - \\
& 3402*a^4*b*c*d - 675*a^5*e^2)*\text{sqrt}(-(((-I*\text{sqrt}(3) + 1)*(81*d^2/a^6 - (81*d \\
& ^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366 \\
& *(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^{10}*b) - 1/393 \\
& 66*(2744*b^2*c^3 - 125*a^2*e^3)/(a^{10}*b))^{(1/3)} + 729*(I*\text{sqrt}(3) + 1)*(-1/2 \\
& 7*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^ \\
& 2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^{10}*b) - 1/39366*(2744*b^2*c^3 - 125* \\
& a^2*e^3)/(a^{10}*b))^{(1/3)} + 486*d/a^3)^2*a^6 - 972*((-I*\text{sqrt}(3) + 1)*(81*d^2 \\
& /a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a
\end{aligned}$$

$$\begin{aligned} &^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^{10*b}) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^{10*b})^{(1/3)} + 729*(I*\sqrt{3} + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^{10*b}) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^{10*b})^{(1/3)} + 486*d/a^3)*a^3*d + 236196*d^2 - 816480*c*e/a^6) - 2916*(b^2*d*x^7 + 2*a*b*d*x^4 + a^2*d*x)*\log(x))/(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x) \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.11718, size = 377, normalized size = 1.41

$$-\frac{d \log(|bx^3 + a|)}{3a^3} + \frac{d \log(|x|)}{a^3} + \frac{\left(5(-ab^2)^{\frac{1}{3}}ae - 14(-ab^2)^{\frac{2}{3}}c\right) \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54a^4b} + \frac{\sqrt{3}\left(5(-ab^2)^{\frac{1}{3}}ab^2e + 1\right)}{54a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="giac")

[Out]
$$-1/3*d*\log(\text{abs}(b*x^3 + a))/a^3 + d*\log(\text{abs}(x))/a^3 + 1/54*(5*(-a*b^2)^{(1/3)}*a*e - 14*(-a*b^2)^{(2/3)}*c)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a^4*b) + 1/27*\sqrt{3}*(5*(-a*b^2)^{(1/3)}*a*b^2*e + 14*(-a*b^2)^{(2/3)}*b^2*c)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(a^4*b^3) - 1/18*(28*b^2*c*x^6 - 5*a*b*x^5*e - 6*a*b*d*x^4 + 49*a*b*c*x^3 - 8*a^2*x^2*e - 9*a^2*d*x + 18*a^2*c)/((b*x^3 + a)^2*a^3*x) + 1/27*(14*a^3*b^2*c*(-a/b)^{(1/3)} - 5*a^4*b*e)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/a^7*b$$

$$3.356 \quad \int \frac{c+dx+ex^2}{x^3(a+bx^3)^3} dx$$

Optimal. Leaf size=276

$$\frac{\sqrt[3]{b}(10\sqrt[3]{bc} - 7\sqrt[3]{ad}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{27a^{11/3}} - \frac{x(11bc + 10bdx + 9bex^2)}{18a^3(a + bx^3)} - \frac{x(bc + bdx + bex^2)}{6a^2(a + bx^3)^2} - \frac{2\sqrt[3]{b}(10\sqrt[3]{bc} - 7\sqrt[3]{ad})}{27a^{11/3}}$$

[Out] $-c/(2*a^3*x^2) - d/(a^3*x) - (x*(b*c + b*d*x + b*e*x^2))/(6*a^2*(a + b*x^3)^2) - (x*(11*b*c + 10*b*d*x + 9*b*e*x^2))/(18*a^3*(a + b*x^3)) + (2*b^(1/3)*(10*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(11/3)) + (e*Log[x])/a^3 - (2*b^(1/3)*(10*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x]/(27*a^(11/3)) + (b^(1/3)*(10*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(27*a^(11/3)) - (e*Log[a + b*x^3])/(3*a^3)$

Rubi [A] time = 0.499915, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\sqrt[3]{b}(10\sqrt[3]{bc} - 7\sqrt[3]{ad}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{27a^{11/3}} - \frac{x(11bc + 10bdx + 9bex^2)}{18a^3(a + bx^3)} - \frac{x(bc + bdx + bex^2)}{6a^2(a + bx^3)^2} - \frac{2\sqrt[3]{b}(10\sqrt[3]{bc} - 7\sqrt[3]{ad})}{27a^{11/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^3), x]

[Out] $-c/(2*a^3*x^2) - d/(a^3*x) - (x*(b*c + b*d*x + b*e*x^2))/(6*a^2*(a + b*x^3)^2) - (x*(11*b*c + 10*b*d*x + 9*b*e*x^2))/(18*a^3*(a + b*x^3)) + (2*b^(1/3)*(10*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(11/3)) + (e*Log[x])/a^3 - (2*b^(1/3)*(10*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x]/(27*a^(11/3)) + (b^(1/3)*(10*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(27*a^(11/3)) - (e*Log[a + b*x^3])/(3*a^3)$

Rule 1829

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^

```
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_)*(x_))^(m_))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^( -1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
```

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 204

$\text{Int}[\frac{(a_+) + (b_+)(x_+)^2}{(x_+)^{-1}}, x_Symbol] \ :> \ -\text{Simp}[\frac{\text{ArcTan}[\frac{\text{Rt}[-b, 2]x}{\text{Rt}[-a, 2] - \text{Rt}[-a, 2] \text{Rt}[-b, 2]}}{\text{Rt}[-a, 2] \text{Rt}[-b, 2]}], x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 628

$\text{Int}[\frac{(d_+) + (e_+)(x_+)}{(a_+) + (b_+)(x_+) + (c_+)(x_+)^2}, x_Symbol] \ :> \ \text{Simp}[\frac{d \cdot \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]}{b}, x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2cd - be, 0]$

Rule 260

$\text{Int}[\frac{(x_+)^{m_+}}{(a_+) + (b_+)(x_+)^{n_+}}, x_Symbol] \ :> \ \text{Simp}[\frac{\text{Log}[\text{RemoveContent}[a + bx^n, x]]}{(b \cdot n)}, x] \ /; \ \text{FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2}{x^3 (a + bx^3)^3} dx &= -\frac{x(bc + bdx + bex^2)}{6a^2 (a + bx^3)^2} - \frac{\int \frac{-6bc - 6bdx - 6bex^2 + \frac{5b^2cx^3}{a} + \frac{4b^2dx^4}{a} + \frac{3b^2ex^5}{a}}{x^3(a+bx^3)^2} dx}{6ab} \\
&= -\frac{x(bc + bdx + bex^2)}{6a^2 (a + bx^3)^2} - \frac{x(11bc + 10bdx + 9bex^2)}{18a^3 (a + bx^3)} + \frac{\int \frac{18b^3c + 18b^3dx + 18b^3ex^2 - \frac{22b^4cx^3}{a} - \frac{10b^4dx^4}{a}}{x^3(a+bx^3)} dx}{18a^2b^3} \\
&= -\frac{x(bc + bdx + bex^2)}{6a^2 (a + bx^3)^2} - \frac{x(11bc + 10bdx + 9bex^2)}{18a^3 (a + bx^3)} + \frac{\int \left(\frac{18b^3c}{ax^3} + \frac{18b^3d}{ax^2} + \frac{18b^3e}{ax} - \frac{2b^4(20c+14dx+9ex^2)}{a(a+bx^3)} \right) dx}{18a^2b^3} \\
&= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc + bdx + bex^2)}{6a^2 (a + bx^3)^2} - \frac{x(11bc + 10bdx + 9bex^2)}{18a^3 (a + bx^3)} + \frac{e \log(x)}{a^3} - \frac{b \int \frac{20c+14dx+9ex^2}{a+bx^3} dx}{9a^3} \\
&= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc + bdx + bex^2)}{6a^2 (a + bx^3)^2} - \frac{x(11bc + 10bdx + 9bex^2)}{18a^3 (a + bx^3)} + \frac{e \log(x)}{a^3} - \frac{b \int \frac{20c+14dx}{a+bx^3} dx}{9a^3} \\
&= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc + bdx + bex^2)}{6a^2 (a + bx^3)^2} - \frac{x(11bc + 10bdx + 9bex^2)}{18a^3 (a + bx^3)} + \frac{e \log(x)}{a^3} - \frac{e \log(a + bx^3)}{3a^3} \\
&= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc + bdx + bex^2)}{6a^2 (a + bx^3)^2} - \frac{x(11bc + 10bdx + 9bex^2)}{18a^3 (a + bx^3)} + \frac{e \log(x)}{a^3} - \frac{2\sqrt[3]{b}(10\sqrt[3]{bc} - 7\sqrt[3]{ad})}{27a^3} \\
&= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc + bdx + bex^2)}{6a^2 (a + bx^3)^2} - \frac{x(11bc + 10bdx + 9bex^2)}{18a^3 (a + bx^3)} + \frac{e \log(x)}{a^3} - \frac{2\sqrt[3]{b}(10\sqrt[3]{bc} - 7\sqrt[3]{ad})}{27a^3} \\
&= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc + bdx + bex^2)}{6a^2 (a + bx^3)^2} - \frac{x(11bc + 10bdx + 9bex^2)}{18a^3 (a + bx^3)} + \frac{2\sqrt[3]{b}(10\sqrt[3]{bc} + 7\sqrt[3]{ad}) \tan^{-1}}{9\sqrt{3}a^{11/3}}
\end{aligned}$$

Mathematica [A] time = 0.209617, size = 253, normalized size = 0.92

$$2\sqrt[3]{b}(10\sqrt[3]{a}\sqrt[3]{bc} - 7a^{2/3}d) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2) + \frac{9a^2(ae - bx(c+dx))}{(a+bx^3)^2} + 4\sqrt[3]{b}(7a^{2/3}d - 10\sqrt[3]{a}\sqrt[3]{bc}) \log(\sqrt[3]{a} + \sqrt[3]{bx}) + \frac{3e \log(a+bx^3)}{a^3}$$

54a⁴

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^3), x]

[Out]
$$\begin{aligned} &((-27*a*c)/x^2 - (54*a*d)/x + (9*a^2*(a*e - b*x*(c + d*x)))/(a + b*x^3)^2 + \\ &(3*a*(6*a*e - b*x*(11*c + 10*d*x)))/(a + b*x^3) + 4*\sqrt{3}*a^{1/3}*b^{1/3} \\ &*(10*b^{1/3}*c + 7*a^{1/3}*d)*\text{ArcTan}[(1 - (2*b^{1/3}*x)/a^{1/3})/\sqrt{3}] \\ &+ 54*a*e*\text{Log}[x] + 4*b^{1/3}*(-10*a^{1/3}*b^{1/3}*c + 7*a^{2/3}*d)*\text{Log}[a^{1/3} \\ &+ b^{1/3}*x] + 2*b^{1/3}*(10*a^{1/3}*b^{1/3}*c - 7*a^{2/3}*d)*\text{Log}[a^{2/3} \\ &- a^{1/3}*b^{1/3}*x + b^{2/3}*x^2] - 18*a*e*\text{Log}[a + b*x^3]/(54*a^4) \end{aligned}$$

Maple [A] time = 0.016, size = 337, normalized size = 1.2

$$-\frac{5b^2dx^5}{9a^3(bx^3+a)^2} - \frac{11b^2cx^4}{18a^3(bx^3+a)^2} + \frac{bex^3}{3a^2(bx^3+a)^2} - \frac{13bdx^2}{18a^2(bx^3+a)^2} - \frac{7bcx}{9a^2(bx^3+a)^2} + \frac{e}{2a(bx^3+a)^2} - \frac{20c}{27a^3} \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)/x^3/(b*x^3+a)^3, x)

[Out]
$$\begin{aligned} &-5/9/a^3/(b*x^3+a)^2*x^5*b^2*d-11/18/a^3/(b*x^3+a)^2*b^2*x^4*c+1/3*b/a^2/(b \\ &*x^3+a)^2*e*x^3-13/18/a^2/(b*x^3+a)^2*b*x^2*d-7/9/a^2/(b*x^3+a)^2*b*x*c+1/2 \\ &/a/(b*x^3+a)^2*e-20/27/a^3/(1/b*a)^{2/3}*ln(x+(1/b*a)^{1/3})*c+10/27/a^3/(1 \\ &/b*a)^{2/3}*ln(x^2-(1/b*a)^{1/3}*x+(1/b*a)^{2/3})*c-20/27/a^3/(1/b*a)^{2/3} \\ &*3^{1/2}*arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1))*c+14/27/a^3/(1/b*a)^{1/3} \\ &)*ln(x+(1/b*a)^{1/3})*d-7/27/a^3/(1/b*a)^{1/3}*ln(x^2-(1/b*a)^{1/3}*x+(1/b* \\ &a)^{2/3})*d-14/27/a^3*3^{1/2}/(1/b*a)^{1/3}*arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3} \\ &)*x-1))*d-1/3*e*ln(b*x^3+a)/a^3-d/a^3/x-1/2*c/a^3/x^2+e*ln(x)/a^3 \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^3, x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [C] time = 9.17148, size = 13346, normalized size = 48.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/2916*(4536*b^2*d*x^7 + 3240*b^2*c*x^6 - 972*a*b*e*x^5 + 7938*a*b*d*x^4 + \\ & 5184*a*b*c*x^3 - 1458*a^2*e*x^2 + 2916*a^2*d*x + 1458*a^2*c + 2*(a^3*b^2*x \\ & ^8 + 2*a^4*b*x^5 + a^5*x^2)*((-I*\sqrt{3}) + 1)*(81*e^2/a^6 - (280*b*c*d + 81 \\ & *a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^{10} + 4/1968 \\ & 3*(1000*b*c^3 + 343*a*d^3)*b/a^{11} - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 5 \\ & 6*(49*d^3 - 135*c*d*e)*a*b)/a^{11})^{1/3} + 729*(I*\sqrt{3}) + 1)*(-1/27*e^3/a^ \\ & 9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^{10} + 4/19683*(1000*b*c^3 + 343*a*d^3) \\ & *b/a^{11} - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b \\ &)/a^{11})^{1/3} + 486*e/a^3)*\log(7/2916*((-I*\sqrt{3}) + 1)*(81*e^2/a^6 - (280* \\ & b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^{1 \\ & 0 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^{11} - 1/39366*(8000*b^2*c^3 + 729*a \\ & ^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^{11})^{1/3} + 729*(I*\sqrt{3}) + 1)*(-1 \\ & /27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^{10} + 4/19683*(1000*b*c^3 + \\ & 343*a*d^3)*b/a^{11} - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135* \\ & c*d*e)*a*b)/a^{11})^{1/3} + 486*e/a^3)^2*a^8*d + 3920*a*b*c*d^2 - 1800*a*b*c^ \\ & 2*e + 567*a^2*d*e^2 + 1/27*(100*a^4*b*c^2 - 63*a^5*d*e)*((-I*\sqrt{3}) + 1)* \\ & (81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d \\ & + 81*a*e^2)*e/a^{10} + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^{11} - 1/39366*(80 \\ & 00*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^{11})^{1/3} + 729*(\\ & I*\sqrt{3}) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^{10} + 4/19 \\ & 683*(1000*b*c^3 + 343*a*d^3)*b/a^{11} - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - \\ & 56*(49*d^3 - 135*c*d*e)*a*b)/a^{11})^{1/3} + 486*e/a^3) + 4*(1000*b^2*c^3 + \\ & 343*a*b*d^3)*x) + (1458*b^2*e*x^8 + 2916*a*b*e*x^5 + 1458*a^2*e*x^2 - (a^3*b^ \\ & 2*x^8 + 2*a^4*b*x^5 + a^5*x^2)*((-I*\sqrt{3}) + 1)*(81*e^2/a^6 - (280*b*c*d \\ & + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^{10} + 4 \\ & /19683*(1000*b*c^3 + 343*a*d^3)*b/a^{11} - 1/39366*(8000*b^2*c^3 + 729*a^2*e^ \\ & 3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^{11})^{1/3} + 729*(I*\sqrt{3}) + 1)*(-1/27*e \\ & ^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^{10} + 4/19683*(1000*b*c^3 + 343*a \\ & *d^3)*b/a^{11} - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e) \\ &)*a*b)/a^{11})^{1/3} + 486*e/a^3) - 3*\sqrt{1/3}*(a^3*b^2*x^8 + 2*a^4*b*x^5 + \\ & a^5*x^2)*\sqrt{-(((I*\sqrt{3}) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a^7) \\ & /(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^{10} + 4/19683*(1000*b*c^ \\ & 3 + 343*a*d^3)*b/a^{11} - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - \\ & 135*c*d*e)*a*b)/a^{11})^{1/3} + 729*(I*\sqrt{3}) + 1)*(-1/27*e^3/a^9 + 1/1458*(\\ & 280*b*c*d + 81*a*e^2)*e/a^{10} + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^{11} - 1/ \\ & 39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^{11})^{1/3} \end{aligned}$$

$$\begin{aligned}
&) + 486e/a^3)^2a^7 - 972*((-I\sqrt{3} + 1)*(81e^2/a^6 - (280b*c*d + 81a*a*e^2)/a^7)/(-1/27e^3/a^9 + 1/1458*(280b*c*d + 81a*a*e^2)*e/a^{10} + 4/19683 \\
& *(1000b*c^3 + 343a*d^3)*b/a^{11} - 1/39366*(8000b^2*c^3 + 729a^2*e^3 - 56 \\
& *(49d^3 - 135c*d*e)*a*b)/a^{11})^{(1/3)} + 729*(I\sqrt{3} + 1)*(-1/27e^3/a^9 \\
& + 1/1458*(280b*c*d + 81a*a*e^2)*e/a^{10} + 4/19683*(1000b*c^3 + 343a*d^3)* \\
& b/a^{11} - 1/39366*(8000b^2*c^3 + 729a^2*e^3 - 56*(49d^3 - 135c*d*e)*a*b) \\
& /a^{11})^{(1/3)} + 486e/a^3)*a^4e + 3265920*b*c*d + 236196*a*e^2)/a^7))*\log(- \\
& 7/2916*((-I\sqrt{3} + 1)*(81e^2/a^6 - (280b*c*d + 81a*a*e^2)/a^7)/(-1/27e \\
& ^3/a^9 + 1/1458*(280b*c*d + 81a*a*e^2)*e/a^{10} + 4/19683*(1000b*c^3 + 343a \\
& *d^3)*b/a^{11} - 1/39366*(8000b^2*c^3 + 729a^2*e^3 - 56*(49d^3 - 135c*d*e \\
&)*a*b)/a^{11})^{(1/3)} + 729*(I\sqrt{3} + 1)*(-1/27e^3/a^9 + 1/1458*(280b*c*d \\
& + 81a*a*e^2)*e/a^{10} + 4/19683*(1000b*c^3 + 343a*d^3)*b/a^{11} - 1/39366*(80 \\
& 00b^2*c^3 + 729a^2*e^3 - 56*(49d^3 - 135c*d*e)*a*b)/a^{11})^{(1/3)} + 486e \\
& /a^3)^2a^8*d - 3920*a*b*c*d^2 + 1800*a*b*c^2*e - 567*a^2*d*e^2 - 1/27*(100 \\
& *a^4*b*c^2 - 63a^5*d*e)*((-I\sqrt{3} + 1)*(81e^2/a^6 - (280b*c*d + 81a*a \\
& e^2)/a^7)/(-1/27e^3/a^9 + 1/1458*(280b*c*d + 81a*a*e^2)*e/a^{10} + 4/19683*(\\
& 1000b*c^3 + 343a*d^3)*b/a^{11} - 1/39366*(8000b^2*c^3 + 729a^2*e^3 - 56*(\\
& 49d^3 - 135c*d*e)*a*b)/a^{11})^{(1/3)} + 729*(I\sqrt{3} + 1)*(-1/27e^3/a^9 + \\
& 1/1458*(280b*c*d + 81a*a*e^2)*e/a^{10} + 4/19683*(1000b*c^3 + 343a*d^3)*b/ \\
& a^{11} - 1/39366*(8000b^2*c^3 + 729a^2*e^3 - 56*(49d^3 - 135c*d*e)*a*b)/a \\
& ^{11})^{(1/3)} + 486e/a^3) + 8*(1000b^2*c^3 + 343a*b*d^3)*x + 1/972*\sqrt{1/3} \\
&)*(7*((-I\sqrt{3} + 1)*(81e^2/a^6 - (280b*c*d + 81a*a*e^2)/a^7)/(-1/27e^3 \\
& /a^9 + 1/1458*(280b*c*d + 81a*a*e^2)*e/a^{10} + 4/19683*(1000b*c^3 + 343a*d \\
& ^3)*b/a^{11} - 1/39366*(8000b^2*c^3 + 729a^2*e^3 - 56*(49d^3 - 135c*d*e)* \\
& a*b)/a^{11})^{(1/3)} + 729*(I\sqrt{3} + 1)*(-1/27e^3/a^9 + 1/1458*(280b*c*d + \\
& 81a*a*e^2)*e/a^{10} + 4/19683*(1000b*c^3 + 343a*d^3)*b/a^{11} - 1/39366*(8000 \\
& *b^2*c^3 + 729a^2*e^3 - 56*(49d^3 - 135c*d*e)*a*b)/a^{11})^{(1/3)} + 486e/a \\
& ^3)*a^8*d - 10800*a^4*b*c^2 - 3402*a^5*d*e)*\sqrt{-(((-I\sqrt{3} + 1)*(81e^ \\
& 2/a^6 - (280b*c*d + 81a*a*e^2)/a^7)/(-1/27e^3/a^9 + 1/1458*(280b*c*d + 81 \\
& *a*a*e^2)*e/a^{10} + 4/19683*(1000b*c^3 + 343a*d^3)*b/a^{11} - 1/39366*(8000b^ \\
& 2*c^3 + 729a^2*e^3 - 56*(49d^3 - 135c*d*e)*a*b)/a^{11})^{(1/3)} + 729*(I\sqrt{ \\
& t(3) + 1)*(-1/27e^3/a^9 + 1/1458*(280b*c*d + 81a*a*e^2)*e/a^{10} + 4/19683*(\\
& 1000b*c^3 + 343a*d^3)*b/a^{11} - 1/39366*(8000b^2*c^3 + 729a^2*e^3 - 56*(\\
& 49d^3 - 135c*d*e)*a*b)/a^{11})^{(1/3)} + 486e/a^3)^2a^7 - 972*((-I\sqrt{3} \\
& + 1)*(81e^2/a^6 - (280b*c*d + 81a*a*e^2)/a^7)/(-1/27e^3/a^9 + 1/1458*(280 \\
& *b*c*d + 81a*a*e^2)*e/a^{10} + 4/19683*(1000b*c^3 + 343a*d^3)*b/a^{11} - 1/393 \\
& 66*(8000b^2*c^3 + 729a^2*e^3 - 56*(49d^3 - 135c*d*e)*a*b)/a^{11})^{(1/3)} + \\
& 729*(I\sqrt{3} + 1)*(-1/27e^3/a^9 + 1/1458*(280b*c*d + 81a*a*e^2)*e/a^{10} \\
& + 4/19683*(1000b*c^3 + 343a*d^3)*b/a^{11} - 1/39366*(8000b^2*c^3 + 729a^2 \\
& *e^3 - 56*(49d^3 - 135c*d*e)*a*b)/a^{11})^{(1/3)} + 486e/a^3)*a^4e + 326592 \\
& 0*b*c*d + 236196*a*e^2)/a^7)) + (1458*b^2*e*x^8 + 2916*a*b*e*x^5 + 1458*a^2 \\
& *e*x^2 - (a^3*b^2*x^8 + 2*a^4*b*x^5 + a^5*x^2))*((-I\sqrt{3} + 1)*(81e^2/a^ \\
& 6 - (280b*c*d + 81a*a*e^2)/a^7)/(-1/27e^3/a^9 + 1/1458*(280b*c*d + 81a*a \\
& e^2)*e/a^{10} + 4/19683*(1000b*c^3 + 343a*d^3)*b/a^{11} - 1/39366*(8000b^2*c^ \\
& 3 + 729a^2*e^3 - 56*(49d^3 - 135c*d*e)*a*b)/a^{11})^{(1/3)} + 729*(I\sqrt{3})
\end{aligned}$$

$$\begin{aligned} & b/a^{11} - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b) \\ & /a^{11})^{(1/3)} + 729*(I*\sqrt{3} + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81* \\ & a*e^2)*e/a^{10} + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^{11} - 1/39366*(8000*b^2 \\ & *c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^{11})^{(1/3)} + 486*e/a^3)* \\ & a^4*e + 3265920*b*c*d + 236196*a*e^2)/a^7)) - 2916*(b^2*e*x^8 + 2*a*b*e*x^5 \\ & + a^2*e*x^2)*\log(x))/(a^3*b^2*x^8 + 2*a^4*b*x^5 + a^5*x^2) \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.09321, size = 390, normalized size = 1.41

$$-\frac{e \log(|bx^3 + a|)}{3a^3} + \frac{e \log(|x|)}{a^3} - \frac{\left(10(-ab^2)^{\frac{1}{3}}bc + 7(-ab^2)^{\frac{2}{3}}d\right) \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27a^4b} - \frac{28b^2dx^7 + 20b^2cx^6 - 6a^2b^2x^5 + 49a^2b^2dx^4 + 32a^2b^2c^2x^3 - 9a^2x^2e + 18a^2d^2x + 9a^2c^2}{(b^2x^4 + a^2x^2 + a^2)^{2/3}} - \frac{2/27*\sqrt{3}*(10*(-a*b^2)^{1/3}*a*b^3*c - 7*(-a*b^2)^{2/3}*a*b^2*d)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{1/3})/(-a/b)^{1/3})/(a^5*b^3) + 2/27*(7*a^3*b^2*d*(-a/b)^{1/3} + 10*a^3*b^2*c)*(-a/b)^{1/3}*\log(\text{abs}(x - (-a/b)^{1/3}))}{a^7*b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="giac")

[Out]
$$-1/3*e*\log(\text{abs}(b*x^3 + a))/a^3 + e*\log(\text{abs}(x))/a^3 - 1/27*(10*(-a*b^2)^{(1/3)}*b*c + 7*(-a*b^2)^{(2/3)}*d)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a^4*b) - 1/18*(28*b^2*d*x^7 + 20*b^2*c*x^6 - 6*a*b*x^5*e + 49*a*b*d*x^4 + 32*a*b*c*x^3 - 9*a^2*x^2*e + 18*a^2*d*x + 9*a^2*c)/((b*x^4 + a*x)^2*a^3) - 2/27*\sqrt{3}*(10*(-a*b^2)^{(1/3)}*a*b^3*c - 7*(-a*b^2)^{(2/3)}*a*b^2*d)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(a^5*b^3) + 2/27*(7*a^3*b^2*d*(-a/b)^{(1/3)} + 10*a^3*b^2*c)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/a^7*b$$

$$3.357 \quad \int \frac{c+dx+ex^2}{x^4(a+bx^3)^3} dx$$

Optimal. Leaf size=298

$$\frac{x\left(-\frac{15b^2cx^2}{a} + 11bd + 10bex\right)}{18a^3(a+bx^3)} - \frac{x\left(-\frac{b^2cx^2}{a} + bd + bex\right)}{6a^2(a+bx^3)^2} + \frac{\sqrt[3]{b}(10\sqrt[3]{bd} - 7\sqrt[3]{ae})\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{27a^{11/3}} + \frac{bc\log(a+bx^3)}{a^4}$$

[Out] $-c/(3*a^3*x^3) - d/(2*a^3*x^2) - e/(a^3*x) - (x*(b*d + b*e*x - (b^2*c*x^2)/a))/(6*a^2*(a + b*x^3)^2) - (x*(11*b*d + 10*b*e*x - (15*b^2*c*x^2)/a))/(18*a^3*(a + b*x^3)) + (2*b^(1/3)*(10*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(11/3)) - (3*b*c*Log[x])/a^4 - (2*b^(1/3)*(10*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x]/(27*a^(11/3)) + (b^(1/3)*(10*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(27*a^(11/3)) + (b*c*Log[a + b*x^3])/a^4$

Rubi [A] time = 0.589456, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{x\left(-\frac{15b^2cx^2}{a} + 11bd + 10bex\right)}{18a^3(a+bx^3)} - \frac{x\left(-\frac{b^2cx^2}{a} + bd + bex\right)}{6a^2(a+bx^3)^2} + \frac{\sqrt[3]{b}(10\sqrt[3]{bd} - 7\sqrt[3]{ae})\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{27a^{11/3}} + \frac{bc\log(a+bx^3)}{a^4}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^3), x]

[Out] $-c/(3*a^3*x^3) - d/(2*a^3*x^2) - e/(a^3*x) - (x*(b*d + b*e*x - (b^2*c*x^2)/a))/(6*a^2*(a + b*x^3)^2) - (x*(11*b*d + 10*b*e*x - (15*b^2*c*x^2)/a))/(18*a^3*(a + b*x^3)) + (2*b^(1/3)*(10*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(11/3)) - (3*b*c*Log[x])/a^4 - (2*b^(1/3)*(10*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x]/(27*a^(11/3)) + (b^(1/3)*(10*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(27*a^(11/3)) + (b*c*Log[a + b*x^3])/a^4$

Rule 1829

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^

```
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_)*(x_))^(m_))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^( -1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
```

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 204

$\text{Int}[\frac{(a_1 + (b_1)x^2)^{-1}}{x}, x_Symbol] \rightarrow -\text{Simp}[\frac{\text{ArcTan}[\frac{\text{Rt}[-b, 2]x}{\text{Rt}[-a, 2] - \text{Rt}[-a, 2]\text{Rt}[-b, 2]}}{x}], x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 628

$\text{Int}[\frac{(d_1 + (e_1)x)}{(a_1 + (b_1)x + (c_1)x^2)}, x_Symbol] \rightarrow \text{Simp}[\frac{d \cdot \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]}{b}, x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2cd - be, 0]$

Rule 260

$\text{Int}[x^{(m_1)} / ((a_1 + (b_1)x^n)), x_Symbol] \rightarrow \text{Simp}[\frac{\text{Log}[\text{RemoveContent}[a + bx^n, x]]}{bn}, x] \ /; \ \text{FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2}{x^4 (a + bx^3)^3} dx &= -\frac{x \left(bd + bex - \frac{b^2 cx^2}{a} \right)}{6a^2 (a + bx^3)^2} - \frac{\int \frac{-6bc - 6bdx - 6bex^2 + \frac{6b^2 cx^3}{a} + \frac{5b^2 dx^4}{a} + \frac{4b^2 ex^5}{a} - \frac{3b^3 cx^6}{a^2}}{x^4 (a + bx^3)^2} dx}{6ab} \\
&= -\frac{x \left(bd + bex - \frac{b^2 cx^2}{a} \right)}{6a^2 (a + bx^3)^2} - \frac{x \left(11bd + 10bex - \frac{15b^2 cx^2}{a} \right)}{18a^3 (a + bx^3)} + \frac{\int \frac{18b^3 c + 18b^3 dx + 18b^3 ex^2 - \frac{36b^4 cx^3}{a} - \frac{22b^4 dx^4}{a} - \frac{10b^4 ex^5}{a}}{x^4 (a + bx^3)}}{18a^2 b^3} \\
&= -\frac{x \left(bd + bex - \frac{b^2 cx^2}{a} \right)}{6a^2 (a + bx^3)^2} - \frac{x \left(11bd + 10bex - \frac{15b^2 cx^2}{a} \right)}{18a^3 (a + bx^3)} + \frac{\int \left(\frac{18b^3 c}{ax^4} + \frac{18b^3 d}{ax^3} + \frac{18b^3 e}{ax^2} - \frac{54b^4 c}{a^2 x} - \frac{2b^4 (20ad + 14ae)}{a^2 (a + bx^3)} \right)}{18a^2 b^3} \\
&= -\frac{c}{3a^3 x^3} - \frac{d}{2a^3 x^2} - \frac{e}{a^3 x} - \frac{x \left(bd + bex - \frac{b^2 cx^2}{a} \right)}{6a^2 (a + bx^3)^2} - \frac{x \left(11bd + 10bex - \frac{15b^2 cx^2}{a} \right)}{18a^3 (a + bx^3)} - \frac{3bc \log(x)}{a^4} - \frac{b \int \frac{20ad + 14ae}{a^2 (a + bx^3)}}{9} \\
&= -\frac{c}{3a^3 x^3} - \frac{d}{2a^3 x^2} - \frac{e}{a^3 x} - \frac{x \left(bd + bex - \frac{b^2 cx^2}{a} \right)}{6a^2 (a + bx^3)^2} - \frac{x \left(11bd + 10bex - \frac{15b^2 cx^2}{a} \right)}{18a^3 (a + bx^3)} - \frac{3bc \log(x)}{a^4} - \frac{b \int \frac{20ad + 14ae}{a^2 (a + bx^3)}}{9} \\
&= -\frac{c}{3a^3 x^3} - \frac{d}{2a^3 x^2} - \frac{e}{a^3 x} - \frac{x \left(bd + bex - \frac{b^2 cx^2}{a} \right)}{6a^2 (a + bx^3)^2} - \frac{x \left(11bd + 10bex - \frac{15b^2 cx^2}{a} \right)}{18a^3 (a + bx^3)} - \frac{3bc \log(x)}{a^4} + \frac{bc \log \left(\frac{a + bx^3}{a} \right)}{a^4} \\
&= -\frac{c}{3a^3 x^3} - \frac{d}{2a^3 x^2} - \frac{e}{a^3 x} - \frac{x \left(bd + bex - \frac{b^2 cx^2}{a} \right)}{6a^2 (a + bx^3)^2} - \frac{x \left(11bd + 10bex - \frac{15b^2 cx^2}{a} \right)}{18a^3 (a + bx^3)} - \frac{3bc \log(x)}{a^4} - \frac{2\sqrt[3]{b} (10\sqrt[3]{a} \sqrt[3]{bd} - 7a^{2/3} e)}{9\sqrt{3}a} \\
&= -\frac{c}{3a^3 x^3} - \frac{d}{2a^3 x^2} - \frac{e}{a^3 x} - \frac{x \left(bd + bex - \frac{b^2 cx^2}{a} \right)}{6a^2 (a + bx^3)^2} - \frac{x \left(11bd + 10bex - \frac{15b^2 cx^2}{a} \right)}{18a^3 (a + bx^3)} - \frac{3bc \log(x)}{a^4} - \frac{2\sqrt[3]{b} (10\sqrt[3]{a} \sqrt[3]{bd} - 7a^{2/3} e)}{9\sqrt{3}a} \\
&= -\frac{c}{3a^3 x^3} - \frac{d}{2a^3 x^2} - \frac{e}{a^3 x} - \frac{x \left(bd + bex - \frac{b^2 cx^2}{a} \right)}{6a^2 (a + bx^3)^2} - \frac{x \left(11bd + 10bex - \frac{15b^2 cx^2}{a} \right)}{18a^3 (a + bx^3)} + \frac{2\sqrt[3]{b} (10\sqrt[3]{a} \sqrt[3]{bd} + 7\sqrt[3]{a} e)}{9\sqrt{3}a}
\end{aligned}$$

Mathematica [A] time = 0.370094, size = 255, normalized size = 0.86

$$-2\sqrt[3]{b} (10\sqrt[3]{a} \sqrt[3]{bd} - 7a^{2/3} e) \log \left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2 \right) + \frac{9a^2 b(c + x(d + ex))}{(a + bx^3)^2} + 4\sqrt[3]{b} (10\sqrt[3]{a} \sqrt[3]{bd} - 7a^{2/3} e) \log \left(\sqrt[3]{a} + \sqrt[3]{bx} \right) + \frac{bc \log \left(\frac{a + bx^3}{a} \right)}{a^4}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^3), x]

[Out] $-\left(\frac{18ac}{x^3} + \frac{27ad}{x^2} + \frac{54ae}{x} + \frac{9a^2b(c + x(d + ex))}{(a + bx^3)^2} + \frac{3ab(12c + x(11d + 10ex))}{(a + bx^3)} - 4\sqrt{3}a^{1/3}b^{1/3}(10b^{1/3}d + 7a^{1/3}e)\operatorname{ArcTan}\left[\frac{1 - (2b^{1/3}x)/a^{1/3}}{\sqrt{3}}\right] + 162b^2c\operatorname{Log}[x] + 4b^{1/3}(10a^{1/3}b^{1/3}d - 7a^{2/3}e)\operatorname{Log}[a^{1/3} + b^{1/3}x] - 2b^{1/3}(10a^{1/3}b^{1/3}d - 7a^{2/3}e)\operatorname{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2] - 54b^2c\operatorname{Log}[a + bx^3]\right)/(54a^4)$

Maple [A] time = 0.017, size = 351, normalized size = 1.2

$$\frac{5x^5eb^2}{9a^3(bx^3+a)^2} - \frac{11x^4b^2d}{18a^3(bx^3+a)^2} - \frac{2b^2cx^3}{3a^3(bx^3+a)^2} - \frac{13bex^2}{18a^2(bx^3+a)^2} - \frac{7bdx}{9a^2(bx^3+a)^2} - \frac{5bc}{6a^2(bx^3+a)^2} - \frac{20d}{27a^3} \ln$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)/x^4/(b*x^3+a)^3, x)

[Out] $-5/9/a^3/(bx^3+a)^2x^5e*b^2 - 11/18/a^3/(bx^3+a)^2x^4b^2*d - 2/3/a^3*b^2/(bx^3+a)^2*c*x^3 - 13/18/a^2/(bx^3+a)^2*x^2*b*e - 7/9/a^2/(bx^3+a)^2*b*x*d - 5/6/a^2*b/(bx^3+a)^2*c - 20/27/a^3/(1/b*a)^{2/3}*\ln(x+(1/b*a)^{1/3})*d + 10/27/a^3/(1/b*a)^{2/3}*\ln(x^2-(1/b*a)^{1/3}*x+(1/b*a)^{2/3})*d - 20/27/a^3/(1/b*a)^{2/3}*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1))*d + 14/27/a^3*e/(1/b*a)^{1/3}*\ln(x+(1/b*a)^{1/3}) - 7/27/a^3*e/(1/b*a)^{1/3}*\ln(x^2-(1/b*a)^{1/3})*x + (1/b*a)^{2/3} - 14/27/a^3*e*3^{1/2}/(1/b*a)^{1/3}*\arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1)) + b*c*\ln(bx^3+a)/a^4 - 1/3*c/a^3/x^3 - 1/2*d/a^3/x^2 - e/a^3/x - 3*b*c*\ln(x)/a^4$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^3, x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [C] time = 12.2192, size = 14256, normalized size = 47.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="fricas")

[Out]
$$-1/108*(168*a*b^2*e*x^8 + 120*a*b^2*d*x^7 + 108*a*b^2*c*x^6 + 294*a^2*b*e*x^5 + 192*a^2*b*d*x^4 + 162*a^2*b*c*x^3 + 108*a^3*e*x^2 + 54*a^3*d*x + 36*a^3*c + 2*(a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)*(2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^{12} + 8*(1000*b*d^3 + 343*a*e^3)*b/a^{11} - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^{12} + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^{12})^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(39366*b^3*c^3/a^{12} + 8*(1000*b*d^3 + 343*a*e^3)*b/a^{11} - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^{12} + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^{12})^{(1/3)} - 54*b*c/a^4)*\log(7/4*(2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^{12} + 8*(1000*b*d^3 + 343*a*e^3)*b/a^{11} - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^{12} + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^{12})^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(39366*b^3*c^3/a^{12} + 8*(1000*b*d^3 + 343*a*e^3)*b/a^{11} - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^{12} + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^{12})^{(1/3)} - 54*b*c/a^4)^2*a^8*e + 5400*b^2*c*d^2 + 5103*b^2*c^2*e + 3920*a*b*d*e^2 + (100*a^4*b*d^2 + 189*a^4*b*c*e)*(2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^{12} + 8*(1000*b*d^3 + 343*a*e^3)*b/a^{11} - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^{12} + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^{12})^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(39366*b^3*c^3/a^{12} + 8*(1000*b*d^3 + 343*a*e^3)*b/a^{11} - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^{12} + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^{12})^{(1/3)} - 54*b*c/a^4) + 4*(1000*b^2*d^3 + 343*a*b*e^3)*x) - (162*b^3*c*x^9 + 324*a*b^2*c*x^6 + 162*a^2*b*c*x^3 + (a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)*(2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^{12} + 8*(1000*b*d^3 + 343*a*e^3)*b/a^{11} - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^{12} + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^{12})^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(39366*b^3*c^3/a^{12} + 8*(1000*b*d^3 + 343*a*e^3)*b/a^{11} - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^{12} + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^{12})^{(1/3)} - 54*b*c/a^4) + 3*\sqrt{1/3}*(a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)*\sqrt{-((2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^{12} + 8*(1000*b*d^3 + 343*a*e^3)*b/a^{11} - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^{12} + (19683*b^3*c^3 + 274$$

$$\begin{aligned}
& 4*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2/a^{12})^{(1/3)} + (1/2)^{(1/3)}*(I* \\
& \text{sqrt}(3) + 1)*(39366*b^3*c^3/a^{12} + 8*(1000*b*d^3 + 343*a*e^3)*b/a^{11} - 81*(\\
& 729*b^2*c^2 + 280*a*b*d*e)*b*c/a^{12} + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40* \\
& (200*d^3 - 567*c*d*e)*a*b^2/a^{12})^{(1/3)} - 54*b*c/a^4)^2*a^8 + 108*(2*(1/2) \\
& ^{(2/3)}*(-I*\text{sqrt}(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8) \\
& /(39366*b^3*c^3/a^{12} + 8*(1000*b*d^3 + 343*a*e^3)*b/a^{11} - 81*(729*b^2*c^2 \\
& + 280*a*b*d*e)*b*c/a^{12} + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 5 \\
& 67*c*d*e)*a*b^2/a^{12})^{(1/3)} + (1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*(39366*b^3*c^3/a \\
& ^{12} + 8*(1000*b*d^3 + 343*a*e^3)*b/a^{11} - 81*(729*b^2*c^2 + 280*a*b*d*e)*b* \\
& c/a^{12} + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/ \\
& a^{12})^{(1/3)} - 54*b*c/a^4)*a^4*b*c + 2916*b^2*c^2 + 4480*a*b*d*e)/a^8))*\log(\\
& -7/4*(2*(1/2)^{(2/3)}*(-I*\text{sqrt}(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280* \\
& a*b*d*e)/a^8)/(39366*b^3*c^3/a^{12} + 8*(1000*b*d^3 + 343*a*e^3)*b/a^{11} - 81* \\
& (729*b^2*c^2 + 280*a*b*d*e)*b*c/a^{12} + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40 \\
& *(200*d^3 - 567*c*d*e)*a*b^2/a^{12})^{(1/3)} + (1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*(39 \\
& 366*b^3*c^3/a^{12} + 8*(1000*b*d^3 + 343*a*e^3)*b/a^{11} - 81*(729*b^2*c^2 + 28 \\
& 0*a*b*d*e)*b*c/a^{12} + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c \\
& *d*e)*a*b^2/a^{12})^{(1/3)} - 54*b*c/a^4)^2*a^8*e - 5400*b^2*c*d^2 - 5103*b^2* \\
& c^2*e - 3920*a*b*d*e^2 - (100*a^4*b*d^2 + 189*a^4*b*c*e)*(2*(1/2)^{(2/3)}*(-I \\
& *\text{sqrt}(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^ \\
& 3*c^3/a^{12} + 8*(1000*b*d^3 + 343*a*e^3)*b/a^{11} - 81*(729*b^2*c^2 + 280*a*b \\
& *d*e)*b*c/a^{12} + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)* \\
& a*b^2/a^{12})^{(1/3)} + (1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*(39366*b^3*c^3/a^{12} + 8*(1 \\
& 000*b*d^3 + 343*a*e^3)*b/a^{11} - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^{12} + (\\
& 19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2/a^{12})^{(1/3} \\
&) - 54*b*c/a^4) + 8*(1000*b^2*d^3 + 343*a*b*e^3)*x + 3/4*\text{sqrt}(1/3)*(7*(2*(1 \\
& /2)^{(2/3)}*(-I*\text{sqrt}(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a \\
& ^8)/(39366*b^3*c^3/a^{12} + 8*(1000*b*d^3 + 343*a*e^3)*b/a^{11} - 81*(729*b^2*c \\
& ^2 + 280*a*b*d*e)*b*c/a^{12} + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 \\
& - 567*c*d*e)*a*b^2/a^{12})^{(1/3)} + (1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*(39366*b^3*c^ \\
& 3/a^{12} + 8*(1000*b*d^3 + 343*a*e^3)*b/a^{11} - 81*(729*b^2*c^2 + 280*a*b*d*e) \\
& *b*c/a^{12} + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^ \\
& 2)/a^{12})^{(1/3)} - 54*b*c/a^4)*a^8*e - 400*a^4*b*d^2 + 378*a^4*b*c*e)*\text{sqrt}(- \\
& (2*(1/2)^{(2/3)}*(-I*\text{sqrt}(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d \\
& *e)/a^8)/(39366*b^3*c^3/a^{12} + 8*(1000*b*d^3 + 343*a*e^3)*b/a^{11} - 81*(729* \\
& b^2*c^2 + 280*a*b*d*e)*b*c/a^{12} + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200 \\
& *d^3 - 567*c*d*e)*a*b^2/a^{12})^{(1/3)} + (1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*(39366*b \\
& ^3*c^3/a^{12} + 8*(1000*b*d^3 + 343*a*e^3)*b/a^{11} - 81*(729*b^2*c^2 + 280*a*b \\
& *d*e)*b*c/a^{12} + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e) \\
& *a*b^2/a^{12})^{(1/3)} - 54*b*c/a^4)^2*a^8 + 108*(2*(1/2)^{(2/3)}*(-I*\text{sqrt}(3) + \\
& 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^{12} \\
& + 8*(1000*b*d^3 + 343*a*e^3)*b/a^{11} - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^ \\
& ^{12} + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2/a^{12} \\
&)^{(1/3)} + (1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*(39366*b^3*c^3/a^{12} + 8*(1000*b*d^3 + \\
& 343*a*e^3)*b/a^{11} - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^{12} + (19683*b^3*c
\end{aligned}$$

$$\begin{aligned}
&^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^{12})^{(1/3)} - 54*b*c/ \\
&a^4)*a^4*b*c + 2916*b^2*c^2 + 4480*a*b*d*e)/a^8)) - (162*b^3*c*x^9 + 324*a* \\
&b^2*c*x^6 + 162*a^2*b*c*x^3 + (a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)*(2*(1/2 \\
&)^{(2/3)}*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8 \\
&)/(39366*b^3*c^3/a^{12} + 8*(1000*b*d^3 + 343*a*e^3)*b/a^{11} - 81*(729*b^2*c^2 \\
& + 280*a*b*d*e)*b*c/a^{12} + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - \\
&567*c*d*e)*a*b^2)/a^{12})^{(1/3)} + (1/2)^{(1/3)}*(I*sqrt(3) + 1)*(39366*b^3*c^3/ \\
&a^{12} + 8*(1000*b*d^3 + 343*a*e^3)*b/a^{11} - 81*(729*b^2*c^2 + 280*a*b*d*e)*b \\
&*c/a^{12} + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2) \\
&/a^{12})^{(1/3)} - 54*b*c/a^4) - 3*sqrt(1/3)*(a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x \\
&^3)*sqrt(-((2*(1/2)^{(2/3)}*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 \\
& + 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^{12} + 8*(1000*b*d^3 + 343*a*e^3)*b/a^{11} \\
&- 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^{12} + (19683*b^3*c^3 + 2744*a^2*b*e^ \\
&3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^{12})^{(1/3)} + (1/2)^{(1/3)}*(I*sqrt(3) + \\
&1)*(39366*b^3*c^3/a^{12} + 8*(1000*b*d^3 + 343*a*e^3)*b/a^{11} - 81*(729*b^2*c^ \\
&2 + 280*a*b*d*e)*b*c/a^{12} + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - \\
&567*c*d*e)*a*b^2)/a^{12})^{(1/3)} - 54*b*c/a^4)^2*a^8 + 108*(2*(1/2)^{(2/3)}*(-I \\
&*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^ \\
&3*c^3/a^{12} + 8*(1000*b*d^3 + 343*a*e^3)*b/a^{11} - 81*(729*b^2*c^2 + 280*a*b* \\
&d*e)*b*c/a^{12} + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)* \\
&a*b^2)/a^{12})^{(1/3)} + (1/2)^{(1/3)}*(I*sqrt(3) + 1)*(39366*b^3*c^3/a^{12} + 8*(1 \\
&000*b*d^3 + 343*a*e^3)*b/a^{11} - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^{12} + (\\
&19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^{12})^{(1/3} \\
&)) - 54*b*c/a^4)*a^4*b*c + 2916*b^2*c^2 + 4480*a*b*d*e)/a^8))*log(-7/4*(2*(1 \\
&/2)^{(2/3)}*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a \\
&^8)/(39366*b^3*c^3/a^{12} + 8*(1000*b*d^3 + 343*a*e^3)*b/a^{11} - 81*(729*b^2*c \\
&^2 + 280*a*b*d*e)*b*c/a^{12} + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 \\
&- 567*c*d*e)*a*b^2)/a^{12})^{(1/3)} + (1/2)^{(1/3)}*(I*sqrt(3) + 1)*(39366*b^3*c^ \\
&3/a^{12} + 8*(1000*b*d^3 + 343*a*e^3)*b/a^{11} - 81*(729*b^2*c^2 + 280*a*b*d*e) \\
&*b*c/a^{12} + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^ \\
&2)/a^{12})^{(1/3)} - 54*b*c/a^4)^2*a^8*e - 5400*b^2*c*d^2 - 5103*b^2*c^2*e - 39 \\
&20*a*b*d*e^2 - (100*a^4*b*d^2 + 189*a^4*b*c*e)*(2*(1/2)^{(2/3)}*(-I*sqrt(3) + \\
&1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^{12} \\
& + 8*(1000*b*d^3 + 343*a*e^3)*b/a^{11} - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a \\
&^{12} + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^1 \\
&2)^{(1/3)} + (1/2)^{(1/3)}*(I*sqrt(3) + 1)*(39366*b^3*c^3/a^{12} + 8*(1000*b*d^3 \\
& + 343*a*e^3)*b/a^{11} - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^{12} + (19683*b^3* \\
&c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^{12})^{(1/3)} - 54*b*c \\
&/a^4) + 8*(1000*b^2*d^3 + 343*a*b*e^3)*x - 3/4*sqrt(1/3)*(7*(2*(1/2)^{(2/3)}* \\
&(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366 \\
&*b^3*c^3/a^{12} + 8*(1000*b*d^3 + 343*a*e^3)*b/a^{11} - 81*(729*b^2*c^2 + 280*a \\
&*b*d*e)*b*c/a^{12} + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d* \\
&e)*a*b^2)/a^{12})^{(1/3)} + (1/2)^{(1/3)}*(I*sqrt(3) + 1)*(39366*b^3*c^3/a^{12} + 8 \\
&*(1000*b*d^3 + 343*a*e^3)*b/a^{11} - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^{12} \\
& + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^{12})^{(
\end{aligned}$$

$$\frac{1}{3} - 54bc/a^4 * a^8 e - 400a^4 b d^2 + 378a^4 b c e) * \sqrt{-((2*(1/2)^{(2/3)} * (-I*\sqrt{3} + 1) * (729b^2 c^2/a^8 - (729b^2 c^2 + 280a b d e)/a^8) / (39366b^3 c^3/a^{12} + 8*(1000b d^3 + 343a e^3) * b/a^{11} - 81*(729b^2 c^2 + 280a b d e) * b c/a^{12} + (19683b^3 c^3 + 2744a^2 b e^3 - 40*(200d^3 - 567 * c d e) * a b^2)/a^{12})^{(1/3)} + (1/2)^{(1/3)} * (I*\sqrt{3} + 1) * (39366b^3 c^3/a^{12} + 8*(1000b d^3 + 343a e^3) * b/a^{11} - 81*(729b^2 c^2 + 280a b d e) * b c/a^{12} + (19683b^3 c^3 + 2744a^2 b e^3 - 40*(200d^3 - 567 * c d e) * a b^2)/a^{12})^{(1/3)} - 54bc/a^4)^2 a^8 + 108*(2*(1/2)^{(2/3)} * (-I*\sqrt{3} + 1) * (729b^2 c^2/a^8 - (729b^2 c^2 + 280a b d e)/a^8) / (39366b^3 c^3/a^{12} + 8*(1000b d^3 + 343a e^3) * b/a^{11} - 81*(729b^2 c^2 + 280a b d e) * b c/a^{12} + (19683b^3 c^3 + 2744a^2 b e^3 - 40*(200d^3 - 567 * c d e) * a b^2)/a^{12})^{(1/3)} + (1/2)^{(1/3)} * (I*\sqrt{3} + 1) * (39366b^3 c^3/a^{12} + 8*(1000b d^3 + 343a e^3) * b/a^{11} - 81*(729b^2 c^2 + 280a b d e) * b c/a^{12} + (19683b^3 c^3 + 2744a^2 b e^3 - 40*(200d^3 - 567 * c d e) * a b^2)/a^{12})^{(1/3)} - 54bc/a^4) * a^4 b c + 2916b^2 c^2 + 4480a b d e) / a^8)) + 324*(b^3 c x^9 + 2a b^2 c x^6 + a^2 b c x^3) * \log(x)) / (a^4 b^2 x^9 + 2a^5 b x^6 + a^6 x^3)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/x**4/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.08487, size = 421, normalized size = 1.41

$$\frac{bc \log(|bx^3 + a|)}{a^4} - \frac{3bc \log(|x|)}{a^4} - \frac{\left(10(-ab^2)^{\frac{1}{3}}bd + 7(-ab^2)^{\frac{2}{3}}e\right) \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27a^4b} - \frac{2\sqrt{3}\left(10(-ab^2)^{\frac{1}{3}}ab^3\right)}{27a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="giac")

```
[Out] b*c*log(abs(b*x^3 + a))/a^4 - 3*b*c*log(abs(x))/a^4 - 1/27*(10*(-a*b^2)^(1/3)*b*d + 7*(-a*b^2)^(2/3)*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b) - 2/27*sqrt(3)*(10*(-a*b^2)^(1/3)*a*b^3*d - 7*(-a*b^2)^(2/3)*a*b^2*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^5*b^3) + 2/27*(7*a^5*b^2*(-a/b)^(1/3)*e + 10*a^5*b^2*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^9*b) - 1/18*(28*a*b^2*x^8*e + 20*a*b^2*d*x^7 + 18*a*b^2*c*x^6 + 49*a^2*b*x^5*e + 32*a^2*b*d*x^4 + 27*a^2*b*c*x^3 + 18*a^3*x^2*e + 9*a^3*d*x + 6*a^3*c)/((b*x^3 + a)^2*a^4*x^3)
```

$$3.358 \quad \int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^4} dx$$

Optimal. Leaf size=248

$$\frac{(5\sqrt[3]{bd} - 4\sqrt[3]{ae}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{486a^{8/3}b^{5/3}} + \frac{(5\sqrt[3]{bd} - 4\sqrt[3]{ae}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{243a^{8/3}b^{5/3}} - \frac{(4\sqrt[3]{ae} + 5\sqrt[3]{bd}) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{8/3}b^{5/3}} +$$

[Out] $-(c + d*x + e*x^2)/(9*b*(a + b*x^3)^3) + (x*(d + 2*e*x))/(54*a*b*(a + b*x^3)^2) + (x*(5*d + 8*e*x))/(162*a^2*b*(a + b*x^3)) - ((5*b^{(1/3)}*d + 4*a^{(1/3)}*e)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(81*Sqrt[3]*a^{(8/3)}*b^{(5/3)}) + ((5*b^{(1/3)}*d - 4*a^{(1/3)}*e)*Log[a^{(1/3)} + b^{(1/3)}*x])/(243*a^{(8/3)}*b^{(5/3)}) - ((5*b^{(1/3)}*d - 4*a^{(1/3)}*e)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(486*a^{(8/3)}*b^{(5/3)})$

Rubi [A] time = 0.242314, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {1823, 1855, 1860, 31, 634, 617, 204, 628}

$$\frac{(5\sqrt[3]{bd} - 4\sqrt[3]{ae}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{486a^{8/3}b^{5/3}} + \frac{(5\sqrt[3]{bd} - 4\sqrt[3]{ae}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{243a^{8/3}b^{5/3}} - \frac{(4\sqrt[3]{ae} + 5\sqrt[3]{bd}) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{8/3}b^{5/3}} +$$

Antiderivative was successfully verified.

[In] Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^4, x]

[Out] $-(c + d*x + e*x^2)/(9*b*(a + b*x^3)^3) + (x*(d + 2*e*x))/(54*a*b*(a + b*x^3)^2) + (x*(5*d + 8*e*x))/(162*a^2*b*(a + b*x^3)) - ((5*b^{(1/3)}*d + 4*a^{(1/3)}*e)*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(81*Sqrt[3]*a^{(8/3)}*b^{(5/3)}) + ((5*b^{(1/3)}*d - 4*a^{(1/3)}*e)*Log[a^{(1/3)} + b^{(1/3)}*x])/(243*a^{(8/3)}*b^{(5/3)}) - ((5*b^{(1/3)}*d - 4*a^{(1/3)}*e)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(486*a^{(8/3)}*b^{(5/3)})$

Rule 1823

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Pq*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq

$Q[m - n + 1, 0] \ \&\& \ \text{LtQ}[p, -1]$

Rule 1855

$\text{Int}[(Pq_)*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow -\text{Simp}[(x*Pq*(a + b*x^n)^{(p + 1)})/(a*n*(p + 1)), x] + \text{Dist}[1/(a*n*(p + 1)), \text{Int}[\text{ExpandToSum}[n*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^{(p + 1)}, x], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{LtQ}[\text{Expon}[Pq, x], n - 1]$

Rule 1860

$\text{Int}[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] \rightarrow \text{With}[\{r = \text{Numerator}[\text{Rt}[a/b, 3]], s = \text{Denominator}[\text{Rt}[a/b, 3]]\}, -\text{Dist}[(r*(B*r - A*s))/(3*a*s), \text{Int}[1/(r + s*x), x], x] + \text{Dist}[r/(3*a*s), \text{Int}[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; \text{FreeQ}[\{a, b, A, B\}, x] \ \&\& \ \text{NeQ}[a*B^3 - b*A^3, 0] \ \&\& \ \text{PosQ}[a/b]$

Rule 31

$\text{Int}[((a_) + (b_)*(x_))^{(-1)}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}[\{a, b\}, x]$

Rule 634

$\text{Int}[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] \rightarrow \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rule 617

$\text{Int}[((a_) + (b_)*(x_) + (c_)*(x_)^2)^{(-1)}, x_Symbol] \rightarrow \text{With}[\{q = 1 - 4*S\text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ \|\ \text{!RationalQ}[b^2 - 4*a*c])] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 204

$\text{Int}[((a_) + (b_)*(x_)^2)^{(-1)}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ \|\ \text{LtQ}[b, 0])$

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^4} dx &= -\frac{c+dx+ex^2}{9b(a+bx^3)^3} + \frac{\int \frac{d+2ex}{(a+bx^3)^3} dx}{9b} \\
&= -\frac{c+dx+ex^2}{9b(a+bx^3)^3} + \frac{x(d+2ex)}{54ab(a+bx^3)^2} - \frac{\int \frac{-5d-8ex}{(a+bx^3)^2} dx}{54ab} \\
&= -\frac{c+dx+ex^2}{9b(a+bx^3)^3} + \frac{x(d+2ex)}{54ab(a+bx^3)^2} + \frac{x(5d+8ex)}{162a^2b(a+bx^3)} + \frac{\int \frac{10d+8ex}{a+bx^3} dx}{162a^2b} \\
&= -\frac{c+dx+ex^2}{9b(a+bx^3)^3} + \frac{x(d+2ex)}{54ab(a+bx^3)^2} + \frac{x(5d+8ex)}{162a^2b(a+bx^3)} + \frac{\int \frac{\sqrt[3]{a}(20\sqrt[3]{bd}+8\sqrt[3]{ae})+\sqrt[3]{b}(-10\sqrt[3]{bd}+8\sqrt[3]{ae})x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2} dx}{486a^{8/3}b^{4/3}} \\
&= -\frac{c+dx+ex^2}{9b(a+bx^3)^3} + \frac{x(d+2ex)}{54ab(a+bx^3)^2} + \frac{x(5d+8ex)}{162a^2b(a+bx^3)} + \frac{(5\sqrt[3]{bd}-4\sqrt[3]{ae})\log(\sqrt[3]{a}+\sqrt[3]{bx})}{243a^{8/3}b^{5/3}} \\
&= -\frac{c+dx+ex^2}{9b(a+bx^3)^3} + \frac{x(d+2ex)}{54ab(a+bx^3)^2} + \frac{x(5d+8ex)}{162a^2b(a+bx^3)} + \frac{(5\sqrt[3]{bd}-4\sqrt[3]{ae})\log(\sqrt[3]{a}+\sqrt[3]{bx})}{243a^{8/3}b^{5/3}} \\
&= -\frac{c+dx+ex^2}{9b(a+bx^3)^3} + \frac{x(d+2ex)}{54ab(a+bx^3)^2} + \frac{x(5d+8ex)}{162a^2b(a+bx^3)} - \frac{(5\sqrt[3]{bd}+4\sqrt[3]{ae})\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{8/3}b^{5/3}}
\end{aligned}$$

Mathematica [A] time = 0.232971, size = 230, normalized size = 0.93

$$\frac{\frac{3b^{2/3}x(5d+8ex)}{a^2(a+bx^3)} + \frac{(4\sqrt[3]{ae}-5\sqrt[3]{bd})\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2)}{a^{8/3}} + \frac{2(5\sqrt[3]{bd}-4\sqrt[3]{ae})\log(\sqrt[3]{a}+\sqrt[3]{bx})}{a^{8/3}} - \frac{2\sqrt{3}(4\sqrt[3]{ae}+5\sqrt[3]{bd})\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{8/3}} - \frac{54b^{2/3}(c+x(d+ex))}{(a+bx^3)^3}}{486b^{5/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^4,x]

[Out] ((9*b^(2/3)*x*(d + 2*e*x))/(a*(a + b*x^3)^2) + (3*b^(2/3)*x*(5*d + 8*e*x))/(a^2*(a + b*x^3)) - (54*b^(2/3)*(c + x*(d + e*x)))/(a + b*x^3)^3 - (2*sqrt[3]*(5*b^(1/3)*d + 4*a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]])/a^(8/3) + (2*(5*b^(1/3)*d - 4*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/a^(8/3) + ((-5*b^(1/3)*d + 4*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(8/3))/(486*b^(5/3))

Maple [A] time = 0.01, size = 275, normalized size = 1.1

$$\frac{1}{(bx^3 + a)^3} \left(\frac{4bex^8}{81a^2} + \frac{5bdx^7}{162a^2} + \frac{11ex^5}{81a} + \frac{13dx^4}{162a} - \frac{2ex^2}{81b} - \frac{5dx}{81b} - \frac{c}{9b} \right) + \frac{5d}{243b^2a^2} \ln \left(x + \sqrt[3]{\frac{a}{b}} \right) \left(\frac{a}{b} \right)^{-\frac{2}{3}} - \frac{5d}{486b^2a^2} \ln \left(x^2 - \left(\frac{a}{b} \right)^{\frac{1}{3}} x + \left(\frac{a}{b} \right)^{\frac{2}{3}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^4,x)

[Out] (4/81/a^2*b*e*x^8+5/162/a^2*d*b*x^7+11/81/a*e*x^5+13/162*d/a*x^4-2/81*e*x^2/b-5/81*d*x/b-1/9*c/b)/(b*x^3+a)^3+5/243/b^2/a^2*d/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-5/486/b^2/a^2*d/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+5/243/b^2/a^2*d/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-4/243/b^2/a^2*e/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+2/243/b^2/a^2*e/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+4/243/b^2/a^2*e*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

$$\begin{aligned} & \left(\frac{1}{2} \right)^{1/3} (I\sqrt{3} + 1) \left(\frac{125bd^3 + 64ae^3}{a^8b^5} + \frac{125bd^3 - 64ae^3}{a^8b^5} \right)^{1/3} - 40 \left(\frac{1}{2} \right)^{2/3} d e (-I\sqrt{3} + 1) / (a^5b^3) \\ & * \left(\frac{125bd^3 + 64ae^3}{a^8b^5} + \frac{125bd^3 - 64ae^3}{a^8b^5} \right)^{1/3} - 3\sqrt{1/3} (a^2b^4x^9 + 3a^3b^3x^6 + 3a^4b^2x^3 + a^5b) \sqrt{-\left(\left(\frac{1}{2} \right)^{1/3} (I\sqrt{3} + 1) \left(\frac{125bd^3 + 64ae^3}{a^8b^5} + \frac{125bd^3 - 64ae^3}{a^8b^5} \right)^{1/3} - 40 \left(\frac{1}{2} \right)^{2/3} d e (-I\sqrt{3} + 1) / (a^5b^3) \left(\frac{125bd^3 + 64ae^3}{a^8b^5} + \frac{125bd^3 - 64ae^3}{a^8b^5} \right)^{1/3} \right)^2 a^5b^3 + 320d e} / (a^5b^3)) * \log\left(-\left(\frac{1}{2} \right)^{1/3} (I\sqrt{3} + 1) \left(\frac{125bd^3 + 64ae^3}{a^8b^5} + \frac{125bd^3 - 64ae^3}{a^8b^5} \right)^{1/3} - 40 \left(\frac{1}{2} \right)^{2/3} d e (-I\sqrt{3} + 1) / (a^5b^3) \left(\frac{125bd^3 + 64ae^3}{a^8b^5} + \frac{125bd^3 - 64ae^3}{a^8b^5} \right)^{1/3} \right)^2 a^6b^3 e + 25/2 \left(\frac{1}{2} \right)^{1/3} (I\sqrt{3} + 1) \left(\frac{125bd^3 + 64ae^3}{a^8b^5} + \frac{125bd^3 - 64ae^3}{a^8b^5} \right)^{1/3} - 40 \left(\frac{1}{2} \right)^{2/3} d e (-I\sqrt{3} + 1) / (a^5b^3) \left(\frac{125bd^3 + 64ae^3}{a^8b^5} + \frac{125bd^3 - 64ae^3}{a^8b^5} \right)^{1/3} \right) \\ & * a^3b^2d^2 - 160a d e^2 + 2(125bd^3 + 64ae^3)x - 3/2\sqrt{1/3} \left(\left(\frac{1}{2} \right)^{1/3} (I\sqrt{3} + 1) \left(\frac{125bd^3 + 64ae^3}{a^8b^5} + \frac{125bd^3 - 64ae^3}{a^8b^5} \right)^{1/3} - 40 \left(\frac{1}{2} \right)^{2/3} d e (-I\sqrt{3} + 1) / (a^5b^3) \left(\frac{125bd^3 + 64ae^3}{a^8b^5} + \frac{125bd^3 - 64ae^3}{a^8b^5} \right)^{1/3} \right) \\ & * a^6b^3 e + 25a^3b^2d^2) \sqrt{-\left(\left(\frac{1}{2} \right)^{1/3} (I\sqrt{3} + 1) \left(\frac{125bd^3 + 64ae^3}{a^8b^5} + \frac{125bd^3 - 64ae^3}{a^8b^5} \right)^{1/3} - 40 \left(\frac{1}{2} \right)^{2/3} d e (-I\sqrt{3} + 1) / (a^5b^3) \left(\frac{125bd^3 + 64ae^3}{a^8b^5} + \frac{125bd^3 - 64ae^3}{a^8b^5} \right)^{1/3} \right)^2 a^5b^3 + 320d e} / (a^5b^3) \\ & \left. \right) / (a^2b^4x^9 + 3a^3b^3x^6 + 3a^4b^2x^3 + a^5b) \end{aligned}$$

Sympy [A] time = 13.5946, size = 201, normalized size = 0.81

$$\text{RootSum}\left(14348907t^3a^8b^5 + 14580ta^3b^2de + 64ae^3 - 125bd^3, \left(t \mapsto t \log\left(x + \frac{236196t^2a^6b^3e + 6075ta^3b^2d^2 + 160ade^2}{64ae^3 + 125bd^3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**4,x)

[Out] RootSum(14348907*_t**3*a**8*b**5 + 14580*_t*a**3*b**2*d*e + 64*a*e**3 - 125*b*d**3, Lambda(_t, _t*log(x + (236196*_t**2*a**6*b**3*e + 6075*_t*a**3*b**2*d**2 + 160*a*d*e**2)/(64*a*e**3 + 125*b*d**3)))) + (-18*a**2*c - 10*a**2*d*x - 4*a**2*e*x**2 + 13*a*b*d*x**4 + 22*a*b*e*x**5 + 5*b**2*d*x**7 + 8*b**2*e*x**8)/(162*a**5*b + 486*a**4*b**2*x**3 + 486*a**3*b**3*x**6 + 162*a**2*b**4*x**9)

Giac [A] time = 1.07474, size = 333, normalized size = 1.34

$$\frac{\left(4\left(-\frac{a}{b}\right)^{\frac{1}{3}}e + 5d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243a^3b} + \frac{\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}bd - 4\left(-ab^2\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243a^3b^3} + \frac{8b^2x^8e + 5b^2x^7e + 22abx^5e + 13a^2bx^4e - 4a^2x^2e - 10a^2dx - 18a^2c}{(bx^3 + a)^3a^2b} + \frac{1}{486}\left(5\left(-ab^2\right)^{\frac{1}{3}}a^2b^2d + 4\left(-ab^2\right)^{\frac{2}{3}}a^2b^2e\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)/(a^4b^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="giac")

[Out] -1/243*(4*(-a/b)^(1/3)*e + 5*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b) + 1/243*sqrt(3)*(5*(-a*b^2)^(1/3)*b*d - 4*(-a*b^2)^(2/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^3) + 1/162*(8*b^2*x^8*e + 5*b^2*d*x^7 + 22*a*b*x^5*e + 13*a*b*d*x^4 - 4*a^2*x^2*e - 10*a^2*d*x - 18*a^2*c)/((b*x^3 + a)^3*a^2*b) + 1/486*(5*(-a*b^2)^(1/3)*a^2*b^2*d + 4*(-a*b^2)^(2/3)*a^2*b^2*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b^4)

$$3.359 \quad \int \frac{x(c+dx+ex^2)}{(a+bx^3)^4} dx$$

Optimal. Leaf size=270

$$\frac{(14b^{2/3}c - 5a^{2/3}e) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{486a^{10/3}b^{4/3}} - \frac{(14b^{2/3}c - 5a^{2/3}e) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{243a^{10/3}b^{4/3}} - \frac{(5a^{2/3}e + 14b^{2/3}c) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{10/3}b^{4/3}}$$

[Out] $-(x*(a*e - b*c*x - b*d*x^2))/(9*a*b*(a + b*x^3)^3) + (x*(5*a*e + 28*b*c*x))/(162*a^3*b*(a + b*x^3)) - (6*a*d - x*(a*e + 7*b*c*x))/(54*a^2*b*(a + b*x^3)^2) - ((14*b^(2/3)*c + 5*a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(81*Sqrt[3]*a^(10/3)*b^(4/3)) - ((14*b^(2/3)*c - 5*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x]/(243*a^(10/3)*b^(4/3)) + ((14*b^(2/3)*c - 5*a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(486*a^(10/3)*b^(4/3))$

Rubi [A] time = 0.25355, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1828, 1854, 1855, 1860, 31, 634, 617, 204, 628}

$$\frac{(14b^{2/3}c - 5a^{2/3}e) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{486a^{10/3}b^{4/3}} - \frac{(14b^{2/3}c - 5a^{2/3}e) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{243a^{10/3}b^{4/3}} - \frac{(5a^{2/3}e + 14b^{2/3}c) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{10/3}b^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[(x*(c + d*x + e*x^2))/(a + b*x^3)^4, x]

[Out] $-(x*(a*e - b*c*x - b*d*x^2))/(9*a*b*(a + b*x^3)^3) + (x*(5*a*e + 28*b*c*x))/(162*a^3*b*(a + b*x^3)) - (6*a*d - x*(a*e + 7*b*c*x))/(54*a^2*b*(a + b*x^3)^2) - ((14*b^(2/3)*c + 5*a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(81*Sqrt[3]*a^(10/3)*b^(4/3)) - ((14*b^(2/3)*c - 5*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x]/(243*a^(10/3)*b^(4/3)) + ((14*b^(2/3)*c - 5*a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(486*a^(10/3)*b^(4/3))$

Rule 1828

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)

```
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
+ b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq,
x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1})*(a + b*x^n)^(p
+ 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
+ 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(p_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(c + dx + ex^2)}{(a + bx^3)^4} dx &= -\frac{x(ae - bcx - bdx^2)}{9ab(a + bx^3)^3} - \frac{\int \frac{-ae - 7bcx - 6bdx^2}{(a + bx^3)^3} dx}{9ab} \\
&= -\frac{x(ae - bcx - bdx^2)}{9ab(a + bx^3)^3} - \frac{6ad - x(ae + 7bcx)}{54a^2b(a + bx^3)^2} + \frac{\int \frac{5ae + 28bcx}{(a + bx^3)^2} dx}{54a^2b} \\
&= -\frac{x(ae - bcx - bdx^2)}{9ab(a + bx^3)^3} + \frac{x(5ae + 28bcx)}{162a^3b(a + bx^3)} - \frac{6ad - x(ae + 7bcx)}{54a^2b(a + bx^3)^2} - \frac{\int \frac{-10ae - 28bcx}{a + bx^3} dx}{162a^3b} \\
&= -\frac{x(ae - bcx - bdx^2)}{9ab(a + bx^3)^3} + \frac{x(5ae + 28bcx)}{162a^3b(a + bx^3)} - \frac{6ad - x(ae + 7bcx)}{54a^2b(a + bx^3)^2} - \frac{\int \frac{\sqrt[3]{a}(-28\sqrt[3]{abc} - 20a\sqrt[3]{be}) + \sqrt[3]{b}(-28a^{2/3} - 20\sqrt[3]{a}\sqrt[3]{bx + b^{2/3}x^2})}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx + b^{2/3}x^2}} dx}{486a^{11/3}b^{4/3}} \\
&= -\frac{x(ae - bcx - bdx^2)}{9ab(a + bx^3)^3} + \frac{x(5ae + 28bcx)}{162a^3b(a + bx^3)} - \frac{6ad - x(ae + 7bcx)}{54a^2b(a + bx^3)^2} - \frac{(14b^{2/3}c - 5a^{2/3}e) \log(\sqrt[3]{a} + \sqrt[3]{bx + b^{2/3}x^2})}{243a^{10/3}b^{4/3}} \\
&= -\frac{x(ae - bcx - bdx^2)}{9ab(a + bx^3)^3} + \frac{x(5ae + 28bcx)}{162a^3b(a + bx^3)} - \frac{6ad - x(ae + 7bcx)}{54a^2b(a + bx^3)^2} - \frac{(14b^{2/3}c - 5a^{2/3}e) \log(\sqrt[3]{a} + \sqrt[3]{bx + b^{2/3}x^2})}{243a^{10/3}b^{4/3}} \\
&= -\frac{x(ae - bcx - bdx^2)}{9ab(a + bx^3)^3} + \frac{x(5ae + 28bcx)}{162a^3b(a + bx^3)} - \frac{6ad - x(ae + 7bcx)}{54a^2b(a + bx^3)^2} - \frac{(14b^{2/3}c + 5a^{2/3}e) \tan^{-1}\left(\frac{\sqrt[3]{a} + \sqrt[3]{bx + b^{2/3}x^2}}{\sqrt[3]{a} - \sqrt[3]{bx + b^{2/3}x^2}}\right)}{81\sqrt{3}a^{10/3}b^{4/3}}
\end{aligned}$$

Mathematica [A] time = 0.345652, size = 241, normalized size = 0.89

$$\frac{3ab^{2/3}(a^2bx^2(67c+13ex^2)-2a^3(9d+5ex)+ab^2x^5(77c+5ex^2)+28b^3cx^8)}{(a+bx^3)^3} + a^{2/3}\sqrt[3]{b}(14b^{2/3}c - 5a^{2/3}e) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx + b^{2/3}x^2}) - 2\sqrt{3}a^{2/3}$$

$$486a^4b^{5/3}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3)^4,x]

[Out] ((3*a*b^(2/3)*(28*b^3*c*x^8 - 2*a^3*(9*d + 5*e*x) + a*b^2*x^5*(77*c + 5*e*x^2) + a^2*b*x^2*(67*c + 13*e*x^2)))/(a + b*x^3)^3 - 2*sqrt[3]*a^(2/3)*b^(1/3)*(14*b^(2/3)*c + 5*a^(2/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]] + 2*(-14*a^(2/3)*b*c + 5*a^(4/3)*b^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x] + a^(

$$\frac{2/3 * b^{1/3} * (14 * b^{2/3} * c - 5 * a^{2/3} * e) * \text{Log}[a^{2/3} - a^{1/3} * b^{1/3} * x + b^{2/3} * x^2]}{(486 * a^4 * b^{5/3})}$$

Maple [A] time = 0.012, size = 278, normalized size = 1.

$$\frac{1}{(bx^3 + a)^3} \left(\frac{14b^2cx^8}{81a^3} + \frac{5bex^7}{162a^2} + \frac{77bcx^5}{162a^2} + \frac{13ex^4}{162a} + \frac{67cx^2}{162a} - \frac{5ex}{81b} - \frac{d}{9b} \right) + \frac{5e}{243b^2a^2} \ln \left(x + \sqrt[3]{\frac{a}{b}} \right) \left(\frac{a}{b} \right)^{-\frac{2}{3}} - \frac{5e}{486b^2a^2} \ln$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(e*x^2+d*x+c)/(b*x^3+a)^4,x)

[Out] (14/81*c/a^3*b^2*x^8+5/162/a^2*b*e*x^7+77/162/a^2*c*b*x^5+13/162/a*e*x^4+67/162*c/a*x^2-5/81*e*x/b-1/9/b*d)/(b*x^3+a)^3+5/243/b^2/a^2*e/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-5/486/b^2/a^2*e/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+5/243/b^2/a^2*e/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-14/243/b/a^3*c/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+7/243/b/a^3*c/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+14/243/b/a^3*c*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [C] time = 8.83698, size = 6597, normalized size = 24.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="fricas")

[Out] $\frac{1}{972} \cdot (168b^3cx^8 + 30ab^2ex^7 + 462ab^2cx^5 + 78a^2b^2ex^4 + 402a^2b^2cx^2 - 60a^3ex - 108a^3d - 2(a^3b^4x^9 + 3a^4b^3x^6 + 3a^5b^2x^3 + a^6b) \cdot ((\frac{1}{2})^{1/3} \cdot (I\sqrt{3} + 1) \cdot ((2744b^2c^3 + 125a^2e^3)/(a^{10}b^4) - (2744b^2c^3 - 125a^2e^3)/(a^{10}b^4))^{1/3} - 140 \cdot (\frac{1}{2})^{2/3} \cdot c \cdot e \cdot (-I\sqrt{3} + 1)/(a^6b^2 \cdot ((2744b^2c^3 + 125a^2e^3)/(a^{10}b^4) - (2744b^2c^3 - 125a^2e^3)/(a^{10}b^4))^{1/3})) \cdot \log(7/2 \cdot ((\frac{1}{2})^{1/3} \cdot (I\sqrt{3} + 1) \cdot ((2744b^2c^3 + 125a^2e^3)/(a^{10}b^4) - (2744b^2c^3 - 125a^2e^3)/(a^{10}b^4))^{1/3} - 140 \cdot (\frac{1}{2})^{2/3} \cdot c \cdot e \cdot (-I\sqrt{3} + 1)/(a^6b^2 \cdot ((2744b^2c^3 + 125a^2e^3)/(a^{10}b^4) - (2744b^2c^3 - 125a^2e^3)/(a^{10}b^4))^{1/3}))^2 \cdot a^7b^3c - 25/2 \cdot ((\frac{1}{2})^{1/3} \cdot (I\sqrt{3} + 1) \cdot ((2744b^2c^3 + 125a^2e^3)/(a^{10}b^4) - (2744b^2c^3 - 125a^2e^3)/(a^{10}b^4))^{1/3} - 140 \cdot (\frac{1}{2})^{2/3} \cdot c \cdot e \cdot (-I\sqrt{3} + 1)/(a^6b^2 \cdot ((2744b^2c^3 + 125a^2e^3)/(a^{10}b^4) - (2744b^2c^3 - 125a^2e^3)/(a^{10}b^4))^{1/3})) \cdot a^5b^2e^2 + 1960 \cdot ab^2c^2e + (2744b^2c^3 + 125a^2e^3) \cdot x) + ((a^3b^4x^9 + 3a^4b^3x^6 + 3a^5b^2x^3 + a^6b) \cdot ((\frac{1}{2})^{1/3} \cdot (I\sqrt{3} + 1) \cdot ((2744b^2c^3 + 125a^2e^3)/(a^{10}b^4) - (2744b^2c^3 - 125a^2e^3)/(a^{10}b^4))^{1/3} - 140 \cdot (\frac{1}{2})^{2/3} \cdot c \cdot e \cdot (-I\sqrt{3} + 1)/(a^6b^2 \cdot ((2744b^2c^3 + 125a^2e^3)/(a^{10}b^4) - (2744b^2c^3 - 125a^2e^3)/(a^{10}b^4))^{1/3})) + 3 \cdot \sqrt{1/3} \cdot (a^3b^4x^9 + 3a^4b^3x^6 + 3a^5b^2x^3 + a^6b) \cdot \sqrt{-(((\frac{1}{2})^{1/3} \cdot (I\sqrt{3} + 1) \cdot ((2744b^2c^3 + 125a^2e^3)/(a^{10}b^4) - (2744b^2c^3 - 125a^2e^3)/(a^{10}b^4))^{1/3} - 140 \cdot (\frac{1}{2})^{2/3} \cdot c \cdot e \cdot (-I\sqrt{3} + 1)/(a^6b^2 \cdot ((2744b^2c^3 + 125a^2e^3)/(a^{10}b^4) - (2744b^2c^3 - 125a^2e^3)/(a^{10}b^4))^{1/3}))^2 \cdot a^6b^2 + 1120 \cdot c \cdot e)/(a^6b^2))} \cdot \log(-7/2 \cdot ((\frac{1}{2})^{1/3} \cdot (I\sqrt{3} + 1) \cdot ((2744b^2c^3 + 125a^2e^3)/(a^{10}b^4) - (2744b^2c^3 - 125a^2e^3)/(a^{10}b^4))^{1/3} - 140 \cdot (\frac{1}{2})^{2/3} \cdot c \cdot e \cdot (-I\sqrt{3} + 1)/(a^6b^2 \cdot ((2744b^2c^3 + 125a^2e^3)/(a^{10}b^4) - (2744b^2c^3 - 125a^2e^3)/(a^{10}b^4))^{1/3}))^2 \cdot a^7b^3c + 25/2 \cdot ((\frac{1}{2})^{1/3} \cdot (I\sqrt{3} + 1) \cdot ((2744b^2c^3 + 125a^2e^3)/(a^{10}b^4) - (2744b^2c^3 - 125a^2e^3)/(a^{10}b^4))^{1/3} - 140 \cdot (\frac{1}{2})^{2/3} \cdot c \cdot e \cdot (-I\sqrt{3} + 1)/(a^6b^2 \cdot ((2744b^2c^3 + 125a^2e^3)/(a^{10}b^4) - (2744b^2c^3 - 125a^2e^3)/(a^{10}b^4))^{1/3})) \cdot a^5b^2e^2 - 1960 \cdot ab^2c^2e + 2 \cdot (2744b^2c^3 + 125a^2e^3) \cdot x + 3/2 \cdot \sqrt{1/3} \cdot (7 \cdot ((\frac{1}{2})^{1/3} \cdot (I\sqrt{3} + 1) \cdot ((2744b^2c^3 + 125a^2e^3)/(a^{10}b^4) - (2744b^2c^3 - 125a^2e^3)/(a^{10}b^4))^{1/3} - 140 \cdot (\frac{1}{2})^{2/3} \cdot c \cdot e \cdot (-I\sqrt{3} + 1)/(a^6b^2 \cdot ((2744b^2c^3 + 125a^2e^3)/(a^{10}b^4) - (2744b^2c^3 - 125a^2e^3)/(a^{10}b^4))^{1/3})) \cdot a^7b^3c + 25 \cdot a^5b^2e^2) \cdot \sqrt{-(((\frac{1}{2})^{1/3} \cdot (I\sqrt{3} + 1) \cdot ((2744b^2c^3 + 125a^2e^3)/(a^{10}b^4) - (2744b^2c^3 - 125a^2e^3)/(a^{10}b^4))^{1/3} - 140 \cdot (\frac{1}{2})^{2/3} \cdot c \cdot e \cdot (-I\sqrt{3} + 1)/(a^6b^2 \cdot ((2744b^2c^3 + 125a^2e^3)/(a^{10}b^4) - (2744b^2c^3 - 125a^2e^3)/(a^{10}b^4))^{1/3}))^2 \cdot a^6b^2 + 1120 \cdot c \cdot e)/(a^6b^2))} + ((a^3b^4x^9 + 3a^4b^3x^6 + 3a^5b^2x^3 + a^6b) \cdot ((\frac{1}{2})^{1/3} \cdot (I\sqrt{3} + 1) \cdot ((2744b^2c^3 + 125a^2e^3)/(a^{10}b^4) - (2744b^2c^3 - 125a^2e^3)/(a^{10}b^4))^{1/3} - 140 \cdot (\frac{1}{2})^{2/3} \cdot c \cdot e \cdot (-I\sqrt{3} + 1)/$

$$\begin{aligned}
& (a^6 b^2 ((2744 b^2 c^3 + 125 a^2 e^3) / (a^{10} b^4) - (2744 b^2 c^3 - 125 a^2 e^3) / (a^{10} b^4))^{(1/3)}) - 3 \sqrt{1/3} (a^3 b^4 x^9 + 3 a^4 b^3 x^6 + 3 a^5 b^2 x^3 + a^6 b) \sqrt{-((1/2)^{(1/3)} (I \sqrt{3} + 1) ((2744 b^2 c^3 + 125 a^2 e^3) / (a^{10} b^4) - (2744 b^2 c^3 - 125 a^2 e^3) / (a^{10} b^4))^{(1/3)} - 140 (1/2)^{(2/3)} c e (-I \sqrt{3} + 1) / (a^6 b^2 ((2744 b^2 c^3 + 125 a^2 e^3) / (a^{10} b^4) - (2744 b^2 c^3 - 125 a^2 e^3) / (a^{10} b^4))^{(1/3)}))^{2 a^6 b^2 + 1120 c e} / (a^6 b^2))} \log(-7/2 ((1/2)^{(1/3)} (I \sqrt{3} + 1) ((2744 b^2 c^3 + 125 a^2 e^3) / (a^{10} b^4) - (2744 b^2 c^3 - 125 a^2 e^3) / (a^{10} b^4))^{(1/3)} - 140 (1/2)^{(2/3)} c e (-I \sqrt{3} + 1) / (a^6 b^2 ((2744 b^2 c^3 + 125 a^2 e^3) / (a^{10} b^4) - (2744 b^2 c^3 - 125 a^2 e^3) / (a^{10} b^4))^{(1/3)}))^{2 a^7 b^3 c + 25/2 ((1/2)^{(1/3)} (I \sqrt{3} + 1) ((2744 b^2 c^3 + 125 a^2 e^3) / (a^{10} b^4) - (2744 b^2 c^3 - 125 a^2 e^3) / (a^{10} b^4))^{(1/3)} - 140 (1/2)^{(2/3)} c e (-I \sqrt{3} + 1) / (a^6 b^2 ((2744 b^2 c^3 + 125 a^2 e^3) / (a^{10} b^4) - (2744 b^2 c^3 - 125 a^2 e^3) / (a^{10} b^4))^{(1/3)})) a^5 b e^2 - 1960 a b c^2 e + 2 (2744 b^2 c^3 + 125 a^2 e^3) x - 3/2 \sqrt{1/3} (7 ((1/2)^{(1/3)} (I \sqrt{3} + 1) ((2744 b^2 c^3 + 125 a^2 e^3) / (a^{10} b^4) - (2744 b^2 c^3 - 125 a^2 e^3) / (a^{10} b^4))^{(1/3)} - 140 (1/2)^{(2/3)} c e (-I \sqrt{3} + 1) / (a^6 b^2 ((2744 b^2 c^3 + 125 a^2 e^3) / (a^{10} b^4) - (2744 b^2 c^3 - 125 a^2 e^3) / (a^{10} b^4))^{(1/3)})) a^7 b^3 c + 25 a^5 b e^2) \sqrt{-((1/2)^{(1/3)} (I \sqrt{3} + 1) ((2744 b^2 c^3 + 125 a^2 e^3) / (a^{10} b^4) - (2744 b^2 c^3 - 125 a^2 e^3) / (a^{10} b^4))^{(1/3)} - 140 (1/2)^{(2/3)} c e (-I \sqrt{3} + 1) / (a^6 b^2 ((2744 b^2 c^3 + 125 a^2 e^3) / (a^{10} b^4) - (2744 b^2 c^3 - 125 a^2 e^3) / (a^{10} b^4))^{(1/3)}))^{2 a^6 b^2 + 1120 c e} / (a^6 b^2))} / (a^3 b^4 x^9 + 3 a^4 b^3 x^6 + 3 a^5 b^2 x^3 + a^6 b)
\end{aligned}$$

Sympy [A] time = 7.04865, size = 214, normalized size = 0.79

$$\text{RootSum}\left(14348907 t^3 a^{10} b^4 + 51030 t a^4 b^2 c e - 125 a^2 e^3 + 2744 b^2 c^3, \left(t \mapsto t \log\left(x + \frac{826686 t^2 a^7 b^3 c + 6075 t a^5 b e^2 + 1960}{125 a^2 e^3 + 2744 b^2 c^3}\right)\right)
\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**4,x)

[Out] RootSum(14348907*_t**3*a**10*b**4 + 51030*_t*a**4*b**2*c*e - 125*a**2*e**3 + 2744*b**2*c**3, Lambda(_t, _t*log(x + (826686*_t**2*a**7*b**3*c + 6075*_t*a**5*b*e**2 + 1960*a*b*c**2*e)/(125*a**2*e**3 + 2744*b**2*c**3)))) + (-18*a**3*d - 10*a**3*e*x + 67*a**2*b*c*x**2 + 13*a**2*b*e*x**4 + 77*a*b**2*c*x**5 + 5*a*b**2*e*x**7 + 28*b**3*c*x**8)/(162*a**6*b + 486*a**5*b**2*x**3 + 486*a**4*b**3*x**6 + 162*a**3*b**4*x**9)

Giac [A] time = 1.08782, size = 346, normalized size = 1.28

$$\frac{\left(14bc\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 5ae\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243a^4b} + \frac{\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}ae - 14\left(-ab^2\right)^{\frac{2}{3}}c\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243a^4b^2} + \frac{28b^3cx^8}{243a^4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/243*(14*b*c*(-a/b)^{(1/3)} + 5*a*e)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)})) \\ &)/(a^4*b) + 1/243*\text{sqrt}(3)*(5*(-a*b^2)^{(1/3)}*a*e - 14*(-a*b^2)^{(2/3)}*c)*\text{arctan} \\ & \text{an}(1/3*\text{sqrt}(3)*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(a^4*b^2) + 1/162*(28*b^3 \\ & *c*x^8 + 5*a*b^2*x^7*e + 77*a*b^2*c*x^5 + 13*a^2*b*x^4*e + 67*a^2*b*c*x^2 - \\ & 10*a^3*x*e - 18*a^3*d)/((b*x^3 + a)^3*a^3*b) + 1/486*(5*(-a*b^2)^{(1/3)}*a*b \\ & ^2*e + 14*(-a*b^2)^{(2/3)}*b^2*c)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a \\ & ^4*b^4) \end{aligned}$$

$$3.360 \quad \int \frac{c+dx+ex^2}{(a+bx^3)^4} dx$$

Optimal. Leaf size=250

$$-\frac{(20\sqrt[3]{bc} - 7\sqrt[3]{ad}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{243a^{11/3}b^{2/3}} + \frac{2(20\sqrt[3]{bc} - 7\sqrt[3]{ad}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{243a^{11/3}b^{2/3}} - \frac{2(7\sqrt[3]{ad} + 20\sqrt[3]{bc}) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt[3]{3}\sqrt[3]{a+bx^3}}\right)}{81\sqrt{3}a^{11/3}b^{2/3}}$$

[Out] (x*(8*c + 7*d*x))/(54*a^2*(a + b*x^3)^2) + (2*x*(10*c + 7*d*x))/(81*a^3*(a + b*x^3)) - (a*e - b*x*(c + d*x))/(9*a*b*(a + b*x^3)^3) - (2*(20*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(81*Sqrt[3]*a^(11/3)*b^(2/3)) + (2*(20*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x]/(243*a^(11/3)*b^(2/3)) - ((20*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(243*a^(11/3)*b^(2/3))

Rubi [A] time = 0.222237, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {1854, 1855, 1860, 31, 634, 617, 204, 628}

$$-\frac{(20\sqrt[3]{bc} - 7\sqrt[3]{ad}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{243a^{11/3}b^{2/3}} + \frac{2(20\sqrt[3]{bc} - 7\sqrt[3]{ad}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{243a^{11/3}b^{2/3}} - \frac{2(7\sqrt[3]{ad} + 20\sqrt[3]{bc}) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt[3]{3}\sqrt[3]{a+bx^3}}\right)}{81\sqrt{3}a^{11/3}b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/(a + b*x^3)^4, x]

[Out] (x*(8*c + 7*d*x))/(54*a^2*(a + b*x^3)^2) + (2*x*(10*c + 7*d*x))/(81*a^3*(a + b*x^3)) - (a*e - b*x*(c + d*x))/(9*a*b*(a + b*x^3)^3) - (2*(20*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(81*Sqrt[3]*a^(11/3)*b^(2/3)) + (2*(20*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x]/(243*a^(11/3)*b^(2/3)) - ((20*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(243*a^(11/3)*b^(2/3))

Rule 1854

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int

```
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1})*(a + b*x^n)^(p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2}{(a + bx^3)^4} dx &= -\frac{ae - bx(c + dx)}{9ab(a + bx^3)^3} - \frac{\int \frac{-8c - 7dx}{(a + bx^3)^3} dx}{9a} \\
&= \frac{x(8c + 7dx)}{54a^2(a + bx^3)^2} - \frac{ae - bx(c + dx)}{9ab(a + bx^3)^3} + \frac{\int \frac{40c + 28dx}{(a + bx^3)^2} dx}{54a^2} \\
&= \frac{x(8c + 7dx)}{54a^2(a + bx^3)^2} + \frac{2x(10c + 7dx)}{81a^3(a + bx^3)} - \frac{ae - bx(c + dx)}{9ab(a + bx^3)^3} - \frac{\int \frac{-80c - 28dx}{a + bx^3} dx}{162a^3} \\
&= \frac{x(8c + 7dx)}{54a^2(a + bx^3)^2} + \frac{2x(10c + 7dx)}{81a^3(a + bx^3)} - \frac{ae - bx(c + dx)}{9ab(a + bx^3)^3} - \frac{\int \frac{\sqrt[3]{a}(-160\sqrt[3]{bc} - 28\sqrt[3]{ad}) + \sqrt[3]{b}(80\sqrt[3]{bc} - 28\sqrt[3]{ad})x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2} dx}{486a^{11/3}\sqrt[3]{b}} + \dots \\
&= \frac{x(8c + 7dx)}{54a^2(a + bx^3)^2} + \frac{2x(10c + 7dx)}{81a^3(a + bx^3)} - \frac{ae - bx(c + dx)}{9ab(a + bx^3)^3} + \frac{2(20\sqrt[3]{bc} - 7\sqrt[3]{ad}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{243a^{11/3}b^{2/3}} - \frac{(20\sqrt[3]{b})}{\dots} \\
&= \frac{x(8c + 7dx)}{54a^2(a + bx^3)^2} + \frac{2x(10c + 7dx)}{81a^3(a + bx^3)} - \frac{ae - bx(c + dx)}{9ab(a + bx^3)^3} + \frac{2(20\sqrt[3]{bc} - 7\sqrt[3]{ad}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{243a^{11/3}b^{2/3}} - \frac{(20\sqrt[3]{b})}{\dots} \\
&= \frac{x(8c + 7dx)}{54a^2(a + bx^3)^2} + \frac{2x(10c + 7dx)}{81a^3(a + bx^3)} - \frac{ae - bx(c + dx)}{9ab(a + bx^3)^3} - \frac{2(20\sqrt[3]{bc} + 7\sqrt[3]{ad}) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{11/3}b^{2/3}} + \frac{2(20\sqrt[3]{b})}{\dots}
\end{aligned}$$

Mathematica [A] time = 0.231645, size = 239, normalized size = 0.96

$$\frac{2(7a^{2/3}d - 20\sqrt[3]{a}\sqrt[3]{bc}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{b^{2/3}} + \frac{4(20\sqrt[3]{a}\sqrt[3]{bc} - 7a^{2/3}d) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{b^{2/3}} - \frac{54a^3(ae - bx(c + dx))}{b(a + bx^3)^3} + \frac{9a^2x(8c + 7dx)}{(a + bx^3)^2} - \frac{4\sqrt{3}\sqrt[3]{a}(7\sqrt[3]{ad} + 20\sqrt[3]{bc}) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{2/3}}$$

$486a^4$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(a + b*x^3)^4, x]

[Out]
$$\left(\frac{9a^2x(8c + 7dx)}{(a + bx^3)^2} + \frac{12ax(10c + 7dx)}{(a + bx^3)^3} - \frac{54a^3(ae - bxc + dx)}{b(a + bx^3)^3} - \frac{4\sqrt[3]{a} \left(20b^{1/3}c + 7a^{1/3}d \right) \operatorname{ArcTan}\left[\frac{1 - (2b^{1/3}x)/a^{1/3}}{\sqrt[3]{a + bx^3}} \right]}{b^{2/3}} + \frac{4(20a^{1/3}b^{1/3}c - 7a^{2/3}d) \operatorname{Log}[a^{1/3} + b^{1/3}x]}{b^{2/3}} + \frac{2(-20a^{1/3}b^{1/3}c + 7a^{2/3}d) \operatorname{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2]}{b^{2/3}} \right) / (486a^4)$$

Maple [A] time = 0.004, size = 360, normalized size = 1.4

$$\frac{cx}{9a(bx^3 + a)^3} + \frac{4cx}{27a^2(bx^3 + a)^2} + \frac{20cx}{81a^3(bx^3 + a)} + \frac{40c}{243a^3b} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{20c}{243a^3b} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)/(b*x^3+a)^4, x)

[Out]
$$\frac{1}{9} \frac{c}{a} \frac{x}{(bx^3+a)^3} + \frac{4}{27} \frac{c}{a^2} \frac{x}{(bx^3+a)^2} + \frac{20}{81} \frac{c}{a^3} \frac{x}{(bx^3+a)} + \frac{40}{243} \frac{c}{a^3b} \frac{1}{(1/ba)^{2/3}} \ln\left(x + (1/ba)^{1/3}\right) - \frac{20}{243} \frac{c}{a^3b} \frac{1}{(1/ba)^{2/3}} \ln\left(x^2 - (1/ba)^{1/3}x + (1/ba)^{2/3}\right) + \frac{40}{243} \frac{c}{a^3b} \frac{1}{(1/ba)^{2/3}} \operatorname{arctan}\left(\frac{1}{3} \sqrt[3]{(1/ba)^{1/3}} \frac{2}{(1/ba)^{1/3}} \frac{x-1}{x}\right) + \frac{1}{9} \frac{d}{a} \frac{x^2}{(bx^3+a)^3} + \frac{7}{54} \frac{d}{a^2} \frac{x^2}{(bx^3+a)^2} + \frac{14}{81} \frac{d}{a^3} \frac{x^2}{(bx^3+a)} - \frac{14}{243} \frac{d}{a^3b} \frac{1}{(1/ba)^{1/3}} \ln\left(x + (1/ba)^{1/3}\right) + \frac{7}{243} \frac{d}{a^3b} \frac{1}{(1/ba)^{1/3}} \ln\left(x^2 - (1/ba)^{1/3}x + (1/ba)^{2/3}\right) + \frac{14}{243} \frac{d}{a^3} \sqrt[3]{(1/ba)^{1/3}} \frac{1}{(1/ba)^{1/3}} \operatorname{arctan}\left(\frac{1}{3} \sqrt[3]{(1/ba)^{1/3}} \frac{2}{(1/ba)^{1/3}} \frac{x-1}{x}\right) + \frac{1}{9} \frac{e}{a} \frac{x^3}{(bx^3+a)^3} + \frac{1}{9} \frac{e}{a^2} \frac{x^3}{(bx^3+a)^2} - \frac{1}{9} \frac{e}{a^2b} \frac{1}{(bx^3+a)}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^4, x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [C] time = 6.22442, size = 6020, normalized size = 24.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="fricas")

[Out]
$$\frac{1}{972} \cdot (168b^3d^2x^8 + 240b^3c^2x^7 + 462a^2b^2d^2x^5 + 624a^2b^2c^2x^4 + 402a^2b^2d^2x^2 + 492a^2b^2c^2x - 108a^3e - 2(a^3b^4x^9 + 3a^4b^3x^6 + 3a^5b^2x^3 + a^6b) \cdot (4^{1/3} \cdot (I\sqrt{3} + 1) \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2))^{1/3} - 140 \cdot 4^{2/3} \cdot c \cdot d \cdot (-I\sqrt{3} + 1)/(a^7b \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2))^{1/3})) \cdot \log(7/4 \cdot (4^{1/3} \cdot (I\sqrt{3} + 1) \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2))^{1/3} - 140 \cdot 4^{2/3} \cdot c \cdot d \cdot (-I\sqrt{3} + 1)/(a^7b \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2))^{1/3}))^2 \cdot a^8 \cdot b \cdot d - 400 \cdot (4^{1/3} \cdot (I\sqrt{3} + 1) \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2))^{1/3} - 140 \cdot 4^{2/3} \cdot c \cdot d \cdot (-I\sqrt{3} + 1)/(a^7b \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2))^{1/3})) \cdot a^4 \cdot b \cdot c^2 + 7840 \cdot a \cdot c \cdot d^2 + 4 \cdot (8000b^3c^3 + 343ad^3) \cdot x) + ((a^3b^4x^9 + 3a^4b^3x^6 + 3a^5b^2x^3 + a^6b) \cdot (4^{1/3} \cdot (I\sqrt{3} + 1) \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2))^{1/3} - 140 \cdot 4^{2/3} \cdot c \cdot d \cdot (-I\sqrt{3} + 1)/(a^7b \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2))^{1/3})) + 3 \cdot \sqrt{1/3} \cdot (a^3b^4x^9 + 3a^4b^3x^6 + 3a^5b^2x^3 + a^6b) \cdot \sqrt{-((4^{1/3} \cdot (I\sqrt{3} + 1) \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2))^{1/3} - 140 \cdot 4^{2/3} \cdot c \cdot d \cdot (-I\sqrt{3} + 1)/(a^7b \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2))^{1/3}))^2 \cdot a^7 \cdot b + 8960 \cdot c \cdot d)/(a^7 \cdot b)) \cdot \log(-7/4 \cdot (4^{1/3} \cdot (I\sqrt{3} + 1) \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2))^{1/3} - 140 \cdot 4^{2/3} \cdot c \cdot d \cdot (-I\sqrt{3} + 1)/(a^7b \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2))^{1/3}))^2 \cdot a^8 \cdot b \cdot d + 400 \cdot (4^{1/3} \cdot (I\sqrt{3} + 1) \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2))^{1/3} - 140 \cdot 4^{2/3} \cdot c \cdot d \cdot (-I\sqrt{3} + 1)/(a^7b \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2))^{1/3})) \cdot a^4 \cdot b \cdot c^2 - 7840 \cdot a \cdot c \cdot d^2 + 8 \cdot (8000b^3c^3 + 343ad^3) \cdot x + 3/4 \cdot \sqrt{1/3} \cdot (7 \cdot (4^{1/3} \cdot (I\sqrt{3} + 1) \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2))^{1/3} - 140 \cdot 4^{2/3} \cdot c \cdot d \cdot (-I\sqrt{3} + 1)/(a^7b \cdot ((8000b^3c^3 + 343ad^3)/(a^{11}b^2) + (8000b^3c^3 - 343ad^3)/(a^{11}b^2))^{1/3})) \cdot a^8 \cdot b \cdot d + 1600 \cdot a$$

$$\begin{aligned}
&^4*b*c^2)*\sqrt{-((4^{(1/3)}*(I*\sqrt{3}) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^{11}*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^{11}*b^2))^{(1/3)} - 140*4^{(2/3)}*c*d*(-I*\sqrt{3} + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^{11}*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^{11}*b^2))^{(1/3)}))^{2*a^7*b + 8960*c*d)/(a^7*b))} + ((a^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*(4^{(1/3)}*(I*\sqrt{3}) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^{11}*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^{11}*b^2))^{(1/3)} - 140*4^{(2/3)}*c*d*(-I*\sqrt{3} + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^{11}*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^{11}*b^2))^{(1/3)})) - 3*\sqrt{1/3}*(a^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*\sqrt{-((4^{(1/3)}*(I*\sqrt{3}) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^{11}*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^{11}*b^2))^{(1/3)} - 140*4^{(2/3)}*c*d*(-I*\sqrt{3} + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^{11}*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^{11}*b^2))^{(1/3)}))^{2*a^7*b + 8960*c*d)/(a^7*b))})*\log(-7/4*(4^{(1/3)}*(I*\sqrt{3}) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^{11}*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^{11}*b^2))^{(1/3)} - 140*4^{(2/3)}*c*d*(-I*\sqrt{3} + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^{11}*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^{11}*b^2))^{(1/3)}))^{2*a^8*b*d + 400*(4^{(1/3)}*(I*\sqrt{3}) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^{11}*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^{11}*b^2))^{(1/3)} - 140*4^{(2/3)}*c*d*(-I*\sqrt{3} + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^{11}*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^{11}*b^2))^{(1/3)})))*a^4*b*c^2 - 7840*a*c*d^2 + 8*(8000*b*c^3 + 343*a*d^3)*x - 3/4*\sqrt{1/3}*(7*(4^{(1/3)}*(I*\sqrt{3}) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^{11}*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^{11}*b^2))^{(1/3)} - 140*4^{(2/3)}*c*d*(-I*\sqrt{3} + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^{11}*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^{11}*b^2))^{(1/3)})))*a^8*b*d + 1600*a^4*b*c^2)*\sqrt{-((4^{(1/3)}*(I*\sqrt{3}) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^{11}*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^{11}*b^2))^{(1/3)} - 140*4^{(2/3)}*c*d*(-I*\sqrt{3} + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^{11}*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^{11}*b^2))^{(1/3)}))^{2*a^7*b + 8960*c*d)/(a^7*b))}))/((a^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)
\end{aligned}$$

Sympy [A] time = 4.04936, size = 202, normalized size = 0.81

$$\text{RootSum}\left(14348907t^3a^{11}b^2 + 408240ta^4bcd + 2744ad^3 - 64000bc^3, \left(t \mapsto t \log\left(x + \frac{413343t^2a^8bd + 194400ta^4bc^2 + 1372ad^3 + 32000bc^3}{1372ad^3 + 32000bc^3}\right)\right)
\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/(b*x**3+a)**4,x)

[Out] RootSum(14348907*_t**3*a**11*b**2 + 408240*_t*a**4*b*c*d + 2744*a*d**3 - 64000*b*c**3, Lambda(_t, _t*log(x + (413343*_t**2*a**8*b*d + 194400*_t*a**4*b*c**2 + 7840*a*c*d**2)/(1372*a*d**3 + 32000*b*c**3)))) + (-18*a**3*e + 82*a**2*b*c*x + 67*a**2*b*d*x**2 + 104*a*b**2*c*x**4 + 77*a*b**2*d*x**5 + 40*b*

$*3*c*x**7 + 28*b**3*d*x**8)/(162*a**6*b + 486*a**5*b**2*x**3 + 486*a**4*b**3*x**6 + 162*a**3*b**4*x**9)$

Giac [A] time = 1.08396, size = 333, normalized size = 1.33

$$-\frac{2\left(7d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 20c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243 a^4} + \frac{2\sqrt{3}\left(20(-ab^2)^{\frac{1}{3}}bc - 7(-ab^2)^{\frac{2}{3}}d\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243 a^4 b^2} + \frac{28 b^3 dx^8}{243 a^4 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="giac")

[Out] $-2/243*(7*d*(-a/b)^{(1/3)} + 20*c)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/a^4 + 2/243*\text{sqrt}(3)*(20*(-a*b^2)^{(1/3)}*b*c - 7*(-a*b^2)^{(2/3)}*d)*\arctan(1/3*\text{sqrt}(3)*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(a^4*b^2) + 1/162*(28*b^3*d*x^8 + 40*b^3*c*x^7 + 77*a*b^2*d*x^5 + 104*a*b^2*c*x^4 + 67*a^2*b*d*x^2 + 82*a^2*b*c*x - 18*a^3*e)/((b*x^3 + a)^3*a^3*b) + 1/243*(20*(-a*b^2)^{(1/3)}*a*b^3*c + 7*(-a*b^2)^{(2/3)}*a*b^2*d)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a^5*b^4)$

$$3.361 \quad \int \frac{c+dx+ex^2}{x(a+bx^3)^4} dx$$

Optimal. Leaf size=291

$$\frac{(20\sqrt[3]{bd} - 7\sqrt[3]{ae}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{243a^{11/3}b^{2/3}} + \frac{2(20\sqrt[3]{bd} - 7\sqrt[3]{ae}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{243a^{11/3}b^{2/3}} - \frac{2(7\sqrt[3]{ae} + 20\sqrt[3]{bd}) \tan^{-1}\left(\frac{\sqrt[3]{a}-2}{\sqrt{3}}\right)}{81\sqrt{3}a^{11/3}b^{2/3}}$$

[Out] (x*(a*d + a*e*x - b*c*x^2))/(9*a^2*(a + b*x^3)^3) + (x*(8*a*d + 7*a*e*x - 15*b*c*x^2))/(54*a^3*(a + b*x^3)^2) + (x*(40*a*d + 28*a*e*x - 99*b*c*x^2))/(162*a^4*(a + b*x^3)) - (2*(20*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(81*Sqrt[3]*a^(11/3)*b^(2/3)) + (c*Log[x])/a^4 + (2*(20*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x]/(243*a^(11/3)*b^(2/3)) - ((20*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(243*a^(11/3)*b^(2/3)) - (c*Log[a + b*x^3])/(3*a^4)

Rubi [A] time = 0.516882, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{(20\sqrt[3]{bd} - 7\sqrt[3]{ae}) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{243a^{11/3}b^{2/3}} + \frac{2(20\sqrt[3]{bd} - 7\sqrt[3]{ae}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{243a^{11/3}b^{2/3}} - \frac{2(7\sqrt[3]{ae} + 20\sqrt[3]{bd}) \tan^{-1}\left(\frac{\sqrt[3]{a}-2}{\sqrt{3}}\right)}{81\sqrt{3}a^{11/3}b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/(x*(a + b*x^3)^4), x]

[Out] (x*(a*d + a*e*x - b*c*x^2))/(9*a^2*(a + b*x^3)^3) + (x*(8*a*d + 7*a*e*x - 15*b*c*x^2))/(54*a^3*(a + b*x^3)^2) + (x*(40*a*d + 28*a*e*x - 99*b*c*x^2))/(162*a^4*(a + b*x^3)) - (2*(20*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(81*Sqrt[3]*a^(11/3)*b^(2/3)) + (c*Log[x])/a^4 + (2*(20*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x]/(243*a^(11/3)*b^(2/3)) - ((20*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(243*a^(11/3)*b^(2/3)) - (c*Log[a + b*x^3])/(3*a^4)

Rule 1829

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^

```
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_)*(x_))^(m_))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[Ex
pandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^( -1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
```

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 204

$\text{Int}[\frac{(a_1 + (b_1)x^2)^{-1}}{x}, x_Symbol] \rightarrow -\text{Simp}[\frac{\text{ArcTan}[\frac{\text{Rt}[-b, 2]x}{\text{Rt}[-a, 2] - \text{Rt}[-a, 2]\text{Rt}[-b, 2]}}{x}]}{\text{Rt}[-a, 2] - \text{Rt}[-a, 2]\text{Rt}[-b, 2]}, x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 628

$\text{Int}[\frac{(d_1 + (e_1)x)}{(a_1 + (b_1)x + (c_1)x^2)}, x_Symbol] \rightarrow \text{Simp}[\frac{d \cdot \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]}{b}, x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2cd - be, 0]$

Rule 260

$\text{Int}[\frac{x^{(m_1)}}{(a_1 + (b_1)x^n)}, x_Symbol] \rightarrow \text{Simp}[\frac{\text{Log}[\text{RemoveContent}[a + bx^n, x]]}{bn}, x] \ /; \ \text{FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2}{x(a + bx^3)^4} dx &= \frac{x(ad + aex - bcx^2)}{9a^2(a + bx^3)^3} - \frac{\int \frac{-9bc - 8bdx - 7bex^2 + \frac{6b^2cx^3}{a}}{x(a + bx^3)^3} dx}{9ab} \\
&= \frac{x(ad + aex - bcx^2)}{9a^2(a + bx^3)^3} + \frac{x(8ad + 7aex - 15bcx^2)}{54a^3(a + bx^3)^2} + \frac{\int \frac{54b^2c + 40b^2dx + 28b^2ex^2 - \frac{45b^3cx^3}{a}}{x(a + bx^3)^2} dx}{54a^2b^2} \\
&= \frac{x(ad + aex - bcx^2)}{9a^2(a + bx^3)^3} + \frac{x(8ad + 7aex - 15bcx^2)}{54a^3(a + bx^3)^2} + \frac{x(40ad + 28aex - 99bcx^2)}{162a^4(a + bx^3)} - \frac{\int \frac{-162b^3c - 80b^3dx - 28b^3ex^2}{x(a + bx^3)}}{162a^3b^3} \\
&= \frac{x(ad + aex - bcx^2)}{9a^2(a + bx^3)^3} + \frac{x(8ad + 7aex - 15bcx^2)}{54a^3(a + bx^3)^2} + \frac{x(40ad + 28aex - 99bcx^2)}{162a^4(a + bx^3)} - \frac{\int \left(-\frac{162b^3c}{ax} - \frac{2b^3(40ad + 28aex - 99bcx^2)}{a + bx^3} \right) dx}{162a^3b^3} \\
&= \frac{x(ad + aex - bcx^2)}{9a^2(a + bx^3)^3} + \frac{x(8ad + 7aex - 15bcx^2)}{54a^3(a + bx^3)^2} + \frac{x(40ad + 28aex - 99bcx^2)}{162a^4(a + bx^3)} + \frac{c \log(x)}{a^4} + \frac{\int \frac{40ad + 28aex - 99bcx^2}{a + bx^3} dx}{81a^3b^3} \\
&= \frac{x(ad + aex - bcx^2)}{9a^2(a + bx^3)^3} + \frac{x(8ad + 7aex - 15bcx^2)}{54a^3(a + bx^3)^2} + \frac{x(40ad + 28aex - 99bcx^2)}{162a^4(a + bx^3)} + \frac{c \log(x)}{a^4} + \frac{\int \frac{40ad + 28aex - 99bcx^2}{a + bx^3} dx}{81a^3b^3} \\
&= \frac{x(ad + aex - bcx^2)}{9a^2(a + bx^3)^3} + \frac{x(8ad + 7aex - 15bcx^2)}{54a^3(a + bx^3)^2} + \frac{x(40ad + 28aex - 99bcx^2)}{162a^4(a + bx^3)} + \frac{c \log(x)}{a^4} - \frac{c \log(a)}{3a} \\
&= \frac{x(ad + aex - bcx^2)}{9a^2(a + bx^3)^3} + \frac{x(8ad + 7aex - 15bcx^2)}{54a^3(a + bx^3)^2} + \frac{x(40ad + 28aex - 99bcx^2)}{162a^4(a + bx^3)} + \frac{c \log(x)}{a^4} + \frac{2(20\sqrt[3]{b})}{81a^3b^3} \\
&= \frac{x(ad + aex - bcx^2)}{9a^2(a + bx^3)^3} + \frac{x(8ad + 7aex - 15bcx^2)}{54a^3(a + bx^3)^2} + \frac{x(40ad + 28aex - 99bcx^2)}{162a^4(a + bx^3)} + \frac{c \log(x)}{a^4} + \frac{2(20\sqrt[3]{b})}{81a^3b^3} \\
&= \frac{x(ad + aex - bcx^2)}{9a^2(a + bx^3)^3} + \frac{x(8ad + 7aex - 15bcx^2)}{54a^3(a + bx^3)^2} + \frac{x(40ad + 28aex - 99bcx^2)}{162a^4(a + bx^3)} - \frac{2(20\sqrt[3]{bd} + 7\sqrt[3]{ae})}{81\sqrt[3]{3a^1}}
\end{aligned}$$

Mathematica [A] time = 0.227502, size = 259, normalized size = 0.89

$$\frac{2\left(7a^{2/3}e-20\sqrt[3]{a}\sqrt[3]{bd}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx+b^{2/3}x^2}\right)}{b^{2/3}} + \frac{4\left(20\sqrt[3]{a}\sqrt[3]{bd}-7a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{b^{2/3}} + \frac{54a^3(c+xd+ex)}{(a+bx^3)^3} + \frac{9a^2(9c+x(8d+7ex))}{(a+bx^3)^2} - \frac{4\sqrt{3}\sqrt[3]{a}\left(7\sqrt[3]{ae}+20\sqrt[3]{b}\right)}{486a^4}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)^4), x]

[Out] ((54*a^3*(c + x*(d + e*x)))/(a + b*x^3)^3 + (9*a^2*(9*c + x*(8*d + 7*e*x)))/(a + b*x^3)^2 + (6*a*(27*c + 2*x*(10*d + 7*e*x)))/(a + b*x^3) - (4*sqrt[3]*a^(1/3)*(20*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]])/b^(2/3) + 486*c*Log[x] + (4*(20*a^(1/3)*b^(1/3)*d - 7*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (2*(-20*a^(1/3)*b^(1/3)*d + 7*a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) - 162*c*Log[a + b*x^3))/(486*a^4)

Maple [A] time = 0.016, size = 394, normalized size = 1.4

$$\frac{14b^2ex^8}{81a^3(bx^3+a)^3} + \frac{20b^2dx^7}{81a^3(bx^3+a)^3} + \frac{b^2cx^6}{3a^3(bx^3+a)^3} + \frac{77bex^5}{162a^2(bx^3+a)^3} + \frac{52bdx^4}{81a^2(bx^3+a)^3} + \frac{5bcx^3}{6a^2(bx^3+a)^3} + \frac{6}{162a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)/x/(b*x^3+a)^4, x)

[Out] 14/81/a^3/(b*x^3+a)^3*b^2*e*x^8+20/81/a^3/(b*x^3+a)^3*b^2*d*x^7+1/3/a^3/(b*x^3+a)^3*c*b^2*x^6+77/162/a^2/(b*x^3+a)^3*b*e*x^5+52/81/a^2/(b*x^3+a)^3*b*d*x^4+5/6/a^2/(b*x^3+a)^3*b*c*x^3+67/162/a/(b*x^3+a)^3*e*x^2+41/81/a/(b*x^3+a)^3*d*x+11/18/a/(b*x^3+a)^3*c+40/243/a^3*d/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-20/243/a^3*d/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+40/243/a^3*d/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-14/243/a^3*e/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+7/243/a^3*e/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+14/243/a^3*e*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/3*c*ln(b*x^3+a)/a^4+c*ln(x)/a^4

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [C] time = 7.40697, size = 15555, normalized size = 53.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^4,x, algorithm="fricas")

[Out]
$$\begin{aligned} & 1/236196*(40824*a*b^2*e*x^8 + 58320*a*b^2*d*x^7 + 78732*a*b^2*c*x^6 + 11226 \\ & 6*a^2*b*e*x^5 + 151632*a^2*b*d*x^4 + 196830*a^2*b*c*x^3 + 97686*a^3*e*x^2 + \\ & 119556*a^3*d*x + 144342*a^3*c - 2*(a^4*b^3*x^9 + 3*a^5*b^2*x^6 + 3*a^6*b*x \\ & ^3 + a^7)*((-I*\sqrt{3}) + 1)*(6561*c^2/a^8 - (6561*b*c^2 + 560*a*d*e)/(a^8*b \\ &))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348 \\ & 907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2744 \\ & *a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 59049*(I*\sqrt{3} \\ & + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4 \\ & /14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c^3 \\ & + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 39366*c \\ & /a^4)*\log(7/236196*((-I*\sqrt{3}) + 1)*(6561*c^2/a^8 - (6561*b*c^2 + 560*a*d* \\ & e)/(a^8*b)))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) \\ & + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c \\ & ^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 5904 \\ & 9*(I*\sqrt{3}) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^ \\ & 12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441 \\ & *b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) \\ & + 39366*c/a^4)^2*a^8*b*e + 64800*b*c*d^2 + 45927*b*c^2*e + 7840*a*d*e^2 - 1 \\ & /243*(400*a^4*b*d^2 + 567*a^4*b*c*e)*((-I*\sqrt{3}) + 1)*(6561*c^2/a^8 - (656 \\ & 1*b*c^2 + 560*a*d*e)/(a^8*b)))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560* \\ & a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/2869 \\ & 7814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12* \end{aligned}$$

$$\begin{aligned}
& b^2)^{1/3} + 59049*(I*\sqrt{3} + 1)*(-1/27*c^3/a^{12} + 1/118098*(6561*b*c^2 \\
& + 560*a*d*e)*c/(a^{12}*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^{11}*b^2) - \\
& 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/ \\
& (a^{12}*b^2)^{1/3} + 39366*c/a^4 + 4*(8000*b*d^3 + 343*a*e^3)*x - (118098* \\
& b^3*c*x^9 + 354294*a*b^2*c*x^6 + 354294*a^2*b*c*x^3 + 118098*a^3*c - (a^4*b \\
& ^3*x^9 + 3*a^5*b^2*x^6 + 3*a^6*b*x^3 + a^7))*((-I*\sqrt{3} + 1)*(6561*c^2/a^8 \\
& - (6561*b*c^2 + 560*a*d*e)/(a^8*b)))/(-1/27*c^3/a^{12} + 1/118098*(6561*b*c^2 \\
& + 560*a*d*e)*c/(a^{12}*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^{11}*b^2) - \\
& 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b) \\
& / (a^{12}*b^2)^{1/3} + 59049*(I*\sqrt{3} + 1)*(-1/27*c^3/a^{12} + 1/118098*(6561 \\
& *b*c^2 + 560*a*d*e)*c/(a^{12}*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^{11}* \\
& b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e) \\
&)*a*b)/(a^{12}*b^2)^{1/3} + 39366*c/a^4) - 3*\sqrt{1/3}*(a^4*b^3*x^9 + 3*a^5* \\
& b^2*x^6 + 3*a^6*b*x^3 + a^7)*\sqrt{-(((-I*\sqrt{3} + 1)*(6561*c^2/a^8 - (6561 \\
& *b*c^2 + 560*a*d*e)/(a^8*b)))/(-1/27*c^3/a^{12} + 1/118098*(6561*b*c^2 + 560*a \\
& *d*e)*c/(a^{12}*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^{11}*b^2) - 1/28697 \\
& 814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^{12}*b \\
& ^2))^{1/3} + 59049*(I*\sqrt{3} + 1)*(-1/27*c^3/a^{12} + 1/118098*(6561*b*c^2 + \\
& 560*a*d*e)*c/(a^{12}*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^{11}*b^2) - 1 \\
& /28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(\\
& a^{12}*b^2)^{1/3} + 39366*c/a^4)^2*a^8*b - 78732*((-I*\sqrt{3} + 1)*(6561*c^2 \\
& /a^8 - (6561*b*c^2 + 560*a*d*e)/(a^8*b)))/(-1/27*c^3/a^{12} + 1/118098*(6561*b \\
& *c^2 + 560*a*d*e)*c/(a^{12}*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^{11}*b^ \\
& 2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)* \\
& a*b)/(a^{12}*b^2)^{1/3} + 59049*(I*\sqrt{3} + 1)*(-1/27*c^3/a^{12} + 1/118098*(\\
& 6561*b*c^2 + 560*a*d*e)*c/(a^{12}*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a \\
& ^{11}*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c \\
& *d*e)*a*b)/(a^{12}*b^2)^{1/3} + 39366*c/a^4)*a^4*b*c + 1549681956*b*c^2 + 52 \\
& 9079040*a*d*e)/(a^8*b)))*\log(-7/236196*((-I*\sqrt{3} + 1)*(6561*c^2/a^8 - (6 \\
& 561*b*c^2 + 560*a*d*e)/(a^8*b)))/(-1/27*c^3/a^{12} + 1/118098*(6561*b*c^2 + 56 \\
& 0*a*d*e)*c/(a^{12}*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^{11}*b^2) - 1/28 \\
& 697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^{1 \\
& 2}*b^2))^{1/3} + 59049*(I*\sqrt{3} + 1)*(-1/27*c^3/a^{12} + 1/118098*(6561*b*c^ \\
& 2 + 560*a*d*e)*c/(a^{12}*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^{11}*b^2) \\
& - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b) \\
&)/(a^{12}*b^2)^{1/3} + 39366*c/a^4)^2*a^8*b*e - 64800*b*c*d^2 - 45927*b*c^2* \\
& e - 7840*a*d*e^2 + 1/243*(400*a^4*b*d^2 + 567*a^4*b*c*e))*((-I*\sqrt{3} + 1)* \\
& (6561*c^2/a^8 - (6561*b*c^2 + 560*a*d*e)/(a^8*b)))/(-1/27*c^3/a^{12} + 1/11809 \\
& 8*(6561*b*c^2 + 560*a*d*e)*c/(a^{12}*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3) \\
& / (a^{11}*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 170 \\
& 1*c*d*e)*a*b)/(a^{12}*b^2)^{1/3} + 59049*(I*\sqrt{3} + 1)*(-1/27*c^3/a^{12} + 1 \\
& /118098*(6561*b*c^2 + 560*a*d*e)*c/(a^{12}*b) + 4/14348907*(8000*b*d^3 + 343* \\
& a*e^3)/(a^{11}*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 \\
& - 1701*c*d*e)*a*b)/(a^{12}*b^2)^{1/3} + 39366*c/a^4) + 8*(8000*b*d^3 + 343* \\
& a*e^3)*x + 1/78732*\sqrt{1/3}*(7*((-I*\sqrt{3} + 1)*(6561*c^2/a^8 - (6561*b*c
\end{aligned}$$

$$\begin{aligned}
&^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e) \\
&)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814* \\
&(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2)) \\
&^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560 \\
&*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/286 \\
&97814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12 \\
&*b^2))^(1/3) + 39366*c/a^4)*a^8*b*e + 388800*a^4*b*d^2 - 275562*a^4*b*c*e)* \\
&sqrt(-(((-I*sqrt(3) + 1)*(6561*c^2/a^8 - (6561*b*c^2 + 560*a*d*e)/(a^8*b)))/ \\
&(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907 \\
&*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^ \\
&2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 59049*(I*sqrt(3) \\
&+ 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14 \\
&348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2 \\
&744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 39366*c/a^ \\
&4)^2*a^8*b - 78732*((-I*sqrt(3) + 1)*(6561*c^2/a^8 - (6561*b*c^2 + 560*a*d* \\
&e)/(a^8*b)))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) \\
&+ 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c \\
&^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 5904 \\
&9*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^ \\
&12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441 \\
&*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) \\
&+ 39366*c/a^4)*a^4*b*c + 1549681956*b*c^2 + 529079040*a*d*e)/(a^8*b))) - (1 \\
&18098*b^3*c*x^9 + 354294*a*b^2*c*x^6 + 354294*a^2*b*c*x^3 + 118098*a^3*c - \\
&(a^4*b^3*x^9 + 3*a^5*b^2*x^6 + 3*a^6*b*x^3 + a^7))*((-I*sqrt(3) + 1)*(6561*c \\
&^2/a^8 - (6561*b*c^2 + 560*a*d*e)/(a^8*b)))/(-1/27*c^3/a^12 + 1/118098*(6561 \\
&*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11* \\
&b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e) \\
&)*a*b)/(a^12*b^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098 \\
&*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/ \\
&(a^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701 \\
&*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 39366*c/a^4) + 3*sqrt(1/3)*(a^4*b^3*x^9 + \\
&3*a^5*b^2*x^6 + 3*a^6*b*x^3 + a^7)*sqrt(-(((-I*sqrt(3) + 1)*(6561*c^2/a^8 - \\
&(6561*b*c^2 + 560*a*d*e)/(a^8*b)))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + \\
&560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1 \\
&/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(\\
&a^12*b^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b \\
&*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^ \\
&2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)* \\
&a*b)/(a^12*b^2))^(1/3) + 39366*c/a^4)^2*a^8*b - 78732*((-I*sqrt(3) + 1)*(65 \\
&61*c^2/a^8 - (6561*b*c^2 + 560*a*d*e)/(a^8*b)))/(-1/27*c^3/a^12 + 1/118098*(\\
&6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a \\
&^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c \\
&*d*e)*a*b)/(a^12*b^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/11 \\
&8098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e \\
&^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 -
\end{aligned}$$

$$\begin{aligned}
& 1701*c*d*e)*a*b)/(a^{12}*b^2)^{(1/3)} + 39366*c/a^4)*a^4*b*c + 1549681956*b*c^2 \\
& + 529079040*a*d*e)/(a^8*b)))*\log(-7/236196*((-I*\sqrt{3}) + 1)*(6561*c^2/a^8 \\
& - (6561*b*c^2 + 560*a*d*e)/(a^8*b)))/(-1/27*c^3/a^{12} + 1/118098*(6561*b*c^2 \\
& + 560*a*d*e)*c/(a^{12}*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^{11}*b^2) \\
& - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b) \\
&)/(a^{12}*b^2)^{(1/3)} + 59049*(I*\sqrt{3}) + 1)*(-1/27*c^3/a^{12} + 1/118098*(656 \\
& 1*b*c^2 + 560*a*d*e)*c/(a^{12}*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^{11} \\
& *b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d* \\
& e)*a*b)/(a^{12}*b^2)^{(1/3)} + 39366*c/a^4)^2*a^8*b*e - 64800*b*c*d^2 - 45927* \\
& b*c^2*e - 7840*a*d*e^2 + 1/243*(400*a^4*b*d^2 + 567*a^4*b*c*e)*((-I*\sqrt{3}) \\
& + 1)*(6561*c^2/a^8 - (6561*b*c^2 + 560*a*d*e)/(a^8*b)))/(-1/27*c^3/a^{12} + 1 \\
& /118098*(6561*b*c^2 + 560*a*d*e)*c/(a^{12}*b) + 4/14348907*(8000*b*d^3 + 343* \\
& a*e^3)/(a^{11}*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 \\
& - 1701*c*d*e)*a*b)/(a^{12}*b^2)^{(1/3)} + 59049*(I*\sqrt{3}) + 1)*(-1/27*c^3/a^ \\
& 12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^{12}*b) + 4/14348907*(8000*b*d^3 \\
& + 343*a*e^3)/(a^{11}*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(8 \\
& 00*d^3 - 1701*c*d*e)*a*b)/(a^{12}*b^2)^{(1/3)} + 39366*c/a^4) + 8*(8000*b*d^3 \\
& + 343*a*e^3)*x - 1/78732*\sqrt{1/3}*(7*((-I*\sqrt{3}) + 1)*(6561*c^2/a^8 - (65 \\
& 61*b*c^2 + 560*a*d*e)/(a^8*b)))/(-1/27*c^3/a^{12} + 1/118098*(6561*b*c^2 + 560 \\
& *a*d*e)*c/(a^{12}*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^{11}*b^2) - 1/286 \\
& 97814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^{12} \\
& *b^2)^{(1/3)} + 59049*(I*\sqrt{3}) + 1)*(-1/27*c^3/a^{12} + 1/118098*(6561*b*c^2 \\
& + 560*a*d*e)*c/(a^{12}*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^{11}*b^2) - \\
& 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b) \\
&)/(a^{12}*b^2)^{(1/3)} + 39366*c/a^4)*a^8*b*e + 388800*a^4*b*d^2 - 275562*a^4*b \\
& *c*e)*\sqrt{-(((-I*\sqrt{3}) + 1)*(6561*c^2/a^8 - (6561*b*c^2 + 560*a*d*e)/(a^ \\
& 8*b)))/(-1/27*c^3/a^{12} + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^{12}*b) + 4/14 \\
& 348907*(8000*b*d^3 + 343*a*e^3)/(a^{11}*b^2) - 1/28697814*(531441*b^2*c^3 + 2 \\
& 744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^{12}*b^2)^{(1/3)} + 59049*(I*s \\
& qrt(3) + 1)*(-1/27*c^3/a^{12} + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^{12}*b) \\
& + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^{11}*b^2) - 1/28697814*(531441*b^2*c \\
& ^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^{12}*b^2)^{(1/3)} + 3936 \\
& 6*c/a^4)^2*a^8*b - 78732*((-I*\sqrt{3}) + 1)*(6561*c^2/a^8 - (6561*b*c^2 + 56 \\
& 0*a*d*e)/(a^8*b)))/(-1/27*c^3/a^{12} + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^ \\
& 12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^{11}*b^2) - 1/28697814*(531441 \\
& *b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^{12}*b^2)^{(1/3)} \\
& + 59049*(I*\sqrt{3}) + 1)*(-1/27*c^3/a^{12} + 1/118098*(6561*b*c^2 + 560*a*d*e) \\
& *c/(a^{12}*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^{11}*b^2) - 1/28697814*(\\
& 531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^{12}*b^2)^{ \\
& (1/3)} + 39366*c/a^4)*a^4*b*c + 1549681956*b*c^2 + 529079040*a*d*e)/(a^8*b) \\
&) + 236196*(b^3*c*x^9 + 3*a*b^2*c*x^6 + 3*a^2*b*c*x^3 + a^3*c)*\log(x))/(a^4 \\
& *b^3*x^9 + 3*a^5*b^2*x^6 + 3*a^6*b*x^3 + a^7)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/x/(b*x**3+a)**4,x)

[Out] Timed out

Giac [A] time = 1.09416, size = 409, normalized size = 1.41

$$-\frac{c \log(|bx^3 + a|)}{3a^4} + \frac{c \log(|x|)}{a^4} + \frac{2\sqrt{3}\left(20(-ab^2)^{\frac{1}{3}}bd - 7(-ab^2)^{\frac{2}{3}}e\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243a^4b^2} + \frac{28ab^2x^8e + 40ab^2dx^7 + 54a^2c^2x^6 + 77a^2b^2x^5e + 104a^2b^2dx^4 + 135a^2b^2cx^3 + 67a^3x^2e + 82a^3dx + 99a^3c}{(bx^3 + a)^3a^4} + \frac{1}{243} \frac{20(-ab^2)^{\frac{1}{3}}ab^3d + 7(-ab^2)^{\frac{2}{3}}ab^2e \log(x^2 + x(-a/b)^{\frac{1}{3}} + (-a/b)^{\frac{2}{3}})}{(a^5b^4) - 2/243(7a^5b(-a/b)^{\frac{1}{3}}e + 20a^5bd)(-a/b)^{\frac{1}{3}} \log(\text{abs}(x - (-a/b)^{\frac{1}{3}}))}{(a^9b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^4,x, algorithm="giac")

[Out]
$$-1/3*c*\log(\text{abs}(b*x^3 + a))/a^4 + c*\log(\text{abs}(x))/a^4 + 2/243*\text{sqrt}(3)*(20*(-a*b^2)^{(1/3)}*b*d - 7*(-a*b^2)^{(2/3)}*e)*\arctan(1/3*\text{sqrt}(3)*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(a^4*b^2) + 1/162*(28*a*b^2*x^8*e + 40*a*b^2*d*x^7 + 54*a*b^2*c*x^6 + 77*a^2*b*x^5*e + 104*a^2*b*d*x^4 + 135*a^2*b*c*x^3 + 67*a^3*x^2*e + 82*a^3*d*x + 99*a^3*c)/((b*x^3 + a)^3*a^4) + 1/243*(20*(-a*b^2)^{(1/3)}*a*b^3*d + 7*(-a*b^2)^{(2/3)}*a*b^2*e)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a^5*b^4) - 2/243*(7*a^5*b*(-a/b)^{(1/3)}*e + 20*a^5*b*d)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/a^9*b$$

$$3.362 \quad \int \frac{c+dx+ex^2}{x^2(a+bx^3)^4} dx$$

Optimal. Leaf size=301

$$-\frac{10(2a^{2/3}e + 7b^{2/3}c) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{243a^{13/3}\sqrt[3]{b}} + \frac{20(2a^{2/3}e + 7b^{2/3}c) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{243a^{13/3}\sqrt[3]{b}} + \frac{20(7b^{2/3}c - 2a^{2/3}e) \tan^{-1}\left(\frac{a^{1/3} - 2b^{1/3}x}{\sqrt[3]{a^2 + b^2x^2}}\right)}{81\sqrt{3}a^{13/3}\sqrt[3]{b}}$$

[Out] $-(c/(a^4x)) + (x*(a*e - b*c*x - b*d*x^2))/(9*a^2*(a + b*x^3)^3) + (x*(8*a*e - 16*b*c*x - 15*b*d*x^2))/(54*a^3*(a + b*x^3)^2) + (x*(40*a*e - 118*b*c*x - 99*b*d*x^2))/(162*a^4*(a + b*x^3)) + (20*(7*b^(2/3)*c - 2*a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(81*Sqrt[3]*a^(13/3)*b^(1/3)) + (d*Log[x])/a^4 + (20*(7*b^(2/3)*c + 2*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x]/(243*a^(13/3)*b^(1/3)) - (10*(7*b^(2/3)*c + 2*a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(243*a^(13/3)*b^(1/3)) - (d*Log[a + b*x^3])/(3*a^4)$

Rubi [A] time = 0.601175, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{10(2a^{2/3}e + 7b^{2/3}c) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{243a^{13/3}\sqrt[3]{b}} + \frac{20(2a^{2/3}e + 7b^{2/3}c) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{243a^{13/3}\sqrt[3]{b}} + \frac{20(7b^{2/3}c - 2a^{2/3}e) \tan^{-1}\left(\frac{a^{1/3} - 2b^{1/3}x}{\sqrt[3]{a^2 + b^2x^2}}\right)}{81\sqrt{3}a^{13/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^4), x]

[Out] $-(c/(a^4x)) + (x*(a*e - b*c*x - b*d*x^2))/(9*a^2*(a + b*x^3)^3) + (x*(8*a*e - 16*b*c*x - 15*b*d*x^2))/(54*a^3*(a + b*x^3)^2) + (x*(40*a*e - 118*b*c*x - 99*b*d*x^2))/(162*a^4*(a + b*x^3)) + (20*(7*b^(2/3)*c - 2*a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(81*Sqrt[3]*a^(13/3)*b^(1/3)) + (d*Log[x])/a^4 + (20*(7*b^(2/3)*c + 2*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x]/(243*a^(13/3)*b^(1/3)) - (10*(7*b^(2/3)*c + 2*a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(243*a^(13/3)*b^(1/3)) - (d*Log[a + b*x^3])/(3*a^4)$

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2}{x^2(a + bx^3)^4} dx &= \frac{x(ae - bcx - bdx^2)}{9a^2(a + bx^3)^3} - \frac{\int \frac{-9bc - 9bdx - 8bex^2 + \frac{7b^2cx^3}{a} + \frac{6b^2dx^4}{a}}{x^2(a + bx^3)^3} dx}{9ab} \\
&= \frac{x(ae - bcx - bdx^2)}{9a^2(a + bx^3)^3} + \frac{x(8ae - 16bcx - 15bdx^2)}{54a^3(a + bx^3)^2} + \frac{\int \frac{54b^3c + 54b^3dx + 40b^3ex^2 - \frac{64b^4cx^3}{a} - \frac{45b^4dx^4}{a}}{x^2(a + bx^3)^2} dx}{54a^2b^3} \\
&= \frac{x(ae - bcx - bdx^2)}{9a^2(a + bx^3)^3} + \frac{x(8ae - 16bcx - 15bdx^2)}{54a^3(a + bx^3)^2} + \frac{x(40ae - 118bcx - 99bdx^2)}{162a^4(a + bx^3)} - \frac{\int \frac{-162b^5c - 162b^5dx}{x^2(a + bx^3)}}{162a^4} \\
&= \frac{x(ae - bcx - bdx^2)}{9a^2(a + bx^3)^3} + \frac{x(8ae - 16bcx - 15bdx^2)}{54a^3(a + bx^3)^2} + \frac{x(40ae - 118bcx - 99bdx^2)}{162a^4(a + bx^3)} - \frac{\int \left(-\frac{162b^5c}{ax^2} - \frac{162b^5dx}{ax} \right)}{162a^4} \\
&= -\frac{c}{a^4x} + \frac{x(ae - bcx - bdx^2)}{9a^2(a + bx^3)^3} + \frac{x(8ae - 16bcx - 15bdx^2)}{54a^3(a + bx^3)^2} + \frac{x(40ae - 118bcx - 99bdx^2)}{162a^4(a + bx^3)} + \frac{d \log(x)}{a^4} \\
&= -\frac{c}{a^4x} + \frac{x(ae - bcx - bdx^2)}{9a^2(a + bx^3)^3} + \frac{x(8ae - 16bcx - 15bdx^2)}{54a^3(a + bx^3)^2} + \frac{x(40ae - 118bcx - 99bdx^2)}{162a^4(a + bx^3)} + \frac{d \log(x)}{a^4} \\
&= -\frac{c}{a^4x} + \frac{x(ae - bcx - bdx^2)}{9a^2(a + bx^3)^3} + \frac{x(8ae - 16bcx - 15bdx^2)}{54a^3(a + bx^3)^2} + \frac{x(40ae - 118bcx - 99bdx^2)}{162a^4(a + bx^3)} + \frac{d \log(x)}{a^4} \\
&= -\frac{c}{a^4x} + \frac{x(ae - bcx - bdx^2)}{9a^2(a + bx^3)^3} + \frac{x(8ae - 16bcx - 15bdx^2)}{54a^3(a + bx^3)^2} + \frac{x(40ae - 118bcx - 99bdx^2)}{162a^4(a + bx^3)} + \frac{d \log(x)}{a^4} \\
&= -\frac{c}{a^4x} + \frac{x(ae - bcx - bdx^2)}{9a^2(a + bx^3)^3} + \frac{x(8ae - 16bcx - 15bdx^2)}{54a^3(a + bx^3)^2} + \frac{x(40ae - 118bcx - 99bdx^2)}{162a^4(a + bx^3)} + \frac{d \log(x)}{a^4} \\
&= -\frac{c}{a^4x} + \frac{x(ae - bcx - bdx^2)}{9a^2(a + bx^3)^3} + \frac{x(8ae - 16bcx - 15bdx^2)}{54a^3(a + bx^3)^2} + \frac{x(40ae - 118bcx - 99bdx^2)}{162a^4(a + bx^3)} + \frac{20(7b^{2/3})}{162a^4(a + bx^3)} + \frac{d \log(x)}{a^4}
\end{aligned}$$

Mathematica [A] time = 0.29171, size = 279, normalized size = 0.93

$$\frac{\frac{20(7a^{2/3}b^{2/3}c+2a^{4/3}e)\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2)}{\sqrt[3]{b}} + \frac{40(7a^{2/3}b^{2/3}c+2a^{4/3}e)\log(\sqrt[3]{a}+\sqrt[3]{bx})}{\sqrt[3]{b}} - \frac{40\sqrt[3]{3}a^{2/3}(2a^{2/3}e-7b^{2/3}c)\tan^{-1}\left(\frac{1-2\sqrt[3]{bx}}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}} + \frac{54a^3(a(dx)-b)}{(a+bx^3)^3}}{486a^5}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^4), x]

[Out] ((-486*a*c)/x + (9*a^2*(9*a*d + 8*a*e*x - 16*b*c*x^2))/(a + b*x^3)^2 + (6*a*(27*a*d + 20*a*e*x - 59*b*c*x^2))/(a + b*x^3) + (54*a^3*(-(b*c*x^2) + a*(d + e*x)))/(a + b*x^3)^3 - (40*sqrt[3]*a^(2/3)*(-7*b^(2/3)*c + 2*a^(2/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]]/b^(1/3) + 486*a*d*Log[x] + (40*(7*a^(2/3)*b^(2/3)*c + 2*a^(4/3)*e)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (20*(7*a^(2/3)*b^(2/3)*c + 2*a^(4/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3) - 162*a*d*Log[a + b*x^3))/(486*a^5)

Maple [A] time = 0.018, size = 397, normalized size = 1.3

$$-\frac{59b^3cx^8}{81a^4(bx^3+a)^3} + \frac{20b^2ex^7}{81a^3(bx^3+a)^3} + \frac{b^2dx^6}{3a^3(bx^3+a)^3} - \frac{142b^2cx^5}{81a^3(bx^3+a)^3} + \frac{52bex^4}{81a^2(bx^3+a)^3} + \frac{5bdx^3}{6a^2(bx^3+a)^3} - \frac{9}{81a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)/x^2/(b*x^3+a)^4, x)

[Out] -59/81/a^4/(b*x^3+a)^3*b^3*c*x^8+20/81/a^3/(b*x^3+a)^3*x^7*b^2*e+1/3/a^3/(b*x^3+a)^3*b^2*d*x^6-142/81/a^3/(b*x^3+a)^3*c*b^2*x^5+52/81/a^2/(b*x^3+a)^3*b*e*x^4+5/6/a^2/(b*x^3+a)^3*b*d*x^3-92/81/a^2/(b*x^3+a)^3*b*c*x^2+41/81/a/(b*x^3+a)^3*e*x+11/18/a/(b*x^3+a)^3*d+40/243/a^3*e/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-20/243/a^3*e/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+40/243/a^3*e/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+140/243/a^4*c/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-70/243/a^4*c/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-140/243/a^4*c*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/3*d*ln(b*x^3+a)/a^4-c/a^4/x+d*ln(x)/a^4

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [C] time = 7.53068, size = 15244, normalized size = 50.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^4,x, algorithm="fricas")

[Out]
$$-1/236196*(408240*b^3*c*x^9 - 58320*a*b^2*e*x^8 - 78732*a*b^2*d*x^7 + 112260*a*b^2*c*x^6 - 151632*a^2*b*e*x^5 - 196830*a^2*b*d*x^4 + 976860*a^2*b*c*x^3 - 119556*a^3*e*x^2 - 144342*a^3*d*x + 236196*a^3*c + 2*(a^4*b^3*x^{10} + 3*a^5*b^2*x^7 + 3*a^6*b*x^4 + a^7*x)*((-I*\sqrt{3}) + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^{12} + 1/118098*(6561*d^2 - 5600*c*e)*d/a^{12} + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^{13}*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^{13}*b))^{1/3} + 59049*(I*\sqrt{3}) + 1)*(-1/27*d^3/a^{12} + 1/118098*(6561*d^2 - 5600*c*e)*d/a^{12} + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^{13}*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^{13}*b))^{1/3} + 39366*d/a^4)*\log(-7/236196*((-I*\sqrt{3}) + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^{12} + 1/118098*(6561*d^2 - 5600*c*e)*d/a^{12} + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^{13}*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^{13}*b))^{1/3} + 59049*(I*\sqrt{3}) + 1)*(-1/27*d^3/a^{12} + 1/118098*(6561*d^2 - 5600*c*e)*d/a^{12} + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^{13}*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^{13}*b))^{1/3} + 39366*d/a^4)^2*a^9*b*c - 45927*a*b*c*d^2 + 78400*a*b*c^2*e + 6480*a^2*d*e^2 + 1/243*(567*a^5*b*c*d - 40*a^6*e^2)*((-I*\sqrt{3}) + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^{12} + 1/118098*(6561*d^2 - 5600*c*e)*d/a^{12} + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^{13}*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^{13}*b))^{1/3}$$

$$\begin{aligned}
& /a^{12} + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^{13}*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^{13}*b))^{(1/3)} \\
& + 59049*(I*\sqrt{3} + 1)*(-1/27*d^3/a^{12} + 1/118098*(6561*d^2 - 5600*c*e)*d/a^{12} + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^{13}*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^{13}*b))^{(1/3)} \\
& + 39366*d/a^4)*a^9*b*c - 275562*a^5*b*c*d - 38880*a^6*e^2)*\sqrt{-(((-I*\sqrt{3} + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^{12} + 1/118098*(6561*d^2 - 5600*c*e)*d/a^{12} + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^{13}*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^{13}*b))^{(1/3)} + 59049*(I*\sqrt{3} + 1)*(-1/27*d^3/a^{12} + 1/118098*(6561*d^2 - 5600*c*e)*d/a^{12} + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^{13}*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^{13}*b))^{(1/3)} + 39366*d/a^4)^2*a^8 - 78732*((-I*\sqrt{3} + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^{12} + 1/118098*(6561*d^2 - 5600*c*e)*d/a^{12} + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^{13}*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^{13}*b))^{(1/3)} + 59049*(I*\sqrt{3} + 1)*(-1/27*d^3/a^{12} + 1/118098*(6561*d^2 - 5600*c*e)*d/a^{12} + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^{13}*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^{13}*b))^{(1/3)} + 39366*d/a^4)*a^4*d + 1549681956*d^2 - 5290790400*c*e/a^8)) + (118098*b^3*d*x^{10} + 354294*a*b^2*d*x^7 + 354294*a^2*b*d*x^4 + 118098*a^3*d*x - (a^4*b^3*x^{10} + 3*a^5*b^2*x^7 + 3*a^6*b*x^4 + a^7*x))*((-I*\sqrt{3} + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^{12} + 1/118098*(6561*d^2 - 5600*c*e)*d/a^{12} + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^{13}*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^{13}*b))^{(1/3)} + 59049*(I*\sqrt{3} + 1)*(-1/27*d^3/a^{12} + 1/118098*(6561*d^2 - 5600*c*e)*d/a^{12} + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^{13}*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^{13}*b))^{(1/3)} + 39366*d/a^4) - 3*\sqrt{3}*(a^4*b^3*x^{10} + 3*a^5*b^2*x^7 + 3*a^6*b*x^4 + a^7*x)*\sqrt{-(((-I*\sqrt{3} + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^{12} + 1/118098*(6561*d^2 - 5600*c*e)*d/a^{12} + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^{13}*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^{13}*b))^{(1/3)} + 59049*(I*\sqrt{3} + 1)*(-1/27*d^3/a^{12} + 1/118098*(6561*d^2 - 5600*c*e)*d/a^{12} + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^{13}*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^{13}*b))^{(1/3)} + 39366*d/a^4)^2*a^8 - 78732*((-I*\sqrt{3} + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^{12} + 1/118098*(6561*d^2 - 5600*c*e)*d/a^{12} + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^{13}*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^{13}*b))^{(1/3)} + 59049*(I*\sqrt{3} + 1)*(-1/27*d^3/a^{12} + 1/118098*(6561*d^2 - 5600*c*e)*d/a^{12} + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^{13}*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^{13}*b))^{(1/3)} + 39366*d/a^4)*a^4*d + 1549681956*d^2 - 5290790400*c*e/a^8))*\log(7/236196*((-I*\sqrt{3} + 1)*(6561*d^2/a^8 - (6561*d^2
\end{aligned}$$

$$\begin{aligned}
& - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + \\
& 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)* \\
& a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^{(1/3)} + 5 \\
& 9049*(I*\sqrt{3} + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^1 \\
& 2 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d* \\
& e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^{(1/3)} \\
& + 39366*d/a^4)^2*a^9*b*c + 45927*a*b*c*d^2 - 78400*a*b*c^2*e - 6480*a^2*d*e \\
& ^2 - 1/243*(567*a^5*b*c*d - 40*a^6*e^2)*((-I*\sqrt{3} + 1)*(6561*d^2/a^8 - (\\
& 6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)* \\
& d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600 \\
& *c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^{(\\
& 1/3)} + 59049*(I*\sqrt{3} + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c* \\
& e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5 \\
& 600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b) \\
&)^{(1/3)} + 39366*d/a^4) - 800*(343*b^2*c^3 - 8*a^2*e^3)*x - 1/78732*\sqrt{1/3} \\
&)*(7*((-I*\sqrt{3} + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^ \\
& 3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^ \\
& 3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/143489 \\
& 07*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^{(1/3)} + 59049*(I*\sqrt{3} + 1)*(-1/27 \\
& *d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2 \\
& *c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/143 \\
& 48907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^{(1/3)} + 39366*d/a^4)*a^9*b*c - 27 \\
& 5562*a^5*b*c*d - 38880*a^6*e^2)*\sqrt{-(((-I*\sqrt{3} + 1)*(6561*d^2/a^8 - (6 \\
& 561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d \\
& /a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600* \\
& c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^{(1 \\
& /3)} + 59049*(I*\sqrt{3} + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e \\
&)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 56 \\
& 00*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b)) \\
& ^{(1/3)} + 39366*d/a^4)^2*a^8 - 78732*((-I*\sqrt{3} + 1)*(6561*d^2/a^8 - (6561 \\
& *d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^ \\
& 12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d \\
& *e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^{(1/3)} \\
& + 59049*(I*\sqrt{3} + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d \\
& /a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600* \\
& c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^{(1 \\
& /3)} + 39366*d/a^4)*a^4*d + 1549681956*d^2 - 5290790400*c*e)/a^8)) - 236196* \\
& (b^3*d*x^10 + 3*a*b^2*d*x^7 + 3*a^2*b*d*x^4 + a^3*d*x)*\log(x))/(a^4*b^3*x^1 \\
& 0 + 3*a^5*b^2*x^7 + 3*a^6*b*x^4 + a^7*x)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**4,x)

[Out] Timed out

Giac [A] time = 1.07262, size = 427, normalized size = 1.42

$$-\frac{d \log(|bx^3 + a|)}{3a^4} + \frac{d \log(|x|)}{a^4} + \frac{10 \left(2 (-ab^2)^{\frac{1}{3}} ae - 7 (-ab^2)^{\frac{2}{3}} c \right) \log \left(x^2 + x \left(-\frac{a}{b} \right)^{\frac{1}{3}} + \left(-\frac{a}{b} \right)^{\frac{2}{3}} \right)}{243 a^5 b} + \frac{20 \sqrt{3} \left(2 (-ab^2)^{\frac{1}{3}} ab^2 e \right)}{243 a^5 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^4,x, algorithm="giac")

[Out] $-\frac{1}{3}d \log(\text{abs}(bx^3 + a))/a^4 + d \log(\text{abs}(x))/a^4 + \frac{10}{243} \frac{(2(-ab^2)^{1/3}ae - 7(-ab^2)^{2/3}c) \log(x^2 + x(-a/b)^{1/3} + (-a/b)^{2/3})}{a^5 b} + \frac{20}{243} \frac{\sqrt{3} (2(-ab^2)^{1/3}ab^2e + 7(-ab^2)^{2/3}b^2c) \arctan(1/3 \sqrt{3} (2x + (-a/b)^{1/3}) / (-a/b)^{1/3})}{a^5 b^3} - \frac{1}{162} \frac{(280b^3cx^9 - 40ab^2x^8e - 54a^2b^2dx^7 + 770ab^2cx^6 - 104a^2bx^5e - 135a^2bdx^4 + 670a^2b^2cx^3 - 82a^3x^2e - 99a^3dx + 162a^3c)}{(bx^3 + a)^3 a^4 x} + \frac{20}{243} \frac{(7a^4b^2c(-a/b)^{1/3} - 2a^5be) (-a/b)^{1/3} \log(\text{abs}(x - (-a/b)^{1/3}))}{a^9 b}$

$$3.363 \quad \int \frac{c+dx+ex^2}{x^3(a+bx^3)^4} dx$$

Optimal. Leaf size=310

$$\frac{10\sqrt[3]{b}(11\sqrt[3]{bc}-7\sqrt[3]{ad})\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2)}{243a^{14/3}} - \frac{x(139bc+118bdx+99bex^2)}{162a^4(a+bx^3)} - \frac{x(17bc+16bdx+15bex^2)}{54a^3(a+bx^3)^2} - \frac{x(11bc+10bdx+9bex^2)}{9a^2(a+bx^3)^3}$$

[Out] $-c/(2*a^4*x^2) - d/(a^4*x) - (x*(b*c + b*d*x + b*e*x^2))/(9*a^2*(a + b*x^3)^3) - (x*(17*b*c + 16*b*d*x + 15*b*e*x^2))/(54*a^3*(a + b*x^3)^2) - (x*(139*b*c + 118*b*d*x + 99*b*e*x^2))/(162*a^4*(a + b*x^3)) + (20*b^(1/3)*(11*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(8*1*Sqrt[3]*a^(14/3)) + (e*Log[x])/a^4 - (20*b^(1/3)*(11*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x]/(243*a^(14/3)) + (10*b^(1/3)*(11*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(243*a^(14/3)) - (e*Log[a + b*x^3])/(3*a^4)$

Rubi [A] time = 0.65622, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{10\sqrt[3]{b}(11\sqrt[3]{bc}-7\sqrt[3]{ad})\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2)}{243a^{14/3}} - \frac{x(139bc+118bdx+99bex^2)}{162a^4(a+bx^3)} - \frac{x(17bc+16bdx+15bex^2)}{54a^3(a+bx^3)^2} - \frac{x(11bc+10bdx+9bex^2)}{9a^2(a+bx^3)^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^4), x]

[Out] $-c/(2*a^4*x^2) - d/(a^4*x) - (x*(b*c + b*d*x + b*e*x^2))/(9*a^2*(a + b*x^3)^3) - (x*(17*b*c + 16*b*d*x + 15*b*e*x^2))/(54*a^3*(a + b*x^3)^2) - (x*(139*b*c + 118*b*d*x + 99*b*e*x^2))/(162*a^4*(a + b*x^3)) + (20*b^(1/3)*(11*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(8*1*Sqrt[3]*a^(14/3)) + (e*Log[x])/a^4 - (20*b^(1/3)*(11*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x]/(243*a^(14/3)) + (10*b^(1/3)*(11*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(243*a^(14/3)) - (e*Log[a + b*x^3])/(3*a^4)$

Rule 1829

```

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]

```

Rule 1834

```

Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Int[Ex
pandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]

```

Rule 1871

```

Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

```

Rule 1860

```

Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]

```

Rule 31

```

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]

```

Rule 634

```

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

```

Rule 617


```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

Mathematica [A] time = 0.286433, size = 284, normalized size = 0.92

$$20\sqrt[3]{b}\left(11\sqrt[3]{a}\sqrt[3]{bc}-7a^{2/3}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2\right)+\frac{54a^3(ae-bx(c+dx))}{(a+bx^3)^3}+\frac{9a^2(9ae-bx(17c+16dx))}{(a+bx^3)^2}+40\sqrt[3]{b}\left(7a^{2/3}d-11\sqrt[3]{a}\sqrt[3]{bc}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^4), x]

[Out] $\frac{(-243ac)/x^2 - (486ad)/x + (54a^3(ae - bxc + dx))}{(a + bx^3)^3} + \frac{9a^2(9ae - bxc + dx)}{(a + bx^3)^2} + \frac{3a(54ae - bxc + 139c + 118dx)}{(a + bx^3)} + 40\sqrt[3]{3}a^{1/3}b^{1/3}(11b^{1/3}c + 7a^{1/3}d)\operatorname{ArcTan}\left[\frac{1 - (2b^{1/3}x)/a^{1/3}}{\sqrt[3]{3}}\right] + 486ae\operatorname{Log}[x] + 40b^{1/3}(-11a^{1/3}b^{1/3}c + 7a^{2/3}d)\operatorname{Log}[a^{1/3} + b^{1/3}x] + 20b^{1/3}(11a^{1/3}b^{1/3}c - 7a^{2/3}d)\operatorname{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2] - 162ae\operatorname{Log}[a + bx^3]/(486a^5)$

Maple [A] time = 0.019, size = 400, normalized size = 1.3

$$\frac{59b^3dx^8}{81a^4(bx^3+a)^3} - \frac{139b^3cx^7}{162a^4(bx^3+a)^3} + \frac{b^2ex^6}{3a^3(bx^3+a)^3} - \frac{142b^2dx^5}{81a^3(bx^3+a)^3} - \frac{329b^2cx^4}{162a^3(bx^3+a)^3} + \frac{5bex^3}{6a^2(bx^3+a)^3} - \frac{c}{81a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)/x^3/(b*x^3+a)^4, x)

[Out] $-59/81*b^3/a^4/(b*x^3+a)^3*d*x^8-139/162*b^3/a^4/(b*x^3+a)^3*c*x^7+1/3*b^2/a^3/(b*x^3+a)^3*e*x^6-142/81*b^2/a^3/(b*x^3+a)^3*d*x^5-329/162*b^2/a^3/(b*x^3+a)^3*c*x^4+5/6*b/a^2/(b*x^3+a)^3*e*x^3-92/81*b/a^2/(b*x^3+a)^3*d*x^2-104/81*b/a^2/(b*x^3+a)^3*c*x+11/18/a/(b*x^3+a)^3*e-220/243/a^4*c/(1/b*a)^(2/3)*\ln(x+(1/b*a)^(1/3))+110/243/a^4*c/(1/b*a)^(2/3)*\ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-220/243/a^4*c/(1/b*a)^(2/3)*3^(1/2)*\arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+140/243/a^4*d/(1/b*a)^(1/3)*\ln(x+(1/b*a)^(1/3))-70/243/a^4*d/(1/b*a)^(1/3)*\ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-140/243/a^4*d*3^(1/2)/(1/b*a)^(1/3)*\arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/3*e*\ln(b*x^3+a)/a^4-1/2*c/a^4/x^2-d/a^4/x+e*\ln(x)/a^4$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [C] time = 7.40401, size = 15485, normalized size = 49.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^4,x, algorithm="fricas")

[Out]
$$-1/236196*(408240*b^3*d*x^{10} + 320760*b^3*c*x^9 - 78732*a*b^2*e*x^8 + 112260*a*b^2*d*x^7 + 833976*a*b^2*c*x^6 - 196830*a^2*b*e*x^5 + 976860*a^2*b*d*x^4 + 657558*a^2*b*c*x^3 - 144342*a^3*e*x^2 + 236196*a^3*d*x + 118098*a^3*c + 2*(a^4*b^3*x^{11} + 3*a^5*b^2*x^8 + 3*a^6*b*x^5 + a^7*x^2)*((-I*\sqrt{3} + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^{12} + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^{13} + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^{14} - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^{14})^{1/3} + 59049*(I*\sqrt{3} + 1)*(-1/27*e^3/a^{12} + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^{13} + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^{14} - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^{14})^{1/3} + 39366*e/a^4)*\log(7/236196*((-I*\sqrt{3} + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^{12} + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^{13} + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^{14} - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^{14})^{1/3} + 59049*(I*\sqrt{3} + 1)*(-1/27*e^3/a^{12} + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^{13} + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^{14} - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^{14})^{1/3} + 39366*e/a^4)^2*a^{10}*d + 431200*a*b*c*d^2 - 196020*a*b*c^2*e + 45927*a^2*d*e^2 + 1/243*(1210*a^5*b*c^2 - 567*a^6*d*e)*((-I*\sqrt{3} + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^{12} + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^{13} + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^{14} - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^{14})^{1/3} + 59049*(I*\sqrt{3} + 1)*(-1/27*e^3/a^{12} + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^{13} + 4000/14348907*(1$$

$$\begin{aligned}
& 331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e \\
& ^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^{(1/3)} + 39366*e/a^4) + 400*(133 \\
& 1*b^2*c^3 + 343*a*b*d^3)*x) + (118098*b^3*e*x^11 + 354294*a*b^2*e*x^8 + 354 \\
& 294*a^2*b*e*x^5 + 118098*a^3*e*x^2 - (a^4*b^3*x^11 + 3*a^5*b^2*x^8 + 3*a^6* \\
& b*x^5 + a^7*x^2))*((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^ \\
& 2)/a^9)/(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000 \\
& /14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + \\
& 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^{(1/3)} + 59049*(I*sq \\
& rt(3) + 1)*(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4 \\
& 000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 \\
& + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^{(1/3)} + 39366*e/ \\
& a^4) - 3*sqrt(1/3)*(a^4*b^3*x^11 + 3*a^5*b^2*x^8 + 3*a^6*b*x^5 + a^7*x^2)*s \\
& qrt(-(((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1 \\
& /27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(\\
& 1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2* \\
& e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^{(1/3)} + 59049*(I*sqrt(3) + 1)* \\
& (-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/1434890 \\
& 7*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a \\
& ^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^{(1/3)} + 39366*e/a^4)^2*a^9 \\
& - 78732*((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(- \\
& -1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907 \\
& *(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^ \\
& 2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^{(1/3)} + 59049*(I*sqrt(3) + 1 \\
&)*(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348 \\
& 907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441 \\
& *a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^{(1/3)} + 39366*e/a^4)*a^5* \\
& e + 29099347200*b*c*d + 1549681956*a*e^2)/a^9))*log(-7/236196*((-I*sqrt(3) \\
& + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^12 + 1/11 \\
& 8098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a* \\
& d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 \\
& - 2673*c*d*e)*a*b)/a^14)^{(1/3)} + 59049*(I*sqrt(3) + 1)*(-1/27*e^3/a^12 + 1 \\
& /118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343 \\
& *a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980* \\
& d^3 - 2673*c*d*e)*a*b)/a^14)^{(1/3)} + 39366*e/a^4)^2*a^10*d - 431200*a*b*c*d \\
& ^2 + 196020*a*b*c^2*e - 45927*a^2*d*e^2 - 1/243*(1210*a^5*b*c^2 - 567*a^6*d \\
& *e))*((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/2 \\
& 7*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(13 \\
& 31*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^ \\
& 3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^{(1/3)} + 59049*(I*sqrt(3) + 1)*(- \\
& 1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907* \\
& (1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2 \\
& *e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^{(1/3)} + 39366*e/a^4) + 800*(1 \\
& 331*b^2*c^3 + 343*a*b*d^3)*x + 1/78732*sqrt(1/3)*(7*((-I*sqrt(3) + 1)*(6561 \\
& *e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^12 + 1/118098*(3080 \\
& 0*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^1
\end{aligned}$$

$$\begin{aligned}
& 4 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^{14})^{(1/3)} + 59049*(I*\sqrt{3} + 1)*(-1/27*e^3/a^{12} + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^{13} + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^{14} - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^{14})^{(1/3)} + 39366*e/a^4)*a^{10}*d - 1176120*a^5*b*c^2 - 275562*a^6*d*e)*\sqrt{-(((-I*\sqrt{3} + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^{12} + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^{13} + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^{14} - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^{14})^{(1/3)} + 59049*(I*\sqrt{3} + 1)*(-1/27*e^3/a^{12} + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^{13} + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^{14} - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^{14})^{(1/3)} + 39366*e/a^4)^2*a^9 - 78732*((-I*\sqrt{3} + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^{12} + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^{13} + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^{14} - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^{14})^{(1/3)} + 59049*(I*\sqrt{3} + 1)*(-1/27*e^3/a^{12} + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^{13} + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^{14} - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^{14})^{(1/3)} + 39366*e/a^4)*a^5*e + 29099347200*b*c*d + 1549681956*a*e^2)/a^9)) + (118098*b^3*e*x^{11} + 354294*a*b^2*e*x^8 + 354294*a^2*b*e*x^5 + 118098*a^3*e*x^2 - (a^4*b^3*x^{11} + 3*a^5*b^2*x^8 + 3*a^6*b*x^5 + a^7*x^2))*((-I*\sqrt{3} + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^{12} + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^{13} + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^{14} - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^{14})^{(1/3)} + 59049*(I*\sqrt{3} + 1)*(-1/27*e^3/a^{12} + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^{13} + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^{14} - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^{14})^{(1/3)} + 39366*e/a^4) + 3*\sqrt{1/3}*(a^4*b^3*x^{11} + 3*a^5*b^2*x^8 + 3*a^6*b*x^5 + a^7*x^2)*\sqrt{-(((-I*\sqrt{3} + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^{12} + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^{13} + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^{14} - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^{14})^{(1/3)} + 59049*(I*\sqrt{3} + 1)*(-1/27*e^3/a^{12} + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^{13} + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^{14} - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^{14})^{(1/3)} + 39366*e/a^4)^2*a^9 - 78732*((-I*\sqrt{3} + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^{12} + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^{13} + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^{14} - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^{14})^{(1/3)} + 59049*(I*\sqrt{3} + 1)*(-1/27*e^3/a^{12} + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^{13} + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^{14} - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^{14})^{(1/3)} + 39366*e/a^4)*a^5*e + 29099347200*b*c*d + 1549681956*a*e^2)/a^9))*\log(-7/236196*((-I*\sqrt{3} + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^{12} + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^{13} + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^{14} - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^{14})^{(1/3)} + 59049*(I*\sqrt{3} + 1)*(-1/27*e^3/a^{12} + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^{13} + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^{14} - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^{14})^{(1/3)} + 39366*e/a^4)
\end{aligned}$$

$$\begin{aligned}
&^2/a^9)/(-1/27*e^3/a^{12} + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^{13} + 400 \\
&0/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^{14} - 1/28697814*(10648000*b^2*c^3 + \\
&531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^{14})^{(1/3)} + 59049*(I*s \\
&qrt(3) + 1)*(-1/27*e^3/a^{12} + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^{13} + \\
&4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^{14} - 1/28697814*(10648000*b^2*c^ \\
&3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^{14})^{(1/3)} + 39366*e \\
&/a^4)^2*a^{10}*d - 431200*a*b*c*d^2 + 196020*a*b*c^2*e - 45927*a^2*d*e^2 - 1/ \\
&243*(1210*a^5*b*c^2 - 567*a^6*d*e)*((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800 \\
&*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^{12} + 1/118098*(30800*b*c*d + 6561*a* \\
&e^2)*e/a^{13} + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^{14} - 1/28697814*(1 \\
&0648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^{14})^{(\\
&1/3)} + 59049*(I*sqrt(3) + 1)*(-1/27*e^3/a^{12} + 1/118098*(30800*b*c*d + 6561 \\
&*a*e^2)*e/a^{13} + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^{14} - 1/28697814 \\
&*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^{14} \\
&)^{(1/3)} + 39366*e/a^4) + 800*(1331*b^2*c^3 + 343*a*b*d^3)*x - 1/78732*sqrt(\\
&1/3)*(7*((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(- \\
&-1/27*e^3/a^{12} + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^{13} + 4000/14348907 \\
&*(1331*b*c^3 + 343*a*d^3)*b/a^{14} - 1/28697814*(10648000*b^2*c^3 + 531441*a^ \\
&2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^{14})^{(1/3)} + 59049*(I*sqrt(3) + 1 \\
&)*(-1/27*e^3/a^{12} + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^{13} + 4000/14348 \\
&907*(1331*b*c^3 + 343*a*d^3)*b/a^{14} - 1/28697814*(10648000*b^2*c^3 + 531441 \\
&*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^{14})^{(1/3)} + 39366*e/a^4)*a^{10} \\
&*d - 1176120*a^5*b*c^2 - 275562*a^6*d*e)*sqrt(-(((-I*sqrt(3) + 1)*(6561*e^2 \\
&/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^{12} + 1/118098*(30800*b* \\
&c*d + 6561*a*e^2)*e/a^{13} + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^{14} - \\
&1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e) \\
&)*a*b)/a^{14})^{(1/3)} + 59049*(I*sqrt(3) + 1)*(-1/27*e^3/a^{12} + 1/118098*(30800 \\
&*b*c*d + 6561*a*e^2)*e/a^{13} + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^{14} \\
&- 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d \\
&e)*a*b)/a^{14})^{(1/3)} + 39366*e/a^4)^2*a^9 - 78732*((-I*sqrt(3) + 1)*(6561*e \\
&^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^{12} + 1/118098*(30800* \\
&b*c*d + 6561*a*e^2)*e/a^{13} + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^{14} \\
&- 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d* \\
&e)*a*b)/a^{14})^{(1/3)} + 59049*(I*sqrt(3) + 1)*(-1/27*e^3/a^{12} + 1/118098*(308 \\
&00*b*c*d + 6561*a*e^2)*e/a^{13} + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^ \\
&14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c \\
&*d*e)*a*b)/a^{14})^{(1/3)} + 39366*e/a^4)*a^5*e + 29099347200*b*c*d + 154968195 \\
&6*a*e^2)/a^9)) - 236196*(b^3*e*x^{11} + 3*a*b^2*e*x^8 + 3*a^2*b*e*x^5 + a^3*e \\
&*x^2)*log(x))/(a^4*b^3*x^{11} + 3*a^5*b^2*x^8 + 3*a^6*b*x^5 + a^7*x^2)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**4,x)

[Out] Timed out

Giac [A] time = 1.07115, size = 441, normalized size = 1.42

$$-\frac{e \log(|bx^3 + a|)}{3a^4} + \frac{e \log(|x|)}{a^4} - \frac{10 \left(11 (-ab^2)^{\frac{1}{3}} bc + 7 (-ab^2)^{\frac{2}{3}} d \right) \log \left(x^2 + x \left(-\frac{a}{b} \right)^{\frac{1}{3}} + \left(-\frac{a}{b} \right)^{\frac{2}{3}} \right)}{243 a^5 b} - \frac{20 \sqrt{3} \left(11 (-ab^2)^{\frac{1}{3}} ab^3 c \right)}{243 a^5 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^4,x, algorithm="giac")

[Out]
$$-1/3 * e * \log(\text{abs}(b*x^3 + a)) / a^4 + e * \log(\text{abs}(x)) / a^4 - 10/243 * (11 * (-a*b^2)^{(1/3)} * b*c + 7 * (-a*b^2)^{(2/3)} * d) * \log(x^2 + x * (-a/b)^{(1/3)} + (-a/b)^{(2/3)}) / (a^5 * b) - 20/243 * \text{sqrt}(3) * (11 * (-a*b^2)^{(1/3)} * a*b^3*c - 7 * (-a*b^2)^{(2/3)} * a*b^2*d) * \arctan(1/3 * \text{sqrt}(3) * (2*x + (-a/b)^{(1/3)}) / (-a/b)^{(1/3)}) / (a^6 * b^3) + 20/243 * (7 * a^4 * b^2 * d * (-a/b)^{(1/3)} + 11 * a^4 * b^2 * c) * (-a/b)^{(1/3)} * \log(\text{abs}(x - (-a/b)^{(1/3)}) / (a^9 * b) - 1/162 * (280 * b^3 * d * x^{10} + 220 * b^3 * c * x^9 - 54 * a * b^2 * x^8 * e + 77 * 0 * a * b^2 * d * x^7 + 572 * a * b^2 * c * x^6 - 135 * a^2 * b * x^5 * e + 670 * a^2 * b * d * x^4 + 451 * a^2 * b * c * x^3 - 99 * a^3 * x^2 * e + 162 * a^3 * d * x + 81 * a^3 * c) / ((b*x^3 + a)^3 * a^4 * x^2)$$

$$3.364 \quad \int \frac{c+dx+ex^2}{x^4(a+bx^3)^4} dx$$

Optimal. Leaf size=340

$$\frac{x\left(-\frac{234b^2cx^2}{a} + 139bd + 118bex\right)}{162a^4(a+bx^3)} - \frac{x\left(-\frac{24b^2cx^2}{a} + 17bd + 16bex\right)}{54a^3(a+bx^3)^2} - \frac{x\left(-\frac{b^2cx^2}{a} + bd + bex\right)}{9a^2(a+bx^3)^3} + \frac{10\sqrt[3]{b}(11\sqrt[3]{bd} - 7\sqrt[3]{ae})\log\left(\frac{x^2 + bx^3}{a}\right)}{243a^{14}}$$

[Out] $-c/(3*a^4*x^3) - d/(2*a^4*x^2) - e/(a^4*x) - (x*(b*d + b*e*x - (b^2*c*x^2)/a))/(9*a^2*(a + b*x^3)^3) - (x*(17*b*d + 16*b*e*x - (24*b^2*c*x^2)/a))/(54*a^3*(a + b*x^3)^2) - (x*(139*b*d + 118*b*e*x - (234*b^2*c*x^2)/a))/(162*a^4*(a + b*x^3)) + (20*b^(1/3)*(11*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(81*Sqrt[3]*a^(14/3)) - (4*b*c*Log[x])/a^5 - (20*b^(1/3)*(11*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x]/(243*a^(14/3)) + (10*b^(1/3)*(11*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(243*a^(14/3)) + (4*b*c*Log[a + b*x^3])/(3*a^5)$

Rubi [A] time = 0.7727, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{x\left(-\frac{234b^2cx^2}{a} + 139bd + 118bex\right)}{162a^4(a+bx^3)} - \frac{x\left(-\frac{24b^2cx^2}{a} + 17bd + 16bex\right)}{54a^3(a+bx^3)^2} - \frac{x\left(-\frac{b^2cx^2}{a} + bd + bex\right)}{9a^2(a+bx^3)^3} + \frac{10\sqrt[3]{b}(11\sqrt[3]{bd} - 7\sqrt[3]{ae})\log\left(\frac{x^2 + bx^3}{a}\right)}{243a^{14}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^4), x]

[Out] $-c/(3*a^4*x^3) - d/(2*a^4*x^2) - e/(a^4*x) - (x*(b*d + b*e*x - (b^2*c*x^2)/a))/(9*a^2*(a + b*x^3)^3) - (x*(17*b*d + 16*b*e*x - (24*b^2*c*x^2)/a))/(54*a^3*(a + b*x^3)^2) - (x*(139*b*d + 118*b*e*x - (234*b^2*c*x^2)/a))/(162*a^4*(a + b*x^3)) + (20*b^(1/3)*(11*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(81*Sqrt[3]*a^(14/3)) - (4*b*c*Log[x])/a^5 - (20*b^(1/3)*(11*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x]/(243*a^(14/3)) + (10*b^(1/3)*(11*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(243*a^(14/3)) + (4*b*c*Log[a + b*x^3])/(3*a^5)$

Rule 1829

```

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]

```

Rule 1834

```

Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[Ex
pandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]

```

Rule 1871

```

Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

```

Rule 1860

```

Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]

```

Rule 31

```

Int[((a_) + (b_.)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]

```

Rule 634

```

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2}{x^4(a + bx^3)^4} dx &= -\frac{x\left(bd + bex - \frac{b^2cx^2}{a}\right)}{9a^2(a + bx^3)^3} - \frac{\int \frac{-9bc - 9bdx - 9bex^2 + \frac{9b^2cx^3}{a} + \frac{8b^2dx^4}{a} + \frac{7b^2ex^5}{a} - \frac{6b^3cx^6}{a^2}}{x^4(a + bx^3)^3} dx}{9ab} \\
&= -\frac{x\left(bd + bex - \frac{b^2cx^2}{a}\right)}{9a^2(a + bx^3)^3} - \frac{x\left(17bd + 16bex - \frac{24b^2cx^2}{a}\right)}{54a^3(a + bx^3)^2} + \frac{\int \frac{54b^3c + 54b^3dx + 54b^3ex^2 - \frac{108b^4cx^3}{a} - \frac{85b^4dx^4}{a} - \frac{64b^4ex^5}{a} + 72}{x^4(a + bx^3)^2}}{54a^2b^3} \\
&= -\frac{x\left(bd + bex - \frac{b^2cx^2}{a}\right)}{9a^2(a + bx^3)^3} - \frac{x\left(17bd + 16bex - \frac{24b^2cx^2}{a}\right)}{54a^3(a + bx^3)^2} - \frac{x\left(139bd + 118bex - \frac{234b^2cx^2}{a}\right)}{162a^4(a + bx^3)} - \frac{\int -162b^5c - 162}{ax^4} \\
&= -\frac{x\left(bd + bex - \frac{b^2cx^2}{a}\right)}{9a^2(a + bx^3)^3} - \frac{x\left(17bd + 16bex - \frac{24b^2cx^2}{a}\right)}{54a^3(a + bx^3)^2} - \frac{x\left(139bd + 118bex - \frac{234b^2cx^2}{a}\right)}{162a^4(a + bx^3)} - \frac{\int \left(-\frac{162b^5c}{ax^4} - 162\right)}{ax^4} \\
&= -\frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(bd + bex - \frac{b^2cx^2}{a}\right)}{9a^2(a + bx^3)^3} - \frac{x\left(17bd + 16bex - \frac{24b^2cx^2}{a}\right)}{54a^3(a + bx^3)^2} - \frac{x\left(139bd + 118bex - \frac{234b^2cx^2}{a}\right)}{162a^4(a + bx^3)} \\
&= -\frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(bd + bex - \frac{b^2cx^2}{a}\right)}{9a^2(a + bx^3)^3} - \frac{x\left(17bd + 16bex - \frac{24b^2cx^2}{a}\right)}{54a^3(a + bx^3)^2} - \frac{x\left(139bd + 118bex - \frac{234b^2cx^2}{a}\right)}{162a^4(a + bx^3)} \\
&= -\frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(bd + bex - \frac{b^2cx^2}{a}\right)}{9a^2(a + bx^3)^3} - \frac{x\left(17bd + 16bex - \frac{24b^2cx^2}{a}\right)}{54a^3(a + bx^3)^2} - \frac{x\left(139bd + 118bex - \frac{234b^2cx^2}{a}\right)}{162a^4(a + bx^3)} \\
&= -\frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(bd + bex - \frac{b^2cx^2}{a}\right)}{9a^2(a + bx^3)^3} - \frac{x\left(17bd + 16bex - \frac{24b^2cx^2}{a}\right)}{54a^3(a + bx^3)^2} - \frac{x\left(139bd + 118bex - \frac{234b^2cx^2}{a}\right)}{162a^4(a + bx^3)} \\
&= -\frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(bd + bex - \frac{b^2cx^2}{a}\right)}{9a^2(a + bx^3)^3} - \frac{x\left(17bd + 16bex - \frac{24b^2cx^2}{a}\right)}{54a^3(a + bx^3)^2} - \frac{x\left(139bd + 118bex - \frac{234b^2cx^2}{a}\right)}{162a^4(a + bx^3)} \\
&= -\frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(bd + bex - \frac{b^2cx^2}{a}\right)}{9a^2(a + bx^3)^3} - \frac{x\left(17bd + 16bex - \frac{24b^2cx^2}{a}\right)}{54a^3(a + bx^3)^2} - \frac{x\left(139bd + 118bex - \frac{234b^2cx^2}{a}\right)}{162a^4(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.47782, size = 284, normalized size = 0.84

$$-20\sqrt[3]{b}\left(11\sqrt[3]{a}\sqrt[3]{bd}-7a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2\right)+\frac{54a^3b(c+x(d+ex))}{(a+bx^3)^3}+\frac{9a^2b(18c+x(17d+16ex))}{(a+bx^3)^2}+40\sqrt[3]{b}\left(11\sqrt[3]{a}\sqrt[3]{bd}-7\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^4), x]

[Out] $-\left(\frac{162ac}{x^3} + \frac{243ad}{x^2} + \frac{486ae}{x} + 54a^3b(c + x(d + ex))\right) / (a + bx^3)^3 + \left(\frac{9a^2b(18c + x(17d + 16ex))}{(a + bx^3)^2} + \frac{3a^3b(162c + x(139d + 118ex))}{(a + bx^3)} - 40\sqrt[3]{3}a^{1/3}b^{1/3}\right) \operatorname{ArcTan}\left[\frac{1 - (2b^{1/3}x)/a^{1/3}}{\sqrt[3]{3}}\right] + 1944b^3c \operatorname{Log}[x] + 40b^{1/3}(11a^{1/3}b^{1/3}d - 7a^{2/3}e) \operatorname{Log}[a^{1/3} + b^{1/3}x] - 20b^{1/3}(11a^{1/3}b^{1/3}d - 7a^{2/3}e) \operatorname{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2] - 648b^3c \operatorname{Log}[a + bx^3] / (486a^5)$

Maple [A] time = 0.019, size = 415, normalized size = 1.2

$$\frac{59b^3ex^8}{81a^4(bx^3+a)^3} - \frac{139b^3dx^7}{162a^4(bx^3+a)^3} - \frac{b^3cx^6}{a^4(bx^3+a)^3} - \frac{142b^2ex^5}{81a^3(bx^3+a)^3} - \frac{329b^2dx^4}{162a^3(bx^3+a)^3} - \frac{7b^2cx^3}{3a^3(bx^3+a)^3} - \frac{9}{81a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)/x^4/(b*x^3+a)^4, x)

[Out] $-\frac{59}{81} \frac{b^3 e x^8}{a^4 (b x^3 + a)^3} - \frac{139}{162} \frac{b^3 d x^7}{a^4 (b x^3 + a)^3} - \frac{b^3 c x^6}{a^4 (b x^3 + a)^3} - \frac{142}{81} \frac{b^2 e x^5}{a^3 (b x^3 + a)^3} - \frac{329}{162} \frac{b^2 d x^4}{a^3 (b x^3 + a)^3} - \frac{7}{3} \frac{b^2 c x^3}{a^3 (b x^3 + a)^3} - \frac{9}{81 a^2} - \frac{110}{243} \frac{d}{a^4} \frac{1}{(b a)^{2/3}} \ln(x^2 - (1/b a)^{1/3} x + (1/b a)^{2/3}) - \frac{220}{243} \frac{d}{a^4} \frac{1}{(b a)^{2/3}} \operatorname{arctan}\left(\frac{1}{3} \sqrt[3]{\frac{1}{2}} \frac{2}{(1/b a)^{1/3} x - 1}\right) + \frac{140}{243} \frac{e}{a^4} \frac{1}{(b a)^{1/3}} \ln(x + (1/b a)^{1/3}) - \frac{70}{243} \frac{e}{a^4} \frac{1}{(b a)^{1/3}} \ln(x^2 - (1/b a)^{1/3} x + (1/b a)^{2/3}) - \frac{140}{243} \frac{e}{a^4} \frac{1}{(b a)^{1/3}} \operatorname{arctan}\left(\frac{1}{3} \sqrt[3]{\frac{1}{2}} \frac{2}{(1/b a)^{1/3} x - 1}\right) + \frac{4}{3} \frac{b^3 c \ln(b x^3 + a)}{a^5} - \frac{1}{3} \frac{c}{a^4} \frac{1}{x^3} - \frac{1}{2} \frac{d}{a^4} \frac{1}{x^2} - \frac{e}{a^4} \frac{1}{x} - \frac{b^3 c \ln(x)}{a^5}$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [C] time = 12.2973, size = 15115, normalized size = 44.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^4,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/486*(840*a*b^3*e*x^{11} + 660*a*b^3*d*x^{10} + 648*a*b^3*c*x^9 + 2310*a^2*b^2 * e*x^8 + 1716*a^2*b^2*d*x^7 + 1620*a^2*b^2*c*x^6 + 2010*a^3*b*e*x^5 + 1353 * a^3*b*d*x^4 + 1188*a^3*b*c*x^3 + 486*a^4*e*x^2 + 243*a^4*d*x + 162*a^4*c + \\ & 2*(a^5*b^3*x^{12} + 3*a^6*b^2*x^9 + 3*a^7*b*x^6 + a^8*x^3)*(4^{(2/3)}*(-I*\sqrt{3} + 1)*(6561*b^2*c^2/a^{10} - (6561*b^2*c^2 + 1925*a*b*d*e)/a^{10})/(1062882 * b^3*c^3/a^{15} + 125*(1331*b*d^3 + 343*a*e^3)*b/a^{14} - 243*(6561*b^2*c^2 + 19 \\ & 25*a*b*d*e)*b*c/a^{15} + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1 \\ & 701*c*d*e)*a*b^2)/a^{15})^{(1/3)} + 4^{(1/3)}*(I*\sqrt{3} + 1)*(1062882*b^3*c^3/a^{15} + 125 * (1331*b*d^3 + 343*a*e^3)*b/a^{14} - 243*(6561*b^2*c^2 + 1925*a*b*d*e) * b*c/a^{15} + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e) * a*b^2)/a^{15})^{(1/3)} - 324*b*c/a^5)*\log(7*(4^{(2/3)}*(-I*\sqrt{3} + 1)*(6561*b^2 * c^2/a^{10} - (6561*b^2*c^2 + 1925*a*b*d*e)/a^{10})/(1062882*b^3*c^3/a^{15} + 12 \\ & 5*(1331*b*d^3 + 343*a*e^3)*b/a^{14} - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^{15} + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2) /a^{15})^{(1/3)} + 4^{(1/3)}*(I*\sqrt{3} + 1)*(1062882*b^3*c^3/a^{15} + 125*(1331*b * d^3 + 343*a*e^3)*b/a^{14} - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^{15} + (531 \\ & 441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^{15})^{(1/3)} - 324*b*c/a^5)^2*a^{10}*e + 784080*b^2*c*d^2 + 734832*b^2*c^2*e + 431200*a * b*d*e^2 + 4*(605*a^5*b*d^2 + 1134*a^5*b*c*e)*(4^{(2/3)}*(-I*\sqrt{3} + 1)*(65 \\ & 61*b^2*c^2/a^{10} - (6561*b^2*c^2 + 1925*a*b*d*e)/a^{10})/(1062882*b^3*c^3/a^{15} + 125 * (1331*b*d^3 + 343*a*e^3)*b/a^{14} - 243*(6561*b^2*c^2 + 1925*a*b*d*e)* \\ & b*c/a^{15} + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a * b^2)/a^{15})^{(1/3)} + 4^{(1/3)}*(I*\sqrt{3} + 1)*(1062882*b^3*c^3/a^{15} + 125*(13 \\ & 31*b*d^3 + 343*a*e^3)*b/a^{14} - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^{15} + \end{aligned}$$

$$\begin{aligned}
& (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^{15} \\
&)^{(1/3)} - 324*b*c/a^5 + 400*(1331*b^2*d^3 + 343*a*b*e^3)*x) - (972*b^4*c*x \\
& ^{12} + 2916*a*b^3*c*x^9 + 2916*a^2*b^2*c*x^6 + 972*a^3*b*c*x^3 + (a^5*b^3*x^ \\
& 12 + 3*a^6*b^2*x^9 + 3*a^7*b*x^6 + a^8*x^3)*(4^{(2/3)}*(-I*sqrt(3) + 1)*(6561 \\
& *b^2*c^2/a^{10} - (6561*b^2*c^2 + 1925*a*b*d*e)/a^{10}))/((1062882*b^3*c^3/a^{15} + \\
& 125*(1331*b*d^3 + 343*a*e^3)*b/a^{14} - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b* \\
& c/a^{15} + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b \\
& ^2)/a^{15})^{(1/3)} + 4^{(1/3)}*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^{15} + 125*(1331 \\
& *b*d^3 + 343*a*e^3)*b/a^{14} - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^{15} + (\\
& 531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^{15})^{(1/3)} - 324*b*c/a^5) \\
& + 3*sqrt(1/3)*(a^5*b^3*x^{12} + 3*a^6*b^2*x^9 + 3*a^7*b* \\
& x^6 + a^8*x^3)*sqrt(-((4^{(2/3)}*(-I*sqrt(3) + 1)*(6561*b^2*c^2/a^{10} - (6561* \\
& b^2*c^2 + 1925*a*b*d*e)/a^{10}))/((1062882*b^3*c^3/a^{15} + 125*(1331*b*d^3 + 343 \\
& *a*e^3)*b/a^{14} - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^{15} + (531441*b^3*c \\
& ^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^{15})^{(1/3)} + 4^{(1 \\
& /3)}*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^{15} + 125*(1331*b*d^3 + 343*a*e^3)*b/ \\
& a^{14} - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^{15} + (531441*b^3*c^3 + 42875 \\
& *a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^{15})^{(1/3)} - 324*b*c/a^5)^2 \\
& *a^{10} + 648*(4^{(2/3)}*(-I*sqrt(3) + 1)*(6561*b^2*c^2/a^{10} - (6561*b^2*c^2 + \\
& 1925*a*b*d*e)/a^{10}))/((1062882*b^3*c^3/a^{15} + 125*(1331*b*d^3 + 343*a*e^3)*b/ \\
& a^{14} - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^{15} + (531441*b^3*c^3 + 42875 \\
& *a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^{15})^{(1/3)} + 4^{(1/3)}*(I*sq \\
& r t(3) + 1)*(1062882*b^3*c^3/a^{15} + 125*(1331*b*d^3 + 343*a*e^3)*b/a^{14} - 243 \\
& *(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^{15} + (531441*b^3*c^3 + 42875*a^2*b*e^3 \\
& - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^{15})^{(1/3)} - 324*b*c/a^5)*a^5*b*c + 1 \\
& 04976*b^2*c^2 + 123200*a*b*d*e)/a^{10}))*log(-7*(4^{(2/3)}*(-I*sqrt(3) + 1)*(65 \\
& 61*b^2*c^2/a^{10} - (6561*b^2*c^2 + 1925*a*b*d*e)/a^{10}))/((1062882*b^3*c^3/a^{15} \\
& + 125*(1331*b*d^3 + 343*a*e^3)*b/a^{14} - 243*(6561*b^2*c^2 + 1925*a*b*d*e)* \\
& b*c/a^{15} + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a \\
& *b^2)/a^{15})^{(1/3)} + 4^{(1/3)}*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^{15} + 125*(13 \\
& 31*b*d^3 + 343*a*e^3)*b/a^{14} - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^{15} + \\
& (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^{15} \\
&)^{(1/3)} - 324*b*c/a^5)^2*a^{10}*e - 784080*b^2*c*d^2 - 734832*b^2*c^2*e - 431 \\
& 200*a*b*d*e^2 - 4*(605*a^5*b*d^2 + 1134*a^5*b*c*e)*(4^{(2/3)}*(-I*sqrt(3) + 1 \\
&)*(6561*b^2*c^2/a^{10} - (6561*b^2*c^2 + 1925*a*b*d*e)/a^{10}))/((1062882*b^3*c^3 \\
& /a^{15} + 125*(1331*b*d^3 + 343*a*e^3)*b/a^{14} - 243*(6561*b^2*c^2 + 1925*a*b* \\
& d*e)*b*c/a^{15} + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d \\
& *e)*a*b^2)/a^{15})^{(1/3)} + 4^{(1/3)}*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^{15} + 12 \\
& 5*(1331*b*d^3 + 343*a*e^3)*b/a^{14} - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a \\
& ^{15} + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2) \\
& /a^{15})^{(1/3)} - 324*b*c/a^5) + 800*(1331*b^2*d^3 + 343*a*b*e^3)*x + 3*sqrt(1 \\
& /3)*(7*(4^{(2/3)}*(-I*sqrt(3) + 1)*(6561*b^2*c^2/a^{10} - (6561*b^2*c^2 + 1925* \\
& a*b*d*e)/a^{10}))/((1062882*b^3*c^3/a^{15} + 125*(1331*b*d^3 + 343*a*e^3)*b/a^{14} \\
& - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^{15} + (531441*b^3*c^3 + 42875*a^2* \\
& b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^{15})^{(1/3)} + 4^{(1/3)}*(I*sqrt(3)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{1}{3} + 4^{\frac{1}{3}} \cdot (I\sqrt{3} + 1) \cdot (1062882b^3c^3/a^{15} + 125 \cdot (1331bd^3 + 343a^2e^3) \cdot b/a^{14} - 243 \cdot (6561b^2c^2 + 1925a^2bde) \cdot bc/a^{15} + (531441b^3c^3 + 42875a^2b^2e^3 - 275 \cdot (605d^3 - 1701cde) \cdot ab^2)/a^{15})^{\frac{1}{3}} - 324bc/a^5 \right)^2 a^{10}e - 784080b^2c^2d^2 - 734832b^2c^2e - 431200abde^2 - 4 \cdot (605a^5bd^2 + 1134a^5b^2c^2e) \cdot (4^{\frac{2}{3}} \cdot (-I\sqrt{3} + 1) \cdot (6561b^2c^2/a^{10} - (6561b^2c^2 + 1925a^2bde)/a^{10}) / (1062882b^3c^3/a^{15} + 125 \cdot (1331bd^3 + 343a^2e^3) \cdot b/a^{14} - 243 \cdot (6561b^2c^2 + 1925a^2bde) \cdot bc/a^{15} + (531441b^3c^3 + 42875a^2b^2e^3 - 275 \cdot (605d^3 - 1701cde) \cdot ab^2)/a^{15})^{\frac{1}{3}} + 4^{\frac{1}{3}} \cdot (I\sqrt{3} + 1) \cdot (1062882b^3c^3/a^{15} + 125 \cdot (1331bd^3 + 343a^2e^3) \cdot b/a^{14} - 243 \cdot (6561b^2c^2 + 1925a^2bde) \cdot bc/a^{15} + (531441b^3c^3 + 42875a^2b^2e^3 - 275 \cdot (605d^3 - 1701cde) \cdot ab^2)/a^{15})^{\frac{1}{3}} - 324bc/a^5 + 800 \cdot (1331b^2d^3 + 343a^2b^2e^3) \cdot x - 3\sqrt{3} \cdot (7 \cdot (4^{\frac{2}{3}} \cdot (-I\sqrt{3} + 1) \cdot (6561b^2c^2/a^{10} - (6561b^2c^2 + 1925a^2bde)/a^{10}) / (1062882b^3c^3/a^{15} + 125 \cdot (1331bd^3 + 343a^2e^3) \cdot b/a^{14} - 243 \cdot (6561b^2c^2 + 1925a^2bde) \cdot bc/a^{15} + (531441b^3c^3 + 42875a^2b^2e^3 - 275 \cdot (605d^3 - 1701cde) \cdot ab^2)/a^{15})^{\frac{1}{3}} + 4^{\frac{1}{3}} \cdot (I\sqrt{3} + 1) \cdot (1062882b^3c^3/a^{15} + 125 \cdot (1331bd^3 + 343a^2e^3) \cdot b/a^{14} - 243 \cdot (6561b^2c^2 + 1925a^2bde) \cdot bc/a^{15} + (531441b^3c^3 + 42875a^2b^2e^3 - 275 \cdot (605d^3 - 1701cde) \cdot ab^2)/a^{15})^{\frac{1}{3}} - 324bc/a^5) \cdot a^{10}e - 2420a^5bd^2 + 2268a^5b^2c^2e) \cdot \sqrt{-((4^{\frac{2}{3}} \cdot (-I\sqrt{3} + 1) \cdot (6561b^2c^2/a^{10} - (6561b^2c^2 + 1925a^2bde)/a^{10}) / (1062882b^3c^3/a^{15} + 125 \cdot (1331bd^3 + 343a^2e^3) \cdot b/a^{14} - 243 \cdot (6561b^2c^2 + 1925a^2bde) \cdot bc/a^{15} + (531441b^3c^3 + 42875a^2b^2e^3 - 275 \cdot (605d^3 - 1701cde) \cdot ab^2)/a^{15})^{\frac{1}{3}} + 4^{\frac{1}{3}} \cdot (I\sqrt{3} + 1) \cdot (1062882b^3c^3/a^{15} + 125 \cdot (1331bd^3 + 343a^2e^3) \cdot b/a^{14} - 243 \cdot (6561b^2c^2 + 1925a^2bde) \cdot bc/a^{15} + (531441b^3c^3 + 42875a^2b^2e^3 - 275 \cdot (605d^3 - 1701cde) \cdot ab^2)/a^{15})^{\frac{1}{3}} - 324bc/a^5)^2 a^{10} + 648 \cdot (4^{\frac{2}{3}} \cdot (-I\sqrt{3} + 1) \cdot (6561b^2c^2/a^{10} - (6561b^2c^2 + 1925a^2bde)/a^{10}) / (1062882b^3c^3/a^{15} + 125 \cdot (1331bd^3 + 343a^2e^3) \cdot b/a^{14} - 243 \cdot (6561b^2c^2 + 1925a^2bde) \cdot bc/a^{15} + (531441b^3c^3 + 42875a^2b^2e^3 - 275 \cdot (605d^3 - 1701cde) \cdot ab^2)/a^{15})^{\frac{1}{3}} + 4^{\frac{1}{3}} \cdot (I\sqrt{3} + 1) \cdot (1062882b^3c^3/a^{15} + 125 \cdot (1331bd^3 + 343a^2e^3) \cdot b/a^{14} - 243 \cdot (6561b^2c^2 + 1925a^2bde) \cdot bc/a^{15} + (531441b^3c^3 + 42875a^2b^2e^3 - 275 \cdot (605d^3 - 1701cde) \cdot ab^2)/a^{15})^{\frac{1}{3}} - 324bc/a^5) \cdot a^5bc + 104976b^2c^2 + 123200abde)/a^{10}} + 1944 \cdot (b^4cx^{12} + 3a^2b^3cx^9 + 3a^2b^2c^2x^6 + a^3b^2cx^3) \cdot \log(x) / (a^5b^3x^{12} + 3a^6b^2x^9 + 3a^7b^2x^6 + a^8x^3)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/x**4/(b*x**3+a)**4,x)

[Out] Timed out

Giac [A] time = 1.06862, size = 459, normalized size = 1.35

$$\frac{4bc \log(|bx^3 + a|)}{3a^5} - \frac{4bc \log(|x|)}{a^5} - \frac{10 \left(11 (-ab^2)^{\frac{1}{3}} bd + 7 (-ab^2)^{\frac{2}{3}} e \right) \log \left(x^2 + x \left(-\frac{a}{b} \right)^{\frac{1}{3}} + \left(-\frac{a}{b} \right)^{\frac{2}{3}} \right)}{243 a^5 b} - \frac{280 b^3 x^{11} e + 220 b^3 d x^{10} + 216 b^3 c x^9 + 770 a b^2 x^8 e + 572 a b^2 d x^7 + 540 a b^2 c x^6 + 670 a^2 b x^5 e + 451 a^2 b d x^4 + 396 a^2 b c x^3 + 162 a^3 x^2 e + 81 a^3 d x + 54 a^3 c}{(b x^4 + a x)^3 a^4} - \frac{20}{243} \sqrt{3} \frac{(11 (-ab^2)^{\frac{1}{3}} a b^3 d - 7 (-ab^2)^{\frac{2}{3}} a b^2 e) \arctan \left(\frac{1}{3} \sqrt{3} (2x + (-a/b)^{\frac{1}{3}}) / (-a/b)^{\frac{1}{3}} \right)}{(a^6 b^3) + 20/243 (7 a^6 b^2 (-a/b)^{\frac{1}{3}} e + 11 a^6 b^2 d) (-a/b)^{\frac{1}{3}} \log(\text{abs}(x - (-a/b)^{\frac{1}{3}}))}{a^{11} b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^4,x, algorithm="giac")

[Out] $\frac{4}{3} b c \log(\text{abs}(b x^3 + a)) / a^5 - 4 b c \log(\text{abs}(x)) / a^5 - \frac{10}{243} (11 (-a b^2)^{\frac{1}{3}} b d + 7 (-a b^2)^{\frac{2}{3}} e) \log \left(x^2 + x \left(-\frac{a}{b} \right)^{\frac{1}{3}} + \left(-\frac{a}{b} \right)^{\frac{2}{3}} \right) / (a^5 b) - \frac{1}{162} (280 b^3 x^{11} e + 220 b^3 d x^{10} + 216 b^3 c x^9 + 770 a b^2 x^8 e + 572 a b^2 d x^7 + 540 a b^2 c x^6 + 670 a^2 b x^5 e + 451 a^2 b d x^4 + 396 a^2 b c x^3 + 162 a^3 x^2 e + 81 a^3 d x + 54 a^3 c) / ((b x^4 + a x)^3 a^4) - \frac{20}{243} \sqrt{3} \frac{(11 (-a b^2)^{\frac{1}{3}} a b^3 d - 7 (-a b^2)^{\frac{2}{3}} a b^2 e) \arctan \left(\frac{1}{3} \sqrt{3} (2 x + (-a/b)^{\frac{1}{3}}) / (-a/b)^{\frac{1}{3}} \right)}{(a^6 b^3) + \frac{20}{243} (7 a^6 b^2 (-a/b)^{\frac{1}{3}} e + 11 a^6 b^2 d) (-a/b)^{\frac{1}{3}} \log(\text{abs}(x - (-a/b)^{\frac{1}{3}}))} / (a^{11} b)$

$$3.365 \quad \int \frac{2ax-x^2}{a^3+x^3} dx$$

Optimal. Leaf size=29

$$-\log(a+x) - \frac{2 \tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}}$$

[Out] $(-2*\text{ArcTan}[(a - 2*x)/(\text{Sqrt}[3]*a)])/\text{Sqrt}[3] - \text{Log}[a + x]$

Rubi [A] time = 0.0578282, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1593, 1868, 31, 617, 204}

$$-\log(a+x) - \frac{2 \tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2*a*x - x^2)/(a^3 + x^3), x]$

[Out] $(-2*\text{ArcTan}[(a - 2*x)/(\text{Sqrt}[3]*a)])/\text{Sqrt}[3] - \text{Log}[a + x]$

Rule 1593

$\text{Int}[(u_.)*((a_.)*(x_)^{(p_.)} + (b_.)*(x_)^{(q_.)})^{(n_.)}, x_Symbol] \rightarrow \text{Int}[u*x^{(n*p)}*(a + b*x^{(q-p)})^n, x] /;$ FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1868

$\text{Int}[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] \rightarrow \text{With}[\{A = \text{Coeff}[P2, x, 0], B = \text{Coeff}[P2, x, 1], C = \text{Coeff}[P2, x, 2]\}, \text{With}[\{q = \text{Rt}[a/b, 3]\}, \text{Dist}[C/b, \text{Int}[1/(q+x), x], x] + \text{Dist}[(B + C*q)/b, \text{Int}[1/(q^2 - q*x + x^2), x], x]] /;$ EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

Rule 31

$\text{Int}[(a_.) + (b_.)*(x_)^{(-1)}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /;$ FreeQ[{a, b}, x]

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{2ax - x^2}{a^3 + x^3} dx &= \int \frac{(2a - x)x}{a^3 + x^3} dx \\ &= a \int \frac{1}{a^2 - ax + x^2} dx - \int \frac{1}{a + x} dx \\ &= -\log(a + x) + 2 \operatorname{Subst} \left(\int \frac{1}{-3 - x^2} dx, x, 1 - \frac{2x}{a} \right) \\ &= -\frac{2 \tan^{-1} \left(\frac{a-2x}{\sqrt{3}a} \right)}{\sqrt{3}} - \log(a + x) \end{aligned}$$

Mathematica [A] time = 0.0138302, size = 57, normalized size = 1.97

$$\frac{1}{3} \left(\log(a^2 - ax + x^2) - \log(a^3 + x^3) - 2 \log(a + x) + 2\sqrt{3} \tan^{-1} \left(\frac{2x - a}{\sqrt{3}a} \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a*x - x^2)/(a^3 + x^3), x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] - 2*Log[a + x] + Log[a^2 - a*x +
x^2] - Log[a^3 + x^3])/3
```

Maple [A] time = 0.007, size = 29, normalized size = 1.

$$-\ln(a + x) + \frac{2\sqrt{3}}{3} \arctan \left(\frac{(2x - a)\sqrt{3}}{3a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*a*x-x^2)/(a^3+x^3),x)`

[Out] `-ln(a+x)+2/3*3^(1/2)*arctan(1/3*(2*x-a)/a*3^(1/2))`

Maxima [A] time = 1.41899, size = 35, normalized size = 1.21

$$\frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)-\log(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*a*x-x^2)/(a^3+x^3),x, algorithm="maxima")`

[Out] `2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(a + x)`

Fricas [A] time = 0.991842, size = 80, normalized size = 2.76

$$\frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)-\log(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*a*x-x^2)/(a^3+x^3),x, algorithm="fricas")`

[Out] `2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(a + x)`

Sympy [C] time = 0.287255, size = 54, normalized size = 1.86

$$-\log(a+x)-\frac{\sqrt{3}i\log\left(-\frac{a}{2}-\frac{\sqrt{3}ia}{2}+x\right)}{3}+\frac{\sqrt{3}i\log\left(-\frac{a}{2}+\frac{\sqrt{3}ia}{2}+x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*a*x-x**2)/(a**3+x**3),x)`

[Out] $-\log(a + x) - \sqrt{3} \operatorname{arctan}\left(-\frac{\sqrt{3}(a - 2x)}{3a}\right) - \log(|a + x|)$

Giac [A] time = 1.06164, size = 36, normalized size = 1.24

$$\frac{2}{3} \sqrt{3} \operatorname{arctan}\left(-\frac{\sqrt{3}(a - 2x)}{3a}\right) - \log(|a + x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*a*x-x^2)/(a^3+x^3),x, algorithm="giac")`

[Out] $2/3*\sqrt{3}*\operatorname{arctan}(-1/3*\sqrt{3}*(a - 2*x)/a) - \log(\operatorname{abs}(a + x))$

$$3.366 \quad \int \frac{(2a-x)x}{a^3+x^3} dx$$

Optimal. Leaf size=29

$$-\log(a+x) - \frac{2 \tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}}$$

[Out] $(-2*\text{ArcTan}[(a - 2*x)/(\text{Sqrt}[3]*a)])/\text{Sqrt}[3] - \text{Log}[a + x]$

Rubi [A] time = 0.0347537, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1868, 31, 617, 204}

$$-\log(a+x) - \frac{2 \tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2*a - x)*x/(a^3 + x^3), x]$

[Out] $(-2*\text{ArcTan}[(a - 2*x)/(\text{Sqrt}[3]*a)])/\text{Sqrt}[3] - \text{Log}[a + x]$

Rule 1868

$\text{Int}[(P2_)/((a_)+(b_)*(x_)^3), x_Symbol] \rightarrow \text{With}[\{A = \text{Coeff}[P2, x, 0], B = \text{Coeff}[P2, x, 1], C = \text{Coeff}[P2, x, 2]\}, \text{With}[\{q = \text{Rt}[a/b, 3]\}, \text{Dist}[C/b, \text{Int}[1/(q+x), x], x] + \text{Dist}[(B + C*q)/b, \text{Int}[1/(q^2 - q*x + x^2), x], x]] / ; \text{EqQ}[A - \text{Rt}[a/b, 3]*B - 2*\text{Rt}[a/b, 3]^2*C, 0]] / ; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[P2, x, 2]$

Rule 31

$\text{Int}[(a_)+(b_)*(x_)^(-1), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] / ; \text{FreeQ}[\{a, b\}, x]$

Rule 617

$\text{Int}[(a_)+(b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] \rightarrow \text{With}[\{q = 1 - 4*S\text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] / ; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \|\ !\text{RationalQ}[b^2 - 4*a*c]) / ; \text{Free}$

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 204

$\text{Int}[\frac{(a_.) + (b_.)(x_)^2}{a^3 + x^3}, x_Symbol] \ :> \ -\text{Simp}[\text{ArcTan}[\frac{\text{Rt}[-b, 2]*x}{\text{Rt}[-a, 2]}] / (\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \frac{(2a-x)x}{a^3+x^3} dx &= a \int \frac{1}{a^2-ax+x^2} dx - \int \frac{1}{a+x} dx \\ &= -\log(a+x) + 2 \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1 - \frac{2x}{a} \right) \\ &= -\frac{2 \tan^{-1} \left(\frac{a-2x}{\sqrt{3}a} \right)}{\sqrt{3}} - \log(a+x) \end{aligned}$$

Mathematica [A] time = 0.0052049, size = 57, normalized size = 1.97

$$\frac{1}{3} \left(\log(a^2 - ax + x^2) - \log(a^3 + x^3) - 2 \log(a + x) + 2\sqrt{3} \tan^{-1} \left(\frac{2x - a}{\sqrt{3}a} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((2*a - x)*x)/(a^3 + x^3), x]

[Out] (2*Sqrt[3]*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] - 2*Log[a + x] + Log[a^2 - a*x + x^2] - Log[a^3 + x^3])/3

Maple [A] time = 0.006, size = 29, normalized size = 1.

$$-\ln(a+x) + \frac{2\sqrt{3}}{3} \arctan\left(\frac{(2x-a)\sqrt{3}}{3a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*a-x)*x/(a^3+x^3), x)

[Out] $-\ln(a+x)+2/3\cdot 3^{(1/2)}\cdot \arctan(1/3\cdot(2\cdot x-a)/a\cdot 3^{(1/2)})$

Maxima [A] time = 1.42322, size = 35, normalized size = 1.21

$$\frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)-\log(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*a-x)*x/(a^3+x^3),x, algorithm="maxima")`

[Out] $2/3\cdot \sqrt{3}\cdot \arctan(-1/3\cdot \sqrt{3}\cdot (a-2\cdot x)/a)-\log(a+x)$

Fricas [A] time = 1.00505, size = 80, normalized size = 2.76

$$\frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)-\log(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*a-x)*x/(a^3+x^3),x, algorithm="fricas")`

[Out] $2/3\cdot \sqrt{3}\cdot \arctan(-1/3\cdot \sqrt{3}\cdot (a-2\cdot x)/a)-\log(a+x)$

Sympy [C] time = 0.286357, size = 54, normalized size = 1.86

$$-\log(a+x)-\frac{\sqrt{3}i\log\left(-\frac{a}{2}-\frac{\sqrt{3}ia}{2}+x\right)}{3}+\frac{\sqrt{3}i\log\left(-\frac{a}{2}+\frac{\sqrt{3}ia}{2}+x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*a-x)*x/(a**3+x**3),x)`

[Out] $-\log(a+x)-\sqrt{3}\cdot I\cdot \log(-a/2-\sqrt{3}\cdot I\cdot a/2+x)/3+\sqrt{3}\cdot I\cdot \log(-a/2+\sqrt{3}\cdot I\cdot a/2+x)/3$

Giac [A] time = 1.07708, size = 36, normalized size = 1.24

$$\frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right) - \log(|a+x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a-x)*x/(a^3+x^3),x, algorithm="giac")

[Out] 2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(abs(a + x))

$$3.367 \quad \int \frac{2ax+x^2}{a^3-x^3} dx$$

Optimal. Leaf size=31

$$-\log(a-x) - \frac{2 \tan^{-1}\left(\frac{a+2x}{\sqrt{3}a}\right)}{\sqrt{3}}$$

[Out] $(-2*\text{ArcTan}[(a + 2*x)/(\text{Sqrt}[3]*a)])/\text{Sqrt}[3] - \text{Log}[a - x]$

Rubi [A] time = 0.0564489, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1593, 1868, 31, 617, 204}

$$-\log(a-x) - \frac{2 \tan^{-1}\left(\frac{a+2x}{\sqrt{3}a}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2*a*x + x^2)/(a^3 - x^3), x]$

[Out] $(-2*\text{ArcTan}[(a + 2*x)/(\text{Sqrt}[3]*a)])/\text{Sqrt}[3] - \text{Log}[a - x]$

Rule 1593

$\text{Int}[(u_.)*((a_.)*(x_)^{(p_.)} + (b_.)*(x_)^{(q_.)})^{(n_.)}, x_Symbol] \rightarrow \text{Int}[u*x^{(n*p)}*(a + b*x^{(q-p)})^n, x] \text{ ; FreeQ}\{a, b, p, q\}, x \ \&\& \ \text{IntegerQ}[n] \ \&\& \ \text{PosQ}[q-p]$

Rule 1868

$\text{Int}[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] \rightarrow \text{With}\{A = \text{Coeff}[P2, x, 0], B = \text{Coeff}[P2, x, 1], C = \text{Coeff}[P2, x, 2]\}, \text{With}\{q = \text{Rt}[a/b, 3]\}, \text{Dist}[C/b, \text{Int}[1/(q+x), x], x] + \text{Dist}[(B + C*q)/b, \text{Int}[1/(q^2 - q*x + x^2), x], x] \text{ ; EqQ}[A - \text{Rt}[a/b, 3]*B - 2*\text{Rt}[a/b, 3]^2*C, 0] \text{ ; FreeQ}\{a, b\}, x \ \&\& \ \text{PolyQ}[P2, x, 2]$

Rule 31

$\text{Int}[(a_) + (b_.)*(x_)^{(-1)}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] \text{ ; FreeQ}\{a, b\}, x]$

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{2ax + x^2}{a^3 - x^3} dx &= \int \frac{x(2a + x)}{a^3 - x^3} dx \\ &= -\left(a \int \frac{1}{a^2 + ax + x^2} dx\right) - \int \frac{1}{-a + x} dx \\ &= -\log(a - x) + 2 \operatorname{Subst}\left(\int \frac{1}{-3 - x^2} dx, x, 1 + \frac{2x}{a}\right) \\ &= -\frac{2 \tan^{-1}\left(\frac{a+2x}{\sqrt{3}a}\right)}{\sqrt{3}} - \log(a - x) \end{aligned}$$

Mathematica [A] time = 0.0130022, size = 58, normalized size = 1.87

$$\frac{1}{3} \left(\log(a^2 + ax + x^2) - \log(x^3 - a^3) - 2 \log(x - a) - 2\sqrt{3} \tan^{-1}\left(\frac{a + 2x}{\sqrt{3}a}\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a*x + x^2)/(a^3 - x^3), x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(a + 2*x)/(Sqrt[3]*a)] - 2*Log[-a + x] + Log[a^2 + a*x +
x^2] - Log[-a^3 + x^3])/3
```

Maple [A] time = 0.006, size = 29, normalized size = 0.9

$$-\frac{2\sqrt{3}}{3} \arctan\left(\frac{(a + 2x)\sqrt{3}}{3a}\right) - \ln(-a + x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*a*x+x^2)/(a^3-x^3),x)`

[Out] $-2/3*\arctan(1/3*(a+2*x)/a*3^{(1/2)})*3^{(1/2)}-\ln(-a+x)$

Maxima [A] time = 1.43221, size = 38, normalized size = 1.23

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right)-\log(-a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*a*x+x^2)/(a^3-x^3),x, algorithm="maxima")`

[Out] $-2/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(a+2*x)/a)-\log(-a+x)$

Fricas [A] time = 1.02411, size = 81, normalized size = 2.61

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right)-\log(-a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*a*x+x^2)/(a^3-x^3),x, algorithm="fricas")`

[Out] $-2/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(a+2*x)/a)-\log(-a+x)$

Sympy [C] time = 0.287166, size = 54, normalized size = 1.74

$$-\log(-a+x)+\frac{\sqrt{3}i\log\left(\frac{a}{2}-\frac{\sqrt{3}ia}{2}+x\right)}{3}-\frac{\sqrt{3}i\log\left(\frac{a}{2}+\frac{\sqrt{3}ia}{2}+x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*a*x+x**2)/(a**3-x**3),x)`

[Out] $-\log(-a + x) + \sqrt{3} \cdot I \cdot \log(a/2 - \sqrt{3} \cdot I \cdot a/2 + x)/3 - \sqrt{3} \cdot I \cdot \log(a/2 + \sqrt{3} \cdot I \cdot a/2 + x)/3$

Giac [A] time = 1.05652, size = 39, normalized size = 1.26

$$-\frac{2}{3} \sqrt{3} \arctan\left(\frac{\sqrt{3}(a + 2x)}{3a}\right) - \log(|-a + x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*a*x+x^2)/(a^3-x^3),x, algorithm="giac")`

[Out] $-2/3 \cdot \sqrt{3} \cdot \arctan(1/3 \cdot \sqrt{3} \cdot (a + 2 \cdot x)/a) - \log(\text{abs}(-a + x))$

$$3.368 \quad \int \frac{x(2a+x)}{a^3-x^3} dx$$

Optimal. Leaf size=31

$$-\log(a-x) - \frac{2 \tan^{-1}\left(\frac{a+2x}{\sqrt{3}a}\right)}{\sqrt{3}}$$

[Out] $(-2*\text{ArcTan}[(a + 2*x)/(\text{Sqrt}[3]*a)])/\text{Sqrt}[3] - \text{Log}[a - x]$

Rubi [A] time = 0.0367465, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1868, 31, 617, 204}

$$-\log(a-x) - \frac{2 \tan^{-1}\left(\frac{a+2x}{\sqrt{3}a}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x*(2*a + x))/(a^3 - x^3), x]$

[Out] $(-2*\text{ArcTan}[(a + 2*x)/(\text{Sqrt}[3]*a)])/\text{Sqrt}[3] - \text{Log}[a - x]$

Rule 1868

$\text{Int}[(P2_)/((a_ + (b_)*(x_)^3), x_Symbol] := \text{With}[\{A = \text{Coeff}[P2, x, 0], B = \text{Coeff}[P2, x, 1], C = \text{Coeff}[P2, x, 2]\}, \text{With}[\{q = \text{Rt}[a/b, 3]\}, \text{Dist}[C/b, \text{Int}[1/(q + x), x], x] + \text{Dist}[(B + C*q)/b, \text{Int}[1/(q^2 - q*x + x^2), x], x]] / ; \text{EqQ}[A - \text{Rt}[a/b, 3]*B - 2*\text{Rt}[a/b, 3]^2*C, 0]] / ; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[P2, x, 2]$

Rule 31

$\text{Int}[(a_ + (b_)*(x_))^{(-1)}, x_Symbol] := \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] / ; \text{FreeQ}[\{a, b\}, x]$

Rule 617

$\text{Int}[(a_ + (b_)*(x_ + (c_)*(x_)^2))^{(-1)}, x_Symbol] := \text{With}[\{q = 1 - 4*S\text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] / ; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \|\ !\text{RationalQ}[b^2 - 4*a*c]) / ; \text{Free}$

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 204

$\text{Int}[(a_ + (b_ .)*(x_)^2)^{-1}, x_Symbol] \ :> \ -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \frac{x(2a+x)}{a^3-x^3} dx &= -\left(a \int \frac{1}{a^2+ax+x^2} dx\right) - \int \frac{1}{-a+x} dx \\ &= -\log(a-x) + 2 \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 + \frac{2x}{a}\right) \\ &= -\frac{2 \tan^{-1}\left(\frac{a+2x}{\sqrt{3}a}\right)}{\sqrt{3}} - \log(a-x) \end{aligned}$$

Mathematica [A] time = 0.0057692, size = 58, normalized size = 1.87

$$\frac{1}{3} \left(\log(a^2 + ax + x^2) - \log(x^3 - a^3) - 2 \log(x - a) - 2\sqrt{3} \tan^{-1}\left(\frac{a+2x}{\sqrt{3}a}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x*(2*a + x))/(a^3 - x^3), x]

[Out] (-2*Sqrt[3]*ArcTan[(a + 2*x)/(Sqrt[3]*a)] - 2*Log[-a + x] + Log[a^2 + a*x + x^2] - Log[-a^3 + x^3])/3

Maple [A] time = 0.005, size = 29, normalized size = 0.9

$$-\frac{2\sqrt{3}}{3} \arctan\left(\frac{(a+2x)\sqrt{3}}{3a}\right) - \ln(-a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(2*a+x)/(a^3-x^3), x)

[Out] $-2/3*\arctan(1/3*(a+2*x)/a*3^{(1/2)})*3^{(1/2)}-\ln(-a+x)$

Maxima [A] time = 1.41686, size = 38, normalized size = 1.23

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right)-\log(-a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(2*a+x)/(a^3-x^3),x, algorithm="maxima")`

[Out] $-2/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(a+2*x)/a)-\log(-a+x)$

Fricas [A] time = 0.941276, size = 81, normalized size = 2.61

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right)-\log(-a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(2*a+x)/(a^3-x^3),x, algorithm="fricas")`

[Out] $-2/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(a+2*x)/a)-\log(-a+x)$

Sympy [C] time = 0.288601, size = 54, normalized size = 1.74

$$-\log(-a+x) + \frac{\sqrt{3}i \log\left(\frac{a}{2} - \frac{\sqrt{3}ia}{2} + x\right)}{3} - \frac{\sqrt{3}i \log\left(\frac{a}{2} + \frac{\sqrt{3}ia}{2} + x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(2*a+x)/(a**3-x**3),x)`

[Out] $-\log(-a+x) + \sqrt{3}(3)*I*\log(a/2 - \sqrt{3}(3)*I*a/2 + x)/3 - \sqrt{3}(3)*I*\log(a/2 + \sqrt{3}(3)*I*a/2 + x)/3$

Giac [A] time = 1.05676, size = 39, normalized size = 1.26

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right) - \log(|-a+x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*a+x)/(a^3-x^3),x, algorithm="giac")

[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(abs(-a + x))

$$3.369 \quad \int \frac{x \left(-2 \sqrt[3]{\frac{a}{b}} C + Cx \right)}{a + bx^3} dx$$

Optimal. Leaf size=50

$$\frac{C \log \left(\sqrt[3]{\frac{a}{b}} + x \right)}{b} + \frac{2C \tan^{-1} \left(\frac{1 - \frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}} \right)}{\sqrt{3}b}$$

[Out] (2*C*ArcTan[(1 - (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C*Log[(a/b)^(1/3) + x])/b

Rubi [A] time = 0.086742, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1867, 31, 617, 204}

$$\frac{C \log \left(\sqrt[3]{\frac{a}{b}} + x \right)}{b} + \frac{2C \tan^{-1} \left(\frac{1 - \frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}} \right)}{\sqrt{3}b}$$

Antiderivative was successfully verified.

[In] Int[(x*(-2*(a/b)^(1/3)*C + C*x))/(a + b*x^3), x]

[Out] (2*C*ArcTan[(1 - (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C*Log[(a/b)^(1/3) + x])/b

Rule 1867

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[C/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x \left(-2\sqrt[3]{\frac{a}{b}}C + Cx \right)}{a + bx^3} dx &= \frac{C \int \frac{1}{\sqrt[3]{\frac{a}{b} + x}} dx}{b} - \frac{\left(\sqrt[3]{\frac{a}{b}}C \right) \int \frac{1}{\left(\frac{a}{b} \right)^{2/3} - \sqrt[3]{\frac{a}{b}}x + x^2} dx}{b} \\ &= \frac{C \log \left(\sqrt[3]{\frac{a}{b}} + x \right)}{b} - \frac{(2C) \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1 - \frac{2x}{\sqrt[3]{\frac{a}{b}}} \right)}{b} \\ &= \frac{2C \tan^{-1} \left(\frac{1 - \frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}} \right)}{\sqrt{3}b} + \frac{C \log \left(\sqrt[3]{\frac{a}{b}} + x \right)}{b} \end{aligned}$$

Mathematica [B] time = 0.0410268, size = 146, normalized size = 2.92

$$\frac{C \left(-\sqrt[3]{b} \sqrt[3]{\frac{a}{b}} \log \left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2 \right) + \sqrt[3]{a} \log \left(a + bx^3 \right) + 2\sqrt[3]{b} \sqrt[3]{\frac{a}{b}} \log \left(\sqrt[3]{a} + \sqrt[3]{bx} \right) + 2\sqrt{3} \sqrt[3]{b} \sqrt[3]{\frac{a}{b}} \tan^{-1} \left(\frac{1 - \frac{2\sqrt[3]{bx}}{\sqrt[3]{a}}}{\sqrt{3}} \right) \right)}{3\sqrt[3]{ab}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(-2*(a/b)^(1/3)*C + C*x))/(a + b*x^3), x]
```

```
[Out] (C*(2*Sqrt[3]*(a/b)^(1/3)*b^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]] + 2*(a/b)^(1/3)*b^(1/3)*Log[a^(1/3) + b^(1/3)*x] - (a/b)^(1/3)*b^(1/3)*L
og[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(1/3)*Log[a + b*x^3]))/(3
```

$*a^{(1/3)*b}$

Maple [A] time = 0.004, size = 87, normalized size = 1.7

$$\frac{2C}{3b} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) - \frac{C}{3b} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) - \frac{2C\sqrt{3}}{3b} \arctan\left(\frac{\sqrt{3}}{3} \left(2x \frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) + \frac{C \ln(bx^3 + a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(-2*(1/b*a)^(1/3)*C+C*x)/(b*x^3+a),x)`

[Out] `2/3*C*ln(x+(1/b*a)^(1/3))/b-1/3*C/b*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-2/3*C/b*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/3*C/b*ln(b*x^3+a)`

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(-2*(a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 1.08227, size = 139, normalized size = 2.78

$$\frac{2\sqrt{3}C \arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}} - \sqrt{3}a}{3a}\right) - 3C \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(-2*(a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="fricas")`

[Out] $-1/3*(2*\sqrt{3}*C*\arctan(1/3*(2*\sqrt{3})*b*x*(a/b)^{(2/3)} - \sqrt{3}*a)/a) - 3*C*\log(x + (a/b)^{(1/3)))/b$

Sympy [C] time = 0.413596, size = 100, normalized size = 2.

$$C \left(\log \left(\frac{a}{b \left(\frac{a}{b} \right)^{\frac{2}{3}}} + x \right) + \frac{\sqrt{3}i \log \left(-\frac{a}{2b \left(\frac{a}{b} \right)^{\frac{2}{3}}} - \frac{\sqrt{3}ia}{2b \left(\frac{a}{b} \right)^{\frac{2}{3}}} + x \right)}{3} - \frac{\sqrt{3}i \log \left(-\frac{a}{2b \left(\frac{a}{b} \right)^{\frac{2}{3}}} + \frac{\sqrt{3}ia}{2b \left(\frac{a}{b} \right)^{\frac{2}{3}}} + x \right)}{3} \right) / b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(-2*(a/b)**(1/3)*C+C*x)/(b*x**3+a),x)`

[Out] $C*(\log(a/(b*(a/b)**(2/3)) + x) + \sqrt{3}*I*\log(-a/(2*b*(a/b)**(2/3)) - \sqrt{3}*I*a/(2*b*(a/b)**(2/3)) + x)/3 - \sqrt{3}*I*\log(-a/(2*b*(a/b)**(2/3)) + \sqrt{3}*I*a/(2*b*(a/b)**(2/3)) + x)/3)/b$

Giac [B] time = 1.11064, size = 235, normalized size = 4.7

$$\frac{\left(Cb \left(-\frac{a}{b} \right)^{\frac{2}{3}} - 2 \left(ab^2 \right)^{\frac{1}{3}} C \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right) \left(-\frac{a}{b} \right)^{\frac{1}{3}} \log \left(\left| x - \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right| \right)}{3ab} - \frac{\sqrt{3} \left(ab^2 - \sqrt{3} \sqrt{a^2 b^4} i \right) C \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{3ab^3} + \frac{\left(3ab^2 - \sqrt{3} \sqrt{a^2 b^4} i \right) C \log \left(x^2 + x \left(-\frac{a}{b} \right)^{\frac{1}{3}} + \left(-\frac{a}{b} \right)^{\frac{2}{3}} \right)}{3ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(-2*(a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="giac")`

[Out] $-1/3*(C*b*(-a/b)^{(2/3)} - 2*(a*b^2)^{(1/3)*C*(-a/b)^{(1/3)}*(-a/b)^{(1/3)*\log(a*b*(x - (-a/b)^{(1/3)))/(a*b) - 1/3*\sqrt{3}*(a*b^2 - \sqrt{3}*\sqrt{a^2*b^4}*i)*C*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)))/(-a/b)^{(1/3)))/(a*b^3) + 1/6*(3*a*b^2 - \sqrt{3}*\sqrt{a^2*b^4}*i)*C*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)))/(a*b^3)$

$$3.370 \quad \int \frac{x \left(-2 \sqrt[3]{-\frac{a}{b}} C + Cx \right)}{a - bx^3} dx$$

Optimal. Leaf size=53

$$\frac{C \log \left(\sqrt[3]{-\frac{a}{b}} + x \right)}{b} - \frac{2C \tan^{-1} \left(\frac{1 - \frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}} \right)}{\sqrt{3}b}$$

[Out] $(-2*C*ArcTan[(1 - (2*x)/(-a/b))^(1/3))/Sqrt[3]]/(Sqrt[3]*b) - (C*Log[(-a/b)^(1/3) + x])/b$

Rubi [A] time = 0.0931049, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1867, 31, 617, 204}

$$\frac{C \log \left(\sqrt[3]{-\frac{a}{b}} + x \right)}{b} - \frac{2C \tan^{-1} \left(\frac{1 - \frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}} \right)}{\sqrt{3}b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x*(-2*(-a/b))^(1/3)*C + C*x))/(a - b*x^3), x]$

[Out] $(-2*C*ArcTan[(1 - (2*x)/(-a/b))^(1/3))/Sqrt[3]]/(Sqrt[3]*b) - (C*Log[(-a/b)^(1/3) + x])/b$

Rule 1867

$\text{Int}[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] \rightarrow \text{With}[\{A = \text{Coeff}[P2, x, 0], B = \text{Coeff}[P2, x, 1], C = \text{Coeff}[P2, x, 2]\}, \text{With}[\{q = (a/b)^{(1/3)}\}, \text{Dist}[C/b, \text{Int}[1/(q + x), x], x] + \text{Dist}[(B + C*q)/b, \text{Int}[1/(q^2 - q*x + x^2), x], x]] /; \text{EqQ}[A - (a/b)^{(1/3)*B} - 2*(a/b)^{(2/3)*C}, 0]] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[P2, x, 2]$

Rule 31

$\text{Int}[(a_) + (b_)*(x_)^(-1), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}[\{a, b\}, x]$

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{x \left(-2\sqrt[3]{-\frac{a}{b}}C + Cx \right)}{a - bx^3} dx &= -\frac{C \int \frac{1}{\sqrt[3]{-\frac{a}{b}+x}} dx}{b} + \frac{\left(\sqrt[3]{-\frac{a}{b}}C \right) \int \frac{1}{\left(-\frac{a}{b}\right)^{2/3} - \sqrt[3]{-\frac{a}{b}}x + x^2} dx}{b} \\ &= -\frac{C \log \left(\sqrt[3]{-\frac{a}{b}} + x \right)}{b} + \frac{(2C) \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1 - \frac{2x}{\sqrt[3]{-\frac{a}{b}}} \right)}{b} \\ &= -\frac{2C \tan^{-1} \left(\frac{1 - \frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}} \right)}{\sqrt{3}b} - \frac{C \log \left(\sqrt[3]{-\frac{a}{b}} + x \right)}{b} \end{aligned}$$

Mathematica [B] time = 0.0692506, size = 149, normalized size = 2.81

$$\frac{C \left(\sqrt[3]{b} \sqrt[3]{-\frac{a}{b}} \log \left(a^{2/3} + \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2 \right) + \sqrt[3]{a} \log \left(a - bx^3 \right) - 2\sqrt[3]{b} \sqrt[3]{-\frac{a}{b}} \log \left(\sqrt[3]{a} - \sqrt[3]{bx} \right) - 2\sqrt{3} \sqrt[3]{b} \sqrt[3]{-\frac{a}{b}} \tan^{-1} \left(\frac{2\sqrt[3]{bx} + 1}{\sqrt{3}} \right) \right)}{3\sqrt[3]{ab}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(-2*(-(a/b))^(1/3)*C + C*x))/(a - b*x^3), x]

[Out] -(C*(-2*Sqrt[3]*(-(a/b))^(1/3)*b^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3)]/Sqrt[3]] - 2*(-(a/b))^(1/3)*b^(1/3)*Log[a^(1/3) - b^(1/3)*x] + (-(a/b))^(1/3)*b^(1/3)*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(1/3)*Log[a -

$b*x^3)))/(3*a^{(1/3)*b})$

Maple [B] time = 0.006, size = 135, normalized size = 2.6

$$\frac{2C}{3b} \sqrt[3]{-\frac{a}{b}} \ln\left(x - \sqrt[3]{\frac{a}{b}}\right) \frac{1}{\sqrt[3]{\frac{a}{b}}} - \frac{C}{3b} \sqrt[3]{-\frac{a}{b}} \ln\left(x^2 + \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \frac{1}{\sqrt[3]{\frac{a}{b}}} + \frac{2C\sqrt{3}}{3b} \sqrt[3]{-\frac{a}{b}} \arctan\left(\frac{\sqrt{3}}{3} \left(1 + 2x \frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\right) \frac{1}{\sqrt[3]{\frac{a}{b}}} - \frac{C}{3b} \sqrt[3]{-\frac{a}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(-2*(-1/b*a)^(1/3)*C+C*x)/(-b*x^3+a), x)`

[Out] $\frac{2}{3}C*(-1/b*a)^{(1/3)}/b/(1/b*a)^{(1/3)}*\ln(x-(1/b*a)^{(1/3)})-1/3*C*(-1/b*a)^{(1/3)}/b/(1/b*a)^{(1/3)}*\ln(x^2+(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})+2/3*C*(-1/b*a)^{(1/3)}*3^{(1/2)}/b/(1/b*a)^{(1/3)}*\arctan(1/3*(1+2/(1/b*a)^{(1/3)}*x)*3^{(1/2)})-1/3*C/b*\ln(b*x^3-a)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(-2*(-a/b)^(1/3)*C+C*x)/(-b*x^3+a), x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 1.10121, size = 142, normalized size = 2.68

$$\frac{2\sqrt{3}C \arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}+\sqrt{3}a}{3a}\right) + 3C \log\left(x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(-2*(-a/b)^(1/3)*C+C*x)/(-b*x^3+a), x, algorithm="fricas")`

[Out] $-1/3*(2*\sqrt{3}*C*\arctan(1/3*(2*\sqrt{3})*b*x*(-a/b)^{(2/3)} + \sqrt{3}*a)/a) + 3*C*\log(x + (-a/b)^{(1/3)})/b$

Sympy [C] time = 0.435654, size = 110, normalized size = 2.08

$$\frac{C \left(\log \left(-\frac{a}{b \left(-\frac{a}{b} \right)^{\frac{2}{3}}} + x \right) - \frac{\sqrt{3}i \log \left(\frac{a}{2b \left(-\frac{a}{b} \right)^{\frac{2}{3}}} - \frac{\sqrt{3}ia}{2} + x \right)}{3} + \frac{\sqrt{3}i \log \left(\frac{a}{2b \left(-\frac{a}{b} \right)^{\frac{2}{3}}} + \frac{\sqrt{3}ia}{2} + x \right)}{3} \right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(-2*(-a/b)**(1/3)*C+C*x)/(-b*x**3+a),x)`

[Out] $-C*(\log(-a/(b*(-a/b)**(2/3)) + x) - \sqrt{3}*I*\log(a/(2*b*(-a/b)**(2/3)) - \sqrt{3}*I*a/(2*b*(-a/b)**(2/3)) + x)/3 + \sqrt{3}*I*\log(a/(2*b*(-a/b)**(2/3)) + \sqrt{3}*I*a/(2*b*(-a/b)**(2/3)) + x)/3)/b$

Giac [B] time = 1.09418, size = 223, normalized size = 4.21

$$\frac{\left(Cb \left(\frac{a}{b} \right)^{\frac{2}{3}} - 2(-ab^2)^{\frac{1}{3}} C \left(\frac{a}{b} \right)^{\frac{1}{3}} \right) \left(\frac{a}{b} \right)^{\frac{1}{3}} \log \left(\left| x - \left(\frac{a}{b} \right)^{\frac{1}{3}} \right| \right)}{3ab} + \frac{\sqrt{3}(ab^2 + \sqrt{3}\sqrt{a^2b^4}i)C \arctan \left(\frac{\sqrt{3} \left(2x + \left(\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{3ab^3} - \frac{(3ab^2 + \sqrt{3}\sqrt{a^2b^4}i)C}{3ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(-2*(-a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="giac")`

[Out] $-1/3*(C*b*(a/b)^{(2/3)} - 2*(-a*b^2)^{(1/3)}*C*(a/b)^{(1/3)})*(a/b)^{(1/3)}*\log(\text{abs}(x - (a/b)^{(1/3)}))/(a*b) + 1/3*\sqrt{3}*(a*b^2 + \sqrt{3}*\sqrt{a^2*b^4}*i)*C*\arctan(1/3*\sqrt{3}*(2*x + (a/b)^{(1/3)})/(a/b)^{(1/3)})/(a*b^3) - 1/6*(3*a*b^2 + \sqrt{3}*\sqrt{a^2*b^4}*i)*C*\log(x^2 + x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(a*b^3)$

$$3.371 \quad \int \frac{x \left(2 \sqrt[3]{-\frac{a}{b}} C + Cx \right)}{a + bx^3} dx$$

Optimal. Leaf size=54

$$\frac{C \log \left(\sqrt[3]{-\frac{a}{b}} - x \right)}{b} + \frac{2C \tan^{-1} \left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}} + 1}{\sqrt{3}} \right)}{\sqrt{3}b}$$

[Out] (2*C*ArcTan[(1 + (2*x)/(-a/b))^(1/3)]/Sqrt[3])/(Sqrt[3]*b) + (C*Log[(-a/b))^(1/3) - x])/b

Rubi [A] time = 0.0772552, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1869, 31, 617, 204}

$$\frac{C \log \left(\sqrt[3]{-\frac{a}{b}} - x \right)}{b} + \frac{2C \tan^{-1} \left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}} + 1}{\sqrt{3}} \right)}{\sqrt{3}b}$$

Antiderivative was successfully verified.

[In] Int[(x*(2*(-a/b))^(1/3)*C + C*x))/(a + b*x^3), x]

[Out] (2*C*ArcTan[(1 + (2*x)/(-a/b))^(1/3)]/Sqrt[3])/(Sqrt[3]*b) + (C*Log[(-a/b))^(1/3) - x])/b

Rule 1869

Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a/b)^(1/3)}, -Dist[C/b, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2), x], x]] /; EqQ[A + (-a/b)^(1/3)*B - 2*(-a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

Rule 31

Int[((a_) + (b_.)*(x_))^(1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x \left(2\sqrt[3]{-\frac{a}{b}}C + Cx \right)}{a + bx^3} dx &= \frac{C \int \frac{1}{\sqrt[3]{-\frac{a}{b}} - x} dx}{b} + \frac{\left(\sqrt[3]{-\frac{a}{b}}C \right) \int \frac{1}{\left(-\frac{a}{b}\right)^{2/3} + \sqrt[3]{-\frac{a}{b}}x + x^2} dx}{b} \\ &= \frac{C \log\left(\sqrt[3]{-\frac{a}{b}} - x\right)}{b} - \frac{(2C) \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 + \frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)}{b} \\ &= \frac{2C \tan^{-1}\left(\frac{1 + \frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b} + \frac{C \log\left(\sqrt[3]{-\frac{a}{b}} - x\right)}{b} \end{aligned}$$

Mathematica [B] time = 0.0415237, size = 148, normalized size = 2.74

$$\frac{C \left(\sqrt[3]{b} \sqrt[3]{-\frac{a}{b}} \log\left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3}x^2\right) + \sqrt[3]{a} \log\left(a + bx^3\right) - 2\sqrt[3]{b} \sqrt[3]{-\frac{a}{b}} \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right) - 2\sqrt{3} \sqrt[3]{b} \sqrt[3]{-\frac{a}{b}} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{bx}}{\sqrt{3}}\right) \right)}{3\sqrt[3]{ab}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2*(-(a/b))^(1/3)*C + C*x))/(a + b*x^3), x]
```

```
[Out] (C*(-2*Sqrt[3]*(-(a/b))^(1/3)*b^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*(-(a/b))^(1/3)*b^(1/3)*Log[a^(1/3) + b^(1/3)*x] + (-(a/b))^(1/3)*b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(1/3)*Log[a + b
```

$*x^3)))/(3*a^{(1/3)*b}$

Maple [B] time = 0.003, size = 132, normalized size = 2.4

$$-\frac{2C}{3b} \sqrt[3]{-\frac{a}{b}} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \frac{1}{\sqrt[3]{\frac{a}{b}}} + \frac{C}{3b} \sqrt[3]{-\frac{a}{b}} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \frac{1}{\sqrt[3]{\frac{a}{b}}} + \frac{2C\sqrt{3}}{3b} \sqrt[3]{-\frac{a}{b}} \arctan\left(\frac{\sqrt{3}}{3} \left(2x \frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) \frac{1}{\sqrt[3]{\frac{a}{b}}} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(2*(-1/b*a)^(1/3)*C+C*x)/(b*x^3+a),x)`

[Out] $-2/3*C*(-1/b*a)^{(1/3)}/b/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})+1/3*C*(-1/b*a)^{(1/3)}/b/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})+2/3*C*(-1/b*a)^{(1/3)}*3^{(1/2)}/b/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))+1/3*C/b*\ln(b*x^3+a)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(2*(-a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 1.1221, size = 142, normalized size = 2.63

$$\frac{2\sqrt{3}C \arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}} - \sqrt{3}a}{3a}\right) - 3C \log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(2*(-a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="fricas")`

[Out] $-1/3*(2*\sqrt{3}*C*\arctan(1/3*(2*\sqrt{3})*b*x*(-a/b)^{(2/3)} - \sqrt{3}*a)/a) - 3*C*\log(x - (-a/b)^{(1/3)})/b$

Sympy [C] time = 0.421174, size = 109, normalized size = 2.02

$$C \left(\log \left(\frac{a}{b(-\frac{a}{b})^{\frac{2}{3}}} + x \right) + \frac{\sqrt{3}i \log \left(-\frac{a}{2b(-\frac{a}{b})^{\frac{2}{3}}} - \frac{\sqrt{3}ia}{2} + x \right)}{3} - \frac{\sqrt{3}i \log \left(-\frac{a}{2b(-\frac{a}{b})^{\frac{2}{3}}} + \frac{\sqrt{3}ia}{2} + x \right)}{3} \right) / b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(2*(-a/b)**(1/3)*C+C*x)/(b*x**3+a),x)`

[Out] $C*(\log(a/(b*(-a/b)**(2/3)) + x) + \sqrt{3}*I*\log(-a/(2*b*(-a/b)**(2/3)) - \sqrt{3}*I*a/(2*b*(-a/b)**(2/3)) + x)/3 - \sqrt{3}*I*\log(-a/(2*b*(-a/b)**(2/3)) + \sqrt{3}*I*a/(2*b*(-a/b)**(2/3)) + x)/3)/b$

Giac [B] time = 1.06755, size = 131, normalized size = 2.43

$$\frac{2\sqrt{3}C \arctan \left(\frac{\sqrt{3} \left(2x + (-\frac{a}{b})^{\frac{1}{3}} \right)}{3(-\frac{a}{b})^{\frac{1}{3}}} \right)}{3b} - \frac{\left(Cb(-\frac{a}{b})^{\frac{2}{3}} + 2(-ab^2)^{\frac{1}{3}} C(-\frac{a}{b})^{\frac{1}{3}} \right) (-\frac{a}{b})^{\frac{1}{3}} \log \left(\left| x - (-\frac{a}{b})^{\frac{1}{3}} \right| \right)}{3ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(2*(-a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="giac")`

[Out] $2/3*\sqrt{3}*C*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/b - 1/3*(C*b*(-a/b)^{(2/3)} + 2*(-a*b^2)^{(1/3)}*C*(-a/b)^{(1/3)})*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/(a*b)$

$$3.372 \quad \int \frac{x \left(2 \sqrt[3]{\frac{a}{b}} C + Cx \right)}{a - bx^3} dx$$

Optimal. Leaf size=53

$$\frac{C \log \left(\sqrt[3]{\frac{a}{b}} - x \right)}{b} - \frac{2C \tan^{-1} \left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}} + 1}{\sqrt{3}} \right)}{\sqrt{3}b}$$

[Out] $(-2*C*ArcTan[(1 + (2*x)/(a/b)^{(1/3)})/Sqrt[3]])/(Sqrt[3]*b) - (C*Log[(a/b)^{(1/3)} - x])/b$

Rubi [A] time = 0.0755327, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1869, 31, 617, 204}

$$\frac{C \log \left(\sqrt[3]{\frac{a}{b}} - x \right)}{b} - \frac{2C \tan^{-1} \left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}} + 1}{\sqrt{3}} \right)}{\sqrt{3}b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x*(2*(a/b)^{(1/3)}*C + C*x))/(a - b*x^3), x]$

[Out] $(-2*C*ArcTan[(1 + (2*x)/(a/b)^{(1/3)})/Sqrt[3]])/(Sqrt[3]*b) - (C*Log[(a/b)^{(1/3)} - x])/b$

Rule 1869

$\text{Int}[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] \rightarrow \text{With}[\{A = \text{Coeff}[P2, x, 0], B = \text{Coeff}[P2, x, 1], C = \text{Coeff}[P2, x, 2]\}, \text{With}[\{q = (-a/b)^{(1/3)}\}, -\text{Dist}[C/b, \text{Int}[1/(q - x), x], x] + \text{Dist}[(B - C*q)/b, \text{Int}[1/(q^2 + q*x + x^2), x], x]] \text{ /; EqQ}[A + (-a/b)^{(1/3)}*B - 2*(-a/b)^{(2/3)}*C, 0]] \text{ /; FreeQ}[\{a, b\}, x] \ \&\& \ \text{PolyQ}[P2, x, 2]$

Rule 31

$\text{Int}[(a_) + (b_)*(x_)^(-1), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] \text{ /; FreeQ}[\{a, b\}, x]$

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x \left(2\sqrt[3]{\frac{a}{b}}C + Cx \right)}{a - bx^3} dx &= \frac{C \int \frac{1}{\sqrt[3]{\frac{a}{b}} - x} dx}{b} - \frac{\left(\sqrt[3]{\frac{a}{b}}C \right) \int \frac{1}{\left(\frac{a}{b}\right)^{2/3} + \sqrt[3]{\frac{a}{b}}x + x^2} dx}{b} \\ &= -\frac{C \log\left(\sqrt[3]{\frac{a}{b}} - x\right)}{b} + \frac{(2C) \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 + \frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)}{b} \\ &= -\frac{2C \tan^{-1}\left(\frac{1 + \frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b} - \frac{C \log\left(\sqrt[3]{\frac{a}{b}} - x\right)}{b} \end{aligned}$$

Mathematica [B] time = 0.0533153, size = 147, normalized size = 2.77

$$\frac{C \left(-\sqrt[3]{b} \sqrt[3]{\frac{a}{b}} \log\left(a^{2/3} + \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2\right) + \sqrt[3]{a} \log\left(a - bx^3\right) + 2\sqrt[3]{b} \sqrt[3]{\frac{a}{b}} \log\left(\sqrt[3]{a} - \sqrt[3]{bx}\right) + 2\sqrt{3} \sqrt[3]{b} \sqrt[3]{\frac{a}{b}} \tan^{-1}\left(\frac{2\sqrt[3]{bx} + 1}{\sqrt{3}}\right) \right)}{3\sqrt[3]{ab}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2*(a/b)^(1/3)*C + C*x))/(a - b*x^3), x]
```

```
[Out] -(C*(2*Sqrt[3]*(a/b)^(1/3)*b^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqrt[
3]] + 2*(a/b)^(1/3)*b^(1/3)*Log[a^(1/3) - b^(1/3)*x] - (a/b)^(1/3)*b^(1/3)*
Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(1/3)*Log[a - b*x^3]))/(
```


$$3a^{1/3}b$$

Maple [A] time = 0.005, size = 90, normalized size = 1.7

$$-\frac{2C}{3b} \ln\left(x - \sqrt[3]{\frac{a}{b}}\right) + \frac{C}{3b} \ln\left(x^2 + \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) - \frac{2C\sqrt{3}}{3b} \arctan\left(\frac{\sqrt{3}}{3} \left(1 + 2x \frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\right) - \frac{C \ln(bx^3 - a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(2*(1/b*a)^(1/3)*C+C*x)/(-b*x^3+a),x)`

[Out] `-2/3*C/b*ln(x-(1/b*a)^(1/3))+1/3*C/b*ln(x^2+(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-2/3*C*arctan(1/3*(1+2/(1/b*a)^(1/3)*x)*3^(1/2))/b*3^(1/2)-1/3*C/b*ln(b*x^3-a)`

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(2*(a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 1.03239, size = 139, normalized size = 2.62

$$\frac{2\sqrt{3}C \arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}} + \sqrt{3}a}{3a}\right) + 3C \log\left(x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(2*(a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="fricas")`

[Out] $-1/3*(2*\sqrt{3}*C*\arctan(1/3*(2*\sqrt{3})*b*x*(a/b)^{(2/3)} + \sqrt{3}*a)/a) + 3*C*\log(x - (a/b)^{(1/3)))/b$

Sympy [C] time = 0.430672, size = 102, normalized size = 1.92

$$\frac{C \left(\log \left(-\frac{a}{b \left(\frac{a}{b} \right)^{\frac{2}{3}}} + x \right) - \frac{\sqrt{3}i \log \left(\frac{\frac{a}{2b \left(\frac{a}{b} \right)^{\frac{2}{3}}} - \frac{\sqrt{3}ia}{2b \left(\frac{a}{b} \right)^{\frac{2}{3}}} + x}{3} \right) + \frac{\sqrt{3}i \log \left(\frac{\frac{a}{2b \left(\frac{a}{b} \right)^{\frac{2}{3}}} + \frac{\sqrt{3}ia}{2b \left(\frac{a}{b} \right)^{\frac{2}{3}}} + x}{3} \right)}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(2*(a/b)**(1/3)*C+C*x)/(-b*x**3+a),x)`

[Out] $-C*(\log(-a/(b*(a/b)**(2/3)) + x) - \sqrt{3}*I*\log(a/(2*b*(a/b)**(2/3)) - \sqrt{3}*I*a/(2*b*(a/b)**(2/3)) + x)/3 + \sqrt{3}*I*\log(a/(2*b*(a/b)**(2/3)) + \sqrt{3}*I*a/(2*b*(a/b)**(2/3)) + x)/3)/b$

Giac [A] time = 1.09216, size = 122, normalized size = 2.3

$$\frac{2\sqrt{3}C \arctan \left(\frac{\sqrt{3} \left(2x + \left(\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{3b} - \frac{\left(Cb \left(\frac{a}{b} \right)^{\frac{2}{3}} + 2 \left(ab^2 \right)^{\frac{1}{3}} C \left(\frac{a}{b} \right)^{\frac{1}{3}} \right) \left(\frac{a}{b} \right)^{\frac{1}{3}} \log \left(\left| x - \left(\frac{a}{b} \right)^{\frac{1}{3}} \right| \right)}{3ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(2*(a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="giac")`

[Out] $-2/3*\sqrt{3}*C*\arctan(1/3*\sqrt{3}*(2*x + (a/b)^{(1/3)))/(a/b)^{(1/3)))/b - 1/3*(C*b*(a/b)^{(2/3)} + 2*(a*b^2)^{(1/3)}*C*(a/b)^{(1/3)))*(a/b)^{(1/3)}*\log(\text{abs}(x - (a/b)^{(1/3)))/(a*b)$

$$3.373 \quad \int x^4 (a + bx^3) (c + dx + ex^2 + fx^3 + gx^4 + hx^5) dx$$

Optimal. Leaf size=97

$$\frac{1}{8}x^8(af + bc) + \frac{1}{9}x^9(ag + bd) + \frac{1}{10}x^{10}(ah + be) + \frac{1}{5}acx^5 + \frac{1}{6}adx^6 + \frac{1}{7}aex^7 + \frac{1}{11}bfx^{11} + \frac{1}{12}bgx^{12} + \frac{1}{13}bhx^{13}$$

[Out] (a*c*x^5)/5 + (a*d*x^6)/6 + (a*e*x^7)/7 + ((b*c + a*f)*x^8)/8 + ((b*d + a*g)*x^9)/9 + ((b*e + a*h)*x^10)/10 + (b*f*x^11)/11 + (b*g*x^12)/12 + (b*h*x^13)/13

Rubi [A] time = 0.121369, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {1820}

$$\frac{1}{8}x^8(af + bc) + \frac{1}{9}x^9(ag + bd) + \frac{1}{10}x^{10}(ah + be) + \frac{1}{5}acx^5 + \frac{1}{6}adx^6 + \frac{1}{7}aex^7 + \frac{1}{11}bfx^{11} + \frac{1}{12}bgx^{12} + \frac{1}{13}bhx^{13}$$

Antiderivative was successfully verified.

[In] Int[x^4*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]

[Out] (a*c*x^5)/5 + (a*d*x^6)/6 + (a*e*x^7)/7 + ((b*c + a*f)*x^8)/8 + ((b*d + a*g)*x^9)/9 + ((b*e + a*h)*x^10)/10 + (b*f*x^11)/11 + (b*g*x^12)/12 + (b*h*x^13)/13

Rule 1820

Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int x^4 (a + bx^3) (c + dx + ex^2 + fx^3 + gx^4 + hx^5) dx &= \int (acx^4 + adx^5 + aex^6 + (bc + af)x^7 + (bd + ag)x^8 + (be + ah)x^9) dx \\ &= \frac{1}{5}acx^5 + \frac{1}{6}adx^6 + \frac{1}{7}aex^7 + \frac{1}{8}(bc + af)x^8 + \frac{1}{9}(bd + ag)x^9 + \frac{1}{10}(be + ah)x^{10} \end{aligned}$$

Mathematica [A] time = 0.0280747, size = 97, normalized size = 1.

$$\frac{1}{8}x^8(af + bc) + \frac{1}{9}x^9(ag + bd) + \frac{1}{10}x^{10}(ah + be) + \frac{1}{5}acx^5 + \frac{1}{6}adx^6 + \frac{1}{7}aex^7 + \frac{1}{11}bfx^{11} + \frac{1}{12}bgx^{12} + \frac{1}{13}bhx^{13}$$

Antiderivative was successfully verified.

[In] Integrate[x^4*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]

[Out] (a*c*x^5)/5 + (a*d*x^6)/6 + (a*e*x^7)/7 + ((b*c + a*f)*x^8)/8 + ((b*d + a*g)*x^9)/9 + ((b*e + a*h)*x^10)/10 + (b*f*x^11)/11 + (b*g*x^12)/12 + (b*h*x^13)/13

Maple [A] time = 0.001, size = 80, normalized size = 0.8

$$\frac{acx^5}{5} + \frac{adx^6}{6} + \frac{aex^7}{7} + \frac{(af + bc)x^8}{8} + \frac{(ag + bd)x^9}{9} + \frac{(ah + be)x^{10}}{10} + \frac{bfx^{11}}{11} + \frac{bgx^{12}}{12} + \frac{bhx^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)

[Out] 1/5*a*c*x^5+1/6*a*d*x^6+1/7*a*e*x^7+1/8*(a*f+b*c)*x^8+1/9*(a*g+b*d)*x^9+1/10*(a*h+b*e)*x^10+1/11*b*f*x^11+1/12*b*g*x^12+1/13*b*h*x^13

Maxima [A] time = 0.954434, size = 107, normalized size = 1.1

$$\frac{1}{13}bhx^{13} + \frac{1}{12}bgx^{12} + \frac{1}{11}bfx^{11} + \frac{1}{10}(be + ah)x^{10} + \frac{1}{9}(bd + ag)x^9 + \frac{1}{7}aex^7 + \frac{1}{8}(bc + af)x^8 + \frac{1}{6}adx^6 + \frac{1}{5}acx^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima")

[Out] 1/13*b*h*x^13 + 1/12*b*g*x^12 + 1/11*b*f*x^11 + 1/10*(b*e + a*h)*x^10 + 1/9*(b*d + a*g)*x^9 + 1/7*a*e*x^7 + 1/8*(b*c + a*f)*x^8 + 1/6*a*d*x^6 + 1/5*a*c*x^5

Fricas [A] time = 0.844004, size = 239, normalized size = 2.46

$$\frac{1}{13}x^{13}hb + \frac{1}{12}x^{12}gb + \frac{1}{11}x^{11}fb + \frac{1}{10}x^{10}eb + \frac{1}{10}x^{10}ha + \frac{1}{9}x^9db + \frac{1}{9}x^9ga + \frac{1}{8}x^8cb + \frac{1}{8}x^8fa + \frac{1}{7}x^7ea + \frac{1}{6}x^6da + \frac{1}{5}x^5ca$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")

[Out] 1/13*x^13*h*b + 1/12*x^12*g*b + 1/11*x^11*f*b + 1/10*x^10*e*b + 1/10*x^10*h*a + 1/9*x^9*d*b + 1/9*x^9*g*a + 1/8*x^8*c*b + 1/8*x^8*f*a + 1/7*x^7*e*a + 1/6*x^6*d*a + 1/5*x^5*c*a

Sympy [A] time = 0.070904, size = 90, normalized size = 0.93

$$\frac{acx^5}{5} + \frac{adx^6}{6} + \frac{aex^7}{7} + \frac{bfx^{11}}{11} + \frac{bgx^{12}}{12} + \frac{bhx^{13}}{13} + x^{10}\left(\frac{ah}{10} + \frac{be}{10}\right) + x^9\left(\frac{ag}{9} + \frac{bd}{9}\right) + x^8\left(\frac{af}{8} + \frac{bc}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)

[Out] a*c*x**5/5 + a*d*x**6/6 + a*e*x**7/7 + b*f*x**11/11 + b*g*x**12/12 + b*h*x**13/13 + x**10*(a*h/10 + b*e/10) + x**9*(a*g/9 + b*d/9) + x**8*(a*f/8 + b*c/8)

Giac [A] time = 1.09469, size = 117, normalized size = 1.21

$$\frac{1}{13}bhx^{13} + \frac{1}{12}bgx^{12} + \frac{1}{11}bfx^{11} + \frac{1}{10}ahx^{10} + \frac{1}{10}bx^{10}e + \frac{1}{9}bdx^9 + \frac{1}{9}agx^9 + \frac{1}{8}bcx^8 + \frac{1}{8}afx^8 + \frac{1}{7}ax^7e + \frac{1}{6}adx^6 + \frac{1}{5}a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")

[Out] 1/13*b*h*x^13 + 1/12*b*g*x^12 + 1/11*b*f*x^11 + 1/10*a*h*x^10 + 1/10*b*x^10*e + 1/9*b*d*x^9 + 1/9*a*g*x^9 + 1/8*b*c*x^8 + 1/8*a*f*x^8 + 1/7*a*x^7*e + 1/6*a*d*x^6 + 1/5*a*c*x^5

$$3.374 \quad \int x^3 (a + bx^3) (c + dx + ex^2 + fx^3 + gx^4 + hx^5) dx$$

Optimal. Leaf size=97

$$\frac{1}{7}x^7(af + bc) + \frac{1}{8}x^8(ag + bd) + \frac{1}{9}x^9(ah + be) + \frac{1}{4}acx^4 + \frac{1}{5}adx^5 + \frac{1}{6}aex^6 + \frac{1}{10}bfx^{10} + \frac{1}{11}bgx^{11} + \frac{1}{12}bhx^{12}$$

[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + ((b*c + a*f)*x^7)/7 + ((b*d + a*g)*x^8)/8 + ((b*e + a*h)*x^9)/9 + (b*f*x^10)/10 + (b*g*x^11)/11 + (b*h*x^12)/12

Rubi [A] time = 0.100829, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {1820}

$$\frac{1}{7}x^7(af + bc) + \frac{1}{8}x^8(ag + bd) + \frac{1}{9}x^9(ah + be) + \frac{1}{4}acx^4 + \frac{1}{5}adx^5 + \frac{1}{6}aex^6 + \frac{1}{10}bfx^{10} + \frac{1}{11}bgx^{11} + \frac{1}{12}bhx^{12}$$

Antiderivative was successfully verified.

[In] Int[x^3*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]

[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + ((b*c + a*f)*x^7)/7 + ((b*d + a*g)*x^8)/8 + ((b*e + a*h)*x^9)/9 + (b*f*x^10)/10 + (b*g*x^11)/11 + (b*h*x^12)/12

Rule 1820

Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\int x^3 (a + bx^3) (c + dx + ex^2 + fx^3 + gx^4 + hx^5) dx = \int (acx^3 + adx^4 + aex^5 + (bc + af)x^6 + (bd + ag)x^7 + (be + ah)x^8 + \frac{1}{4}acx^4 + \frac{1}{5}adx^5 + \frac{1}{6}aex^6 + \frac{1}{7}(bc + af)x^7 + \frac{1}{8}(bd + ag)x^8 + \frac{1}{9}(be + ah)x^9) dx$$

Mathematica [A] time = 0.027894, size = 97, normalized size = 1.

$$\frac{1}{7}x^7(af + bc) + \frac{1}{8}x^8(ag + bd) + \frac{1}{9}x^9(ah + be) + \frac{1}{4}acx^4 + \frac{1}{5}adx^5 + \frac{1}{6}aex^6 + \frac{1}{10}bfx^{10} + \frac{1}{11}bgx^{11} + \frac{1}{12}bhx^{12}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]

[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + ((b*c + a*f)*x^7)/7 + ((b*d + a*g)*x^8)/8 + ((b*e + a*h)*x^9)/9 + (b*f*x^10)/10 + (b*g*x^11)/11 + (b*h*x^12)/12

Maple [A] time = 0.001, size = 80, normalized size = 0.8

$$\frac{acx^4}{4} + \frac{adx^5}{5} + \frac{aex^6}{6} + \frac{(af + bc)x^7}{7} + \frac{(ag + bd)x^8}{8} + \frac{(ah + be)x^9}{9} + \frac{bfx^{10}}{10} + \frac{bgx^{11}}{11} + \frac{bhx^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)

[Out] 1/4*a*c*x^4+1/5*a*d*x^5+1/6*a*e*x^6+1/7*(a*f+b*c)*x^7+1/8*(a*g+b*d)*x^8+1/9*(a*h+b*e)*x^9+1/10*b*f*x^10+1/11*b*g*x^11+1/12*b*h*x^12

Maxima [A] time = 0.936407, size = 107, normalized size = 1.1

$$\frac{1}{12}bhx^{12} + \frac{1}{11}bgx^{11} + \frac{1}{10}bfx^{10} + \frac{1}{9}(be + ah)x^9 + \frac{1}{8}(bd + ag)x^8 + \frac{1}{6}aex^6 + \frac{1}{7}(bc + af)x^7 + \frac{1}{5}adx^5 + \frac{1}{4}acx^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima")

[Out] 1/12*b*h*x^12 + 1/11*b*g*x^11 + 1/10*b*f*x^10 + 1/9*(b*e + a*h)*x^9 + 1/8*(b*d + a*g)*x^8 + 1/6*a*e*x^6 + 1/7*(b*c + a*f)*x^7 + 1/5*a*d*x^5 + 1/4*a*c*x^4

Fricas [A] time = 0.875431, size = 234, normalized size = 2.41

$$\frac{1}{12}x^{12}hb + \frac{1}{11}x^{11}gb + \frac{1}{10}x^{10}fb + \frac{1}{9}x^9eb + \frac{1}{9}x^9ha + \frac{1}{8}x^8db + \frac{1}{8}x^8ga + \frac{1}{7}x^7cb + \frac{1}{7}x^7fa + \frac{1}{6}x^6ea + \frac{1}{5}x^5da + \frac{1}{4}x^4ca$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")

[Out] 1/12*x^12*h*b + 1/11*x^11*g*b + 1/10*x^10*f*b + 1/9*x^9*e*b + 1/9*x^9*h*a + 1/8*x^8*d*b + 1/8*x^8*g*a + 1/7*x^7*c*b + 1/7*x^7*f*a + 1/6*x^6*e*a + 1/5*x^5*d*a + 1/4*x^4*c*a

Sympy [A] time = 0.072214, size = 90, normalized size = 0.93

$$\frac{acx^4}{4} + \frac{adx^5}{5} + \frac{aex^6}{6} + \frac{bfx^{10}}{10} + \frac{bgx^{11}}{11} + \frac{bhx^{12}}{12} + x^9\left(\frac{ah}{9} + \frac{be}{9}\right) + x^8\left(\frac{ag}{8} + \frac{bd}{8}\right) + x^7\left(\frac{af}{7} + \frac{bc}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)

[Out] a*c*x**4/4 + a*d*x**5/5 + a*e*x**6/6 + b*f*x**10/10 + b*g*x**11/11 + b*h*x**12/12 + x**9*(a*h/9 + b*e/9) + x**8*(a*g/8 + b*d/8) + x**7*(a*f/7 + b*c/7)

Giac [A] time = 1.06642, size = 117, normalized size = 1.21

$$\frac{1}{12}b hx^{12} + \frac{1}{11}bgx^{11} + \frac{1}{10}bfx^{10} + \frac{1}{9}ahx^9 + \frac{1}{9}bx^9e + \frac{1}{8}bdx^8 + \frac{1}{8}agx^8 + \frac{1}{7}bcx^7 + \frac{1}{7}afx^7 + \frac{1}{6}ax^6e + \frac{1}{5}adx^5 + \frac{1}{4}acx^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")

[Out] 1/12*b*h*x^12 + 1/11*b*g*x^11 + 1/10*b*f*x^10 + 1/9*a*h*x^9 + 1/9*b*x^9*e + 1/8*b*d*x^8 + 1/8*a*g*x^8 + 1/7*b*c*x^7 + 1/7*a*f*x^7 + 1/6*a*x^6*e + 1/5*a*d*x^5 + 1/4*a*c*x^4

$$3.375 \quad \int x^2 (a + bx^3) (c + dx + ex^2 + fx^3 + gx^4 + hx^5) dx$$

Optimal. Leaf size=97

$$\frac{1}{6}x^6(af + bc) + \frac{1}{7}x^7(ag + bd) + \frac{1}{8}x^8(ah + be) + \frac{1}{3}acx^3 + \frac{1}{4}adx^4 + \frac{1}{5}aex^5 + \frac{1}{9}bfx^9 + \frac{1}{10}bgx^{10} + \frac{1}{11}bhx^{11}$$

[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + ((b*c + a*f)*x^6)/6 + ((b*d + a*g)*x^7)/7 + ((b*e + a*h)*x^8)/8 + (b*f*x^9)/9 + (b*g*x^10)/10 + (b*h*x^11)/11
1

Rubi [A] time = 0.0914639, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {1820}

$$\frac{1}{6}x^6(af + bc) + \frac{1}{7}x^7(ag + bd) + \frac{1}{8}x^8(ah + be) + \frac{1}{3}acx^3 + \frac{1}{4}adx^4 + \frac{1}{5}aex^5 + \frac{1}{9}bfx^9 + \frac{1}{10}bgx^{10} + \frac{1}{11}bhx^{11}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]

[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + ((b*c + a*f)*x^6)/6 + ((b*d + a*g)*x^7)/7 + ((b*e + a*h)*x^8)/8 + (b*f*x^9)/9 + (b*g*x^10)/10 + (b*h*x^11)/11
1

Rule 1820

Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :=
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int x^2 (a + bx^3) (c + dx + ex^2 + fx^3 + gx^4 + hx^5) dx &= \int (acx^2 + adx^3 + aex^4 + (bc + af)x^5 + (bd + ag)x^6 + (be + ah)x^7) dx \\ &= \frac{1}{3}acx^3 + \frac{1}{4}adx^4 + \frac{1}{5}aex^5 + \frac{1}{6}(bc + af)x^6 + \frac{1}{7}(bd + ag)x^7 + \frac{1}{8}(be + ah)x^8 \end{aligned}$$

Mathematica [A] time = 0.0242234, size = 97, normalized size = 1.

$$\frac{1}{6}x^6(af + bc) + \frac{1}{7}x^7(ag + bd) + \frac{1}{8}x^8(ah + be) + \frac{1}{3}acx^3 + \frac{1}{4}adx^4 + \frac{1}{5}aex^5 + \frac{1}{9}bfx^9 + \frac{1}{10}bgx^{10} + \frac{1}{11}bhx^{11}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5), x]

[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + ((b*c + a*f)*x^6)/6 + ((b*d + a*g)*x^7)/7 + ((b*e + a*h)*x^8)/8 + (b*f*x^9)/9 + (b*g*x^10)/10 + (b*h*x^11)/11

Maple [A] time = 0.001, size = 80, normalized size = 0.8

$$\frac{acx^3}{3} + \frac{adx^4}{4} + \frac{aex^5}{5} + \frac{(af + bc)x^6}{6} + \frac{(ag + bd)x^7}{7} + \frac{(ah + be)x^8}{8} + \frac{bfx^9}{9} + \frac{bgx^{10}}{10} + \frac{bhx^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c), x)

[Out] 1/3*a*c*x^3+1/4*a*d*x^4+1/5*a*e*x^5+1/6*(a*f+b*c)*x^6+1/7*(a*g+b*d)*x^7+1/8*(a*h+b*e)*x^8+1/9*b*f*x^9+1/10*b*g*x^10+1/11*b*h*x^11

Maxima [A] time = 0.941309, size = 107, normalized size = 1.1

$$\frac{1}{11}bhx^{11} + \frac{1}{10}bgx^{10} + \frac{1}{9}bfx^9 + \frac{1}{8}(be + ah)x^8 + \frac{1}{7}(bd + ag)x^7 + \frac{1}{5}aex^5 + \frac{1}{6}(bc + af)x^6 + \frac{1}{4}adx^4 + \frac{1}{3}acx^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c), x, algorithm="maxima")

[Out] 1/11*b*h*x^11 + 1/10*b*g*x^10 + 1/9*b*f*x^9 + 1/8*(b*e + a*h)*x^8 + 1/7*(b*d + a*g)*x^7 + 1/5*a*e*x^5 + 1/6*(b*c + a*f)*x^6 + 1/4*a*d*x^4 + 1/3*a*c*x^3

Fricas [A] time = 0.838869, size = 231, normalized size = 2.38

$$\frac{1}{11}x^{11}hb + \frac{1}{10}x^{10}gb + \frac{1}{9}x^9fb + \frac{1}{8}x^8eb + \frac{1}{8}x^8ha + \frac{1}{7}x^7db + \frac{1}{7}x^7ga + \frac{1}{6}x^6cb + \frac{1}{6}x^6fa + \frac{1}{5}x^5ea + \frac{1}{4}x^4da + \frac{1}{3}x^3ca$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")

[Out] 1/11*x^11*h*b + 1/10*x^10*g*b + 1/9*x^9*f*b + 1/8*x^8*e*b + 1/8*x^8*h*a + 1/7*x^7*d*b + 1/7*x^7*g*a + 1/6*x^6*c*b + 1/6*x^6*f*a + 1/5*x^5*e*a + 1/4*x^4*d*a + 1/3*x^3*c*a

Sympy [A] time = 0.071216, size = 90, normalized size = 0.93

$$\frac{acx^3}{3} + \frac{adx^4}{4} + \frac{aex^5}{5} + \frac{bfx^9}{9} + \frac{bgx^{10}}{10} + \frac{bhx^{11}}{11} + x^8 \left(\frac{ah}{8} + \frac{be}{8} \right) + x^7 \left(\frac{ag}{7} + \frac{bd}{7} \right) + x^6 \left(\frac{af}{6} + \frac{bc}{6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)

[Out] a*c*x**3/3 + a*d*x**4/4 + a*e*x**5/5 + b*f*x**9/9 + b*g*x**10/10 + b*h*x**11/11 + x**8*(a*h/8 + b*e/8) + x**7*(a*g/7 + b*d/7) + x**6*(a*f/6 + b*c/6)

Giac [A] time = 1.32095, size = 117, normalized size = 1.21

$$\frac{1}{11}bhx^{11} + \frac{1}{10}bgx^{10} + \frac{1}{9}bfx^9 + \frac{1}{8}ahx^8 + \frac{1}{8}bx^8e + \frac{1}{7}bdx^7 + \frac{1}{7}agx^7 + \frac{1}{6}bcx^6 + \frac{1}{6}afx^6 + \frac{1}{5}ax^5e + \frac{1}{4}adx^4 + \frac{1}{3}acx^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")

[Out] 1/11*b*h*x^11 + 1/10*b*g*x^10 + 1/9*b*f*x^9 + 1/8*a*h*x^8 + 1/8*b*x^8*e + 1/7*b*d*x^7 + 1/7*a*g*x^7 + 1/6*b*c*x^6 + 1/6*a*f*x^6 + 1/5*a*x^5*e + 1/4*a*d*x^4 + 1/3*a*c*x^3

3.376 $\int x(a + bx^3)(c + dx + ex^2 + fx^3 + gx^4 + hx^5) dx$

Optimal. Leaf size=97

$$\frac{1}{5}x^5(af + bc) + \frac{1}{6}x^6(ag + bd) + \frac{1}{7}x^7(ah + be) + \frac{1}{2}acx^2 + \frac{1}{3}adx^3 + \frac{1}{4}aex^4 + \frac{1}{8}bfx^8 + \frac{1}{9}bgx^9 + \frac{1}{10}bhx^{10}$$

[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + ((b*c + a*f)*x^5)/5 + ((b*d + a*g)*x^6)/6 + ((b*e + a*h)*x^7)/7 + (b*f*x^8)/8 + (b*g*x^9)/9 + (b*h*x^10)/10

Rubi [A] time = 0.0802172, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {1820}

$$\frac{1}{5}x^5(af + bc) + \frac{1}{6}x^6(ag + bd) + \frac{1}{7}x^7(ah + be) + \frac{1}{2}acx^2 + \frac{1}{3}adx^3 + \frac{1}{4}aex^4 + \frac{1}{8}bfx^8 + \frac{1}{9}bgx^9 + \frac{1}{10}bhx^{10}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5), x]

[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + ((b*c + a*f)*x^5)/5 + ((b*d + a*g)*x^6)/6 + ((b*e + a*h)*x^7)/7 + (b*f*x^8)/8 + (b*g*x^9)/9 + (b*h*x^10)/10

Rule 1820

Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int x(a + bx^3)(c + dx + ex^2 + fx^3 + gx^4 + hx^5) dx &= \int (acx + adx^2 + aex^3 + (bc + af)x^4 + (bd + ag)x^5 + (be + ah)x^6 + \\ &= \frac{1}{2}acx^2 + \frac{1}{3}adx^3 + \frac{1}{4}aex^4 + \frac{1}{5}(bc + af)x^5 + \frac{1}{6}(bd + ag)x^6 + \frac{1}{7}(be + \end{aligned}$$

Mathematica [A] time = 0.0173656, size = 97, normalized size = 1.

$$\frac{1}{5}x^5(af + bc) + \frac{1}{6}x^6(ag + bd) + \frac{1}{7}x^7(ah + be) + \frac{1}{2}acx^2 + \frac{1}{3}adx^3 + \frac{1}{4}aex^4 + \frac{1}{8}bfx^8 + \frac{1}{9}bgx^9 + \frac{1}{10}bhx^{10}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]

[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + ((b*c + a*f)*x^5)/5 + ((b*d + a*g)*x^6)/6 + ((b*e + a*h)*x^7)/7 + (b*f*x^8)/8 + (b*g*x^9)/9 + (b*h*x^10)/10

Maple [A] time = 0.002, size = 80, normalized size = 0.8

$$\frac{acx^2}{2} + \frac{adx^3}{3} + \frac{aex^4}{4} + \frac{(af + bc)x^5}{5} + \frac{(ag + bd)x^6}{6} + \frac{(ah + be)x^7}{7} + \frac{bfx^8}{8} + \frac{bgx^9}{9} + \frac{bhx^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)

[Out] 1/2*a*c*x^2+1/3*a*d*x^3+1/4*a*e*x^4+1/5*(a*f+b*c)*x^5+1/6*(a*g+b*d)*x^6+1/7*(a*h+b*e)*x^7+1/8*b*f*x^8+1/9*b*g*x^9+1/10*b*h*x^10

Maxima [A] time = 0.951365, size = 107, normalized size = 1.1

$$\frac{1}{10}bhx^{10} + \frac{1}{9}bgx^9 + \frac{1}{8}bfx^8 + \frac{1}{7}(be + ah)x^7 + \frac{1}{6}(bd + ag)x^6 + \frac{1}{4}aex^4 + \frac{1}{5}(bc + af)x^5 + \frac{1}{3}adx^3 + \frac{1}{2}acx^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima")

[Out] 1/10*b*h*x^10 + 1/9*b*g*x^9 + 1/8*b*f*x^8 + 1/7*(b*e + a*h)*x^7 + 1/6*(b*d + a*g)*x^6 + 1/4*a*e*x^4 + 1/5*(b*c + a*f)*x^5 + 1/3*a*d*x^3 + 1/2*a*c*x^2

Fricas [A] time = 0.893111, size = 228, normalized size = 2.35

$$\frac{1}{10}x^{10}hb + \frac{1}{9}x^9gb + \frac{1}{8}x^8fb + \frac{1}{7}x^7eb + \frac{1}{7}x^7ha + \frac{1}{6}x^6db + \frac{1}{6}x^6ga + \frac{1}{5}x^5cb + \frac{1}{5}x^5fa + \frac{1}{4}x^4ea + \frac{1}{3}x^3da + \frac{1}{2}x^2ca$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")

[Out] 1/10*x^10*h*b + 1/9*x^9*g*b + 1/8*x^8*f*b + 1/7*x^7*e*b + 1/7*x^7*h*a + 1/6*x^6*d*b + 1/6*x^6*g*a + 1/5*x^5*c*b + 1/5*x^5*f*a + 1/4*x^4*e*a + 1/3*x^3*d*a + 1/2*x^2*c*a

Sympy [A] time = 0.070509, size = 90, normalized size = 0.93

$$\frac{acx^2}{2} + \frac{adx^3}{3} + \frac{aex^4}{4} + \frac{bfx^8}{8} + \frac{bgx^9}{9} + \frac{bhx^{10}}{10} + x^7\left(\frac{ah}{7} + \frac{be}{7}\right) + x^6\left(\frac{ag}{6} + \frac{bd}{6}\right) + x^5\left(\frac{af}{5} + \frac{bc}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)

[Out] a*c*x**2/2 + a*d*x**3/3 + a*e*x**4/4 + b*f*x**8/8 + b*g*x**9/9 + b*h*x**10/10 + x**7*(a*h/7 + b*e/7) + x**6*(a*g/6 + b*d/6) + x**5*(a*f/5 + b*c/5)

Giac [A] time = 1.05363, size = 117, normalized size = 1.21

$$\frac{1}{10}bhx^{10} + \frac{1}{9}bgx^9 + \frac{1}{8}bfx^8 + \frac{1}{7}ahx^7 + \frac{1}{7}bx^7e + \frac{1}{6}bdx^6 + \frac{1}{6}agx^6 + \frac{1}{5}bcx^5 + \frac{1}{5}afx^5 + \frac{1}{4}ax^4e + \frac{1}{3}adx^3 + \frac{1}{2}acx^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")

[Out] 1/10*b*h*x^10 + 1/9*b*g*x^9 + 1/8*b*f*x^8 + 1/7*a*h*x^7 + 1/7*b*x^7*e + 1/6*b*d*x^6 + 1/6*a*g*x^6 + 1/5*b*c*x^5 + 1/5*a*f*x^5 + 1/4*a*x^4*e + 1/3*a*d*x^3 + 1/2*a*c*x^2

$$3.377 \quad \int (a + bx^3)(c + dx + ex^2 + fx^3 + gx^4 + hx^5) dx$$

Optimal. Leaf size=92

$$\frac{1}{4}x^4(af + bc) + \frac{1}{5}x^5(ag + bd) + \frac{1}{6}x^6(ah + be) + acx + \frac{1}{2}adx^2 + \frac{1}{3}aex^3 + \frac{1}{7}bfx^7 + \frac{1}{8}bgx^8 + \frac{1}{9}bhx^9$$

[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + ((b*c + a*f)*x^4)/4 + ((b*d + a*g)*x^5)/5 + ((b*e + a*h)*x^6)/6 + (b*f*x^7)/7 + (b*g*x^8)/8 + (b*h*x^9)/9

Rubi [A] time = 0.0734568, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.03$, Rules used = {1850}

$$\frac{1}{4}x^4(af + bc) + \frac{1}{5}x^5(ag + bd) + \frac{1}{6}x^6(ah + be) + acx + \frac{1}{2}adx^2 + \frac{1}{3}aex^3 + \frac{1}{7}bfx^7 + \frac{1}{8}bgx^8 + \frac{1}{9}bhx^9$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5), x]

[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + ((b*c + a*f)*x^4)/4 + ((b*d + a*g)*x^5)/5 + ((b*e + a*h)*x^6)/6 + (b*f*x^7)/7 + (b*g*x^8)/8 + (b*h*x^9)/9

Rule 1850

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int (a + bx^3)(c + dx + ex^2 + fx^3 + gx^4 + hx^5) dx &= \int (ac + adx + aex^2 + (bc + af)x^3 + (bd + ag)x^4 + (be + ah)x^5 + bfx^6 + bgx^7 + bgx^8 + bhx^9) dx \\ &= acx + \frac{1}{2}adx^2 + \frac{1}{3}aex^3 + \frac{1}{4}(bc + af)x^4 + \frac{1}{5}(bd + ag)x^5 + \frac{1}{6}(be + ah)x^6 + \frac{1}{7}bfx^7 + \frac{1}{8}bgx^8 + \frac{1}{9}bhx^9 \end{aligned}$$

Mathematica [A] time = 0.0132528, size = 92, normalized size = 1.

$$\frac{1}{4}x^4(af + bc) + \frac{1}{5}x^5(ag + bd) + \frac{1}{6}x^6(ah + be) + acx + \frac{1}{2}adx^2 + \frac{1}{3}aex^3 + \frac{1}{7}bfx^7 + \frac{1}{8}bgx^8 + \frac{1}{9}bhx^9$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]

[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + ((b*c + a*f)*x^4)/4 + ((b*d + a*g)*x^5)/5 + ((b*e + a*h)*x^6)/6 + (b*f*x^7)/7 + (b*g*x^8)/8 + (b*h*x^9)/9

Maple [A] time = 0., size = 77, normalized size = 0.8

$$acx + \frac{adx^2}{2} + \frac{aex^3}{3} + \frac{(af + bc)x^4}{4} + \frac{(ag + bd)x^5}{5} + \frac{(ah + be)x^6}{6} + \frac{bfx^7}{7} + \frac{bgx^8}{8} + \frac{bhx^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)

[Out] a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*(a*f+b*c)*x^4+1/5*(a*g+b*d)*x^5+1/6*(a*h+b*e)*x^6+1/7*b*f*x^7+1/8*b*g*x^8+1/9*b*h*x^9

Maxima [A] time = 0.933405, size = 103, normalized size = 1.12

$$\frac{1}{9}bhx^9 + \frac{1}{8}bgx^8 + \frac{1}{7}bfx^7 + \frac{1}{6}(be + ah)x^6 + \frac{1}{5}(bd + ag)x^5 + \frac{1}{3}aex^3 + \frac{1}{4}(bc + af)x^4 + \frac{1}{2}adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima")

[Out] 1/9*b*h*x^9 + 1/8*b*g*x^8 + 1/7*b*f*x^7 + 1/6*(b*e + a*h)*x^6 + 1/5*(b*d + a*g)*x^5 + 1/3*a*e*x^3 + 1/4*(b*c + a*f)*x^4 + 1/2*a*d*x^2 + a*c*x

Fricas [A] time = 0.846603, size = 217, normalized size = 2.36

$$\frac{1}{9}x^9hb + \frac{1}{8}x^8gb + \frac{1}{7}x^7fb + \frac{1}{6}x^6eb + \frac{1}{6}x^6ha + \frac{1}{5}x^5db + \frac{1}{5}x^5ga + \frac{1}{4}x^4cb + \frac{1}{4}x^4fa + \frac{1}{3}x^3ea + \frac{1}{2}x^2da + xca$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")

[Out] $\frac{1}{9}x^9hb + \frac{1}{8}x^8gb + \frac{1}{7}x^7fb + \frac{1}{6}x^6eb + \frac{1}{6}x^6ha + \frac{1}{5}x^5db + \frac{1}{5}x^5ga + \frac{1}{4}x^4cb + \frac{1}{4}x^4fa + \frac{1}{3}x^3ea + \frac{1}{2}x^2da + xca$

Sympy [A] time = 0.069543, size = 87, normalized size = 0.95

$$acx + \frac{adx^2}{2} + \frac{aex^3}{3} + \frac{bfx^7}{7} + \frac{bgx^8}{8} + \frac{bhx^9}{9} + x^6\left(\frac{ah}{6} + \frac{be}{6}\right) + x^5\left(\frac{ag}{5} + \frac{bd}{5}\right) + x^4\left(\frac{af}{4} + \frac{bc}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)

[Out] $a*c*x + a*d*x**2/2 + a*e*x**3/3 + b*f*x**7/7 + b*g*x**8/8 + b*h*x**9/9 + x**6*(a*h/6 + b*e/6) + x**5*(a*g/5 + b*d/5) + x**4*(a*f/4 + b*c/4)$

Giac [A] time = 1.06718, size = 113, normalized size = 1.23

$$\frac{1}{9}bhx^9 + \frac{1}{8}bgx^8 + \frac{1}{7}bfx^7 + \frac{1}{6}ahx^6 + \frac{1}{6}bx^6e + \frac{1}{5}bdx^5 + \frac{1}{5}agx^5 + \frac{1}{4}bcx^4 + \frac{1}{4}afx^4 + \frac{1}{3}ax^3e + \frac{1}{2}adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")

[Out] $\frac{1}{9}b*h*x^9 + \frac{1}{8}b*g*x^8 + \frac{1}{7}b*f*x^7 + \frac{1}{6}a*h*x^6 + \frac{1}{6}b*x^6*e + \frac{1}{5}b*d*x^5 + \frac{1}{5}a*g*x^5 + \frac{1}{4}b*c*x^4 + \frac{1}{4}a*f*x^4 + \frac{1}{3}a*x^3*e + \frac{1}{2}a*d*x^2 + a*c*x$

$$3.378 \quad \int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x} dx$$

Optimal. Leaf size=88

$$\frac{1}{3}x^3(af+bc) + \frac{1}{4}x^4(ag+bd) + \frac{1}{5}x^5(ah+be) + ac \log(x) + adx + \frac{1}{2}aex^2 + \frac{1}{6}bfx^6 + \frac{1}{7}bgx^7 + \frac{1}{8}bhx^8$$

[Out] a*d*x + (a*e*x^2)/2 + ((b*c + a*f)*x^3)/3 + ((b*d + a*g)*x^4)/4 + ((b*e + a*h)*x^5)/5 + (b*f*x^6)/6 + (b*g*x^7)/7 + (b*h*x^8)/8 + a*c*Log[x]

Rubi [A] time = 0.0578865, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {1820}

$$\frac{1}{3}x^3(af+bc) + \frac{1}{4}x^4(ag+bd) + \frac{1}{5}x^5(ah+be) + ac \log(x) + adx + \frac{1}{2}aex^2 + \frac{1}{6}bfx^6 + \frac{1}{7}bgx^7 + \frac{1}{8}bhx^8$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]

[Out] a*d*x + (a*e*x^2)/2 + ((b*c + a*f)*x^3)/3 + ((b*d + a*g)*x^4)/4 + ((b*e + a*h)*x^5)/5 + (b*f*x^6)/6 + (b*g*x^7)/7 + (b*h*x^8)/8 + a*c*Log[x]

Rule 1820

Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x} dx = \int \left(ad + \frac{ac}{x} + aex + (bc+af)x^2 + (bd+ag)x^3 + (be+ah)x^4 + bfx^5 \right) dx$$

$$= adx + \frac{1}{2}aex^2 + \frac{1}{3}(bc+af)x^3 + \frac{1}{4}(bd+ag)x^4 + \frac{1}{5}(be+ah)x^5 + \frac{1}{6}bfx^6$$

Mathematica [A] time = 0.0287393, size = 88, normalized size = 1.

$$\frac{1}{3}x^3(af+bc) + \frac{1}{4}x^4(ag+bd) + \frac{1}{5}x^5(ah+be) + ac \log(x) + adx + \frac{1}{2}aex^2 + \frac{1}{6}bfx^6 + \frac{1}{7}bgx^7 + \frac{1}{8}bhx^8$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]

[Out] a*d*x + (a*e*x^2)/2 + ((b*c + a*f)*x^3)/3 + ((b*d + a*g)*x^4)/4 + ((b*e + a*h)*x^5)/5 + (b*f*x^6)/6 + (b*g*x^7)/7 + (b*h*x^8)/8 + a*c*Log[x]

Maple [A] time = 0.003, size = 81, normalized size = 0.9

$$\frac{bhx^8}{8} + \frac{bgx^7}{7} + \frac{bfx^6}{6} + \frac{x^5ah}{5} + \frac{bex^5}{5} + \frac{x^4ag}{4} + \frac{bdx^4}{4} + \frac{x^3af}{3} + \frac{bcx^3}{3} + \frac{aex^2}{2} + adx + ac \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x)

[Out] 1/8*b*h*x^8+1/7*b*g*x^7+1/6*b*f*x^6+1/5*x^5*a*h+1/5*b*e*x^5+1/4*x^4*a*g+1/4*b*d*x^4+1/3*x^3*a*f+1/3*b*c*x^3+1/2*a*e*x^2+a*d*x+a*c*ln(x)

Maxima [A] time = 0.943872, size = 100, normalized size = 1.14

$$\frac{1}{8}bhx^8 + \frac{1}{7}bgx^7 + \frac{1}{6}bfx^6 + \frac{1}{5}(be + ah)x^5 + \frac{1}{4}(bd + ag)x^4 + \frac{1}{2}aex^2 + \frac{1}{3}(bc + af)x^3 + adx + ac \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="maxima")

[Out] 1/8*b*h*x^8 + 1/7*b*g*x^7 + 1/6*b*f*x^6 + 1/5*(b*e + a*h)*x^5 + 1/4*(b*d + a*g)*x^4 + 1/2*a*e*x^2 + 1/3*(b*c + a*f)*x^3 + a*d*x + a*c*log(x)

Fricas [A] time = 0.992809, size = 192, normalized size = 2.18

$$\frac{1}{8}bhx^8 + \frac{1}{7}bgx^7 + \frac{1}{6}bfx^6 + \frac{1}{5}(be + ah)x^5 + \frac{1}{4}(bd + ag)x^4 + \frac{1}{2}aex^2 + \frac{1}{3}(bc + af)x^3 + adx + ac \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="fricas")

[Out] $\frac{1}{8}bhx^8 + \frac{1}{7}b^2gx^7 + \frac{1}{6}b^2fx^6 + \frac{1}{5}(b^2e + a^2h)x^5 + \frac{1}{4}(b^2d + a^2g)x^4 + \frac{1}{2}a^2ex^2 + \frac{1}{3}(b^2c + a^2f)x^3 + a^2dx + a^2c \log(x)$

Sympy [A] time = 0.339668, size = 85, normalized size = 0.97

$$ac \log(x) + adx + \frac{aex^2}{2} + \frac{bfx^6}{6} + \frac{bgx^7}{7} + \frac{bhx^8}{8} + x^5 \left(\frac{ah}{5} + \frac{be}{5} \right) + x^4 \left(\frac{ag}{4} + \frac{bd}{4} \right) + x^3 \left(\frac{af}{3} + \frac{bc}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x,x)

[Out] $a^2c \log(x) + a^2dx + a^2ex^{2/2} + b^2fx^{6/6} + b^2gx^{7/7} + b^2hx^{8/8} + x^{*5}(a^2h/5 + b^2e/5) + x^{*4}(a^2g/4 + b^2d/4) + x^{*3}(a^2f/3 + b^2c/3)$

Giac [A] time = 1.05718, size = 112, normalized size = 1.27

$$\frac{1}{8}bhx^8 + \frac{1}{7}b^2gx^7 + \frac{1}{6}b^2fx^6 + \frac{1}{5}ahx^5 + \frac{1}{5}bx^5e + \frac{1}{4}bdx^4 + \frac{1}{4}agx^4 + \frac{1}{3}bcx^3 + \frac{1}{3}afx^3 + \frac{1}{2}ax^2e + adx + ac \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="giac")

[Out] $\frac{1}{8}bhx^8 + \frac{1}{7}b^2gx^7 + \frac{1}{6}b^2fx^6 + \frac{1}{5}a^2hx^5 + \frac{1}{5}b^2x^5e + \frac{1}{4}b^2dx^4 + \frac{1}{4}a^2gx^4 + \frac{1}{3}b^2cx^3 + \frac{1}{3}a^2fx^3 + \frac{1}{2}a^2x^2e + a^2dx + a^2c \log(\text{abs}(x))$

$$3.379 \quad \int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2} dx$$

Optimal. Leaf size=86

$$\frac{1}{2}x^2(af+bc) + \frac{1}{3}x^3(ag+bd) + \frac{1}{4}x^4(ah+be) - \frac{ac}{x} + ad \log(x) + aex + \frac{1}{5}bfx^5 + \frac{1}{6}bgx^6 + \frac{1}{7}bhx^7$$

[Out] $-\frac{(a*c)}{x} + a*e*x + \frac{(b*c + a*f)*x^2}{2} + \frac{(b*d + a*g)*x^3}{3} + \frac{(b*e + a*h)*x^4}{4} + \frac{(b*f*x^5)}{5} + \frac{(b*g*x^6)}{6} + \frac{(b*h*x^7)}{7} + a*d*\text{Log}[x]$

Rubi [A] time = 0.0662083, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {1820}

$$\frac{1}{2}x^2(af+bc) + \frac{1}{3}x^3(ag+bd) + \frac{1}{4}x^4(ah+be) - \frac{ac}{x} + ad \log(x) + aex + \frac{1}{5}bfx^5 + \frac{1}{6}bgx^6 + \frac{1}{7}bhx^7$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/x^2, x]$

[Out] $-\frac{(a*c)}{x} + a*e*x + \frac{(b*c + a*f)*x^2}{2} + \frac{(b*d + a*g)*x^3}{3} + \frac{(b*e + a*h)*x^4}{4} + \frac{(b*f*x^5)}{5} + \frac{(b*g*x^6)}{6} + \frac{(b*h*x^7)}{7} + a*d*\text{Log}[x]$

Rule 1820

$\text{Int}[(Pq_*)*((c_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x_Symbol] :>$
 $\text{Int}[\text{ExpandIntegrand}[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /;$ FreeQ[{a, b, c, m, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2} dx = \int \left(ae + \frac{ac}{x^2} + \frac{ad}{x} + (bc+af)x + (bd+ag)x^2 + (be+ah)x^3 + bfx^4 \right. \\ \left. = -\frac{ac}{x} + aex + \frac{1}{2}(bc+af)x^2 + \frac{1}{3}(bd+ag)x^3 + \frac{1}{4}(be+ah)x^4 + \frac{1}{5}bfx^5 \right) dx$$

Mathematica [A] time = 0.0412215, size = 86, normalized size = 1.

$$\frac{1}{2}x^2(af+bc) + \frac{1}{3}x^3(ag+bd) + \frac{1}{4}x^4(ah+be) - \frac{ac}{x} + ad \log(x) + aex + \frac{1}{5}bfx^5 + \frac{1}{6}bgx^6 + \frac{1}{7}bhx^7$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]

[Out] -((a*c)/x) + a*e*x + ((b*c + a*f)*x^2)/2 + ((b*d + a*g)*x^3)/3 + ((b*e + a*h)*x^4)/4 + (b*f*x^5)/5 + (b*g*x^6)/6 + (b*h*x^7)/7 + a*d*Log[x]

Maple [A] time = 0.004, size = 81, normalized size = 0.9

$$\frac{bhx^7}{7} + \frac{bgx^6}{6} + \frac{bfx^5}{5} + \frac{x^4ah}{4} + \frac{bex^4}{4} + \frac{x^3ag}{3} + \frac{bdx^3}{3} + \frac{afx^2}{2} + \frac{bcx^2}{2} + aex + ad \ln(x) - \frac{ac}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x)

[Out] 1/7*b*h*x^7+1/6*b*g*x^6+1/5*b*f*x^5+1/4*x^4*a*h+1/4*b*e*x^4+1/3*x^3*a*g+1/3*b*d*x^3+1/2*a*f*x^2+1/2*b*c*x^2+a*e*x+a*d*ln(x)-a*c/x

Maxima [A] time = 0.9478, size = 100, normalized size = 1.16

$$\frac{1}{7}bhx^7 + \frac{1}{6}bgx^6 + \frac{1}{5}bfx^5 + \frac{1}{4}(be + ah)x^4 + \frac{1}{3}(bd + ag)x^3 + aex + \frac{1}{2}(bc + af)x^2 + ad \log(x) - \frac{ac}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="maxima")

[Out] 1/7*b*h*x^7 + 1/6*b*g*x^6 + 1/5*b*f*x^5 + 1/4*(b*e + a*h)*x^4 + 1/3*(b*d + a*g)*x^3 + a*e*x + 1/2*(b*c + a*f)*x^2 + a*d*log(x) - a*c/x

Fricas [A] time = 1.02038, size = 212, normalized size = 2.47

$$\frac{60bhx^8 + 70bgx^7 + 84bfx^6 + 105(be + ah)x^5 + 140(bd + ag)x^4 + 420aex^2 + 210(bc + af)x^3 + 420adx \log(x) - 420ac}{420x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="fricas")

[Out] 1/420*(60*b*h*x^8 + 70*b*g*x^7 + 84*b*f*x^6 + 105*(b*e + a*h)*x^5 + 140*(b*d + a*g)*x^4 + 420*a*e*x^2 + 210*(b*c + a*f)*x^3 + 420*a*d*x*log(x) - 420*a*c)/x

Sympy [A] time = 0.347942, size = 82, normalized size = 0.95

$$-\frac{ac}{x} + ad \log(x) + aex + \frac{bfx^5}{5} + \frac{bgx^6}{6} + \frac{bhx^7}{7} + x^4 \left(\frac{ah}{4} + \frac{be}{4} \right) + x^3 \left(\frac{ag}{3} + \frac{bd}{3} \right) + x^2 \left(\frac{af}{2} + \frac{bc}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)

[Out] -a*c/x + a*d*log(x) + a*e*x + b*f*x**5/5 + b*g*x**6/6 + b*h*x**7/7 + x**4*(a*h/4 + b*e/4) + x**3*(a*g/3 + b*d/3) + x**2*(a*f/2 + b*c/2)

Giac [A] time = 1.07097, size = 112, normalized size = 1.3

$$\frac{1}{7}bhx^7 + \frac{1}{6}bgx^6 + \frac{1}{5}bfx^5 + \frac{1}{4}ahx^4 + \frac{1}{4}bx^4e + \frac{1}{3}bdx^3 + \frac{1}{3}agx^3 + \frac{1}{2}bcx^2 + \frac{1}{2}afx^2 + axe + ad \log(|x|) - \frac{ac}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="giac")

[Out] 1/7*b*h*x^7 + 1/6*b*g*x^6 + 1/5*b*f*x^5 + 1/4*a*h*x^4 + 1/4*b*x^4*e + 1/3*b*d*x^3 + 1/3*a*g*x^3 + 1/2*b*c*x^2 + 1/2*a*f*x^2 + a*x*e + a*d*log(abs(x)) - a*c/x

$$3.380 \quad \int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3} dx$$

Optimal. Leaf size=86

$$x(af+bc) + \frac{1}{2}x^2(ag+bd) + \frac{1}{3}x^3(ah+be) - \frac{ac}{2x^2} - \frac{ad}{x} + ae \log(x) + \frac{1}{4}bfx^4 + \frac{1}{5}bgx^5 + \frac{1}{6}bhx^6$$

[Out] $-(a*c)/(2*x^2) - (a*d)/x + (b*c + a*f)*x + ((b*d + a*g)*x^2)/2 + ((b*e + a*h)*x^3)/3 + (b*f*x^4)/4 + (b*g*x^5)/5 + (b*h*x^6)/6 + a*e*\text{Log}[x]$

Rubi [A] time = 0.0724331, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {1820}

$$x(af+bc) + \frac{1}{2}x^2(ag+bd) + \frac{1}{3}x^3(ah+be) - \frac{ac}{2x^2} - \frac{ad}{x} + ae \log(x) + \frac{1}{4}bfx^4 + \frac{1}{5}bgx^5 + \frac{1}{6}bhx^6$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/x^3, x]$

[Out] $-(a*c)/(2*x^2) - (a*d)/x + (b*c + a*f)*x + ((b*d + a*g)*x^2)/2 + ((b*e + a*h)*x^3)/3 + (b*f*x^4)/4 + (b*g*x^5)/5 + (b*h*x^6)/6 + a*e*\text{Log}[x]$

Rule 1820

$\text{Int}[(Pq_)*((c_)*(x_))^{(m_)}*((a_)+(b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow$
 $\text{Int}[\text{ExpandIntegrand}[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /;$ FreeQ[{a, b, c, m, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3} dx &= \int \left(bc \left(1 + \frac{af}{bc} \right) + \frac{ac}{x^3} + \frac{ad}{x^2} + \frac{ae}{x} + (bd+ag)x + (be+ah)x^2 + bfx^3 \right. \\ &= -\frac{ac}{2x^2} - \frac{ad}{x} + (bc+af)x + \frac{1}{2}(bd+ag)x^2 + \frac{1}{3}(be+ah)x^3 + \frac{1}{4}bfx^4 + \end{aligned}$$

Mathematica [A] time = 0.0597417, size = 78, normalized size = 0.91

$$\frac{a(-3c-6dx+6fx^3+3gx^4+2hx^5)}{6x^2} + ae \log(x) + bcx + \frac{1}{60}bx^2(30d+x(20e+15fx+12gx^2+10hx^3))$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]

[Out] b*c*x + (a*(-3*c - 6*d*x + 6*f*x^3 + 3*g*x^4 + 2*h*x^5))/(6*x^2) + (b*x^2*(30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x^3)))/60 + a*e*Log[x]

Maple [A] time = 0.005, size = 78, normalized size = 0.9

$$\frac{bhx^6}{6} + \frac{bgx^5}{5} + \frac{bfx^4}{4} + \frac{x^3ah}{3} + \frac{bex^3}{3} + \frac{x^2ag}{2} + \frac{bdx^2}{2} + afx + bcx + ae \ln(x) - \frac{ac}{2x^2} - \frac{ad}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x)

[Out] 1/6*b*h*x^6+1/5*b*g*x^5+1/4*b*f*x^4+1/3*x^3*a*h+1/3*b*e*x^3+1/2*x^2*a*g+1/2*b*d*x^2+a*f*x+b*c*x+a*e*ln(x)-1/2*a*c/x^2-a*d/x

Maxima [A] time = 0.940463, size = 100, normalized size = 1.16

$$\frac{1}{6}bhx^6 + \frac{1}{5}bgx^5 + \frac{1}{4}bfx^4 + \frac{1}{3}(be + ah)x^3 + \frac{1}{2}(bd + ag)x^2 + ae \log(x) + (bc + af)x - \frac{2adx + ac}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="maxima")

[Out] 1/6*b*h*x^6 + 1/5*b*g*x^5 + 1/4*b*f*x^4 + 1/3*(b*e + a*h)*x^3 + 1/2*(b*d + a*g)*x^2 + a*e*log(x) + (b*c + a*f)*x - 1/2*(2*a*d*x + a*c)/x^2

Fricas [A] time = 1.11308, size = 205, normalized size = 2.38

$$\frac{10bhx^8 + 12bgx^7 + 15bfx^6 + 20(be + ah)x^5 + 30(bd + ag)x^4 + 60aex^2 \log(x) + 60(bc + af)x^3 - 60adx - 30ac}{60x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="fricas")

[Out] 1/60*(10*b*h*x^8 + 12*b*g*x^7 + 15*b*f*x^6 + 20*(b*e + a*h)*x^5 + 30*(b*d + a*g)*x^4 + 60*a*e*x^2*log(x) + 60*(b*c + a*f)*x^3 - 60*a*d*x - 30*a*c)/x^2

Sympy [A] time = 0.456412, size = 82, normalized size = 0.95

$$ae \log(x) + \frac{bf x^4}{4} + \frac{bg x^5}{5} + \frac{bh x^6}{6} + x^3 \left(\frac{ah}{3} + \frac{be}{3} \right) + x^2 \left(\frac{ag}{2} + \frac{bd}{2} \right) + x(af + bc) - \frac{ac + 2adx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)

[Out] a*e*log(x) + b*f*x**4/4 + b*g*x**5/5 + b*h*x**6/6 + x**3*(a*h/3 + b*e/3) + x**2*(a*g/2 + b*d/2) + x*(a*f + b*c) - (a*c + 2*a*d*x)/(2*x**2)

Giac [A] time = 1.06629, size = 108, normalized size = 1.26

$$\frac{1}{6}bhx^6 + \frac{1}{5}bgx^5 + \frac{1}{4}bfx^4 + \frac{1}{3}ahx^3 + \frac{1}{3}bx^3e + \frac{1}{2}bdx^2 + \frac{1}{2}agx^2 + bcx + afx + ae \log(|x|) - \frac{2adx + ac}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="giac")

[Out] 1/6*b*h*x^6 + 1/5*b*g*x^5 + 1/4*b*f*x^4 + 1/3*a*h*x^3 + 1/3*b*x^3*e + 1/2*b*d*x^2 + 1/2*a*g*x^2 + b*c*x + a*f*x + a*e*log(abs(x)) - 1/2*(2*a*d*x + a*c)/x^2

$$3.381 \quad \int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4} dx$$

Optimal. Leaf size=86

$$\log(x)(af + bc) + x(ag + bd) + \frac{1}{2}x^2(ah + be) - \frac{ac}{3x^3} - \frac{ad}{2x^2} - \frac{ae}{x} + \frac{1}{3}bfx^3 + \frac{1}{4}bgx^4 + \frac{1}{5}bhx^5$$

[Out] $-(a*c)/(3*x^3) - (a*d)/(2*x^2) - (a*e)/x + (b*d + a*g)*x + ((b*e + a*h)*x^2)/2 + (b*f*x^3)/3 + (b*g*x^4)/4 + (b*h*x^5)/5 + (b*c + a*f)*\text{Log}[x]$

Rubi [A] time = 0.070949, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {1820}

$$\log(x)(af + bc) + x(ag + bd) + \frac{1}{2}x^2(ah + be) - \frac{ac}{3x^3} - \frac{ad}{2x^2} - \frac{ae}{x} + \frac{1}{3}bfx^3 + \frac{1}{4}bgx^4 + \frac{1}{5}bhx^5$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/x^4, x]$

[Out] $-(a*c)/(3*x^3) - (a*d)/(2*x^2) - (a*e)/x + (b*d + a*g)*x + ((b*e + a*h)*x^2)/2 + (b*f*x^3)/3 + (b*g*x^4)/4 + (b*h*x^5)/5 + (b*c + a*f)*\text{Log}[x]$

Rule 1820

$\text{Int}[(Pq_*)*((c_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}), x_Symbol] \rightarrow$
 $\text{Int}[\text{ExpandIntegrand}[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /;$ FreeQ[{a, b, c, m, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)(c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{x^4} dx &= \int \left(bd \left(1 + \frac{ag}{bd} \right) + \frac{ac}{x^4} + \frac{ad}{x^3} + \frac{ae}{x^2} + \frac{bc + af}{x} + (be + ah)x + bfx^2 + \right. \\ &= -\frac{ac}{3x^3} - \frac{ad}{2x^2} - \frac{ae}{x} + (bd + ag)x + \frac{1}{2}(be + ah)x^2 + \frac{1}{3}bfx^3 + \frac{1}{4}bgx^4 + \end{aligned}$$

Mathematica [A] time = 0.0605947, size = 76, normalized size = 0.88

$$\log(x)(af + bc) - \frac{a(2c + 3x(d + 2ex + x^3(-2g + hx)))}{6x^3} + \frac{1}{60}bx(60d + x(30e + x(20f + 15gx + 12hx^2)))$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]

[Out] -(a*(2*c + 3*x*(d + 2*e*x - x^3*(2*g + h*x))))/(6*x^3) + (b*x*(60*d + x*(30*e + x*(20*f + 15*g*x + 12*h*x^2)))/60 + (b*c + a*f)*Log[x]

Maple [A] time = 0.006, size = 76, normalized size = 0.9

$$\frac{bhx^5}{5} + \frac{bgx^4}{4} + \frac{bfx^3}{3} + \frac{x^2ah}{2} + \frac{bex^2}{2} + agx + bdx + \ln(x)af + \ln(x)bc - \frac{ac}{3x^3} - \frac{ad}{2x^2} - \frac{ae}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x)

[Out] 1/5*b*h*x^5+1/4*b*g*x^4+1/3*b*f*x^3+1/2*x^2*a*h+1/2*b*e*x^2+a*g*x+b*d*x+ln(x)*a*f+ln(x)*b*c-1/3*a*c/x^3-1/2*a*d/x^2-a*e/x

Maxima [A] time = 0.942655, size = 101, normalized size = 1.17

$$\frac{1}{5}bhx^5 + \frac{1}{4}bgx^4 + \frac{1}{3}bfx^3 + \frac{1}{2}(be + ah)x^2 + (bd + ag)x + (bc + af)\log(x) - \frac{6aex^2 + 3adx + 2ac}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="maxima")

[Out] 1/5*b*h*x^5 + 1/4*b*g*x^4 + 1/3*b*f*x^3 + 1/2*(b*e + a*h)*x^2 + (b*d + a*g)*x + (b*c + a*f)*log(x) - 1/6*(6*a*e*x^2 + 3*a*d*x + 2*a*c)/x^3

Fricas [A] time = 1.24613, size = 205, normalized size = 2.38

$$\frac{12bhx^8 + 15bgx^7 + 20bfx^6 + 30(be + ah)x^5 + 60(bd + ag)x^4 + 60(bc + af)x^3 \log(x) - 60aex^2 - 30adx - 20ac}{60x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="fricas")

[Out] 1/60*(12*b*h*x^8 + 15*b*g*x^7 + 20*b*f*x^6 + 30*(b*e + a*h)*x^5 + 60*(b*d + a*g)*x^4 + 60*(b*c + a*f)*x^3*log(x) - 60*a*e*x^2 - 30*a*d*x - 20*a*c)/x^3

Sympy [A] time = 0.732298, size = 82, normalized size = 0.95

$$\frac{bfx^3}{3} + \frac{bgx^4}{4} + \frac{bhx^5}{5} + x^2 \left(\frac{ah}{2} + \frac{be}{2} \right) + x(ag + bd) + (af + bc) \log(x) - \frac{2ac + 3adx + 6aex^2}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)

[Out] b*f*x**3/3 + b*g*x**4/4 + b*h*x**5/5 + x**2*(a*h/2 + b*e/2) + x*(a*g + b*d) + (a*f + b*c)*log(x) - (2*a*c + 3*a*d*x + 6*a*e*x**2)/(6*x**3)

Giac [A] time = 1.05191, size = 107, normalized size = 1.24

$$\frac{1}{5}bhx^5 + \frac{1}{4}bgx^4 + \frac{1}{3}bfx^3 + \frac{1}{2}ahx^2 + \frac{1}{2}bx^2e + bdx + agx + (bc + af) \log(|x|) - \frac{6ax^2e + 3adx + 2ac}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="giac")

[Out] 1/5*b*h*x^5 + 1/4*b*g*x^4 + 1/3*b*f*x^3 + 1/2*a*h*x^2 + 1/2*b*x^2*e + b*d*x + a*g*x + (b*c + a*f)*log(abs(x)) - 1/6*(6*a*x^2*e + 3*a*d*x + 2*a*c)/x^3

$$3.382 \quad \int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5} dx$$

Optimal. Leaf size=86

$$-\frac{af+bc}{x} + \log(x)(ag+bd) + x(ah+be) - \frac{ac}{4x^4} - \frac{ad}{3x^3} - \frac{ae}{2x^2} + \frac{1}{2}bfx^2 + \frac{1}{3}bgx^3 + \frac{1}{4}bhx^4$$

[Out] $-(a*c)/(4*x^4) - (a*d)/(3*x^3) - (a*e)/(2*x^2) - (b*c + a*f)/x + (b*e + a*h)*x + (b*f*x^2)/2 + (b*g*x^3)/3 + (b*h*x^4)/4 + (b*d + a*g)*\text{Log}[x]$

Rubi [A] time = 0.0729936, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {1820}

$$-\frac{af+bc}{x} + \log(x)(ag+bd) + x(ah+be) - \frac{ac}{4x^4} - \frac{ad}{3x^3} - \frac{ae}{2x^2} + \frac{1}{2}bfx^2 + \frac{1}{3}bgx^3 + \frac{1}{4}bhx^4$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/x^5, x]$

[Out] $-(a*c)/(4*x^4) - (a*d)/(3*x^3) - (a*e)/(2*x^2) - (b*c + a*f)/x + (b*e + a*h)*x + (b*f*x^2)/2 + (b*g*x^3)/3 + (b*h*x^4)/4 + (b*d + a*g)*\text{Log}[x]$

Rule 1820

$\text{Int}[(Pq_*)*((c_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow$
 $\text{Int}[\text{ExpandIntegrand}[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; \text{FreeQ}[\{a, b, c, m, n\}, x] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ (\text{IGtQ}[p, 0] \ || \ \text{EqQ}[n, 1])$

Rubi steps

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5} dx = \int \left(be \left(1 + \frac{ah}{be} \right) + \frac{ac}{x^5} + \frac{ad}{x^4} + \frac{ae}{x^3} + \frac{bc+af}{x^2} + \frac{bd+ag}{x} + bfx + bgx^2 \right) dx$$

$$= -\frac{ac}{4x^4} - \frac{ad}{3x^3} - \frac{ae}{2x^2} - \frac{bc+af}{x} + (be+ah)x + \frac{1}{2}bfx^2 + \frac{1}{3}bgx^3 + \frac{1}{4}bhx^4$$

Mathematica [A] time = 0.0607921, size = 77, normalized size = 0.9

$$\log(x)(ag+bd) - \frac{a(3c+4dx+6x^2(e+2fx-2hx^3))}{12x^4} + b \left(-\frac{c}{x} + ex + \frac{1}{12}x^2(6f+4gx+3hx^2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]

[Out] $b*(-(c/x) + e*x + (x^2*(6*f + 4*g*x + 3*h*x^2))/12) - (a*(3*c + 4*d*x + 6*x^2*(e + 2*f*x - 2*h*x^3)))/(12*x^4) + (b*d + a*g)*\text{Log}[x]$

Maple [A] time = 0.005, size = 76, normalized size = 0.9

$$\frac{bhx^4}{4} + \frac{bgx^3}{3} + \frac{bf x^2}{2} + ahx + bxe + \ln(x)ag + \ln(x)bd - \frac{ad}{3x^3} - \frac{ac}{4x^4} - \frac{ae}{2x^2} - \frac{af}{x} - \frac{bc}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x)

[Out] $1/4*b*h*x^4 + 1/3*b*g*x^3 + 1/2*b*f*x^2 + a*h*x + b*x*e + \ln(x)*a*g + \ln(x)*b*d - 1/3*a*d/x^3 - 1/4*a*c/x^4 - 1/2*a*e/x^2 - 1/x*a*f - 1/x*b*c$

Maxima [A] time = 1.20498, size = 101, normalized size = 1.17

$$\frac{1}{4}bhx^4 + \frac{1}{3}bgx^3 + \frac{1}{2}bf x^2 + (be + ah)x + (bd + ag)\log(x) - \frac{6aex^2 + 12(bc + af)x^3 + 4adx + 3ac}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="maxima")

[Out] $1/4*b*h*x^4 + 1/3*b*g*x^3 + 1/2*b*f*x^2 + (b*e + a*h)*x + (b*d + a*g)*\log(x) - 1/12*(6*a*e*x^2 + 12*(b*c + a*f)*x^3 + 4*a*d*x + 3*a*c)/x^4$

Fricas [A] time = 1.23532, size = 197, normalized size = 2.29

$$\frac{3bhx^8 + 4bgx^7 + 6bf x^6 + 12(be + ah)x^5 + 12(bd + ag)x^4 \log(x) - 6aex^2 - 12(bc + af)x^3 - 4adx - 3ac}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="fricas")

[Out] 1/12*(3*b*h*x^8 + 4*b*g*x^7 + 6*b*f*x^6 + 12*(b*e + a*h)*x^5 + 12*(b*d + a*g)*x^4*log(x) - 6*a*e*x^2 - 12*(b*c + a*f)*x^3 - 4*a*d*x - 3*a*c)/x^4

Sympy [A] time = 2.88505, size = 82, normalized size = 0.95

$$\frac{bfx^2}{2} + \frac{bgx^3}{3} + \frac{bhx^4}{4} + x(ah + be) + (ag + bd)\log(x) - \frac{3ac + 4adx + 6aex^2 + x^3(12af + 12bc)}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)

[Out] b*f*x**2/2 + b*g*x**3/3 + b*h*x**4/4 + x*(a*h + b*e) + (a*g + b*d)*log(x) - (3*a*c + 4*a*d*x + 6*a*e*x**2 + x**3*(12*a*f + 12*b*c))/(12*x**4)

Giac [A] time = 1.07568, size = 104, normalized size = 1.21

$$\frac{1}{4}bhx^4 + \frac{1}{3}bgx^3 + \frac{1}{2}bfx^2 + ahx + bxe + (bd + ag)\log(|x|) - \frac{12(bc + af)x^3 + 6ax^2e + 4adx + 3ac}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="giac")

[Out] 1/4*b*h*x^4 + 1/3*b*g*x^3 + 1/2*b*f*x^2 + a*h*x + b*x*e + (b*d + a*g)*log(abs(x)) - 1/12*(12*(b*c + a*f)*x^3 + 6*a*x^2*e + 4*a*d*x + 3*a*c)/x^4

$$3.383 \quad \int x^4 (a + bx^3)^2 (c + dx + ex^2 + fx^3 + gx^4 + hx^5) dx$$

Optimal. Leaf size=163

$$\frac{1}{5}a^2cx^5 + \frac{1}{6}a^2dx^6 + \frac{1}{7}a^2ex^7 + \frac{1}{11}bx^{11}(2af + bc) + \frac{1}{8}ax^8(af + 2bc) + \frac{1}{12}bx^{12}(2ag + bd) + \frac{1}{9}ax^9(ag + 2bd) + \frac{1}{13}bx^{13}(2ah + bc)$$

[Out] (a^2*c*x^5)/5 + (a^2*d*x^6)/6 + (a^2*e*x^7)/7 + (a*(2*b*c + a*f)*x^8)/8 + (a*(2*b*d + a*g)*x^9)/9 + (a*(2*b*e + a*h)*x^10)/10 + (b*(b*c + 2*a*f)*x^11)/11 + (b*(b*d + 2*a*g)*x^12)/12 + (b*(b*e + 2*a*h)*x^13)/13 + (b^2*f*x^14)/14 + (b^2*g*x^15)/15 + (b^2*h*x^16)/16

Rubi [A] time = 0.210111, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1820}

$$\frac{1}{5}a^2cx^5 + \frac{1}{6}a^2dx^6 + \frac{1}{7}a^2ex^7 + \frac{1}{11}bx^{11}(2af + bc) + \frac{1}{8}ax^8(af + 2bc) + \frac{1}{12}bx^{12}(2ag + bd) + \frac{1}{9}ax^9(ag + 2bd) + \frac{1}{13}bx^{13}(2ah + bc)$$

Antiderivative was successfully verified.

[In] Int[x^4*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]

[Out] (a^2*c*x^5)/5 + (a^2*d*x^6)/6 + (a^2*e*x^7)/7 + (a*(2*b*c + a*f)*x^8)/8 + (a*(2*b*d + a*g)*x^9)/9 + (a*(2*b*e + a*h)*x^10)/10 + (b*(b*c + 2*a*f)*x^11)/11 + (b*(b*d + 2*a*g)*x^12)/12 + (b*(b*e + 2*a*h)*x^13)/13 + (b^2*f*x^14)/14 + (b^2*g*x^15)/15 + (b^2*h*x^16)/16

Rule 1820

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\int x^4 (a + bx^3)^2 (c + dx + ex^2 + fx^3 + gx^4 + hx^5) dx = \int (a^2cx^4 + a^2dx^5 + a^2ex^6 + a(2bc + af)x^7 + a(2bd + ag)x^8 + a^2fx^9 + a^2gx^{10} + a^2hx^{11}) dx$$

$$= \frac{1}{5}a^2cx^5 + \frac{1}{6}a^2dx^6 + \frac{1}{7}a^2ex^7 + \frac{1}{8}a(2bc + af)x^8 + \frac{1}{9}a(2bd + ag)x^9 + \frac{1}{10}a^2fx^{10} + \frac{1}{11}a^2gx^{11} + \frac{1}{12}a^2hx^{12}$$

Mathematica [A] time = 0.0395111, size = 163, normalized size = 1.

$$\frac{1}{5}a^2cx^5 + \frac{1}{6}a^2dx^6 + \frac{1}{7}a^2ex^7 + \frac{1}{11}bx^{11}(2af + bc) + \frac{1}{8}ax^8(af + 2bc) + \frac{1}{12}bx^{12}(2ag + bd) + \frac{1}{9}ax^9(ag + 2bd) + \frac{1}{13}bx^{13}(2ah + bc)$$

Antiderivative was successfully verified.

[In] Integrate[x^4*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5), x]

[Out] (a^2*c*x^5)/5 + (a^2*d*x^6)/6 + (a^2*e*x^7)/7 + (a*(2*b*c + a*f)*x^8)/8 + (a*(2*b*d + a*g)*x^9)/9 + (a*(2*b*e + a*h)*x^10)/10 + (b*(b*c + 2*a*f)*x^11)/11 + (b*(b*d + 2*a*g)*x^12)/12 + (b*(b*e + 2*a*h)*x^13)/13 + (b^2*f*x^14)/14 + (b^2*g*x^15)/15 + (b^2*h*x^16)/16

Maple [A] time = 0.001, size = 152, normalized size = 0.9

$$\frac{b^2hx^{16}}{16} + \frac{b^2gx^{15}}{15} + \frac{b^2fx^{14}}{14} + \frac{(2abh + b^2e)x^{13}}{13} + \frac{(2abg + b^2d)x^{12}}{12} + \frac{(2abf + b^2c)x^{11}}{11} + \frac{(a^2h + 2aeb)x^{10}}{10} + \frac{(a^2g + 2abf)x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c), x)

[Out] 1/16*b^2*h*x^16+1/15*b^2*g*x^15+1/14*b^2*f*x^14+1/13*(2*a*b*h+b^2*e)*x^13+1/12*(2*a*b*g+b^2*d)*x^12+1/11*(2*a*b*f+b^2*c)*x^11+1/10*(a^2*h+2*a*b*e)*x^10+1/9*(a^2*g+2*a*b*d)*x^9+1/8*(a^2*f+2*a*b*c)*x^8+1/7*a^2*e*x^7+1/6*a^2*d*x^6+1/5*a^2*c*x^5

Maxima [A] time = 0.956301, size = 204, normalized size = 1.25

$$\frac{1}{16}b^2hx^{16} + \frac{1}{15}b^2gx^{15} + \frac{1}{14}b^2fx^{14} + \frac{1}{13}(b^2e + 2abh)x^{13} + \frac{1}{12}(b^2d + 2abg)x^{12} + \frac{1}{11}(b^2c + 2abf)x^{11} + \frac{1}{10}(2abe + a^2h)x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c), x, algorithm="maxima")

[Out] 1/16*b^2*h*x^16 + 1/15*b^2*g*x^15 + 1/14*b^2*f*x^14 + 1/13*(b^2*e + 2*a*b*h)*x^13 + 1/12*(b^2*d + 2*a*b*g)*x^12 + 1/11*(b^2*c + 2*a*b*f)*x^11 + 1/10*(

$$2*a*b*e + a^2*h)*x^{10} + 1/7*a^2*e*x^7 + 1/9*(2*a*b*d + a^2*g)*x^9 + 1/6*a^2*d*x^6 + 1/8*(2*a*b*c + a^2*f)*x^8 + 1/5*a^2*c*x^5$$

Fricas [A] time = 1.11735, size = 414, normalized size = 2.54

$$\frac{1}{16}x^{16}hb^2 + \frac{1}{15}x^{15}gb^2 + \frac{1}{14}x^{14}fb^2 + \frac{1}{13}x^{13}eb^2 + \frac{2}{13}x^{13}hba + \frac{1}{12}x^{12}db^2 + \frac{1}{6}x^{12}gba + \frac{1}{11}x^{11}cb^2 + \frac{2}{11}x^{11}fba + \frac{1}{5}x^{10}eba +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")

[Out] 1/16*x^16*h*b^2 + 1/15*x^15*g*b^2 + 1/14*x^14*f*b^2 + 1/13*x^13*e*b^2 + 2/13*x^13*h*b*a + 1/12*x^12*d*b^2 + 1/6*x^12*g*b*a + 1/11*x^11*c*b^2 + 2/11*x^11*f*b*a + 1/5*x^10*e*b*a + 1/10*x^10*h*a^2 + 2/9*x^9*d*b*a + 1/9*x^9*g*a^2 + 1/4*x^8*c*b*a + 1/8*x^8*f*a^2 + 1/7*x^7*e*a^2 + 1/6*x^6*d*a^2 + 1/5*x^5*c*a^2

Sympy [A] time = 0.095411, size = 167, normalized size = 1.02

$$\frac{a^2cx^5}{5} + \frac{a^2dx^6}{6} + \frac{a^2ex^7}{7} + \frac{b^2fx^{14}}{14} + \frac{b^2gx^{15}}{15} + \frac{b^2hx^{16}}{16} + x^{13} \left(\frac{2abh}{13} + \frac{b^2e}{13} \right) + x^{12} \left(\frac{abg}{6} + \frac{b^2d}{12} \right) + x^{11} \left(\frac{2abf}{11} + \frac{b^2c}{11} \right) + x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)

[Out] a**2*c*x**5/5 + a**2*d*x**6/6 + a**2*e*x**7/7 + b**2*f*x**14/14 + b**2*g*x**15/15 + b**2*h*x**16/16 + x**13*(2*a*b*h/13 + b**2*e/13) + x**12*(a*b*g/6 + b**2*d/12) + x**11*(2*a*b*f/11 + b**2*c/11) + x**10*(a**2*h/10 + a*b*e/5) + x**9*(a**2*g/9 + 2*a*b*d/9) + x**8*(a**2*f/8 + a*b*c/4)

Giac [A] time = 1.07411, size = 216, normalized size = 1.33

$$\frac{1}{16}b^2hx^{16} + \frac{1}{15}b^2gx^{15} + \frac{1}{14}b^2fx^{14} + \frac{2}{13}abhx^{13} + \frac{1}{13}b^2x^{13}e + \frac{1}{12}b^2dx^{12} + \frac{1}{6}abgx^{12} + \frac{1}{11}b^2cx^{11} + \frac{2}{11}abfx^{11} + \frac{1}{10}a^2h$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giasic")
```

```
[Out] 1/16*b^2*h*x^16 + 1/15*b^2*g*x^15 + 1/14*b^2*f*x^14 + 2/13*a*b*h*x^13 + 1/13*b^2*x^13*e + 1/12*b^2*d*x^12 + 1/6*a*b*g*x^12 + 1/11*b^2*c*x^11 + 2/11*a*b*f*x^11 + 1/10*a^2*h*x^10 + 1/5*a*b*x^10*e + 2/9*a*b*d*x^9 + 1/9*a^2*g*x^9 + 1/4*a*b*c*x^8 + 1/8*a^2*f*x^8 + 1/7*a^2*x^7*e + 1/6*a^2*d*x^6 + 1/5*a^2*c*x^5
```

$$3.384 \quad \int x^3 (a + bx^3)^2 (c + dx + ex^2 + fx^3 + gx^4 + hx^5) dx$$

Optimal. Leaf size=163

$$\frac{1}{4}a^2cx^4 + \frac{1}{5}a^2dx^5 + \frac{1}{6}a^2ex^6 + \frac{1}{10}bx^{10}(2af + bc) + \frac{1}{7}ax^7(af + 2bc) + \frac{1}{11}bx^{11}(2ag + bd) + \frac{1}{8}ax^8(ag + 2bd) + \frac{1}{12}bx^{12}(2ah + bc)$$

[Out] (a^2*c*x^4)/4 + (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a*(2*b*c + a*f)*x^7)/7 + (a*(2*b*d + a*g)*x^8)/8 + (a*(2*b*e + a*h)*x^9)/9 + (b*(b*c + 2*a*f)*x^10)/10 + (b*(b*d + 2*a*g)*x^11)/11 + (b*(b*e + 2*a*h)*x^12)/12 + (b^2*f*x^13)/13 + (b^2*g*x^14)/14 + (b^2*h*x^15)/15

Rubi [A] time = 0.15901, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1820}

$$\frac{1}{4}a^2cx^4 + \frac{1}{5}a^2dx^5 + \frac{1}{6}a^2ex^6 + \frac{1}{10}bx^{10}(2af + bc) + \frac{1}{7}ax^7(af + 2bc) + \frac{1}{11}bx^{11}(2ag + bd) + \frac{1}{8}ax^8(ag + 2bd) + \frac{1}{12}bx^{12}(2ah + bc)$$

Antiderivative was successfully verified.

[In] Int[x^3*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]

[Out] (a^2*c*x^4)/4 + (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a*(2*b*c + a*f)*x^7)/7 + (a*(2*b*d + a*g)*x^8)/8 + (a*(2*b*e + a*h)*x^9)/9 + (b*(b*c + 2*a*f)*x^10)/10 + (b*(b*d + 2*a*g)*x^11)/11 + (b*(b*e + 2*a*h)*x^12)/12 + (b^2*f*x^13)/13 + (b^2*g*x^14)/14 + (b^2*h*x^15)/15

Rule 1820

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\int x^3 (a + bx^3)^2 (c + dx + ex^2 + fx^3 + gx^4 + hx^5) dx = \int (a^2cx^3 + a^2dx^4 + a^2ex^5 + a(2bc + af)x^6 + a(2bd + ag)x^7 + a^2fx^8 + a^2gx^9 + a^2hx^{10}) dx$$

$$= \frac{1}{4}a^2cx^4 + \frac{1}{5}a^2dx^5 + \frac{1}{6}a^2ex^6 + \frac{1}{7}a(2bc + af)x^7 + \frac{1}{8}a(2bd + ag)x^8 + \frac{1}{9}a^2fx^9 + \frac{1}{10}a^2gx^{10} + \frac{1}{11}a^2hx^{11}$$

Mathematica [A] time = 0.027851, size = 163, normalized size = 1.

$$\frac{1}{4}a^2cx^4 + \frac{1}{5}a^2dx^5 + \frac{1}{6}a^2ex^6 + \frac{1}{10}bx^{10}(2af + bc) + \frac{1}{7}ax^7(af + 2bc) + \frac{1}{11}bx^{11}(2ag + bd) + \frac{1}{8}ax^8(ag + 2bd) + \frac{1}{12}bx^{12}(2ah + 2bd)$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5), x]

[Out] (a^2*c*x^4)/4 + (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a*(2*b*c + a*f)*x^7)/7 + (a*(2*b*d + a*g)*x^8)/8 + (a*(2*b*e + a*h)*x^9)/9 + (b*(b*c + 2*a*f)*x^10)/10 + (b*(b*d + 2*a*g)*x^11)/11 + (b*(b*e + 2*a*h)*x^12)/12 + (b^2*f*x^13)/13 + (b^2*g*x^14)/14 + (b^2*h*x^15)/15

Maple [A] time = 0.001, size = 152, normalized size = 0.9

$$\frac{b^2hx^{15}}{15} + \frac{b^2gx^{14}}{14} + \frac{b^2fx^{13}}{13} + \frac{(2abh + b^2e)x^{12}}{12} + \frac{(2abg + b^2d)x^{11}}{11} + \frac{(2abf + b^2c)x^{10}}{10} + \frac{(a^2h + 2aeb)x^9}{9} + \frac{(a^2g + 2bae)x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c), x)

[Out] 1/15*b^2*h*x^15+1/14*b^2*g*x^14+1/13*b^2*f*x^13+1/12*(2*a*b*h+b^2*e)*x^12+1/11*(2*a*b*g+b^2*d)*x^11+1/10*(2*a*b*f+b^2*c)*x^10+1/9*(a^2*h+2*a*b*e)*x^9+1/8*(a^2*g+2*a*b*d)*x^8+1/7*(a^2*f+2*a*b*c)*x^7+1/6*a^2*e*x^6+1/5*a^2*d*x^5+1/4*a^2*c*x^4

Maxima [A] time = 0.935679, size = 204, normalized size = 1.25

$$\frac{1}{15}b^2hx^{15} + \frac{1}{14}b^2gx^{14} + \frac{1}{13}b^2fx^{13} + \frac{1}{12}(b^2e + 2abh)x^{12} + \frac{1}{11}(b^2d + 2abg)x^{11} + \frac{1}{10}(b^2c + 2abf)x^{10} + \frac{1}{9}(2abe + a^2h)x^9$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c), x, algorithm="maxima")

[Out] 1/15*b^2*h*x^15 + 1/14*b^2*g*x^14 + 1/13*b^2*f*x^13 + 1/12*(b^2*e + 2*a*b*h)*x^12 + 1/11*(b^2*d + 2*a*b*g)*x^11 + 1/10*(b^2*c + 2*a*b*f)*x^10 + 1/9*(2

$$*a*b*e + a^2*h)*x^9 + 1/6*a^2*e*x^6 + 1/8*(2*a*b*d + a^2*g)*x^8 + 1/5*a^2*d*x^5 + 1/7*(2*a*b*c + a^2*f)*x^7 + 1/4*a^2*c*x^4$$

Fricas [A] time = 1.0839, size = 409, normalized size = 2.51

$$\frac{1}{15}x^{15}hb^2 + \frac{1}{14}x^{14}gb^2 + \frac{1}{13}x^{13}fb^2 + \frac{1}{12}x^{12}eb^2 + \frac{1}{6}x^{12}hba + \frac{1}{11}x^{11}db^2 + \frac{2}{11}x^{11}gba + \frac{1}{10}x^{10}cb^2 + \frac{1}{5}x^{10}fba + \frac{2}{9}x^9eba + \frac{1}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")

[Out] 1/15*x^15*h*b^2 + 1/14*x^14*g*b^2 + 1/13*x^13*f*b^2 + 1/12*x^12*e*b^2 + 1/6*x^12*h*b*a + 1/11*x^11*d*b^2 + 2/11*x^11*g*b*a + 1/10*x^10*c*b^2 + 1/5*x^10*f*b*a + 2/9*x^9*e*b*a + 1/9*x^9*h*a^2 + 1/4*x^8*d*b*a + 1/8*x^8*g*a^2 + 2/7*x^7*c*b*a + 1/7*x^7*f*a^2 + 1/6*x^6*e*a^2 + 1/5*x^5*d*a^2 + 1/4*x^4*c*a^2

Sympy [A] time = 0.13967, size = 167, normalized size = 1.02

$$\frac{a^2cx^4}{4} + \frac{a^2dx^5}{5} + \frac{a^2ex^6}{6} + \frac{b^2fx^{13}}{13} + \frac{b^2gx^{14}}{14} + \frac{b^2hx^{15}}{15} + x^{12} \left(\frac{abh}{6} + \frac{b^2e}{12} \right) + x^{11} \left(\frac{2abg}{11} + \frac{b^2d}{11} \right) + x^{10} \left(\frac{abf}{5} + \frac{b^2c}{10} \right) + x^9 \left(\frac{2abf}{9} + \frac{b^2c}{9} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)

[Out] a**2*c*x**4/4 + a**2*d*x**5/5 + a**2*e*x**6/6 + b**2*f*x**13/13 + b**2*g*x**14/14 + b**2*h*x**15/15 + x**12*(a*b*h/6 + b**2*e/12) + x**11*(2*a*b*g/11 + b**2*d/11) + x**10*(a*b*f/5 + b**2*c/10) + x**9*(a**2*h/9 + 2*a*b*e/9) + x**8*(a**2*g/8 + a*b*d/4) + x**7*(a**2*f/7 + 2*a*b*c/7)

Giac [A] time = 1.06638, size = 216, normalized size = 1.33

$$\frac{1}{15}b^2hx^{15} + \frac{1}{14}b^2gx^{14} + \frac{1}{13}b^2fx^{13} + \frac{1}{6}abhx^{12} + \frac{1}{12}b^2x^{12}e + \frac{1}{11}b^2dx^{11} + \frac{2}{11}abgx^{11} + \frac{1}{10}b^2cx^{10} + \frac{1}{5}abfx^{10} + \frac{1}{9}a^2hx^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gia  
c")
```

```
[Out] 1/15*b^2*h*x^15 + 1/14*b^2*g*x^14 + 1/13*b^2*f*x^13 + 1/6*a*b*h*x^12 + 1/12  
*b^2*x^12*e + 1/11*b^2*d*x^11 + 2/11*a*b*g*x^11 + 1/10*b^2*c*x^10 + 1/5*a*b  
*f*x^10 + 1/9*a^2*h*x^9 + 2/9*a*b*x^9*e + 1/4*a*b*d*x^8 + 1/8*a^2*g*x^8 + 2  
/7*a*b*c*x^7 + 1/7*a^2*f*x^7 + 1/6*a^2*x^6*e + 1/5*a^2*d*x^5 + 1/4*a^2*c*x^  
4
```


$$3.385 \quad \int x^2 (a + bx^3)^2 (c + dx + ex^2 + fx^3 + gx^4 + hx^5) dx$$

Optimal. Leaf size=158

$$\frac{1}{4}a^2dx^4 + \frac{1}{5}a^2ex^5 + \frac{1}{6}a^2fx^6 + \frac{c(a+bx^3)^3}{9b} + \frac{1}{10}bx^{10}(2ag+bd) + \frac{1}{7}ax^7(ag+2bd) + \frac{1}{11}bx^{11}(2ah+be) + \frac{1}{8}ax^8(ah+2be)$$

[Out] (a^2*d*x^4)/4 + (a^2*e*x^5)/5 + (a^2*f*x^6)/6 + (a*(2*b*d + a*g)*x^7)/7 + (a*(2*b*e + a*h)*x^8)/8 + (2*a*b*f*x^9)/9 + (b*(b*d + 2*a*g)*x^10)/10 + (b*(b*e + 2*a*h)*x^11)/11 + (b^2*f*x^12)/12 + (b^2*g*x^13)/13 + (b^2*h*x^14)/14 + (c*(a + b*x^3)^3)/(9*b)

Rubi [A] time = 0.125777, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1582, 1850}

$$\frac{1}{4}a^2dx^4 + \frac{1}{5}a^2ex^5 + \frac{1}{6}a^2fx^6 + \frac{c(a+bx^3)^3}{9b} + \frac{1}{10}bx^{10}(2ag+bd) + \frac{1}{7}ax^7(ag+2bd) + \frac{1}{11}bx^{11}(2ah+be) + \frac{1}{8}ax^8(ah+2be)$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]

[Out] (a^2*d*x^4)/4 + (a^2*e*x^5)/5 + (a^2*f*x^6)/6 + (a*(2*b*d + a*g)*x^7)/7 + (a*(2*b*e + a*h)*x^8)/8 + (2*a*b*f*x^9)/9 + (b*(b*d + 2*a*g)*x^10)/10 + (b*(b*e + 2*a*h)*x^11)/11 + (b^2*f*x^12)/12 + (b^2*g*x^13)/13 + (b^2*h*x^14)/14 + (c*(a + b*x^3)^3)/(9*b)

Rule 1582

Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_)] /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a + b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]

Rule 1850

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned} \int x^2 (a + bx^3)^2 (c + dx + ex^2 + fx^3 + gx^4 + hx^5) dx &= \frac{c(a + bx^3)^3}{9b} + \int (a + bx^3)^2 (-cx^2 + x^2(c + dx + ex^2 + fx^3 + gx^4 + hx^5)) dx \\ &= \frac{c(a + bx^3)^3}{9b} + \int (a^2 dx^3 + a^2 ex^4 + a^2 fx^5 + a(2bd + ag)x^6 + a(2cd + 2bx^3d + 2bx^3e + 2bx^3f + 2bx^3g + 2bx^3h)) dx \\ &= \frac{1}{4}a^2 dx^4 + \frac{1}{5}a^2 ex^5 + \frac{1}{6}a^2 fx^6 + \frac{1}{7}a(2bd + ag)x^7 + \frac{1}{8}a(2be + ah)x^8 \end{aligned}$$

Mathematica [A] time = 0.0759694, size = 150, normalized size = 0.95

$$a^2 \left(\frac{cx^3}{3} + \frac{dx^4}{4} + \frac{ex^5}{5} + \frac{fx^6}{6} + \frac{gx^7}{7} + \frac{hx^8}{8} \right) + ab \left(\frac{cx^6}{3} + \frac{2dx^7}{7} + \frac{ex^8}{4} + \frac{2fx^9}{9} + \frac{gx^{10}}{5} + \frac{2hx^{11}}{11} \right) + \frac{b^2 x^9 (20020c + 3x(6006d + 5460e + 55x(91f + 84gx + 78hx^2)))}{180180}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5), x]
```

```
[Out] a^2*((c*x^3)/3 + (d*x^4)/4 + (e*x^5)/5 + (f*x^6)/6 + (g*x^7)/7 + (h*x^8)/8)
+ a*b*((c*x^6)/3 + (2*d*x^7)/7 + (e*x^8)/4 + (2*f*x^9)/9 + (g*x^10)/5 + (2
*h*x^11)/11) + (b^2*x^9*(20020*c + 3*x*(6006*d + 5460*e*x + 55*x^2*(91*f +
84*g*x + 78*h*x^2))))/180180
```

Maple [A] time = 0.001, size = 152, normalized size = 1.

$$\frac{b^2 h x^{14}}{14} + \frac{b^2 g x^{13}}{13} + \frac{b^2 f x^{12}}{12} + \frac{(2 a b h + b^2 e) x^{11}}{11} + \frac{(2 a b g + b^2 d) x^{10}}{10} + \frac{(2 a b f + b^2 c) x^9}{9} + \frac{(a^2 h + 2 a e b) x^8}{8} + \frac{(a^2 g + 2 b d a) x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c), x)
```

```
[Out] 1/14*b^2*h*x^14+1/13*b^2*g*x^13+1/12*b^2*f*x^12+1/11*(2*a*b*h+b^2*e)*x^11+1
/10*(2*a*b*g+b^2*d)*x^10+1/9*(2*a*b*f+b^2*c)*x^9+1/8*(a^2*h+2*a*b*e)*x^8+1/
```

$$7*(a^2*g+2*a*b*d)*x^7+1/6*(a^2*f+2*a*b*c)*x^6+1/5*a^2*e*x^5+1/4*a^2*d*x^4+1/3*a^2*c*x^3$$

Maxima [A] time = 0.959115, size = 204, normalized size = 1.29

$$\frac{1}{14}b^2hx^{14} + \frac{1}{13}b^2gx^{13} + \frac{1}{12}b^2fx^{12} + \frac{1}{11}(b^2e + 2abh)x^{11} + \frac{1}{10}(b^2d + 2abg)x^{10} + \frac{1}{9}(b^2c + 2abf)x^9 + \frac{1}{8}(2abe + a^2h)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima")

[Out] 1/14*b^2*h*x^14 + 1/13*b^2*g*x^13 + 1/12*b^2*f*x^12 + 1/11*(b^2*e + 2*a*b*h)*x^11 + 1/10*(b^2*d + 2*a*b*g)*x^10 + 1/9*(b^2*c + 2*a*b*f)*x^9 + 1/8*(2*a*b*e + a^2*h)*x^8 + 1/5*a^2*e*x^5 + 1/7*(2*a*b*d + a^2*g)*x^7 + 1/4*a^2*d*x^4 + 1/6*(2*a*b*c + a^2*f)*x^6 + 1/3*a^2*c*x^3

Fricas [A] time = 0.861869, size = 405, normalized size = 2.56

$$\frac{1}{14}x^{14}hb^2 + \frac{1}{13}x^{13}gb^2 + \frac{1}{12}x^{12}fb^2 + \frac{1}{11}x^{11}eb^2 + \frac{2}{11}x^{11}hba + \frac{1}{10}x^{10}db^2 + \frac{1}{5}x^{10}gba + \frac{1}{9}x^9cb^2 + \frac{2}{9}x^9fba + \frac{1}{4}x^8eba + \frac{1}{8}x^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")

[Out] 1/14*x^14*h*b^2 + 1/13*x^13*g*b^2 + 1/12*x^12*f*b^2 + 1/11*x^11*e*b^2 + 2/11*x^11*h*b*a + 1/10*x^10*d*b^2 + 1/5*x^10*g*b*a + 1/9*x^9*c*b^2 + 2/9*x^9*f*b*a + 1/4*x^8*e*b*a + 1/8*x^8*h*a^2 + 2/7*x^7*d*b*a + 1/7*x^7*g*a^2 + 1/3*x^6*c*b*a + 1/6*x^6*f*a^2 + 1/5*x^5*e*a^2 + 1/4*x^4*d*a^2 + 1/3*x^3*c*a^2

Sympy [A] time = 0.100332, size = 167, normalized size = 1.06

$$\frac{a^2cx^3}{3} + \frac{a^2dx^4}{4} + \frac{a^2ex^5}{5} + \frac{b^2fx^{12}}{12} + \frac{b^2gx^{13}}{13} + \frac{b^2hx^{14}}{14} + x^{11}\left(\frac{2abh}{11} + \frac{b^2e}{11}\right) + x^{10}\left(\frac{abg}{5} + \frac{b^2d}{10}\right) + x^9\left(\frac{2abf}{9} + \frac{b^2c}{9}\right) + x^8\left(\frac{2abe}{8} + \frac{a^2h}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)

[Out] a**2*c*x**3/3 + a**2*d*x**4/4 + a**2*e*x**5/5 + b**2*f*x**12/12 + b**2*g*x**13/13 + b**2*h*x**14/14 + x**11*(2*a*b*h/11 + b**2*e/11) + x**10*(a*b*g/5 + b**2*d/10) + x**9*(2*a*b*f/9 + b**2*c/9) + x**8*(a**2*h/8 + a*b*e/4) + x**7*(a**2*g/7 + 2*a*b*d/7) + x**6*(a**2*f/6 + a*b*c/3)

Giac [A] time = 1.07267, size = 216, normalized size = 1.37

$$\frac{1}{14} b^2 h x^{14} + \frac{1}{13} b^2 g x^{13} + \frac{1}{12} b^2 f x^{12} + \frac{2}{11} a b h x^{11} + \frac{1}{11} b^2 x^{11} e + \frac{1}{10} b^2 d x^{10} + \frac{1}{5} a b g x^{10} + \frac{1}{9} b^2 c x^9 + \frac{2}{9} a b f x^9 + \frac{1}{8} a^2 h x^8 + \frac{1}{4} a^2 b x^8 e + \frac{2}{7} a^2 b d x^7 + \frac{1}{7} a^2 g x^7 + \frac{1}{3} a^2 b c x^6 + \frac{1}{6} a^2 f x^6 + \frac{1}{5} a^2 x^5 e + \frac{1}{4} a^2 d x^4 + \frac{1}{3} a^2 c x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")

[Out] 1/14*b^2*h*x^14 + 1/13*b^2*g*x^13 + 1/12*b^2*f*x^12 + 2/11*a*b*h*x^11 + 1/11*b^2*x^11*e + 1/10*b^2*d*x^10 + 1/5*a*b*g*x^10 + 1/9*b^2*c*x^9 + 2/9*a*b*f*x^9 + 1/8*a^2*h*x^8 + 1/4*a*b*x^8*e + 2/7*a*b*d*x^7 + 1/7*a^2*g*x^7 + 1/3*a^2*b*c*x^6 + 1/6*a^2*f*x^6 + 1/5*a^2*x^5*e + 1/4*a^2*d*x^4 + 1/3*a^2*c*x^3

$$3.386 \quad \int x (a + bx^3)^2 (c + dx + ex^2 + fx^3 + gx^4 + hx^5) dx$$

Optimal. Leaf size=158

$$\frac{1}{2}a^2cx^2 + \frac{1}{4}a^2ex^4 + \frac{1}{6}a^2gx^6 + \frac{1}{8}bx^8(2af + bc) + \frac{1}{5}ax^5(af + 2bc) + \frac{d(a + bx^3)^3}{9b} + \frac{1}{10}bx^{10}(2ah + be) + \frac{1}{7}ax^7(ah + 2be)$$

[Out] (a^2*c*x^2)/2 + (a^2*e*x^4)/4 + (a*(2*b*c + a*f)*x^5)/5 + (a^2*g*x^6)/6 + (a*(2*b*e + a*h)*x^7)/7 + (b*(b*c + 2*a*f)*x^8)/8 + (2*a*b*g*x^9)/9 + (b*(b*e + 2*a*h)*x^10)/10 + (b^2*f*x^11)/11 + (b^2*g*x^12)/12 + (b^2*h*x^13)/13 + (d*(a + b*x^3)^3)/(9*b)

Rubi [A] time = 0.129048, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1582, 1850}

$$\frac{1}{2}a^2cx^2 + \frac{1}{4}a^2ex^4 + \frac{1}{6}a^2gx^6 + \frac{1}{8}bx^8(2af + bc) + \frac{1}{5}ax^5(af + 2bc) + \frac{d(a + bx^3)^3}{9b} + \frac{1}{10}bx^{10}(2ah + be) + \frac{1}{7}ax^7(ah + 2be)$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]

[Out] (a^2*c*x^2)/2 + (a^2*e*x^4)/4 + (a*(2*b*c + a*f)*x^5)/5 + (a^2*g*x^6)/6 + (a*(2*b*e + a*h)*x^7)/7 + (b*(b*c + 2*a*f)*x^8)/8 + (2*a*b*g*x^9)/9 + (b*(b*e + 2*a*h)*x^10)/10 + (b^2*f*x^11)/11 + (b^2*g*x^12)/12 + (b^2*h*x^13)/13 + (d*(a + b*x^3)^3)/(9*b)

Rule 1582

Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_)] /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a + b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]

Rule 1850

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned} \int x(a + bx^3)^2(c + dx + ex^2 + fx^3 + gx^4 + hx^5) dx &= \frac{d(a + bx^3)^3}{9b} + \int (a + bx^3)^2(-dx^2 + x(c + dx + ex^2 + fx^3 + gx^4 \\ &= \frac{d(a + bx^3)^3}{9b} + \int (a^2cx + a^2ex^3 + a(2bc + af)x^4 + a^2gx^5 + a(2be \\ &= \frac{1}{2}a^2cx^2 + \frac{1}{4}a^2ex^4 + \frac{1}{5}a(2bc + af)x^5 + \frac{1}{6}a^2gx^6 + \frac{1}{7}a(2be + ah)x^7 + \end{aligned}$$

Mathematica [A] time = 0.0238715, size = 163, normalized size = 1.03

$$\frac{1}{2}a^2cx^2 + \frac{1}{3}a^2dx^3 + \frac{1}{4}a^2ex^4 + \frac{1}{8}bx^8(2af + bc) + \frac{1}{5}ax^5(af + 2bc) + \frac{1}{9}bx^9(2ag + bd) + \frac{1}{6}ax^6(ag + 2bd) + \frac{1}{10}bx^{10}(2ah + be)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5), x]
```

```
[Out] (a^2*c*x^2)/2 + (a^2*d*x^3)/3 + (a^2*e*x^4)/4 + (a*(2*b*c + a*f)*x^5)/5 + (
a*(2*b*d + a*g)*x^6)/6 + (a*(2*b*e + a*h)*x^7)/7 + (b*(b*c + 2*a*f)*x^8)/8
+ (b*(b*d + 2*a*g)*x^9)/9 + (b*(b*e + 2*a*h)*x^10)/10 + (b^2*f*x^11)/11 + (
b^2*g*x^12)/12 + (b^2*h*x^13)/13
```

Maple [A] time = 0.001, size = 152, normalized size = 1.

$$\frac{b^2hx^{13}}{13} + \frac{b^2gx^{12}}{12} + \frac{b^2fx^{11}}{11} + \frac{(2abh + b^2e)x^{10}}{10} + \frac{(2abg + b^2d)x^9}{9} + \frac{(2abf + b^2c)x^8}{8} + \frac{(a^2h + 2aeb)x^7}{7} + \frac{(a^2g + 2bda)x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c), x)
```

```
[Out] 1/13*b^2*h*x^13+1/12*b^2*g*x^12+1/11*b^2*f*x^11+1/10*(2*a*b*h+b^2*e)*x^10+1
/9*(2*a*b*g+b^2*d)*x^9+1/8*(2*a*b*f+b^2*c)*x^8+1/7*(a^2*h+2*a*b*e)*x^7+1/6*
```

$$(a^2g+2a*b*d)*x^6+1/5*(a^2*f+2*a*b*c)*x^5+1/4*a^2*e*x^4+1/3*a^2*d*x^3+1/2*a^2*c*x^2$$

Maxima [A] time = 0.947235, size = 204, normalized size = 1.29

$$\frac{1}{13}b^2hx^{13} + \frac{1}{12}b^2gx^{12} + \frac{1}{11}b^2fx^{11} + \frac{1}{10}(b^2e + 2abh)x^{10} + \frac{1}{9}(b^2d + 2abg)x^9 + \frac{1}{8}(b^2c + 2abf)x^8 + \frac{1}{7}(2abe + a^2h)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima")

[Out] 1/13*b^2*h*x^13 + 1/12*b^2*g*x^12 + 1/11*b^2*f*x^11 + 1/10*(b^2*e + 2*a*b*h)*x^10 + 1/9*(b^2*d + 2*a*b*g)*x^9 + 1/8*(b^2*c + 2*a*b*f)*x^8 + 1/7*(2*a*b*e + a^2*h)*x^7 + 1/4*a^2*e*x^4 + 1/6*(2*a*b*d + a^2*g)*x^6 + 1/3*a^2*d*x^3 + 1/5*(2*a*b*c + a^2*f)*x^5 + 1/2*a^2*c*x^2

Fricas [A] time = 0.862469, size = 400, normalized size = 2.53

$$\frac{1}{13}x^{13}hb^2 + \frac{1}{12}x^{12}gb^2 + \frac{1}{11}x^{11}fb^2 + \frac{1}{10}x^{10}eb^2 + \frac{1}{5}x^{10}hba + \frac{1}{9}x^9db^2 + \frac{2}{9}x^9gba + \frac{1}{8}x^8cb^2 + \frac{1}{4}x^8fba + \frac{2}{7}x^7eba + \frac{1}{7}x^7ha^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")

[Out] 1/13*x^13*h*b^2 + 1/12*x^12*g*b^2 + 1/11*x^11*f*b^2 + 1/10*x^10*e*b^2 + 1/5*x^10*h*b*a + 1/9*x^9*d*b^2 + 2/9*x^9*g*b*a + 1/8*x^8*c*b^2 + 1/4*x^8*f*b*a + 2/7*x^7*e*b*a + 1/7*x^7*h*a^2 + 1/3*x^6*d*b*a + 1/6*x^6*g*a^2 + 2/5*x^5*c*b*a + 1/5*x^5*f*a^2 + 1/4*x^4*e*a^2 + 1/3*x^3*d*a^2 + 1/2*x^2*c*a^2

Sympy [A] time = 0.10222, size = 167, normalized size = 1.06

$$\frac{a^2cx^2}{2} + \frac{a^2dx^3}{3} + \frac{a^2ex^4}{4} + \frac{b^2fx^{11}}{11} + \frac{b^2gx^{12}}{12} + \frac{b^2hx^{13}}{13} + x^{10}\left(\frac{abh}{5} + \frac{b^2e}{10}\right) + x^9\left(\frac{2abg}{9} + \frac{b^2d}{9}\right) + x^8\left(\frac{abf}{4} + \frac{b^2c}{8}\right) + x^7\left(\frac{a^2h}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)

[Out] a**2*c*x**2/2 + a**2*d*x**3/3 + a**2*e*x**4/4 + b**2*f*x**11/11 + b**2*g*x**12/12 + b**2*h*x**13/13 + x**10*(a*b*h/5 + b**2*e/10) + x**9*(2*a*b*g/9 + b**2*d/9) + x**8*(a*b*f/4 + b**2*c/8) + x**7*(a**2*h/7 + 2*a*b*e/7) + x**6*(a**2*g/6 + a*b*d/3) + x**5*(a**2*f/5 + 2*a*b*c/5)

Giac [A] time = 1.05239, size = 216, normalized size = 1.37

$$\frac{1}{13} b^2 h x^{13} + \frac{1}{12} b^2 g x^{12} + \frac{1}{11} b^2 f x^{11} + \frac{1}{5} a b h x^{10} + \frac{1}{10} b^2 x^{10} e + \frac{1}{9} b^2 d x^9 + \frac{2}{9} a b g x^9 + \frac{1}{8} b^2 c x^8 + \frac{1}{4} a b f x^8 + \frac{1}{7} a^2 h x^7 + \frac{2}{7} a b e x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")

[Out] 1/13*b^2*h*x^13 + 1/12*b^2*g*x^12 + 1/11*b^2*f*x^11 + 1/5*a*b*h*x^10 + 1/10*b^2*x^10*e + 1/9*b^2*d*x^9 + 2/9*a*b*g*x^9 + 1/8*b^2*c*x^8 + 1/4*a*b*f*x^8 + 1/7*a^2*h*x^7 + 2/7*a*b*e*x^7 + 1/3*a*b*d*x^6 + 1/6*a^2*g*x^6 + 2/5*a*b*c*x^5 + 1/5*a^2*f*x^5 + 1/4*a^2*x^4*e + 1/3*a^2*d*x^3 + 1/2*a^2*c*x^2

$$3.387 \quad \int (a + bx^3)^2 (c + dx + ex^2 + fx^3 + gx^4 + hx^5) dx$$

Optimal. Leaf size=153

$$a^2cx + \frac{1}{2}a^2dx^2 + \frac{1}{6}a^2hx^6 + \frac{1}{7}bx^7(2af + bc) + \frac{1}{4}ax^4(af + 2bc) + \frac{1}{8}bx^8(2ag + bd) + \frac{1}{5}ax^5(ag + 2bd) + \frac{e(a + bx^3)^3}{9b} + \frac{2}{9}a^2$$

[Out] $a^2c*x + (a^2*d*x^2)/2 + (a*(2*b*c + a*f)*x^4)/4 + (a*(2*b*d + a*g)*x^5)/5 + (a^2*h*x^6)/6 + (b*(b*c + 2*a*f)*x^7)/7 + (b*(b*d + 2*a*g)*x^8)/8 + (2*a*b*h*x^9)/9 + (b^2*f*x^10)/10 + (b^2*g*x^11)/11 + (b^2*h*x^12)/12 + (e*(a + b*x^3)^3)/(9*b)$

Rubi [A] time = 0.12658, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1582, 1850}

$$a^2cx + \frac{1}{2}a^2dx^2 + \frac{1}{6}a^2hx^6 + \frac{1}{7}bx^7(2af + bc) + \frac{1}{4}ax^4(af + 2bc) + \frac{1}{8}bx^8(2ag + bd) + \frac{1}{5}ax^5(ag + 2bd) + \frac{e(a + bx^3)^3}{9b} + \frac{2}{9}a^2$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5), x]$

[Out] $a^2c*x + (a^2*d*x^2)/2 + (a*(2*b*c + a*f)*x^4)/4 + (a*(2*b*d + a*g)*x^5)/5 + (a^2*h*x^6)/6 + (b*(b*c + 2*a*f)*x^7)/7 + (b*(b*d + 2*a*g)*x^8)/8 + (2*a*b*h*x^9)/9 + (b^2*f*x^10)/10 + (b^2*g*x^11)/11 + (b^2*h*x^12)/12 + (e*(a + b*x^3)^3)/(9*b)$

Rule 1582

$\text{Int}[(P_x) * ((a_) + (b_) * (x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(\text{Coeff}[P_x, x, n - 1] * (a + b*x^n)^{(p + 1)}) / (b*n*(p + 1)), x] + \text{Int}[(P_x - \text{Coeff}[P_x, x, n - 1] * x^{(n - 1)}) * (a + b*x^n)^p, x] /;$ FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1] * x^{(n - 1)}] && !MatchQ[Px, (Qx_)*((c_) + (d_) * x^{(m_)})^{(q_)} /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a + b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]

Rule 1850

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned} \int (a + bx^3)^2 (c + dx + ex^2 + fx^3 + gx^4 + hx^5) dx &= \frac{e(a + bx^3)^3}{9b} + \int (a + bx^3)^2 (c + dx + fx^3 + gx^4 + hx^5) dx \\ &= \frac{e(a + bx^3)^3}{9b} + \int (a^2c + a^2dx + a(2bc + af)x^3 + a(2bd + ag)x^4 + a^2ex^5) dx \\ &= a^2cx + \frac{1}{2}a^2dx^2 + \frac{1}{4}a(2bc + af)x^4 + \frac{1}{5}a(2bd + ag)x^5 + \frac{1}{6}a^2hx^6 + \frac{1}{7}a^2ex^7 \end{aligned}$$

Mathematica [A] time = 0.0742271, size = 125, normalized size = 0.82

$$\frac{462a^2x(60c + x(30d + x(20e + 15fx + 12gx^2 + 10hx^3))) + 22abx^4(630c + x(504d + 5x(84e + x(72f + 7x(9g + 8hx))))}{27720}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5), x]
```

```
[Out] (b^2*x^7*(3960*c + 7*x*(495*d + 440*e*x + 6*x^2*(66*f + 60*g*x + 55*h*x^2))
) + 462*a^2*x*(60*c + x*(30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x^3))) +
22*a*b*x^4*(630*c + x*(504*d + 5*x*(84*e + x*(72*f + 7*x*(9*g + 8*h*x))))
)/27720
```

Maple [A] time = 0.002, size = 149, normalized size = 1.

$$\frac{b^2hx^{12}}{12} + \frac{b^2gx^{11}}{11} + \frac{b^2fx^{10}}{10} + \frac{(2abh + b^2e)x^9}{9} + \frac{(2abg + b^2d)x^8}{8} + \frac{(2abf + b^2c)x^7}{7} + \frac{(a^2h + 2aeb)x^6}{6} + \frac{(a^2g + 2bda)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c), x)
```

```
[Out] 1/12*b^2*h*x^12+1/11*b^2*g*x^11+1/10*b^2*f*x^10+1/9*(2*a*b*h+b^2*e)*x^9+1/8
*(2*a*b*g+b^2*d)*x^8+1/7*(2*a*b*f+b^2*c)*x^7+1/6*(a^2*h+2*a*b*e)*x^6+1/5*(a
```

$\frac{1}{2}g+2a*b*d)*x^5+1/4*(a^2*f+2*a*b*c)*x^4+1/3*a^2*e*x^3+1/2*a^2*d*x^2+a^2*c*x$

Maxima [A] time = 0.947005, size = 200, normalized size = 1.31

$$\frac{1}{12}b^2hx^{12} + \frac{1}{11}b^2gx^{11} + \frac{1}{10}b^2fx^{10} + \frac{1}{9}(b^2e + 2abh)x^9 + \frac{1}{8}(b^2d + 2abg)x^8 + \frac{1}{7}(b^2c + 2abf)x^7 + \frac{1}{6}(2abe + a^2h)x^6 + \frac{1}{5}a^2ex^5 + \frac{1}{4}a^2dix^4 + \frac{1}{3}a^2cix^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima")

[Out] 1/12*b^2*h*x^12 + 1/11*b^2*g*x^11 + 1/10*b^2*f*x^10 + 1/9*(b^2*e + 2*a*b*h)*x^9 + 1/8*(b^2*d + 2*a*b*g)*x^8 + 1/7*(b^2*c + 2*a*b*f)*x^7 + 1/6*(2*a*b*e + a^2*h)*x^6 + 1/5*a^2*e*x^5 + 1/5*(2*a*b*d + a^2*g)*x^5 + 1/2*a^2*d*x^2 + 1/4*(2*a*b*c + a^2*f)*x^4 + a^2*c*x

Fricas [A] time = 0.900123, size = 387, normalized size = 2.53

$$\frac{1}{12}x^{12}hb^2 + \frac{1}{11}x^{11}gb^2 + \frac{1}{10}x^{10}fb^2 + \frac{1}{9}x^9eb^2 + \frac{2}{9}x^9hba + \frac{1}{8}x^8db^2 + \frac{1}{4}x^8gba + \frac{1}{7}x^7cb^2 + \frac{2}{7}x^7fba + \frac{1}{3}x^6eba + \frac{1}{6}x^6ha^2 + \frac{1}{5}a^2ex^5 + \frac{1}{5}(2abdx^5 + a^2gx^5) + \frac{1}{2}a^2dix^2 + \frac{1}{4}(2abcx^4 + a^2fx^4) + a^2cix^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")

[Out] 1/12*x^12*h*b^2 + 1/11*x^11*g*b^2 + 1/10*x^10*f*b^2 + 1/9*x^9*e*b^2 + 2/9*x^9*h*b*a + 1/8*x^8*d*b^2 + 1/4*x^8*g*b*a + 1/7*x^7*c*b^2 + 2/7*x^7*f*b*a + 1/3*x^6*e*b*a + 1/6*x^6*h*a^2 + 2/5*x^5*d*b*a + 1/5*x^5*g*a^2 + 1/2*x^4*c*b*a + 1/4*x^4*f*a^2 + 1/3*x^3*e*a^2 + 1/2*x^2*d*a^2 + x*c*a^2

Sympy [A] time = 0.087431, size = 163, normalized size = 1.07

$$a^2cx + \frac{a^2dx^2}{2} + \frac{a^2ex^3}{3} + \frac{b^2fx^{10}}{10} + \frac{b^2gx^{11}}{11} + \frac{b^2hx^{12}}{12} + x^9\left(\frac{2abh}{9} + \frac{b^2e}{9}\right) + x^8\left(\frac{abg}{4} + \frac{b^2d}{8}\right) + x^7\left(\frac{2abf}{7} + \frac{b^2c}{7}\right) + x^6\left(\frac{a^2h}{6} + \frac{2abe}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)

[Out] a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + b**2*f*x**10/10 + b**2*g*x**11/11 + b**2*h*x**12/12 + x**9*(2*a*b*h/9 + b**2*e/9) + x**8*(a*b*g/4 + b**2*d/8) + x**7*(2*a*b*f/7 + b**2*c/7) + x**6*(a**2*h/6 + a*b*e/3) + x**5*(a**2*g/5 + 2*a*b*d/5) + x**4*(a**2*f/4 + a*b*c/2)

Giac [A] time = 1.05691, size = 212, normalized size = 1.39

$$\frac{1}{12} b^2 h x^{12} + \frac{1}{11} b^2 g x^{11} + \frac{1}{10} b^2 f x^{10} + \frac{2}{9} a b h x^9 + \frac{1}{9} b^2 x^9 e + \frac{1}{8} b^2 d x^8 + \frac{1}{4} a b g x^8 + \frac{1}{7} b^2 c x^7 + \frac{2}{7} a b f x^7 + \frac{1}{6} a^2 h x^6 + \frac{1}{3} a b x^6 e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")

[Out] 1/12*b^2*h*x^12 + 1/11*b^2*g*x^11 + 1/10*b^2*f*x^10 + 2/9*a*b*h*x^9 + 1/9*b^2*x^9*e + 1/8*b^2*d*x^8 + 1/4*a*b*g*x^8 + 1/7*b^2*c*x^7 + 2/7*a*b*f*x^7 + 1/6*a^2*h*x^6 + 1/3*a*b*x^6*e + 2/5*a*b*d*x^5 + 1/5*a^2*g*x^5 + 1/2*a*b*c*x^4 + 1/4*a^2*f*x^4 + 1/3*a^2*x^3*e + 1/2*a^2*d*x^2 + a^2*c*x

$$3.388 \quad \int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x} dx$$

Optimal. Leaf size=149

$$a^2c \log(x) + a^2dx + \frac{1}{2}a^2ex^2 + \frac{2}{3}abcx^3 + \frac{1}{7}bx^7(2ag + bd) + \frac{1}{4}ax^4(ag + 2bd) + \frac{1}{8}bx^8(2ah + be) + \frac{1}{5}ax^5(ah + 2be) + \frac{f(a - b^3x^3)}{9b^3}$$

[Out] $a^2d*x + (a^2e*x^2)/2 + (2*a*b*c*x^3)/3 + (a*(2*b*d + a*g)*x^4)/4 + (a*(2*b*e + a*h)*x^5)/5 + (b^2*c*x^6)/6 + (b*(b*d + 2*a*g)*x^7)/7 + (b*(b*e + 2*a*h)*x^8)/8 + (b^2*g*x^10)/10 + (b^2*h*x^11)/11 + (f*(a + b*x^3)^3)/(9*b) + a^2*c*Log[x]$

Rubi [A] time = 0.105703, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1583, 1820}

$$a^2c \log(x) + a^2dx + \frac{1}{2}a^2ex^2 + \frac{2}{3}abcx^3 + \frac{1}{7}bx^7(2ag + bd) + \frac{1}{4}ax^4(ag + 2bd) + \frac{1}{8}bx^8(2ah + be) + \frac{1}{5}ax^5(ah + 2be) + \frac{f(a - b^3x^3)}{9b^3}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]

[Out] $a^2d*x + (a^2e*x^2)/2 + (2*a*b*c*x^3)/3 + (a*(2*b*d + a*g)*x^4)/4 + (a*(2*b*e + a*h)*x^5)/5 + (b^2*c*x^6)/6 + (b*(b*d + 2*a*g)*x^7)/7 + (b*(b*e + 2*a*h)*x^8)/8 + (b^2*g*x^10)/10 + (b^2*h*x^11)/11 + (f*(a + b*x^3)^3)/(9*b) + a^2*c*Log[x]$

Rule 1583

Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p_, x_Symbol] := Simp[(Coeff[Px, x, n - m - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x, n - m - 1], 0]

Rule 1820

Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p_, x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,

n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)^2 (c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{x} dx &= \frac{f(a + bx^3)^3}{9b} + \int \frac{(a + bx^3)^2 (c + dx + ex^2 + gx^4 + hx^5)}{x} dx \\ &= \frac{f(a + bx^3)^3}{9b} + \int \left(a^2d + \frac{a^2c}{x} + a^2ex + 2abcx^2 + a(2bd + ag)x^3 + a \right. \\ &\quad \left. + a^2dx + \frac{1}{2}a^2ex^2 + \frac{2}{3}abcx^3 + \frac{1}{4}a(2bd + ag)x^4 + \frac{1}{5}a(2be + ah)x^5 + \frac{1}{6}a \right) dx \end{aligned}$$

Mathematica [A] time = 0.0438508, size = 154, normalized size = 1.03

$$a^2c \log(x) + a^2dx + \frac{1}{2}a^2ex^2 + \frac{1}{6}bx^6(2af + bc) + \frac{1}{3}ax^3(af + 2bc) + \frac{1}{7}bx^7(2ag + bd) + \frac{1}{4}ax^4(ag + 2bd) + \frac{1}{8}bx^8(2ah + be) + \dots$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]

[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*c + a*f)*x^3)/3 + (a*(2*b*d + a*g)*x^4)/4 + (a*(2*b*e + a*h)*x^5)/5 + (b*(b*c + 2*a*f)*x^6)/6 + (b*(b*d + 2*a*g)*x^7)/7 + (b*(b*e + 2*a*h)*x^8)/8 + (b^2*f*x^9)/9 + (b^2*g*x^10)/10 + (b^2*h*x^11)/11 + a^2*c*Log[x]

Maple [A] time = 0.003, size = 153, normalized size = 1.

$$\frac{b^2hx^{11}}{11} + \frac{b^2gx^{10}}{10} + \frac{x^9b^2f}{9} + \frac{x^8abh}{4} + \frac{b^2ex^8}{8} + \frac{2x^7abg}{7} + \frac{b^2dx^7}{7} + \frac{x^6abf}{3} + \frac{b^2cx^6}{6} + \frac{x^5a^2h}{5} + \frac{2abex^5}{5} + \frac{x^4a^2g}{4} + \frac{abdx^4}{2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x)

[Out] 1/11*b^2*h*x^11+1/10*b^2*g*x^10+1/9*x^9*b^2*f+1/4*x^8*a*b*h+1/8*b^2*e*x^8+2/7*x^7*a*b*g+1/7*b^2*d*x^7+1/3*x^6*a*b*f+1/6*b^2*c*x^6+1/5*x^5*a^2*h+2/5*a*b*e*x^5+1/4*x^4*a^2*g+1/2*a*b*d*x^4+1/3*a^2*f*x^3+2/3*a*b*c*x^3+1/2*a^2*e*x^2+1/2*a^2*c*log(x)

$$^2+a^2*d*x+a^2*c*\ln(x)$$

Maxima [A] time = 0.934238, size = 197, normalized size = 1.32

$$\frac{1}{11} b^2 h x^{11} + \frac{1}{10} b^2 g x^{10} + \frac{1}{9} b^2 f x^9 + \frac{1}{8} (b^2 e + 2 a b h) x^8 + \frac{1}{7} (b^2 d + 2 a b g) x^7 + \frac{1}{6} (b^2 c + 2 a b f) x^6 + \frac{1}{5} (2 a b e + a^2 h) x^5 + \frac{1}{4} (2 a b d + a^2 g) x^4 + a^2 d x + \frac{1}{3} (2 a b c + a^2 f) x^3 + a^2 c \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="maxima")

[Out] 1/11*b^2*h*x^11 + 1/10*b^2*g*x^10 + 1/9*b^2*f*x^9 + 1/8*(b^2*e + 2*a*b*h)*x^8 + 1/7*(b^2*d + 2*a*b*g)*x^7 + 1/6*(b^2*c + 2*a*b*f)*x^6 + 1/5*(2*a*b*e + a^2*h)*x^5 + 1/4*(2*a*b*d + a^2*g)*x^4 + a^2*d*x + 1/3*(2*a*b*c + a^2*f)*x^3 + a^2*c*log(x)

Fricas [A] time = 0.937602, size = 351, normalized size = 2.36

$$\frac{1}{11} b^2 h x^{11} + \frac{1}{10} b^2 g x^{10} + \frac{1}{9} b^2 f x^9 + \frac{1}{8} (b^2 e + 2 a b h) x^8 + \frac{1}{7} (b^2 d + 2 a b g) x^7 + \frac{1}{6} (b^2 c + 2 a b f) x^6 + \frac{1}{5} (2 a b e + a^2 h) x^5 + \frac{1}{4} (2 a b d + a^2 g) x^4 + a^2 d x + \frac{1}{3} (2 a b c + a^2 f) x^3 + a^2 c \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="fricas")

[Out] 1/11*b^2*h*x^11 + 1/10*b^2*g*x^10 + 1/9*b^2*f*x^9 + 1/8*(b^2*e + 2*a*b*h)*x^8 + 1/7*(b^2*d + 2*a*b*g)*x^7 + 1/6*(b^2*c + 2*a*b*f)*x^6 + 1/5*(2*a*b*e + a^2*h)*x^5 + 1/4*(2*a*b*d + a^2*g)*x^4 + a^2*d*x + 1/3*(2*a*b*c + a^2*f)*x^3 + a^2*c*log(x)

Sympy [A] time = 0.47338, size = 162, normalized size = 1.09

$$a^2 c \log(x) + a^2 d x + \frac{a^2 e x^2}{2} + \frac{b^2 f x^9}{9} + \frac{b^2 g x^{10}}{10} + \frac{b^2 h x^{11}}{11} + x^8 \left(\frac{a b h}{4} + \frac{b^2 e}{8} \right) + x^7 \left(\frac{2 a b g}{7} + \frac{b^2 d}{7} \right) + x^6 \left(\frac{a b f}{3} + \frac{b^2 c}{6} \right) + x^5 \left(\frac{2 a b e}{5} + \frac{a^2 h}{5} \right) + \frac{1}{4} (2 a b d + a^2 g) x^4 + a^2 d x + \frac{1}{3} (2 a b c + a^2 f) x^3 + a^2 c \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x,x)

[Out] a**2*c*log(x) + a**2*d*x + a**2*e*x**2/2 + b**2*f*x**9/9 + b**2*g*x**10/10 + b**2*h*x**11/11 + x**8*(a*b*h/4 + b**2*e/8) + x**7*(2*a*b*g/7 + b**2*d/7) + x**6*(a*b*f/3 + b**2*c/6) + x**5*(a**2*h/5 + 2*a*b*e/5) + x**4*(a**2*g/4 + a*b*d/2) + x**3*(a**2*f/3 + 2*a*b*c/3)

Giac [A] time = 1.06798, size = 211, normalized size = 1.42

$$\frac{1}{11} b^2 h x^{11} + \frac{1}{10} b^2 g x^{10} + \frac{1}{9} b^2 f x^9 + \frac{1}{4} a b h x^8 + \frac{1}{8} b^2 x^8 e + \frac{1}{7} b^2 d x^7 + \frac{2}{7} a b g x^7 + \frac{1}{6} b^2 c x^6 + \frac{1}{3} a b f x^6 + \frac{1}{5} a^2 h x^5 + \frac{2}{5} a b x^5 e +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="giac")

[Out] 1/11*b^2*h*x^11 + 1/10*b^2*g*x^10 + 1/9*b^2*f*x^9 + 1/4*a*b*h*x^8 + 1/8*b^2*x^8*e + 1/7*b^2*d*x^7 + 2/7*a*b*g*x^7 + 1/6*b^2*c*x^6 + 1/3*a*b*f*x^6 + 1/5*a^2*h*x^5 + 2/5*a*b*x^5*e + 1/2*a*b*d*x^4 + 1/4*a^2*g*x^4 + 2/3*a*b*c*x^3 + 1/3*a^2*f*x^3 + 1/2*a^2*x^2*e + a^2*d*x + a^2*c*log(abs(x))

$$3.389 \quad \int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2} dx$$

Optimal. Leaf size=147

$$-\frac{a^2c}{x} + a^2d \log(x) + a^2ex + \frac{1}{5}bx^5(2af + bc) + \frac{1}{2}ax^2(af + 2bc) + \frac{2}{3}abdx^3 + \frac{1}{7}bx^7(2ah + be) + \frac{1}{4}ax^4(ah + 2be) + \frac{g(a + b^2c)}{9b}$$

[Out] $-\frac{a^2c}{x} + a^2e*x + \frac{a*(2*b*c + a*f)*x^2}{2} + \frac{2*a*b*d*x^3}{3} + \frac{a*(2*b*e + a*h)*x^4}{4} + \frac{b*(b*c + 2*a*f)*x^5}{5} + \frac{b^2*d*x^6}{6} + \frac{b*(b*e + 2*a*h)*x^7}{7} + \frac{b^2*f*x^8}{8} + \frac{b^2*h*x^{10}}{10} + \frac{g*(a + b*x^3)^3}{9*b} + a^2*d*\text{Log}[x]$

Rubi [A] time = 0.127544, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1583, 1820}

$$-\frac{a^2c}{x} + a^2d \log(x) + a^2ex + \frac{1}{5}bx^5(2af + bc) + \frac{1}{2}ax^2(af + 2bc) + \frac{2}{3}abdx^3 + \frac{1}{7}bx^7(2ah + be) + \frac{1}{4}ax^4(ah + 2be) + \frac{g(a + b^2c)}{9b}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]

[Out] $-\frac{a^2c}{x} + a^2e*x + \frac{a*(2*b*c + a*f)*x^2}{2} + \frac{2*a*b*d*x^3}{3} + \frac{a*(2*b*e + a*h)*x^4}{4} + \frac{b*(b*c + 2*a*f)*x^5}{5} + \frac{b^2*d*x^6}{6} + \frac{b*(b*e + 2*a*h)*x^7}{7} + \frac{b^2*f*x^8}{8} + \frac{b^2*h*x^{10}}{10} + \frac{g*(a + b*x^3)^3}{9*b} + a^2*d*\text{Log}[x]$

Rule 1583

Int[(Px_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(Coeff[Px, x, n - m - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x, n - m - 1], 0]

Rule 1820

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,

n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)^2 (c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{x^2} dx &= \frac{g(a + bx^3)^3}{9b} + \int \frac{(a + bx^3)^2 (c + dx + ex^2 + fx^3 + hx^5)}{x^2} dx \\ &= \frac{g(a + bx^3)^3}{9b} + \int \left(a^2e + \frac{a^2c}{x^2} + \frac{a^2d}{x} + a(2bc + af)x + 2abdx^2 + a(2bc + af)x + 2abdx^2 + a(2bc + af)x + 2abdx^2 \right) dx \\ &= -\frac{a^2c}{x} + a^2ex + \frac{1}{2}a(2bc + af)x^2 + \frac{2}{3}abdx^3 + \frac{1}{4}a(2bc + af)x^4 + \frac{1}{5}a(2bc + af)x^5 \end{aligned}$$

Mathematica [A] time = 0.0605502, size = 152, normalized size = 1.03

$$-\frac{a^2c}{x} + a^2d \log(x) + a^2ex + \frac{1}{5}bx^5(2af + bc) + \frac{1}{2}ax^2(af + 2bc) + \frac{1}{6}bx^6(2ag + bd) + \frac{1}{3}ax^3(ag + 2bd) + \frac{1}{7}bx^7(2ah + be) + \frac{1}{8}bx^8(2fg + dh) + \frac{1}{9}bx^9(2gh + eh) + \frac{1}{10}bx^{10}(2hg + fh)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]

[Out] -((a^2*c)/x) + a^2*e*x + (a*(2*b*c + a*f)*x^2)/2 + (a*(2*b*d + a*g)*x^3)/3 + (a*(2*b*e + a*h)*x^4)/4 + (b*(b*c + 2*a*f)*x^5)/5 + (b*(b*d + 2*a*g)*x^6)/6 + (b*(b*e + 2*a*h)*x^7)/7 + (b^2*f*x^8)/8 + (b^2*g*x^9)/9 + (b^2*h*x^10)/10 + a^2*d*Log[x]

Maple [A] time = 0.006, size = 152, normalized size = 1.

$$\frac{b^2hx^{10}}{10} + \frac{b^2gx^9}{9} + \frac{b^2fx^8}{8} + \frac{2x^7abh}{7} + \frac{b^2ex^7}{7} + \frac{x^6abg}{3} + \frac{b^2dx^6}{6} + \frac{2x^5abf}{5} + \frac{b^2cx^5}{5} + \frac{x^4a^2h}{4} + \frac{abex^4}{2} + \frac{x^3a^2g}{3} + \frac{2abdx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x)

[Out] 1/10*b^2*h*x^10+1/9*b^2*g*x^9+1/8*b^2*f*x^8+2/7*x^7*a*b*h+1/7*b^2*e*x^7+1/3*x^6*a*b*g+1/6*b^2*d*x^6+2/5*x^5*a*b*f+1/5*b^2*c*x^5+1/4*x^4*a^2*h+1/2*a*b*e*x^4+1/3*x^3*a^2*g+2/3*a*b*d*x^3+1/2*a^2*f*x^2+a*b*c*x^2+a^2*e*x+a^2*d*ln(x)

$x) - a^2 c/x$

Maxima [A] time = 0.958382, size = 197, normalized size = 1.34

$$\frac{1}{10} b^2 h x^{10} + \frac{1}{9} b^2 g x^9 + \frac{1}{8} b^2 f x^8 + \frac{1}{7} (b^2 e + 2 a b h) x^7 + \frac{1}{6} (b^2 d + 2 a b g) x^6 + \frac{1}{5} (b^2 c + 2 a b f) x^5 + \frac{1}{4} (2 a b e + a^2 h) x^4 + a^2 e x^3 + a^2 d \log(x) + \frac{1}{2} (2 a b c + a^2 f) x^2 - a^2 c/x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="maxima")

[Out] 1/10*b^2*h*x^10 + 1/9*b^2*g*x^9 + 1/8*b^2*f*x^8 + 1/7*(b^2*e + 2*a*b*h)*x^7 + 1/6*(b^2*d + 2*a*b*g)*x^6 + 1/5*(b^2*c + 2*a*b*f)*x^5 + 1/4*(2*a*b*e + a^2*h)*x^4 + a^2*e*x + 1/3*(2*a*b*d + a^2*g)*x^3 + a^2*d*log(x) + 1/2*(2*a*b*c + a^2*f)*x^2 - a^2*c/x

Fricas [A] time = 0.992344, size = 379, normalized size = 2.58

$$\frac{252 b^2 h x^{11} + 280 b^2 g x^{10} + 315 b^2 f x^9 + 360 (b^2 e + 2 a b h) x^8 + 420 (b^2 d + 2 a b g) x^7 + 504 (b^2 c + 2 a b f) x^6 + 630 (2 a b e + a^2 h) x^5 + 2520 a^2 e x^4 + 840 (2 a b d + a^2 g) x^3 + 2520 a^2 d x^2 \log(x) + 1260 (2 a b c + a^2 f) x - 2520 a^2 c}{2520 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="fricas")

[Out] 1/2520*(252*b^2*h*x^11 + 280*b^2*g*x^10 + 315*b^2*f*x^9 + 360*(b^2*e + 2*a*b*h)*x^8 + 420*(b^2*d + 2*a*b*g)*x^7 + 504*(b^2*c + 2*a*b*f)*x^6 + 630*(2*a*b*e + a^2*h)*x^5 + 2520*a^2*e*x^4 + 840*(2*a*b*d + a^2*g)*x^3 + 2520*a^2*d*x*log(x) + 1260*(2*a*b*c + a^2*f)*x^2 - 2520*a^2*c)/x

Sympy [A] time = 0.499578, size = 156, normalized size = 1.06

$$-\frac{a^2 c}{x} + a^2 d \log(x) + a^2 e x + \frac{b^2 f x^8}{8} + \frac{b^2 g x^9}{9} + \frac{b^2 h x^{10}}{10} + x^7 \left(\frac{2 a b h}{7} + \frac{b^2 e}{7} \right) + x^6 \left(\frac{a b g}{3} + \frac{b^2 d}{6} \right) + x^5 \left(\frac{2 a b f}{5} + \frac{b^2 c}{5} \right) + x^4 \left(\frac{2 a b e}{4} + \frac{a^2 h}{4} \right) + a^2 d \log(x) + \frac{1}{2} (2 a b c + a^2 f) x - a^2 c/x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)

[Out] -a**2*c/x + a**2*d*log(x) + a**2*e*x + b**2*f*x**8/8 + b**2*g*x**9/9 + b**2*h*x**10/10 + x**7*(2*a*b*h/7 + b**2*e/7) + x**6*(a*b*g/3 + b**2*d/6) + x**5*(2*a*b*f/5 + b**2*c/5) + x**4*(a**2*h/4 + a*b*e/2) + x**3*(a**2*g/3 + 2*a*b*d/3) + x**2*(a**2*f/2 + a*b*c)

Giac [A] time = 1.05546, size = 209, normalized size = 1.42

$$\frac{1}{10} b^2 h x^{10} + \frac{1}{9} b^2 g x^9 + \frac{1}{8} b^2 f x^8 + \frac{2}{7} a b h x^7 + \frac{1}{7} b^2 x^7 e + \frac{1}{6} b^2 d x^6 + \frac{1}{3} a b g x^6 + \frac{1}{5} b^2 c x^5 + \frac{2}{5} a b f x^5 + \frac{1}{4} a^2 h x^4 + \frac{1}{2} a b x^4 e + \frac{2}{3} a^2 f x^4 + \frac{1}{2} a^2 x^4 e + \frac{2}{3} a^2 d x^3 + \frac{1}{3} a^2 g x^3 + a b c x^2 + \frac{1}{2} a^2 f x^2 + a^2 x e + a^2 d \log(\text{abs}(x)) - a^2 c/x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="giac")

[Out] 1/10*b^2*h*x^10 + 1/9*b^2*g*x^9 + 1/8*b^2*f*x^8 + 2/7*a*b*h*x^7 + 1/7*b^2*x^7*e + 1/6*b^2*d*x^6 + 1/3*a*b*g*x^6 + 1/5*b^2*c*x^5 + 2/5*a*b*f*x^5 + 1/4*a^2*h*x^4 + 1/2*a*b*x^4*e + 2/3*a*b*d*x^3 + 1/3*a^2*g*x^3 + a*b*c*x^2 + 1/2*a^2*f*x^2 + a^2*x*e + a^2*d*log(abs(x)) - a^2*c/x

$$3.390 \quad \int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3} dx$$

Optimal. Leaf size=147

$$-\frac{a^2c}{2x^2} - \frac{a^2d}{x} + a^2e \log(x) + \frac{1}{4}bx^4(2af + bc) + ax(af + 2bc) + \frac{1}{5}bx^5(2ag + bd) + \frac{1}{2}ax^2(ag + 2bd) + \frac{2}{3}abex^3 + \frac{h(a + bx^3)}{9b}$$

[Out] $-(a^2c)/(2x^2) - (a^2d)/x + a(2bc + af)x + (a(2bd + ag)x^2)/2 + (2abex^3)/3 + (b(bc + 2af)x^4)/4 + (b(bd + 2ag)x^5)/5 + (b^2ex^6)/6 + (b^2fx^7)/7 + (b^2gx^8)/8 + (h(a + bx^3)^3)/(9b) + a^2e \operatorname{Log}[x]$

Rubi [A] time = 0.128372, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1583, 1820}

$$-\frac{a^2c}{2x^2} - \frac{a^2d}{x} + a^2e \log(x) + \frac{1}{4}bx^4(2af + bc) + ax(af + 2bc) + \frac{1}{5}bx^5(2ag + bd) + \frac{1}{2}ax^2(ag + 2bd) + \frac{2}{3}abex^3 + \frac{h(a + bx^3)}{9b}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + bx^3)^2(c + dx + ex^2 + fx^3 + gx^4 + hx^5)/x^3, x]$

[Out] $-(a^2c)/(2x^2) - (a^2d)/x + a(2bc + af)x + (a(2bd + ag)x^2)/2 + (2abex^3)/3 + (b(bc + 2af)x^4)/4 + (b(bd + 2ag)x^5)/5 + (b^2ex^6)/6 + (b^2fx^7)/7 + (b^2gx^8)/8 + (h(a + bx^3)^3)/(9b) + a^2e \operatorname{Log}[x]$

Rule 1583

$\operatorname{Int}[(Px_*)(x_*)^{(m_*)}((a_*) + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Coeff}[Px, x, n - m - 1](a + bx^n)^{(p + 1)})/(b^n(p + 1)), x] + \operatorname{Int}[(Px - \operatorname{Coeff}[Px, x, n - m - 1]x^{(n - m - 1)})x^m(a + bx^n)^p, x] /;$ $\operatorname{FreeQ}\{a, b, m, n\}, x] \&\& \operatorname{PolyQ}[Px, x] \&\& \operatorname{IGtQ}[p, 1] \&\& \operatorname{IGtQ}[n - m, 0] \&\& \operatorname{NeQ}[\operatorname{Coeff}[Px, x, n - m - 1], 0]$

Rule 1820

$\operatorname{Int}[(Pq_*)((c_*)(x_*)^{(m_*)}((a_*) + (b_*)(x_*)^{(n_*)})^{(p_*)}), x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(c*x)^m Pq(a + bx^n)^p, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, m,$

n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)^2 (c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{x^3} dx &= \frac{h(a + bx^3)^3}{9b} + \int \frac{(a + bx^3)^2 (c + dx + ex^2 + fx^3 + gx^4)}{x^3} dx \\ &= \frac{h(a + bx^3)^3}{9b} + \int \left(a(2bc + af) + \frac{a^2c}{x^3} + \frac{a^2d}{x^2} + \frac{a^2e}{x} + a(2bd + ag)x \right) dx \\ &= -\frac{a^2c}{2x^2} - \frac{a^2d}{x} + a(2bc + af)x + \frac{1}{2}a(2bd + ag)x^2 + \frac{2}{3}abex^3 + \frac{1}{4}b(bc + ag)x^4 \end{aligned}$$

Mathematica [A] time = 0.0706433, size = 127, normalized size = 0.86

$$\frac{a^2(-3c - 6dx + x^3(6f + 3gx + 2hx^2))}{6x^2} + a^2e \log(x) + \frac{1}{30}abx(60c + x(30d + x(20e + 15fx + 12gx^2 + 10hx^3))) + \frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]

[Out] (a^2*(-3*c - 6*d*x + x^3*(6*f + 3*g*x + 2*h*x^2)))/(6*x^2) + (a*b*x*(60*c + x*(30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x^3))))/30 + (b^2*x^4*(630*c + x*(504*d + 5*x*(84*e + x*(72*f + 7*x*(9*g + 8*h*x))))))/2520 + a^2*e*Log[x]

Maple [A] time = 0.006, size = 150, normalized size = 1.

$$\frac{b^2hx^9}{9} + \frac{b^2gx^8}{8} + \frac{b^2fx^7}{7} + \frac{x^6abh}{3} + \frac{b^2ex^6}{6} + \frac{2x^5abg}{5} + \frac{b^2dx^5}{5} + \frac{x^4abf}{2} + \frac{b^2cx^4}{4} + \frac{x^3a^2h}{3} + \frac{2abex^3}{3} + \frac{x^2a^2g}{2} + abdx^2 + a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x)

[Out] 1/9*b^2*h*x^9+1/8*b^2*g*x^8+1/7*b^2*f*x^7+1/3*x^6*a*b*h+1/6*b^2*e*x^6+2/5*x^5*a*b*g+1/5*b^2*d*x^5+1/2*x^4*a*b*f+1/4*b^2*c*x^4+1/3*x^3*a^2*h+2/3*a*b*e*x^3+1/2*x^2*a^2*g+a*b*d*x^2+a^2*f*x+2*a*b*c*x+a^2*e*ln(x)-1/2*a^2*c/x^2-a^2

*d/x

Maxima [A] time = 0.939303, size = 197, normalized size = 1.34

$$\frac{1}{9} b^2 h x^9 + \frac{1}{8} b^2 g x^8 + \frac{1}{7} b^2 f x^7 + \frac{1}{6} (b^2 e + 2 a b h) x^6 + \frac{1}{5} (b^2 d + 2 a b g) x^5 + \frac{1}{4} (b^2 c + 2 a b f) x^4 + \frac{1}{3} (2 a b e + a^2 h) x^3 + a^2 e \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="maxima")

[Out] 1/9*b^2*h*x^9 + 1/8*b^2*g*x^8 + 1/7*b^2*f*x^7 + 1/6*(b^2*e + 2*a*b*h)*x^6 + 1/5*(b^2*d + 2*a*b*g)*x^5 + 1/4*(b^2*c + 2*a*b*f)*x^4 + 1/3*(2*a*b*e + a^2*h)*x^3 + a^2*e*log(x) + 1/2*(2*a*b*d + a^2*g)*x^2 + (2*a*b*c + a^2*f)*x - 1/2*(2*a^2*d*x + a^2*c)/x^2

Fricas [A] time = 1.00032, size = 383, normalized size = 2.61

$$\frac{280 b^2 h x^{11} + 315 b^2 g x^{10} + 360 b^2 f x^9 + 420 (b^2 e + 2 a b h) x^8 + 504 (b^2 d + 2 a b g) x^7 + 630 (b^2 c + 2 a b f) x^6 + 840 (2 a b e + a^2 h) x^5 + 2520 a^2 e x^4 \log(x) + 1260 (2 a b d + a^2 g) x^3 - 2520 a^2 d x + 2520 (2 a b c + a^2 f) x^2 - 1260 a^2 c}{2520 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="fricas")

[Out] 1/2520*(280*b^2*h*x^11 + 315*b^2*g*x^10 + 360*b^2*f*x^9 + 420*(b^2*e + 2*a*b*h)*x^8 + 504*(b^2*d + 2*a*b*g)*x^7 + 630*(b^2*c + 2*a*b*f)*x^6 + 840*(2*a*b*e + a^2*h)*x^5 + 2520*a^2*e*x^4*log(x) + 1260*(2*a*b*d + a^2*g)*x^3 - 2520*a^2*d*x + 2520*(2*a*b*c + a^2*f)*x^2 - 1260*a^2*c)/x^2

Sympy [A] time = 0.550674, size = 156, normalized size = 1.06

$$a^2 e \log(x) + \frac{b^2 f x^7}{7} + \frac{b^2 g x^8}{8} + \frac{b^2 h x^9}{9} + x^6 \left(\frac{a b h}{3} + \frac{b^2 e}{6} \right) + x^5 \left(\frac{2 a b g}{5} + \frac{b^2 d}{5} \right) + x^4 \left(\frac{a b f}{2} + \frac{b^2 c}{4} \right) + x^3 \left(\frac{a^2 h}{3} + \frac{2 a b e}{3} \right) + x^2 \left(\frac{2 a b d}{2} + a^2 g \right) - \frac{2520 a^2 d x + 2520 (2 a b c + a^2 f) x^2 - 1260 a^2 c}{2520 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)

[Out] a**2*e*log(x) + b**2*f*x**7/7 + b**2*g*x**8/8 + b**2*h*x**9/9 + x**6*(a*b*h/3 + b**2*e/6) + x**5*(2*a*b*g/5 + b**2*d/5) + x**4*(a*b*f/2 + b**2*c/4) + x**3*(a**2*h/3 + 2*a*b*e/3) + x**2*(a**2*g/2 + a*b*d) + x*(a**2*f + 2*a*b*c) - (a**2*c + 2*a**2*d*x)/(2*x**2)

Giac [A] time = 1.07399, size = 207, normalized size = 1.41

$$\frac{1}{9}b^2hx^9 + \frac{1}{8}b^2gx^8 + \frac{1}{7}b^2fx^7 + \frac{1}{3}abhx^6 + \frac{1}{6}b^2x^6e + \frac{1}{5}b^2dx^5 + \frac{2}{5}abgx^5 + \frac{1}{4}b^2cx^4 + \frac{1}{2}abfx^4 + \frac{1}{3}a^2hx^3 + \frac{2}{3}abx^3e + abd$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="giac")

[Out] 1/9*b^2*h*x^9 + 1/8*b^2*g*x^8 + 1/7*b^2*f*x^7 + 1/3*a*b*h*x^6 + 1/6*b^2*x^6*e + 1/5*b^2*d*x^5 + 2/5*a*b*g*x^5 + 1/4*b^2*c*x^4 + 1/2*a*b*f*x^4 + 1/3*a^2*h*x^3 + 2/3*a*b*x^3*e + a*b*d*x^2 + 1/2*a^2*g*x^2 + 2*a*b*c*x + a^2*f*x + a^2*e*log(abs(x)) - 1/2*(2*a^2*d*x + a^2*c)/x^2

$$3.391 \quad \int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4} dx$$

Optimal. Leaf size=152

$$-\frac{a^2c}{3x^3} - \frac{a^2d}{2x^2} - \frac{a^2e}{x} + \frac{1}{3}bx^3(2af+bc) + a \log(x)(af+2bc) + \frac{1}{4}bx^4(2ag+bd) + ax(ag+2bd) + \frac{1}{5}bx^5(2ah+be) + \frac{1}{2}ax^2(b^2c+af)$$

[Out] $-(a^2c)/(3x^3) - (a^2d)/(2x^2) - (a^2e)/x + a(2bd+ag)x + (a(2be+ah)x^2)/2 + (b(b^2c+2af)x^3)/3 + (b(b^2d+2ag)x^4)/4 + (b(b^2e+2ah)x^5)/5 + (b^2fx^6)/6 + (b^2gx^7)/7 + (b^2hx^8)/8 + a(2b^2c+af)*\text{Log}[x]$

Rubi [A] time = 0.118, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1820}

$$-\frac{a^2c}{3x^3} - \frac{a^2d}{2x^2} - \frac{a^2e}{x} + \frac{1}{3}bx^3(2af+bc) + a \log(x)(af+2bc) + \frac{1}{4}bx^4(2ag+bd) + ax(ag+2bd) + \frac{1}{5}bx^5(2ah+be) + \frac{1}{2}ax^2(b^2c+af)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + bx^3)^2(c + dx + ex^2 + fx^3 + gx^4 + hx^5)/x^4, x]$

[Out] $-(a^2c)/(3x^3) - (a^2d)/(2x^2) - (a^2e)/x + a(2bd+ag)x + (a(2be+ah)x^2)/2 + (b(b^2c+2af)x^3)/3 + (b(b^2d+2ag)x^4)/4 + (b(b^2e+2ah)x^5)/5 + (b^2fx^6)/6 + (b^2gx^7)/7 + (b^2hx^8)/8 + a(2b^2c+af)*\text{Log}[x]$

Rule 1820

$\text{Int}[(Pq_*)((c_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /;$ FreeQ[{a, b, c, m, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4} dx = \int \left(a(2bd+ag) + \frac{a^2c}{x^4} + \frac{a^2d}{x^3} + \frac{a^2e}{x^2} + \frac{a(2bc+af)}{x} + a(2be+ah)x \right) dx$$

$$= -\frac{a^2c}{3x^3} - \frac{a^2d}{2x^2} - \frac{a^2e}{x} + a(2bd+ag)x + \frac{1}{2}a(2be+ah)x^2 + \frac{1}{3}b(bc+2af)x^3$$

Mathematica [A] time = 0.0806839, size = 123, normalized size = 0.81

$$-\frac{a^2(2c + 3x(d + 2ex + x^3(-(2g + hx))))}{6x^3} + a \log(x)(af + 2bc) + \frac{1}{30}abx(60d + x(30e + x(20f + 15gx + 12hx^2))) + \frac{1}{84}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]

[Out] -(a^2*(2*c + 3*x*(d + 2*e*x - x^3*(2*g + h*x)))/(6*x^3) + (a*b*x*(60*d + x*(30*e + x*(20*f + 15*g*x + 12*h*x^2)))/30 + (b^2*x^3*(280*c + x*(210*d + x*(168*e + 140*f*x + 120*g*x^2 + 105*h*x^3)))/840 + a*(2*b*c + a*f)*Log[x]

Maple [A] time = 0.005, size = 149, normalized size = 1.

$$\frac{b^2hx^8}{8} + \frac{b^2gx^7}{7} + \frac{b^2fx^6}{6} + \frac{2x^5abh}{5} + \frac{x^5b^2e}{5} + \frac{x^4abg}{2} + \frac{b^2dx^4}{4} + \frac{2x^3abf}{3} + \frac{b^2cx^3}{3} + \frac{x^2a^2h}{2} + aebx^2 + a^2gx + 2bdax + \ln$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x)

[Out] 1/8*b^2*h*x^8+1/7*b^2*g*x^7+1/6*b^2*f*x^6+2/5*x^5*a*b*h+1/5*x^5*b^2*e+1/2*x^4*a*b*g+1/4*b^2*d*x^4+2/3*x^3*a*b*f+1/3*b^2*c*x^3+1/2*x^2*a^2*h+a*e*b*x^2+a^2*g*x+2*b*d*a*x+ln(x)*a^2*f+2*ln(x)*a*b*c-1/3*a^2*c/x^3-1/2*a^2*d/x^2-a^2*e/x

Maxima [A] time = 0.9255, size = 198, normalized size = 1.3

$$\frac{1}{8}b^2hx^8 + \frac{1}{7}b^2gx^7 + \frac{1}{6}b^2fx^6 + \frac{1}{5}(b^2e + 2abh)x^5 + \frac{1}{4}(b^2d + 2abg)x^4 + \frac{1}{3}(b^2c + 2abf)x^3 + \frac{1}{2}(2abe + a^2h)x^2 + (2abd$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="maxima")

[Out] 1/8*b^2*h*x^8 + 1/7*b^2*g*x^7 + 1/6*b^2*f*x^6 + 1/5*(b^2*e + 2*a*b*h)*x^5 + 1/4*(b^2*d + 2*a*b*g)*x^4 + 1/3*(b^2*c + 2*a*b*f)*x^3 + 1/2*(2*a*b*e + a^2

$$*h)*x^2 + (2*a*b*d + a^2*g)*x + (2*a*b*c + a^2*f)*\log(x) - 1/6*(6*a^2*e*x^2 + 3*a^2*d*x + 2*a^2*c)/x^3$$

Fricas [A] time = 0.991457, size = 375, normalized size = 2.47

$$\frac{105b^2hx^{11} + 120b^2gx^{10} + 140b^2fx^9 + 168(b^2e + 2abh)x^8 + 210(b^2d + 2abg)x^7 + 280(b^2c + 2abf)x^6 + 420(2abe + a^2h)x^5 - 840a^2e*x^2 + 840(2a*b*d + a^2*g)*x^4 + 840(2a*b*c + a^2*f)*x^3*\log(x) - 420a^2*d*x - 280a^2*c}{840x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="fricas")

[Out] 1/840*(105*b^2*h*x^11 + 120*b^2*g*x^10 + 140*b^2*f*x^9 + 168*(b^2*e + 2*a*b*h)*x^8 + 210*(b^2*d + 2*a*b*g)*x^7 + 280*(b^2*c + 2*a*b*f)*x^6 + 420*(2*a*b*e + a^2*h)*x^5 - 840*a^2*e*x^2 + 840*(2*a*b*d + a^2*g)*x^4 + 840*(2*a*b*c + a^2*f)*x^3*log(x) - 420*a^2*d*x - 280*a^2*c)/x^3

Sympy [A] time = 0.872923, size = 156, normalized size = 1.03

$$a(af + 2bc)\log(x) + \frac{b^2fx^6}{6} + \frac{b^2gx^7}{7} + \frac{b^2hx^8}{8} + x^5\left(\frac{2abh}{5} + \frac{b^2e}{5}\right) + x^4\left(\frac{abg}{2} + \frac{b^2d}{4}\right) + x^3\left(\frac{2abf}{3} + \frac{b^2c}{3}\right) + x^2\left(\frac{a^2h}{2} + a^2e\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)

[Out] a*(a*f + 2*b*c)*log(x) + b**2*f*x**6/6 + b**2*g*x**7/7 + b**2*h*x**8/8 + x**5*(2*a*b*h/5 + b**2*e/5) + x**4*(a*b*g/2 + b**2*d/4) + x**3*(2*a*b*f/3 + b**2*c/3) + x**2*(a**2*h/2 + a*b*e) + x*(a**2*g + 2*a*b*d) - (2*a**2*c + 3*a**2*d*x + 6*a**2*e*x**2)/(6*x**3)

Giac [A] time = 1.07006, size = 207, normalized size = 1.36

$$\frac{1}{8}b^2hx^8 + \frac{1}{7}b^2gx^7 + \frac{1}{6}b^2fx^6 + \frac{2}{5}abhx^5 + \frac{1}{5}b^2x^5e + \frac{1}{4}b^2dx^4 + \frac{1}{2}abgx^4 + \frac{1}{3}b^2cx^3 + \frac{2}{3}abfx^3 + \frac{1}{2}a^2hx^2 + abx^2e + 2abx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="gia  
c")
```

```
[Out] 1/8*b^2*h*x^8 + 1/7*b^2*g*x^7 + 1/6*b^2*f*x^6 + 2/5*a*b*h*x^5 + 1/5*b^2*x^5  
*e + 1/4*b^2*d*x^4 + 1/2*a*b*g*x^4 + 1/3*b^2*c*x^3 + 2/3*a*b*f*x^3 + 1/2*a^  
2*h*x^2 + a*b*x^2*e + 2*a*b*d*x + a^2*g*x + (2*a*b*c + a^2*f)*log(abs(x)) -  
1/6*(6*a^2*x^2*e + 3*a^2*d*x + 2*a^2*c)/x^3
```

$$3.392 \quad \int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5} dx$$

Optimal. Leaf size=152

$$-\frac{a^2c}{4x^4} - \frac{a^2d}{3x^3} - \frac{a^2e}{2x^2} + \frac{1}{2}bx^2(2af+bc) - \frac{a(af+2bc)}{x} + \frac{1}{3}bx^3(2ag+bd) + a \log(x)(ag+2bd) + \frac{1}{4}bx^4(2ah+be) + ax(ah$$

[Out] $-(a^2c)/(4x^4) - (a^2d)/(3x^3) - (a^2e)/(2x^2) - (a(2bc+af))/x + a(2be+ah)x + (b(b^2c+2af)x^2)/2 + (b(b^2d+2ag)x^3)/3 + (b(b^2e+2ah)x^4)/4 + (b^2fx^5)/5 + (b^2gx^6)/6 + (b^2hx^7)/7 + a(2bd+ag)*\text{Log}[x]$

Rubi [A] time = 0.117274, antiderivative size = 152, normalized size of antiderivative = 1, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1820}

$$-\frac{a^2c}{4x^4} - \frac{a^2d}{3x^3} - \frac{a^2e}{2x^2} + \frac{1}{2}bx^2(2af+bc) - \frac{a(af+2bc)}{x} + \frac{1}{3}bx^3(2ag+bd) + a \log(x)(ag+2bd) + \frac{1}{4}bx^4(2ah+be) + ax(ah$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + bx^3)^2(c + dx + ex^2 + fx^3 + gx^4 + hx^5)/x^5, x]$

[Out] $-(a^2c)/(4x^4) - (a^2d)/(3x^3) - (a^2e)/(2x^2) - (a(2bc+af))/x + a(2be+ah)x + (b(b^2c+2af)x^2)/2 + (b(b^2d+2ag)x^3)/3 + (b(b^2e+2ah)x^4)/4 + (b^2fx^5)/5 + (b^2gx^6)/6 + (b^2hx^7)/7 + a(2bd+ag)*\text{Log}[x]$

Rule 1820

$\text{Int}[(Pq_*)((c_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}), x_Symbol] := \text{Int}[\text{ExpandIntegrand}[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /;$ FreeQ[{a, b, c, m, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5} dx = \int \left(a(2be+ah) + \frac{a^2c}{x^5} + \frac{a^2d}{x^4} + \frac{a^2e}{x^3} + \frac{a(2bc+af)}{x^2} + \frac{a(2bd+ag)}{x} \right. \\ \left. - \frac{a^2c}{4x^4} - \frac{a^2d}{3x^3} - \frac{a^2e}{2x^2} - \frac{a(2bc+af)}{x} + a(2be+ah)x + \frac{1}{2}b(bc+2af) \right) dx$$

Mathematica [A] time = 0.0725245, size = 125, normalized size = 0.82

$$-\frac{a^2(3c + 4dx + 6x^2(e + 2fx - 2hx^3))}{12x^4} - \frac{2abc}{x} + a \log(x)(ag + 2bd) + \frac{1}{6}abx(12e + x(6f + x(4g + 3hx))) + \frac{1}{420}b^2x^2(210$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]

[Out] (-2*a*b*c)/x - (a^2*(3*c + 4*d*x + 6*x^2*(e + 2*f*x - 2*h*x^3)))/(12*x^4) + (a*b*x*(12*e + x*(6*f + x*(4*g + 3*h*x)))/6 + (b^2*x^2*(210*c + x*(140*d + x*(105*e + 84*f*x + 70*g*x^2 + 60*h*x^3))))/420 + a*(2*b*d + a*g)*Log[x]

Maple [A] time = 0.006, size = 149, normalized size = 1.

$$\frac{b^2hx^7}{7} + \frac{b^2gx^6}{6} + \frac{b^2fx^5}{5} + \frac{x^4abh}{2} + \frac{x^4b^2e}{4} + \frac{2x^3abg}{3} + \frac{b^2dx^3}{3} + x^2abf + \frac{x^2b^2c}{2} + a^2hx + 2aebx + \ln(x)a^2g + 2 \ln(x)ab$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x)

[Out] 1/7*b^2*h*x^7+1/6*b^2*g*x^6+1/5*b^2*f*x^5+1/2*x^4*a*b*h+1/4*x^4*b^2*e+2/3*x^3*a*b*g+1/3*b^2*d*x^3+x^2*a*b*f+1/2*x^2*b^2*c+a^2*h*x+2*a*e*b*x+ln(x)*a^2*g+2*ln(x)*a*b*d-1/3*a^2*d/x^3-1/4*a^2*c/x^4-1/2*a^2*e/x^2-a^2/x*f-2*a/x*b*c

Maxima [A] time = 0.947725, size = 198, normalized size = 1.3

$$\frac{1}{7}b^2hx^7 + \frac{1}{6}b^2gx^6 + \frac{1}{5}b^2fx^5 + \frac{1}{4}(b^2e + 2abh)x^4 + \frac{1}{3}(b^2d + 2abg)x^3 + \frac{1}{2}(b^2c + 2abf)x^2 + (2abe + a^2h)x + (2abd + a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="maxima")

[Out] 1/7*b^2*h*x^7 + 1/6*b^2*g*x^6 + 1/5*b^2*f*x^5 + 1/4*(b^2*e + 2*a*b*h)*x^4 + 1/3*(b^2*d + 2*a*b*g)*x^3 + 1/2*(b^2*c + 2*a*b*f)*x^2 + (2*a*b*e + a^2*h)*x + (2*a*b*d + a^2*g)*log(x) - 1/12*(6*a^2*e*x^2 + 4*a^2*d*x + 12*(2*a*b*c

$$+ a^2 f x^3 + 3 a^2 c) / x^4$$

Fricas [A] time = 0.990554, size = 371, normalized size = 2.44

$$\frac{60 b^2 h x^{11} + 70 b^2 g x^{10} + 84 b^2 f x^9 + 105 (b^2 e + 2 a b h) x^8 + 140 (b^2 d + 2 a b g) x^7 + 210 (b^2 c + 2 a b f) x^6 + 420 (2 a b e + a^2 h) x^5 + 420 (2 a b d + a^2 g) x^4 \log(x) - 210 a^2 e x^2 - 140 a^2 d x - 420 (2 a b c + a^2 f) x^3 - 105 a^2 c) / x^4}{420 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="fricas")

[Out] 1/420*(60*b^2*h*x^11 + 70*b^2*g*x^10 + 84*b^2*f*x^9 + 105*(b^2*e + 2*a*b*h)*x^8 + 140*(b^2*d + 2*a*b*g)*x^7 + 210*(b^2*c + 2*a*b*f)*x^6 + 420*(2*a*b*e + a^2*h)*x^5 + 420*(2*a*b*d + a^2*g)*x^4*log(x) - 210*a^2*e*x^2 - 140*a^2*d*x - 420*(2*a*b*c + a^2*f)*x^3 - 105*a^2*c)/x^4

Sympy [A] time = 2.7749, size = 155, normalized size = 1.02

$$a (a g + 2 b d) \log(x) + \frac{b^2 f x^5}{5} + \frac{b^2 g x^6}{6} + \frac{b^2 h x^7}{7} + x^4 \left(\frac{a b h}{2} + \frac{b^2 e}{4} \right) + x^3 \left(\frac{2 a b g}{3} + \frac{b^2 d}{3} \right) + x^2 \left(a b f + \frac{b^2 c}{2} \right) + x (a^2 h + 2 a b e)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)

[Out] a*(a*g + 2*b*d)*log(x) + b**2*f*x**5/5 + b**2*g*x**6/6 + b**2*h*x**7/7 + x**4*(a*b*h/2 + b**2*e/4) + x**3*(2*a*b*g/3 + b**2*d/3) + x**2*(a*b*f + b**2*c/2) + x*(a**2*h + 2*a*b*e) - (3*a**2*c + 4*a**2*d*x + 6*a**2*e*x**2 + x**3*(12*a**2*f + 24*a*b*c))/(12*x**4)

Giac [A] time = 1.06372, size = 205, normalized size = 1.35

$$\frac{1}{7} b^2 h x^7 + \frac{1}{6} b^2 g x^6 + \frac{1}{5} b^2 f x^5 + \frac{1}{2} a b h x^4 + \frac{1}{4} b^2 x^4 e + \frac{1}{3} b^2 d x^3 + \frac{2}{3} a b g x^3 + \frac{1}{2} b^2 c x^2 + a b f x^2 + a^2 h x + 2 a b e + (2 a b d +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="gia  
c")
```

```
[Out] 1/7*b^2*h*x^7 + 1/6*b^2*g*x^6 + 1/5*b^2*f*x^5 + 1/2*a*b*h*x^4 + 1/4*b^2*x^4  
*e + 1/3*b^2*d*x^3 + 2/3*a*b*g*x^3 + 1/2*b^2*c*x^2 + a*b*f*x^2 + a^2*h*x +  
2*a*b*x*e + (2*a*b*d + a^2*g)*log(abs(x)) - 1/12*(6*a^2*x^2*e + 4*a^2*d*x +  
12*(2*a*b*c + a^2*f)*x^3 + 3*a^2*c)/x^4
```


3.393 $\int x^4 (a + bx^3)^3 (c + dx + ex^2 + fx^3 + gx^4 + hx^5) dx$

Optimal. Leaf size=223

$$\frac{1}{8}a^2x^8(af + 3bc) + \frac{1}{9}a^2x^9(ag + 3bd) + \frac{1}{10}a^2x^{10}(ah + 3be) + \frac{1}{5}a^3cx^5 + \frac{1}{6}a^3dx^6 + \frac{1}{7}a^3ex^7 + \frac{1}{14}b^2x^{14}(3af + bc) + \frac{1}{15}b^2x^{15}$$

$$\begin{aligned} \text{[Out]} & (a^3c*x^5)/5 + (a^3d*x^6)/6 + (a^3e*x^7)/7 + (a^2*(3*b*c + a*f)*x^8)/8 + \\ & (a^2*(3*b*d + a*g)*x^9)/9 + (a^2*(3*b*e + a*h)*x^{10})/10 + (3*a*b*(b*c + a* \\ & f)*x^{11})/11 + (a*b*(b*d + a*g)*x^{12})/4 + (3*a*b*(b*e + a*h)*x^{13})/13 + (b^2 \\ & *(b*c + 3*a*f)*x^{14})/14 + (b^2*(b*d + 3*a*g)*x^{15})/15 + (b^2*(b*e + 3*a*h)* \\ & x^{16})/16 + (b^3*f*x^{17})/17 + (b^3*g*x^{18})/18 + (b^3*h*x^{19})/19 \end{aligned}$$

Rubi [A] time = 0.292612, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1820}

$$\frac{1}{8}a^2x^8(af + 3bc) + \frac{1}{9}a^2x^9(ag + 3bd) + \frac{1}{10}a^2x^{10}(ah + 3be) + \frac{1}{5}a^3cx^5 + \frac{1}{6}a^3dx^6 + \frac{1}{7}a^3ex^7 + \frac{1}{14}b^2x^{14}(3af + bc) + \frac{1}{15}b^2x^{15}$$

Antiderivative was successfully verified.

[In] Int[x^4*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]

$$\begin{aligned} \text{[Out]} & (a^3c*x^5)/5 + (a^3d*x^6)/6 + (a^3e*x^7)/7 + (a^2*(3*b*c + a*f)*x^8)/8 + \\ & (a^2*(3*b*d + a*g)*x^9)/9 + (a^2*(3*b*e + a*h)*x^{10})/10 + (3*a*b*(b*c + a* \\ & f)*x^{11})/11 + (a*b*(b*d + a*g)*x^{12})/4 + (3*a*b*(b*e + a*h)*x^{13})/13 + (b^2 \\ & *(b*c + 3*a*f)*x^{14})/14 + (b^2*(b*d + 3*a*g)*x^{15})/15 + (b^2*(b*e + 3*a*h)* \\ & x^{16})/16 + (b^3*f*x^{17})/17 + (b^3*g*x^{18})/18 + (b^3*h*x^{19})/19 \end{aligned}$$

Rule 1820

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :=
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int x^4 (a + bx^3)^3 (c + dx + ex^2 + fx^3 + gx^4 + hx^5) dx &= \int (a^3cx^4 + a^3dx^5 + a^3ex^6 + a^2(3bc + af)x^7 + a^2(3bd + ag)x^8 + \\ &= \frac{1}{5}a^3cx^5 + \frac{1}{6}a^3dx^6 + \frac{1}{7}a^3ex^7 + \frac{1}{8}a^2(3bc + af)x^8 + \frac{1}{9}a^2(3bd + ag)x^9 \end{aligned}$$

Mathematica [A] time = 0.0509596, size = 223, normalized size = 1.

$$\frac{1}{8}a^2x^8(af + 3bc) + \frac{1}{9}a^2x^9(ag + 3bd) + \frac{1}{10}a^2x^{10}(ah + 3be) + \frac{1}{5}a^3cx^5 + \frac{1}{6}a^3dx^6 + \frac{1}{7}a^3ex^7 + \frac{1}{14}b^2x^{14}(3af + bc) + \frac{1}{15}b^2x^{15}(3ag + bd)$$

Antiderivative was successfully verified.

[In] Integrate[x^4*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5), x]

[Out] (a^3*c*x^5)/5 + (a^3*d*x^6)/6 + (a^3*e*x^7)/7 + (a^2*(3*b*c + a*f)*x^8)/8 + (a^2*(3*b*d + a*g)*x^9)/9 + (a^2*(3*b*e + a*h)*x^10)/10 + (3*a*b*(b*c + a*f)*x^11)/11 + (a*b*(b*d + a*g)*x^12)/4 + (3*a*b*(b*e + a*h)*x^13)/13 + (b^2*(b*c + 3*a*f)*x^14)/14 + (b^2*(b*d + 3*a*g)*x^15)/15 + (b^2*(b*e + 3*a*h)*x^16)/16 + (b^3*f*x^17)/17 + (b^3*g*x^18)/18 + (b^3*h*x^19)/19

Maple [A] time = 0.001, size = 224, normalized size = 1.

$$\frac{b^3hx^{19}}{19} + \frac{b^3gx^{18}}{18} + \frac{b^3fx^{17}}{17} + \frac{(3b^2ah + b^3e)x^{16}}{16} + \frac{(3b^2ag + b^3d)x^{15}}{15} + \frac{(3b^2af + b^3c)x^{14}}{14} + \frac{(3ba^2h + 3aeb^2)x^{13}}{13} + \frac{(3ba^2g + 3aeb)x^{12}}{12} + \frac{(3ba^2f + 3aeb^2)x^{11}}{11} + \frac{(3ba^2d + 3aeb^2)x^{10}}{10} + \frac{(3ba^2c + 3aeb^2)x^9}{9} + \frac{(3ba^2b + 3aeb^2)x^8}{8} + \frac{(3ba^2a + 3aeb^2)x^7}{7} + \frac{(3ba^2e + 3aeb^2)x^6}{6} + \frac{(3ba^2d + 3aeb^2)x^5}{5} + \frac{(3ba^2c + 3aeb^2)x^4}{4} + \frac{(3ba^2b + 3aeb^2)x^3}{3} + \frac{(3ba^2a + 3aeb^2)x^2}{2} + \frac{(3ba^2e + 3aeb^2)x}{1} + \frac{(3ba^2d + 3aeb^2)}{0}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c), x)

[Out] 1/19*b^3*h*x^19+1/18*b^3*g*x^18+1/17*b^3*f*x^17+1/16*(3*a*b^2*h+b^3*e)*x^16+1/15*(3*a*b^2*g+b^3*d)*x^15+1/14*(3*a*b^2*f+b^3*c)*x^14+1/13*(3*a^2*b*h+3*a*b^2*e)*x^13+1/12*(3*a^2*b*g+3*a*b^2*d)*x^12+1/11*(3*a^2*b*f+3*a*b^2*c)*x^11+1/10*(a^3*h+3*a^2*b*e)*x^10+1/9*(a^3*g+3*a^2*b*d)*x^9+1/8*(a^3*f+3*a^2*b*c)*x^8+1/7*a^3*e*x^7+1/6*a^3*d*x^6+1/5*a^3*c*x^5

Maxima [A] time = 0.957263, size = 293, normalized size = 1.31

$$\frac{1}{19}b^3hx^{19} + \frac{1}{18}b^3gx^{18} + \frac{1}{17}b^3fx^{17} + \frac{1}{16}(b^3e + 3ab^2h)x^{16} + \frac{1}{15}(b^3d + 3ab^2g)x^{15} + \frac{1}{14}(b^3c + 3ab^2f)x^{14} + \frac{3}{13}(ab^2e + a^3h)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c), x, algorithm="maxima")

[Out] $\frac{1}{19}b^3hx^{19} + \frac{1}{18}b^3gx^{18} + \frac{1}{17}b^3fx^{17} + \frac{1}{16}(b^3e + 3a^2b^2h)x^{16} + \frac{1}{15}(b^3d + 3a^2b^2g)x^{15} + \frac{1}{14}(b^3c + 3a^2b^2f)x^{14} + \frac{3}{13}(a^2b^2e + a^2b^2h)x^{13} + \frac{1}{4}(a^2b^2d + a^2b^2g)x^{12} + \frac{3}{11}(a^2b^2c + a^2b^2f)x^{11} + \frac{1}{7}a^3ex^7 + \frac{1}{10}(3a^2b^2e + a^3h)x^{10} + \frac{1}{6}a^3d^2x^6 + \frac{1}{9}(3a^2b^2d + a^3g)x^9 + \frac{1}{5}a^3cx^5 + \frac{1}{8}(3a^2b^2c + a^3f)x^8$

Fricas [A] time = 0.873177, size = 591, normalized size = 2.65

$$\frac{1}{19}x^{19}hb^3 + \frac{1}{18}x^{18}gb^3 + \frac{1}{17}x^{17}fb^3 + \frac{1}{16}x^{16}eb^3 + \frac{3}{16}x^{16}hb^2a + \frac{1}{15}x^{15}db^3 + \frac{1}{5}x^{15}gb^2a + \frac{1}{14}x^{14}cb^3 + \frac{3}{14}x^{14}fb^2a + \frac{3}{13}x^{13}e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate($x^4(bx^3+a)^3(hx^5+gx^4+fx^3+ex^2+dx+c)$, x, algorithm="fricas")

[Out] $\frac{1}{19}x^{19}hb^3 + \frac{1}{18}x^{18}gb^3 + \frac{1}{17}x^{17}fb^3 + \frac{1}{16}x^{16}eb^3 + \frac{3}{16}x^{16}hb^2a + \frac{1}{15}x^{15}db^3 + \frac{1}{5}x^{15}gb^2a + \frac{1}{14}x^{14}cb^3 + \frac{3}{14}x^{14}fb^2a + \frac{3}{13}x^{13}e$

Sympy [A] time = 0.099741, size = 246, normalized size = 1.1

$$\frac{a^3cx^5}{5} + \frac{a^3dx^6}{6} + \frac{a^3ex^7}{7} + \frac{b^3fx^{17}}{17} + \frac{b^3gx^{18}}{18} + \frac{b^3hx^{19}}{19} + x^{16}\left(\frac{3ab^2h}{16} + \frac{b^3e}{16}\right) + x^{15}\left(\frac{ab^2g}{5} + \frac{b^3d}{15}\right) + x^{14}\left(\frac{3ab^2f}{14} + \frac{b^3c}{14}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate($x**4*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)$, x)

[Out] $a**3*c*x**5/5 + a**3*d*x**6/6 + a**3*e*x**7/7 + b**3*f*x**17/17 + b**3*g*x**18/18 + b**3*h*x**19/19 + x**16*(3*a*b**2*h/16 + b**3*e/16) + x**15*(a*b**2*g/5 + b**3*d/15) + x**14*(3*a*b**2*f/14 + b**3*c/14) + x**13*(3*a**2*b*h/13 + 3*a*b**2*e/13) + x**12*(a**2*b*g/4 + a*b**2*d/4) + x**11*(3*a**2*b*f/11 + 3*a*b**2*c/11) + x**10*(a**3*h/10 + 3*a**2*b*e/10) + x**9*(a**3*g/9 + a**2*b*d/3) + x**8*(a**3*f/8 + 3*a**2*b*c/8)$

Giac [A] time = 1.06827, size = 315, normalized size = 1.41

$$\frac{1}{19} b^3 h x^{19} + \frac{1}{18} b^3 g x^{18} + \frac{1}{17} b^3 f x^{17} + \frac{3}{16} a b^2 h x^{16} + \frac{1}{16} b^3 x^{16} e + \frac{1}{15} b^3 d x^{15} + \frac{1}{5} a b^2 g x^{15} + \frac{1}{14} b^3 c x^{14} + \frac{3}{14} a b^2 f x^{14} + \frac{3}{13} a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")

[Out] 1/19*b^3*h*x^19 + 1/18*b^3*g*x^18 + 1/17*b^3*f*x^17 + 3/16*a*b^2*h*x^16 + 1/16*b^3*x^16*e + 1/15*b^3*d*x^15 + 1/5*a*b^2*g*x^15 + 1/14*b^3*c*x^14 + 3/14*a*b^2*f*x^14 + 3/13*a^2*b*h*x^13 + 3/13*a*b^2*x^13*e + 1/4*a*b^2*d*x^12 + 1/4*a^2*b*g*x^12 + 3/11*a*b^2*c*x^11 + 3/11*a^2*b*f*x^11 + 1/10*a^3*h*x^10 + 3/10*a^2*b*x^10*e + 1/3*a^2*b*d*x^9 + 1/9*a^3*g*x^9 + 3/8*a^2*b*c*x^8 + 1/8*a^3*f*x^8 + 1/7*a^3*x^7*e + 1/6*a^3*d*x^6 + 1/5*a^3*c*x^5

$$3.394 \quad \int x^3 (a + bx^3)^3 (c + dx + ex^2 + fx^3 + gx^4 + hx^5) dx$$

Optimal. Leaf size=223

$$\frac{1}{7}a^2x^7(af + 3bc) + \frac{1}{8}a^2x^8(ag + 3bd) + \frac{1}{9}a^2x^9(ah + 3be) + \frac{1}{4}a^3cx^4 + \frac{1}{5}a^3dx^5 + \frac{1}{6}a^3ex^6 + \frac{1}{13}b^2x^{13}(3af + bc) + \frac{1}{14}b^2x^{14}(3ag + bd) + \frac{1}{15}b^2x^{15}(3ah + be) + \frac{1}{16}b^3fx^{16} + \frac{1}{17}b^3gx^{17} + \frac{1}{18}b^3hx^{18}$$

[Out] $(a^3cx^4)/4 + (a^3dx^5)/5 + (a^3ex^6)/6 + (a^2(3bc + af)x^7)/7 + (a^2(3bd + ag)x^8)/8 + (a^2(3be + ah)x^9)/9 + (3ab(bc + af)x^{10})/10 + (3ab(bd + ag)x^{11})/11 + (ab(be + ah)x^{12})/4 + (b^2(bc + 3af)x^{13})/13 + (b^2(bd + 3ag)x^{14})/14 + (b^2(be + 3ah)x^{15})/15 + (b^3fx^{16})/16 + (b^3gx^{17})/17 + (b^3hx^{18})/18$

Rubi [A] time = 0.227574, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1820}

$$\frac{1}{7}a^2x^7(af + 3bc) + \frac{1}{8}a^2x^8(ag + 3bd) + \frac{1}{9}a^2x^9(ah + 3be) + \frac{1}{4}a^3cx^4 + \frac{1}{5}a^3dx^5 + \frac{1}{6}a^3ex^6 + \frac{1}{13}b^2x^{13}(3af + bc) + \frac{1}{14}b^2x^{14}(3ag + bd) + \frac{1}{15}b^2x^{15}(3ah + be) + \frac{1}{16}b^3fx^{16} + \frac{1}{17}b^3gx^{17} + \frac{1}{18}b^3hx^{18}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^3(a + bx^3)^3(c + dx + ex^2 + fx^3 + gx^4 + hx^5), x]$

[Out] $(a^3cx^4)/4 + (a^3dx^5)/5 + (a^3ex^6)/6 + (a^2(3bc + af)x^7)/7 + (a^2(3bd + ag)x^8)/8 + (a^2(3be + ah)x^9)/9 + (3ab(bc + af)x^{10})/10 + (3ab(bd + ag)x^{11})/11 + (ab(be + ah)x^{12})/4 + (b^2(bc + 3af)x^{13})/13 + (b^2(bd + 3ag)x^{14})/14 + (b^2(be + 3ah)x^{15})/15 + (b^3fx^{16})/16 + (b^3gx^{17})/17 + (b^3hx^{18})/18$

Rule 1820

$\text{Int}[(Pq_*)((c_*)*(x_*)^{(m_*)}((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x_Symbol] :>$
 $\text{Int}[\text{ExpandIntegrand}[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /;$ FreeQ[{a, b, c, m, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\int x^3 (a + bx^3)^3 (c + dx + ex^2 + fx^3 + gx^4 + hx^5) dx = \int (a^3cx^3 + a^3dx^4 + a^3ex^5 + a^2(3bc + af)x^6 + a^2(3bd + ag)x^7 + a^2(3be + ah)x^8 + a^2(3af + bc)x^9 + a^2(3ag + bd)x^{10} + a^2(3ah + be)x^{11} + b^2fx^{12} + b^2gx^{13} + b^2hx^{14}) dx$$

$$= \frac{1}{4}a^3cx^4 + \frac{1}{5}a^3dx^5 + \frac{1}{6}a^3ex^6 + \frac{1}{7}a^2(3bc + af)x^7 + \frac{1}{8}a^2(3bd + ag)x^8 + \frac{1}{9}a^2(3be + ah)x^9 + \frac{3}{10}ab(bc + af)x^{10} + \frac{3}{11}ab(bd + ag)x^{11} + \frac{1}{4}ab(be + ah)x^{12} + \frac{1}{13}b^2(bc + 3af)x^{13} + \frac{1}{14}b^2(bd + 3ag)x^{14} + \frac{1}{15}b^2(be + 3ah)x^{15} + \frac{1}{16}b^3fx^{16} + \frac{1}{17}b^3gx^{17} + \frac{1}{18}b^3hx^{18}$$

Mathematica [A] time = 0.0468082, size = 223, normalized size = 1.

$$\frac{1}{7}a^2x^7(af + 3bc) + \frac{1}{8}a^2x^8(ag + 3bd) + \frac{1}{9}a^2x^9(ah + 3be) + \frac{1}{4}a^3cx^4 + \frac{1}{5}a^3dx^5 + \frac{1}{6}a^3ex^6 + \frac{1}{13}b^2x^{13}(3af + bc) + \frac{1}{14}b^2x^{14}(3a$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5), x]

[Out] (a^3*c*x^4)/4 + (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^2*(3*b*c + a*f)*x^7)/7 + (a^2*(3*b*d + a*g)*x^8)/8 + (a^2*(3*b*e + a*h)*x^9)/9 + (3*a*b*(b*c + a*f)*x^10)/10 + (3*a*b*(b*d + a*g)*x^11)/11 + (a*b*(b*e + a*h)*x^12)/4 + (b^2*(b*c + 3*a*f)*x^13)/13 + (b^2*(b*d + 3*a*g)*x^14)/14 + (b^2*(b*e + 3*a*h)*x^15)/15 + (b^3*f*x^16)/16 + (b^3*g*x^17)/17 + (b^3*h*x^18)/18

Maple [A] time = 0.002, size = 224, normalized size = 1.

$$\frac{b^3hx^{18}}{18} + \frac{b^3gx^{17}}{17} + \frac{b^3fx^{16}}{16} + \frac{(3b^2ah + b^3e)x^{15}}{15} + \frac{(3b^2ag + b^3d)x^{14}}{14} + \frac{(3b^2af + b^3c)x^{13}}{13} + \frac{(3ba^2h + 3aeb^2)x^{12}}{12} + \frac{(3a^2b^2h + 3a^2b^2e)x^{11}}{11} + \frac{(3a^2b^2g + 3a^2b^2d)x^{10}}{10} + \frac{(3a^2b^2f + 3a^2b^2c)x^9}{9} + \frac{(a^3h + 3a^2b^2e)x^8}{8} + \frac{(a^3g + 3a^2b^2d)x^7}{7} + \frac{(a^3f + 3a^2b^2c)x^6}{6} + \frac{a^3d^2x^5}{5} + \frac{a^3c^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c), x)

[Out] 1/18*b^3*h*x^18+1/17*b^3*g*x^17+1/16*b^3*f*x^16+1/15*(3*a*b^2*h+b^3*e)*x^15+1/14*(3*a*b^2*g+b^3*d)*x^14+1/13*(3*a*b^2*f+b^3*c)*x^13+1/12*(3*a^2*b*h+3*a*b^2*e)*x^12+1/11*(3*a^2*b*g+3*a*b^2*d)*x^11+1/10*(3*a^2*b*f+3*a*b^2*c)*x^10+1/9*(a^3*h+3*a^2*b^2*e)*x^9+1/8*(a^3*g+3*a^2*b^2*d)*x^8+1/7*(a^3*f+3*a^2*b^2*c)*x^7+1/6*a^3*e*x^6+1/5*a^3*d*x^5+1/4*a^3*c*x^4

Maxima [A] time = 0.931833, size = 293, normalized size = 1.31

$$\frac{1}{18}b^3hx^{18} + \frac{1}{17}b^3gx^{17} + \frac{1}{16}b^3fx^{16} + \frac{1}{15}(b^3e + 3ab^2h)x^{15} + \frac{1}{14}(b^3d + 3ab^2g)x^{14} + \frac{1}{13}(b^3c + 3ab^2f)x^{13} + \frac{1}{4}(ab^2e + a^2b^2h)x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c), x, algorithm="maxima")

[Out] $\frac{1}{18}b^3hx^{18} + \frac{1}{17}b^3gx^{17} + \frac{1}{16}b^3fx^{16} + \frac{1}{15}(b^3e + 3ab^2h)x^{15} + \frac{1}{14}(b^3d + 3ab^2g)x^{14} + \frac{1}{13}(b^3c + 3ab^2f)x^{13} + \frac{1}{4}(ab^2e + a^2bh)x^{12} + \frac{3}{11}(ab^2d + a^2bg)x^{11} + \frac{3}{10}(ab^2c + a^2bf)x^{10} + \frac{1}{6}a^3ex^6 + \frac{1}{9}(3a^2be + a^3h)x^9 + \frac{1}{5}a^3dx^5 + \frac{1}{8}(3a^2bd + a^3g)x^8 + \frac{1}{4}a^3cx^4 + \frac{1}{7}(3a^2bc + a^3f)x^7$

Fricas [A] time = 0.869605, size = 586, normalized size = 2.63

$$\frac{1}{18}x^{18}hb^3 + \frac{1}{17}x^{17}gb^3 + \frac{1}{16}x^{16}fb^3 + \frac{1}{15}x^{15}eb^3 + \frac{1}{5}x^{15}hb^2a + \frac{1}{14}x^{14}db^3 + \frac{3}{14}x^{14}gb^2a + \frac{1}{13}x^{13}cb^3 + \frac{3}{13}x^{13}fb^2a + \frac{1}{4}x^{12}eb^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")`

[Out] $\frac{1}{18}x^{18}hb^3 + \frac{1}{17}x^{17}gb^3 + \frac{1}{16}x^{16}fb^3 + \frac{1}{15}x^{15}eb^3 + \frac{1}{5}x^{15}hb^2a + \frac{1}{14}x^{14}db^3 + \frac{3}{14}x^{14}gb^2a + \frac{1}{13}x^{13}cb^3 + \frac{3}{13}x^{13}fb^2a + \frac{1}{4}x^{12}eb^3 + \frac{1}{4}x^{12}eab^2a + \frac{1}{4}x^{12}hba^2 + \frac{3}{11}x^{11}db^2a + \frac{3}{11}x^{11}gba^2 + \frac{3}{10}x^{10}c b^2a + \frac{3}{10}x^{10}fba^2 + \frac{1}{3}x^9e b a^2 + \frac{1}{9}x^9h a^3 + \frac{3}{8}x^8d b a^2 + \frac{1}{8}x^8g a^3 + \frac{3}{7}x^7c b a^2 + \frac{1}{7}x^7f a^3 + \frac{1}{6}x^6e a^3 + \frac{1}{5}x^5d a^3 + \frac{1}{4}x^4c a^3$

Sympy [A] time = 0.100897, size = 246, normalized size = 1.1

$$\frac{a^3cx^4}{4} + \frac{a^3dx^5}{5} + \frac{a^3ex^6}{6} + \frac{b^3fx^{16}}{16} + \frac{b^3gx^{17}}{17} + \frac{b^3hx^{18}}{18} + x^{15}\left(\frac{ab^2h}{5} + \frac{b^3e}{15}\right) + x^{14}\left(\frac{3ab^2g}{14} + \frac{b^3d}{14}\right) + x^{13}\left(\frac{3ab^2f}{13} + \frac{b^3c}{13}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)`

[Out] $a**3*c*x**4/4 + a**3*d*x**5/5 + a**3*e*x**6/6 + b**3*f*x**16/16 + b**3*g*x**17/17 + b**3*h*x**18/18 + x**15*(a*b**2*h/5 + b**3*e/15) + x**14*(3*a*b**2*g/14 + b**3*d/14) + x**13*(3*a*b**2*f/13 + b**3*c/13) + x**12*(a**2*b*h/4 + a*b**2*e/4) + x**11*(3*a**2*b*g/11 + 3*a*b**2*d/11) + x**10*(3*a**2*b*f/10 + 3*a*b**2*c/10) + x**9*(a**3*h/9 + a**2*b*e/3) + x**8*(a**3*g/8 + 3*a**2*b*d/8) + x**7*(a**3*f/7 + 3*a**2*b*c/7)$

Giac [A] time = 1.07661, size = 315, normalized size = 1.41

$$\frac{1}{18} b^3 h x^{18} + \frac{1}{17} b^3 g x^{17} + \frac{1}{16} b^3 f x^{16} + \frac{1}{5} a b^2 h x^{15} + \frac{1}{15} b^3 x^{15} e + \frac{1}{14} b^3 d x^{14} + \frac{3}{14} a b^2 g x^{14} + \frac{1}{13} b^3 c x^{13} + \frac{3}{13} a b^2 f x^{13} + \frac{1}{4} a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")

[Out] 1/18*b^3*h*x^18 + 1/17*b^3*g*x^17 + 1/16*b^3*f*x^16 + 1/5*a*b^2*h*x^15 + 1/15*b^3*x^15*e + 1/14*b^3*d*x^14 + 3/14*a*b^2*g*x^14 + 1/13*b^3*c*x^13 + 3/13*a*b^2*f*x^13 + 1/4*a^2*b*h*x^12 + 1/4*a*b^2*x^12*e + 3/11*a*b^2*d*x^11 + 3/11*a^2*b*g*x^11 + 3/10*a*b^2*c*x^10 + 3/10*a^2*b*f*x^10 + 1/9*a^3*h*x^9 + 1/3*a^2*b*x^9*e + 3/8*a^2*b*d*x^8 + 1/8*a^3*g*x^8 + 3/7*a^2*b*c*x^7 + 1/7*a^3*f*x^7 + 1/6*a^3*x^6*e + 1/5*a^3*d*x^5 + 1/4*a^3*c*x^4

$$3.395 \quad \int x^2 (a + bx^3)^3 (c + dx + ex^2 + fx^3 + gx^4 + hx^5) dx$$

Optimal. Leaf size=212

$$\frac{1}{7}a^2x^7(ag + 3bd) + \frac{1}{8}a^2x^8(ah + 3be) + \frac{1}{3}a^2bfx^9 + \frac{1}{4}a^3dx^4 + \frac{1}{5}a^3ex^5 + \frac{1}{6}a^3fx^6 + \frac{1}{13}b^2x^{13}(3ag + bd) + \frac{1}{14}b^2x^{14}(3ah + b$$

$$\begin{aligned} \text{[Out]} \quad & (a^3d*x^4)/4 + (a^3e*x^5)/5 + (a^3f*x^6)/6 + (a^2*(3*b*d + a*g)*x^7)/7 + \\ & (a^2*(3*b*e + a*h)*x^8)/8 + (a^2*b*f*x^9)/3 + (3*a*b*(b*d + a*g)*x^{10})/10 \\ & + (3*a*b*(b*e + a*h)*x^{11})/11 + (a*b^2*f*x^{12})/4 + (b^2*(b*d + 3*a*g)*x^{13}) \\ & /13 + (b^2*(b*e + 3*a*h)*x^{14})/14 + (b^3*f*x^{15})/15 + (b^3*g*x^{16})/16 + (b^ \\ & 3*h*x^{17})/17 + (c*(a + b*x^3)^4)/(12*b) \end{aligned}$$

Rubi [A] time = 0.179245, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1582, 1850}

$$\frac{1}{7}a^2x^7(ag + 3bd) + \frac{1}{8}a^2x^8(ah + 3be) + \frac{1}{3}a^2bfx^9 + \frac{1}{4}a^3dx^4 + \frac{1}{5}a^3ex^5 + \frac{1}{6}a^3fx^6 + \frac{1}{13}b^2x^{13}(3ag + bd) + \frac{1}{14}b^2x^{14}(3ah + b$$

Antiderivative was successfully verified.

$$\text{[In]} \quad \text{Int}[x^2*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5), x]$$

$$\begin{aligned} \text{[Out]} \quad & (a^3d*x^4)/4 + (a^3e*x^5)/5 + (a^3f*x^6)/6 + (a^2*(3*b*d + a*g)*x^7)/7 + \\ & (a^2*(3*b*e + a*h)*x^8)/8 + (a^2*b*f*x^9)/3 + (3*a*b*(b*d + a*g)*x^{10})/10 \\ & + (3*a*b*(b*e + a*h)*x^{11})/11 + (a*b^2*f*x^{12})/4 + (b^2*(b*d + 3*a*g)*x^{13}) \\ & /13 + (b^2*(b*e + 3*a*h)*x^{14})/14 + (b^3*f*x^{15})/15 + (b^3*g*x^{16})/16 + (b^ \\ & 3*h*x^{17})/17 + (c*(a + b*x^3)^4)/(12*b) \end{aligned}$$

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
- 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned} \int x^2 (a + bx^3)^3 (c + dx + ex^2 + fx^3 + gx^4 + hx^5) dx &= \frac{c(a + bx^3)^4}{12b} + \int (a + bx^3)^3 (-cx^2 + x^2(c + dx + ex^2 + fx^3 + gx^4 + hx^5)) dx \\ &= \frac{c(a + bx^3)^4}{12b} + \int (a^3 dx^3 + a^3 ex^4 + a^3 fx^5 + a^2(3bd + ag)x^6 + a^2(3cd + ah)x^7 + a^2(3bd + ag)x^8 + a^2(3cd + ah)x^9) dx \\ &= \frac{1}{4}a^3 dx^4 + \frac{1}{5}a^3 ex^5 + \frac{1}{6}a^3 fx^6 + \frac{1}{7}a^2(3bd + ag)x^7 + \frac{1}{8}a^2(3cd + ah)x^8 + \frac{1}{9}a^2(3bd + ag)x^9 + \frac{1}{10}a^2(3cd + ah)x^{10} \end{aligned}$$

Mathematica [A] time = 0.0480482, size = 223, normalized size = 1.05

$$\frac{1}{6}a^2x^6(af + 3bc) + \frac{1}{7}a^2x^7(ag + 3bd) + \frac{1}{8}a^2x^8(ah + 3be) + \frac{1}{3}a^3cx^3 + \frac{1}{4}a^3dx^4 + \frac{1}{5}a^3ex^5 + \frac{1}{12}b^2x^{12}(3af + bc) + \frac{1}{13}b^2x^{13}(3ag + bd)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5), x]
```

```
[Out] (a^3*c*x^3)/3 + (a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (a^2*(3*b*c + a*f)*x^6)/6 +
(a^2*(3*b*d + a*g)*x^7)/7 + (a^2*(3*b*e + a*h)*x^8)/8 + (a*b*(b*c + a*f)*x^9)/3 +
(3*a*b*(b*d + a*g)*x^10)/10 + (3*a*b*(b*e + a*h)*x^11)/11 + (b^2*(b*c + 3*a*f)*x^12)/12 +
(b^2*(b*d + 3*a*g)*x^13)/13 + (b^2*(b*e + 3*a*h)*x^14)/14 + (b^3*f*x^15)/15 +
(b^3*g*x^16)/16 + (b^3*h*x^17)/17
```

Maple [A] time = 0.001, size = 224, normalized size = 1.1

$$\frac{b^3hx^{17}}{17} + \frac{b^3gx^{16}}{16} + \frac{b^3fx^{15}}{15} + \frac{(3b^2ah + b^3e)x^{14}}{14} + \frac{(3b^2ag + b^3d)x^{13}}{13} + \frac{(3b^2af + b^3c)x^{12}}{12} + \frac{(3ba^2h + 3aeb^2)x^{11}}{11} + \frac{(3ba^2g + 3aebd)x^{10}}{10} + \frac{(3ba^2f + 3aebc)x^9}{9} + \frac{a^3cx^3}{3} + \frac{a^3dx^4}{4} + \frac{a^3ex^5}{5} + \frac{a^2(3bd + ag)x^6}{6} + \frac{a^2(3cd + ah)x^7}{7} + \frac{a^2(3bd + ag)x^8}{8} + \frac{a^2(3cd + ah)x^9}{9} + \frac{a^2(3bd + ag)x^{10}}{10} + \frac{a^2(3cd + ah)x^{11}}{11} + \frac{a^2(3bd + ag)x^{12}}{12} + \frac{a^2(3cd + ah)x^{13}}{13} + \frac{a^2(3bd + ag)x^{14}}{14} + \frac{a^2(3cd + ah)x^{15}}{15} + \frac{a^2(3bd + ag)x^{16}}{16} + \frac{a^2(3cd + ah)x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c), x)
```

[Out] $\frac{1}{17}b^3hx^{17} + \frac{1}{16}b^3gx^{16} + \frac{1}{15}b^3fx^{15} + \frac{1}{14}(3ab^2h + b^3e)x^{14} + \frac{1}{13}(3ab^2g + b^3d)x^{13} + \frac{1}{12}(3ab^2f + b^3c)x^{12} + \frac{1}{11}(3a^2bh + 3ab^2e)x^{11} + \frac{1}{10}(3a^2bg + 3ab^2d)x^{10} + \frac{1}{9}(3a^2bf + 3ab^2c)x^9 + \frac{1}{8}(a^3h + 3a^2be)x^8 + \frac{1}{7}(a^3g + 3a^2bd)x^7 + \frac{1}{6}(a^3f + 3a^2bc)x^6 + \frac{1}{5}a^3ex^5 + \frac{1}{4}a^3dx^4 + \frac{1}{3}a^3cx^3$

Maxima [A] time = 0.942734, size = 293, normalized size = 1.38

$$\frac{1}{17}b^3hx^{17} + \frac{1}{16}b^3gx^{16} + \frac{1}{15}b^3fx^{15} + \frac{1}{14}(b^3e + 3ab^2h)x^{14} + \frac{1}{13}(b^3d + 3ab^2g)x^{13} + \frac{1}{12}(b^3c + 3ab^2f)x^{12} + \frac{3}{11}(ab^2e +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima")

[Out] $\frac{1}{17}b^3hx^{17} + \frac{1}{16}b^3gx^{16} + \frac{1}{15}b^3fx^{15} + \frac{1}{14}(b^3e + 3ab^2h)x^{14} + \frac{1}{13}(b^3d + 3ab^2g)x^{13} + \frac{1}{12}(b^3c + 3ab^2f)x^{12} + \frac{3}{11}(ab^2e + a^2bh)x^{11} + \frac{3}{10}(ab^2d + a^2bg)x^{10} + \frac{1}{3}(ab^2c + a^2bf)x^9 + \frac{1}{5}a^3ex^5 + \frac{1}{8}(3a^2be + a^3h)x^8 + \frac{1}{4}a^3dx^4 + \frac{1}{7}(3a^2bd + a^3g)x^7 + \frac{1}{3}a^3cx^3 + \frac{1}{6}(3a^2bc + a^3f)x^6$

Fricas [A] time = 0.828252, size = 583, normalized size = 2.75

$$\frac{1}{17}x^{17}hb^3 + \frac{1}{16}x^{16}gb^3 + \frac{1}{15}x^{15}fb^3 + \frac{1}{14}x^{14}eb^3 + \frac{3}{14}x^{14}hb^2a + \frac{1}{13}x^{13}db^3 + \frac{3}{13}x^{13}gb^2a + \frac{1}{12}x^{12}cb^3 + \frac{1}{4}x^{12}fb^2a + \frac{3}{11}x^{11}e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")

[Out] $\frac{1}{17}x^{17}hb^3 + \frac{1}{16}x^{16}gb^3 + \frac{1}{15}x^{15}fb^3 + \frac{1}{14}x^{14}eb^3 + \frac{3}{14}x^{14}hb^2a + \frac{1}{13}x^{13}db^3 + \frac{3}{13}x^{13}gb^2a + \frac{1}{12}x^{12}cb^3 + \frac{1}{4}x^{12}fb^2a + \frac{3}{11}x^{11}eab^2 + \frac{3}{11}x^{11}hba^2 + \frac{3}{10}x^{10}dab^2 + \frac{3}{10}x^{10}gba^2 + \frac{1}{3}x^9cab^2 + \frac{1}{3}x^9fba^2 + \frac{3}{8}x^8eab^2 + \frac{1}{8}x^8hba^3 + \frac{3}{7}x^7dab^2 + \frac{1}{7}x^7gaa^3 + \frac{1}{2}x^6caba^2 + \frac{1}{6}x^6fab^3 + \frac{1}{5}x^5eaa^3 + \frac{1}{4}x^4daa^3 + \frac{1}{3}x^3caa^3$

Sympy [A] time = 0.098678, size = 246, normalized size = 1.16

$$\frac{a^3cx^3}{3} + \frac{a^3dx^4}{4} + \frac{a^3ex^5}{5} + \frac{b^3fx^{15}}{15} + \frac{b^3gx^{16}}{16} + \frac{b^3hx^{17}}{17} + x^{14} \left(\frac{3ab^2h}{14} + \frac{b^3e}{14} \right) + x^{13} \left(\frac{3ab^2g}{13} + \frac{b^3d}{13} \right) + x^{12} \left(\frac{ab^2f}{4} + \frac{b^3c}{12} \right) + x^{11} \left(\frac{3a^2bh}{11} + \frac{3a^2be}{11} \right) + x^{10} \left(\frac{3a^2bg}{10} + \frac{3a^2bd}{10} \right) + x^9 \left(\frac{a^2bf}{3} + \frac{a^2bc}{3} \right) + x^8 \left(\frac{a^3h}{8} + \frac{3a^2be}{8} \right) + x^7 \left(\frac{a^3g}{7} + \frac{3a^2bd}{7} \right) + x^6 \left(\frac{a^3f}{6} + \frac{a^2bc}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c), x)

[Out] a**3*c*x**3/3 + a**3*d*x**4/4 + a**3*e*x**5/5 + b**3*f*x**15/15 + b**3*g*x**16/16 + b**3*h*x**17/17 + x**14*(3*a*b**2*h/14 + b**3*e/14) + x**13*(3*a*b**2*g/13 + b**3*d/13) + x**12*(a*b**2*f/4 + b**3*c/12) + x**11*(3*a**2*b*h/11 + 3*a**2*b*e/11) + x**10*(3*a**2*b*g/10 + 3*a**2*b*d/10) + x**9*(a**2*b*f/3 + a**2*b*c/3) + x**8*(a**3*h/8 + 3*a**2*b*e/8) + x**7*(a**3*g/7 + 3*a**2*b*d/7) + x**6*(a**3*f/6 + a**2*b*c/2)

Giac [A] time = 1.07915, size = 315, normalized size = 1.49

$$\frac{1}{17} b^3 h x^{17} + \frac{1}{16} b^3 g x^{16} + \frac{1}{15} b^3 f x^{15} + \frac{3}{14} a b^2 h x^{14} + \frac{1}{14} b^3 x^{14} e + \frac{1}{13} b^3 d x^{13} + \frac{3}{13} a b^2 g x^{13} + \frac{1}{12} b^3 c x^{12} + \frac{1}{4} a b^2 f x^{12} + \frac{3}{11} a^2 b h x^{11} + \frac{3}{11} a^2 b e x^{11} + \frac{3}{10} a^2 b g x^{10} + \frac{1}{3} a^2 b d x^{10} + \frac{1}{3} a^2 b f x^9 + \frac{1}{8} a^3 h x^8 + \frac{3}{8} a^2 b x^8 e + \frac{3}{7} a^2 b d x^7 + \frac{1}{7} a^3 g x^7 + \frac{1}{2} a^2 b c x^6 + \frac{1}{6} a^3 f x^6 + \frac{1}{5} a^3 x^5 e + \frac{1}{4} a^3 d x^4 + \frac{1}{3} a^3 c x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c), x, algorithm="giac")

[Out] 1/17*b^3*h*x^17 + 1/16*b^3*g*x^16 + 1/15*b^3*f*x^15 + 3/14*a*b^2*h*x^14 + 1/14*b^3*x^14*e + 1/13*b^3*d*x^13 + 3/13*a*b^2*g*x^13 + 1/12*b^3*c*x^12 + 1/4*a*b^2*f*x^12 + 3/11*a^2*b*h*x^11 + 3/11*a*b^2*x^11*e + 3/10*a*b^2*d*x^10 + 3/10*a^2*b*g*x^10 + 1/3*a*b^2*c*x^9 + 1/3*a^2*b*f*x^9 + 1/8*a^3*h*x^8 + 3/8*a^2*b*x^8*e + 3/7*a^2*b*d*x^7 + 1/7*a^3*g*x^7 + 1/2*a^2*b*c*x^6 + 1/6*a^3*f*x^6 + 1/5*a^3*x^5*e + 1/4*a^3*d*x^4 + 1/3*a^3*c*x^3

$$3.396 \quad \int x (a + bx^3)^3 (c + dx + ex^2 + fx^3 + gx^4 + hx^5) dx$$

Optimal. Leaf size=212

$$\frac{1}{5}a^2x^5(af + 3bc) + \frac{1}{7}a^2x^7(ah + 3be) + \frac{1}{3}a^2bgx^9 + \frac{1}{2}a^3cx^2 + \frac{1}{4}a^3ex^4 + \frac{1}{6}a^3gx^6 + \frac{1}{11}b^2x^{11}(3af + bc) + \frac{1}{13}b^2x^{13}(3ah + be)$$

$$\begin{aligned} \text{[Out]} \quad & (a^3cx^2)/2 + (a^3ex^4)/4 + (a^2(3bc + af)x^5)/5 + (a^3gx^6)/6 + \\ & (a^2(3be + ah)x^7)/7 + (3ab(b^2c + a^2f)x^8)/8 + (a^2b^2gx^9)/3 + \\ & (3ab^2(b^2e + a^2h)x^{10})/10 + (b^2(b^2c + 3a^2f)x^{11})/11 + (ab^2g^2x^{12})/ \\ & 4 + (b^2(b^2e + 3a^2h)x^{13})/13 + (b^3f^2x^{14})/14 + (b^3g^2x^{15})/15 + (b^3h^2x^{16})/16 + \\ & (d(a + bx^3)^4)/(12b) \end{aligned}$$

Rubi [A] time = 0.177018, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1582, 1850}

$$\frac{1}{5}a^2x^5(af + 3bc) + \frac{1}{7}a^2x^7(ah + 3be) + \frac{1}{3}a^2bgx^9 + \frac{1}{2}a^3cx^2 + \frac{1}{4}a^3ex^4 + \frac{1}{6}a^3gx^6 + \frac{1}{11}b^2x^{11}(3af + bc) + \frac{1}{13}b^2x^{13}(3ah + be)$$

Antiderivative was successfully verified.

$$\text{[In]} \quad \text{Int}[x(a + bx^3)^3(c + dx + ex^2 + fx^3 + gx^4 + hx^5), x]$$

$$\begin{aligned} \text{[Out]} \quad & (a^3cx^2)/2 + (a^3ex^4)/4 + (a^2(3bc + af)x^5)/5 + (a^3gx^6)/6 + \\ & (a^2(3be + ah)x^7)/7 + (3ab(b^2c + a^2f)x^8)/8 + (a^2b^2gx^9)/3 + \\ & (3ab^2(b^2e + a^2h)x^{10})/10 + (b^2(b^2c + 3a^2f)x^{11})/11 + (ab^2g^2x^{12})/ \\ & 4 + (b^2(b^2e + 3a^2h)x^{13})/13 + (b^3f^2x^{14})/14 + (b^3g^2x^{15})/15 + (b^3h^2x^{16})/16 + \\ & (d(a + bx^3)^4)/(12b) \end{aligned}$$

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(Coeff[Px, x, n - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a + b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned} \int x(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)dx &= \frac{d(a+bx^3)^4}{12b} + \int (a+bx^3)^3(-dx^2+x(c+dx+ex^2+fx^3+gx^4 \\ &= \frac{d(a+bx^3)^4}{12b} + \int (a^3cx+a^3ex^3+a^2(3bc+af)x^4+a^3gx^5+a^2(3b \\ &= \frac{1}{5}a^3cx^2 + \frac{1}{4}a^3ex^4 + \frac{1}{5}a^2(3bc+af)x^5 + \frac{1}{6}a^3gx^6 + \frac{1}{7}a^2(3be+ah)x^7 \end{aligned}$$

Mathematica [A] time = 0.0334001, size = 223, normalized size = 1.05

$$\frac{1}{5}a^2x^5(af+3bc) + \frac{1}{6}a^2x^6(ag+3bd) + \frac{1}{7}a^2x^7(ah+3be) + \frac{1}{2}a^3cx^2 + \frac{1}{3}a^3dx^3 + \frac{1}{4}a^3ex^4 + \frac{1}{11}b^2x^{11}(3af+bc) + \frac{1}{12}b^2x^{12}(3a$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5), x]
```

```
[Out] (a^3*c*x^2)/2 + (a^3*d*x^3)/3 + (a^3*e*x^4)/4 + (a^2*(3*b*c + a*f)*x^5)/5 +
(a^2*(3*b*d + a*g)*x^6)/6 + (a^2*(3*b*e + a*h)*x^7)/7 + (3*a*b*(b*c + a*f)
*x^8)/8 + (a*b*(b*d + a*g)*x^9)/3 + (3*a*b*(b*e + a*h)*x^10)/10 + (b^2*(b*c
+ 3*a*f)*x^11)/11 + (b^2*(b*d + 3*a*g)*x^12)/12 + (b^2*(b*e + 3*a*h)*x^13)
/13 + (b^3*f*x^14)/14 + (b^3*g*x^15)/15 + (b^3*h*x^16)/16
```

Maple [A] time = 0., size = 224, normalized size = 1.1

$$\frac{b^3hx^{16}}{16} + \frac{b^3gx^{15}}{15} + \frac{b^3fx^{14}}{14} + \frac{(3b^2ah+b^3e)x^{13}}{13} + \frac{(3b^2ag+b^3d)x^{12}}{12} + \frac{(3b^2af+b^3c)x^{11}}{11} + \frac{(3ba^2h+3aeb^2)x^{10}}{10} + \frac{(3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c), x)
```

[Out] $\frac{1}{16}b^3hx^{16} + \frac{1}{15}b^3gx^{15} + \frac{1}{14}b^3fx^{14} + \frac{1}{13}(3ab^2h + b^3e)x^{13} + \frac{1}{12}(3ab^2g + b^3d)x^{12} + \frac{1}{11}(3ab^2f + b^3c)x^{11} + \frac{1}{10}(3a^2bh + 3ab^2e)x^{10} + \frac{1}{9}(3a^2bg + 3ab^2d)x^9 + \frac{1}{8}(3a^2bf + 3ab^2c)x^8 + \frac{1}{7}(a^3h + 3a^2be)x^7 + \frac{1}{6}(a^3g + 3a^2bd)x^6 + \frac{1}{5}(a^3f + 3a^2bc)x^5 + \frac{1}{4}a^3ex^4 + \frac{1}{3}a^3dx^3 + \frac{1}{2}a^3cx^2$

Maxima [A] time = 0.954929, size = 293, normalized size = 1.38

$$\frac{1}{16}b^3hx^{16} + \frac{1}{15}b^3gx^{15} + \frac{1}{14}b^3fx^{14} + \frac{1}{13}(b^3e + 3ab^2h)x^{13} + \frac{1}{12}(b^3d + 3ab^2g)x^{12} + \frac{1}{11}(b^3c + 3ab^2f)x^{11} + \frac{3}{10}(ab^2e +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima")

[Out] $\frac{1}{16}b^3hx^{16} + \frac{1}{15}b^3gx^{15} + \frac{1}{14}b^3fx^{14} + \frac{1}{13}(b^3e + 3ab^2h)x^{13} + \frac{1}{12}(b^3d + 3ab^2g)x^{12} + \frac{1}{11}(b^3c + 3ab^2f)x^{11} + \frac{3}{10}(ab^2e + a^2bh)x^{10} + \frac{1}{3}(ab^2d + a^2bg)x^9 + \frac{3}{8}(ab^2c + a^2bf)x^8 + \frac{1}{4}a^3ex^4 + \frac{1}{7}(3a^2be + a^3h)x^7 + \frac{1}{3}a^3dx^3 + \frac{1}{6}(3a^2bd + a^3g)x^6 + \frac{1}{2}a^3cx^2 + \frac{1}{5}(3a^2bc + a^3f)x^5$

Fricas [A] time = 0.876503, size = 578, normalized size = 2.73

$$\frac{1}{16}x^{16}hb^3 + \frac{1}{15}x^{15}gb^3 + \frac{1}{14}x^{14}fb^3 + \frac{1}{13}x^{13}eb^3 + \frac{3}{13}x^{13}hb^2a + \frac{1}{12}x^{12}db^3 + \frac{1}{4}x^{12}gb^2a + \frac{1}{11}x^{11}cb^3 + \frac{3}{11}x^{11}fb^2a + \frac{3}{10}x^{10}e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")

[Out] $\frac{1}{16}x^{16}hb^3 + \frac{1}{15}x^{15}gb^3 + \frac{1}{14}x^{14}fb^3 + \frac{1}{13}x^{13}eb^3 + \frac{3}{13}x^{13}hb^2a + \frac{1}{12}x^{12}db^3 + \frac{1}{4}x^{12}gb^2a + \frac{1}{11}x^{11}cb^3 + \frac{3}{11}x^{11}fb^2a + \frac{3}{10}x^{10}eab^2 + \frac{3}{10}x^{10}hba^2 + \frac{1}{3}x^9db^2a + \frac{1}{3}x^9gba^2 + \frac{3}{8}x^8c b^2a + \frac{3}{8}x^8fba^2 + \frac{3}{7}x^7e b a^2 + \frac{1}{7}x^7h a^3 + \frac{1}{2}x^6d b a^2 + \frac{1}{6}x^6g a^3 + \frac{3}{5}x^5c b a^2 + \frac{1}{5}x^5f a^3 + \frac{1}{4}x^4e a^3 + \frac{1}{3}x^3d a^3 + \frac{1}{2}x^2c a^3$

Sympy [A] time = 0.099559, size = 246, normalized size = 1.16

$$\frac{a^3cx^2}{2} + \frac{a^3dx^3}{3} + \frac{a^3ex^4}{4} + \frac{b^3fx^{14}}{14} + \frac{b^3gx^{15}}{15} + \frac{b^3hx^{16}}{16} + x^{13} \left(\frac{3ab^2h}{13} + \frac{b^3e}{13} \right) + x^{12} \left(\frac{ab^2g}{4} + \frac{b^3d}{12} \right) + x^{11} \left(\frac{3ab^2f}{11} + \frac{b^3c}{11} \right) + x^{10} \left(\frac{3a^2bh}{10} + \frac{3a^2be}{10} \right) + x^9 \left(\frac{a^2bg}{3} + \frac{a^2bd}{3} \right) + x^8 \left(\frac{3a^2bf}{8} + \frac{3a^2bc}{8} \right) + x^7 \left(\frac{a^3h}{7} + \frac{3a^2be}{7} \right) + x^6 \left(\frac{a^3g}{6} + \frac{a^2bd}{2} \right) + x^5 \left(\frac{a^3f}{5} + \frac{3a^2bc}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c), x)

[Out] a**3*c*x**2/2 + a**3*d*x**3/3 + a**3*e*x**4/4 + b**3*f*x**14/14 + b**3*g*x**15/15 + b**3*h*x**16/16 + x**13*(3*a*b**2*h/13 + b**3*e/13) + x**12*(a*b**2*g/4 + b**3*d/12) + x**11*(3*a*b**2*f/11 + b**3*c/11) + x**10*(3*a**2*b*h/10 + 3*a**2*b*e/10) + x**9*(a**2*b*g/3 + a**2*b*d/3) + x**8*(3*a**2*b*f/8 + 3*a**2*b*c/8) + x**7*(a**3*h/7 + 3*a**2*b*e/7) + x**6*(a**3*g/6 + a**2*b*d/2) + x**5*(a**3*f/5 + 3*a**2*b*c/5)

Giac [A] time = 1.0781, size = 315, normalized size = 1.49

$$\frac{1}{16} b^3 h x^{16} + \frac{1}{15} b^3 g x^{15} + \frac{1}{14} b^3 f x^{14} + \frac{3}{13} a b^2 h x^{13} + \frac{1}{13} b^3 x^{13} e + \frac{1}{12} b^3 d x^{12} + \frac{1}{4} a b^2 g x^{12} + \frac{1}{11} b^3 c x^{11} + \frac{3}{11} a b^2 f x^{11} + \frac{3}{10} a^2 b h x^{10} + \frac{3}{10} a^2 b e x^{10} + \frac{1}{3} a^2 b g x^9 + \frac{3}{8} a^2 b c x^8 + \frac{3}{8} a^2 b f x^8 + \frac{1}{7} a^3 h x^7 + \frac{3}{7} a^2 b x^7 e + \frac{1}{2} a^2 b d x^6 + \frac{1}{6} a^3 g x^6 + \frac{3}{5} a^2 b c x^5 + \frac{1}{5} a^3 f x^5 + \frac{1}{4} a^3 x^4 e + \frac{1}{3} a^3 d x^3 + \frac{1}{2} a^3 c x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c), x, algorithm="giac")

[Out] 1/16*b^3*h*x^16 + 1/15*b^3*g*x^15 + 1/14*b^3*f*x^14 + 3/13*a*b^2*h*x^13 + 1/13*b^3*x^13*e + 1/12*b^3*d*x^12 + 1/4*a*b^2*g*x^12 + 1/11*b^3*c*x^11 + 3/11*a*b^2*f*x^11 + 3/10*a^2*b*h*x^10 + 3/10*a^2*b*e*x^10 + 1/3*a^2*b*g*x^9 + 1/3*a^2*b*c*x^8 + 3/8*a^2*b*f*x^8 + 1/7*a^3*h*x^7 + 3/7*a^2*b*x^7*e + 1/2*a^2*b*d*x^6 + 1/6*a^3*g*x^6 + 3/5*a^2*b*c*x^5 + 1/5*a^3*f*x^5 + 1/4*a^3*x^4*e + 1/3*a^3*d*x^3 + 1/2*a^3*c*x^2

$$3.397 \quad \int (a + bx^3)^3 (c + dx + ex^2 + fx^3 + gx^4 + hx^5) dx$$

Optimal. Leaf size=207

$$\frac{1}{4}a^2x^4(af + 3bc) + \frac{1}{5}a^2x^5(ag + 3bd) + \frac{1}{3}a^2bhx^9 + a^3cx + \frac{1}{2}a^3dx^2 + \frac{1}{6}a^3hx^6 + \frac{1}{10}b^2x^{10}(3af + bc) + \frac{1}{11}b^2x^{11}(3ag + bd)$$

[Out] $a^3cx + (a^3d*x^2)/2 + (a^2*(3*b*c + a*f)*x^4)/4 + (a^2*(3*b*d + a*g)*x^5)/5 + (a^3*h*x^6)/6 + (3*a*b*(b*c + a*f)*x^7)/7 + (3*a*b*(b*d + a*g)*x^8)/8 + (a^2*b*h*x^9)/3 + (b^2*(b*c + 3*a*f)*x^{10})/10 + (b^2*(b*d + 3*a*g)*x^{11})/11 + (a*b^2*h*x^{12})/4 + (b^3*f*x^{13})/13 + (b^3*g*x^{14})/14 + (b^3*h*x^{15})/15 + (e*(a + b*x^3)^4)/(12*b)$

Rubi [A] time = 0.177319, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1582, 1850}

$$\frac{1}{4}a^2x^4(af + 3bc) + \frac{1}{5}a^2x^5(ag + 3bd) + \frac{1}{3}a^2bhx^9 + a^3cx + \frac{1}{2}a^3dx^2 + \frac{1}{6}a^3hx^6 + \frac{1}{10}b^2x^{10}(3af + bc) + \frac{1}{11}b^2x^{11}(3ag + bd)$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]

[Out] $a^3cx + (a^3d*x^2)/2 + (a^2*(3*b*c + a*f)*x^4)/4 + (a^2*(3*b*d + a*g)*x^5)/5 + (a^3*h*x^6)/6 + (3*a*b*(b*c + a*f)*x^7)/7 + (3*a*b*(b*d + a*g)*x^8)/8 + (a^2*b*h*x^9)/3 + (b^2*(b*c + 3*a*f)*x^{10})/10 + (b^2*(b*d + 3*a*g)*x^{11})/11 + (a*b^2*h*x^{12})/4 + (b^3*f*x^{13})/13 + (b^3*g*x^{14})/14 + (b^3*h*x^{15})/15 + (e*(a + b*x^3)^4)/(12*b)$

Rule 1582

Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(Coeff[Px, x, n - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a + b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]

Rule 1850

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned} \int (a + bx^3)^3 (c + dx + ex^2 + fx^3 + gx^4 + hx^5) dx &= \frac{e(a + bx^3)^4}{12b} + \int (a + bx^3)^3 (c + dx + fx^3 + gx^4 + hx^5) dx \\ &= \frac{e(a + bx^3)^4}{12b} + \int (a^3c + a^3dx + a^2(3bc + af)x^3 + a^2(3bd + ag)x^4 + \\ &= a^3cx + \frac{1}{2}a^3dx^2 + \frac{1}{4}a^2(3bc + af)x^4 + \frac{1}{5}a^2(3bd + ag)x^5 + \frac{1}{6}a^3hx^6 + \frac{3}{7} \end{aligned}$$

Mathematica [A] time = 0.101428, size = 170, normalized size = 0.82

$$x(143a^2bx^3(630c + x(504d + 5x(84e + x(72f + 7x(9g + 8hx)))))) + 2002a^3(60c + x(30d + x(20e + 15fx + 12gx^2 + 10hx^3)))$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (x*(13*a*b^2*x^6*(3960*c + 7*x*(495*d + 440*e*x + 6*x^2*(66*f + 60*g*x + 55
*h*x^2))) + 2002*a^3*(60*c + x*(30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x
^3))) + 2*b^3*x^9*(6006*c + x*(5460*d + 11*x*(455*e + 420*f*x + 390*g*x^2 +
364*h*x^3))) + 143*a^2*b*x^3*(630*c + x*(504*d + 5*x*(84*e + x*(72*f + 7*x
*(9*g + 8*h*x)))))))/120120
```

Maple [A] time = 0.001, size = 221, normalized size = 1.1

$$\frac{b^3hx^{15}}{15} + \frac{b^3gx^{14}}{14} + \frac{b^3fx^{13}}{13} + \frac{(3b^2ah + b^3e)x^{12}}{12} + \frac{(3b^2ag + b^3d)x^{11}}{11} + \frac{(3b^2af + b^3c)x^{10}}{10} + \frac{(3ba^2h + 3aeb^2)x^9}{9} + \frac{(3ba^2g + 3aeb^2)x^8}{8} + \frac{(3ba^2f + 3aeb^2)x^7}{7} + \frac{(3ba^2d + 3aeb^2)x^6}{6} + \frac{(3ba^2c + 3aeb^2)x^5}{5} + \frac{(3ba^2e + 3aeb^2)x^4}{4} + \frac{(3ba^2f + 3aeb^2)x^3}{3} + \frac{(3ba^2g + 3aeb^2)x^2}{2} + \frac{(3ba^2h + 3aeb^2)x}{1} + \frac{(3ba^2i + 3aeb^2)}{0}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/15*b^3*h*x^15+1/14*b^3*g*x^14+1/13*b^3*f*x^13+1/12*(3*a*b^2*h+b^3*e)*x^12
+1/11*(3*a*b^2*g+b^3*d)*x^11+1/10*(3*a*b^2*f+b^3*c)*x^10+1/9*(3*a^2*b*h+3*a
*b^2*e)*x^9+1/8*(3*a^2*b*g+3*a*b^2*d)*x^8+1/7*(3*a^2*b*f+3*a*b^2*c)*x^7+1/6
*(a^3*h+3*a^2*b*e)*x^6+1/5*(a^3*g+3*a^2*b*d)*x^5+1/4*(a^3*f+3*a^2*b*c)*x^4+
1/3*a^3*e*x^3+1/2*a^3*d*x^2+a^3*c*x
```

Maxima [A] time = 0.948258, size = 289, normalized size = 1.4

$$\frac{1}{15} b^3 h x^{15} + \frac{1}{14} b^3 g x^{14} + \frac{1}{13} b^3 f x^{13} + \frac{1}{12} (b^3 e + 3 a b^2 h) x^{12} + \frac{1}{11} (b^3 d + 3 a b^2 g) x^{11} + \frac{1}{10} (b^3 c + 3 a b^2 f) x^{10} + \frac{1}{3} (a b^2 e + a^2 b h) x^9 + \frac{3}{8} (a b^2 d + a^2 b g) x^8 + \frac{3}{7} (a b^2 c + a^2 b f) x^7 + \frac{1}{3} a^3 e x^3 + \frac{1}{6} (3 a^2 b e + a^3 h) x^6 + \frac{1}{2} a^3 d x^2 + \frac{1}{5} (3 a^2 b d + a^3 g) x^5 + a^3 c x + \frac{1}{4} (3 a^2 b c + a^3 f) x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima"
)
```

```
[Out] 1/15*b^3*h*x^15 + 1/14*b^3*g*x^14 + 1/13*b^3*f*x^13 + 1/12*(b^3*e + 3*a*b^2
*h)*x^12 + 1/11*(b^3*d + 3*a*b^2*g)*x^11 + 1/10*(b^3*c + 3*a*b^2*f)*x^10 +
1/3*(a*b^2*e + a^2*b*h)*x^9 + 3/8*(a*b^2*d + a^2*b*g)*x^8 + 3/7*(a*b^2*c +
a^2*b*f)*x^7 + 1/3*a^3*e*x^3 + 1/6*(3*a^2*b*e + a^3*h)*x^6 + 1/2*a^3*d*x^2
+ 1/5*(3*a^2*b*d + a^3*g)*x^5 + a^3*c*x + 1/4*(3*a^2*b*c + a^3*f)*x^4
```

Fricas [A] time = 0.832962, size = 564, normalized size = 2.72

$$\frac{1}{15} x^{15} h b^3 + \frac{1}{14} x^{14} g b^3 + \frac{1}{13} x^{13} f b^3 + \frac{1}{12} x^{12} e b^3 + \frac{1}{4} x^{12} h b^2 a + \frac{1}{11} x^{11} d b^3 + \frac{3}{11} x^{11} g b^2 a + \frac{1}{10} x^{10} c b^3 + \frac{3}{10} x^{10} f b^2 a + \frac{1}{3} x^9 e b^2 a + \frac{1}{3} x^9 h b a^2 + \frac{3}{8} x^8 d b^2 a + \frac{3}{8} x^8 g b a^2 + \frac{3}{7} x^7 c b^2 a + \frac{3}{7} x^7 f b a^2 + \frac{1}{2} x^6 e b a^2 + \frac{1}{6} x^6 h a^3 + \frac{3}{5} x^5 d b a^2 + \frac{1}{5} x^5 g a^3 + \frac{3}{4} x^4 c b a^2 + \frac{1}{4} x^4 f a^3 + \frac{1}{3} x^3 e a^3 + \frac{1}{2} x^2 d a^3 + x c a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas"
)
```

```
[Out] 1/15*x^15*h*b^3 + 1/14*x^14*g*b^3 + 1/13*x^13*f*b^3 + 1/12*x^12*e*b^3 + 1/4
*x^12*h*b^2*a + 1/11*x^11*d*b^3 + 3/11*x^11*g*b^2*a + 1/10*x^10*c*b^3 + 3/1
0*x^10*f*b^2*a + 1/3*x^9*e*b^2*a + 1/3*x^9*h*b*a^2 + 3/8*x^8*d*b^2*a + 3/8*
x^8*g*b*a^2 + 3/7*x^7*c*b^2*a + 3/7*x^7*f*b*a^2 + 1/2*x^6*e*b*a^2 + 1/6*x^6
*h*a^3 + 3/5*x^5*d*b*a^2 + 1/5*x^5*g*a^3 + 3/4*x^4*c*b*a^2 + 1/4*x^4*f*a^3
+ 1/3*x^3*e*a^3 + 1/2*x^2*d*a^3 + x*c*a^3
```

Sympy [A] time = 0.095226, size = 243, normalized size = 1.17

$$a^3cx + \frac{a^3dx^2}{2} + \frac{a^3ex^3}{3} + \frac{b^3fx^{13}}{13} + \frac{b^3gx^{14}}{14} + \frac{b^3hx^{15}}{15} + x^{12} \left(\frac{ab^2h}{4} + \frac{b^3e}{12} \right) + x^{11} \left(\frac{3ab^2g}{11} + \frac{b^3d}{11} \right) + x^{10} \left(\frac{3ab^2f}{10} + \frac{b^3c}{10} \right) + x^9$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c), x)

[Out] a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + b**3*f*x**13/13 + b**3*g*x**14/14 + b**3*h*x**15/15 + x**12*(a*b**2*h/4 + b**3*e/12) + x**11*(3*a*b**2*g/11 + b**3*d/11) + x**10*(3*a*b**2*f/10 + b**3*c/10) + x**9*(a**2*b*h/3 + a*b**2*e/3) + x**8*(3*a**2*b*g/8 + 3*a*b**2*d/8) + x**7*(3*a**2*b*f/7 + 3*a*b**2*c/7) + x**6*(a**3*h/6 + a**2*b*e/2) + x**5*(a**3*g/5 + 3*a**2*b*d/5) + x**4*(a**3*f/4 + 3*a**2*b*c/4)

Giac [A] time = 1.06531, size = 311, normalized size = 1.5

$$\frac{1}{15} b^3 h x^{15} + \frac{1}{14} b^3 g x^{14} + \frac{1}{13} b^3 f x^{13} + \frac{1}{4} a b^2 h x^{12} + \frac{1}{12} b^3 x^{12} e + \frac{1}{11} b^3 d x^{11} + \frac{3}{11} a b^2 g x^{11} + \frac{1}{10} b^3 c x^{10} + \frac{3}{10} a b^2 f x^{10} + \frac{1}{3} a^2 b^2 h x^9$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c), x, algorithm="giac")

[Out] 1/15*b^3*h*x^15 + 1/14*b^3*g*x^14 + 1/13*b^3*f*x^13 + 1/4*a*b^2*h*x^12 + 1/12*b^3*x^12*e + 1/11*b^3*d*x^11 + 3/11*a*b^2*g*x^11 + 1/10*b^3*c*x^10 + 3/10*a*b^2*f*x^10 + 1/3*a^2*b*h*x^9 + 1/3*a*b^2*x^9*e + 3/8*a*b^2*d*x^8 + 3/8*a^2*b*g*x^8 + 3/7*a*b^2*c*x^7 + 3/7*a^2*b*f*x^7 + 1/6*a^3*h*x^6 + 1/2*a^2*b*x^6*e + 3/5*a^2*b*d*x^5 + 1/5*a^3*g*x^5 + 3/4*a^2*b*c*x^4 + 1/4*a^3*f*x^4 + 1/3*a^3*x^3*e + 1/2*a^3*d*x^2 + a^3*c*x

$$3.398 \quad \int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x} dx$$

Optimal. Leaf size=200

$$a^2bcx^3 + \frac{1}{4}a^2x^4(ag + 3bd) + \frac{1}{5}a^2x^5(ah + 3be) + a^3c \log(x) + a^3dx + \frac{1}{2}a^3ex^2 + \frac{1}{2}ab^2cx^6 + \frac{1}{10}b^2x^{10}(3ag + bd) + \frac{1}{11}b^2x^{11}$$

[Out] $a^3d*x + (a^3e*x^2)/2 + a^2*b*c*x^3 + (a^2*(3*b*d + a*g)*x^4)/4 + (a^2*(3*b*e + a*h)*x^5)/5 + (a*b^2*c*x^6)/2 + (3*a*b*(b*d + a*g)*x^7)/7 + (3*a*b*(b*e + a*h)*x^8)/8 + (b^3*c*x^9)/9 + (b^2*(b*d + 3*a*g)*x^{10})/10 + (b^2*(b*e + 3*a*h)*x^{11})/11 + (b^3*g*x^{13})/13 + (b^3*h*x^{14})/14 + (f*(a + b*x^3)^4)/(12*b) + a^3*c*Log[x]$

Rubi [A] time = 0.145609, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1583, 1820}

$$a^2bcx^3 + \frac{1}{4}a^2x^4(ag + 3bd) + \frac{1}{5}a^2x^5(ah + 3be) + a^3c \log(x) + a^3dx + \frac{1}{2}a^3ex^2 + \frac{1}{2}ab^2cx^6 + \frac{1}{10}b^2x^{10}(3ag + bd) + \frac{1}{11}b^2x^{11}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]

[Out] $a^3d*x + (a^3e*x^2)/2 + a^2*b*c*x^3 + (a^2*(3*b*d + a*g)*x^4)/4 + (a^2*(3*b*e + a*h)*x^5)/5 + (a*b^2*c*x^6)/2 + (3*a*b*(b*d + a*g)*x^7)/7 + (3*a*b*(b*e + a*h)*x^8)/8 + (b^3*c*x^9)/9 + (b^2*(b*d + 3*a*g)*x^{10})/10 + (b^2*(b*e + 3*a*h)*x^{11})/11 + (b^3*g*x^{13})/13 + (b^3*h*x^{14})/14 + (f*(a + b*x^3)^4)/(12*b) + a^3*c*Log[x]$

Rule 1583

Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(Coeff[Px, x, n - m - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x, n - m - 1], 0]

Rule 1820

```
Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)^3 (c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{x} dx &= \frac{f(a + bx^3)^4}{12b} + \int \frac{(a + bx^3)^3 (c + dx + ex^2 + gx^4 + hx^5)}{x} dx \\ &= \frac{f(a + bx^3)^4}{12b} + \int \left(a^3d + \frac{a^3c}{x} + a^3ex + 3a^2bcx^2 + a^2(3bd + ag)x^3 + \right. \\ &\quad \left. + a^3dx + \frac{1}{2}a^3ex^2 + a^2bcx^3 + \frac{1}{4}a^2(3bd + ag)x^4 + \frac{1}{5}a^2(3be + ah)x^5 + \frac{1}{2} \right) dx \end{aligned}$$

Mathematica [A] time = 0.0777477, size = 214, normalized size = 1.07

$$\frac{1}{3}a^2x^3(af + 3bc) + \frac{1}{4}a^2x^4(ag + 3bd) + \frac{1}{5}a^2x^5(ah + 3be) + a^3c \log(x) + a^3dx + \frac{1}{2}a^3ex^2 + \frac{1}{9}b^2x^9(3af + bc) + \frac{1}{10}b^2x^{10}(3ag$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

```
[Out] a^3*d*x + (a^3*e*x^2)/2 + (a^2*(3*b*c + a*f)*x^3)/3 + (a^2*(3*b*d + a*g)*x^4)/4 + (a^2*(3*b*e + a*h)*x^5)/5 + (a*b*(b*c + a*f)*x^6)/2 + (3*a*b*(b*d + a*g)*x^7)/7 + (3*a*b*(b*e + a*h)*x^8)/8 + (b^2*(b*c + 3*a*f)*x^9)/9 + (b^2*(b*d + 3*a*g)*x^10)/10 + (b^2*(b*e + 3*a*h)*x^11)/11 + (b^3*f*x^12)/12 + (b^3*g*x^13)/13 + (b^3*h*x^14)/14 + a^3*c*Log[x]
```

Maple [A] time = 0.004, size = 224, normalized size = 1.1

$$\frac{b^3hx^{14}}{14} + \frac{b^3gx^{13}}{13} + \frac{fx^{12}b^3}{12} + \frac{3x^{11}ab^2h}{11} + \frac{b^3ex^{11}}{11} + \frac{3x^{10}ab^2g}{10} + \frac{b^3dx^{10}}{10} + \frac{x^9ab^2f}{3} + \frac{b^3cx^9}{9} + \frac{3x^8a^2bh}{8} + \frac{3ab^2ex^8}{8} + \frac{3x^7a^3c}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x)
```

[Out] $1/14*b^3*h*x^{14}+1/13*b^3*g*x^{13}+1/12*f*x^{12}*b^3+3/11*x^{11}*a*b^2*h+1/11*b^3*e*x^{11}+3/10*x^{10}*a*b^2*g+1/10*b^3*d*x^{10}+1/3*x^9*a*b^2*f+1/9*b^3*c*x^9+3/8*x^8*a^2*b*h+3/8*a*b^2*e*x^8+3/7*x^7*a^2*b*g+3/7*a*b^2*d*x^7+1/2*x^6*a^2*b*f+1/2*a*b^2*c*x^6+1/5*x^5*a^3*h+3/5*a^2*b*e*x^5+1/4*x^4*a^3*g+3/4*a^2*b*d*x^4+1/3*a^3*f*x^3+a^2*b*c*x^3+1/2*a^3*e*x^2+a^3*d*x+a^3*c*\ln(x)$

Maxima [A] time = 0.960838, size = 286, normalized size = 1.43

$$\frac{1}{14} b^3 h x^{14} + \frac{1}{13} b^3 g x^{13} + \frac{1}{12} b^3 f x^{12} + \frac{1}{11} (b^3 e + 3 a b^2 h) x^{11} + \frac{1}{10} (b^3 d + 3 a b^2 g) x^{10} + \frac{1}{9} (b^3 c + 3 a b^2 f) x^9 + \frac{3}{8} (a b^2 e + a^2 b h) x^8 + \frac{3}{8} (a b^2 e + a^2 b h) x^8 + \frac{3}{7} (a b^2 d + a^2 b g) x^7 + \frac{1}{2} (a b^2 c + a^2 b f) x^6 + \frac{1}{2} a^3 e x^2 + \frac{1}{5} (3 a^2 b e + a^3 h) x^5 + a^3 d x + \frac{1}{4} (3 a^2 b d + a^3 g) x^4 + a^3 c \log(x) + \frac{1}{3} (3 a^2 b c + a^3 f) x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="maxima")

[Out] $1/14*b^3*h*x^{14} + 1/13*b^3*g*x^{13} + 1/12*b^3*f*x^{12} + 1/11*(b^3*e + 3*a*b^2*h)*x^{11} + 1/10*(b^3*d + 3*a*b^2*g)*x^{10} + 1/9*(b^3*c + 3*a*b^2*f)*x^9 + 3/8*(a*b^2*e + a^2*b*h)*x^8 + 3/7*(a*b^2*d + a^2*b*g)*x^7 + 1/2*(a*b^2*c + a^2*b*f)*x^6 + 1/2*a^3*e*x^2 + 1/5*(3*a^2*b*e + a^3*h)*x^5 + a^3*d*x + 1/4*(3*a^2*b*d + a^3*g)*x^4 + a^3*c*\log(x) + 1/3*(3*a^2*b*c + a^3*f)*x^3$

Fricas [A] time = 0.997019, size = 497, normalized size = 2.48

$$\frac{1}{14} b^3 h x^{14} + \frac{1}{13} b^3 g x^{13} + \frac{1}{12} b^3 f x^{12} + \frac{1}{11} (b^3 e + 3 a b^2 h) x^{11} + \frac{1}{10} (b^3 d + 3 a b^2 g) x^{10} + \frac{1}{9} (b^3 c + 3 a b^2 f) x^9 + \frac{3}{8} (a b^2 e + a^2 b h) x^8 + \frac{3}{8} (a b^2 e + a^2 b h) x^8 + \frac{3}{7} (a b^2 d + a^2 b g) x^7 + \frac{1}{2} (a b^2 c + a^2 b f) x^6 + \frac{1}{2} a^3 e x^2 + \frac{1}{5} (3 a^2 b e + a^3 h) x^5 + a^3 d x + \frac{1}{4} (3 a^2 b d + a^3 g) x^4 + a^3 c \log(x) + \frac{1}{3} (3 a^2 b c + a^3 f) x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="fricas")

[Out] $1/14*b^3*h*x^{14} + 1/13*b^3*g*x^{13} + 1/12*b^3*f*x^{12} + 1/11*(b^3*e + 3*a*b^2*h)*x^{11} + 1/10*(b^3*d + 3*a*b^2*g)*x^{10} + 1/9*(b^3*c + 3*a*b^2*f)*x^9 + 3/8*(a*b^2*e + a^2*b*h)*x^8 + 3/7*(a*b^2*d + a^2*b*g)*x^7 + 1/2*(a*b^2*c + a^2*b*f)*x^6 + 1/2*a^3*e*x^2 + 1/5*(3*a^2*b*e + a^3*h)*x^5 + a^3*d*x + 1/4*(3*a^2*b*d + a^3*g)*x^4 + a^3*c*\log(x) + 1/3*(3*a^2*b*c + a^3*f)*x^3$

Sympy [A] time = 0.604787, size = 240, normalized size = 1.2

$$a^3c \log(x) + a^3dx + \frac{a^3ex^2}{2} + \frac{b^3fx^{12}}{12} + \frac{b^3gx^{13}}{13} + \frac{b^3hx^{14}}{14} + x^{11} \left(\frac{3ab^2h}{11} + \frac{b^3e}{11} \right) + x^{10} \left(\frac{3ab^2g}{10} + \frac{b^3d}{10} \right) + x^9 \left(\frac{ab^2f}{3} + \frac{b^3c}{9} \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x,x)

[Out] a**3*c*log(x) + a**3*d*x + a**3*e*x**2/2 + b**3*f*x**12/12 + b**3*g*x**13/13 + b**3*h*x**14/14 + x**11*(3*a*b**2*h/11 + b**3*e/11) + x**10*(3*a*b**2*g/10 + b**3*d/10) + x**9*(a*b**2*f/3 + b**3*c/9) + x**8*(3*a**2*b*h/8 + 3*a*b**2*e/8) + x**7*(3*a**2*b*g/7 + 3*a*b**2*d/7) + x**6*(a**2*b*f/2 + a*b**2*c/2) + x**5*(a**3*h/5 + 3*a**2*b*e/5) + x**4*(a**3*g/4 + 3*a**2*b*d/4) + x**3*(a**3*f/3 + a**2*b*c)

Giac [A] time = 1.06867, size = 308, normalized size = 1.54

$$\frac{1}{14} b^3 h x^{14} + \frac{1}{13} b^3 g x^{13} + \frac{1}{12} b^3 f x^{12} + \frac{3}{11} a b^2 h x^{11} + \frac{1}{11} b^3 x^{11} e + \frac{1}{10} b^3 d x^{10} + \frac{3}{10} a b^2 g x^{10} + \frac{1}{9} b^3 c x^9 + \frac{1}{3} a b^2 f x^9 + \frac{3}{8} a^2 b h x^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="giac")

[Out] 1/14*b^3*h*x^14 + 1/13*b^3*g*x^13 + 1/12*b^3*f*x^12 + 3/11*a*b^2*h*x^11 + 1/11*b^3*x^11*e + 1/10*b^3*d*x^10 + 3/10*a*b^2*g*x^10 + 1/9*b^3*c*x^9 + 1/3*a*b^2*f*x^9 + 3/8*a^2*b*h*x^8 + 3/8*a*b^2*x^8*e + 3/7*a*b^2*d*x^7 + 3/7*a^2*b*g*x^7 + 1/2*a*b^2*c*x^6 + 1/2*a^2*b*f*x^6 + 1/5*a^3*h*x^5 + 3/5*a^2*b*x^5*e + 3/4*a^2*b*d*x^4 + 1/4*a^3*g*x^4 + a^2*b*c*x^3 + 1/3*a^3*f*x^3 + 1/2*a^3*x^2*e + a^3*d*x + a^3*c*log(abs(x))

$$3.399 \quad \int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2} dx$$

Optimal. Leaf size=198

$$\frac{1}{2}a^2x^2(af+3bc)+a^2bdx^3+\frac{1}{4}a^2x^4(ah+3be)-\frac{a^3c}{x}+a^3d\log(x)+a^3ex+\frac{1}{8}b^2x^8(3af+bc)+\frac{1}{2}ab^2dx^6+\frac{1}{10}b^2x^{10}(3ah$$

[Out] $-\frac{a^3c}{x}+a^3e*x+(a^2*(3*b*c+a*f)*x^2)/2+a^2*b*d*x^3+(a^2*(3*b*e+a*h)*x^4)/4+(3*a*b*(b*c+a*f)*x^5)/5+(a*b^2*d*x^6)/2+(3*a*b*(b*e+a*h)*x^7)/7+(b^2*(b*c+3*a*f)*x^8)/8+(b^3*d*x^9)/9+(b^2*(b*e+3*a*h)*x^{10})/10+(b^3*f*x^{11})/11+(b^3*h*x^{13})/13+(g*(a+b*x^3)^4)/(12*b)+a^3*d*\text{Log}[x]$

Rubi [A] time = 0.182106, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1583, 1820}

$$\frac{1}{2}a^2x^2(af+3bc)+a^2bdx^3+\frac{1}{4}a^2x^4(ah+3be)-\frac{a^3c}{x}+a^3d\log(x)+a^3ex+\frac{1}{8}b^2x^8(3af+bc)+\frac{1}{2}ab^2dx^6+\frac{1}{10}b^2x^{10}(3ah$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a+b*x^3)^3*(c+d*x+e*x^2+f*x^3+g*x^4+h*x^5)/x^2,x]$

[Out] $-\frac{a^3c}{x}+a^3e*x+(a^2*(3*b*c+a*f)*x^2)/2+a^2*b*d*x^3+(a^2*(3*b*e+a*h)*x^4)/4+(3*a*b*(b*c+a*f)*x^5)/5+(a*b^2*d*x^6)/2+(3*a*b*(b*e+a*h)*x^7)/7+(b^2*(b*c+3*a*f)*x^8)/8+(b^3*d*x^9)/9+(b^2*(b*e+3*a*h)*x^{10})/10+(b^3*f*x^{11})/11+(b^3*h*x^{13})/13+(g*(a+b*x^3)^4)/(12*b)+a^3*d*\text{Log}[x]$

Rule 1583

$\text{Int}[(P_x)_*(x_)^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(\text{Coeff}[P_x, x, n - m - 1]*(a + b*x^n)^{(p + 1)})/(b*n*(p + 1)), x] + \text{Int}[(P_x - \text{Coeff}[P_x, x, n - m - 1]*x^{(n - m - 1)})*x^m*(a + b*x^n)^p, x] /;$ FreeQ[{a, b, m, n}, x] && PolyQ[P_x, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[P_x, x, n - m - 1], 0]

Rule 1820

```
Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :=
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)^3 (c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{x^2} dx &= \frac{g(a + bx^3)^4}{12b} + \int \frac{(a + bx^3)^3 (c + dx + ex^2 + fx^3 + hx^5)}{x^2} dx \\ &= \frac{g(a + bx^3)^4}{12b} + \int \left(a^3e + \frac{a^3c}{x^2} + \frac{a^3d}{x} + a^2(3bc + af)x + 3a^2bdx^2 + a \right. \\ &\quad \left. - \frac{a^3c}{x} + a^3ex + \frac{1}{2}a^2(3bc + af)x^2 + a^2bdx^3 + \frac{1}{4}a^2(3be + ah)x^4 + \frac{3}{5}ab \right) dx \end{aligned}$$

Mathematica [A] time = 0.136937, size = 172, normalized size = 0.87

$$\frac{1}{140}a^2bx^2(210c + x(140d + x(105e + 84fx + 70gx^2 + 60hx^3))) + a^3\left(-\frac{c}{x} + ex + \frac{1}{12}x^2(6f + 4gx + 3hx^2)\right) + a^3d \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

```
[Out] a^3*(-(c/x) + e*x + (x^2*(6*f + 4*g*x + 3*h*x^2))/12) + (b^3*x^8*(6435*c +
5720*d*x + 6*x^2*(858*e + 780*f*x + 715*g*x^2 + 660*h*x^3)))/51480 + (a^2*b
*x^2*(210*c + x*(140*d + x*(105*e + 84*f*x + 70*g*x^2 + 60*h*x^3)))/140 +
(a*b^2*x^5*(504*c + x*(420*d + x*(360*e + 315*f*x + 280*g*x^2 + 252*h*x^3))
)/840 + a^3*d*Log[x]
```

Maple [A] time = 0.007, size = 224, normalized size = 1.1

$$\frac{b^3hx^{13}}{13} + \frac{b^3gx^{12}}{12} + \frac{b^3fx^{11}}{11} + \frac{3x^{10}ab^2h}{10} + \frac{b^3ex^{10}}{10} + \frac{x^9ab^2g}{3} + \frac{b^3dx^9}{9} + \frac{3x^8ab^2f}{8} + \frac{b^3cx^8}{8} + \frac{3x^7a^2bh}{7} + \frac{3ab^2ex^7}{7} + \frac{x^6a^2b}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x)
```

[Out] $1/13*b^3*h*x^{13}+1/12*b^3*g*x^{12}+1/11*b^3*f*x^{11}+3/10*x^{10}*a*b^2*h+1/10*b^3*e*x^{10}+1/3*x^9*a*b^2*g+1/9*b^3*d*x^9+3/8*x^8*a*b^2*f+1/8*b^3*c*x^8+3/7*x^7*a^2*b*h+3/7*a*b^2*e*x^7+1/2*x^6*a^2*b*g+1/2*a*b^2*d*x^6+3/5*x^5*a^2*b*f+3/5*a*b^2*c*x^5+1/4*x^4*a^3*h+3/4*a^2*b*e*x^4+1/3*x^3*a^3*g+a^2*b*d*x^3+1/2*a^3*f*x^2+3/2*a^2*b*c*x^2+a^3*e*x+a^3*d*\ln(x)-a^3*c/x$

Maxima [A] time = 0.945867, size = 286, normalized size = 1.44

$$\frac{1}{13} b^3 h x^{13} + \frac{1}{12} b^3 g x^{12} + \frac{1}{11} b^3 f x^{11} + \frac{1}{10} (b^3 e + 3 a b^2 h) x^{10} + \frac{1}{9} (b^3 d + 3 a b^2 g) x^9 + \frac{1}{8} (b^3 c + 3 a b^2 f) x^8 + \frac{3}{7} (a b^2 e + a^2 b h) x^7 + \frac{1}{2} (a b^2 d + a^2 b g) x^6 + \frac{3}{5} (a b^2 c + a^2 b f) x^5 + a^3 e x + \frac{1}{4} (3 a^2 b e + a^3 h) x^4 + a^3 d \log(x) + \frac{1}{3} (3 a^2 b d + a^3 g) x^3 - a^3 c/x + \frac{1}{2} (3 a^2 b c + a^3 f) x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="maxima")

[Out] $1/13*b^3*h*x^{13} + 1/12*b^3*g*x^{12} + 1/11*b^3*f*x^{11} + 1/10*(b^3*e + 3*a*b^2*h)*x^{10} + 1/9*(b^3*d + 3*a*b^2*g)*x^9 + 1/8*(b^3*c + 3*a*b^2*f)*x^8 + 3/7*(a*b^2*e + a^2*b*h)*x^7 + 1/2*(a*b^2*d + a^2*b*g)*x^6 + 3/5*(a*b^2*c + a^2*b*f)*x^5 + a^3*e*x + 1/4*(3*a^2*b*e + a^3*h)*x^4 + a^3*d*\log(x) + 1/3*(3*a^2*b*d + a^3*g)*x^3 - a^3*c/x + 1/2*(3*a^2*b*c + a^3*f)*x^2$

Fricas [A] time = 1.02106, size = 570, normalized size = 2.88

$$27720 b^3 h x^{14} + 30030 b^3 g x^{13} + 32760 b^3 f x^{12} + 36036 (b^3 e + 3 a b^2 h) x^{11} + 40040 (b^3 d + 3 a b^2 g) x^{10} + 45045 (b^3 c + 3 a b^2 f) x^9 + 154440 (a b^2 e + a^2 b h) x^8 + 180180 (a b^2 d + a^2 b g) x^7 + 216216 (a b^2 c + a^2 b f) x^6 + 360360 a^3 e x^2 + 90090 (3 a^2 b e + a^3 h) x^5 + 360360 a^3 d x \log(x) + 120120 (3 a^2 b d + a^3 g) x^4 - 360360 a^3 c + 180180 (3 a^2 b c + a^3 f) x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="fricas")

[Out] $1/360360*(27720*b^3*h*x^{14} + 30030*b^3*g*x^{13} + 32760*b^3*f*x^{12} + 36036*(b^3*e + 3*a*b^2*h)*x^{11} + 40040*(b^3*d + 3*a*b^2*g)*x^{10} + 45045*(b^3*c + 3*a*b^2*f)*x^9 + 154440*(a*b^2*e + a^2*b*h)*x^8 + 180180*(a*b^2*d + a^2*b*g)*x^7 + 216216*(a*b^2*c + a^2*b*f)*x^6 + 360360*a^3*e*x^2 + 90090*(3*a^2*b*e + a^3*h)*x^5 + 360360*a^3*d*x*\log(x) + 120120*(3*a^2*b*d + a^3*g)*x^4 - 360360*a^3*c + 180180*(3*a^2*b*c + a^3*f)*x^3)/x$

Sympy [A] time = 0.633058, size = 236, normalized size = 1.19

$$-\frac{a^3c}{x} + a^3d \log(x) + a^3ex + \frac{b^3fx^{11}}{11} + \frac{b^3gx^{12}}{12} + \frac{b^3hx^{13}}{13} + x^{10} \left(\frac{3ab^2h}{10} + \frac{b^3e}{10} \right) + x^9 \left(\frac{ab^2g}{3} + \frac{b^3d}{9} \right) + x^8 \left(\frac{3ab^2f}{8} + \frac{b^3c}{8} \right) + x^7 \left(\frac{3a^2bh}{7} + \frac{3a^2be}{7} \right) + x^6 \left(\frac{a^2bg}{2} + \frac{a^2bd}{2} \right) + x^5 \left(\frac{3a^2bf}{5} + \frac{3a^2bc}{5} \right) + x^4 \left(\frac{a^3h}{4} + \frac{3a^2be}{4} \right) + x^3 \left(\frac{a^3g}{3} + a^2bd \right) + x^2 \left(\frac{a^3f}{2} + \frac{3a^2bc}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)

[Out] -a**3*c/x + a**3*d*log(x) + a**3*e*x + b**3*f*x**11/11 + b**3*g*x**12/12 + b**3*h*x**13/13 + x**10*(3*a*b**2*h/10 + b**3*e/10) + x**9*(a*b**2*g/3 + b**3*d/9) + x**8*(3*a*b**2*f/8 + b**3*c/8) + x**7*(3*a**2*b*h/7 + 3*a*b**2*e/7) + x**6*(a**2*b*g/2 + a*b**2*d/2) + x**5*(3*a**2*b*f/5 + 3*a*b**2*c/5) + x**4*(a**3*h/4 + 3*a**2*b*e/4) + x**3*(a**3*g/3 + a**2*b*d) + x**2*(a**3*f/2 + 3*a**2*b*c/2)

Giac [A] time = 1.06084, size = 308, normalized size = 1.56

$$\frac{1}{13} b^3 h x^{13} + \frac{1}{12} b^3 g x^{12} + \frac{1}{11} b^3 f x^{11} + \frac{3}{10} a b^2 h x^{10} + \frac{1}{10} b^3 x^{10} e + \frac{1}{9} b^3 d x^9 + \frac{1}{3} a b^2 g x^9 + \frac{1}{8} b^3 c x^8 + \frac{3}{8} a b^2 f x^8 + \frac{3}{7} a^2 b h x^7 + \frac{3}{7} a^2 b e x^7 + \frac{1}{2} a^2 b g x^6 + \frac{1}{2} a^2 b d x^6 + \frac{3}{5} a^2 b f x^5 + \frac{3}{5} a^2 b c x^5 + \frac{1}{4} a^3 h x^4 + \frac{3}{4} a^2 b x^4 e + a^2 b d x^3 + \frac{1}{3} a^3 g x^3 + \frac{3}{2} a^2 b c x^2 + \frac{1}{2} a^3 f x^2 + a^3 x e + a^3 d \log(\text{abs}(x)) - a^3 c/x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="giac")

[Out] 1/13*b^3*h*x^13 + 1/12*b^3*g*x^12 + 1/11*b^3*f*x^11 + 3/10*a*b^2*h*x^10 + 1/10*b^3*x^10*e + 1/9*b^3*d*x^9 + 1/3*a*b^2*g*x^9 + 1/8*b^3*c*x^8 + 3/8*a*b^2*f*x^8 + 3/7*a^2*b*h*x^7 + 3/7*a*b^2*x^7*e + 1/2*a*b^2*d*x^6 + 1/2*a^2*b*g*x^6 + 3/5*a*b^2*c*x^5 + 3/5*a^2*b*f*x^5 + 1/4*a^3*h*x^4 + 3/4*a^2*b*x^4*e + a^2*b*d*x^3 + 1/3*a^3*g*x^3 + 3/2*a^2*b*c*x^2 + 1/2*a^3*f*x^2 + a^3*x*e + a^3*d*log(abs(x)) - a^3*c/x

$$3.400 \quad \int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3} dx$$

Optimal. Leaf size=198

$$a^2x(af + 3bc) + \frac{1}{2}a^2x^2(ag + 3bd) + a^2bex^3 - \frac{a^3c}{2x^2} - \frac{a^3d}{x} + a^3e \log(x) + \frac{1}{7}b^2x^7(3af + bc) + \frac{1}{8}b^2x^8(3ag + bd) + \frac{1}{2}ab^2ex^6$$

[Out] $-(a^3c)/(2*x^2) - (a^3d)/x + a^2*(3*b*c + a*f)*x + (a^2*(3*b*d + a*g))*x^2$
 $/2 + a^2*b*e*x^3 + (3*a*b*(b*c + a*f))*x^4/4 + (3*a*b*(b*d + a*g))*x^5/5 +$
 $(a*b^2*e*x^6)/2 + (b^2*(b*c + 3*a*f))*x^7/7 + (b^2*(b*d + 3*a*g))*x^8/8 +$
 $(b^3*e*x^9)/9 + (b^3*f*x^10)/10 + (b^3*g*x^11)/11 + (h*(a + b*x^3)^4)/(12*b$
 $) + a^3*e*Log[x]$

Rubi [A] time = 0.196774, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1583, 1820}

$$a^2x(af + 3bc) + \frac{1}{2}a^2x^2(ag + 3bd) + a^2bex^3 - \frac{a^3c}{2x^2} - \frac{a^3d}{x} + a^3e \log(x) + \frac{1}{7}b^2x^7(3af + bc) + \frac{1}{8}b^2x^8(3ag + bd) + \frac{1}{2}ab^2ex^6$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/x^3, x]$

[Out] $-(a^3c)/(2*x^2) - (a^3d)/x + a^2*(3*b*c + a*f)*x + (a^2*(3*b*d + a*g))*x^2$
 $/2 + a^2*b*e*x^3 + (3*a*b*(b*c + a*f))*x^4/4 + (3*a*b*(b*d + a*g))*x^5/5 +$
 $(a*b^2*e*x^6)/2 + (b^2*(b*c + 3*a*f))*x^7/7 + (b^2*(b*d + 3*a*g))*x^8/8 +$
 $(b^3*e*x^9)/9 + (b^3*f*x^10)/10 + (b^3*g*x^11)/11 + (h*(a + b*x^3)^4)/(12*b$
 $) + a^3*e*Log[x]$

Rule 1583

$\text{Int}[(P_x)_*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] \rightarrow \text{Simp}[(\text{Coeff}[P_x, x, n - m - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + \text{Int}[(P_x - \text{Coeff}[P_x, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /;$ FreeQ[{a, b, m, n}, x] && PolyQ[P_x, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[P_x, x, n - m - 1], 0]

Rule 1820

```
Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)^3 (c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{x^3} dx &= \frac{h(a + bx^3)^4}{12b} + \int \frac{(a + bx^3)^3 (c + dx + ex^2 + fx^3 + gx^4)}{x^3} dx \\ &= \frac{h(a + bx^3)^4}{12b} + \int \left(a^2(3bc + af) + \frac{a^3c}{x^3} + \frac{a^3d}{x^2} + \frac{a^3e}{x} + a^2(3bd + ag) \right) dx \\ &= -\frac{a^3c}{2x^2} - \frac{a^3d}{x} + a^2(3bc + af)x + \frac{1}{2}a^2(3bd + ag)x^2 + a^2bex^3 + \frac{3}{4}ab(bc + af)x^4 \end{aligned}$$

Mathematica [A] time = 0.117362, size = 174, normalized size = 0.88

$$\frac{1}{20}a^2bx(60c + x(30d + x(20e + 15fx + 12gx^2 + 10hx^3))) + \frac{a^3(-3c - 6dx + x^3(6f + 3gx + 2hx^2))}{6x^2} + a^3e \log(x) + \frac{1}{840}a^3e \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3, x]
```

```
[Out] (a^3*(-3*c - 6*d*x + x^3*(6*f + 3*g*x + 2*h*x^2)))/(6*x^2) + (b^3*x^7*(3960*c + 7*x*(495*d + 440*e*x + 6*x^2*(66*f + 60*g*x + 55*h*x^2)))/27720 + (a^2*b*x*(60*c + x*(30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x^3)))/20 + (a*b^2*x^4*(630*c + x*(504*d + 5*x*(84*e + x*(72*f + 7*x*(9*g + 8*h*x)))))/840 + a^3*e*Log[x]
```

Maple [A] time = 0.008, size = 222, normalized size = 1.1

$$\frac{b^3hx^{12}}{12} + \frac{b^3gx^{11}}{11} + \frac{b^3fx^{10}}{10} + \frac{x^9ab^2h}{3} + \frac{b^3ex^9}{9} + \frac{3x^8ab^2g}{8} + \frac{b^3dx^8}{8} + \frac{3x^7ab^2f}{7} + \frac{b^3cx^7}{7} + \frac{x^6a^2bh}{2} + \frac{ab^2ex^6}{2} + \frac{3x^5a^2bg}{5} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3, x)
```

```
[Out] 1/12*b^3*h*x^12+1/11*b^3*g*x^11+1/10*b^3*f*x^10+1/3*x^9*a*b^2*h+1/9*b^3*e*x^9+3/8*x^8*a*b^2*g+1/8*b^3*d*x^8+3/7*x^7*a*b^2*f+1/7*b^3*c*x^7+1/2*x^6*a^2*b*h+1/2*a*b^2*e*x^6+3/5*x^5*a^2*b*g+3/5*a*b^2*d*x^5+3/4*x^4*a^2*b*f+3/4*a*b^2*c*x^4+1/3*x^3*a^3*h+a^2*b*e*x^3+1/2*x^2*a^3*g+3/2*a^2*b*d*x^2+a^3*f*x+a^2*b*c*x+a^3*e*log(x)-1/2*a^3*c/x^2-a^3*d/x
```

Maxima [A] time = 0.951091, size = 286, normalized size = 1.44

$$\frac{1}{12} b^3 h x^{12} + \frac{1}{11} b^3 g x^{11} + \frac{1}{10} b^3 f x^{10} + \frac{1}{9} (b^3 e + 3 a b^2 h) x^9 + \frac{1}{8} (b^3 d + 3 a b^2 g) x^8 + \frac{1}{7} (b^3 c + 3 a b^2 f) x^7 + \frac{1}{2} (a b^2 e + a^2 b h) x^6 + \frac{3}{5} (a b^2 d + a^2 b g) x^5 + \frac{3}{4} (a b^2 c + a^2 b f) x^4 + a^3 e \log(x) + \frac{1}{3} (3 a^2 b e + a^3 h) x^3 + \frac{1}{2} (3 a^2 b d + a^3 g) x^2 + (3 a^2 b c + a^3 f) x - \frac{1}{2} (2 a^3 d x + a^3 c) / x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*b^3*h*x^12 + 1/11*b^3*g*x^11 + 1/10*b^3*f*x^10 + 1/9*(b^3*e + 3*a*b^2*h)*x^9 + 1/8*(b^3*d + 3*a*b^2*g)*x^8 + 1/7*(b^3*c + 3*a*b^2*f)*x^7 + 1/2*(a*b^2*e + a^2*b*h)*x^6 + 3/5*(a*b^2*d + a^2*b*g)*x^5 + 3/4*(a*b^2*c + a^2*b*f)*x^4 + a^3*e*log(x) + 1/3*(3*a^2*b*e + a^3*h)*x^3 + 1/2*(3*a^2*b*d + a^3*g)*x^2 + (3*a^2*b*c + a^3*f)*x - 1/2*(2*a^3*d*x + a^3*c)/x^2
```

Fricas [A] time = 1.01098, size = 551, normalized size = 2.78

$$2310 b^3 h x^{14} + 2520 b^3 g x^{13} + 2772 b^3 f x^{12} + 3080 (b^3 e + 3 a b^2 h) x^{11} + 3465 (b^3 d + 3 a b^2 g) x^{10} + 3960 (b^3 c + 3 a b^2 f) x^9 + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] 1/27720*(2310*b^3*h*x^14 + 2520*b^3*g*x^13 + 2772*b^3*f*x^12 + 3080*(b^3*e + 3*a*b^2*h)*x^11 + 3465*(b^3*d + 3*a*b^2*g)*x^10 + 3960*(b^3*c + 3*a*b^2*f)*x^9 + 13860*(a*b^2*e + a^2*b*h)*x^8 + 16632*(a*b^2*d + a^2*b*g)*x^7 + 20790*(a*b^2*c + a^2*b*f)*x^6 + 27720*a^3*e*x^2*log(x) + 9240*(3*a^2*b*e + a^3*h)*x^5 - 27720*a^3*d*x + 13860*(3*a^2*b*d + a^3*g)*x^4 - 13860*a^3*c + 27720*(3*a^2*b*c + a^3*f)*x^3)/x^2
```

Sympy [A] time = 0.716463, size = 236, normalized size = 1.19

$$a^3 e \log(x) + \frac{b^3 f x^{10}}{10} + \frac{b^3 g x^{11}}{11} + \frac{b^3 h x^{12}}{12} + x^9 \left(\frac{ab^2 h}{3} + \frac{b^3 e}{9} \right) + x^8 \left(\frac{3ab^2 g}{8} + \frac{b^3 d}{8} \right) + x^7 \left(\frac{3ab^2 f}{7} + \frac{b^3 c}{7} \right) + x^6 \left(\frac{a^2 b h}{2} + \frac{ab^2 e}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)

[Out] a**3*e*log(x) + b**3*f*x**10/10 + b**3*g*x**11/11 + b**3*h*x**12/12 + x**9*(a*b**2*h/3 + b**3*e/9) + x**8*(3*a*b**2*g/8 + b**3*d/8) + x**7*(3*a*b**2*f/7 + b**3*c/7) + x**6*(a**2*b*h/2 + a*b**2*e/2) + x**5*(3*a**2*b*g/5 + 3*a*b**2*d/5) + x**4*(3*a**2*b*f/4 + 3*a*b**2*c/4) + x**3*(a**3*h/3 + a**2*b*e) + x**2*(a**3*g/2 + 3*a**2*b*d/2) + x*(a**3*f + 3*a**2*b*c) - (a**3*c + 2*a**3*d*x)/(2*x**2)

Giac [A] time = 1.06294, size = 305, normalized size = 1.54

$$\frac{1}{12} b^3 h x^{12} + \frac{1}{11} b^3 g x^{11} + \frac{1}{10} b^3 f x^{10} + \frac{1}{3} a b^2 h x^9 + \frac{1}{9} b^3 x^9 e + \frac{1}{8} b^3 d x^8 + \frac{3}{8} a b^2 g x^8 + \frac{1}{7} b^3 c x^7 + \frac{3}{7} a b^2 f x^7 + \frac{1}{2} a^2 b h x^6 + \frac{1}{2} a b^2 e x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="giac")

[Out] 1/12*b^3*h*x^12 + 1/11*b^3*g*x^11 + 1/10*b^3*f*x^10 + 1/3*a*b^2*h*x^9 + 1/9*b^3*x^9*e + 1/8*b^3*d*x^8 + 3/8*a*b^2*g*x^8 + 1/7*b^3*c*x^7 + 3/7*a*b^2*f*x^7 + 1/2*a^2*b*h*x^6 + 1/2*a*b^2*x^6*e + 3/5*a*b^2*d*x^5 + 3/5*a^2*b*g*x^5 + 3/4*a*b^2*c*x^4 + 3/4*a^2*b*f*x^4 + 1/3*a^3*h*x^3 + a^2*b*x^3*e + 3/2*a^2*b*d*x^2 + 1/2*a^3*g*x^2 + 3*a^2*b*c*x + a^3*f*x + a^3*e*log(abs(x)) - 1/2*(2*a^3*d*x + a^3*c)/x^2

$$3.401 \quad \int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4} dx$$

Optimal. Leaf size=209

$$a^2 \log(x)(af + 3bc) + a^2x(ag + 3bd) + \frac{1}{2}a^2x^2(ah + 3be) - \frac{a^3c}{3x^3} - \frac{a^3d}{2x^2} - \frac{a^3e}{x} + \frac{1}{6}b^2x^6(3af + bc) + \frac{1}{7}b^2x^7(3ag + bd) + \frac{1}{8}b^2x^8(3ah + 3bc)$$

[Out] $-(a^3c)/(3x^3) - (a^3d)/(2x^2) - (a^3e)/x + a^2(3bd + a^2g)x + (a^2(3be + a^2h)x^2)/2 + a^2b(bc + af)x^3 + (3a^2b(bd + ag)x^4)/4 + (3a^2b(be + ah)x^5)/5 + (b^2(b^2c + 3a^2f)x^6)/6 + (b^2(b^2d + 3a^2g)x^7)/7 + (b^2(b^2e + 3a^2h)x^8)/8 + (b^3f)x^9/9 + (b^3g)x^{10}/10 + (b^3h)x^{11}/11 + a^2(3bc + af)*\text{Log}[x]$

Rubi [A] time = 0.179548, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1820}

$$a^2 \log(x)(af + 3bc) + a^2x(ag + 3bd) + \frac{1}{2}a^2x^2(ah + 3be) - \frac{a^3c}{3x^3} - \frac{a^3d}{2x^2} - \frac{a^3e}{x} + \frac{1}{6}b^2x^6(3af + bc) + \frac{1}{7}b^2x^7(3ag + bd) + \frac{1}{8}b^2x^8(3ah + 3bc)$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4, x]

[Out] $-(a^3c)/(3x^3) - (a^3d)/(2x^2) - (a^3e)/x + a^2(3bd + a^2g)x + (a^2(3be + a^2h)x^2)/2 + a^2b(bc + af)x^3 + (3a^2b(bd + ag)x^4)/4 + (3a^2b(be + ah)x^5)/5 + (b^2(b^2c + 3a^2f)x^6)/6 + (b^2(b^2d + 3a^2g)x^7)/7 + (b^2(b^2e + 3a^2h)x^8)/8 + (b^3f)x^9/9 + (b^3g)x^{10}/10 + (b^3h)x^{11}/11 + a^2(3bc + af)*\text{Log}[x]$

Rule 1820

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\int \frac{(a + bx^3)^3 (c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{x^4} dx = \int \left(a^2(3bd + ag) + \frac{a^3c}{x^4} + \frac{a^3d}{x^3} + \frac{a^3e}{x^2} + \frac{a^2(3bc + af)}{x} + a^2(3be + ah) \right. \\ \left. - \frac{a^3c}{3x^3} - \frac{a^3d}{2x^2} - \frac{a^3e}{x} + a^2(3bd + ag)x + \frac{1}{2}a^2(3be + ah)x^2 + ab(bc + a^2) \right) dx$$

Mathematica [A] time = 0.126123, size = 172, normalized size = 0.82

$$a^2 \log(x)(af + 3bc) + \frac{1}{20}a^2bx(60d + x(30e + x(20f + 15gx + 12hx^2))) - \frac{a^3(2c + 3x(d + 2ex + x^3(-2g + hx)))}{6x^3} + \frac{1}{28}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]

[Out] -(a^3*(2*c + 3*x*(d + 2*e*x - x^3*(2*g + h*x)))/(6*x^3) + (a^2*b*x*(60*d + x*(30*e + x*(20*f + 15*g*x + 12*h*x^2)))/20 + (a*b^2*x^3*(280*c + x*(210*d + x*(168*e + 140*f*x + 120*g*x^2 + 105*h*x^3)))/280 + (b^3*x^6*(4620*c + x*(3960*d + 7*x*(495*e + 4*x*(110*f + 99*g*x + 90*h*x^2))))/27720 + a^2*(3*b*c + a*f)*Log[x]

Maple [A] time = 0.006, size = 220, normalized size = 1.1

$$\frac{b^3hx^{11}}{11} + \frac{b^3gx^{10}}{10} + \frac{b^3fx^9}{9} + \frac{3x^8ab^2h}{8} + \frac{x^8b^3e}{8} + \frac{3x^7ab^2g}{7} + \frac{x^7b^3d}{7} + \frac{x^6ab^2f}{2} + \frac{x^6b^3c}{6} + \frac{3x^5a^2bh}{5} + \frac{3x^5ab^2e}{5} + \frac{3x^4a^2bg}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x)

[Out] 1/11*b^3*h*x^11+1/10*b^3*g*x^10+1/9*b^3*f*x^9+3/8*x^8*a*b^2*h+1/8*x^8*b^3*e+3/7*x^7*a*b^2*g+1/7*x^7*b^3*d+1/2*x^6*a*b^2*f+1/6*x^6*b^3*c+3/5*x^5*a^2*b*h+3/5*x^5*a*b^2*e+3/4*x^4*a^2*b*g+3/4*a*b^2*d*x^4+x^3*a^2*b*f+x^3*a*b^2*c+1/2*x^2*a^3*h+3/2*a^2*b*e*x^2+a^3*g*x+3*a^2*b*d*x+ln(x)*a^3*f+3*ln(x)*a^2*b*c-1/3*a^3*c/x^3-1/2*a^3*d/x^2-a^3*e/x

Maxima [A] time = 0.946772, size = 286, normalized size = 1.37

$$\frac{1}{11} b^3 h x^{11} + \frac{1}{10} b^3 g x^{10} + \frac{1}{9} b^3 f x^9 + \frac{1}{8} (b^3 e + 3 a b^2 h) x^8 + \frac{1}{7} (b^3 d + 3 a b^2 g) x^7 + \frac{1}{6} (b^3 c + 3 a b^2 f) x^6 + \frac{3}{5} (a b^2 e + a^2 b h) x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="maxima")

[Out] 1/11*b^3*h*x^11 + 1/10*b^3*g*x^10 + 1/9*b^3*f*x^9 + 1/8*(b^3*e + 3*a*b^2*h)*x^8 + 1/7*(b^3*d + 3*a*b^2*g)*x^7 + 1/6*(b^3*c + 3*a*b^2*f)*x^6 + 3/5*(a*b^2*e + a^2*b*h)*x^5 + 3/4*(a*b^2*d + a^2*b*g)*x^4 + (a*b^2*c + a^2*b*f)*x^3 + 1/2*(3*a^2*b*e + a^3*h)*x^2 + (3*a^2*b*d + a^3*g)*x + (3*a^2*b*c + a^3*f)*log(x) - 1/6*(6*a^3*e*x^2 + 3*a^3*d*x + 2*a^3*c)/x^3

Fricas [A] time = 1.00583, size = 551, normalized size = 2.64

$$2520 b^3 h x^{14} + 2772 b^3 g x^{13} + 3080 b^3 f x^{12} + 3465 (b^3 e + 3 a b^2 h) x^{11} + 3960 (b^3 d + 3 a b^2 g) x^{10} + 4620 (b^3 c + 3 a b^2 f) x^9 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="fricas")

[Out] 1/27720*(2520*b^3*h*x^14 + 2772*b^3*g*x^13 + 3080*b^3*f*x^12 + 3465*(b^3*e + 3*a*b^2*h)*x^11 + 3960*(b^3*d + 3*a*b^2*g)*x^10 + 4620*(b^3*c + 3*a*b^2*f)*x^9 + 16632*(a*b^2*e + a^2*b*h)*x^8 + 20790*(a*b^2*d + a^2*b*g)*x^7 + 27720*(a*b^2*c + a^2*b*f)*x^6 - 27720*a^3*e*x^2 + 13860*(3*a^2*b*e + a^3*h)*x^5 - 13860*a^3*d*x + 27720*(3*a^2*b*d + a^3*g)*x^4 + 27720*(3*a^2*b*c + a^3*f)*x^3*log(x) - 9240*a^3*c)/x^3

Sympy [A] time = 1.06122, size = 235, normalized size = 1.12

$$a^2 (a f + 3 b c) \log(x) + \frac{b^3 f x^9}{9} + \frac{b^3 g x^{10}}{10} + \frac{b^3 h x^{11}}{11} + x^8 \left(\frac{3 a b^2 h}{8} + \frac{b^3 e}{8} \right) + x^7 \left(\frac{3 a b^2 g}{7} + \frac{b^3 d}{7} \right) + x^6 \left(\frac{a b^2 f}{2} + \frac{b^3 c}{6} \right) + x^5 \left(3 a \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)

[Out] a**2*(a*f + 3*b*c)*log(x) + b**3*f*x**9/9 + b**3*g*x**10/10 + b**3*h*x**11/11 + x**8*(3*a*b**2*h/8 + b**3*e/8) + x**7*(3*a*b**2*g/7 + b**3*d/7) + x**6*(a*b**2*f/2 + b**3*c/6) + x**5*(3*a**2*b*h/5 + 3*a*b**2*e/5) + x**4*(3*a**2*b*g/4 + 3*a*b**2*d/4) + x**3*(a**2*b*f + a*b**2*c) + x**2*(a**3*h/2 + 3*a**2*b*e/2) + x*(a**3*g + 3*a**2*b*d) - (2*a**3*c + 3*a**3*d*x + 6*a**3*e*x**2)/(6*x**3)

Giac [A] time = 1.06246, size = 304, normalized size = 1.45

$$\frac{1}{11} b^3 h x^{11} + \frac{1}{10} b^3 g x^{10} + \frac{1}{9} b^3 f x^9 + \frac{3}{8} a b^2 h x^8 + \frac{1}{8} b^3 x^8 e + \frac{1}{7} b^3 d x^7 + \frac{3}{7} a b^2 g x^7 + \frac{1}{6} b^3 c x^6 + \frac{1}{2} a b^2 f x^6 + \frac{3}{5} a^2 b h x^5 + \frac{3}{5} a b^2 e x^5 + \frac{3}{4} a^2 b g x^4 + \frac{3}{4} a b^2 d x^4 + a b^2 c x^3 + a^2 b f x^3 + \frac{1}{2} a^3 h x^2 + \frac{3}{2} a^2 b e x^2 + 3 a^2 b d x + a^3 g x + (3 a^2 b c + a^3 f) \log(\operatorname{abs}(x)) - \frac{1}{6} (6 a^3 x^2 e + 3 a^3 d x + 2 a^3 c) / x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="giac")

[Out] 1/11*b^3*h*x^11 + 1/10*b^3*g*x^10 + 1/9*b^3*f*x^9 + 3/8*a*b^2*h*x^8 + 1/8*b^3*x^8*e + 1/7*b^3*d*x^7 + 3/7*a*b^2*g*x^7 + 1/6*b^3*c*x^6 + 1/2*a*b^2*f*x^6 + 3/5*a^2*b*h*x^5 + 3/5*a*b^2*x^5*e + 3/4*a*b^2*d*x^4 + 3/4*a^2*b*g*x^4 + a*b^2*c*x^3 + a^2*b*f*x^3 + 1/2*a^3*h*x^2 + 3/2*a^2*b*x^2*e + 3*a^2*b*d*x + a^3*g*x + (3*a^2*b*c + a^3*f)*log(abs(x)) - 1/6*(6*a^3*x^2*e + 3*a^3*d*x + 2*a^3*c)/x^3

$$3.402 \quad \int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5} dx$$

Optimal. Leaf size=209

$$-\frac{a^2(af+3bc)}{x} + a^2 \log(x)(ag+3bd) + a^2x(ah+3be) - \frac{a^3c}{4x^4} - \frac{a^3d}{3x^3} - \frac{a^3e}{2x^2} + \frac{1}{5}b^2x^5(3af+bc) + \frac{1}{6}b^2x^6(3ag+bd) + \frac{1}{7}b^2x^7(3af+bc) + \frac{1}{8}b^2x^8(3ag+bd) + \frac{1}{9}b^2x^9(3af+bc) + \frac{1}{10}b^2x^{10}(3ag+bd)$$

[Out] $-(a^3c)/(4x^4) - (a^3d)/(3x^3) - (a^3e)/(2x^2) - (a^2(3bc+af))/x + a^2(3be+ah)x + (3ab(bc+af)x^2)/2 + ab(bd+ag)x^3 + (3ab(be+ah)x^4)/4 + (b^2(bc+3af)x^5)/5 + (b^2(bd+3ag)x^6)/6 + (b^2(be+3ah)x^7)/7 + (b^3f)x^8/8 + (b^3g)x^9/9 + (b^3h)x^{10}/10 + a^2(3bd+ag)*\text{Log}[x]$

Rubi [A] time = 0.176742, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1820}

$$-\frac{a^2(af+3bc)}{x} + a^2 \log(x)(ag+3bd) + a^2x(ah+3be) - \frac{a^3c}{4x^4} - \frac{a^3d}{3x^3} - \frac{a^3e}{2x^2} + \frac{1}{5}b^2x^5(3af+bc) + \frac{1}{6}b^2x^6(3ag+bd) + \frac{1}{7}b^2x^7(3af+bc) + \frac{1}{8}b^2x^8(3ag+bd) + \frac{1}{9}b^2x^9(3af+bc) + \frac{1}{10}b^2x^{10}(3ag+bd)$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5, x]

[Out] $-(a^3c)/(4x^4) - (a^3d)/(3x^3) - (a^3e)/(2x^2) - (a^2(3bc+af))/x + a^2(3be+ah)x + (3ab(bc+af)x^2)/2 + ab(bd+ag)x^3 + (3ab(be+ah)x^4)/4 + (b^2(bc+3af)x^5)/5 + (b^2(bd+3ag)x^6)/6 + (b^2(be+3ah)x^7)/7 + (b^3f)x^8/8 + (b^3g)x^9/9 + (b^3h)x^{10}/10 + a^2(3bd+ag)*\text{Log}[x]$

Rule 1820

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\int \frac{(a + bx^3)^3 (c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{x^5} dx = \int \left(a^2(3be + ah) + \frac{a^3c}{x^5} + \frac{a^3d}{x^4} + \frac{a^3e}{x^3} + \frac{a^2(3bc + af)}{x^2} + \frac{a^2(3bd + ag)}{x} \right. \\ \left. = -\frac{a^3c}{4x^4} - \frac{a^3d}{3x^3} - \frac{a^3e}{2x^2} - \frac{a^2(3bc + af)}{x} + a^2(3be + ah)x + \frac{3}{2}ab(bc + af) \right) dx$$

Mathematica [A] time = 0.120628, size = 170, normalized size = 0.81

$$\frac{630a^2bx^3(x^2(12e + 6fx + 4gx^2 + 3hx^3) - 12c) - 210a^3(3c + 4dx + 6x^2(e + 2fx - 2hx^3)) + 18ab^2x^6(210c + x(140d + 105ex + 84fx^2 + 70gx^3 + 60hx^4)) + b^3x^9(504c + x(420d + 360ex + 315fx^2 + 280gx^3 + 252hx^4))}{2520x^4} + a^2(3bd + ag) \operatorname{Log}[x]$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]

[Out] (-210*a^3*(3*c + 4*d*x + 6*x^2*(e + 2*f*x - 2*h*x^3)) + 630*a^2*b*x^3*(-12*c + x^2*(12*e + 6*f*x + 4*g*x^2 + 3*h*x^3)) + 18*a*b^2*x^6*(210*c + x*(140*d + 105*e*x + 84*f*x^2 + 70*g*x^3 + 60*h*x^4)) + b^3*x^9*(504*c + x*(420*d + 360*e*x + 315*f*x^2 + 280*g*x^3 + 252*h*x^4)))/(2520*x^4) + a^2*(3*b*d + a*g)*Log[x]

Maple [A] time = 0.007, size = 220, normalized size = 1.1

$$\frac{b^3hx^{10}}{10} + \frac{b^3gx^9}{9} + \frac{b^3fx^8}{8} + \frac{3x^7ab^2h}{7} + \frac{x^7b^3e}{7} + \frac{x^6ab^2g}{2} + \frac{x^6b^3d}{6} + \frac{3x^5ab^2f}{5} + \frac{x^5b^3c}{5} + \frac{3x^4a^2bh}{4} + \frac{3x^4ab^2e}{4} + x^3a^2bg +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x)

[Out] 1/10*b^3*h*x^10+1/9*b^3*g*x^9+1/8*b^3*f*x^8+3/7*x^7*a*b^2*h+1/7*x^7*b^3*e+1/2*x^6*a*b^2*g+1/6*x^6*b^3*d+3/5*x^5*a*b^2*f+1/5*x^5*b^3*c+3/4*x^4*a^2*b*h+3/4*x^4*a*b^2*e+x^3*a^2*b*g+a*b^2*d*x^3+3/2*x^2*a^2*b*f+3/2*x^2*a*b^2*c+a^3*h*x+3*a^2*b*e*x+ln(x)*a^3*g+3*ln(x)*a^2*b*d-1/3*a^3*d/x^3-1/4*a^3*c/x^4-1/2*a^3*e/x^2-a^3/x*f-3*a^2/x*b*c

Maxima [A] time = 0.947768, size = 286, normalized size = 1.37

$$\frac{1}{10} b^3 h x^{10} + \frac{1}{9} b^3 g x^9 + \frac{1}{8} b^3 f x^8 + \frac{1}{7} (b^3 e + 3 a b^2 h) x^7 + \frac{1}{6} (b^3 d + 3 a b^2 g) x^6 + \frac{1}{5} (b^3 c + 3 a b^2 f) x^5 + \frac{3}{4} (a b^2 e + a^2 b h) x^4 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="maxima")

[Out] 1/10*b^3*h*x^10 + 1/9*b^3*g*x^9 + 1/8*b^3*f*x^8 + 1/7*(b^3*e + 3*a*b^2*h)*x^7 + 1/6*(b^3*d + 3*a*b^2*g)*x^6 + 1/5*(b^3*c + 3*a*b^2*f)*x^5 + 3/4*(a*b^2*e + a^2*b*h)*x^4 + (a*b^2*d + a^2*b*g)*x^3 + 3/2*(a*b^2*c + a^2*b*f)*x^2 + (3*a^2*b*e + a^3*h)*x + (3*a^2*b*d + a^3*g)*log(x) - 1/12*(6*a^3*e*x^2 + 4*a^3*d*x + 3*a^3*c + 12*(3*a^2*b*c + a^3*f)*x^3)/x^4

Fricas [A] time = 0.97709, size = 528, normalized size = 2.53

$$252 b^3 h x^{14} + 280 b^3 g x^{13} + 315 b^3 f x^{12} + 360 (b^3 e + 3 a b^2 h) x^{11} + 420 (b^3 d + 3 a b^2 g) x^{10} + 504 (b^3 c + 3 a b^2 f) x^9 + 1890 (a b^2 e + a^2 b h) x^8 + 2520 (a b^2 d + a^2 b g) x^7 + 3780 (a b^2 c + a^2 b f) x^6 - 1260 a^3 e x^2 + 2520 (3 a^2 b e + a^3 h) x^5 + 2520 (3 a^2 b d + a^3 g) x^4 \log(x) - 840 a^3 d x - 630 a^3 c - 2520 (3 a^2 b c + a^3 f) x^3 / x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="fricas")

[Out] 1/2520*(252*b^3*h*x^14 + 280*b^3*g*x^13 + 315*b^3*f*x^12 + 360*(b^3*e + 3*a*b^2*h)*x^11 + 420*(b^3*d + 3*a*b^2*g)*x^10 + 504*(b^3*c + 3*a*b^2*f)*x^9 + 1890*(a*b^2*e + a^2*b*h)*x^8 + 2520*(a*b^2*d + a^2*b*g)*x^7 + 3780*(a*b^2*c + a^2*b*f)*x^6 - 1260*a^3*e*x^2 + 2520*(3*a^2*b*e + a^3*h)*x^5 + 2520*(3*a^2*b*d + a^3*g)*x^4*log(x) - 840*a^3*d*x - 630*a^3*c - 2520*(3*a^2*b*c + a^3*f)*x^3)/x^4

Sympy [A] time = 2.97899, size = 233, normalized size = 1.11

$$a^2 (ag + 3bd) \log(x) + \frac{b^3 f x^8}{8} + \frac{b^3 g x^9}{9} + \frac{b^3 h x^{10}}{10} + x^7 \left(\frac{3 a b^2 h}{7} + \frac{b^3 e}{7} \right) + x^6 \left(\frac{a b^2 g}{2} + \frac{b^3 d}{6} \right) + x^5 \left(\frac{3 a b^2 f}{5} + \frac{b^3 c}{5} \right) + x^4 \left(\frac{3 a^2 b h}{4} + \frac{a^2 b g}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)

[Out] a**2*(a*g + 3*b*d)*log(x) + b**3*f*x**8/8 + b**3*g*x**9/9 + b**3*h*x**10/10 + x**7*(3*a*b**2*h/7 + b**3*e/7) + x**6*(a*b**2*g/2 + b**3*d/6) + x**5*(3*a*b**2*f/5 + b**3*c/5) + x**4*(3*a**2*b*h/4 + 3*a*b**2*e/4) + x**3*(a**2*b*g + a*b**2*d) + x**2*(3*a**2*b*f/2 + 3*a*b**2*c/2) + x*(a**3*h + 3*a**2*b*e) - (3*a**3*c + 4*a**3*d*x + 6*a**3*e*x**2 + x**3*(12*a**3*f + 36*a**2*b*c))/(12*x**4)

Giac [A] time = 1.07367, size = 302, normalized size = 1.44

$$\frac{1}{10} b^3 h x^{10} + \frac{1}{9} b^3 g x^9 + \frac{1}{8} b^3 f x^8 + \frac{3}{7} a b^2 h x^7 + \frac{1}{7} b^3 x^7 e + \frac{1}{6} b^3 d x^6 + \frac{1}{2} a b^2 g x^6 + \frac{1}{5} b^3 c x^5 + \frac{3}{5} a b^2 f x^5 + \frac{3}{4} a^2 b h x^4 + \frac{3}{4} a b^2 x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="giac")

[Out] 1/10*b^3*h*x^10 + 1/9*b^3*g*x^9 + 1/8*b^3*f*x^8 + 3/7*a*b^2*h*x^7 + 1/7*b^3*x^7*e + 1/6*b^3*d*x^6 + 1/2*a*b^2*g*x^6 + 1/5*b^3*c*x^5 + 3/5*a*b^2*f*x^5 + 3/4*a^2*b*h*x^4 + 3/4*a*b^2*x^4*e + a*b^2*d*x^3 + a^2*b*g*x^3 + 3/2*a*b^2*c*x^2 + 3/2*a^2*b*f*x^2 + a^3*h*x + 3*a^2*b*x*e + (3*a^2*b*d + a^3*g)*log(abs(x)) - 1/12*(6*a^3*x^2*e + 4*a^3*d*x + 3*a^3*c + 12*(3*a^2*b*c + a^3*f)*x^3)/x^4

$$3.403 \quad \int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3} dx$$

Optimal. Leaf size=331

$$\frac{a^{2/3} \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(a^{2/3}(be-ah) + b^{2/3}(bc-af)\right)}{6b^{10/3}} + \frac{a^{2/3} \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(a^{2/3}(be-ah) + b^{2/3}(bc-af)\right)}{3b^{10/3}}$$

[Out] $-\left(\frac{a(b^3e - a^3h)x}{b^3} + \frac{(b^3c - a^3f)x^2}{2b^2} + \frac{(b^3d - a^3g)x^3}{3b^2} + \frac{(b^3e - a^3h)x^4}{4b^2} + \frac{f^3x^5}{5b} + \frac{g^3x^6}{6b} + \frac{h^3x^7}{7b} + \frac{a^{2/3}(b^{5/3}c - a^{2/3}b^3e - a^3b^{2/3}f + a^{5/3}h) \operatorname{ArcTan}\left[\frac{a^{1/3} - 2b^{1/3}x}{\sqrt[3]{a^{1/3}}}\right]}{\sqrt[3]{b^{10/3}}} + \frac{a^{2/3}(b^{2/3}(b^3c - a^3f) + a^{2/3}(b^3e - a^3h)) \operatorname{Log}[a^{1/3} + b^{1/3}x]}{3b^{10/3}} - \frac{a^{2/3}(b^{2/3}(b^3c - a^3f) + a^{2/3}(b^3e - a^3h)) \operatorname{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2]}{6b^{10/3}} - \frac{a(b^3d - a^3g) \operatorname{Log}[a + b^3x^3]}{3b^3}\right)$

Rubi [A] time = 1.06967, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1836, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{a^{2/3} \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(a^{2/3}(be-ah) + b^{2/3}(bc-af)\right)}{6b^{10/3}} + \frac{a^{2/3} \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(a^{2/3}(be-ah) + b^{2/3}(bc-af)\right)}{3b^{10/3}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x^4(c + dx + ex^2 + fx^3 + gx^4 + hx^5))/(a + b^3x^3), x]$

[Out] $-\left(\frac{a(b^3e - a^3h)x}{b^3} + \frac{(b^3c - a^3f)x^2}{2b^2} + \frac{(b^3d - a^3g)x^3}{3b^2} + \frac{(b^3e - a^3h)x^4}{4b^2} + \frac{f^3x^5}{5b} + \frac{g^3x^6}{6b} + \frac{h^3x^7}{7b} + \frac{a^{2/3}(b^{5/3}c - a^{2/3}b^3e - a^3b^{2/3}f + a^{5/3}h) \operatorname{ArcTan}\left[\frac{a^{1/3} - 2b^{1/3}x}{\sqrt[3]{a^{1/3}}}\right]}{\sqrt[3]{b^{10/3}}} + \frac{a^{2/3}(b^{2/3}(b^3c - a^3f) + a^{2/3}(b^3e - a^3h)) \operatorname{Log}[a^{1/3} + b^{1/3}x]}{3b^{10/3}} - \frac{a^{2/3}(b^{2/3}(b^3c - a^3f) + a^{2/3}(b^3e - a^3h)) \operatorname{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2]}{6b^{10/3}} - \frac{a(b^3d - a^3g) \operatorname{Log}[a + b^3x^3]}{3b^3}\right)$

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m + q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Pq/(a + b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Dist[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
```

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 204

$\text{Int}[\frac{(a_1 + (b_1)x^2)^{-1}}{x}, x_Symbol] \rightarrow -\text{Simp}[\frac{\text{ArcTan}[\frac{\text{Rt}[-b, 2]x}{\text{Rt}[-a, 2] - \text{Rt}[-a, 2]\text{Rt}[-b, 2]}}{x}], x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 628

$\text{Int}[\frac{(d_1 + (e_1)x)}{(a_1 + (b_1)x + (c_1)x^2)}, x_Symbol] \rightarrow \text{Simp}[\frac{d \cdot \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]}{b}, x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2cd - be, 0]$

Rule 260

$\text{Int}[\frac{x^{(m_1)}}{(a_1 + (b_1)x^n)}, x_Symbol] \rightarrow \text{Simp}[\frac{\text{Log}[\text{RemoveContent}[a + bx^n, x]]}{bn}, x] \ /; \ \text{FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$

Rubi steps

$$\begin{aligned}
\int \frac{x^4(c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{a + bx^3} dx &= \frac{hx^7}{7b} + \frac{\int \frac{x^4(7bc+7bdx+7(be-ah)x^2+7bfx^3+7bgx^4)}{a+bx^3} dx}{7b} \\
&= \frac{gx^6}{6b} + \frac{hx^7}{7b} + \frac{\int \frac{x^4(42b^2c+42b(bd-ag)x+42b(be-ah)x^2+42b^2fx^3)}{a+bx^3} dx}{42b^2} \\
&= \frac{fx^5}{5b} + \frac{gx^6}{6b} + \frac{hx^7}{7b} + \frac{\int \frac{x^4(210b^2(bc-af)+210b^2(bd-ag)x+210b^2(be-ah)x^2)}{a+bx^3} dx}{210b^3} \\
&= \frac{fx^5}{5b} + \frac{gx^6}{6b} + \frac{hx^7}{7b} + \frac{\int \left(-210a(be-ah) + 210b(bc-af)x + 210b(bd-ag)x^2 \right)}{210b^3} dx \\
&= -\frac{a(be-ah)x}{b^3} + \frac{(bc-af)x^2}{2b^2} + \frac{(bd-ag)x^3}{3b^2} + \frac{(be-ah)x^4}{4b^2} + \frac{fx^5}{5b} + \frac{gx^6}{6b} + \frac{hx^7}{7b} \\
&= -\frac{a(be-ah)x}{b^3} + \frac{(bc-af)x^2}{2b^2} + \frac{(bd-ag)x^3}{3b^2} + \frac{(be-ah)x^4}{4b^2} + \frac{fx^5}{5b} + \frac{gx^6}{6b} + \frac{hx^7}{7b} \\
&= -\frac{a(be-ah)x}{b^3} + \frac{(bc-af)x^2}{2b^2} + \frac{(bd-ag)x^3}{3b^2} + \frac{(be-ah)x^4}{4b^2} + \frac{fx^5}{5b} + \frac{gx^6}{6b} + \frac{hx^7}{7b} \\
&= -\frac{a(be-ah)x}{b^3} + \frac{(bc-af)x^2}{2b^2} + \frac{(bd-ag)x^3}{3b^2} + \frac{(be-ah)x^4}{4b^2} + \frac{fx^5}{5b} + \frac{gx^6}{6b} + \frac{hx^7}{7b} \\
&= -\frac{a(be-ah)x}{b^3} + \frac{(bc-af)x^2}{2b^2} + \frac{(bd-ag)x^3}{3b^2} + \frac{(be-ah)x^4}{4b^2} + \frac{fx^5}{5b} + \frac{gx^6}{6b} + \frac{hx^7}{7b} \\
&= -\frac{a(be-ah)x}{b^3} + \frac{(bc-af)x^2}{2b^2} + \frac{(bd-ag)x^3}{3b^2} + \frac{(be-ah)x^4}{4b^2} + \frac{fx^5}{5b} + \frac{gx^6}{6b} + \frac{hx^7}{7b}
\end{aligned}$$

Mathematica [A] time = 0.438062, size = 334, normalized size = 1.01

$$\frac{a^{2/3} \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(-a^{2/3}be + a^{5/3}h + ab^{2/3}f - b^{5/3}c\right)}{6b^{10/3}} + \frac{a^{2/3} \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(a^{2/3}be + a^{5/3}(-h) - ab^{2/3}f + b^{5/3}c\right)}{3b^{10/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3), x]

```
[Out] (a*(-(b*e) + a*h)*x)/b^3 + ((b*c - a*f)*x^2)/(2*b^2) + ((b*d - a*g)*x^3)/(3
*b^2) + ((b*e - a*h)*x^4)/(4*b^2) + (f*x^5)/(5*b) + (g*x^6)/(6*b) + (h*x^7)
/(7*b) + (a^(2/3)*(b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*ArcTa
n[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(10/3)) + (a^(2/3)*(b^(5
/3)*c + a^(2/3)*b*e - a*b^(2/3)*f - a^(5/3)*h)*Log[a^(1/3) + b^(1/3)*x]/(3
*b^(10/3)) + (a^(2/3)*(-(b^(5/3)*c) - a^(2/3)*b*e + a*b^(2/3)*f + a^(5/3)*h
)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*b^(10/3)) + (a*(-(b*d)
+ a*g)*Log[a + b*x^3])/(3*b^3)
```

Maple [B] time = 0.006, size = 533, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a), x)
```

```
[Out] 1/3/b*d*x^3+1/6*g*x^6/b+1/7*h*x^7/b-1/3*a/b^2*3^(1/2)/(1/b*a)^(1/3)*arctan(
1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c-1/4/b^2*x^4*a*h+1/3*a^2/b^3*ln(b*x^3+a
)*g+1/3/b^3/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*a^2*e-1/6/b^3/(1/b*a)^(2/3)*l
n(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*a^2*e-1/3*a^3/b^4/(1/b*a)^(2/3)*ln(x+(
1/b*a)^(1/3))*h+1/6*a^3/b^4/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2
/3))*h-1/3/b^3*a^2/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*f+1/2/b*x^2*c+1/6/b^3*
a^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f+1/4/b*x^4*e-1/6*a
/b^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+1/3*a/b^2/(1/b*a
)^(1/3)*ln(x+(1/b*a)^(1/3))*c-1/3/b^2*ln(b*x^3+a)*a*d-1/2/b^2*x^2*a*f-1/3/b
^2*x^3*a*g+1/b^3*a^2*h*x+1/5*f*x^5/b-1/b^2*a*e*x-1/3*a^3/b^4/(1/b*a)^(2/3)*
3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*h+1/3/b^3/(1/b*a)^(2/3)*3
^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*a^2*e+1/3/b^3*a^2*3^(1/2)/
(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a), x, algorithm="maxim
a")
```

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")

[Out] Timed out

Sympy [B] time = 29.1659, size = 874, normalized size = 2.64

RootSum($27t^3b^{10} + t^2(-27a^2b^7g + 27ab^8d) + t(-9a^4b^4fh + 9a^4b^4g^2 + 9a^3b^5ch - 18a^3b^5dg + 9a^3b^5ef - 9a^2b^6ce + 9a^2b^6d^2) + a^7h^3 - 3a^6b^2e^2 + 3a^6b^2fg^2 - a^6b^2g^3 - 3a^5b^2c^2g^2 - 3a^5b^2d^2fh + 3a^5b^2d^2g^2 + 3a^5b^2e^2h - 3a^5b^2efg + a^5b^2f^3 + 3a^4b^3cdh + 3a^4b^3c^2eg - 3a^4b^3c^2f^2 - 3a^4b^3d^2g + 3a^4b^3d^2ef - a^4b^3e^3 + 3a^3b^4c^2f - 3a^3b^4c^2d^2e + a^3b^4d^3 - a^2b^5c^3$, Lambda(_t, _t*log(x + (-9*_t^2*a*b^7*f + 9*_t^2*b^8*c - 3*_t*a^4*b^3*h^2 + 6*_t*a^3*b^4*e*h + 6*_t*a^3*b^4*f*g - 6*_t*a^2*b^5*c*g - 6*_t*a^2*b^5*d*f - 3*_t*a^2*b^5*e^2 + 6*_t*a*b^6*c*d + a^6*g^2h^2 - a^5*b*d^2h^2 - 2*a^5*b^2e*g^2h + 2*a^5*b^2f^2h - a^5*b^2fg^2 - 4*a^4*b^2c^2fh + a^4*b^2c^2g^2 + 2*a^4*b^2d^2eh + 2*a^4*b^2d^2fg + a^4*b^2e^2g - 2*a^4*b^2ef^2 + 2*a^3*b^3c^2h - 2*a^3*b^3c^2dg + 4*a^3*b^3c^2ef - a^3*b^3d^2f - a^3*b^3d^2e^2 - 2*a^2*b^4c^2e + a^2*b^4c^2d^2)/(a^6*h^3 - 3*a^5*b^2e^2h + 3*a^4*b^2e^2h - a^4*b^2f^3 + 3*a^3*b^3c^2f^2 - a^3*b^3e^3 - 3*a^2*b^4c^2f + a*b^5c^3)))) + f*x^5/(5*b)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)

[Out] RootSum($27*_t^3*b^{10} + _t^2*(-27*a^{**2}*b^{**7}*g + 27*a*b^{**8}*d) + _t*(-9*a^{**4}*b^{**4}*f*h + 9*a^{**4}*b^{**4}*g^{**2} + 9*a^{**3}*b^{**5}*c*h - 18*a^{**3}*b^{**5}*d*g + 9*a^{**3}*b^{**5}*e*f - 9*a^{**2}*b^{**6}*c*e + 9*a^{**2}*b^{**6}*d^{**2}) + a^{**7}*h^{**3} - 3*a^{**6}*b^2*e^2 + 3*a^{**6}*b^2*f*g^2 - a^{**6}*b^2*g^3 - 3*a^{**5}*b^{**2}*c^2*g^2 - 3*a^{**5}*b^{**2}*d^2*f*h + 3*a^{**5}*b^{**2}*d^2*g^2 + 3*a^{**5}*b^{**2}*e^2*h - 3*a^{**5}*b^{**2}*e*f*g + a^{**5}*b^{**2}*f^3 + 3*a^{**4}*b^{**3}*c*d*h + 3*a^{**4}*b^{**3}*c^2*e*g - 3*a^{**4}*b^{**3}*c^2*f^2 - 3*a^{**4}*b^{**3}*d^2*g + 3*a^{**4}*b^{**3}*d^2*e*f - a^{**4}*b^{**3}*e^3 + 3*a^{**3}*b^{**4}*c^2*f - 3*a^{**3}*b^{**4}*c^2*d^2*e + a^{**3}*b^{**4}*d^3 - a^{**2}*b^{**5}*c^3$, Lambda(_t, _t*log(x + (-9*_t^2*a*b^7*f + 9*_t^2*b^8*c - 3*_t*a^4*b^3*h^2 + 6*_t*a^3*b^4*e*h + 6*_t*a^3*b^4*f*g - 6*_t*a^2*b^5*c*g - 6*_t*a^2*b^5*d*f - 3*_t*a^2*b^5*e^2 + 6*_t*a*b^6*c*d + a^6*g^2h^2 - a^5*b*d^2h^2 - 2*a^5*b^2e*g^2h + 2*a^5*b^2f^2h - a^5*b^2fg^2 - 4*a^4*b^2c^2fh + a^4*b^2c^2g^2 + 2*a^4*b^2d^2eh + 2*a^4*b^2d^2fg + a^4*b^2e^2g - 2*a^4*b^2ef^2 + 2*a^3*b^3c^2h - 2*a^3*b^3c^2dg + 4*a^3*b^3c^2ef - a^3*b^3d^2f - a^3*b^3d^2e^2 - 2*a^2*b^4c^2e + a^2*b^4c^2d^2)/(a^6*h^3 - 3*a^5*b^2e^2h + 3*a^4*b^2e^2h - a^4*b^2f^3 + 3*a^3*b^3c^2f^2 - a^3*b^3e^3 - 3*a^2*b^4c^2f + a*b^5c^3)))) + f*x^5/(5*b)

$$+ g*x**6/(6*b) + h*x**7/(7*b) - x**4*(a*h - b*e)/(4*b**2) - x**3*(a*g - b*d)/(3*b**2) - x**2*(a*f - b*c)/(2*b**2) + x*(a**2*h - a*b*e)/b**3$$

Giac [A] time = 1.07807, size = 513, normalized size = 1.55

$$\frac{(abd - a^2g) \log(|bx^3 + a|)}{3b^3} - \frac{\sqrt{3} \left((-ab^2)^{\frac{1}{3}} a^2h - (-ab^2)^{\frac{1}{3}} abe - (-ab^2)^{\frac{2}{3}} bc + (-ab^2)^{\frac{2}{3}} af \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{3b^4} - \left(\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*(a*b*d - a^2*g)*log(abs(b*x^3 + a))/b^3 - 1/3*sqrt(3)*((-a*b^2)^(1/3)*
a^2*h - (-a*b^2)^(1/3)*a*b*e - (-a*b^2)^(2/3)*b*c + (-a*b^2)^(2/3)*a*f)*arc
tan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/6*((-a*b^2)^(1/3)
)*a^2*h - (-a*b^2)^(1/3)*a*b*e + (-a*b^2)^(2/3)*b*c - (-a*b^2)^(2/3)*a*f)*l
og(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 1/420*(60*b^6*h*x^7 + 70*b^6*
g*x^6 + 84*b^6*f*x^5 - 105*a*b^5*h*x^4 + 105*b^6*x^4*e + 140*b^6*d*x^3 - 14
0*a*b^5*g*x^3 + 210*b^6*c*x^2 - 210*a*b^5*f*x^2 + 420*a^2*b^4*h*x - 420*a*b
^5*x*e)/b^7 + 1/3*(a*b^14*c*(-a/b)^(1/3) - a^2*b^13*f*(-a/b)^(1/3) + a^3*b^
12*h - a^2*b^13*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^15)
```

$$3.404 \quad \int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3} dx$$

Optimal. Leaf size=313

$$\frac{\sqrt[3]{a} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2) (\sqrt[3]{b}(bc - af) - \sqrt[3]{a}(bd - ag))}{6b^{8/3}} + \frac{\sqrt[3]{a} \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right) (a^{4/3}(-g) + \sqrt[3]{abd} - a\sqrt[3]{bf} + b^{4/3}c)}{\sqrt{3}b^{8/3}}$$

[Out] ((b*c - a*f)*x)/b^2 + ((b*d - a*g)*x^2)/(2*b^2) + ((b*e - a*h)*x^3)/(3*b^2) + (f*x^4)/(4*b) + (g*x^5)/(5*b) + (h*x^6)/(6*b) + (a^(1/3)*(b^(4/3)*c + a^(1/3)*b*d - a*b^(1/3)*f - a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*b^(8/3)) - (a^(1/3)*(b^(1/3)*(b*c - a*f) - a^(1/3)*(b*d - a*g))*Log[a^(1/3) + b^(1/3)*x]/(3*b^(8/3)) + (a^(1/3)*(b^(1/3)*(b*c - a*f) - a^(1/3)*(b*d - a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*b^(8/3)) - (a*(b*e - a*h)*Log[a + b*x^3]/(3*b^3))

Rubi [A] time = 0.988245, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1836, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\sqrt[3]{a} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2) (\sqrt[3]{b}(bc - af) - \sqrt[3]{a}(bd - ag))}{6b^{8/3}} + \frac{\sqrt[3]{a} \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right) (a^{4/3}(-g) + \sqrt[3]{abd} - a\sqrt[3]{bf} + b^{4/3}c)}{\sqrt{3}b^{8/3}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3), x]

[Out] ((b*c - a*f)*x)/b^2 + ((b*d - a*g)*x^2)/(2*b^2) + ((b*e - a*h)*x^3)/(3*b^2) + (f*x^4)/(4*b) + (g*x^5)/(5*b) + (h*x^6)/(6*b) + (a^(1/3)*(b^(4/3)*c + a^(1/3)*b*d - a*b^(1/3)*f - a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*b^(8/3)) - (a^(1/3)*(b^(1/3)*(b*c - a*f) - a^(1/3)*(b*d - a*g))*Log[a^(1/3) + b^(1/3)*x]/(3*b^(8/3)) + (a^(1/3)*(b^(1/3)*(b*c - a*f) - a^(1/3)*(b*d - a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*b^(8/3)) - (a*(b*e - a*h)*Log[a + b*x^3]/(3*b^3))

Rule 1836

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*

$(m + q - n + 1)x^{(q - n)}, x] * (a + b*x^n)^p, x], x] + \text{Simp}[(\text{Pqq}*(c*x)^{(m + q - n + 1)}*(a + b*x^n)^{(p + 1)})/(b*c^{(q - n + 1)}*(m + q + n*p + 1)), x]] /;$
 $\text{NeQ}[m + q + n*p + 1, 0] \ \&\& \ q - n \geq 0 \ \&\& \ (\text{IntegerQ}[2*p] \ || \ \text{IntegerQ}[p + (q + 1)/(2*n)])] /;$ FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]

Rule 1887

$\text{Int}[(\text{Pq}_)/((a_) + (b_)*(x_)^n), x_Symbol] \ :> \ \text{Int}[\text{ExpandIntegrand}[\text{Pq}/(a + b*x^n), x], x] /;$ FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]

Rule 1871

$\text{Int}[(\text{P2}_)/((a_) + (b_)*(x_)^3), x_Symbol] \ :> \ \text{With}[\{A = \text{Coeff}[\text{P2}, x, 0], B = \text{Coeff}[\text{P2}, x, 1], C = \text{Coeff}[\text{P2}, x, 2]\}, \text{Int}[(A + B*x)/(a + b*x^3), x] + \text{Dist}[C, \text{Int}[x^2/(a + b*x^3), x], x] /;$ EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

Rule 1860

$\text{Int}[(\text{A}_) + (\text{B}_)*(x_)]/((a_) + (b_)*(x_)^3), x_Symbol] \ :> \ \text{With}[\{r = \text{Numerator}[\text{Rt}[a/b, 3]], s = \text{Denominator}[\text{Rt}[a/b, 3]]\}, -\text{Dist}[(r*(B*r - A*s))/(3*a*s), \text{Int}[1/(r + s*x), x], x] + \text{Dist}[r/(3*a*s), \text{Int}[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /;$ FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]

Rule 31

$\text{Int}[(\text{A}_) + (\text{B}_)*(x_)^{-1}), x_Symbol] \ :> \ \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /;$ FreeQ[{a, b}, x]

Rule 634

$\text{Int}[(\text{d}_) + (\text{e}_)*(x_)]/((a_) + (b_)*(x_) + (\text{c}_)*(x_)^2), x_Symbol] \ :> \ \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /;$ FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 617

$\text{Int}[(\text{A}_) + (\text{B}_)*(x_) + (\text{C}_)*(x_)^2]^{-1}, x_Symbol] \ :> \ \text{With}[\{q = 1 - 4*S\text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /;$ RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{a + bx^3} dx &= \frac{hx^6}{6b} + \frac{\int \frac{x^3(6bc + 6bdx + 6(be-ah)x^2 + 6bfx^3 + 6bgx^4)}{a+bx^3} dx}{6b} \\
&= \frac{gx^5}{5b} + \frac{hx^6}{6b} + \frac{\int \frac{x^3(30b^2c + 30b(bd-ag)x + 30b(be-ah)x^2 + 30b^2fx^3)}{a+bx^3} dx}{30b^2} \\
&= \frac{fx^4}{4b} + \frac{gx^5}{5b} + \frac{hx^6}{6b} + \frac{\int \frac{x^3(120b^2(bc-af) + 120b^2(bd-ag)x + 120b^2(be-ah)x^2)}{a+bx^3} dx}{120b^3} \\
&= \frac{fx^4}{4b} + \frac{gx^5}{5b} + \frac{hx^6}{6b} + \frac{\int (120b(bc-af) + 120b(bd-ag)x + 120b(be-ah)x^2)}{120b^3} \\
&= \frac{(bc-af)x}{b^2} + \frac{(bd-ag)x^2}{2b^2} + \frac{(be-ah)x^3}{3b^2} + \frac{fx^4}{4b} + \frac{gx^5}{5b} + \frac{hx^6}{6b} - \frac{\int \frac{ab(bc-af)}{a+bx^3} dx}{120b^3} \\
&= \frac{(bc-af)x}{b^2} + \frac{(bd-ag)x^2}{2b^2} + \frac{(be-ah)x^3}{3b^2} + \frac{fx^4}{4b} + \frac{gx^5}{5b} + \frac{hx^6}{6b} - \frac{\int \frac{ab(bc-af)}{a+bx^3} dx}{120b^3} \\
&= \frac{(bc-af)x}{b^2} + \frac{(bd-ag)x^2}{2b^2} + \frac{(be-ah)x^3}{3b^2} + \frac{fx^4}{4b} + \frac{gx^5}{5b} + \frac{hx^6}{6b} - \frac{a(be-ah)}{120b^3} \\
&= \frac{(bc-af)x}{b^2} + \frac{(bd-ag)x^2}{2b^2} + \frac{(be-ah)x^3}{3b^2} + \frac{fx^4}{4b} + \frac{gx^5}{5b} + \frac{hx^6}{6b} - \frac{\sqrt[3]{a}(\sqrt[3]{b}(bc-af))}{120b^3} \\
&= \frac{(bc-af)x}{b^2} + \frac{(bd-ag)x^2}{2b^2} + \frac{(be-ah)x^3}{3b^2} + \frac{fx^4}{4b} + \frac{gx^5}{5b} + \frac{hx^6}{6b} - \frac{\sqrt[3]{a}(\sqrt[3]{b}(bc-af))}{120b^3} \\
&= \frac{(bc-af)x}{b^2} + \frac{(bd-ag)x^2}{2b^2} + \frac{(be-ah)x^3}{3b^2} + \frac{fx^4}{4b} + \frac{gx^5}{5b} + \frac{hx^6}{6b} + \frac{\sqrt[3]{a}(b^{4/3}c - a^2)}{120b^3}
\end{aligned}$$

Mathematica [A] time = 0.238224, size = 299, normalized size = 0.96

$$10\sqrt[3]{a}\sqrt[3]{b}\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^{4/3}g - \sqrt[3]{abd} - a\sqrt[3]{bf} + b^{4/3}c) - 20\sqrt[3]{a}\sqrt[3]{b}\log(\sqrt[3]{a} + \sqrt[3]{bx})(a^{4/3}g - \sqrt[3]{abd} - a\sqrt[3]{bf})$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3), x]

```
[Out] (60*b*(b*c - a*f)*x + 30*b*(b*d - a*g)*x^2 + 20*b*(b*e - a*h)*x^3 + 15*b^2*
f*x^4 + 12*b^2*g*x^5 + 10*b^2*h*x^6 - 20*Sqrt[3]*a^(1/3)*b^(1/3)*(-(b^(4/3)
*c) - a^(1/3)*b*d + a*b^(1/3)*f + a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]] - 20*a^(1/3)*b^(1/3)*(b^(4/3)*c - a^(1/3)*b*d - a*b^(1/3)*f +
a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x] + 10*a^(1/3)*b^(1/3)*(b^(4/3)*c - a^(1
/3)*b*d - a*b^(1/3)*f + a^(4/3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2] + 20*a*(-(b*e) + a*h)*Log[a + b*x^3])/(60*b^3)
```

Maple [B] time = 0.005, size = 505, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a), x)
```

```
[Out] 1/6*h*x^6/b+1/5*g*x^5/b+1/4*f*x^4/b-1/3/b^2*x^3*a*h+1/3*e*x^3/b-1/2/b^2*x^2
*a*g+1/2*d*x^2/b-1/b^2*a*f*x+c*x/b+1/3/b^3*a^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(
1/3))*f-1/3*a/b^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c-1/6/b^3*a^2/(1/b*a)^(
2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f+1/6*a/b^2/(1/b*a)^(2/3)*ln(x^2
-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+1/3/b^3*a^2/(1/b*a)^(2/3)*3^(1/2)*arctan(
1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f-1/3*a/b^2/(1/b*a)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c-1/3/b^3*a^2/(1/b*a)^(1/3)*ln(x+(1/b*a)
^(1/3))*g+1/3/b^2/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*a*d+1/6/b^3*a^2/(1/b*a)
^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*g-1/6/b^2/(1/b*a)^(1/3)*ln(x^
2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*a*d+1/3/b^3*a^2*3^(1/2)/(1/b*a)^(1/3)*arct
an(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*g-1/3/b^2*3^(1/2)/(1/b*a)^(1/3)*arcta
n(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*a*d+1/3/b^3*a^2*ln(b*x^3+a)*h-1/3*a*e*
ln(b*x^3+a)/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a), x, algorithm="maxim
a")
```

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")

[Out] Timed out

Sympy [B] time = 29.5259, size = 842, normalized size = 2.69

RootSum($27t^3b^9 + t^2(-27a^2b^6h + 27ab^7e) + t(9a^4b^3h^2 - 18a^3b^4eh + 9a^3b^4fg - 9a^2b^5cg - 9a^2b^5df + 9a^2b^5e^2 + 9ab^6$)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)

[Out] RootSum($27*_t**3*b**9 + *_t**2*(-27*a**2*b**6*h + 27*a*b**7*e) + *_t*(9*a**4*b**3*h**2 - 18*a**3*b**4*e*h + 9*a**3*b**4*f*g - 9*a**2*b**5*c*g - 9*a**2*b**5*d*f + 9*a**2*b**5*e**2 + 9*a*b**6*c*d) - a**6*h**3 + 3*a**5*b*e*h**2 - 3*a**5*b*f*g*h + a**5*b*g**3 + 3*a**4*b**2*c*g*h + 3*a**4*b**2*d*f*h - 3*a**4*b**2*d*g**2 - 3*a**4*b**2*e**2*h + 3*a**4*b**2*e*f*g - a**4*b**2*f**3 - 3*a**3*b**3*c*d*h - 3*a**3*b**3*c*e*g + 3*a**3*b**3*c*f**2 + 3*a**3*b**3*d**2*g - 3*a**3*b**3*d*e*f + a**3*b**3*e**3 - 3*a**2*b**4*c**2*f + 3*a**2*b**4*c*d*e - a**2*b**4*d**3 + a*b**5*c**3, Lambda(_t, *_t*log(x + (9*_t**2*a*b**6*g - 9*_t**2*b**7*d - 6*_t*a**3*b**3*g*h + 6*_t*a**2*b**4*d*h + 6*_t*a**2*b**4*e*g + 3*_t*a**2*b**4*f**2 - 6*_t*a*b**5*c*f - 6*_t*a*b**5*d*e + 3*_t*b**6*c**2 + a**5*g*h**2 - a**4*b*d*h**2 - 2*a**4*b*e*g*h - a**4*b*f**2*h + 2*a**4*b*f*g**2 + 2*a**3*b**2*c*f*h - 2*a**3*b**2*c*g**2 + 2*a**3*b**2*d*e*h - 4*a**3*b**2*d*f*g + a**3*b**2*e**2*g + a**3*b**2*e*f**2 - a**2*b**3*c**2*h + 4*a**2*b**3*c*d*g - 2*a**2*b**3*c*e*f + 2*a**2*b**3*d**2*f - a**2*b**3*d*e**2 + a*b**4*c**2*e - 2*a*b**4*c*d**2))/(a**4*b*g**3 - 3*a**3*b**2*d*g**2 + a**3*b**2*f**3 - 3*a**2*b**3*c*f**2 + 3*a**2*b**3*d**2*g + 3*a*b**4*c**2*f - a*b**4*d**3 - b**5*c**3))) + f*x**4/(4*b) + g*x**5/(5*b) + h*x**6/($

$$6*b) - x**3*(a*h - b*e)/(3*b**2) - x**2*(a*g - b*d)/(2*b**2) - x*(a*f - b*c)/b**2$$

Giac [A] time = 1.09472, size = 477, normalized size = 1.52

$$\frac{(a^2h - abe) \log(|bx^3 + a|)}{3b^3} - \frac{\sqrt{3} \left((-ab^2)^{\frac{1}{3}} b^2c - (-ab^2)^{\frac{1}{3}} abf - (-ab^2)^{\frac{2}{3}} bd + (-ab^2)^{\frac{2}{3}} ag \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}} \right)}{3b^4} - \left((-a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")

[Out] 1/3*(a^2*h - a*b*e)*log(abs(b*x^3 + a))/b^3 - 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^2*c - (-a*b^2)^(1/3)*a*b*f - (-a*b^2)^(2/3)*b*d + (-a*b^2)^(2/3)*a*g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/6*((-a*b^2)^(1/3)*b^2*c - (-a*b^2)^(1/3)*a*b*f + (-a*b^2)^(2/3)*b*d - (-a*b^2)^(2/3)*a*g)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 1/60*(10*b^5*h*x^6 + 12*b^5*g*x^5 + 15*b^5*f*x^4 - 20*a*b^4*h*x^3 + 20*b^5*x^3*e + 30*b^5*d*x^2 - 30*a*b^4*g*x^2 + 60*b^5*c*x - 60*a*b^4*f*x)/b^6 + 1/3*(a*b^12*d*(-a/b)^(1/3) - a^2*b^11*g*(-a/b)^(1/3) + a*b^12*c - a^2*b^11*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^13)

$$3.405 \quad \int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3} dx$$

Optimal. Leaf size=294

$$\frac{\sqrt[3]{a} \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(\sqrt[3]{b}(bd-ag) - \sqrt[3]{a}(be-ah)\right)}{6b^{8/3}} + \frac{\sqrt[3]{a} \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-h) + \sqrt[3]{abe} - a\sqrt[3]{bg} + b^{4/3}d\right)}{\sqrt{3}b^{8/3}}$$

[Out] $((b*d - a*g)*x)/b^2 + ((b*e - a*h)*x^2)/(2*b^2) + (f*x^3)/(3*b) + (g*x^4)/(4*b) + (h*x^5)/(5*b) + (a^{1/3}*(b^{4/3}*d + a^{1/3}*b*e - a*b^{1/3}*g - a^{4/3}*h)*\text{ArcTan}[(a^{1/3} - 2*b^{1/3}*x)/(\text{Sqrt}[3]*a^{1/3})]) / (\text{Sqrt}[3]*b^{8/3}) - (a^{1/3}*(b^{1/3}*(b*d - a*g) - a^{1/3}*(b*e - a*h))*\text{Log}[a^{1/3} + b^{1/3}*x]) / (3*b^{8/3}) + (a^{1/3}*(b^{1/3}*(b*d - a*g) - a^{1/3}*(b*e - a*h))*\text{Log}[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2]) / (6*b^{8/3}) + ((b*c - a*f)*\text{Log}[a + b*x^3]) / (3*b^2)$

Rubi [A] time = 0.976263, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1836, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\sqrt[3]{a} \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(\sqrt[3]{b}(bd-ag) - \sqrt[3]{a}(be-ah)\right)}{6b^{8/3}} + \frac{\sqrt[3]{a} \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-h) + \sqrt[3]{abe} - a\sqrt[3]{bg} + b^{4/3}d\right)}{\sqrt{3}b^{8/3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3), x]$

[Out] $((b*d - a*g)*x)/b^2 + ((b*e - a*h)*x^2)/(2*b^2) + (f*x^3)/(3*b) + (g*x^4)/(4*b) + (h*x^5)/(5*b) + (a^{1/3}*(b^{4/3}*d + a^{1/3}*b*e - a*b^{1/3}*g - a^{4/3}*h)*\text{ArcTan}[(a^{1/3} - 2*b^{1/3}*x)/(\text{Sqrt}[3]*a^{1/3})]) / (\text{Sqrt}[3]*b^{8/3}) - (a^{1/3}*(b^{1/3}*(b*d - a*g) - a^{1/3}*(b*e - a*h))*\text{Log}[a^{1/3} + b^{1/3}*x]) / (3*b^{8/3}) + (a^{1/3}*(b^{1/3}*(b*d - a*g) - a^{1/3}*(b*e - a*h))*\text{Log}[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2]) / (6*b^{8/3}) + ((b*c - a*f)*\text{Log}[a + b*x^3]) / (3*b^2)$

Rule 1836

$\text{Int}[(Pq_)*((c_)*(x_))^{\wedge}(m_)*((a_)+(b_)*(x_)^{\wedge}(n_))^{\wedge}(p_), x_Symbol] \rightarrow \text{With}[\{q = \text{Expon}[Pq, x]\}, \text{With}[\{Pqq = \text{Coeff}[Pq, x, q]\}, \text{Dist}[1/(b*(m + q + n*p + 1)), \text{Int}[(c*x)^{\wedge}m*\text{ExpandToSum}[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq^{\wedge}q]$

$(m + q - n + 1)x^{(q - n), x] * (a + b*x^n)^p, x], x] + \text{Simp}[(\text{Pqq}*(c*x)^{(m + q - n + 1)*(a + b*x^n)^{(p + 1))}/(b*c^{(q - n + 1)*(m + q + n*p + 1)}), x]] /;$
 $\text{NeQ}[m + q + n*p + 1, 0] \ \&\& \ q - n \geq 0 \ \&\& \ (\text{IntegerQ}[2*p] \ || \ \text{IntegerQ}[p + (q + 1)/(2*n)])] /;$
 $\text{FreeQ}\{a, b, c, m, p\}, x] \ \&\& \ \text{PolyQ}[\text{Pq}, x] \ \&\& \ \text{IGtQ}[n, 0]$

Rule 1887

$\text{Int}[(\text{Pq}_)/((a_) + (b_)*(x_)^{(n_)}), x_Symbol] \ := \ \text{Int}[\text{ExpandIntegrand}[\text{Pq}/(a + b*x^n), x], x] /;$
 $\text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PolyQ}[\text{Pq}, x] \ \&\& \ \text{IntegerQ}[n]$

Rule 1871

$\text{Int}[(\text{P2}_)/((a_) + (b_)*(x_)^3), x_Symbol] \ := \ \text{With}\{A = \text{Coeff}[\text{P2}, x, 0], B = \text{Coeff}[\text{P2}, x, 1], C = \text{Coeff}[\text{P2}, x, 2]\}, \text{Int}[(A + B*x)/(a + b*x^3), x] + \text{Dist}[C, \text{Int}[x^2/(a + b*x^3), x], x] /;$
 $\text{EqQ}[a*B^3 - b*A^3, 0] \ || \ \text{!RationalQ}[a/b]] /;$
 $\text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PolyQ}[\text{P2}, x, 2]$

Rule 1860

$\text{Int}[(A_) + (B_)*(x_)]/((a_) + (b_)*(x_)^3), x_Symbol] \ := \ \text{With}\{r = \text{Numerator}[\text{Rt}[a/b, 3]], s = \text{Denominator}[\text{Rt}[a/b, 3]]\}, -\text{Dist}[(r*(B*r - A*s))/(3*a*s), \text{Int}[1/(r + s*x), x], x] + \text{Dist}[r/(3*a*s), \text{Int}[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x] /;$
 $\text{FreeQ}\{a, b, A, B\}, x] \ \&\& \ \text{NeQ}[a*B^3 - b*A^3, 0] \ \&\& \ \text{PosQ}[a/b]$

Rule 31

$\text{Int}[(a_) + (b_)*(x_)]^{(-1)}, x_Symbol] \ := \ \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /;$
 $\text{FreeQ}\{a, b\}, x]$

Rule 634

$\text{Int}[(d_) + (e_)*(x_)]/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] \ := \ \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /;$
 $\text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rule 617

$\text{Int}[(a_) + (b_)*(x_) + (c_)*(x_)^2]^{(-1)}, x_Symbol] \ := \ \text{With}\{q = 1 - 4*S\text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /;$
 $\text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ \text{!RationalQ}[b^2 - 4*a*c]) /;$
 $\text{FreeQ}\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{a + bx^3} dx &= \frac{hx^5}{5b} + \frac{\int \frac{x^2(5bc + 5bdx + 5(be-ah)x^2 + 5bf x^3 + 5bgx^4)}{a + bx^3} dx}{5b} \\
&= \frac{gx^4}{4b} + \frac{hx^5}{5b} + \frac{\int \frac{x^2(20b^2c + 20b(bd-ag)x + 20b(be-ah)x^2 + 20b^2fx^3)}{a + bx^3} dx}{20b^2} \\
&= \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} + \frac{\int \frac{x^2(60b^2(bc-af) + 60b^2(bd-ag)x + 60b^2(be-ah)x^2)}{a + bx^3} dx}{60b^3} \\
&= \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} + \frac{\int \left(60b(bd-ag) + 60b(be-ah)x - \frac{60(ab(bd-ag) + ab(be-ah)x)}{a + bx^3} \right) dx}{60b^3} \\
&= \frac{(bd-ag)x}{b^2} + \frac{(be-ah)x^2}{2b^2} + \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} - \frac{\int \frac{ab(bd-ag) + ab(be-ah)x - b^2(bc-af)}{a + bx^3} dx}{b^3} \\
&= \frac{(bd-ag)x}{b^2} + \frac{(be-ah)x^2}{2b^2} + \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} - \frac{\int \frac{ab(bd-ag) + ab(be-ah)x}{a + bx^3} dx}{b^3} + \frac{(bc-af)}{3b^2} \log(a + bx^3) \\
&= \frac{(bd-ag)x}{b^2} + \frac{(be-ah)x^2}{2b^2} + \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} + \frac{(bc-af) \log(a + bx^3)}{3b^2} - \frac{\int \frac{ab(bd-ag) + ab(be-ah)x}{a + bx^3} dx}{b^3} \\
&= \frac{(bd-ag)x}{b^2} + \frac{(be-ah)x^2}{2b^2} + \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} - \frac{\sqrt[3]{a} \left(\sqrt[3]{b}(bd-ag) - \sqrt[3]{a}(be-ah) \right)}{3b^{8/3}} \\
&= \frac{(bd-ag)x}{b^2} + \frac{(be-ah)x^2}{2b^2} + \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} - \frac{\sqrt[3]{a} \left(\sqrt[3]{b}(bd-ag) - \sqrt[3]{a}(be-ah) \right)}{3b^{8/3}} \\
&= \frac{(bd-ag)x}{b^2} + \frac{(be-ah)x^2}{2b^2} + \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} + \frac{\sqrt[3]{a} \left(b^{4/3}d + \sqrt[3]{abe} - a\sqrt[3]{bg} \right)}{\sqrt{3b}}
\end{aligned}$$

Mathematica [A] time = 0.26219, size = 290, normalized size = 0.99

$$10\sqrt[3]{a} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2) (a^{4/3}h - \sqrt[3]{abe} - a\sqrt[3]{bg} + b^{4/3}d) + 20\sqrt[3]{a} \log(\sqrt[3]{a} + \sqrt[3]{bx}) (a^{4/3}(-h) + \sqrt[3]{abe} + a\sqrt[3]{bg} - b^{4/3}d)$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3), x]

```
[Out] (60*b^(2/3)*(b*d - a*g)*x + 30*b^(2/3)*(b*e - a*h)*x^2 + 20*b^(5/3)*f*x^3 +
  15*b^(5/3)*g*x^4 + 12*b^(5/3)*h*x^5 - 20*sqrt[3]*a^(1/3)*(-(b^(4/3)*d) - a
  ^^(1/3)*b*e + a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
  rt[3]] + 20*a^(1/3)*(-(b^(4/3)*d) + a^(1/3)*b*e + a*b^(1/3)*g - a^(4/3)*h)*
  Log[a^(1/3) + b^(1/3)*x] + 10*a^(1/3)*(b^(4/3)*d - a^(1/3)*b*e - a*b^(1/3)*
  g + a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 20*b^(2/3)*
  (b*c - a*f)*Log[a + b*x^3])/(60*b^(8/3))
```

Maple [B] time = 0.003, size = 483, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a), x)
```

```
[Out] 1/5*h*x^5/b+1/4*g*x^4/b+1/3*f*x^3/b-1/2/b^2*x^2*a*h+1/2*e*x^2/b-1/b^2*a*g*x
+d*x/b+1/3/b^3/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*a^2*g-1/3/b^2/(1/b*a)^(2/3
)*ln(x+(1/b*a)^(1/3))*a*d-1/6/b^3/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b
*a)^(2/3))*a^2*g+1/6/b^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3
))*a*d+1/3/b^3/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1
))*a^2*g-1/3/b^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*
x-1))*a*d-1/3/b^3/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*a^2*h+1/3/b^2*a/(1/b*a)
^(1/3)*ln(x+(1/b*a)^(1/3))*e+1/6/b^3/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(
1/b*a)^(2/3))*a^2*h-1/6/b^2*a/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(
2/3))*e+1/3/b^3*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*
x-1))*a^2*h-1/3/b^2*a*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(
1/3)*x-1))*e-1/3*a/b^2*ln(b*x^3+a)*f+1/3*c*ln(b*x^3+a)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a), x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")

[Out] Timed out

Sympy [B] time = 63.1816, size = 789, normalized size = 2.68

RootSum($27t^3b^8 + t^2(27ab^6f - 27b^7c) + t(9a^3b^3gh - 9a^2b^4dh - 9a^2b^4eg + 9a^2b^4f^2 - 18ab^5cf + 9ab^5de + 9b^6c^2) + a^5$)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)

[Out] RootSum(27*_t**3*b**8 + _t**2*(27*a*b**6*f - 27*b**7*c) + _t*(9*a**3*b**3*g*h - 9*a**2*b**4*d*h - 9*a**2*b**4*e*g + 9*a**2*b**4*f**2 - 18*a*b**5*c*f + 9*a*b**5*d*e + 9*b**6*c**2) + a**5*h**3 - 3*a**4*b*e*h**2 + 3*a**4*b*f*g*h - a**4*b*g**3 - 3*a**3*b**2*c*g*h - 3*a**3*b**2*d*f*h + 3*a**3*b**2*d*g**2 + 3*a**3*b**2*e**2*h - 3*a**3*b**2*e*f*g + a**3*b**2*f**3 + 3*a**2*b**3*c*d*h + 3*a**2*b**3*c*e*g - 3*a**2*b**3*c*f**2 - 3*a**2*b**3*d**2*g + 3*a**2*b**3*d*e*f - a**2*b**3*e**3 + 3*a*b**4*c**2*f - 3*a*b**4*c*d*e + a*b**4*d**3 - b**5*c**3, Lambda(_t, _t*log(x + (9*_t**2*a*b**5*h - 9*_t**2*b**6*e + 6*_t*a**2*b**3*f*h + 3*_t*a**2*b**3*g**2 - 6*_t*a*b**4*c*h - 6*_t*a*b**4*d*g - 6*_t*a*b**4*e*f + 6*_t*b**5*c*e + 3*_t*b**5*d**2 + 2*a**4*g*h**2 - 2*a**3*b*d*h**2 - 4*a**3*b*e*g*h + a**3*b*f**2*h + a**3*b*f*g**2 - 2*a**2*b**2*c*f*h - a**2*b**2*c*g**2 + 4*a**2*b**2*d*e*h - 2*a**2*b**2*d*f*g + 2*a**2*b**2*e**2*g - a**2*b**2*e*f**2 + a*b**3*c**2*h + 2*a*b**3*c*d*g + 2*a*b**3*c*e*f + a*b**3*d**2*f - 2*a*b**3*d*e**2 - b**4*c**2*e - b**4*c*d**2)/(a**4*h**3 - 3*a**3*b*e*h**2 + a**3*b*g**3 - 3*a**2*b**2*d*g**2 + 3*a**2*b**2*e**2*h + 3*a*b**3*d**2*g - a*b**3*e**3 - b**4*d**3))) + f*x**3/(3*b) + g*x**4/(4*b) + h*x**5/(5*b) - x**2*(a*h - b*e)/(2*b**2) - x*(a*g - b*d)/b**2

Giac [A] time = 1.08309, size = 450, normalized size = 1.53

$$\frac{(bc - af) \log(|bx^3 + a|)}{3b^2} - \frac{\sqrt{3} \left((-ab^2)^{\frac{1}{3}} b^2 d - (-ab^2)^{\frac{1}{3}} abg + (-ab^2)^{\frac{2}{3}} ah - (-ab^2)^{\frac{2}{3}} be \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{3b^4} - \frac{\left((-ab^2)^{\frac{1}{3}} \right)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*(b*c - a*f)*log(abs(b*x^3 + a))/b^2 - 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^2*d
- (-a*b^2)^(1/3)*a*b*g + (-a*b^2)^(2/3)*a*h - (-a*b^2)^(2/3)*b*e)*arctan(1
/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/6*((-a*b^2)^(1/3)*b^2
*d - (-a*b^2)^(1/3)*a*b*g - (-a*b^2)^(2/3)*a*h + (-a*b^2)^(2/3)*b*e)*log(x^
2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 1/60*(12*b^4*h*x^5 + 15*b^4*g*x^4
+ 20*b^4*f*x^3 - 30*a*b^3*h*x^2 + 30*b^4*x^2*e + 60*b^4*d*x - 60*a*b^3*g*x)
/b^5 - 1/3*(a^2*b^9*h*(-a/b)^(1/3) - a*b^10*(-a/b)^(1/3)*e - a*b^10*d + a^2
*b^9*g)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^11)
```

$$3.406 \quad \int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3} dx$$

Optimal. Leaf size=275

$$\frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^{2/3}(be-ah) + b^{2/3}(bc-af))}{6\sqrt[3]{ab^{7/3}}} - \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(a^{2/3}(be-ah) + b^{2/3}(bc-af))}{3\sqrt[3]{ab^{7/3}}} - \tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{bx}}{\sqrt[3]{ab^{7/3}}}\right)$$

[Out] ((b*e - a*h)*x)/b^2 + (f*x^2)/(2*b) + (g*x^3)/(3*b) + (h*x^4)/(4*b) - ((b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3)*b^(7/3)) - ((b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*b^(7/3)) + ((b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(7/3)) + ((b*d - a*g)*Log[a + b*x^3])/(3*b^2)

Rubi [A] time = 0.921478, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1836, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^{2/3}(be-ah) + b^{2/3}(bc-af))}{6\sqrt[3]{ab^{7/3}}} - \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(a^{2/3}(be-ah) + b^{2/3}(bc-af))}{3\sqrt[3]{ab^{7/3}}} - \tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{bx}}{\sqrt[3]{ab^{7/3}}}\right)$$

Antiderivative was successfully verified.

[In] Int[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3), x]

[Out] ((b*e - a*h)*x)/b^2 + (f*x^2)/(2*b) + (g*x^3)/(3*b) + (h*x^4)/(4*b) - ((b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3)*b^(7/3)) - ((b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*b^(7/3)) + ((b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(7/3)) + ((b*d - a*g)*Log[a + b*x^3])/(3*b^2)

Rule 1836

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m + q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;

NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]

Rule 1887

Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Pq/(a + b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]

Rule 1871

Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Dist[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

Rule 1860

Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]

Rule 31

Int[((a_) + (b_)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{a + bx^3} dx &= \frac{hx^4}{4b} + \frac{\int \frac{x(4bc + 4bdx + 4(be-ah)x^2 + 4bfx^3 + 4bgx^4)}{a+bx^3} dx}{4b} \\
&= \frac{gx^3}{3b} + \frac{hx^4}{4b} + \frac{\int \frac{x(12b^2c + 12b(bd-ag)x + 12b(be-ah)x^2 + 12b^2fx^3)}{a+bx^3} dx}{12b^2} \\
&= \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} + \frac{\int \frac{x(24b^2(bc-af) + 24b^2(bd-ag)x + 24b^2(be-ah)x^2)}{a+bx^3} dx}{24b^3} \\
&= \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} + \frac{\int \left(24b(be-ah) - \frac{24(ab(be-ah) - b^2(bc-af)x - b^2(bd-ag)x^2)}{a+bx^3} \right) dx}{24b^3} \\
&= \frac{(be-ah)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} - \frac{\int \frac{ab(be-ah) - b^2(bc-af)x - b^2(bd-ag)x^2}{a+bx^3} dx}{b^3} \\
&= \frac{(be-ah)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} - \frac{\int \frac{ab(be-ah) - b^2(bc-af)x}{a+bx^3} dx}{b^3} + \frac{(bd-ag) \int \frac{x^2}{a+bx^3} dx}{b} \\
&= \frac{(be-ah)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} + \frac{(bd-ag) \log(a+bx^3)}{3b^2} - \frac{\int \frac{\sqrt[3]{a}(-\sqrt[3]{ab^2(bc-af)} - \sqrt[3]{ab^2(bc-af)})}{a+bx^3} dx}{3b^2} \\
&= \frac{(be-ah)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} - \frac{(b^{2/3}(bc-af) + a^{2/3}(be-ah)) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3\sqrt[3]{ab^{7/3}}} \\
&= \frac{(be-ah)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} - \frac{(b^{2/3}(bc-af) + a^{2/3}(be-ah)) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3\sqrt[3]{ab^{7/3}}} \\
&= \frac{(be-ah)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} - \frac{(b^{5/3}c - a^{2/3}be - ab^{2/3}f + a^{5/3}h) \tan^{-1}\left(\frac{\sqrt[3]{bx}}{\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{ab^{7/3}}}
\end{aligned}$$

Mathematica [A] time = 0.372748, size = 272, normalized size = 0.99

$$\frac{2 \log\left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2\right) \left(a^{2/3} be + a^{5/3}(-h) - ab^{2/3} f + b^{5/3} c\right)}{\sqrt[3]{a}} + \frac{4 \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right) \left(-a^{2/3} be + a^{5/3} h + ab^{2/3} f - b^{5/3} c\right)}{\sqrt[3]{a}} - \frac{4\sqrt{3} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{bx}}{\sqrt[3]{a}}\right) \left(-a^{2/3} be + a^{5/3} h - ab^{2/3} f + b^{5/3} c\right)}{\sqrt[3]{a}}$$

12b^{7/3}

Antiderivative was successfully verified.

[In] Integrate[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3), x]

```
[Out] (12*b^(1/3)*(b*e - a*h)*x + 6*b^(4/3)*f*x^2 + 4*b^(4/3)*g*x^3 + 3*b^(4/3)*h*x^4 - (4*sqrt[3]*(b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]])/a^(1/3) + (4*(-(b^(5/3)*c) - a^(2/3)*b*e + a*b^(2/3)*f + a^(5/3)*h)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + (2*(b^(5/3)*c + a^(2/3)*b*e - a*b^(2/3)*f - a^(5/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(1/3) + 4*b^(1/3)*(b*d - a*g)*Log[a + b*x^3]/(12*b^(7/3))
```

Maple [B] time = 0.003, size = 455, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a), x)
```

```
[Out] 1/4*h*x^4/b+1/3*g*x^3/b+1/2*f*x^2/b-1/b^2*a*h*x+e*x/b+1/3/b^3/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*a^2*h-1/3/b^2*a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*e-1/6/b^3/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*a^2*h+1/6/b^2*a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e+1/3/b^3/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*a^2*h-1/3/b^2*a/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e+1/3/b^2*a/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*f-1/3/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*c-1/6/b^2*a/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f+1/6/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c-1/3/b^2*a*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f+1/3/b*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c-1/3/b^2*ln(b*x^3+a)*a*g+1/3*d*ln(b*x^3+a)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a), x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")

[Out] Timed out

Sympy [B] time = 33.0156, size = 811, normalized size = 2.95

RootSum($27t^3ab^7 + t^2(27a^2b^5g - 27ab^6d) + t(-9a^3b^3fh + 9a^3b^3g^2 + 9a^2b^4ch - 18a^2b^4dg + 9a^2b^4ef - 9ab^5ce + 9ab^5d)$)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)

[Out] RootSum($27_t**3*a*b**7 + _t**2*(27*a**2*b**5*g - 27*a*b**6*d) + _t*(-9*a**3*b**3*f*h + 9*a**3*b**3*g**2 + 9*a**2*b**4*c*h - 18*a**2*b**4*d*g + 9*a**2*b**4*e*f - 9*a*b**5*c*e + 9*a*b**5*d**2) - a**5*h**3 + 3*a**4*b*e*h**2 - 3*a**4*b*f*g*h + a**4*b*g**3 + 3*a**3*b**2*c*g*h + 3*a**3*b**2*d*f*h - 3*a**3*b**2*d*g**2 - 3*a**3*b**2*e**2*h + 3*a**3*b**2*e*f*g - a**3*b**2*f**3 - 3*a**2*b**3*c*d*h - 3*a**2*b**3*c*e*g + 3*a**2*b**3*c*f**2 + 3*a**2*b**3*d**2*g - 3*a**2*b**3*d*e*f + a**2*b**3*e**3 - 3*a*b**4*c**2*f + 3*a*b**4*c*d*e - a*b**4*d**3 + b**5*c**3, Lambda(_t, _t*log(x + (-9*_t**2*a**2*b**5*f + 9*_t**2*a*b**6*c + 3*_t*a**4*b**2*h**2 - 6*_t*a**3*b**3*e*h - 6*_t*a**3*b**3*f*g + 6*_t*a**2*b**4*c*g + 6*_t*a**2*b**4*d*f + 3*_t*a**2*b**4*e**2 - 6*_t*a*b**5*c*d + a**5*g*h**2 - a**4*b*d*h**2 - 2*a**4*b*e*g*h + 2*a**4*b*f**2*h - a**4*b*f*g**2 - 4*a**3*b**2*c*f*h + a**3*b**2*c*g**2 + 2*a**3*b**2*d*e*h + 2*a**3*b**2*d*f*g + a**3*b**2*e**2*g - 2*a**3*b**2*e*f**2 + 2*a**2*b**3*c**2*h - 2*a**2*b**3*c*d*g + 4*a**2*b**3*c*e*f - a**2*b**3*d**2*f - a**2*b**3*d*e**2 - 2*a*b**4*c**2*e + a*b**4*c*d**2)/(a**5*h**3 - 3*a**4*b*e*h**2 + 3*a**3*b**2*e**2*h - a**3*b**2*f**3 + 3*a**2*b**3*c*f**2 - a**2*b**3*e**3 - 3*a*b**4*c**2*f + b**5*c**3))) + f*x**2/(2*b) + g*x**3/(3*b) + h*x**4/(4*b) - x*(a*h - b*e)/b**2$)

Giac [A] time = 1.08277, size = 428, normalized size = 1.56

$$\frac{(bd - ag) \log(|bx^3 + a|)}{3b^2} + \frac{\sqrt{3} \left((-ab^2)^{\frac{1}{3}} a^2 h - (-ab^2)^{\frac{1}{3}} a b e - (-ab^2)^{\frac{2}{3}} b c + (-ab^2)^{\frac{2}{3}} a f \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{3ab^3} + \frac{\left((-ab^2)^{\frac{1}{3}} \right)}{3ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")

[Out] 1/3*(b*d - a*g)*log(abs(b*x^3 + a))/b^2 + 1/3*sqrt(3)*((-a*b^2)^(1/3)*a^2*h - (-a*b^2)^(1/3)*a*b*e - (-a*b^2)^(2/3)*b*c + (-a*b^2)^(2/3)*a*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^3) + 1/6*((-a*b^2)^(1/3)*a^2*h - (-a*b^2)^(1/3)*a*b*e + (-a*b^2)^(2/3)*b*c - (-a*b^2)^(2/3)*a*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^3) + 1/12*(3*b^3*h*x^4 + 4*b^3*g*x^3 + 6*b^3*f*x^2 - 12*a*b^2*h*x + 12*b^3*x*e)/b^4 - 1/3*(b^9*c*(-a/b)^(1/3) - a*b^8*f*(-a/b)^(1/3) + a^2*b^7*h - a*b^8*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^9)

$$3.407 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^3} dx$$

Optimal. Leaf size=259

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(\sqrt[3]{b}(bc - af) - \sqrt[3]{a}(bd - ag)\right)}{6a^{2/3}b^{5/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(\sqrt[3]{b}(bc - af) - \sqrt[3]{a}(bd - ag)\right)}{3a^{2/3}b^{5/3}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{bx}}{\sqrt[3]{a} + \sqrt[3]{bx}}\right)}{\sqrt[3]{a}}$$

[Out] (f*x)/b + (g*x^2)/(2*b) + (h*x^3)/(3*b) - ((b^(4/3)*c + a^(1/3)*b*d - a*b^(1/3)*f - a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*b^(5/3)) + ((b^(1/3)*(b*c - a*f) - a^(1/3)*(b*d - a*g))*Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(5/3)) - ((b^(1/3)*(b*c - a*f) - a^(1/3)*(b*d - a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*a^(2/3)*b^(5/3)) + ((b*e - a*h)*Log[a + b*x^3]/(3*b^2))

Rubi [A] time = 0.373397, antiderivative size = 257, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}} - af + bc\right)}{6a^{2/3}b^{4/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(\sqrt[3]{b}(bc - af) - \sqrt[3]{a}(bd - ag)\right)}{3a^{2/3}b^{5/3}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{bx}}{\sqrt[3]{a} + \sqrt[3]{bx}}\right)}{\sqrt[3]{a}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3), x]

[Out] (f*x)/b + (g*x^2)/(2*b) + (h*x^3)/(3*b) - ((b^(4/3)*c + a^(1/3)*b*d - a*b^(1/3)*f - a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*b^(5/3)) + ((b^(1/3)*(b*c - a*f) - a^(1/3)*(b*d - a*g))*Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(5/3)) - ((b*c - a*f - (a^(1/3)*(b*d - a*g))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*a^(2/3)*b^(4/3)) + ((b*e - a*h)*Log[a + b*x^3]/(3*b^2))

Rule 1887

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a + b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]

Rule 1871

```
Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned}
 \int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{a + bx^3} dx &= \int \left(\frac{f}{b} + \frac{gx}{b} + \frac{hx^2}{b} + \frac{bc - af + (bd - ag)x + (be - ah)x^2}{b(a + bx^3)} \right) dx \\
 &= \frac{fx}{b} + \frac{gx^2}{2b} + \frac{hx^3}{3b} + \frac{\int \frac{bc - af + (bd - ag)x + (be - ah)x^2}{a + bx^3} dx}{b} \\
 &= \frac{fx}{b} + \frac{gx^2}{2b} + \frac{hx^3}{3b} + \frac{\int \frac{bc - af + (bd - ag)x}{a + bx^3} dx}{b} + \frac{(be - ah) \int \frac{x^2}{a + bx^3} dx}{b} \\
 &= \frac{fx}{b} + \frac{gx^2}{2b} + \frac{hx^3}{3b} + \frac{(be - ah) \log(a + bx^3)}{3b^2} + \frac{\int \frac{\sqrt[3]{a}(2\sqrt[3]{b}(bc - af) + \sqrt[3]{a}(bd - ag)) + \sqrt[3]{b}(-\sqrt[3]{b}c)}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}} dx}{3a^{2/3}b^{4/3}} \\
 &= \frac{fx}{b} + \frac{gx^2}{2b} + \frac{hx^3}{3b} + \frac{(bc - af - \frac{\sqrt[3]{a}(bd - ag)}{\sqrt[3]{b}}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{2/3}b^{4/3}} + \frac{(be - ah) \log(a + bx^3)}{3b^2} \\
 &= \frac{fx}{b} + \frac{gx^2}{2b} + \frac{hx^3}{3b} + \frac{(bc - af - \frac{\sqrt[3]{a}(bd - ag)}{\sqrt[3]{b}}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{2/3}b^{4/3}} - \frac{(bc - af - \frac{\sqrt[3]{a}(bd - ag)}{\sqrt[3]{b}}) \log(a + bx^3)}{3b^2} \\
 &= \frac{fx}{b} + \frac{gx^2}{2b} + \frac{hx^3}{3b} - \frac{(b^{4/3}c + \sqrt[3]{abd} - a\sqrt[3]{b}f - a^{4/3}g) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{5/3}} + \frac{(bc - af - \frac{\sqrt[3]{a}(bd - ag)}{\sqrt[3]{b}}) \log(a + bx^3)}{3b^2}
 \end{aligned}$$

Mathematica [A] time = 0.293717, size = 254, normalized size = 0.98

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}\right) \left(a^{4/3}g - \sqrt[3]{a}bd - a\sqrt[3]{b}f + b^{4/3}c\right)}{a^{2/3}} + \frac{2 \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right) \left(a^{4/3}g - \sqrt[3]{a}bd - a\sqrt[3]{b}f + b^{4/3}c\right)}{a^{2/3}} + \frac{2\sqrt{3} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right) \left(a^{4/3}g - \sqrt[3]{a}bd + a\sqrt[3]{b}f - b^{4/3}c\right)}{a^{2/3}}}{6b^{5/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3), x]

[Out] (6*b^(2/3)*f*x + 3*b^(2/3)*g*x^2 + 2*b^(2/3)*h*x^3 + (2*sqrt(3)*(-b^(4/3)*c) - a^(1/3)*b*d + a*b^(1/3)*f + a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))]

$$\begin{aligned} & 3)) / \text{Sqrt}[3]]) / a^{(2/3)} + (2 * (b^{(4/3)} * c - a^{(1/3)} * b * d - a * b^{(1/3)} * f + a^{(4/3)} \\ & * g) * \text{Log}[a^{(1/3)} + b^{(1/3)} * x]) / a^{(2/3)} - ((b^{(4/3)} * c - a^{(1/3)} * b * d - a * b^{(1/3)} \\ & * f + a^{(4/3)} * g) * \text{Log}[a^{(2/3)} - a^{(1/3)} * b^{(1/3)} * x + b^{(2/3)} * x^2]) / a^{(2/3)} + \\ & (2 * (b * e - a * h) * \text{Log}[a + b * x^3]) / b^{(1/3)}) / (6 * b^{(5/3)}) \end{aligned}$$

Maple [B] time = 0.003, size = 429, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a), x)$

[Out] $\frac{1}{3} h x^3 / b + \frac{1}{2} g x^2 / b + f x / b - \frac{1}{3} b^2 a / (b a)^{(2/3)} * \ln(x + (b a)^{(1/3)}) * f + \frac{1}{3} c / b / (b a)^{(2/3)} * \ln(x + (b a)^{(1/3)}) + \frac{1}{6} b^2 a / (b a)^{(2/3)} * \ln(x^2 - (b a)^{(1/3)} * x + (b a)^{(2/3)}) * f - \frac{1}{6} c / b / (b a)^{(2/3)} * \ln(x^2 - (b a)^{(1/3)} * x + (b a)^{(2/3)}) - \frac{1}{3} b^2 a / (b a)^{(2/3)} * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (2 / (b a)^{(1/3)} * x - 1)) * f + \frac{1}{3} c / b / (b a)^{(2/3)} * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (2 / (b a)^{(1/3)} * x - 1)) + \frac{1}{3} b^2 / (b a)^{(1/3)} * \ln(x + (b a)^{(1/3)}) * a * g - \frac{1}{3} d / b / (b a)^{(1/3)} * \ln(x + (b a)^{(1/3)}) - \frac{1}{6} b^2 / (b a)^{(1/3)} * \ln(x^2 - (b a)^{(1/3)} * x + (b a)^{(2/3)}) * a * g + \frac{1}{6} d / b / (b a)^{(1/3)} * \ln(x^2 - (b a)^{(1/3)} * x + (b a)^{(2/3)}) - \frac{1}{3} b^2 * 3^{(1/2)} / (b a)^{(1/3)} * \arctan(1/3 * 3^{(1/2)} * (2 / (b a)^{(1/3)} * x - 1)) * a * g + \frac{1}{3} d * 3^{(1/2)} / b / (b a)^{(1/3)} * \arctan(1/3 * 3^{(1/2)} * (2 / (b a)^{(1/3)} * x - 1)) - \frac{1}{3} b^2 * \ln(b * x^3 + a) * a * h + \frac{1}{3} e * \ln(b * x^3 + a) / b$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a), x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] time = 28.6818, size = 804, normalized size = 3.1

$$\text{RootSum}\left(27t^3a^2b^6 + t^2(27a^3b^4h - 27a^2b^5e) + t(9a^4b^2h^2 - 18a^3b^3eh + 9a^3b^3fg - 9a^2b^4cg - 9a^2b^4df + 9a^2b^4e^2 + 9ab^5c^2) + d^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**6 + _t**2*(27*a**3*b**4*h - 27*a**2*b**5*e) + _t*(
9*a**4*b**2*h**2 - 18*a**3*b**3*e*h + 9*a**3*b**3*f*g - 9*a**2*b**4*c*g - 9
*a**2*b**4*d*f + 9*a**2*b**4*e**2 + 9*a*b**5*c*d) + a**5*h**3 - 3*a**4*b**e
h**2 + 3*a**4*b*f*g*h - a**4*b*g**3 - 3*a**3*b**2*c*g*h - 3*a**3*b**2*d*f*h
+ 3*a**3*b**2*d*g**2 + 3*a**3*b**2*e**2*h - 3*a**3*b**2*e*f*g + a**3*b**2*
f**3 + 3*a**2*b**3*c*d*h + 3*a**2*b**3*c*e*g - 3*a**2*b**3*c*f**2 - 3*a**2*
b**3*d**2*g + 3*a**2*b**3*d*e*f - a**2*b**3*e**3 + 3*a*b**4*c**2*f - 3*a*b
**4*c*d*e + a*b**4*d**3 - b**5*c**3, Lambda(_t, _t*log(x + (9*_t**2*a**3*b**
4*g - 9*_t**2*a**2*b**5*d + 6*_t*a**4*b**2*g*h - 6*_t*a**3*b**3*d*h - 6*_t
a**3*b**3*e*g - 3*_t*a**3*b**3*f**2 + 6*_t*a**2*b**4*c*f + 6*_t*a**2*b**4*d
*e - 3*_t*a*b**5*c**2 + a**5*g*h**2 - a**4*b*d*h**2 - 2*a**4*b*e*g*h - a**4
*b*f**2*h + 2*a**4*b*f*g**2 + 2*a**3*b**2*c*f*h - 2*a**3*b**2*c*g**2 + 2*a
**3*b**2*d*e*h - 4*a**3*b**2*d*f*g + a**3*b**2*e**2*g + a**3*b**2*e*f**2 - a
**2*b**3*c**2*h + 4*a**2*b**3*c*d*g - 2*a**2*b**3*c*e*f + 2*a**2*b**3*d**2*
f - a**2*b**3*d*e**2 + a*b**4*c**2*e - 2*a*b**4*c*d**2)/(a**4*b*g**3 - 3*a
**3*b**2*d*g**2 + a**3*b**2*f**3 - 3*a**2*b**3*c*f**2 + 3*a**2*b**3*d**2*g +
3*a*b**4*c**2*f - a*b**4*d**3 - b**5*c**3)))) + f*x/b + g*x**2/(2*b) + h*x
**3/(3*b)
```

Giac [A] time = 1.07956, size = 397, normalized size = 1.53

$$\frac{(ah - be) \log(|bx^3 + a|)}{3b^2} + \frac{\sqrt{3} \left((-ab^2)^{\frac{1}{3}} b^2c - (-ab^2)^{\frac{1}{3}} abf - (-ab^2)^{\frac{2}{3}} bd + (-ab^2)^{\frac{2}{3}} ag \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{3ab^3} + \frac{2b^2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")`

[Out]
$$-1/3*(a*h - b*e)*\log(\text{abs}(b*x^3 + a))/b^2 + 1/3*\sqrt{3}*((-a*b^2)^{(1/3)}*b^2*c - (-a*b^2)^{(1/3)}*a*b*f - (-a*b^2)^{(2/3)}*b*d + (-a*b^2)^{(2/3)}*a*g)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(a*b^3) + 1/6*(2*b^2*h*x^3 + 3*b^2*g*x^2 + 6*b^2*f*x)/b^3 + 1/6*((-a*b^2)^{(1/3)}*b^2*c - (-a*b^2)^{(1/3)}*a*b*f + (-a*b^2)^{(2/3)}*b*d - (-a*b^2)^{(2/3)}*a*g)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a*b^3) - 1/3*(b^7*d*(-a/b)^{(1/3)} - a*b^6*g*(-a/b)^{(1/3)} + b^7*c - a*b^6*f)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/a*b^7$$

$$3.408 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)} dx$$

Optimal. Leaf size=258

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(\sqrt[3]{b}(bd - ag) - \sqrt[3]{a}(be - ah)\right)}{6a^{2/3}b^{5/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(\sqrt[3]{b}(bd - ag) - \sqrt[3]{a}(be - ah)\right)}{3a^{2/3}b^{5/3}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}$$

[Out] (g*x)/b + (h*x^2)/(2*b) - ((b^(4/3)*d + a^(1/3)*b*e - a*b^(1/3)*g - a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*b^(5/3)) + (c*Log[x])/a + ((b^(1/3)*(b*d - a*g) - a^(1/3)*(b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(5/3)) - ((b^(1/3)*(b*d - a*g) - a^(1/3)*(b*e - a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(5/3)) - ((b*c - a*f)*Log[a + b*x^3])/(3*a*b)

Rubi [A] time = 0.470611, antiderivative size = 256, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$, Rules used = {1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}} - ag + bd\right)}{6a^{2/3}b^{4/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(\sqrt[3]{b}(bd - ag) - \sqrt[3]{a}(be - ah)\right)}{3a^{2/3}b^{5/3}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)), x]

[Out] (g*x)/b + (h*x^2)/(2*b) - ((b^(4/3)*d + a^(1/3)*b*e - a*b^(1/3)*g - a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*b^(5/3)) + (c*Log[x])/a + ((b^(1/3)*(b*d - a*g) - a^(1/3)*(b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(5/3)) - ((b*d - a*g - (a^(1/3)*(b*e - a*h))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(4/3)) - ((b*c - a*f)*Log[a + b*x^3])/(3*a*b)

Rule 1834

Int[((Pq_)*((c_)*(x_))^(m_))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^( -1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^( -1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d},
```

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned}
 \int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{x(a + bx^3)} dx &= \int \left(\frac{g}{b} + \frac{c}{ax} + \frac{hx}{b} + \frac{a(bd - ag) + a(be - ah)x - b(bc - af)x^2}{ab(a + bx^3)} \right) dx \\
 &= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{c \log(x)}{a} + \frac{\int \frac{a(bd - ag) + a(be - ah)x - b(bc - af)x^2}{a + bx^3} dx}{ab} \\
 &= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{c \log(x)}{a} + \frac{\int \frac{a(bd - ag) + a(be - ah)x}{a + bx^3} dx}{ab} - \frac{(bc - af) \int \frac{x^2}{a + bx^3} dx}{a} \\
 &= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{c \log(x)}{a} - \frac{(bc - af) \log(a + bx^3)}{3ab} + \frac{\int \frac{\sqrt[3]{a}(2a\sqrt[3]{b}(bd - ag) + a^{4/3}(be - ah)) + \sqrt[3]{b}}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}}}{3a^{5/3}} \\
 &= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{c \log(x)}{a} + \frac{\left(bd - ag - \frac{\sqrt[3]{a}(be - ah)}{\sqrt[3]{b}} \right) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{2/3}b^{4/3}} - \frac{(bc - af) \log(a + bx^3)}{3ab} \\
 &= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{c \log(x)}{a} + \frac{\left(bd - ag - \frac{\sqrt[3]{a}(be - ah)}{\sqrt[3]{b}} \right) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{2/3}b^{4/3}} - \frac{\left(bd - ag - \frac{\sqrt[3]{a}(be - ah)}{\sqrt[3]{b}} \right) \log(a + bx^3)}{3ab} \\
 &= \frac{gx}{b} + \frac{hx^2}{2b} - \frac{\left(b^{4/3}d + \sqrt[3]{abe} - a\sqrt[3]{bg} - a^{4/3}h \right) \tan^{-1} \left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}} \right)}{\sqrt{3}a^{2/3}b^{5/3}} + \frac{c \log(x)}{a} + \frac{\left(bd - ag - \frac{\sqrt[3]{a}(be - ah)}{\sqrt[3]{b}} \right) \log(a + bx^3)}{3ab}
 \end{aligned}$$

Mathematica [A] time = 0.212392, size = 258, normalized size = 1.

$$-\sqrt[3]{a} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2) (a^{4/3}h - \sqrt[3]{abe} - a\sqrt[3]{bg} + b^{4/3}d) + 2\sqrt[3]{a} \log(\sqrt[3]{a} + \sqrt[3]{bx}) (a^{4/3}h - \sqrt[3]{abe} - a\sqrt[3]{bg} + b^{4/3}d)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)), x]

```
[Out] (6*a*b^(2/3)*g*x + 3*a*b^(2/3)*h*x^2 + 2*Sqrt[3]*a^(1/3)*(-(b^(4/3)*d) - a^(1/3)*b*e + a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 6*b^(5/3)*c*Log[x] + 2*a^(1/3)*(b^(4/3)*d - a^(1/3)*b*e - a*b^(1/3)*g + a^(4/3)*h)*Log[a^(1/3) + b^(1/3)*x] - a^(1/3)*(b^(4/3)*d - a^(1/3)*b*e - a*b^(1/3)*g + a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*b^(2/3)*(b*c - a*f)*Log[a + b*x^3])/(6*a*b^(5/3))
```

Maple [B] time = 0.006, size = 426, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a), x)
```

```
[Out] 1/2*h*x^2/b+g*x/b-1/3/b^2*a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*g+1/3/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d+1/6/b^2*a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*g-1/6/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d-1/3/b^2*a/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*g+1/3/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d+1/3/b^2*a/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*h-1/3/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*e-1/6/b^2*a/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*h+1/6/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e-1/3/b^2*a*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*h+1/3/b*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e+1/3/b*ln(b*x^3+a)*f-1/3*c*ln(b*x^3+a)/a+c*ln(x)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a), x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x/(b*x**3+a),x)

[Out] Timed out

Giac [A] time = 1.06272, size = 409, normalized size = 1.59

$$\frac{c \log(|x|)}{a} - \frac{(bc - af) \log(|bx^3 + a|)}{3ab} + \frac{bhx^2 + 2bgx}{2b^2} + \frac{\sqrt{3} \left((-ab^2)^{\frac{1}{3}} b^2 d - (-ab^2)^{\frac{1}{3}} abg + (-ab^2)^{\frac{2}{3}} ah - (-ab^2)^{\frac{2}{3}} be \right) \arctan\left(\frac{bx + (-a/b)^{1/3}}{(-a/b)^{1/3}}\right)}{3ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="giac")

[Out] c*log(abs(x))/a - 1/3*(b*c - a*f)*log(abs(b*x^3 + a))/(a*b) + 1/2*(b*h*x^2 + 2*b*g*x)/b^2 + 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^2*d - (-a*b^2)^(1/3)*a*b*g + (-a*b^2)^(2/3)*a*h - (-a*b^2)^(2/3)*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^3) + 1/6*((-a*b^2)^(1/3)*b^2*d - (-a*b^2)^(1/3)*a*b*g - (-a*b^2)^(2/3)*a*h + (-a*b^2)^(2/3)*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^3) + 1/3*(a^3*b^2*h*(-a/b)^(1/3) - a^2*b^3*(-a/b)^(1/3)*e - a^2*b^3*d + a^3*b^2*g)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^3)

$$3.409 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)} dx$$

Optimal. Leaf size=253

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{6a^{4/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{3a^{4/3}b^{4/3}}+\frac{\tan^{-1}\left(\frac{a^{1/3}-2b^{1/3}x}{\sqrt[3]{a+bx^3}}\right)}{1}$$

[Out] $-(c/(a*x)) + (h*x)/b + ((b^{(5/3)*c} - a^{(2/3)*b*e} - a*b^{(2/3)*f} + a^{(5/3)*h}) * \text{ArcTan}[(a^{(1/3)} - 2*b^{(1/3)*x})/(\text{Sqrt}[3]*a^{(1/3)})]/(\text{Sqrt}[3]*a^{(4/3)*b^{(4/3)}}) + (d*\text{Log}[x])/a + ((b^{(2/3)*(b*c - a*f)} + a^{(2/3)*(b*e - a*h)}) * \text{Log}[a^{(1/3)} + b^{(1/3)*x}]/(3*a^{(4/3)*b^{(4/3)}}) - ((b^{(2/3)*(b*c - a*f)} + a^{(2/3)*(b*e - a*h)}) * \text{Log}[a^{(2/3)} - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2}]/(6*a^{(4/3)*b^{(4/3)}}) - ((b*d - a*g) * \text{Log}[a + b*x^3])/ (3*a*b)$

Rubi [A] time = 0.453954, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$, Rules used = {1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{6a^{4/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{3a^{4/3}b^{4/3}}+\frac{\tan^{-1}\left(\frac{a^{1/3}-2b^{1/3}x}{\sqrt[3]{a+bx^3}}\right)}{1}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)), x]

[Out] $-(c/(a*x)) + (h*x)/b + ((b^{(5/3)*c} - a^{(2/3)*b*e} - a*b^{(2/3)*f} + a^{(5/3)*h}) * \text{ArcTan}[(a^{(1/3)} - 2*b^{(1/3)*x})/(\text{Sqrt}[3]*a^{(1/3)})]/(\text{Sqrt}[3]*a^{(4/3)*b^{(4/3)}}) + (d*\text{Log}[x])/a + ((b^{(2/3)*(b*c - a*f)} + a^{(2/3)*(b*e - a*h)}) * \text{Log}[a^{(1/3)} + b^{(1/3)*x}]/(3*a^{(4/3)*b^{(4/3)}}) - ((b^{(2/3)*(b*c - a*f)} + a^{(2/3)*(b*e - a*h)}) * \text{Log}[a^{(2/3)} - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2}]/(6*a^{(4/3)*b^{(4/3)}}) - ((b*d - a*g) * \text{Log}[a + b*x^3])/ (3*a*b)$

Rule 1834

Int[((Pq_)*((c_)*(x_))^(m_))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] & & PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]

Rule 1871

Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Dist[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

Rule 1860

Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^-1, x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d},

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned}
 \int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{x^2(a + bx^3)} dx &= \int \left(\frac{h}{b} + \frac{c}{ax^2} + \frac{d}{ax} + \frac{a(be - ah) - b(bc - af)x - b(bd - ag)x^2}{ab(a + bx^3)} \right) dx \\
 &= -\frac{c}{ax} + \frac{hx}{b} + \frac{d \log(x)}{a} + \frac{\int \frac{a(be - ah) - b(bc - af)x - b(bd - ag)x^2}{a + bx^3} dx}{ab} \\
 &= -\frac{c}{ax} + \frac{hx}{b} + \frac{d \log(x)}{a} + \frac{\int \frac{a(be - ah) - b(bc - af)x}{a + bx^3} dx}{ab} - \frac{(bd - ag) \int \frac{x^2}{a + bx^3} dx}{a} \\
 &= -\frac{c}{ax} + \frac{hx}{b} + \frac{d \log(x)}{a} - \frac{(bd - ag) \log(a + bx^3)}{3ab} + \frac{\int \frac{\sqrt[3]{a}(-\sqrt[3]{ab}(bc - af) + 2a\sqrt[3]{b}(be - ah)) + \dots}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx}} dx}{3a^{5/3}b^{4/3}} \\
 &= -\frac{c}{ax} + \frac{hx}{b} + \frac{d \log(x)}{a} + \frac{(b^{2/3}(bc - af) + a^{2/3}(be - ah)) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{4/3}b^{4/3}} - \frac{(bd - ag) \log(a + bx^3)}{3ab} \\
 &= -\frac{c}{ax} + \frac{hx}{b} + \frac{d \log(x)}{a} + \frac{(b^{2/3}(bc - af) + a^{2/3}(be - ah)) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{4/3}b^{4/3}} - \frac{(b^{2/3}(bc - af) + a^{2/3}(be - ah)) \log(a + bx^3)}{3ab} \\
 &= -\frac{c}{ax} + \frac{hx}{b} + \frac{(b^{5/3}c - a^{2/3}be - ab^{2/3}f + a^{5/3}h) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}b^{4/3}} + \frac{d \log(x)}{a} + \frac{(b^{2/3}(bc - af) + a^{2/3}(be - ah)) \log(a + bx^3)}{3ab}
 \end{aligned}$$

Mathematica [A] time = 0.280359, size = 257, normalized size = 1.02

$$\frac{1}{6} \left(\frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2) (-a^{2/3}be + a^{5/3}h + ab^{2/3}f - b^{5/3}c)}{a^{4/3}b^{4/3}} + \frac{2 \log(\sqrt[3]{a} + \sqrt[3]{bx}) (a^{2/3}be + a^{5/3}(-h) - ab^{2/3}f + b^{5/3}c)}{a^{4/3}b^{4/3}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)),x]

[Out]
$$\begin{aligned} &((-6*c)/(a*x) + (6*h*x)/b + (2*\text{Sqrt}[3]*(b^{(5/3)*c} - a^{(2/3)*b*e} - a*b^{(2/3)} \\ &*f + a^{(5/3)*h})*\text{ArcTan}[(1 - (2*b^{(1/3)*x})/a^{(1/3)})/\text{Sqrt}[3]])/(a^{(4/3)*b^{(4/3)}}) \\ &+ (6*d*\text{Log}[x])/a + (2*(b^{(5/3)*c} + a^{(2/3)*b*e} - a*b^{(2/3)*f} - a^{(5/3)*h})*\text{Log}[a^{(1/3)} + b^{(1/3)*x}]) \\ &/(a^{(4/3)*b^{(4/3)}}) + ((-(b^{(5/3)*c}) - a^{(2/3)*b*e} + a*b^{(2/3)*f} + a^{(5/3)*h})*\text{Log}[a^{(2/3)} - a^{(1/3)*b^{(1/3)*x}} + b^{(2/3)*x^2}]) \\ &/(a^{(4/3)*b^{(4/3)}}) + (2*(-(b*d) + a*g)*\text{Log}[a + b*x^3])/(a*b))/6 \end{aligned}$$

Maple [B] time = 0.007, size = 423, normalized size = 1.7

$$\frac{hx}{b} - \frac{ah}{3b^2} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{e}{3b} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{ah}{6b^2} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{e}{6b} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a),x)

[Out]
$$\begin{aligned} &h*x/b-1/3/b^2*a/(1/b*a)^{(2/3)*\ln(x+(1/b*a)^{(1/3)})}*h+1/3/b/(1/b*a)^{(2/3)*\ln(x+(1/b*a)^{(1/3)})} \\ &*e+1/6/b^2*a/(1/b*a)^{(2/3)*\ln(x^2-(1/b*a)^{(1/3)*x+(1/b*a)^{(2/3)})}*h-1/6/b/(1/b*a)^{(2/3)*\ln(x^2-(1/b*a)^{(1/3)*x+(1/b*a)^{(2/3)})} \\ &*e-1/3/b^2*a/(1/b*a)^{(2/3)*3^{(1/2)*\arctan(1/3*3^{(1/2)*(2/(1/b*a)^{(1/3)*x-1})})*h+1/3/b/(1/b*a)^{(2/3)*3^{(1/2)*\arctan(1/3*3^{(1/2)*(2/(1/b*a)^{(1/3)*x-1})})*e-1/3/b/(1/b*a)^{(1/3)*\ln(x+(1/b*a)^{(1/3)})} \\ &*f+1/3/a/(1/b*a)^{(1/3)*\ln(x+(1/b*a)^{(1/3)})}*c+1/6/b/(1/b*a)^{(1/3)*\ln(x^2-(1/b*a)^{(1/3)*x+(1/b*a)^{(2/3)})} \\ &*f-1/6/a/(1/b*a)^{(1/3)*\ln(x^2-(1/b*a)^{(1/3)*x+(1/b*a)^{(2/3)})}*c+1/3*3^{(1/2)}/b/(1/b*a)^{(1/3)*\arctan(1/3*3^{(1/2)*(2/(1/b*a)^{(1/3)*x-1})})*f-1/3/a*3^{(1/2)}/(1/b*a)^{(1/3)*\arctan(1/3*3^{(1/2)*(2/(1/b*a)^{(1/3)*x-1})})*c+1/3/b*\ln(b*x^3+a)*g-1/3*d*\ln(b*x^3+a) \\ &)/a+d*\ln(x)/a-c/a/x \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2/(b*x**3+a),x)

[Out] Timed out

Giac [A] time = 1.09191, size = 404, normalized size = 1.6

$$\frac{hx}{b} + \frac{d \log(|x|)}{a} - \frac{(bd - ag) \log(|bx^3 + a|)}{3ab} - \frac{c}{ax} - \frac{\sqrt{3} \left((-ab^2)^{\frac{1}{3}} a^2 h - (-ab^2)^{\frac{1}{3}} a b e - (-ab^2)^{\frac{2}{3}} b c + (-ab^2)^{\frac{2}{3}} a f \right) \arctan \left(\frac{bx^3 + a}{\sqrt{3} a b^2} \right)}{3a^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="giac")

[Out] h*x/b + d*log(abs(x))/a - 1/3*(b*d - a*g)*log(abs(b*x^3 + a))/(a*b) - c/(a*x) - 1/3*sqrt(3)*((-a*b^2)^(1/3)*a^2*h - (-a*b^2)^(1/3)*a*b*e - (-a*b^2)^(2/3)*b*c + (-a*b^2)^(2/3)*a*f)*arctan((b*x^3 + a)/sqrt(3)*a/b^2)

$$\begin{aligned} & /3)*b*c + (-a*b^2)^{(2/3)}*a*f)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b) \\ &)^{(1/3)})/(a^2*b^2) - 1/6*((-a*b^2)^{(1/3)}*a^2*h - (-a*b^2)^{(1/3)}*a*b*e + (-a \\ & *b^2)^{(2/3)}*b*c - (-a*b^2)^{(2/3)}*a*f)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/ \\ & 3)})/(a^2*b^2) + 1/3*(a*b^4*c*(-a/b)^{(1/3)} - a^2*b^3*f*(-a/b)^{(1/3)} + a^3*b^ \\ & 2*h - a^2*b^3*e)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/(a^3*b^3) \end{aligned}$$

$$3.410 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)} dx$$

Optimal. Leaf size=260

$$\frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(\sqrt[3]{b}(bc - af) - \sqrt[3]{a}(bd - ag))}{6a^{5/3}b^{2/3}} - \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(\sqrt[3]{b}(bc - af) - \sqrt[3]{a}(bd - ag))}{3a^{5/3}b^{2/3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{1}$$

[Out] $-c/(2*a*x^2) - d/(a*x) + ((b^{(4/3)*c} + a^{(1/3)*b*d} - a*b^{(1/3)*f} - a^{(4/3)*g}) * \text{ArcTan}[(a^{(1/3)} - 2*b^{(1/3)*x}) / (\text{Sqrt}[3]*a^{(1/3)})] / (\text{Sqrt}[3]*a^{(5/3)*b^{(2/3)}}) + (e*\text{Log}[x])/a - ((b^{(1/3)*(b*c - a*f)} - a^{(1/3)*(b*d - a*g)}) * \text{Log}[a^{(1/3)} + b^{(1/3)*x}] / (3*a^{(5/3)*b^{(2/3)}}) + ((b^{(1/3)*(b*c - a*f)} - a^{(1/3)*(b*d - a*g)}) * \text{Log}[a^{(2/3)} - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2}] / (6*a^{(5/3)*b^{(2/3)}}) - ((b*e - a*h) * \text{Log}[a + b*x^3]) / (3*a*b)$

Rubi [A] time = 0.380025, antiderivative size = 258, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$, Rules used = {1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)\left(-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}} - af + bc\right)}{6a^{5/3}\sqrt[3]{b}} - \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(\sqrt[3]{b}(bc - af) - \sqrt[3]{a}(bd - ag))}{3a^{5/3}b^{2/3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{1}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)), x]

[Out] $-c/(2*a*x^2) - d/(a*x) + ((b^{(4/3)*c} + a^{(1/3)*b*d} - a*b^{(1/3)*f} - a^{(4/3)*g}) * \text{ArcTan}[(a^{(1/3)} - 2*b^{(1/3)*x}) / (\text{Sqrt}[3]*a^{(1/3)})] / (\text{Sqrt}[3]*a^{(5/3)*b^{(2/3)}}) + (e*\text{Log}[x])/a - ((b^{(1/3)*(b*c - a*f)} - a^{(1/3)*(b*d - a*g)}) * \text{Log}[a^{(1/3)} + b^{(1/3)*x}] / (3*a^{(5/3)*b^{(2/3)}}) + ((b*c - a*f - (a^{(1/3)*(b*d - a*g)}) / b^{(1/3)}) * \text{Log}[a^{(2/3)} - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2}] / (6*a^{(5/3)*b^{(1/3)}}) - ((b*e - a*h) * \text{Log}[a + b*x^3]) / (3*a*b)$

Rule 1834

Int[((Pq_)*((c_)*(x_))^(m_))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] & & PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned}
 \int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{x^3(a + bx^3)} dx &= \int \left(\frac{c}{ax^3} + \frac{d}{ax^2} + \frac{e}{ax} + \frac{-bc + af - (bd - ag)x - (be - ah)x^2}{a(a + bx^3)} \right) dx \\
 &= -\frac{c}{2ax^2} - \frac{d}{ax} + \frac{e \log(x)}{a} + \frac{\int \frac{-bc + af - (bd - ag)x - (be - ah)x^2}{a + bx^3} dx}{a} \\
 &= -\frac{c}{2ax^2} - \frac{d}{ax} + \frac{e \log(x)}{a} + \frac{\int \frac{-bc + af + (-bd + ag)x}{a + bx^3} dx}{a} + \frac{(-be + ah) \int \frac{x^2}{a + bx^3} dx}{a} \\
 &= -\frac{c}{2ax^2} - \frac{d}{ax} + \frac{e \log(x)}{a} - \frac{(be - ah) \log(a + bx^3)}{3ab} + \frac{\int \frac{\sqrt[3]{a}(2\sqrt[3]{b}(-bc + af) + \sqrt[3]{a}(-bd + ag))}{a^{2/3} - \sqrt[3]{a}} dx}{3a^5} \\
 &= -\frac{c}{2ax^2} - \frac{d}{ax} + \frac{e \log(x)}{a} - \frac{(bc - af - \frac{\sqrt[3]{a}(bd - ag)}{\sqrt[3]{b}}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{5/3} \sqrt[3]{b}} - \frac{(be - ah) \log(a + bx^3)}{3ab} \\
 &= -\frac{c}{2ax^2} - \frac{d}{ax} + \frac{e \log(x)}{a} - \frac{(bc - af - \frac{\sqrt[3]{a}(bd - ag)}{\sqrt[3]{b}}) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{5/3} \sqrt[3]{b}} + \frac{(bc - af - \frac{\sqrt[3]{a}(bd - ag)}{\sqrt[3]{b}}) \log(a + bx^3)}{3ab} \\
 &= -\frac{c}{2ax^2} - \frac{d}{ax} + \frac{(b^{4/3}c + \sqrt[3]{abd} - a\sqrt[3]{bf} - a^{4/3}g) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right) + e \log(x)}{\sqrt{3}a^{5/3}b^{2/3}} - \frac{(be - ah) \log(a + bx^3)}{3ab}
 \end{aligned}$$

Mathematica [A] time = 0.32134, size = 257, normalized size = 0.99

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(a^{4/3}g - \sqrt[3]{abd} - a\sqrt[3]{bf} + b^{4/3}c\right)}{b^{2/3}} - \frac{2 \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(a^{4/3}g - \sqrt[3]{abd} - a\sqrt[3]{bf} + b^{4/3}c\right)}{b^{2/3}} + \frac{2\sqrt{3} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-g) + \sqrt[3]{abd} - a\sqrt[3]{bf} + b^{4/3}c\right)}{6a^{5/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)), x]


```
[Out] ((-3*a^(2/3)*c)/x^2 - (6*a^(2/3)*d)/x + (2*Sqrt[3]*(b^(4/3)*c + a^(1/3)*b*d
- a*b^(1/3)*f - a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(
(2/3) + 6*a^(2/3)*e*Log[x] - (2*(b^(4/3)*c - a^(1/3)*b*d - a*b^(1/3)*f + a^(
(4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((b^(4/3)*c - a^(1/3)*b*d - a*
b^(1/3)*f + a^(4/3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2
/3) + (2*a^(2/3)*(-(b*e) + a*h)*Log[a + b*x^3])/b/(6*a^(5/3))
```

Maple [B] time = 0.007, size = 423, normalized size = 1.6

$$\frac{f}{3b} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{c}{3a} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{f}{6b} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{c}{6a} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a), x)
```

```
[Out] 1/3/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*f-1/3/a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(
1/3))*c-1/6/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f+1/6/a/
(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+1/3/b/(1/b*a)^(2/3)*3
^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f-1/3/a/(1/b*a)^(2/3)*3^(1
/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c-1/3/b/(1/b*a)^(1/3)*ln(x+(1
/b*a)^(1/3))*g+1/3/a/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*d+1/6/b/(1/b*a)^(1/3
)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*g-1/6/a/(1/b*a)^(1/3)*ln(x^2-(1/b*a
)^(1/3)*x+(1/b*a)^(2/3))*d+1/3*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(
2/(1/b*a)^(1/3)*x-1))*g-1/3/a*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(
1/b*a)^(1/3)*x-1))*d+1/3/b*ln(b*x^3+a)*h-1/3*e*ln(b*x^3+a)/a-d/a/x+e*ln(x)/
a-1/2*c/a/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a), x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3/(b*x**3+a),x)

[Out] Timed out

Giac [A] time = 1.08235, size = 393, normalized size = 1.51

$$\frac{e \log(|x|)}{a} + \frac{(ah - be) \log(|bx^3 + a|)}{3ab} - \frac{\sqrt{3} \left((-ab^2)^{\frac{1}{3}} b^2 c - (-ab^2)^{\frac{1}{3}} abf - (-ab^2)^{\frac{2}{3}} bd + (-ab^2)^{\frac{2}{3}} ag \right) \arctan \left(\frac{\sqrt{3} \left(2x + (-\frac{a}{b})^{\frac{1}{3}} \right)}{3(-\frac{a}{b})^{\frac{1}{3}}} \right)}{3a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="giac")

[Out] e*log(abs(x))/a + 1/3*(a*h - b*e)*log(abs(b*x^3 + a))/(a*b) - 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^2*c - (-a*b^2)^(1/3)*a*b*f - (-a*b^2)^(2/3)*b*d + (-a*b^2)^(2/3)*a*g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^2) + 1/3*(a*b^2*d*(-a/b)^(1/3) - a^2*b*g*(-a/b)^(1/3) + a*b^2*c - a^2*b*f)*(-

$$\frac{a/b^{1/3} \log(\text{abs}(x - (-a/b)^{1/3}))}{a^3 b} - \frac{1}{2} \frac{(2dx + c)}{a x^2} - \frac{1}{6} \frac{(-a b^2)^{1/3} b^2 c - (-a b^2)^{1/3} a b f + (-a b^2)^{2/3} b d - (-a b^2)^{2/3} a g}{a^2 b^2} \log(x^2 + x(-a/b)^{1/3} + (-a/b)^{2/3})$$

$$3.411 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)} dx$$

Optimal. Leaf size=276

$$\frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(\sqrt[3]{b}(bd - ag) - \sqrt[3]{a}(be - ah))}{6a^{5/3}b^{2/3}} - \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(\sqrt[3]{b}(bd - ag) - \sqrt[3]{a}(be - ah))}{3a^{5/3}b^{2/3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{bx}}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] $-c/(3*a*x^3) - d/(2*a*x^2) - e/(a*x) + ((b^{(4/3)}*d + a^{(1/3)}*b*e - a*b^{(1/3)})*g - a^{(4/3)}*h)*\text{ArcTan}[(a^{(1/3)} - 2*b^{(1/3)}*x)/(\text{Sqrt}[3]*a^{(1/3)})]/(\text{Sqrt}[3]*a^{(5/3)}*b^{(2/3)}) - ((b*c - a*f)*\text{Log}[x])/a^2 - ((b^{(1/3)}*(b*d - a*g) - a^{(1/3)}*(b*e - a*h))*\text{Log}[a^{(1/3)} + b^{(1/3)}*x])/(3*a^{(5/3)}*b^{(2/3)}) + ((b^{(1/3)}*(b*d - a*g) - a^{(1/3)}*(b*e - a*h))*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(6*a^{(5/3)}*b^{(2/3)}) + ((b*c - a*f)*\text{Log}[a + b*x^3])/(3*a^2)$

Rubi [A] time = 0.435676, antiderivative size = 274, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$, Rules used = {1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)\left(-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}} - ag + bd\right)}{6a^{5/3}\sqrt[3]{b}} - \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(\sqrt[3]{b}(bd - ag) - \sqrt[3]{a}(be - ah))}{3a^{5/3}b^{2/3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)), x]$

[Out] $-c/(3*a*x^3) - d/(2*a*x^2) - e/(a*x) + ((b^{(4/3)}*d + a^{(1/3)}*b*e - a*b^{(1/3)})*g - a^{(4/3)}*h)*\text{ArcTan}[(a^{(1/3)} - 2*b^{(1/3)}*x)/(\text{Sqrt}[3]*a^{(1/3)})]/(\text{Sqrt}[3]*a^{(5/3)}*b^{(2/3)}) - ((b*c - a*f)*\text{Log}[x])/a^2 - ((b^{(1/3)}*(b*d - a*g) - a^{(1/3)}*(b*e - a*h))*\text{Log}[a^{(1/3)} + b^{(1/3)}*x])/(3*a^{(5/3)}*b^{(2/3)}) + ((b*d - a*g - (a^{(1/3)}*(b*e - a*h))/b^{(1/3)})*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/(6*a^{(5/3)}*b^{(1/3)}) + ((b*c - a*f)*\text{Log}[a + b*x^3])/(3*a^2)$

Rule 1834

$\text{Int}[(Pq_)*((c_)*(x_))^{(m_)}]/((a_) + (b_)*(x_)^{(n_)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*Pq]/(a + b*x^n), x] /; \text{FreeQ}[\{a, b, c, m\}, x] \& \& \text{PolyQ}[Pq, x] \&\& \text{IntegerQ}[n] \&\& !\text{IGTQ}[m, 0]$

Rule 1871

Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Dist[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

Rule 1860

Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 634

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^-1, x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d},

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned}
 \int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{x^4(a + bx^3)} dx &= \int \left(\frac{c}{ax^4} + \frac{d}{ax^3} + \frac{e}{ax^2} + \frac{-bc + af}{a^2x} + \frac{-a(bd - ag) - a(be - ah)x + b(bc - af)x^2}{a^2(a + bx^3)} \right) dx \\
 &= -\frac{c}{3ax^3} - \frac{d}{2ax^2} - \frac{e}{ax} - \frac{(bc - af) \log(x)}{a^2} + \frac{\int \frac{-a(bd - ag) - a(be - ah)x + b(bc - af)x^2}{a + bx^3} dx}{a^2} \\
 &= -\frac{c}{3ax^3} - \frac{d}{2ax^2} - \frac{e}{ax} - \frac{(bc - af) \log(x)}{a^2} + \frac{\int \frac{-a(bd - ag) - a(be - ah)x}{a + bx^3} dx}{a^2} + \frac{b(bc - af)}{a^2} \\
 &= -\frac{c}{3ax^3} - \frac{d}{2ax^2} - \frac{e}{ax} - \frac{(bc - af) \log(x)}{a^2} + \frac{(bc - af) \log(a + bx^3)}{3a^2} + \frac{\int \frac{\sqrt[3]{a}(-2a\sqrt[3]{b})}{\sqrt[3]{a} + \sqrt[3]{bx}} dx}{\sqrt[3]{a} + \sqrt[3]{bx}} \\
 &= -\frac{c}{3ax^3} - \frac{d}{2ax^2} - \frac{e}{ax} - \frac{(bc - af) \log(x)}{a^2} - \frac{\left(bd - ag - \frac{\sqrt[3]{a}(be - ah)}{\sqrt[3]{b}} \right) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{5/3}\sqrt[3]{b}} \\
 &= -\frac{c}{3ax^3} - \frac{d}{2ax^2} - \frac{e}{ax} - \frac{(bc - af) \log(x)}{a^2} - \frac{\left(bd - ag - \frac{\sqrt[3]{a}(be - ah)}{\sqrt[3]{b}} \right) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{5/3}\sqrt[3]{b}} \\
 &= -\frac{c}{3ax^3} - \frac{d}{2ax^2} - \frac{e}{ax} + \frac{(b^{4/3}d + \sqrt[3]{abe} - a\sqrt[3]{bg} - a^{4/3}h) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}b^{2/3}} - \frac{b(bc - af)}{a^2}
 \end{aligned}$$

Mathematica [A] time = 0.439707, size = 264, normalized size = 0.96

$$\frac{-\frac{\sqrt[3]{a} \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(a^{4/3}h - \sqrt[3]{abe} - a\sqrt[3]{bg} + b^{4/3}d\right)}{b^{2/3}} + \frac{2\sqrt[3]{a} \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(a^{4/3}h - \sqrt[3]{abe} - a\sqrt[3]{bg} + b^{4/3}d\right)}{b^{2/3}} + \frac{2\sqrt{3}\sqrt[3]{a} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}h - \sqrt[3]{abe} + a\sqrt[3]{bg} - b^{4/3}d\right)}{b^{2/3}}}{6a^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)), x]

```
[Out] -((2*a*c)/x^3 + (3*a*d)/x^2 + (6*a*e)/x + (2*sqrt[3]*a^(1/3)*(-(b^(4/3)*d)
- a^(1/3)*b*e + a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))
/sqrt[3]])/b^(2/3) + 6*(b*c - a*f)*Log[x] + (2*a^(1/3)*(b^(4/3)*d - a^(1/3)
*b*e - a*b^(1/3)*g + a^(4/3)*h)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) - (a^(1/3)
)*(b^(4/3)*d - a^(1/3)*b*e - a*b^(1/3)*g + a^(4/3)*h)*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) - 2*(b*c - a*f)*Log[a + b*x^3])/(6*a^2)
```

Maple [B] time = 0.006, size = 442, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a),x)
```

```
[Out] 1/3/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*g-1/3/a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d-1/6/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*g+1/6/a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d+1/3/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*g-1/3/a/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d-1/3/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*h+1/3/a/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*e+1/6/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*h-1/6/a/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e+1/3*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*h-1/3/a*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e-1/3/a*ln(b*x^3+a)*f+1/3/a^2*b*ln(b*x^3+a)*c-e/a/x-1/3*c/a/x^3-1/2*d/a/x^2+1/a*ln(x)*f-1/a^2*ln(x)*b*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4/(b*x**3+a),x)

[Out] Timed out

Giac [A] time = 1.0965, size = 423, normalized size = 1.53

$$\frac{(bc - af) \log(|bx^3 + a|)}{3a^2} - \frac{(bc - af) \log(|x|)}{a^2} - \frac{\sqrt{3} \left((-ab^2)^{\frac{1}{3}} b^2 d - (-ab^2)^{\frac{1}{3}} abg + (-ab^2)^{\frac{2}{3}} ah - (-ab^2)^{\frac{2}{3}} be \right) \arctan \left(\frac{\sqrt{3} (2x + (-a/b)^{\frac{1}{3}})}{3} \right)}{3a^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a),x, algorithm="giac")

[Out] 1/3*(b*c - a*f)*log(abs(b*x^3 + a))/a^2 - (b*c - a*f)*log(abs(x))/a^2 - 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^2*d - (-a*b^2)^(1/3)*a*b*g + (-a*b^2)^(2/3)*a*h - (-a*b^2)^(2/3)*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^2) - 1/6*((-a*b^2)^(1/3)*b^2*d - (-a*b^2)^(1/3)*a*b*g - (-a*b^2)^(2/3)*a*h + (-a*b^2)^(2/3)*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2

$$\begin{aligned} & *b^2) - 1/3*(a^4*b*h*(-a/b)^{(1/3)} - a^3*b^2*(-a/b)^{(1/3)}*e - a^3*b^2*d + a^4*b*g)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)})))/(a^5*b) - 1/6*(6*a*x^2*e + 3 \\ & *a*d*x + 2*a*c)/(a^2*x^3) \end{aligned}$$

$$3.412 \quad \int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=337

$$\frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^{2/3}(4be - 7ah) + b^{2/3}(2bc - 5af))}{18\sqrt[3]{ab^{10/3}}} - \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(a^{2/3}(4be - 7ah) + b^{2/3}(2bc - 5af))}{9\sqrt[3]{ab^{10/3}}}$$

[Out] ((b*e - 2*a*h)*x)/b^3 + (f*x^2)/(2*b^2) + (g*x^3)/(3*b^2) + (h*x^4)/(4*b^2) + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(3*b^3*(a + b*x^3)) - ((2*b^(5/3)*c - 4*a^(2/3)*b*e - 5*a*b^(2/3)*f + 7*a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(1/3)*b^(10/3)) - ((b^(2/3)*(2*b*c - 5*a*f) + a^(2/3)*(4*b*e - 7*a*h))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(1/3)*b^(10/3)) + ((b^(2/3)*(2*b*c - 5*a*f) + a^(2/3)*(4*b*e - 7*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(1/3)*b^(10/3)) + ((b*d - 2*a*g)*Log[a + b*x^3])/(3*b^3)

Rubi [A] time = 0.717088, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1828, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^{2/3}(4be - 7ah) + b^{2/3}(2bc - 5af))}{18\sqrt[3]{ab^{10/3}}} - \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(a^{2/3}(4be - 7ah) + b^{2/3}(2bc - 5af))}{9\sqrt[3]{ab^{10/3}}}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]

[Out] ((b*e - 2*a*h)*x)/b^3 + (f*x^2)/(2*b^2) + (g*x^3)/(3*b^2) + (h*x^4)/(4*b^2) + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(3*b^3*(a + b*x^3)) - ((2*b^(5/3)*c - 4*a^(2/3)*b*e - 5*a*b^(2/3)*f + 7*a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(1/3)*b^(10/3)) - ((b^(2/3)*(2*b*c - 5*a*f) + a^(2/3)*(4*b*e - 7*a*h))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(1/3)*b^(10/3)) + ((b^(2/3)*(2*b*c - 5*a*f) + a^(2/3)*(4*b*e - 7*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(1/3)*b^(10/3)) + ((b*d - 2*a*g)*Log[a + b*x^3])/(3*b^3)

Rule 1828

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q =
m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
+ b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4(c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{(a + bx^3)^2} dx &= \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{3b^3(a + bx^3)} - \int \frac{a^2(be - ah) - 2ab(bc - af)x - 3ab(bd - ag)}{3b^3(a + bx^3)^2} dx \\
&= \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{3b^3(a + bx^3)} - \frac{\int (-3a(be - 2ah) - 3abfx - 3ab^2)}{3b^3(a + bx^3)^2} dx \\
&= \frac{(be - 2ah)x}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2} + \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{3b^3(a + bx^3)} \\
&= \frac{(be - 2ah)x}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2} + \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{3b^3(a + bx^3)} \\
&= \frac{(be - 2ah)x}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2} + \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{3b^3(a + bx^3)} \\
&= \frac{(be - 2ah)x}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2} + \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{3b^3(a + bx^3)} \\
&= \frac{(be - 2ah)x}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2} + \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{3b^3(a + bx^3)} \\
&= \frac{(be - 2ah)x}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2} + \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{3b^3(a + bx^3)} \\
&= \frac{(be - 2ah)x}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2} + \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{3b^3(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.397708, size = 334, normalized size = 0.99

$$\frac{-\frac{12b^{2/3}(a^2(g+hx) - ab(d+xe+fx)) + b^2cx^2}{a+bx^3} + \frac{2 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx+b^{2/3}x^2}) (4a^{2/3}b^{4/3}e - 7a^{5/3} \sqrt[3]{bh} - 5abf + 2b^2c)}{\sqrt[3]{a}} + \frac{4 \log(\sqrt[3]{a} + \sqrt[3]{bx}) (-4a^{2/3}b^{4/3}e + 7a^{5/3} \sqrt[3]{bh} + 5abf - 2b^2c)}{\sqrt[3]{a}}}{3b^3(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]

[Out] (36*b^(2/3)*(b*e - 2*a*h)*x + 18*b^(5/3)*f*x^2 + 12*b^(5/3)*g*x^3 + 9*b^(5/3)*h*x^4 - (12*b^(2/3)*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e + f*x))))/(3*b^3*(a + b*x^3)^2)

$$\frac{1}{(a + b*x^3) - (4*\sqrt{3}*(2*b^2*c - 4*a^{(2/3)}*b^{(4/3)}*e - 5*a*b*f + 7*a^{(5/3)}*b^{(1/3)}*h)*\text{ArcTan}[(1 - (2*b^{(1/3)}*x)/a^{(1/3)})/\sqrt{3}])/a^{(1/3)} + (4*(-2*b^2*c - 4*a^{(2/3)}*b^{(4/3)}*e + 5*a*b*f + 7*a^{(5/3)}*b^{(1/3)}*h)*\text{Log}[a^{(1/3)} + b^{(1/3)}*x])/a^{(1/3)} + (2*(2*b^2*c + 4*a^{(2/3)}*b^{(4/3)}*e - 5*a*b*f - 7*a^{(5/3)}*b^{(1/3)}*h)*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/a^{(1/3)} + 12*b^{(2/3)}*(b*d - 2*a*g)*\text{Log}[a + b*x^3]}/(36*b^{(11/3)})$$

Maple [B] time = 0.011, size = 562, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x)`

[Out]
$$\begin{aligned} & -7/18/b^4*a^2*h/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})+1/3*g*x \\ & ^3/b^2+1/4*h*x^4/b^2+1/2*f*x^2/b^2-2/b^3*a*h*x-1/3/b*x^2/(b*x^3+a)*c-2/9/b^2*c/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})+1/9/b^2*c/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})+2/9/b^3*a/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*e-1/3/b^3/(b*x^3+a)*a^2*g+1/3/b^2/(b*x^3+a)*d*a-2/3/b^3*\ln(b*x^3+a)*a*g+5/9/b^3*a*f/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})-5/18/b^3*a*f/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})+1/3/b^2*a*x^2/(b*x^3+a)*f+1/3/b^2*x*a/(b*x^3+a)*e-4/9/b^3*a/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*e-4/9/b^3*a/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*e+1/b^2*x*e+1/3/b^2*\ln(b*x^3+a)*d+2/9/b^2*c*3^{(1/2)}/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))+7/9/b^4*a^2*h/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})-5/9/b^3*a*f*3^{(1/2)}/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))-1/3/b^3/(b*x^3+a)*a^2*h*x+7/9/b^4*a^2*h/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1)) \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)

[Out] Timed out

Giac [A] time = 1.09132, size = 512, normalized size = 1.52

$$\frac{(bd - 2ag) \log(|bx^3 + a|)}{3b^3} + \frac{\sqrt{3} \left(7(-ab^2)^{\frac{1}{3}} a^2 h - 4(-ab^2)^{\frac{1}{3}} a b e - 2(-ab^2)^{\frac{2}{3}} b c + 5(-ab^2)^{\frac{2}{3}} a f \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}} \right)}{9ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac")

[Out] 1/3*(b*d - 2*a*g)*log(abs(b*x^3 + a))/b^3 + 1/9*sqrt(3)*(7*(-a*b^2)^(1/3)*a^2*h - 4*(-a*b^2)^(1/3)*a*b*e - 2*(-a*b^2)^(2/3)*b*c + 5*(-a*b^2)^(2/3)*a*f

$$\begin{aligned}
&) * \arctan\left(\frac{1}{3} \sqrt{3} (2x + (-a/b)^{1/3}) / (-a/b)^{1/3}\right) / (a*b^4) + \frac{1}{3} (a*b*d - a^2*g - (b^2*c - a*b*f)*x^2 - (a^2*h - a*b*e)*x) / ((b*x^3 + a)*b^3) + \frac{1}{18} (7*(-a*b^2)^{1/3}*a^2*h - 4*(-a*b^2)^{1/3}*a*b*e + 2*(-a*b^2)^{2/3}*b*c - 5*(-a*b^2)^{2/3}*a*f) * \log(x^2 + x*(-a/b)^{1/3} + (-a/b)^{2/3}) / (a*b^4) - \\
& \frac{1}{9} (2*b^6*c*(-a/b)^{1/3} - 5*a*b^5*f*(-a/b)^{1/3} + 7*a^2*b^4*h - 4*a*b^5*e) * (-a/b)^{1/3} * \log(\text{abs}(x - (-a/b)^{1/3})) / (a*b^7) + \frac{1}{12} (3*b^6*h*x^4 + 4*b^6*g*x^3 + 6*b^6*f*x^2 - 24*a*b^5*h*x + 12*b^6*x*e) / b^8
\end{aligned}$$

$$3.413 \quad \int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=311

$$\frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(\sqrt[3]{b}(bc - 4af) - \sqrt[3]{a}(2bd - 5ag))}{18a^{2/3}b^{8/3}} + \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(\sqrt[3]{b}(bc - 4af) - \sqrt[3]{a}(2bd - 5ag))}{9a^{2/3}b^{8/3}}$$

[Out] (f*x)/b^2 + (g*x^2)/(2*b^2) + (h*x^3)/(3*b^2) - (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(3*b^2*(a + b*x^3)) - ((b^(4/3)*c + 2*a^(1/3)*b*d - 4*a*b^(1/3)*f - 5*a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)))]/(3*Sqrt[3]*a^(2/3)*b^(8/3)) + ((b^(1/3)*(b*c - 4*a*f) - a^(1/3)*(2*b*d - 5*a*g))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(2/3)*b^(8/3)) - ((b^(1/3)*(b*c - 4*a*f) - a^(1/3)*(2*b*d - 5*a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(2/3)*b^(8/3)) + ((b*e - 2*a*h)*Log[a + b*x^3])/(3*b^3)

Rubi [A] time = 0.639639, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1828, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(\sqrt[3]{b}(bc - 4af) - \sqrt[3]{a}(2bd - 5ag))}{18a^{2/3}b^{8/3}} + \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(\sqrt[3]{b}(bc - 4af) - \sqrt[3]{a}(2bd - 5ag))}{9a^{2/3}b^{8/3}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]

[Out] (f*x)/b^2 + (g*x^2)/(2*b^2) + (h*x^3)/(3*b^2) - (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(3*b^2*(a + b*x^3)) - ((b^(4/3)*c + 2*a^(1/3)*b*d - 4*a*b^(1/3)*f - 5*a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)))]/(3*Sqrt[3]*a^(2/3)*b^(8/3)) + ((b^(1/3)*(b*c - 4*a*f) - a^(1/3)*(2*b*d - 5*a*g))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(2/3)*b^(8/3)) - ((b^(1/3)*(b*c - 4*a*f) - a^(1/3)*(2*b*d - 5*a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(2/3)*b^(8/3)) + ((b*e - 2*a*h)*Log[a + b*x^3])/(3*b^3)

Rule 1828

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)

```
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
+ b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
```

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 204

$\text{Int}[\frac{(a_1 + (b_1)x^2)^{-1}}{x}, x_Symbol] \rightarrow -\text{Simp}[\frac{\text{ArcTan}[\frac{\text{Rt}[-b, 2]x}{\text{Rt}[-a, 2] - \text{Rt}[-a, 2]\text{Rt}[-b, 2]}}{x}], x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 628

$\text{Int}[\frac{(d_1 + (e_1)x)}{(a_1 + (b_1)x + (c_1)x^2)}, x_Symbol] \rightarrow \text{Simp}[\frac{d \cdot \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]}{b}, x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2cd - be, 0]$

Rule 260

$\text{Int}[\frac{x^{(m_1)}}{(a_1 + (b_1)x^n)}, x_Symbol] \rightarrow \text{Simp}[\frac{\text{Log}[\text{RemoveContent}[a + bx^n, x]]}{(bn)}, x] \ /; \ \text{FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$

Rubi steps

$$\begin{aligned}
\int \frac{x^3(c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{(a + bx^3)^2} dx &= -\frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{3b^2(a + bx^3)} - \frac{\int \frac{-ab(bc-af)-2ab(bd-ag)x-3ab(be-ah)x^2}{a+bx^3}}{3ab^3} \\
&= -\frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{3b^2(a + bx^3)} - \frac{\int (-3abf - 3abgx - 3abhx^2 - \frac{ab}{3})}{3} \\
&= \frac{fx}{b^2} + \frac{gx^2}{2b^2} + \frac{hx^3}{3b^2} - \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{3b^2(a + bx^3)} + \frac{\int \frac{ab(bc-4af)+ab}{a+bx^3}}{3ab^3} \\
&= \frac{fx}{b^2} + \frac{gx^2}{2b^2} + \frac{hx^3}{3b^2} - \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{3b^2(a + bx^3)} + \frac{\int \frac{ab(bc-4af)+ab}{a+bx^3}}{3ab^3} \\
&= \frac{fx}{b^2} + \frac{gx^2}{2b^2} + \frac{hx^3}{3b^2} - \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{3b^2(a + bx^3)} + \frac{(be - 2ah) \log}{3b^3} \\
&= \frac{fx}{b^2} + \frac{gx^2}{2b^2} + \frac{hx^3}{3b^2} - \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{3b^2(a + bx^3)} + \frac{(\sqrt[3]{b}(bc - 4af))}{3b^3} \\
&= \frac{fx}{b^2} + \frac{gx^2}{2b^2} + \frac{hx^3}{3b^2} - \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{3b^2(a + bx^3)} + \frac{(\sqrt[3]{b}(bc - 4af))}{3b^3} \\
&= \frac{fx}{b^2} + \frac{gx^2}{2b^2} + \frac{hx^3}{3b^2} - \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{3b^2(a + bx^3)} - \frac{(b^{4/3}c + 2\sqrt[3]{abc})}{18b^3}
\end{aligned}$$

Mathematica [A] time = 0.212374, size = 294, normalized size = 0.95

$$\frac{-\frac{6(a^2h-ab(e+x(f+gx))+b^2x(c+dx))}{a+bx^3} - \frac{\sqrt[3]{b} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx+b^{2/3}x^2}) (5a^{4/3}g - 2\sqrt[3]{abd} - 4a\sqrt[3]{bf} + b^{4/3}c)}{a^{2/3}} + \frac{2\sqrt[3]{b} \log(\sqrt[3]{a} + \sqrt[3]{bx}) (5a^{4/3}g - 2\sqrt[3]{abd} - 4a\sqrt[3]{bf} + b^{4/3}c)}{a^{2/3}}}{18b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]

[Out] (18*b*f*x + 9*b*g*x^2 + 6*b*h*x^3 - (6*(a^2*h + b^2*x*(c + d*x) - a*b*(e + x*(f + g*x))))/(a + b*x^3) + (2*sqrt[3]*b^(1/3)*(-(b^(4/3)*c) - 2*a^(1/3)*b

$$\begin{aligned} & *d + 4*a*b^{(1/3)*f + 5*a^{(4/3)*g})*\text{ArcTan}[(1 - (2*b^{(1/3)*x})/a^{(1/3)})/\text{Sqrt}[3 \\ &]])/a^{(2/3)} + (2*b^{(1/3)*(b^{(4/3)*c} - 2*a^{(1/3)*b*d} - 4*a*b^{(1/3)*f + 5*a^{(4/3)*g})} \\ & *Log[a^{(1/3)} + b^{(1/3)*x}])/a^{(2/3)} - (b^{(1/3)*(b^{(4/3)*c} - 2*a^{(1/3)*b*d} - 4*a*b^{(1/3)*f + 5*a^{(4/3)*g})} \\ & *Log[a^{(2/3)} - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2}])/a^{(2/3)} + 6*(b*e - 2*a*h)*Log[a + b*x^3]/(18*b^3) \end{aligned}$$

Maple [B] time = 0.011, size = 533, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x)`

[Out] $\frac{1}{3}hx^3/b^2 + \frac{1}{2}gx^2/b^2 + fx/b^2 + \frac{1}{3}b^2/(b^3x^3+a)x^2ag - \frac{1}{3}x^2d/(b^3x^3+a)/b + \frac{1}{3}b^2ax/(b^3x^3+a) + \frac{1}{3}b^2x/(b^3x^3+a) + \frac{1}{3}b^2c/(b^3x^3+a) + \frac{1}{3}b^2a^2h + \frac{1}{3}b^2a/(b^3x^3+a) + \frac{1}{3}b^2e - \frac{4}{9}b^3af/(1/b^2a)^{(2/3)} * \ln(x + (1/b^2a)^{(1/3)}) + \frac{2}{9}b^3af/(1/b^2a)^{(2/3)} * \ln(x^2 - (1/b^2a)^{(1/3)}x + (1/b^2a)^{(2/3)}) - \frac{4}{9}b^3af/(1/b^2a)^{(2/3)} * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (2/(1/b^2a)^{(1/3)}x - 1)) + \frac{1}{9}b^2c/(1/b^2a)^{(2/3)} * \ln(x + (1/b^2a)^{(1/3)}) - \frac{1}{18}b^2c/(1/b^2a)^{(2/3)} * \ln(x^2 - (1/b^2a)^{(1/3)}x + (1/b^2a)^{(2/3)}) + \frac{1}{9}b^2c/(1/b^2a)^{(2/3)} * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (2/(1/b^2a)^{(1/3)}x - 1)) + \frac{5}{9}b^3ag/(1/b^2a)^{(1/3)} * \ln(x + (1/b^2a)^{(1/3)}) - \frac{5}{18}b^3ag/(1/b^2a)^{(1/3)} * \ln(x^2 - (1/b^2a)^{(1/3)}x + (1/b^2a)^{(2/3)}) - \frac{5}{9}b^3ag * 3^{(1/2)} / (1/b^2a)^{(1/3)} * \arctan(1/3 * 3^{(1/2)} * (2/(1/b^2a)^{(1/3)}x - 1)) - \frac{2}{9}b^2/(1/b^2a)^{(1/3)} * \ln(x + (1/b^2a)^{(1/3)}) * d + \frac{1}{9}b^2/(1/b^2a)^{(1/3)} * \ln(x^2 - (1/b^2a)^{(1/3)}x + (1/b^2a)^{(2/3)}) * d + \frac{2}{9}b^2 * 3^{(1/2)} / (1/b^2a)^{(1/3)} * \arctan(1/3 * 3^{(1/2)} * (2/(1/b^2a)^{(1/3)}x - 1)) * d - \frac{2}{3}b^3 * \ln(b^3x^3+a) * ah + \frac{1}{3}b^2 * \ln(b^3x^3+a) * e$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)

[Out] Timed out

Giac [A] time = 1.08641, size = 475, normalized size = 1.53

$$-\frac{(2ah - be) \log(|bx^3 + a|)}{3b^3} + \frac{\sqrt{3} \left((-ab^2)^{\frac{1}{3}} b^2c - 4 (-ab^2)^{\frac{1}{3}} abf - 2 (-ab^2)^{\frac{2}{3}} bd + 5 (-ab^2)^{\frac{2}{3}} ag \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{9ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac")

[Out] -1/3*(2*a*h - b*e)*log(abs(b*x^3 + a))/b^3 + 1/9*sqrt(3)*((-a*b^2)^(1/3)*b^2*c - 4*(-a*b^2)^(1/3)*a*b*f - 2*(-a*b^2)^(2/3)*b*d + 5*(-a*b^2)^(2/3)*a*g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^4) - 1/3*(a^2*h

$$\begin{aligned}
& + (b^2*d - a*b*g)*x^2 - a*b*e + (b^2*c - a*b*f)*x)/((b*x^3 + a)*b^3) + 1/1 \\
& 8*((-a*b^2)^{(1/3)}*b^2*c - 4*(-a*b^2)^{(1/3)}*a*b*f + 2*(-a*b^2)^{(2/3)}*b*d - 5 \\
& *(-a*b^2)^{(2/3)}*a*g)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a*b^4) - 1/9 \\
& *(2*b^4*d*(-a/b)^{(1/3)} - 5*a*b^3*g*(-a/b)^{(1/3)} + b^4*c - 4*a*b^3*f)*(-a/b) \\
& ^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/(a*b^5) + 1/6*(2*b^4*h*x^3 + 3*b^4*g*x^2 \\
& + 6*b^4*f*x)/b^6
\end{aligned}$$

$$3.414 \quad \int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=290

$$\frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(\sqrt[3]{b}(bd - 4ag) - \sqrt[3]{a}(2be - 5ah))}{18a^{2/3}b^{8/3}} + \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(\sqrt[3]{b}(bd - 4ag) - \sqrt[3]{a}(2be - 5ah))}{9a^{2/3}b^{8/3}} - \tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{b}x}{\sqrt[3]{a} + \sqrt[3]{b}x}\right)$$

[Out] $(4*g*x)/(3*b^2) + (5*h*x^2)/(6*b^2) - (c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(3*b*(a + b*x^3)) - ((b^{(4/3)*d} + 2*a^{(1/3)*b*e} - 4*a*b^{(1/3)*g} - 5*a^{(4/3)*h})*\text{ArcTan}[(a^{(1/3)} - 2*b^{(1/3)*x})/(\text{Sqrt}[3]*a^{(1/3)})]/(3*\text{Sqrt}[3]*a^{(2/3)*b^{(8/3)}}) + ((b^{(1/3)*b*d} - 4*a*g) - a^{(1/3)*(2*b*e - 5*a*h)})*\text{Log}[a^{(1/3)} + b^{(1/3)*x}]/(9*a^{(2/3)*b^{(8/3)}}) - ((b^{(1/3)*b*d} - 4*a*g) - a^{(1/3)*(2*b*e - 5*a*h)})*\text{Log}[a^{(2/3)} - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2}]/(18*a^{(2/3)*b^{(8/3)}}) + (f*\text{Log}[a + b*x^3])/(3*b^2)$

Rubi [A] time = 0.497935, antiderivative size = 288, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1823, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)\left(-\frac{\sqrt[3]{a}(2be-5ah)}{\sqrt[3]{b}} - 4ag + bd\right)}{18a^{2/3}b^{7/3}} + \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(\sqrt[3]{b}(bd - 4ag) - \sqrt[3]{a}(2be - 5ah))}{9a^{2/3}b^{8/3}} - \tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{b}x}{\sqrt[3]{a} + \sqrt[3]{b}x}\right)$$

Antiderivative was successfully verified.

[In] Int[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]

[Out] $(4*g*x)/(3*b^2) + (5*h*x^2)/(6*b^2) - (c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(3*b*(a + b*x^3)) - ((b^{(4/3)*d} + 2*a^{(1/3)*b*e} - 4*a*b^{(1/3)*g} - 5*a^{(4/3)*h})*\text{ArcTan}[(a^{(1/3)} - 2*b^{(1/3)*x})/(\text{Sqrt}[3]*a^{(1/3)})]/(3*\text{Sqrt}[3]*a^{(2/3)*b^{(8/3)}}) + ((b^{(1/3)*b*d} - 4*a*g) - a^{(1/3)*(2*b*e - 5*a*h)})*\text{Log}[a^{(1/3)} + b^{(1/3)*x}]/(9*a^{(2/3)*b^{(8/3)}}) - ((b*d - 4*a*g - (a^{(1/3)*(2*b*e - 5*a*h)})/b^{(1/3)})*\text{Log}[a^{(2/3)} - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2}]/(18*a^{(2/3)*b^{(8/3)}}) + (f*\text{Log}[a + b*x^3])/(3*b^2)$

Rule 1823

Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Pq*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*

$(a + b*x^n)^{(p + 1), x], x] /;$ FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && EqQ[m - n + 1, 0] && LtQ[p, -1]

Rule 1887

Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Pq/(a + b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]

Rule 1871

Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Dist[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

Rule 1860

Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 (c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{(a + bx^3)^2} dx &= -\frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{3b(a + bx^3)} + \frac{\int \frac{d+2ex+3fx^2+4gx^3+5hx^4}{a+bx^3} dx}{3b} \\
&= -\frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{3b(a + bx^3)} + \frac{\int \left(\frac{4g}{b} + \frac{5hx}{b} + \frac{bd-4ag+(2be-5ah)x+3bfx^2}{b(a+bx^3)} \right) dx}{3b} \\
&= \frac{4gx}{3b^2} + \frac{5hx^2}{6b^2} - \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{3b(a + bx^3)} + \frac{\int \frac{bd-4ag+(2be-5ah)x+3bfx^2}{a+bx^3} dx}{3b^2} \\
&= \frac{4gx}{3b^2} + \frac{5hx^2}{6b^2} - \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{3b(a + bx^3)} + \frac{\int \frac{bd-4ag+(2be-5ah)x}{a+bx^3} dx}{3b^2} + \frac{\int \frac{3\sqrt[3]{a}(bd-4ag+(2be-5ah)x+3bfx^2)}{a+bx^3} dx}{3b^2} \\
&= \frac{4gx}{3b^2} + \frac{5hx^2}{6b^2} - \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{3b(a + bx^3)} + \frac{(bd - 4ag - \frac{\sqrt[3]{a}(2be-5ah)}{\sqrt[3]{b}})}{9a^{2/3}b^{7/3}} \\
&= \frac{4gx}{3b^2} + \frac{5hx^2}{6b^2} - \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{3b(a + bx^3)} + \frac{(bd - 4ag - \frac{\sqrt[3]{a}(2be-5ah)}{\sqrt[3]{b}})}{9a^{2/3}b^{7/3}} \\
&= \frac{4gx}{3b^2} + \frac{5hx^2}{6b^2} - \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{3b(a + bx^3)} - \frac{(b^{4/3}d + 2\sqrt[3]{abe} - 4a\sqrt[3]{bg})}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.198048, size = 280, normalized size = 0.97

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx+b^{2/3}x^2}\right)\left(5a^{4/3}h-2\sqrt[3]{abe}-4a\sqrt[3]{bg+b^{4/3}d}\right)}{a^{2/3}} + \frac{2\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(5a^{4/3}h-2\sqrt[3]{abe}-4a\sqrt[3]{bg+b^{4/3}d}\right)}{a^{2/3}} + \frac{2\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{bx}}{\sqrt{3}}\right)\left(5a^{4/3}h-2\sqrt[3]{abe}+4a\sqrt[3]{bg+b^{4/3}d}\right)}{a^{2/3}}}{18b^{8/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]

[Out] (18*b^(2/3)*g*x + 9*b^(2/3)*h*x^2 - (6*b^(2/3)*(b*(c + x*(d + e*x)) - a*(f + x*(g + h*x))))/(a + b*x^3) + (2*Sqrt[3]*(-(b^(4/3)*d) - 2*a^(1/3)*b*e + 4

$$\begin{aligned} & *a*b^{(1/3)*g} + 5*a^{(4/3)*h}*ArcTan[(1 - (2*b^{(1/3)*x})/a^{(1/3)})/Sqrt[3]]/a^{(2/3)} \\ & + (2*(b^{(4/3)*d} - 2*a^{(1/3)*b}*e - 4*a*b^{(1/3)*g} + 5*a^{(4/3)*h})*Log[a^{(1/3)} + b^{(1/3)*x}]/a^{(2/3)} \\ & - ((b^{(4/3)*d} - 2*a^{(1/3)*b}*e - 4*a*b^{(1/3)*g} + 5*a^{(4/3)*h})*Log[a^{(2/3)} - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2}]/a^{(2/3)} + 6*b^{(2/3)*f}*Log[a + b*x^3]/(18*b^{(8/3)}) \end{aligned}$$

Maple [B] time = 0.01, size = 506, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x)`

[Out] $\frac{1}{2}h*x^2/b^2 + g*x/b^2 + 1/3/b^2/(b*x^3+a)*x^2*a*h - 1/3/b*x^2/(b*x^3+a)*e + 1/3/b^2/(b*x^3+a)*a*g*x - 1/3/b*x/(b*x^3+a)*d + 1/3*a/b^2/(b*x^3+a)*f - 1/3/b/(b*x^3+a)*c - 4/9/b^3*a*g/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)}) + 2/9/b^3*a*g/(1/b*a)^{(2/3)}*\ln(x^2 - (1/b*a)^{(1/3)*x} + (1/b*a)^{(2/3)}) - 4/9/b^3*a*g/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)*x} - 1)) + 1/9/b^2/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*d - 1/18/b^2/(1/b*a)^{(2/3)}*\ln(x^2 - (1/b*a)^{(1/3)*x} + (1/b*a)^{(2/3)})*d + 1/9/b^2/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)*x} - 1))*d + 5/9/b^3*a*h/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)}) - 5/18/b^3*a*h/(1/b*a)^{(1/3)}*\ln(x^2 - (1/b*a)^{(1/3)*x} + (1/b*a)^{(2/3)}) - 5/9/b^3*a*h*3^{(1/2)}/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)*x} - 1)) - 2/9/b^2*e/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)}) + 1/9/b^2*e/(1/b*a)^{(1/3)}*\ln(x^2 - (1/b*a)^{(1/3)*x} + (1/b*a)^{(2/3)}) + 2/9/b^2*e*3^{(1/2)}/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)*x} - 1)) + 1/3*f*\ln(b*x^3+a)/b^2$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [C] time = 11.4732, size = 27702, normalized size = 95.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(18*b*h*x^5 + 36*b*g*x^4 - 6*(2*b*e - 5*a*h)*x^2 - 2*(b^3*x^3 + a*b^2)
*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5)))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) - 6*f/b^2*log(-8*a*b^3*d*e^2 + 3*a*b^3*d^2*f - 18*a^2*b^2*e*f^2 + 48*a^3*b*f*g^2 - 1/4*(2*a^2*b^6*e - 5*a^3*b^5*h)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5)))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) - 6*f/b^2)^2 - 50*(a^3*b*d - 4*a^4*g)*h^2 + 1/2*(a*b^5*d^2 - 12*a^2*b^4*e*f - 8*a^2*b^4*d*g + 16*a^3*b^3*g^2 + 30*a^3*b^3*f*h)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5)))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) - 6*f/b^2)
```

$$\begin{aligned}
& 4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8)^{(1/3)} + (1/2)^{(1/3)}*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8)^{(1/3)} - 6*f/b^2) + 8*(4*a^2*b^2*e^2 - 3*a^2*b^2*d*f)*g + 5*(8*a^2*b^2*d*e + 9*a^3*b*f^2 - 32*a^3*b*e*g)*h - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)*x - 12*b*c + 12*a*f - 12*(b*d - 4*a*g)*x + (18*b*f*x^3 + (b^3*x^3 + a*b^2)*(2*(1/2)^{(2/3)}*(-I*sqrt(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5)))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8)^{(1/3)} + (1/2)^{(1/3)}*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8)^{(1/3)} - 6*f/b^2) + 18*a*f - 3*sqrt(1/3)*(b^3*x^3 + a*b^2)*sqrt(-((2*(1/2)^{(2/3)}*(-I*sqrt(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5)))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8)^{(1/3)} + (1/2)^{(1/3)}*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8)^{(1/3)} - 6*f/b^2)^2*a*b^5 + 12*(2*(1/2)^{(2/3)}*(-I*sqrt(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5)))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d
\end{aligned}$$

$$\begin{aligned}
& *e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^{(1/3)} + (1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) \\
& - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 \\
& + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g) \\
& *a*b^3)/(a^2*b^8))^{(1/3)} - 6*f/b^2)*a*b^3*f + 32*b^2*d*e + 36*a*b*f^2 - 128 \\
& *a*b*e*g - 80*(a*b*d - 4*a^2*g)*h)/(a*b^5)))*\log(8*a*b^3*d*e^2 - 3*a*b^3*d^2*f + 18*a^2*b^2*e*f^2 - 48*a^3*b*f*g^2 + 1/4*(2*a^2*b^6*e - 5*a^3*b^5*h)*(\\
& 2*(1/2)^{(2/3)}*(-I*\text{sqrt}(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5)))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + \\
& (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3) \\
& / (a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g) \\
& *a*b^3)/(a^2*b^8))^{(1/3)} + (1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2* \\
& b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3 \\
& *b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^{(1/3)} - 6*f/b^2)^2 + 50*(a^3*b*d \\
& - 4*a^4*g)*h^2 - 1/2*(a*b^5*d^2 - 12*a^2*b^4*e*f - 8*a^2*b^4*d*g + 16*a^3*b^3*g^2 + 30*a^3*b^3*f*h)*(2*(1/2)^{(2/3)}*(-I*\text{sqrt}(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5)))/(54*f^3/b^6 - 9* \\
& (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3) \\
& / (a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g) \\
& *a*b^3)/(a^2*b^8))^{(1/3)} + (1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 \\
& - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 1 \\
& 5*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^{(1/3)} - \\
& 6*f/b^2) - 8*(4*a^2*b^2*e^2 - 3*a^2*b^2*d*f)*g - 5*(8*a^2*b^2*d*e + 9*a^3*b*f^2 - 32*a^3*b*e*g)*h - 2*(b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3) \\
& *x + 3/4*\text{sqrt}(1/3)*(2*a*b^5*d^2 + 12*a^2*b^4*e*f - 16*a^2*b^4*d*g + 32*a^3*b^3*g^2 - 30*a^3*b^3*f*h + (2*a^2*b^6*e - 5*a^3*b^5*h)*(2*(1/2)^{(2/3)}*(-I*\text{sqrt}(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5)))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)
\end{aligned}$$

$$\begin{aligned}
& ^2b^2dg^2 - 64a^3b^3g^3 - 60a^2b^2e^2h + 150a^3b^3e^2h^2 - 125a^4h^3)/(a^2b^8) + (b^4d^3 + 125a^4h^3 - 2*(32g^3 - 90f^2g^2 + 75e^2h^2)* \\
& a^3b + 3*(9f^3 - 24e^2fg + 20e^2h + (16g^2 - 15f^2h)*d)*a^2b^2 - 2*(4e^3 - 9d^2ef + 6d^2g)*a^2b^3)/(a^2b^8))^{1/3} - 6f/b^2 + 18af + 3* \\
& \text{sqrt}(1/3)*(b^3x^3 + a^2b^2)*\text{sqrt}(-((2*(1/2)^{2/3})*(-I*\text{sqrt}(3) + 1)*(9f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5)))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) \\
& - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b^3*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b^3*e^2*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + \\
& 125*a^4*h^3 - 2*(32*g^3 - 90*f^2*g^2 + 75*e^2*h^2)*a^3*b + 3*(9*f^3 - 24*e^2*f*g + 20*e^2*h + (16*g^2 - 15*f^2*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d^2*ef + 6*d^2*g)* \\
& a^2*b^3)/(a^2*b^8))^{1/3} + (1/2)^{1/3}*(I*\text{sqrt}(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8* \\
& a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b^3*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b^3*e^2*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - \\
& 2*(32*g^3 - 90*f^2*g^2 + 75*e^2*h^2)*a^3*b + 3*(9*f^3 - 24*e^2*f*g + 20*e^2*h + (16*g^2 - 15*f^2*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d^2*ef + 6*d^2*g)*a^2*b^3)/(a^2*b^8) \\
&))^{1/3} - 6f/b^2)^2*a*b^5 + 12*(2*(1/2)^{2/3})*(-I*\text{sqrt}(3) + 1)*(9f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5)))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - \\
& (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b^3*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b^3*e^2*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 1 \\
& 25*a^4*h^3 - 2*(32*g^3 - 90*f^2*g^2 + 75*e^2*h^2)*a^3*b + 3*(9*f^3 - 24*e^2*f*g + 20*e^2*h + (16*g^2 - 15*f^2*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d^2*ef + 6*d^2*g)*a^2*b^3)/(a^2*b^8) \\
&))^{1/3} + (1/2)^{1/3}*(I*\text{sqrt}(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a* \\
& b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b^3*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b^3*e^2*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2* \\
& (32*g^3 - 90*f^2*g^2 + 75*e^2*h^2)*a^3*b + 3*(9*f^3 - 24*e^2*f*g + 20*e^2*h + (16*g^2 - 15*f^2*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d^2*ef + 6*d^2*g)*a^2*b^3)/(a^2*b^8) \\
&))^{1/3} - 6f/b^2)*a*b^3*f + 32*b^2*d*e + 36*a*b*f^2 - 128*a*b^2*e*g - 80*(a*b \\
& *d - 4*a^2*g)*h)/(a*b^5)))*\log(8*a*b^3*d^2*e^2 - 3*a*b^3*d^2*f + 18*a^2*b^2*e \\
& *f^2 - 48*a^3*b^2*f^2*g^2 + 1/4*(2*a^2*b^6*e - 5*a^3*b^5*h)*(2*(1/2)^{2/3})*(-I* \\
& \text{sqrt}(3) + 1)*(9f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h) \\
& *a*b)/(a*b^5)))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5 \\
& *d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2 \\
& *d*g^2 - 64*a^3*b^3*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b^3*e^2*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f^2*g^2 + 75*e^2*h^2)*a^3*b \\
& + 3*(9*f^3 - 24*e^2*f*g + 20*e^2*h + (16*g^2 - 15*f^2*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d^2*ef + 6*d^2*g)*a^2*b^3)/(a^2*b^8))^{1/3} + (1/2)^{1/3}*(I*\text{sqrt}(3) + 1)* \\
& (54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a \\
& *b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3 \\
& *b^3*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b^3*e^2*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4 \\
& *d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f^2*g^2 + 75*e^2*h^2)*a^3*b + 3*(9*f^3 - 24 \\
& *e^2*f*g + 20*e^2*h + (16*g^2 - 15*f^2*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d^2*ef + 6*d
\end{aligned}$$

$$\begin{aligned}
& ^2*g)*a*b^3)/(a^2*b^8))^{(1/3)} - 6*f/b^2)^2 + 50*(a^3*b*d - 4*a^4*g)*h^2 - 1 \\
& /2*(a*b^5*d^2 - 12*a^2*b^4*e*f - 8*a^2*b^4*d*g + 16*a^3*b^3*g^2 + 30*a^3*b^3 \\
& *f*h)*(2*(1/2)^{(2/3)}*(-I*\text{sqrt}(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h \\
& + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5)))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2 \\
& *g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 1 \\
& 2*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3 \\
& *b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 9 \\
& 0*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f \\
& *h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^{(1/3)} + (1 \\
& /2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 \\
& - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + \\
& 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125 \\
& *a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e \\
& h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 \\
& - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^{(1/3)} - 6*f/b^2) - 8*(4*a \\
& ^2*b^2*e^2 - 3*a^2*b^2*d*f)*g - 5*(8*a^2*b^2*d*e + 9*a^3*b*f^2 - 32*a^3*b*e \\
& *g)*h - 2*(b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a \\
& ^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)*x - 3/4*\text{sqrt}(1 \\
& /3)*(2*a*b^5*d^2 + 12*a^2*b^4*e*f - 16*a^2*b^4*d*g + 32*a^3*b^3*g^2 - 30*a^3 \\
& *b^3*f*h + (2*a^2*b^6*e - 5*a^3*b^5*h)*(2*(1/2)^{(2/3)}*(-I*\text{sqrt}(3) + 1)*(9* \\
& f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5)))/(\\
& 54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a* \\
& b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3* \\
& b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d \\
& ^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24* \\
& e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2 \\
& *g)*a*b^3)/(a^2*b^8))^{(1/3)} + (1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*(54*f^3/b^6 - 9* \\
& (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 \\
& + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2* \\
& b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h \\
& ^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2* \\
& h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^ \\
& 2*b^8))^{(1/3)} - 6*f/b^2))*\text{sqrt}(-((2*(1/2)^{(2/3)}*(-I*\text{sqrt}(3) + 1)*(9*f^2/b^4 \\
& - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5)))/(54*f^3/ \\
& b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - \\
& (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - \\
& 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 1 \\
& 25*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + \\
& 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a* \\
& b^3)/(a^2*b^8))^{(1/3)} + (1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*(54*f^3/b^6 - 9*(2*b^2* \\
& d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a* \\
& b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2 \\
& *h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2* \\
& (32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16 \\
& *g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))
\end{aligned}$$

$$\begin{aligned} & \left(\frac{1}{3} - 6f/b^2 \right)^2 a^2 b^5 + 12 \left(\frac{1}{2} \right)^{2/3} (-I\sqrt{3} + 1) (9f^2/b^4 - \\ & (2b^2 d e + 20a^2 g h + (9f^2 - 8e g - 5d h) a b) / (a^2 b^5)) / (54f^3/b^6 - \\ & 9(2b^2 d e + 20a^2 g h + (9f^2 - 8e g - 5d h) a b) f / (a^2 b^7) - (b^4 d^3 + \\ & 8a^3 b^3 e^3 - 12a^2 b^3 d^2 g + 48a^2 b^2 d g^2 - 64a^3 b g^3 - 60a^2 b^2 e^2 h + \\ & 150a^3 b e h^2 - 125a^4 h^3) / (a^2 b^8) + (b^4 d^3 + 125a^4 h^3 - 2(32g^3 - \\ & 90f g h + 75e h^2) a^3 b + 3(9f^3 - 24e f g + 20e^2 h + (16g^2 - 15f h) d) a^2 b^2 - \\ & 2(4e^3 - 9d e f + 6d^2 g) a b^3) / (a^2 b^8) \Big)^{1/3} + \left(\frac{1}{2} \right)^{1/3} (I\sqrt{3} + 1) (54f^3/b^6 - \\ & 9(2b^2 d e + 20a^2 g h + (9f^2 - 8e g - 5d h) a b) f / (a^2 b^7) - (b^4 d^3 + 8a^3 b^3 e^3 - \\ & 12a^2 b^3 d^2 g + 48a^2 b^2 d g^2 - 64a^3 b g^3 - 60a^2 b^2 e^2 h + 150a^3 b e h^2 - \\ & 125a^4 h^3) / (a^2 b^8) + (b^4 d^3 + 125a^4 h^3 - 2(32g^3 - 90f g h + 75e h^2) a^3 b + \\ & 3(9f^3 - 24e f g + 20e^2 h + (16g^2 - 15f h) d) a^2 b^2 - 2(4e^3 - 9d e f + 6d^2 g) a b^3) / (a^2 b^8) \Big)^{1/3} - \\ & 6f/b^2 \Big) a^2 b^3 f + 32b^2 d e + 36a^2 b f^2 - 128a^2 b e g - 80(a^2 b d - 4a^2 g) h) / (a^2 b^5) \Big) / (b^3 x^3 + a^2 b^2) \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)

[Out] Timed out

Giac [A] time = 1.09203, size = 444, normalized size = 1.53

$$\frac{f \log(|bx^3 + a|)}{3b^2} + \frac{(ah - be)x^2 - bc + af - (bd - ag)x}{3(bx^3 + a)b^2} + \frac{\sqrt{3} \left((-ab^2)^{\frac{1}{3}} b^2 d - 4(-ab^2)^{\frac{1}{3}} abg + 5(-ab^2)^{\frac{2}{3}} ah - 2(-ab^2)^{\frac{2}{3}} \right)}{9ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac")

```
[Out] 1/3*f*log(abs(b*x^3 + a))/b^2 + 1/3*((a*h - b*e)*x^2 - b*c + a*f - (b*d - a
*g)*x)/((b*x^3 + a)*b^2) + 1/9*sqrt(3)*((-a*b^2)^(1/3)*b^2*d - 4*(-a*b^2)^(
1/3)*a*b*g + 5*(-a*b^2)^(2/3)*a*h - 2*(-a*b^2)^(2/3)*b*e)*arctan(1/3*sqrt(3
)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^4) + 1/2*(b^2*h*x^2 + 2*b^2*g*x)/
b^4 + 1/18*((-a*b^2)^(1/3)*b^2*d - 4*(-a*b^2)^(1/3)*a*b*g - 5*(-a*b^2)^(2/3
)*a*h + 2*(-a*b^2)^(2/3)*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b
^4) + 1/9*(5*a*b^3*h*(-a/b)^(1/3) - 2*b^4*(-a/b)^(1/3)*e - b^4*d + 4*a*b^3*
g)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5)
```

$$3.415 \quad \int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2} dx$$

Optimal. Leaf size=289

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(b^{2/3}(2af+bc) - a^{2/3}(be-4ah)\right)}{18a^{4/3}b^{7/3}} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(b^{2/3}(2af+bc) - a^{2/3}(be-4ah)\right)}{9a^{4/3}b^{7/3}} - \text{tan}$$

[Out] (h*x)/b^2 - (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(3*a*b^2*(a + b*x^3)) - ((b^(5/3)*c + a^(2/3)*b*e + 2*a*b^(2/3)*f - 4*a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(4/3)*b^(7/3)) - ((b^(2/3)*(b*c + 2*a*f) - a^(2/3)*(b*e - 4*a*h))*Log[a^(1/3) + b^(1/3)*x]/(9*a^(4/3)*b^(7/3)) + ((b^(2/3)*(b*c + 2*a*f) - a^(2/3)*(b*e - 4*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(18*a^(4/3)*b^(7/3)) + (g*Log[a + b*x^3])/(3*b^2)

Rubi [A] time = 0.505355, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1828, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(b^{2/3}(2af+bc) - a^{2/3}(be-4ah)\right)}{18a^{4/3}b^{7/3}} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(b^{2/3}(2af+bc) - a^{2/3}(be-4ah)\right)}{9a^{4/3}b^{7/3}} - \text{tan}$$

Antiderivative was successfully verified.

[In] Int[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]

[Out] (h*x)/b^2 - (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(3*a*b^2*(a + b*x^3)) - ((b^(5/3)*c + a^(2/3)*b*e + 2*a*b^(2/3)*f - 4*a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(4/3)*b^(7/3)) - ((b^(2/3)*(b*c + 2*a*f) - a^(2/3)*(b*e - 4*a*h))*Log[a^(1/3) + b^(1/3)*x]/(9*a^(4/3)*b^(7/3)) + ((b^(2/3)*(b*c + 2*a*f) - a^(2/3)*(b*e - 4*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(18*a^(4/3)*b^(7/3)) + (g*Log[a + b*x^3])/(3*b^2)

Rule 1828

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)

```
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
+ b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
```

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 204

$\text{Int}[\frac{(a_1 + (b_1)x^2)^{-1}}{x}, x_Symbol] \rightarrow -\text{Simp}[\frac{\text{ArcTan}[\frac{\text{Rt}[-b, 2]x}{\text{Rt}[-a, 2] - \text{Rt}[-a, 2]\text{Rt}[-b, 2]}}{x}]}{\text{Rt}[-a, 2] - \text{Rt}[-a, 2]\text{Rt}[-b, 2]}, x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 628

$\text{Int}[\frac{(d_1 + (e_1)x)}{(a_1 + (b_1)x + (c_1)x^2)}, x_Symbol] \rightarrow \text{Simp}[\frac{d \cdot \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]}{b}, x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2cd - be, 0]$

Rule 260

$\text{Int}[x^{(m_1)} / (a_1 + (b_1)x^{(n_1)}), x_Symbol] \rightarrow \text{Simp}[\frac{\text{Log}[\text{RemoveContent}[a + bx^n, x]]}{(b \cdot n)}, x] \ /; \ \text{FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$

Rubi steps

$$\begin{aligned}
\int \frac{x(c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{(a + bx^3)^2} dx &= -\frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{3ab^2(a + bx^3)} - \frac{\int \frac{-a(be - ah) - b(bc + 2af)x - 3abgx^2 - 3a^2}{a + bx^3}}{3ab^2} \\
&= -\frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{3ab^2(a + bx^3)} - \frac{\int \left(-3ah - \frac{a(be - 4ah) + b(bc + 2af)x}{a + bx^3}\right)}{3ab^2} \\
&= \frac{hx}{b^2} - \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{3ab^2(a + bx^3)} + \frac{\int \frac{a(be - 4ah) + b(bc + 2af)x + 3abg}{a + bx^3}}{3ab^2} \\
&= \frac{hx}{b^2} - \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{3ab^2(a + bx^3)} + \frac{\int \frac{a(be - 4ah) + b(bc + 2af)x}{a + bx^3} dx}{3ab^2} \\
&= \frac{hx}{b^2} - \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{3ab^2(a + bx^3)} + \frac{g \log(a + bx^3)}{3b^2} + \frac{\int \frac{\sqrt[3]{a}}{a + bx^3}}{3b^2} \\
&= \frac{hx}{b^2} - \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{3ab^2(a + bx^3)} - \frac{(b^{2/3}(bc + 2af) - a^{2/3}(be - ah))}{9a^{4/3}} \\
&= \frac{hx}{b^2} - \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{3ab^2(a + bx^3)} - \frac{(b^{2/3}(bc + 2af) - a^{2/3}(be - ah))}{9a^{4/3}} \\
&= \frac{hx}{b^2} - \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{3ab^2(a + bx^3)} - \frac{(b^{5/3}c + a^{2/3}be + 2ab^{2/3}f - a^{4/3}g)}{3\sqrt[3]{3a^4}}
\end{aligned}$$

Mathematica [A] time = 0.196368, size = 285, normalized size = 0.99

$$\frac{6b^{2/3}(a^2(g+hx) - ab(d+x(e+fx)) + b^2cx^2)}{a(a+bx^3)} + \frac{\log\left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx + b^{2/3}x^2}\right) \left(-a^{2/3}b^{4/3}e + 4a^{5/3} \sqrt[3]{bh + 2abf + b^2c}\right)}{a^{4/3}} - \frac{2 \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right) \left(-a^{2/3}b^{4/3}e + 4a^{5/3} \sqrt[3]{bh + 2abf + b^2c}\right)}{a^{4/3}}}{18b^{8/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2, x]

[Out] (18*b^(2/3)*h*x + (6*b^(2/3)*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e + f*x))))/(a*(a + b*x^3)) - (2*sqrt[3]*(b^2*c + a^(2/3)*b^(4/3)*e + 2*a*b*f -

$$4a^{5/3}b^{1/3}h \operatorname{ArcTan}\left[\frac{1 - (2b^{1/3}x)/a^{1/3}}{\sqrt{3}}\right]/a^{4/3} - (2(b^2c - a^{2/3}b^{4/3})e + 2abf + 4a^{5/3}b^{1/3}h) \operatorname{Log}[a^{1/3} + b^{1/3}x]/a^{4/3} + ((b^2c - a^{2/3}b^{4/3})e + 2abf + 4a^{5/3}b^{1/3}h) \operatorname{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2]/a^{4/3} + 6b^{2/3}g \operatorname{Log}[a + b^3x^3]/(18b^{8/3})$$

Maple [B] time = 0.01, size = 502, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x)`

[Out]
$$h*x/b^2 - 1/3/b*x^2/(b*x^3+a)*f + 1/3*x^2/a/(b*x^3+a)*c + 1/3/b^2/(b*x^3+a)*a*h*x - 1/3/b*x/(b*x^3+a)*e + 1/3/b^2/(b*x^3+a)*a*g - 1/3/b/(b*x^3+a)*d - 4/9/b^3*a/(1/b*a)^{2/3}*\ln(x+(1/b*a)^{1/3})*h + 1/9/b^2*e/(1/b*a)^{2/3}*\ln(x+(1/b*a)^{1/3}) + 2/9/b^3*a/(1/b*a)^{2/3}*\ln(x^2-(1/b*a)^{1/3}*x+(1/b*a)^{2/3})*h - 1/18/b^2*e/(1/b*a)^{2/3}*\ln(x^2-(1/b*a)^{1/3}*x+(1/b*a)^{2/3}) - 4/9/b^3*a/(1/b*a)^{2/3})*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1))*h + 1/9/b^2*e/(1/b*a)^{2/3})*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1)) - 2/9/b^2/(1/b*a)^{1/3}*\ln(x+(1/b*a)^{1/3})*f - 1/9/b/a/(1/b*a)^{1/3}*\ln(x+(1/b*a)^{1/3})*c + 1/9/b^2/(1/b*a)^{1/3}*\ln(x^2-(1/b*a)^{1/3}*x+(1/b*a)^{2/3})*f + 1/18/b/a/(1/b*a)^{1/3}*\ln(x^2-(1/b*a)^{1/3}*x+(1/b*a)^{2/3})*c + 2/9/b^2*3^{1/2}/(1/b*a)^{1/3}*\arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1))*f + 1/9/b/a*3^{1/2}/(1/b*a)^{1/3}*\arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1))*c + 1/3*g*\ln(b*x^3+a)/b^2$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [C] time = 13.0278, size = 27753, normalized size = 96.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")

[Out]
$$\frac{1}{36} \cdot (36 \cdot a \cdot b \cdot h \cdot x^4 - 12 \cdot a \cdot b \cdot d + 12 \cdot a^2 \cdot g + 12 \cdot (b^2 \cdot c - a \cdot b \cdot f) \cdot x^2 - 2 \cdot (a \cdot b^3 \cdot x^3 + a^2 \cdot b^2) \cdot (2 \cdot (1/2)^{2/3} \cdot (-\sqrt{3}) + 1) \cdot (9 \cdot g^2/b^4 - (b^2 \cdot c \cdot e + (9 \cdot g^2 - 8 \cdot f \cdot h) \cdot a^2 + 2 \cdot (e \cdot f - 2 \cdot c \cdot h) \cdot a \cdot b) / (a^2 \cdot b^4))) / (54 \cdot g^3/b^6 - 9 \cdot (b^2 \cdot c \cdot e + (9 \cdot g^2 - 8 \cdot f \cdot h) \cdot a^2 + 2 \cdot (e \cdot f - 2 \cdot c \cdot h) \cdot a \cdot b) \cdot g / (a^2 \cdot b^6) - (b^5 \cdot c^3 + a^2 \cdot b^3 \cdot e^3 + 6 \cdot a \cdot b^4 \cdot c^2 \cdot f + 12 \cdot a^2 \cdot b^3 \cdot c \cdot f^2 + 8 \cdot a^3 \cdot b^2 \cdot f^3 - 12 \cdot a^3 \cdot b^2 \cdot e^2 \cdot h + 48 \cdot a^4 \cdot b \cdot e \cdot h^2 - 64 \cdot a^5 \cdot h^3) / (a^4 \cdot b^7) - (b^5 \cdot c^3 + 6 \cdot a \cdot b^4 \cdot c^2 \cdot f + 64 \cdot a^5 \cdot h^3 - 3 \cdot (9 \cdot g^3 - 24 \cdot f \cdot g \cdot h + 16 \cdot e \cdot h^2) \cdot a^4 \cdot b + 2 \cdot (4 \cdot f^3 - 9 \cdot e \cdot f \cdot g + 6 \cdot e^2 \cdot h + 18 \cdot c \cdot g \cdot h) \cdot a^3 \cdot b^2 - (e^3 - 3 \cdot (4 \cdot f^2 - 3 \cdot e \cdot g) \cdot c) \cdot a^2 \cdot b^3) / (a^4 \cdot b^7))^{1/3} + (1/2)^{1/3} \cdot (I \cdot \sqrt{3}) + 1) \cdot (54 \cdot g^3/b^6 - 9 \cdot (b^2 \cdot c \cdot e + (9 \cdot g^2 - 8 \cdot f \cdot h) \cdot a^2 + 2 \cdot (e \cdot f - 2 \cdot c \cdot h) \cdot a \cdot b) \cdot g / (a^2 \cdot b^6) - (b^5 \cdot c^3 + a^2 \cdot b^3 \cdot e^3 + 6 \cdot a \cdot b^4 \cdot c^2 \cdot f + 12 \cdot a^2 \cdot b^3 \cdot c \cdot f^2 + 8 \cdot a^3 \cdot b^2 \cdot f^3 - 12 \cdot a^3 \cdot b^2 \cdot e^2 \cdot h + 48 \cdot a^4 \cdot b \cdot e \cdot h^2 - 64 \cdot a^5 \cdot h^3) / (a^4 \cdot b^7) - (b^5 \cdot c^3 + 6 \cdot a \cdot b^4 \cdot c^2 \cdot f + 64 \cdot a^5 \cdot h^3 - 3 \cdot (9 \cdot g^3 - 24 \cdot f \cdot g \cdot h + 16 \cdot e \cdot h^2) \cdot a^4 \cdot b + 2 \cdot (4 \cdot f^3 - 9 \cdot e \cdot f \cdot g + 6 \cdot e^2 \cdot h + 18 \cdot c \cdot g \cdot h) \cdot a^3 \cdot b^2 - (e^3 - 3 \cdot (4 \cdot f^2 - 3 \cdot e \cdot g) \cdot c) \cdot a^2 \cdot b^3) / (a^4 \cdot b^7))^{1/3} - 6 \cdot g / b^2) \cdot \log(-2 \cdot a \cdot b^4 \cdot c^2 \cdot e - 8 \cdot a^2 \cdot b^3 \cdot c \cdot e \cdot f - 8 \cdot a^3 \cdot b^2 \cdot e \cdot f^2 + 3 \cdot a^3 \cdot b^2 \cdot e^2 \cdot g + 48 \cdot a^5 \cdot g \cdot h^2 - 1/4 \cdot (a^3 \cdot b^6 \cdot c + 2 \cdot a^4 \cdot b^5 \cdot f) \cdot (2 \cdot (1/2)^{2/3} \cdot (-\sqrt{3}) + 1) \cdot (9 \cdot g^2/b^4 - (b^2 \cdot c \cdot e + (9 \cdot g^2 - 8 \cdot f \cdot h) \cdot a^2 + 2 \cdot (e \cdot f - 2 \cdot c \cdot h) \cdot a \cdot b) / (a^2 \cdot b^4))) / (54 \cdot g^3/b^6 - 9 \cdot (b^2 \cdot c \cdot e + (9 \cdot g^2 - 8 \cdot f \cdot h) \cdot a^2 + 2 \cdot (e \cdot f - 2 \cdot c \cdot h) \cdot a \cdot b) \cdot g / (a^2 \cdot b^6) - (b^5 \cdot c^3 + a^2 \cdot b^3 \cdot e^3 + 6 \cdot a \cdot b^4 \cdot c^2 \cdot f + 12 \cdot a^2 \cdot b^3 \cdot c \cdot f^2 + 8 \cdot a^3 \cdot b^2 \cdot f^3 - 12 \cdot a^3 \cdot b^2 \cdot e^2 \cdot h + 48 \cdot a^4 \cdot b \cdot e \cdot h^2 - 64 \cdot a^5 \cdot h^3) / (a^4 \cdot b^7) - (b^5 \cdot c^3 + 6 \cdot a \cdot b^4 \cdot c^2 \cdot f + 64 \cdot a^5 \cdot h^3 - 3 \cdot (9 \cdot g^3 - 24 \cdot f \cdot g \cdot h + 16 \cdot e \cdot h^2) \cdot a^4 \cdot b + 2 \cdot (4 \cdot f^3 - 9 \cdot e \cdot f \cdot g + 6 \cdot e^2 \cdot h + 18 \cdot c \cdot g \cdot h) \cdot a^3 \cdot b^2 - (e^3 - 3 \cdot (4 \cdot f^2 - 3 \cdot e \cdot g) \cdot c) \cdot a^2 \cdot b^3) / (a^4 \cdot b^7))^{1/3} + (1/2)^{1/3} \cdot (I \cdot \sqrt{3}) + 1) \cdot (54 \cdot g^3/b^6 - 9 \cdot (b^2 \cdot c \cdot e + (9 \cdot g^2 - 8 \cdot f \cdot h) \cdot a^2 + 2 \cdot (e \cdot f - 2 \cdot c \cdot h) \cdot a \cdot b) \cdot g / (a^2 \cdot b^6) - (b^5 \cdot c^3 + a^2 \cdot b^3 \cdot e^3 + 6 \cdot a \cdot b^4 \cdot c^2 \cdot f + 12 \cdot a^2 \cdot b^3 \cdot c \cdot f^2 + 8 \cdot a^3 \cdot b^2 \cdot f^3 - 12 \cdot a^3 \cdot b^2 \cdot e^2 \cdot h + 48 \cdot a^4 \cdot b \cdot e \cdot h^2 - 64 \cdot a^5 \cdot h^3) / (a^4 \cdot b^7) - (b^5 \cdot c^3 + 6 \cdot a \cdot b^4 \cdot c^2 \cdot f + 64 \cdot a^5 \cdot h^3 - 3 \cdot (9 \cdot g^3 - 24 \cdot f \cdot g \cdot h + 16 \cdot e \cdot h^2) \cdot a^4 \cdot b + 2 \cdot (4 \cdot f^3 - 9 \cdot e \cdot f \cdot g + 6 \cdot e^2 \cdot h + 18 \cdot c \cdot g \cdot h) \cdot a^3 \cdot b^2 - (e^3 - 3 \cdot (4 \cdot f^2 - 3 \cdot e \cdot g) \cdot c) \cdot a^2 \cdot b^3) / (a^4 \cdot b^7))^{1/3} - 9 \cdot (a^3 \cdot b^2 \cdot c + 2 \cdot a^4 \cdot b \cdot f) \cdot g^2 + 1/2 \cdot (a^3 \cdot b^4 \cdot e^2 - 8 \cdot a^4 \cdot b^3 \cdot e \cdot h + 16 \cdot a^5 \cdot b^2 \cdot h^2 - 6 \cdot (a^3 \cdot b^4 \cdot c + 2 \cdot a^4 \cdot b^3 \cdot f) \cdot g) \cdot (2 \cdot (1/2)^{2/3} \cdot (-\sqrt{3}) + 1) \cdot (9 \cdot g^2/b^4 - (b^2 \cdot c \cdot e + (9 \cdot g^2 - 8 \cdot f \cdot h) \cdot a^2 + 2 \cdot (e \cdot f - 2 \cdot c \cdot h) \cdot a \cdot b) / (a^2 \cdot b^4))) / (54 \cdot g^3/b^6 - 9 \cdot (b^2 \cdot c \cdot e + (9 \cdot g^2 - 8 \cdot f \cdot h) \cdot a^2 + 2 \cdot (e \cdot f - 2 \cdot c \cdot h) \cdot a \cdot b) \cdot g / (a^2 \cdot b^6) - (b^5 \cdot c^3 + a^2 \cdot b^3 \cdot e^3 + 6 \cdot a \cdot b^4 \cdot c^2 \cdot f + 12 \cdot a^2 \cdot b^3 \cdot c \cdot f^2 + 8 \cdot a^3 \cdot b^2 \cdot f^3 - 12 \cdot a^3 \cdot b^2 \cdot e^2 \cdot h + 48 \cdot a^4 \cdot b \cdot e \cdot h^2 - 64 \cdot a^5 \cdot h^3) / (a^4 \cdot b^7) - (b^5 \cdot c^3 + 6 \cdot a \cdot b^4 \cdot c^2 \cdot f + 64 \cdot a^5 \cdot h^3 - 3 \cdot (9 \cdot g^3 - 24 \cdot f \cdot g \cdot h + 16 \cdot e \cdot h^2) \cdot a^4 \cdot b + 2 \cdot (4 \cdot f^3 - 9 \cdot e \cdot f \cdot g + 6 \cdot e^2 \cdot h + 18 \cdot c \cdot g \cdot h) \cdot a^3 \cdot b^2 - (e^3 - 3 \cdot (4 \cdot f^2 - 3 \cdot e \cdot g) \cdot c) \cdot a^2 \cdot b^3) / (a^4 \cdot b^7))^{1/3} - 6 \cdot g / b^2)^2$$

$$\begin{aligned}
& 3 - 9*ef*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2 \\
& *b^3)/(a^4*b^7))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(54*g^3/b^6 - 9*(b^2*c \\
& *e + (9*g^2 - 8*f*h)*a^2 + 2*(ef - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^ \\
& 2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e \\
& ^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + \\
& 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*ef*g + 6 \\
& *e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7) \\
&)^{(1/3)} - 6*g/b^2) + 6*(a^3*b^4*c + 2*a^4*b^3*f)*g)*\sqrt{-((2*(1/2)^{(2/3)}*(\\
& -I*\sqrt{3} + 1)*(9*g^2/b^4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(ef - 2*c \\
& h)*a*b)/(a^2*b^4)))/(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(ef \\
& - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2 \\
& *b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3 \\
&)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + \\
& 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*ef*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 \\
& - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + \\
& 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(ef - 2*c*h)*a*b)*g \\
& / (a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8* \\
& a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (\\
& b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4* \\
& b + 2*(4*f^3 - 9*ef*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3* \\
& e*g)*c)*a^2*b^3)/(a^4*b^7))^{(1/3)} - 6*g/b^2)^2*a^2*b^4 + 12*(2*(1/2)^{(2/3)}* \\
& (-I*\sqrt{3} + 1)*(9*g^2/b^4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(ef - 2*c \\
& h)*a*b)/(a^2*b^4)))/(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(ef \\
& - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^ \\
& 2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^ \\
& 3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h \\
& + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*ef*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 \\
& - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} \\
& + 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(ef - 2*c*h)*a*b)* \\
& g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8 \\
& *a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - \\
& (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4 \\
& *b + 2*(4*f^3 - 9*ef*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3 \\
& *e*g)*c)*a^2*b^3)/(a^4*b^7))^{(1/3)} - 6*g/b^2)*a^2*b^2*g + 16*b^2*c*e + 32*a \\
& *b*e*f + 36*a^2*g^2 - 64*(a*b*c + 2*a^2*f)*h)/(a^2*b^4))) + (18*a*b*g*x^3 + \\
& 18*a^2*g + (a*b^3*x^3 + a^2*b^2)*(2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*(9*g^2/b^ \\
& 4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(ef - 2*c*h)*a*b)/(a^2*b^4)))/(54*g^ \\
& 3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(ef - 2*c*h)*a*b)*g/(a^2*b^6) \\
& - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^ \\
& 3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + \\
& 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f \\
& ^3 - 9*ef*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^ \\
& 2*b^3)/(a^4*b^7))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(54*g^3/b^6 - 9*(b^2* \\
& c*e + (9*g^2 - 8*f*h)*a^2 + 2*(ef - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a \\
& ^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*
\end{aligned}$$

$$\begin{aligned}
& e^2 h + 48 a^4 b e h^2 - 64 a^5 h^3 / (a^4 b^7) - (b^5 c^3 + 6 a b^4 c^2 f + 64 a^5 h^3 - 3(9 g^3 - 24 f g h + 16 e h^2) a^4 b + 2(4 f^3 - 9 e f g + 6 e^2 h + 18 c g h) a^3 b^2 - (e^3 - 3(4 f^2 - 3 e g) c) a^2 b^3) / (a^4 b^7)^{1/3} - 6 g / b^2 + 3 \sqrt{1/3} (a b^3 x^3 + a^2 b^2) \sqrt{-((2(1/2)^{2/3}) (-I \sqrt{3}) + 1) (9 g^2 / b^4 - (b^2 c e + (9 g^2 - 8 f h) a^2 + 2(e f - 2 c h) a b) / (a^2 b^4)) / (54 g^3 / b^6 - 9(b^2 c e + (9 g^2 - 8 f h) a^2 + 2(e f - 2 c h) a b) g / (a^2 b^6) - (b^5 c^3 + a^2 b^3 e^3 + 6 a b^4 c^2 f + 12 a^2 b^3 c f^2 + 8 a^3 b^2 f^3 - 12 a^3 b^2 e^2 h + 48 a^4 b e h^2 - 64 a^5 h^3) / (a^4 b^7) - (b^5 c^3 + 6 a b^4 c^2 f + 64 a^5 h^3 - 3(9 g^3 - 24 f g h + 16 e h^2) a^4 b + 2(4 f^3 - 9 e f g + 6 e^2 h + 18 c g h) a^3 b^2 - (e^3 - 3(4 f^2 - 3 e g) c) a^2 b^3) / (a^4 b^7))^{1/3} + (1/2)^{1/3} (I \sqrt{3}) + 1) (54 g^3 / b^6 - 9(b^2 c e + (9 g^2 - 8 f h) a^2 + 2(e f - 2 c h) a b) g / (a^2 b^6) - (b^5 c^3 + a^2 b^3 e^3 + 6 a b^4 c^2 f + 12 a^2 b^3 c f^2 + 8 a^3 b^2 f^3 - 12 a^3 b^2 e^2 h + 48 a^4 b e h^2 - 64 a^5 h^3) / (a^4 b^7) - (b^5 c^3 + 6 a b^4 c^2 f + 64 a^5 h^3 - 3(9 g^3 - 24 f g h + 16 e h^2) a^4 b + 2(4 f^3 - 9 e f g + 6 e^2 h + 18 c g h) a^3 b^2 - (e^3 - 3(4 f^2 - 3 e g) c) a^2 b^3) / (a^4 b^7))^{1/3} - 6 g / b^2)^2 a^2 b^4 + 12(2(1/2)^{2/3}) (-I \sqrt{3}) + 1) (9 g^2 / b^4 - (b^2 c e + (9 g^2 - 8 f h) a^2 + 2(e f - 2 c h) a b) / (a^2 b^4)) / (54 g^3 / b^6 - 9(b^2 c e + (9 g^2 - 8 f h) a^2 + 2(e f - 2 c h) a b) g / (a^2 b^6) - (b^5 c^3 + a^2 b^3 e^3 + 6 a b^4 c^2 f + 12 a^2 b^3 c f^2 + 8 a^3 b^2 f^3 - 12 a^3 b^2 e^2 h + 48 a^4 b e h^2 - 64 a^5 h^3) / (a^4 b^7) - (b^5 c^3 + 6 a b^4 c^2 f + 64 a^5 h^3 - 3(9 g^3 - 24 f g h + 16 e h^2) a^4 b + 2(4 f^3 - 9 e f g + 6 e^2 h + 18 c g h) a^3 b^2 - (e^3 - 3(4 f^2 - 3 e g) c) a^2 b^3) / (a^4 b^7))^{1/3} + (1/2)^{1/3} (I \sqrt{3}) + 1) (54 g^3 / b^6 - 9(b^2 c e + (9 g^2 - 8 f h) a^2 + 2(e f - 2 c h) a b) g / (a^2 b^6) - (b^5 c^3 + a^2 b^3 e^3 + 6 a b^4 c^2 f + 12 a^2 b^3 c f^2 + 8 a^3 b^2 f^3 - 12 a^3 b^2 e^2 h + 48 a^4 b e h^2 - 64 a^5 h^3) / (a^4 b^7) - (b^5 c^3 + 6 a b^4 c^2 f + 64 a^5 h^3 - 3(9 g^3 - 24 f g h + 16 e h^2) a^4 b + 2(4 f^3 - 9 e f g + 6 e^2 h + 18 c g h) a^3 b^2 - (e^3 - 3(4 f^2 - 3 e g) c) a^2 b^3) / (a^4 b^7))^{1/3} - 6 g / b^2) a^2 b^2 g + 16 b^2 c e + 32 a b e f + 36 a^2 g^2 - 64(a b c + 2 a^2 f) h) / (a^2 b^4)) * \log(2 a b^4 c^2 e + 8 a^2 b^3 c e f + 8 a^3 b^2 e f^2 - 3 a^3 b^2 e^2 g - 48 a^5 g h^2 + 1/4(a^3 b^6 c + 2 a^4 b^5 f) * (2(1/2)^{2/3}) (-I \sqrt{3}) + 1) (9 g^2 / b^4 - (b^2 c e + (9 g^2 - 8 f h) a^2 + 2(e f - 2 c h) a b) / (a^2 b^4)) / (54 g^3 / b^6 - 9(b^2 c e + (9 g^2 - 8 f h) a^2 + 2(e f - 2 c h) a b) g / (a^2 b^6) - (b^5 c^3 + a^2 b^3 e^3 + 6 a b^4 c^2 f + 12 a^2 b^3 c f^2 + 8 a^3 b^2 f^3 - 12 a^3 b^2 e^2 h + 48 a^4 b e h^2 - 64 a^5 h^3) / (a^4 b^7) - (b^5 c^3 + 6 a b^4 c^2 f + 64 a^5 h^3 - 3(9 g^3 - 24 f g h + 16 e h^2) a^4 b + 2(4 f^3 - 9 e f g + 6 e^2 h + 18 c g h) a^3 b^2 - (e^3 - 3(4 f^2 - 3 e g) c) a^2 b^3) / (a^4 b^7))^{1/3} + (1/2)^{1/3} (I \sqrt{3}) + 1) (54 g^3 / b^6 - 9(b^2 c e + (9 g^2 - 8 f h) a^2 + 2(e f - 2 c h) a b) g / (a^2 b^6) - (b^5 c^3 + a^2 b^3 e^3 + 6 a b^4 c^2 f + 12 a^2 b^3 c f^2 + 8 a^3 b^2 f^3 - 12 a^3 b^2 e^2 h + 48 a^4 b e h^2 - 64 a^5 h^3) / (a^4 b^7) - (b^5 c^3 + 6 a b^4 c^2 f + 64 a^5 h^3 - 3(9 g^3 - 24 f g h + 16 e h^2) a^4 b + 2(4 f^3 - 9 e f g + 6 e^2 h + 18 c g h) a^3 b^2 - (e^3 - 3(4 f^2 - 3 e g) c) a^2 b^3) / (a^4 b^7))^{1/3}
\end{aligned}$$

$$\begin{aligned}
& (1/3) - 6*g/b^2)^2 + 9*(a^3*b^2*c + 2*a^4*b*f)*g^2 - 1/2*(a^3*b^4*e^2 - 8*a^4*b^3*e*h + 16*a^5*b^2*h^2 - 6*(a^3*b^4*c + 2*a^4*b^3*f)*g)*(2*(1/2)^{(2/3)} \\
& *(-I*\sqrt{3} + 1)*(9*g^2/b^4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)/(a^2*b^4)))/(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^{(1/3)} - 6*g/b^2) - 8*(a^2*b^3*c^2 + 4*a^3*b^2*c*f + 4*a^4*b*f^2 - 3*a^4*b*e*g)*h - 2*(b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)*x - 3/4*\sqrt{1/3}*(2*a^3*b^4*e^2 - 16*a^4*b^3*e*h + 32*a^5*b^2*h^2 + (a^3*b^6*c + 2*a^4*b^5*f)*(2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*(9*g^2/b^4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)/(a^2*b^4)))/(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^{(1/3)} - 6*g/b^2) + 6*(a^3*b^4*c + 2*a^4*b^3*f)*g)*\sqrt{-((2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*(9*g^2/b^4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)/(a^2*b^4)))/(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^{(1/3)} - 6*g/b^2)^2*a^2*b^4 + 12*(2*(1/2)^{(2/3)}
\end{aligned}$$

$$\begin{aligned} & *(-I\sqrt{3} + 1)*(9g^2/b^4 - (b^2ce + (9g^2 - 8fh)a^2 + 2(ef - 2ch)ab)/(a^2b^4))/(54g^3/b^6 - 9(b^2ce + (9g^2 - 8fh)a^2 + 2(ef - 2ch)ab) * g/(a^2b^6) - (b^5c^3 + a^2b^3e^3 + 6ab^4c^2f + 12a^2b^3cf^2 + 8a^3b^2f^3 - 12a^3b^2e^2h + 48a^4b*eh^2 - 64a^5h^3)/(a^4b^7) - (b^5c^3 + 6ab^4c^2f + 64a^5h^3 - 3(9g^3 - 24f*gh + 16eh^2)a^4b + 2(4f^3 - 9efg + 6e^2h + 18c*gh)a^3b^2 - (e^3 - 3(4f^2 - 3eg)c)a^2b^3)/(a^4b^7))^{1/3} + (1/2)^{1/3}*(I\sqrt{3} + 1)*(54g^3/b^6 - 9(b^2ce + (9g^2 - 8fh)a^2 + 2(ef - 2ch)ab) * g/(a^2b^6) - (b^5c^3 + a^2b^3e^3 + 6ab^4c^2f + 12a^2b^3cf^2 + 8a^3b^2f^3 - 12a^3b^2e^2h + 48a^4b*eh^2 - 64a^5h^3)/(a^4b^7) - (b^5c^3 + 6ab^4c^2f + 64a^5h^3 - 3(9g^3 - 24f*gh + 16eh^2)a^4b + 2(4f^3 - 9efg + 6e^2h + 18c*gh)a^3b^2 - (e^3 - 3(4f^2 - 3eg)c)a^2b^3)/(a^4b^7))^{1/3} - 6g/b^2)a^2b^2g + 16b^2ce + 32ab*ef + 36a^2g^2 - 64(ab*c + 2a^2f)h)/(a^2b^4)))/(ab^3x^3 + a^2b^2) \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)

[Out] Timed out

Giac [A] time = 1.08823, size = 451, normalized size = 1.56

$$\frac{hx}{b^2} + \frac{g \log(|bx^3 + a|)}{3b^2} - \frac{\sqrt{3} \left(4(-ab^2)^{\frac{1}{3}} a^2 h - (-ab^2)^{\frac{1}{3}} a b e + (-ab^2)^{\frac{2}{3}} b c + 2(-ab^2)^{\frac{2}{3}} a f \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}} \right)}{9a^2b^3} - \frac{abd}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac")


```
[Out] h*x/b^2 + 1/3*g*log(abs(b*x^3 + a))/b^2 - 1/9*sqrt(3)*(4*(-a*b^2)^(1/3)*a^2
*h - (-a*b^2)^(1/3)*a*b*e + (-a*b^2)^(2/3)*b*c + 2*(-a*b^2)^(2/3)*a*f)*arct
an(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^3) - 1/3*(a*b*d -
a^2*g - (b^2*c - a*b*f)*x^2 - (a^2*h - a*b*e)*x)/((b*x^3 + a)*a*b^2) - 1/18
*(4*(-a*b^2)^(1/3)*a^2*h - (-a*b^2)^(1/3)*a*b*e - (-a*b^2)^(2/3)*b*c - 2*(-
a*b^2)^(2/3)*a*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^3) - 1/9*
(a*b^5*c*(-a/b)^(1/3) + 2*a^2*b^4*f*(-a/b)^(1/3) - 4*a^3*b^3*h + a^2*b^4*e)
*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^5)
```

$$3.416 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^2} dx$$

Optimal. Leaf size=276

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(\sqrt[3]{b}(af + 2bc) - \sqrt[3]{a}(2ag + bd)\right)}{18a^{5/3}b^{5/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(\sqrt[3]{b}(af + 2bc) - \sqrt[3]{a}(2ag + bd)\right)}{9a^{5/3}b^{5/3}} - \frac{\tan^{-1}\left(\frac{a^{1/3} - 2b^{1/3}x}{\sqrt[3]{a}}\right)}{3\sqrt[3]{a}b^{5/3}}$$

[Out] (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(3*a*b*(a + b*x^3)) - ((2*b^(4/3)*c + a^(1/3)*b*d + a*b^(1/3)*f + 2*a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(5/3)*b^(5/3)) + ((b^(1/3)*(2*b*c + a*f) - a^(1/3)*(b*d + 2*a*g))*Log[a^(1/3) + b^(1/3)*x]/(9*a^(5/3)*b^(5/3)) - ((b^(1/3)*(2*b*c + a*f) - a^(1/3)*(b*d + 2*a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(18*a^(5/3)*b^(5/3)) + (h*Log[a + b*x^3])/(3*b^2)

Rubi [A] time = 0.369973, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1858, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(\sqrt[3]{b}(af + 2bc) - \sqrt[3]{a}(2ag + bd)\right)}{18a^{5/3}b^{5/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(\sqrt[3]{b}(af + 2bc) - \sqrt[3]{a}(2ag + bd)\right)}{9a^{5/3}b^{5/3}} - \frac{\tan^{-1}\left(\frac{a^{1/3} - 2b^{1/3}x}{\sqrt[3]{a}}\right)}{3\sqrt[3]{a}b^{5/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^2,x]

[Out] (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(3*a*b*(a + b*x^3)) - ((2*b^(4/3)*c + a^(1/3)*b*d + a*b^(1/3)*f + 2*a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(5/3)*b^(5/3)) + ((b^(1/3)*(2*b*c + a*f) - a^(1/3)*(b*d + 2*a*g))*Log[a^(1/3) + b^(1/3)*x]/(9*a^(5/3)*b^(5/3)) - ((b^(1/3)*(2*b*c + a*f) - a^(1/3)*(b*d + 2*a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(18*a^(5/3)*b^(5/3)) + (h*Log[a + b*x^3])/(3*b^2)

Rule 1858

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,

```

x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x], D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a +
b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]

```

Rule 1871

```

Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

```

Rule 1860

```

Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]

```

Rule 31

```

Int[((a_) + (b_)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]

```

Rule 634

```

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

```

Rule 617

```

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^( -1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

```

Rule 204

```

Int[((a_) + (b_)*(x_)^2)^( -1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

```

a, 0] || LtQ[b, 0])

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned} \int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{(a + bx^3)^2} dx &= \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{3ab(a + bx^3)} - \frac{\int \frac{-b(2bc+af) - b(bd+2ag)x - 3abhx^2}{a+bx^3} dx}{3ab^2} \\ &= \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{3ab(a + bx^3)} - \frac{\int \frac{-b(2bc+af) - b(bd+2ag)x}{a+bx^3} dx}{3ab^2} + \frac{h \int \frac{x^2}{a+bx^3} dx}{b} \\ &= \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{3ab(a + bx^3)} + \frac{h \log(a + bx^3)}{3b^2} - \frac{\int \frac{\sqrt[3]{a}(-2b^{4/3}(2bc+af) - \sqrt[3]{a})}{a+bx^3} dx}{3ab^2} \\ &= \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{3ab(a + bx^3)} + \frac{(\sqrt[3]{b}(2bc + af) - \sqrt[3]{a}(bd + 2ag)) \log(\sqrt[3]{a+bx^3})}{9a^{5/3}b^{5/3}} \\ &= \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{3ab(a + bx^3)} + \frac{(\sqrt[3]{b}(2bc + af) - \sqrt[3]{a}(bd + 2ag)) \log(\sqrt[3]{a+bx^3})}{9a^{5/3}b^{5/3}} \\ &= \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{3ab(a + bx^3)} - \frac{(2b^{4/3}c + \sqrt[3]{abd} + a\sqrt[3]{bf} + 2a^{4/3}g) \tan^{-1}\left(\frac{\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{3\sqrt[3]{3}a^{5/3}b^{5/3}} \end{aligned}$$

Mathematica [A] time = 0.17552, size = 268, normalized size = 0.97

$$\frac{6(a^2h - ab(e + x(f + gx)) + b^2x(c + dx))}{a(a + bx^3)} + \frac{\sqrt[3]{b} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx + b^{2/3}x^2}) (2a^{4/3}g + \sqrt[3]{abd} - a\sqrt[3]{bf} - 2b^{4/3}c)}{a^{5/3}} + \frac{2\sqrt[3]{b} \log(\sqrt[3]{a} + \sqrt[3]{bx}) (-2a^{4/3}g - \sqrt[3]{abd} + a\sqrt[3]{bf} + 2b^{4/3}c)}{a^{5/3}} - \frac{(2b^{4/3}c + \sqrt[3]{abd} + a\sqrt[3]{bf} + 2a^{4/3}g) \tan^{-1}\left(\frac{\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{3\sqrt[3]{3}a^{5/3}b^{5/3}}$$

18b²

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^2,x]
```

```
[Out] ((6*(a^2*h + b^2*x*(c + d*x) - a*b*(e + x*(f + g*x)))/(a*(a + b*x^3)) - (2
*Sqrt[3]*b^(1/3)*(2*b^(4/3)*c + a^(1/3)*b*d + a*b^(1/3)*f + 2*a^(4/3)*g)*Ar
cTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(5/3) + (2*b^(1/3)*(2*b^(4/3)*
c - a^(1/3)*b*d + a*b^(1/3)*f - 2*a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/a^(5
/3) + (b^(1/3)*(-2*b^(4/3)*c + a^(1/3)*b*d - a*b^(1/3)*f + 2*a^(4/3)*g)*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3) + 6*h*Log[a + b*x^3])/
(18*b^2)
```

Maple [B] time = 0.008, size = 462, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] (-1/3*(a*g-b*d)/a/b*x^2-1/3*(a*f-b*c)/a/b*x+1/3*(a*h-b*e)/b^2)/(b*x^3+a)+1/
9/b^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*f+2/9*c/a/b/(1/b*a)^(2/3)*ln(x+(1/b
*a)^(1/3))-1/18/b^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f-1
/9*c/a/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/9/b^2/(1/b*a
)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f+2/9*c/a/b/(1/b*
a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-2/9/b^2/(1/b*a)^(
1/3)*ln(x+(1/b*a)^(1/3))*g-1/9*d/a/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/9
/b^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*g+1/18*d/a/b/(1/b*
a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2/9/b^2*3^(1/2)/(1/b*a)^(1/3
)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*g+1/9*d/a*3^(1/2)/b/(1/b*a)^(1/
3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/3*h*ln(b*x^3+a)/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxima"
)
```

[Out] Exception raised: ValueError

Fricas [C] time = 12.3701, size = 27752, normalized size = 100.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")

[Out]
$$-1/36*(12*a*b*e - 12*a^2*h - 12*(b^2*d - a*b*g)*x^2 + 2*(a*b^3*x^3 + a^2*b^2)*(2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*(9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a^3*b^4)))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^{(1/3)} - 6*h/b^2)*\log(4*a*b^4*c*d^2 + 2*a^2*b^3*d^2*f + 1/4*(a^4*b^5*d + 2*a^5*b^4*g)*(2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*(9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a^3*b^4)))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^{(1/3)} - 6*h/b^2)^2 + 8*(2*a^3*b^2*c + a^4*b*f)*g^2 + 9*(a^4*b*d + 2*a^5*g)*h^2 - 1/2*(4*a^2*b^5*c^2 + 4*a^3*b^4*c*f + a^4*b^3*f^2 - 6*(a^4*b^3*d + 2*a^5*b^2*g)*h)*(2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*(9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a^3*b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a^3*b^4)))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^{(1/3)} - 6*h/b^2)*\log(4*a*b^4*c*d^2 + 2*a^2*b^3*d^2*f + 1/4*(a^4*b^5*d + 2*a^5*b^4*g)*(2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*(9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a^3*b^4)))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^{(1/3)} - 6*h/b^2)$$

$$\begin{aligned}
&^4)) / (54h^3/b^6 - 9(2b^3cd + 2a^2bfg + 9a^3h^2 + (df + 4cg)ab^2)h / (a^3b^6) + (8b^4c^3 + ab^3d^3 + 12ab^3c^2f + 6a^2b^2c^2f^2 + a^3b^2f^3 + 6a^2b^2d^2g + 12a^3b^2d^2g^2 + 8a^4g^3) / (a^5b^5) + \\
&(8b^5c^3 + 27a^5h^3 - 2(4g^3 - 9fgh)a^4b + (f^3 + 36cgh - 3(4g^2 - 3fh)d)a^3b^2 - 6(d^2g - (f^2 + 3dh)c)a^2b^3 - (d^3 - 12c^2f)ab^4) / (a^5b^6))^{1/3} + (1/2)^{1/3}(I\sqrt{3} + 1)(54h^3/b^6 - \\
&9(2b^3cd + 2a^2bfg + 9a^3h^2 + (df + 4cg)ab^2)h / (a^3b^6) + (8b^4c^3 + ab^3d^3 + 12ab^3c^2f + 6a^2b^2c^2f^2 + a^3b^2f^3 + 6 \\
&a^2b^2d^2g + 12a^3b^2d^2g^2 + 8a^4g^3) / (a^5b^5) + (8b^5c^3 + 27a^5h^3 - 2(4g^3 - 9fgh)a^4b + (f^3 + 36cgh - 3(4g^2 - 3fh)d) \\
&a^3b^2 - 6(d^2g - (f^2 + 3dh)c)a^2b^3 - (d^3 - 12c^2f)ab^4) / (a^5b^6))^{1/3} - 6h/b^2) + 8(2a^2b^3cd + a^3b^2df)g - 3(4a^2b^3 \\
&c^2 + 4a^3b^2cf + a^4b^2f^2)h + (8b^5c^3 + ab^4d^3 + 12ab^4c^2f + 6a^2b^3c^2f^2 + a^3b^2f^3 + 6a^2b^3d^2g + 12a^3b^2d^2g^2 + 8 \\
&a^4b^2g^3)x - 12(b^2c - abf)x - (18abhx^3 + 18a^2h + (ab^3x^3 + a^2b^2)(2(1/2)^{2/3}(-I\sqrt{3} + 1)(9h^2/b^4 - (2b^3cd + 2a^2bfg + 9a^3h^2 + (df + 4cg)ab^2) / (a^3b^4)) / (54h^3/b^6 - 9(2b^3cd + 2a^2bfg + 9a^3h^2 + (df + 4cg)ab^2)h / (a^3b^6) + (8b^4c^3 + ab^3d^3 + 12ab^3c^2f + 6a^2b^2c^2f^2 + a^3b^2f^3 + 6a^2b^2d^2g + 12a^3b^2d^2g^2 + 8a^4g^3) / (a^5b^5) + (8b^5c^3 + 27a^5h^3 - 2(4g^3 - 9fgh)a^4b + (f^3 + 36cgh - 3(4g^2 - 3fh)d)a^3b^2 - 6(d^2g - (f^2 + 3dh)c)a^2b^3 - (d^3 - 12c^2f)ab^4) / (a^5b^6))^{1/3} + (1/2)^{1/3}(I\sqrt{3} + 1)(54h^3/b^6 - 9(2b^3cd + 2a^2bfg + 9a^3h^2 + (df + 4cg)ab^2)h / (a^3b^6) + (8b^4c^3 + ab^3d^3 + 12ab^3c^2f + 6a^2b^2c^2f^2 + a^3b^2f^3 + 6a^2b^2d^2g + 12a^3b^2d^2g^2 + 8a^4g^3) / (a^5b^5) + (8b^5c^3 + 27a^5h^3 - 2(4g^3 - 9fgh)a^4b + (f^3 + 36cgh - 3(4g^2 - 3fh)d)a^3b^2 - 6(d^2g - (f^2 + 3dh)c)a^2b^3 - (d^3 - 12c^2f)ab^4) / (a^5b^6))^{1/3} - 6h/b^2) + 3\sqrt{1/3}(ab^3x^3 + a^2b^2)\sqrt{-((2(1/2)^{2/3}(-I\sqrt{3} + 1)(9h^2/b^4 - (2b^3cd + 2a^2bfg + 9a^3h^2 + (df + 4cg)ab^2) / (a^3b^4)) / (54h^3/b^6 - 9(2b^3cd + 2a^2bfg + 9a^3h^2 + (df + 4cg)ab^2)h / (a^3b^6) + (8b^4c^3 + ab^3d^3 + 12ab^3c^2f + 6a^2b^2c^2f^2 + a^3b^2f^3 + 6a^2b^2d^2g + 12a^3b^2d^2g^2 + 8a^4g^3) / (a^5b^5) + (8b^5c^3 + 27a^5h^3 - 2(4g^3 - 9fgh)a^4b + (f^3 + 36cgh - 3(4g^2 - 3fh)d)a^3b^2 - 6(d^2g - (f^2 + 3dh)c)a^2b^3 - (d^3 - 12c^2f)ab^4) / (a^5b^6))^{1/3} + (1/2)^{1/3}(I\sqrt{3} + 1)(54h^3/b^6 - 9(2b^3cd + 2a^2bfg + 9a^3h^2 + (df + 4cg)ab^2)h / (a^3b^6) + (8b^4c^3 + ab^3d^3 + 12ab^3c^2f + 6a^2b^2c^2f^2 + a^3b^2f^3 + 6a^2b^2d^2g + 12a^3b^2d^2g^2 + 8a^4g^3) / (a^5b^5) + (8b^5c^3 + 27a^5h^3 - 2(4g^3 - 9fgh)a^4b + (f^3 + 36cgh - 3(4g^2 - 3fh)d)a^3b^2 - 6(d^2g - (f^2 + 3dh)c)a^2b^3 - (d^3 - 12c^2f)ab^4) / (a^5b^6))^{1/3} - 6h/b^2) + 12(2(1/2)^{2/3}(-I\sqrt{3} + 1)(9h^2/b^4 - (2b^3cd + 2a^2bfg + 9a^3h^2 + (df + 4cg)ab^2) / (a^3b^4)) / (54h^3/b^6 - 9(2b^3cd + 2a^2bfg + 9a^3h^2 + (df + 4cg)ab^2)h / (a^3b^6) + (8b^4c^3 + ab^3d^3 + 12ab^3c^2f + 6a^2b^2c^2f^2 + a^3b^2f^3 + 6a^2b^2d^2g + 12a^3b^2d^2g^2 + 8a^4g^3) / (a^5b^5) + (8b^5c^3 + 27a^5h^3 - 2(4g^3 - 9fgh)a^4b + (f^3 + 36cgh - 3(4g^2 - 3fh)d)a^3b^2 - 6(d^2g - (f^2 + 3dh)c)a^2b^3 - (d^3 - 12c^2f)ab^4) / (a^5b^6))^{1/3} - 6h/b^2)
\end{aligned}$$

$$\begin{aligned}
& 2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h \\
& - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 \\
& - 12*c^2*f)*a*b^4)/(a^5*b^6))^{(1/3)} + (1/2)^{(1/3)}*(I*sqrt(3) + 1)*(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) \\
& + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + \\
& 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h) \\
&)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4 \\
&)/(a^5*b^6))^{(1/3)} - 6*h/b^2)*a^3*b^2*h + 32*b^3*c*d + 16*a*b^2*d*f + 36*a^3*h^2 + 32*(2*a*b^2*c + a^2*b*f)*g)/(a^3*b^4)))*log(-4*a*b^4*c*d^2 - 2*a^2*b^3*d^2*f - 1/4*(a^4*b^5*d + 2*a^5*b^4*g)*(2*(1/2)^{(2/3)}*(-I*sqrt(3) + 1)*(\\
& 9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a^3*b^4)))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g) \\
&)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) \\
& + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)* \\
& d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/ \\
& (a^5*b^6))^{(1/3)} + (1/2)^{(1/3)}*(I*sqrt(3) + 1)*(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) \\
& + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27 \\
& *a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)* \\
& d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/ \\
& (a^5*b^6))^{(1/3)} - 6*h/b^2)^2 - 8*(2*a^3*b^2*c + a^4*b*f)*g^2 - 9*(a^4*b*d \\
& + 2*a^5*g)*h^2 + 1/2*(4*a^2*b^5*c^2 + 4*a^3*b^4*c*f + a^4*b^3*f^2 - 6*(a^4*b^3*d + 2*a^5*b^2*g)*h)*(2*(1/2)^{(2/3)}*(-I*sqrt(3) + 1)*(9*h^2/b^4 - (2*b^3 \\
& *c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a^3*b^4)))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) \\
& + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27 \\
& *a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)* \\
& d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/ \\
& (a^5*b^6))^{(1/3)} + (1/2)^{(1/3)}*(I*sqrt(3) + 1)*(54*h^3/b^6 - 9*(2*b^3*c*d + \\
& 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + \\
& a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g \\
& + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 \\
& - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2 \\
& *g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^{(1/3)} - \\
& 6*h/b^2) - 8*(2*a^2*b^3*c*d + a^3*b^2*d*f)*g + 3*(4*a^2*b^3*c^2 + 4*a^3*b^2 \\
& *c*f + a^4*b*f^2)*h + 2*(8*b^5*c^3 + a*b^4*d^3 + 12*a*b^4*c^2*f + 6*a^2*b^3 \\
& *c*f^2 + a^3*b^2*f^3 + 6*a^2*b^3*d^2*g + 12*a^3*b^2*d*g^2 + 8*a^4*b*g^3)*x \\
& + 3/4*sqrt(1/3)*(8*a^2*b^5*c^2 + 8*a^3*b^4*c*f + 2*a^4*b^3*f^2 + (a^4*b^5*d \\
& + 2*a^5*b^4*g)*(2*(1/2)^{(2/3)}*(-I*sqrt(3) + 1)*(9*h^2/b^4 - (2*b^3*c*d + \\
& 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a^3*b^4)))/(54*h^3/b^6 - 9*(
\end{aligned}$$

$$\begin{aligned}
& 2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8 \\
& *b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2 \\
& *b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^ \\
& 3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3* \\
& b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^ \\
& 6))^{(1/3)} + (1/2)^{(1/3)}*(I*sqrt(3) + 1)*(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2* \\
& b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d \\
& ^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^ \\
& 3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f \\
& *g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (\\
& f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^{(1/3)} - 6*h/b^ \\
& 2) + 6*(a^4*b^3*d + 2*a^5*b^2*g)*h)*sqrt(-((2*(1/2)^{(2/3)}*(-I*sqrt(3) + 1)* \\
& (9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a \\
& ^3*b^4)))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c* \\
& g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2 \\
& *c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5 \\
&) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - \\
& 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 \\
& - 12*c^2*f)*a*b^4)/(a^5*b^6))^{(1/3)} + (1/2)^{(1/3)}*(I*sqrt(3) + 1)*(54*h^3/b \\
& ^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b \\
& ^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 \\
& + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 2 \\
& 7*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h) \\
& *d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4) \\
& /(a^5*b^6))^{(1/3)} - 6*h/b^2)^2*a^3*b^4 + 12*(2*(1/2)^{(2/3)}*(-I*sqrt(3) + 1) \\
& *(9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(\\
& a^3*b^4)))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c \\
& *g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^ \\
& 2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^ \\
& 5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h \\
& - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 \\
& - 12*c^2*f)*a*b^4)/(a^5*b^6))^{(1/3)} + (1/2)^{(1/3)}*(I*sqrt(3) + 1)*(54*h^3/ \\
& b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3* \\
& b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^ \\
& 3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + \\
& 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h) \\
&)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4 \\
&)/(a^5*b^6))^{(1/3)} - 6*h/b^2)*a^3*b^2*h + 32*b^3*c*d + 16*a*b^2*d*f + 36*a^ \\
& 3*h^2 + 32*(2*a*b^2*c + a^2*b*f)*g)/(a^3*b^4))) - (18*a*b*h*x^3 + 18*a^2*h \\
& + (a*b^3*x^3 + a^2*b^2)*(2*(1/2)^{(2/3)}*(-I*sqrt(3) + 1)*(9*h^2/b^4 - (2*b^3 \\
& *c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a^3*b^4)))/(54*h^3/b^ \\
& 6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^ \\
& 6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 \\
& + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27 \\
& *a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*
\end{aligned}$$

$$\begin{aligned}
& d) * a^3 * b^2 - 6 * (d^2 * g - (f^2 + 3 * d * h) * c) * a^2 * b^3 - (d^3 - 12 * c^2 * f) * a * b^4 / \\
& (a^5 * b^6))^{(1/3)} + (1/2)^{(1/3)} * (I * \text{sqrt}(3) + 1) * (54 * h^3 / b^6 - 9 * (2 * b^3 * c * d + \\
& 2 * a^2 * b * f * g + 9 * a^3 * h^2 + (d * f + 4 * c * g) * a * b^2) * h / (a^3 * b^6) + (8 * b^4 * c^3 + \\
& a * b^3 * d^3 + 12 * a * b^3 * c^2 * f + 6 * a^2 * b^2 * c * f^2 + a^3 * b * f^3 + 6 * a^2 * b^2 * d^2 * g \\
& + 12 * a^3 * b * d * g^2 + 8 * a^4 * g^3) / (a^5 * b^5) + (8 * b^5 * c^3 + 27 * a^5 * h^3 - 2 * (4 * g^3 \\
& - 9 * f * g * h) * a^4 * b + (f^3 + 36 * c * g * h - 3 * (4 * g^2 - 3 * f * h) * d) * a^3 * b^2 - 6 * (d^2 * g \\
& - (f^2 + 3 * d * h) * c) * a^2 * b^3 - (d^3 - 12 * c^2 * f) * a * b^4) / (a^5 * b^6))^{(1/3)} - \\
& 6 * h / b^2) - 3 * \text{sqrt}(1/3) * (a * b^3 * x^3 + a^2 * b^2) * \text{sqrt}(-((2 * (1/2)^{(2/3)} * (-I * \text{sqrt} \\
& \text{t}(3) + 1) * (9 * h^2 / b^4 - (2 * b^3 * c * d + 2 * a^2 * b * f * g + 9 * a^3 * h^2 + (d * f + 4 * c * g) \\
& * a * b^2) / (a^3 * b^4)) / (54 * h^3 / b^6 - 9 * (2 * b^3 * c * d + 2 * a^2 * b * f * g + 9 * a^3 * h^2 + (\\
& d * f + 4 * c * g) * a * b^2) * h / (a^3 * b^6) + (8 * b^4 * c^3 + a * b^3 * d^3 + 12 * a * b^3 * c^2 * f + \\
& 6 * a^2 * b^2 * c * f^2 + a^3 * b * f^3 + 6 * a^2 * b^2 * d^2 * g + 12 * a^3 * b * d * g^2 + 8 * a^4 * g^3 \\
&) / (a^5 * b^5) + (8 * b^5 * c^3 + 27 * a^5 * h^3 - 2 * (4 * g^3 - 9 * f * g * h) * a^4 * b + (f^3 + \\
& 36 * c * g * h - 3 * (4 * g^2 - 3 * f * h) * d) * a^3 * b^2 - 6 * (d^2 * g - (f^2 + 3 * d * h) * c) * a^2 * b^3 \\
& - (d^3 - 12 * c^2 * f) * a * b^4) / (a^5 * b^6))^{(1/3)} + (1/2)^{(1/3)} * (I * \text{sqrt}(3) + 1) \\
& * (54 * h^3 / b^6 - 9 * (2 * b^3 * c * d + 2 * a^2 * b * f * g + 9 * a^3 * h^2 + (d * f + 4 * c * g) * a * b^2 \\
&) * h / (a^3 * b^6) + (8 * b^4 * c^3 + a * b^3 * d^3 + 12 * a * b^3 * c^2 * f + 6 * a^2 * b^2 * c * f^2 + \\
& a^3 * b * f^3 + 6 * a^2 * b^2 * d^2 * g + 12 * a^3 * b * d * g^2 + 8 * a^4 * g^3) / (a^5 * b^5) + (8 * b \\
& ^5 * c^3 + 27 * a^5 * h^3 - 2 * (4 * g^3 - 9 * f * g * h) * a^4 * b + (f^3 + 36 * c * g * h - 3 * (4 * g^2 \\
& - 3 * f * h) * d) * a^3 * b^2 - 6 * (d^2 * g - (f^2 + 3 * d * h) * c) * a^2 * b^3 - (d^3 - 12 * c^2 \\
& * f) * a * b^4) / (a^5 * b^6))^{(1/3)} - 6 * h / b^2)^2 * a^3 * b^4 + 12 * (2 * (1/2)^{(2/3)} * (-I * \text{sqrt} \\
& \text{rt}(3) + 1) * (9 * h^2 / b^4 - (2 * b^3 * c * d + 2 * a^2 * b * f * g + 9 * a^3 * h^2 + (d * f + 4 * c * g) \\
&) * a * b^2) / (a^3 * b^4)) / (54 * h^3 / b^6 - 9 * (2 * b^3 * c * d + 2 * a^2 * b * f * g + 9 * a^3 * h^2 + \\
& (d * f + 4 * c * g) * a * b^2) * h / (a^3 * b^6) + (8 * b^4 * c^3 + a * b^3 * d^3 + 12 * a * b^3 * c^2 * f \\
& + 6 * a^2 * b^2 * c * f^2 + a^3 * b * f^3 + 6 * a^2 * b^2 * d^2 * g + 12 * a^3 * b * d * g^2 + 8 * a^4 * g^3 \\
&) / (a^5 * b^5) + (8 * b^5 * c^3 + 27 * a^5 * h^3 - 2 * (4 * g^3 - 9 * f * g * h) * a^4 * b + (f^3 + 36 * c * g * h \\
& - 3 * (4 * g^2 - 3 * f * h) * d) * a^3 * b^2 - 6 * (d^2 * g - (f^2 + 3 * d * h) * c) * a^2 * b^3 - (d^3 - 12 * c^2 \\
& * f) * a * b^4) / (a^5 * b^6))^{(1/3)} + (1/2)^{(1/3)} * (I * \text{sqrt}(3) + 1) \\
&) * (54 * h^3 / b^6 - 9 * (2 * b^3 * c * d + 2 * a^2 * b * f * g + 9 * a^3 * h^2 + (d * f + 4 * c * g) * a * b^2 \\
&) * h / (a^3 * b^6) + (8 * b^4 * c^3 + a * b^3 * d^3 + 12 * a * b^3 * c^2 * f + 6 * a^2 * b^2 * c * f^2 \\
& + a^3 * b * f^3 + 6 * a^2 * b^2 * d^2 * g + 12 * a^3 * b * d * g^2 + 8 * a^4 * g^3) / (a^5 * b^5) + (8 * \\
& b^5 * c^3 + 27 * a^5 * h^3 - 2 * (4 * g^3 - 9 * f * g * h) * a^4 * b + (f^3 + 36 * c * g * h - 3 * (4 * g^2 \\
& - 3 * f * h) * d) * a^3 * b^2 - 6 * (d^2 * g - (f^2 + 3 * d * h) * c) * a^2 * b^3 - (d^3 - 12 * c^2 \\
& * f) * a * b^4) / (a^5 * b^6))^{(1/3)} - 6 * h / b^2) * a^3 * b^2 * h + 32 * b^3 * c * d + 16 * a * b^2 * d \\
& * f + 36 * a^3 * h^2 + 32 * (2 * a * b^2 * c + a^2 * b * f) * g) / (a^3 * b^4))) * \log(-4 * a * b^4 * c * d^2 \\
& - 2 * a^2 * b^3 * d^2 * f - 1/4 * (a^4 * b^5 * d + 2 * a^5 * b^4 * g) * (2 * (1/2)^{(2/3)} * (-I * \text{sqrt} \\
& (3) + 1) * (9 * h^2 / b^4 - (2 * b^3 * c * d + 2 * a^2 * b * f * g + 9 * a^3 * h^2 + (d * f + 4 * c * g) * \\
& a * b^2) / (a^3 * b^4)) / (54 * h^3 / b^6 - 9 * (2 * b^3 * c * d + 2 * a^2 * b * f * g + 9 * a^3 * h^2 + (d \\
& * f + 4 * c * g) * a * b^2) * h / (a^3 * b^6) + (8 * b^4 * c^3 + a * b^3 * d^3 + 12 * a * b^3 * c^2 * f + \\
& 6 * a^2 * b^2 * c * f^2 + a^3 * b * f^3 + 6 * a^2 * b^2 * d^2 * g + 12 * a^3 * b * d * g^2 + 8 * a^4 * g^3) \\
& / (a^5 * b^5) + (8 * b^5 * c^3 + 27 * a^5 * h^3 - 2 * (4 * g^3 - 9 * f * g * h) * a^4 * b + (f^3 + 3 \\
& 6 * c * g * h - 3 * (4 * g^2 - 3 * f * h) * d) * a^3 * b^2 - 6 * (d^2 * g - (f^2 + 3 * d * h) * c) * a^2 * b^3 \\
& - (d^3 - 12 * c^2 * f) * a * b^4) / (a^5 * b^6))^{(1/3)} + (1/2)^{(1/3)} * (I * \text{sqrt}(3) + 1) * \\
& (54 * h^3 / b^6 - 9 * (2 * b^3 * c * d + 2 * a^2 * b * f * g + 9 * a^3 * h^2 + (d * f + 4 * c * g) * a * b^2) \\
&) * h / (a^3 * b^6) + (8 * b^4 * c^3 + a * b^3 * d^3 + 12 * a * b^3 * c^2 * f + 6 * a^2 * b^2 * c * f^2 +
\end{aligned}$$

$$\begin{aligned} & a^3 b f^3 + 6 a^2 b^2 d^2 g + 12 a^3 b d g^2 + 8 a^4 g^3 / (a^5 b^5) + (8 b^5 c^3 + 27 a^5 h^3 - 2(4 g^3 - 9 f g h) a^4 b + (f^3 + 36 c g h - 3(4 g^2 - 3 f h) d) a^3 b^2 - 6(d^2 g - (f^2 + 3 d h) c) a^2 b^3 - (d^3 - 12 c^2 f) a b^4) / (a^5 b^6)^{(1/3)} - 6 h / b^2)^2 a^3 b^4 + 12(2(1/2)^{(2/3)} (-I \operatorname{sqrt}(3) + 1) (9 h^2 / b^4 - (2 b^3 c d + 2 a^2 b f g + 9 a^3 h^2 + (d f + 4 c g) a b^2) / (a^3 b^4)) / (54 h^3 / b^6 - 9(2 b^3 c d + 2 a^2 b f g + 9 a^3 h^2 + (d f + 4 c g) a b^2) h / (a^3 b^6) + (8 b^4 c^3 + a b^3 d^3 + 12 a b^3 c^2 f + 6 a^2 b^2 c f^2 + a^3 b f^3 + 6 a^2 b^2 d^2 g + 12 a^3 b d g^2 + 8 a^4 g^3) / (a^5 b^5) + (8 b^5 c^3 + 27 a^5 h^3 - 2(4 g^3 - 9 f g h) a^4 b + (f^3 + 36 c g h - 3(4 g^2 - 3 f h) d) a^3 b^2 - 6(d^2 g - (f^2 + 3 d h) c) a^2 b^3 - (d^3 - 12 c^2 f) a b^4) / (a^5 b^6)^{(1/3)} + (1/2)^{(1/3)} (I \operatorname{sqrt}(3) + 1) (54 h^3 / b^6 - 9(2 b^3 c d + 2 a^2 b f g + 9 a^3 h^2 + (d f + 4 c g) a b^2) h / (a^3 b^6) + (8 b^4 c^3 + a b^3 d^3 + 12 a b^3 c^2 f + 6 a^2 b^2 c f^2 + a^3 b f^3 + 6 a^2 b^2 d^2 g + 12 a^3 b d g^2 + 8 a^4 g^3) / (a^5 b^5) + (8 b^5 c^3 + 27 a^5 h^3 - 2(4 g^3 - 9 f g h) a^4 b + (f^3 + 36 c g h - 3(4 g^2 - 3 f h) d) a^3 b^2 - 6(d^2 g - (f^2 + 3 d h) c) a^2 b^3 - (d^3 - 12 c^2 f) a b^4) / (a^5 b^6)^{(1/3)} - 6 h / b^2) a^3 b^2 h + 32 b^3 c d + 16 a b^2 d f + 36 a^3 h^2 + 32(2 a b^2 c + a^2 b f) g) / (a^3 b^4) / (a b^3 x^3 + a^2 b^2) \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)

[Out] Timed out

Giac [A] time = 1.08394, size = 429, normalized size = 1.55

$$\frac{h \log(|bx^3 + a|)}{3b^2} + \frac{(bd - ag)x^2 + (bc - af)x + \frac{a^2h - abe}{b}}{3(bx^3 + a)ab} + \frac{\sqrt{3} \left(2(-ab^2)^{\frac{1}{3}} b^2c + (-ab^2)^{\frac{1}{3}} abf - (-ab^2)^{\frac{2}{3}} bd - 2(-ab^2)^{\frac{2}{3}} ag \right)}{9a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac")

[Out] $\frac{1}{3}h \log(\text{abs}(b x^3 + a)) / b^2 + \frac{1}{3}((b d - a g) x^2 + (b c - a f) x + (a^2 h - a b e) / b) / ((b x^3 + a) a b) + \frac{1}{9} \sqrt{3} (2 (-a b^2)^{1/3} b^2 c + (-a b^2)^{1/3} a b f - (-a b^2)^{2/3} b d - 2 (-a b^2)^{2/3} a g) \arctan\left(\frac{1}{3} \sqrt{3} (2 x + (-a/b)^{1/3}) / (-a/b)^{1/3}\right) / (a^2 b^3) + \frac{1}{18} (2 (-a b^2)^{1/3} b^2 c + (-a b^2)^{1/3} a b f + (-a b^2)^{2/3} b d + 2 (-a b^2)^{2/3} a g) \log(x^2 + x (-a/b)^{1/3} + (-a/b)^{2/3}) / (a^2 b^3) - \frac{1}{9} (a b^3 d (-a/b)^{1/3} + 2 a^2 b^2 g (-a/b)^{1/3} + 2 a b^3 c + a^2 b^2 f) (-a/b)^{1/3} \log(\text{abs}(x - (-a/b)^{1/3})) / (a^3 b^3)$

$$3.417 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)^2} dx$$

Optimal. Leaf size=289

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(\sqrt[3]{b}(ag + 2bd) - \sqrt[3]{a}(2ah + be)\right)}{18a^{5/3}b^{5/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(\sqrt[3]{b}(ag + 2bd) - \sqrt[3]{a}(2ah + be)\right)}{9a^{5/3}b^{5/3}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{bx}}{\sqrt[3]{a} + \sqrt[3]{bx}}\right)}{\sqrt[3]{a} + \sqrt[3]{bx}}$$

[Out] (x*(a*(b*d - a*g) + a*(b*e - a*h)*x - b*(b*c - a*f)*x^2))/(3*a^2*b*(a + b*x^3)) - ((2*b^(4/3)*d + a^(1/3)*b*e + a*b^(1/3)*g + 2*a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(5/3)*b^(5/3)) + (c*Log[x])/a^2 + ((b^(1/3)*(2*b*d + a*g) - a^(1/3)*(b*e + 2*a*h))*Log[a^(1/3) + b^(1/3)*x]/(9*a^(5/3)*b^(5/3)) - ((b^(1/3)*(2*b*d + a*g) - a^(1/3)*(b*e + 2*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(18*a^(5/3)*b^(5/3))) - (c*Log[a + b*x^3]/(3*a^2))

Rubi [A] time = 0.559415, antiderivative size = 287, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(2ah+be)}{\sqrt[3]{b}} + ag + 2bd\right)}{18a^{5/3}b^{4/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(\sqrt[3]{b}(ag + 2bd) - \sqrt[3]{a}(2ah + be)\right)}{9a^{5/3}b^{5/3}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{a} - \sqrt[3]{bx}}{\sqrt[3]{a} + \sqrt[3]{bx}}\right)}{\sqrt[3]{a} + \sqrt[3]{bx}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^2), x]

[Out] (x*(a*(b*d - a*g) + a*(b*e - a*h)*x - b*(b*c - a*f)*x^2))/(3*a^2*b*(a + b*x^3)) - ((2*b^(4/3)*d + a^(1/3)*b*e + a*b^(1/3)*g + 2*a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(5/3)*b^(5/3)) + (c*Log[x])/a^2 + ((b^(1/3)*(2*b*d + a*g) - a^(1/3)*(b*e + 2*a*h))*Log[a^(1/3) + b^(1/3)*x]/(9*a^(5/3)*b^(5/3)) - ((2*b*d + a*g - (a^(1/3)*(b*e + 2*a*h))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(18*a^(5/3)*b^(4/3))) - (c*Log[a + b*x^3]/(3*a^2))

Rule 1829

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^

```
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_)*(x_))^(m_))/((a_) + (b_)*(x_)^n), x_Symbol] := Int[ExpandIntegrand[
((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^( -1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
```

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 204

$\text{Int}[\frac{(a_+) + (b_+)(x_+)^2}{(x_+)^{-1}}, x_Symbol] \ :> \ -\text{Simp}[\frac{\text{ArcTan}[\frac{\text{Rt}[-b, 2]x}{\text{Rt}[-a, 2] - \text{Rt}[-a, 2] \text{Rt}[-b, 2]}]}{\text{Rt}[-a, 2] \text{Rt}[-b, 2]}, x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 628

$\text{Int}[\frac{(d_+) + (e_+)(x_+)}{(a_+) + (b_+)(x_+) + (c_+)(x_+)^2}, x_Symbol] \ :> \ \text{Simp}[\frac{d \cdot \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]}{b}, x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2cd - be, 0]$

Rule 260

$\text{Int}[\frac{(x_+)^{m_+}}{(a_+) + (b_+)(x_+)^{n_+}}, x_Symbol] \ :> \ \text{Simp}[\frac{\text{Log}[\text{RemoveContent}[a + bx^n, x]]}{(b \cdot n)}, x] \ /; \ \text{FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{x(a + bx^3)^2} dx &= \frac{x(a(bd - ag) + a(be - ah)x - b(bc - af)x^2)}{3a^2b(a + bx^3)} - \frac{\int \frac{-3b^2c - b(2bd + ag)x - b(be + 2ah)x^2}{x(a + bx^3)} dx}{3ab^2} \\
&= \frac{x(a(bd - ag) + a(be - ah)x - b(bc - af)x^2)}{3a^2b(a + bx^3)} - \frac{\int \left(-\frac{3b^2c}{ax} + \frac{b(-a(2bd + ag) - a(be + 2ah)x)}{a(a + bx^3)} \right) dx}{3ab^2} \\
&= \frac{x(a(bd - ag) + a(be - ah)x - b(bc - af)x^2)}{3a^2b(a + bx^3)} + \frac{c \log(x)}{a^2} - \frac{\int \frac{-a(2bd + ag) - a(be + 2ah)x}{a + bx^3} dx}{3a^2b} \\
&= \frac{x(a(bd - ag) + a(be - ah)x - b(bc - af)x^2)}{3a^2b(a + bx^3)} + \frac{c \log(x)}{a^2} - \frac{\int \frac{-a(2bd + ag) - a(be + 2ah)x}{a + bx^3} dx}{3a^2b} \\
&= \frac{x(a(bd - ag) + a(be - ah)x - b(bc - af)x^2)}{3a^2b(a + bx^3)} + \frac{c \log(x)}{a^2} - \frac{c \log(a + bx^3)}{3a^2} - \int \frac{\sqrt[3]{a}}{a + bx^3} dx \\
&= \frac{x(a(bd - ag) + a(be - ah)x - b(bc - af)x^2)}{3a^2b(a + bx^3)} + \frac{c \log(x)}{a^2} + \frac{\left(2bd + ag - \frac{\sqrt[3]{a}(be + 2ah)}{\sqrt[3]{b}}\right)}{9a^{5/3}} \\
&= \frac{x(a(bd - ag) + a(be - ah)x - b(bc - af)x^2)}{3a^2b(a + bx^3)} + \frac{c \log(x)}{a^2} + \frac{\left(2bd + ag - \frac{\sqrt[3]{a}(be + 2ah)}{\sqrt[3]{b}}\right)}{9a^{5/3}} \\
&= \frac{x(a(bd - ag) + a(be - ah)x - b(bc - af)x^2)}{3a^2b(a + bx^3)} - \frac{\left(2b^{4/3}d + \sqrt[3]{abe} + a\sqrt[3]{bg} + 2a^{4/3}h\right)}{3\sqrt[3]{3a^{5/3}b^{5/3}}}
\end{aligned}$$

Mathematica [A] time = 0.197893, size = 269, normalized size = 0.93

$$\frac{\sqrt[3]{a} \log\left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2\right) \left(2a^{4/3} h + \sqrt[3]{a} b e - a \sqrt[3]{b} g - 2b^{4/3} d\right)}{b^{5/3}} + \frac{2 \sqrt[3]{a} \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right) \left(-2a^{4/3} h - \sqrt[3]{a} b e + a \sqrt[3]{b} g + 2b^{4/3} d\right)}{b^{5/3}} - \frac{2 \sqrt{3} \sqrt[3]{a} \tan^{-1}\left(\frac{1 - 2 \sqrt[3]{bx}}{\sqrt{3} \sqrt[3]{a}}\right) \left(2a^{4/3} h + \sqrt[3]{a} b e\right)}{b^{5/3}}$$

$$18a^2$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^2), x]

[Out] ((-6*a*(-(b*(c + x*(d + e*x))) + a*(f + x*(g + h*x))))/(b*(a + b*x^3)) - (2*sqrt[3]*a^(1/3)*(2*b^(4/3)*d + a^(1/3)*b*e + a*b^(1/3)*g + 2*a^(4/3)*h)*Ar

$$c \operatorname{Tan}\left[\frac{1 - (2b^{1/3}x)/a^{1/3}}{\sqrt{3}}\right]/b^{5/3} + 18c \operatorname{Log}[x] + (2a^{1/3}) \cdot (2b^{4/3}d - a^{1/3}b^e + ab^{1/3}g - 2a^{4/3}h) \operatorname{Log}[a^{1/3} + b^{1/3}x]/b^{5/3} + (a^{1/3}(-2b^{4/3}d + a^{1/3}b^e - ab^{1/3}g + 2a^{4/3}h) \operatorname{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2])/b^{5/3} - 6c \operatorname{Log}[a + bx^3]/(18a^2)$$

Maple [B] time = 0.012, size = 507, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^2,x)`

[Out]
$$-1/3/(b*x^3+a)/b*x^2*h+1/3/a*x^2/(b*x^3+a)*e-1/3/(b*x^3+a)/b*x*g+1/3/a*x/(b*x^3+a)*d-1/3/b/(b*x^3+a)*f+1/3/a/(b*x^3+a)*c+1/9/b^2/(1/b*a)^{2/3}*\ln(x+(1/b*a)^{1/3})*g+2/9/b/a*d/(1/b*a)^{2/3}*\ln(x+(1/b*a)^{1/3})-1/18/b^2/(1/b*a)^{2/3}*\ln(x^2-(1/b*a)^{1/3}*x+(1/b*a)^{2/3})*g-1/9/b/a*d/(1/b*a)^{2/3}*\ln(x^2-(1/b*a)^{1/3}*x+(1/b*a)^{2/3})+1/9/b^2/(1/b*a)^{2/3}*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1))*g+2/9/b/a*d/(1/b*a)^{2/3}*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1))-2/9/b^2/(1/b*a)^{1/3}*\ln(x+(1/b*a)^{1/3})*h-1/9/a/b/(1/b*a)^{1/3}*\ln(x+(1/b*a)^{1/3})*e+1/9/b^2/(1/b*a)^{1/3}*\ln(x^2-(1/b*a)^{1/3}*x+(1/b*a)^{2/3})*h+1/18/a/b/(1/b*a)^{1/3}*\ln(x^2-(1/b*a)^{1/3}*x+(1/b*a)^{2/3})*e+2/9/b^2*3^{1/2}/(1/b*a)^{1/3}*\arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1))*h+1/9/a/b*3^{1/2}/(1/b*a)^{1/3}*\arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1))*e-1/3*c*\ln(b*x^3+a)/a^2+c*\ln(x)/a^2$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [C] time = 156.896, size = 28260, normalized size = 97.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="fricas")

[Out]
$$\frac{1}{324} \cdot (108 \cdot a \cdot b \cdot c - 108 \cdot a^2 \cdot f + 108 \cdot (a \cdot b \cdot e - a^2 \cdot h) \cdot x^2 - 2 \cdot (a^2 \cdot b^2 \cdot x^3 + a^3 \cdot b) \cdot ((-I \cdot \sqrt{3}) + 1) \cdot (9 \cdot c^2 / a^4 - (9 \cdot b^3 \cdot c^2 + 2 \cdot a \cdot b^2 \cdot d \cdot e + 2 \cdot a^3 \cdot g \cdot h + (e \cdot g + 4 \cdot d \cdot h) \cdot a^2 \cdot b) / (a^4 \cdot b^3))) / (-1/27 \cdot c^3 / a^6 + 1/162 \cdot (9 \cdot b^3 \cdot c^2 + 2 \cdot a \cdot b^2 \cdot d \cdot e + 2 \cdot a^3 \cdot g \cdot h + (e \cdot g + 4 \cdot d \cdot h) \cdot a^2 \cdot b) \cdot c / (a^6 \cdot b^3) + 1/1458 \cdot (8 \cdot b^4 \cdot d^3 + a \cdot b^3 \cdot e^3 + 12 \cdot a \cdot b^3 \cdot d^2 \cdot g + 6 \cdot a^2 \cdot b^2 \cdot d \cdot g^2 + a^3 \cdot b \cdot g^3 + 6 \cdot a^2 \cdot b^2 \cdot e^2 \cdot h + 12 \cdot a^3 \cdot b \cdot e \cdot h^2 + 8 \cdot a^4 \cdot h^3) / (a^5 \cdot b^5) - 1/1458 \cdot (27 \cdot b^5 \cdot c^3 + 8 \cdot a^5 \cdot h^3 - (g^3 - 12 \cdot e \cdot h^2) \cdot a^4 \cdot b - 6 \cdot (d \cdot g^2 - e^2 \cdot h - 3 \cdot c \cdot g \cdot h) \cdot a^3 \cdot b^2 + (e^3 - 12 \cdot d^2 \cdot g + 9 \cdot (e \cdot g + 4 \cdot d \cdot h) \cdot c) \cdot a^2 \cdot b^3 - 2 \cdot (4 \cdot d^3 - 9 \cdot c \cdot d \cdot e) \cdot a \cdot b^4) / (a^6 \cdot b^5))^{1/3} + 81 \cdot (I \cdot \sqrt{3}) + 1) \cdot (-1/27 \cdot c^3 / a^6 + 1/162 \cdot (9 \cdot b^3 \cdot c^2 + 2 \cdot a \cdot b^2 \cdot d \cdot e + 2 \cdot a^3 \cdot g \cdot h + (e \cdot g + 4 \cdot d \cdot h) \cdot a^2 \cdot b) \cdot c / (a^6 \cdot b^3) + 1/1458 \cdot (8 \cdot b^4 \cdot d^3 + a \cdot b^3 \cdot e^3 + 12 \cdot a \cdot b^3 \cdot d^2 \cdot g + 6 \cdot a^2 \cdot b^2 \cdot d \cdot g^2 + a^3 \cdot b \cdot g^3 + 6 \cdot a^2 \cdot b^2 \cdot e^2 \cdot h + 12 \cdot a^3 \cdot b \cdot e \cdot h^2 + 8 \cdot a^4 \cdot h^3) / (a^5 \cdot b^5) - 1/1458 \cdot (27 \cdot b^5 \cdot c^3 + 8 \cdot a^5 \cdot h^3 - (g^3 - 12 \cdot e \cdot h^2) \cdot a^4 \cdot b - 6 \cdot (d \cdot g^2 - e^2 \cdot h - 3 \cdot c \cdot g \cdot h) \cdot a^3 \cdot b^2 + (e^3 - 12 \cdot d^2 \cdot g + 9 \cdot (e \cdot g + 4 \cdot d \cdot h) \cdot c) \cdot a^2 \cdot b^3 - 2 \cdot (4 \cdot d^3 - 9 \cdot c \cdot d \cdot e) \cdot a \cdot b^4) / (a^6 \cdot b^5))^{1/3} + 54 \cdot c / a^2) \cdot \log(12 \cdot b^4 \cdot c \cdot d^2 + 9 \cdot b^4 \cdot c^2 \cdot e + 4 \cdot a \cdot b^3 \cdot d \cdot e^2 + 3 \cdot a^2 \cdot b^2 \cdot c \cdot g^2 + 1/324 \cdot (a^4 \cdot b^4 \cdot e + 2 \cdot a^5 \cdot b^3 \cdot h) \cdot ((-I \cdot \sqrt{3}) + 1) \cdot (9 \cdot c^2 / a^4 - (9 \cdot b^3 \cdot c^2 + 2 \cdot a \cdot b^2 \cdot d \cdot e + 2 \cdot a^3 \cdot g \cdot h + (e \cdot g + 4 \cdot d \cdot h) \cdot a^2 \cdot b) / (a^4 \cdot b^3))) / (-1/27 \cdot c^3 / a^6 + 1/162 \cdot (9 \cdot b^3 \cdot c^2 + 2 \cdot a \cdot b^2 \cdot d \cdot e + 2 \cdot a^3 \cdot g \cdot h + (e \cdot g + 4 \cdot d \cdot h) \cdot a^2 \cdot b) \cdot c / (a^6 \cdot b^3) + 1/1458 \cdot (8 \cdot b^4 \cdot d^3 + a \cdot b^3 \cdot e^3 + 12 \cdot a \cdot b^3 \cdot d^2 \cdot g + 6 \cdot a^2 \cdot b^2 \cdot d \cdot g^2 + a^3 \cdot b \cdot g^3 + 6 \cdot a^2 \cdot b^2 \cdot e^2 \cdot h + 12 \cdot a^3 \cdot b \cdot e \cdot h^2 + 8 \cdot a^4 \cdot h^3) / (a^5 \cdot b^5) - 1/1458 \cdot (27 \cdot b^5 \cdot c^3 + 8 \cdot a^5 \cdot h^3 - (g^3 - 12 \cdot e \cdot h^2) \cdot a^4 \cdot b - 6 \cdot (d \cdot g^2 - e^2 \cdot h - 3 \cdot c \cdot g \cdot h) \cdot a^3 \cdot b^2 + (e^3 - 12 \cdot d^2 \cdot g + 9 \cdot (e \cdot g + 4 \cdot d \cdot h) \cdot c) \cdot a^2 \cdot b^3 - 2 \cdot (4 \cdot d^3 - 9 \cdot c \cdot d \cdot e) \cdot a \cdot b^4) / (a^6 \cdot b^5))^{1/3} + 81 \cdot (I \cdot \sqrt{3}) + 1) \cdot (-1/27 \cdot c^3 / a^6 + 1/162 \cdot (9 \cdot b^3 \cdot c^2 + 2 \cdot a \cdot b^2 \cdot d \cdot e + 2 \cdot a^3 \cdot g \cdot h + (e \cdot g + 4 \cdot d \cdot h) \cdot a^2 \cdot b) \cdot c / (a^6 \cdot b^3) + 1/1458 \cdot (8 \cdot b^4 \cdot d^3 + a \cdot b^3 \cdot e^3 + 12 \cdot a \cdot b^3 \cdot d^2 \cdot g + 6 \cdot a^2 \cdot b^2 \cdot d \cdot g^2 + a^3 \cdot b \cdot g^3 + 6 \cdot a^2 \cdot b^2 \cdot e^2 \cdot h + 12 \cdot a^3 \cdot b \cdot e \cdot h^2 + 8 \cdot a^4 \cdot h^3) / (a^5 \cdot b^5) - 1/1458 \cdot (27 \cdot b^5 \cdot c^3 + 8 \cdot a^5 \cdot h^3 - (g^3 - 12 \cdot e \cdot h^2) \cdot a^4 \cdot b - 6 \cdot (d \cdot g^2 - e^2 \cdot h - 3 \cdot c \cdot g \cdot h) \cdot a^3 \cdot b^2 + (e^3 - 12 \cdot d^2 \cdot g + 9 \cdot (e \cdot g + 4 \cdot d \cdot h) \cdot c) \cdot a^2 \cdot b^3 - 2 \cdot (4 \cdot d^3 - 9 \cdot c \cdot d \cdot e) \cdot a \cdot b^4) / (a^6 \cdot b^5))^{1/3} + 54 \cdot c / a^2)^2 + 8 \cdot (2 \cdot a^3 \cdot b \cdot d + a^4 \cdot g) \cdot h^2 - 1/18 \cdot (4 \cdot a^2 \cdot b^4 \cdot d^2 + 6 \cdot a^2 \cdot b^4 \cdot c \cdot e + 4 \cdot a^3 \cdot b^3 \cdot d \cdot g + a^4 \cdot b^2 \cdot g^2 + 12 \cdot a^3 \cdot b^3 \cdot c \cdot h) \cdot ((-I \cdot \sqrt{3}) + 1) \cdot (9 \cdot c^2 / a^4 - (9 \cdot b^3 \cdot c^2 + 2 \cdot a \cdot b^2 \cdot d \cdot e + 2 \cdot a^3 \cdot g \cdot h + (e \cdot g + 4 \cdot d \cdot h) \cdot a^2 \cdot b) / (a^4 \cdot b^3))) / (-1/27 \cdot c^3 / a^6 + 1/162 \cdot (9 \cdot b^3 \cdot c^2 + 2 \cdot a \cdot b^2 \cdot d \cdot e + 2 \cdot a^3 \cdot g \cdot h + (e \cdot g + 4 \cdot d \cdot h) \cdot a^2 \cdot b) \cdot c / (a^6 \cdot b^3) + 1/1458 \cdot (8 \cdot b^4 \cdot d^3 + a \cdot b^3 \cdot e^3 + 12 \cdot a \cdot b^3 \cdot d^2 \cdot g + 6 \cdot a^2 \cdot b^2 \cdot d \cdot g^2 + a^3 \cdot b \cdot g^3 + 6 \cdot a^2 \cdot b^2 \cdot e^2 \cdot h + 12 \cdot a^3 \cdot b \cdot e \cdot h^2 + 8 \cdot a^4 \cdot h^3) / (a^5 \cdot b^5) - 1/1458 \cdot (27 \cdot b^5 \cdot c^3 + 8 \cdot a^5 \cdot h^3 - (g^3 -$$

$$\begin{aligned}
& 12e^h^2)a^4b - 6(dg^2 - e^2h - 3c*g*h)a^3b^2 + (e^3 - 12d^2g + 9*(eg + 4d*h)*c)a^2b^3 - 2*(4d^3 - 9c*d*e)a*b^4)/(a^6b^5))^{1/3} + \\
& 81*(I*\text{sqrt}(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3* \\
& g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12 \\
& *a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h \\
& ^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^ \\
& 2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + \\
& 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^{1/3} + 54*c/a^2 \\
&) + 2*(6*a*b^3*c*d + a^2*b^2*e^2)*g + 2*(9*a*b^3*c^2 + 8*a^2*b^2*d*e + 4*a^ \\
& 3*b*e*g)*h + (8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^ \\
& 3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)*x) + 108*(a*b*d - a \\
& ^2*g)*x - (162*b^2*c*x^3 + 162*a*b*c - (a^2*b^2*x^3 + a^3*b))*((-I*\text{sqrt}(3) + \\
& 1)*(9*c^2/a^4 - (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b) \\
&))/(a^4*b^3))/(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + \\
& (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3 \\
& *d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8 \\
& *a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4 \\
& *b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h) \\
&)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^{1/3} + 81*(I*\text{sqrt}(3) \\
& + 1)*(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4 \\
& *d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + \\
& 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3) \\
&))/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(\\
& d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2 \\
& *b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^{1/3} + 54*c/a^2) - 3*\text{sqrt}(1/3) \\
&)*(a^2*b^2*x^3 + a^3*b)*\text{sqrt}(-(((I*\text{sqrt}(3) + 1)*(9*c^2/a^4 - (9*b^3*c^2 + \\
& 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)/(a^4*b^3))/(-1/27*c^3/a^6 + \\
& 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^ \\
& 3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3 \\
& *b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(\\
& 27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g* \\
& h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c* \\
& d*e)*a*b^4)/(a^6*b^5))^{1/3} + 81*(I*\text{sqrt}(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9 \\
& *b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/ \\
& 458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + \\
& 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c \\
& ^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b \\
& ^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b \\
& ^4)/(a^6*b^5))^{1/3} + 54*c/a^2)^2*a^4*b^3 - 108*((I*\text{sqrt}(3) + 1)*(9*c^2/a \\
& ^4 - (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)/(a^4*b^3)) \\
&)/(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h) \\
&)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a \\
& ^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a \\
& ^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^ \\
& 2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3
\end{aligned}$$

$$\begin{aligned}
& - 2*(4*d^3 - 9*c*d*e)*a*b^4/(a^6*b^5))^{(1/3)} + 81*(I*\text{sqrt}(3) + 1)*(-1/27* \\
& c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b) \\
& *c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d \\
& *g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) \\
& - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2* \\
& h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4* \\
& d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^{(1/3)} + 54*c/a^2)*a^2*b^3*c + 2916*b^3*c^2 \\
& + 2592*a*b^2*d*e + 1296*a^2*b*e*g + 2592*(2*a^2*b*d + a^3*g)*h)/(a^4*b^3)) \\
&)*\log(-12*b^4*c*d^2 - 9*b^4*c^2*e - 4*a*b^3*d*e^2 - 3*a^2*b^2*c*g^2 - 1/324 \\
& *(a^4*b^4*e + 2*a^5*b^3*h))*((-I*\text{sqrt}(3) + 1)*(9*c^2/a^4 - (9*b^3*c^2 + 2*a* \\
& b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)/(a^4*b^3)))/(-1/27*c^3/a^6 + 1/16 \\
& 2*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + \\
& 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g \\
& ^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b \\
& ^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a \\
& ^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e) \\
& *a*b^4)/(a^6*b^5))^{(1/3)} + 81*(I*\text{sqrt}(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b^3 \\
& *c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458* \\
& (8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a \\
& ^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + \\
& 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + \\
& (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/ \\
& (a^6*b^5))^{(1/3)} + 54*c/a^2)^2 - 8*(2*a^3*b*d + a^4*g)*h^2 + 1/18*(4*a^2*b^ \\
& 4*d^2 + 6*a^2*b^4*c*e + 4*a^3*b^3*d*g + a^4*b^2*g^2 + 12*a^3*b^3*c*h))*((-I* \\
& \text{sqrt}(3) + 1)*(9*c^2/a^4 - (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d \\
& *h)*a^2*b)/(a^4*b^3)))/(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a \\
& ^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + \\
& 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b* \\
& e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e \\
& *h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e* \\
& g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^{(1/3)} + 81*(I \\
& *\text{sqrt}(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + \\
& (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^ \\
& 3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + \\
& 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^ \\
& 4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d* \\
& h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^{(1/3)} + 54*c/a^2) - 2 \\
& *(6*a*b^3*c*d + a^2*b^2*e^2)*g - 2*(9*a*b^3*c^2 + 8*a^2*b^2*d*e + 4*a^3*b*e \\
& *g)*h + 2*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b \\
& *g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)*x + 1/108*\text{sqrt}(1/3)*(7 \\
& 2*a^2*b^4*d^2 - 54*a^2*b^4*c*e + 72*a^3*b^3*d*g + 18*a^4*b^2*g^2 - 108*a^3* \\
& b^3*c*h + (a^4*b^4*e + 2*a^5*b^3*h))*((-I*\text{sqrt}(3) + 1)*(9*c^2/a^4 - (9*b^3*c \\
& ^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)/(a^4*b^3)))/(-1/27*c^3/a \\
& ^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a \\
& ^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2
\end{aligned}$$

$$\begin{aligned}
& + a^3 b g^3 + 6 a^2 b^2 e^2 h + 12 a^3 b e e h^2 + 8 a^4 h^3) / (a^5 b^5) - 1 / 1458 * (27 b^5 c^3 + 8 a^5 h^3 - (g^3 - 12 e e h^2) a^4 b - 6 (d g^2 - e^2 h - 3 c g h) a^3 b^2 + (e^3 - 12 d^2 g + 9 (e g + 4 d h) c) a^2 b^3 - 2 (4 d^3 - 9 c d e) a b^4) / (a^6 b^5)^{(1/3)} + 81 * (I \sqrt{3} + 1) * (-1 / 27 c^3 / a^6 + 1 / 62 * (9 b^3 c^2 + 2 a b^2 d e + 2 a^3 g h + (e g + 4 d h) a^2 b) c / (a^6 b^3) \\
& + 1 / 1458 * (8 b^4 d^3 + a b^3 e^3 + 12 a b^3 d^2 g + 6 a^2 b^2 d g^2 + a^3 b g^3 + 6 a^2 b^2 e^2 h + 12 a^3 b e e h^2 + 8 a^4 h^3) / (a^5 b^5) - 1 / 1458 * (27 b^5 c^3 + 8 a^5 h^3 - (g^3 - 12 e e h^2) a^4 b - 6 (d g^2 - e^2 h - 3 c g h) a^3 b^2 + (e^3 - 12 d^2 g + 9 (e g + 4 d h) c) a^2 b^3 - 2 (4 d^3 - 9 c d e) a b^4) / (a^6 b^5)^{(1/3)} + 54 c / a^2) * \sqrt{-(((-I \sqrt{3} + 1) * (9 c^2 / a^4 - (9 b^3 c^2 + 2 a b^2 d e + 2 a^3 g h + (e g + 4 d h) a^2 b) / (a^4 b^3))) / (-1 / 27 c^3 / a^6 + 1 / 162 * (9 b^3 c^2 + 2 a b^2 d e + 2 a^3 g h + (e g + 4 d h) a^2 b) c / (a^6 b^3) + 1 / 1458 * (8 b^4 d^3 + a b^3 e^3 + 12 a b^3 d^2 g + 6 a^2 b^2 d g^2 + a^3 b g^3 + 6 a^2 b^2 e^2 h + 12 a^3 b e e h^2 + 8 a^4 h^3) / (a^5 b^5) - 1 / 1458 * (27 b^5 c^3 + 8 a^5 h^3 - (g^3 - 12 e e h^2) a^4 b - 6 (d g^2 - e^2 h - 3 c g h) a^3 b^2 + (e^3 - 12 d^2 g + 9 (e g + 4 d h) c) a^2 b^3 - 2 (4 d^3 - 9 c d e) a b^4) / (a^6 b^5)^{(1/3)} + 81 * (I \sqrt{3} + 1) * (-1 / 27 c^3 / a^6 + 1 / 162 * (9 b^3 c^2 + 2 a b^2 d e + 2 a^3 g h + (e g + 4 d h) a^2 b) c / (a^6 b^3) + 1 / 1458 * (8 b^4 d^3 + a b^3 e^3 + 12 a b^3 d^2 g + 6 a^2 b^2 d g^2 + a^3 b g^3 + 6 a^2 b^2 e^2 h + 12 a^3 b e e h^2 + 8 a^4 h^3) / (a^5 b^5) - 1 / 1458 * (27 b^5 c^3 + 8 a^5 h^3 - (g^3 - 12 e e h^2) a^4 b - 6 (d g^2 - e^2 h - 3 c g h) a^3 b^2 + (e^3 - 12 d^2 g + 9 (e g + 4 d h) c) a^2 b^3 - 2 (4 d^3 - 9 c d e) a b^4) / (a^6 b^5)^{(1/3)} + 54 c / a^2)^2 a^4 b^3 - 108 * ((-I \sqrt{3} + 1) * (9 c^2 / a^4 - (9 b^3 c^2 + 2 a b^2 d e + 2 a^3 g h + (e g + 4 d h) a^2 b) / (a^4 b^3))) / (-1 / 27 c^3 / a^6 + 1 / 162 * (9 b^3 c^2 + 2 a b^2 d e + 2 a^3 g h + (e g + 4 d h) a^2 b) c / (a^6 b^3) + 1 / 1458 * (8 b^4 d^3 + a b^3 e^3 + 12 a b^3 d^2 g + 6 a^2 b^2 d g^2 + a^3 b g^3 + 6 a^2 b^2 e^2 h + 12 a^3 b e e h^2 + 8 a^4 h^3) / (a^5 b^5) - 1 / 1458 * (27 b^5 c^3 + 8 a^5 h^3 - (g^3 - 12 e e h^2) a^4 b - 6 (d g^2 - e^2 h - 3 c g h) a^3 b^2 + (e^3 - 12 d^2 g + 9 (e g + 4 d h) c) a^2 b^3 - 2 (4 d^3 - 9 c d e) a b^4) / (a^6 b^5)^{(1/3)} + 81 * (I \sqrt{3} + 1) * (-1 / 27 c^3 / a^6 + 1 / 162 * (9 b^3 c^2 + 2 a b^2 d e + 2 a^3 g h + (e g + 4 d h) a^2 b) c / (a^6 b^3) + 1 / 1458 * (8 b^4 d^3 + a b^3 e^3 + 12 a b^3 d^2 g + 6 a^2 b^2 d g^2 + a^3 b g^3 + 6 a^2 b^2 e^2 h + 12 a^3 b e e h^2 + 8 a^4 h^3) / (a^5 b^5) - 1 / 1458 * (27 b^5 c^3 + 8 a^5 h^3 - (g^3 - 12 e e h^2) a^4 b - 6 (d g^2 - e^2 h - 3 c g h) a^3 b^2 + (e^3 - 12 d^2 g + 9 (e g + 4 d h) c) a^2 b^3 - 2 (4 d^3 - 9 c d e) a b^4) / (a^6 b^5)^{(1/3)} + 81 * (I \sqrt{3} + 1) * (-1 / 27 c^3 / a^6 + 1 / 162 * (9 b^3 c^2 + 2 a b^2 d e + 2 a^3 g h + (e g + 4 d h) a^2 b) c / (a^6 b^3) + 1 / 1458 * (8 b^4 d^3 + a b^3 e^3 + 12 a b^3 d^2 g + 6 a^2 b^2 d g^2 + a^3 b g^3 + 6 a^2 b^2 e^2 h + 12 a^3 b e e h^2 + 8 a^4 h^3) / (a^5 b^5) - 1 / 1458 * (27 b^5 c^3 + 8 a^5 h^3 - (g^3 - 12 e e h^2) a^4 b - 6 (d g^2 - e^2 h - 3 c g h) a^3 b^2 + (e^3 - 12 d^2 g + 9 (e g + 4 d h) c) a^2 b^3 - 2 (4 d^3 - 9 c d e) a b^4) / (a^6 b^5)^{(1/3)} + 54 c / a^2) * a^2 b^3 c + 2916 b^3 c^2 + 2592 a b^2 d e + 1296 a^2 b e e g + 2592 * (2 a^2 b d + a^3 g h) / (a^4 b^3))) - (162 b^2 c x^3 + 162 a b c - (a^2 b^2 x^3 + a^3 b) * ((-I \sqrt{3} + 1) * (9 c^2 / a^4 - (9 b^3 c^2 + 2 a b^2 d e + 2 a^3 g h + (e g + 4 d h) a^2 b) / (a^4 b^3))) / (-1 / 27 c^3 / a^6 + 1 / 162 * (9 b^3 c^2 + 2 a b^2 d e + 2 a^3 g h + (e g + 4 d h) a^2 b) c / (a^6 b^3) + 1 / 1458 * (8 b^4 d^3 + a b^3 e^3 + 12 a b^3 d^2 g + 6 a^2 b^2 d g^2 + a^3 b g^3 + 6 a^2 b^2 e^2 h + 12 a^3 b e e h^2 + 8 a^4 h^3) / (a^5 b^5) - 1 / 1458 * (27 b^5 c^3 + 8 a^5 h^3 - (g^3 - 12 e e h^2) a^4 b - 6 (d g^2 - e^2 h - 3 c g h) a^3 b^2 + (e^3 - 12 d^2 g + 9 (e g + 4 d h) c) a^2 b^3 - 2 (4 d^3 - 9 c d e) a b^4) / (a^6 b^5)^{(1/3)} + 81 * (I
\end{aligned}$$

$$\begin{aligned}
& g^3 + 6a^2b^2e^2h + 12a^3b^2e^2h + 8a^4h^3)/(a^5b^5) - 1/1458(27b^5c^3 + 8a^5h^3 - (g^3 - 12e^2h^2)a^4b - 6(dg^2 - e^2h - 3c*g*h)* \\
& a^3b^2 + (e^3 - 12d^2g + 9(e*g + 4d*h)*c)a^2b^3 - 2(4d^3 - 9c*d*e) \\
&)a^2b^4)/(a^6b^5))^{(1/3)} + 54c/a^2)^2 - 8(2a^3b*d + a^4g)*h^2 + 1/18* \\
& (4a^2b^4d^2 + 6a^2b^4c*e + 4a^3b^3d*g + a^4b^2g^2 + 12a^3b^3c \\
& *h)*((-I*sqrt(3) + 1)*(9c^2/a^4 - (9b^3c^2 + 2a*b^2*d*e + 2a^3*g*h + (\\
& e*g + 4d*h)*a^2*b)/(a^4*b^3)))/(-1/27*c^3/a^6 + 1/162*(9b^3*c^2 + 2a*b^2* \\
& d*e + 2a^3*g*h + (e*g + 4d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8b^4*d^3 + a* \\
& b^3*e^3 + 12a*b^3*d^2*g + 6a^2*b^2*d*g^2 + a^3*b*g^3 + 6a^2*b^2*e^2*h + \\
& 12a^3*b*e*h^2 + 8a^4*h^3)/(a^5*b^5) - 1/1458*(27b^5*c^3 + 8a^5*h^3 - (g \\
& ^3 - 12e^2h^2)a^4*b - 6(dg^2 - e^2h - 3c*g*h)a^3*b^2 + (e^3 - 12d^2* \\
& g + 9(e*g + 4d*h)*c)a^2*b^3 - 2(4d^3 - 9c*d*e)a^2*b^4)/(a^6*b^5))^{(1/3)} \\
&) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9b^3*c^2 + 2a*b^2*d*e + 2* \\
& a^3*g*h + (e*g + 4d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8b^4*d^3 + a*b^3*e^3 \\
& + 12a*b^3*d^2*g + 6a^2*b^2*d*g^2 + a^3*b*g^3 + 6a^2*b^2*e^2*h + 12a^3*b \\
& *e*h^2 + 8a^4*h^3)/(a^5*b^5) - 1/1458*(27b^5*c^3 + 8a^5*h^3 - (g^3 - 12* \\
& e^2h^2)a^4*b - 6(dg^2 - e^2h - 3c*g*h)a^3*b^2 + (e^3 - 12d^2*g + 9(e \\
& *g + 4d*h)*c)a^2*b^3 - 2(4d^3 - 9c*d*e)a^2*b^4)/(a^6*b^5))^{(1/3)} + 54*c \\
& /a^2) - 2*(6a*b^3*c*d + a^2*b^2*e^2)*g - 2*(9a*b^3*c^2 + 8a^2*b^2*d*e + \\
& 4a^3*b*e*g)*h + 2*(8b^4*d^3 + a*b^3*e^3 + 12a*b^3*d^2*g + 6a^2*b^2*d*g^ \\
& 2 + a^3*b*g^3 + 6a^2*b^2*e^2*h + 12a^3*b*e*h^2 + 8a^4*h^3)*x - 1/108*sqrt \\
& t(1/3)*(72a^2b^4d^2 - 54a^2b^4c*e + 72a^3b^3d*g + 18a^4b^2g^2 - \\
& 108a^3b^3c*h + (a^4b^4e + 2a^5b^3h)*((-I*sqrt(3) + 1)*(9c^2/a^4 - \\
& (9b^3c^2 + 2a*b^2*d*e + 2a^3*g*h + (e*g + 4d*h)*a^2*b)/(a^4*b^3)))/(-1 \\
& /27*c^3/a^6 + 1/162*(9b^3*c^2 + 2a*b^2*d*e + 2a^3*g*h + (e*g + 4d*h)*a^ \\
& 2*b)*c/(a^6*b^3) + 1/1458*(8b^4*d^3 + a*b^3*e^3 + 12a*b^3*d^2*g + 6a^2*b \\
& ^2*d*g^2 + a^3*b*g^3 + 6a^2*b^2*e^2*h + 12a^3*b*e*h^2 + 8a^4*h^3)/(a^5*b \\
& ^5) - 1/1458*(27b^5*c^3 + 8a^5*h^3 - (g^3 - 12e^2h^2)a^4*b - 6(dg^2 - \\
& e^2h - 3c*g*h)a^3*b^2 + (e^3 - 12d^2*g + 9(e*g + 4d*h)*c)a^2*b^3 - 2 \\
& *(4d^3 - 9c*d*e)a^2*b^4)/(a^6*b^5))^{(1/3)} + 81*(I*sqrt(3) + 1)*(-1/27*c^3/ \\
& a^6 + 1/162*(9b^3*c^2 + 2a*b^2*d*e + 2a^3*g*h + (e*g + 4d*h)*a^2*b)*c/(\\
& a^6*b^3) + 1/1458*(8b^4*d^3 + a*b^3*e^3 + 12a*b^3*d^2*g + 6a^2*b^2*d*g^2 \\
& + a^3*b*g^3 + 6a^2*b^2*e^2*h + 12a^3*b*e*h^2 + 8a^4*h^3)/(a^5*b^5) - 1/ \\
& 1458*(27b^5*c^3 + 8a^5*h^3 - (g^3 - 12e^2h^2)a^4*b - 6(dg^2 - e^2h - \\
& 3c*g*h)a^3*b^2 + (e^3 - 12d^2*g + 9(e*g + 4d*h)*c)a^2*b^3 - 2(4d^3 \\
& - 9c*d*e)a^2*b^4)/(a^6*b^5))^{(1/3)} + 54*c/a^2))*sqrt(-(((I*sqrt(3) + 1)*(9 \\
& *c^2/a^4 - (9b^3c^2 + 2a*b^2*d*e + 2a^3*g*h + (e*g + 4d*h)*a^2*b)/(a^4 \\
& *b^3)))/(-1/27*c^3/a^6 + 1/162*(9b^3*c^2 + 2a*b^2*d*e + 2a^3*g*h + (e*g + \\
& 4d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8b^4*d^3 + a*b^3*e^3 + 12a*b^3*d^2*g \\
& + 6a^2*b^2*d*g^2 + a^3*b*g^3 + 6a^2*b^2*e^2*h + 12a^3*b*e*h^2 + 8a^4*h \\
& ^3)/(a^5*b^5) - 1/1458*(27b^5*c^3 + 8a^5*h^3 - (g^3 - 12e^2h^2)a^4*b - 6 \\
& *(dg^2 - e^2h - 3c*g*h)a^3*b^2 + (e^3 - 12d^2*g + 9(e*g + 4d*h)*c)a \\
& ^2*b^3 - 2(4d^3 - 9c*d*e)a^2*b^4)/(a^6*b^5))^{(1/3)} + 81*(I*sqrt(3) + 1)* \\
& (-1/27*c^3/a^6 + 1/162*(9b^3*c^2 + 2a*b^2*d*e + 2a^3*g*h + (e*g + 4d*h)* \\
& a^2*b)*c/(a^6*b^3) + 1/1458*(8b^4*d^3 + a*b^3*e^3 + 12a*b^3*d^2*g + 6a^2
\end{aligned}$$

$$\begin{aligned}
& *b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5 \\
& *b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 \\
& - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - \\
& 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^{(1/3)} + 54*c/a^2)^2*a^4*b^3 - 108*((\\
& -I*\sqrt{3} + 1)*(9*c^2/a^4 - (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + \\
& 4*d*h)*a^2*b)/(a^4*b^3)))/(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + \\
& 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^ \\
& 3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3 \\
& *b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 1 \\
& 2*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9* \\
& (e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^{(1/3)} + 81 \\
& *(I*\sqrt{3} + 1)*(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g* \\
& h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a \\
& *b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 \\
& + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2) \\
& *a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4 \\
& *d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^{(1/3)} + 54*c/a^2)* \\
& a^2*b^3*c + 2916*b^3*c^2 + 2592*a*b^2*d*e + 1296*a^2*b*e*g + 2592*(2*a^2*b* \\
& d + a^3*g)*h)/(a^4*b^3))) + 324*(b^2*c*x^3 + a*b*c)*\log(x))/(a^2*b^2*x^3 + \\
& a^3*b)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x/(b*x**3+a)**2,x)

[Out] Timed out

Giac [A] time = 1.08568, size = 452, normalized size = 1.56

$$-\frac{c \log(|bx^3 + a|)}{3a^2} + \frac{c \log(|x|)}{a^2} + \frac{\sqrt{3} \left(2 (-ab^2)^{\frac{1}{3}} b^2 d + (-ab^2)^{\frac{1}{3}} abg - 2 (-ab^2)^{\frac{2}{3}} ah - (-ab^2)^{\frac{2}{3}} be \right) \arctan \left(\frac{\sqrt{3} \left(2x + (-\frac{a}{b})^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{9a^2 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*c*log(abs(b*x^3 + a))/a^2 + c*log(abs(x))/a^2 + 1/9*sqrt(3)*(2*(-a*b^2)^(1/3)*b^2*d + (-a*b^2)^(1/3)*a*b*g - 2*(-a*b^2)^(2/3)*a*h - (-a*b^2)^(2/3)*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^3) + 1/3*(a*b*c - a^2*f - (a^2*h - a*b*e)*x^2 + (a*b*d - a^2*g)*x)/((b*x^3 + a)*a^2*b) + 1/18*(2*(-a*b^2)^(1/3)*b^2*d + (-a*b^2)^(1/3)*a*b*g + 2*(-a*b^2)^(2/3)*a*h + (-a*b^2)^(2/3)*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^3) - 1/9*(2*a^4*b^2*h*(-a/b)^(1/3) + a^3*b^3*(-a/b)^(1/3)*e + 2*a^3*b^3*d + a^4*b^2*g)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^5*b^3)
```

$$3.418 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)^2} dx$$

Optimal. Leaf size=301

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(a^{2/3}(ah + 2be) + b^{2/3}(4bc - af)\right)}{18a^{7/3}b^{4/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(a^{2/3}(ah + 2be) + b^{2/3}(4bc - af)\right)}{9a^{7/3}b^{4/3}} + \dots$$

[Out] $-(c/(a^2x)) + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(3*a^2*b*(a + b*x^3)) + ((4*b^(5/3)*c - 2*a^(2/3)*b*e - a*b^(2/3)*f - a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(7/3)*b^(4/3)) + (d*Log[x])/a^2 + ((b^(2/3)*(4*b*c - a*f) + a^(2/3)*(2*b*e + a*h))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(7/3)*b^(4/3)) - ((b^(2/3)*(4*b*c - a*f) + a^(2/3)*(2*b*e + a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(7/3)*b^(4/3)) - (d*Log[a + b*x^3])/(3*a^2)$

Rubi [A] time = 0.593431, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(a^{2/3}(ah + 2be) + b^{2/3}(4bc - af)\right)}{18a^{7/3}b^{4/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(a^{2/3}(ah + 2be) + b^{2/3}(4bc - af)\right)}{9a^{7/3}b^{4/3}} + \dots$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^2), x]

[Out] $-(c/(a^2x)) + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(3*a^2*b*(a + b*x^3)) + ((4*b^(5/3)*c - 2*a^(2/3)*b*e - a*b^(2/3)*f - a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(7/3)*b^(4/3)) + (d*Log[x])/a^2 + ((b^(2/3)*(4*b*c - a*f) + a^(2/3)*(2*b*e + a*h))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(7/3)*b^(4/3)) - ((b^(2/3)*(4*b*c - a*f) + a^(2/3)*(2*b*e + a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(7/3)*b^(4/3)) - (d*Log[a + b*x^3])/(3*a^2)$

Rule 1829

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^

```
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_)*(x_))^(m_))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[Ex
pandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^( -1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
```

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 204

$\text{Int}[\frac{(a_1 + (b_1)x^2)^{-1}}{x}, x_Symbol] \rightarrow -\text{Simp}[\frac{\text{ArcTan}[\frac{\text{Rt}[-b, 2]x}{\text{Rt}[-a, 2] - \text{Rt}[-a, 2]\text{Rt}[-b, 2]}}{x}], x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 628

$\text{Int}[\frac{(d_1 + (e_1)x)}{(a_1 + (b_1)x + (c_1)x^2)}, x_Symbol] \rightarrow \text{Simp}[\frac{d \cdot \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]}{b}, x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2cd - be, 0]$

Rule 260

$\text{Int}[x^{(m_1)} / (a_1 + (b_1)x^n), x_Symbol] \rightarrow \text{Simp}[\frac{\text{Log}[\text{RemoveContent}[a + bx^n, x]]}{bn}, x] \ /; \ \text{FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{x^2(a + bx^3)^2} dx &= \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{3a^2b(a + bx^3)} - \frac{\int \frac{-3b^2c - 3b^2dx - b(2be + ah)x^2 + b^2\left(\frac{bc}{a} - f\right)x^3}{x^2(a + bx^3)}}{3ab^2} \\
&= \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{3a^2b(a + bx^3)} - \frac{\int \left(-\frac{3b^2c}{ax^2} - \frac{3b^2d}{ax} + \frac{b(-a(2be + ah) + b(4bc - af))}{a(a + bx^3)}\right)}{3ab^2} \\
&= -\frac{c}{a^2x} + \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{3a^2b(a + bx^3)} + \frac{d \log(x)}{a^2} - \frac{\int \frac{-a(2be + ah) + b(4bc - af)}{a + bx^3}}{3a^2b} \\
&= -\frac{c}{a^2x} + \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{3a^2b(a + bx^3)} + \frac{d \log(x)}{a^2} - \frac{\int \frac{-a(2be + ah) + b(4bc - af)}{a + bx^3}}{3a^2b} \\
&= -\frac{c}{a^2x} + \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{3a^2b(a + bx^3)} + \frac{d \log(x)}{a^2} - \frac{d \log(a + bx^3)}{3a^2} \\
&= -\frac{c}{a^2x} + \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{3a^2b(a + bx^3)} + \frac{d \log(x)}{a^2} + \frac{(b^{2/3}(4bc - af))}{3a^2} \\
&= -\frac{c}{a^2x} + \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{3a^2b(a + bx^3)} + \frac{d \log(x)}{a^2} + \frac{(b^{2/3}(4bc - af))}{3a^2} \\
&= -\frac{c}{a^2x} + \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{3a^2b(a + bx^3)} + \frac{(4b^{5/3}c - 2a^{2/3}be - ab^{2/3}f)}{3\sqrt{3}a}
\end{aligned}$$

Mathematica [A] time = 0.298581, size = 285, normalized size = 0.95

$$\frac{6a(a^2(g+hx) - ab(d+x(e+fx)) + b^2cx^2)}{b(a+bx^3)} + \frac{a^{2/3} \log\left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx + b^{2/3}x^2}\right) (2a^{2/3}be + a^{5/3}h - ab^{2/3}f + 4b^{5/3}c)}{b^{4/3}} - \frac{2a^{2/3} \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right) (2a^{2/3}be + a^{5/3}h - ab^{2/3}f + 4b^{5/3}c)}{b^{4/3}}$$

$$18a^3$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^2), x]

```
[Out] -((18*a*c)/x + (6*a*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e + f*x)))/(b
*(a + b*x^3)) + (2*Sqrt[3]*a^(2/3)*(-4*b^(5/3)*c + 2*a^(2/3)*b*e + a*b^(2/3
)*f + a^(5/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(4/3) - 18*
a*d*Log[x] - (2*a^(2/3)*(4*b^(5/3)*c + 2*a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3
)*h)*Log[a^(1/3) + b^(1/3)*x])/b^(4/3) + (a^(2/3)*(4*b^(5/3)*c + 2*a^(2/3)*
b*e - a*b^(2/3)*f + a^(5/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/b^(4/3) + 6*a*d*Log[a + b*x^3])/(18*a^3)
```

Maple [B] time = 0.014, size = 517, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x)
```

```
[Out] 1/3/a*x^2/(b*x^3+a)*f-1/3*b/a^2*x^2/(b*x^3+a)*c-1/3/(b*x^3+a)/b*x*h+1/3/a*x
/(b*x^3+a)*e-1/3/(b*x^3+a)/b*g+1/3/a/(b*x^3+a)*d+1/9/b^2/(1/b*a)^(2/3)*ln(x
+(1/b*a)^(1/3))*h+2/9/a/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*e-1/18/b^2/(1/b
*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*h-1/9/a/b/(1/b*a)^(2/3)*ln(
x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e+1/9/b^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*h+2/9/a/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e-1/9/a*f/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/
3))+4/9/a^2*c/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/18/a*f/b/(1/b*a)^(1/3)*ln
(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-2/9/a^2*c/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(
1/3)*x+(1/b*a)^(2/3))+1/9/a*f*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(
2/(1/b*a)^(1/3)*x-1))-4/9/a^2*c*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2
/(1/b*a)^(1/3)*x-1))-1/3*d*ln(b*x^3+a)/a^2+d*ln(x)/a^2-1/a^2*c/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2/(b*x**3+a)**2,x)

[Out] Timed out

Giac [A] time = 1.10244, size = 473, normalized size = 1.57

$$-\frac{d \log(|bx^3 + a|)}{3a^2} + \frac{d \log(|x|)}{a^2} + \frac{\sqrt{3} \left((-ab^2)^{\frac{1}{3}} a^2 h + 2 (-ab^2)^{\frac{1}{3}} a b e + 4 (-ab^2)^{\frac{2}{3}} b c - (-ab^2)^{\frac{2}{3}} a f \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}} \right)}{9a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="giac")

[Out] -1/3*d*log(abs(b*x^3 + a))/a^2 + d*log(abs(x))/a^2 + 1/9*sqrt(3)*((-a*b^2)^(1/3)*a^2*h + 2*(-a*b^2)^(1/3)*a*b*e + 4*(-a*b^2)^(2/3)*b*c - (-a*b^2)^(2/3)*a*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^2) - 1/

$$\frac{3(4b^2cx^3 - abfx^3 + a^2hx^2 - abx^2e - abd*x + a^2gx + 3abc)}{(bx^4 + ax)a^2b} + \frac{1}{18} \frac{(-ab^2)^{1/3}a^2h + 2(-ab^2)^{1/3}abe - 4(-ab^2)^{2/3}bc + (-ab^2)^{2/3}af \log(x^2 + x(-a/b)^{1/3} + (-a/b)^{2/3})}{a^3b^2} + \frac{1}{9} \frac{4a^2b^4c(-a/b)^{1/3} - a^3b^3f(-a/b)^{1/3} - a^4b^2h - 2a^3b^3e}{a^5b^3} (-a/b)^{1/3} \log(\text{abs}(x - (-a/b)^{1/3}))$$

$$3.419 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)^2} dx$$

Optimal. Leaf size=306

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(\sqrt[3]{b}(5bc - 2af) - \sqrt[3]{a}(4bd - ag)\right)}{18a^{8/3}b^{2/3}} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(\sqrt[3]{b}(5bc - 2af) - \sqrt[3]{a}(4bd - ag)\right)}{9a^{8/3}b^{2/3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt[3]{a} + \sqrt[3]{bx}}\right)}{3\sqrt[3]{a}}$$

[Out] $-c/(2*a^2*x^2) - d/(a^2*x) - (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(3*a^2*(a + b*x^3)) + ((5*b^(4/3)*c + 4*a^(1/3)*b*d - 2*a*b^(1/3)*f - a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(8/3)*b^(2/3)) + (e*Log[x])/a^2 - ((b^(1/3)*(5*b*c - 2*a*f) - a^(1/3)*(4*b*d - a*g))*Log[a^(1/3) + b^(1/3)*x]/(9*a^(8/3)*b^(2/3)) + ((b^(1/3)*(5*b*c - 2*a*f) - a^(1/3)*(4*b*d - a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(18*a^(8/3)*b^(2/3)) - (e*Log[a + b*x^3])/(3*a^2)$

Rubi [A] time = 0.576867, antiderivative size = 304, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(4bd-ag)}{\sqrt[3]{b}} - 2af + 5bc\right)}{18a^{8/3}\sqrt[3]{b}} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(\sqrt[3]{b}(5bc - 2af) - \sqrt[3]{a}(4bd - ag)\right)}{9a^{8/3}b^{2/3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{bx}}{\sqrt[3]{a} + \sqrt[3]{bx}}\right)}{3\sqrt[3]{a}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^2), x]

[Out] $-c/(2*a^2*x^2) - d/(a^2*x) - (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(3*a^2*(a + b*x^3)) + ((5*b^(4/3)*c + 4*a^(1/3)*b*d - 2*a*b^(1/3)*f - a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(8/3)*b^(2/3)) + (e*Log[x])/a^2 - ((b^(1/3)*(5*b*c - 2*a*f) - a^(1/3)*(4*b*d - a*g))*Log[a^(1/3) + b^(1/3)*x]/(9*a^(8/3)*b^(2/3)) + ((5*b*c - 2*a*f - (a^(1/3)*(4*b*d - a*g))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(18*a^(8/3)*b^(1/3)) - (e*Log[a + b*x^3])/(3*a^2)$

Rule 1829

Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^

```
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_)*(x_))^(m_))/((a_) + (b_)*(x_)^n), x_Symbol] := Int[ExpandIntegrand[
((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^( -1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
```

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 204

$\text{Int}[\frac{(a_+) + (b_+)(x_+)^2}{(a_+ + (b_+)(x_+)^2)^{-1}}, x_Symbol] \ :> \ -\text{Simp}[\frac{\text{ArcTan}[\frac{\text{Rt}[-b, 2]x}{\text{Rt}[-a, 2] + \text{Rt}[-a, 2]\text{Rt}[-b, 2]}}{\text{Rt}[-a, 2] + \text{Rt}[-a, 2]\text{Rt}[-b, 2]}], x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 628

$\text{Int}[\frac{(d_+) + (e_+)(x_+)}{(a_+) + (b_+)(x_+) + (c_+)(x_+)^2}, x_Symbol] \ :> \ \text{Simp}[\frac{d \cdot \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]}{b}, x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2cd - be, 0]$

Rule 260

$\text{Int}[\frac{(x_+)^{m_+}}{(a_+) + (b_+)(x_+)^{n_+}}, x_Symbol] \ :> \ \text{Simp}[\frac{\text{Log}[\text{RemoveContent}[a + bx^n, x]]}{(b \cdot n)}, x] \ /; \ \text{FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{x^3(a + bx^3)^2} dx &= \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{3a^2(a + bx^3)} - \frac{\int \frac{-3b^2c - 3b^2dx - 3b^2ex^2 + 2b^2\left(\frac{bc}{a} - f\right)x^3 + b^2\left(\frac{bd}{a} - g\right)x^4}{x^3(a + bx^3)} dx}{3ab^2} \\
&= \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{3a^2(a + bx^3)} - \frac{\int \left(-\frac{3b^2c}{ax^3} - \frac{3b^2d}{ax^2} - \frac{3b^2e}{ax} + \frac{b^2(5bc - 2af + (bd - ag)x + (be - ah)x^2)}{a(a + bx^3)}\right) dx}{3ab^2} \\
&= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{3a^2(a + bx^3)} + \frac{e \log(x)}{a^2} - \frac{\int \frac{5bc - 2af + (bd - ag)x + (be - ah)x^2}{a(a + bx^3)} dx}{3ab^2} \\
&= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{3a^2(a + bx^3)} + \frac{e \log(x)}{a^2} - \frac{\int \frac{5bc - 2af + (bd - ag)x + (be - ah)x^2}{a(a + bx^3)} dx}{3ab^2} \\
&= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{3a^2(a + bx^3)} + \frac{e \log(x)}{a^2} - \frac{e \log(a + bx^3)}{3a^2} \\
&= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{3a^2(a + bx^3)} + \frac{e \log(x)}{a^2} - \frac{(5bc - 2af + (bd - ag)x + (be - ah)x^2)}{3ab^2} \\
&= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{3a^2(a + bx^3)} + \frac{e \log(x)}{a^2} - \frac{(5bc - 2af + (bd - ag)x + (be - ah)x^2)}{3ab^2} \\
&= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{3a^2(a + bx^3)} + \frac{(5b^{4/3}c + 4\sqrt[3]{abd} - 2af + (bd - ag)x + (be - ah)x^2)}{3ab^2}
\end{aligned}$$

Mathematica [A] time = 0.488182, size = 292, normalized size = 0.95

$$\frac{6a(a^2h - ab(e + x(f + gx)) + b^2x(c + dx))}{b(a + bx^3)} - \frac{\sqrt[3]{a} \log\left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx + b^{2/3}x^2}\right) \left(a^{4/3}g - 4\sqrt[3]{abd} - 2a\sqrt[3]{bf} + 5b^{4/3}c\right)}{b^{2/3}} + \frac{2\sqrt[3]{a} \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right) \left(a^{4/3}g - 4\sqrt[3]{abd} - 2a\sqrt[3]{bf} + 5b^{4/3}c\right)}{b^{2/3}}$$

18a³

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^2), x]

```
[Out] -((9*a*c)/x^2 + (18*a*d)/x + (6*a*(a^2*h + b^2*x*(c + d*x) - a*b*(e + x*(f + g*x))))/(b*(a + b*x^3)) + (2*sqrt[3]*a^(1/3)*(-5*b^(4/3)*c - 4*a^(1/3)*b*d + 2*a*b^(1/3)*f + a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]]/b^(2/3) - 18*a*e*Log[x] + (2*a^(1/3)*(5*b^(4/3)*c - 4*a^(1/3)*b*d - 2*a*b^(1/3)*f + a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) - (a^(1/3)*(5*b^(4/3)*c - 4*a^(1/3)*b*d - 2*a*b^(1/3)*f + a^(4/3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) + 6*a*e*Log[a + b*x^3])/(18*a^3)
```

Maple [B] time = 0.013, size = 527, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x)
```

```
[Out] 1/3/a/(b*x^3+a)*x^2*g-1/3/a^2*b*x^2/(b*x^3+a)*d+1/3/a*x/(b*x^3+a)*f-1/3*b/a^2*x/(b*x^3+a)*c-1/3/(b*x^3+a)/b*h+1/3/a/(b*x^3+a)*e-5/9/a^2*c/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+5/18/a^2*c/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-5/9/a^2*c/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+2/9/a*f/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/9/a*f/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2/9/a*f/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+4/9/a^2/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*d-2/9/a^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d-4/9/a^2*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d-1/9/a*g/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/18/a*g/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/9/a*g*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/3*e*ln(b*x^3+a)/a^2-d/a^2/x-1/2*c/a^2/x^2+e*ln(x)/a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [C] time = 121.35, size = 28467, normalized size = 93.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/324*(108*(4*b^2*d - a*b*g)*x^4 + 324*a*b*d*x + 54*(5*b^2*c - 2*a*b*f)*x^3 \\ & + 162*a*b*c - 108*(a*b*e - a^2*h)*x^2 + 2*(a^2*b^2*x^5 + a^3*b*x^2)*((-I*\sqrt{3} + 1)* \\ & (9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b)) / (-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^{1/3} + 81*(I*\sqrt{3} + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^{1/3} + 54*e/a^2)* \\ & \log(-160*a*b^3*c*d^2 + 75*a*b^3*c^2*e - 36*a^2*b^2*d*e^2 + 12*a^3*b*e*f^2 - 1/324*(4*a^6*b^2*d - a^7*b*g)*((-I*\sqrt{3} + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b)) / (-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^{1/3} + 81*(I*\sqrt{3} + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^{1/3} + 54*e/a^2)^2 - 2*(5*a^3*b*c - 2*a^4*f)*g^2 - 1/18*(25*a^3*b^3*c^2 - 24*a^4*b^2*d*e - 20*a^4*b^2*c*f + 4*a^5*b*f^2 + 6*a^5*b*e \end{aligned}$$

$$\begin{aligned}
& *g) * ((-I\sqrt{3} + 1) * (9e^2/a^4 - (20b^2cd + 2a^2f*g + (9e^2 - 8d*f - 5c*g) * a*b)) / (a^5*b)) / (-1/27e^3/a^6 + 1/162 * (20b^2cd + 2a^2f*g + (9e^2 - 8d*f - 5c*g) * a*b) * e / (a^7*b) - 1/1458 * (125b^4c^3 + 64a*b^3d^3 - 150a*b^3c^2f + 60a^2b^2c*f^2 - 8a^3b*f^3 - 48a^2b^2d^2g + 12a^3b*d*g^2 - a^4g^3) / (a^8b^2) - 1/1458 * (125b^4c^3 + a^4g^3 - 2*(4f^3 - 9e*f*g + 6d*g^2) * a^3b + 3*(9e^3 - 24d*e*f + 16d^2g + 5*(4f^2 - 3e*g) * c) * a^2b^2 - 2*(32d^3 - 90c*d*e + 75c^2f) * a*b^3) / (a^8b^2))^(1/3) \\
& + 81 * (I\sqrt{3} + 1) * (-1/27e^3/a^6 + 1/162 * (20b^2cd + 2a^2f*g + (9e^2 - 8d*f - 5c*g) * a*b) * e / (a^7*b) - 1/1458 * (125b^4c^3 + 64a*b^3d^3 - 150a*b^3c^2f + 60a^2b^2c*f^2 - 8a^3b*f^3 - 48a^2b^2d^2g + 12a^3b*d*g^2 - a^4g^3) / (a^8b^2) - 1/1458 * (125b^4c^3 + a^4g^3 - 2*(4f^3 - 9e*f*g + 6d*g^2) * a^3b + 3*(9e^3 - 24d*e*f + 16d^2g + 5*(4f^2 - 3e*g) * c) * a^2b^2 - 2*(32d^3 - 90c*d*e + 75c^2f) * a*b^3) / (a^8b^2))^(1/3) + 5 \\
& 4 * e / a^2) + 4 * (16a^2b^2d^2 - 15a^2b^2c*e) * f + (80a^2b^2c*d + 9a^3b * e^2 - 32a^3b*d*f) * g - (125b^4c^3 + 64a*b^3d^3 - 150a*b^3c^2f + 60a^2b^2c*f^2 - 8a^3b*f^3 - 48a^2b^2d^2g + 12a^3b*d*g^2 - a^4g^3) * x) + (162b^2e*x^5 + 162a*b*e*x^2 - (a^2b^2*x^5 + a^3b*x^2) * ((-I\sqrt{3} + 1) * (9e^2/a^4 - (20b^2cd + 2a^2f*g + (9e^2 - 8d*f - 5c*g) * a*b)) / (a^5*b)) / (-1/27e^3/a^6 + 1/162 * (20b^2cd + 2a^2f*g + (9e^2 - 8d*f - 5c*g) * a*b) * e / (a^7*b) - 1/1458 * (125b^4c^3 + 64a*b^3d^3 - 150a*b^3c^2f + 60a^2b^2c*f^2 - 8a^3b*f^3 - 48a^2b^2d^2g + 12a^3b*d*g^2 - a^4g^3) / (a^8b^2) - 1/1458 * (125b^4c^3 + a^4g^3 - 2*(4f^3 - 9e*f*g + 6d*g^2) * a^3b + 3*(9e^3 - 24d*e*f + 16d^2g + 5*(4f^2 - 3e*g) * c) * a^2b^2 - 2*(32d^3 - 90c*d*e + 75c^2f) * a*b^3) / (a^8b^2))^(1/3) + 81 * (I\sqrt{3} + 1) * (-1/27e^3/a^6 + 1/162 * (20b^2cd + 2a^2f*g + (9e^2 - 8d*f - 5c*g) * a*b) * e / (a^7*b) - 1/1458 * (125b^4c^3 + 64a*b^3d^3 - 150a*b^3c^2f + 60a^2b^2c*f^2 - 8a^3b*f^3 - 48a^2b^2d^2g + 12a^3b*d*g^2 - a^4g^3) / (a^8b^2) - 1/1458 * (125b^4c^3 + a^4g^3 - 2*(4f^3 - 9e*f*g + 6d*g^2) * a^3b + 3*(9e^3 - 24d*e*f + 16d^2g + 5*(4f^2 - 3e*g) * c) * a^2b^2 - 2*(32d^3 - 90c*d*e + 75c^2f) * a*b^3) / (a^8b^2))^(1/3) + 54 * e / a^2) - 3 * \\
& \sqrt{1/3} * (a^2b^2*x^5 + a^3b*x^2) * \sqrt{-(((-I\sqrt{3} + 1) * (9e^2/a^4 - (20b^2cd + 2a^2f*g + (9e^2 - 8d*f - 5c*g) * a*b)) / (a^5*b)) / (-1/27e^3/a^6 + 1/162 * (20b^2cd + 2a^2f*g + (9e^2 - 8d*f - 5c*g) * a*b) * e / (a^7*b) - 1/1458 * (125b^4c^3 + 64a*b^3d^3 - 150a*b^3c^2f + 60a^2b^2c*f^2 - 8a^3b*f^3 - 48a^2b^2d^2g + 12a^3b*d*g^2 - a^4g^3) / (a^8b^2) - 1/1458 * (125b^4c^3 + a^4g^3 - 2*(4f^3 - 9e*f*g + 6d*g^2) * a^3b + 3*(9e^3 - 24d*e*f + 16d^2g + 5*(4f^2 - 3e*g) * c) * a^2b^2 - 2*(32d^3 - 90c*d*e + 75c^2f) * a*b^3) / (a^8b^2))^(1/3) + 81 * (I\sqrt{3} + 1) * (-1/27e^3/a^6 + 1/162 * (20b^2cd + 2a^2f*g + (9e^2 - 8d*f - 5c*g) * a*b) * e / (a^7*b) - 1/1458 * (125b^4c^3 + 64a*b^3d^3 - 150a*b^3c^2f + 60a^2b^2c*f^2 - 8a^3b*f^3 - 48a^2b^2d^2g + 12a^3b*d*g^2 - a^4g^3) / (a^8b^2) - 1/1458 * (125b^4c^3 + a^4g^3 - 2*(4f^3 - 9e*f*g + 6d*g^2) * a^3b + 3*(9e^3 - 24d*e*f + 16d^2g + 5*(4f^2 - 3e*g) * c) * a^2b^2 - 2*(32d^3 - 90c*d*e + 75c^2f) * a*b^3) / (a^8b^2))^(1/3) + 54 * e / a^2)^2 * a^5 * b - 108 * ((-I\sqrt{3} + 1) * (9e^2/a^4 - (20b^2cd + 2a^2f*g + (9e^2 - 8d*f - 5c*g) * a*b)) / (a
\end{aligned}$$

$$\begin{aligned}
& ^5b)) / (-1/27 * e^3 / a^6 + 1/162 * (20 * b^2 * c * d + 2 * a^2 * f * g + (9 * e^2 - 8 * d * f - 5 * c * g) * a * b) * e / (a^7 * b) - 1/1458 * (125 * b^4 * c^3 + 64 * a * b^3 * d^3 - 150 * a * b^3 * c^2 * f \\
& + 60 * a^2 * b^2 * c * f^2 - 8 * a^3 * b * f^3 - 48 * a^2 * b^2 * d^2 * g + 12 * a^3 * b * d * g^2 - a^4 * g^3) / (a^8 * b^2) - 1/1458 * (125 * b^4 * c^3 + a^4 * g^3 - 2 * (4 * f^3 - 9 * e * f * g + 6 * d * g \\
& ^2) * a^3 * b + 3 * (9 * e^3 - 24 * d * e * f + 16 * d^2 * g + 5 * (4 * f^2 - 3 * e * g) * c) * a^2 * b^2 - 2 * (32 * d^3 - 90 * c * d * e + 75 * c^2 * f) * a * b^3) / (a^8 * b^2))^{(1/3)} + 81 * (I * \text{sqrt}(3) + \\
& 1) * (-1/27 * e^3 / a^6 + 1/162 * (20 * b^2 * c * d + 2 * a^2 * f * g + (9 * e^2 - 8 * d * f - 5 * c * g) * a * b) * e / (a^7 * b) - 1/1458 * (125 * b^4 * c^3 + 64 * a * b^3 * d^3 - 150 * a * b^3 * c^2 * f + 6 \\
& 0 * a^2 * b^2 * c * f^2 - 8 * a^3 * b * f^3 - 48 * a^2 * b^2 * d^2 * g + 12 * a^3 * b * d * g^2 - a^4 * g^3) / (a^8 * b^2) - 1/1458 * (125 * b^4 * c^3 + a^4 * g^3 - 2 * (4 * f^3 - 9 * e * f * g + 6 * d * g^2) \\
&) * a^3 * b + 3 * (9 * e^3 - 24 * d * e * f + 16 * d^2 * g + 5 * (4 * f^2 - 3 * e * g) * c) * a^2 * b^2 - 2 * (32 * d^3 - 90 * c * d * e + 75 * c^2 * f) * a * b^3) / (a^8 * b^2))^{(1/3)} + 54 * e / a^2) * a^3 * b * e \\
& + 25920 * b^2 * c * d + 2916 * a * b * e^2 - 10368 * a * b * d * f - 1296 * (5 * a * b * c - 2 * a^2 * f) * g) / (a^5 * b)) * \log(160 * a * b^3 * c * d^2 - 75 * a * b^3 * c^2 * e + 36 * a^2 * b^2 * d * e^2 - 12 * a^ \\
& 3 * b * e * f^2 + 1/324 * (4 * a^6 * b^2 * d - a^7 * b * g) * ((-I * \text{sqrt}(3) + 1) * (9 * e^2 / a^4 - (2 \\
& 0 * b^2 * c * d + 2 * a^2 * f * g + (9 * e^2 - 8 * d * f - 5 * c * g) * a * b) / (a^5 * b)) / (-1/27 * e^3 / a^ \\
& 6 + 1/162 * (20 * b^2 * c * d + 2 * a^2 * f * g + (9 * e^2 - 8 * d * f - 5 * c * g) * a * b) * e / (a^7 * b) \\
& - 1/1458 * (125 * b^4 * c^3 + 64 * a * b^3 * d^3 - 150 * a * b^3 * c^2 * f + 60 * a^2 * b^2 * c * f^2 - \\
& 8 * a^3 * b * f^3 - 48 * a^2 * b^2 * d^2 * g + 12 * a^3 * b * d * g^2 - a^4 * g^3) / (a^8 * b^2) - 1/1 \\
& 458 * (125 * b^4 * c^3 + a^4 * g^3 - 2 * (4 * f^3 - 9 * e * f * g + 6 * d * g^2) * a^3 * b + 3 * (9 * e^3 \\
& - 24 * d * e * f + 16 * d^2 * g + 5 * (4 * f^2 - 3 * e * g) * c) * a^2 * b^2 - 2 * (32 * d^3 - 90 * c * d * \\
& e + 75 * c^2 * f) * a * b^3) / (a^8 * b^2))^{(1/3)} + 81 * (I * \text{sqrt}(3) + 1) * (-1/27 * e^3 / a^6 + \\
& 1/162 * (20 * b^2 * c * d + 2 * a^2 * f * g + (9 * e^2 - 8 * d * f - 5 * c * g) * a * b) * e / (a^7 * b) - 1 \\
& /1458 * (125 * b^4 * c^3 + 64 * a * b^3 * d^3 - 150 * a * b^3 * c^2 * f + 60 * a^2 * b^2 * c * f^2 - 8 * \\
& a^3 * b * f^3 - 48 * a^2 * b^2 * d^2 * g + 12 * a^3 * b * d * g^2 - a^4 * g^3) / (a^8 * b^2) - 1/1458 \\
& * (125 * b^4 * c^3 + a^4 * g^3 - 2 * (4 * f^3 - 9 * e * f * g + 6 * d * g^2) * a^3 * b + 3 * (9 * e^3 - \\
& 24 * d * e * f + 16 * d^2 * g + 5 * (4 * f^2 - 3 * e * g) * c) * a^2 * b^2 - 2 * (32 * d^3 - 90 * c * d * e + \\
& 75 * c^2 * f) * a * b^3) / (a^8 * b^2))^{(1/3)} + 54 * e / a^2)^2 + 2 * (5 * a^3 * b * c - 2 * a^4 * f) * \\
& g^2 + 1/18 * (25 * a^3 * b^3 * c^2 - 24 * a^4 * b^2 * d * e - 20 * a^4 * b^2 * c * f + 4 * a^5 * b * f^2 \\
& + 6 * a^5 * b * e * g) * ((-I * \text{sqrt}(3) + 1) * (9 * e^2 / a^4 - (20 * b^2 * c * d + 2 * a^2 * f * g + (9 * \\
& e^2 - 8 * d * f - 5 * c * g) * a * b) / (a^5 * b)) / (-1/27 * e^3 / a^6 + 1/162 * (20 * b^2 * c * d + 2 * a \\
& ^2 * f * g + (9 * e^2 - 8 * d * f - 5 * c * g) * a * b) * e / (a^7 * b) - 1/1458 * (125 * b^4 * c^3 + 64 * \\
& a * b^3 * d^3 - 150 * a * b^3 * c^2 * f + 60 * a^2 * b^2 * c * f^2 - 8 * a^3 * b * f^3 - 48 * a^2 * b^2 * d \\
& ^2 * g + 12 * a^3 * b * d * g^2 - a^4 * g^3) / (a^8 * b^2) - 1/1458 * (125 * b^4 * c^3 + a^4 * g^3 - 2 \\
& * (4 * f^3 - 9 * e * f * g + 6 * d * g^2) * a^3 * b + 3 * (9 * e^3 - 24 * d * e * f + 16 * d^2 * g + 5 * \\
& (4 * f^2 - 3 * e * g) * c) * a^2 * b^2 - 2 * (32 * d^3 - 90 * c * d * e + 75 * c^2 * f) * a * b^3) / (a^8 * b \\
& ^2))^{(1/3)} + 81 * (I * \text{sqrt}(3) + 1) * (-1/27 * e^3 / a^6 + 1/162 * (20 * b^2 * c * d + 2 * a^2 * \\
& f * g + (9 * e^2 - 8 * d * f - 5 * c * g) * a * b) * e / (a^7 * b) - 1/1458 * (125 * b^4 * c^3 + 64 * a * b \\
& ^3 * d^3 - 150 * a * b^3 * c^2 * f + 60 * a^2 * b^2 * c * f^2 - 8 * a^3 * b * f^3 - 48 * a^2 * b^2 * d^2 * \\
& g + 12 * a^3 * b * d * g^2 - a^4 * g^3) / (a^8 * b^2) - 1/1458 * (125 * b^4 * c^3 + a^4 * g^3 - 2 \\
& * (4 * f^3 - 9 * e * f * g + 6 * d * g^2) * a^3 * b + 3 * (9 * e^3 - 24 * d * e * f + 16 * d^2 * g + 5 * (4 * \\
& f^2 - 3 * e * g) * c) * a^2 * b^2 - 2 * (32 * d^3 - 90 * c * d * e + 75 * c^2 * f) * a * b^3) / (a^8 * b^2) \\
&)^{(1/3)} + 54 * e / a^2) - 4 * (16 * a^2 * b^2 * d^2 - 15 * a^2 * b^2 * c * e) * f - (80 * a^2 * b^2 * c \\
& * d + 9 * a^3 * b * e^2 - 32 * a^3 * b * d * f) * g - 2 * (125 * b^4 * c^3 + 64 * a * b^3 * d^3 - 150 * a * \\
& b^3 * c^2 * f + 60 * a^2 * b^2 * c * f^2 - 8 * a^3 * b * f^3 - 48 * a^2 * b^2 * d^2 * g + 12 * a^3 * b * d *
\end{aligned}$$

$$\begin{aligned}
& g^2 - a^4 g^3) x + 1/108 \sqrt{1/3} (450 a^3 b^3 c^2 + 216 a^4 b^2 d e - 360 \\
& a^4 b^2 c f + 72 a^5 b f^2 - 54 a^5 b e g - (4 a^6 b^2 d - a^7 b g) ((-I \sqrt{3} \\
& \sqrt{3} + 1) (9 e^2 / a^4 - (20 b^2 c d + 2 a^2 f g + (9 e^2 - 8 d f - 5 c g) * \\
& a b) / (a^5 b)) / (-1/27 e^3 / a^6 + 1/162 (20 b^2 c d + 2 a^2 f g + (9 e^2 - 8 d \\
& f - 5 c g) * a b) * e / (a^7 b) - 1/1458 (125 b^4 c^3 + 64 a b^3 d^3 - 150 a b^3 \\
& c^2 f + 60 a^2 b^2 c f^2 - 8 a^3 b f^3 - 48 a^2 b^2 d^2 g + 12 a^3 b d g^2 \\
& - a^4 g^3) / (a^8 b^2) - 1/1458 (125 b^4 c^3 + a^4 g^3 - 2 (4 f^3 - 9 e f g \\
& + 6 d g^2) * a^3 b + 3 (9 e^3 - 24 d e f + 16 d^2 g + 5 (4 f^2 - 3 e g) * c) * a^2 \\
& b^2 - 2 (32 d^3 - 90 c d e + 75 c^2 f) * a b^3) / (a^8 b^2))^{1/3} + 81 (I \sqrt{3} \\
& \sqrt{3} + 1) (-1/27 e^3 / a^6 + 1/162 (20 b^2 c d + 2 a^2 f g + (9 e^2 - 8 d f \\
& - 5 c g) * a b) * e / (a^7 b) - 1/1458 (125 b^4 c^3 + 64 a b^3 d^3 - 150 a b^3 c^2 \\
& f + 60 a^2 b^2 c f^2 - 8 a^3 b f^3 - 48 a^2 b^2 d^2 g + 12 a^3 b d g^2 - \\
& a^4 g^3) / (a^8 b^2) - 1/1458 (125 b^4 c^3 + a^4 g^3 - 2 (4 f^3 - 9 e f g + 6 \\
& d g^2) * a^3 b + 3 (9 e^3 - 24 d e f + 16 d^2 g + 5 (4 f^2 - 3 e g) * c) * a^2 b \\
& ^2 - 2 (32 d^3 - 90 c d e + 75 c^2 f) * a b^3) / (a^8 b^2))^{1/3} + 54 e / a^2) * \\
& \sqrt{-(((-I \sqrt{3} + 1) (9 e^2 / a^4 - (20 b^2 c d + 2 a^2 f g + (9 e^2 - 8 d \\
& f - 5 c g) * a b) / (a^5 b)) / (-1/27 e^3 / a^6 + 1/162 (20 b^2 c d + 2 a^2 f g + \\
& (9 e^2 - 8 d f - 5 c g) * a b) * e / (a^7 b) - 1/1458 (125 b^4 c^3 + 64 a b^3 d^3 \\
& - 150 a b^3 c^2 f + 60 a^2 b^2 c f^2 - 8 a^3 b f^3 - 48 a^2 b^2 d^2 g + 1 \\
& 2 a^3 b d g^2 - a^4 g^3) / (a^8 b^2) - 1/1458 (125 b^4 c^3 + a^4 g^3 - 2 (4 f^3 \\
& - 9 e f g + 6 d g^2) * a^3 b + 3 (9 e^3 - 24 d e f + 16 d^2 g + 5 (4 f^2 - 3 \\
& e g) * c) * a^2 b^2 - 2 (32 d^3 - 90 c d e + 75 c^2 f) * a b^3) / (a^8 b^2))^{1/3} \\
& + 81 (I \sqrt{3} + 1) (-1/27 e^3 / a^6 + 1/162 (20 b^2 c d + 2 a^2 f g + (9 \\
& e^2 - 8 d f - 5 c g) * a b) * e / (a^7 b) - 1/1458 (125 b^4 c^3 + 64 a b^3 d^3 - \\
& 150 a b^3 c^2 f + 60 a^2 b^2 c f^2 - 8 a^3 b f^3 - 48 a^2 b^2 d^2 g + 12 a \\
& ^3 b d g^2 - a^4 g^3) / (a^8 b^2) - 1/1458 (125 b^4 c^3 + a^4 g^3 - 2 (4 f^3 \\
& - 9 e f g + 6 d g^2) * a^3 b + 3 (9 e^3 - 24 d e f + 16 d^2 g + 5 (4 f^2 - 3 \\
& e g) * c) * a^2 b^2 - 2 (32 d^3 - 90 c d e + 75 c^2 f) * a b^3) / (a^8 b^2))^{1/3} \\
& + 54 e / a^2)^2 a^5 b - 108 ((-I \sqrt{3} + 1) (9 e^2 / a^4 - (20 b^2 c d + 2 a^2 \\
& f g + (9 e^2 - 8 d f - 5 c g) * a b) / (a^5 b)) / (-1/27 e^3 / a^6 + 1/162 (20 b^2 \\
& c d + 2 a^2 f g + (9 e^2 - 8 d f - 5 c g) * a b) * e / (a^7 b) - 1/1458 (125 b^4 \\
& c^3 + 64 a b^3 d^3 - 150 a b^3 c^2 f + 60 a^2 b^2 c f^2 - 8 a^3 b f^3 - 4 \\
& 8 a^2 b^2 d^2 g + 12 a^3 b d g^2 - a^4 g^3) / (a^8 b^2) - 1/1458 (125 b^4 c^3 \\
& + a^4 g^3 - 2 (4 f^3 - 9 e f g + 6 d g^2) * a^3 b + 3 (9 e^3 - 24 d e f + 16 \\
& d^2 g + 5 (4 f^2 - 3 e g) * c) * a^2 b^2 - 2 (32 d^3 - 90 c d e + 75 c^2 f) * a \\
& b^3) / (a^8 b^2))^{1/3} + 81 (I \sqrt{3} + 1) (-1/27 e^3 / a^6 + 1/162 (20 b^2 c \\
& d + 2 a^2 f g + (9 e^2 - 8 d f - 5 c g) * a b) * e / (a^7 b) - 1/1458 (125 b^4 c \\
& ^3 + 64 a b^3 d^3 - 150 a b^3 c^2 f + 60 a^2 b^2 c f^2 - 8 a^3 b f^3 - 48 a \\
& ^2 b^2 d^2 g + 12 a^3 b d g^2 - a^4 g^3) / (a^8 b^2) - 1/1458 (125 b^4 c^3 + \\
& a^4 g^3 - 2 (4 f^3 - 9 e f g + 6 d g^2) * a^3 b + 3 (9 e^3 - 24 d e f + 16 d^2 \\
& g + 5 (4 f^2 - 3 e g) * c) * a^2 b^2 - 2 (32 d^3 - 90 c d e + 75 c^2 f) * a b^3 \\
&) / (a^8 b^2))^{1/3} + 54 e / a^2) * a^3 b e + 25920 b^2 c d + 2916 a b e^2 - 103 \\
& 68 a b d f - 1296 (5 a b c - 2 a^2 f) g) / (a^5 b)) + (162 b^2 e x^5 + 162 a \\
& b e x^2 - (a^2 b^2 x^5 + a^3 b x^2) * ((-I \sqrt{3} + 1) (9 e^2 / a^4 - (20 b^2 \\
& c d + 2 a^2 f g + (9 e^2 - 8 d f - 5 c g) * a b) / (a^5 b)) / (-1/27 e^3 / a^6 + 1
\end{aligned}$$

$$\begin{aligned}
& /162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2) + 3*sqrt(1/3)*(a^2*b^2*x^5 + a^3*b*x^2)*sqrt(-(((I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b)))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2)^2*a^5*b - 108*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b)))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2)*a^3*b*e + 25920*b^2*c*d + 2916*a*b*e^2 - 10368*a*b*d*f - 1296*(5*a*b*c - 2*a^2*f)*g)/(a^5*b))*log(160*a*b^3*c*d^2 - 75*a*b^3*c^2*e + 36*a^2*b^2*d*e^2 - 12*a^3*b*e*f^2 + 1/324*(4*a^6*b^2*d - a^7*b*g)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b)))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2)
\end{aligned}$$

$$\begin{aligned}
& *g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - \\
& 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 \\
& - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2 \\
&))^{(1/3)} + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g \\
& + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3 \\
& *d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g \\
& + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(\\
& 4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 \\
& - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^{(\\
& 1/3)} + 54*e/a^2)^2 + 2*(5*a^3*b*c - 2*a^4*f)*g^2 + 1/18*(25*a^3*b^3*c^2 - \\
& 24*a^4*b^2*d*e - 20*a^4*b^2*c*f + 4*a^5*b*f^2 + 6*a^5*b*e*g)*((-I*sqrt(3) + \\
& 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b)) \\
&)/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c \\
& *g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + \\
& 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g \\
& ^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2) \\
&)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - \\
& 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^{(1/3)} + 81*(I*sqrt(3) + \\
& 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g) \\
&)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60 \\
& *a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3) \\
&)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)* \\
& a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(\\
& 32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^{(1/3)} + 54*e/a^2) - 4*(16*a \\
& ^2*b^2*d^2 - 15*a^2*b^2*c*e)*f - (80*a^2*b^2*c*d + 9*a^3*b*e^2 - 32*a^3*b*d \\
& *f)*g - 2*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 \\
& - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)*x - 1/108*sqrt \\
& (1/3)*(450*a^3*b^3*c^2 + 216*a^4*b^2*d*e - 360*a^4*b^2*c*f + 72*a^5*b*f^2 - \\
& 54*a^5*b*e*g - (4*a^6*b^2*d - a^7*b*g)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20* \\
& b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b)))/(-1/27*e^3/a^6 \\
& + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - \\
& 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8 \\
& *a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/145 \\
& 8*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - \\
& 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e \\
& + 75*c^2*f)*a*b^3)/(a^8*b^2))^{(1/3)} + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1 \\
& /162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/ \\
& 1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^ \\
& 3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(\\
& 125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24 \\
& *d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 7 \\
& 5*c^2*f)*a*b^3)/(a^8*b^2))^{(1/3)} + 54*e/a^2))*sqrt(-(((I*sqrt(3) + 1)*(9*e \\
& ^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b)))/(- \\
& 1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b) \\
&)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*
\end{aligned}$$

$$\begin{aligned}
& b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8 \\
& *b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b \\
& + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 \\
& 3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^{(1/3)} + 81*(I*\sqrt{3} + 1)*(-1/2 \\
& 7*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/ \\
& (a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2 \\
& *c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2 \\
& 2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + \\
& 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - \\
& 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^{(1/3)} + 54*e/a^2)^2*a^5*b - 108*((- \\
& I*\sqrt{3} + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c* \\
& g)*a*b)/(a^5*b)))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - \\
& 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a* \\
& b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d* \\
& g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f* \\
& g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c) \\
& *a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^{(1/3)} + 81*(I \\
& *\sqrt{3} + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d \\
& *f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3 \\
& *c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 \\
& - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g \\
& + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^ \\
& 2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^{(1/3)} + 54*e/a^2 \\
&)*a^3*b*e + 25920*b^2*c*d + 2916*a*b*e^2 - 10368*a*b*d*f - 1296*(5*a*b*c - \\
& 2*a^2*f)*g)/(a^5*b))) - 324*(b^2*e*x^5 + a*b*e*x^2)*log(x))/(a^2*b^2*x^5 + \\
& a^3*b*x^2)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3/(b*x**3+a)**2,x)

[Out] Timed out

Giac [A] time = 1.07838, size = 483, normalized size = 1.58

$$-\frac{e \log(|bx^3 + a|)}{3a^2} + \frac{e \log(|x|)}{a^2} - \frac{\sqrt{3} \left(5 (-ab^2)^{\frac{1}{3}} b^2 c - 2 (-ab^2)^{\frac{1}{3}} abf - 4 (-ab^2)^{\frac{2}{3}} bd + (-ab^2)^{\frac{2}{3}} ag \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}} \right)}{9a^3 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="giac")

[Out] -1/3*e*log(abs(b*x^3 + a))/a^2 + e*log(abs(x))/a^2 - 1/9*sqrt(3)*(5*(-a*b^2)^(1/3)*b^2*c - 2*(-a*b^2)^(1/3)*a*b*f - 4*(-a*b^2)^(2/3)*b*d + (-a*b^2)^(2/3)*a*g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^2) - 1/18*(5*(-a*b^2)^(1/3)*b^2*c - 2*(-a*b^2)^(1/3)*a*b*f + 4*(-a*b^2)^(2/3)*b*d - (-a*b^2)^(2/3)*a*g)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^2) + 1/9*(4*a^2*b^2*d*(-a/b)^(1/3) - a^3*b*g*(-a/b)^(1/3) + 5*a^2*b^2*c - 2*a^3*b*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^5*b) - 1/6*(2*(4*b^2*d - a*b*g)*x^4 + 6*a*b*d*x + (5*b^2*c - 2*a*b*f)*x^3 + 3*a*b*c + 2*(a^2*h - a*b*e)*x^2)/((b*x^3 + a)*a^2*b*x^2)

$$3.420 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)^2} dx$$

Optimal. Leaf size=338

$$\frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2) \left(\sqrt[3]{b}(5bd - 2ag) - \sqrt[3]{a}(4be - ah) \right)}{18a^{8/3}b^{2/3}} - \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx}) \left(\sqrt[3]{b}(5bd - 2ag) - \sqrt[3]{a}(4be - ah) \right)}{9a^{8/3}b^{2/3}} + \tan^{-1} \left(\frac{\sqrt[3]{a(4be - ah)}}{\sqrt[3]{b}} - 2ag + 5bd \right)$$

[Out] $-c/(3*a^2*x^3) - d/(2*a^2*x^2) - e/(a^2*x) - (x*(b*d - a*g + (b*e - a*h)*x - b*((b*c)/a - f)*x^2))/(3*a^2*(a + b*x^3)) + ((5*b^(4/3)*d + 4*a^(1/3)*b*e - 2*a*b^(1/3)*g - a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(8/3)*b^(2/3)) - ((2*b*c - a*f)*Log[x])/a^3 - ((b^(1/3)*(5*b*d - 2*a*g) - a^(1/3)*(4*b*e - a*h))*Log[a^(1/3) + b^(1/3)*x]/(9*a^(8/3)*b^(2/3)) + ((b^(1/3)*(5*b*d - 2*a*g) - a^(1/3)*(4*b*e - a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(18*a^(8/3)*b^(2/3)) + ((2*b*c - a*f)*Log[a + b*x^3]/(3*a^3))$

Rubi [A] time = 0.727317, antiderivative size = 336, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2) \left(-\frac{\sqrt[3]{a(4be - ah)}}{\sqrt[3]{b}} - 2ag + 5bd \right)}{18a^{8/3}\sqrt[3]{b}} - \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx}) \left(\sqrt[3]{b}(5bd - 2ag) - \sqrt[3]{a}(4be - ah) \right)}{9a^{8/3}b^{2/3}} + \tan^{-1} \left(\frac{\sqrt[3]{a(4be - ah)}}{\sqrt[3]{b}} - 2ag + 5bd \right)$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^2), x]

[Out] $-c/(3*a^2*x^3) - d/(2*a^2*x^2) - e/(a^2*x) - (x*(b*d - a*g + (b*e - a*h)*x - b*((b*c)/a - f)*x^2))/(3*a^2*(a + b*x^3)) + ((5*b^(4/3)*d + 4*a^(1/3)*b*e - 2*a*b^(1/3)*g - a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(8/3)*b^(2/3)) - ((2*b*c - a*f)*Log[x])/a^3 - ((b^(1/3)*(5*b*d - 2*a*g) - a^(1/3)*(4*b*e - a*h))*Log[a^(1/3) + b^(1/3)*x]/(9*a^(8/3)*b^(2/3)) + ((5*b*d - 2*a*g - (a^(1/3)*(4*b*e - a*h))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(18*a^(8/3)*b^(1/3)) + ((2*b*c - a*f)*Log[a + b*x^3]/(3*a^3))$

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Int[Ex
pandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[A*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[A*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617


```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{x^4(a + bx^3)^2} dx &= \frac{x \left(bd - ag + (be - ah)x - b \left(\frac{bc}{a} - f \right) x^2 \right)}{3a^2(a + bx^3)} - \frac{\int \frac{-3b^2c - 3b^2dx - 3b^2ex^2 + 3b^2 \left(\frac{bc}{a} - f \right) x^3 + 2b^2 \left(\frac{bd}{a} - \frac{bc}{a} \right) x^4}{x^4(a + bx^3)} dx}{3ab^2} \\
&= \frac{x \left(bd - ag + (be - ah)x - b \left(\frac{bc}{a} - f \right) x^2 \right)}{3a^2(a + bx^3)} - \int \left(\frac{3b^2c}{ax^4} - \frac{3b^2d}{ax^3} - \frac{3b^2e}{ax^2} - \frac{3b^2(-2bc+af)}{a^2x} \right) dx \\
&= -\frac{c}{3a^2x^3} - \frac{d}{2a^2x^2} - \frac{e}{a^2x} - \frac{x \left(bd - ag + (be - ah)x - b \left(\frac{bc}{a} - f \right) x^2 \right)}{3a^2(a + bx^3)} - \frac{(2bc - af) \log(a + bx^3)}{a^3} \\
&= -\frac{c}{3a^2x^3} - \frac{d}{2a^2x^2} - \frac{e}{a^2x} - \frac{x \left(bd - ag + (be - ah)x - b \left(\frac{bc}{a} - f \right) x^2 \right)}{3a^2(a + bx^3)} - \frac{(2bc - af) \log(a + bx^3)}{a^3} \\
&= -\frac{c}{3a^2x^3} - \frac{d}{2a^2x^2} - \frac{e}{a^2x} - \frac{x \left(bd - ag + (be - ah)x - b \left(\frac{bc}{a} - f \right) x^2 \right)}{3a^2(a + bx^3)} - \frac{(2bc - af) \log(a + bx^3)}{a^3} \\
&= -\frac{c}{3a^2x^3} - \frac{d}{2a^2x^2} - \frac{e}{a^2x} - \frac{x \left(bd - ag + (be - ah)x - b \left(\frac{bc}{a} - f \right) x^2 \right)}{3a^2(a + bx^3)} - \frac{(2bc - af) \log(a + bx^3)}{a^3} \\
&= -\frac{c}{3a^2x^3} - \frac{d}{2a^2x^2} - \frac{e}{a^2x} - \frac{x \left(bd - ag + (be - ah)x - b \left(\frac{bc}{a} - f \right) x^2 \right)}{3a^2(a + bx^3)} - \frac{(2bc - af) \log(a + bx^3)}{a^3} \\
&= -\frac{c}{3a^2x^3} - \frac{d}{2a^2x^2} - \frac{e}{a^2x} - \frac{x \left(bd - ag + (be - ah)x - b \left(\frac{bc}{a} - f \right) x^2 \right)}{3a^2(a + bx^3)} + \frac{(5b^{4/3}d + 4b^{1/3}e) \log(a + bx^3)}{18a^3}
\end{aligned}$$

Mathematica [A] time = 0.454282, size = 303, normalized size = 0.9

$$\frac{\sqrt[3]{a} \log\left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2\right) \left(a^{4/3} h - 4 \sqrt[3]{abe} - 2a \sqrt[3]{bg} + 5b^{4/3} d\right)}{b^{2/3}} - \frac{2 \sqrt[3]{a} \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right) \left(a^{4/3} h - 4 \sqrt[3]{abe} - 2a \sqrt[3]{bg} + 5b^{4/3} d\right)}{b^{2/3}} - \frac{2 \sqrt{3} \sqrt[3]{a} \tan^{-1}\left(\frac{1 - 2 \sqrt[3]{bx}}{\sqrt{3}}\right) \left(a^{4/3} h - 4 \sqrt[3]{abe} - 2a \sqrt[3]{bg} + 5b^{4/3} d\right)}{18a^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^2), x]

```
[Out] ((-6*a*c)/x^3 - (9*a*d)/x^2 - (18*a*e)/x + (a*(-6*b*(c + x*(d + e*x)) + 6*a*(f + x*(g + h*x))))/(a + b*x^3) - (2*sqrt[3]*a^(1/3)*(-5*b^(4/3)*d - 4*a^(1/3)*b*e + 2*a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]]/b^(2/3) + 18*(-2*b*c + a*f)*Log[x] - (2*a^(1/3)*(5*b^(4/3)*d - 4*a^(1/3)*b*e - 2*a*b^(1/3)*g + a^(4/3)*h)*Log[a^(1/3) + b^(1/3)*x]/b^(2/3) + (a^(1/3)*(5*b^(4/3)*d - 4*a^(1/3)*b*e - 2*a*b^(1/3)*g + a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/b^(2/3) + 6*(2*b*c - a*f)*Log[a + b*x^3]/(18*a^3)
```

Maple [B] time = 0.014, size = 561, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x)
```

```
[Out] -1/3/a^2*b*x/(b*x^3+a)*d+4/9/a^2*e/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-2/9/a^2*e/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-1/2*d/a^2/x^2-e/a^2/x+2/9/a*g/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/3/a^2*x^2/(b*x^3+a)*b*e-1/3/a^2*ln(b*x^3+a)*f+1/3/a/(b*x^3+a)*f+1/a^2*ln(x)*f+1/3/a/(b*x^3+a)*x^2*h+1/3/a/(b*x^3+a)*g*x-1/3*c/x^3/a^2-1/9/a*g/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-1/9/a*h/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/18/a*h/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-5/9/a^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d+1/9/a*h*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+2/9/a*g/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-4/9/a^2*e*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-5/9/a^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d+5/18/a^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d-1/3/a^2*b/(b*x^3+a)*c-2*b*c*ln(x)/a^3+2/3*b*c*ln(b*x^3+a)/a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4/(b*x**3+a)**2,x)

[Out] Timed out

Giac [A] time = 1.09594, size = 520, normalized size = 1.54

$$\frac{(2bc - af) \log(|bx^3 + a|)}{3a^3} - \frac{(2bc - af) \log(|x|)}{a^3} - \frac{\sqrt{3} \left(5(-ab^2)^{\frac{1}{3}} b^2 d - 2(-ab^2)^{\frac{1}{3}} abg + (-ab^2)^{\frac{2}{3}} ah - 4(-ab^2)^{\frac{2}{3}} be \right) \arcsin\left(\frac{x}{\sqrt{bx^3 + a}}\right)}{9a^3 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="giac")

[Out] 1/3*(2*b*c - a*f)*log(abs(b*x^3 + a))/a^3 - (2*b*c - a*f)*log(abs(x))/a^3 - 1/9*sqrt(3)*(5*(-a*b^2)^(1/3)*b^2*d - 2*(-a*b^2)^(1/3)*a*b*g + (-a*b^2)^(2/3)*a*h - 4*(-a*b^2)^(2/3)*b*e)*arcsin(x/sqrt(b*x^3 + a))/9*a^3*b^2

$$\begin{aligned}
& /3)*a*h - 4*(-a*b^2)^{(2/3)}*b*e)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a \\
& /b)^{(1/3)})/(a^3*b^2) - 1/18*(5*(-a*b^2)^{(1/3)}*b^2*d - 2*(-a*b^2)^{(1/3)}*a*b* \\
& g - (-a*b^2)^{(2/3)}*a*h + 4*(-a*b^2)^{(2/3)}*b*e)*\log(x^2 + x*(-a/b)^{(1/3)} + (\\
& -a/b)^{(2/3)})/(a^3*b^2) - 1/9*(a^5*b*h*(-a/b)^{(1/3)} - 4*a^4*b^2*(-a/b)^{(1/3)} \\
& *e - 5*a^4*b^2*d + 2*a^5*b*g)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/(a^7* \\
& b) + 1/6*(2*(a^2*h - 4*a*b*e)*x^5 - (5*a*b*d - 2*a^2*g)*x^4 - 6*a^2*x^2*e - \\
& 3*a^2*d*x - 2*(2*a*b*c - a^2*f)*x^3 - 2*a^2*c)/((b*x^3 + a)*a^3*x^3)
\end{aligned}$$

$$3.421 \quad \int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=345

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(b^{2/3}(5af+bc) - 2a^{2/3}(be-7ah)\right)}{54a^{4/3}b^{10/3}} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(b^{2/3}(5af+bc) - 2a^{2/3}(be-7ah)\right)}{27a^{4/3}b^{10/3}} - \text{ta}$$

[Out] (h*x)/b^3 + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(6*b^3*(a + b*x^3)^2) - (x*(a*(7*b*e - 13*a*h) - 2*b*(b*c - 4*a*f)*x - 3*b*(b*d - 3*a*g)*x^2))/(18*a*b^3*(a + b*x^3)) - ((b^(5/3)*c + 2*a^(2/3)*b*e + 5*a*b^(2/3)*f - 14*a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(4/3)*b^(10/3)) - ((b^(2/3)*(b*c + 5*a*f) - 2*a^(2/3)*(b*e - 7*a*h))*Log[a^(1/3) + b^(1/3)*x]/(27*a^(4/3)*b^(10/3)) + ((b^(2/3)*(b*c + 5*a*f) - 2*a^(2/3)*(b*e - 7*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(54*a^(4/3)*b^(10/3)) + (g*Log[a + b*x^3])/(3*b^3)

Rubi [A] time = 0.891265, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.29$, Rules used = {1828, 1858, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(b^{2/3}(5af+bc) - 2a^{2/3}(be-7ah)\right)}{54a^{4/3}b^{10/3}} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(b^{2/3}(5af+bc) - 2a^{2/3}(be-7ah)\right)}{27a^{4/3}b^{10/3}} - \text{ta}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]

[Out] (h*x)/b^3 + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(6*b^3*(a + b*x^3)^2) - (x*(a*(7*b*e - 13*a*h) - 2*b*(b*c - 4*a*f)*x - 3*b*(b*d - 3*a*g)*x^2))/(18*a*b^3*(a + b*x^3)) - ((b^(5/3)*c + 2*a^(2/3)*b*e + 5*a*b^(2/3)*f - 14*a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(4/3)*b^(10/3)) - ((b^(2/3)*(b*c + 5*a*f) - 2*a^(2/3)*(b*e - 7*a*h))*Log[a^(1/3) + b^(1/3)*x]/(27*a^(4/3)*b^(10/3)) + ((b^(2/3)*(b*c + 5*a*f) - 2*a^(2/3)*(b*e - 7*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(54*a^(4/3)*b^(10/3)) + (g*Log[a + b*x^3])/(3*b^3)

Rule 1828

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q =
m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
+ b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a +
b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(p_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_.) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4 (c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{(a + bx^3)^3} dx &= \frac{x (a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{6b^3 (a + bx^3)^2} - \int \frac{a^2 (be - ah) - 2ab(bc - af)x - 3ab(bd - ag)x^2}{6b^3 (a + bx^3)^3} dx \\
&= \frac{x (a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{6b^3 (a + bx^3)^2} - \frac{x (a(7be - 13ah) - 2b(bc - af)x - b(bd - ag)x^2)}{18ab^3 (a + bx^3)^3} \\
&= \frac{x (a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{6b^3 (a + bx^3)^2} - \frac{x (a(7be - 13ah) - 2b(bc - af)x - b(bd - ag)x^2)}{18ab^3 (a + bx^3)^3} \\
&= \frac{hx}{b^3} + \frac{x (a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{6b^3 (a + bx^3)^2} - \frac{x (a(7be - 13ah) - 2b(bc - af)x - b(bd - ag)x^2)}{18ab^3 (a + bx^3)^3} \\
&= \frac{hx}{b^3} + \frac{x (a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{6b^3 (a + bx^3)^2} - \frac{x (a(7be - 13ah) - 2b(bc - af)x - b(bd - ag)x^2)}{18ab^3 (a + bx^3)^3} \\
&= \frac{hx}{b^3} + \frac{x (a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{6b^3 (a + bx^3)^2} - \frac{x (a(7be - 13ah) - 2b(bc - af)x - b(bd - ag)x^2)}{18ab^3 (a + bx^3)^3} \\
&= \frac{hx}{b^3} + \frac{x (a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{6b^3 (a + bx^3)^2} - \frac{x (a(7be - 13ah) - 2b(bc - af)x - b(bd - ag)x^2)}{18ab^3 (a + bx^3)^3} \\
&= \frac{hx}{b^3} + \frac{x (a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{6b^3 (a + bx^3)^2} - \frac{x (a(7be - 13ah) - 2b(bc - af)x - b(bd - ag)x^2)}{18ab^3 (a + bx^3)^3} \\
&= \frac{hx}{b^3} + \frac{x (a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{6b^3 (a + bx^3)^2} - \frac{x (a(7be - 13ah) - 2b(bc - af)x - b(bd - ag)x^2)}{18ab^3 (a + bx^3)^3}
\end{aligned}$$

Mathematica [A] time = 0.305199, size = 342, normalized size = 0.99

$$\frac{9b^{2/3}(a^2(g+hx) - ab(d+x(e+fx)) + b^2cx^2)}{(a+bx^3)^2} + \frac{3b^{2/3}(a^2(12g+13hx) - ab(6d+x(7e+8fx)) + 2b^2cx^2)}{a(a+bx^3)} + \frac{\log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx+b^{2/3}x^2}) \left(-2a^{2/3}b^{4/3}e + 14a^{5/3} \sqrt[3]{bh} + 5abf + \dots \right)}{a^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]

[Out]
$$\frac{(54*b^{2/3}*h*x - (9*b^{2/3}*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e + f*x))))/(a + b*x^3)^2 + (3*b^{2/3}*(2*b^2*c*x^2 + a^2*(12*g + 13*h*x) - a*b*(6*d + x*(7*e + 8*f*x))))/(a*(a + b*x^3)) - (2*\sqrt{3}*(b^2*c + 2*a^{2/3}*b^{4/3}*e + 5*a*b*f - 14*a^{5/3}*b^{1/3}*h)*\text{ArcTan}[(1 - (2*b^{1/3}*x)/a^{1/3})/\sqrt{3}])/a^{4/3} - (2*(b^2*c - 2*a^{2/3}*b^{4/3}*e + 5*a*b*f + 14*a^{5/3}*b^{1/3}*h)*\text{Log}[a^{1/3} + b^{1/3}*x])/a^{4/3} + ((b^2*c - 2*a^{2/3}*b^{4/3}*e + 5*a*b*f + 14*a^{5/3}*b^{1/3}*h)*\text{Log}[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2])/a^{4/3} + 18*b^{2/3}*g*\text{Log}[a + b*x^3])/(54*b^{11/3})$$

Maple [B] time = 0.011, size = 619, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x)

[Out]
$$\begin{aligned} & -4/9/(b*x^3+a)^2/b*x^5*f-5/27/b^3/(1/b*a)^{1/3}*\ln(x+(1/b*a)^{1/3})*f+5/54/ \\ & b^3/(1/b*a)^{1/3}*\ln(x^2-(1/b*a)^{1/3}*x+(1/b*a)^{2/3})*f+13/18/b^2/(b*x^3+ \\ & a)^2*x^4*a*h+2/3/b^2/(b*x^3+a)^2*x^3*a*g+2/27/b^3/(1/b*a)^{2/3}*3^{1/2}*\text{arc} \\ & \text{tan}(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1))*e+5/9/b^3/(b*x^3+a)^2*a^2*h*x-14/27/ \\ & b^4*a/(1/b*a)^{2/3}*\ln(x+(1/b*a)^{1/3})*h-2/9/b^2/(b*x^3+a)^2*a*e*x+h*x/b^3 \\ & -1/3/b/(b*x^3+a)^2*d*x^3+1/2/b^3/(b*x^3+a)^2*a^2*g-1/6/b^2/(b*x^3+a)^2*a*d+ \\ & 5/27/b^3*3^{1/2}/(1/b*a)^{1/3}*\arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1))*f- \\ & 5/18*a/(b*x^3+a)^2/b^2*x^2*f+7/27/b^4*a/(1/b*a)^{2/3}*\ln(x^2-(1/b*a)^{1/3}* \\ & x+(1/b*a)^{2/3})*h-1/27/b^2/a/(1/b*a)^{1/3}*\ln(x+(1/b*a)^{1/3})*c+1/54/b^2/ \\ & a/(1/b*a)^{1/3}*\ln(x^2-(1/b*a)^{1/3}*x+(1/b*a)^{2/3})*c-14/27/b^4*a/(1/b*a) \\ & ^{2/3}*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1))*h+1/3*g*\ln(b*x^3+a \\ &)/b^3-7/18/b/(b*x^3+a)^2*x^4*e+2/27/b^3/(1/b*a)^{2/3}*\ln(x+(1/b*a)^{1/3})*e \\ & -1/27/b^3/(1/b*a)^{2/3}*\ln(x^2-(1/b*a)^{1/3}*x+(1/b*a)^{2/3})*e+1/9/(b*x^3+ \\ & a)^2/a*x^5*c-1/18/b/(b*x^3+a)^2*x^2*c+1/27/b^2/a*3^{1/2}/(1/b*a)^{1/3}*\arct \\ & \text{an}(1/3*3^{1/2}*(2/(1/b*a)^{1/3}*x-1))*c \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [C] time = 34.1865, size = 30286, normalized size = 87.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] $1/108*(108*a*b^2*h*x^7 + 12*(b^3*c - 4*a*b^2*f)*x^5 - 42*(a*b^2*e - 7*a^2*b*h)*x^4 - 18*a^2*b*d + 54*a^3*g - 36*(a*b^2*d - 2*a^2*b*g)*x^3 - 6*(a*b^2*c + 5*a^2*b*f)*x^2 - 2*(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3)*(2*(1/2)^(2/3)*(-I*\sqrt{3} + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)/(a^2*b^6)))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) + (1/2)^(1/3)*(I*\sqrt{3} + 1)*(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3)*log(-4*a*b^4*c^2*e - 40*a^2*b^3*c*e*f - 100*a^3*b^2*e*f^2 + 36*a^3*b^2*e^2*g + 1764*a^5*g*h^2 - 1/4*(a^3*b^8*c + 5*a^4*b^7*f)*(2*(1/2)^(2/3)*(-I*\sqrt{3} + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)/(a^2*b^6)))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) + (1/2)^(1/3)*(I*\sqrt{3} + 1)*(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2$

$$\begin{aligned}
& *f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e* \\
& h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - \\
& 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2 \\
& *h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^1 \\
& 0))^(1/3) - 18*g/b^3)^2 - 81*(a^3*b^2*c + 5*a^4*b*f)*g^2 + (2*a^3*b^5*e^2 - \\
& 28*a^4*b^4*e*h + 98*a^5*b^3*h^2 - 9*(a^3*b^5*c + 5*a^4*b^4*f)*g)*(2*(1/2)^ \\
& (2/3)*(-I*sqrt(3) + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2 \\
& *(5*e*f - 7*c*h)*a*b)/(a^2*b^6)))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - \\
& 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 \\
& + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h \\
& + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + \\
& 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270* \\
& e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^ \\
& 2*b^3)/(a^4*b^10))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(\\
& 2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b \\
& ^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^ \\
& 3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5* \\
& c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a \\
& ^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(\\
& 25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3) + 28*(a^2*b^3*c^ \\
& 2 + 10*a^3*b^2*c*f + 25*a^4*b*f^2 - 18*a^4*b*e*g)*h - (b^5*c^3 + 8*a^2*b^3* \\
& e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2 \\
& *h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)*x) - 24*(a^2*b*e - 7*a^3*h)*x + (54*a \\
& *b^2*g*x^6 + 108*a^2*b*g*x^3 + 54*a^3*g + (a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3* \\
& b^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 7 \\
& 0*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)/(a^2*b^6)))/(1458*g^3/b^9 - 27*(2*b^2*c* \\
& e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + \\
& 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168* \\
& a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15 \\
& *a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (\\
& 125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - \\
& 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458* \\
& g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/ \\
& (a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + \\
& 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4 \\
& *b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h \\
& + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 \\
& - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3) - \\
& 3*sqrt(1/3)*(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3)*sqrt(-((2*(1/2)^(2/3)*(-I \\
& *sqrt(3) + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - \\
& 7*c*h)*a*b)/(a^2*b^6)))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a \\
& ^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b \\
& ^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^ \\
& 4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5 \\
& *h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 1
\end{aligned}$$

$$\begin{aligned}
& 68e^{2h} + 378c*g*h)a^3b^2 - (8e^3 - 3(25f^2 - 18e*g)*c)a^2b^3)/(a \\
& ^4b^{10})^{(1/3)} + (1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*(1458g^3/b^9 - 27*(2b^2*c*e \\
& + (81g^2 - 70f*h)a^2 + 2*(5e*f - 7c*h)*a*b)*g/(a^2b^9) - (b^5c^3 + \\
& 8a^2b^3e^3 + 15a*b^4c^2*f + 75a^2b^3c*f^2 + 125a^3b^2*f^3 - 168a \\
& ^3b^2e^2*h + 1176a^4b*e*h^2 - 2744a^5h^3)/(a^4b^{10}) - (b^5c^3 + 15* \\
& a*b^4c^2*f + 2744a^5h^3 - 3*(243g^3 - 630f*g*h + 392e*h^2)*a^4*b + (1 \\
& 25f^3 - 270e*f*g + 168e^2*h + 378c*g*h)a^3b^2 - (8e^3 - 3(25f^2 - \\
& 18e*g)*c)a^2b^3)/(a^4b^{10})^{(1/3)} - 18g/b^3)^2*a^2b^6 + 36*(2*(1/2)^{(\\
& 2/3)}*(-I*\text{sqrt}(3) + 1)*(81g^2/b^6 - (2b^2*c*e + (81g^2 - 70f*h)a^2 + 2* \\
& (5e*f - 7c*h)*a*b)/(a^2b^6)))/(1458g^3/b^9 - 27*(2b^2*c*e + (81g^2 - 7 \\
& 0f*h)a^2 + 2*(5e*f - 7c*h)*a*b)*g/(a^2b^9) - (b^5c^3 + 8a^2b^3e^3 \\
& + 15a*b^4c^2*f + 75a^2b^3c*f^2 + 125a^3b^2*f^3 - 168a^3b^2e^2*h + \\
& 1176a^4b*e*h^2 - 2744a^5h^3)/(a^4b^{10}) - (b^5c^3 + 15a*b^4c^2*f + \\
& 2744a^5h^3 - 3*(243g^3 - 630f*g*h + 392e*h^2)*a^4*b + (125f^3 - 270e \\
& *f*g + 168e^2*h + 378c*g*h)a^3b^2 - (8e^3 - 3(25f^2 - 18e*g)*c)a^2 \\
& *b^3)/(a^4b^{10})^{(1/3)} + (1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*(1458g^3/b^9 - 27*(2 \\
& *b^2*c*e + (81g^2 - 70f*h)a^2 + 2*(5e*f - 7c*h)*a*b)*g/(a^2b^9) - (b^ \\
& 5c^3 + 8a^2b^3e^3 + 15a*b^4c^2*f + 75a^2b^3c*f^2 + 125a^3b^2*f^3 \\
& - 168a^3b^2e^2*h + 1176a^4b*e*h^2 - 2744a^5h^3)/(a^4b^{10}) - (b^5c \\
& ^3 + 15a*b^4c^2*f + 2744a^5h^3 - 3*(243g^3 - 630f*g*h + 392e*h^2)*a^ \\
& 4*b + (125f^3 - 270e*f*g + 168e^2*h + 378c*g*h)a^3b^2 - (8e^3 - 3(2 \\
& 5f^2 - 18e*g)*c)a^2b^3)/(a^4b^{10})^{(1/3)} - 18g/b^3)*a^2b^3*g + 32*b^ \\
& 2*c*e + 160a*b*e*f + 324a^2*g^2 - 224*(a*b*c + 5a^2*f)*h)/(a^2b^6))*lo \\
& g(4a*b^4c^2*e + 40a^2b^3c*e*f + 100a^3b^2e*f^2 - 36a^3b^2e^2*g - \\
& 1764a^5g*h^2 + 1/4*(a^3b^8*c + 5a^4b^7*f)*(2*(1/2)^{(2/3)}*(-I*\text{sqrt}(3) \\
& + 1)*(81g^2/b^6 - (2b^2*c*e + (81g^2 - 70f*h)a^2 + 2*(5e*f - 7c*h)*a \\
& *b)/(a^2b^6)))/(1458g^3/b^9 - 27*(2b^2*c*e + (81g^2 - 70f*h)a^2 + 2*(5 \\
& *e*f - 7c*h)*a*b)*g/(a^2b^9) - (b^5c^3 + 8a^2b^3e^3 + 15a*b^4c^2*f \\
& + 75a^2b^3c*f^2 + 125a^3b^2*f^3 - 168a^3b^2e^2*h + 1176a^4b*e*h^2 \\
& - 2744a^5h^3)/(a^4b^{10}) - (b^5c^3 + 15a*b^4c^2*f + 2744a^5h^3 - 3* \\
& (243g^3 - 630f*g*h + 392e*h^2)*a^4*b + (125f^3 - 270e*f*g + 168e^2*h \\
& + 378c*g*h)a^3b^2 - (8e^3 - 3(25f^2 - 18e*g)*c)a^2b^3)/(a^4b^{10}) \\
& ^{(1/3)} + (1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*(1458g^3/b^9 - 27*(2b^2*c*e + (81g^ \\
& 2 - 70f*h)a^2 + 2*(5e*f - 7c*h)*a*b)*g/(a^2b^9) - (b^5c^3 + 8a^2b^3 \\
& *e^3 + 15a*b^4c^2*f + 75a^2b^3c*f^2 + 125a^3b^2*f^3 - 168a^3b^2e^ \\
& 2*h + 1176a^4b*e*h^2 - 2744a^5h^3)/(a^4b^{10}) - (b^5c^3 + 15a*b^4c^2 \\
& *f + 2744a^5h^3 - 3*(243g^3 - 630f*g*h + 392e*h^2)*a^4*b + (125f^3 - \\
& 270e*f*g + 168e^2*h + 378c*g*h)a^3b^2 - (8e^3 - 3(25f^2 - 18e*g)*c \\
&)a^2b^3)/(a^4b^{10})^{(1/3)} - 18g/b^3)^2 + 81*(a^3b^2*c + 5a^4b*f)*g^2 \\
& - (2a^3b^5e^2 - 28a^4b^4e*h + 98a^5b^3h^2 - 9*(a^3b^5*c + 5a^4* \\
& b^4*f)*g)*(2*(1/2)^{(2/3)}*(-I*\text{sqrt}(3) + 1)*(81g^2/b^6 - (2b^2*c*e + (81g^ \\
& 2 - 70f*h)a^2 + 2*(5e*f - 7c*h)*a*b)/(a^2b^6)))/(1458g^3/b^9 - 27*(2b \\
& ^2*c*e + (81g^2 - 70f*h)a^2 + 2*(5e*f - 7c*h)*a*b)*g/(a^2b^9) - (b^5c \\
& ^3 + 8a^2b^3e^3 + 15a*b^4c^2*f + 75a^2b^3c*f^2 + 125a^3b^2*f^3 - \\
& 168a^3b^2e^2*h + 1176a^4b*e*h^2 - 2744a^5h^3)/(a^4b^{10}) - (b^5c^3
\end{aligned}$$

$$\begin{aligned}
& + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4* \\
& b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25* \\
& f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10)^{(1/3)} + (1/2)^{(1/3)}*(I*sqrt(3) + 1)*(\\
& 1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a* \\
& b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f \\
& ^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3) \\
& / (a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f* \\
& g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3* \\
& b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10)^{(1/3)} - 18*g/b^3) \\
& - 28*(a^2*b^3*c^2 + 10*a^3*b^2*c*f + 25*a^4*b*f^2 - 18*a^4*b*e*g)*h - 2* \\
& (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 \\
& - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)*x + 3/4*sqrt(1/3) \\
&)*(8*a^3*b^5*e^2 - 112*a^4*b^4*e*h + 392*a^5*b^3*h^2 + (a^3*b^8*c + 5*a^4*b \\
& ^7*f)*(2*(1/2)^{(2/3)}*(-I*sqrt(3) + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - \\
& 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)/(a^2*b^6)))/(1458*g^3/b^9 - 27*(2*b^2*c \\
& e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 \\
& + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168 \\
& *a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 1 \\
& 5*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + \\
& (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 \\
& - 18*e*g)*c)*a^2*b^3)/(a^4*b^10)^{(1/3)} + (1/2)^{(1/3)}*(I*sqrt(3) + 1)*(1458 \\
& *g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g \\
& / (a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + \\
& 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4* \\
& b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h \\
& + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 \\
& - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10)^{(1/3)} - 18*g/b^3) + \\
& 18*(a^3*b^5*c + 5*a^4*b^4*f)*g)*sqrt(-((2*(1/2)^{(2/3)}*(-I*sqrt(3) + 1)*(81 \\
& *g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)/(a^2 \\
& *b^6)))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7 \\
& *c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2 \\
& *b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744* \\
& a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 \\
& - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c* \\
& g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10)^{(1/3)} + \\
& (1/2)^{(1/3)}*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f \\
& h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 1 \\
& 5*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 11 \\
& 76*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 274 \\
& 4*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f* \\
& g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^ \\
& 3)/(a^4*b^10)^{(1/3)} - 18*g/b^3)^2*a^2*b^6 + 36*(2*(1/2)^{(2/3)}*(-I*sqrt(3) \\
& + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a \\
& *b)/(a^2*b^6)))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5 \\
& *e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f
\end{aligned}$$

$$\begin{aligned}
& + 75a^2b^3c^2f^2 + 125a^3b^2f^3 - 168a^3b^2e^2h + 1176a^4b^2e^2h^2 \\
& - 2744a^5h^3)/(a^4b^{10}) - (b^5c^3 + 15a^2b^4c^2f + 2744a^5h^3 - 3 \\
& (243g^3 - 630f^2g^2h + 392e^2h^2)a^4b + (125f^3 - 270e^2f^2g + 168e^2h^2 \\
& + 378c^2g^2h)a^3b^2 - (8e^3 - 3(25f^2 - 18e^2g)c)a^2b^3)/(a^4b^{10})) \\
& ^{(1/3)} + (1/2)^{(1/3)}(I\sqrt{3} + 1)(1458g^3/b^9 - 27(2b^2c^2e + (81g^2 \\
& - 70f^2h)a^2 + 2(5e^2f - 7c^2h)a^2b)g/(a^2b^9) - (b^5c^3 + 8a^2b^3 \\
& e^3 + 15a^2b^4c^2f + 75a^2b^3c^2f^2 + 125a^3b^2f^3 - 168a^3b^2e^2 \\
& h + 1176a^4b^2e^2h^2 - 2744a^5h^3)/(a^4b^{10}) - (b^5c^3 + 15a^2b^4c^2 \\
& f + 2744a^5h^3 - 3(243g^3 - 630f^2g^2h + 392e^2h^2)a^4b + (125f^3 - \\
& 270e^2f^2g + 168e^2h^2 + 378c^2g^2h)a^3b^2 - (8e^3 - 3(25f^2 - 18e^2g)c \\
&)a^2b^3)/(a^4b^{10}))^{(1/3)} - 18g/b^3)a^2b^3g + 32b^2c^2e + 160a^2b^2e \\
& f + 324a^2g^2 - 224(a^2bc + 5a^2f)h)/(a^2b^6))) + (54a^2b^2g^2x^6 + \\
& 108a^2b^2g^2x^3 + 54a^3g^2 + (a^2b^5x^6 + 2a^2b^4x^3 + a^3b^3)(2*(1/2 \\
&)^{(2/3)}*(-I\sqrt{3} + 1)(81g^2/b^6 - (2b^2c^2e + (81g^2 - 70f^2h)a^2 + \\
& 2(5e^2f - 7c^2h)a^2b)/(a^2b^6)))/(1458g^3/b^9 - 27(2b^2c^2e + (81g^2 \\
& - 70f^2h)a^2 + 2(5e^2f - 7c^2h)a^2b)g/(a^2b^9) - (b^5c^3 + 8a^2b^3e \\
& ^3 + 15a^2b^4c^2f + 75a^2b^3c^2f^2 + 125a^3b^2f^3 - 168a^3b^2e^2h \\
& + 1176a^4b^2e^2h^2 - 2744a^5h^3)/(a^4b^{10}) - (b^5c^3 + 15a^2b^4c^2f \\
& + 2744a^5h^3 - 3(243g^3 - 630f^2g^2h + 392e^2h^2)a^4b + (125f^3 - 27 \\
& 0e^2f^2g + 168e^2h^2 + 378c^2g^2h)a^3b^2 - (8e^3 - 3(25f^2 - 18e^2g)c) \\
&)a^2b^3)/(a^4b^{10}))^{(1/3)} + (1/2)^{(1/3)}(I\sqrt{3} + 1)(1458g^3/b^9 - 27 \\
& (2b^2c^2e + (81g^2 - 70f^2h)a^2 + 2(5e^2f - 7c^2h)a^2b)g/(a^2b^9) - \\
& (b^5c^3 + 8a^2b^3e^3 + 15a^2b^4c^2f + 75a^2b^3c^2f^2 + 125a^3b^2f^3 - \\
& 168a^3b^2e^2h + 1176a^4b^2e^2h^2 - 2744a^5h^3)/(a^4b^{10}) - (b^5 \\
& c^3 + 15a^2b^4c^2f + 2744a^5h^3 - 3(243g^3 - 630f^2g^2h + 392e^2h^2) \\
&)a^4b + (125f^3 - 270e^2f^2g + 168e^2h^2 + 378c^2g^2h)a^3b^2 - (8e^3 - 3 \\
& (25f^2 - 18e^2g)c)a^2b^3)/(a^4b^{10}))^{(1/3)} - 18g/b^3) + 3\sqrt{1/3} \\
& (a^2b^5x^6 + 2a^2b^4x^3 + a^3b^3)\sqrt{-((2*(1/2)^{(2/3)}*(-I\sqrt{3} + 1) \\
&)*(81g^2/b^6 - (2b^2c^2e + (81g^2 - 70f^2h)a^2 + 2(5e^2f - 7c^2h)a^2b) \\
&)/(a^2b^6)))/(1458g^3/b^9 - 27(2b^2c^2e + (81g^2 - 70f^2h)a^2 + 2(5e^2 \\
& f - 7c^2h)a^2b)g/(a^2b^9) - (b^5c^3 + 8a^2b^3e^3 + 15a^2b^4c^2f + 7 \\
& 5a^2b^3c^2f^2 + 125a^3b^2f^3 - 168a^3b^2e^2h + 1176a^4b^2e^2h^2 - \\
& 2744a^5h^3)/(a^4b^{10}) - (b^5c^3 + 15a^2b^4c^2f + 2744a^5h^3 - 3(24 \\
& 3g^3 - 630f^2g^2h + 392e^2h^2)a^4b + (125f^3 - 270e^2f^2g + 168e^2h^2 + 3 \\
& 78c^2g^2h)a^3b^2 - (8e^3 - 3(25f^2 - 18e^2g)c)a^2b^3)/(a^4b^{10}))^{(1 \\
& /3)} + (1/2)^{(1/3)}(I\sqrt{3} + 1)(1458g^3/b^9 - 27(2b^2c^2e + (81g^2 - \\
& 70f^2h)a^2 + 2(5e^2f - 7c^2h)a^2b)g/(a^2b^9) - (b^5c^3 + 8a^2b^3e^ \\
& ^3 + 15a^2b^4c^2f + 75a^2b^3c^2f^2 + 125a^3b^2f^3 - 168a^3b^2e^2h \\
& + 1176a^4b^2e^2h^2 - 2744a^5h^3)/(a^4b^{10}) - (b^5c^3 + 15a^2b^4c^2f \\
& + 2744a^5h^3 - 3(243g^3 - 630f^2g^2h + 392e^2h^2)a^4b + (125f^3 - 270 \\
& e^2f^2g + 168e^2h^2 + 378c^2g^2h)a^3b^2 - (8e^3 - 3(25f^2 - 18e^2g)c) \\
&)a^2b^3)/(a^4b^{10}))^{(1/3)} - 18g/b^3)^2a^2b^6 + 36(2*(1/2)^{(2/3)}*(-I\sqrt{3} \\
& + 1)(81g^2/b^6 - (2b^2c^2e + (81g^2 - 70f^2h)a^2 + 2(5e^2f - 7c^2 \\
& h)a^2b)/(a^2b^6)))/(1458g^3/b^9 - 27(2b^2c^2e + (81g^2 - 70f^2h)a^2 + \\
& 2(5e^2f - 7c^2h)a^2b)g/(a^2b^9) - (b^5c^3 + 8a^2b^3e^3 + 15a^2b^4c^2
\end{aligned}$$

$$\begin{aligned}
& ^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b* \\
& e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 \\
& - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e \\
& ^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b \\
& ^10))^{(1/3)} + (1/2)^{(1/3)}*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(2*b^2*c*e + (\\
& 81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^ \\
& 2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b \\
& ^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^ \\
& 4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f \\
& ^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e \\
& *g)*c)*a^2*b^3)/(a^4*b^10))^{(1/3)} - 18*g/b^3)*a^2*b^3*g + 32*b^2*c*e + 160* \\
& a*b*e*f + 324*a^2*g^2 - 224*(a*b*c + 5*a^2*f)*h)/(a^2*b^6))*log(4*a*b^4*c^ \\
& 2*e + 40*a^2*b^3*c*e*f + 100*a^3*b^2*e*f^2 - 36*a^3*b^2*e^2*g - 1764*a^5*g* \\
& h^2 + 1/4*(a^3*b^8*c + 5*a^4*b^7*f)*(2*(1/2)^{(2/3)}*(-I*sqrt(3) + 1)*(81*g^2 \\
& /b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)/(a^2*b^6 \\
&))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h \\
&)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3 \\
& *c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5* \\
& h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 6 \\
& 30*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h) \\
& *a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^{(1/3)} + (1/ \\
& 2)^{(1/3)}*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)* \\
& a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a* \\
& b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a \\
& ^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^ \\
& 5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + \\
& 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(\\
& a^4*b^10))^{(1/3)} - 18*g/b^3)^2 + 81*(a^3*b^2*c + 5*a^4*b*f)*g^2 - (2*a^3*b^ \\
& 5*e^2 - 28*a^4*b^4*e*h + 98*a^5*b^3*h^2 - 9*(a^3*b^5*c + 5*a^4*b^4*f)*g)*(2 \\
& *(1/2)^{(2/3)}*(-I*sqrt(3) + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)* \\
& a^2 + 2*(5*e*f - 7*c*h)*a*b)/(a^2*b^6)))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81 \\
& *g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2* \\
& b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2 \\
& *e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4* \\
& c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 \\
& - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g \\
&)*c)*a^2*b^3)/(a^4*b^10))^{(1/3)} + (1/2)^{(1/3)}*(I*sqrt(3) + 1)*(1458*g^3/b^9 \\
& - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^ \\
& 9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3 \\
& *b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) \\
& - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e \\
& *h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^ \\
& 3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^{(1/3)} - 18*g/b^3) - 28*(a^2 \\
& *b^3*c^2 + 10*a^3*b^2*c*f + 25*a^4*b*f^2 - 18*a^4*b*e*g)*h - 2*(b^5*c^3 + 8 \\
& *a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^
\end{aligned}$$

$$\begin{aligned}
& 3b^2e^2h + 1176a^4b^4e^2h^2 - 2744a^5h^3)x - 3/4\sqrt{1/3}*(8a^3b^5 \\
& *e^2 - 112a^4b^4e^2h + 392a^5b^3h^2 + (a^3b^8c + 5a^4b^7f)*(2*(1/ \\
& 2)^{(2/3)}*(-I\sqrt{3}) + 1)*(81g^2/b^6 - (2b^2c^2e + (81g^2 - 70f^2h)*a^2 \\
& + 2*(5e^2f - 7c^2h)*ab)/(a^2b^6)))/(1458g^3/b^9 - 27*(2b^2c^2e + (81g^2 \\
& - 70f^2h)*a^2 + 2*(5e^2f - 7c^2h)*ab)*g/(a^2b^9) - (b^5c^3 + 8a^2b^3* \\
& e^3 + 15ab^4c^2f + 75a^2b^3c^2f^2 + 125a^3b^2f^3 - 168a^3b^2e^2 \\
& *h + 1176a^4b^4e^2h^2 - 2744a^5h^3)/(a^4b^10) - (b^5c^3 + 15ab^4c^2* \\
& f + 2744a^5h^3 - 3*(243g^3 - 630f^2g^2h + 392e^2h^2)*a^4b + (125f^3 - 2 \\
& 70e^2f^2g + 168e^2h^2 + 378c^2g^2h)*a^3b^2 - (8e^3 - 3*(25f^2 - 18e^2g)*c) \\
& *a^2b^3)/(a^4b^10)^{(1/3)} + (1/2)^{(1/3)}*(I\sqrt{3}) + 1)*(1458g^3/b^9 - 2 \\
& 7*(2b^2c^2e + (81g^2 - 70f^2h)*a^2 + 2*(5e^2f - 7c^2h)*ab)*g/(a^2b^9) - \\
& (b^5c^3 + 8a^2b^3e^3 + 15ab^4c^2f + 75a^2b^3c^2f^2 + 125a^3b^2 \\
& *f^3 - 168a^3b^2e^2h + 1176a^4b^4e^2h^2 - 2744a^5h^3)/(a^4b^10) - (b \\
& ^5c^3 + 15ab^4c^2f + 2744a^5h^3 - 3*(243g^3 - 630f^2g^2h + 392e^2h^2) \\
&)*a^4b + (125f^3 - 270e^2f^2g + 168e^2h^2 + 378c^2g^2h)*a^3b^2 - (8e^3 - \\
& 3*(25f^2 - 18e^2g)*c)*a^2b^3)/(a^4b^10)^{(1/3)} - 18g/b^3) + 18*(a^3b^5 \\
& *c + 5a^4b^4f)*g*\sqrt{-((2*(1/2)^{(2/3)}*(-I\sqrt{3}) + 1)*(81g^2/b^6 - (\\
& 2b^2c^2e + (81g^2 - 70f^2h)*a^2 + 2*(5e^2f - 7c^2h)*ab)/(a^2b^6)))/(1458 \\
& *g^3/b^9 - 27*(2b^2c^2e + (81g^2 - 70f^2h)*a^2 + 2*(5e^2f - 7c^2h)*ab)*g \\
& /(a^2b^9) - (b^5c^3 + 8a^2b^3e^3 + 15ab^4c^2f + 75a^2b^3c^2f^2 + \\
& 125a^3b^2f^3 - 168a^3b^2e^2h + 1176a^4b^4e^2h^2 - 2744a^5h^3)/(a^ \\
& 4b^10) - (b^5c^3 + 15ab^4c^2f + 2744a^5h^3 - 3*(243g^3 - 630f^2g^2h \\
& + 392e^2h^2)*a^4b + (125f^3 - 270e^2f^2g + 168e^2h^2 + 378c^2g^2h)*a^3b^2 \\
& - (8e^3 - 3*(25f^2 - 18e^2g)*c)*a^2b^3)/(a^4b^10)^{(1/3)} + (1/2)^{(1/3)} \\
& *(I\sqrt{3}) + 1)*(1458g^3/b^9 - 27*(2b^2c^2e + (81g^2 - 70f^2h)*a^2 + 2* \\
& (5e^2f - 7c^2h)*ab)*g/(a^2b^9) - (b^5c^3 + 8a^2b^3e^3 + 15ab^4c^2* \\
& f + 75a^2b^3c^2f^2 + 125a^3b^2f^3 - 168a^3b^2e^2h + 1176a^4b^4e^2h^2 \\
& - 2744a^5h^3)/(a^4b^10) - (b^5c^3 + 15ab^4c^2f + 2744a^5h^3 - \\
& 3*(243g^3 - 630f^2g^2h + 392e^2h^2)*a^4b + (125f^3 - 270e^2f^2g + 168e^2* \\
& h + 378c^2g^2h)*a^3b^2 - (8e^3 - 3*(25f^2 - 18e^2g)*c)*a^2b^3)/(a^4b^10 \\
&)^{(1/3)} - 18g/b^3)^2*a^2b^6 + 36*(2*(1/2)^{(2/3)}*(-I\sqrt{3}) + 1)*(81g^2 \\
& /b^6 - (2b^2c^2e + (81g^2 - 70f^2h)*a^2 + 2*(5e^2f - 7c^2h)*ab)/(a^2b^6 \\
&))/(1458g^3/b^9 - 27*(2b^2c^2e + (81g^2 - 70f^2h)*a^2 + 2*(5e^2f - 7c^2h) \\
&)*ab)*g/(a^2b^9) - (b^5c^3 + 8a^2b^3e^3 + 15ab^4c^2f + 75a^2b^3 \\
& *c^2f^2 + 125a^3b^2f^3 - 168a^3b^2e^2h + 1176a^4b^4e^2h^2 - 2744a^5* \\
& h^3)/(a^4b^10) - (b^5c^3 + 15ab^4c^2f + 2744a^5h^3 - 3*(243g^3 - 6 \\
& 30f^2g^2h + 392e^2h^2)*a^4b + (125f^3 - 270e^2f^2g + 168e^2h^2 + 378c^2g^2h) \\
& *a^3b^2 - (8e^3 - 3*(25f^2 - 18e^2g)*c)*a^2b^3)/(a^4b^10)^{(1/3)} + (1/ \\
& 2)^{(1/3)}*(I\sqrt{3}) + 1)*(1458g^3/b^9 - 27*(2b^2c^2e + (81g^2 - 70f^2h)* \\
& a^2 + 2*(5e^2f - 7c^2h)*ab)*g/(a^2b^9) - (b^5c^3 + 8a^2b^3e^3 + 15a* \\
& b^4c^2f + 75a^2b^3c^2f^2 + 125a^3b^2f^3 - 168a^3b^2e^2h + 1176a^ \\
& ^4b^4e^2h^2 - 2744a^5h^3)/(a^4b^10) - (b^5c^3 + 15ab^4c^2f + 2744a^ \\
& 5h^3 - 3*(243g^3 - 630f^2g^2h + 392e^2h^2)*a^4b + (125f^3 - 270e^2f^2g + \\
& 168e^2h^2 + 378c^2g^2h)*a^3b^2 - (8e^3 - 3*(25f^2 - 18e^2g)*c)*a^2b^3)/(\\
& a^4b^10)^{(1/3)} - 18g/b^3)*a^2b^3*g + 32b^2c^2e + 160ab^2e^2f + 324a^2
\end{aligned}$$

$*g^2 - 224*(a*b*c + 5*a^2*f)*h)/(a^2*b^6))))/(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.08362, size = 541, normalized size = 1.57

$$\frac{hx}{b^3} + \frac{g \log(|bx^3 + a|)}{3b^3} - \frac{\sqrt{3} \left(14 (-ab^2)^{\frac{1}{3}} a^2 h - 2 (-ab^2)^{\frac{1}{3}} a b e + (-ab^2)^{\frac{2}{3}} b c + 5 (-ab^2)^{\frac{2}{3}} a f \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}} \right)}{27 a^2 b^4} - \left(1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac")

[Out] $h*x/b^3 + 1/3*g*\log(\text{abs}(b*x^3 + a))/b^3 - 1/27*\text{sqrt}(3)*(14*(-a*b^2)^{(1/3)}*a^2*h - 2*(-a*b^2)^{(1/3)}*a*b*e + (-a*b^2)^{(2/3)}*b*c + 5*(-a*b^2)^{(2/3)}*a*f)*\arctan(1/3*\text{sqrt}(3)*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(a^2*b^4) - 1/54*(14*(-a*b^2)^{(1/3)}*a^2*h - 2*(-a*b^2)^{(1/3)}*a*b*e - (-a*b^2)^{(2/3)}*b*c - 5*(-a*b^2)^{(2/3)}*a*f)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a^2*b^4) + 1/18*(2*(b^3*c - 4*a*b^2*f)*x^5 + (13*a^2*b*h - 7*a*b^2*e)*x^4 - 3*a^2*b*d + 9*a^3*g - 6*(a*b^2*d - 2*a^2*b*g)*x^3 - (a*b^2*c + 5*a^2*b*f)*x^2 + 2*(5*a^3*h - 2*a^2*b*e)*x)/((b*x^3 + a)^2*a*b^3) - 1/27*(a*b^6*c*(-a/b)^{(1/3)} + 5*a^2*b^5*f*(-a/b)^{(1/3)} - 14*a^3*b^4*h + 2*a^2*b^5*e)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/a^3*b^7)$

$$3.422 \quad \int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=325

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(\sqrt[3]{b}(2af+bc) - \sqrt[3]{a}(5ag+bd)\right)}{54a^{5/3}b^{8/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(\sqrt[3]{b}(2af+bc) - \sqrt[3]{a}(5ag+bd)\right)}{27a^{5/3}b^{8/3}} - \text{ta}$$

[Out] $-(x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(6*b^2*(a + b*x^3)^2) + (x*(b*c - 7*a*f + 2*(b*d - 4*a*g)*x + 3*(b*e - 3*a*h)*x^2))/(18*a*b^2*(a + b*x^3)) - ((b^{(4/3)*c} + a^{(1/3)*b*d} + 2*a*b^{(1/3)*f} + 5*a^{(4/3)*g})*\text{ArcTan}[(a^{(1/3)} - 2*b^{(1/3)*x})/(\text{Sqrt}[3]*a^{(1/3)})])/(9*\text{Sqrt}[3]*a^{(5/3)*b^{(8/3)}}) + ((b^{(1/3)*(b*c + 2*a*f)} - a^{(1/3)*(b*d + 5*a*g)})*\text{Log}[a^{(1/3)} + b^{(1/3)*x}])/(2*7*a^{(5/3)*b^{(8/3)}}) - ((b^{(1/3)*(b*c + 2*a*f)} - a^{(1/3)*(b*d + 5*a*g)})*\text{Log}[a^{(2/3)} - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2}])/(54*a^{(5/3)*b^{(8/3)}}) + (h*\text{Log}[a + b*x^3])/(3*b^3)$

Rubi [A] time = 0.642445, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1828, 1858, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(\sqrt[3]{b}(2af+bc) - \sqrt[3]{a}(5ag+bd)\right)}{54a^{5/3}b^{8/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(\sqrt[3]{b}(2af+bc) - \sqrt[3]{a}(5ag+bd)\right)}{27a^{5/3}b^{8/3}} - \text{ta}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]

[Out] $-(x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(6*b^2*(a + b*x^3)^2) + (x*(b*c - 7*a*f + 2*(b*d - 4*a*g)*x + 3*(b*e - 3*a*h)*x^2))/(18*a*b^2*(a + b*x^3)) - ((b^{(4/3)*c} + a^{(1/3)*b*d} + 2*a*b^{(1/3)*f} + 5*a^{(4/3)*g})*\text{ArcTan}[(a^{(1/3)} - 2*b^{(1/3)*x})/(\text{Sqrt}[3]*a^{(1/3)})])/(9*\text{Sqrt}[3]*a^{(5/3)*b^{(8/3)}}) + ((b^{(1/3)*(b*c + 2*a*f)} - a^{(1/3)*(b*d + 5*a*g)})*\text{Log}[a^{(1/3)} + b^{(1/3)*x}])/(2*7*a^{(5/3)*b^{(8/3)}}) - ((b^{(1/3)*(b*c + 2*a*f)} - a^{(1/3)*(b*d + 5*a*g)})*\text{Log}[a^{(2/3)} - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2}])/(54*a^{(5/3)*b^{(8/3)}}) + (h*\text{Log}[a + b*x^3])/(3*b^3)$

Rule 1828

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q =
m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
+ b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(p_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

$[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rule 617

$\text{Int}[\frac{(a_.) + (b_.)(x_) + (c_.)(x_)^2}{(x_)^2}, x_Symbol] \rightarrow \text{With}[\{q = 1 - 4*S\text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \|\| \text{!RationalQ}[b^2 - 4*a*c])] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 204

$\text{Int}[\frac{(a_.) + (b_.)(x_)^2}{(x_)^2}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[\frac{\text{Rt}[-b, 2]*x}{\text{Rt}[-a, 2]}] / \frac{\text{Rt}[-a, 2]*\text{Rt}[-b, 2]}{x}], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{LtQ}[a, 0] \|\| \text{LtQ}[b, 0])$

Rule 628

$\text{Int}[\frac{(d_.) + (e_.)(x_)}{(a_.) + (b_.)(x_) + (c_.)(x_)^2}, x_Symbol] \rightarrow \text{Simp}[\frac{d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]}{b}, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

Rule 260

$\text{Int}[\frac{(x_)^m}{(a_.) + (b_.)(x_)^n}, x_Symbol] \rightarrow \text{Simp}[\frac{\text{Log}[\text{RemoveContent}[a + b*x^n, x]]}{b*n}, x] /; \text{FreeQ}[\{a, b, m, n\}, x] \&\& \text{EqQ}[m, n - 1]$

Rubi steps

$$\begin{aligned}
\int \frac{x^3(c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{(a + bx^3)^3} dx &= -\frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{6b^2(a + bx^3)^2} - \frac{\int \frac{-ab(bc-af)-2ab(bd-ag)x-3ab(be-ah)x^2}{(a+bx^3)^2} dx}{6ab^3} \\
&= -\frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{6b^2(a + bx^3)^2} + \frac{x(bc - 7af + 2(bd - 4ag)x + 3(bd - ag)x^2)}{18ab^2(a + bx^3)} \\
&= -\frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{6b^2(a + bx^3)^2} + \frac{x(bc - 7af + 2(bd - 4ag)x + 3(bd - ag)x^2)}{18ab^2(a + bx^3)} \\
&= -\frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{6b^2(a + bx^3)^2} + \frac{x(bc - 7af + 2(bd - 4ag)x + 3(bd - ag)x^2)}{18ab^2(a + bx^3)} \\
&= -\frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{6b^2(a + bx^3)^2} + \frac{x(bc - 7af + 2(bd - 4ag)x + 3(bd - ag)x^2)}{18ab^2(a + bx^3)} \\
&= -\frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{6b^2(a + bx^3)^2} + \frac{x(bc - 7af + 2(bd - 4ag)x + 3(bd - ag)x^2)}{18ab^2(a + bx^3)} \\
&= -\frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{6b^2(a + bx^3)^2} + \frac{x(bc - 7af + 2(bd - 4ag)x + 3(bd - ag)x^2)}{18ab^2(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.253258, size = 315, normalized size = 0.97

$$\frac{\frac{9(a^2h - ab(e + x(f + gx)) + b^2x(c + dx))}{(a + bx^3)^2} + \frac{36a^2h - 3ab(6e + x(7f + 8gx)) + 3b^2x(c + 2dx)}{a(a + bx^3)} + \frac{\sqrt[3]{b} \log\left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx + b^{2/3}x^2}\right) \left(5a^{4/3}g + \sqrt[3]{abd} - 2a \sqrt[3]{bf - b^{4/3}c}\right)}{a^{5/3}}}{54b^3} + \frac{2 \sqrt[3]{b} \log\left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx + b^{2/3}x^2}\right)}{54b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]

[Out] ((-9*(a^2*h + b^2*x*(c + d*x) - a*b*(e + x*(f + g*x)))/(a + b*x^3)^2 + (36*a^2*h + 3*b^2*x*(c + 2*d*x) - 3*a*b*(6*e + x*(7*f + 8*g*x)))/(a*(a + b*x^3)) - (2*sqrt[3]*b^(1/3)*(b^(4/3)*c + a^(1/3)*b*d + 2*a*b^(1/3)*f + 5*a^(4/3

```
) * g) * ArcTan[(1 - (2 * b^(1/3) * x) / a^(1/3)) / Sqrt[3]] / a^(5/3) + (2 * b^(1/3) * (b^(4/3) * c - a^(1/3) * b * d + 2 * a * b^(1/3) * f - 5 * a^(4/3) * g) * Log[a^(1/3) + b^(1/3) * x]) / a^(5/3) + (b^(1/3) * (-(b^(4/3) * c) + a^(1/3) * b * d - 2 * a * b^(1/3) * f + 5 * a^(4/3) * g) * Log[a^(2/3) - a^(1/3) * b^(1/3) * x + b^(2/3) * x^2]) / a^(5/3) + 18 * h * Log[a + b * x^3]) / (54 * b^3)
```

Maple [A] time = 0.012, size = 515, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] (-1/9*(4*a*g-b*d)/a/b*x^5-1/18*(7*a*f-b*c)/a/b*x^4+1/3*(2*a*h-b*e)/b^2*x^3-1/18*(5*a*g+b*d)/b^2*x^2-1/9*(2*a*f+b*c)/b^2*x+1/6*a*(3*a*h-b*e)/b^3)/(b*x^3+a)^2+2/27/b^3/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*f+1/27/b^2/a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c-1/27/b^3/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f-1/54/b^2/a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+2/27/b^3/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f+1/27/b^2/a/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c-5/27/b^3/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*g-1/27/b^2/a/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*d+5/54/b^3/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*g+1/54/b^2/a/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d+1/27/b^3*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*g+1/27/b^2/a*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d+1/3*h*ln(b*x^3+a)/b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 28.7256, size = 29083, normalized size = 89.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")

[Out]
$$\frac{1}{108} \cdot (12 \cdot (b^3 \cdot d - 4 \cdot a \cdot b^2 \cdot g) \cdot x^5 + 6 \cdot (b^3 \cdot c - 7 \cdot a \cdot b^2 \cdot f) \cdot x^4 - 18 \cdot a^2 \cdot b \cdot e + 54 \cdot a^3 \cdot h - 36 \cdot (a \cdot b^2 \cdot e - 2 \cdot a^2 \cdot b \cdot h) \cdot x^3 - 6 \cdot (a \cdot b^2 \cdot d + 5 \cdot a^2 \cdot b \cdot g) \cdot x^2 - 2 \cdot (a \cdot b^5 \cdot x^6 + 2 \cdot a^2 \cdot b^4 \cdot x^3 + a^3 \cdot b^3) \cdot (2 \cdot (1/2)^{(2/3)} \cdot (-I \cdot \sqrt{3}) + 1) \cdot (81 \cdot h^2/b^6 - (b^3 \cdot c \cdot d + 10 \cdot a^2 \cdot b \cdot f \cdot g + 81 \cdot a^3 \cdot h^2 + (2 \cdot d \cdot f + 5 \cdot c \cdot g) \cdot a \cdot b^2) / (a^3 \cdot b^6))) / (1458 \cdot h^3/b^9 - 27 \cdot (b^3 \cdot c \cdot d + 10 \cdot a^2 \cdot b \cdot f \cdot g + 81 \cdot a^3 \cdot h^2 + (2 \cdot d \cdot f + 5 \cdot c \cdot g) \cdot a \cdot b^2) \cdot h / (a^3 \cdot b^9) + (b^4 \cdot c^3 + a \cdot b^3 \cdot d^3 + 6 \cdot a \cdot b^3 \cdot c^2 \cdot f + 12 \cdot a^2 \cdot b^2 \cdot c \cdot f^2 + 8 \cdot a^3 \cdot b \cdot f^3 + 15 \cdot a^2 \cdot b^2 \cdot d^2 \cdot g + 75 \cdot a^3 \cdot b \cdot d \cdot g^2 + 125 \cdot a^4 \cdot g^3) / (a^5 \cdot b^8) + (b^5 \cdot c^3 + 729 \cdot a^5 \cdot h^3 - 5 \cdot (25 \cdot g^3 - 54 \cdot f \cdot g \cdot h) \cdot a^4 \cdot b + (8 \cdot f^3 + 135 \cdot c \cdot g \cdot h - 3 \cdot (25 \cdot g^2 - 18 \cdot f \cdot h) \cdot d) \cdot a^3 \cdot b^2 - 3 \cdot (5 \cdot d^2 \cdot g - (4 \cdot f^2 + 9 \cdot d \cdot h) \cdot c) \cdot a^2 \cdot b^3 - (d^3 - 6 \cdot c^2 \cdot f) \cdot a \cdot b^4) / (a^5 \cdot b^9))^{(1/3)} + (1/2)^{(1/3)} \cdot (I \cdot \sqrt{3}) + 1) \cdot (1458 \cdot h^3/b^9 - 27 \cdot (b^3 \cdot c \cdot d + 10 \cdot a^2 \cdot b \cdot f \cdot g + 81 \cdot a^3 \cdot h^2 + (2 \cdot d \cdot f + 5 \cdot c \cdot g) \cdot a \cdot b^2) \cdot h / (a^3 \cdot b^9) + (b^4 \cdot c^3 + a \cdot b^3 \cdot d^3 + 6 \cdot a \cdot b^3 \cdot c^2 \cdot f + 12 \cdot a^2 \cdot b^2 \cdot c \cdot f^2 + 8 \cdot a^3 \cdot b \cdot f^3 + 15 \cdot a^2 \cdot b^2 \cdot d^2 \cdot g + 75 \cdot a^3 \cdot b \cdot d \cdot g^2 + 125 \cdot a^4 \cdot g^3) / (a^5 \cdot b^8) + (b^5 \cdot c^3 + 729 \cdot a^5 \cdot h^3 - 5 \cdot (25 \cdot g^3 - 54 \cdot f \cdot g \cdot h) \cdot a^4 \cdot b + (8 \cdot f^3 + 135 \cdot c \cdot g \cdot h - 3 \cdot (25 \cdot g^2 - 18 \cdot f \cdot h) \cdot d) \cdot a^3 \cdot b^2 - 3 \cdot (5 \cdot d^2 \cdot g - (4 \cdot f^2 + 9 \cdot d \cdot h) \cdot c) \cdot a^2 \cdot b^3 - (d^3 - 6 \cdot c^2 \cdot f) \cdot a \cdot b^4) / (a^5 \cdot b^9))^{(1/3)} - 18 \cdot h / b^3) \cdot \log(2 \cdot a \cdot b^4 \cdot c \cdot d^2 + 4 \cdot a^2 \cdot b^3 \cdot d^2 \cdot f + 1/4 \cdot (a^4 \cdot b^7 \cdot d + 5 \cdot a^5 \cdot b^6 \cdot g) \cdot (2 \cdot (1/2)^{(2/3)} \cdot (-I \cdot \sqrt{3}) + 1) \cdot (81 \cdot h^2/b^6 - (b^3 \cdot c \cdot d + 10 \cdot a^2 \cdot b \cdot f \cdot g + 81 \cdot a^3 \cdot h^2 + (2 \cdot d \cdot f + 5 \cdot c \cdot g) \cdot a \cdot b^2) / (a^3 \cdot b^6))) / (1458 \cdot h^3/b^9 - 27 \cdot (b^3 \cdot c \cdot d + 10 \cdot a^2 \cdot b \cdot f \cdot g + 81 \cdot a^3 \cdot h^2 + (2 \cdot d \cdot f + 5 \cdot c \cdot g) \cdot a \cdot b^2) \cdot h / (a^3 \cdot b^9) + (b^4 \cdot c^3 + a \cdot b^3 \cdot d^3 + 6 \cdot a \cdot b^3 \cdot c^2 \cdot f + 12 \cdot a^2 \cdot b^2 \cdot c \cdot f^2 + 8 \cdot a^3 \cdot b \cdot f^3 + 15 \cdot a^2 \cdot b^2 \cdot d^2 \cdot g + 75 \cdot a^3 \cdot b \cdot d \cdot g^2 + 125 \cdot a^4 \cdot g^3) / (a^5 \cdot b^8) + (b^5 \cdot c^3 + 729 \cdot a^5 \cdot h^3 - 5 \cdot (25 \cdot g^3 - 54 \cdot f \cdot g \cdot h) \cdot a^4 \cdot b + (8 \cdot f^3 + 135 \cdot c \cdot g \cdot h - 3 \cdot (25 \cdot g^2 - 18 \cdot f \cdot h) \cdot d) \cdot a^3 \cdot b^2 - 3 \cdot (5 \cdot d^2 \cdot g - (4 \cdot f^2 + 9 \cdot d \cdot h) \cdot c) \cdot a^2 \cdot b^3 - (d^3 - 6 \cdot c^2 \cdot f) \cdot a \cdot b^4) / (a^5 \cdot b^9))^{(1/3)} + (1/2)^{(1/3)} \cdot (I \cdot \sqrt{3}) + 1) \cdot (1458 \cdot h^3/b^9 - 27 \cdot (b^3 \cdot c \cdot d + 10 \cdot a^2 \cdot b \cdot f \cdot g + 81 \cdot a^3 \cdot h^2 + (2 \cdot d \cdot f + 5 \cdot c \cdot g) \cdot a \cdot b^2) \cdot h / (a^3 \cdot b^9) + (b^4 \cdot c^3 + a \cdot b^3 \cdot d^3 + 6 \cdot a \cdot b^3 \cdot c^2 \cdot f + 12 \cdot a^2 \cdot b^2 \cdot c \cdot f^2 + 8 \cdot a^3 \cdot b \cdot f^3 + 15 \cdot a^2 \cdot b^2 \cdot d^2 \cdot g + 75 \cdot a^3 \cdot b \cdot d \cdot g^2 + 125 \cdot a^4 \cdot g^3) / (a^5 \cdot b^8) + (b^5 \cdot c^3 + 729 \cdot a^5 \cdot h^3 - 5 \cdot (25 \cdot g^3 - 54 \cdot f \cdot g \cdot h) \cdot a^4 \cdot b + (8 \cdot f^3 + 135 \cdot c \cdot g \cdot h - 3 \cdot (25 \cdot g^2 - 18 \cdot f \cdot h) \cdot d) \cdot a^3 \cdot b^2 - 3 \cdot (5 \cdot d^2 \cdot g - (4 \cdot f^2 + 9 \cdot d \cdot h) \cdot c) \cdot a^2 \cdot b^3 - (d^3 - 6 \cdot c^2 \cdot f) \cdot a \cdot b^4) / (a^5 \cdot b^9))^{(1/3)} - 18 \cdot h / b^3)^2 + 50 \cdot (a^3 \cdot b^2 \cdot c + 2 \cdot a^4 \cdot b \cdot f) \cdot g^2 + 81 \cdot (a^4 \cdot b \cdot d + 5 \cdot a^5 \cdot g) \cdot h^2 - 1/2 \cdot (a^2 \cdot b^6 \cdot c^2 + 4 \cdot a^3 \cdot b^5 \cdot c \cdot f + 4 \cdot a^4 \cdot b^4 \cdot f^2 - 18 \cdot (a^4 \cdot b^4 \cdot d + 5 \cdot a^5 \cdot b^3 \cdot g) \cdot h) \cdot (2 \cdot (1/2)^{(2/3)} \cdot (-I \cdot \sqrt{3}) + 1) \cdot (81 \cdot h^2/b^6 - (b^3 \cdot c \cdot d + 10 \cdot a^2 \cdot b \cdot f \cdot g + 81 \cdot a^3 \cdot h^2 + (2 \cdot d \cdot f + 5 \cdot c \cdot g) \cdot a \cdot b^2) / (a^3 \cdot b^6))) / (1458 \cdot h^3/b^9 - 27 \cdot (b^3 \cdot c \cdot d$$

$$\begin{aligned}
& + 10a^2b^3fg + 81a^3h^2 + (2df + 5cg)ab^2)h/(a^3b^9) + (b^4c^3 + a^2b^3d^3 + 6ab^3c^2f + 12a^2b^2c^2f^2 + 8a^3b^2f^3 + 15a^2b^2d^2g + 75a^3b^2d^2g^2 + 125a^4g^3)/(a^5b^8) + (b^5c^3 + 729a^5h^3 - 5(25g^3 - 54fgh)a^4b + (8f^3 + 135cgh - 3(25g^2 - 18fh)d)a^3b^2 - 3(5d^2g - (4f^2 + 9dh)c)a^2b^3 - (d^3 - 6c^2f)ab^4)/(a^5b^9))^{1/3} + (1/2)^{1/3}(I\sqrt{3} + 1)(1458h^3/b^9 - 27(b^3cd + 10a^2b^3fg + 81a^3h^2 + (2df + 5cg)ab^2)h/(a^3b^9) + (b^4c^3 + a^2b^3d^3 + 6ab^3c^2f + 12a^2b^2c^2f^2 + 8a^3b^2f^3 + 15a^2b^2d^2g + 75a^3b^2d^2g^2 + 125a^4g^3)/(a^5b^8) + (b^5c^3 + 729a^5h^3 - 5(25g^3 - 54fgh)a^4b + (8f^3 + 135cgh - 3(25g^2 - 18fh)d)a^3b^2 - 3(5d^2g - (4f^2 + 9dh)c)a^2b^3 - (d^3 - 6c^2f)ab^4)/(a^5b^9))^{1/3} - 18h/b^3) + 20(a^2b^3cd + 2a^3b^2df)g - 9(a^2b^3c^2 + 4a^3b^2cf + 4a^4b^2f^2)h + (b^5c^3 + a^2b^4d^3 + 6a^2b^4c^2f + 12a^2b^3c^2f^2 + 8a^3b^2f^3 + 15a^2b^3d^2g + 75a^3b^2d^2g^2 + 125a^4b^2g^3)x) - 12(ab^2c + 2a^2b^2f)x + (54ab^2hx^6 + 108a^2b^2hx^3 + 54a^3h + (ab^5x^6 + 2a^2b^4x^3 + a^3b^3)(2(1/2)^{2/3})(-I\sqrt{3} + 1)(81h^2/b^6 - (b^3cd + 10a^2b^3fg + 81a^3h^2 + (2df + 5cg)ab^2)/(a^3b^6)))/(1458h^3/b^9 - 27(b^3cd + 10a^2b^3fg + 81a^3h^2 + (2df + 5cg)ab^2)h/(a^3b^9) + (b^4c^3 + a^2b^3d^3 + 6ab^3c^2f + 12a^2b^2c^2f^2 + 8a^3b^2f^3 + 15a^2b^2d^2g + 75a^3b^2d^2g^2 + 125a^4g^3)/(a^5b^8) + (b^5c^3 + 729a^5h^3 - 5(25g^3 - 54fgh)a^4b + (8f^3 + 135cgh - 3(25g^2 - 18fh)d)a^3b^2 - 3(5d^2g - (4f^2 + 9dh)c)a^2b^3 - (d^3 - 6c^2f)ab^4)/(a^5b^9))^{1/3} + (1/2)^{1/3}(I\sqrt{3} + 1)(1458h^3/b^9 - 27(b^3cd + 10a^2b^3fg + 81a^3h^2 + (2df + 5cg)ab^2)h/(a^3b^9) + (b^4c^3 + a^2b^3d^3 + 6ab^3c^2f + 12a^2b^2c^2f^2 + 8a^3b^2f^3 + 15a^2b^2d^2g + 75a^3b^2d^2g^2 + 125a^4g^3)/(a^5b^8) + (b^5c^3 + 729a^5h^3 - 5(25g^3 - 54fgh)a^4b + (8f^3 + 135cgh - 3(25g^2 - 18fh)d)a^3b^2 - 3(5d^2g - (4f^2 + 9dh)c)a^2b^3 - (d^3 - 6c^2f)ab^4)/(a^5b^9))^{1/3} - 18h/b^3) + 3\sqrt{1/3}(ab^5x^6 + 2a^2b^4x^3 + a^3b^3)\sqrt{-((2(1/2)^{2/3})(-I\sqrt{3} + 1)(81h^2/b^6 - (b^3cd + 10a^2b^3fg + 81a^3h^2 + (2df + 5cg)ab^2)/(a^3b^6)))/(1458h^3/b^9 - 27(b^3cd + 10a^2b^3fg + 81a^3h^2 + (2df + 5cg)ab^2)h/(a^3b^9) + (b^4c^3 + a^2b^3d^3 + 6ab^3c^2f + 12a^2b^2c^2f^2 + 8a^3b^2f^3 + 15a^2b^2d^2g + 75a^3b^2d^2g^2 + 125a^4g^3)/(a^5b^8) + (b^5c^3 + 729a^5h^3 - 5(25g^3 - 54fgh)a^4b + (8f^3 + 135cgh - 3(25g^2 - 18fh)d)a^3b^2 - 3(5d^2g - (4f^2 + 9dh)c)a^2b^3 - (d^3 - 6c^2f)ab^4)/(a^5b^9))^{1/3} + (1/2)^{1/3}(I\sqrt{3} + 1)(1458h^3/b^9 - 27(b^3cd + 10a^2b^3fg + 81a^3h^2 + (2df + 5cg)ab^2)h/(a^3b^9) + (b^4c^3 + a^2b^3d^3 + 6ab^3c^2f + 12a^2b^2c^2f^2 + 8a^3b^2f^3 + 15a^2b^2d^2g + 75a^3b^2d^2g^2 + 125a^4g^3)/(a^5b^8) + (b^5c^3 + 729a^5h^3 - 5(25g^3 - 54fgh)a^4b + (8f^3 + 135cgh - 3(25g^2 - 18fh)d)a^3b^2 - 3(5d^2g - (4f^2 + 9dh)c)a^2b^3 - (d^3 - 6c^2f)ab^4)/(a^5b^9))^{1/3} - 18h/b^3)^2a^3b^6 + 36(2(1/2)^{2/3})(-I\sqrt{3} + 1)(81h^2/b^6 - (b^3cd + 10a^2b^3fg + 81a^3h^2 + (2df + 5cg)ab^2)/(a^3b^6)
\end{aligned}$$

$$\begin{aligned}
& 6)) / (1458h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g) * a*b^2) * h / (a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3) / (a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h) * a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h) * d) * a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h) * c) * a^2*b^3 - (d^3 - 6*c^2*f) * a*b^4) / (a^5*b^9))^{(1/3)} + (1/2)^{(1/3)} * (I*sqrt(3) + 1) * (1458h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g) * a*b^2) * h / (a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3) / (a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h) * a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h) * d) * a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h) * c) * a^2*b^3 - (d^3 - 6*c^2*f) * a*b^4) / (a^5*b^9))^{(1/3)} - 18*h/b^3) * a^3*b^3*h + 16*b^3*c*d + 32*a*b^2*d*f + 324*a^3*h^2 + 80*(a*b^2*c + 2*a^2*b*f) * g) / (a^3*b^6) \\
&)) * \log(-2*a*b^4*c*d^2 - 4*a^2*b^3*d^2*f - 1/4*(a^4*b^7*d + 5*a^5*b^6*g) * (2*(1/2)^{(2/3)} * (-I*sqrt(3) + 1) * (81*h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g) * a*b^2) / (a^3*b^6)) / (1458h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g) * a*b^2) * h / (a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3) / (a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h) * a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h) * d) * a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h) * c) * a^2*b^3 - (d^3 - 6*c^2*f) * a*b^4) / (a^5*b^9))^{(1/3)} + (1/2)^{(1/3)} * (I*sqrt(3) + 1) * (1458h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g) * a*b^2) * h / (a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3) / (a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h) * a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h) * d) * a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h) * c) * a^2*b^3 - (d^3 - 6*c^2*f) * a*b^4) / (a^5*b^9))^{(1/3)} - 18*h/b^3)^2 - 50*(a^3*b^2*c + 2*a^4*b*f) * g^2 - 81*(a^4*b*d + 5*a^5*g) * h^2 + 1/2*(a^2*b^6*c^2 + 4*a^3*b^5*c*f + 4*a^4*b^4*f^2 - 18*(a^4*b^4*d + 5*a^5*b^3*g) * h) * (2*(1/2)^{(2/3)} * (-I*sqrt(3) + 1) * (81*h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g) * a*b^2) / (a^3*b^6)) / (1458h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g) * a*b^2) * h / (a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3) / (a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h) * a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h) * d) * a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h) * c) * a^2*b^3 - (d^3 - 6*c^2*f) * a*b^4) / (a^5*b^9))^{(1/3)} + (1/2)^{(1/3)} * (I*sqrt(3) + 1) * (1458h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g) * a*b^2) * h / (a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3) / (a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h) * a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h) * d) * a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h) * c) * a^2*b^3 - (d^3 - 6*c^2*f) * a*b^4) / (a^5*b^9))^{(1/3)} - 18*h/b^3) - 20*(a^2*b^3*c*d + 2*a^3*b^2*d*f) * g + 9*(a^2*b^3*c^2 + 4*a^3*b^2*c*f + 4*a^4*b*f^2) * h + 2*(b^5*c^3 + a*b^4*d^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 + 15*a^2*b^3*d^2*g
\end{aligned}$$

$$\begin{aligned}
& + 75a^3b^2d^2g^2 + 125a^4b^2g^3)x + 3/4\sqrt{1/3}(2a^2b^6c^2 + 8a^3b^5c^2f + 8a^4b^4f^2 + (a^4b^7d + 5a^5b^6g)(2(1/2)^{(2/3)}(-I\sqrt{3} + 1)(81h^2/b^6 - (b^3cd + 10a^2bfg + 81a^3h^2 + (2df + 5c^2g)ab^2)/(a^3b^6)))/(1458h^3/b^9 - 27(b^3cd + 10a^2bfg + 81a^3h^2 + (2df + 5c^2g)ab^2)h/(a^3b^9) + (b^4c^3 + ab^3d^3 + 6ab^3c^2f + 12a^2b^2c^2f^2 + 8a^3b^2f^3 + 15a^2b^2d^2g + 75a^3b^2d^2g^2 + 125a^4g^3)/(a^5b^8) + (b^5c^3 + 729a^5h^3 - 5(25g^3 - 54fgh)a^4b + (8f^3 + 135cgh - 3(25g^2 - 18fh)d)a^3b^2 - 3(5d^2g - (4f^2 + 9d^2h)c)a^2b^3 - (d^3 - 6c^2f)ab^4)/(a^5b^9))^{(1/3)} + (1/2)^{(1/3)}(I\sqrt{3} + 1)(1458h^3/b^9 - 27(b^3cd + 10a^2bfg + 81a^3h^2 + (2df + 5c^2g)ab^2)h/(a^3b^9) + (b^4c^3 + ab^3d^3 + 6ab^3c^2f + 12a^2b^2c^2f^2 + 8a^3b^2f^3 + 15a^2b^2d^2g + 75a^3b^2d^2g^2 + 125a^4g^3)/(a^5b^8) + (b^5c^3 + 729a^5h^3 - 5(25g^3 - 54fgh)a^4b + (8f^3 + 135cgh - 3(25g^2 - 18fh)d)a^3b^2 - 3(5d^2g - (4f^2 + 9d^2h)c)a^2b^3 - (d^3 - 6c^2f)ab^4)/(a^5b^9))^{(1/3)} - 18h/b^3) + 18(a^4b^4d + 5a^5b^3g)h)\sqrt{-((2(1/2)^{(2/3)}(-I\sqrt{3} + 1)(81h^2/b^6 - (b^3cd + 10a^2bfg + 81a^3h^2 + (2df + 5c^2g)ab^2)/(a^3b^6)))/(1458h^3/b^9 - 27(b^3cd + 10a^2bfg + 81a^3h^2 + (2df + 5c^2g)ab^2)h/(a^3b^9) + (b^4c^3 + ab^3d^3 + 6ab^3c^2f + 12a^2b^2c^2f^2 + 8a^3b^2f^3 + 15a^2b^2d^2g + 75a^3b^2d^2g^2 + 125a^4g^3)/(a^5b^8) + (b^5c^3 + 729a^5h^3 - 5(25g^3 - 54fgh)a^4b + (8f^3 + 135cgh - 3(25g^2 - 18fh)d)a^3b^2 - 3(5d^2g - (4f^2 + 9d^2h)c)a^2b^3 - (d^3 - 6c^2f)ab^4)/(a^5b^9))^{(1/3)} + (1/2)^{(1/3)}(I\sqrt{3} + 1)(1458h^3/b^9 - 27(b^3cd + 10a^2bfg + 81a^3h^2 + (2df + 5c^2g)ab^2)h/(a^3b^9) + (b^4c^3 + ab^3d^3 + 6ab^3c^2f + 12a^2b^2c^2f^2 + 8a^3b^2f^3 + 15a^2b^2d^2g + 75a^3b^2d^2g^2 + 125a^4g^3)/(a^5b^8) + (b^5c^3 + 729a^5h^3 - 5(25g^3 - 54fgh)a^4b + (8f^3 + 135cgh - 3(25g^2 - 18fh)d)a^3b^2 - 3(5d^2g - (4f^2 + 9d^2h)c)a^2b^3 - (d^3 - 6c^2f)ab^4)/(a^5b^9))^{(1/3)} - 18h/b^3)^2a^3b^6 + 36(2(1/2)^{(2/3)}(-I\sqrt{3} + 1)(81h^2/b^6 - (b^3cd + 10a^2bfg + 81a^3h^2 + (2df + 5c^2g)ab^2)/(a^3b^6)))/(1458h^3/b^9 - 27(b^3cd + 10a^2bfg + 81a^3h^2 + (2df + 5c^2g)ab^2)h/(a^3b^9) + (b^4c^3 + ab^3d^3 + 6ab^3c^2f + 12a^2b^2c^2f^2 + 8a^3b^2f^3 + 15a^2b^2d^2g + 75a^3b^2d^2g^2 + 125a^4g^3)/(a^5b^8) + (b^5c^3 + 729a^5h^3 - 5(25g^3 - 54fgh)a^4b + (8f^3 + 135cgh - 3(25g^2 - 18fh)d)a^3b^2 - 3(5d^2g - (4f^2 + 9d^2h)c)a^2b^3 - (d^3 - 6c^2f)ab^4)/(a^5b^9))^{(1/3)} + (1/2)^{(1/3)}(I\sqrt{3} + 1)(1458h^3/b^9 - 27(b^3cd + 10a^2bfg + 81a^3h^2 + (2df + 5c^2g)ab^2)h/(a^3b^9) + (b^4c^3 + ab^3d^3 + 6ab^3c^2f + 12a^2b^2c^2f^2 + 8a^3b^2f^3 + 15a^2b^2d^2g + 75a^3b^2d^2g^2 + 125a^4g^3)/(a^5b^8) + (b^5c^3 + 729a^5h^3 - 5(25g^3 - 54fgh)a^4b + (8f^3 + 135cgh - 3(25g^2 - 18fh)d)a^3b^2 - 3(5d^2g - (4f^2 + 9d^2h)c)a^2b^3 - (d^3 - 6c^2f)ab^4)/(a^5b^9))^{(1/3)} - 18h/b^3)a^3b^3h + 16b^3cd + 32ab^2df + 324a^3h^2 + 80(ab^2c + 2a^2bf)g)/(a^3b^6)) + (54ab^2hx^6 + 108a^2bhx^3 + 54a^3h + (ab^5x^6 + 2a^2b^4x^3 + a^3b^3)(2(1/2)^{(2/3)}(-I\sqrt{3} + 1)(81h^2/b^6 - (b^3cd + 10a^2bfg + 81a^3h^2 + (2df + 5c^2g)ab^2)/(a^3b^6)))/(1458h^3/b^9 - 27(b^3cd + 10a^2bfg + 81a^3h^2 + (2df + 5c^2g)ab^2)h/(a^3b^9) + (b^4c^3 + ab^3d^3 + 6ab^3c^2f + 12a^2b^2c^2f^2 + 8a^3b^2f^3 + 15a^2b^2d^2g + 75a^3b^2d^2g^2 + 125a^4g^3)/(a^5b^8) + (b^5c^3 + 729a^5h^3 - 5(25g^3 - 54fgh)a^4b + (8f^3 + 135cgh - 3(25g^2 - 18fh)d)a^3b^2 - 3(5d^2g - (4f^2 + 9d^2h)c)a^2b^3 - (d^3 - 6c^2f)ab^4)/(a^5b^9))^{(1/3)} + (1/2)^{(1/3)}(I\sqrt{3} + 1)(1458h^3/b^9 - 27(b^3cd + 10a^2bfg + 81a^3h^2 + (2df + 5c^2g)ab^2)h/(a^3b^9) + (b^4c^3 + ab^3d^3 + 6ab^3c^2f + 12a^2b^2c^2f^2 + 8a^3b^2f^3 + 15a^2b^2d^2g + 75a^3b^2d^2g^2 + 125a^4g^3)/(a^5b^8) + (b^5c^3 + 729a^5h^3 - 5(25g^3 - 54fgh)a^4b + (8f^3 + 135cgh - 3(25g^2 - 18fh)d)a^3b^2 - 3(5d^2g - (4f^2 + 9d^2h)c)a^2b^3 - (d^3 - 6c^2f)ab^4)/(a^5b^9))^{(1/3)} - 18h/b^3)
\end{aligned}$$

$$\begin{aligned}
& /3)*(-I*\sqrt{3} + 1)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (\\
& 2*d*f + 5*c*g)*a*b^2)/(a^3*b^6)))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g \\
& + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + \\
& 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3 \\
& *b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54 \\
& *f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5* \\
& d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3 \\
&) + (1/2)^(1/3)*(I*\sqrt{3} + 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g \\
& + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + \\
& 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3* \\
& b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54* \\
& f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d \\
& ^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) \\
& - 18*h/b^3) - 3*\sqrt{1/3}*(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3)*\sqrt{-((2* \\
& (1/2)^(2/3)*(-I*\sqrt{3} + 1)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3 \\
& *h^2 + (2*d*f + 5*c*g)*a*b^2)/(a^3*b^6)))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a \\
& ^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b \\
& ^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g \\
& + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25* \\
& g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 \\
& - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^ \\
& 9))^(1/3) + (1/2)^(1/3)*(I*\sqrt{3} + 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^ \\
& 2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^ \\
& 3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + \\
& 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g \\
& ^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 \\
& - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9 \\
&))^(1/3) - 18*h/b^3)^2*a^3*b^6 + 36*(2*(1/2)^(2/3)*(-I*\sqrt{3} + 1)*(81*h^2 \\
& /b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)/(a^3*b \\
& ^6)))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c \\
& *g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2* \\
& c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5 \\
& *b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135 \\
& *c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a \\
& ^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) + (1/2)^(1/3)*(I*\sqrt{3} + \\
& 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c* \\
& g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c \\
& *f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5* \\
& b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135* \\
& c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^ \\
& 2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) - 18*h/b^3)*a^3*b^3*h + 16* \\
& b^3*c*d + 32*a*b^2*d*f + 324*a^3*h^2 + 80*(a*b^2*c + 2*a^2*b*f)*g)/(a^3*b^6 \\
&))*\log(-2*a*b^4*c*d^2 - 4*a^2*b^3*d^2*f - 1/4*(a^4*b^7*d + 5*a^5*b^6*g)*(2 \\
& *(1/2)^(2/3)*(-I*\sqrt{3} + 1)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^ \\
& 3*h^2 + (2*d*f + 5*c*g)*a*b^2)/(a^3*b^6)))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*
\end{aligned}$$

$$\begin{aligned}
& a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a* \\
& b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g \\
& + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25* \\
& *g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^ \\
& 2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b \\
& ^9))^{(1/3)} + (1/2)^{(1/3)}*(I*sqrt(3) + 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a \\
& ^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b \\
& ^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g \\
& + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25* \\
& g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 \\
& - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^ \\
& 9))^{(1/3)} - 18*h/b^3)^2 - 50*(a^3*b^2*c + 2*a^4*b*f)*g^2 - 81*(a^4*b*d + 5* \\
& a^5*g)*h^2 + 1/2*(a^2*b^6*c^2 + 4*a^3*b^5*c*f + 4*a^4*b^4*f^2 - 18*(a^4*b^4 \\
& *d + 5*a^5*b^3*g)*h)*(2*(1/2)^{(2/3)}*(-I*sqrt(3) + 1)*(81*h^2/b^6 - (b^3*c*d \\
& + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)/(a^3*b^6)))/(1458*h^3/ \\
& b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a \\
& ^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b* \\
& f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 \\
& + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g \\
& ^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - \\
& 6*c^2*f)*a*b^4)/(a^5*b^9))^{(1/3)} + (1/2)^{(1/3)}*(I*sqrt(3) + 1)*(1458*h^3/b \\
& ^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^ \\
& 3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f \\
& f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 \\
& + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g \\
& ^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - \\
& 6*c^2*f)*a*b^4)/(a^5*b^9))^{(1/3)} - 18*h/b^3) - 20*(a^2*b^3*c*d + 2*a^3*b^2* \\
& d*f)*g + 9*(a^2*b^3*c^2 + 4*a^3*b^2*c*f + 4*a^4*b*f^2)*h + 2*(b^5*c^3 + a*b \\
& ^4*d^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 + 15*a^2*b^3*d^2* \\
& g + 75*a^3*b^2*d*g^2 + 125*a^4*b*g^3)*x - 3/4*sqrt(1/3)*(2*a^2*b^6*c^2 + 8* \\
& a^3*b^5*c*f + 8*a^4*b^4*f^2 + (a^4*b^7*d + 5*a^5*b^6*g)*(2*(1/2)^{(2/3)}*(-I* \\
& sqrt(3) + 1)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + \\
& 5*c*g)*a*b^2)/(a^3*b^6)))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^ \\
& 3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3 \\
& *c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 \\
& + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)* \\
& a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - \\
& (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^{(1/3)} + (1/2 \\
&)^{(1/3)}*(I*sqrt(3) + 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3 \\
& *h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3* \\
& c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 \\
& + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a \\
& ^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (\\
& 4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^{(1/3)} - 18*h/ \\
& b^3) + 18*(a^4*b^4*d + 5*a^5*b^3*g)*h)*sqrt(-((2*(1/2)^{(2/3)}*(-I*sqrt(3) +
\end{aligned}$$

$$\begin{aligned}
& 1) \cdot (81 \cdot h^2 / b^6 - (b^3 \cdot c \cdot d + 10 \cdot a^2 \cdot b \cdot f \cdot g + 81 \cdot a^3 \cdot h^2 + (2 \cdot d \cdot f + 5 \cdot c \cdot g) \cdot a \cdot b^2) / (a^3 \cdot b^6)) / (1458 \cdot h^3 / b^9 - 27 \cdot (b^3 \cdot c \cdot d + 10 \cdot a^2 \cdot b \cdot f \cdot g + 81 \cdot a^3 \cdot h^2 + (2 \cdot d \cdot f + 5 \cdot c \cdot g) \cdot a \cdot b^2) \cdot h / (a^3 \cdot b^9) + (b^4 \cdot c^3 + a \cdot b^3 \cdot d^3 + 6 \cdot a \cdot b^3 \cdot c^2 \cdot f + 12 \cdot a^2 \cdot b^2 \cdot c \cdot f^2 + 8 \cdot a^3 \cdot b \cdot f^3 + 15 \cdot a^2 \cdot b^2 \cdot d^2 \cdot g + 75 \cdot a^3 \cdot b \cdot d \cdot g^2 + 125 \cdot a^4 \cdot g^3) / (a^5 \cdot b^8) + (b^5 \cdot c^3 + 729 \cdot a^5 \cdot h^3 - 5 \cdot (25 \cdot g^3 - 54 \cdot f \cdot g \cdot h) \cdot a^4 \cdot b + (8 \cdot f^3 + 135 \cdot c \cdot g \cdot h - 3 \cdot (25 \cdot g^2 - 18 \cdot f \cdot h) \cdot d) \cdot a^3 \cdot b^2 - 3 \cdot (5 \cdot d^2 \cdot g - (4 \cdot f^2 + 9 \cdot d \cdot h) \cdot c) \cdot a^2 \cdot b^3 - (d^3 - 6 \cdot c^2 \cdot f) \cdot a \cdot b^4) / (a^5 \cdot b^9))^{1/3} + (1/2)^{1/3} \cdot (I \cdot \sqrt{3} + 1) \cdot (1458 \cdot h^3 / b^9 - 27 \cdot (b^3 \cdot c \cdot d + 10 \cdot a^2 \cdot b \cdot f \cdot g + 81 \cdot a^3 \cdot h^2 + (2 \cdot d \cdot f + 5 \cdot c \cdot g) \cdot a \cdot b^2) \cdot h / (a^3 \cdot b^9) + (b^4 \cdot c^3 + a \cdot b^3 \cdot d^3 + 6 \cdot a \cdot b^3 \cdot c^2 \cdot f + 12 \cdot a^2 \cdot b^2 \cdot c \cdot f^2 + 8 \cdot a^3 \cdot b \cdot f^3 + 15 \cdot a^2 \cdot b^2 \cdot d^2 \cdot g + 75 \cdot a^3 \cdot b \cdot d \cdot g^2 + 125 \cdot a^4 \cdot g^3) / (a^5 \cdot b^8) + (b^5 \cdot c^3 + 729 \cdot a^5 \cdot h^3 - 5 \cdot (25 \cdot g^3 - 54 \cdot f \cdot g \cdot h) \cdot a^4 \cdot b + (8 \cdot f^3 + 135 \cdot c \cdot g \cdot h - 3 \cdot (25 \cdot g^2 - 18 \cdot f \cdot h) \cdot d) \cdot a^3 \cdot b^2 - 3 \cdot (5 \cdot d^2 \cdot g - (4 \cdot f^2 + 9 \cdot d \cdot h) \cdot c) \cdot a^2 \cdot b^3 - (d^3 - 6 \cdot c^2 \cdot f) \cdot a \cdot b^4) / (a^5 \cdot b^9))^{1/3} - 18 \cdot h / b^3)^{2/3} \cdot a^3 \cdot b^6 + 36 \cdot (2 \cdot (1/2)^{2/3} \cdot (-I \cdot \sqrt{3} + 1) \cdot (81 \cdot h^2 / b^6 - (b^3 \cdot c \cdot d + 10 \cdot a^2 \cdot b \cdot f \cdot g + 81 \cdot a^3 \cdot h^2 + (2 \cdot d \cdot f + 5 \cdot c \cdot g) \cdot a \cdot b^2) / (a^3 \cdot b^6)) / (1458 \cdot h^3 / b^9 - 27 \cdot (b^3 \cdot c \cdot d + 10 \cdot a^2 \cdot b \cdot f \cdot g + 81 \cdot a^3 \cdot h^2 + (2 \cdot d \cdot f + 5 \cdot c \cdot g) \cdot a \cdot b^2) \cdot h / (a^3 \cdot b^9) + (b^4 \cdot c^3 + a \cdot b^3 \cdot d^3 + 6 \cdot a \cdot b^3 \cdot c^2 \cdot f + 12 \cdot a^2 \cdot b^2 \cdot c \cdot f^2 + 8 \cdot a^3 \cdot b \cdot f^3 + 15 \cdot a^2 \cdot b^2 \cdot d^2 \cdot g + 75 \cdot a^3 \cdot b \cdot d \cdot g^2 + 125 \cdot a^4 \cdot g^3) / (a^5 \cdot b^8) + (b^5 \cdot c^3 + 729 \cdot a^5 \cdot h^3 - 5 \cdot (25 \cdot g^3 - 54 \cdot f \cdot g \cdot h) \cdot a^4 \cdot b + (8 \cdot f^3 + 135 \cdot c \cdot g \cdot h - 3 \cdot (25 \cdot g^2 - 18 \cdot f \cdot h) \cdot d) \cdot a^3 \cdot b^2 - 3 \cdot (5 \cdot d^2 \cdot g - (4 \cdot f^2 + 9 \cdot d \cdot h) \cdot c) \cdot a^2 \cdot b^3 - (d^3 - 6 \cdot c^2 \cdot f) \cdot a \cdot b^4) / (a^5 \cdot b^9))^{1/3} + (1/2)^{1/3} \cdot (I \cdot \sqrt{3} + 1) \cdot (1458 \cdot h^3 / b^9 - 27 \cdot (b^3 \cdot c \cdot d + 10 \cdot a^2 \cdot b \cdot f \cdot g + 81 \cdot a^3 \cdot h^2 + (2 \cdot d \cdot f + 5 \cdot c \cdot g) \cdot a \cdot b^2) \cdot h / (a^3 \cdot b^9) + (b^4 \cdot c^3 + a \cdot b^3 \cdot d^3 + 6 \cdot a \cdot b^3 \cdot c^2 \cdot f + 12 \cdot a^2 \cdot b^2 \cdot c \cdot f^2 + 8 \cdot a^3 \cdot b \cdot f^3 + 15 \cdot a^2 \cdot b^2 \cdot d^2 \cdot g + 75 \cdot a^3 \cdot b \cdot d \cdot g^2 + 125 \cdot a^4 \cdot g^3) / (a^5 \cdot b^8) + (b^5 \cdot c^3 + 729 \cdot a^5 \cdot h^3 - 5 \cdot (25 \cdot g^3 - 54 \cdot f \cdot g \cdot h) \cdot a^4 \cdot b + (8 \cdot f^3 + 135 \cdot c \cdot g \cdot h - 3 \cdot (25 \cdot g^2 - 18 \cdot f \cdot h) \cdot d) \cdot a^3 \cdot b^2 - 3 \cdot (5 \cdot d^2 \cdot g - (4 \cdot f^2 + 9 \cdot d \cdot h) \cdot c) \cdot a^2 \cdot b^3 - (d^3 - 6 \cdot c^2 \cdot f) \cdot a \cdot b^4) / (a^5 \cdot b^9))^{1/3} - 18 \cdot h / b^3) \cdot a^3 \cdot b^3 \cdot h + 16 \cdot b^3 \cdot c \cdot d + 32 \cdot a \cdot b^2 \cdot d \cdot f + 3 \cdot 24 \cdot a^3 \cdot h^2 + 80 \cdot (a \cdot b^2 \cdot c + 2 \cdot a^2 \cdot b \cdot f) \cdot g) / (a^3 \cdot b^6)) / (a \cdot b^5 \cdot x^6 + 2 \cdot a^2 \cdot b^4 \cdot x^3 + a^3 \cdot b^3)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.08958, size = 512, normalized size = 1.58

$$\frac{h \log(|bx^3 + a|)}{3b^3} + \frac{\sqrt{3} \left((-ab^2)^{\frac{1}{3}} b^2 c + 2 (-ab^2)^{\frac{1}{3}} abf - (-ab^2)^{\frac{2}{3}} bd - 5 (-ab^2)^{\frac{2}{3}} ag \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{27 a^2 b^4} + \frac{2(b^2 d - 4 a^2 g)}{18 a^2 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac")

[Out] 1/3*h*log(abs(b*x^3 + a))/b^3 + 1/27*sqrt(3)*((-a*b^2)^(1/3)*b^2*c + 2*(-a*b^2)^(1/3)*a*b*f - (-a*b^2)^(2/3)*b*d - 5*(-a*b^2)^(2/3)*a*g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^4) + 1/18*(2*(b^2*d - 4*a*b*g)*x^5 + (b^2*c - 7*a*b*f)*x^4 + 6*(2*a^2*h - a*b*e)*x^3 - (a*b*d + 5*a^2*g)*x^2 - 2*(a*b*c + 2*a^2*f)*x + 3*(3*a^3*h - a^2*b*e)/b)/((b*x^3 + a)^2*a*b^2) + 1/54*((-a*b^2)^(1/3)*b^2*c + 2*(-a*b^2)^(1/3)*a*b*f + (-a*b^2)^(2/3)*b*d + 5*(-a*b^2)^(2/3)*a*g)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^4) - 1/27*(a*b^4*d*(-a/b)^(1/3) + 5*a^2*b^3*g*(-a/b)^(1/3) + a*b^4*c + 2*a^2*b^3*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^5)

$$3.423 \quad \int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=297

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(\sqrt[3]{b}(2ag+bd) - \sqrt[3]{a}(5ah+be)\right)}{54a^{5/3}b^{8/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(\sqrt[3]{b}(2ag+bd) - \sqrt[3]{a}(5ah+be)\right)}{27a^{5/3}b^{8/3}} - \frac{\tan^{-1}\left(\frac{a^{1/3} - 2b^{1/3}x}{\sqrt[3]{a} + \sqrt[3]{bx}}\right)}{\sqrt[3]{a} + \sqrt[3]{bx}}$$

[Out] (x*(b*d - 4*a*g + (2*b*e - 5*a*h)*x + 3*b*f*x^2))/(18*a*b^2*(a + b*x^3)) - (c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(6*b*(a + b*x^3)^2) - ((b^(4/3)*d + a^(1/3)*b*e + 2*a*b^(1/3)*g + 5*a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(5/3)*b^(8/3)) + ((b^(1/3)*(b*d + 2*a*g) - a^(1/3)*(b*e + 5*a*h))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(5/3)*b^(8/3)) - ((b^(1/3)*(b*d + 2*a*g) - a^(1/3)*(b*e + 5*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(8/3))

Rubi [A] time = 0.430108, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {1823, 1858, 1860, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(\sqrt[3]{b}(2ag+bd) - \sqrt[3]{a}(5ah+be)\right)}{54a^{5/3}b^{8/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(\sqrt[3]{b}(2ag+bd) - \sqrt[3]{a}(5ah+be)\right)}{27a^{5/3}b^{8/3}} - \frac{\tan^{-1}\left(\frac{a^{1/3} - 2b^{1/3}x}{\sqrt[3]{a} + \sqrt[3]{bx}}\right)}{\sqrt[3]{a} + \sqrt[3]{bx}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3, x]

[Out] (x*(b*d - 4*a*g + (2*b*e - 5*a*h)*x + 3*b*f*x^2))/(18*a*b^2*(a + b*x^3)) - (c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(6*b*(a + b*x^3)^2) - ((b^(4/3)*d + a^(1/3)*b*e + 2*a*b^(1/3)*g + 5*a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(5/3)*b^(8/3)) + ((b^(1/3)*(b*d + 2*a*g) - a^(1/3)*(b*e + 5*a*h))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(5/3)*b^(8/3)) - ((b^(1/3)*(b*d + 2*a*g) - a^(1/3)*(b*e + 5*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(8/3))

Rule 1823

Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Pq*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*

$(a + b*x^n)^{(p + 1)}, x], x] /;$ FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && EqQ[m - n + 1, 0] && LtQ[p, -1]

Rule 1858

Int[(Pq_)*((a_) + (b_)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]

Rule 1860

Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]

Rule 31

Int[((a_) + (b_)*(x_))^(p_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
 imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\int \frac{x^2 (c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{(a + bx^3)^3} dx = -\frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{6b(a + bx^3)^2} + \frac{\int \frac{d+2ex+3fx^2+4gx^3+5hx^4}{(a+bx^3)^2} dx}{6b}$$

$$= \frac{x (bd - 4ag + (2be - 5ah)x + 3bfx^2)}{18ab^2 (a + bx^3)} - \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{6b (a + bx^3)^2} - \frac{\int \frac{d+2ex+3fx^2+4gx^3+5hx^4}{(a+bx^3)^2} dx}{6b}$$

$$= \frac{x (bd - 4ag + (2be - 5ah)x + 3bfx^2)}{18ab^2 (a + bx^3)} - \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{6b (a + bx^3)^2} - \frac{\int \frac{d+2ex+3fx^2+4gx^3+5hx^4}{(a+bx^3)^2} dx}{6b}$$

$$= \frac{x (bd - 4ag + (2be - 5ah)x + 3bfx^2)}{18ab^2 (a + bx^3)} - \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{6b (a + bx^3)^2} + \frac{\int \frac{d+2ex+3fx^2+4gx^3+5hx^4}{(a+bx^3)^2} dx}{6b}$$

$$= \frac{x (bd - 4ag + (2be - 5ah)x + 3bfx^2)}{18ab^2 (a + bx^3)} - \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{6b (a + bx^3)^2} + \frac{\int \frac{d+2ex+3fx^2+4gx^3+5hx^4}{(a+bx^3)^2} dx}{6b}$$

$$= \frac{x (bd - 4ag + (2be - 5ah)x + 3bfx^2)}{18ab^2 (a + bx^3)} - \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{6b (a + bx^3)^2} - \frac{\int \frac{d+2ex+3fx^2+4gx^3+5hx^4}{(a+bx^3)^2} dx}{6b}$$

Mathematica [A] time = 0.234061, size = 287, normalized size = 0.97

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2\right) \left(5a^{4/3} h + \sqrt[3]{abe} - 2a \sqrt[3]{bg} - b^{4/3} d\right)}{a^{5/3}} + \frac{2 \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right) \left(-5a^{4/3} h - \sqrt[3]{abe} + 2a \sqrt[3]{bg} + b^{4/3} d\right)}{a^{5/3}} - \frac{2\sqrt{3} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{bx}}{\sqrt[3]{a}}\right) \left(5a^{4/3} h + \sqrt[3]{abe} + 2a \sqrt[3]{bg} + b^{4/3} d\right)}{a^{5/3}}$$

$$54b^{8/3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]

[Out]
$$\begin{aligned} &((-9*b^{(2/3)}*(b*(c + x*(d + e*x)) - a*(f + x*(g + h*x))))/(a + b*x^3)^2 + (\\ &3*b^{(2/3)}*(b*x*(d + 2*e*x) - a*(6*f + x*(7*g + 8*h*x)))/(a*(a + b*x^3)) - \\ &(2*sqrt[3]*(b^{(4/3)}*d + a^{(1/3)}*b*e + 2*a*b^{(1/3)}*g + 5*a^{(4/3)}*h)*ArcTan[(\\ &1 - (2*b^{(1/3)}*x)/a^{(1/3)})/sqrt[3]])/a^{(5/3)} + (2*(b^{(4/3)}*d - a^{(1/3)}*b*e \\ &+ 2*a*b^{(1/3)}*g - 5*a^{(4/3)}*h)*Log[a^{(1/3)} + b^{(1/3)}*x])/a^{(5/3)} + ((- (b^{(4/3)}*d) \\ &+ a^{(1/3)}*b*e - 2*a*b^{(1/3)}*g + 5*a^{(4/3)}*h)*Log[a^{(2/3)} - a^{(1/3)}*b \\ &^{(1/3)}*x + b^{(2/3)}*x^2])/a^{(5/3)})/(54*b^{(8/3)}) \end{aligned}$$

Maple [A] time = 0.011, size = 490, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x)

[Out]
$$\begin{aligned} &(-1/9*(4*a*h-b*e)/a/b*x^5-1/18*(7*a*g-b*d)/a/b*x^4-1/3*f*x^3/b-1/18*(5*a*h+ \\ &b*e)/b^2*x^2-1/9*(2*a*g+b*d)/b^2*x-1/6*(a*f+b*c)/b^2)/(b*x^3+a)^2+2/27/b^3/ \\ &(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*g+1/27/b^2/a/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)}) \\ &)*d-1/27/b^3/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*g-1/54 \\ &/b^2/a/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*d+2/27/b^3/(1/b* \\ &a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*g+1/27/b^2/a/(1/ \\ &b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*d-5/27/b^3/(1/ \\ &b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})*h-1/27/a/b^2/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)} \\ &))*e+5/54/b^3/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*h+1/54/a/ \\ &b^2/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*e+5/27/b^3*3^{(1/2)}/ \\ &(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*h+1/27/a/b^2*3^{(1/2)} \\ &)/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*e \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [C] time = 19.2287, size = 15354, normalized size = 51.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/108*(36*a*b*f*x^3 - 12*(b^2*e - 4*a*b*h)*x^5 - 6*(b^2*d - 7*a*b*g)*x^4 + \\ & 18*a*b*c + 18*a^2*f + 6*(a*b*e + 5*a^2*h)*x^2 + 2*(a*b^4*x^6 + 2*a^2*b^3*x \\ & ^3 + a^3*b^2)*((1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g \\ & + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 \\ & + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b \\ & + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^{(1/3)} \\ & - 2*(1/2)^{(2/3)}*(b^2*d*e + 10*a^2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*\text{sqrt}(3) + \\ & 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 \\ & + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 \\ & + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^{(1/3)})) \\ & * \log(2*a*b^3*d*e^2 + 4*a^2*b^2*e^2*g + 1/4*(a^4*b^6*e + 5*a^5*b^5*h)*((1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*((b^4 \\ & *d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h \\ & + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 \\ & + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^{(1/3)} \\ & - 2*(1/2)^{(2/3)}*(b^2*d*e + 10*a^2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*\text{sqrt}(3) + 1)/(a^3*b^5*((b^4*d^3 \\ & + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 1 \\ & 25*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 \\ & - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^{(1/3)}))^{(1/3)} \\ & + 50*(a^3*b*d + 2*a^4*g)*h^2 - 1/2*(a^2*b^5*d^2 + 4*a^3*b^4*d*g + 4*a^4*b^3*g^2)*((1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g \\ & + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 \\ & + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^{(1/3)} \\ & - 2*(1/2)^{(2/3)}*(b^2*d*e + 10*a^2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*\text{sqrt}(3) + 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 \\ & + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 \\ & + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^{(1/3)})) \\ & + 20*(a^2*b^2*d*e + 2*a^3*b*e*g)*h + (b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + \end{aligned}$$

$$\begin{aligned}
& 15a^2b^2e^2h + 75a^3b^2e^2h^2 + 125a^4h^3)x) + 12*(a*b*d + 2*a^2*g) \\
& *x - ((a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)*((1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*((\\
& b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a \\
& ^2*b^2*e^2*h + 75*a^3*b^2*e^2*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4 \\
& *h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6* \\
& d^2*g)*a*b^3)/(a^5*b^8))^{(1/3)} - 2*(1/2)^{(2/3)}*(b^2*d*e + 10*a^2*g*h + (2*e \\
& *g + 5*d*h)*a*b)*(-I*\text{sqrt}(3) + 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3* \\
& d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b^2*e^2*h^2 \\
& + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b \\
& + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^{(1/3)} \\
&)) + 3*\text{sqrt}(1/3)*(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)*\text{sqrt}(-(((1/2)^{(1/3)}* \\
& (I*\text{sqrt}(3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + \\
& 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b^2*e^2*h^2 + 125*a^4*h^3)/(a^5*b^8) + \\
& (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a \\
& ^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^{(1/3)} - 2*(1/2)^{(2/3)}*(b^2*d*e + \\
& 10*a^2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*\text{sqrt}(3) + 1)/(a^3*b^5*((b^4*d^3 + a* \\
& b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h \\
& + 75*a^3*b^2*e^2*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^ \\
& 3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3 \\
&))/(a^5*b^8))^{(1/3)})^2*a^3*b^5 + 16*b^2*d*e + 32*a*b*e*g + 80*(a*b*d + 2*a^ \\
& 2*g)*h)/(a^3*b^5))*\log(-2*a*b^3*d*e^2 - 4*a^2*b^2*e^2*g - 1/4*(a^4*b^6*e + \\
& 5*a^5*b^5*h)*((1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3* \\
& d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b^2*e^2*h^2 \\
& + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b \\
& + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^{(1/3)} \\
& - 2*(1/2)^{(2/3)}*(b^2*d*e + 10*a^2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*\text{sqrt}(3) + \\
& 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a \\
& ^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b^2*e^2*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^ \\
& 4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2* \\
& b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^{(1/3)})^2 - 50*(a^3*b*d + 2*a^4*g)* \\
& h^2 + 1/2*(a^2*b^5*d^2 + 4*a^3*b^4*d*g + 4*a^4*b^3*g^2)*((1/2)^{(1/3)}*(I*\text{sq} \\
& r\text{t}(3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3* \\
& b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b^2*e^2*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d \\
& ^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 \\
& - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^{(1/3)} - 2*(1/2)^{(2/3)}*(b^2*d*e + 10*a^ \\
& 2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*\text{sqrt}(3) + 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^ \\
& 3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75* \\
& a^3*b^2*e^2*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75 \\
& *e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5 \\
& *b^8))^{(1/3)}) - 20*(a^2*b^2*d*e + 2*a^3*b^2*e*g)*h + 2*(b^4*d^3 + a*b^3*e^3 \\
& + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^ \\
& 3*b^2*e^2*h^2 + 125*a^4*h^3)*x + 3/4*\text{sqrt}(1/3)*(2*a^2*b^5*d^2 + 8*a^3*b^4*d*g + \\
& 8*a^4*b^3*g^2 + (a^4*b^6*e + 5*a^5*b^5*h)*((1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*((b \\
& ^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^ \\
& 2*b^2*e^2*h + 75*a^3*b^2*e^2*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*
\end{aligned}$$

$$\begin{aligned}
& h^3 + (8g^3 - 75e^2h^2)a^3b + 3(4d^2g^2 - 5e^2h)a^2b^2 - (e^3 - 6d^2g^2)ab^3 / (a^5b^8)^{1/3} - 2(1/2)^{2/3}(b^2de + 10a^2g^2h + (2eg + 5d^2h)ab) \cdot (-I\sqrt{3} + 1) / (a^3b^5((b^4d^3 + a^3e^3 + 6a^2b^3d^2g^2 + 12a^2b^2d^2g^2 + 8a^3b^2g^3 + 15a^2b^2e^2h + 75a^3b^2e^2h^2 + 125a^4h^3) / (a^5b^8) + (b^4d^3 - 125a^4h^3 + (8g^3 - 75e^2h^2)a^3b + 3(4d^2g^2 - 5e^2h)a^2b^2 - (e^3 - 6d^2g^2)ab^3) / (a^5b^8))^{1/3}) \\
&) \cdot \sqrt{-((1/2)^{1/3}(I\sqrt{3} + 1) \cdot ((b^4d^3 + a^3e^3 + 6a^2b^3d^2g^2 + 12a^2b^2d^2g^2 + 8a^3b^2g^3 + 15a^2b^2e^2h + 75a^3b^2e^2h^2 + 125a^4h^3) / (a^5b^8) + (b^4d^3 - 125a^4h^3 + (8g^3 - 75e^2h^2)a^3b + 3(4d^2g^2 - 5e^2h)a^2b^2 - (e^3 - 6d^2g^2)ab^3) / (a^5b^8))^{1/3} - 2(1/2)^{2/3}(b^2de + 10a^2g^2h + (2eg + 5d^2h)ab) \cdot (-I\sqrt{3} + 1) / (a^3b^5((b^4d^3 + a^3e^3 + 6a^2b^3d^2g^2 + 12a^2b^2d^2g^2 + 8a^3b^2g^3 + 15a^2b^2e^2h + 75a^3b^2e^2h^2 + 125a^4h^3) / (a^5b^8) + (b^4d^3 - 125a^4h^3 + (8g^3 - 75e^2h^2)a^3b + 3(4d^2g^2 - 5e^2h)a^2b^2 - (e^3 - 6d^2g^2)ab^3) / (a^5b^8))^{1/3}) \\
&)^2 a^3b^5 + 16b^2de + 32abeg + 80(abd + 2a^2g)h / (a^3b^5)) - ((ab^4x^6 + 2a^2b^3x^3 + a^3b^2) \cdot ((1/2)^{1/3}(I\sqrt{3} + 1) \cdot ((b^4d^3 + a^3e^3 + 6a^2b^3d^2g^2 + 12a^2b^2d^2g^2 + 8a^3b^2g^3 + 15a^2b^2e^2h + 75a^3b^2e^2h^2 + 125a^4h^3) / (a^5b^8) + (b^4d^3 - 125a^4h^3 + (8g^3 - 75e^2h^2)a^3b + 3(4d^2g^2 - 5e^2h)a^2b^2 - (e^3 - 6d^2g^2)ab^3) / (a^5b^8))^{1/3} - 2(1/2)^{2/3}(b^2de + 10a^2g^2h + (2eg + 5d^2h)ab) \cdot (-I\sqrt{3} + 1) / (a^3b^5((b^4d^3 + a^3e^3 + 6a^2b^3d^2g^2 + 12a^2b^2d^2g^2 + 8a^3b^2g^3 + 15a^2b^2e^2h + 75a^3b^2e^2h^2 + 125a^4h^3) / (a^5b^8) + (b^4d^3 - 125a^4h^3 + (8g^3 - 75e^2h^2)a^3b + 3(4d^2g^2 - 5e^2h)a^2b^2 - (e^3 - 6d^2g^2)ab^3) / (a^5b^8))^{1/3}) \\
&) - 3\sqrt{1/3} \cdot (ab^4x^6 + 2a^2b^3x^3 + a^3b^2) \cdot \sqrt{-((1/2)^{1/3}(I\sqrt{3} + 1) \cdot ((b^4d^3 + a^3e^3 + 6a^2b^3d^2g^2 + 12a^2b^2d^2g^2 + 8a^3b^2g^3 + 15a^2b^2e^2h + 75a^3b^2e^2h^2 + 125a^4h^3) / (a^5b^8) + (b^4d^3 - 125a^4h^3 + (8g^3 - 75e^2h^2)a^3b + 3(4d^2g^2 - 5e^2h)a^2b^2 - (e^3 - 6d^2g^2)ab^3) / (a^5b^8))^{1/3} - 2(1/2)^{2/3}(b^2de + 10a^2g^2h + (2eg + 5d^2h)ab) \cdot (-I\sqrt{3} + 1) / (a^3b^5((b^4d^3 + a^3e^3 + 6a^2b^3d^2g^2 + 12a^2b^2d^2g^2 + 8a^3b^2g^3 + 15a^2b^2e^2h + 75a^3b^2e^2h^2 + 125a^4h^3) / (a^5b^8) + (b^4d^3 - 125a^4h^3 + (8g^3 - 75e^2h^2)a^3b + 3(4d^2g^2 - 5e^2h)a^2b^2 - (e^3 - 6d^2g^2)ab^3) / (a^5b^8))^{1/3}) \\
&) - 2(1/2)^{2/3}(b^2de + 10a^2g^2h + (2eg + 5d^2h)ab) \cdot (-I\sqrt{3} + 1) / (a^3b^5((b^4d^3 + a^3e^3 + 6a^2b^3d^2g^2 + 12a^2b^2d^2g^2 + 8a^3b^2g^3 + 15a^2b^2e^2h + 75a^3b^2e^2h^2 + 125a^4h^3) / (a^5b^8) + (b^4d^3 - 125a^4h^3 + (8g^3 - 75e^2h^2)a^3b + 3(4d^2g^2 - 5e^2h)a^2b^2 - (e^3 - 6d^2g^2)ab^3) / (a^5b^8))^{1/3}) \\
&)^2 a^3b^5 + 16b^2de + 32abeg + 80(abd + 2a^2g)h / (a^3b^5)) \cdot \log(-2a^2b^3de^2 - 4a^2b^2e^2g - 1/4(a^4b^6e + 5a^5b^5h) \cdot ((1/2)^{1/3}(I\sqrt{3} + 1) \cdot ((b^4d^3 + a^3e^3 + 6a^2b^3d^2g^2 + 12a^2b^2d^2g^2 + 8a^3b^2g^3 + 15a^2b^2e^2h + 75a^3b^2e^2h^2 + 125a^4h^3) / (a^5b^8) + (b^4d^3 - 125a^4h^3 + (8g^3 - 75e^2h^2)a^3b + 3(4d^2g^2 - 5e^2h)a^2b^2 - (e^3 - 6d^2g^2)ab^3) / (a^5b^8))^{1/3} - 2(1/2)^{2/3}(b^2de + 10a^2g^2h + (2eg + 5d^2h)ab) \cdot (-I\sqrt{3} + 1) / (a^3b^5((b^4d^3 + a^3e^3 + 6a^2b^3d^2g^2 + 12a^2b^2d^2g^2 + 8a^3b^2g^3 + 15a^2b^2e^2h + 75a^3b^2e^2h^2 + 125a^4h^3) / (a^5b^8) + (b^4d^3 - 125a^4h^3 + (8g^3 - 75e^2h^2)a^3b + 3(4d^2g^2 - 5e^2h)a^2b^2 - (e^3 - 6d^2g^2)ab^3) / (a^5b^8))^{1/3}) \\
&)^2 - 50(a^3bd + 2a^4g)h^2 + 1/2(a^2b^5d^2 + 4a^3b^4*
\end{aligned}$$

$$\begin{aligned}
& d*g + 4*a^4*b^3*g^2)*((1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6 \\
& *a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b \\
& *e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2) \\
& *a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8) \\
&)^{(1/3)} - 2*(1/2)^{(2/3)}*(b^2*d*e + 10*a^2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*\text{sq} \\
& \text{rt}(3) + 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 \\
& + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) \\
&) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2* \\
& h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^{(1/3))) - 20*(a^2*b^2*d*e + \\
& 2*a^3*b*e*g)*h + 2*(b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 \\
& + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)*x - 3/4*\text{sq} \\
& \text{rt}(1/3)*(2*a^2*b^5*d^2 + 8*a^3*b^4*d*g + 8*a^4*b^3*g^2 + (a^4*b^6*e + 5*a^5 \\
& *b^5*h))*((1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g \\
& + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125* \\
& a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3* \\
& (4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^{(1/3)} - 2*(\\
& 1/2)^{(2/3)}*(b^2*d*e + 10*a^2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*\text{sqrt}(3) + 1)/(a \\
& ^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g \\
& ^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 \\
& - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - \\
& (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^{(1/3)))*\text{sqrt}(-(((1/2)^{(1/3)}*(I*\text{sqrt}(3) + \\
& 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 \\
& + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 1 \\
& 25*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^ \\
& 3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^{(1/3)} - 2*(1/2)^{(2/3)}*(b^2*d*e + 10*a^2*g*h \\
& + (2*e*g + 5*d*h)*a*b)*(-I*\text{sqrt}(3) + 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6* \\
& a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b* \\
& e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2) \\
&)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8)) \\
& ^{(1/3))})^2*a^3*b^5 + 16*b^2*d*e + 32*a*b*e*g + 80*(a*b*d + 2*a^2*g)*h)/(a^3 \\
& *b^5)))/(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.12213, size = 454, normalized size = 1.53

$$\frac{\left(5ah\left(-\frac{a}{b}\right)^{\frac{1}{3}} + b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e + bd + 2ag\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^2b^2} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2d + 2\left(-ab^2\right)^{\frac{1}{3}}abg - 5\left(-ab^2\right)^{\frac{2}{3}}ah - \left(-ab^2\right)^{\frac{2}{3}}b^2e\right)}{27a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac")

[Out] -1/27*(5*a*h*(-a/b)^(1/3) + b*(-a/b)^(1/3)*e + b*d + 2*a*g)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^2) + 1/27*sqrt(3)*((-a*b^2)^(1/3)*b^2*d + 2*(-a*b^2)^(1/3)*a*b*g - 5*(-a*b^2)^(2/3)*a*h - (-a*b^2)^(2/3)*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^4) - 1/18*(8*a*b*h*x^5 - 2*b^2*x^5*e - b^2*d*x^4 + 7*a*b*g*x^4 + 6*a*b*f*x^3 + 5*a^2*h*x^2 + a*b*x^2*e + 2*a*b*d*x + 4*a^2*g*x + 3*a*b*c + 3*a^2*f)/((b*x^3 + a)^2*a*b^2) + 1/54*((-a*b^2)^(1/3)*b^2*d + 2*(-a*b^2)^(1/3)*a*b*g + 5*(-a*b^2)^(2/3)*a*h + (-a*b^2)^(2/3)*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^4)

$$3.424 \quad \int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3} dx$$

Optimal. Leaf size=323

$$\frac{x(2bx(af+2bc)+3bx^2(ag+bd)+a(be-7ah))}{18a^2b^2(a+bx^3)} + \frac{\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2)(b^{2/3}(af+2bc)-a^{2/3}(2ah+be))}{54a^{7/3}b^{7/3}} - \log$$

[Out] $-(x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(6*a*b^2*(a + b*x^3)^2) + (x*(a*(b*e - 7*a*h) + 2*b*(2*b*c + a*f)*x + 3*b*(b*d + a*g)*x^2))/(18*a^2*b^2*(a + b*x^3)) - ((2*b^(5/3)*c + a^(2/3)*b*e + a*b^(2/3)*f + 2*a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(7/3)*b^(7/3)) - ((b^(2/3)*(2*b*c + a*f) - a^(2/3)*(b*e + 2*a*h))*Log[a^(1/3) + b^(1/3)*x]/(27*a^(7/3)*b^(7/3)) + ((b^(2/3)*(2*b*c + a*f) - a^(2/3)*(b*e + 2*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(54*a^(7/3)*b^(7/3)))$

Rubi [A] time = 0.481828, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1828, 1858, 1860, 31, 634, 617, 204, 628}

$$\frac{x(2bx(af+2bc)+3bx^2(ag+bd)+a(be-7ah))}{18a^2b^2(a+bx^3)} + \frac{\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2)(b^{2/3}(af+2bc)-a^{2/3}(2ah+be))}{54a^{7/3}b^{7/3}} - \log$$

Antiderivative was successfully verified.

[In] Int[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]

[Out] $-(x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(6*a*b^2*(a + b*x^3)^2) + (x*(a*(b*e - 7*a*h) + 2*b*(2*b*c + a*f)*x + 3*b*(b*d + a*g)*x^2))/(18*a^2*b^2*(a + b*x^3)) - ((2*b^(5/3)*c + a^(2/3)*b*e + a*b^(2/3)*f + 2*a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(7/3)*b^(7/3)) - ((b^(2/3)*(2*b*c + a*f) - a^(2/3)*(b*e + 2*a*h))*Log[a^(1/3) + b^(1/3)*x]/(27*a^(7/3)*b^(7/3)) + ((b^(2/3)*(2*b*c + a*f) - a^(2/3)*(b*e + 2*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(54*a^(7/3)*b^(7/3)))$

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q =
m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
+ b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(p_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
```

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 204

$\text{Int}[\{(a_)+(b_)*(x_)^2\}^{-1}, x_Symbol] \ :> \ -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 628

$\text{Int}[\{(d_)+(e_)*(x_)\}/\{(a_)+(b_)*(x_)+(c_)*(x_)^2\}, x_Symbol] \ :> \ \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rubi steps

$$\begin{aligned} \int \frac{x(c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{(a + bx^3)^3} dx &= -\frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{6ab^2(a + bx^3)^2} - \frac{\int \frac{-a(be - ah) - 2b(2bc + af)x - 3b(bd + ag)x^2}{(a + bx^3)^2} dx}{6ab^2} \\ &= -\frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{6ab^2(a + bx^3)^2} + \frac{x(a(be - 7ah) + 2b(2bc + ag))}{18a^2b^2(a + bx^3)} \\ &= -\frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{6ab^2(a + bx^3)^2} + \frac{x(a(be - 7ah) + 2b(2bc + ag))}{18a^2b^2(a + bx^3)} \\ &= -\frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{6ab^2(a + bx^3)^2} + \frac{x(a(be - 7ah) + 2b(2bc + ag))}{18a^2b^2(a + bx^3)} \\ &= -\frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{6ab^2(a + bx^3)^2} + \frac{x(a(be - 7ah) + 2b(2bc + ag))}{18a^2b^2(a + bx^3)} \\ &= -\frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{6ab^2(a + bx^3)^2} + \frac{x(a(be - 7ah) + 2b(2bc + ag))}{18a^2b^2(a + bx^3)} \end{aligned}$$

Mathematica [A] time = 0.299634, size = 297, normalized size = 0.92

$$\frac{9a^{4/3} \sqrt[3]{b(a^2(g+hx) - ab(d+x(e+fx)) + b^2cx^2)}}{(a+bx^3)^2} - \frac{3 \sqrt[3]{a} \sqrt[3]{b(a^2(6g+7hx) - abx(e+2fx) - 4b^2cx^2)}}{a+bx^3} + \log\left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3}x^2\right) \left(-a^{2/3}be - 2a^{5/3}h + ab\right)$$

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Antiderivative was successfully verified.

[In] Integrate[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3, x]

[Out]
$$\begin{aligned} &((-3*a^{(1/3)}*b^{(1/3)}*(-4*b^2*c*x^2 - a*b*x*(e + 2*f*x) + a^2*(6*g + 7*h*x)) \\ &)/(a + b*x^3) + (9*a^{(4/3)}*b^{(1/3)}*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x* \\ &(e + f*x))))/(a + b*x^3)^2 - 2*sqrt[3]*(2*b^{(5/3)}*c + a^{(2/3)}*b*e + a*b^{(2/3)}*f \\ &+ 2*a^{(5/3)}*h)*ArcTan[(1 - (2*b^{(1/3)}*x)/a^{(1/3)})/sqrt[3]] + 2*(-2*b^{(5/3)}*c \\ &+ a^{(2/3)}*b*e - a*b^{(2/3)}*f + 2*a^{(5/3)}*h)*Log[a^{(1/3)} + b^{(1/3)}*x] \\ &+ (2*b^{(5/3)}*c - a^{(2/3)}*b*e + a*b^{(2/3)}*f - 2*a^{(5/3)}*h)*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x \\ &+ b^{(2/3)}*x^2]/(54*a^{(7/3)}*b^{(7/3)}) \end{aligned}$$

Maple [A] time = 0.011, size = 498, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3, x)

[Out]
$$\begin{aligned} &(1/9*(a*f+2*b*c)/a^2*x^5-1/18*(7*a*h-b*e)/a/b*x^4-1/3*g*x^3/b-1/18*(a*f-7*b \\ &*c)/a/b*x^2-1/9*(2*a*h+b*e)/b^2*x-1/6*(a*g+b*d)/b^2)/(b*x^3+a)^2+2/27/b^3/(\\ &1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*h+1/27/a/b^2/(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)}) \\ &*e-1/27/b^3/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*h-1/54/a/b^2/(1/b*a)^{(2/3)} \\ &*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*e+2/27/b^3/(1/b*a)^{(2/3)}*3^{(1/2)}*arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1)) \\ &*h+1/27/a/b^2/(1/b*a)^{(2/3)}*3^{(1/2)}*arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*e-1/27/a/b^2/(1/b*a)^{(1/3)} \\ &*\ln(x+(1/b*a)^{(1/3)})*f-2/27/b/a^2/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})*c+1/54/a/b^2/(1/b*a)^{(1/3)} \\ &*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*f+1/27/b/a^2/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)}) \\ &*c+1/27/a/b^2*3^{(1/2)}/(1/b*a)^{(1/3)}*arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*f+2/27/b/a^2*3^{(1/2)}/(1/b*a)^{(1/3)} \\ &*arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*c \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [C] time = 22.2304, size = 15553, normalized size = 48.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/108*(36*a^2*b*g*x^3 - 12*(2*b^3*c + a*b^2*f)*x^5 - 6*(a*b^2*e - 7*a^2*b*h)*x^4 + 18*a^2*b*d + 18*a^3*g - 6*(7*a*b^2*c - a^2*b*f)*x^2 + 2*(a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2)*((1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^{(1/3)} - 2*(1/2)^{(2/3)}*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)*a*b)*(-I*\sqrt{3} + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^{(1/3)})) * \log(8*a*b^4*c^2*e + 8*a^2*b^3*c*e*f + 2*a^3*b^2*e*f^2 + 1/4*(2*a^5*b^6*c + a^6*b^5*f)*((1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^{(1/3)} - 2*(1/2)^{(2/3)}*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)*a*b)*(-I*\sqrt{3} + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^{(1/3)}))^{-2} - 1/2*(a^4*b^4*e^2 + 4*a^5*b^3*e*h + 4*a^6*b^2*h^2)*((1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^{(1/3)})) \end{aligned}$$

$$\begin{aligned}
& ^3e^3 + 12ab^4c^2f + 6a^2b^3c^2f^2 + a^3b^2f^3 + 6a^3b^2e^2h + \\
& 12a^4b^2e^2h^2 + 8a^5h^3)/(a^7b^7) - (8b^5c^3 + 12ab^4c^2f - 12a^4b^2e^2h^2 - 8a^5h^3 + (f^3 - 6e^2h)a^3b^2 - (e^3 - 6c^2f^2)a^2b^3) \\
& / (a^7b^7)^{(1/3)} - 2(1/2)^{(2/3)}(2b^2c^2e + 2a^2f^2h + (ef + 4ch)ab) * (-I\sqrt{3} + 1) / (a^4b^4 * ((8b^5c^3 + a^2b^3e^3 + 12ab^4c^2f + 6 \\
& a^2b^3c^2f^2 + a^3b^2f^3 + 6a^3b^2e^2h + 12a^4b^2e^2h^2 + 8a^5h^3) / (a^7b^7) - (8b^5c^3 + 12ab^4c^2f - 12a^4b^2e^2h^2 - 8a^5h^3 + (f \\
& ^3 - 6e^2h)a^3b^2 - (e^3 - 6c^2f^2)a^2b^3) / (a^7b^7))^{(1/3)}) + 4(4a^2b^3c^2 + 4a^3b^2c^2f + a^4b^2f^2) * h + (8b^5c^3 + a^2b^3e^3 + 12 \\
& ab^4c^2f + 6a^2b^3c^2f^2 + a^3b^2f^3 + 6a^3b^2e^2h + 12a^4b^2e^2h^2 + 8a^5h^3) * x + 12(a^2b^2e + 2a^3h) * x - ((a^2b^4x^6 + 2a^3b^3x \\
& x^3 + a^4b^2) * ((1/2)^{(1/3)} * (I\sqrt{3} + 1) * ((8b^5c^3 + a^2b^3e^3 + 12ab^4c^2f + 6a^2b^3c^2f^2 + a^3b^2f^3 + 6a^3b^2e^2h + 12a^4b^2e^2h^2 + 8a^5h^3) / (a^7b^7) - (8b^5c^3 + 12ab^4c^2f - 12a^4b^2e^2h^2 - 8a^5h^3 + (f^3 - 6e^2h)a^3b^2 - (e^3 - 6c^2f^2)a^2b^3) / (a^7b^7))^{(1/3)} - 2(1/2)^{(2/3)}(2b^2c^2e + 2a^2f^2h + (ef + 4ch)ab) * (-I\sqrt{3} + 1) / (a^4b^4 * ((8b^5c^3 + a^2b^3e^3 + 12ab^4c^2f + 6a^2b^3c^2f^2 + a^3b^2f^3 + 6a^3b^2e^2h + 12a^4b^2e^2h^2 + 8a^5h^3) / (a^7b^7) - (8b^5c^3 + 12ab^4c^2f - 12a^4b^2e^2h^2 - 8a^5h^3 + (f^3 - 6e^2h)a^3b^2 - (e^3 - 6c^2f^2)a^2b^3) / (a^7b^7))^{(1/3)})) + 3\sqrt{1/3} * (a^2b^4x^6 + 2a^3b^3x^3 + a^4b^2) * \sqrt{-(((1/2)^{(1/3)} * (I\sqrt{3} + 1) * ((8b^5c^3 + a^2b^3e^3 + 12ab^4c^2f + 6a^2b^3c^2f^2 + a^3b^2f^3 + 6a^3b^2e^2h + 12a^4b^2e^2h^2 + 8a^5h^3) / (a^7b^7) - (8b^5c^3 + 12ab^4c^2f - 12a^4b^2e^2h^2 - 8a^5h^3 + (f^3 - 6e^2h)a^3b^2 - (e^3 - 6c^2f^2)a^2b^3) / (a^7b^7))^{(1/3)} - 2(1/2)^{(2/3)}(2b^2c^2e + 2a^2f^2h + (ef + 4ch)ab) * (-I\sqrt{3} + 1) / (a^4b^4 * ((8b^5c^3 + a^2b^3e^3 + 12ab^4c^2f + 6a^2b^3c^2f^2 + a^3b^2f^3 + 6a^3b^2e^2h + 12a^4b^2e^2h^2 + 8a^5h^3) / (a^7b^7) - (8b^5c^3 + 12ab^4c^2f - 12a^4b^2e^2h^2 - 8a^5h^3 + (f^3 - 6e^2h)a^3b^2 - (e^3 - 6c^2f^2)a^2b^3) / (a^7b^7))^{(1/3)}))} \\
& ^2a^4b^4 + 32b^2c^2e + 16ab^2ef + 32(2ab^2c + a^2f) * h) / (a^4b^4)) * \log(-8ab^4c^2e - 8a^2b^3c^2ef - 2a^3b^2e^2f^2 - 1/4(2a^5b^6c + a^6b^5f) * ((1/2)^{(1/3)} * (I\sqrt{3} + 1) * ((8b^5c^3 + a^2b^3e^3 + 12ab^4c^2f + 6a^2b^3c^2f^2 + a^3b^2f^3 + 6a^3b^2e^2h + 12a^4b^2e^2h^2 + 8a^5h^3) / (a^7b^7) - (8b^5c^3 + 12ab^4c^2f - 12a^4b^2e^2h^2 - 8a^5h^3 + (f^3 - 6e^2h)a^3b^2 - (e^3 - 6c^2f^2)a^2b^3) / (a^7b^7))^{(1/3)} - 2(1/2)^{(2/3)}(2b^2c^2e + 2a^2f^2h + (ef + 4ch)ab) * (-I\sqrt{3} + 1) / (a^4b^4 * ((8b^5c^3 + a^2b^3e^3 + 12ab^4c^2f + 6a^2b^3c^2f^2 + a^3b^2f^3 + 6a^3b^2e^2h + 12a^4b^2e^2h^2 + 8a^5h^3) / (a^7b^7) - (8b^5c^3 + 12ab^4c^2f - 12a^4b^2e^2h^2 - 8a^5h^3 + (f^3 - 6e^2h)a^3b^2 - (e^3 - 6c^2f^2)a^2b^3) / (a^7b^7))^{(1/3)}))} \\
& ^2 + 1/2(a^4b^4e^2 + 4a^5b^3e^2h + 4a^6b^2h^2) * ((1/2)^{(1/3)} * (I\sqrt{3} + 1) * ((8b^5c^3 + a^2b^3e^3 + 12ab^4c^2f + 6a^2b^3c^2f^2 + a^3b^2f^3 + 6a^3b^2e^2h + 12a^4b^2e^2h^2 + 8a^5h^3) / (a^7b^7) - (8b^5c^3 + 12ab^4c^2f - 12a^4b^2e^2h^2 - 8a^5h^3 + (f^3 - 6e^2h)a^3b^2 - (e^3 - 6c^2f^2)a^2b^3) / (a^7b^7))^{(1/3)} - 2(1/2)^{(2/3)}(2b^2c^2e + 2a^2f^2h + (ef + 4ch)ab) * (-I\sqrt{3} + 1) / (a^4b^4 * ((8b^5c^3 + a^2b^3e^3 + 12ab^4c^2f + 6a^2b^3c^2f^2 + a^3b^2f^3 + 6a^3b^2e^2h + 12a^4b^2e^2h^2 + 8a^5h^3) / (a^7b^7) - (8b^5c^3 + 12ab^4c^2f - 12a^4b^2e^2h^2 - 8a^5h^3 + (f^3 - 6e^2h)a^3b^2 - (e^3 - 6c^2f^2)a^2b^3) / (a^7b^7))^{(1/3)}))} \\
& ^2) * a^2b^3) / (a^7b^7))^{(1/3)} - 2(1/2)^{(2/3)}(2b^2c^2e + 2a^2f^2h + (ef + 4ch)ab) * (-I\sqrt{3} + 1) / (a^4b^4 * ((8b^5c^3 + a^2b^3e^3 + 12ab^4c^2f + 6a^2b^3c^2f^2 + a^3b^2f^3 + 6a^3b^2e^2h + 12a^4b^2e^2h^2 + 8a^5h^3) / (a^7b^7) - (8b^5c^3 + 12ab^4c^2f - 12a^4b^2e^2h^2 - 8a^5h^3 + (f^3 - 6e^2h)a^3b^2 - (e^3 - 6c^2f^2)a^2b^3) / (a^7b^7))^{(1/3)}))}
\end{aligned}$$

$$\begin{aligned}
& + 4*c*h)*a*b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3))) - 4*(4*a^2*b^3*c^2 + 4*a^3*b^2*c*f + a^4*b*f^2)*h + 2*(8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)*x + 3/4*sqrt(1/3)*(2*a^4*b^4*e^2 + 8*a^5*b^3*e*h + 8*a^6*b^2*h^2 + (2*a^5*b^6*c + a^6*b^5*f)*((1/2)^(1/3)*(I*sqrt(3) + 1))*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)*a*b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3))))*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1))*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)*a*b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3)))^2*a^4*b^4 + 32*b^2*c*e + 16*a*b*e*f + 32*(2*a*b*c + a^2*f)*h)/(a^4*b^4)) - ((a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1))*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)*a*b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3))) - 3*sqrt(1/3)*(a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1))*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)*a*b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3)))^2*a^4*b^4 + 32*b^2*c*e + 16*a*b*e*f + 32*(2*a*b*c +
\end{aligned}$$

$$\begin{aligned}
& a^2*f*h)/(a^4*b^4))) * \log(-8*a*b^4*c^2*e - 8*a^2*b^3*c*e*f - 2*a^3*b^2*e*f^2 - 1/4*(2*a^5*b^6*c + a^6*b^5*f)*((1/2)^{(1/3)}*(I*\sqrt{3}) + 1)*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^{(1/3)} - 2*(1/2)^{(2/3)}*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)*a*b)*(-I*\sqrt{3}) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^{(1/3)})) \\
& ^2 + 1/2*(a^4*b^4*e^2 + 4*a^5*b^3*e*h + 4*a^6*b^2*h^2)*((1/2)^{(1/3)}*(I*\sqrt{3}) + 1)*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^{(1/3)} - 2*(1/2)^{(2/3)}*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)*a*b)*(-I*\sqrt{3}) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^{(1/3)})) - 4*(4*a^2*b^3*c^2 + 4*a^3*b^2*c*f + a^4*b*f^2)*h + 2*(8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)*x - 3/4*\sqrt{1/3}*(2*a^4*b^4*e^2 + 8*a^5*b^3*e*h + 8*a^6*b^2*h^2 + (2*a^5*b^6*c + a^6*b^5*f)*((1/2)^{(1/3)}*(I*\sqrt{3}) + 1)*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^{(1/3)} - 2*(1/2)^{(2/3)}*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)*a*b)*(-I*\sqrt{3}) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^{(1/3)})) * \sqrt{-(((1/2)^{(1/3)}*(I*\sqrt{3}) + 1)*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^{(1/3)} - 2*(1/2)^{(2/3)}*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)*a*b)*(-I*\sqrt{3}) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^{(1/3)})) \\
& ^2*a^4*b^4 + 32*b^2*c*e + 16*a*b*e*f + 32*(2*a*b*c + a^2*f)*h)/(a^4*b^4)))/((a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.09139, size = 481, normalized size = 1.49

$$\frac{\left(2b^2c\left(-\frac{a}{b}\right)^{\frac{1}{3}} + abf\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 2a^2h + abe\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right) + \sqrt{3}\left(2\left(-ab^2\right)^{\frac{1}{3}}a^2h + \left(-ab^2\right)^{\frac{1}{3}}abe - 2\left(-ab^2\right)^{\frac{2}{3}}bc\right)}{27a^3b^2} + \frac{\sqrt{3}\left(2\left(-ab^2\right)^{\frac{1}{3}}a^2h + \left(-ab^2\right)^{\frac{1}{3}}abe - 2\left(-ab^2\right)^{\frac{2}{3}}bc\right)}{27a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac")

[Out]
$$\frac{-1/27*(2*b^2*c*(-a/b)^{(1/3)} + a*b*f*(-a/b)^{(1/3)} + 2*a^2*h + a*b*e)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))}{(a^3*b^2)} + \frac{1/27*\sqrt{3}*(2*(-a*b^2)^{(1/3)}*a^2*h + (-a*b^2)^{(1/3)}*a*b*e - 2*(-a*b^2)^{(2/3)}*b*c - (-a*b^2)^{(2/3)}*a*f)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})}{(a^3*b^3)} + \frac{1/54*(2*(-a*b^2)^{(1/3)}*a^2*h + (-a*b^2)^{(1/3)}*a*b*e + 2*(-a*b^2)^{(2/3)}*b*c + (-a*b^2)^{(2/3)}*a*f)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})}{(a^3*b^3)} + \frac{1/18*(4*b^3*c*x^5 + 2*a*b^2*f*x^5 - 7*a^2*b*h*x^4 + a*b^2*x^4*e - 6*a^2*b*g*x^3 + 7*a*b^2*c*x^2 - a^2*b*f*x^2 - 4*a^3*h*x - 2*a^2*b*x*e - 3*a^2*b*d - 3*a^3*g)}{(b*x^3 + a)^2*a^2*b^2}$$

$$3.425 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^3} dx$$

Optimal. Leaf size=313

$$\frac{3a(ah+be)-bx(2x(ag+2bd)+af+5bc)}{18a^2b^2(a+bx^3)} - \frac{\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2)(\sqrt[3]{b}(af+5bc)-\sqrt[3]{a}(ag+2bd))}{54a^{8/3}b^{5/3}} + \frac{\log(\sqrt[3]{a}+...)}{...}$$

[Out] (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(6*a*b*(a + b*x^3)^2) - (3*a*(b*e + a*h) - b*x*(5*b*c + a*f + 2*(2*b*d + a*g)*x))/(18*a^2*b^2*(a + b*x^3)) - ((5*b^(4/3)*c + 2*a^(1/3)*b*d + a*b^(1/3)*f + a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(8/3)*b^(5/3)) + ((b^(1/3)*(5*b*c + a*f) - a^(1/3)*(2*b*d + a*g))*Log[a^(1/3) + b^(1/3)*x]/(27*a^(8/3)*b^(5/3)) - ((b^(1/3)*(5*b*c + a*f) - a^(1/3)*(2*b*d + a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(5/3))

Rubi [A] time = 0.429386, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {1858, 1854, 1860, 31, 634, 617, 204, 628}

$$\frac{3a(ah+be)-bx(2x(ag+2bd)+af+5bc)}{18a^2b^2(a+bx^3)} - \frac{\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2)(\sqrt[3]{b}(af+5bc)-\sqrt[3]{a}(ag+2bd))}{54a^{8/3}b^{5/3}} + \frac{\log(\sqrt[3]{a}+...)}{...}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^3,x]

[Out] (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(6*a*b*(a + b*x^3)^2) - (3*a*(b*e + a*h) - b*x*(5*b*c + a*f + 2*(2*b*d + a*g)*x))/(18*a^2*b^2*(a + b*x^3)) - ((5*b^(4/3)*c + 2*a^(1/3)*b*d + a*b^(1/3)*f + a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(8/3)*b^(5/3)) + ((b^(1/3)*(5*b*c + a*f) - a^(1/3)*(2*b*d + a*g))*Log[a^(1/3) + b^(1/3)*x]/(27*a^(8/3)*b^(5/3)) - ((b^(1/3)*(5*b*c + a*f) - a^(1/3)*(2*b*d + a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(5/3))

Rule 1858

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,

```
x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x], Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x] /; GeQ[q, n] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1})*(a + b*x^n)^(p + 1), x], x] /; q == n - 1 /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(p_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{(a + bx^3)^3} dx &= \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{6ab(a + bx^3)^2} - \frac{\int \frac{-b(5bc + af) - 2b(2bd + ag)x - 3b(be + ah)x^2}{(a + bx^3)^2} dx}{6ab^2} \\ &= \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{6ab(a + bx^3)^2} - \frac{3a(be + ah) - bx(5bc + af + 2(2bd + ag))}{18a^2b^2(a + bx^3)} \\ &= \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{6ab(a + bx^3)^2} - \frac{3a(be + ah) - bx(5bc + af + 2(2bd + ag))}{18a^2b^2(a + bx^3)} \\ &= \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{6ab(a + bx^3)^2} - \frac{3a(be + ah) - bx(5bc + af + 2(2bd + ag))}{18a^2b^2(a + bx^3)} \\ &= \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{6ab(a + bx^3)^2} - \frac{3a(be + ah) - bx(5bc + af + 2(2bd + ag))}{18a^2b^2(a + bx^3)} \\ &= \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{6ab(a + bx^3)^2} - \frac{3a(be + ah) - bx(5bc + af + 2(2bd + ag))}{18a^2b^2(a + bx^3)} \end{aligned}$$

Mathematica [A] time = 0.237521, size = 295, normalized size = 0.94

$$\frac{9a^{5/3}(a^2h - ab(e + x(f + gx)) + b^2x(c + dx))}{(a + bx^3)^2} + \frac{3a^{2/3}(-6a^2h + abx(f + 2gx) + b^2x(5c + 4dx))}{a + bx^3} + \sqrt[3]{b} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(a^{4/3}g + 2\sqrt[3]{abd} - a\sqrt[3]{b})$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^3,x]

[Out]
$$\frac{\left(3a^{2/3}(-6a^2h + b^2x(5c + 4dx)) + abx(f + 2gx)\right)/(a + bx^3) + (9a^{5/3}(a^2h + b^2x(c + dx) - ab(e + x(f + gx))))/(a + bx^3)^2 - 2\sqrt{3}b^{1/3}(5b^{4/3}c + 2a^{1/3}bd + ab^{1/3}f + a^{4/3}g)\operatorname{ArcTan}\left[\frac{1 - (2b^{1/3}x)/a^{1/3}}{\sqrt{3}}\right] + 2b^{1/3}(5b^{4/3}c - 2a^{1/3}bd + ab^{1/3}f - a^{4/3}g)\operatorname{Log}[a^{1/3} + b^{1/3}x] + b^{1/3}(-5b^{4/3}c + 2a^{1/3}bd - ab^{1/3}f + a^{4/3}g)\operatorname{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2]}{(54a^{8/3}b^2)}$$

Maple [A] time = 0.008, size = 506, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x)

[Out]
$$\begin{aligned} & (1/9*(a*g+2*b*d)/a^2*x^5+1/18*(a*f+5*b*c)/a^2*x^4-1/3*h*x^3/b-1/18*(a*g-7*b \\ & *d)/a/b*x^2-1/9*(a*f-4*b*c)/a/b*x-1/6*(a*h+b*e)/b^2)/(b*x^3+a)^2+1/27/a/b^2 \\ & /(1/b*a)^{(2/3)}*\ln(x+(1/b*a)^{(1/3)})*f+5/27*c/a^2/b/(1/b*a)^{(2/3)}*\ln(x+(1/b*a) \\ &)^{(1/3)}-1/54/a/b^2/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})*f-5 \\ & /54*c/a^2/b/(1/b*a)^{(2/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})+1/27/a/b^2/ \\ & (1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*f+5/27*c/a^ \\ & 2/b/(1/b*a)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))-1/27/b^ \\ & 2/a/(1/b*a)^{(1/3)}*\ln(x+(1/b*a)^{(1/3)})*g-2/27*d/a^2/b/(1/b*a)^{(1/3)}*\ln(x+(1/ \\ & b*a)^{(1/3)})+1/54/b^2/a/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})* \\ & g+1/27*d/a^2/b/(1/b*a)^{(1/3)}*\ln(x^2-(1/b*a)^{(1/3)}*x+(1/b*a)^{(2/3)})+1/27/b^2 \\ & /a*3^{(1/2)}/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1))*g+2/27*d \\ & /a^2*3^{(1/2)}/b/(1/b*a)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(1/b*a)^{(1/3)}*x-1)) \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [C] time = 20.8148, size = 15406, normalized size = 49.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/108*(36*a^2*b*h*x^3 - 12*(2*b^3*d + a*b^2*g)*x^5 - 6*(5*b^3*c + a*b^2*f) \\ & *x^4 + 18*a^2*b*e + 18*a^3*h - 6*(7*a*b^2*d - a^2*b*g)*x^2 + 2*(a^2*b^4*x^6 \\ & + 2*a^3*b^3*x^3 + a^4*b^2)*((1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1))*((125*b^4*c^3 + 8* \\ & a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2* \\ & g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6* \\ & d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a \\ & ^8*b^5))^{(1/3)} - 2*(1/2)^{(2/3)}*(10*b^2*c*d + a^2*f*g + (2*d*f + 5*c*g)*a*b) \\ & *(-I*\text{sqrt}(3) + 1)/(a^5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 1 \\ & 5*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(\\ & a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4* \\ & d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^{(1/3)})) * \log(40*a*b^3* \\ & c*d^2 + 8*a^2*b^2*d^2*f + 1/4*(2*a^6*b^4*d + a^7*b^3*g)*((1/2)^{(1/3)}*(I*\text{sqrt} \\ & (3) + 1))*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + \\ & a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b \\ & ^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - \\ & (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^{(1/3)} - 2*(1/2)^{(2/3)}*(10*b^2*c*d + a^ \\ & 2*f*g + (2*d*f + 5*c*g)*a*b)*(-I*\text{sqrt}(3) + 1)/(a^5*b^3*((125*b^4*c^3 + 8*a* \\ & b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g \\ & + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d* \\ & g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8 \\ & *b^5))^{(1/3)}))^2 + 2*(5*a^3*b*c + a^4*f)*g^2 - 1/2*(25*a^3*b^4*c^2 + 10*a^4 \\ & *b^3*c*f + a^5*b^2*f^2)*((1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1))*((125*b^4*c^3 + 8*a*b^ \\ & 3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + \\ & 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^ \\ & 2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b \\ & ^5))^{(1/3)} - 2*(1/2)^{(2/3)}*(10*b^2*c*d + a^2*f*g + (2*d*f + 5*c*g)*a*b)*(-I \\ & * \text{sqrt}(3) + 1)/(a^5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^ \\ & 2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8* \\ & b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2* \\ & g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^{(1/3)})) + 8*(5*a^2*b^2*c* \\ & d + a^3*b*d*f)*g + (125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2 \end{aligned}$$

$$\begin{aligned}
& *c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)*x) - 12*(4 \\
& *a*b^2*c - a^2*b*f)*x - ((a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2)*((1/2)^(1/ \\
& 3)*(I*sqrt(3) + 1)*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^ \\
& 2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) \\
& + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a \\
& ^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3) - 2*(1/2)^(2/3)*(10*b^2 \\
& *c*d + a^2*f*g + (2*d*f + 5*c*g)*a*b)*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b^4*c \\
& ^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b \\
& ^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f \\
& ^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a \\
& b^3)/(a^8*b^5))^(1/3))) + 3*sqrt(1/3)*(a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^ \\
& 2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b \\
& ^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 \\
& + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(\\
& 5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3) - 2 \\
& *(1/2)^(2/3)*(10*b^2*c*d + a^2*f*g + (2*d*f + 5*c*g)*a*b)*(-I*sqrt(3) + 1)/ \\
& (a^5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + \\
& a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^ \\
& 4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (\\
& 8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3)))^2*a^5*b^3 + 160*b^2*c*d + 32*a \\
& b*d*f + 16*(5*a*b*c + a^2*f)*g)/(a^5*b^3))*log(-40*a*b^3*c*d^2 - 8*a^2*b^2 \\
& *d^2*f - 1/4*(2*a^6*b^4*d + a^7*b^3*g)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b \\
& ^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a \\
& ^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 \\
& + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f \\
&)*a*b^3)/(a^8*b^5))^(1/3) - 2*(1/2)^(2/3)*(10*b^2*c*d + a^2*f*g + (2*d*f + \\
& 5*c*g)*a*b)*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^ \\
& 3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + \\
& a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5 \\
& *c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3)))^2 \\
& - 2*(5*a^3*b*c + a^4*f)*g^2 + 1/2*(25*a^3*b^4*c^2 + 10*a^4*b^3*c*f + a^5*b^ \\
& 2*f^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3* \\
& c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a \\
& ^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c \\
& *f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3) - 2*(1 \\
& /2)^(2/3)*(10*b^2*c*d + a^2*f*g + (2*d*f + 5*c*g)*a*b)*(-I*sqrt(3) + 1)/(a^ \\
& 5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3 \\
& *b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c \\
& ^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d \\
& ^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3))) - 8*(5*a^2*b^2*c*d + a^3*b*d*f)*g \\
& + 2*(125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b* \\
& f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)*x + 3/4*sqrt(1/3)*(50*a^3 \\
& *b^4*c^2 + 20*a^4*b^3*c*f + 2*a^5*b^2*f^2 + (2*a^6*b^4*d + a^7*b^3*g)*((1/2) \\
&)^(1/3)*(I*sqrt(3) + 1)*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a \\
& ^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8
\end{aligned}$$

$$\begin{aligned}
& *b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2* \\
& *g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^{(1/3)} - 2*(1/2)^{(2/3)}*(1 \\
& 0*b^2*c*d + a^2*f*g + (2*d*f + 5*c*g)*a*b)*(-I*\sqrt{3} + 1)/(a^5*b^3*((125* \\
& b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12* \\
& a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 \\
& + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2* \\
& f)*a*b^3)/(a^8*b^5))^{(1/3)})))*\sqrt{-(((1/2)^{(1/3)}*(I*\sqrt{3} + 1))*((125*b^4 \\
& *c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2 \\
& *b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + \\
& (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)* \\
& a*b^3)/(a^8*b^5))^{(1/3)} - 2*(1/2)^{(2/3)}*(10*b^2*c*d + a^2*f*g + (2*d*f + 5* \\
& c*g)*a*b)*(-I*\sqrt{3} + 1)/(a^5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3* \\
& c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a \\
& ^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c \\
& *f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^{(1/3)}))^2*a^ \\
& 5*b^3 + 160*b^2*c*d + 32*a*b*d*f + 16*(5*a*b*c + a^2*f)*g)/(a^5*b^3))) - ((\\
& a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2)*((1/2)^{(1/3)}*(I*\sqrt{3} + 1))*((125*b \\
& ^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a \\
& ^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 \\
& + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f \\
&)*a*b^3)/(a^8*b^5))^{(1/3)} - 2*(1/2)^{(2/3)}*(10*b^2*c*d + a^2*f*g + (2*d*f + \\
& 5*c*g)*a*b)*(-I*\sqrt{3} + 1)/(a^5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^ \\
& 3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + \\
& a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5 \\
& *c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^{(1/3)})) - \\
& 3*\sqrt{1/3}*(a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2)*\sqrt{-(((1/2)^{(1/3)}*(I* \\
& \sqrt{3} + 1))*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^ \\
& 2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (12 \\
& 5*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 \\
& - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^{(1/3)} - 2*(1/2)^{(2/3)}*(10*b^2*c*d + \\
& a^2*f*g + (2*d*f + 5*c*g)*a*b)*(-I*\sqrt{3} + 1)/(a^5*b^3*((125*b^4*c^3 + 8 \\
& *a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2 \\
& *g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6 \\
& *d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(\\
& a^8*b^5))^{(1/3)}))^2*a^5*b^3 + 160*b^2*c*d + 32*a*b*d*f + 16*(5*a*b*c + a^2* \\
& f)*g)/(a^5*b^3))) * \log(-40*a*b^3*c*d^2 - 8*a^2*b^2*d^2*f - 1/4*(2*a^6*b^4*d \\
& + a^7*b^3*g)*((1/2)^{(1/3)}*(I*\sqrt{3} + 1))*((125*b^4*c^3 + 8*a*b^3*d^3 + 75* \\
& a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^ \\
& ^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + \\
& 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^{(1/3)} \\
& - 2*(1/2)^{(2/3)}*(10*b^2*c*d + a^2*f*g + (2*d*f + 5*c*g)*a*b)*(-I*\sqrt{3} + \\
& 1)/(a^5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 \\
& + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125 \\
& *b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 \\
& - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^{(1/3)}))^2 - 2*(5*a^3*b*c + a^4*f)*g^
\end{aligned}$$

$$\begin{aligned}
& 2 + \frac{1}{2} * (25 * a^3 * b^4 * c^2 + 10 * a^4 * b^3 * c * f + a^5 * b^2 * f^2) * \left(\frac{1}{2} \right)^{\frac{1}{3}} * (I * \sqrt[3]{3} + 1) * \left((125 * b^4 * c^3 + 8 * a * b^3 * d^3 + 75 * a * b^3 * c^2 * f + 15 * a^2 * b^2 * c * f^2 + \right. \\
& a^3 * b * f^3 + 12 * a^2 * b^2 * d^2 * g + 6 * a^3 * b * d * g^2 + a^4 * g^3) / (a^8 * b^5) + (125 * b^4 * c^3 - a^4 * g^3 + (f^3 - 6 * d * g^2) * a^3 * b + 3 * (5 * c * f^2 - 4 * d^2 * g) * a^2 * b^2 - \\
& (8 * d^3 - 75 * c^2 * f) * a * b^3) / (a^8 * b^5) \left. \right)^{\frac{1}{3}} - 2 * \left(\frac{1}{2} \right)^{\frac{2}{3}} * (10 * b^2 * c * d + a^2 * f * g + (2 * d * f + 5 * c * g) * a * b) * (-I * \sqrt[3]{3} + 1) / (a^5 * b^3 * \left((125 * b^4 * c^3 + 8 * a * \right. \\
& b^3 * d^3 + 75 * a * b^3 * c^2 * f + 15 * a^2 * b^2 * c * f^2 + a^3 * b * f^3 + 12 * a^2 * b^2 * d^2 * g + 6 * a^3 * b * d * g^2 + a^4 * g^3) / (a^8 * b^5) + (125 * b^4 * c^3 - a^4 * g^3 + (f^3 - 6 * d * \\
& g^2) * a^3 * b + 3 * (5 * c * f^2 - 4 * d^2 * g) * a^2 * b^2 - (8 * d^3 - 75 * c^2 * f) * a * b^3) / (a^8 * b^5) \left. \right)^{\frac{1}{3}}) - 8 * (5 * a^2 * b^2 * c * d + a^3 * b * d * f) * g + 2 * (125 * b^4 * c^3 + 8 * a * b^3 \\
& * d^3 + 75 * a * b^3 * c^2 * f + 15 * a^2 * b^2 * c * f^2 + a^3 * b * f^3 + 12 * a^2 * b^2 * d^2 * g + 6 * a^3 * b * d * g^2 + a^4 * g^3) * x - \frac{3}{4} * \sqrt[3]{1/3} * (50 * a^3 * b^4 * c^2 + 20 * a^4 * b^3 * c * f \\
& + 2 * a^5 * b^2 * f^2 + (2 * a^6 * b^4 * d + a^7 * b^3 * g) * \left(\frac{1}{2} \right)^{\frac{1}{3}} * (I * \sqrt[3]{3} + 1) * \left((125 * b^4 * c^3 + 8 * a * b^3 * d^3 + 75 * a * b^3 * c^2 * f + 15 * a^2 * b^2 * c * f^2 + a^3 * b * f^3 + \right. \\
& 12 * a^2 * b^2 * d^2 * g + 6 * a^3 * b * d * g^2 + a^4 * g^3) / (a^8 * b^5) + (125 * b^4 * c^3 - a^4 * g^3 + (f^3 - 6 * d * g^2) * a^3 * b + 3 * (5 * c * f^2 - 4 * d^2 * g) * a^2 * b^2 - (8 * d^3 - 75 * \\
& c^2 * f) * a * b^3) / (a^8 * b^5) \left. \right)^{\frac{1}{3}} - 2 * \left(\frac{1}{2} \right)^{\frac{2}{3}} * (10 * b^2 * c * d + a^2 * f * g + (2 * d * f + 5 * c * g) * a * b) * (-I * \sqrt[3]{3} + 1) / (a^5 * b^3 * \left((125 * b^4 * c^3 + 8 * a * b^3 * d^3 + 75 * \\
& a * b^3 * c^2 * f + 15 * a^2 * b^2 * c * f^2 + a^3 * b * f^3 + 12 * a^2 * b^2 * d^2 * g + 6 * a^3 * b * d * g^2 + a^4 * g^3) / (a^8 * b^5) + (125 * b^4 * c^3 - a^4 * g^3 + (f^3 - 6 * d * g^2) * a^3 * b + \right. \\
& 3 * (5 * c * f^2 - 4 * d^2 * g) * a^2 * b^2 - (8 * d^3 - 75 * c^2 * f) * a * b^3) / (a^8 * b^5) \left. \right)^{\frac{1}{3}}) \left. \right) * \sqrt{-\left(\left(\frac{1}{2} \right)^{\frac{1}{3}} * (I * \sqrt[3]{3} + 1) * \left((125 * b^4 * c^3 + 8 * a * b^3 * d^3 + 75 * a * \right. \right. \\
& b^3 * c^2 * f + 15 * a^2 * b^2 * c * f^2 + a^3 * b * f^3 + 12 * a^2 * b^2 * d^2 * g + 6 * a^3 * b * d * g^2 + a^4 * g^3) / (a^8 * b^5) + (125 * b^4 * c^3 - a^4 * g^3 + (f^3 - 6 * d * g^2) * a^3 * b + 3 * \\
& (5 * c * f^2 - 4 * d^2 * g) * a^2 * b^2 - (8 * d^3 - 75 * c^2 * f) * a * b^3) / (a^8 * b^5) \left. \right)^{\frac{1}{3}} - 2 * \left(\frac{1}{2} \right)^{\frac{2}{3}} * (10 * b^2 * c * d + a^2 * f * g + (2 * d * f + 5 * c * g) * a * b) * (-I * \sqrt[3]{3} + 1) \\
& / (a^5 * b^3 * \left((125 * b^4 * c^3 + 8 * a * b^3 * d^3 + 75 * a * b^3 * c^2 * f + 15 * a^2 * b^2 * c * f^2 + a^3 * b * f^3 + 12 * a^2 * b^2 * d^2 * g + 6 * a^3 * b * d * g^2 + a^4 * g^3) / (a^8 * b^5) + (125 * b^4 * c^3 - a^4 * g^3 + (f^3 - 6 * d * g^2) * a^3 * b + \right. \\
& (8 * d^3 - 75 * c^2 * f) * a * b^3) / (a^8 * b^5) \left. \right)^{\frac{1}{3}}) \left. \right)^2 * a^5 * b^3 + 160 * b^2 * c * d + 32 * a * b * d * f + 16 * (5 * a * b * c + a^2 * f) * g) / (a^5 * b^3) \left. \right) \left. \right) / (a^2 * b^4 * x^6 + 2 * a^3 * b^3 * x^3 + a^4 * b^2)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.08976, size = 467, normalized size = 1.49

$$\frac{\left(2bd\left(-\frac{a}{b}\right)^{\frac{1}{3}} + ag\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 5bc + af\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^3b} + \frac{\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}b^2c + \left(-ab^2\right)^{\frac{1}{3}}abf - 2\left(-ab^2\right)^{\frac{2}{3}}bd - \left(-ab^2\right)^{\frac{2}{3}}b^2d\right)}{27a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac")

[Out] -1/27*(2*b*d*(-a/b)^(1/3) + a*g*(-a/b)^(1/3) + 5*b*c + a*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b) + 1/27*sqrt(3)*(5*(-a*b^2)^(1/3)*b^2*c + (-a*b^2)^(1/3)*a*b*f - 2*(-a*b^2)^(2/3)*b*d - (-a*b^2)^(2/3)*a*g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^3) + 1/54*(5*(-a*b^2)^(1/3)*b^2*c + (-a*b^2)^(1/3)*a*b*f + 2*(-a*b^2)^(2/3)*b*d + (-a*b^2)^(2/3)*a*g)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^3) + 1/18*(4*b^3*d*x^5 + 2*a*b^2*g*x^5 + 5*b^3*c*x^4 + a*b^2*f*x^4 - 6*a^2*b*h*x^3 + 7*a*b^2*d*x^2 - a^2*b*g*x^2 + 8*a*b^2*c*x - 2*a^2*b*f*x - 3*a^3*h - 3*a^2*b*e)/((b*x^3 + a)^2*a^2*b^2)

$$3.426 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)^3} dx$$

Optimal. Leaf size=347

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(\sqrt[3]{b}(ag + 5bd) - \sqrt[3]{a}(ah + 2be)\right)}{54a^{8/3}b^{5/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(\sqrt[3]{b}(ag + 5bd) - \sqrt[3]{a}(ah + 2be)\right)}{27a^{8/3}b^{5/3}} - \tan^{-1}\left(\frac{\sqrt[3]{a}(ah + 2be) - \sqrt[3]{b}(ag + 5bd)}{\sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}\right)$$

[Out] (x*(a*(b*d - a*g) + a*(b*e - a*h)*x - b*(b*c - a*f)*x^2))/(6*a^2*b*(a + b*x^3)^2) + (x*(a*(5*b*d + a*g) + 2*a*(2*b*e + a*h)*x - 3*b*(3*b*c - a*f)*x^2))/(18*a^3*b*(a + b*x^3)) - ((5*b^(4/3)*d + 2*a^(1/3)*b*e + a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(8/3)*b^(5/3)) + (c*Log[x])/a^3 + ((b^(1/3)*(5*b*d + a*g) - a^(1/3)*(2*b*e + a*h))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(5/3)) - ((b^(1/3)*(5*b*d + a*g) - a^(1/3)*(2*b*e + a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(5/3)) - (c*Log[a + b*x^3])/(3*a^3)

Rubi [A] time = 0.723069, antiderivative size = 345, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(ah+2be)}{\sqrt[3]{b}} + ag + 5bd\right)}{54a^{8/3}b^{4/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(\sqrt[3]{b}(ag + 5bd) - \sqrt[3]{a}(ah + 2be)\right)}{27a^{8/3}b^{5/3}} - \tan^{-1}\left(\frac{\sqrt[3]{a}(ah + 2be) - \sqrt[3]{b}(ag + 5bd)}{\sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}\right)$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^3), x]

[Out] (x*(a*(b*d - a*g) + a*(b*e - a*h)*x - b*(b*c - a*f)*x^2))/(6*a^2*b*(a + b*x^3)^2) + (x*(a*(5*b*d + a*g) + 2*a*(2*b*e + a*h)*x - 3*b*(3*b*c - a*f)*x^2))/(18*a^3*b*(a + b*x^3)) - ((5*b^(4/3)*d + 2*a^(1/3)*b*e + a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(8/3)*b^(5/3)) + (c*Log[x])/a^3 + ((b^(1/3)*(5*b*d + a*g) - a^(1/3)*(2*b*e + a*h))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(5/3)) - ((5*b*d + a*g - (a^(1/3)*(2*b*e + a*h))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(4/3)) - (c*Log[a + b*x^3])/(3*a^3)

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Int[Ex
pandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[A*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[A*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{x(a + bx^3)^3} dx &= \frac{x(a(bd - ag) + a(be - ah)x - b(bc - af)x^2)}{6a^2b(a + bx^3)^2} - \int \frac{-6b^2c - b(5bd + ag)x - 2b(2be + ah)x^2 + 3b^2\left(\frac{bc}{a}\right)}{x(a + bx^3)^2} \\
&= \frac{x(a(bd - ag) + a(be - ah)x - b(bc - af)x^2)}{6a^2b(a + bx^3)^2} + \frac{x(a(5bd + ag) + 2a(2be + ah)x -}{18a^3b(a + bx^3)} \\
&= \frac{x(a(bd - ag) + a(be - ah)x - b(bc - af)x^2)}{6a^2b(a + bx^3)^2} + \frac{x(a(5bd + ag) + 2a(2be + ah)x -}{18a^3b(a + bx^3)} \\
&= \frac{x(a(bd - ag) + a(be - ah)x - b(bc - af)x^2)}{6a^2b(a + bx^3)^2} + \frac{x(a(5bd + ag) + 2a(2be + ah)x -}{18a^3b(a + bx^3)} \\
&= \frac{x(a(bd - ag) + a(be - ah)x - b(bc - af)x^2)}{6a^2b(a + bx^3)^2} + \frac{x(a(5bd + ag) + 2a(2be + ah)x -}{18a^3b(a + bx^3)} \\
&= \frac{x(a(bd - ag) + a(be - ah)x - b(bc - af)x^2)}{6a^2b(a + bx^3)^2} + \frac{x(a(5bd + ag) + 2a(2be + ah)x -}{18a^3b(a + bx^3)} \\
&= \frac{x(a(bd - ag) + a(be - ah)x - b(bc - af)x^2)}{6a^2b(a + bx^3)^2} + \frac{x(a(5bd + ag) + 2a(2be + ah)x -}{18a^3b(a + bx^3)} \\
&= \frac{x(a(bd - ag) + a(be - ah)x - b(bc - af)x^2)}{6a^2b(a + bx^3)^2} + \frac{x(a(5bd + ag) + 2a(2be + ah)x -}{18a^3b(a + bx^3)} \\
&= \frac{x(a(bd - ag) + a(be - ah)x - b(bc - af)x^2)}{6a^2b(a + bx^3)^2} + \frac{x(a(5bd + ag) + 2a(2be + ah)x -}{18a^3b(a + bx^3)} \\
&= \frac{x(a(bd - ag) + a(be - ah)x - b(bc - af)x^2)}{6a^2b(a + bx^3)^2} + \frac{x(a(5bd + ag) + 2a(2be + ah)x -}{18a^3b(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.274023, size = 311, normalized size = 0.9

$$\frac{\sqrt[3]{a} \log\left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2\right) \left(a^{4/3} h + 2 \sqrt[3]{abe} - a \sqrt[3]{bg} - 5b^{4/3} d\right)}{b^{5/3}} + \frac{2 \sqrt[3]{a} \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right) \left(a^{4/3} (-h) - 2 \sqrt[3]{abe} + a \sqrt[3]{bg} + 5b^{4/3} d\right)}{b^{5/3}} - \frac{2 \sqrt{3} \sqrt[3]{a} \tan^{-1}\left(\frac{1 - \frac{2 \sqrt[3]{bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) \left(a^{4/3} h + 2 \sqrt[3]{ab}\right)}{b^{5/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^3),x]

[Out]
$$\frac{((3*a*(6*b*c + b*x*(5*d + 4*e*x)) + a*x*(g + 2*h*x)))/(b*(a + b*x^3)) - (9*a^2*(-(b*(c + x*(d + e*x))) + a*(f + x*(g + h*x))))/(b*(a + b*x^3)^2) - (2*\text{Sqrt}[3]*a^{1/3}*(5*b^{4/3}*d + 2*a^{1/3}*b*e + a*b^{1/3}*g + a^{4/3}*h)*\text{ArcTan}[(1 - (2*b^{1/3}*x)/a^{1/3})/\text{Sqrt}[3]])/b^{5/3} + 54*c*\text{Log}[x] + (2*a^{1/3}*(5*b^{4/3}*d - 2*a^{1/3}*b*e + a*b^{1/3}*g - a^{4/3}*h)*\text{Log}[a^{1/3} + b^{1/3}*x])/b^{5/3} + (a^{1/3}*(-5*b^{4/3}*d + 2*a^{1/3}*b*e - a*b^{1/3}*g + a^{4/3}*h)*\text{Log}[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2])/b^{5/3} - 18*c*\text{Log}[a + b*x^3])/(54*a^3)$$

Maple [B] time = 0.017, size = 618, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^3,x)

[Out]
$$\frac{1}{27} \frac{a}{b^2} \frac{1}{(1/b*a)^{2/3}} * 3^{1/2} * \arctan(1/3 * 3^{1/2} * (2/(1/b*a)^{1/3} * x - 1)) * g - 1/3 * c * \ln(b*x^3+a) / a^3 + 1/2 * a / (b*x^3+a)^2 * c + 2/9 * a^2 / (b*x^3+a)^2 * x^5 * b * e + 7/18 * a / (b*x^3+a)^2 * x^2 * e + 1/9 * a / (b*x^3+a)^2 * x^5 * h + 1/18 * a / (b*x^3+a)^2 * x^4 * g - 1/18 / (b*x^3+a)^2 / b * x^2 * h - 1/9 / (b*x^3+a)^2 / b * x * g + 5/18 * a^2 / (b*x^3+a)^2 * b * x^4 * d + 1/27 * a / b^2 / (1/b*a)^{2/3} * \ln(x + (1/b*a)^{1/3}) * g - 1/54 * a / b^2 / (1/b*a)^{2/3} * \ln(x^2 - (1/b*a)^{1/3} * x + (1/b*a)^{2/3}) * g - 1/27 * a / b^2 / (1/b*a)^{1/3} * \ln(x + (1/b*a)^{1/3}) * h + 1/54 * a / b^2 / (1/b*a)^{1/3} * \ln(x^2 - (1/b*a)^{1/3} * x + (1/b*a)^{2/3}) * h + 1/27 / a / b^2 * 3^{1/2} / (1/b*a)^{1/3} * \arctan(1/3 * 3^{1/2} * (2/(1/b*a)^{1/3} * x - 1)) * h - 1/6 * b / (b*x^3+a)^2 * f - 5/54 * a^2 / b / (1/b*a)^{2/3} * \ln(x^2 - (1/b*a)^{1/3} * x + (1/b*a)^{2/3}) * d + 5/27 * a^2 / b / (1/b*a)^{2/3} * \ln(x + (1/b*a)^{1/3}) * d + 4/9 * a / (b*x^3+a)^2 * x * d - 2/27 * a^2 / b / (1/b*a)^{1/3} * \ln(x + (1/b*a)^{1/3}) * e + 2/27 * a^2 / b * 3^{1/2} / (1/b*a)^{1/3} * \arctan(1/3 * 3^{1/2} * (2/(1/b*a)^{1/3} * x - 1)) * e + 1/3 * a^2 / (b*x^3+a)^2 * b * c * x^3 + c * \ln(x) / a^3 + 5/27 * a^2 / b / (1/b*a)^{2/3} * 3^{1/2} * \arctan(1/3 * 3^{1/2} * (2/(1/b*a)^{1/3} * x - 1)) * d + 1/27 * a^2 / b / (1/b*a)^{1/3} * \ln(x^2 - (1/b*a)^{1/3} * x + (1/b*a)^{2/3}) * e$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.09791, size = 529, normalized size = 1.52

$$-\frac{c \log(|bx^3 + a|)}{3a^3} + \frac{c \log(|x|)}{a^3} + \frac{\sqrt{3} \left(5 (-ab^2)^{\frac{1}{3}} b^2 d + (-ab^2)^{\frac{1}{3}} abg - (-ab^2)^{\frac{2}{3}} ah - 2 (-ab^2)^{\frac{2}{3}} be \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}} \right)}{27 a^3 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/3*c*log(abs(b*x^3 + a))/a^3 + c*log(abs(x))/a^3 + 1/27*sqrt(3)*(5*(-a*b^2)^(1/3)*b^2*d + (-a*b^2)^(1/3)*a*b*g - (-a*b^2)^(2/3)*a*h - 2*(-a*b^2)^(2/3)*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^3) + 1/54*(5*(-a*b^2)^(1/3)*b^2*d + (-a*b^2)^(1/3)*a*b*g + (-a*b^2)^(2/3)*a*h + 2*(-a*b^2)^(2/3)*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^3) + 1/18*(6*a*b^2*c*x^3 + 2*(a^2*b*h + 2*a*b^2*e)*x^5 + (5*a*b^2*d + a^2*b*g)*x^4 + 9*a^2*b*c - 3*a^3*f - (a^3*h - 7*a^2*b*e)*x^2 + 2*(4*a^2*b*d - a^3*g)*x)/((b*x^3 + a)^2*a^3*b) - 1/27*(a^5*b^2*h*(-a/b)^(1/3) + 2*a^4*b^3*(-a/b)^(1/3)*e + 5*a^4*b^3*d + a^5*b^2*g)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b^3)
```

$$3.427 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)^3} dx$$

Optimal. Leaf size=362

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2\right)\left(a^{2/3}(ah+5be)+2b^{2/3}(7bc-af)\right)}{54a^{10/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)\left(a^{2/3}(ah+5be)+2b^{2/3}(7bc-af)\right)}{27a^{10/3}b^{4/3}}+$$

[Out] $-(c/(a^3x)) + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(6*a^2*b*(a + b*x^3)^2) + (x*(a*(5*b*e + a*h) - 2*b*(5*b*c - 2*a*f)*x - 3*b*(3*b*d - a*g)*x^2))/(18*a^3*b*(a + b*x^3)) + ((14*b^(5/3)*c - 5*a^(2/3)*b*e - 2*a*b^(2/3)*f - a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)))]/(9*Sqrt[3]*a^(10/3)*b^(4/3)) + (d*Log[x])/a^3 + ((2*b^(2/3)*(7*b*c - a*f) + a^(2/3)*(5*b*e + a*h))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*b^(4/3)) - ((2*b^(2/3)*(7*b*c - a*f) + a^(2/3)*(5*b*e + a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(10/3)*b^(4/3)) - (d*Log[a + b*x^3])/(3*a^3)$

Rubi [A] time = 0.829961, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2\right)\left(a^{2/3}(ah+5be)+2b^{2/3}(7bc-af)\right)}{54a^{10/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)\left(a^{2/3}(ah+5be)+2b^{2/3}(7bc-af)\right)}{27a^{10/3}b^{4/3}}+$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^3), x]

[Out] $-(c/(a^3x)) + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(6*a^2*b*(a + b*x^3)^2) + (x*(a*(5*b*e + a*h) - 2*b*(5*b*c - 2*a*f)*x - 3*b*(3*b*d - a*g)*x^2))/(18*a^3*b*(a + b*x^3)) + ((14*b^(5/3)*c - 5*a^(2/3)*b*e - 2*a*b^(2/3)*f - a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)))]/(9*Sqrt[3]*a^(10/3)*b^(4/3)) + (d*Log[x])/a^3 + ((2*b^(2/3)*(7*b*c - a*f) + a^(2/3)*(5*b*e + a*h))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*b^(4/3)) - ((2*b^(2/3)*(7*b*c - a*f) + a^(2/3)*(5*b*e + a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(10/3)*b^(4/3)) - (d*Log[a + b*x^3])/(3*a^3)$

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_)^(m_.)))/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(p_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{x^2(a + bx^3)^3} dx &= \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{6a^2b(a + bx^3)^2} - \int \frac{-6b^2c - 6b^2dx - b(5be + ah)x^2 + 4b^2\left(\frac{bc}{a} - f\right)}{x^2(a + bx^3)^2} \\
&= \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{6a^2b(a + bx^3)^2} + \frac{x(a(5be + ah) - 2b(5bc - 2af)x)}{18a^3b(a + bx^3)} \\
&= \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{6a^2b(a + bx^3)^2} + \frac{x(a(5be + ah) - 2b(5bc - 2af)x)}{18a^3b(a + bx^3)} \\
&= -\frac{c}{a^3x} + \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{6a^2b(a + bx^3)^2} + \frac{x(a(5be + ah) - 2b(5bc - 2af)x)}{18a^3b} \\
&= -\frac{c}{a^3x} + \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{6a^2b(a + bx^3)^2} + \frac{x(a(5be + ah) - 2b(5bc - 2af)x)}{18a^3b} \\
&= -\frac{c}{a^3x} + \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{6a^2b(a + bx^3)^2} + \frac{x(a(5be + ah) - 2b(5bc - 2af)x)}{18a^3b} \\
&= -\frac{c}{a^3x} + \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{6a^2b(a + bx^3)^2} + \frac{x(a(5be + ah) - 2b(5bc - 2af)x)}{18a^3b} \\
&= -\frac{c}{a^3x} + \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{6a^2b(a + bx^3)^2} + \frac{x(a(5be + ah) - 2b(5bc - 2af)x)}{18a^3b} \\
&= -\frac{c}{a^3x} + \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{6a^2b(a + bx^3)^2} + \frac{x(a(5be + ah) - 2b(5bc - 2af)x)}{18a^3b} \\
&= -\frac{c}{a^3x} + \frac{x(a(be - ah) - b(bc - af)x - b(bd - ag)x^2)}{6a^2b(a + bx^3)^2} + \frac{x(a(5be + ah) - 2b(5bc - 2af)x)}{18a^3b}
\end{aligned}$$

Mathematica [A] time = 0.569985, size = 336, normalized size = 0.93

$$\frac{9a^2(a^2(g+hx) - ab(d+x(e+fx)) + b^2cx^2)}{b(a+bx^3)^2} - \frac{3a(a^2hx + ab(6d+x(5e+4fx)) - 10b^2cx^2)}{b(a+bx^3)} + \frac{a^{2/3} \log\left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx + b^{2/3}x^2}\right) (5a^{2/3}be + a^{5/3}h - 2ab^{2/3}f + 14b^{5/3}c)}{b^{4/3}} - 2a^2$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^3),x]

[Out]
$$\begin{aligned} & -\left(\frac{54ac}{x} + (9a^2(b^2cx^2 + a^2(g + hx) - ab(d + x(e + fx))))\right) / \\ & (b(a + b^3x^3)^2 - (3a(a^2hx - 10b^2cx^2 + ab(6d + x(5e + 4fx)))) / (b(a + b^3x^3)) + (2\sqrt{3}a^{2/3}(-14b^{5/3}c + 5a^{2/3}be + \\ & 2ab^{2/3}f + a^{5/3}h)\operatorname{ArcTan}\left[\frac{1 - (2b^{1/3}x)/a^{1/3}}{\sqrt{3}}\right]) / b^{4/3} - \\ & 54ad\operatorname{Log}[x] - (2a^{2/3}(14b^{5/3}c + 5a^{2/3}be - 2ab^{2/3}f + a^{5/3}h)\operatorname{Log}[a^{1/3} + b^{1/3}x]) / b^{4/3} + \\ & (a^{2/3}(14b^{5/3}c + 5a^{2/3}be - 2ab^{2/3}f + a^{5/3}h)\operatorname{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2]) / b^{4/3} + 18ad\operatorname{Log}[a + b^3x^3]) / (54a^4) \end{aligned}$$

Maple [B] time = 0.016, size = 622, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x)

[Out]
$$\begin{aligned} & 1/27/a/b^2/(1/b^3a)^{2/3}*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(1/b^3a)^{1/3}*x-1))* \\ & h+14/27/a^3/(1/b^3a)^{1/3}*\ln(x+(1/b^3a)^{1/3})*c-7/27/a^3/(1/b^3a)^{1/3}*\ln(x \\ & ^2-(1/b^3a)^{1/3}*x+(1/b^3a)^{2/3})*c+d*\ln(x)/a^3-1/3*d*\ln(b^3x^3+a)/a^3-5/9/a \\ & ^3/(b^3x^3+a)^2*b^2*x^5*c+5/18/a^2/(b^3x^3+a)^2*x^4*b*e+2/9/a^2/(b^3x^3+a)^2*x \\ & ^5*b*f-5/54/a^2/b/(1/b^3a)^{2/3}*\ln(x^2-(1/b^3a)^{1/3}*x+(1/b^3a)^{2/3})*e-c/a \\ & ^3/x+5/27/a^2/b/(1/b^3a)^{2/3}*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(1/b^3a)^{1/3}*x \\ & -1))*e+5/27/a^2/b/(1/b^3a)^{2/3}*\ln(x+(1/b^3a)^{1/3})*e-2/27/a^2*f/b/(1/b^3a)^{1/3} \\ & *\ln(x+(1/b^3a)^{1/3})+1/27/a^2*f/b/(1/b^3a)^{1/3}*\ln(x^2-(1/b^3a)^{1/3}*x \\ & +(1/b^3a)^{2/3}))+7/18/a/(b^3x^3+a)^2*x^2*f-1/9/(b^3x^3+a)^2/b*x*h+1/18/a/(b^3x^3 \\ & +a)^2*x^4*h-1/6/(b^3x^3+a)^2/b*g+4/9/a/(b^3x^3+a)^2*x*e+1/2/a/(b^3x^3+a)^2*d- \\ & 14/27/a^3*3^{1/2}/(1/b^3a)^{1/3}*\arctan(1/3*3^{1/2}*(2/(1/b^3a)^{1/3}*x-1))*c \\ & -13/18/a^2/(b^3x^3+a)^2*b*x^2*c+2/27/a^2*f*3^{1/2}/b/(1/b^3a)^{1/3}*\arctan(1/ \\ & 3*3^{1/2}*(2/(1/b^3a)^{1/3}*x-1))-1/54/a/b^2/(1/b^3a)^{2/3}*\ln(x^2-(1/b^3a)^{1/3} \\ & *x+(1/b^3a)^{2/3})*h+1/27/a/b^2/(1/b^3a)^{2/3}*\ln(x+(1/b^3a)^{1/3})*h+1/3/a \\ & ^2/(b^3x^3+a)^2*b*d*x^3 \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.09166, size = 556, normalized size = 1.54

$$-\frac{d \log(|bx^3 + a|)}{3a^3} + \frac{d \log(|x|)}{a^3} + \frac{\sqrt{3} \left((-ab^2)^{\frac{1}{3}} a^2 h + 5 (-ab^2)^{\frac{1}{3}} a b e + 14 (-ab^2)^{\frac{2}{3}} b c - 2 (-ab^2)^{\frac{2}{3}} a f \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}} \right)}{27 a^4 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="giac")

[Out]
$$-1/3*d*\log(\text{abs}(b*x^3 + a))/a^3 + d*\log(\text{abs}(x))/a^3 + 1/27*\sqrt{3}*((-a*b^2)^{1/3}*a^2*h + 5*(-a*b^2)^{1/3}*a*b*e + 14*(-a*b^2)^{2/3}*b*c - 2*(-a*b^2)^{2/3}*a*f)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{1/3})/(-a/b)^{1/3})/(a^4*b^2) + 1/54*((-a*b^2)^{1/3}*a^2*h + 5*(-a*b^2)^{1/3}*a*b*e - 14*(-a*b^2)^{2/3}*b*c + 2*(-a*b^2)^{2/3}*a*f)*\log(x^2 + x*(-a/b)^{1/3} + (-a/b)^{2/3})/(a^4*b^2) + 1/18*(6*a*b^2*d*x^4 - 4*(7*b^3*c - a*b^2*f)*x^6 + (a^2*b*h + 5*a*b^2*e)*x^5 - 18*a^2*b*c - 7*(7*a*b^2*c - a^2*b*f)*x^3 - 2*(a^3*h - 4*a^2*b*e)*x^2 + 3*(3*a^2*b*d - a^3*g)*x)/((b*x^3 + a)^2*a^3*b*x) + 1/27*(14*a^3*b^4*c*(-a/b)^{1/3} - 2*a^4*b^3*f*(-a/b)^{1/3} - a^5*b^2*h - 5*a^4*b^3*e)*(-a/b)^{1/3}*\log(\text{abs}(x - (-a/b)^{1/3}))/a^7*b^3$$

$$3.428 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)^3} dx$$

Optimal. Leaf size=360

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(5\sqrt[3]{b}(4bc - af) - 2\sqrt[3]{a}(7bd - ag)\right)}{54a^{11/3}b^{2/3}} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(5\sqrt[3]{b}(4bc - af) - 2\sqrt[3]{a}(7bd - ag)\right)}{27a^{11/3}b^{2/3}} +$$

[Out] $-c/(2*a^3*x^2) - d/(a^3*x) - (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(6*a^2*(a + b*x^3)^2) - (x*(11*b*c - 5*a*f + 2*(5*b*d - 2*a*g)*x + 3*(3*b*e - a*h)*x^2))/(18*a^3*(a + b*x^3)) + ((20*b^(4/3)*c + 14*a^(1/3)*b*d - 5*a*b^(1/3)*f - 2*a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)))]/(9*Sqrt[3]*a^(11/3)*b^(2/3)) + (e*Log[x])/a^3 - ((5*b^(1/3)*(4*b*c - a*f) - 2*a^(1/3)*(7*b*d - a*g))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*b^(2/3)) + ((5*b^(1/3)*(4*b*c - a*f) - 2*a^(1/3)*(7*b*d - a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(11/3)*b^(2/3)) - (e*Log[a + b*x^3])/(3*a^3)$

Rubi [A] time = 0.813874, antiderivative size = 357, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)\left(-\frac{2\sqrt[3]{a}(7bd-ag)}{\sqrt[3]{b}} - 5af + 20bc\right)}{54a^{11/3}\sqrt[3]{b}} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)\left(5\sqrt[3]{b}(4bc - af) - 2\sqrt[3]{a}(7bd - ag)\right)}{27a^{11/3}b^{2/3}} + \frac{\tan^{-1}\left(\frac{a^{1/3} - 2b^{1/3}x}{\sqrt[3]{3}a^{1/3}}\right)}{\sqrt[3]{3}a^{11/3}b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^3), x]

[Out] $-c/(2*a^3*x^2) - d/(a^3*x) - (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(6*a^2*(a + b*x^3)^2) - (x*(11*b*c - 5*a*f + 2*(5*b*d - 2*a*g)*x + 3*(3*b*e - a*h)*x^2))/(18*a^3*(a + b*x^3)) + ((20*b^(4/3)*c + 14*a^(1/3)*b*d - 5*a*b^(1/3)*f - 2*a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)))]/(9*Sqrt[3]*a^(11/3)*b^(2/3)) + (e*Log[x])/a^3 - ((5*b^(1/3)*(4*b*c - a*f) - 2*a^(1/3)*(7*b*d - a*g))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*b^(2/3)) + ((20*b*c - 5*a*f - (2*a^(1/3)*(7*b*d - a*g))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(11/3)*b^(1/3)) - (e*Log[a + b*x^3])/(3*a^3)$

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_)^(m_.)))/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{x^3(a + bx^3)^3} dx &= -\frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{6a^2(a + bx^3)^2} - \frac{\int \frac{-6b^2c - 6b^2dx - 6b^2ex^2 + 5b^2\left(\frac{bc}{a} - f\right)x^3 + 4b^2\left(\frac{bd}{a} - g\right)x^4 - 3b^2\left(\frac{be}{a} - h\right)x^5}{x^3(a + bx^3)^2} dx}{6ab^2} \\
&= -\frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{6a^2(a + bx^3)^2} - \frac{x(11bc - 5af + 2(5bd - 2ag)x + 3(3be - 2ah)x^2)}{18a^3(a + bx^3)} \\
&= -\frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{6a^2(a + bx^3)^2} - \frac{x(11bc - 5af + 2(5bd - 2ag)x + 3(3be - 2ah)x^2)}{18a^3(a + bx^3)} \\
&= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{6a^2(a + bx^3)^2} - \frac{x(11bc - 5af + 2(5bd - 2ag)x + 3(3be - 2ah)x^2)}{18a^3} \\
&= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{6a^2(a + bx^3)^2} - \frac{x(11bc - 5af + 2(5bd - 2ag)x + 3(3be - 2ah)x^2)}{18a^3} \\
&= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{6a^2(a + bx^3)^2} - \frac{x(11bc - 5af + 2(5bd - 2ag)x + 3(3be - 2ah)x^2)}{18a^3} \\
&= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{6a^2(a + bx^3)^2} - \frac{x(11bc - 5af + 2(5bd - 2ag)x + 3(3be - 2ah)x^2)}{18a^3} \\
&= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{6a^2(a + bx^3)^2} - \frac{x(11bc - 5af + 2(5bd - 2ag)x + 3(3be - 2ah)x^2)}{18a^3} \\
&= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{6a^2(a + bx^3)^2} - \frac{x(11bc - 5af + 2(5bd - 2ag)x + 3(3be - 2ah)x^2)}{18a^3} \\
&= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x(bc - af + (bd - ag)x + (be - ah)x^2)}{6a^2(a + bx^3)^2} - \frac{x(11bc - 5af + 2(5bd - 2ag)x + 3(3be - 2ah)x^2)}{18a^3}
\end{aligned}$$

Mathematica [A] time = 0.606096, size = 337, normalized size = 0.94

$$\frac{9a^2(a^2h - ab(e + x(f + gx)) + b^2x(c + dx))}{b(a + bx^3)^2} - \frac{\sqrt[3]{a} \log\left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx + b^{2/3}x^2}\right) \left(2a^{4/3}g - 14\sqrt[3]{abd} - 5a\sqrt[3]{bf} + 20b^{4/3}c\right)}{b^{2/3}} + \frac{2\sqrt[3]{a} \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right) \left(2a^{4/3}g - 14\sqrt[3]{abd} - 5a\sqrt[3]{bf} + 20b^{4/3}c\right)}{b^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^3),x]

[Out]
$$-\left(\frac{27ac}{x^2} + \frac{54ad}{x} - \frac{3a(6ae - bx(11c + 10dx) + ax(5f + 4gx))}{(a + bx^3)} + \frac{9a^2(a^2h + b^2x(c + dx) - ab(e + x(f + gx)))}{(b(a + bx^3)^2)} + \frac{2\sqrt{3}a^{1/3}(-20b^{4/3}c - 14a^{1/3}bd + 5ab^{1/3}f + 2a^{4/3}g)\operatorname{ArcTan}\left[\frac{1 - (2b^{1/3}x)/a^{1/3}}{\sqrt{3}}\right]}{b^{2/3}} - \frac{54ae\operatorname{Log}[x] + (2a^{1/3}(20b^{4/3}c - 14a^{1/3}bd - 5ab^{1/3}f + 2a^{4/3}g)\operatorname{Log}[a^{1/3} + b^{1/3}x])/b^{2/3} - (a^{1/3}(20b^{4/3}c - 14a^{1/3}bd - 5ab^{1/3}f + 2a^{4/3}g)\operatorname{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2])/b^{2/3} + 18ae\operatorname{Log}[a + bx^3]}{54a^4}\right)$$

Maple [B] time = 0.017, size = 626, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x)

[Out]
$$-20/27/a^3/(1/ba)^{2/3}*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(1/ba)^{1/3}*x-1))*c-11/18/a^3/(b*x^3+a)^2*b^2*x^4*c+e*\ln(x)/a^3-1/3*e*\ln(b*x^3+a)/a^3+2/27/a^2*g*3^{1/2}/b/(1/ba)^{1/3}*\arctan(1/3*3^{1/2}*(2/(1/ba)^{1/3}*x-1))-5/9/a^3/(b*x^3+a)^2*x^5*b^2*d-7/9/a^2/(b*x^3+a)^2*b*x*c-14/27/a^3*3^{1/2}/(1/ba)^{1/3}*\arctan(1/3*3^{1/2}*(2/(1/ba)^{1/3}*x-1))*d-d/a^3/x-1/2*c/a^3/x^2-13/18/a^2/(b*x^3+a)^2*b*x^2*d-2/27/a^2*g/b/(1/ba)^{1/3}*\ln(x+(1/ba)^{1/3})+1/27/a^2*g/b/(1/ba)^{1/3}*\ln(x^2-(1/ba)^{1/3}*x+(1/ba)^{2/3})-20/27/a^3/(1/ba)^{2/3}*\ln(x+(1/ba)^{1/3})*c+10/27/a^3/(1/ba)^{2/3}*\ln(x^2-(1/ba)^{1/3}*x+(1/ba)^{2/3})*c+14/27/a^3/(1/ba)^{1/3}*\ln(x+(1/ba)^{1/3})*d-7/27/a^3/(1/ba)^{1/3}*\ln(x^2-(1/ba)^{1/3}*x+(1/ba)^{2/3})*d-1/6/(b*x^3+a)^2/b*h+2/9/a^2/(b*x^3+a)^2*x^5*b*g+4/9/a/(b*x^3+a)^2*f*x+7/18/a/(b*x^3+a)^2*x^2*g+1/2/a/(b*x^3+a)^2*e+5/18/a^2/(b*x^3+a)^2*x^4*b*f+5/27/a^2*f/b/(1/ba)^{2/3}*\ln(x+(1/ba)^{1/3})-5/54/a^2*f/b/(1/ba)^{2/3}*\ln(x^2-(1/ba)^{1/3}*x+(1/ba)^{2/3})+5/27/a^2*f/b/(1/ba)^{2/3}*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(1/ba)^{1/3}*x-1))+1/3*b/a^2/(b*x^3+a)^2*e*x^3$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 139.296, size = 31293, normalized size = 86.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2916*(972*a*b^2*e*x^5 - 648*(7*b^3*d - a*b^2*g)*x^7 - 810*(4*b^3*c - a*b^2*f)*x^6 - 2916*a^2*b*d*x - 1134*(7*a*b^2*d - a^2*b*g)*x^4 - 1458*a^2*b*c - 1296*(4*a*b^2*c - a^2*b*f)*x^3 + 486*(3*a^2*b*e - a^3*h)*x^2 - 2*(a^3*b^3*x^8 + 2*a^4*b^2*x^5 + a^5*b*x^2)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b)))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 486*e/a^3)*log(-7840*a*b^3*c*d^2 + 3600*a*b^3*c^2*e - 1134*a^2*b^2*d*e^2 + 225*a^3*b*e*f^2 - 1/1458*(7*a^8*b^2*d - a^9*b*g)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b)))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*
```

$$\begin{aligned}
& (343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^{11}*b^2))^{(1/3)} + 729*(I*\text{sqrt}(3) \\
& + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f \\
& - 40*c*g)*a*b)*e/(a^{10}*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a \\
& *b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168* \\
& a^3*b*d*g^2 - 8*a^4*g^3)/(a^{11}*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (\\
& 125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g \\
& + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a* \\
& b^3)/(a^{11}*b^2))^{(1/3)} + 486*e/a^3)^2 - 40*(4*a^3*b*c - a^4*f)*g^2 - 1/54*(\\
& 400*a^4*b^3*c^2 - 252*a^5*b^2*d*e - 200*a^5*b^2*c*f + 25*a^6*b*f^2 + 36*a^6 \\
& *b*e*g)*((-I*\text{sqrt}(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2 \\
& - 70*d*f - 40*c*g)*a*b)/(a^7*b)))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10 \\
& *a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^{10}*b) - 1/39366*(8000*b^4*c \\
& ^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 \\
& - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^{11}*b^2) - 1/39366*(\\
& 8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243 \\
& *e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 \\
& - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^{11}*b^2))^{(1/3)} + 729*(I*\text{sqrt}(3) + 1)*(- \\
& 1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g) \\
&)*a*b)*e/(a^{10}*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2 \\
& *f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d* \\
& g^2 - 8*a^4*g^3)/(a^{11}*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 \\
& - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(2 \\
& 5*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^ \\
& 11*b^2))^{(1/3)} + 486*e/a^3) + 40*(49*a^2*b^2*d^2 - 45*a^2*b^2*c*e)*f + 2*(1 \\
& 120*a^2*b^2*c*d + 81*a^3*b*e^2 - 280*a^3*b*d*f)*g - (8000*b^4*c^3 + 2744*a* \\
& b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2* \\
& b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)*x) - (1458*b^3*e*x^8 + 2916*a*b^2* \\
& e*x^5 + 1458*a^2*b*e*x^2 - (a^3*b^3*x^8 + 2*a^4*b^2*x^5 + a^5*b*x^2))*((-I*s \\
& \text{qrt}(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40 \\
& *c*g)*a*b)/(a^7*b)))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81 \\
& *e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^{10}*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^ \\
& 3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^ \\
& 2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^{11}*b^2) - 1/39366*(8000*b^4*c^3 + \\
& 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e \\
& *f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + \\
& 750*c^2*f)*a*b^3)/(a^{11}*b^2))^{(1/3)} + 729*(I*\text{sqrt}(3) + 1)*(-1/27*e^3/a^9 + \\
& 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^{10} \\
& *b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2* \\
& b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^ \\
& 3)/(a^{11}*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + \\
& 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g) \\
&)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^{11}*b^2))^{(1/3)} \\
& + 486*e/a^3) - 3*\text{sqrt}(1/3)*(a^3*b^3*x^8 + 2*a^4*b^2*x^5 + a^5*b*x^2)*\text{sqrt}(\\
& -(((-I*\text{sqrt}(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70* \\
& d*f - 40*c*g)*a*b)/(a^7*b)))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f
\end{aligned}$$

$$\begin{aligned}
& *g + (81e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^{10}*b) - 1/39366*(8000*b^4*c^3 + 2 \\
& 744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 117 \\
& 6*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^{11}*b^2) - 1/39366*(8000*b \\
& ^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - \\
& 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945 \\
& *c*d*e + 750*c^2*f)*a*b^3)/(a^{11}*b^2))^{(1/3)} + 729*(I*sqrt(3) + 1)*(-1/27*e \\
& ^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b) \\
& *e/(a^{10}*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1 \\
& 500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - \\
& 8*a^4*g^3)/(a^{11}*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270* \\
& e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 \\
& - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^{11}*b^2 \\
&))^{(1/3)} + 486*e/a^3)^2*a^7*b - 972*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^ \\
& 2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b)))/(-1/27*e^3/a^ \\
& 9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a \\
& ^{10}*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a \\
& ^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4 \\
& *g^3)/(a^{11}*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g \\
& + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18* \\
& e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^{11}*b^2))^{(1 \\
& /3)} + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g \\
& + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^{10}*b) - 1/39366*(8000*b^4*c^3 + 274 \\
& 4*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176* \\
& a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^{11}*b^2) - 1/39366*(8000*b^4 \\
& *c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 6 \\
& 30*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c \\
& *d*e + 750*c^2*f)*a*b^3)/(a^{11}*b^2))^{(1/3)} + 486*e/a^3)*a^4*b*e + 3265920*b \\
& ^2*c*d + 236196*a*b*e^2 - 816480*a*b*d*f - 116640*(4*a*b*c - a^2*f)*g)/(a^7 \\
& *b))*log(7840*a*b^3*c*d^2 - 3600*a*b^3*c^2*e + 1134*a^2*b^2*d*e^2 - 225*a^ \\
& 3*b*e*f^2 + 1/1458*(7*a^8*b^2*d - a^9*b*g))*((-I*sqrt(3) + 1)*(81*e^2/a^6 - \\
& (280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b)))/(-1/27 \\
& *e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a* \\
& b)*e/(a^{10}*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + \\
& 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 \\
& - 8*a^4*g^3)/(a^{11}*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 27 \\
& 0*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^ \\
& 2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^{11}*b \\
& ^2))^{(1/3)} + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10* \\
& a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^{10}*b) - 1/39366*(8000*b^4*c^ \\
& 3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 \\
& - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^{11}*b^2) - 1/39366*(8 \\
& 000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243* \\
& e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 \\
& - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^{11}*b^2))^{(1/3)} + 486*e/a^3)^2 + 40*(4*a^ \\
& 3*b*c - a^4*f)*g^2 + 1/54*(400*a^4*b^3*c^2 - 252*a^5*b^2*d*e - 200*a^5*b^2*
\end{aligned}$$

$$\begin{aligned}
& c*f + 25*a^6*b*f^2 + 36*a^6*b*e*g)*((-I*\sqrt{3}) + 1)*(81*e^2/a^6 - (280*b^2 \\
& *c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b)))/(-1/27*e^3/a^9 \\
& + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^ \\
& 10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^ \\
& 2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4* \\
& g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g \\
& + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e \\
& *g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/ \\
& 3) + 729*(I*\sqrt{3}) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g \\
& + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744 \\
& *a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a \\
& ^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4* \\
& c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 63 \\
& 0*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c* \\
& d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 486*e/a^3) - 40*(49*a^2*b^2*d^2 \\
& - 45*a^2*b^2*c*e)*f - 2*(1120*a^2*b^2*c*d + 81*a^3*b*e^2 - 280*a^3*b*d*f)* \\
& g - 2*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^ \\
& 2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)*x + 1 \\
& /486*\sqrt{1/3)*(10800*a^4*b^3*c^2 + 3402*a^5*b^2*d*e - 5400*a^5*b^2*c*f + 6 \\
& 75*a^6*b*f^2 - 486*a^6*b*e*g - (7*a^8*b^2*d - a^9*b*g)*((-I*\sqrt{3}) + 1)*(8 \\
& 1*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^ \\
& 7*b)))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f \\
& - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a \\
& *b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168* \\
& a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (\\
& 125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g \\
& + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a* \\
& b^3)/(a^11*b^2))^(1/3) + 729*(I*\sqrt{3}) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b \\
& ^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366* \\
& (8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 12 \\
& 5*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) \\
& - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3 \\
& *b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - \\
& 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 486*e/a^3)) \\
& *sqrt(-(((I*\sqrt{3}) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2 \\
& - 70*d*f - 40*c*g)*a*b)/(a^7*b)))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10 \\
& *a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c \\
& ^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 \\
& - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(\\
& 8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243 \\
& *e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 \\
& - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 729*(I*\sqrt{3}) + 1)*(- \\
& 1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g \\
&)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2 \\
& *f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*
\end{aligned}$$

$$\begin{aligned}
&g^2 - 8a^4g^3)/(a^{11}b^2) - 1/39366*(8000b^4c^3 + 8a^4g^3 - (125f^3 - \\
&270e*fg + 168d*g^2)*a^3b + 3*(243e^3 - 630d*ef + 392d^2*g + 20*(25f^2 - \\
&18e*g)*c)*a^2b^2 - 8*(343d^3 - 945c*d*ef + 750c^2*f)*ab^3)/(a^{11}b^2))^{(1/3)} + 486e/a^3)^2a^7b - 972*((-I*sqrt(3) + 1)*(81e^2/a^6 - (\\
&280b^2*c*d + 10a^2*f*g + (81e^2 - 70d*f - 40c*g)*ab)/(a^7b)))/(-1/27* \\
&e^3/a^9 + 1/1458*(280b^2*c*d + 10a^2*f*g + (81e^2 - 70d*f - 40c*g)*ab \\
&)*e/(a^{10}b) - 1/39366*(8000b^4c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + \\
&1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - \\
&8a^4g^3)/(a^{11}b^2) - 1/39366*(8000b^4c^3 + 8a^4g^3 - (125f^3 - 270 \\
&*e*fg + 168d*g^2)*a^3b + 3*(243e^3 - 630d*ef + 392d^2*g + 20*(25f^2 - \\
&18e*g)*c)*a^2b^2 - 8*(343d^3 - 945c*d*ef + 750c^2*f)*ab^3)/(a^{11}b^2 \\
&2))^{(1/3)} + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280b^2*c*d + 10a \\
&^2*f*g + (81e^2 - 70d*f - 40c*g)*ab)*e/(a^{10}b) - 1/39366*(8000b^4c^3 \\
&+ 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - \\
&1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8a^4g^3)/(a^{11}b^2) - 1/39366*(80 \\
&00b^4c^3 + 8a^4g^3 - (125f^3 - 270e*fg + 168d*g^2)*a^3b + 3*(243e \\
&^3 - 630d*ef + 392d^2*g + 20*(25f^2 - 18e*g)*c)*a^2b^2 - 8*(343d^3 - \\
&945c*d*ef + 750c^2*f)*ab^3)/(a^{11}b^2))^{(1/3)} + 486e/a^3)*a^4b*e + 326 \\
&5920b^2*c*d + 236196*a*b*e^2 - 816480*a*b*d*f - 116640*(4*a*b*c - a^2*f)*g \\
&)/(a^7b))) - (1458*b^3*e*x^8 + 2916*a*b^2*e*x^5 + 1458*a^2*b*e*x^2 - (a^3* \\
&b^3*x^8 + 2a^4*b^2*x^5 + a^5*b*x^2))*((-I*sqrt(3) + 1)*(81e^2/a^6 - (280b \\
&^2*c*d + 10a^2*f*g + (81e^2 - 70d*f - 40c*g)*ab)/(a^7b)))/(-1/27*e^3/a \\
&^9 + 1/1458*(280b^2*c*d + 10a^2*f*g + (81e^2 - 70d*f - 40c*g)*ab)*e/(\\
&a^{10}b) - 1/39366*(8000b^4c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500* \\
&a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8a^ \\
&4g^3)/(a^{11}b^2) - 1/39366*(8000b^4c^3 + 8a^4g^3 - (125f^3 - 270e*fg* \\
&g + 168d*g^2)*a^3b + 3*(243e^3 - 630d*ef + 392d^2*g + 20*(25f^2 - 18 \\
&*e*g)*c)*a^2b^2 - 8*(343d^3 - 945c*d*ef + 750c^2*f)*ab^3)/(a^{11}b^2))^{(\\
&1/3)} + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280b^2*c*d + 10a^2*f* \\
&g + (81e^2 - 70d*f - 40c*g)*ab)*e/(a^{10}b) - 1/39366*(8000b^4c^3 + 27 \\
&44*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176 \\
&*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8a^4g^3)/(a^{11}b^2) - 1/39366*(8000b^ \\
&4c^3 + 8a^4g^3 - (125f^3 - 270e*fg + 168d*g^2)*a^3b + 3*(243e^3 - \\
&630d*ef + 392d^2*g + 20*(25f^2 - 18e*g)*c)*a^2b^2 - 8*(343d^3 - 945* \\
&c*d*ef + 750c^2*f)*ab^3)/(a^{11}b^2))^{(1/3)} + 486e/a^3) + 3*sqrt(1/3)*(a^3 \\
&*b^3*x^8 + 2a^4*b^2*x^5 + a^5*b*x^2)*sqrt(-(((I*sqrt(3) + 1)*(81e^2/a^6 \\
&- (280b^2*c*d + 10a^2*f*g + (81e^2 - 70d*f - 40c*g)*ab)/(a^7b)))/(-1/ \\
&27*e^3/a^9 + 1/1458*(280b^2*c*d + 10a^2*f*g + (81e^2 - 70d*f - 40c*g)* \\
&a*b)*e/(a^{10}b) - 1/39366*(8000b^4c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f \\
&+ 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^ \\
&2 - 8a^4g^3)/(a^{11}b^2) - 1/39366*(8000b^4c^3 + 8a^4g^3 - (125f^3 - \\
&270e*fg + 168d*g^2)*a^3b + 3*(243e^3 - 630d*ef + 392d^2*g + 20*(25f \\
&^2 - 18e*g)*c)*a^2b^2 - 8*(343d^3 - 945c*d*ef + 750c^2*f)*ab^3)/(a^{11} \\
&*b^2))^{(1/3)} + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280b^2*c*d + 1 \\
&0a^2*f*g + (81e^2 - 70d*f - 40c*g)*ab)*e/(a^{10}b) - 1/39366*(8000b^4*
\end{aligned}$$

$$\begin{aligned}
& c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 \\
& - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^{11}*b^2) - 1/39366* \\
& (8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(24 \\
& 3*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^ \\
& 3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^{11}*b^2))^{(1/3)} + 486*e/a^3)^2*a^7*b - \\
& 972*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 7 \\
& 0*d*f - 40*c*g)*a*b)/(a^7*b)))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2 \\
& *f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^{10}*b) - 1/39366*(8000*b^4*c^3 + \\
& 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1 \\
& 176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^{11}*b^2) - 1/39366*(8000 \\
& *b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 \\
& - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 9 \\
& 45*c*d*e + 750*c^2*f)*a*b^3)/(a^{11}*b^2))^{(1/3)} + 729*(I*sqrt(3) + 1)*(-1/27 \\
& *e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a* \\
& b)*e/(a^{10}*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + \\
& 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 \\
& - 8*a^4*g^3)/(a^{11}*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 27 \\
& 0*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^ \\
& 2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^{11}*b \\
& ^2))^{(1/3)} + 486*e/a^3)*a^4*b*e + 3265920*b^2*c*d + 236196*a*b*e^2 - 816480 \\
& *a*b*d*f - 116640*(4*a*b*c - a^2*f)*g)/(a^7*b))) * log(7840*a*b^3*c*d^2 - 360 \\
& 0*a*b^3*c^2*e + 1134*a^2*b^2*d*e^2 - 225*a^3*b*e*f^2 + 1/1458*(7*a^8*b^2*d \\
& - a^9*b*g)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81* \\
& e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b)))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + \\
& 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^{10}*b) - 1/39366*(8000*b^ \\
& 4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b* \\
& f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^{11}*b^2) - 1/3936 \\
& 6*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(\\
& 243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^ \\
& 3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^{11}*b^2))^{(1/3)} + 729*(I*sqrt(3) + 1) \\
& *(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40* \\
& c*g)*a*b)*e/(a^{10}*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3* \\
& c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b \\
& *d*g^2 - 8*a^4*g^3)/(a^{11}*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f \\
& ^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20 \\
& *(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/ \\
& (a^{11}*b^2))^{(1/3)} + 486*e/a^3)^2 + 40*(4*a^3*b*c - a^4*f)*g^2 + 1/54*(400*a \\
& ^4*b^3*c^2 - 252*a^5*b^2*d*e - 200*a^5*b^2*c*f + 25*a^6*b*f^2 + 36*a^6*b*e*g) * \\
& ((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70 \\
& *d*f - 40*c*g)*a*b)/(a^7*b)))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2* \\
& f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^{10}*b) - 1/39366*(8000*b^4*c^3 + \\
& 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 11 \\
& 76*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^{11}*b^2) - 1/39366*(8000* \\
& b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 \\
& - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 94
\end{aligned}$$

$$\begin{aligned}
& 5*c*d*e + 750*c^2*f)*a*b^3)/(a^{11}*b^2))^{(1/3)} + 729*(I*\sqrt{3} + 1)*(-1/27* \\
& e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b \\
&)*e/(a^{10}*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + \\
& 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - \\
& 8*a^4*g^3)/(a^{11}*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270 \\
& *e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 \\
& - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^{11}*b^2) \\
&)^{(1/3)} + 486*e/a^3) - 40*(49*a^2*b^2*d^2 - 45*a^2*b^2*c*e)*f - 2*(1120*a \\
& ^2*b^2*c*d + 81*a^3*b*e^2 - 280*a^3*b*d*f)*g - 2*(8000*b^4*c^3 + 2744*a*b^3 \\
& *d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2 \\
& *d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)*x - 1/486*\sqrt{1/3}*(10800*a^4*b^3*c^2 \\
& + 3402*a^5*b^2*d*e - 5400*a^5*b^2*c*f + 675*a^6*b*f^2 - 486*a^6*b*e*g - (\\
& 7*a^8*b^2*d - a^9*b*g)*((-I*\sqrt{3} + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2 \\
& *f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b)))/(-1/27*e^3/a^9 + 1/1458*(2 \\
& 80*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^{10}*b) - 1/39 \\
& 366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 \\
& - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^{11}*b \\
& ^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2) \\
& *a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b \\
& ^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^{11}*b^2))^{(1/3)} + 729*(I* \\
& \sqrt{3} + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - \\
& 70*d*f - 40*c*g)*a*b)*e/(a^{10}*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - \\
& 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g \\
& + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^{11}*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4* \\
& g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 39 \\
& 2*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^ \\
& 2*f)*a*b^3)/(a^{11}*b^2))^{(1/3)} + 486*e/a^3))*\sqrt{-(((I*\sqrt{3} + 1)*(81*e^ \\
& 2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b) \\
&))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40 \\
& *c*g)*a*b)*e/(a^{10}*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3 \\
& *c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3* \\
& b*d*g^2 - 8*a^4*g^3)/(a^{11}*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125* \\
& f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 2 \\
& 0*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3) \\
& / (a^{11}*b^2))^{(1/3)} + 729*(I*\sqrt{3} + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c \\
& *d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^{10}*b) - 1/39366*(800 \\
& 0*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^ \\
& 3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^{11}*b^2) - 1/ \\
& 39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + \\
& 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(\\
& 343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^{11}*b^2))^{(1/3)} + 486*e/a^3)^2*a^ \\
& 7*b - 972*((-I*\sqrt{3} + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e \\
& ^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b)))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + \\
& 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^{10}*b) - 1/39366*(8000*b^4 \\
& *c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f
\end{aligned}$$

$$\begin{aligned} &^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^{11}*b^2) - 1/39366 \\ &*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(2 \\ &43*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d \\ &^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^{11}*b^2))^{(1/3)} + 729*(I*sqrt(3) + 1)* \\ &(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c \\ &*g)*a*b)*e/(a^{10}*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c \\ &^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b* \\ &d*g^2 - 8*a^4*g^3)/(a^{11}*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^ \\ &3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20* \\ &(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(\\ &a^{11}*b^2))^{(1/3)} + 486*e/a^3)*a^4*b*e + 3265920*b^2*c*d + 236196*a*b*e^2 - \\ &816480*a*b*d*f - 116640*(4*a*b*c - a^2*f)*g)/(a^7*b))) + 2916*(b^3*e*x^8 + \\ &2*a*b^2*e*x^5 + a^2*b*e*x^2)*log(x))/(a^3*b^3*x^8 + 2*a^4*b^2*x^5 + a^5*b*x \\ &^2) \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.11512, size = 568, normalized size = 1.58

$$-\frac{e \log(|bx^3 + a|)}{3a^3} + \frac{e \log(|x|)}{a^3} - \frac{\sqrt{3} \left(20 (-ab^2)^{\frac{1}{3}} b^2c - 5 (-ab^2)^{\frac{1}{3}} abf - 14 (-ab^2)^{\frac{2}{3}} bd + 2 (-ab^2)^{\frac{2}{3}} ag \right) \arctan \left(\frac{\sqrt{3} (2x + 3(-ab^2)^{\frac{1}{3}})}{3(-ab^2)^{\frac{1}{3}}} \right)}{27a^4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="giac")

[Out] -1/3*e*log(abs(b*x^3 + a))/a^3 + e*log(abs(x))/a^3 - 1/27*sqrt(3)*(20*(-a*b^2)^(1/3)*b^2*c - 5*(-a*b^2)^(1/3)*a*b*f - 14*(-a*b^2)^(2/3)*b*d + 2*(-a*b^2)^(2/3)*a*g) * arctan(sqrt(3)*(2*x + 3*(-a*b^2)^(1/3))/(3*(-a*b^2)^(1/3)))/27*a^4*b^2

$$\begin{aligned}
& 2)^{(2/3)} * a * g) * \arctan(1/3 * \sqrt{3} * (2 * x + (-a/b)^{(1/3)}) / (-a/b)^{(1/3)}) / (a^4 * b^2) \\
& - 1/54 * (20 * (-a * b^2)^{(1/3)} * b^2 * c - 5 * (-a * b^2)^{(1/3)} * a * b * f + 14 * (-a * b^2)^{(2/3)} * b * d \\
& - 2 * (-a * b^2)^{(2/3)} * a * g) * \log(x^2 + x * (-a/b)^{(1/3)} + (-a/b)^{(2/3)}) / (a^4 * b^2) \\
& - 1/18 * (28 * b^3 * d * x^7 - 4 * a * b^2 * g * x^7 + 20 * b^3 * c * x^6 - 5 * a * b^2 * f * x^6 \\
& - 6 * a * b^2 * x^5 * e + 49 * a * b^2 * d * x^4 - 7 * a^2 * b * g * x^4 + 32 * a * b^2 * c * x^3 - 8 * a^2 * b * f * x^3 \\
& + 3 * a^3 * h * x^2 - 9 * a^2 * b * x^2 * e + 18 * a^2 * b * d * x + 9 * a^2 * b * c) / ((b * x^4 + a * x)^2 * a^3 * b) \\
& + 1/27 * (14 * a^3 * b^2 * d * (-a/b)^{(1/3)} - 2 * a^4 * b * g * (-a/b)^{(1/3)} + 20 * a^3 * b^2 * c - 5 * a^4 * b * f) * (-a/b)^{(1/3)} * \log(\text{abs}(x - (-a/b)^{(1/3)})) / (a^7 * b)
\end{aligned}$$

$$3.429 \quad \int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)^3} dx$$

Optimal. Leaf size=395

$$\frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)(5\sqrt[3]{b}(4bd - ag) - 2\sqrt[3]{a}(7be - ah))}{54a^{11/3}b^{2/3}} - \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(5\sqrt[3]{b}(4bd - ag) - 2\sqrt[3]{a}(7be - ah))}{27a^{11/3}b^{2/3}} +$$

[Out] $-c/(3*a^3*x^3) - d/(2*a^3*x^2) - e/(a^3*x) - (x*(b*d - a*g + (b*e - a*h)*x - b*((b*c)/a - f)*x^2))/(6*a^2*(a + b*x^3)^2) - (x*(11*b*d - 5*a*g + 2*(5*b*e - 2*a*h)*x - 3*b*((5*b*c)/a - 3*f)*x^2))/(18*a^3*(a + b*x^3)) + ((20*b^(4/3)*d + 14*a^(1/3)*b*e - 5*a*b^(1/3)*g - 2*a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(11/3)*b^(2/3)) - ((3*b*c - a*f)*Log[x])/a^4 - ((5*b^(1/3)*(4*b*d - a*g) - 2*a^(1/3)*(7*b*e - a*h))*Log[a^(1/3) + b^(1/3)*x]/(27*a^(11/3)*b^(2/3)) + ((5*b^(1/3)*(4*b*d - a*g) - 2*a^(1/3)*(7*b*e - a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(54*a^(11/3)*b^(2/3)) + ((3*b*c - a*f)*Log[a + b*x^3]/(3*a^4))$

Rubi [A] time = 1.00616, antiderivative size = 392, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)\left(-\frac{2\sqrt[3]{a}(7be-ah)}{\sqrt[3]{b}} - 5ag + 20bd\right)}{54a^{11/3}\sqrt[3]{b}} - \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})(5\sqrt[3]{b}(4bd - ag) - 2\sqrt[3]{a}(7be - ah))}{27a^{11/3}b^{2/3}} + \frac{\tan^{-1}\left(\frac{a^{1/3} - 2b^{1/3}x}{\sqrt[3]{a} + \sqrt[3]{bx}}\right)}{3\sqrt[3]{a}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^3), x]

[Out] $-c/(3*a^3*x^3) - d/(2*a^3*x^2) - e/(a^3*x) - (x*(b*d - a*g + (b*e - a*h)*x - b*((b*c)/a - f)*x^2))/(6*a^2*(a + b*x^3)^2) - (x*(11*b*d - 5*a*g + 2*(5*b*e - 2*a*h)*x - 3*b*((5*b*c)/a - 3*f)*x^2))/(18*a^3*(a + b*x^3)) + ((20*b^(4/3)*d + 14*a^(1/3)*b*e - 5*a*b^(1/3)*g - 2*a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(11/3)*b^(2/3)) - ((3*b*c - a*f)*Log[x])/a^4 - ((5*b^(1/3)*(4*b*d - a*g) - 2*a^(1/3)*(7*b*e - a*h))*Log[a^(1/3) + b^(1/3)*x]/(27*a^(11/3)*b^(2/3)) + ((20*b*d - 5*a*g - (2*a^(1/3)*(7*b*e - a*h))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(54*a^(11/3)*b^(2/3)) + ((3*b*c - a*f)*Log[a + b*x^3]/(3*a^4))$

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_)^(m_.)))/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617


```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{x^4(a + bx^3)^3} dx &= -\frac{x\left(bd - ag + (be - ah)x - b\left(\frac{bc}{a} - f\right)x^2\right)}{6a^2(a + bx^3)^2} - \frac{\int \frac{-6b^2c - 6b^2dx - 6b^2ex^2 + 6b^2\left(\frac{bc}{a} - f\right)x^3 + 5b^2\left(\frac{bd}{a} - f\right)x^4}{x^4(a + bx^3)^2} dx}{6ab^2} \\
&= -\frac{x\left(bd - ag + (be - ah)x - b\left(\frac{bc}{a} - f\right)x^2\right)}{6a^2(a + bx^3)^2} - \frac{x\left(11bd - 5ag + 2(5be - 2ah)x - 3b\left(\frac{bc}{a} - f\right)x^2\right)}{18a^3(a + bx^3)} \\
&= -\frac{x\left(bd - ag + (be - ah)x - b\left(\frac{bc}{a} - f\right)x^2\right)}{6a^2(a + bx^3)^2} - \frac{x\left(11bd - 5ag + 2(5be - 2ah)x - 3b\left(\frac{bc}{a} - f\right)x^2\right)}{18a^3(a + bx^3)} \\
&= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd - ag + (be - ah)x - b\left(\frac{bc}{a} - f\right)x^2\right)}{6a^2(a + bx^3)^2} - \frac{x\left(11bd - 5ag + 2(5be - 2ah)x - 3b\left(\frac{bc}{a} - f\right)x^2\right)}{18a^3(a + bx^3)} \\
&= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd - ag + (be - ah)x - b\left(\frac{bc}{a} - f\right)x^2\right)}{6a^2(a + bx^3)^2} - \frac{x\left(11bd - 5ag + 2(5be - 2ah)x - 3b\left(\frac{bc}{a} - f\right)x^2\right)}{18a^3(a + bx^3)} \\
&= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd - ag + (be - ah)x - b\left(\frac{bc}{a} - f\right)x^2\right)}{6a^2(a + bx^3)^2} - \frac{x\left(11bd - 5ag + 2(5be - 2ah)x - 3b\left(\frac{bc}{a} - f\right)x^2\right)}{18a^3(a + bx^3)} \\
&= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd - ag + (be - ah)x - b\left(\frac{bc}{a} - f\right)x^2\right)}{6a^2(a + bx^3)^2} - \frac{x\left(11bd - 5ag + 2(5be - 2ah)x - 3b\left(\frac{bc}{a} - f\right)x^2\right)}{18a^3(a + bx^3)} \\
&= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd - ag + (be - ah)x - b\left(\frac{bc}{a} - f\right)x^2\right)}{6a^2(a + bx^3)^2} - \frac{x\left(11bd - 5ag + 2(5be - 2ah)x - 3b\left(\frac{bc}{a} - f\right)x^2\right)}{18a^3(a + bx^3)} \\
&= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd - ag + (be - ah)x - b\left(\frac{bc}{a} - f\right)x^2\right)}{6a^2(a + bx^3)^2} - \frac{x\left(11bd - 5ag + 2(5be - 2ah)x - 3b\left(\frac{bc}{a} - f\right)x^2\right)}{18a^3(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.590882, size = 352, normalized size = 0.89

$$\frac{\sqrt[3]{a} \log\left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3}x^2\right) \left(2a^{4/3}h - 14\sqrt[3]{a}be - 5a\sqrt[3]{b}g + 20b^{4/3}d\right)}{b^{2/3}} - \frac{2\sqrt[3]{a} \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right) \left(2a^{4/3}h - 14\sqrt[3]{a}be - 5a\sqrt[3]{b}g + 20b^{4/3}d\right)}{b^{2/3}} + \frac{2\sqrt{3} \sqrt[3]{a} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{bx}}{\sqrt{3}}\right) \left(-2a^{4/3}\right)}{b^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^3),x]

[Out]
$$\begin{aligned} &((-18*a*c)/x^3 - (27*a*d)/x^2 - (54*a*e)/x + (3*a*(-12*b*c + 6*a*f - b*x*(1 \\ &1*d + 10*e*x) + a*x*(5*g + 4*h*x)))/(a + b*x^3) + (a^2*(-9*b*(c + x*(d + e* \\ &x) + 9*a*(f + x*(g + h*x))))/(a + b*x^3)^2 + (2*sqrt[3]*a^(1/3)*(20*b^(4/3) \\ &)*d + 14*a^(1/3)*b*e - 5*a*b^(1/3)*g - 2*a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)* \\ &x)/a^(1/3))/sqrt[3]]/b^(2/3) + 54*(-3*b*c + a*f)*Log[x] - (2*a^(1/3)*(20*b \\ &^(4/3)*d - 14*a^(1/3)*b*e - 5*a*b^(1/3)*g + 2*a^(4/3)*h)*Log[a^(1/3) + b^(1 \\ &/3)*x]/b^(2/3) + (a^(1/3)*(20*b^(4/3)*d - 14*a^(1/3)*b*e - 5*a*b^(1/3)*g + \\ &2*a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/b^(2/3) + 18* \\ &(3*b*c - a*f)*Log[a + b*x^3]/(54*a^4) \end{aligned}$$

Maple [B] time = 0.017, size = 680, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x)

[Out]
$$\begin{aligned} &1/2/a/(b*x^3+a)^2*f-1/3/a^3*\ln(b*x^3+a)*f+1/a^3*\ln(x)*f+2/27/a^2*h*3^(1/2)/ \\ &b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+5/27/a^2*g/b/(1/b \\ &*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/3*c/a^3/x^3+5 \\ &/27/a^2*g/b/(1/b*a)^(2/3)*\ln(x+(1/b*a)^(1/3))-5/54/a^2*g/b/(1/b*a)^(2/3)*\ln \\ &(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-5/9/a^3/(b*x^3+a)^2*x^5*e*b^2-20/27/a^3 \\ &/ (1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d-11/18/a^ \\ &3/(b*x^3+a)^2*x^4*b^2*d-1/2*d/a^3/x^2-e/a^3/x-5/6/a^2*b/(b*x^3+a)^2*c-7/9/a \\ &^2/(b*x^3+a)^2*b*x*d-13/18/a^2/(b*x^3+a)^2*x^2*b*e-14/27/a^3*e*3^(1/2)/(1/b \\ &*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+14/27/a^3*e/(1/b*a)^(1/ \\ &3)*\ln(x+(1/b*a)^(1/3))-7/27/a^3*e/(1/b*a)^(1/3)*\ln(x^2-(1/b*a)^(1/3)*x+(1/b \\ &*a)^(2/3))-20/27/a^3/(1/b*a)^(2/3)*\ln(x+(1/b*a)^(1/3))*d+10/27/a^3/(1/b*a)^(\\ &2/3)*\ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d+7/18/a/(b*x^3+a)^2*x^2*h+4/9/ \\ &a/(b*x^3+a)^2*g*x+5/18/a^2/(b*x^3+a)^2*x^4*b*g+2/9/a^2/(b*x^3+a)^2*x^5*b*h- \\ &2/27/a^2*h/b/(1/b*a)^(1/3)*\ln(x+(1/b*a)^(1/3))+1/3/a^2/(b*x^3+a)^2*x^3*b*f+ \\ &1/27/a^2*h/b/(1/b*a)^(1/3)*\ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-3*b*c*\ln(x) \\ &/a^4+b*c*\ln(b*x^3+a)/a^4-2/3/a^3*b^2/(b*x^3+a)^2*c*x^3 \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.09566, size = 612, normalized size = 1.55

$$\frac{(3bc - af) \log(|bx^3 + a|)}{3a^4} - \frac{(3bc - af) \log(|x|)}{a^4} - \frac{\sqrt{3} \left(20 (-ab^2)^{\frac{1}{3}} b^2 d - 5 (-ab^2)^{\frac{1}{3}} abg + 2 (-ab^2)^{\frac{2}{3}} ah - 14 (-ab^2)^{\frac{2}{3}} be \right)}{27 a^4 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="giac")

[Out] $\frac{1}{3}(3bc - af)\log(\text{abs}(bx^3 + a))/a^4 - (3bc - af)\log(\text{abs}(x))/a^4 - \frac{1}{27}\sqrt{3}(20(-ab^2)^{1/3}b^2d - 5(-ab^2)^{1/3}abg + 2(-ab^2)^{2/3}ah - 14(-ab^2)^{2/3}b^2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x + (-a/b)^{1/3})/(-a/b)^{1/3}\right)/(a^4b^2) - \frac{1}{54}(20(-ab^2)^{1/3}b^2d - 5(-ab^2)^{1/3}abg - 2(-ab^2)^{2/3}ah + 14(-ab^2)^{2/3}b^2e)\log(x^2 + x(-a/b)^{1/3} + (-a/b)^{2/3})/(a^4b^2) - \frac{1}{27}(2a^6bh(-a/b)^{1/3} - 14a^5b^2(-a/b)^{1/3}e - 20a^5b^2d + 5a^6b^2g)(-a/b)^{1/3}\log(\text{abs}(x - (-a/b)^{1/3}))/a^9b + \frac{1}{18}(4(a^2bh - 7a^2b^2e)x^8 - 5(4a^2b^2d - a^2bg)x^7 - 6(3a^2b^2c - a^2b^2f)x^6 + 7(a^3h - 7a^2b^2e)x^5 - 18a^3x^2e - 9a^3d^2x - 8(4a^2b^2d - a^3g)x^4 - 6a^3c - 9(3a^2b^2c - a^3f)x^3)/(b^2x^3 + a)^2a^4x^3$

$$3.430 \quad \int \frac{x^3(c+dx+ex^2)}{\sqrt{a+bx^3}} dx$$

Optimal. Leaf size=583

$$\frac{4\sqrt{2+\sqrt{3}}a(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(7\sqrt[3]{bc}-10(1-\sqrt{3})\sqrt[3]{ad})\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)}{35^4\sqrt[3]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}}$$

[Out] $(-4*a*e*\text{Sqrt}[a + b*x^3])/(9*b^2) + (2*c*x*\text{Sqrt}[a + b*x^3])/(5*b) + (2*d*x^2*\text{Sqrt}[a + b*x^3])/(7*b) + (2*e*x^3*\text{Sqrt}[a + b*x^3])/(9*b) - (8*a*d*\text{Sqrt}[a + b*x^3])/(7*b^{5/3}*((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)) + (4*3^{1/4}*\text{Sqrt}[2 - \text{Sqrt}[3]]*a^{4/3}*d*(a^{1/3} + b^{1/3}*x)*\text{Sqrt}[(a^{2/3} - a^{1/3}*b^{1/3})*x + b^{2/3}*x^2]/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2)*\text{EllipticE}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x]/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)], -7 - 4*\text{Sqrt}[3]]/(7*b^{5/3}*\text{Sqrt}[(a^{1/3}*(a^{1/3} + b^{1/3}*x))/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\text{Sqrt}[a + b*x^3]) - (4*\text{Sqrt}[2 + \text{Sqrt}[3]]*a*(7*b^{1/3}*c - 10*(1 - \text{Sqrt}[3])*a^{1/3}*d)*(a^{1/3} + b^{1/3}*x)*\text{Sqrt}[(a^{2/3} - a^{1/3}*b^{1/3})*x + b^{2/3}*x^2]/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2)*\text{EllipticF}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x]/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)], -7 - 4*\text{Sqrt}[3]]/(35*3^{1/4}*b^{5/3}*\text{Sqrt}[(a^{1/3}*(a^{1/3} + b^{1/3}*x))/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\text{Sqrt}[a + b*x^3])$

Rubi [A] time = 0.733102, antiderivative size = 583, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {1888, 1594, 1886, 261, 1878, 218, 1877}

$$\frac{4\sqrt{2+\sqrt{3}}a(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(7\sqrt[3]{bc}-10(1-\sqrt{3})\sqrt[3]{ad})F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{35^4\sqrt[3]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(c + d*x + e*x^2))/Sqrt[a + b*x^3], x]

[Out] $(-4*a*e*\text{Sqrt}[a + b*x^3])/(9*b^2) + (2*c*x*\text{Sqrt}[a + b*x^3])/(5*b) + (2*d*x^2*\text{Sqrt}[a + b*x^3])/(7*b) + (2*e*x^3*\text{Sqrt}[a + b*x^3])/(9*b) - (8*a*d*\text{Sqrt}[a +$

$$\begin{aligned} & b*x^3)/(7*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (4*3^(1/4)*Sqrt[\\ & 2 - Sqrt[3]]*a^(4/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3) \\ &)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticE[ArcSin[\\ & ((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], - \\ & 7 - 4*Sqrt[3]])/(7*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[\\ & 3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (4*Sqrt[2 + Sqrt[3]]*a*(7*b^(\\ & 1/3)*c - 10*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - \\ & a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*El \\ & lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + \\ & b^(1/3)*x)], -7 - 4*Sqrt[3]])/(35*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + \\ & b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) \end{aligned}$$
Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)]) /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[Coeff[Pq, x, n -
1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
```

```
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(c + dx + ex^2)}{\sqrt{a + bx^3}} dx &= \frac{2ex^3\sqrt{a + bx^3}}{9b} + \frac{2 \int \frac{-3aex^2 + \frac{9}{2}bcx^3 + \frac{9}{2}bdx^4}{\sqrt{a + bx^3}} dx}{9b} \\
&= \frac{2ex^3\sqrt{a + bx^3}}{9b} + \frac{2 \int \frac{x^2(-3ae + \frac{9bcx}{2} + \frac{9bdx^2}{2})}{\sqrt{a + bx^3}} dx}{9b} \\
&= \frac{2dx^2\sqrt{a + bx^3}}{7b} + \frac{2ex^3\sqrt{a + bx^3}}{9b} + \frac{4 \int \frac{-9abdx - \frac{21}{2}abex^2 + \frac{63}{4}b^2cx^3}{\sqrt{a + bx^3}} dx}{63b^2} \\
&= \frac{2dx^2\sqrt{a + bx^3}}{7b} + \frac{2ex^3\sqrt{a + bx^3}}{9b} + \frac{4 \int \frac{x(-9abd - \frac{21}{2}abex + \frac{63}{4}b^2cx^2)}{\sqrt{a + bx^3}} dx}{63b^2} \\
&= \frac{2cx\sqrt{a + bx^3}}{5b} + \frac{2dx^2\sqrt{a + bx^3}}{7b} + \frac{2ex^3\sqrt{a + bx^3}}{9b} + \frac{8 \int \frac{-\frac{63}{4}ab^2c - \frac{45}{2}ab^2dx - \frac{105}{4}ab^2ex^2}{\sqrt{a + bx^3}} dx}{315b^3} \\
&= \frac{2cx\sqrt{a + bx^3}}{5b} + \frac{2dx^2\sqrt{a + bx^3}}{7b} + \frac{2ex^3\sqrt{a + bx^3}}{9b} + \frac{8 \int \frac{-\frac{63}{4}ab^2c - \frac{45}{2}ab^2dx}{\sqrt{a + bx^3}} dx}{315b^3} - \frac{(2ae) \int \frac{x^2}{\sqrt{a + bx^3}} dx}{3b} \\
&= -\frac{4ae\sqrt{a + bx^3}}{9b^2} + \frac{2cx\sqrt{a + bx^3}}{5b} + \frac{2dx^2\sqrt{a + bx^3}}{7b} + \frac{2ex^3\sqrt{a + bx^3}}{9b} - \frac{(4ad) \int \frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt{a + bx^3}} dx}{7b^{4/3}} \\
&= -\frac{4ae\sqrt{a + bx^3}}{9b^2} + \frac{2cx\sqrt{a + bx^3}}{5b} + \frac{2dx^2\sqrt{a + bx^3}}{7b} + \frac{2ex^3\sqrt{a + bx^3}}{9b} - \frac{8ad\sqrt{a + bx^3}}{7b^{5/3}((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{b})}
\end{aligned}$$

Mathematica [C] time = 0.128637, size = 132, normalized size = 0.23

$$\frac{-2(a + bx^3)(70ae - bx(63c + 5x(9d + 7ex))) - 126abcx\sqrt{\frac{bx^3}{a} + 1} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, -\frac{bx^3}{a}\right) - 90abdx^2\sqrt{\frac{bx^3}{a} + 1} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, -\frac{bx^3}{a}\right)}{315b^2\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(c + d*x + e*x^2))/Sqrt[a + b*x^3], x]

[Out] (-2*(a + b*x^3)*(70*a*e - b*x*(63*c + 5*x*(9*d + 7*e*x))) - 126*a*b*c*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] - 90*a*b*d*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(31

$5*b^2*\text{Sqrt}[a + b*x^3]$

Maple [A] time = 0.016, size = 793, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^3*(e*x^2+d*x+c)/(b*x^3+a)^{(1/2)}, x)$

[Out] $e*(2/9/b*x^3*(b*x^3+a)^{(1/2)}-4/9/b^2*a*(b*x^3+a)^{(1/2)})+d*(2/7/b*x^2*(b*x^3+a)^{(1/2)}+8/21*I/b^2*a*3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*EllipticE(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)})+1/b*(-b^2*a)^{(1/3)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)})))+c*(2/5/b*x*(b*x^3+a)^{(1/2)}+4/15*I/b^2*a*3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/2)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}))^{(1/2))}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + dx + c)x^3}{\sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^3*(e*x^2+d*x+c)/(b*x^3+a)^{(1/2)}, x, \text{algorithm}="maxima")$

[Out] integrate((e*x^2 + d*x + c)*x^3/sqrt(b*x^3 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^5 + dx^4 + cx^3}{\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] integral((e*x^5 + d*x^4 + c*x^3)/sqrt(b*x^3 + a), x)

Sympy [A] time = 3.13196, size = 129, normalized size = 0.22

$$e^{\left(\begin{cases} -\frac{4a\sqrt{a+bx^3}}{9b^2} + \frac{2x^3\sqrt{a+bx^3}}{9b} & \text{for } b \neq 0 \\ \frac{x^6}{6\sqrt{a}} & \text{otherwise} \end{cases}\right)} + \frac{cx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{4}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)} + \frac{dx^5\Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{5}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(e*x**2+d*x+c)/(b*x**3+a)**(1/2), x)

[Out] e*Piecewise((-4*a*sqrt(a + b*x**3)/(9*b**2) + 2*x**3*sqrt(a + b*x**3)/(9*b), Ne(b, 0)), (x**6/(6*sqrt(a)), True)) + c*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(7/3)) + d*x**5*gamma(5/3)*hyper((1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(8/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + dx + c)x^3}{\sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)*x^3/sqrt(b*x^3 + a), x)
```

$$3.431 \quad \int \frac{x^2(c+dx+ex^2)}{\sqrt{a+bx^3}} dx$$

Optimal. Leaf size=560

$$4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}\left(7\sqrt[3]{bd}-10(1-\sqrt{3})\sqrt[3]{ae}\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)$$

$$35\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}\sqrt{a+bx^3}$$

[Out] $(2*c*\text{Sqrt}[a + b*x^3])/(3*b) + (2*d*x*\text{Sqrt}[a + b*x^3])/(5*b) + (2*e*x^2*\text{Sqrt}[a + b*x^3])/(7*b) - (8*a*e*\text{Sqrt}[a + b*x^3])/(7*b^{5/3}*((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)) + (4*3^{1/4}*\text{Sqrt}[2 - \text{Sqrt}[3]]*a^{4/3}*e*(a^{1/3} + b^{1/3}*x)*\text{Sqrt}[(a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2)/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\text{EllipticE}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x]/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)], -7 - 4*\text{Sqrt}[3])/(7*b^{5/3}*\text{Sqrt}[(a^{1/3}*(a^{1/3} + b^{1/3}*x))/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\text{Sqrt}[a + b*x^3]) - (4*\text{Sqrt}[2 + \text{Sqrt}[3]]*a*(7*b^{1/3}*d - 10*(1 - \text{Sqrt}[3])*a^{1/3}*e)*(a^{1/3} + b^{1/3}*x)*\text{Sqrt}[(a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2)/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\text{EllipticF}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x]/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)], -7 - 4*\text{Sqrt}[3])/(35*3^{1/4}*b^{5/3}*\text{Sqrt}[(a^{1/3}*(a^{1/3} + b^{1/3}*x))/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\text{Sqrt}[a + b*x^3])$

Rubi [A] time = 0.484352, antiderivative size = 560, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {1888, 1594, 1886, 261, 1878, 218, 1877}

$$4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}\left(7\sqrt[3]{bd}-10(1-\sqrt{3})\sqrt[3]{ae}\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right) 4\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}\sqrt{a+bx^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^2*(c + d*x + e*x^2))/\text{Sqrt}[a + b*x^3], x]$

[Out] $(2*c*\text{Sqrt}[a + b*x^3])/(3*b) + (2*d*x*\text{Sqrt}[a + b*x^3])/(5*b) + (2*e*x^2*\text{Sqrt}[a + b*x^3])/(7*b) - (8*a*e*\text{Sqrt}[a + b*x^3])/(7*b^{5/3}*((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x))$

$$\begin{aligned} & /3) + b^{(1/3)*x}) + (4*3^{(1/4)*\text{Sqrt}[2 - \text{Sqrt}[3]]*a^{(4/3)*e*(a^{(1/3)} + b^{(1/3)} \\ & *x)*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 + \text{Sqrt}[3])*a^{(1/3)} \\ & + b^{(1/3)*x})^2]*\text{EllipticE}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})/((1 \\ & + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})], -7 - 4*\text{Sqrt}[3]])/(7*b^{(5/3)*\text{Sqrt}[(a^{(1/3)} \\ & *(a^{(1/3)} + b^{(1/3)*x})/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2]*\text{Sqrt}[a + b* \\ & x^3]) - (4*\text{Sqrt}[2 + \text{Sqrt}[3]]*a*(7*b^{(1/3)*d} - 10*(1 - \text{Sqrt}[3])*a^{(1/3)*e}*(\\ & a^{(1/3)} + b^{(1/3)*x})*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 + \\ & \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2]*\text{EllipticF}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{(1/3)} + \\ & b^{(1/3)*x})/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})], -7 - 4*\text{Sqrt}[3]))/(35*3^{(1 \\ & /4)*b^{(5/3)*\text{Sqrt}[(a^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x})/((1 + \text{Sqrt}[3])*a^{(1/3)} + b \\ & ^{(1/3)*x})^2]*\text{Sqrt}[a + b*x^3]) \end{aligned}$$
Rule 1888

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)]) /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1594

```
Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x
_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[Coeff[Pq, x, n -
1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
```

$[a + b*x^3], x], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{PosQ}[a] \ \&\& \ \text{NeQ}[b*c^3 - 2*(5 - 3*\text{Sqrt}[3])*a*d^3, 0]$

Rule 218

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^3], x_Symbol] \text{ :> With}[\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Simp}[(2*\text{Sqrt}[2 + \text{Sqrt}[3]]*(s + r*x)*\text{Sqrt}[(s^2 - r*s*x + r^2*x^2)/((1 + \text{Sqrt}[3])*s + r*x)^2]*\text{EllipticF}[\text{ArcSin}[(1 - \text{Sqrt}[3])*s + r*x]/((1 + \text{Sqrt}[3])*s + r*x)], -7 - 4*\text{Sqrt}[3]])/(3^{1/4}*r*\text{Sqrt}[a + b*x^3]*\text{Sqrt}[(s*(s + r*x))/((1 + \text{Sqrt}[3])*s + r*x)^2]), x]] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a]$

Rule 1877

$\text{Int}[(c_) + (d_)*(x_)]/\text{Sqrt}[(a_) + (b_)*(x_)^3], x_Symbol] \text{ :> With}[\{r = \text{Numer}[\text{Simplify}[(1 - \text{Sqrt}[3])*d]/c], s = \text{Denom}[\text{Simplify}[(1 - \text{Sqrt}[3])*d]/c]]\}, \text{Simp}[(2*d*s^3*\text{Sqrt}[a + b*x^3])/(a*r^2*((1 + \text{Sqrt}[3])*s + r*x)), x] - \text{Simp}[(3^{1/4}*\text{Sqrt}[2 - \text{Sqrt}[3]]*d*s*(s + r*x)*\text{Sqrt}[(s^2 - r*s*x + r^2*x^2)/((1 + \text{Sqrt}[3])*s + r*x)^2]*\text{EllipticE}[\text{ArcSin}[(1 - \text{Sqrt}[3])*s + r*x]/((1 + \text{Sqrt}[3])*s + r*x)], -7 - 4*\text{Sqrt}[3]])/(r^2*\text{Sqrt}[a + b*x^3]*\text{Sqrt}[(s*(s + r*x))/((1 + \text{Sqrt}[3])*s + r*x)^2]), x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{PosQ}[a] \ \&\& \ \text{EqQ}[b*c^3 - 2*(5 - 3*\text{Sqrt}[3])*a*d^3, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{x^2(c + dx + ex^2)}{\sqrt{a + bx^3}} dx &= \frac{2ex^2\sqrt{a + bx^3}}{7b} + \frac{2 \int \frac{-2aex + \frac{7}{2}bcx^2 + \frac{7}{2}bdx^3}{\sqrt{a+bx^3}} dx}{7b} \\
&= \frac{2ex^2\sqrt{a + bx^3}}{7b} + \frac{2 \int \frac{x(-2ae + \frac{7bcx}{2} + \frac{7}{2}bdx^2)}{\sqrt{a+bx^3}} dx}{7b} \\
&= \frac{2dx\sqrt{a + bx^3}}{5b} + \frac{2ex^2\sqrt{a + bx^3}}{7b} + \frac{4 \int \frac{-\frac{7}{2}abd - 5abex + \frac{35}{4}b^2cx^2}{\sqrt{a+bx^3}} dx}{35b^2} \\
&= \frac{2dx\sqrt{a + bx^3}}{5b} + \frac{2ex^2\sqrt{a + bx^3}}{7b} + \frac{4 \int \frac{-\frac{7}{2}abd - 5abex}{\sqrt{a+bx^3}} dx}{35b^2} + c \int \frac{x^2}{\sqrt{a + bx^3}} dx \\
&= \frac{2c\sqrt{a + bx^3}}{3b} + \frac{2dx\sqrt{a + bx^3}}{5b} + \frac{2ex^2\sqrt{a + bx^3}}{7b} - \frac{(4ae) \int \frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt{a+bx^3}} dx}{7b^{4/3}} - \frac{(2a(7\sqrt[3]{bd} - 10(1-\sqrt{3})\sqrt[3]{a}))}{7b^{4/3}} \\
&= \frac{2c\sqrt{a + bx^3}}{3b} + \frac{2dx\sqrt{a + bx^3}}{5b} + \frac{2ex^2\sqrt{a + bx^3}}{7b} - \frac{8ae\sqrt{a + bx^3}}{7b^{5/3}((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})} + \frac{4^4\sqrt{3}\sqrt{2 - \sqrt{3}a^4}}{7b^{4/3}}
\end{aligned}$$

Mathematica [C] time = 0.0790605, size = 121, normalized size = 0.22

$$\frac{2(a + bx^3)(35c + 3x(7d + 5ex)) - 42adx\sqrt{\frac{bx^3}{a}} + {}_2F_1\left(\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, -\frac{bx^3}{a}\right) - 30aex^2\sqrt{\frac{bx^3}{a}} + {}_2F_1\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, -\frac{bx^3}{a}\right)}{105b\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(c + d*x + e*x^2))/Sqrt[a + b*x^3], x]

[Out] (2*(a + b*x^3)*(35*c + 3*x*(7*d + 5*e*x)) - 42*a*d*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] - 30*a*e*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(105*b*Sqrt[a + b*x^3])

Maple [A] time = 0.007, size = 773, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^2*(e*x^2+d*x+c)/(b*x^3+a)^{(1/2)}, x)$

[Out] $e*(2/7/b*x^2*(b*x^3+a)^{(1/2)}+8/21*I/b^2*a*3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*EllipticE(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}))+1/b*(-b^2*a)^{(1/3)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)})))+d*(2/5/b*x*(b*x^3+a)^{(1/2)}+4/15*I/b^2*a*3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/2)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}))+2/3*c*(b*x^3+a)^{(1/2)}/b$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\sqrt{bx^3+ac}}{3b} + \int \frac{ex^4+dx^3}{\sqrt{bx^3+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^2*(e*x^2+d*x+c)/(b*x^3+a)^{(1/2)}, x, \text{algorithm}="maxima")$

[Out] $2/3*\text{sqrt}(b*x^3+a)*c/b + \text{integrate}((e*x^4+d*x^3)/\text{sqrt}(b*x^3+a), x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^4+dx^3+cx^2}{\sqrt{bx^3+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] integral((e*x^4 + d*x^3 + c*x^2)/sqrt(b*x^3 + a), x)

Sympy [A] time = 2.87018, size = 107, normalized size = 0.19

$$c \left(\begin{cases} \frac{x^3}{3\sqrt{a}} & \text{for } b = 0 \\ \frac{2\sqrt{a+bx^3}}{3b} & \text{otherwise} \end{cases} \right) + \frac{dx^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{4}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a} \Gamma\left(\frac{7}{3}\right)} + \frac{ex^5 \Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{5}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a} \Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)

[Out] c*Piecewise((x**3/(3*sqrt(a)), Eq(b, 0)), (2*sqrt(a + b*x**3)/(3*b), True)) + d*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(7/3)) + e*x**5*gamma(5/3)*hyper((1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(8/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + dx + c)x^2}{\sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")

[Out] integrate((e*x^2 + d*x + c)*x^2/sqrt(b*x^3 + a), x)

$$3.432 \quad \int \frac{x(c+dx+ex^2)}{\sqrt{a+bx^3}} dx$$

Optimal. Leaf size=537

$$\frac{2\sqrt{2+\sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(2a^{2/3}e+5(1-\sqrt{3})b^{2/3}c)\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}}$$

[Out] $(2*d*\text{Sqrt}[a + b*x^3])/(3*b) + (2*e*x*\text{Sqrt}[a + b*x^3])/(5*b) + (2*c*\text{Sqrt}[a + b*x^3])/(b^{2/3}*((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)) - (3^{1/4}*\text{Sqrt}[2 - \text{Sqrt}[3]]*a^{1/3}*c*(a^{1/3} + b^{1/3}*x)*\text{Sqrt}[(a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2)/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\text{EllipticE}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x]/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)], -7 - 4*\text{Sqrt}[3])/(b^{2/3}*\text{Sqrt}[(a^{1/3}*(a^{1/3} + b^{1/3}*x))/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\text{Sqrt}[a + b*x^3]) - (2*\text{Sqrt}[2 + \text{Sqrt}[3]]*a^{1/3}*(5*(1 - \text{Sqrt}[3])*b^{2/3}*c + 2*a^{2/3}*e)*(a^{1/3} + b^{1/3}*x)*\text{Sqrt}[(a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2)/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\text{EllipticF}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x]/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)], -7 - 4*\text{Sqrt}[3])/(5*3^{1/4}*b^{4/3}*\text{Sqrt}[(a^{1/3}*(a^{1/3} + b^{1/3}*x))/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\text{Sqrt}[a + b*x^3])$

Rubi [A] time = 0.322531, antiderivative size = 537, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1888, 1886, 261, 1878, 218, 1877}

$$\frac{2\sqrt{2+\sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(2a^{2/3}e+5(1-\sqrt{3})b^{2/3}c)F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x*(c + d*x + e*x^2))/\text{Sqrt}[a + b*x^3], x]$

[Out] $(2*d*\text{Sqrt}[a + b*x^3])/(3*b) + (2*e*x*\text{Sqrt}[a + b*x^3])/(5*b) + (2*c*\text{Sqrt}[a + b*x^3])/(b^{2/3}*((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)) - (3^{1/4}*\text{Sqrt}[2 - \text{Sqrt}[3]]*a^{1/3}*c*(a^{1/3} + b^{1/3}*x)*\text{Sqrt}[(a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2)/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\text{EllipticE}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x]/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)], -7 - 4*\text{Sqrt}[3])/(b^{2/3}*\text{Sqrt}[(a^{1/3}*(a^{1/3} + b^{1/3}*x))/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\text{Sqrt}[a + b*x^3]) - (2*\text{Sqrt}[2 + \text{Sqrt}[3]]*a^{1/3}*(5*(1 - \text{Sqrt}[3])*b^{2/3}*c + 2*a^{2/3}*e)*(a^{1/3} + b^{1/3}*x)*\text{Sqrt}[(a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2)/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\text{EllipticF}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x]/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)], -7 - 4*\text{Sqrt}[3])/(5*3^{1/4}*b^{4/3}*\text{Sqrt}[(a^{1/3}*(a^{1/3} + b^{1/3}*x))/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\text{Sqrt}[a + b*x^3])$

$$+ b^{(2/3)}x^2)/((1 + \text{Sqrt}[3])a^{(1/3)} + b^{(1/3)}x)^2 * \text{EllipticE}[\text{ArcSin}[\frac{(1 - \text{Sqrt}[3])a^{(1/3)} + b^{(1/3)}x}{(1 + \text{Sqrt}[3])a^{(1/3)} + b^{(1/3)}x}], -7 - 4\text{Sqrt}[3]]]/(b^{(2/3)}\text{Sqrt}[(a^{(1/3)}(a^{(1/3)} + b^{(1/3)}x))/(1 + \text{Sqrt}[3])a^{(1/3)} + b^{(1/3)}x]^2 * \text{Sqrt}[a + b*x^3]) - (2*\text{Sqrt}[2 + \text{Sqrt}[3]]*a^{(1/3)}*(5*(1 - \text{Sqrt}[3])*b^{(2/3)}*c + 2*a^{(2/3)}*e)*(a^{(1/3)} + b^{(1/3)}x)*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)}x + b^{(2/3)}*x^2)/((1 + \text{Sqrt}[3])a^{(1/3)} + b^{(1/3)}x)^2] * \text{EllipticF}[\text{ArcSin}[\frac{(1 - \text{Sqrt}[3])a^{(1/3)} + b^{(1/3)}x}{(1 + \text{Sqrt}[3])a^{(1/3)} + b^{(1/3)}x}], -7 - 4*\text{Sqrt}[3]])/(5*3^{(1/4)}*b^{(4/3)}*\text{Sqrt}[(a^{(1/3)}(a^{(1/3)} + b^{(1/3)}x))/(1 + \text{Sqrt}[3])a^{(1/3)} + b^{(1/3)}x]^2 * \text{Sqrt}[a + b*x^3])$$

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*n)]) /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[Coeff[Pq, x, n - 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2] * EllipticF[ArcSin[((1 - Sqrt[3])*s
```

+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] & PosQ[a]

Rule 1877

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

Rubi steps

$$\begin{aligned} \int \frac{x(c + dx + ex^2)}{\sqrt{a + bx^3}} dx &= \frac{2ex\sqrt{a + bx^3}}{5b} + \frac{2 \int \frac{-ae + \frac{5bcx}{2} + \frac{5}{2}bdx^2}{\sqrt{a + bx^3}} dx}{5b} \\ &= \frac{2ex\sqrt{a + bx^3}}{5b} + \frac{2 \int \frac{-ae + \frac{5bcx}{2}}{\sqrt{a + bx^3}} dx}{5b} + d \int \frac{x^2}{\sqrt{a + bx^3}} dx \\ &= \frac{2d\sqrt{a + bx^3}}{3b} + \frac{2ex\sqrt{a + bx^3}}{5b} + \frac{c \int \frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt{a + bx^3}} dx}{\sqrt[3]{b}} - \frac{(\sqrt[3]{a}(5(1-\sqrt{3})b^{2/3}c + 2a^{2/3}e)) \int \frac{1}{\sqrt{a + bx^3}} dx}{5b} \\ &= \frac{2d\sqrt{a + bx^3}}{3b} + \frac{2ex\sqrt{a + bx^3}}{5b} + \frac{2c\sqrt{a + bx^3}}{b^{2/3}((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})} - \frac{\sqrt[4]{3}\sqrt{2 - \sqrt{3}}\sqrt[3]{ac}(\sqrt[3]{a} + \sqrt[3]{bx})}{b^{2/3} \sqrt{\left(\frac{a^2}{(1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}\right)}} \end{aligned}$$

Mathematica [C] time = 0.0621461, size = 114, normalized size = 0.21

$$\frac{15bcx^2\sqrt{\frac{bx^3}{a}} + {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^3}{a}\right) + 4(a + bx^3)(5d + 3ex) - 12aex\sqrt{\frac{bx^3}{a}} + {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{30b\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]

[Out] (4*(5*d + 3*e*x)*(a + b*x^3) - 12*a*e*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 15*b*c*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(30*b*Sqrt[a + b*x^3])

Maple [A] time = 0.006, size = 753, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x)

[Out] e*(2/5/b*x*(b*x^3+a)^(1/2)+4/15*I/b^2*a*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+2/3*d*(b*x^3+a)^(1/2)/b-2/3*I*c*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + dx + c)x}{\sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] integrate((e*x^2 + d*x + c)*x/sqrt(b*x^3 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^3 + dx^2 + cx}{\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] integral((e*x^3 + d*x^2 + c*x)/sqrt(b*x^3 + a), x)

Sympy [A] time = 2.68915, size = 107, normalized size = 0.2

$$d \left(\begin{cases} \frac{x^3}{3\sqrt{a}} & \text{for } b = 0 \\ \frac{2\sqrt{a+bx^3}}{3b} & \text{otherwise} \end{cases} \right) + \frac{cx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} + \frac{ex^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{4}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)

[Out] d*Piecewise((x**3/(3*sqrt(a)), Eq(b, 0)), (2*sqrt(a + b*x**3)/(3*b), True)) + c*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3)) + e*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(7/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + dx + c)x}{\sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)*x/sqrt(b*x^3 + a), x)
```


$$3.433 \quad \int \frac{c+dx+ex^2}{\sqrt{a+bx^3}} dx$$

Optimal. Leaf size=509

$$\frac{2\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(\sqrt[3]{bc}-(1-\sqrt{3})\sqrt[3]{ad})\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}}$$

[Out] (2*e*Sqrt[a + b*x^3])/(3*b) + (2*d*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3]) * a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*c - (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 0.173025, antiderivative size = 509, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1886, 261, 1878, 218, 1877}

$$\frac{2\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(\sqrt[3]{bc}-(1-\sqrt{3})\sqrt[3]{ad})F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/Sqrt[a + b*x^3], x]

[Out] (2*e*Sqrt[a + b*x^3])/(3*b) + (2*d*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3]) * a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*c - (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

$$\begin{aligned} & (1/3) + b^{(1/3)*x}^2 * \text{EllipticE}[\text{ArcSin}[\frac{(1 - \sqrt{3})a^{(1/3)} + b^{(1/3)*x}}{(1 + \sqrt{3})a^{(1/3)} + b^{(1/3)*x}}], -7 - 4\sqrt{3}]] / (b^{(2/3)*\sqrt{a + b*x^3}}) \\ & + (2*\sqrt{2 + \sqrt{3}}*(b^{(1/3)*c} - (1 - \sqrt{3})a^{(1/3)*d})*(a^{(1/3)} + b^{(1/3)*x})*\sqrt{a^2 - a^{(1/3)*b^{(1/3)*x}} + b^{(2/3)*x^2}}) / ((1 + \sqrt{3})a^{(1/3)} + b^{(1/3)*x})^2 * \text{EllipticF}[\text{ArcSin}[\frac{(1 - \sqrt{3})a^{(1/3)} + b^{(1/3)*x}}{(1 + \sqrt{3})a^{(1/3)} + b^{(1/3)*x}}], -7 - 4\sqrt{3}]] / (3^{(1/4)*b^{(2/3)*\sqrt{a + b*x^3}}}) \end{aligned}$$

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[Coeff[Pq, x, n - 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[(((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x))], -7 - 4*Sqrt[3]]]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
```

```
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{c + dx + ex^2}{\sqrt{a + bx^3}} dx &= e \int \frac{x^2}{\sqrt{a + bx^3}} dx + \int \frac{c + dx}{\sqrt{a + bx^3}} dx \\ &= \frac{2e\sqrt{a + bx^3}}{3b} + \frac{d \int \frac{(1-\sqrt{3})\sqrt[3]{a+\sqrt[3]{bx^3}}}{\sqrt{a+bx^3}} dx}{\sqrt[3]{b}} + \left(c - \frac{(1-\sqrt{3})\sqrt[3]{ad}}{\sqrt[3]{b}} \right) \int \frac{1}{\sqrt{a + bx^3}} dx \\ &= \frac{2e\sqrt{a + bx^3}}{3b} + \frac{2d\sqrt{a + bx^3}}{b^{2/3}((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx^3})} - \frac{\sqrt[4]{3}\sqrt{2 - \sqrt{3}}\sqrt[3]{ad}(\sqrt[3]{a} + \sqrt[3]{bx^3}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx^3} + b^{2/3}x^2}{((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx^3})^2}} E\left(\sin^{-1}\left(\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx^3})}{((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx^3})}\right)\right)}{b^{2/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx^3})}{((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx^3})^2}} \sqrt{a - bx^3}} \end{aligned}$$

Mathematica [C] time = 0.045915, size = 107, normalized size = 0.21

$$\frac{6bcx\sqrt{\frac{bx^3}{a}} + 1 {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a}\right) + 3bdx^2\sqrt{\frac{bx^3}{a}} + 1 {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^3}{a}\right) + 4e(a + bx^3)}{6b\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/Sqrt[a + b*x^3], x]

[Out] (4*e*(a + b*x^3) + 6*b*c*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 3*b*d*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(6*b*Sqrt[a + b*x^3])

Maple [A] time = 0.004, size = 735, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)/(b*x^3+a)^(1/2),x)

[Out] $\frac{2}{3}e(bx^3+a)^{1/2}/b - \frac{2}{3}I^3d^{3/2}/b(-b^2a)^{1/3} * (I(x+1/2/b(-b^2a)^{1/3}) - 1/2I^3)^{1/2} * b/(-b^2a)^{1/3} * ((x-1/b(-b^2a)^{1/3})/(-3/2/b(-b^2a)^{1/3} + 1/2I^3)^{1/2} * b/(-b^2a)^{1/3})^{1/2} * (-I(x+1/2/b(-b^2a)^{1/3}) + 1/2I^3)^{3/2} * b/(-b^2a)^{1/3} * (bx^3+a)^{1/2} * ((-3/2/b(-b^2a)^{1/3} + 1/2I^3)^{1/2} * b/(-b^2a)^{1/3}) * \text{EllipticE}(1/3, 3^{1/2} * (I(x+1/2/b(-b^2a)^{1/3}) - 1/2I^3)^{1/2} * b/(-b^2a)^{1/3})^{3/2} * b/(-b^2a)^{1/3})^{1/2}, (I^3)^{1/2} * b/(-b^2a)^{1/3} / (-3/2/b(-b^2a)^{1/3} + 1/2I^3)^{1/2} * b/(-b^2a)^{1/3})^{1/2} + 1/b(-b^2a)^{1/3} * \text{EllipticF}(1/3, 3^{1/2} * (I(x+1/2/b(-b^2a)^{1/3}) - 1/2I^3)^{1/2} * b/(-b^2a)^{1/3})^{3/2} * b/(-b^2a)^{1/3})^{1/2}, (I^3)^{1/2} * b/(-b^2a)^{1/3} / (-3/2/b(-b^2a)^{1/3} + 1/2I^3)^{1/2} * b/(-b^2a)^{1/3})^{1/2} - 2/3I^3c^{3/2}/b(-b^2a)^{1/3} * (I(x+1/2/b(-b^2a)^{1/3}) - 1/2I^3)^{1/2} * b/(-b^2a)^{1/3} * 3^{1/2} * b/(-b^2a)^{1/3})^{1/2} * ((x-1/b(-b^2a)^{1/3})/(-3/2/b(-b^2a)^{1/3} + 1/2I^3)^{1/2} * b/(-b^2a)^{1/3})^{1/2} * (-I(x+1/2/b(-b^2a)^{1/3}) + 1/2I^3)^{3/2} * b/(-b^2a)^{1/3})^{1/2} / (bx^3+a)^{1/2} * \text{EllipticF}(1/3, 3^{1/2} * (I(x+1/2/b(-b^2a)^{1/3}) - 1/2I^3)^{1/2} * b/(-b^2a)^{1/3})^{3/2} * b/(-b^2a)^{1/3})^{1/2}, (I^3)^{1/2} * b/(-b^2a)^{1/3} / (-3/2/b(-b^2a)^{1/3} + 1/2I^3)^{1/2} * b/(-b^2a)^{1/3})^{1/2}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{\sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] integrate((e*x^2 + d*x + c)/sqrt(b*x^3 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2 + dx + c}{\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] integral((e*x^2 + d*x + c)/sqrt(b*x^3 + a), x)

Sympy [A] time = 1.98094, size = 105, normalized size = 0.21

$$e^{\begin{cases} \frac{x^3}{3\sqrt{a}} & \text{for } b = 0 \\ \frac{2\sqrt{a+bx^3}}{3b} & \text{otherwise} \end{cases}} + \frac{cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{2}{3}, \frac{2}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/(b*x**3+a)**(1/2), x)

[Out] e*Piecewise((x**3/(3*sqrt(a)), Eq(b, 0)), (2*sqrt(a + b*x**3)/(3*b), True)) + c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{\sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^(1/2), x, algorithm="giac")

[Out] integrate((e*x^2 + d*x + c)/sqrt(b*x^3 + a), x)

$$3.434 \quad \int \frac{c+dx+ex^2}{x\sqrt{a+bx^3}} dx$$

Optimal. Leaf size=518

$$2\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(\sqrt[3]{bd}-(1-\sqrt{3})\sqrt[3]{ae})\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)\sqrt[4]{3}\sqrt{2}$$

$$\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}$$

[Out] (2*e*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*d - (1 - Sqrt[3])*a^(1/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 0.204874, antiderivative size = 518, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {1832, 266, 63, 208, 1878, 218, 1877}

$$2\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(\sqrt[3]{bd}-(1-\sqrt{3})\sqrt[3]{ae})F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}$$

$$\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/(x*Sqrt[a + b*x^3]),x]

[Out] (2*e*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*d - (1 - Sqrt[3])*a^(1/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

$$\begin{aligned} & *x^2)/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2 * \text{EllipticE}[\text{ArcSin}[\frac{(1 - \text{Sqrt}[3]) * a^{(1/3)} + b^{(1/3)*x}}{(1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x}}], -7 - 4*\text{Sqrt}[3]]] / (b^{(2/3)} * \text{Sqrt}[(a^{(1/3)} * (a^{(1/3)} + b^{(1/3)*x})) / ((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2] * \text{Sqrt}[a + b*x^3]) + (2*\text{Sqrt}[2 + \text{Sqrt}[3]] * (b^{(1/3)} * d - (1 - \text{Sqrt}[3]) * a^{(1/3)} * e) * (a^{(1/3)} + b^{(1/3)*x}) * \text{Sqrt}[(a^{(2/3)} - a^{(1/3)} * b^{(1/3)*x} + b^{(2/3)} * x^2) / ((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2] * \text{EllipticF}[\text{ArcSin}[\frac{(1 - \text{Sqrt}[3]) * a^{(1/3)} + b^{(1/3)*x}}{(1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x}}], -7 - 4*\text{Sqrt}[3]]) / (3^{(1/4)} * b^{(2/3)} * \text{Sqrt}[(a^{(1/3)} * (a^{(1/3)} + b^{(1/3)*x})) / ((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2] * \text{Sqrt}[a + b*x^3]) \end{aligned}$$

Rule 1832

$$\text{Int}[(\text{Pq}_)/((\text{x}_)*\text{Sqrt}[(\text{a}_) + (\text{b}_.)(\text{x}_)^{(\text{n}_)}])], \text{x_Symbol}] \text{:>} \text{Dist}[\text{Coeff}[\text{Pq}, \text{x}, 0], \text{Int}[1/(\text{x}*\text{Sqrt}[a + b*x^n]), \text{x}], \text{x}] + \text{Int}[\text{ExpandToSum}[(\text{Pq} - \text{Coeff}[\text{Pq}, \text{x}, 0])/x, \text{x}]/\text{Sqrt}[a + b*x^n], \text{x}] /; \text{FreeQ}[\{a, b\}, \text{x}] \&\& \text{PolyQ}[\text{Pq}, \text{x}] \&\& \text{IGtQ}[\text{n}, 0] \&\& \text{NeQ}[\text{Coeff}[\text{Pq}, \text{x}, 0], 0]$$

Rule 266

$$\text{Int}[(\text{x}_)^{(\text{m}_.)*((\text{a}_) + (\text{b}_.)(\text{x}_)^{(\text{n}_)})^{(\text{p}_)}], \text{x_Symbol}] \text{:>} \text{Dist}[1/\text{n}, \text{Subst}[\text{Int}[\text{x}^{(\text{Simplify}[(\text{m} + 1)/\text{n}] - 1)*(a + b*x)^p}, \text{x}], \text{x}, \text{x}^n], \text{x}] /; \text{FreeQ}[\{a, b, m, n, p\}, \text{x}] \&\& \text{IntegerQ}[\text{Simplify}[(\text{m} + 1)/\text{n}]]$$

Rule 63

$$\text{Int}[(\text{a}_.) + (\text{b}_.)(\text{x}_)]^{(\text{m}_)*((\text{c}_.) + (\text{d}_.)(\text{x}_))^{(\text{n}_)}], \text{x_Symbol}] \text{:>} \text{With}[\{p = \text{Denominator}[\text{m}]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[\text{x}^{(p*(\text{m} + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n}, \text{x}], \text{x}, (a + b*x)^{(1/p)}], \text{x}]] /; \text{FreeQ}[\{a, b, c, d\}, \text{x}] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, \text{m}, 0] \&\& \text{LeQ}[-1, \text{n}, 0] \&\& \text{LeQ}[\text{Denominator}[\text{n}], \text{Denominator}[\text{m}]] \&\& \text{IntLinearQ}[a, b, c, d, \text{m}, \text{n}, \text{x}]$$

Rule 208

$$\text{Int}[(\text{a}_) + (\text{b}_.)(\text{x}_)^2)^{-1}, \text{x_Symbol}] \text{:>} \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[\text{x}/\text{Rt}[-(a/b), 2]])/a, \text{x}] /; \text{FreeQ}[\{a, b\}, \text{x}] \&\& \text{NegQ}[a/b]$$

Rule 1878

$$\text{Int}[(\text{c}_) + (\text{d}_.)(\text{x}_)]/\text{Sqrt}[(\text{a}_) + (\text{b}_.)(\text{x}_)^3], \text{x_Symbol}] \text{:>} \text{With}[\{r = \text{Numerator}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Dist}[(\text{c}*r - (1 - \text{Sqrt}[3])*d*s)/r, \text{Int}[1/\text{Sqrt}[a + b*x^3], \text{x}], \text{x}] + \text{Dist}[d/r, \text{Int}[(1 - \text{Sqrt}[3])*s + r*x]/\text{Sqrt}[a + b*x^3], \text{x}], \text{x}]] /; \text{FreeQ}[\{a, b, c, d\}, \text{x}] \&\& \text{PosQ}[a] \&\& \text{NeQ}[b*c^3 - 2*(5 - 3*\text{Sqrt}[3])*a*d^3, 0]$$

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]]/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]]]/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2}{x\sqrt{a + bx^3}} dx &= c \int \frac{1}{x\sqrt{a + bx^3}} dx + \int \frac{d + ex}{\sqrt{a + bx^3}} dx \\
&= \frac{1}{3}c \operatorname{Subst} \left(\int \frac{1}{x\sqrt{a + bx}} dx, x, x^3 \right) + \frac{e \int \frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt{a+bx^3}} dx}{\sqrt[3]{b}} + \left(d - \frac{(1-\sqrt{3})\sqrt[3]{ae}}{\sqrt[3]{b}} \right) \int \frac{1}{\sqrt{a + bx^3}} dx \\
&= \frac{2e\sqrt{a + bx^3}}{b^{2/3} \left((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx} \right)} - \frac{\sqrt[4]{3} \sqrt{2 - \sqrt{3}} \sqrt[3]{ae} \left(\sqrt[3]{a} + \sqrt[3]{bx} \right) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{\left((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx} \right)^2}} E \left(\sin^{-1} \left(\frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}} \right) \right)}{b^{2/3} \sqrt{\frac{\sqrt[3]{a} \left(\sqrt[3]{a} + \sqrt[3]{bx} \right)}{\left((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx} \right)^2}} \sqrt{a + bx^3}} \\
&= \frac{2e\sqrt{a + bx^3}}{b^{2/3} \left((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx} \right)} - \frac{2c \tanh^{-1} \left(\frac{\sqrt{a+bx^3}}{\sqrt{a}} \right)}{3\sqrt{a}} - \frac{\sqrt[4]{3} \sqrt{2 - \sqrt{3}} \sqrt[3]{ae} \left(\sqrt[3]{a} + \sqrt[3]{bx} \right) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{\left((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx} \right)^2}}}{b^{2/3} \sqrt{\frac{\sqrt[3]{a} \left(\sqrt[3]{a} + \sqrt[3]{bx} \right)}{\left((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx} \right)^2}}}
\end{aligned}$$

Mathematica [C] time = 0.141124, size = 128, normalized size = 0.25

$$-\frac{2c \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}} + \frac{dx\sqrt{\frac{bx^3}{a}} + {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{\sqrt{a+bx^3}} + \frac{ex^2\sqrt{\frac{bx^3}{a}} + {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^3}{a}\right)}{2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(x*sqrt[a + b*x^3]),x]

[Out] $(-2*c*\text{ArcTanh}[\text{Sqrt}[a + b*x^3]/\text{Sqrt}[a]])/(3*\text{Sqrt}[a]) + (d*x*\text{Sqrt}[1 + (b*x^3)/a]*\text{Hypergeometric2F1}[1/3, 1/2, 4/3, -((b*x^3)/a)]/\text{Sqrt}[a + b*x^3] + (e*x^2*\text{Sqrt}[1 + (b*x^3)/a]*\text{Hypergeometric2F1}[1/2, 2/3, 5/3, -((b*x^3)/a)]/(2*\text{Sqrt}[a + b*x^3]))$

Maple [A] time = 0.007, size = 740, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)/x/(b*x^3+a)^(1/2),x)

[Out] $-2/3*I*e*3^{(1/2)}/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}/(b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*EllipticE(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)},(I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}))+1/b*(-b^2*a)^{(1/3)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)},(I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}))-2/3*I*d*3^{(1/2)}/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}/(b*x^3+a)^{(1/2)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)},(I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}))-2/3*c*arctanh((b*x^3+a)^{(1/2)}}$

$1/2)/a^{(1/2)})/a^{(1/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{\sqrt{bx^3 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + a}(ex^2 + dx + c)}{bx^4 + ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^3 + a)*(e*x^2 + d*x + c)/(b*x^4 + a*x), x)

Sympy [A] time = 3.33502, size = 105, normalized size = 0.2

$$-\frac{2c \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx^2}}\right)}{3\sqrt{a}} + \frac{dx \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \left| \frac{bx^3 e^{i\pi}}{a} \right.\right)}{3\sqrt{a} \Gamma\left(\frac{4}{3}\right)} + \frac{ex^2 \Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{2}{3} \left| \frac{bx^3 e^{i\pi}}{a} \right.\right)}{3\sqrt{a} \Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/x/(b*x**3+a)**(1/2),x)

[Out] $-2*c*\operatorname{asinh}(\sqrt{a}/(\sqrt{b}*x**(3/2)))/(3*\sqrt{a}) + d*x*\operatorname{gamma}(1/3)*\operatorname{hyper}((1/3, 1/2), (4/3,), b*x**3*\exp_polar(I*pi)/a)/(3*\sqrt{a}*\operatorname{gamma}(4/3)) + e*x**$

```
2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)
*gamma(5/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{\sqrt{bx^3 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x), x)
```

$$3.435 \quad \int \frac{c+dx+ex^2}{x^2\sqrt{a+bx^3}} dx$$

Optimal. Leaf size=547

$$\frac{\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}((1-\sqrt{3})b^{2/3}c-2a^{2/3}e)\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}a^{2/3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}}$$

[Out] $-\left(\frac{c\sqrt{a+bx^3}}{ax}\right) + \frac{b^{1/3}c\sqrt{a+bx^3}}{a((1+\sqrt{3})a^{1/3}+b^{1/3}x)} - \frac{2d\operatorname{ArcTanh}\left[\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right]}{3\sqrt{a}} - \frac{3^{1/4}\sqrt{2-\sqrt{3}}b^{1/3}c(a^{1/3}+b^{1/3}x)\sqrt{(a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2)/((1+\sqrt{3})a^{1/3}+b^{1/3}x)^2}}{(1+\sqrt{3})a^{1/3}+b^{1/3}x)^2} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{(1-\sqrt{3})a^{1/3}+b^{1/3}x}{(1+\sqrt{3})a^{1/3}+b^{1/3}x}\right], -7-4\sqrt{3}\right] - \frac{2a^{2/3}\sqrt{(a^{1/3}(a^{1/3}+b^{1/3}x))}}{(1+\sqrt{3})a^{1/3}+b^{1/3}x)^2} \sqrt{a+bx^3} - \frac{\sqrt{2+\sqrt{3}}((1-\sqrt{3})b^{2/3}c-2a^{2/3}e)(a^{1/3}+b^{1/3}x)\sqrt{(a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2)/((1+\sqrt{3})a^{1/3}+b^{1/3}x)^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1-\sqrt{3})a^{1/3}+b^{1/3}x}{(1+\sqrt{3})a^{1/3}+b^{1/3}x}\right], -7-4\sqrt{3}\right]}{3^{1/4}a^{2/3}b^{1/3}\sqrt{(a^{1/3}(a^{1/3}+b^{1/3}x))}} \sqrt{a+bx^3}$

Rubi [A] time = 0.342628, antiderivative size = 547, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.32$, Rules used = {1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$\frac{\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}((1-\sqrt{3})b^{2/3}c-2a^{2/3}e)F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}a^{2/3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}\left[\frac{c+dx+ex^2}{x^2\sqrt{a+bx^3}}, x\right]$

[Out] $-\left(\frac{c\sqrt{a+bx^3}}{ax}\right) + \frac{b^{1/3}c\sqrt{a+bx^3}}{a((1+\sqrt{3})a^{1/3}+b^{1/3}x)} - \frac{2d\operatorname{ArcTanh}\left[\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right]}{3\sqrt{a}}$

$$\begin{aligned} & - (3^{1/4} \sqrt{2 - \sqrt{3}} b^{1/3} c (a^{1/3} + b^{1/3} x) \sqrt{(a^{2/3} \\ & - a^{1/3} b^{1/3} x + b^{2/3} x^2) / ((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2} \\ & * \text{EllipticE}[\text{ArcSin}[(1 - \sqrt{3}) a^{1/3} + b^{1/3} x] / ((1 + \sqrt{3}) a^{1/3} \\ & + b^{1/3} x)], -7 - 4\sqrt{3}]) / (2 a^{2/3} \sqrt{(a^{1/3} (a^{1/3} + b^{1/3} \\ & x)) / ((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2} \sqrt{a + b x^3}) - (\sqrt{2 + \sqrt{3}} \\ & * ((1 - \sqrt{3}) b^{2/3} c - 2 a^{2/3} e) (a^{1/3} + b^{1/3} x) \sqrt{ \\ & (a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2) / ((1 + \sqrt{3}) a^{1/3} + b^{1/3} \\ & x)^2} * \text{EllipticF}[\text{ArcSin}[(1 - \sqrt{3}) a^{1/3} + b^{1/3} x] / ((1 + \sqrt{3}) \\ &) a^{1/3} + b^{1/3} x)], -7 - 4\sqrt{3}]) / (3^{1/4} a^{2/3} b^{1/3} \sqrt{(a^{1/3} \\ & (a^{1/3} + b^{1/3} x)) / ((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2} \sqrt{a \\ & + b x^3}) \end{aligned}$$

Rule 1835

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_)*(x_)^(n_)]), x_Symbol] := Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
```

$\text{Rt}[-(a/b), 2]]/a, x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{NegQ}[a/b]$

Rule 1878

$\text{Int}[\frac{(c_.) + (d_.)(x_.)}{\text{Sqrt}[a_ + (b_.)(x_)^3]}, x_Symbol] := \text{With}\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Dist}[(c*r - (1 - \text{Sqrt}[3])*d*s)/r, \text{Int}[1/\text{Sqrt}[a + b*x^3], x], x] + \text{Dist}[d/r, \text{Int}[\frac{(1 - \text{Sqrt}[3])*s + r*x}{\text{Sqrt}[a + b*x^3]}, x], x]] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{PosQ}[a] \&\& \text{NeQ}[b*c^3 - 2*(5 - 3*\text{Sqrt}[3])*a*d^3, 0]$

Rule 218

$\text{Int}[1/\text{Sqrt}[(a_.) + (b_.)(x_)^3], x_Symbol] := \text{With}\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Simp}[(2*\text{Sqrt}[2 + \text{Sqrt}[3]]*(s + r*x)*\text{Sqrt}[(s^2 - r*s*x + r^2*x^2)/((1 + \text{Sqrt}[3])*s + r*x)^2]*\text{EllipticF}[\text{ArcSin}[\frac{(1 - \text{Sqrt}[3])*s + r*x}{(1 + \text{Sqrt}[3])*s + r*x}], -7 - 4*\text{Sqrt}[3]])/(3^{1/4}*r*\text{Sqrt}[a + b*x^3]*\text{Sqrt}[(s*(s + r*x))/((1 + \text{Sqrt}[3])*s + r*x)^2]), x]] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{PosQ}[a]$

Rule 1877

$\text{Int}[\frac{(c_.) + (d_.)(x_.)}{\text{Sqrt}[a_ + (b_.)(x_)^3]}, x_Symbol] := \text{With}\{r = \text{Numer}[\text{Simplify}[\frac{(1 - \text{Sqrt}[3])*d}{c}], s = \text{Denom}[\text{Simplify}[\frac{(1 - \text{Sqrt}[3])*d}{c}]]\}, \text{Simp}[(2*d*s^3*\text{Sqrt}[a + b*x^3])/(a*r^2*((1 + \text{Sqrt}[3])*s + r*x)), x] - \text{Simp}[(3^{1/4}*\text{Sqrt}[2 - \text{Sqrt}[3]]*d*s*(s + r*x)*\text{Sqrt}[(s^2 - r*s*x + r^2*x^2)/((1 + \text{Sqrt}[3])*s + r*x)^2]*\text{EllipticE}[\text{ArcSin}[\frac{(1 - \text{Sqrt}[3])*s + r*x}{(1 + \text{Sqrt}[3])*s + r*x}], -7 - 4*\text{Sqrt}[3]])/(r^2*\text{Sqrt}[a + b*x^3]*\text{Sqrt}[(s*(s + r*x))/((1 + \text{Sqrt}[3])*s + r*x)^2]), x]] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{PosQ}[a] \&\& \text{EqQ}[b*c^3 - 2*(5 - 3*\text{Sqrt}[3])*a*d^3, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2}{x^2\sqrt{a + bx^3}} dx &= -\frac{c\sqrt{a + bx^3}}{ax} - \frac{\int \frac{-2ad - 2aex - bcx^2}{x\sqrt{a + bx^3}} dx}{2a} \\
&= -\frac{c\sqrt{a + bx^3}}{ax} - \frac{\int \frac{-2ae - bcx}{\sqrt{a + bx^3}} dx}{2a} + d \int \frac{1}{x\sqrt{a + bx^3}} dx \\
&= -\frac{c\sqrt{a + bx^3}}{ax} + \frac{(b^{2/3}c) \int \frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt{a + bx^3}} dx}{2a} + \frac{1}{3}d \operatorname{Subst} \left(\int \frac{1}{x\sqrt{a + bx}} dx, x, x^3 \right) - \frac{1}{2} \left(\frac{(1-\sqrt{3})b^{2/3}c}{a^{2/3}} \right. \\
&= -\frac{c\sqrt{a + bx^3}}{ax} + \frac{\sqrt[3]{bc}\sqrt{a + bx^3}}{a((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{bc}(\sqrt[3]{a} + \sqrt[3]{bx})}{\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}}} E \left(\sin^{-1} \right. \\
&= -\frac{c\sqrt{a + bx^3}}{ax} + \frac{\sqrt[3]{bc}\sqrt{a + bx^3}}{a((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})} - \frac{2d \tanh^{-1} \left(\frac{\sqrt{a + bx^3}}{\sqrt{a}} \right)}{3\sqrt{a}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{bc}(\sqrt[3]{a} + \sqrt[3]{bx})}{\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}}} \sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}} \\
&= -\frac{c\sqrt{a + bx^3}}{ax} + \frac{\sqrt[3]{bc}\sqrt{a + bx^3}}{a((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})} - \frac{2d \tanh^{-1} \left(\frac{\sqrt{a + bx^3}}{\sqrt{a}} \right)}{3\sqrt{a}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{bc}(\sqrt[3]{a} + \sqrt[3]{bx})}{\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}}} \sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}}
\end{aligned}$$

Mathematica [C] time = 0.123586, size = 126, normalized size = 0.23

$$-\frac{c\sqrt{\frac{bx^3}{a}} + {}_2F_1\left(-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, -\frac{bx^3}{a}\right)}{x\sqrt{a + bx^3}} - \frac{2d \tanh^{-1}\left(\frac{\sqrt{a + bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}} + \frac{ex\sqrt{\frac{bx^3}{a}} + {}_2F_1\left(\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, -\frac{bx^3}{a}\right)}{\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(x^2*sqrt[a + b*x^3]),x]

[Out] (-2*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]/(3*Sqrt[a]) - (c*Sqrt[1 + (b*x^3)/a])*Hypergeometric2F1[-1/3, 1/2, 2/3, -((b*x^3)/a)]/(x*Sqrt[a + b*x^3]) + (e*x*Sqrt[1 + (b*x^3)/a])*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]/Sqrt[a + b*x^3]

Maple [A] time = 0.007, size = 759, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d*x+c)/x^2/(b*x^3+a)^(1/2),x)`

[Out]
$$\begin{aligned} & -2/3*I*e*3^{(1/2)}/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/ \\ & b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(\\ & -3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}*(-I*(x+1/2/b*(\\ & -b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}/ \\ & (b*x^3+a)^{(1/2)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/ \\ & b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}, (I*3^{(1/2)}/b*(-b^2 \\ & *a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)})-2/ \\ & 3*d*arctanh((b*x^3+a)^{(1/2)}/a^{(1/2)})/a^{(1/2)}+c*(-1/a*(b*x^3+a)^{(1/2)}/x-1/3* \\ & I/a*3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2 \\ & *a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b* \\ & (-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}*(-I*(x+1/2/b*(-b^2*a) \\ & ^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}/(b*x \\ & ^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*Ellipti \\ & cE(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3 \\ & ^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a) \\ & ^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)})+1/b*(-b^2*a)^{(1/3)}*EllipticF \\ & (1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(\\ & 1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(\\ & 1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)})) \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{\sqrt{bx^3 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(1/2),x, algorithm="maxima")`

[Out] `integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x^2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + a}(ex^2 + dx + c)}{bx^5 + ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^3 + a)*(e*x^2 + d*x + c)/(b*x^5 + a*x^2), x)

Sympy [A] time = 2.94802, size = 107, normalized size = 0.2

$$\frac{c\Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(-\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{ax}\Gamma\left(\frac{2}{3}\right)} - \frac{2d \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx^2}}\right)}{3\sqrt{a}} + \frac{ex\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**(1/2),x)

[Out] c*gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*x*gamma(2/3)) - 2*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a)) + e*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{\sqrt{bx^3 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(1/2),x, algorithm="giac")

[Out] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x^2), x)

$$3.436 \quad \int \frac{c+dx+ex^2}{x^3\sqrt{a+bx^3}} dx$$

Optimal. Leaf size=569

$$\frac{\sqrt{2+\sqrt{3}}\sqrt[3]{b}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(2(1-\sqrt{3})\sqrt[3]{ad}+\sqrt[3]{bc})\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}}$$

[Out] $-(c*\text{Sqrt}[a + b*x^3])/(2*a*x^2) - (d*\text{Sqrt}[a + b*x^3])/(a*x) + (b^{(1/3)}*d*\text{Sqrt}[a + b*x^3])/(a*((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)) - (2*e*\text{ArcTanh}[\text{Sqrt}[a + b*x^3]/\text{Sqrt}[a]])/(3*\text{Sqrt}[a]) - (3^{(1/4)}*\text{Sqrt}[2 - \text{Sqrt}[3]]*b^{(1/3)}*d*(a^{(1/3)} + b^{(1/3)}*x)*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2)/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)^2]*\text{EllipticE}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x]/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)], -7 - 4*\text{Sqrt}[3]))/(2*a^{(2/3)}*\text{Sqrt}[(a^{(1/3)}*(a^{(1/3)} + b^{(1/3)}*x))/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)^2]*\text{Sqrt}[a + b*x^3]) - (\text{Sqrt}[2 + \text{Sqrt}[3]]*b^{(1/3)}*(b^{(1/3)}*c + 2*(1 - \text{Sqrt}[3])*a^{(1/3)}*d)*(a^{(1/3)} + b^{(1/3)}*x)*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2)/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)^2]*\text{EllipticF}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x]/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)], -7 - 4*\text{Sqrt}[3]))/(2*3^{(1/4)}*a*\text{Sqrt}[(a^{(1/3)}*(a^{(1/3)} + b^{(1/3)}*x))/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)^2]*\text{Sqrt}[a + b*x^3])$

Rubi [A] time = 0.463536, antiderivative size = 569, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.32$, Rules used = {1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$\frac{\sqrt{2+\sqrt{3}}\sqrt[3]{b}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(2(1-\sqrt{3})\sqrt[3]{ad}+\sqrt[3]{bc})F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x + e*x^2)/(x^3*\text{Sqrt}[a + b*x^3]),x]$

[Out] $-(c*\text{Sqrt}[a + b*x^3])/(2*a*x^2) - (d*\text{Sqrt}[a + b*x^3])/(a*x) + (b^{(1/3)}*d*\text{Sqrt}[a + b*x^3])/(a*((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)) - (2*e*\text{ArcTanh}[\text{Sqrt}[a$

$$\begin{aligned} & + b*x^3/\text{Sqrt}[a]]/(3*\text{Sqrt}[a]) - (3^{(1/4)}*\text{Sqrt}[2 - \text{Sqrt}[3]]*b^{(1/3)}*d*(a^{(1/3)} \\ & + b^{(1/3)}*x)*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2)/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)^2] \\ & * \text{EllipticE}[\text{ArcSin}[\frac{(1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x}{(1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x}], -7 - 4*\text{Sqrt}[3]])/(2*a^{(2/3)}* \\ & \text{Sqrt}[(a^{(1/3)}*(a^{(1/3)} + b^{(1/3)}*x))/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)^2] \\ & * \text{Sqrt}[a + b*x^3]) - (\text{Sqrt}[2 + \text{Sqrt}[3]]*b^{(1/3)}*(b^{(1/3)}*c + 2*(1 - \text{Sqrt}[3])*a^{(1/3)}*d) \\ & *(a^{(1/3)} + b^{(1/3)}*x)*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2)/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)^2] \\ & * \text{EllipticF}[\text{ArcSin}[\frac{(1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x}{(1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x}], -7 - 4*\text{Sqrt}[3]]) \\ & / (2*3^{(1/4)}*a*\text{Sqrt}[(a^{(1/3)}*(a^{(1/3)} + b^{(1/3)}*x))/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)}*x)^2] * \text{Sqrt}[a + b*x^3]) \end{aligned}$$

Rule 1835

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[
  {Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x^n)^p, x], x] /; NeQ[Pq0, 0] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_)*(x_)^(n_)]), x_Symbol] := Dist[Coeff[Pq, x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq, x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
```

$\text{Rt}[-(a/b), 2]]/a, x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{NegQ}[a/b]$

Rule 1878

$\text{Int}[\frac{(c_.) + (d_.)(x_.)}{\text{Sqrt}[a_ + (b_.)(x_)^3]}, x_Symbol] := \text{With}\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Dist}[(c*r - (1 - \text{Sqrt}[3])*d*s)/r, \text{Int}[1/\text{Sqrt}[a + b*x^3], x], x] + \text{Dist}[d/r, \text{Int}[\frac{(1 - \text{Sqrt}[3])*s + r*x}{\text{Sqrt}[a + b*x^3]}, x], x]] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{PosQ}[a] \&\& \text{NeQ}[b*c^3 - 2*(5 - 3*\text{Sqrt}[3])*a*d^3, 0]$

Rule 218

$\text{Int}[1/\text{Sqrt}[(a_.) + (b_.)(x_)^3], x_Symbol] := \text{With}\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Simp}[(2*\text{Sqrt}[2 + \text{Sqrt}[3]]*(s + r*x)*\text{Sqrt}[(s^2 - r*s*x + r^2*x^2)/((1 + \text{Sqrt}[3])*s + r*x)^2]*\text{EllipticF}[\text{ArcSin}[\frac{(1 - \text{Sqrt}[3])*s + r*x}{(1 + \text{Sqrt}[3])*s + r*x}], -7 - 4*\text{Sqrt}[3]])/(3^{1/4}*r*\text{Sqrt}[a + b*x^3]*\text{Sqrt}[(s*(s + r*x))/((1 + \text{Sqrt}[3])*s + r*x)^2]), x]] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{PosQ}[a]$

Rule 1877

$\text{Int}[\frac{(c_.) + (d_.)(x_.)}{\text{Sqrt}[a_ + (b_.)(x_)^3]}, x_Symbol] := \text{With}\{r = \text{Numer}[\text{Simplify}[\frac{(1 - \text{Sqrt}[3])*d}{c}], s = \text{Denom}[\text{Simplify}[\frac{(1 - \text{Sqrt}[3])*d}{c}]]\}, \text{Simp}[(2*d*s^3*\text{Sqrt}[a + b*x^3])/(a*r^2*((1 + \text{Sqrt}[3])*s + r*x)), x] - \text{Simp}[(3^{1/4}*\text{Sqrt}[2 - \text{Sqrt}[3]]*d*s*(s + r*x)*\text{Sqrt}[(s^2 - r*s*x + r^2*x^2)/((1 + \text{Sqrt}[3])*s + r*x)^2]*\text{EllipticE}[\text{ArcSin}[\frac{(1 - \text{Sqrt}[3])*s + r*x}{(1 + \text{Sqrt}[3])*s + r*x}], -7 - 4*\text{Sqrt}[3]])/(r^2*\text{Sqrt}[a + b*x^3]*\text{Sqrt}[(s*(s + r*x))/((1 + \text{Sqrt}[3])*s + r*x)^2]), x]] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{PosQ}[a] \&\& \text{EqQ}[b*c^3 - 2*(5 - 3*\text{Sqrt}[3])*a*d^3, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2}{x^3 \sqrt{a + bx^3}} dx &= -\frac{c\sqrt{a + bx^3}}{2ax^2} - \frac{\int \frac{-4ad - 4aex + bcx^2}{x^2 \sqrt{a + bx^3}} dx}{4a} \\
&= -\frac{c\sqrt{a + bx^3}}{2ax^2} - \frac{d\sqrt{a + bx^3}}{ax} + \frac{\int \frac{8a^2e - 2abcx + 4abdx^2}{x\sqrt{a + bx^3}} dx}{8a^2} \\
&= -\frac{c\sqrt{a + bx^3}}{2ax^2} - \frac{d\sqrt{a + bx^3}}{ax} + \frac{\int \frac{-2abc + 4abdx}{\sqrt{a + bx^3}} dx}{8a^2} + e \int \frac{1}{x\sqrt{a + bx^3}} dx \\
&= -\frac{c\sqrt{a + bx^3}}{2ax^2} - \frac{d\sqrt{a + bx^3}}{ax} + \frac{(b^{2/3}d) \int \frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt{a + bx^3}} dx}{2a} - \frac{(b^{2/3}(\sqrt[3]{bc} + 2(1-\sqrt{3})\sqrt[3]{ad})) \int \frac{1}{\sqrt{a + bx^3}} dx}{4a} \\
&= -\frac{c\sqrt{a + bx^3}}{2ax^2} - \frac{d\sqrt{a + bx^3}}{ax} + \frac{\sqrt[3]{bd}\sqrt{a + bx^3}}{a((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})} - \frac{\sqrt[4]{3}\sqrt{2 - \sqrt{3}}\sqrt[3]{bd}(\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx}}{((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}}}{2a^{2/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}}} \\
&= -\frac{c\sqrt{a + bx^3}}{2ax^2} - \frac{d\sqrt{a + bx^3}}{ax} + \frac{\sqrt[3]{bd}\sqrt{a + bx^3}}{a((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})} - \frac{2e \tanh^{-1}\left(\frac{\sqrt{a + bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}} - \frac{\sqrt[4]{3}\sqrt{2 - \sqrt{3}}\sqrt[3]{bd}(\sqrt[3]{a} + \sqrt[3]{bx})}{3\sqrt{a}}
\end{aligned}$$

Mathematica [C] time = 0.193753, size = 131, normalized size = 0.23

$$-\frac{c\sqrt{\frac{bx^3}{a}} + {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}, \frac{1}{3}, -\frac{bx^3}{a}\right)}{2x^2\sqrt{a + bx^3}} - \frac{d\sqrt{\frac{bx^3}{a}} + {}_2F_1\left(-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, -\frac{bx^3}{a}\right)}{x\sqrt{a + bx^3}} - \frac{2e \tanh^{-1}\left(\frac{\sqrt{a + bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(x^3*Sqrt[a + b*x^3]), x]

[Out] (-2*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]/(3*Sqrt[a]) - (c*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-2/3, 1/2, 1/3, -((b*x^3)/a)]/(2*x^2*Sqrt[a + b*x^3]) - (d*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/3, 1/2, 2/3, -((b*x^3)/a)]/(x*Sqrt[a + b*x^3]))

Maple [A] time = 0.009, size = 778, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d*x+c)/x^3/(b*x^3+a)^(1/2),x)`

[Out]
$$-2/3*e*\operatorname{arctanh}\left(\frac{(b*x^3+a)^{1/2}}{a^{1/2}}\right)/a^{1/2}+c*\left(-1/2/a*(b*x^3+a)^{1/2}/x^{2+1/6}*I/a*3^{1/2}*(-b^2*a)^{1/3}*(I*(x+1/2/b*(-b^2*a)^{1/3})-1/2*I*3^{1/2}/b*(-b^2*a)^{1/3})\right)*3^{1/2}*b/(-b^2*a)^{1/3})^{1/2}*((x-1/b*(-b^2*a)^{1/3})/(-3/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2}/b*(-b^2*a)^{1/3}))^{1/2}*(-I*(x+1/2/b*(-b^2*a)^{1/3})+1/2*I*3^{1/2}/b*(-b^2*a)^{1/3})\right)*3^{1/2}*b/(-b^2*a)^{1/3})^{1/2}/(b*x^3+a)^{1/2}*EllipticF\left(1/3*3^{1/2}*(I*(x+1/2/b*(-b^2*a)^{1/3})-1/2*I*3^{1/2}/b*(-b^2*a)^{1/3})\right)*3^{1/2}*b/(-b^2*a)^{1/3})^{1/2},(I*3^{1/2}/b*(-b^2*a)^{1/3})/(-3/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2}/b*(-b^2*a)^{1/3})\right)^{1/2})+d*(-1/a*(b*x^3+a)^{1/2}/x-1/3*I/a*3^{1/2}*(-b^2*a)^{1/3}*(I*(x+1/2/b*(-b^2*a)^{1/3})-1/2*I*3^{1/2}/b*(-b^2*a)^{1/3})\right)*3^{1/2}*b/(-b^2*a)^{1/3})^{1/2}*((x-1/b*(-b^2*a)^{1/3})/(-3/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2}/b*(-b^2*a)^{1/3}))^{1/2}*(-I*(x+1/2/b*(-b^2*a)^{1/3})+1/2*I*3^{1/2}/b*(-b^2*a)^{1/3})\right)*3^{1/2}*b/(-b^2*a)^{1/3})^{1/2}/(b*x^3+a)^{1/2}*((-3/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2}/b*(-b^2*a)^{1/3})\right)*EllipticE\left(1/3*3^{1/2}*(I*(x+1/2/b*(-b^2*a)^{1/3})-1/2*I*3^{1/2}/b*(-b^2*a)^{1/3})\right)*3^{1/2}*b/(-b^2*a)^{1/3})^{1/2},(I*3^{1/2}/b*(-b^2*a)^{1/3})/(-3/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2}/b*(-b^2*a)^{1/3})\right)^{1/2})+1/b*(-b^2*a)^{1/3}*EllipticF\left(1/3*3^{1/2}*(I*(x+1/2/b*(-b^2*a)^{1/3})-1/2*I*3^{1/2}/b*(-b^2*a)^{1/3})\right)*3^{1/2}*b/(-b^2*a)^{1/3})^{1/2},(I*3^{1/2}/b*(-b^2*a)^{1/3})/(-3/2/b*(-b^2*a)^{1/3}+1/2*I*3^{1/2}/b*(-b^2*a)^{1/3})\right)^{1/2})\right)$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{\sqrt{bx^3 + ax^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^(1/2),x, algorithm="maxima")`

[Out] `integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x^3), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + a}(ex^2 + dx + c)}{bx^6 + ax^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^3 + a)*(e*x^2 + d*x + c)/(b*x^6 + a*x^3), x)

Sympy [A] time = 2.96634, size = 112, normalized size = 0.2

$$\frac{c\Gamma\left(-\frac{2}{3}\right) {}_2F_1\left(-\frac{2}{3}, \frac{1}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{ax^2}\Gamma\left(\frac{1}{3}\right)} + \frac{d\Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(-\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{ax}\Gamma\left(\frac{2}{3}\right)} - \frac{2e \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx^{\frac{3}{2}}}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**(1/2),x)

[Out] c*gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*x**2*gamma(1/3)) + d*gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*x*gamma(2/3)) - 2*e*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{\sqrt{bx^3 + ax^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^(1/2),x, algorithm="giac")

[Out] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x^3), x)

$$3.437 \quad \int \frac{x^5(c+dx+ex^2)}{(a+bx^3)^{3/2}} dx$$

Optimal. Leaf size=594

$$16\sqrt{2+\sqrt{3}}a(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(14\sqrt[3]{bd}-25(1-\sqrt{3})\sqrt[3]{ae})\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)$$

$$105\sqrt[4]{3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}$$

[Out] (2*x*(a*d + a*e*x - b*c*x^2))/(3*b^2*Sqrt[a + b*x^3]) + (4*c*Sqrt[a + b*x^3])/((3*b^2) + (2*d*x*Sqrt[a + b*x^3])/(5*b^2) + (2*e*x^2*Sqrt[a + b*x^3])/(7*b^2) - (80*a*e*Sqrt[a + b*x^3])/(21*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (40*Sqrt[2 - Sqrt[3])*a^(4/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(3/4)*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (16*Sqrt[2 + Sqrt[3])*a*(14*b^(1/3)*d - 25*(1 - Sqrt[3])*a^(1/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(105*3^(1/4)*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 0.640867, antiderivative size = 594, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {1828, 1888, 1886, 261, 1878, 218, 1877}

$$16\sqrt{2+\sqrt{3}}a(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(14\sqrt[3]{bd}-25(1-\sqrt{3})\sqrt[3]{ae})F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right),-7-4\sqrt{3}\right) + 40\sqrt{2+\sqrt{3}}a(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(14\sqrt[3]{bd}-25(1-\sqrt{3})\sqrt[3]{ae})$$

$$105\sqrt[4]{3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(c + d*x + e*x^2))/(a + b*x^3)^(3/2), x]


```
[Out] (2*x*(a*d + a*e*x - b*c*x^2))/(3*b^2*Sqrt[a + b*x^3]) + (4*c*Sqrt[a + b*x^3])/(3*b^2) + (2*d*x*Sqrt[a + b*x^3])/(5*b^2) + (2*e*x^2*Sqrt[a + b*x^3])/(7*b^2) - (80*a*e*Sqrt[a + b*x^3])/(21*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (40*Sqrt[2 - Sqrt[3]]*a^(4/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2 *EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(3/4)*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (16*Sqrt[2 + Sqrt[3]]*a*(14*b^(1/3)*d - 25*(1 - Sqrt[3])*a^(1/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2 *EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(105*3^(1/4)*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*n)]) /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[Coeff[Pq, x, n - 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(a + b*x^n)
```

$^{(p+1)}/(b*n*(p+1)), x] /; \text{FreeQ}\{a, b, m, n, p\}, x\} \&\& \text{EqQ}[m, n-1] \&\& \text{NeQ}[p, -1]$

Rule 1878

$\text{Int}[\frac{(c_.) + (d_.)(x_.)}{\text{Sqrt}[a_ + (b_.)(x_)^3]}, x_Symbol] := \text{With}\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Dist}[(c*r - (1 - \text{Sqrt}[3])*d*s)/r, \text{Int}[1/\text{Sqrt}[a + b*x^3], x], x] + \text{Dist}[d/r, \text{Int}[\frac{(1 - \text{Sqrt}[3])*s + r*x}{\text{Sqrt}[a + b*x^3]}, x], x] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{PosQ}[a] \&\& \text{NeQ}[b*c^3 - 2*(5 - 3*\text{Sqrt}[3])*a*d^3, 0]$

Rule 218

$\text{Int}[1/\text{Sqrt}[(a_ + (b_.)(x_)^3)], x_Symbol] := \text{With}\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Simp}[(2*\text{Sqrt}[2 + \text{Sqrt}[3]]*(s + r*x)*\text{Sqrt}[(s^2 - r*s*x + r^2*x^2)/((1 + \text{Sqrt}[3])*s + r*x)^2]*\text{EllipticF}[\text{ArcSin}[\frac{(1 - \text{Sqrt}[3])*s + r*x}{(1 + \text{Sqrt}[3])*s + r*x}], -7 - 4*\text{Sqrt}[3]])/(3^{1/4}*r*\text{Sqrt}[a + b*x^3]*\text{Sqrt}[(s*(s + r*x))/((1 + \text{Sqrt}[3])*s + r*x)^2]), x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{PosQ}[a]$

Rule 1877

$\text{Int}[\frac{(c_.) + (d_.)(x_.)}{\text{Sqrt}[a_ + (b_.)(x_)^3]}, x_Symbol] := \text{With}\{r = \text{Numer}[\text{Simplify}[\frac{(1 - \text{Sqrt}[3])*d}{c}], s = \text{Denom}[\text{Simplify}[\frac{(1 - \text{Sqrt}[3])*d}{c}]]\}, \text{Simp}[(2*d*s^3*\text{Sqrt}[a + b*x^3])/(a*r^2*((1 + \text{Sqrt}[3])*s + r*x)), x] - \text{Simp}[(3^{1/4}*\text{Sqrt}[2 - \text{Sqrt}[3]]*d*s*(s + r*x)*\text{Sqrt}[(s^2 - r*s*x + r^2*x^2)/((1 + \text{Sqrt}[3])*s + r*x)^2]*\text{EllipticE}[\text{ArcSin}[\frac{(1 - \text{Sqrt}[3])*s + r*x}{(1 + \text{Sqrt}[3])*s + r*x}], -7 - 4*\text{Sqrt}[3]])/(r^2*\text{Sqrt}[a + b*x^3]*\text{Sqrt}[(s*(s + r*x))/((1 + \text{Sqrt}[3])*s + r*x)^2]), x] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{PosQ}[a] \&\& \text{EqQ}[b*c^3 - 2*(5 - 3*\text{Sqrt}[3])*a*d^3, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{x^5(c + dx + ex^2)}{(a + bx^3)^{3/2}} dx &= \frac{2x(ad + aex - bcx^2)}{3b^2\sqrt{a + bx^3}} - \frac{2 \int \frac{a^2bd + 2a^2bex - 3ab^2cx^2 - \frac{3}{2}ab^2dx^3 - \frac{3}{2}ab^2ex^4}{\sqrt{a + bx^3}} dx}{3ab^3} \\
&= \frac{2x(ad + aex - bcx^2)}{3b^2\sqrt{a + bx^3}} + \frac{2ex^2\sqrt{a + bx^3}}{7b^2} - \frac{4 \int \frac{\frac{7}{2}a^2b^2d + 10a^2b^2ex - \frac{21}{2}ab^3cx^2 - \frac{21}{4}ab^3dx^3}{\sqrt{a + bx^3}} dx}{21ab^4} \\
&= \frac{2x(ad + aex - bcx^2)}{3b^2\sqrt{a + bx^3}} + \frac{2dx\sqrt{a + bx^3}}{5b^2} + \frac{2ex^2\sqrt{a + bx^3}}{7b^2} - \frac{8 \int \frac{14a^2b^3d + 25a^2b^3ex - \frac{105}{4}ab^4cx^2}{\sqrt{a + bx^3}} dx}{105ab^5} \\
&= \frac{2x(ad + aex - bcx^2)}{3b^2\sqrt{a + bx^3}} + \frac{2dx\sqrt{a + bx^3}}{5b^2} + \frac{2ex^2\sqrt{a + bx^3}}{7b^2} - \frac{8 \int \frac{14a^2b^3d + 25a^2b^3ex}{\sqrt{a + bx^3}} dx}{105ab^5} + \frac{(2c) \int \frac{x^2}{\sqrt{a + bx^3}} dx}{b} \\
&= \frac{2x(ad + aex - bcx^2)}{3b^2\sqrt{a + bx^3}} + \frac{4c\sqrt{a + bx^3}}{3b^2} + \frac{2dx\sqrt{a + bx^3}}{5b^2} + \frac{2ex^2\sqrt{a + bx^3}}{7b^2} - \frac{(40ae) \int \frac{(1-\sqrt{3})\sqrt[3]{a + bx^3}}{\sqrt{a + bx^3}} dx}{21b^{7/3}} \\
&= \frac{2x(ad + aex - bcx^2)}{3b^2\sqrt{a + bx^3}} + \frac{4c\sqrt{a + bx^3}}{3b^2} + \frac{2dx\sqrt{a + bx^3}}{5b^2} + \frac{2ex^2\sqrt{a + bx^3}}{7b^2} - \frac{80ae\sqrt{a + bx^3}}{21b^{8/3}((1 + \sqrt{3})\sqrt[3]{a})}
\end{aligned}$$

Mathematica [C] time = 0.111454, size = 134, normalized size = 0.23

$$\frac{2 \left(-56adx \sqrt{\frac{bx^3}{a}} + 1 {}_2F_1 \left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a} \right) + 150aex^2 \sqrt{\frac{bx^3}{a}} + 1 {}_2F_1 \left(\frac{2}{3}, \frac{3}{2}; \frac{5}{3}; -\frac{bx^3}{a} \right) + 70ac + 56adx - 150aex^2 + 35bcx^3 + 21b^2 \sqrt{a + bx^3} \right)}{105b^2 \sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(c + d*x + e*x^2))/(a + b*x^3)^(3/2), x]

[Out] (2*(70*a*c + 56*a*d*x - 150*a*e*x^2 + 35*b*c*x^3 + 21*b*d*x^4 + 15*b*e*x^5 - 56*a*d*x*sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]) + 150*a*e*x^2*sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)])/(105*b^2*sqrt[a + b*x^3])

Maple [A] time = 0.02, size = 836, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^5*(e*x^2+d*x+c)/(b*x^3+a)^{(3/2)}, x)$

[Out]
$$e*(2/3/b^2*x^2*a/((x^3+1/b*a)*b)^{(1/2)}+2/7/b^2*x^2*(b*x^3+a)^{(1/2)}+80/63*I/b^3*a*3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*EllipticE(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}))+1/b*(-b^2*a)^{(1/3)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)})))+d*(2/3/b^2*x*a/((x^3+1/b*a)*b)^{(1/2)}+2/5/b^2*x*(b*x^3+a)^{(1/2)}+32/45*I/b^3*a*3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/2)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}))+c*(2/3/b^2*a/((x^3+1/b*a)*b)^{(1/2)}+2/3/b^2*(b*x^3+a)^{(1/2)})$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2}{3}c\left(\frac{\sqrt{bx^3+a}}{b^2} + \frac{a}{\sqrt{bx^3+ab^2}}\right) + \int \frac{(ex^7+dx^6)\sqrt{bx^3+a}}{b^2x^6+2abx^3+a^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^5*(e*x^2+d*x+c)/(b*x^3+a)^{(3/2)}, x, \text{algorithm}="maxima")$

[Out] $2/3*c*(\text{sqrt}(b*x^3+a)/b^2+a/(\text{sqrt}(b*x^3+a)*b^2))+\text{integrate}((e*x^7+d*x^6)*\text{sqrt}(b*x^3+a)/(b^2*x^6+2*a*b*x^3+a^2), x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^7 + dx^6 + cx^5)\sqrt{bx^3 + a}}{b^2x^6 + 2abx^3 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] integral((e*x^7 + d*x^6 + c*x^5)*sqrt(b*x^3 + a)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)

Sympy [A] time = 27.0308, size = 129, normalized size = 0.22

$$c \left(\begin{cases} \frac{4a}{3b^2\sqrt{a+bx^3}} + \frac{2x^3}{3b\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise} \end{cases} \right) + \frac{dx^7\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\frac{7}{3}, \frac{7}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)} + \frac{ex^8\Gamma\left(\frac{8}{3}\right) {}_2F_1\left(\frac{8}{3}, \frac{8}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)

[Out] c*Piecewise((4*a/(3*b**2*sqrt(a + b*x**3)) + 2*x**3/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**6/(6*a**(3/2)), True)) + d*x**7*gamma(7/3)*hyper((3/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(10/3)) + e*x**8*gamma(8/3)*hyper((3/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(11/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + dx + c)x^5}{(bx^3 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)*x^5/(b*x^3 + a)^(3/2), x)
```

$$3.438 \quad \int \frac{x^4(c+dx+ex^2)}{(a+bx^3)^{3/2}} dx$$

Optimal. Leaf size=574

$$8\sqrt{2+\sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(4a^{2/3}e+5(1-\sqrt{3})b^{2/3}c)\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)$$

$$15\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}$$

```
[Out] (2*x*(a*e - b*c*x - b*d*x^2))/(3*b^2*Sqrt[a + b*x^3]) + (4*d*Sqrt[a + b*x^3
])/ (3*b^2) + (2*e*x*Sqrt[a + b*x^3])/ (5*b^2) + (8*c*Sqrt[a + b*x^3])/ (3*b^(
5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (4*Sqrt[2 - Sqrt[3]]*a^(1/3)*c
(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3)
+ b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4
)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*Sqrt[a + b*x^3]) - (8*Sqrt[2 + Sqrt[3]]*a^(1/3)*(5*(1 - Sqrt[3])
*b^(2/3)*c + 4*a^(2/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcS
in[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
, -7 - 4*Sqrt[3]])/(15*3^(1/4)*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x)
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.46911, antiderivative size = 574, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {1828, 1888, 1886, 261, 1878, 218, 1877}

$$8\sqrt{2+\sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(4a^{2/3}e+5(1-\sqrt{3})b^{2/3}c)F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right)\middle| -7-4\sqrt{3}\right) 4\sqrt{2}$$

$$15\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(c + d*x + e*x^2))/(a + b*x^3)^(3/2), x]

```
[Out] (2*x*(a*e - b*c*x - b*d*x^2))/(3*b^2*Sqrt[a + b*x^3]) + (4*d*Sqrt[a + b*x^3
])/ (3*b^2) + (2*e*x*Sqrt[a + b*x^3])/ (5*b^2) + (8*c*Sqrt[a + b*x^3])/ (3*b^(
```

$$\begin{aligned} & 5/3 * ((1 + \sqrt{3}) * a^{1/3} + b^{1/3} * x) - (4 * \sqrt{2 - \sqrt{3}} * a^{1/3} * c * \\ & (a^{1/3} + b^{1/3} * x) * \sqrt{(a^{2/3} - a^{1/3} * b^{1/3} * x + b^{2/3} * x^2)} / ((1 + \sqrt{3}) * a^{1/3} + b^{1/3} * x)^2 * \text{EllipticE}[\text{ArcSin}[\frac{(1 - \sqrt{3}) * a^{1/3} + b^{1/3} * x}{(1 + \sqrt{3}) * a^{1/3} + b^{1/3} * x}], -7 - 4 * \sqrt{3}]) / (3^{3/4} * b^{5/3} * \sqrt{(a^{1/3} * (a^{1/3} + b^{1/3} * x)) / ((1 + \sqrt{3}) * a^{1/3} + b^{1/3} * x)^2} * \sqrt{a + b * x^3}) - (8 * \sqrt{2 + \sqrt{3}} * a^{1/3} * (5 * (1 - \sqrt{3}) * b^{2/3} * c + 4 * a^{2/3} * e) * (a^{1/3} + b^{1/3} * x) * \sqrt{(a^{2/3} - a^{1/3} * b^{1/3} * x + b^{2/3} * x^2)} / ((1 + \sqrt{3}) * a^{1/3} + b^{1/3} * x)^2 * \text{EllipticF}[\text{ArcSin}[\frac{(1 - \sqrt{3}) * a^{1/3} + b^{1/3} * x}{(1 + \sqrt{3}) * a^{1/3} + b^{1/3} * x}], -7 - 4 * \sqrt{3}]) / (15 * 3^{1/4} * b^{7/3} * \sqrt{(a^{1/3} * (a^{1/3} + b^{1/3} * x)) / ((1 + \sqrt{3}) * a^{1/3} + b^{1/3} * x)^2} * \sqrt{a + b * x^3}) \end{aligned}$$

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*n)]) /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[Coeff[Pq, x, n - 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```


Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)]/((1 + Sqrt[3])*s + r*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)]/((1 + Sqrt[3])*s + r*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4(c + dx + ex^2)}{(a + bx^3)^{3/2}} dx &= \frac{2x(ae - bcx - bdx^2)}{3b^2\sqrt{a + bx^3}} - \frac{2 \int \frac{a^2e - 2abcx - 3abdx^2 - \frac{3}{2}abex^3}{\sqrt{a+bx^3}} dx}{3ab^2} \\
&= \frac{2x(ae - bcx - bdx^2)}{3b^2\sqrt{a + bx^3}} + \frac{2ex\sqrt{a + bx^3}}{5b^2} - \frac{4 \int \frac{4a^2be - 5ab^2cx - \frac{15}{2}ab^2dx^2}{\sqrt{a+bx^3}} dx}{15ab^3} \\
&= \frac{2x(ae - bcx - bdx^2)}{3b^2\sqrt{a + bx^3}} + \frac{2ex\sqrt{a + bx^3}}{5b^2} - \frac{4 \int \frac{4a^2be - 5ab^2cx}{\sqrt{a+bx^3}} dx}{15ab^3} + \frac{(2d) \int \frac{x^2}{\sqrt{a+bx^3}} dx}{b} \\
&= \frac{2x(ae - bcx - bdx^2)}{3b^2\sqrt{a + bx^3}} + \frac{4d\sqrt{a + bx^3}}{3b^2} + \frac{2ex\sqrt{a + bx^3}}{5b^2} + \frac{(4c) \int \frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt{a+bx^3}} dx}{3b^{4/3}} - \frac{(4\sqrt[3]{a}(5(1 - \sqrt{3})))}{3b^{4/3}} \\
&= \frac{2x(ae - bcx - bdx^2)}{3b^2\sqrt{a + bx^3}} + \frac{4d\sqrt{a + bx^3}}{3b^2} + \frac{2ex\sqrt{a + bx^3}}{5b^2} + \frac{8c\sqrt{a + bx^3}}{3b^{5/3}((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})} - \frac{4\sqrt{2 - \sqrt{3}}}{3b^{4/3}}
\end{aligned}$$

Mathematica [C] time = 0.102964, size = 127, normalized size = 0.22

$$\frac{2\left(-15bcx^2\sqrt{\frac{bx^3}{a}} + {}_2F_1\left(\frac{2}{3}, \frac{3}{2}; \frac{5}{3}; -\frac{bx^3}{a}\right) - 8aex\sqrt{\frac{bx^3}{a}} + {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a}\right) + 10ad + 8aex + 15bcx^2 + 5bdx^3 + 3bex^4\right)}{15b^2\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(c + d*x + e*x^2))/(a + b*x^3)^(3/2), x]

[Out] (2*(10*a*d + 8*a*e*x + 15*b*c*x^2 + 5*b*d*x^3 + 3*b*e*x^4 - 8*a*e*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] - 15*b*c*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)]))/(15*b^2*Sqrt[a + b*x^3])

Maple [A] time = 0.009, size = 817, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^4*(e*x^2+d*x+c)/(b*x^3+a)^{(3/2)}, x)$

[Out] $e*(2/3/b^2*x*a/((x^3+1/b*a)*b)^{(1/2)}+2/5/b^2*x*(b*x^3+a)^{(1/2)}+32/45*I/b^3*a*3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))*3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))*3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/2)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))*3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}))+d*(2/3/b^2*a/((x^3+1/b*a)*b)^{(1/2)}+2/3/b^2*(b*x^3+a)^{(1/2)}+c*(-2/3/b*x^2/((x^3+1/b*a)*b)^{(1/2)}-8/9*I/b^2*3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))*3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))*3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))*3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}))+1/b*(-b^2*a)^{(1/3)}*EllipticE(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))*3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}))+1/b*(-b^2*a)^{(1/3)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))*3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + dx + c)x^4}{(bx^3 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^4*(e*x^2+d*x+c)/(b*x^3+a)^{(3/2)}, x, \text{algorithm}=\text{"maxima"})$

[Out] $\text{integrate}((e*x^2 + d*x + c)*x^4/(b*x^3 + a)^{(3/2)}, x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^6 + dx^5 + cx^4)\sqrt{bx^3 + a}}{b^2x^6 + 2abx^3 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] integral((e*x^6 + d*x^5 + c*x^4)*sqrt(b*x^3 + a)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)

Sympy [A] time = 18.1381, size = 129, normalized size = 0.22

$$d \left(\begin{cases} \frac{4a}{3b^2\sqrt{a+bx^3}} + \frac{2x^3}{3b\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise} \end{cases} \right) + \frac{cx^5\Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\frac{3}{2}, \frac{5}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)} + \frac{ex^7\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\frac{3}{2}, \frac{7}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)

[Out] d*Piecewise((4*a/(3*b**2*sqrt(a + b*x**3)) + 2*x**3/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**6/(6*a**(3/2)), True)) + c*x**5*gamma(5/3)*hyper((3/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(8/3)) + e*x**7*gamma(7/3)*hyper((3/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(10/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + dx + c)x^4}{(bx^3 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")

[Out] integrate((e*x^2 + d*x + c)*x^4/(b*x^3 + a)^(3/2), x)

$$3.439 \quad \int \frac{x^3(c+dx+ex^2)}{(a+bx^3)^{3/2}} dx$$

Optimal. Leaf size=542

$$4\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(\sqrt[3]{bc}-2(1-\sqrt{3})\sqrt[3]{ad})\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)4\sqrt{2-\sqrt{3}}$$

$$3\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}$$

```
[Out] (-2*x*(c + d*x + e*x^2))/(3*b*Sqrt[a + b*x^3]) + (4*e*Sqrt[a + b*x^3])/(3*b^2) + (8*d*Sqrt[a + b*x^3])/(3*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (4*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (4*Sqrt[2 + Sqrt[3]]*(b^(1/3)*c - 2*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.327242, antiderivative size = 542, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {1828, 1886, 261, 1878, 218, 1877}

$$4\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(\sqrt[3]{bc}-2(1-\sqrt{3})\sqrt[3]{ad})F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)4\sqrt{2-\sqrt{3}}$$

$$3\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(c + d*x + e*x^2))/(a + b*x^3)^(3/2), x]

```
[Out] (-2*x*(c + d*x + e*x^2))/(3*b*Sqrt[a + b*x^3]) + (4*e*Sqrt[a + b*x^3])/(3*b^2) + (8*d*Sqrt[a + b*x^3])/(3*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x))
```

$$\begin{aligned}
& - (4\sqrt{2 - \sqrt{3}})a^{1/3}d(a^{1/3} + b^{1/3}x)\sqrt{(a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2)/((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2} \text{EllipticE} \\
& \left[\text{ArcSin}\left[\frac{(1 - \sqrt{3})a^{1/3} + b^{1/3}x}{(1 + \sqrt{3})a^{1/3} + b^{1/3}x}\right], -7 - 4\sqrt{3} \right] / (3^{3/4}b^{5/3}\sqrt{(a^{1/3}(a^{1/3} + b^{1/3}x)))} \\
& \left((1 + \sqrt{3})a^{1/3} + b^{1/3}x \right)^2 \sqrt{a + b^2x^3} + (4\sqrt{2 + \sqrt{3}}) \\
& (b^{1/3}c - 2(1 - \sqrt{3})a^{1/3}d)(a^{1/3} + b^{1/3}x)\sqrt{(a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2)/((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2} \\
& \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1 - \sqrt{3})a^{1/3} + b^{1/3}x}{(1 + \sqrt{3})a^{1/3} + b^{1/3}x}\right], -7 - 4\sqrt{3} \right] / (3^{3/4}b^{5/3}\sqrt{(a^{1/3}(a^{1/3} + b^{1/3}x)))} \\
& \left((1 + \sqrt{3})a^{1/3} + b^{1/3}x \right)^2 \sqrt{a + b^2x^3}
\end{aligned}$$

Rule 1828

```

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q =
m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
+ b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
LtQ[p, -1] && IGtQ[m, 0]

```

Rule 1886

```

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[Coeff[Pq, x, n -
1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1

```

Rule 261

```

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
NeQ[p, -1]

```

Rule 1878

```

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]

```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]]/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]]]/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(c + dx + ex^2)}{(a + bx^3)^{3/2}} dx &= -\frac{2x(c + dx + ex^2)}{3b\sqrt{a + bx^3}} - \frac{2 \int \frac{-abc - 2abdx - 3abex^2}{\sqrt{a + bx^3}} dx}{3ab^2} \\
&= -\frac{2x(c + dx + ex^2)}{3b\sqrt{a + bx^3}} - \frac{2 \int \frac{-abc - 2abdx}{\sqrt{a + bx^3}} dx}{3ab^2} + \frac{(2e) \int \frac{x^2}{\sqrt{a + bx^3}} dx}{b} \\
&= -\frac{2x(c + dx + ex^2)}{3b\sqrt{a + bx^3}} + \frac{4e\sqrt{a + bx^3}}{3b^2} + \frac{(4d) \int \frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt{a + bx^3}} dx}{3b^{4/3}} + \frac{(2(\sqrt[3]{bc} - 2(1 - \sqrt{3})\sqrt[3]{ad})))}{3b^{4/3}} \\
&= -\frac{2x(c + dx + ex^2)}{3b\sqrt{a + bx^3}} + \frac{4e\sqrt{a + bx^3}}{3b^2} + \frac{8d\sqrt{a + bx^3}}{3b^{5/3}((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})} - \frac{4\sqrt{2 - \sqrt{3}}\sqrt[3]{ad}(\sqrt[3]{a} + \sqrt[3]{bx})}{3b^{4/3}}
\end{aligned}$$

Mathematica [C] time = 0.0950857, size = 118, normalized size = 0.22

$$\frac{2\left(bc x \sqrt{\frac{bx^3}{a}} + {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a}\right) - 3bdx^2 \sqrt{\frac{bx^3}{a}} + {}_2F_1\left(\frac{2}{3}, \frac{3}{2}; \frac{5}{3}; -\frac{bx^3}{a}\right) + 2ae - bcx + 3bdx^2 + bex^3\right)}{3b^2\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(2*a*e - b*c*x + 3*b*d*x^2 + b*e*x^3 + b*c*x*sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] - 3*b*d*x^2*sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)])/(3*b^2*sqrt[a + b*x^3])
```

Maple [A] time = 0.007, size = 800, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x)
```

```
[Out] e*(2/3/b^2*a/((x^3+1/b*a)*b)^(1/2)+2/3/b^2*(b*x^3+a)^(1/2))+d*(-2/3/b*x^2/((x^3+1/b*a)*b)^(1/2)-8/9*I/b^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+c*(-2/3/b*x/((x^3+1/b*a)*b)^(1/2)-4/9*I/b^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))
```


Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + dx + c)x^3}{(bx^3 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")

[Out] integrate((e*x^2 + d*x + c)*x^3/(b*x^3 + a)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^5 + dx^4 + cx^3)\sqrt{bx^3 + a}}{b^2x^6 + 2abx^3 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] integral((e*x^5 + d*x^4 + c*x^3)*sqrt(b*x^3 + a)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)

Sympy [A] time = 13.5041, size = 129, normalized size = 0.24

$$e^{\left(\begin{cases} \frac{4a}{3b^2\sqrt{a+bx^3}} + \frac{2x^3}{3b\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right)} + \frac{cx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{4}{3}, \frac{3}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{dx^5\Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\frac{5}{3}, \frac{5}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)

[Out] e*Piecewise((4*a/(3*b**2*sqrt(a + b*x**3)) + 2*x**3/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**6/(6*a**(3/2)), True)) + c*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(7/3)) + d*x**5*gamma(

$5/3) * \text{hyper}((3/2, 5/3), (8/3,), b*x**3*\exp_polar(I*pi)/a)/(3*a**(3/2)*\text{gamma}(8/3))$

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + dx + c)x^3}{(bx^3 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")

[Out] integrate((e*x^2 + d*x + c)*x^3/(b*x^3 + a)^(3/2), x)

$$3.440 \quad \int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^{3/2}} dx$$

Optimal. Leaf size=522

$$4\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(\sqrt[3]{bd}-2(1-\sqrt{3})\sqrt[3]{ae})\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right) - 4\sqrt{2-\sqrt{3}}$$

$$3^4\sqrt{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}$$

[Out] $(-2*(c + d*x + e*x^2))/(3*b*\text{Sqrt}[a + b*x^3]) + (8*e*\text{Sqrt}[a + b*x^3])/(3*b^{5/3}*((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)) - (4*\text{Sqrt}[2 - \text{Sqrt}[3])*a^{1/3}*e*(a^{1/3} + b^{1/3}*x)*\text{Sqrt}[(a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2)/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\text{EllipticE}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x]/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)], -7 - 4*\text{Sqrt}[3])/(3^{3/4}*b^{5/3}*\text{Sqrt}[(a^{1/3}*(a^{1/3} + b^{1/3}*x))/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\text{Sqrt}[a + b*x^3]) + (4*\text{Sqrt}[2 + \text{Sqrt}[3]]*(b^{1/3}*d - 2*(1 - \text{Sqrt}[3])*a^{1/3}*e)*(a^{1/3} + b^{1/3}*x)*\text{Sqrt}[(a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2)/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\text{EllipticF}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x]/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)], -7 - 4*\text{Sqrt}[3])/(3*3^{1/4}*b^{5/3}*\text{Sqrt}[(a^{1/3}*(a^{1/3} + b^{1/3}*x))/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\text{Sqrt}[a + b*x^3])$

Rubi [A] time = 0.26164, antiderivative size = 522, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {1823, 1878, 218, 1877}

$$4\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(\sqrt[3]{bd}-2(1-\sqrt{3})\sqrt[3]{ae})F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right) - 4\sqrt{2-\sqrt{3}}$$

$$3^4\sqrt{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^{(3/2)}, x]$

[Out] $(-2*(c + d*x + e*x^2))/(3*b*\text{Sqrt}[a + b*x^3]) + (8*e*\text{Sqrt}[a + b*x^3])/(3*b^{5/3}*((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)) - (4*\text{Sqrt}[2 - \text{Sqrt}[3])*a^{1/3}*e*$

$$\begin{aligned} & (a^{1/3} + b^{1/3}x) \sqrt{(a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2) / ((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2} \operatorname{EllipticE}[\operatorname{ArcSin}[\frac{(1 - \sqrt{3})a^{1/3} + b^{1/3}x}{(1 + \sqrt{3})a^{1/3} + b^{1/3}x}], -7 - 4\sqrt{3}] / (3^{3/4}) \\ & * b^{5/3} \sqrt{(a^{1/3}(a^{1/3} + b^{1/3}x)) / ((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2} \sqrt{a + b^2x^3} + (4\sqrt{2 + \sqrt{3}})(b^{1/3}d - 2(1 - \sqrt{3})a^{1/3}e) \\ & * (a^{1/3} + b^{1/3}x) \sqrt{(a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2) / ((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2} \operatorname{EllipticF}[\operatorname{ArcSin}[\frac{(1 - \sqrt{3})a^{1/3} + b^{1/3}x}{(1 + \sqrt{3})a^{1/3} + b^{1/3}x}], -7 - 4\sqrt{3}] \\ & / (3^{3/4} * b^{5/3} \sqrt{(a^{1/3}(a^{1/3} + b^{1/3}x)) / ((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2} \sqrt{a + b^2x^3}) \end{aligned}$$
Rule 1823

$$\operatorname{Int}[(Pq_*)(x_)^{(m_*)}((a_) + (b_*)(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \operatorname{Simp}[(Pq*(a + b*x^n)^{(p+1}) / (b*n*(p+1)), x] - \operatorname{Dist}[1/(b*n*(p+1)), \operatorname{Int}[D[Pq, x]*(a + b*x^n)^{(p+1)}, x], x] /; \operatorname{FreeQ}\{a, b, m, n, x\} \&\& \operatorname{PolyQ}[Pq, x] \&\& \operatorname{EqQ}[m - n + 1, 0] \&\& \operatorname{LtQ}[p, -1]$$
Rule 1878

$$\operatorname{Int}[(c_*) + (d_*)(x_*) / \sqrt{(a_*) + (b_*)(x_*)^3}, x_Symbol] \rightarrow \operatorname{With}\{r = \operatorname{Numerator}[Rt[b/a, 3]], s = \operatorname{Denom}[Rt[b/a, 3]]\}, \operatorname{Dist}[(c*r - (1 - \sqrt{3})d*s)/r, \operatorname{Int}[1/\sqrt{a + b*x^3}, x], x] + \operatorname{Dist}[d/r, \operatorname{Int}[(1 - \sqrt{3})s + r*x / \sqrt{a + b*x^3}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, x\} \&\& \operatorname{PosQ}[a] \&\& \operatorname{NeQ}[b*c^3 - 2*(5 - 3*\sqrt{3})a*d^3, 0]$$
Rule 218

$$\operatorname{Int}[1/\sqrt{(a_*) + (b_*)(x_*)^3}, x_Symbol] \rightarrow \operatorname{With}\{r = \operatorname{Numerator}[Rt[b/a, 3]], s = \operatorname{Denom}[Rt[b/a, 3]]\}, \operatorname{Simp}[(2*\sqrt{2 + \sqrt{3}})(s + r*x) \sqrt{(s^2 - r*s*x + r^2*x^2) / ((1 + \sqrt{3})s + r*x)^2} \operatorname{EllipticF}[\operatorname{ArcSin}[\frac{(1 - \sqrt{3})s + r*x}{(1 + \sqrt{3})s + r*x}], -7 - 4\sqrt{3}] / (3^{1/4} * r \sqrt{a + b*x^3}) * \sqrt{(s*(s + r*x)) / ((1 + \sqrt{3})s + r*x)^2}), x] /; \operatorname{FreeQ}\{a, b, x\} \&\& \operatorname{PosQ}[a]$$
Rule 1877

$$\operatorname{Int}[(c_*) + (d_*)(x_*) / \sqrt{(a_*) + (b_*)(x_*)^3}, x_Symbol] \rightarrow \operatorname{With}\{r = \operatorname{Numerator}[\operatorname{Simplify}[(1 - \sqrt{3})d]/c], s = \operatorname{Denom}[\operatorname{Simplify}[(1 - \sqrt{3})d]/c]\}, \operatorname{Simp}[(2*d*s^3 \sqrt{a + b*x^3}) / (a*r^2*((1 + \sqrt{3})s + r*x)), x] - \operatorname{Simp}[(3^{1/4} * \sqrt{2 - \sqrt{3}}) * d*s*(s + r*x) \sqrt{(s^2 - r*s*x + r^2*x^2) / ((1 + \sqrt{3})s + r*x)^2} \operatorname{EllipticE}[\operatorname{ArcSin}[\frac{(1 - \sqrt{3})s + r*x}{(1 + \sqrt{3})s + r*x}], -7 - 4\sqrt{3}] / (r^2 * \sqrt{a + b*x^3} * \sqrt{(s*(s + r*x)) / ((1 + \sqrt{3})s + r*x)^2}), x] /; \operatorname{FreeQ}\{a, b, c, d, x\} \&\& \operatorname{PosQ}[a] \&\& \operatorname{EqQ}[\dots]$$

Q[b*c^3 - 2*(5 - 3*sqrt[3])*a*d^3, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^{3/2}} dx &= -\frac{2(c+dx+ex^2)}{3b\sqrt{a+bx^3}} + \frac{2 \int \frac{d+2ex}{\sqrt{a+bx^3}} dx}{3b} \\ &= -\frac{2(c+dx+ex^2)}{3b\sqrt{a+bx^3}} + \frac{(4e) \int \frac{(1-\sqrt{3})\sqrt[3]{a+\sqrt[3]{bx^3}}}{\sqrt{a+bx^3}} dx}{3b^{4/3}} + \frac{\left(2\left(d - \frac{2(1-\sqrt{3})\sqrt[3]{ae}}{\sqrt[3]{b}}\right)\right) \int \frac{1}{\sqrt{a+bx^3}} dx}{3b} \\ &= -\frac{2(c+dx+ex^2)}{3b\sqrt{a+bx^3}} + \frac{8e\sqrt{a+bx^3}}{3b^{5/3}((1+\sqrt{3})\sqrt[3]{a+\sqrt[3]{bx^3}})} - \frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{ae}(\sqrt[3]{a+\sqrt[3]{bx^3}})}{\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx^3}}{(1+\sqrt{3})\sqrt[3]{a+\sqrt[3]{bx^3}}}}} \sqrt{\frac{\sqrt[3]{a}\sqrt[3]{bx^3}}{(1+\sqrt{3})\sqrt[3]{a+\sqrt[3]{bx^3}}}} \\ &\quad \frac{3^{3/4}b^{5/3}}{\sqrt{\frac{\sqrt[3]{a}\sqrt[3]{bx^3}}{(1+\sqrt{3})\sqrt[3]{a+\sqrt[3]{bx^3}}}}} \end{aligned}$$

Mathematica [C] time = 0.0880882, size = 107, normalized size = 0.2

$$\frac{2dx\sqrt{\frac{bx^3}{a}} + 1 {}_2F_1\left(\frac{1}{3}, \frac{1}{2}, \frac{4}{3}; -\frac{bx^3}{a}\right) - 2\left(3ex^2\sqrt{\frac{bx^3}{a}} + 1 {}_2F_1\left(\frac{2}{3}, \frac{3}{2}, \frac{5}{3}; -\frac{bx^3}{a}\right) + c + x(d - 3ex)\right)}{3b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^(3/2), x]

[Out] (2*d*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] - 2*(c + x*(d - 3*e*x) + 3*e*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)])/(3*b*Sqrt[a + b*x^3])

Maple [B] time = 0.007, size = 779, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(3/2), x)

```
[Out] e*(-2/3/b*x^2/((x^3+1/b*a)*b)^(1/2)-8/9*I/b^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+
1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/
3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-
b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/
3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*
a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I
*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)
^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3
^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3)))^(1/2))))+d*(-2/3/b*x/((x^3+1/b*a)*b)^(1/2)-4/9*I/b^2*3^(1/2)*(-b^2*a)^(
1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/
(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*
3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF
(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(
1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(
1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))-2/3*c/b/(b*x^3+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2c}{3\sqrt{bx^3+ab}} + \int \frac{(ex^4 + dx^3)\sqrt{bx^3+a}}{b^2x^6 + 2abx^3 + a^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/3*c/(sqrt(b*x^3 + a)*b) + integrate((e*x^4 + d*x^3)*sqrt(b*x^3 + a)/(b^2
*x^6 + 2*a*b*x^3 + a^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(ex^4 + dx^3 + cx^2)\sqrt{bx^3+a}}{b^2x^6 + 2abx^3 + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] integral((e*x^4 + d*x^3 + c*x^2)*sqrt(b*x^3 + a)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)

Sympy [A] time = 11.5607, size = 109, normalized size = 0.21

$$c \left(\begin{cases} -\frac{2}{3b\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{x^3}{3a^{\frac{3}{2}}} & \text{otherwise} \end{cases} \right) + \frac{dx^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{4}{3}, \frac{3}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}} \Gamma\left(\frac{7}{3}\right)} + \frac{ex^5 \Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\frac{3}{2}, \frac{5}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}} \Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)

[Out] c*Piecewise((-2/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**3/(3*a**(3/2)), True)) + d*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(7/3)) + e*x**5*gamma(5/3)*hyper((3/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(8/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + dx + c)x^2}{(bx^3 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")

[Out] integrate((e*x^2 + d*x + c)*x^2/(b*x^3 + a)^(3/2), x)

$$3.441 \quad \int \frac{x(c+dx+ex^2)}{(a+bx^3)^{3/2}} dx$$

Optimal. Leaf size=561

$$2\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(2a^{2/3}e+b^{2/3}(c-\sqrt{3}c))\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)\sqrt{2-\sqrt{3}}\sqrt{a+bx^3}}{\frac{3^4\sqrt[3]{3}a^{2/3}b^{4/3}}{\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}}}$$

[Out] $(-2*x*(a*e - b*c*x - b*d*x^2))/(3*a*b*\text{Sqrt}[a + b*x^3]) - (2*d*\text{Sqrt}[a + b*x^3])/(3*a*b) - (2*c*\text{Sqrt}[a + b*x^3])/(3*a*b^{(2/3)*((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})) + (\text{Sqrt}[2 - \text{Sqrt}[3]]*c*(a^{(1/3)} + b^{(1/3)*x})*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2]*\text{EllipticE}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x}/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})], -7 - 4*\text{Sqrt}[3]])/(3^{(3/4)}*a^{(2/3)*b^{(2/3)*x}*\text{Sqrt}[(a^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x})/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2]*\text{Sqrt}[a + b*x^3]) + (2*\text{Sqrt}[2 + \text{Sqrt}[3]]*(b^{(2/3)}*(c - \text{Sqrt}[3]*c) + 2*a^{(2/3)*e}*(a^{(1/3)} + b^{(1/3)*x})*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2]*\text{EllipticF}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x}/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})], -7 - 4*\text{Sqrt}[3]])/(3*3^{(1/4)}*a^{(2/3)*b^{(4/3)*x}*\text{Sqrt}[(a^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x})/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2]*\text{Sqrt}[a + b*x^3])$

Rubi [A] time = 0.321337, antiderivative size = 561, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1828, 1886, 261, 1878, 218, 1877}

$$2\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(2a^{2/3}e+b^{2/3}(c-\sqrt{3}c))F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\sqrt{2-\sqrt{3}}\sqrt{a+bx^3}}{\frac{3^4\sqrt[3]{3}a^{2/3}b^{4/3}}{\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x*(c + d*x + e*x^2))/(a + b*x^3)^{(3/2)}, x]$

[Out] $(-2*x*(a*e - b*c*x - b*d*x^2))/(3*a*b*\text{Sqrt}[a + b*x^3]) - (2*d*\text{Sqrt}[a + b*x^3])/(3*a*b) - (2*c*\text{Sqrt}[a + b*x^3])/(3*a*b^{(2/3)*((1 + \text{Sqrt}[3])*a^{(1/3)} + b$

$$\begin{aligned} &^{(1/3)*x}) + (\text{Sqrt}[2 - \text{Sqrt}[3]]*c*(a^{(1/3)} + b^{(1/3)*x})*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2]*\text{EllipticE}[\text{ArcSin}[\frac{(1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x}}{(1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x}}], -7 - 4*\text{Sqrt}[3]])/(3^{(3/4)}*a^{(2/3)}*b^{(2/3)}*\text{Sqrt}[(a^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x}))/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2]*\text{Sqrt}[a + b*x^3]) + (2*\text{Sqrt}[2 + \text{Sqrt}[3]]*(b^{(2/3)}*(c - \text{Sqrt}[3]*c) + 2*a^{(2/3)*e}*(a^{(1/3)} + b^{(1/3)*x})*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)}*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2]*\text{EllipticF}[\text{ArcSin}[\frac{(1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x}}{(1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x}}], -7 - 4*\text{Sqrt}[3]])/(3*3^{(1/4)}*a^{(2/3)}*b^{(4/3)}*\text{Sqrt}[(a^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x}))/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2]*\text{Sqrt}[a + b*x^3]) \end{aligned}$$
Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p_, x_Symbol] := With[{q = m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x] /; GeQ[q, n] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^p_, x_Symbol] := Dist[Coeff[Pq, x, n - 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p_, x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]]/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]]]/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(c + dx + ex^2)}{(a + bx^3)^{3/2}} dx &= -\frac{2x(ae - bcx - bdx^2)}{3ab\sqrt{a + bx^3}} - \frac{2 \int \frac{-ae + \frac{bcx}{2} + \frac{3}{2}bdx^2}{\sqrt{a + bx^3}} dx}{3ab} \\
&= -\frac{2x(ae - bcx - bdx^2)}{3ab\sqrt{a + bx^3}} - \frac{2 \int \frac{-ae + \frac{bcx}{2}}{\sqrt{a + bx^3}} dx}{3ab} - \frac{d \int \frac{x^2}{\sqrt{a + bx^3}} dx}{a} \\
&= -\frac{2x(ae - bcx - bdx^2)}{3ab\sqrt{a + bx^3}} - \frac{2d\sqrt{a + bx^3}}{3ab} - \frac{c \int \frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt{a + bx^3}} dx}{3a\sqrt[3]{b}} + \frac{\left(\frac{(1-\sqrt{3})b^{2/3}c}{a^{2/3}} + 2e\right) \int \frac{1}{\sqrt{a + bx^3}} dx}{3b} \\
&= -\frac{2x(ae - bcx - bdx^2)}{3ab\sqrt{a + bx^3}} - \frac{2d\sqrt{a + bx^3}}{3ab} - \frac{2c\sqrt{a + bx^3}}{3ab^{2/3}((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})} + \frac{\sqrt{2 - \sqrt{3}}c(\sqrt[3]{a} + \sqrt[3]{bx})}{3b}
\end{aligned}$$

Mathematica [C] time = 0.0642084, size = 108, normalized size = 0.19

$$\frac{3bcx^2\sqrt{\frac{bx^3}{a}} + 1 {}_2F_1\left(\frac{2}{3}, \frac{3}{2}; \frac{5}{3}; -\frac{bx^3}{a}\right) + 4aex\sqrt{\frac{bx^3}{a}} + 1 {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a}\right) - 4a(d + ex)}{6ab\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3)^(3/2), x]

[Out] (-4*a*(d + e*x) + 4*a*e*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 3*b*c*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)])/(6*a*b*Sqrt[a + b*x^3])

Maple [A] time = 0.005, size = 782, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(e*x^2+d*x+c)/(b*x^3+a)^(3/2), x)

[Out] e*(-2/3/b*x/((x^3+1/b*a)*b)^(1/2)-4/9*I/b^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))-2/3*d/b/(b*x^3+a)^(1/2)+c*(2/3*x^2/a/((x^3+1/b*a)*b)^(1/2)+2/9*I/a*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + dx + c)x}{(bx^3 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")

[Out] integrate((e*x^2 + d*x + c)*x/(b*x^3 + a)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + a}(ex^3 + dx^2 + cx)}{b^2x^6 + 2abx^3 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^3 + a)*(e*x^3 + d*x^2 + c*x)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)

Sympy [A] time = 11.1238, size = 109, normalized size = 0.19

$$d \left(\begin{cases} -\frac{2}{3b\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{x^3}{3a^{\frac{3}{2}}} & \text{otherwise} \end{cases} \right) + \frac{cx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{2}{3}, \frac{3}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{ex^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{4}{3}, \frac{3}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)

[Out] d*Piecewise((-2/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**3/(3*a**(3/2)), True)) + c*x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/

```
(3*a**(3/2)*gamma(5/3)) + e*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**
3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(7/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + dx + c)x}{(bx^3 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)*x/(b*x^3 + a)^(3/2), x)
```

$$3.442 \quad \int \frac{c+dx+ex^2}{(a+bx^3)^{3/2}} dx$$

Optimal. Leaf size=532

$$\frac{2\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}((1-\sqrt{3})\sqrt[3]{ad}+\sqrt[3]{bc})\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)\sqrt{2-\sqrt{3}}}{3^4\sqrt{3}ab^{2/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}} + \dots$$

[Out] $(-2*d*\text{Sqrt}[a + b*x^3])/(3*a*b^{(2/3)}*((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x}) - (2*(a*e - b*x*(c + d*x)))/(3*a*b*\text{Sqrt}[a + b*x^3]) + (\text{Sqrt}[2 - \text{Sqrt}[3]]*d*(a^{(1/3)} + b^{(1/3)*x})*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2]*\text{EllipticE}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x}/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})], -7 - 4*\text{Sqrt}[3]])/(3^{(3/4)}*a^{(2/3)*b^{(2/3)*x^2})*\text{Sqrt}[(a^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x})/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2]*\text{Sqrt}[a + b*x^3]) + (2*\text{Sqrt}[2 + \text{Sqrt}[3]]*(b^{(1/3)*c} + (1 - \text{Sqrt}[3])*a^{(1/3)*d}*(a^{(1/3)} + b^{(1/3)*x})*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2]*\text{EllipticF}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x}/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})], -7 - 4*\text{Sqrt}[3]))/(3*3^{(1/4)}*a*b^{(2/3)*x^2})*\text{Sqrt}[(a^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x})/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2]*\text{Sqrt}[a + b*x^3])$

Rubi [A] time = 0.24596, antiderivative size = 532, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1854, 1878, 218, 1877}

$$\frac{2\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}((1-\sqrt{3})\sqrt[3]{ad}+\sqrt[3]{bc})F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\sqrt{2-\sqrt{3}}d(\sqrt[3]{a}+\sqrt[3]{bx})}{3^4\sqrt{3}ab^{2/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}} + \dots$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/(a + b*x^3)^(3/2), x]

[Out] $(-2*d*\text{Sqrt}[a + b*x^3])/(3*a*b^{(2/3)}*((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x}) - (2*(a*e - b*x*(c + d*x)))/(3*a*b*\text{Sqrt}[a + b*x^3]) + (\text{Sqrt}[2 - \text{Sqrt}[3]]*d*(a^{(1/3)} + b^{(1/3)*x})*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2]*\text{EllipticE}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x}/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})], -7 - 4*\text{Sqrt}[3]])/(3^{(3/4)}*a^{(2/3)*b^{(2/3)*x^2})*\text{Sqrt}[(a^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x})/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2]*\text{Sqrt}[a + b*x^3]) + (2*\text{Sqrt}[2 + \text{Sqrt}[3]]*(b^{(1/3)*c} + (1 - \text{Sqrt}[3])*a^{(1/3)*d}*(a^{(1/3)} + b^{(1/3)*x})*\text{Sqrt}[(a^{(2/3)} - a^{(1/3)*b^{(1/3)*x} + b^{(2/3)*x^2})/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2]*\text{EllipticF}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x}/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})], -7 - 4*\text{Sqrt}[3]))/(3*3^{(1/4)}*a*b^{(2/3)*x^2})*\text{Sqrt}[(a^{(1/3)}*(a^{(1/3)} + b^{(1/3)*x})/((1 + \text{Sqrt}[3])*a^{(1/3)} + b^{(1/3)*x})^2]*\text{Sqrt}[a + b*x^3])$

$$\begin{aligned} & \sqrt[3]{a} + \sqrt[3]{b}x) \sqrt{(a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2)/((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2} \text{EllipticE}[\text{ArcSin}[\frac{(1 - \sqrt{3})a^{1/3} + b^{1/3}x}{(1 + \sqrt{3})a^{1/3} + b^{1/3}x}], -7 - 4\sqrt{3}]/(3^{3/4} * \\ & a^{2/3}b^{2/3} \sqrt{(a^{1/3}(a^{1/3} + b^{1/3}x))/(1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2} \sqrt{a + b^3x^3}) + (2\sqrt{2 + \sqrt{3}})(b^{1/3}c + (1 - \sqrt{3})a^{1/3}d) * \\ & (a^{1/3} + b^{1/3}x) \sqrt{(a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2)/((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2} \text{EllipticF}[\text{ArcSin}[\frac{(1 - \sqrt{3})a^{1/3} + b^{1/3}x}{(1 + \sqrt{3})a^{1/3} + b^{1/3}x}], -7 - \\ & 4\sqrt{3}]/(3^{3/4} * a * b^{2/3} \sqrt{(a^{1/3}(a^{1/3} + b^{1/3}x))/(1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2} \sqrt{a + b^3x^3}) \end{aligned}$$

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}](a + b*x^n)^(p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2] * EllipticF[ArcSin[(1 - Sqrt[3])*s + r*x]/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3])]/(3^(1/4)*r*Sqrt[a + b*x^3] * Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2] * EllipticE[ArcSin[(1 - Sqrt[3])*s + r*x]/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3])]/(r^2*Sqrt[a + b*x^3] * Sqrt[(s*(s + r*x))/
```

$((1 + \text{Sqrt}[3]) * s + r * x)^2$], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

Rubi steps

$$\begin{aligned} \int \frac{c + dx + ex^2}{(a + bx^3)^{3/2}} dx &= -\frac{2(ae - bx(c + dx))}{3ab\sqrt{a + bx^3}} - \frac{2 \int \frac{\frac{c}{2} + \frac{dx}{2}}{\sqrt{a + bx^3}} dx}{3a} \\ &= -\frac{2(ae - bx(c + dx))}{3ab\sqrt{a + bx^3}} - \frac{d \int \frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt{a + bx^3}} dx}{3a\sqrt[3]{b}} + \frac{\left(c + \frac{(1-\sqrt{3})\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \int \frac{1}{\sqrt{a + bx^3}} dx}{3a} \\ &= -\frac{2d\sqrt{a + bx^3}}{3ab^{2/3} \left((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}\right)} - \frac{2(ae - bx(c + dx))}{3ab\sqrt{a + bx^3}} + \frac{\sqrt{2 - \sqrt{3}}d \left(\sqrt[3]{a} + \sqrt[3]{bx}\right) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{\left((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}\right)^2}} E\left(\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{\left((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}\right)^2}\right)}{3^{3/4}a^{2/3}b^{2/3} \sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{\left((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}\right)}}} \end{aligned}$$

Mathematica [C] time = 0.0474435, size = 109, normalized size = 0.2

$$\frac{2bcx\sqrt{\frac{bx^3}{a} + 1} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a}\right) + 3bdx^2\sqrt{\frac{bx^3}{a} + 1} {}_2F_1\left(\frac{2}{3}, \frac{3}{2}; \frac{5}{3}; -\frac{bx^3}{a}\right) - 4ae + 4bcx}{6ab\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(a + b*x^3)^(3/2), x]

[Out] $(-4*a*e + 4*b*c*x + 2*b*c*x*\text{Sqrt}[1 + (b*x^3)/a]*\text{Hypergeometric2F1}[1/3, 1/2, 4/3, -((b*x^3)/a)] + 3*b*d*x^2*\text{Sqrt}[1 + (b*x^3)/a]*\text{Hypergeometric2F1}[2/3, 3/2, 5/3, -((b*x^3)/a)])/(6*a*b*\text{Sqrt}[a + b*x^3])$

Maple [A] time = 0.004, size = 785, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)/(b*x^3+a)^(3/2),x)

[Out]
$$\begin{aligned} & -2/3*e/b/(b*x^3+a)^{(1/2)}+d*(2/3*x^2/a/((x^3+1/b*a)*b)^{(1/2)}+2/9*I/a*3^{(1/2)} \\ & /b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)} \\ &)*3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)} \\ & +1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/ \\ & 2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/ \\ & 2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*EllipticE(1/3*3^{(1/2)} \\ & (1/2)*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)}*b/ \\ & (-b^2*a)^{(1/3)})^{(1/2)},(I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/ \\ & 2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}+1/b*(-b^2*a)^{(1/3)}*EllipticF(1/3*3^{(1/2)} \\ & (1/2)*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)}*b/(- \\ & b^2*a)^{(1/3)})^{(1/2)},(I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I \\ & *3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)})))+c*(2/3*x/a/((x^3+1/b*a)*b)^{(1/2)}-2/9*I/a*3^{(1/2)}/b*(-b^2*a)^{(1/3)} \\ & *(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)}*b/(-b^2*a)^{(1/3)} \\ &)*3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)} \\ & +1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)} \\ &)*3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/2)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)} \\ &)/b*(-b^2*a)^{(1/3)})*3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)},(I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)})) \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{(bx^3 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")

[Out] integrate((e*x^2 + d*x + c)/(b*x^3 + a)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + a}(ex^2 + dx + c)}{b^2x^6 + 2abx^3 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^3 + a)*(e*x^2 + d*x + c)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)

Sympy [A] time = 11.2642, size = 107, normalized size = 0.2

$$e^{\left\{ \begin{array}{ll} -\frac{2}{3b\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{x^3}{3a^{\frac{3}{2}}} & \text{otherwise} \end{array} \right\}} + \frac{cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{3}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{2}{3}, \frac{3}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)

[Out] e*Piecewise((-2/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**3/(3*a**(3/2)), True)) + c*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(5/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{(bx^3 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")

[Out] integrate((e*x^2 + d*x + c)/(b*x^3 + a)^(3/2), x)

$$3.443 \quad \int \frac{c+dx+ex^2}{x(a+bx^3)^{3/2}} dx$$

Optimal. Leaf size=579

$$\frac{2\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}((1-\sqrt{3})\sqrt[3]{ae}+\sqrt[3]{bd})\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)\sqrt{2-\sqrt{3}}}{3\sqrt[4]{3}ab^{2/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}} + \dots$$

```
[Out] (2*x*(a*d + a*e*x - b*c*x^2))/(3*a^2*Sqrt[a + b*x^3]) + (2*c*Sqrt[a + b*x^3])/(3*a^2) - (2*e*Sqrt[a + b*x^3])/(3*a*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(3/2)) + (Sqrt[2 - Sqrt[3]]*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]]/(3^(3/4)*a^(2/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*d + (1 - Sqrt[3])*a^(1/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]]/(3*3^(1/4)*a*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.399942, antiderivative size = 579, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {1829, 1832, 266, 63, 208, 1886, 261, 1878, 218, 1877}

$$\frac{2\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}((1-\sqrt{3})\sqrt[3]{ae}+\sqrt[3]{bd})F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\sqrt{2-\sqrt{3}}e(\sqrt[3]{a}+\sqrt[3]{bx})}{3\sqrt[4]{3}ab^{2/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}} + \dots$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/(x*(a + b*x^3)^(3/2)), x]

```
[Out] (2*x*(a*d + a*e*x - b*c*x^2))/(3*a^2*Sqrt[a + b*x^3]) + (2*c*Sqrt[a + b*x^3])/(3*a^2) - (2*e*Sqrt[a + b*x^3])/(3*a*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(3/2)) + (Sqrt[2 - Sqrt[3]]*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]]/(3^(3/4)*a^(2/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*d + (1 - Sqrt[3])*a^(1/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]]/(3*3^(1/4)*a*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

$$\begin{aligned} & (1/3)*x)) - (2*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(3/2)) + (Sqrt[2 - \\ & Sqrt[3]]*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3) \\ &)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticE[ArcSin[((1 - Sqrt[3] \\ &)*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3 \\ &]]/(3^(3/4)*a^(2/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqr \\ & t[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1 \\ & /3)*d + (1 - Sqrt[3])*a^(1/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1 \\ & /3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*Ellipti \\ & cF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1 \\ & /3)*x)], -7 - 4*Sqrt[3]]/(3*3^(1/4)*a*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(\\ & 1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) \end{aligned}$$
Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_.)]), x_Symbol] := Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1886

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[Coeff[Pq, x, n - 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 1878

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

Rule 218

Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)]/((1 + Sqrt[3])*s + r*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b}, x] && PosQ[a]

Rule 1877

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)]/((1 + Sqrt[3])*s + r*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq

Q[b*c^3 - 2*(5 - 3*sqrt[3])*a*d^3, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{c + dx + ex^2}{x(a + bx^3)^{3/2}} dx &= \frac{2x(ad + aex - bcx^2)}{3a^2\sqrt{a + bx^3}} - \frac{2 \int \frac{-\frac{3bc}{2} - \frac{bdx}{2} + \frac{1}{2}bex^2 - \frac{3b^2cx^3}{2a}}{x\sqrt{a+bx^3}} dx}{3ab} \\
 &= \frac{2x(ad + aex - bcx^2)}{3a^2\sqrt{a + bx^3}} - \frac{2 \int \frac{-\frac{bd}{2} + \frac{bex}{2} - \frac{3b^2cx^2}{2a}}{\sqrt{a+bx^3}} dx}{3ab} + \frac{c \int \frac{1}{x\sqrt{a+bx^3}} dx}{a} \\
 &= \frac{2x(ad + aex - bcx^2)}{3a^2\sqrt{a + bx^3}} - \frac{2 \int \frac{-\frac{bd}{2} + \frac{bex}{2}}{\sqrt{a+bx^3}} dx}{3ab} + \frac{c \operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, x^3\right)}{3a} + \frac{(bc) \int \frac{x^2}{\sqrt{a+bx^3}} dx}{a^2} \\
 &= \frac{2x(ad + aex - bcx^2)}{3a^2\sqrt{a + bx^3}} + \frac{2c\sqrt{a + bx^3}}{3a^2} + \frac{(2c) \operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + bx^3}\right)}{3ab} - \frac{e \int \frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{\sqrt{a+bx^3}} dx}{3a\sqrt[3]{b}} \\
 &= \frac{2x(ad + aex - bcx^2)}{3a^2\sqrt{a + bx^3}} + \frac{2c\sqrt{a + bx^3}}{3a^2} - \frac{2e\sqrt{a + bx^3}}{3ab^{2/3}\left((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}\right)} - \frac{2c \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}} + \frac{\sqrt{2}}{3a}
 \end{aligned}$$

Mathematica [C] time = 0.10284, size = 119, normalized size = 0.21

$$\frac{4c {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{bx^3}{a} + 1\right) + x\left(2d\sqrt{\frac{bx^3}{a}} + 1 {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a}\right) + 3ex\sqrt{\frac{bx^3}{a}} + 1 {}_2F_1\left(\frac{2}{3}, \frac{3}{2}; \frac{5}{3}; -\frac{bx^3}{a}\right) + 4d\right)}{6a\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)^(3/2)), x]

[Out] (4*c*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^3)/a] + x*(4*d + 2*d*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 3*e*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)])/(6*a*Sqrt[a + b*x^3])

Maple [A] time = 0.007, size = 810, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d*x+c)/x/(b*x^3+a)^(3/2),x)`

[Out]
$$e*(2/3*x^2/a/((x^3+1/b*a)*b)^(1/2)+2/9*I/a*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+d*(2/3*x/a/((x^3+1/b*a)*b)^(1/2)-2/9*I/a*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+c*(2/3/a/((x^3+1/b*a)*b)^(1/2)-2/3/a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{(bx^3 + a)^{\frac{3}{2}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(3/2),x, algorithm="maxima")`

[Out] `integrate((e*x^2 + d*x + c)/((b*x^3 + a)^(3/2)*x), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3+a}(ex^2+dx+c)}{b^2x^7+2abx^4+a^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^3 + a)*(e*x^2 + d*x + c)/(b^2*x^7 + 2*a*b*x^4 + a^2*x), x)

Sympy [A] time = 22.4442, size = 265, normalized size = 0.46

$$c \left(\frac{2a^3 \sqrt{1 + \frac{bx^3}{a}}}{3a^{\frac{9}{2}} + 3a^{\frac{7}{2}}bx^3} + \frac{a^3 \log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}} + 3a^{\frac{7}{2}}bx^3} - \frac{2a^3 \log\left(\sqrt{1 + \frac{bx^3}{a}} + 1\right)}{3a^{\frac{9}{2}} + 3a^{\frac{7}{2}}bx^3} + \frac{a^2bx^3 \log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}} + 3a^{\frac{7}{2}}bx^3} - \frac{2a^2bx^3 \log\left(\sqrt{1 + \frac{bx^3}{a}} + 1\right)}{3a^{\frac{9}{2}} + 3a^{\frac{7}{2}}bx^3} \right) + \frac{dx\Gamma\left(\frac{1}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/x/(b*x**3+a)**(3/2),x)

[Out] c*(2*a**3*sqrt(1 + b*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**3*log(b*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) - 2*a**3*log(sqrt(1 + b*x**3/a) + 1)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**2*b*x**3*log(b*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) - 2*a**2*b*x**3*log(sqrt(1 + b*x**3/a) + 1)/(3*a**(9/2) + 3*a**(7/2)*b*x**3)) + d*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(4/3)) + e*x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(5/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{(bx^3 + a)^{\frac{3}{2}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^(3/2)*x), x)
```

$$3.444 \quad \int \frac{c+dx+ex^2}{x^2(a+bx^3)^{3/2}} dx$$

Optimal. Leaf size=607

$$\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(5(1-\sqrt{3})b^{2/3}c-2a^{2/3}e)\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)5\sqrt{2-\sqrt{3}}$$

$$3^4\sqrt[3]{3}a^{5/3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}$$

[Out] (2*x*(a*e - b*c*x - b*d*x^2))/(3*a^2*Sqrt[a + b*x^3]) + (2*d*Sqrt[a + b*x^3])/((3*a^2) - (c*Sqrt[a + b*x^3])/(a^2*x) + (5*b^(1/3)*c*Sqrt[a + b*x^3])/(3*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(3/2)) - (5*Sqrt[2 - Sqrt[3]]*b^(1/3)*c*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(3/4)*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (Sqrt[2 + Sqrt[3]]*(5*(1 - Sqrt[3])*b^(2/3)*c - 2*a^(2/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(5/3)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 0.559812, antiderivative size = 607, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.44$, Rules used = {1829, 1835, 1832, 266, 63, 208, 1886, 261, 1878, 218, 1877}

$$\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(5(1-\sqrt{3})b^{2/3}c-2a^{2/3}e)F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)5\sqrt{2-\sqrt{3}}$$

$$3^4\sqrt[3]{3}a^{5/3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^(3/2)), x]

```
[Out] (2*x*(a*e - b*c*x - b*d*x^2))/(3*a^2*Sqrt[a + b*x^3]) + (2*d*Sqrt[a + b*x^3])/
(3*a^2) - (c*Sqrt[a + b*x^3])/(a^2*x) + (5*b^(1/3)*c*Sqrt[a + b*x^3])/(3
*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*d*ArcTanh[Sqrt[a + b*x^3]/Sq
rt[a]])/(3*a^(3/2)) - (5*Sqrt[2 - Sqrt[3]]*b^(1/3)*c*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(3/4)*a^(5/3)*Sqrt[(a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3]) - (Sqrt[2 + Sqrt[3]]*(5*(1 - Sqrt[3])*b^(2/3)*c - 2*a^(2/3)*e)*(a^(
1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*
a^(5/3)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_.)]), x_Symbol] := Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[
```

```
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[Coeff[Pq, x, n - 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)]/((1 + Sqrt[3])*s + r*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
```

& PosQ[a]

Rule 1877

Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{c + dx + ex^2}{x^2(a + bx^3)^{3/2}} dx &= \frac{2x(ae - bcx - bdx^2)}{3a^2\sqrt{a + bx^3}} - \frac{2 \int \frac{\frac{3bc}{2} - \frac{3bdx}{2} - \frac{1}{2}bex^2 - \frac{b^2cx^3}{2a} - \frac{3b^2dx^4}{2a}}{x^2\sqrt{a+bx^3}} dx}{3ab} \\
 &= \frac{2x(ae - bcx - bdx^2)}{3a^2\sqrt{a + bx^3}} - \frac{c\sqrt{a + bx^3}}{a^2x} + \frac{\int \frac{3abd+abex+\frac{5}{2}b^2cx^2+3b^2dx^3}{x\sqrt{a+bx^3}} dx}{3a^2b} \\
 &= \frac{2x(ae - bcx - bdx^2)}{3a^2\sqrt{a + bx^3}} - \frac{c\sqrt{a + bx^3}}{a^2x} + \frac{\int \frac{abe+\frac{5}{2}b^2cx+3b^2dx^2}{\sqrt{a+bx^3}} dx}{3a^2b} + \frac{d \int \frac{1}{x\sqrt{a+bx^3}} dx}{a} \\
 &= \frac{2x(ae - bcx - bdx^2)}{3a^2\sqrt{a + bx^3}} - \frac{c\sqrt{a + bx^3}}{a^2x} + \frac{\int \frac{abe+\frac{5}{2}b^2cx}{\sqrt{a+bx^3}} dx}{3a^2b} + \frac{d \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, x^3\right)}{3a} + \frac{(bd) \int \frac{x}{\sqrt{a+bx^3}} dx}{a^2} \\
 &= \frac{2x(ae - bcx - bdx^2)}{3a^2\sqrt{a + bx^3}} + \frac{2d\sqrt{a + bx^3}}{3a^2} - \frac{c\sqrt{a + bx^3}}{a^2x} + \frac{(5b^{2/3}c) \int \frac{(1-\sqrt{3})\sqrt[3]{a+\sqrt[3]{bx}}}{\sqrt{a+bx^3}} dx}{6a^2} + \frac{(2d) \text{Subst}\left(\int \frac{x}{\sqrt{a+bx}} dx, x, x^3\right)}{3a} \\
 &= \frac{2x(ae - bcx - bdx^2)}{3a^2\sqrt{a + bx^3}} + \frac{2d\sqrt{a + bx^3}}{3a^2} - \frac{c\sqrt{a + bx^3}}{a^2x} + \frac{5\sqrt[3]{bc}\sqrt{a + bx^3}}{3a^2((1 + \sqrt{3})\sqrt[3]{a + \sqrt[3]{bx}})} - \frac{2d \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt[3]{a+bx^3}}\right)}{3a^{3/2}}
 \end{aligned}$$

Mathematica [C] time = 0.0984323, size = 121, normalized size = 0.2

$$\frac{-3c\sqrt{\frac{bx^3}{a}} + {}_2F_1\left(-\frac{1}{3}, \frac{3}{2}; \frac{2}{3}; -\frac{bx^3}{a}\right) + 2dx {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{bx^3}{a} + 1\right) + ex^2\left(\sqrt{\frac{bx^3}{a}} + {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a}\right) + 2\right)}{3ax\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^(3/2)), x]

[Out] (2*d*x*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^3)/a] - 3*c*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/3, 3/2, 2/3, -((b*x^3)/a)] + e*x^2*(2 + Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]))/(3*a*x*Sqrt[a + b*x^3])

Maple [A] time = 0.008, size = 825, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)/x^2/(b*x^3+a)^(3/2), x)

[Out] e*(2/3*x/a/((x^3+1/b*a)*b)^(1/2)-2/9*I/a^3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2)/b*(-b^2*a)^(1/3)-1/2*I^3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I^3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I^3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I^3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I^3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I^3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+d*(2/3/a/((x^3+1/b*a)*b)^(1/2)-2/3/a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+c*(-2/3*b*x^2/a^2/((x^3+1/b*a)*b)^(1/2)-1/a^2*(b*x^3+a)^(1/2)/x-5/9*I/a^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I^3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I^3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I^3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I^3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I^3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I^3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I^3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I^3

$$\frac{1}{b}(-b^2 a)^{1/3} \cdot 3^{1/2} \cdot b / (-b^2 a)^{1/3} \cdot \frac{1}{b}(-b^2 a)^{1/3} \cdot \frac{1}{(-3/2/b(-b^2 a)^{1/3} + 1/2 \cdot 3^{1/2} / b(-b^2 a)^{1/3})^{1/2}}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{(bx^3 + a)^{\frac{3}{2}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(3/2),x, algorithm="maxima")

[Out] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^(3/2)*x^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + a}(ex^2 + dx + c)}{b^2x^8 + 2abx^5 + a^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^3 + a)*(e*x^2 + d*x + c)/(b^2*x^8 + 2*a*b*x^5 + a^2*x^2), x)

Sympy [A] time = 34.6304, size = 267, normalized size = 0.44

$$d \left(\frac{2a^3 \sqrt{1 + \frac{bx^3}{a}}}{3a^{\frac{9}{2}} + 3a^{\frac{7}{2}}bx^3} + \frac{a^3 \log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}} + 3a^{\frac{7}{2}}bx^3} - \frac{2a^3 \log\left(\sqrt{1 + \frac{bx^3}{a}} + 1\right)}{3a^{\frac{9}{2}} + 3a^{\frac{7}{2}}bx^3} + \frac{a^2bx^3 \log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}} + 3a^{\frac{7}{2}}bx^3} - \frac{2a^2bx^3 \log\left(\sqrt{1 + \frac{bx^3}{a}} + 1\right)}{3a^{\frac{9}{2}} + 3a^{\frac{7}{2}}bx^3} \right) + \frac{c\Gamma\left(-\frac{1}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**(3/2),x)

[Out] d*(2*a**3*sqrt(1 + b*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**3*log(b*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) - 2*a**3*log(sqrt(1 + b*x**3/a) + 1)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**2*b*x**3*log(b*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) - 2*a**2*b*x**3*log(sqrt(1 + b*x**3/a) + 1)/(3*a**(9/2) + 3*a**(7/2)*b*x**3)) + c*gamma(-1/3)*hyper((-1/3, 3/2), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*x*gamma(2/3)) + e*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(4/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{(bx^3 + a)^{\frac{3}{2}}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(3/2),x, algorithm="giac")

[Out] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^(3/2)*x^2), x)

3.445 $\int x^3 \sqrt{a + bx^3} (c + dx + ex^2 + fx^3 + gx^4) dx$

Optimal. Leaf size=733

$$4 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}a^2} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} \text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}\right), -7 - 4\sqrt{3}\right) (1729\sqrt[3]{b}(17bc - 8af) -$$

$$1616615b^{8/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

[Out] $(-4a^2e\sqrt{a + bx^3})/(45b^2) + (6a(17bc - 8af)x\sqrt{a + bx^3})/(935b^2) + (6a(19bd - 10ag)x^2\sqrt{a + bx^3})/(1729b^2) + (2aex^3\sqrt{a + bx^3})/(45b) + (6afx^4\sqrt{a + bx^3})/(187b) + (6agx^5\sqrt{a + bx^3})/(247b) - (24a^2(19bd - 10ag)\sqrt{a + bx^3})/(1729b^{8/3}((1 + \sqrt{3})a^{1/3} + b^{1/3}x)) + (2x^3\sqrt{a + bx^3}(62985cx + 53295dx^2 + 46189ex^3 + 40755fx^4 + 36465gx^5))/692835 + (12\sqrt[3]{2 - \sqrt{3}}\sqrt{2 - \sqrt{3}}a^{7/3}(19bd - 10ag)(a^{1/3} + b^{1/3}x)\sqrt{(a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2)/((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2})\text{EllipticE}[\text{ArcSin}[\frac{(1 - \sqrt{3})a^{1/3} + b^{1/3}x}{(1 + \sqrt{3})a^{1/3} + b^{1/3}x}], -7 - 4\sqrt{3}]/(1729b^{8/3}\sqrt{(a^{1/3}(a^{1/3} + b^{1/3}x))/((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2})\sqrt{a + bx^3} - (4\sqrt[3]{2 + \sqrt{3}}\sqrt{2 + \sqrt{3}}a^2(1729b^{1/3}(17bc - 8af) - 1870(1 - \sqrt{3})a^{1/3}(19bd - 10ag))(a^{1/3} + b^{1/3}x)\sqrt{(a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2)/((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2})\text{EllipticF}[\text{ArcSin}[\frac{(1 - \sqrt{3})a^{1/3} + b^{1/3}x}{(1 + \sqrt{3})a^{1/3} + b^{1/3}x}], -7 - 4\sqrt{3}]/(1616615b^{8/3}\sqrt{(a^{1/3}(a^{1/3} + b^{1/3}x))/((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2})\sqrt{a + bx^3}$

Rubi [A] time = 1.90966, antiderivative size = 733, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1826, 1836, 1888, 1594, 1886, 261, 1878, 218, 1877}

$$4 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}a^2} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx} + (1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx} + (1+\sqrt{3})\sqrt[3]{a}}\right) | -7 - 4\sqrt{3}\right) (1729\sqrt[3]{b}(17bc - 8af) - 1870($$

$$1616615b^{8/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

Antiderivative was successfully verified.

[In] Int[x^3*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]

[Out]
$$\begin{aligned} & (-4*a^2*e*\text{Sqrt}[a + b*x^3])/(45*b^2) + (6*a*(17*b*c - 8*a*f)*x*\text{Sqrt}[a + b*x^3])/(935*b^2) \\ & + (6*a*(19*b*d - 10*a*g)*x^2*\text{Sqrt}[a + b*x^3])/(1729*b^2) + (2*a*e*x^3*\text{Sqrt}[a + b*x^3])/(45*b) \\ & + (6*a*f*x^4*\text{Sqrt}[a + b*x^3])/(187*b) + (6*a*g*x^5*\text{Sqrt}[a + b*x^3])/(247*b) - (24*a^2*(19*b*d - 10*a*g)*\text{Sqrt}[a + b*x^3]) \\ & /((1729*b^{8/3}*((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)) + (2*x^3*\text{Sqrt}[a + b*x^3]*(62985*c*x + 53295*d*x^2 + 46189*e*x^3 + 40755*f*x^4 + 36465*g*x^5))/6 \\ & 92835 + (12*3^{1/4}*\text{Sqrt}[2 - \text{Sqrt}[3]]*a^{7/3}*(19*b*d - 10*a*g)*(a^{1/3} + b^{1/3}*x)*\text{Sqrt}[(a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2)/((1 + \text{Sqrt}[3])* \\ & a^{1/3} + b^{1/3}*x)^2]*\text{EllipticE}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x]/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)], -7 - 4*\text{Sqrt}[3]))/(1729*b^{8/3}*\text{Sqrt} \\ & [(a^{1/3}*(a^{1/3} + b^{1/3}*x))/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\text{Sqrt}[a + b*x^3]) - (4*3^{3/4}*\text{Sqrt}[2 + \text{Sqrt}[3]]*a^2*(1729*b^{1/3}*(17*b*c - 8* \\ & a*f) - 1870*(1 - \text{Sqrt}[3])*a^{1/3}*(19*b*d - 10*a*g))*(a^{1/3} + b^{1/3}*x)*\text{Sqrt}[(a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2)/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]* \\ & \text{EllipticF}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x]/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)], -7 - 4*\text{Sqrt}[3]))/(1616615*b^{8/3}*\text{Sqrt}[(a^{1/3} \\ &)*(a^{1/3} + b^{1/3}*x))/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\text{Sqrt}[a + b*x^3]) \end{aligned}$$

Rule 1826

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}], x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]

Rule 1836

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m + q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]

Rule 1888

Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum

```
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1)), x] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*n)]) /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n, x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^p, x_Symbol] := Dist[Coeff[Pq, x, n - 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^p, x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)]/((1 + Sqrt[3])*s + r*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]
```

```

]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

```

Rubi steps

$$\begin{aligned}
\int x^3 \sqrt{a + bx^3} (c + dx + ex^2 + fx^3 + gx^4) dx &= \frac{2x^3 \sqrt{a + bx^3} (62985cx + 53295dx^2 + 46189ex^3 + 40755fx^4 + 36465gx^5)}{692835} \\
&= \frac{6agx^5 \sqrt{a + bx^3}}{247b} + \frac{2x^3 \sqrt{a + bx^3} (62985cx + 53295dx^2 + 46189ex^3 + 40755fx^4 + 36465gx^5)}{692835} \\
&= \frac{6afx^4 \sqrt{a + bx^3}}{187b} + \frac{6agx^5 \sqrt{a + bx^3}}{247b} + \frac{2x^3 \sqrt{a + bx^3} (62985cx + 53295dx^2 + 46189ex^3 + 40755fx^4 + 36465gx^5)}{692835} \\
&= \frac{2aex^3 \sqrt{a + bx^3}}{45b} + \frac{6afx^4 \sqrt{a + bx^3}}{187b} + \frac{6agx^5 \sqrt{a + bx^3}}{247b} + \frac{2x^3 \sqrt{a + bx^3} (62985cx + 53295dx^2 + 46189ex^3 + 40755fx^4 + 36465gx^5)}{692835} \\
&= \frac{2aex^3 \sqrt{a + bx^3}}{45b} + \frac{6afx^4 \sqrt{a + bx^3}}{187b} + \frac{6agx^5 \sqrt{a + bx^3}}{247b} + \frac{2x^3 \sqrt{a + bx^3} (62985cx + 53295dx^2 + 46189ex^3 + 40755fx^4 + 36465gx^5)}{692835} \\
&= \frac{6a(19bd - 10ag)x^2 \sqrt{a + bx^3}}{1729b^2} + \frac{2aex^3 \sqrt{a + bx^3}}{45b} + \frac{6afx^4 \sqrt{a + bx^3}}{187b} + \frac{6agx^5 \sqrt{a + bx^3}}{247b} \\
&= \frac{6a(19bd - 10ag)x^2 \sqrt{a + bx^3}}{1729b^2} + \frac{2aex^3 \sqrt{a + bx^3}}{45b} + \frac{6afx^4 \sqrt{a + bx^3}}{187b} + \frac{6agx^5 \sqrt{a + bx^3}}{247b} \\
&= \frac{6a(17bc - 8af)x \sqrt{a + bx^3}}{935b^2} + \frac{6a(19bd - 10ag)x^2 \sqrt{a + bx^3}}{1729b^2} + \frac{2aex^3 \sqrt{a + bx^3}}{45b} \\
&= \frac{6a(17bc - 8af)x \sqrt{a + bx^3}}{935b^2} + \frac{6a(19bd - 10ag)x^2 \sqrt{a + bx^3}}{1729b^2} + \frac{2aex^3 \sqrt{a + bx^3}}{45b} \\
&= -\frac{4a^2e \sqrt{a + bx^3}}{45b^2} + \frac{6a(17bc - 8af)x \sqrt{a + bx^3}}{935b^2} + \frac{6a(19bd - 10ag)x^2 \sqrt{a + bx^3}}{1729b^2} \\
&= -\frac{4a^2e \sqrt{a + bx^3}}{45b^2} + \frac{6a(17bc - 8af)x \sqrt{a + bx^3}}{935b^2} + \frac{6a(19bd - 10ag)x^2 \sqrt{a + bx^3}}{1729b^2}
\end{aligned}$$

Mathematica [C] time = 0.299319, size = 172, normalized size = 0.23

$$2\sqrt{a + bx^3} \left(- (a + bx^3) \sqrt{\frac{bx^3}{a}} + 1 (a(92378e + 90x(988f + 935gx)) - 3bx (62985c + 11x (4845d + 13x (323e + 285fx + 2078505b^2 \sqrt{\frac{bx^3}{a}} +$$

Antiderivative was successfully verified.

[In] Integrate[x^3*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]

[Out] (2*Sqrt[a + b*x^3]*(-(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*(a*(92378*e + 90*x*(988*f + 935*g*x)) - 3*b*x*(62985*c + 11*x*(4845*d + 13*x*(323*e + 285*f*x + 255*g*x^2)))) + 11115*a*(-17*b*c + 8*a*f)*x*Hypergeometric2F1[-1/2, 1/3, 4/3, -(b*x^3)/a] + 8415*a*(-19*b*d + 10*a*g)*x^2*Hypergeometric2F1[-1/2, 2/3, 5/3, -(b*x^3)/a]))/(2078505*b^2*Sqrt[1 + (b*x^3)/a])

Maple [B] time = 0.019, size = 1674, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x)

[Out] g*(2/19*x^8*(b*x^3+a)^(1/2)+6/247/b*a*x^5*(b*x^3+a)^(1/2)-60/1729*a^2/b^2*x^2*(b*x^3+a)^(1/2)-80/1729*I*a^3/b^3*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2))*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+f*(2/17*x^7*(b*x^3+a)^(1/2)+6/187/b*a*x^4*(b*x^3+a)^(1/2)-48/935*a^2/b^2*x*(b*x^3+a)^(1/2)-32/935*I*a^3/b^3*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2))*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+e*(2/15*x^6*(b*x^3+a)^(1/2)+2/45/b*a*x^3*(b*x^3+a)^(1/2)-4/45*a^2/b^2*(b*x^3+a)^(1/2))+d*(2/13*x^5*(b*x^3+a)^(1/2)+6/91/b*a*x^2*(b*x^3+a)^(1/2)+8/91*I/b^2*a^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*

$$\begin{aligned} & \left(\frac{x-1/b*(-b^2*a)^{(1/3)}}{(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}} \right)^{(1/2)} * (-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}) * 3^{(1/2)} * b / (-b^2*a)^{(1/3)})^{(1/2)} / (b*x^3+a)^{(1/2)} * \left(\frac{-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}}{(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}} \right)^{(1/2)} * \text{EllipticE}\left(\frac{1}{3}, 3^{(1/2)} * (I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}) * 3^{(1/2)} * b / (-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)} / (-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})\right)^{(1/2)} \\ & + 1/b*(-b^2*a)^{(1/3)} * \text{EllipticF}\left(\frac{1}{3}, 3^{(1/2)} * (I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}) * 3^{(1/2)} * b / (-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)} / (-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})\right)^{(1/2)} \\ & \left. + c * (2/11*x^4*(b*x^3+a)^{(1/2)} + 6/55/b*a*x*(b*x^3+a)^{(1/2)} + 4/55*I/b^2*a^2*3^{(1/2)}*(-b^2*a)^{(1/3)} * (I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}) * 3^{(1/2)} * b / (-b^2*a)^{(1/3)})^{(1/2)} * \left(\frac{x-1/b*(-b^2*a)^{(1/3)}}{(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}} \right)^{(1/2)} * (-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}) * 3^{(1/2)} * b / (-b^2*a)^{(1/3)})^{(1/2)} / (b*x^3+a)^{(1/2)} * \text{EllipticF}\left(\frac{1}{3}, 3^{(1/2)} * (I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}) * 3^{(1/2)} * b / (-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)} / (-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})\right)^{(1/2)} \right) \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + ax^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(gx^7 + fx^6 + ex^5 + dx^4 + cx^3\right)\sqrt{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] integral((g*x^7 + f*x^6 + e*x^5 + d*x^4 + c*x^3)*sqrt(b*x^3 + a), x)

Sympy [A] time = 5.10566, size = 238, normalized size = 0.32

$$\frac{\sqrt{ac}x^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(-\frac{1}{2}, \frac{4}{3} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt{ad}x^5\Gamma\left(\frac{5}{3}\right) {}_2F_1\left(-\frac{1}{2}, \frac{5}{3} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{\sqrt{af}x^7\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(-\frac{1}{2}, \frac{7}{3} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{\sqrt{ag}x^8\Gamma\left(\frac{8}{3}\right) {}_2F_1\left(-\frac{1}{2}, \frac{8}{3} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2),x)

[Out] sqrt(a)*c*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*d*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + sqrt(a)*f*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)*g*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) + e*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + ax^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="giac")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x^3, x)

3.446 $\int x^2 \sqrt{a + bx^3} (c + dx + ex^2 + fx^3 + gx^4) dx$

Optimal. Leaf size=681

$$4 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}a^2} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3}x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (-1870(1 - \sqrt{3}) \sqrt[3]{ab^{2/3}e} - 728ag + 1547bd) \operatorname{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3}) \sqrt[3]{a}}{(1+\sqrt{3}) \sqrt[3]{a}}\right)\right)$$

$$85085b^{7/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

```
[Out] (2*a*(5*b*c - 2*a*f)*Sqrt[a + b*x^3])/(45*b^2) + (6*a*(17*b*d - 8*a*g)*x*Sqrt[a + b*x^3])/(935*b^2) + (6*a*e*x^2*Sqrt[a + b*x^3])/(91*b) + (2*a*f*x^3*Sqrt[a + b*x^3])/(45*b) + (6*a*g*x^4*Sqrt[a + b*x^3])/(187*b) - (24*a^2*e*Sqrt[a + b*x^3])/(91*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x^2*Sqrt[a + b*x^3]*(12155*c*x + 9945*d*x^2 + 8415*e*x^3 + 7293*f*x^4 + 6435*g*x^5))/109395 + (12*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(91*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (4*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(1547*b*d - 1870*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e - 728*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticF[ArcSin[(1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(85085*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 1.42152, antiderivative size = 681, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1826, 1836, 1888, 1594, 1886, 261, 1878, 218, 1877}

$$4 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}a^2} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3}x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (-1870(1 - \sqrt{3}) \sqrt[3]{ab^{2/3}e} - 728ag + 1547bd) F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx} + (1-\sqrt{3}) \sqrt[3]{a}}{\sqrt[3]{bx} + (1+\sqrt{3}) \sqrt[3]{a}}\right)\right)$$

$$85085b^{7/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

Antiderivative was successfully verified.

[In] Int[x^2*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4), x]

```
[Out] (2*a*(5*b*c - 2*a*f)*Sqrt[a + b*x^3])/(45*b^2) + (6*a*(17*b*d - 8*a*g)*x*Sqrt[a + b*x^3])/(935*b^2) + (6*a*e*x^2*Sqrt[a + b*x^3])/(91*b) + (2*a*f*x^3*Sqrt[a + b*x^3])/(45*b) + (6*a*g*x^4*Sqrt[a + b*x^3])/(187*b) - (24*a^2*e*Sqrt[a + b*x^3])/(91*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x^2*Sqrt[a + b*x^3]*(12155*c*x + 9945*d*x^2 + 8415*e*x^3 + 7293*f*x^4 + 6435*g*x^5))/109395 + (12*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(91*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (4*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(1547*b*d - 1870*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e - 728*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticF[ArcSin[(1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(85085*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1826

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}], x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1836

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m + q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1594

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n, x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1886

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^p, x_Symbol] := Dist[Coeff[Pq, x, n - 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^p, x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 1878

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

Rule 218

Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)]/((1 + Sqrt[3])*s + r*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b}, x] && PosQ[a]

Rule 1877

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)]/((1 + Sqrt[3])*s + r*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq

Q[b*c^3 - 2*(5 - 3*sqrt[3])*a*d^3, 0]

Rubi steps

$$\begin{aligned}
 \int x^2 \sqrt{a + bx^3} (c + dx + ex^2 + fx^3 + gx^4) dx &= \frac{2x^2 \sqrt{a + bx^3} (12155cx + 9945dx^2 + 8415ex^3 + 7293fx^4 + 6435gx^5)}{109395} + \frac{1}{2} \\
 &= \frac{6agx^4 \sqrt{a + bx^3}}{187b} + \frac{2x^2 \sqrt{a + bx^3} (12155cx + 9945dx^2 + 8415ex^3 + 7293fx^4)}{109395} \\
 &= \frac{2afx^3 \sqrt{a + bx^3}}{45b} + \frac{6agx^4 \sqrt{a + bx^3}}{187b} + \frac{2x^2 \sqrt{a + bx^3} (12155cx + 9945dx^2 + 8415ex^3 + 7293fx^4)}{109395} \\
 &= \frac{6aex^2 \sqrt{a + bx^3}}{91b} + \frac{2afx^3 \sqrt{a + bx^3}}{45b} + \frac{6agx^4 \sqrt{a + bx^3}}{187b} + \frac{2x^2 \sqrt{a + bx^3} (12155cx + 9945dx^2 + 8415ex^3 + 7293fx^4)}{109395} \\
 &= \frac{6aex^2 \sqrt{a + bx^3}}{91b} + \frac{2afx^3 \sqrt{a + bx^3}}{45b} + \frac{6agx^4 \sqrt{a + bx^3}}{187b} + \frac{2x^2 \sqrt{a + bx^3} (12155cx + 9945dx^2 + 8415ex^3 + 7293fx^4)}{109395} \\
 &= \frac{6a(17bd - 8ag)x \sqrt{a + bx^3}}{935b^2} + \frac{6aex^2 \sqrt{a + bx^3}}{91b} + \frac{2afx^3 \sqrt{a + bx^3}}{45b} + \frac{6agx^4 \sqrt{a + bx^3}}{187b} \\
 &= \frac{6a(17bd - 8ag)x \sqrt{a + bx^3}}{935b^2} + \frac{6aex^2 \sqrt{a + bx^3}}{91b} + \frac{2afx^3 \sqrt{a + bx^3}}{45b} + \frac{6agx^4 \sqrt{a + bx^3}}{187b} \\
 &= \frac{2a(5bc - 2af) \sqrt{a + bx^3}}{45b^2} + \frac{6a(17bd - 8ag)x \sqrt{a + bx^3}}{935b^2} + \frac{6aex^2 \sqrt{a + bx^3}}{91b} + \frac{6agx^4 \sqrt{a + bx^3}}{187b} \\
 &= \frac{2a(5bc - 2af) \sqrt{a + bx^3}}{45b^2} + \frac{6a(17bd - 8ag)x \sqrt{a + bx^3}}{935b^2} + \frac{6aex^2 \sqrt{a + bx^3}}{91b} + \frac{6agx^4 \sqrt{a + bx^3}}{187b}
 \end{aligned}$$

Mathematica [C] time = 0.253586, size = 158, normalized size = 0.23

$$\frac{2\sqrt{a + bx^3} \left(- (a + bx^3) \sqrt{\frac{bx^3}{a} + 1} (26a(187f + 180gx) - b(12155c + 9945dx + 33x^2(255e + 13x(17f + 15gx)))) + 585ax^5 \right)}{109395b^2 \sqrt{\frac{bx^3}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]

[Out] (2*Sqrt[a + b*x^3]*(-(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*(26*a*(187*f + 180*g*x) - b*(12155*c + 9945*d*x + 33*x^2*(255*e + 13*x*(17*f + 15*g*x)))) + 585*a*(-17*b*d + 8*a*g)*x*Hypergeometric2F1[-1/2, 1/3, 4/3, -(b*x^3)/a] - 84*15*a*b*e*x^2*Hypergeometric2F1[-1/2, 2/3, 5/3, -(b*x^3)/a]))/(109395*b^2*Sqrt[1 + (b*x^3)/a])

Maple [B] time = 0.008, size = 1197, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x)

[Out] g*(2/17*x^7*(b*x^3+a)^(1/2)+6/187/b*a*x^4*(b*x^3+a)^(1/2)-48/935*a^2/b^2*x*(b*x^3+a)^(1/2)-32/935*I*a^3/b^3*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))) + f*(2/15*x^6*(b*x^3+a)^(1/2)+2/45/b*a*x^3*(b*x^3+a)^(1/2)-4/45*a^2/b^2*(b*x^3+a)^(1/2))+e*(2/13*x^5*(b*x^3+a)^(1/2)+6/91/b*a*x^2*(b*x^3+a)^(1/2)+8/91*I/b^2*a^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))) + d*(2/11*x^4*(b*x^3+a)^(1/2)+6/55/b*a*x*(b*x^3+a)^(1/2)+4/55*I/b^2*a^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)

*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+2/9*c/b*(b*x^3+a)^(3/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2(bx^3 + a)^{\frac{3}{2}}c}{9b} + \int (gx^6 + fx^5 + ex^4 + dx^3)\sqrt{bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] 2/9*(b*x^3 + a)^(3/2)*c/b + integrate((g*x^6 + f*x^5 + e*x^4 + d*x^3)*sqrt(b*x^3 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(gx^6 + fx^5 + ex^4 + dx^3 + cx^2\right)\sqrt{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] integral((g*x^6 + f*x^5 + e*x^4 + d*x^3 + c*x^2)*sqrt(b*x^3 + a), x)

Sympy [A] time = 4.52438, size = 223, normalized size = 0.33

$$\frac{\sqrt{a}dx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{-\frac{1}{2}, \frac{4}{3}}{\frac{7}{3}} \left| \frac{bx^3 e^{i\pi}}{a} \right.\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt{a}ex^5\Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\frac{-\frac{1}{2}, \frac{5}{3}}{\frac{8}{3}} \left| \frac{bx^3 e^{i\pi}}{a} \right.\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{\sqrt{a}gx^7\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\frac{-\frac{1}{2}, \frac{7}{3}}{\frac{10}{3}} \left| \frac{bx^3 e^{i\pi}}{a} \right.\right)}{3\Gamma\left(\frac{10}{3}\right)} + c \left\{ \begin{array}{l} \frac{\sqrt{ax^3}}{3} \\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} \end{array} \right. \begin{array}{l} \text{for } b = \\ \text{other} \end{array}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2),x)
```

```
[Out] sqrt(a)*d*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*e*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + sqrt(a)*g*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + c*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + f*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + ax^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x^2, x)
```

3.447 $\int x\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)dx$

Optimal. Leaf size=667

$$\frac{2 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} a^{4/3} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (182a^{2/3} \sqrt[3]{be} + 55(1-\sqrt{3})(13bc-4af)) \operatorname{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3}) \sqrt[3]{a}}{(1+\sqrt{3}) \sqrt[3]{a}}\right)\right)}{5005b^{5/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a+bx^3}}$$

[Out] (2*a*(5*b*d - 2*a*g)*Sqrt[a + b*x^3])/(45*b^2) + (6*a*e*x*Sqrt[a + b*x^3])/(55*b) + (6*a*f*x^2*Sqrt[a + b*x^3])/(91*b) + (2*a*g*x^3*Sqrt[a + b*x^3])/(45*b) + (6*a*(13*b*c - 4*a*f)*Sqrt[a + b*x^3])/(91*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x*Sqrt[a + b*x^3]*(6435*c*x + 5005*d*x^2 + 4095*e*x^3 + 3465*f*x^4 + 3003*g*x^5))/45045 - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13*b*c - 4*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(182*a^(2/3)*b^(1/3)*e + 55*(1 - Sqrt[3])*(13*b*c - 4*a*f))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5005*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 1.04731, antiderivative size = 667, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {1826, 1836, 1888, 1886, 261, 1878, 218, 1877}

$$\frac{2 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} a^{4/3} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (182a^{2/3} \sqrt[3]{be} + 55(1-\sqrt{3})(13bc-4af)) F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx} + (1-\sqrt{3}) \sqrt[3]{a}}{\sqrt[3]{bx} + (1+\sqrt{3}) \sqrt[3]{a}}\right)\right)}{5005b^{5/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4), x]


```
[Out] (2*a*(5*b*d - 2*a*g)*Sqrt[a + b*x^3])/(45*b^2) + (6*a*e*x*Sqrt[a + b*x^3])/
(55*b) + (6*a*f*x^2*Sqrt[a + b*x^3])/(91*b) + (2*a*g*x^3*Sqrt[a + b*x^3])/(
45*b) + (6*a*(13*b*c - 4*a*f)*Sqrt[a + b*x^3])/(91*b^(5/3)*((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)) + (2*x*Sqrt[a + b*x^3]*(6435*c*x + 5005*d*x^2 + 4095*e
*x^3 + 3465*f*x^4 + 3003*g*x^5))/45045 - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/
3)*(13*b*c - 4*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticE[ArcSin[((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(91*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a
^(4/3)*(182*a^(2/3)*b^(1/3)*e + 55*(1 - Sqrt[3])*(13*b*c - 4*a*f))*(a^(1/3)
+ b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)
)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5005*b^(5/3)*S
qrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a + b*x^3])
```

Rule 1826

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i
]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
+ b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1836

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
+ 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
+ 1)/(2*n)]) /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)]) /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[Coeff[Pq, x, n - 1], Int[x^(n - 1)*(a + b*x^n)^p, x] + Int[ExpandToSum[Pq - Coeff[Pq, x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)]/((1 + Sqrt[3])*s + r*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)]/((1 + Sqrt[3])*s + r*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)dx &= \frac{2x\sqrt{a+bx^3}(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5)}{45045} + \frac{1}{2} \\
&= \frac{2agx^3\sqrt{a+bx^3}}{45b} + \frac{2x\sqrt{a+bx^3}(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5)}{45045} \\
&= \frac{6afx^2\sqrt{a+bx^3}}{91b} + \frac{2agx^3\sqrt{a+bx^3}}{45b} + \frac{2x\sqrt{a+bx^3}(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5)}{45045} \\
&= \frac{6aex\sqrt{a+bx^3}}{55b} + \frac{6afx^2\sqrt{a+bx^3}}{91b} + \frac{2agx^3\sqrt{a+bx^3}}{45b} + \frac{2x\sqrt{a+bx^3}(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5)}{45045} \\
&= \frac{6aex\sqrt{a+bx^3}}{55b} + \frac{6afx^2\sqrt{a+bx^3}}{91b} + \frac{2agx^3\sqrt{a+bx^3}}{45b} + \frac{2x\sqrt{a+bx^3}(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5)}{45045} \\
&= \frac{2a(5bd-2ag)\sqrt{a+bx^3}}{45b^2} + \frac{6aex\sqrt{a+bx^3}}{55b} + \frac{6afx^2\sqrt{a+bx^3}}{91b} + \frac{2agx^3\sqrt{a+bx^3}}{45b} \\
&= \frac{2a(5bd-2ag)\sqrt{a+bx^3}}{45b^2} + \frac{6aex\sqrt{a+bx^3}}{55b} + \frac{6afx^2\sqrt{a+bx^3}}{91b} + \frac{2agx^3\sqrt{a+bx^3}}{45b}
\end{aligned}$$

Mathematica [C] time = 0.193972, size = 143, normalized size = 0.21

$$\frac{\sqrt{a+bx^3}\left(495bx^2(13bc-4af) {}_2F_1\left(-\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^3}{a}\right) - 4(a+bx^3)\sqrt{\frac{bx^3}{a}+1}(286ag-b(715d+585ex+495fx^2+429gx^3))\right)}{12870b^2\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4), x]

[Out] (Sqrt[a + b*x^3]*(-4*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*(286*a*g - b*(715*d + 585*e*x + 495*f*x^2 + 429*g*x^3)) - 2340*a*b*e*x*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^3)/a)] + 495*b*(13*b*c - 4*a*f)*x^2*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^3)/a)])/(12870*b^2*Sqrt[1 + (b*x^3)/a])

Maple [B] time = 0.006, size = 1311, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^{(1/2)}, x)$

[Out] $g*(2/15*x^6*(b*x^3+a)^{(1/2)}+2/45/b*a*x^3*(b*x^3+a)^{(1/2)}-4/45*a^2/b^2*(b*x^3+a)^{(1/2)})+f*(2/13*x^5*(b*x^3+a)^{(1/2)}+6/91/b*a*x^2*(b*x^3+a)^{(1/2)}+8/91*I/b^2*a^2*3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)})/(b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}*EllipticE(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)})+1/b*(-b^2*a)^{(1/3)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)})))+e*(2/11*x^4*(b*x^3+a)^{(1/2)}+6/55/b*a*x*(b*x^3+a)^{(1/2)}+4/55*I/b^2*a^2*3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)})/(b*x^3+a)^{(1/2)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)})))+2/9*d/b*(b*x^3+a)^{(3/2)}+c*(2/7*x^2*(b*x^3+a)^{(1/2)}-2/7*I*a*3^{(1/2)}/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)})/(b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)})+1/b*(-b^2*a)^{(1/3)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)})+1/b*(-b^2*a)^{(1/3)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + ax} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(gx^5 + fx^4 + ex^3 + dx^2 + cx\right)\sqrt{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] integral((g*x^5 + f*x^4 + e*x^3 + d*x^2 + c*x)*sqrt(b*x^3 + a), x)

Sympy [A] time = 4.30689, size = 223, normalized size = 0.33

$$\frac{\sqrt{ac}x^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(-\frac{1}{2}, \frac{2}{3} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt{aex^4}\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(-\frac{1}{2}, \frac{4}{3} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt{afx^5}\Gamma\left(\frac{5}{3}\right) {}_2F_1\left(-\frac{1}{2}, \frac{5}{3} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + d \left(\left\{ \begin{array}{l} \frac{\sqrt{ax^3}}{3} \\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} \end{array} \right. \right. \text{for } b > 0 \\ \text{other} \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2),x)

[Out] sqrt(a)*c*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*e*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*f*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + d*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + g*Piecw

```
ise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(45*b)
+ 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + ax} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x, x)
```

3.448 $\int \sqrt{a + bx^3} (c + dx + ex^2 + fx^3 + gx^4) dx$

Optimal. Leaf size=639

$$2 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} a (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \operatorname{EllipticF} \left(\sin^{-1} \left(\frac{(1 - \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{(1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}} \right), -7 - 4\sqrt{3} \right) (91 \sqrt[3]{b} (11bc - 2af) - 55 \sqrt[3]{a})$$

$$5005b^{5/3} \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

```
[Out] (2*a*e*Sqrt[a + b*x^3])/(9*b) + (6*a*f*x*Sqrt[a + b*x^3])/(55*b) + (6*a*g*x^2*Sqrt[a + b*x^3])/(91*b) + (6*a*(13*b*d - 4*a*g)*Sqrt[a + b*x^3])/(91*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*Sqrt[a + b*x^3]*(9009*c*x + 6435*d*x^2 + 5005*e*x^3 + 4095*f*x^4 + 3465*g*x^5))/45045 - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13*b*d - 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(91*b^(1/3)*(11*b*c - 2*a*f) - 55*(1 - Sqrt[3])*a^(1/3)*(13*b*d - 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5005*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.719342, antiderivative size = 639, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {1853, 1888, 1886, 261, 1878, 218, 1877}

$$2 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} a (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} F \left(\sin^{-1} \left(\frac{\sqrt[3]{bx} + (1 - \sqrt{3}) \sqrt[3]{a}}{\sqrt[3]{bx} + (1 + \sqrt{3}) \sqrt[3]{a}} \right), -7 - 4\sqrt{3} \right) (91 \sqrt[3]{b} (11bc - 2af) - 55 (1 - \sqrt{3}) \sqrt[3]{a})$$

$$5005b^{5/3} \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4), x]

```
[Out] (2*a*e*Sqrt[a + b*x^3])/(9*b) + (6*a*f*x*Sqrt[a + b*x^3])/(55*b) + (6*a*g*x^2*Sqrt[a + b*x^3])/(91*b) + (6*a*(13*b*d - 4*a*g)*Sqrt[a + b*x^3])/(91*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*Sqrt[a + b*x^3]*(9009*c*x + 6435*d*x^2 + 5005*e*x^3 + 4095*f*x^4 + 3465*g*x^5))/45045 - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13*b*d - 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(91*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(91*b^(1/3)*(11*b*c - 2*a*f) - 55*(1 - Sqrt[3])*a^(1/3)*(13*b*d - 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(5005*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1853

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[(a + b*x^n)^p*Sum[(Coeff[Pq, x, i]*x^(i + 1))/(n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(n*p + i + 1), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*n)]) /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[Coeff[Pq, x, n - 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```


NeQ[p, -1]

Rule 1878

```
Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)]/((1 + Sqrt[3])*s + r*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)]/((1 + Sqrt[3])*s + r*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4) dx &= \frac{2\sqrt{a+bx^3}(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5)}{45045} + \frac{1}{2}(3a) \\
&= \frac{6agx^2\sqrt{a+bx^3}}{91b} + \frac{2\sqrt{a+bx^3}(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5)}{45045} \\
&= \frac{6afx\sqrt{a+bx^3}}{55b} + \frac{6agx^2\sqrt{a+bx^3}}{91b} + \frac{2\sqrt{a+bx^3}(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5)}{45045} \\
&= \frac{6afx\sqrt{a+bx^3}}{55b} + \frac{6agx^2\sqrt{a+bx^3}}{91b} + \frac{2\sqrt{a+bx^3}(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5)}{45045} \\
&= \frac{2ae\sqrt{a+bx^3}}{9b} + \frac{6afx\sqrt{a+bx^3}}{55b} + \frac{6agx^2\sqrt{a+bx^3}}{91b} + \frac{2\sqrt{a+bx^3}(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5)}{45045} \\
&= \frac{2ae\sqrt{a+bx^3}}{9b} + \frac{6afx\sqrt{a+bx^3}}{55b} + \frac{6agx^2\sqrt{a+bx^3}}{91b} + \frac{6a(13bd-4ag)\sqrt{a+bx^3}}{91b^{5/3}((1+\sqrt{3})\sqrt[3]{a+bx^3})}
\end{aligned}$$

Mathematica [C] time = 0.148057, size = 135, normalized size = 0.21

$$\frac{\sqrt{a+bx^3} \left(234x(11bc-2af) {}_2F_1\left(-\frac{1}{2}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right) + 99x^2(13bd-4ag) {}_2F_1\left(-\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^3}{a}\right) + 4(a+bx^3)\sqrt{\frac{bx^3}{a}+1}(143e+9g) \right)}{2574b\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]

[Out] (Sqrt[a + b*x^3]*(4*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*(143*e + 9*x*(13*f + 11*g*x)) + 234*(11*b*c - 2*a*f)*x*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^3)/a)] + 99*(13*b*d - 4*a*g)*x^2*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^3)/a)]))/(2574*b*Sqrt[1 + (b*x^3)/a])

Maple [B] time = 0.005, size = 1557, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^{(1/2)}, x)$

[Out]
$$g*(2/13*x^5*(b*x^3+a)^{(1/2)}+6/91/b*a*x^2*(b*x^3+a)^{(1/2)}+8/91*I/b^2*a^2*3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3)^{(1/2)}*b/(-b^2*a)^{(1/3))^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))})^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3)^{(1/2)}*b/(-b^2*a)^{(1/3))^{(1/2)}/(b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*EllipticE(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3)^{(1/2)}*b/(-b^2*a)^{(1/3))^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))})^{(1/2)}+1/b*(-b^2*a)^{(1/3)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3)^{(1/2)}*b/(-b^2*a)^{(1/3))^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))})^{(1/2)})))+f*(2/11*x^4*(b*x^3+a)^{(1/2)}+6/55/b*a*x*(b*x^3+a)^{(1/2)}+4/55*I/b^2*a^2*3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3)^{(1/2)}*b/(-b^2*a)^{(1/3))^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))})^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3)^{(1/2)}*b/(-b^2*a)^{(1/3))^{(1/2)}/(b*x^3+a)^{(1/2)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3)^{(1/2)}*b/(-b^2*a)^{(1/3))^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))})^{(1/2)}))+2/9*e/b*(b*x^3+a)^{(3/2)}+d*(2/7*x^2*(b*x^3+a)^{(1/2)}-2/7*I*a*3^{(1/2)}/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3)^{(1/2)}*b/(-b^2*a)^{(1/3))^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))})^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3)^{(1/2)}*b/(-b^2*a)^{(1/3))^{(1/2)}/(b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*EllipticE(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3)^{(1/2)}*b/(-b^2*a)^{(1/3))^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))})^{(1/2)}+1/b*(-b^2*a)^{(1/3)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3)^{(1/2)}*b/(-b^2*a)^{(1/3))^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))})^{(1/2)})))+c*(2/5*x*(b*x^3+a)^{(1/2)}-2/5*I*a*3^{(1/2)}/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3)^{(1/2)}*b/(-b^2*a)^{(1/3))^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))})^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3)^{(1/2)}*b/(-b^2*a)^{(1/3))^{(1/2)}/(b*x^3+a)^{(1/2)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3)^{(1/2)}*b/(-b^2*a)^{(1/3))^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))})^{(1/2)}))$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a), x)

Sympy [A] time = 3.90194, size = 194, normalized size = 0.3

$$\frac{\sqrt{ac}x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(-\frac{1}{2}, \frac{1}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{ad}x^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(-\frac{1}{2}, \frac{2}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt{a}fx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(-\frac{1}{2}, \frac{4}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt{a}gx^5\Gamma\left(\frac{5}{3}\right) {}_2F_1\left(-\frac{1}{2}, \frac{5}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2),x)

[Out] sqrt(a)*c*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + sqrt(a)*d*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*f*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*g*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + e*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b),

True))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="giac")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a), x)

$$3.449 \quad \int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x} dx$$

Optimal. Leaf size=620

$$2^{3/4} \sqrt{2 + \sqrt{3}} a \left(\sqrt[3]{a} + \sqrt[3]{bx} \right) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{\left((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx} \right)^2}} \left(-55(1 - \sqrt{3}) \sqrt[3]{a} b^{2/3} e - 14ag + 77bd \right) \text{EllipticF} \left(\sin^{-1} \left(\frac{(1 - \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{(1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}} \right) \right) \\ \hline 385b^{4/3} \sqrt{\frac{\sqrt[3]{a} \left(\sqrt[3]{a} + \sqrt[3]{bx} \right)}{\left((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx} \right)^2}} \sqrt{a + bx^3}$$

[Out] (2*a*f*Sqrt[a + b*x^3])/(9*b) + (6*a*g*x*Sqrt[a + b*x^3])/(55*b) + (6*a*e*Sqrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*Sqrt[a + b*x^3]*(1155*c*x + 693*d*x^2 + 495*e*x^3 + 385*f*x^4 + 315*g*x^5))/(3465*x) - (2*Sqrt[a]*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(7*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(77*b*d - 55*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e - 14*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(385*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 0.547109, antiderivative size = 620, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {1826, 1832, 266, 63, 208, 1888, 1886, 261, 1878, 218, 1877}

$$2^{3/4} \sqrt{2 + \sqrt{3}} a \left(\sqrt[3]{a} + \sqrt[3]{bx} \right) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{\left((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx} \right)^2}} \left(-55(1 - \sqrt{3}) \sqrt[3]{a} b^{2/3} e - 14ag + 77bd \right) F \left(\sin^{-1} \left(\frac{\sqrt[3]{bx} + (1 - \sqrt{3}) \sqrt[3]{a}}{\sqrt[3]{bx} + (1 + \sqrt{3}) \sqrt[3]{a}} \right) \right) \\ \hline 385b^{4/3} \sqrt{\frac{\sqrt[3]{a} \left(\sqrt[3]{a} + \sqrt[3]{bx} \right)}{\left((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx} \right)^2}} \sqrt{a + bx^3}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x,x]

```
[Out] (2*a*f*Sqrt[a + b*x^3])/(9*b) + (6*a*g*x*Sqrt[a + b*x^3])/(55*b) + (6*a*e*S
qrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*Sqrt[a
+ b*x^3]*(1155*c*x + 693*d*x^2 + 495*e*x^3 + 385*f*x^4 + 315*g*x^5))/(3465
*x) - (2*Sqrt[a]*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (3*3^(1/4)*Sqrt[2
- Sqrt[3]]*a^(4/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((
1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(7*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a
*(77*b*d - 55*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e - 14*a*g)*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(385*b^(4/3)*Sqrt[(a^(1/3)
*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x
^3])
```

Rule 1826

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i
]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
+ b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_)*(x_)^(n_)]), x_Symbol] := Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^m_*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1888

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*n)]) /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]

Rule 1886

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[Coeff[Pq, x, n - 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 1878

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

Rule 218

Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)]/((1 + Sqrt[3])*s + r*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &

& PosQ[a]

Rule 1877

```
Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x} dx &= \frac{2\sqrt{a+bx^3}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x} + \frac{1}{2}(3a) \int \frac{\sqrt{a+bx^3}}{x} dx \\
 &= \frac{2\sqrt{a+bx^3}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x} + \frac{1}{2}(3a) \int \frac{\sqrt{a+bx^3}}{x} dx \\
 &= \frac{6agx\sqrt{a+bx^3}}{55b} + \frac{2\sqrt{a+bx^3}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x} \\
 &= \frac{6agx\sqrt{a+bx^3}}{55b} + \frac{2\sqrt{a+bx^3}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x} \\
 &= \frac{2af\sqrt{a+bx^3}}{9b} + \frac{6agx\sqrt{a+bx^3}}{55b} + \frac{2\sqrt{a+bx^3}(1155cx+693dx^2+495ex^3)}{3465x} \\
 &= \frac{2af\sqrt{a+bx^3}}{9b} + \frac{6agx\sqrt{a+bx^3}}{55b} + \frac{6ae\sqrt{a+bx^3}}{7b^{2/3}((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})} + \frac{2\sqrt{a+bx^3}}{x}
 \end{aligned}$$

Mathematica [C] time = 0.317007, size = 185, normalized size = 0.3

$$\frac{4\sqrt{\frac{bx^3}{a} + 1} \left(\sqrt{a + bx^3} (11af + 9agx + 33bc + 11bfx^3 + 9bgx^4) - 33\sqrt{abc} \tanh^{-1} \left(\frac{\sqrt{a+bx^3}}{\sqrt{a}} \right) \right) + 18x\sqrt{a + bx^3}(11bd - 2ag)}{198b\sqrt{\frac{bx^3}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x,x]

[Out] (4*Sqrt[1 + (b*x^3)/a]*(Sqrt[a + b*x^3]*(33*b*c + 11*a*f + 9*a*g*x + 11*b*f*x^3 + 9*b*g*x^4) - 33*Sqrt[a]*b*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]) + 18*(11*b*d - 2*a*g)*x*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, 1/3, 4/3, -(b*x^3)/a] + 99*b*e*x^2*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, 2/3, 5/3, -(b*x^3)/a])/(198*b*Sqrt[1 + (b*x^3)/a])

Maple [B] time = 0.008, size = 1118, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x,x)

[Out] g*(2/11*x^4*(b*x^3+a)^(1/2)+6/55/b*a*x*(b*x^3+a)^(1/2)+4/55*I/b^2*a^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+2/9*f/b*(b*x^3+a)^(3/2)+e*(2/7*x^2*(b*x^3+a)^(1/2)-2/7*I*a*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))

$$\frac{1}{3} - \frac{1}{2} I^3 \sqrt{\frac{1}{b(-b^2 a)^{1/3}}} \cdot 3^{1/2} \cdot b / (-b^2 a)^{1/3} \sqrt{\frac{1}{2}}, (I^3 \sqrt{\frac{1}{b(-b^2 a)^{1/3}}} / (-3/2 \sqrt{\frac{1}{b(-b^2 a)^{1/3}}} + 1/2 I^3 \sqrt{\frac{1}{b(-b^2 a)^{1/3}}})^{1/2}) + d \cdot (2/5 x \sqrt{(b x^3 + a)^{1/2}} - 2/5 I^3 \sqrt{\frac{1}{b(-b^2 a)^{1/3}}} \cdot (I^3 (x + 1/2 \sqrt{\frac{1}{b(-b^2 a)^{1/3}}} - 1/2 I^3 \sqrt{\frac{1}{b(-b^2 a)^{1/3}}}) \cdot 3^{1/2} \cdot b / (-b^2 a)^{1/3} \sqrt{\frac{1}{2}}) \cdot ((x - 1/b \sqrt{\frac{1}{b(-b^2 a)^{1/3}}}) / (-3/2 \sqrt{\frac{1}{b(-b^2 a)^{1/3}}} + 1/2 I^3 \sqrt{\frac{1}{b(-b^2 a)^{1/3}}})^{1/2} \cdot (-I^3 (x + 1/2 \sqrt{\frac{1}{b(-b^2 a)^{1/3}}} + 1/2 I^3 \sqrt{\frac{1}{b(-b^2 a)^{1/3}}}) \cdot 3^{1/2} \cdot b / (-b^2 a)^{1/3} \sqrt{\frac{1}{2}}) \cdot (b x^3 + a)^{1/2} \cdot \text{EllipticF}(1/3 \sqrt{\frac{1}{2}} \cdot (I^3 (x + 1/2 \sqrt{\frac{1}{b(-b^2 a)^{1/3}}} - 1/2 I^3 \sqrt{\frac{1}{b(-b^2 a)^{1/3}}}) \cdot 3^{1/2} \cdot b / (-b^2 a)^{1/3} \sqrt{\frac{1}{2}}) \sqrt{\frac{1}{2}}, (I^3 \sqrt{\frac{1}{b(-b^2 a)^{1/3}}} / (-3/2 \sqrt{\frac{1}{b(-b^2 a)^{1/3}}} + 1/2 I^3 \sqrt{\frac{1}{b(-b^2 a)^{1/3}}})^{1/2}) + c \cdot (2/3 \sqrt{(b x^3 + a)^{1/2}} - 2/3 a^{1/2} \cdot \text{arctanh}(\sqrt{(b x^3 + a)^{1/2}} / a^{1/2}))$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g x^4 + f x^3 + e x^2 + d x + c) \sqrt{b x^3 + a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x,x, algorithm="maxima")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(g x^4 + f x^3 + e x^2 + d x + c) \sqrt{b x^3 + a}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x,x, algorithm="fricas")

[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x, x)

Sympy [A] time = 9.2548, size = 235, normalized size = 0.38

$$-\frac{2\sqrt{ac} \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx^2}}\right)}{3} + \frac{\sqrt{adx}\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{-\frac{1}{2}, \frac{1}{3}}{\frac{4}{3}} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{aex^2}\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{-\frac{1}{2}, \frac{2}{3}}{\frac{5}{3}} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt{agx^4}\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{-\frac{1}{2}, \frac{4}{3}}{\frac{7}{3}} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x,x)

[Out] $-2\sqrt{a}c\operatorname{asinh}(\sqrt{a}/(\sqrt{b}x^{3/2}))/3 + \sqrt{a}d\sqrt{x}\Gamma(1/3)\operatorname{hyper}((-1/2, 1/3), (4/3,), b x^3 \exp(\operatorname{I}\pi)/a)/(3\Gamma(4/3)) + \sqrt{a}e x^2 \Gamma(2/3)\operatorname{hyper}((-1/2, 2/3), (5/3,), b x^3 \exp(\operatorname{I}\pi)/a)/(3\Gamma(5/3)) + \sqrt{a}g x^4 \Gamma(4/3)\operatorname{hyper}((-1/2, 4/3), (7/3,), b x^3 \exp(\operatorname{I}\pi)/a)/(3\Gamma(7/3)) + 2ac/(3\sqrt{b}x^{3/2}\sqrt{a/(bx^3+1)}) + 2\sqrt{b}c x^{3/2}/(3\sqrt{a/(bx^3+1)}) + f\operatorname{Piecewise}(\sqrt{a}x^{3/3}, \operatorname{Eq}(b, 0)), (2(a + bx^3)^{3/2}/(9b), \operatorname{True}))$

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x,x, algorithm="giac")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x, x)

$$3.450 \quad \int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^2} dx$$

Optimal. Leaf size=638

$$3^{3/4} \sqrt{2 + \sqrt{3}} \sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (14a^{2/3} \sqrt[3]{be} - 5(1 - \sqrt{3})(2af + 7bc)) \text{EllipticF} \left(\sin^{-1} \left(\frac{(1-\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{(1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}} \right) \right) \\ \frac{35b^{2/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}}$$

[Out] (2*a*g*Sqrt[a + b*x^3])/(9*b) - (3*c*Sqrt[a + b*x^3])/x + (3*(7*b*c + 2*a*f)*Sqrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*Sqrt[a + b*x^3]*(315*c*x + 105*d*x^2 + 63*e*x^3 + 45*f*x^4 + 35*g*x^5))/(315*x^2) - (2*Sqrt[a]*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(7*b*c + 2*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(14*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*Sqrt[a + b*x^3]) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(14*a^(2/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(7*b*c + 2*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(35*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*Sqrt[a + b*x^3])

Rubi [A] time = 0.649673, antiderivative size = 638, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {1826, 1835, 1832, 266, 63, 208, 1886, 261, 1878, 218, 1877}

$$3^{3/4} \sqrt{2 + \sqrt{3}} \sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (14a^{2/3} \sqrt[3]{be} - 5(1 - \sqrt{3})(2af + 7bc)) F \left(\sin^{-1} \left(\frac{\sqrt[3]{bx} + (1-\sqrt{3}) \sqrt[3]{a}}{\sqrt[3]{bx} + (1+\sqrt{3}) \sqrt[3]{a}} \right) \right) - 7 - 4 \\ \frac{35b^{2/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2,x]

```
[Out] (2*a*g*Sqrt[a + b*x^3])/(9*b) - (3*c*Sqrt[a + b*x^3])/x + (3*(7*b*c + 2*a*f)
)*Sqrt[a + b*x^3]/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*Sqr
t[a + b*x^3]*(315*c*x + 105*d*x^2 + 63*e*x^3 + 45*f*x^4 + 35*g*x^5))/(315*x
^2) - (2*Sqrt[a]*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (3*3^(1/4)*Sqrt[2
- Sqrt[3]]*a^(1/3)*(7*b*c + 2*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(14*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x)
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (3^(3/4)*Sqrt[2
+ Sqrt[3]]*a^(1/3)*(14*a^(2/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(7*b*c + 2*a*f)
)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3
) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(35*b
^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1826

```
Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i
]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
+ b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1835

```
Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_)*(x_)^(n_)]), x_Symbol] := Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1886

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[Coeff[Pq, x, n - 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1

Rule 261

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 1878

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

Rule 218

Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b}, x] && PosQ[a]

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)]], -7 - 4*Sqrt[3])]/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^2} dx &= \frac{2\sqrt{a+bx^3}(315cx+105dx^2+63ex^3+45fx^4+35gx^5)}{315x^2} + \frac{1}{2}(3a) \int \frac{2c+\frac{2d}{3}}{x} dx \\ &= -\frac{3c\sqrt{a+bx^3}}{x} + \frac{2\sqrt{a+bx^3}(315cx+105dx^2+63ex^3+45fx^4+35gx^5)}{315x^2} \\ &= -\frac{3c\sqrt{a+bx^3}}{x} + \frac{2\sqrt{a+bx^3}(315cx+105dx^2+63ex^3+45fx^4+35gx^5)}{315x^2} \\ &= -\frac{3c\sqrt{a+bx^3}}{x} + \frac{2\sqrt{a+bx^3}(315cx+105dx^2+63ex^3+45fx^4+35gx^5)}{315x^2} \\ &= \frac{2ag\sqrt{a+bx^3}}{9b} - \frac{3c\sqrt{a+bx^3}}{x} + \frac{2\sqrt{a+bx^3}(315cx+105dx^2+63ex^3+45fx^4+35gx^5)}{315x^2} \\ &= \frac{2ag\sqrt{a+bx^3}}{9b} - \frac{3c\sqrt{a+bx^3}}{x} + \frac{3(7bc+2af)\sqrt{a+bx^3}}{7b^{2/3}((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})} + \frac{2\sqrt{a+bx^3}(315cx+105dx^2+63ex^3+45fx^4+35gx^5)}{315x^2} \end{aligned}$$

Mathematica [C] time = 0.244137, size = 211, normalized size = 0.33

$$-\frac{c\sqrt{a+bx^3} {}_2F_1\left(-\frac{1}{2}, -\frac{1}{3}; \frac{2}{3}; -\frac{bx^3}{a}\right)}{x\sqrt{\frac{bx^3}{a}+1}} + \frac{2}{3}d\left(\sqrt{a+bx^3} - \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right) + \frac{ex\sqrt{a+bx^3} {}_2F_1\left(-\frac{1}{2}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}} + \frac{fx^2\sqrt{a+bx^3}}{x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2,x]

[Out] $(2*g*(a + b*x^3)^{(3/2)})/(9*b) + (2*d*(\text{Sqrt}[a + b*x^3] - \text{Sqrt}[a]*\text{ArcTanh}[\text{Sqrt}[a + b*x^3]/\text{Sqrt}[a]]))/3 - (c*\text{Sqrt}[a + b*x^3]*\text{Hypergeometric2F1}[-1/2, -1/3, 2/3, -((b*x^3)/a)])/(x*\text{Sqrt}[1 + (b*x^3)/a]) + (e*x*\text{Sqrt}[a + b*x^3]*\text{Hypergeometric2F1}[-1/2, 1/3, 4/3, -((b*x^3)/a)])/\text{Sqrt}[1 + (b*x^3)/a] + (f*x^2*\text{Sqrt}[a + b*x^3]*\text{Hypergeometric2F1}[-1/2, 2/3, 5/3, -((b*x^3)/a)])/(2*\text{Sqrt}[1 + (b*x^3)/a])$

Maple [B] time = 0.009, size = 1248, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^2,x)

[Out] $2/9*g/b*(b*x^3+a)^{(3/2)}+f*(2/7*x^2*(b*x^3+a)^{(1/2)}-2/7*I*a*3^{(1/2)}/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*EllipticE(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}))+1/b*(-b^2*a)^{(1/3)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)})))+e*(2/5*x*(b*x^3+a)^{(1/2)}-2/5*I*a*3^{(1/2)}/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/2)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}))+d*(2/3*(b*x^3+a)^{(1/2)}-2/3*a^{(1/2)}*arctanh((b*x^3+a)^{(1/2)}/a^{(1/2)}))+c*(-(b*x^3+a)^{(1/2)}/x-I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*EllipticE(1/3*3^{(1/2)}$

$$\begin{aligned} & (1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/ \\ & (-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/ \\ & 2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1 \\ & /2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(- \\ & b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2* \\ & I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))) \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^2,x, algorithm="maxima")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^2,x, algorithm="fricas")

[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^2, x)

Sympy [A] time = 5.50739, size = 236, normalized size = 0.37

$$\frac{\sqrt{ac}\Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(-\frac{1}{2}, -\frac{1}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \frac{2\sqrt{ad} \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx^3}}\right)}{3} + \frac{\sqrt{aex}\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(-\frac{1}{2}, \frac{1}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a}fx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(-\frac{1}{2}, \frac{2}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**2,x)

[Out] sqrt(a)*c*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*sqrt(a)*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + sqrt(a)*e*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + sqrt(a)*f*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + 2*a*d/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3+1))) + 2*sqrt(b)*d*x**(3/2)/(3*sqrt(a/(b*x**3+1))) + g*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^2,x, algorithm="giac")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^2, x)

$$3.451 \quad \int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^3} dx$$

Optimal. Leaf size=640

$$\frac{3^{3/4} \sqrt{2+\sqrt{3}} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \operatorname{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{(1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}\right), -7-4\sqrt{3}\right) (7\sqrt[3]{b}(4af+5bc) - 10(1-\sqrt{3}) \sqrt[3]{a}(2a+bx))}{70b^{2/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a+bx^3}}$$

[Out] (3*c*Sqrt[a + b*x^3])/(2*x^2) - (3*d*Sqrt[a + b*x^3])/x + (3*(7*b*d + 2*a*g)*Sqrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*Sqrt[a + b*x^3]*(105*c*x - 105*d*x^2 - 35*e*x^3 - 21*f*x^4 - 15*g*x^5))/(105*x^3) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(7*b*d + 2*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(14*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*Sqrt[a + b*x^3]) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*(7*b^(1/3)*(5*b*c + 4*a*f) - 10*(1 - Sqrt[3])*a^(1/3)*(7*b*d + 2*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(70*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*Sqrt[a + b*x^3])

Rubi [A] time = 0.764714, antiderivative size = 640, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1826, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$\frac{3^{3/4} \sqrt{2+\sqrt{3}} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx} + (1-\sqrt{3}) \sqrt[3]{a}}{\sqrt[3]{bx} + (1+\sqrt{3}) \sqrt[3]{a}}\right) \mid -7-4\sqrt{3}\right) (7\sqrt[3]{b}(4af+5bc) - 10(1-\sqrt{3}) \sqrt[3]{a}(2a+bx))}{70b^{2/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3,x]

```
[Out] (3*c*Sqrt[a + b*x^3])/(2*x^2) - (3*d*Sqrt[a + b*x^3])/x + (3*(7*b*d + 2*a*g)
)*Sqrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*Sqr
t[a + b*x^3]*(105*c*x - 105*d*x^2 - 35*e*x^3 - 21*f*x^4 - 15*g*x^5))/(105*x
^3) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (3*3^(1/4)*Sqrt[2
- Sqrt[3]]*a^(1/3)*(7*b*d + 2*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)], -7 - 4*Sqrt[3]])/(14*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x)
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (3^(3/4)*Sqrt[2
+ Sqrt[3]]*(7*b^(1/3)*(5*b*c + 4*a*f) - 10*(1 - Sqrt[3])*a^(1/3)*(7*b*d +
2*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(70*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1826

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^(m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i
]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^(m*(a
+ b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1835

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_)*(x_)^(n_)]), x_Symbol] := Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1878

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

Rule 218

Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)]/((1 + Sqrt[3])*s + r*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b}, x] && PosQ[a]

Rule 1877

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)]/((1 + Sqrt[3])*s + r*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^3} dx &= -\frac{2\sqrt{a+bx^3}(105cx-105dx^2-35ex^3-21fx^4-15gx^5)}{105x^3} + \frac{1}{2}(3a) \int \frac{-2c}{x^3} dx \\
&= \frac{3c\sqrt{a+bx^3}}{2x^2} - \frac{2\sqrt{a+bx^3}(105cx-105dx^2-35ex^3-21fx^4-15gx^5)}{105x^3} - \frac{3a}{8} \\
&= \frac{3c\sqrt{a+bx^3}}{2x^2} - \frac{3d\sqrt{a+bx^3}}{x} - \frac{2\sqrt{a+bx^3}(105cx-105dx^2-35ex^3-21fx^4-15gx^5)}{105x^3} \\
&= \frac{3c\sqrt{a+bx^3}}{2x^2} - \frac{3d\sqrt{a+bx^3}}{x} - \frac{2\sqrt{a+bx^3}(105cx-105dx^2-35ex^3-21fx^4-15gx^5)}{105x^3} \\
&= \frac{3c\sqrt{a+bx^3}}{2x^2} - \frac{3d\sqrt{a+bx^3}}{x} - \frac{2\sqrt{a+bx^3}(105cx-105dx^2-35ex^3-21fx^4-15gx^5)}{105x^3} \\
&= \frac{3c\sqrt{a+bx^3}}{2x^2} - \frac{3d\sqrt{a+bx^3}}{x} + \frac{3(7bd+2ag)\sqrt{a+bx^3}}{7b^{2/3}\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}\right)} - \frac{2\sqrt{a+bx^3}}{x} \\
&= \frac{3c\sqrt{a+bx^3}}{2x^2} - \frac{3d\sqrt{a+bx^3}}{x} + \frac{3(7bd+2ag)\sqrt{a+bx^3}}{7b^{2/3}\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}\right)} - \frac{2\sqrt{a+bx^3}}{x}
\end{aligned}$$

Mathematica [C] time = 0.410574, size = 218, normalized size = 0.34

$$\frac{x \left(x \left(4e\sqrt{\frac{bx^3}{a}} + 1 \left(\sqrt{a+bx^3} - \sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a+bx^3}}{\sqrt{a}} \right) \right) + 6fx\sqrt{a+bx^3} {}_2F_1 \left(-\frac{1}{2}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a} \right) + 3gx^2\sqrt{a+bx^3} {}_2F_1 \left(-\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^3}{a} \right) \right)}{6x^2\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3,x]

[Out] (-3*c*Sqrt[a + b*x^3]*Hypergeometric2F1[-2/3, -1/2, 1/3, -((b*x^3)/a)] + x*(-6*d*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, -1/3, 2/3, -((b*x^3)/a)] + x*(4*e*Sqrt[1 + (b*x^3)/a]*(Sqrt[a + b*x^3] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^3]

$$\frac{1}{\sqrt{a}} + 6fx\sqrt{a+bx^3} \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1}{3}, \frac{4}{3}, -\frac{(bx^3)}{a}\right] + 3g^2x^2\sqrt{a+bx^3} \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, -\frac{(bx^3)}{a}\right] \Big) / (6x^2\sqrt{1+(bx^3)/a})$$

Maple [B] time = 0.009, size = 1529, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}((gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{1/2}/x^3, x)$

[Out] $g(2/7x^2(bx^3+a)^{1/2} - 2/7Ia^{1/2}/b(-b^2a)^{1/3}(I(x+1/2/b(-b^2a)^{1/3} - 1/2I^{3/2}/b(-b^2a)^{1/3}))^{3/2} * b/(-b^2a)^{1/3})^{1/2} * ((x-1/b(-b^2a)^{1/3})/(-3/2/b(-b^2a)^{1/3} + 1/2I^{3/2}/b(-b^2a)^{1/3}))^{1/2} * (-I(x+1/2/b(-b^2a)^{1/3} + 1/2I^{3/2}/b(-b^2a)^{1/3}))^{3/2} * b/(-b^2a)^{1/3})^{1/2} / (bx^3+a)^{1/2} * ((-3/2/b(-b^2a)^{1/3} + 1/2I^{3/2}/b(-b^2a)^{1/3}) * \operatorname{EllipticE}(1/3, 3^{1/2} * (I(x+1/2/b(-b^2a)^{1/3} - 1/2I^{3/2}/b(-b^2a)^{1/3}))^{3/2} * b/(-b^2a)^{1/3})^{1/2}, (I^{3/2}/b(-b^2a)^{1/3})/(-3/2/b(-b^2a)^{1/3} + 1/2I^{3/2}/b(-b^2a)^{1/3}))^{1/2}) + 1/b(-b^2a)^{1/3} * \operatorname{EllipticF}(1/3, 3^{1/2} * (I(x+1/2/b(-b^2a)^{1/3} - 1/2I^{3/2}/b(-b^2a)^{1/3}))^{3/2} * b/(-b^2a)^{1/3})^{1/2}, (I^{3/2}/b(-b^2a)^{1/3})/(-3/2/b(-b^2a)^{1/3} + 1/2I^{3/2}/b(-b^2a)^{1/3}))^{1/2})) + f(2/5x(bx^3+a)^{1/2} - 2/5Ia^{1/2}/b(-b^2a)^{1/3}(I(x+1/2/b(-b^2a)^{1/3} - 1/2I^{3/2}/b(-b^2a)^{1/3}))^{3/2} * b/(-b^2a)^{1/3})^{1/2} * ((x-1/b(-b^2a)^{1/3})/(-3/2/b(-b^2a)^{1/3} + 1/2I^{3/2}/b(-b^2a)^{1/3}))^{1/2} * (-I(x+1/2/b(-b^2a)^{1/3} + 1/2I^{3/2}/b(-b^2a)^{1/3}))^{3/2} * b/(-b^2a)^{1/3})^{1/2} / (bx^3+a)^{1/2} * \operatorname{EllipticF}(1/3, 3^{1/2} * (I(x+1/2/b(-b^2a)^{1/3} - 1/2I^{3/2}/b(-b^2a)^{1/3}))^{3/2} * b/(-b^2a)^{1/3})^{1/2}, (I^{3/2}/b(-b^2a)^{1/3})/(-3/2/b(-b^2a)^{1/3} + 1/2I^{3/2}/b(-b^2a)^{1/3}))^{1/2})) + e(2/3(bx^3+a)^{1/2} - 2/3a^{1/2} * \operatorname{arctanh}((bx^3+a)^{1/2}/a^{1/2})) + c(-1/2(bx^3+a)^{1/2}/x^2 - 1/2I^{3/2} * (-b^2a)^{1/3}(I(x+1/2/b(-b^2a)^{1/3} - 1/2I^{3/2}/b(-b^2a)^{1/3}))^{3/2} * b/(-b^2a)^{1/3})^{1/2} * ((x-1/b(-b^2a)^{1/3})/(-3/2/b(-b^2a)^{1/3} + 1/2I^{3/2}/b(-b^2a)^{1/3}))^{1/2} * (-I(x+1/2/b(-b^2a)^{1/3} + 1/2I^{3/2}/b(-b^2a)^{1/3}))^{3/2} * b/(-b^2a)^{1/3})^{1/2} / (bx^3+a)^{1/2} * \operatorname{EllipticF}(1/3, 3^{1/2} * (I(x+1/2/b(-b^2a)^{1/3} - 1/2I^{3/2}/b(-b^2a)^{1/3}))^{3/2} * b/(-b^2a)^{1/3})^{1/2}, (I^{3/2}/b(-b^2a)^{1/3})/(-3/2/b(-b^2a)^{1/3} + 1/2I^{3/2}/b(-b^2a)^{1/3}))^{1/2})) + d(-(bx^3+a)^{1/2}/x - I^{3/2} * (-b^2a)^{1/3}(I(x+1/2/b(-b^2a)^{1/3} - 1/2I^{3/2}/b(-b^2a)^{1/3}))^{3/2} * b/(-b^2a)^{1/3})^{1/2} * ((x-1/b(-b^2a)^{1/3})/(-3/2/b(-b^2a)^{1/3} + 1/2I^{3/2}/b(-b^2a)^{1/3}))^{1/2} * (-I(x+1/2/b(-b^2a)^{1/3} + 1/2I^{3/2}/b(-b^2a)^{1/3}))^{3/2} * b/(-b^2a)^{1/3})^{1/2} / (bx^3+a)^{1/2} * \operatorname{EllipticE}(1/3, 3^{1/2} * (I(x+1/2/b(-b^2a)^{1/3} - 1/2I^{3/2}/b(-b^2a)^{1/3}))^{3/2} * b/(-b^2a)^{1/3})^{1/2}, (I^{3/2}/b(-b^2a)^{1/3})/(-3/2/b(-b^2a)^{1/3} + 1/2I^{3/2}/b(-b^2a)^{1/3}))^{1/2}))$

$$I \cdot 3^{(1/2)} / b \cdot (-b^2 a)^{(1/3)} \cdot 3^{(1/2)} \cdot b / (-b^2 a)^{(1/3)}^{(1/2)} / (b \cdot x^3 + a)^{(1/2)} \\ \cdot \left((-3/2 / b \cdot (-b^2 a)^{(1/3)} + 1/2 \cdot I \cdot 3^{(1/2)} / b \cdot (-b^2 a)^{(1/3)}) \cdot \text{EllipticE}(1/3 \cdot 3^{(1/2)} \cdot (I \cdot (x + 1/2 / b \cdot (-b^2 a)^{(1/3)} - 1/2 \cdot I \cdot 3^{(1/2)} / b \cdot (-b^2 a)^{(1/3)}) \cdot 3^{(1/2)} \cdot b / (-b^2 a)^{(1/3)})^{(1/2)}, (I \cdot 3^{(1/2)} / b \cdot (-b^2 a)^{(1/3)} / (-3/2 / b \cdot (-b^2 a)^{(1/3)} + 1/2 \cdot I \cdot 3^{(1/2)} / b \cdot (-b^2 a)^{(1/3)})^{(1/2)}) + 1 / b \cdot (-b^2 a)^{(1/3)} \cdot \text{EllipticF}(1/3 \cdot 3^{(1/2)} \cdot (I \cdot (x + 1/2 / b \cdot (-b^2 a)^{(1/3)} - 1/2 \cdot I \cdot 3^{(1/2)} / b \cdot (-b^2 a)^{(1/3)}) \cdot 3^{(1/2)} \cdot b / (-b^2 a)^{(1/3)})^{(1/2)}, (I \cdot 3^{(1/2)} / b \cdot (-b^2 a)^{(1/3)} / (-3/2 / b \cdot (-b^2 a)^{(1/3)} + 1/2 \cdot I \cdot 3^{(1/2)} / b \cdot (-b^2 a)^{(1/3)})^{(1/2)}) \right)$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^3,x, algorithm="maxima")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^3,x, algorithm="fricas")

[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^3, x)

Sympy [A] time = 6.01137, size = 255, normalized size = 0.4

$$\frac{\sqrt{ac} \Gamma\left(-\frac{2}{3}\right) {}_2F_1\left(-\frac{2}{3}, -\frac{1}{2} \left| \frac{bx^3 e^{i\pi}}{a} \right. \right)}{3x^2 \Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{ad} \Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(-\frac{1}{2}, -\frac{1}{3} \left| \frac{bx^3 e^{i\pi}}{a} \right. \right)}{3x \Gamma\left(\frac{2}{3}\right)} - \frac{2\sqrt{ae} \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx^2}}\right)}{3} + \frac{\sqrt{af} x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(-\frac{1}{2}, \frac{1}{3} \left| \frac{bx^3 e^{i\pi}}{a} \right. \right)}{3 \Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**3,x)
```

```
[Out] sqrt(a)*c*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)
/(3*x**2*gamma(1/3)) + sqrt(a)*d*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*
x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*sqrt(a)*e*asinh(sqrt(a)/(sqrt(
b)*x**(3/2)))/3 + sqrt(a)*f*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(4/3)) + sqrt(a)*g*x**2*gamma(2/3)*hyper((-1/2,
2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + 2*a*e/(3*sqrt(b)*x
**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*sqrt(b)*e*x**(3/2)/(3*sqrt(a/(b*x**3) + 1
))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^3,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^3, x)
```

$$3.452 \quad \int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^4} dx$$

Optimal. Leaf size=637

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(-10(1-\sqrt{3})\sqrt[3]{ab^{2/3}}e+4ag+5bd)\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7\right)}{10\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}}$$

[Out] (c*Sqrt[a + b*x^3])/(3*x^3) + (3*d*Sqrt[a + b*x^3])/(2*x^2) - (3*e*Sqrt[a + b*x^3])/x + (3*b^(1/3)*e*Sqrt[a + b*x^3])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x) - (2*Sqrt[a + b*x^3]*(5*c*x + 15*d*x^2 - 15*e*x^3 - 5*f*x^4 - 3*g*x^5))/(15*x^4) - ((b*c + 2*a*f)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(1/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*Sqrt[a + b*x^3]) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*(5*b*d - 10*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e + 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(10*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*Sqrt[a + b*x^3])

Rubi [A] time = 0.844394, antiderivative size = 637, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1826, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}(\sqrt[3]{a}+\sqrt[3]{bx})\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}(-10(1-\sqrt{3})\sqrt[3]{ab^{2/3}}e+4ag+5bd)F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx}+(1+\sqrt{3})\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{10\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx})^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4,x]

```
[Out] (c*Sqrt[a + b*x^3])/(3*x^3) + (3*d*Sqrt[a + b*x^3])/(2*x^2) - (3*e*Sqrt[a +
b*x^3])/x + (3*b^(1/3)*e*Sqrt[a + b*x^3])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x) - (2*Sqrt[a + b*x^3]*(5*c*x + 15*d*x^2 - 15*e*x^3 - 5*f*x^4 - 3*g*x^5))
/(15*x^4) - ((b*c + 2*a*f)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) -
(3*3^(1/4)*Sqrt[2 - Sqrt[3])*a^(1/3)*b^(1/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)
*x))]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*Sqrt[a + b*x^3]) + (3^(3/4)*Sqr
t[2 + Sqrt[3]]*(5*b*d - 10*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e + 4*a*g)*(a^(1/3)
+ b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(10*b^(1/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*S
qrt[a + b*x^3])
```

Rule 1826

```
Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i
]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
+ b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1835

```
Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_)*(x_)^(n_)]), x_Symbol] := Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1878

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

Rule 218

Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)]/((1 + Sqrt[3])*s + r*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b}, x] && PosQ[a]

Rule 1877

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)]/((1 + Sqrt[3])*s + r*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^4} dx &= -\frac{2\sqrt{a+bx^3}(5cx+15dx^2-15ex^3-5fx^4-3gx^5)}{15x^4} + \frac{1}{2}(3a) \int \frac{-\frac{2c}{3}-2dx}{x^4} \\
&= \frac{c\sqrt{a+bx^3}}{3x^3} - \frac{2\sqrt{a+bx^3}(5cx+15dx^2-15ex^3-5fx^4-3gx^5)}{15x^4} - \frac{1}{4} \int \frac{12ad}{x^4} \\
&= \frac{c\sqrt{a+bx^3}}{3x^3} + \frac{3d\sqrt{a+bx^3}}{2x^2} - \frac{2\sqrt{a+bx^3}(5cx+15dx^2-15ex^3-5fx^4-3gx^5)}{15x^4} \\
&= \frac{c\sqrt{a+bx^3}}{3x^3} + \frac{3d\sqrt{a+bx^3}}{2x^2} - \frac{3e\sqrt{a+bx^3}}{x} - \frac{2\sqrt{a+bx^3}(5cx+15dx^2-15ex^3-5fx^4-3gx^5)}{15x^4} \\
&= \frac{c\sqrt{a+bx^3}}{3x^3} + \frac{3d\sqrt{a+bx^3}}{2x^2} - \frac{3e\sqrt{a+bx^3}}{x} - \frac{2\sqrt{a+bx^3}(5cx+15dx^2-15ex^3-5fx^4-3gx^5)}{15x^4} \\
&= \frac{c\sqrt{a+bx^3}}{3x^3} + \frac{3d\sqrt{a+bx^3}}{2x^2} - \frac{3e\sqrt{a+bx^3}}{x} - \frac{2\sqrt{a+bx^3}(5cx+15dx^2-15ex^3-5fx^4-3gx^5)}{15x^4} \\
&= \frac{c\sqrt{a+bx^3}}{3x^3} + \frac{3d\sqrt{a+bx^3}}{2x^2} - \frac{3e\sqrt{a+bx^3}}{x} + \frac{3\sqrt[3]{be}\sqrt{a+bx^3}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}} - \frac{2\sqrt{a+bx^3}}{15x^4} \\
&= \frac{c\sqrt{a+bx^3}}{3x^3} + \frac{3d\sqrt{a+bx^3}}{2x^2} - \frac{3e\sqrt{a+bx^3}}{x} + \frac{3\sqrt[3]{be}\sqrt{a+bx^3}}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{bx}} - \frac{2\sqrt{a+bx^3}}{15x^4}
\end{aligned}$$

Mathematica [C] time = 0.386874, size = 254, normalized size = 0.4

$$\frac{bc\left(\frac{a+bx^3}{bx^3} + \sqrt{\frac{bx^3}{a} + 1} \tanh^{-1}\left(\sqrt{\frac{bx^3}{a} + 1}\right)\right)}{3\sqrt{a+bx^3}} - \frac{d\sqrt{a+bx^3} {}_2F_1\left(-\frac{2}{3}, -\frac{1}{2}; \frac{1}{3}; -\frac{bx^3}{a}\right)}{2x^2\sqrt{\frac{bx^3}{a} + 1}} - \frac{e\sqrt{a+bx^3} {}_2F_1\left(-\frac{1}{2}, -\frac{1}{3}; \frac{2}{3}; -\frac{bx^3}{a}\right)}{x\sqrt{\frac{bx^3}{a} + 1}} + \frac{2}{3}f$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4, x]

```
[Out] (2*f*(Sqrt[a + b*x^3] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]))/3 - (b*c
*((a + b*x^3)/(b*x^3) + Sqrt[1 + (b*x^3)/a]*ArcTanh[Sqrt[1 + (b*x^3)/a]]))/
(3*Sqrt[a + b*x^3]) - (d*Sqrt[a + b*x^3]*Hypergeometric2F1[-2/3, -1/2, 1/3,
-((b*x^3)/a)])/(2*x^2*Sqrt[1 + (b*x^3)/a]) - (e*Sqrt[a + b*x^3]*Hypergeome
tric2F1[-1/2, -1/3, 2/3, -((b*x^3)/a)])/(x*Sqrt[1 + (b*x^3)/a]) + (g*x*Sqrt
[a + b*x^3]*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^3)/a)]/Sqrt[1 + (b*x^
3)/a])
```

Maple [B] time = 0.01, size = 1114, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^4,x)
```

```
[Out] g*(2/5*x*(b*x^3+a)^(1/2)-2/5*I*a*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2
*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*
(x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3
)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/
2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/
b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))
^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b(-
b^2*a)^(1/3))^(1/2))+f*(2/3*(b*x^3+a)^(1/2)-2/3*a^(1/2)*arctanh((b*x^3+a)
^(1/2)/a^(1/2)))+c*(-1/3*(b*x^3+a)^(1/2)/x^3-1/3*b*arctanh((b*x^3+a)^(1/2)/
a^(1/2))/a^(1/2))+d*(-1/2*(b*x^3+a)^(1/2)/x^2-1/2*I*3^(1/2)*(-b^2*a)^(1/3)*
(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*
a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2
)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^
2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3
^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b
/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1
/2*I*3^(1/2)/b*(-b^2*a)^(1/3))^(1/2))+e*(-(b*x^3+a)^(1/2)/x-I*3^(1/2)*(-b
^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1
/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3
^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-
3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*
(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*
a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3
^(1/2)/b*(-b^2*a)^(1/3))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)
```

$$\left(\frac{1}{3} \right)^{\frac{1}{2}}, \left(\frac{1}{b} \cdot \frac{1}{(-b^2 a)^{\frac{1}{3}}} / \left(-\frac{3}{2} \frac{1}{b} \cdot \frac{1}{(-b^2 a)^{\frac{1}{3}}} + \frac{1}{2} \frac{1}{b} \cdot \frac{1}{(-b^2 a)^{\frac{1}{3}}} \right)^{\frac{1}{2}} \right)$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^4,x, algorithm="maxima")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^4,x, algorithm="fricas")

[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^4, x)

Sympy [A] time = 6.89922, size = 265, normalized size = 0.42

$$\frac{\sqrt{a}d\Gamma\left(-\frac{2}{3}\right) {}_2F_1\left(\frac{-\frac{2}{3}, -\frac{1}{2}}{\frac{1}{3}} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{a}e\Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(\frac{-\frac{1}{2}, -\frac{1}{3}}{\frac{2}{3}} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \frac{2\sqrt{a}f \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx^2}}\right)}{3} + \frac{\sqrt{a}gx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{-\frac{1}{2}, \frac{1}{3}}{\frac{4}{3}} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**4,x)

[Out] sqrt(a)*d*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + sqrt(a)*e*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*sqrt(a)*f*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + sqrt(a)*g*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + 2*a*f/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3 + 1))) - sqrt(b)*c*sqrt(a/(b*x**3 + 1))/(3*x**(3/2)) + 2*sqrt(b)*f*x**(3/2)/(3*sqrt(a/(b*x**3 + 1))) - b*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^4,x, algorithm="giac")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^4, x)

$$3.453 \quad \int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^5} dx$$

Optimal. Leaf size=694

$$3^{3/4} \sqrt{2 + \sqrt{3}} \sqrt[3]{b} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (4a^{2/3} \sqrt[3]{be} - (1 - \sqrt{3})(8af + bc)) \text{EllipticF} \left(\sin^{-1} \left(\frac{(1-\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{(1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}} \right), -7 - \right.$$

$$\left. 8a^{2/3} \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3} \right.$$

[Out] (3*c*Sqrt[a + b*x^3])/(20*x^4) + (d*Sqrt[a + b*x^3])/(3*x^3) + (3*e*Sqrt[a + b*x^3])/(2*x^2) - (3*(b*c + 8*a*f)*Sqrt[a + b*x^3])/(8*a*x) + (3*b^(1/3)*(b*c + 8*a*f)*Sqrt[a + b*x^3])/(8*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*Sqrt[a + b*x^3]*(3*c*x + 5*d*x^2 + 15*e*x^3 - 15*f*x^4 - 5*g*x^5))/(15*x^5) - ((b*d + 2*a*g)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(b*c + 8*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(1/3)*(4*a^(2/3)*b^(1/3)*e - (1 - Sqrt[3])*(b*c + 8*a*f))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(8*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 1.08425, antiderivative size = 694, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1826, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$3^{3/4} \sqrt{2 + \sqrt{3}} \sqrt[3]{b} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (4a^{2/3} \sqrt[3]{be} - (1 - \sqrt{3})(8af + bc)) F \left(\sin^{-1} \left(\frac{\sqrt[3]{bx} + (1-\sqrt{3}) \sqrt[3]{a}}{\sqrt[3]{bx} + (1+\sqrt{3}) \sqrt[3]{a}} \right) \middle| -7 - 4\sqrt{3} \right)$$

$$8a^{2/3} \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5,x]

```
[Out] (3*c*Sqrt[a + b*x^3])/(20*x^4) + (d*Sqrt[a + b*x^3])/(3*x^3) + (3*e*Sqrt[a + b*x^3])/(2*x^2) - (3*(b*c + 8*a*f)*Sqrt[a + b*x^3])/(8*a*x) + (3*b^(1/3)*(b*c + 8*a*f)*Sqrt[a + b*x^3])/(8*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*Sqrt[a + b*x^3]*(3*c*x + 5*d*x^2 + 15*e*x^3 - 15*f*x^4 - 5*g*x^5))/(15*x^5) - ((b*d + 2*a*g)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(b*c + 8*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(16*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*Sqrt[a + b*x^3]) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(1/3)*(4*a^(2/3)*b^(1/3)*e - (1 - Sqrt[3])*(b*c + 8*a*f))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(8*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*Sqrt[a + b*x^3])
```

Rule 1826

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}], x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1835

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x^n)^p, x], x] /; NeQ[Pq0, 0] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_)*(x_)^(n_)]), x_Symbol] := Dist[Coeff[Pq, x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq, x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
```

Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1878

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

Rule 218

Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)]/((1 + Sqrt[3])*s + r*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b}, x] && PosQ[a]

Rule 1877

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)]/((1 + Sqrt[3])*s + r*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^5} dx &= -\frac{2\sqrt{a+bx^3}(3cx+5dx^2+15ex^3-15fx^4-5gx^5)}{15x^5} + \frac{1}{2}(3a) \int \frac{-\frac{2c}{5} - \frac{2dx}{3}}{x^5} dx \\
&= \frac{3c\sqrt{a+bx^3}}{20x^4} - \frac{2\sqrt{a+bx^3}(3cx+5dx^2+15ex^3-15fx^4-5gx^5)}{15x^5} - \frac{3}{16} \int \frac{1}{x^5} dx \\
&= \frac{3c\sqrt{a+bx^3}}{20x^4} + \frac{d\sqrt{a+bx^3}}{3x^3} - \frac{2\sqrt{a+bx^3}(3cx+5dx^2+15ex^3-15fx^4-5gx^5)}{15x^5} \\
&= \frac{3c\sqrt{a+bx^3}}{20x^4} + \frac{d\sqrt{a+bx^3}}{3x^3} + \frac{3e\sqrt{a+bx^3}}{2x^2} - \frac{2\sqrt{a+bx^3}(3cx+5dx^2+15ex^3-15fx^4-5gx^5)}{15x^5} \\
&= \frac{3c\sqrt{a+bx^3}}{20x^4} + \frac{d\sqrt{a+bx^3}}{3x^3} + \frac{3e\sqrt{a+bx^3}}{2x^2} - \frac{3(bc+8af)\sqrt{a+bx^3}}{8ax} - \frac{2\sqrt{a+bx^3}}{16x^4} \\
&= \frac{3c\sqrt{a+bx^3}}{20x^4} + \frac{d\sqrt{a+bx^3}}{3x^3} + \frac{3e\sqrt{a+bx^3}}{2x^2} - \frac{3(bc+8af)\sqrt{a+bx^3}}{8ax} - \frac{2\sqrt{a+bx^3}}{16x^4} \\
&= \frac{3c\sqrt{a+bx^3}}{20x^4} + \frac{d\sqrt{a+bx^3}}{3x^3} + \frac{3e\sqrt{a+bx^3}}{2x^2} - \frac{3(bc+8af)\sqrt{a+bx^3}}{8ax} - \frac{2\sqrt{a+bx^3}}{16x^4} \\
&= \frac{3c\sqrt{a+bx^3}}{20x^4} + \frac{d\sqrt{a+bx^3}}{3x^3} + \frac{3e\sqrt{a+bx^3}}{2x^2} - \frac{3(bc+8af)\sqrt{a+bx^3}}{8ax} + \frac{3\sqrt[3]{b}}{8a} \left(\frac{1}{x^4} \right) \\
&= \frac{3c\sqrt{a+bx^3}}{20x^4} + \frac{d\sqrt{a+bx^3}}{3x^3} + \frac{3e\sqrt{a+bx^3}}{2x^2} - \frac{3(bc+8af)\sqrt{a+bx^3}}{8ax} + \frac{3\sqrt[3]{b}}{8a} \left(\frac{1}{x^4} \right)
\end{aligned}$$

Mathematica [C] time = 0.387049, size = 253, normalized size = 0.36

$$\frac{-3c(a+bx^3) {}_2F_1\left(-\frac{4}{3}, -\frac{1}{2}; -\frac{1}{3}; -\frac{bx^3}{a}\right) - 4dx\sqrt{\frac{bx^3}{a}+1} \left(bx^3\sqrt{\frac{bx^3}{a}+1} \tanh^{-1}\left(\sqrt{\frac{bx^3}{a}+1}\right) + a+bx^3 \right) - 6ex^2(a+bx^3) {}_2F_1\left(-\frac{4}{3}, -\frac{1}{2}; -\frac{1}{3}; -\frac{bx^3}{a}\right)}{12x^4\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5,x]

[Out] (8*g*x^4*Sqrt[a + b*x^3]*Sqrt[1 + (b*x^3)/a]*(Sqrt[a + b*x^3] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]) - 4*d*x*Sqrt[1 + (b*x^3)/a]*(a + b*x^3 + b*x^3*Sqrt[1 + (b*x^3)/a]*ArcTanh[Sqrt[1 + (b*x^3)/a]]) - 3*c*(a + b*x^3)*Hypergeometric2F1[-4/3, -1/2, -1/3, -(b*x^3)/a] - 6*e*x^2*(a + b*x^3)*Hypergeometric2F1[-2/3, -1/2, 1/3, -(b*x^3)/a] - 12*f*x^3*(a + b*x^3)*Hypergeometric2F1[-1/2, -1/3, 2/3, -(b*x^3)/a])/(12*x^4*Sqrt[a + b*x^3]*Sqrt[1 + (b*x^3)/a])

Maple [B] time = 0.01, size = 1286, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^5,x)

[Out] g*(2/3*(b*x^3+a)^(1/2)-2/3*a^(1/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+d*(-1/3*(b*x^3+a)^(1/2)/x^3-1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2))+c*(-1/4*(b*x^3+a)^(1/2)/x^4-3/8*b/a*(b*x^3+a)^(1/2)/x-1/8*I/a*b^3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+e*(-1/2*(b*x^3+a)^(1/2)/x^2-1/2*I*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+f*(-(b*x^3+a)^(1/2)/x-I*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3))

$$+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}*3^{(1/2)}*b/(-b^2*a)^{(1/3))^{(1/2)}/(b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*EllipticE(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)}*b/(-b^2*a)^{(1/3))^{(1/2)},(I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))^{(1/2)}))+1/b*(-b^2*a)^{(1/3)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)}*b/(-b^2*a)^{(1/3))^{(1/2)},(I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))^{(1/2)}))$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^5,x, algorithm="maxima")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^5, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^5,x, algorithm="fricas")

[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^5, x)

Sympy [A] time = 7.31502, size = 274, normalized size = 0.39

$$\frac{\sqrt{ac}\Gamma\left(-\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{4}{3}, -\frac{1}{2} \\ -\frac{1}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a} \right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt{ae}\Gamma\left(-\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, -\frac{1}{2} \\ \frac{1}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a} \right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{af}\Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{3} \\ \frac{2}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a} \right)}{3x\Gamma\left(\frac{2}{3}\right)} - \frac{2\sqrt{a}g \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx^3}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**5,x)
```

```
[Out] sqrt(a)*c*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**4*gamma(-1/3)) + sqrt(a)*e*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,),
b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + sqrt(a)*f*gamma(-1/3)*hyper
((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*sqrt(
a)*g*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + 2*a*g/(3*sqrt(b)*x**(3/2)*sqrt(a
/(b*x**3) + 1)) - sqrt(b)*d*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*sqrt(b)*g
*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) - b*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/
(3*sqrt(a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^5,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^5, x)
```


$$3.454 \quad \int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^6} dx$$

Optimal. Leaf size=652

$$\frac{3^{3/4} \sqrt{2 + \sqrt{3}} \sqrt[3]{b} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \operatorname{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{(1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}\right), -7 - 4\sqrt{3}\right) (2\sqrt[3]{b}(bc - 10af) + 5(1 - \sqrt{3}))}{40a \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}}$$

[Out] -(((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2 + (60*g)/x)*Sqrt[a + b*x^3])/60 - (3*b*c*Sqrt[a + b*x^3])/(20*a*x^2) - (3*b*d*Sqrt[a + b*x^3])/(8*a*x) + (3*b^(1/3)*(b*d + 8*a*g)*Sqrt[a + b*x^3])/(8*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (b*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(b*d + 8*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(1/3)*(2*b^(1/3)*(b*c - 10*a*f) + 5*(1 - Sqrt[3])*a^(1/3)*(b*d + 8*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(40*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 0.787092, antiderivative size = 652, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {14, 1825, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$\frac{3^{3/4} \sqrt{2 + \sqrt{3}} \sqrt[3]{b} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx} + (1-\sqrt{3}) \sqrt[3]{a}}{\sqrt[3]{bx} + (1+\sqrt{3}) \sqrt[3]{a}}\right), -7 - 4\sqrt{3}\right) (2\sqrt[3]{b}(bc - 10af) + 5(1 - \sqrt{3}))}{40a \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6, x]

```
[Out] -(((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2 + (60*g)/x)*Sqrt[a + b
*x^3])/60 - (3*b*c*Sqrt[a + b*x^3])/(20*a*x^2) - (3*b*d*Sqrt[a + b*x^3])/(8
*a*x) + (3*b^(1/3)*(b*d + 8*a*g)*Sqrt[a + b*x^3])/(8*a*((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)) - (b*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3*
3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(b*d + 8*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3)
+ b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (3
^(3/4)*Sqrt[2 + Sqrt[3]]*b^(1/3)*(2*b^(1/3)*(b*c - 10*a*f) + 5*(1 - Sqrt[3]
)*a^(1/3)*(b*d + 8*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
-7 - 4*Sqrt[3]])/(40*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Module[{u
= IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_.)]), x_Symbol] := Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
```

$Q[n, 0] \ \&\& \ \text{NeQ}[\text{Coeff}[\text{Pq}, x, 0], 0]$

Rule 266

$\text{Int}[(x_)^m \cdot ((a_) + (b_) \cdot (x_)^n)^p, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n) - 1} \cdot (a + b \cdot x)^p, x], x, x^n], x] /; \text{FreeQ}[\{a, b, m, n, p\}, x] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$

Rule 63

$\text{Int}[(a_) + (b_) \cdot (x_)^m \cdot ((c_) + (d_) \cdot (x_)^n), x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p \cdot (m+1) - 1)} \cdot (c - (a \cdot d)/b + (d \cdot x^p)/b)^n, x], x, (a + b \cdot x)^{1/p}], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 208

$\text{Int}[(a_) + (b_) \cdot (x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2] \cdot \text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

Rule 1878

$\text{Int}[(c_) + (d_) \cdot (x_)]/\text{Sqrt}[(a_) + (b_) \cdot (x_)^3], x_Symbol] \rightarrow \text{With}[\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Dist}[(c \cdot r - (1 - \text{Sqrt}[3]) \cdot d \cdot s)/r, \text{Int}[1/\text{Sqrt}[a + b \cdot x^3], x], x] + \text{Dist}[d/r, \text{Int}[(1 - \text{Sqrt}[3]) \cdot s + r \cdot x]/\text{Sqrt}[a + b \cdot x^3], x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{PosQ}[a] \ \&\& \ \text{NeQ}[b \cdot c^3 - 2 \cdot (5 - 3 \cdot \text{Sqrt}[3]) \cdot a \cdot d^3, 0]$

Rule 218

$\text{Int}[1/\text{Sqrt}[(a_) + (b_) \cdot (x_)^3], x_Symbol] \rightarrow \text{With}[\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Simp}[(2 \cdot \text{Sqrt}[2 + \text{Sqrt}[3]] \cdot (s + r \cdot x) \cdot \text{Sqrt}[(s^2 - r \cdot s \cdot x + r^2 \cdot x^2)/((1 + \text{Sqrt}[3]) \cdot s + r \cdot x)^2] \cdot \text{EllipticF}[\text{ArcSin}[(1 - \text{Sqrt}[3]) \cdot s + r \cdot x]/((1 + \text{Sqrt}[3]) \cdot s + r \cdot x)], -7 - 4 \cdot \text{Sqrt}[3]])/(3^{1/4} \cdot r \cdot \text{Sqrt}[a + b \cdot x^3] \cdot \text{Sqrt}[(s \cdot (s + r \cdot x))/((1 + \text{Sqrt}[3]) \cdot s + r \cdot x)^2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a]$

Rule 1877

$\text{Int}[(c_) + (d_) \cdot (x_)]/\text{Sqrt}[(a_) + (b_) \cdot (x_)^3], x_Symbol] \rightarrow \text{With}[\{r = \text{Numer}[\text{Simplify}[(1 - \text{Sqrt}[3]) \cdot d]/c], s = \text{Denom}[\text{Simplify}[(1 - \text{Sqrt}[3]) \cdot d]/c]\}, \text{Simp}[(2 \cdot d \cdot s^3 \cdot \text{Sqrt}[a + b \cdot x^3])/(a \cdot r^2 \cdot ((1 + \text{Sqrt}[3]) \cdot s + r \cdot x)), x] - \text{Simp}[(3^{1/4} \cdot \text{Sqrt}[2 - \text{Sqrt}[3]] \cdot d \cdot s \cdot (s + r \cdot x) \cdot \text{Sqrt}[(s^2 - r \cdot s \cdot x + r^2 \cdot x^2)/((1 + \text{Sqrt}[3]) \cdot s + r \cdot x)^2] \cdot \text{EllipticE}[\text{ArcSin}[(1 - \text{Sqrt}[3]) \cdot s + r \cdot x]/((1 + \text{Sqrt}[3]) \cdot s + r \cdot x)]), x]$

```
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^6} dx &= -\frac{1}{60} \left(\frac{12c}{x^5} + \frac{15d}{x^4} + \frac{20e}{x^3} + \frac{30f}{x^2} + \frac{60g}{x} \right) \sqrt{a+bx^3} - \frac{1}{2} (3b) \int \frac{-\frac{c}{5} - \frac{dx}{4} - \frac{ex^2}{3}}{x^3 \sqrt{a+bx^3}} dx \\ &= -\frac{1}{60} \left(\frac{12c}{x^5} + \frac{15d}{x^4} + \frac{20e}{x^3} + \frac{30f}{x^2} + \frac{60g}{x} \right) \sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{20ax^2} + \frac{(3b) \int \frac{-\frac{c}{5} - \frac{dx}{4} - \frac{ex^2}{3}}{x^3 \sqrt{a+bx^3}} dx}{2} \\ &= -\frac{1}{60} \left(\frac{12c}{x^5} + \frac{15d}{x^4} + \frac{20e}{x^3} + \frac{30f}{x^2} + \frac{60g}{x} \right) \sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{20ax^2} - \frac{3bd\sqrt{a+bx^3}}{8a} \\ &= -\frac{1}{60} \left(\frac{12c}{x^5} + \frac{15d}{x^4} + \frac{20e}{x^3} + \frac{30f}{x^2} + \frac{60g}{x} \right) \sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{20ax^2} - \frac{3bd\sqrt{a+bx^3}}{8a} \\ &= -\frac{1}{60} \left(\frac{12c}{x^5} + \frac{15d}{x^4} + \frac{20e}{x^3} + \frac{30f}{x^2} + \frac{60g}{x} \right) \sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{20ax^2} - \frac{3bd\sqrt{a+bx^3}}{8a} \\ &= -\frac{1}{60} \left(\frac{12c}{x^5} + \frac{15d}{x^4} + \frac{20e}{x^3} + \frac{30f}{x^2} + \frac{60g}{x} \right) \sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{20ax^2} - \frac{3bd\sqrt{a+bx^3}}{8a} \\ &= -\frac{1}{60} \left(\frac{12c}{x^5} + \frac{15d}{x^4} + \frac{20e}{x^3} + \frac{30f}{x^2} + \frac{60g}{x} \right) \sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{20ax^2} - \frac{3bd\sqrt{a+bx^3}}{8a} \end{aligned}$$

Mathematica [C] time = 0.260194, size = 180, normalized size = 0.28

$$\frac{\sqrt{a+bx^3} \left(12ac {}_2F_1 \left(-\frac{5}{3}, -\frac{1}{2}; -\frac{2}{3}; -\frac{bx^3}{a} \right) + 5x \left(3ad {}_2F_1 \left(-\frac{4}{3}, -\frac{1}{2}; -\frac{1}{3}; -\frac{bx^3}{a} \right) + 2x \left(2ae \sqrt{\frac{bx^3}{a} + 1} + 2bex^3 \tanh^{-1} \left(\sqrt{\frac{bx^3}{a} + 1} \right) \right) \right)}{60ax^5 \sqrt{\frac{bx^3}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6,x]

[Out] $-(\text{Sqrt}[a + b*x^3]*(12*a*c*\text{Hypergeometric2F1}[-5/3, -1/2, -2/3, -((b*x^3)/a)] + 5*x*(3*a*d*\text{Hypergeometric2F1}[-4/3, -1/2, -1/3, -((b*x^3)/a)] + 2*x*(2*a*e*\text{Sqrt}[1 + (b*x^3)/a] + 2*b*e*x^3*\text{ArcTanh}[\text{Sqrt}[1 + (b*x^3)/a]] + 3*a*f*x*\text{Hypergeometric2F1}[-2/3, -1/2, 1/3, -((b*x^3)/a)] + 6*a*g*x^2*\text{Hypergeometric2F1}[-1/2, -1/3, 2/3, -((b*x^3)/a)])))/(60*a*x^5*\text{Sqrt}[1 + (b*x^3)/a])$

Maple [B] time = 0.009, size = 1571, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^6,x)

[Out] $e*(-1/3*(b*x^3+a)^{(1/2)}/x^3-1/3*b*\text{arctanh}((b*x^3+a)^{(1/2)}/a^{(1/2)})/a^{(1/2)}) + c*(-1/5*(b*x^3+a)^{(1/2)}/x^5-3/20*b/a*(b*x^3+a)^{(1/2)}/x^2+1/20*I/a*b^3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/2)})*\text{EllipticF}(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)},(I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)})))+d*(-1/4*(b*x^3+a)^{(1/2)}/x^4-3/8*b/a*(b*x^3+a)^{(1/2)}/x-1/8*I/a*b^3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*\text{EllipticE}(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)},(I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}))+1/b*(-b^2*a)^{(1/3)}*\text{EllipticF}(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)},(I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)})))+f*(-1/2*(b*x^3+a)^{(1/2)}/x^2-1/2*I*3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/2)}*\text{EllipticF}(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b$

$$\begin{aligned} & \sqrt{-2a}^{1/3} \sqrt{-3/2/b(-b^2a)^{1/3} + 1/2I^3^{1/2}/b(-b^2a)^{1/3}}^{1/2} \\ & + g(-bx^3+a)^{1/2}/x - I^3^{1/2}(-b^2a)^{1/3} \\ & + I(x+1/2/b(-b^2a)^{1/3} - 1/2I^3^{1/2}/b(-b^2a)^{1/3})^3^{1/2} \\ & + b/(-b^2a)^{1/3} \sqrt{(x-1/b(-b^2a)^{1/3})/(-3/2/b(-b^2a)^{1/3} + 1/2I^3^{1/2}/b(-b^2a)^{1/3})}^{1/2} \\ & - I(x+1/2/b(-b^2a)^{1/3} + 1/2I^3^{1/2}/b(-b^2a)^{1/3})^3^{1/2} \\ & + b/(-b^2a)^{1/3} \sqrt{(bx^3+a)^{1/2} ((-3/2/b(-b^2a)^{1/3} + 1/2I^3^{1/2}/b(-b^2a)^{1/3})^3^{1/2} \\ & + I(x+1/2/b(-b^2a)^{1/3} - 1/2I^3^{1/2}/b(-b^2a)^{1/3})^3^{1/2} * b/(-b^2a)^{1/3})}^{1/2} \\ & + I^3^{1/2}/b(-b^2a)^{1/3} \sqrt{-3/2/b(-b^2a)^{1/3} + 1/2I^3^{1/2}/b(-b^2a)^{1/3}}^{1/2} \\ & + 1/b(-b^2a)^{1/3} \text{EllipticF}(1/3, 3^{1/2} * I(x+1/2/b(-b^2a)^{1/3} - 1/2I^3^{1/2}/b(-b^2a)^{1/3})^3^{1/2} * b/(-b^2a)^{1/3}) \\ & \sqrt{-3/2/b(-b^2a)^{1/3} + 1/2I^3^{1/2}/b(-b^2a)^{1/3}}^{1/2} \\ & + I^3^{1/2}/b(-b^2a)^{1/3} \sqrt{-3/2/b(-b^2a)^{1/3} + 1/2I^3^{1/2}/b(-b^2a)^{1/3}}^{1/2} \\ & + 1/2I^3^{1/2}/b(-b^2a)^{1/3} \sqrt{-3/2/b(-b^2a)^{1/3} + 1/2I^3^{1/2}/b(-b^2a)^{1/3}}^{1/2} \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^6,x, algorithm="maxima")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^6, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^6,x, algorithm="fricas")

[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^6, x)

Sympy [A] time = 6.55081, size = 240, normalized size = 0.37

$$\frac{\sqrt{ac}\Gamma\left(-\frac{5}{3}\right) {}_2F_1\left(-\frac{5}{3}, -\frac{1}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{\sqrt{ad}\Gamma\left(-\frac{4}{3}\right) {}_2F_1\left(-\frac{4}{3}, -\frac{1}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt{af}\Gamma\left(-\frac{2}{3}\right) {}_2F_1\left(-\frac{2}{3}, \frac{1}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{ag}\Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(-\frac{1}{3}, \frac{2}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**6,x)

[Out] sqrt(a)*c*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*d*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + sqrt(a)*f*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + sqrt(a)*g*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - sqrt(b)*e*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b*e*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^6,x, algorithm="giac")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^6, x)

$$3.455 \quad \int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^7} dx$$

Optimal. Leaf size=659

$$3^{3/4} \sqrt{2 + \sqrt{3}} b^{2/3} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (5(1-\sqrt{3}) \sqrt[3]{ab^{2/3}} e - 20ag + 2bd) \text{EllipticF} \left(\sin^{-1} \left(\frac{(1-\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{(1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}} \right) \right), -$$

$$40a \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

[Out] -(((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3 + (30*g)/x^2)*Sqrt[a + b*x^3])/60 - (b*c*Sqrt[a + b*x^3])/(12*a*x^3) - (3*b*d*Sqrt[a + b*x^3])/(20*a*x^2) - (3*b*e*Sqrt[a + b*x^3])/(8*a*x) + (3*b^(4/3)*e*Sqrt[a + b*x^3])/(8*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (b*(b*c - 4*a*f)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(3/2)) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(16*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(2/3)*(2*b*d + 5*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e - 20*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(40*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 0.979823, antiderivative size = 659, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {14, 1825, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$3^{3/4} \sqrt{2 + \sqrt{3}} b^{2/3} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (5(1-\sqrt{3}) \sqrt[3]{ab^{2/3}} e - 20ag + 2bd) F \left(\sin^{-1} \left(\frac{\sqrt[3]{bx} + (1-\sqrt{3}) \sqrt[3]{a}}{\sqrt[3]{bx} + (1+\sqrt{3}) \sqrt[3]{a}} \right) \right) - 7 - 4\sqrt{3}$$

$$40a \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7,x]


```
[Out] -(((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3 + (30*g)/x^2)*Sqrt[a +
b*x^3])/60 - (b*c*Sqrt[a + b*x^3])/(12*a*x^3) - (3*b*d*Sqrt[a + b*x^3])/(2
0*a*x^2) - (3*b*e*Sqrt[a + b*x^3])/(8*a*x) + (3*b^(4/3)*e*Sqrt[a + b*x^3])/
(8*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (b*(b*c - 4*a*f)*ArcTanh[Sqrt[a
+ b*x^3]/Sqrt[a]])/(12*a^(3/2)) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*e*(
a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]]/(16*a^(2
/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)^2]*Sqrt[a + b*x^3]) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(2/3)*(2*b*d + 5*(1 -
Sqrt[3])*a^(1/3)*b^(2/3)*e - 20*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ell
ipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)], -7 - 4*Sqrt[3]]]/(40*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_
+ (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^((m_)*((a_) + (b_)*(x_)^(n_)))^(p_), x_Symbol] := Module[{u
= IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x] /; FreeQ[{a, b},
x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
0]
```

Rule 1835

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_)*(x_)^(n_)]), x_Symbol] := Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
```

$Q[n, 0] \ \&\& \ \text{NeQ}[\text{Coeff}[\text{Pq}, x, 0], 0]$

Rule 266

$\text{Int}[(x_)^{(m_.)} * ((a_) + (b_.) * (x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1) * (a + b*x)^p, x}], x, x^n], x] \ /; \ \text{FreeQ}[\{a, b, m, n, p\}, x] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 63

$\text{Int}[(a_.) + (b_.) * (x_)^{(m_.)} * ((c_.) + (d_.) * (x_)^{(n_.)}), x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m + 1) - 1)} * (c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] \ /; \ \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 208

$\text{Int}[(a_) + (b_.) * (x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2] * \text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

Rule 1878

$\text{Int}[(c_) + (d_.) * (x_)]/\text{Sqrt}[(a_) + (b_.) * (x_)^3], x_Symbol] \rightarrow \text{With}[\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Dist}[(c*r - (1 - \text{Sqrt}[3])*d*s)/r, \text{Int}[1/\text{Sqrt}[a + b*x^3], x], x] + \text{Dist}[d/r, \text{Int}[(1 - \text{Sqrt}[3])*s + r*x]/\text{Sqrt}[a + b*x^3], x], x]] \ /; \ \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{PosQ}[a] \ \&\& \ \text{NeQ}[b*c^3 - 2*(5 - 3*\text{Sqrt}[3])*a*d^3, 0]$

Rule 218

$\text{Int}[1/\text{Sqrt}[(a_) + (b_.) * (x_)^3], x_Symbol] \rightarrow \text{With}[\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Simp}[(2*\text{Sqrt}[2 + \text{Sqrt}[3]] * (s + r*x) * \text{Sqrt}[(s^2 - r*s*x + r^2*x^2)/((1 + \text{Sqrt}[3])*s + r*x)^2] * \text{EllipticF}[\text{ArcSin}[(1 - \text{Sqrt}[3])*s + r*x]/((1 + \text{Sqrt}[3])*s + r*x)], -7 - 4*\text{Sqrt}[3])]/(3^{(1/4)} * r * \text{Sqrt}[a + b*x^3] * \text{Sqrt}[(s*(s + r*x))/((1 + \text{Sqrt}[3])*s + r*x)^2]), x]] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a]$

Rule 1877

$\text{Int}[(c_) + (d_.) * (x_)]/\text{Sqrt}[(a_) + (b_.) * (x_)^3], x_Symbol] \rightarrow \text{With}[\{r = \text{Numer}[\text{Simplify}[(1 - \text{Sqrt}[3])*d]/c], s = \text{Denom}[\text{Simplify}[(1 - \text{Sqrt}[3])*d]/c]\}, \text{Simp}[(2*d*s^3*\text{Sqrt}[a + b*x^3])/(a*r^2*((1 + \text{Sqrt}[3])*s + r*x)), x] - \text{Simp}[(3^{(1/4)}*\text{Sqrt}[2 - \text{Sqrt}[3]]*d*s*(s + r*x)*\text{Sqrt}[(s^2 - r*s*x + r^2*x^2)/((1 + \text{Sqrt}[3])*s + r*x)^2] * \text{EllipticE}[\text{ArcSin}[(1 - \text{Sqrt}[3])*s + r*x]/((1 + \text{Sqrt}[3])*s + r*x)]), x]] \ /; \ \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{PosQ}[a]$

```
rt[3])*s + r*x]], -7 - 4*Sqrt[3]]/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^7} dx &= -\frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} + \frac{30g}{x^2} \right) \sqrt{a+bx^3} - \frac{1}{2}(3b) \int \frac{-\frac{c}{6} - \frac{dx}{5} - \frac{ex^2}{4}}{x^4 \sqrt{a+bx^3}} dx \\
&= -\frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} + \frac{30g}{x^2} \right) \sqrt{a+bx^3} - \frac{bc\sqrt{a+bx^3}}{12ax^3} + \frac{b \int \frac{6ad}{x^5} dx}{20a} \\
&= -\frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} + \frac{30g}{x^2} \right) \sqrt{a+bx^3} - \frac{bc\sqrt{a+bx^3}}{12ax^3} - \frac{3bd\sqrt{a+bx^3}}{20a} \\
&= -\frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} + \frac{30g}{x^2} \right) \sqrt{a+bx^3} - \frac{bc\sqrt{a+bx^3}}{12ax^3} - \frac{3bd\sqrt{a+bx^3}}{20a} \\
&= -\frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} + \frac{30g}{x^2} \right) \sqrt{a+bx^3} - \frac{bc\sqrt{a+bx^3}}{12ax^3} - \frac{3bd\sqrt{a+bx^3}}{20a} \\
&= -\frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} + \frac{30g}{x^2} \right) \sqrt{a+bx^3} - \frac{bc\sqrt{a+bx^3}}{12ax^3} - \frac{3bd\sqrt{a+bx^3}}{20a} \\
&= -\frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} + \frac{30g}{x^2} \right) \sqrt{a+bx^3} - \frac{bc\sqrt{a+bx^3}}{12ax^3} - \frac{3bd\sqrt{a+bx^3}}{20a} \\
&= -\frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} + \frac{30g}{x^2} \right) \sqrt{a+bx^3} - \frac{bc\sqrt{a+bx^3}}{12ax^3} - \frac{3bd\sqrt{a+bx^3}}{20a}
\end{aligned}$$

Mathematica [C] time = 0.424939, size = 211, normalized size = 0.32

$$\frac{\sqrt{a+bx^3} \left(5x \left(2x \left(6a^2 f \left(a \sqrt{\frac{bx^3}{a}} + 1 + bx^3 \tanh^{-1} \left(\sqrt{\frac{bx^3}{a}} + 1 \right) \right) + 9a^3 g x {}_2F_1 \left(-\frac{2}{3}, -\frac{1}{2}; \frac{1}{3}; -\frac{bx^3}{a} \right) + 4b^2 c x^3 (a + bx^3) \sqrt{\frac{bx^3}{a}} \right) \right)}{180a^3 x^5 \sqrt{\frac{bx^3}{a}} + 1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7,x]
```

```
[Out] -(Sqrt[a + b*x^3]*(36*a^3*d*Hypergeometric2F1[-5/3, -1/2, -2/3, -((b*x^3)/a)] + 5*x*(9*a^3*e*Hypergeometric2F1[-4/3, -1/2, -1/3, -((b*x^3)/a)] + 2*x*(6*a^2*f*(a*Sqrt[1 + (b*x^3)/a] + b*x^3*ArcTanh[Sqrt[1 + (b*x^3)/a]]) + 9*a^3*g*x*Hypergeometric2F1[-2/3, -1/2, 1/3, -((b*x^3)/a)] + 4*b^2*c*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*x^3)/a]))) / (180*a^3*x^5*Sqrt[1 + (b*x^3)/a])
```

Maple [B] time = 0.008, size = 1180, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^7,x)
```

```
[Out] f*(-1/3*(b*x^3+a)^(1/2)/x^3-1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)) + d*(-1/5*(b*x^3+a)^(1/2)/x^5-3/20*b/a*(b*x^3+a)^(1/2)/x^2+1/20*I/a*b^3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))) + e*(-1/4*(b*x^3+a)^(1/2)/x^4-3/8*b/a*(b*x^3+a)^(1/2)/x-1/8*I/a*b^3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)) + 1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))) + g*(-1/2*(b*x^3+a)^(1/2)/x^2-1/2*I*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*
```

$$\begin{aligned} & (-b^2a)^{1/3})^{1/2} * (-I*(x+1/2/b*(-b^2a)^{1/3} + 1/2*I*3^{1/2}/b*(-b^2a)^{1/3}) * 3^{1/2} * b / (-b^2a)^{1/3})^{1/2} / (b*x^3+a)^{1/2} * \text{EllipticF}(1/3*3^{1/2} * (I*(x+1/2/b*(-b^2a)^{1/3} - 1/2*I*3^{1/2}/b*(-b^2a)^{1/3}) * 3^{1/2} * b / (-b^2a)^{1/3})^{1/2}, (I*3^{1/2}/b*(-b^2a)^{1/3} / (-3/2/b*(-b^2a)^{1/3} + 1/2*I*3^{1/2}/b*(-b^2a)^{1/3}))^{1/2})) + c * (-1/6*(b*x^3+a)^{1/2} / x^6 - 1/12*b/a*(b*x^3+a)^{1/2} / x^3 + 1/12/a^{3/2} * b^2 * \text{arctanh}((b*x^3+a)^{1/2} / a^{1/2})) \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^7,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^7,x, algorithm="fricas")

[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^7, x)

Sympy [A] time = 9.38378, size = 304, normalized size = 0.46

$$\frac{\sqrt{a}d\Gamma\left(-\frac{5}{3}\right) {}_2F_1\left(-\frac{5}{3}, -\frac{1}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{\sqrt{a}e\Gamma\left(-\frac{4}{3}\right) {}_2F_1\left(-\frac{4}{3}, -\frac{1}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt{a}g\Gamma\left(-\frac{2}{3}\right) {}_2F_1\left(-\frac{2}{3}, -\frac{1}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} - \frac{ac}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**7,x)
```

```
[Out] sqrt(a)*d*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**5*gamma(-2/3)) + sqrt(a)*e*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + sqrt(a)*g*gamma(-2/3)*hyp
er((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) - a*
c/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - sqrt(b)*c/(4*x**(9/2)*sqrt(a
/(b*x**3) + 1)) - sqrt(b)*f*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b**(3/2)*c/
(12*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b*f*asinh(sqrt(a)/(sqrt(b)*x**(3/2))
)/(3*sqrt(a)) + b**2*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(12*a**(3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^7,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^7, x)
```

$$3.456 \quad \int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^8} dx$$

Optimal. Leaf size=711

$$3^{3/4} \sqrt{2 + \sqrt{3}} b^{4/3} \left(\sqrt[3]{a} + \sqrt[3]{bx} \right) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{\left((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx} \right)^2}} \left(28a^{2/3} \sqrt[3]{be} - 5(1-\sqrt{3})(5bc-14af) \right) \text{EllipticF} \left(\sin^{-1} \left(\frac{(1-\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{(1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}} \right) \right)$$

$$560a^{5/3} \sqrt{\frac{\sqrt[3]{a} \left(\sqrt[3]{a} + \sqrt[3]{bx} \right)}{\left((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx} \right)^2}} \sqrt{a+bx^3}$$

[Out] -(((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4 + (140*g)/x^3)*Sqrt[a + b*x^3])/420 - (3*b*c*Sqrt[a + b*x^3])/(56*a*x^4) - (b*d*Sqrt[a + b*x^3])/(12*a*x^3) - (3*b*e*Sqrt[a + b*x^3])/(20*a*x^2) + (3*b*(5*b*c - 14*a*f)*Sqrt[a + b*x^3])/(112*a^2*x) - (3*b^(4/3)*(5*b*c - 14*a*f)*Sqrt[a + b*x^3])/(112*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (b*(b*d - 4*a*g)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(3/2)) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(5*b*c - 14*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(224*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(4/3)*(28*a^(2/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(5*b*c - 14*a*f))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(560*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 1.12302, antiderivative size = 711, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {14, 1825, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$3^{3/4} \sqrt{2 + \sqrt{3}} b^{4/3} \left(\sqrt[3]{a} + \sqrt[3]{bx} \right) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{\left((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx} \right)^2}} \left(28a^{2/3} \sqrt[3]{be} - 5(1-\sqrt{3})(5bc-14af) \right) F \left(\sin^{-1} \left(\frac{\sqrt[3]{bx} + (1-\sqrt{3}) \sqrt[3]{a}}{\sqrt[3]{bx} + (1+\sqrt{3}) \sqrt[3]{a}} \right) \right) - 7$$

$$560a^{5/3} \sqrt{\frac{\sqrt[3]{a} \left(\sqrt[3]{a} + \sqrt[3]{bx} \right)}{\left((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx} \right)^2}} \sqrt{a+bx^3}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8,x]

[Out] -(((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4 + (140*g)/x^3)*Sqrt[a + b*x^3])/420 - (3*b*c*Sqrt[a + b*x^3])/(56*a*x^4) - (b*d*Sqrt[a + b*x^3])/(12*a*x^3) - (3*b*e*Sqrt[a + b*x^3])/(20*a*x^2) + (3*b*(5*b*c - 14*a*f)*Sqrt[a + b*x^3])/(112*a^2*x) - (3*b^(4/3)*(5*b*c - 14*a*f)*Sqrt[a + b*x^3])/(112*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (b*(b*d - 4*a*g)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(3/2)) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(5*b*c - 14*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(224*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(4/3)*(28*a^(2/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(5*b*c - 14*a*f))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(560*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 1825

Int[(Pq_)*(x_)^m_)*((a_) + (b_)*(x_)^n_)^(p_), x_Symbol] := Module[{u = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1, 0]

Rule 1835

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^n_)^(p_), x_Symbol] := With[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]

Rule 1832


```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]) , x_Symbol] := Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_)^(n_)) , x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
```

```

]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^8} dx &= -\frac{1}{420} \left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} + \frac{140g}{x^3} \right) \sqrt{a+bx^3} - \frac{1}{2}(3b) \int \frac{-\frac{c}{7} - \frac{dx}{6}}{x^5} \\
&= -\frac{1}{420} \left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} + \frac{140g}{x^3} \right) \sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{56ax^4} + \dots \\
&= -\frac{1}{420} \left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} + \frac{140g}{x^3} \right) \sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{56ax^4} - \dots \\
&= -\frac{1}{420} \left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} + \frac{140g}{x^3} \right) \sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{56ax^4} - \dots \\
&= -\frac{1}{420} \left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} + \frac{140g}{x^3} \right) \sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{56ax^4} - \dots \\
&= -\frac{1}{420} \left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} + \frac{140g}{x^3} \right) \sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{56ax^4} - \dots \\
&= -\frac{1}{420} \left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} + \frac{140g}{x^3} \right) \sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{56ax^4} - \dots \\
&= -\frac{1}{420} \left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} + \frac{140g}{x^3} \right) \sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{56ax^4} - \dots \\
&= -\frac{1}{420} \left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} + \frac{140g}{x^3} \right) \sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{56ax^4} - \dots \\
&= -\frac{1}{420} \left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} + \frac{140g}{x^3} \right) \sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{56ax^4} - \dots
\end{aligned}$$

Mathematica [C] time = 0.452092, size = 213, normalized size = 0.3

$$\frac{\sqrt{a+bx^3} \left(7x^2 \left(5x \left(9a^3 f {}_2F_1 \left(-\frac{4}{3}, -\frac{1}{2}; -\frac{1}{3}; -\frac{bx^3}{a} \right) + 12a^2 gx \left(a\sqrt{\frac{bx^3}{a} + 1} + bx^3 \tanh^{-1} \left(\sqrt{\frac{bx^3}{a} + 1} \right) \right) + 8b^2 dx^4 (a+bx^3) \sqrt{a+bx^3} \right) \right)}{1260a^3 x^7 \sqrt{\frac{bx^3}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8,x]

[Out] $-(\text{Sqrt}[a + b*x^3]*(180*a^3*c*\text{Hypergeometric2F1}[-7/3, -1/2, -4/3, -((b*x^3)/a)] + 7*x^2*(36*a^3*e*\text{Hypergeometric2F1}[-5/3, -1/2, -2/3, -((b*x^3)/a)] + 5*x*(12*a^2*g*x*(a*\text{Sqrt}[1 + (b*x^3)/a] + b*x^3*\text{ArcTanh}[\text{Sqrt}[1 + (b*x^3)/a]]) + 9*a^3*f*\text{Hypergeometric2F1}[-4/3, -1/2, -1/3, -((b*x^3)/a)] + 8*b^2*d*x^4*(a + b*x^3)*\text{Sqrt}[1 + (b*x^3)/a]*\text{Hypergeometric2F1}[3/2, 3, 5/2, 1 + (b*x^3)/a])))/(1260*a^3*x^7*\text{Sqrt}[1 + (b*x^3)/a])$

Maple [B] time = 0.01, size = 1376, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^8,x)

[Out] $g*(-1/3*(b*x^3+a)^{(1/2)}/x^3-1/3*b*\text{arctanh}((b*x^3+a)^{(1/2)}/a^{(1/2)})/a^{(1/2)}) + e*(-1/5*(b*x^3+a)^{(1/2)}/x^5-3/20*b/a*(b*x^3+a)^{(1/2)}/x^2+1/20*I/a*b^3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/2)})*\text{EllipticF}(1/3^3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)},(I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}))+f*(-1/4*(b*x^3+a)^{(1/2)}/x^4-3/8*b/a*(b*x^3+a)^{(1/2)}/x-1/8*I/a*b^3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})*\text{EllipticE}(1/3^3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)},(I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}))+1/b*(-b^2*a)^{(1/3)}*\text{EllipticF}(1/3^3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)},(I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)})))+d*(-1/6*(b*x^3+a)^{(1/2)}/x^6-1/12*b/a*(b*x^3+a)^{(1/2)}/x^3+1/12/a^{(3/2)}*b^2*\text{arctanh}((b*x^3+a)^{(1/2)}/a^{(1/2)}))+c*(-1/7/x^7*(b*x^3+a)^{(1/2)}-3/56*b/a*(b*x^3+a)^{(1/2)}/x^4+15/112/a^2*b^2*(b*x^3+a)^{(1/2)}/x+5/112*I/a^2*b^2^3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*$

$$-b^2a)^{1/3}+1/2*I*3^{1/2}/b*(-b^2a)^{1/3}))^{1/2}*(-I*(x+1/2/b*(-b^2a)^{1/3}+1/2*I*3^{1/2}/b*(-b^2a)^{1/3}))*3^{1/2}*b/(-b^2a)^{1/3})^{1/2}/(b*x^3+a)^{1/2}*((-3/2/b*(-b^2a)^{1/3}+1/2*I*3^{1/2}/b*(-b^2a)^{1/3})*\text{EllipticE}(1/3*3^{1/2}*(I*(x+1/2/b*(-b^2a)^{1/3}-1/2*I*3^{1/2}/b*(-b^2a)^{1/3}))*3^{1/2}*b/(-b^2a)^{1/3})^{1/2}, (I*3^{1/2}/b*(-b^2a)^{1/3}/(-3/2/b*(-b^2a)^{1/3}+1/2*I*3^{1/2}/b*(-b^2a)^{1/3}))^{1/2}))+1/b*(-b^2a)^{1/3})*\text{EllipticF}(1/3*3^{1/2}*(I*(x+1/2/b*(-b^2a)^{1/3}-1/2*I*3^{1/2}/b*(-b^2a)^{1/3}))*3^{1/2}*b/(-b^2a)^{1/3})^{1/2}, (I*3^{1/2}/b*(-b^2a)^{1/3}/(-3/2/b*(-b^2a)^{1/3}+1/2*I*3^{1/2}/b*(-b^2a)^{1/3}))^{1/2}))))$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^8,x, algorithm="maxima")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^8, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^8,x, algorithm="fricas")

[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^8, x)

Sympy [A] time = 10.0289, size = 308, normalized size = 0.43

$$\frac{\sqrt{ac}\Gamma\left(-\frac{7}{3}\right) {}_2F_1\left(-\frac{7}{3}, -\frac{1}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)} + \frac{\sqrt{ae}\Gamma\left(-\frac{5}{3}\right) {}_2F_1\left(-\frac{5}{3}, -\frac{1}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{\sqrt{af}\Gamma\left(-\frac{4}{3}\right) {}_2F_1\left(-\frac{4}{3}, -\frac{1}{2} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} - \frac{ad}{6\sqrt{bx^{\frac{15}{2}}}\sqrt{\frac{a}{bx^3} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**8,x)

[Out] sqrt(a)*c*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3)) + sqrt(a)*e*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*f*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) - a*d/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - sqrt(b)*d/(4*x**(9/2)*sqrt(a/(b*x**3) + 1)) - sqrt(b)*g*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b**(3/2)*d/(12*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b*g*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a)) + b**2*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(12*a**(3/2))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^8,x, algorithm="giac")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^8, x)

$$3.457 \quad \int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^9} dx$$

Optimal. Leaf size=743

$$\frac{3^{3/4} \sqrt{2 + \sqrt{3}} b^{4/3} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{(1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}\right), -7 - 4\sqrt{3}\right) (7\sqrt[3]{b}(7bc - 16af) + 20(1 - \sqrt{3}))}{2240a^2 \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}}$$

[Out] -((((105*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5 + (210*g)/x^4)*Sqrt[a + b*x^3])/840 - (3*b*c*Sqrt[a + b*x^3])/(80*a*x^5) - (3*b*d*Sqrt[a + b*x^3])/(56*a*x^4) - (b*e*Sqrt[a + b*x^3])/(12*a*x^3) + (3*b*(7*b*c - 16*a*f)*Sqrt[a + b*x^3])/(320*a^2*x^2) + (3*b*(5*b*d - 14*a*g)*Sqrt[a + b*x^3])/(112*a^2*x) - (3*b^(4/3)*(5*b*d - 14*a*g)*Sqrt[a + b*x^3])/(112*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (b^2*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(3/2)) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(5*b*d - 14*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(224*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(4/3)*(7*b^(1/3)*(7*b*c - 16*a*f) + 20*(1 - Sqrt[3])*a^(1/3)*(5*b*d - 14*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(2240*a^2*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 1.33135, antiderivative size = 743, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {14, 1825, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$\frac{3^{3/4} \sqrt{2 + \sqrt{3}} b^{4/3} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx} + (1-\sqrt{3}) \sqrt[3]{a}}{\sqrt[3]{bx} + (1+\sqrt{3}) \sqrt[3]{a}}\right) | -7 - 4\sqrt{3}\right) (7\sqrt[3]{b}(7bc - 16af) + 20(1 - \sqrt{3}))}{2240a^2 \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9,x]

[Out] -(((105*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5 + (210*g)/x^4)*Sqrt[a + b*x^3])/840 - (3*b*c*Sqrt[a + b*x^3])/(80*a*x^5) - (3*b*d*Sqrt[a + b*x^3])/(56*a*x^4) - (b*e*Sqrt[a + b*x^3])/(12*a*x^3) + (3*b*(7*b*c - 16*a*f)*Sqrt[a + b*x^3])/(320*a^2*x^2) + (3*b*(5*b*d - 14*a*g)*Sqrt[a + b*x^3])/(12*a^2*x) - (3*b^(4/3)*(5*b*d - 14*a*g)*Sqrt[a + b*x^3])/((112*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (b^2*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(3/2)) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(5*b*d - 14*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(4/3)*(7*b^(1/3)*(7*b*c - 16*a*f) + 20*(1 - Sqrt[3])*a^(1/3)*(5*b*d - 14*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2240*a^2*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 1825

Int[(Pq_)*(x_)^ (m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{u = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1, 0]

Rule 1835

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x^n)^p, x], x] /; NeQ[Pq0, 0] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]

Rule 1832


```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] := Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
```

```

]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^9} dx &= -\frac{1}{840} \left(\frac{105c}{x^8} + \frac{120d}{x^7} + \frac{140e}{x^6} + \frac{168f}{x^5} + \frac{210g}{x^4} \right) \sqrt{a+bx^3} - \frac{1}{2}(3b) \int \frac{-\frac{c}{8} -}{x^8} dx \\
&= -\frac{1}{840} \left(\frac{105c}{x^8} + \frac{120d}{x^7} + \frac{140e}{x^6} + \frac{168f}{x^5} + \frac{210g}{x^4} \right) \sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{80ax^5} + \frac{3bc}{80a} \int \frac{1}{x^8} dx \\
&= -\frac{1}{840} \left(\frac{105c}{x^8} + \frac{120d}{x^7} + \frac{140e}{x^6} + \frac{168f}{x^5} + \frac{210g}{x^4} \right) \sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{80ax^5} - \frac{3bc}{160a} \int \frac{1}{x^7} dx \\
&= -\frac{1}{840} \left(\frac{105c}{x^8} + \frac{120d}{x^7} + \frac{140e}{x^6} + \frac{168f}{x^5} + \frac{210g}{x^4} \right) \sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{80ax^5} - \frac{3bc}{112a} \int \frac{1}{x^6} dx \\
&= -\frac{1}{840} \left(\frac{105c}{x^8} + \frac{120d}{x^7} + \frac{140e}{x^6} + \frac{168f}{x^5} + \frac{210g}{x^4} \right) \sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{80ax^5} - \frac{3bc}{112a} \int \frac{1}{x^5} dx \\
&= -\frac{1}{840} \left(\frac{105c}{x^8} + \frac{120d}{x^7} + \frac{140e}{x^6} + \frac{168f}{x^5} + \frac{210g}{x^4} \right) \sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{80ax^5} - \frac{3bc}{224a} \int \frac{1}{x^4} dx \\
&= -\frac{1}{840} \left(\frac{105c}{x^8} + \frac{120d}{x^7} + \frac{140e}{x^6} + \frac{168f}{x^5} + \frac{210g}{x^4} \right) \sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{80ax^5} - \frac{3bc}{224a} \int \frac{1}{x^3} dx \\
&= -\frac{1}{840} \left(\frac{105c}{x^8} + \frac{120d}{x^7} + \frac{140e}{x^6} + \frac{168f}{x^5} + \frac{210g}{x^4} \right) \sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{80ax^5} - \frac{3bc}{112a} \int \frac{1}{x^2} dx \\
&= -\frac{1}{840} \left(\frac{105c}{x^8} + \frac{120d}{x^7} + \frac{140e}{x^6} + \frac{168f}{x^5} + \frac{210g}{x^4} \right) \sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{80ax^5} - \frac{3bc}{112a} \int \frac{1}{x} dx \\
&= -\frac{1}{840} \left(\frac{105c}{x^8} + \frac{120d}{x^7} + \frac{140e}{x^6} + \frac{168f}{x^5} + \frac{210g}{x^4} \right) \sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{80ax^5} - \frac{3bc}{112a} \ln|x| + C
\end{aligned}$$

Mathematica [C] time = 0.190994, size = 192, normalized size = 0.26

$$\frac{\sqrt{a+bx^3} \left(14x^3 \left(36a^3 f {}_2F_1 \left(-\frac{5}{3}, -\frac{1}{2}; -\frac{2}{3}; -\frac{bx^3}{a} \right) + 45a^3 g x {}_2F_1 \left(-\frac{4}{3}, -\frac{1}{2}; -\frac{1}{3}; -\frac{bx^3}{a} \right) + 40b^2 e x^5 (a+bx^3) \sqrt{\frac{bx^3}{a}+1} {}_2F_1 \left(\frac{3}{2}, 3; \right. \right. \right.}{2520a^3 x^8 \sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9,x]

[Out] -(Sqrt[a + b*x^3]*(315*a^3*c*Hypergeometric2F1[-8/3, -1/2, -5/3, -((b*x^3)/a)] + 360*a^3*d*x*Hypergeometric2F1[-7/3, -1/2, -4/3, -((b*x^3)/a)] + 14*x^3*(36*a^3*f*Hypergeometric2F1[-5/3, -1/2, -2/3, -((b*x^3)/a)] + 45*a^3*g*x*Hypergeometric2F1[-4/3, -1/2, -1/3, -((b*x^3)/a)] + 40*b^2*e*x^5*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*x^3)/a]))/(2520*a^3*x^8*Sqrt[1 + (b*x^3)/a])

Maple [B] time = 0.009, size = 1679, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^9,x)

[Out] f*(-1/5*(b*x^3+a)^(1/2)/x^5-3/20*b/a*(b*x^3+a)^(1/2)/x^2+1/20*I/a*b^3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+g*(-1/4*(b*x^3+a)^(1/2)/x^4-3/8*b/a*(b*x^3+a)^(1/2)/x-1/8*I/a*b^3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))^(1/2))

$$\begin{aligned} & /3))^{(1/2)} + 1/b * (-b^2*a)^{(1/3)} * \text{EllipticF}(1/3*3^{(1/2)} * (I*(x+1/2/b*(-b^2*a)^{(1/3)} - 1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}) * 3^{(1/2)} * b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)} + 1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)})) + c * (-1/8*(b*x^3+a)^{(1/2)}/x^8 - 3/80*b/a*(b*x^3+a)^{(1/2)}/x^5 + 21/320/a^2*b^2*(b*x^3+a)^{(1/2)}/x^2 - 7/320*I/a^2*b^2*3^{(1/2)} * (-b^2*a)^{(1/3)} * (I*(x+1/2/b*(-b^2*a)^{(1/3)} - 1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}) * 3^{(1/2)} * b/(-b^2*a)^{(1/3)})^{(1/2)} * ((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)} + 1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)} * (-I*(x+1/2/b*(-b^2*a)^{(1/3)} + 1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}) * 3^{(1/2)} * b/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/2)} * \text{EllipticF}(1/3*3^{(1/2)} * (I*(x+1/2/b*(-b^2*a)^{(1/3)} - 1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}) * 3^{(1/2)} * b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)} + 1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)})) + e * (-1/6*(b*x^3+a)^{(1/2)}/x^6 - 1/12*b/a*(b*x^3+a)^{(1/2)}/x^3 + 1/12/a^{(3/2)} * b^2 * \text{arctanh}((b*x^3+a)^{(1/2)}/a^{(1/2)})) + d * (-1/7/x^7 * (b*x^3+a)^{(1/2)} - 3/56*b/a*(b*x^3+a)^{(1/2)}/x^4 + 15/112/a^2*b^2*(b*x^3+a)^{(1/2)}/x + 5/112*I/a^2*b^2*3^{(1/2)} * (-b^2*a)^{(1/3)} * (I*(x+1/2/b*(-b^2*a)^{(1/3)} - 1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}) * 3^{(1/2)} * b/(-b^2*a)^{(1/3)})^{(1/2)} * ((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)} + 1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)} * (-I*(x+1/2/b*(-b^2*a)^{(1/3)} + 1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}) * 3^{(1/2)} * b/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/2)} * ((-3/2/b*(-b^2*a)^{(1/3)} + 1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}) * \text{EllipticE}(1/3*3^{(1/2)} * (I*(x+1/2/b*(-b^2*a)^{(1/3)} - 1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}) * 3^{(1/2)} * b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)} + 1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)})) + 1/b * (-b^2*a)^{(1/3)} * \text{EllipticF}(1/3*3^{(1/2)} * (I*(x+1/2/b*(-b^2*a)^{(1/3)} - 1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}) * 3^{(1/2)} * b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)} + 1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}))
\end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^9,x, algorithm="maxima")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^9, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^9,x, algorithm="fricas")

[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^9, x)

Sympy [A] time = 9.38585, size = 304, normalized size = 0.41

$$\frac{\sqrt{ac}\Gamma\left(-\frac{8}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{8}{3}, -\frac{1}{2} \\ -\frac{5}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3x^8\Gamma\left(-\frac{5}{3}\right)} + \frac{\sqrt{ad}\Gamma\left(-\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{7}{3}, -\frac{1}{2} \\ -\frac{4}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)} + \frac{\sqrt{af}\Gamma\left(-\frac{5}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{3}, -\frac{1}{2} \\ -\frac{2}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{\sqrt{ag}\Gamma\left(-\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{4}{3} \\ -\frac{1}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**9,x)

[Out] sqrt(a)*c*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**8*gamma(-5/3)) + sqrt(a)*d*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3)) + sqrt(a)*f*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*g*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) - a*e/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - sqrt(b)*e/(4*x**(9/2)*sqrt(a/(b*x**3) + 1)) - b**(3/2)*e/(12*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) + b**2*e*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(12*a**(3/2))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^9, x)
```

$$3.458 \quad \int x^3 (a + bx^3)^{3/2} (c + dx + ex^2 + fx^3 + gx^4) dx$$

Optimal. Leaf size=791

$$36 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} a^3 (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \text{EllipticF} \left(\sin^{-1} \left(\frac{(1 - \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{(1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}} \right), -7 - 4\sqrt{3} \right) (1729 \sqrt[3]{b} (23bc - 8af) -$$

$$37182145 b^{8/3} \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

[Out] $(-4a^3 e \sqrt{a + bx^3}) / (105b^2) + (54a^2 (23bc - 8af) x \sqrt{a + bx^3}) / (21505b^2) + (54a^2 (5bd - 2ag) x^2 \sqrt{a + bx^3}) / (8645b^2) + (2a^2 e x^3 \sqrt{a + bx^3}) / (105b) + (54a^2 f x^4 \sqrt{a + bx^3}) / (4301b) + (54a^2 g x^5 \sqrt{a + bx^3}) / (6175b) - (216a^3 (5bd - 2ag) \sqrt{a + bx^3}) / (8645b^{8/3} ((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)) + (2x^3 (a + bx^3)^{3/2} (229425cx + 205275d x^2 + 185725e x^3 + 169575f x^4 + 156009g x^5)) / 3900225 + (2a x^3 \sqrt{a + bx^3} (8947575cx + 6774075d x^2 + 5311735e x^3 + 4279275f x^4 + 3522519g x^5)) / 185910725 + (108 \cdot 3^{1/4} \sqrt{2 - \sqrt{3}} a^{10/3} (5bd - 2ag) (a^{1/3} + b^{1/3} x) \sqrt{(a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2) / ((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2}) \cdot \text{EllipticE}[\text{ArcSin}(((1 - \sqrt{3}) a^{1/3} + b^{1/3} x) / ((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)), -7 - 4\sqrt{3}]] / (8645b^{8/3} \sqrt{(a^{1/3} (a^{1/3} + b^{1/3} x)) / ((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2}) \sqrt{a + bx^3}) - (36 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} a^3 (1729b^{1/3} (23bc - 8af) - 8602(1 - \sqrt{3}) a^{1/3} (5bd - 2ag)) (a^{1/3} + b^{1/3} x) \sqrt{(a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2) / ((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2}) \cdot \text{EllipticF}[\text{ArcSin}(((1 - \sqrt{3}) a^{1/3} + b^{1/3} x) / ((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)), -7 - 4\sqrt{3}]] / (37182145b^{8/3} \sqrt{(a^{1/3} (a^{1/3} + b^{1/3} x)) / ((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2}) \sqrt{a + bx^3})$

Rubi [A] time = 2.11123, antiderivative size = 791, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1826, 1836, 1888, 1594, 1886, 261, 1878, 218, 1877}

$$36 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} a^3 (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} F \left(\sin^{-1} \left(\frac{\sqrt[3]{bx} + (1 - \sqrt{3}) \sqrt[3]{a}}{\sqrt[3]{bx} + (1 + \sqrt{3}) \sqrt[3]{a}} \right) \middle| -7 - 4\sqrt{3} \right) (1729 \sqrt[3]{b} (23bc - 8af) - 8602 (1 - \sqrt{3}) a^{1/3} (5bd - 2ag)) (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \cdot \text{EllipticF} \left[\text{ArcSin} \left(\frac{\sqrt[3]{bx} + (1 - \sqrt{3}) \sqrt[3]{a}}{\sqrt[3]{bx} + (1 + \sqrt{3}) \sqrt[3]{a}} \right) \middle| -7 - 4\sqrt{3} \right] / (37182145 b^{8/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \sqrt{a + bx^3})$$

$$37182145 b^{8/3} \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

Antiderivative was successfully verified.

[In] Int[x^3*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]

[Out]
$$\begin{aligned} & (-4*a^3*e*\text{Sqrt}[a + b*x^3])/(105*b^2) + (54*a^2*(23*b*c - 8*a*f)*x*\text{Sqrt}[a + \\ & b*x^3])/(21505*b^2) + (54*a^2*(5*b*d - 2*a*g)*x^2*\text{Sqrt}[a + b*x^3])/(8645*b^2) + (2*a^2*e*x^3*\text{Sqrt}[a + b*x^3])/(105*b) + (54*a^2*f*x^4*\text{Sqrt}[a + b*x^3]) \\ & / (4301*b) + (54*a^2*g*x^5*\text{Sqrt}[a + b*x^3])/(6175*b) - (216*a^3*(5*b*d - 2*a \\ & *g)*\text{Sqrt}[a + b*x^3])/(8645*b^{8/3}*((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)) + (\\ & 2*x^3*(a + b*x^3)^{3/2}*(229425*c*x + 205275*d*x^2 + 185725*e*x^3 + 169575* \\ & f*x^4 + 156009*g*x^5))/3900225 + (2*a*x^3*\text{Sqrt}[a + b*x^3]*(8947575*c*x + 67 \\ & 74075*d*x^2 + 5311735*e*x^3 + 4279275*f*x^4 + 3522519*g*x^5))/185910725 + (\\ & 108*3^{1/4}*\text{Sqrt}[2 - \text{Sqrt}[3]]*a^{10/3}*(5*b*d - 2*a*g)*(a^{1/3} + b^{1/3}*x) \\ &)*\text{Sqrt}[(a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2)/((1 + \text{Sqrt}[3])*a^{1/3} + \\ & b^{1/3}*x)^2]*\text{EllipticE}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)/((1 + \text{S} \\ & \text{qrt}[3])*a^{1/3} + b^{1/3}*x)], -7 - 4*\text{Sqrt}[3]]/(8645*b^{8/3}*\text{Sqrt}[(a^{1/3} \\ & *(a^{1/3} + b^{1/3}*x))/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\text{Sqrt}[a + b*x \\ & ^3]) - (36*3^{3/4}*\text{Sqrt}[2 + \text{Sqrt}[3]]*a^3*(1729*b^{1/3}*(23*b*c - 8*a*f) - 8 \\ & 602*(1 - \text{Sqrt}[3])*a^{1/3}*(5*b*d - 2*a*g))*(a^{1/3} + b^{1/3}*x)*\text{Sqrt}[(a^{2/3} \\ & - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2)/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2] \\ & *\text{EllipticF}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)/((1 + \text{Sqrt}[3])*a^{1/3} \\ & + b^{1/3}*x)], -7 - 4*\text{Sqrt}[3]]/(37182145*b^{8/3}*\text{Sqrt}[(a^{1/3}*(a^{1/3} \\ & + b^{1/3}*x))/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\text{Sqrt}[a + b*x^3]) \end{aligned}$$

Rule 1826

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}], x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]

Rule 1836

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m + q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*n)]) /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]

Rule 1888

Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Expon[Pq, x

```

]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]

```

Rule 1594

```

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

```

Rule 1886

```

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[Coeff[Pq, x, n -
1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1

```

Rule 261

```

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
NeQ[p, -1]

```

Rule 1878

```

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]

```

Rule 218

```

Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]

```

Rule 1877

```

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)]/((1 + Sqrt[3])*s + r*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

```

Rubi steps

$$\begin{aligned}
\int x^3 (a + bx^3)^{3/2} (c + dx + ex^2 + fx^3 + gx^4) dx &= \frac{2x^3 (a + bx^3)^{3/2} (229425cx + 205275dx^2 + 185725ex^3 + 169575fx^4 + 149625gx^5)}{3900225} \\
&= \frac{2x^3 (a + bx^3)^{3/2} (229425cx + 205275dx^2 + 185725ex^3 + 169575fx^4 + 149625gx^5)}{3900225} \\
&= \frac{54a^2gx^5\sqrt{a + bx^3}}{6175b} + \frac{2x^3 (a + bx^3)^{3/2} (229425cx + 205275dx^2 + 185725ex^3 + 169575fx^4 + 149625gx^5)}{3900225} \\
&= \frac{54a^2fx^4\sqrt{a + bx^3}}{4301b} + \frac{54a^2gx^5\sqrt{a + bx^3}}{6175b} + \frac{2x^3 (a + bx^3)^{3/2} (229425cx + 205275dx^2 + 185725ex^3 + 169575fx^4 + 149625gx^5)}{3900225} \\
&= \frac{2a^2ex^3\sqrt{a + bx^3}}{105b} + \frac{54a^2fx^4\sqrt{a + bx^3}}{4301b} + \frac{54a^2gx^5\sqrt{a + bx^3}}{6175b} + \frac{2x^3 (a + bx^3)^{3/2} (229425cx + 205275dx^2 + 185725ex^3 + 169575fx^4 + 149625gx^5)}{3900225} \\
&= \frac{2a^2ex^3\sqrt{a + bx^3}}{105b} + \frac{54a^2fx^4\sqrt{a + bx^3}}{4301b} + \frac{54a^2gx^5\sqrt{a + bx^3}}{6175b} + \frac{2x^3 (a + bx^3)^{3/2} (229425cx + 205275dx^2 + 185725ex^3 + 169575fx^4 + 149625gx^5)}{3900225} \\
&= \frac{54a^2(5bd - 2ag)x^2\sqrt{a + bx^3}}{8645b^2} + \frac{2a^2ex^3\sqrt{a + bx^3}}{105b} + \frac{54a^2fx^4\sqrt{a + bx^3}}{4301b} \\
&= \frac{54a^2(5bd - 2ag)x^2\sqrt{a + bx^3}}{8645b^2} + \frac{2a^2ex^3\sqrt{a + bx^3}}{105b} + \frac{54a^2fx^4\sqrt{a + bx^3}}{4301b} \\
&= \frac{54a^2(23bc - 8af)x\sqrt{a + bx^3}}{21505b^2} + \frac{54a^2(5bd - 2ag)x^2\sqrt{a + bx^3}}{8645b^2} + \frac{2a^2ex^3\sqrt{a + bx^3}}{105b} \\
&= \frac{54a^2(23bc - 8af)x\sqrt{a + bx^3}}{21505b^2} + \frac{54a^2(5bd - 2ag)x^2\sqrt{a + bx^3}}{8645b^2} + \frac{2a^2ex^3\sqrt{a + bx^3}}{105b} \\
&= -\frac{4a^3e\sqrt{a + bx^3}}{105b^2} + \frac{54a^2(23bc - 8af)x\sqrt{a + bx^3}}{21505b^2} + \frac{54a^2(5bd - 2ag)x^2\sqrt{a + bx^3}}{8645b^2} \\
&= -\frac{4a^3e\sqrt{a + bx^3}}{105b^2} + \frac{54a^2(23bc - 8af)x\sqrt{a + bx^3}}{21505b^2} + \frac{54a^2(5bd - 2ag)x^2\sqrt{a + bx^3}}{8645b^2}
\end{aligned}$$

Mathematica [C] time = 0.52666, size = 179, normalized size = 0.23

$$\frac{2\sqrt{a+bx^3}\left(9975a^2x(8af-23bc) {}_2F_1\left(-\frac{3}{2}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right) + 41055a^2x^2(2ag-5bd) {}_2F_1\left(-\frac{3}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^3}{a}\right) - (a+bx^3)^2\sqrt{\frac{bx^3}{a}} + 3900225b^2\sqrt{\frac{bx^3}{a}}\right)}{3900225b^2\sqrt{\frac{bx^3}{a}} + \dots}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4), x]

[Out] (2*sqrt[a + b*x^3]*(-(a + b*x^3)^2*sqrt[1 + (b*x^3)/a]*(10*a*(7429*e + 21*x*(380*f + 391*g*x)) - b*x*(229425*c + 17*x*(12075*d + 19*x*(575*e + 525*f*x + 483*g*x^2)))) + 9975*a^2*(-23*b*c + 8*a*f)*x*Hypergeometric2F1[-3/2, 1/3, 4/3, -(b*x^3)/a] + 41055*a^2*(-5*b*d + 2*a*g)*x^2*Hypergeometric2F1[-3/2, 2/3, 5/3, -(b*x^3)/a]))/(3900225*b^2*sqrt[1 + (b*x^3)/a])

Maple [B] time = 0.017, size = 1764, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c), x)

[Out] g*(2/25*b*x^11*(b*x^3+a)^(1/2)+56/475*a*x^8*(b*x^3+a)^(1/2)+54/6175/b*a^2*x^5*(b*x^3+a)^(1/2)-108/8645*a^3/b^2*x^2*(b*x^3+a)^(1/2)-144/8645*I*a^4/b^3*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3))+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+f*(2/23*b*x^10*(b*x^3+a)^(1/2)+52/391*a*x^7*(b*x^3+a)^(1/2)+54/4301/b*a^2*x^4*(b*x^3+a)^(1/2)-432/21505*a^3/b^2*x*(b*x^3+a)^(1/2)-288/21505*I*a^4/b^3*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))

```

*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3
)))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)
^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1
/2)/b*(-b^2*a)^(1/3)))^(1/2)))+e*(2/21*b*x^9*(b*x^3+a)^(1/2)+16/105*a*x^6*(
b*x^3+a)^(1/2)+2/105/b*a^2*x^3*(b*x^3+a)^(1/2)-4/105*a^3/b^2*(b*x^3+a)^(1/2
))+d*(2/19*b*x^8*(b*x^3+a)^(1/2)+44/247*a*x^5*(b*x^3+a)^(1/2)+54/1729/b*a^2
*x^2*(b*x^3+a)^(1/2)+72/1729*I/b^2*a^3*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(
-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/
2))*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3
^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*
I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3
))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2
)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(
1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/
b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1
/2)))))+c*(2/17*b*x^7*(b*x^3+a)^(1/2)+40/187*a*x^4*(b*x^3+a)^(1/2)+54/935/b*
a^2*x*(b*x^3+a)^(1/2)+36/935*I/b^2*a^3*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(
-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/
2))*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3
^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/
3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3)))^(1/2)))

```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bgx^{10} + bfx^9 + bex^8 + (bd + ag)x^7 + aex^5 + (bc + af)x^6 + adx^4 + acx^3\right)\sqrt{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")

[Out] integral((b*g*x^10 + b*f*x^9 + b*e*x^8 + (b*d + a*g)*x^7 + a*e*x^5 + (b*c + a*f)*x^6 + a*d*x^4 + a*c*x^3)*sqrt(b*x^3 + a), x)

Sympy [A] time = 10.7682, size = 512, normalized size = 0.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c),x)

[Out] a**(3/2)*c*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(3/2)*d*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + a**(3/2)*f*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(3/2)*g*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) + sqrt(a)*b*c*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)*b*d*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) + sqrt(a)*b*f*x**10*gamma(10/3)*hyper((-1/2, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(13/3)) + sqrt(a)*b*g*x**11*gamma(11/3)*hyper((-1/2, 11/3), (14/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(14/3)) + a*e*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True)) + b*e*Piecewise((16*a**3*sqrt(a + b*x**3)/(315*b**3) - 8*a**2*x**3*sqrt(a + b*x**3)/(315*b**2) + 2*a*x**6*sqrt(a + b*x**3)/(105*b) + 2*x**9*sqrt(a + b*x**3)/21, Ne(b, 0)), (sqrt(a)*x**9/9, True))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x^3, x)
```


$$3.459 \quad \int x^2 (a + bx^3)^{3/2} (c + dx + ex^2 + fx^3 + gx^4) dx$$

Optimal. Leaf size=742

$$36 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} a^3 (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (43010(1 - \sqrt{3}) \sqrt[3]{ab^{2/3}} e - 1729(23bd - 8ag)) \operatorname{EllipticF}\left(\sin^{-1}\left(\frac{(1 - \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{(1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}\right)\right)$$

$$\frac{37182145 b^{7/3} \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}}$$

[Out] (2*a^2*(7*b*c - 2*a*f)*Sqrt[a + b*x^3])/(105*b^2) + (54*a^2*(23*b*d - 8*a*g)*x*Sqrt[a + b*x^3])/(21505*b^2) + (54*a^2*e*x^2*Sqrt[a + b*x^3])/(1729*b) + (2*a^2*f*x^3*Sqrt[a + b*x^3])/(105*b) + (54*a^2*g*x^4*Sqrt[a + b*x^3])/(4301*b) - (216*a^3*e*Sqrt[a + b*x^3])/(1729*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x^2*(a + b*x^3)^(3/2)*(52003*c*x + 45885*d*x^2 + 41055*e*x^3 + 37145*f*x^4 + 33915*g*x^5))/780045 + (2*a*x^2*Sqrt[a + b*x^3]*(7436429*c*x + 5368545*d*x^2 + 4064445*e*x^3 + 3187041*f*x^4 + 2567565*g*x^5))/111546435 + (108*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(10/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (36*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^3*(43010*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e - 1729*(23*b*d - 8*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(37182145*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 1.55222, antiderivative size = 742, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1826, 1836, 1888, 1594, 1886, 261, 1878, 218, 1877}

$$\frac{2a^2 \sqrt{a + bx^3} (7bc - 2af)}{105b^2} + \frac{36 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} a^3 (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (43010(1 - \sqrt{3}) \sqrt[3]{ab^{2/3}} e - 1729(23bd - 8ag)) \operatorname{EllipticF}\left(\sin^{-1}\left(\frac{(1 - \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{(1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}\right)\right)}{37182145 b^{7/3} \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]

[Out] (2*a^2*(7*b*c - 2*a*f)*Sqrt[a + b*x^3])/(105*b^2) + (54*a^2*(23*b*d - 8*a*g)*x*Sqrt[a + b*x^3])/(21505*b^2) + (54*a^2*e*x^2*Sqrt[a + b*x^3])/(1729*b) + (2*a^2*f*x^3*Sqrt[a + b*x^3])/(105*b) + (54*a^2*g*x^4*Sqrt[a + b*x^3])/(4301*b) - (216*a^3*e*Sqrt[a + b*x^3])/(1729*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x^2*(a + b*x^3)^(3/2)*(52003*c*x + 45885*d*x^2 + 41055*e*x^3 + 37145*f*x^4 + 33915*g*x^5))/780045 + (2*a*x^2*Sqrt[a + b*x^3]*(7436429*c*x + 5368545*d*x^2 + 4064445*e*x^3 + 3187041*f*x^4 + 2567565*g*x^5))/111546435 + (108*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(10/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (36*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^3*(43010*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e - 1729*(23*b*d - 8*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(37182145*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rule 1826

Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}], x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]

Rule 1836

Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m + q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]

Rule 1888

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum

```
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1)), x] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*n)]) /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n, x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^p, x_Symbol] := Dist[Coeff[Pq, x, n - 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^p, x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]
```

$$\text{Simp}[(2*d*s^3*\text{Sqrt}[a + b*x^3])/(a*r^2*((1 + \text{Sqrt}[3])*s + r*x)), x] - \text{Simp}[(3^{(1/4)}*\text{Sqrt}[2 - \text{Sqrt}[3]]*d*s*(s + r*x)*\text{Sqrt}[(s^2 - r*s*x + r^2*x^2)/((1 + \text{Sqrt}[3])*s + r*x)^2]*\text{EllipticE}[\text{ArcSin}[(1 - \text{Sqrt}[3])*s + r*x]/((1 + \text{Sqrt}[3])*s + r*x)], -7 - 4*\text{Sqrt}[3])]/(r^2*\text{Sqrt}[a + b*x^3]*\text{Sqrt}[(s*(s + r*x))/((1 + \text{Sqrt}[3])*s + r*x)^2]), x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{PosQ}[a] \ \&\& \ \text{EqQ}[b*c^3 - 2*(5 - 3*\text{Sqrt}[3])*a*d^3, 0]$$

Rubi steps

$$\begin{aligned}
 \int x^2 (a + bx^3)^{3/2} (c + dx + ex^2 + fx^3 + gx^4) dx &= \frac{2x^2 (a + bx^3)^{3/2} (52003cx + 45885dx^2 + 41055ex^3 + 37145fx^4 + 339c^2)}{780045} \\
 &= \frac{2x^2 (a + bx^3)^{3/2} (52003cx + 45885dx^2 + 41055ex^3 + 37145fx^4 + 339c^2)}{780045} \\
 &= \frac{54a^2gx^4\sqrt{a + bx^3}}{4301b} + \frac{2x^2 (a + bx^3)^{3/2} (52003cx + 45885dx^2 + 41055ex^3 + 37145fx^4 + 339c^2)}{780045} \\
 &= \frac{2a^2fx^3\sqrt{a + bx^3}}{105b} + \frac{54a^2gx^4\sqrt{a + bx^3}}{4301b} + \frac{2x^2 (a + bx^3)^{3/2} (52003cx + 45885dx^2 + 41055ex^3 + 37145fx^4 + 339c^2)}{780045} \\
 &= \frac{54a^2ex^2\sqrt{a + bx^3}}{1729b} + \frac{2a^2fx^3\sqrt{a + bx^3}}{105b} + \frac{54a^2gx^4\sqrt{a + bx^3}}{4301b} + \frac{2x^2 (a + bx^3)^{3/2} (52003cx + 45885dx^2 + 41055ex^3 + 37145fx^4 + 339c^2)}{780045} \\
 &= \frac{54a^2ex^2\sqrt{a + bx^3}}{1729b} + \frac{2a^2fx^3\sqrt{a + bx^3}}{105b} + \frac{54a^2gx^4\sqrt{a + bx^3}}{4301b} + \frac{2x^2 (a + bx^3)^{3/2} (52003cx + 45885dx^2 + 41055ex^3 + 37145fx^4 + 339c^2)}{780045} \\
 &= \frac{54a^2(23bd - 8ag)x\sqrt{a + bx^3}}{21505b^2} + \frac{54a^2ex^2\sqrt{a + bx^3}}{1729b} + \frac{2a^2fx^3\sqrt{a + bx^3}}{105b} \\
 &= \frac{54a^2(23bd - 8ag)x\sqrt{a + bx^3}}{21505b^2} + \frac{54a^2ex^2\sqrt{a + bx^3}}{1729b} + \frac{2a^2fx^3\sqrt{a + bx^3}}{105b} \\
 &= \frac{2a^2(7bc - 2af)\sqrt{a + bx^3}}{105b^2} + \frac{54a^2(23bd - 8ag)x\sqrt{a + bx^3}}{21505b^2} + \frac{54a^2ex^2\sqrt{a + bx^3}}{1729b} \\
 &= \frac{2a^2(7bc - 2af)\sqrt{a + bx^3}}{105b^2} + \frac{54a^2(23bd - 8ag)x\sqrt{a + bx^3}}{21505b^2} + \frac{54a^2ex^2\sqrt{a + bx^3}}{1729b}
 \end{aligned}$$

Mathematica [C] time = 0.349519, size = 162, normalized size = 0.22

$$\frac{2 \left(1995a^3x\sqrt{\frac{bx^3}{a}} + 1(8ag - 23bd) {}_2F_1\left(-\frac{3}{2}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right) - 41055a^3bex^2\sqrt{\frac{bx^3}{a}} + 1 {}_2F_1\left(-\frac{3}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^3}{a}\right) + (a + bx^3)^3(-38a(3) \right)}{780045b^2\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]

[Out] (2*((a + b*x^3)^3*(52003*b*c - 38*a*(391*f + 420*g*x) + 5*b*x*(9177*d + 17*x*(483*e + 19*x*(23*f + 21*g*x)))) + 1995*a^3*(-23*b*d + 8*a*g)*x*sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)] - 41055*a^3*b*e*x^2*sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)]))/(780045*b^2*sqrt[a + b*x^3])

Maple [B] time = 0.008, size = 1269, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x)

[Out] g*(2/23*b*x^10*(b*x^3+a)^(1/2)+52/391*a*x^7*(b*x^3+a)^(1/2)+54/4301/b*a^2*x^4*(b*x^3+a)^(1/2)-432/21505*a^3/b^2*x*(b*x^3+a)^(1/2)-288/21505*I*a^4/b^3*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+f*(2/21*b*x^9*(b*x^3+a)^(1/2)+16/105*a*x^6*(b*x^3+a)^(1/2)+2/105/b*a^2*x^3*(b*x^3+a)^(1/2)-4/105*a^3/b^2*(b*x^3+a)^(1/2))+e*(2/19*b*x^8*(b*x^3+a)^(1/2)+44/247*a*x^5*(b*x^3+a)^(1/2)+54/1729/b*a^2*x^2*(b*x^3+a)^(1/2)+72/1729*I/b^2*a^3*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3

$$\begin{aligned} & \left(\frac{1}{2} \left(I \left(x + \frac{1}{2} \sqrt{b} \sqrt{-b^2 a} \right)^{\frac{1}{3}} - \frac{1}{2} I^3 \left(\frac{1}{2} \sqrt{b} \sqrt{-b^2 a} \right)^{\frac{1}{3}} \right)^{\frac{1}{2}} \sqrt{b} \right. \\ & \left. / \left(-b^2 a \right)^{\frac{1}{3}} \right)^{\frac{1}{2}}, \left(I^3 \left(\frac{1}{2} \sqrt{b} \sqrt{-b^2 a} \right)^{\frac{1}{3}} / \left(-\frac{3}{2} \sqrt{b} \sqrt{-b^2 a} \right)^{\frac{1}{3}} + \right. \\ & \left. \frac{1}{2} I^3 \left(\frac{1}{2} \sqrt{b} \sqrt{-b^2 a} \right)^{\frac{1}{3}} \right)^{\frac{1}{2}} + \frac{1}{b} \sqrt{-b^2 a}^{\frac{1}{3}} \operatorname{EllipticF} \left(\frac{1}{3} \sqrt{3} \left(\right. \right. \\ & \left. \left. \frac{1}{2} \left(I \left(x + \frac{1}{2} \sqrt{b} \sqrt{-b^2 a} \right)^{\frac{1}{3}} - \frac{1}{2} I^3 \left(\frac{1}{2} \sqrt{b} \sqrt{-b^2 a} \right)^{\frac{1}{3}} \right)^{\frac{1}{2}} \sqrt{b} \right. \right. \\ & \left. \left. / \left(-b^2 a \right)^{\frac{1}{3}} \right)^{\frac{1}{2}}, \left(I^3 \left(\frac{1}{2} \sqrt{b} \sqrt{-b^2 a} \right)^{\frac{1}{3}} / \left(-\frac{3}{2} \sqrt{b} \sqrt{-b^2 a} \right)^{\frac{1}{3}} + \right. \right. \\ & \left. \left. \frac{1}{2} I^3 \left(\frac{1}{2} \sqrt{b} \sqrt{-b^2 a} \right)^{\frac{1}{3}} \right)^{\frac{1}{2}} \right) + d \left(\frac{2}{17} b x^7 (b x^3 + a)^{\frac{1}{2}} + \frac{40}{187} \right. \\ & \left. a x^4 (b x^3 + a)^{\frac{1}{2}} + \frac{54}{935} b a^2 x (b x^3 + a)^{\frac{1}{2}} + \frac{36}{935} I / b^2 a^3 \sqrt{3} \left(\right. \right. \\ & \left. \left. \frac{1}{2} \left(-b^2 a \right)^{\frac{1}{3}} \left(I \left(x + \frac{1}{2} \sqrt{b} \sqrt{-b^2 a} \right)^{\frac{1}{3}} - \frac{1}{2} I^3 \left(\frac{1}{2} \sqrt{b} \sqrt{-b^2 a} \right)^{\frac{1}{3}} \right) \right)^{\frac{1}{2}} \right. \\ & \left. \left. \sqrt{b} \right. \right. / \left(-b^2 a \right)^{\frac{1}{3}} \right)^{\frac{1}{2}} \left(\left(x - \frac{1}{b} \sqrt{-b^2 a} \right)^{\frac{1}{3}} / \left(-\frac{3}{2} \sqrt{b} \sqrt{-b^2 a} \right)^{\frac{1}{3}} + \right. \\ & \left. \frac{1}{2} I^3 \left(\frac{1}{2} \sqrt{b} \sqrt{-b^2 a} \right)^{\frac{1}{3}} \right)^{\frac{1}{2}} \left(-I \left(x + \frac{1}{2} \sqrt{b} \sqrt{-b^2 a} \right)^{\frac{1}{3}} + \right. \\ & \left. \frac{1}{2} I^3 \left(\frac{1}{2} \sqrt{b} \sqrt{-b^2 a} \right)^{\frac{1}{3}} \right)^{\frac{1}{2}} \sqrt{b} / \left(-b^2 a \right)^{\frac{1}{3}} \right)^{\frac{1}{2}} / (b x^3 + a)^{\frac{1}{2}} \\ & \left. \left. \operatorname{EllipticF} \left(\frac{1}{3} \sqrt{3} \left(\frac{1}{2} \left(I \left(x + \frac{1}{2} \sqrt{b} \sqrt{-b^2 a} \right)^{\frac{1}{3}} - \frac{1}{2} I^3 \left(\frac{1}{2} \sqrt{b} \sqrt{-b^2 a} \right)^{\frac{1}{3}} \right) \right)^{\frac{1}{2}} \right. \right. \right. \right. \\ & \left. \left. \left. \sqrt{b} \right. \right. / \left(-b^2 a \right)^{\frac{1}{3}} \right)^{\frac{1}{2}}, \left(I^3 \left(\frac{1}{2} \sqrt{b} \sqrt{-b^2 a} \right)^{\frac{1}{3}} / \left(-\frac{3}{2} \sqrt{b} \sqrt{-b^2 a} \right)^{\frac{1}{3}} + \right. \right. \\ & \left. \left. \frac{1}{2} I^3 \left(\frac{1}{2} \sqrt{b} \sqrt{-b^2 a} \right)^{\frac{1}{3}} \right)^{\frac{1}{2}} \right) + \frac{2}{15} c / b (b x^3 + a)^{\frac{5}{2}} \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2(bx^3 + a)^{\frac{5}{2}} c}{15b} + \int (bgx^9 + bfx^8 + bex^7 + afx^5 + (bd + ag)x^6 + aex^4 + adx^3) \sqrt{bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima")

[Out] 2/15*(b*x^3 + a)^(5/2)*c/b + integrate((b*g*x^9 + b*f*x^8 + b*e*x^7 + a*f*x^5 + (b*d + a*g)*x^6 + a*e*x^4 + a*d*x^3)*sqrt(b*x^3 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left((bgx^9 + bfx^8 + bex^7 + (bd + ag)x^6 + aex^4 + (bc + af)x^5 + adx^3 + acx^2) \sqrt{bx^3 + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")

[Out] integral((b*g*x^9 + b*f*x^8 + b*e*x^7 + (b*d + a*g)*x^6 + a*e*x^4 + (b*c + a*f)*x^5 + a*d*x^3 + a*c*x^2)*sqrt(b*x^3 + a), x)

Sympy [A] time = 9.44158, size = 525, normalized size = 0.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c),x)

[Out] a**(3/2)*d*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(3/2)*e*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + a**(3/2)*g*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)*b*d*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)*b*e*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) + sqrt(a)*b*g*x**10*gamma(10/3)*hyper((-1/2, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(13/3)) + a*c*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + a*f*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True)) + b*c*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True)) + b*f*Piecewise((16*a**3*sqrt(a + b*x**3)/(315*b**3) - 8*a**2*x**3*sqrt(a + b*x**3)/(315*b**2) + 2*a*x**6*sqrt(a + b*x**3)/(105*b) + 2*x**9*sqrt(a + b*x**3)/21, Ne(b, 0)), (sqrt(a)*x**9/9, True))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x^2, x)

$$3.460 \quad \int x (a + bx^3)^{3/2} (c + dx + ex^2 + fx^3 + gx^4) dx$$

Optimal. Leaf size=723

$$18 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} a^{7/3} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (3458 a^{2/3} \sqrt[3]{be} + 935 (1 - \sqrt{3}) (19bc - 4af)) \text{EllipticF} \left(\sin^{-1} \left(\frac{(1-\sqrt{3}) \sqrt[3]{a}}{(1+\sqrt{3}) \sqrt[3]{a}} \right) \right)$$

$$1616615 b^{5/3} \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

[Out] (2*a^2*(7*b*d - 2*a*g)*Sqrt[a + b*x^3])/(105*b^2) + (54*a^2*e*x*Sqrt[a + b*x^3])/(935*b) + (54*a^2*f*x^2*Sqrt[a + b*x^3])/(1729*b) + (2*a^2*g*x^3*Sqrt[a + b*x^3])/(105*b) + (54*a^2*(19*b*c - 4*a*f)*Sqrt[a + b*x^3])/(1729*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x*(a + b*x^3)^(3/2)*(33915*c*x + 29393*d*x^2 + 25935*e*x^3 + 23205*f*x^4 + 20995*g*x^5))/440895 + (2*a*x*Sqrt[a + b*x^3]*(479655*c*x + 323323*d*x^2 + 233415*e*x^3 + 176715*f*x^4 + 138567*g*x^5))/4849845 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(19*b*c - 4*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(7/3)*(3458*a^(2/3)*b^(1/3)*e + 935*(1 - Sqrt[3])*(19*b*c - 4*a*f))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1616615*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 1.23865, antiderivative size = 723, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {1826, 1836, 1888, 1886, 261, 1878, 218, 1877}

$$18 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} a^{7/3} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (3458 a^{2/3} \sqrt[3]{be} + 935 (1 - \sqrt{3}) (19bc - 4af)) F \left(\sin^{-1} \left(\frac{\sqrt[3]{bx} + (1-\sqrt{3}) \sqrt[3]{a}}{\sqrt[3]{bx} + (1+\sqrt{3}) \sqrt[3]{a}} \right) \right)$$

$$1616615 b^{5/3} \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]

[Out] (2*a^2*(7*b*d - 2*a*g)*Sqrt[a + b*x^3])/(105*b^2) + (54*a^2*e*x*Sqrt[a + b*x^3])/(935*b) + (54*a^2*f*x^2*Sqrt[a + b*x^3])/(1729*b) + (2*a^2*g*x^3*Sqrt[a + b*x^3])/(105*b) + (54*a^2*(19*b*c - 4*a*f)*Sqrt[a + b*x^3])/(1729*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x*(a + b*x^3)^(3/2)*(33915*c*x + 29393*d*x^2 + 25935*e*x^3 + 23205*f*x^4 + 20995*g*x^5))/440895 + (2*a*x*Sqrt[a + b*x^3]*(479655*c*x + 323323*d*x^2 + 233415*e*x^3 + 176715*f*x^4 + 138567*g*x^5))/4849845 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(19*b*c - 4*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(7/3)*(3458*a^(2/3)*b^(1/3)*e + 935*(1 - Sqrt[3])*(19*b*c - 4*a*f))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1616615*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rule 1826

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}], x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]

Rule 1836

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m + q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]

Rule 1888

Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum

```
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[Coeff[Pq, x, n - 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
```

Q[b*c^3 - 2*(5 - 3*sqrt[3])*a*d^3, 0]

Rubi steps

$$\begin{aligned}
 \int x(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)dx &= \frac{2x(a+bx^3)^{3/2}(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5)}{440895} \\
 &= \frac{2x(a+bx^3)^{3/2}(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5)}{440895} \\
 &= \frac{2a^2gx^3\sqrt{a+bx^3}}{105b} + \frac{2x(a+bx^3)^{3/2}(33915cx+29393dx^2+25935ex^3)}{440895} \\
 &= \frac{54a^2fx^2\sqrt{a+bx^3}}{1729b} + \frac{2a^2gx^3\sqrt{a+bx^3}}{105b} + \frac{2x(a+bx^3)^{3/2}(33915cx+29393dx^2+25935ex^3)}{440895} \\
 &= \frac{54a^2ex\sqrt{a+bx^3}}{935b} + \frac{54a^2fx^2\sqrt{a+bx^3}}{1729b} + \frac{2a^2gx^3\sqrt{a+bx^3}}{105b} + \frac{2x(a+bx^3)^{3/2}(33915cx+29393dx^2+25935ex^3)}{440895} \\
 &= \frac{54a^2ex\sqrt{a+bx^3}}{935b} + \frac{54a^2fx^2\sqrt{a+bx^3}}{1729b} + \frac{2a^2gx^3\sqrt{a+bx^3}}{105b} + \frac{2x(a+bx^3)^{3/2}(33915cx+29393dx^2+25935ex^3)}{440895} \\
 &= \frac{2a^2(7bd-2ag)\sqrt{a+bx^3}}{105b^2} + \frac{54a^2ex\sqrt{a+bx^3}}{935b} + \frac{54a^2fx^2\sqrt{a+bx^3}}{1729b} + \frac{2x(a+bx^3)^{3/2}(33915cx+29393dx^2+25935ex^3)}{440895} \\
 &= \frac{2a^2(7bd-2ag)\sqrt{a+bx^3}}{105b^2} + \frac{54a^2ex\sqrt{a+bx^3}}{935b} + \frac{54a^2fx^2\sqrt{a+bx^3}}{1729b} + \frac{2x(a+bx^3)^{3/2}(33915cx+29393dx^2+25935ex^3)}{440895}
 \end{aligned}$$

Mathematica [C] time = 0.272086, size = 148, normalized size = 0.2

$$\frac{\sqrt{a+bx^3} \left(7980a^2bex {}_2F_1 \left(-\frac{3}{2}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a} \right) + 1785abx^2(4af-19bc) {}_2F_1 \left(-\frac{3}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^3}{a} \right) + 4(a+bx^3)^2 \sqrt{\frac{bx^3}{a} + 1} (646ag + 1785abx^2) \right)}{67830b^2 \sqrt{\frac{bx^3}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4), x]

```
[Out] -(Sqrt[a + b*x^3]*(4*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*(-2261*b*d + 646*a*g
- 5*b*x*(399*e + 17*x*(21*f + 19*g*x))) + 7980*a^2*b*e*x*Hypergeometric2F1
[-3/2, 1/3, 4/3, -((b*x^3)/a)] + 1785*a*b*(-19*b*c + 4*a*f)*x^2*Hypergeomet
ric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)])/(67830*b^2*Sqrt[1 + (b*x^3)/a])
```

Maple [B] time = 0.006, size = 1383, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c), x)
```

```
[Out] g*(2/21*b*x^9*(b*x^3+a)^(1/2)+16/105*a*x^6*(b*x^3+a)^(1/2)+2/105/b*a^2*x^3*
(b*x^3+a)^(1/2)-4/105*a^3/b^2*(b*x^3+a)^(1/2))+f*(2/19*b*x^8*(b*x^3+a)^(1/2
)+44/247*a*x^5*(b*x^3+a)^(1/2)+54/1729/b*a^2*x^2*(b*x^3+a)^(1/2)+72/1729*I/
b^2*a^3*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(
-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/
2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^
2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/
(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*Ell
ipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3
))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^
2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))
*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*
a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+e*(2/17*b*x^7*(b*x^3+a)^(
1/2)+40/187*a*x^4*(b*x^3+a)^(1/2)+54/935/b*a^2*x*(b*x^3+a)^(1/2)+36/935*I/
b^2*a^3*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(
-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/
2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^
2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/
(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1
/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)
^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+2/15
*d/b*(b*x^3+a)^(5/2)+c*(2/13*b*x^5*(b*x^3+a)^(1/2)+32/91*a*x^2*(b*x^3+a)^(1
/2)-18/91*I*a^2*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3
^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(
1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x
+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1
/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b
```

$$\begin{aligned} & ^2*a)^{(1/3)}*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)/(-} \\ & 3/2/b*(-b^2*a)^{(1/3)+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2))}+1/b*(-b^2*a)^{(} \\ & 1/3)*\text{EllipticF}(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2} \\ & *a)^{(1/3)))*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)/(-3/} \\ & 2/b*(-b^2*a)^{(1/3)+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2))} \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bgx^8 + bfx^7 + bex^6 + (bd + ag)x^5 + aex^3 + (bc + af)x^4 + adx^2 + acx\right)\sqrt{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*g*x^8 + b*f*x^7 + b*e*x^6 + (b*d + a*g)*x^5 + a*e*x^3 + (b*c + a*f)*x^4 + a*d*x^2 + a*c*x)*sqrt(b*x^3 + a), x)
```

Sympy [A] time = 8.80445, size = 525, normalized size = 0.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**(3/2)*c*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + a**(3/2)*e*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(3/2)*f*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + sqrt(a)*b*c*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + sqrt(a)*b*e*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)*b*f*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) + a*d*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + a*g*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True)) + b*d*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True)) + b*g*Piecewise((16*a**3*sqrt(a + b*x**3)/(315*b**3) - 8*a**2*x**3*sqrt(a + b*x**3)/(315*b**2) + 2*a*x**6*sqrt(a + b*x**3)/(105*b) + 2*x**9*sqrt(a + b*x**3)/21, Ne(b, 0)), (sqrt(a)*x**9/9, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x, x)
```

3.461 $\int (a + bx^3)^{3/2} (c + dx + ex^2 + fx^3 + gx^4) dx$

Optimal. Leaf size=694

$$18 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} a^2 (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \text{EllipticF} \left(\sin^{-1} \left(\frac{(1 - \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{(1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}} \right), -7 - 4\sqrt{3} \right) (1729 \sqrt[3]{b} (17bc - 2af) -$$

$$1616615 b^{5/3} \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

[Out] $(2a^2e\sqrt{a + bx^3})/(15b) + (54a^2f\sqrt{a + bx^3})/(935b) + (54a^2g\sqrt{a + bx^3})/(1729b) + (54a^2(19bd - 4ag)\sqrt{a + bx^3})/(1729b^{5/3}((1 + \sqrt{3})a^{1/3} + b^{1/3}x)) + (2(a + bx^3)^{3/2}(62985cx + 53295dx^2 + 46189ex^3 + 40755fx^4 + 36465gx^5))/692835 + (2a\sqrt{a + bx^3}(793611cx + 479655dx^2 + 323323ex^3 + 233415fx^4 + 176715gx^5))/4849845 - (27 \cdot 3^{1/4} \sqrt{2 - \sqrt{3}} a^{7/3} (19bd - 4ag) (a^{1/3} + b^{1/3}x) \sqrt{(a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2)/((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2}) \text{EllipticE}[\text{ArcSin}(((1 - \sqrt{3})a^{1/3} + b^{1/3}x)/((1 + \sqrt{3})a^{1/3} + b^{1/3}x))], -7 - 4\sqrt{3}]/(1729b^{5/3} \sqrt{(a^{1/3}(a^{1/3} + b^{1/3}x))/((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2}) \sqrt{a + bx^3} + (18 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} a^2 (1729b^{1/3}(17bc - 2af) - 935(1 - \sqrt{3})a^{1/3}(19bd - 4ag)) (a^{1/3} + b^{1/3}x) \sqrt{(a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2)/((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2}) \text{EllipticF}[\text{ArcSin}(((1 - \sqrt{3})a^{1/3} + b^{1/3}x)/((1 + \sqrt{3})a^{1/3} + b^{1/3}x))], -7 - 4\sqrt{3}]/(1616615b^{5/3} \sqrt{(a^{1/3}(a^{1/3} + b^{1/3}x))/((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2}) \sqrt{a + bx^3}$

Rubi [A] time = 0.899239, antiderivative size = 694, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {1853, 1888, 1886, 261, 1878, 218, 1877}

$$18 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} a^2 (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} F \left(\sin^{-1} \left(\frac{\sqrt[3]{bx} + (1 - \sqrt{3}) \sqrt[3]{a}}{\sqrt[3]{bx} + (1 + \sqrt{3}) \sqrt[3]{a}} \right) | -7 - 4\sqrt{3} \right) (1729 \sqrt[3]{b} (17bc - 2af) - 935 (1$$

$$1616615 b^{5/3} \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]

[Out] (2*a^2*e*Sqrt[a + b*x^3])/(15*b) + (54*a^2*f*x*Sqrt[a + b*x^3])/(935*b) + (54*a^2*g*x^2*Sqrt[a + b*x^3])/(1729*b) + (54*a^2*(19*b*d - 4*a*g)*Sqrt[a + b*x^3])/(1729*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*(a + b*x^3)^(3/2)*(62985*c*x + 53295*d*x^2 + 46189*e*x^3 + 40755*f*x^4 + 36465*g*x^5))/692835 + (2*a*Sqrt[a + b*x^3]*(793611*c*x + 479655*d*x^2 + 323323*e*x^3 + 233415*f*x^4 + 176715*g*x^5))/4849845 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(19*b*d - 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(1729*b^(1/3)*(17*b*c - 2*a*f) - 935*(1 - Sqrt[3])*a^(1/3)*(19*b*d - 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1616615*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rule 1853

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[(a + b*x^n)^p*Sum[(Coeff[Pq, x, i]*x^(i + 1))/(n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(n*p + i + 1), {i, 0, q}], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]

Rule 1888

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*n)]) /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]

Rule 1886

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[Coeff[Pq, x, n - 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int (a + bx^3)^{3/2} (c + dx + ex^2 + fx^3 + gx^4) dx &= \frac{2(a + bx^3)^{3/2} (62985cx + 53295dx^2 + 46189ex^3 + 40755fx^4 + 36465gx^5)}{692835} \\
&= \frac{2(a + bx^3)^{3/2} (62985cx + 53295dx^2 + 46189ex^3 + 40755fx^4 + 36465gx^5)}{692835} \\
&= \frac{54a^2gx^2\sqrt{a + bx^3}}{1729b} + \frac{2(a + bx^3)^{3/2} (62985cx + 53295dx^2 + 46189ex^3 + 40755fx^4 + 36465gx^5)}{692835} \\
&= \frac{54a^2fx\sqrt{a + bx^3}}{935b} + \frac{54a^2gx^2\sqrt{a + bx^3}}{1729b} + \frac{2(a + bx^3)^{3/2} (62985cx + 53295dx^2 + 46189ex^3 + 40755fx^4 + 36465gx^5)}{692835} \\
&= \frac{54a^2fx\sqrt{a + bx^3}}{935b} + \frac{54a^2gx^2\sqrt{a + bx^3}}{1729b} + \frac{2(a + bx^3)^{3/2} (62985cx + 53295dx^2 + 46189ex^3 + 40755fx^4 + 36465gx^5)}{692835} \\
&= \frac{2a^2e\sqrt{a + bx^3}}{15b} + \frac{54a^2fx\sqrt{a + bx^3}}{935b} + \frac{54a^2gx^2\sqrt{a + bx^3}}{1729b} + \frac{2(a + bx^3)^{3/2} (62985cx + 53295dx^2 + 46189ex^3 + 40755fx^4 + 36465gx^5)}{692835} \\
&= \frac{2a^2e\sqrt{a + bx^3}}{15b} + \frac{54a^2fx\sqrt{a + bx^3}}{935b} + \frac{54a^2gx^2\sqrt{a + bx^3}}{1729b} + \frac{54a^2(19bd - 4ac)}{1729b^{5/3}} \left((1 + \frac{bx^3}{a})^{3/2} - 1 \right)
\end{aligned}$$

Mathematica [C] time = 0.177176, size = 139, normalized size = 0.2

$$\frac{\sqrt{a + bx^3} \left(-570ax(2af - 17bc) {}_2F_1 \left(-\frac{3}{2}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a} \right) - 255ax^2(4ag - 19bd) {}_2F_1 \left(-\frac{3}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^3}{a} \right) + 4(a + bx^3)^2 \sqrt{\frac{bx^3}{a}} + 1(32a^2c - 17abx) \sqrt{a + bx^3} \right)}{9690b\sqrt{\frac{bx^3}{a}} + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]

[Out] (Sqrt[a + b*x^3]*(4*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*(323*e + 15*x*(19*f + 17*g*x)) - 570*a*(-17*b*c + 2*a*f)*x*Hypergeometric2F1[-3/2, 1/3, 4/3, -(b*x^3)/a] - 255*a*(-19*b*d + 4*a*g)*x^2*Hypergeometric2F1[-3/2, 2/3, 5/3, -(b*x^3)/a]))/(9690*b*Sqrt[1 + (b*x^3)/a])

Maple [B] time = 0.006, size = 1629, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((b*x^3+a)^{(3/2)}*(g*x^4+f*x^3+e*x^2+d*x+c), x)$

[Out]
$$g*(2/19*b*x^8*(b*x^3+a)^{(1/2)}+44/247*a*x^5*(b*x^3+a)^{(1/2)}+54/1729/b*a^2*x^2*(b*x^3+a)^{(1/2)}+72/1729*I/b^2*a^3*(1/2)*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*EllipticE(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}))+1/b*(-b^2*a)^{(1/3)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)})))+f*(2/17*b*x^7*(b*x^3+a)^{(1/2)}+40/187*a*x^4*(b*x^3+a)^{(1/2)}+54/935/b*a^2*x*(b*x^3+a)^{(1/2)}+36/935*I/b^2*a^3*(1/2)*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/2)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}))+2/15*e/b*(b*x^3+a)^{(5/2)}+d*(2/13*b*x^5*(b*x^3+a)^{(1/2)}+32/91*a*x^2*(b*x^3+a)^{(1/2)}-18/91*I*a^2*3^{(1/2)}/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*EllipticE(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}))+1/b*(-b^2*a)^{(1/3)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)})))+c*(2/11*b*x^4*(b*x^3+a)^{(1/2)}+28/55*a*x*(b*x^3+a)^{(1/2)}-18/55*I*a^2*3^{(1/2)}/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}*(-I*(x+1/2/b*(-b$$

$$\begin{aligned} & \sqrt[2]{a}^{\frac{1}{3}} + \frac{1}{2} I \sqrt[3]{\frac{1}{2}} / b \sqrt[2]{(-b^2 a)^{\frac{1}{3}}} * \sqrt[3]{\frac{1}{2}} * b / (-b^2 a)^{\frac{1}{3}})^{\frac{1}{2}} \\ & / (b x^3 + a)^{\frac{1}{2}} * \text{EllipticF}\left(\frac{1}{3} \sqrt[3]{\frac{1}{2}} * (I * (x + \frac{1}{2} / b \sqrt[2]{(-b^2 a)^{\frac{1}{3}}} - \frac{1}{2} I \sqrt[3]{\frac{1}{2}} / b \sqrt[2]{(-b^2 a)^{\frac{1}{3}}}) * \sqrt[3]{\frac{1}{2}} * b / (-b^2 a)^{\frac{1}{3}})^{\frac{1}{2}}, (I \sqrt[3]{\frac{1}{2}} / b \sqrt[2]{(-b^2 a)^{\frac{1}{3}}})^{\frac{1}{2}} / (-\frac{3}{2} / b \sqrt[2]{(-b^2 a)^{\frac{1}{3}}} + \frac{1}{2} I \sqrt[3]{\frac{1}{2}} / b \sqrt[2]{(-b^2 a)^{\frac{1}{3}}})^{\frac{1}{2}}\right) \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (g x^4 + f x^3 + e x^2 + d x + c) (b x^3 + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b g x^7 + b f x^6 + b e x^5 + (b d + a g) x^4 + a e x^2 + (b c + a f) x^3 + a d x + a c\right) \sqrt{b x^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")

[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2 + (b*c + a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a), x)

Sympy [A] time = 7.32464, size = 444, normalized size = 0.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c),x)

[Out] a**(3/2)*c*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + a**(3/2)*d*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*

```

x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + a**(3/2)*f*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(3/2)*g*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + sqrt(a)*b*c*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*b*d*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + sqrt(a)*b*f*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)*b*g*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) + a*e*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + b*e*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True))

```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2), x)

$$3.462 \quad \int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x} dx$$

Optimal. Leaf size=676

$$18 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} a^2 (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (-935(1 - \sqrt{3}) \sqrt[3]{ab^{2/3}} e - 182ag + 1547bd) \operatorname{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3}) \sqrt[3]{a}}{(1+\sqrt{3}) \sqrt[3]{a}}\right)\right) - \frac{85085b^{4/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}}$$

[Out] $(2a^2f\sqrt{a + bx^3})/(15b) + (54a^2g\sqrt{a + bx^3})/(935b) + (54a^2e\sqrt{a + bx^3})/(91b^{2/3}((1 + \sqrt{3})a^{1/3} + b^{1/3}x)) + (2(a + bx^3)^{3/2}(12155cx + 9945dx^2 + 8415ex^3 + 7293fx^4 + 6435gx^5))/(109395x) + (2a\sqrt{a + bx^3}(85085cx + 41769dx^2 + 25245ex^3 + 17017fx^4 + 12285gx^5))/(255255x) - (2a^{3/2}c\operatorname{ArcTanh}[\sqrt{a + bx^3}/\sqrt{a}])/3 - (27 \cdot 3^{1/4} \sqrt{2 - \sqrt{3}} a^{7/3} e (a^{1/3} + b^{1/3}x) \sqrt{(a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2)/((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2}) \operatorname{EllipticE}[\operatorname{ArcSin}(((1 - \sqrt{3})a^{1/3} + b^{1/3}x)/((1 + \sqrt{3})a^{1/3} + b^{1/3}x))], -7 - 4\sqrt{3}]/(91b^{2/3} \sqrt{(a^{1/3}(a^{1/3} + b^{1/3}x))/((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2}) \sqrt{a + bx^3}) + (18 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} a^2 (1547bd - 935(1 - \sqrt{3})a^{1/3}b^{2/3}e - 182ag) (a^{1/3} + b^{1/3}x) \sqrt{(a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2)/((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2}) \operatorname{EllipticF}[\operatorname{ArcSin}(((1 - \sqrt{3})a^{1/3} + b^{1/3}x)/((1 + \sqrt{3})a^{1/3} + b^{1/3}x))], -7 - 4\sqrt{3}]/(85085b^{4/3} \sqrt{(a^{1/3}(a^{1/3} + b^{1/3}x))/((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2}) \sqrt{a + bx^3})$

Rubi [A] time = 0.707064, antiderivative size = 676, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {1826, 1832, 266, 63, 208, 1888, 1886, 261, 1878, 218, 1877}

$$18 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} a^2 (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (-935(1 - \sqrt{3}) \sqrt[3]{ab^{2/3}} e - 182ag + 1547bd) F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx} + (1-\sqrt{3}) \sqrt[3]{a}}{\sqrt[3]{bx} + (1+\sqrt{3}) \sqrt[3]{a}}\right)\right) - \frac{85085b^{4/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + bx^3)^{3/2}(c + dx + ex^2 + fx^3 + gx^4)/x, x]$

```
[Out] (2*a^2*f*Sqrt[a + b*x^3])/(15*b) + (54*a^2*g*x*Sqrt[a + b*x^3])/(935*b) + (
54*a^2*e*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x))
+ (2*(a + b*x^3)^(3/2)*(12155*c*x + 9945*d*x^2 + 8415*e*x^3 + 7293*f*x^4 +
6435*g*x^5))/(109395*x) + (2*a*Sqrt[a + b*x^3]*(85085*c*x + 41769*d*x^2 + 2
5245*e*x^3 + 17017*f*x^4 + 12285*g*x^5))/(255255*x) - (2*a^(3/2)*c*ArcTanh[
Sqrt[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*e*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(2/3)*
Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*Sqrt[a + b*x^3]) + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(1547*b*d - 935*(1 -
Sqrt[3])*a^(1/3)*b^(2/3)*e - 182*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)], -7 - 4*Sqrt[3]])/(85085*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1826

```
Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i
]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
+ b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_)*(x_)^(n_)]), x_Symbol] := Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1888

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*n)]) /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]

Rule 1886

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[Coeff[Pq, x, n - 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 1878

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

Rule 218

Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)]/((1 + Sqrt[3])*s + r*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &

& PosQ[a]

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{(a + bx^3)^{3/2} (c + dx + ex^2 + fx^3 + gx^4)}{x} dx &= \frac{2(a + bx^3)^{3/2} (12155cx + 9945dx^2 + 8415ex^3 + 7293fx^4 + 6435gx^5)}{109395x} \\
 &= \frac{2(a + bx^3)^{3/2} (12155cx + 9945dx^2 + 8415ex^3 + 7293fx^4 + 6435gx^5)}{109395x} \\
 &= \frac{2(a + bx^3)^{3/2} (12155cx + 9945dx^2 + 8415ex^3 + 7293fx^4 + 6435gx^5)}{109395x} \\
 &= \frac{54a^2gx\sqrt{a + bx^3}}{935b} + \frac{2(a + bx^3)^{3/2} (12155cx + 9945dx^2 + 8415ex^3 + 7293fx^4 + 6435gx^5)}{109395x} \\
 &= \frac{54a^2gx\sqrt{a + bx^3}}{935b} + \frac{2(a + bx^3)^{3/2} (12155cx + 9945dx^2 + 8415ex^3 + 7293fx^4 + 6435gx^5)}{109395x} \\
 &= \frac{2a^2f\sqrt{a + bx^3}}{15b} + \frac{54a^2gx\sqrt{a + bx^3}}{935b} + \frac{2(a + bx^3)^{3/2} (12155cx + 9945dx^2 + 8415ex^3 + 7293fx^4 + 6435gx^5)}{109395x} \\
 &= \frac{2a^2f\sqrt{a + bx^3}}{15b} + \frac{54a^2gx\sqrt{a + bx^3}}{935b} + \frac{54a^2e\sqrt{a + bx^3}}{91b^{2/3}((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})} + \frac{2(a + bx^3)^{3/2} (12155cx + 9945dx^2 + 8415ex^3 + 7293fx^4 + 6435gx^5)}{109395x}
 \end{aligned}$$

Mathematica [C] time = 0.439483, size = 215, normalized size = 0.32

$$\frac{4\sqrt{\frac{bx^3}{a}+1}\left(\sqrt{a+bx^3}\left(a^2(51f+45gx)+2ab(170c+51fx^3+45gx^4)+b^2x^3(85c+51fx^3+45gx^4)\right)-255a^{3/2}bc\tanh^{-1}\right)}{1530b\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x,x]

[Out] (4*sqrt[1 + (b*x^3)/a]*(sqrt[a + b*x^3]*(a^2*(51*f + 45*g*x) + b^2*x^3*(85*c + 51*f*x^3 + 45*g*x^4) + 2*a*b*(170*c + 51*f*x^3 + 45*g*x^4)) - 255*a^(3/2)*b*c*ArcTanh[sqrt[a + b*x^3]/sqrt[a]]) - 90*a*(-17*b*d + 2*a*g)*x*sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)] + 765*a*b*e*x^2*sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)]/(1530*b*sqrt[1 + (b*x^3)/a]))

Maple [B] time = 0.009, size = 1188, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x,x)

[Out] g*(2/17*b*x^7*(b*x^3+a)^(1/2)+40/187*a*x^4*(b*x^3+a)^(1/2)+54/935/b*a^2*x*(b*x^3+a)^(1/2)+36/935*I/b^2*a^3*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3))+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+2/15*f/b*(b*x^3+a)^(5/2)+e*(2/13*b*x^5*(b*x^3+a)^(1/2)+32/91*a*x^2*(b*x^3+a)^(1/2)-18/91*I*a^2*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3))+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))

$$\frac{1}{2})/b*(-b^2*a)^{(1/3)/(-3/2/b*(-b^2*a)^{(1/3)+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2))+1/b*(-b^2*a)^{(1/3)*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)},(I*3^{(1/2)}/b*(-b^2*a)^{(1/3)/(-3/2/b*(-b^2*a)^{(1/3)+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)))+d*(2/11*b*x^4*(b*x^3+a)^{(1/2)+28/55*a*x*(b*x^3+a)^{(1/2)-18/55*I*a^{2*3^{(1/2)}/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)*((x-1/b*(-b^2*a)^{(1/3)/(-3/2/b*(-b^2*a)^{(1/3)+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)*(-I*(x+1/2/b*(-b^2*a)^{(1/3)+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)/(b*x^3+a)^{(1/2)*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)},(I*3^{(1/2)}/b*(-b^2*a)^{(1/3)/(-3/2/b*(-b^2*a)^{(1/3)+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)))+c*(2/9*b*x^3*(b*x^3+a)^{(1/2)+8/9*a*(b*x^3+a)^{(1/2)-2/3*a^{(3/2)*arctanh((b*x^3+a)^{(1/2)/a^{(1/2))}}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="maxima")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac)\sqrt{bx^3 + a}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="fricas")

[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2 + (b*c + a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x, x)

Sympy [A] time = 21.3669, size = 473, normalized size = 0.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x,x)

[Out] $-2*a^{3/2}*c*\operatorname{asinh}(\sqrt{a}/(\sqrt{b}*x^{3/2}))/3 + a^{3/2}*d*x*\Gamma(1/3)*\operatorname{hyper}((-1/2, 1/3), (4/3,), b*x^{3/2}*\exp(\operatorname{I}*\pi)/a)/(3*\Gamma(4/3)) + a^{3/2}*e*x^{2/3}*\Gamma(2/3)*\operatorname{hyper}((-1/2, 2/3), (5/3,), b*x^{3/2}*\exp(\operatorname{I}*\pi)/a)/(3*\Gamma(5/3)) + a^{3/2}*g*x^{4/3}*\Gamma(4/3)*\operatorname{hyper}((-1/2, 4/3), (7/3,), b*x^{3/2}*\exp(\operatorname{I}*\pi)/a)/(3*\Gamma(7/3)) + \sqrt{a}*b*d*x^{4/3}*\Gamma(4/3)*\operatorname{hyper}((-1/2, 4/3), (7/3,), b*x^{3/2}*\exp(\operatorname{I}*\pi)/a)/(3*\Gamma(7/3)) + \sqrt{a}*b*e*x^{5/3}*\Gamma(5/3)*\operatorname{hyper}((-1/2, 5/3), (8/3,), b*x^{3/2}*\exp(\operatorname{I}*\pi)/a)/(3*\Gamma(8/3)) + \sqrt{a}*b*g*x^{7/3}*\Gamma(7/3)*\operatorname{hyper}((-1/2, 7/3), (10/3,), b*x^{3/2}*\exp(\operatorname{I}*\pi)/a)/(3*\Gamma(10/3)) + 2*a^{2/3}*c/(3*\sqrt{b}*x^{3/2}*\sqrt{a/(b*x^{3/2} + 1)}) + 2*a*\sqrt{b}*c*x^{3/2}/(3*\sqrt{a/(b*x^{3/2} + 1)}) + a*f*\operatorname{Piecewise}(\sqrt{a}*x^{3/3}, \operatorname{Eq}(b, 0)), (2*(a + b*x^{3/2})^{3/2}/(9*b), \operatorname{True})) + b*c*\operatorname{Piecewise}(\sqrt{a}*x^{3/3}, \operatorname{Eq}(b, 0)), (2*(a + b*x^{3/2})^{3/2}/(9*b), \operatorname{True})) + b*f*\operatorname{Piecewise}((-4*a^{2/3}*\sqrt{a + b*x^{3/2}}/(45*b^{2/3}) + 2*a*x^{3/2}*\sqrt{a + b*x^{3/2}})/(45*b) + 2*x^{6/3}*\sqrt{a + b*x^{3/2}}/15, \operatorname{Ne}(b, 0)), (\sqrt{a}*x^{6/6}, \operatorname{True}))$

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{3/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="giac")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x, x)

$$3.463 \quad \int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^2} dx$$

Optimal. Leaf size=692

$$9 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} a^{4/3} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (182a^{2/3} \sqrt[3]{be} - 55(1 - \sqrt{3})(2af + 13bc)) \operatorname{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3}) \sqrt[3]{a}}{(1+\sqrt{3}) \sqrt[3]{a}}\right)\right) -$$

$$\frac{5005b^{2/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}}$$

[Out] (2*a^2*g*Sqrt[a + b*x^3])/(15*b) - (27*a*c*Sqrt[a + b*x^3])/(7*x) + (27*a*(13*b*c + 2*a*f)*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*a*Sqrt[a + b*x^3]*(19305*c*x + 5005*d*x^2 + 2457*e*x^3 + 1485*f*x^4 + 1001*g*x^5))/(15015*x^2) + (2*(a + b*x^3)^(3/2)*(6435*c*x + 5005*d*x^2 + 4095*e*x^3 + 3465*f*x^4 + 3003*g*x^5))/(45045*x^2) - (2*a^(3/2)*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13*b*c + 2*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(182*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(182*a^(2/3)*b^(1/3)*e - 55*(1 - Sqrt[3])*(13*b*c + 2*a*f))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(5005*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 0.790546, antiderivative size = 692, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {1826, 1835, 1832, 266, 63, 208, 1886, 261, 1878, 218, 1877}

$$9 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} a^{4/3} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (182a^{2/3} \sqrt[3]{be} - 55(1 - \sqrt{3})(2af + 13bc)) F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx} + (1-\sqrt{3}) \sqrt[3]{a}}{\sqrt[3]{bx} + (1+\sqrt{3}) \sqrt[3]{a}}\right)\right) -$$

$$\frac{5005b^{2/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2,x]

[Out] (2*a^2*g*Sqrt[a + b*x^3])/(15*b) - (27*a*c*Sqrt[a + b*x^3])/(7*x) + (27*a*(13*b*c + 2*a*f)*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*a*Sqrt[a + b*x^3]*(19305*c*x + 5005*d*x^2 + 2457*e*x^3 + 1485*f*x^4 + 1001*g*x^5))/(15015*x^2) + (2*(a + b*x^3)^(3/2)*(6435*c*x + 5005*d*x^2 + 4095*e*x^3 + 3465*f*x^4 + 3003*g*x^5))/(45045*x^2) - (2*a^(3/2)*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13*b*c + 2*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(182*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(182*a^(2/3)*b^(1/3)*e - 55*(1 - Sqrt[3])*(13*b*c + 2*a*f))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5005*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rule 1826

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}], x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]

Rule 1835

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> With[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]

Rule 1832

Int[(Pq_)/((x_)*Sqrt[(a_) + (b_)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq, x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq, x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b,
m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1886

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[Coeff[Pq, x, n -
1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x,
n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq,
x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
NeQ[p, -1]

Rule 1878

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]

Rule 218

Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s

$*x + r^2*x^2)/((1 + \text{Sqrt}[3])*s + r*x)^2]*\text{EllipticF}[\text{ArcSin}[\frac{(1 - \text{Sqrt}[3])*s + r*x}{(1 + \text{Sqrt}[3])*s + r*x}], -7 - 4*\text{Sqrt}[3]]/(3^{1/4}*r*\text{Sqrt}[a + b*x^3])*\text{Sqrt}[\frac{s*(s + r*x)}{(1 + \text{Sqrt}[3])*s + r*x}^2)], x]] /; \text{FreeQ}\{a, b, x\} \& \& \text{PosQ}[a]$

Rule 1877

$\text{Int}[\frac{(c_.) + (d_.)*(x_.)}{\text{Sqrt}[(a_.) + (b_.)*(x_.)^3]}, x_Symbol] := \text{With}\{r = \text{N umer}[\text{Simplify}[\frac{(1 - \text{Sqrt}[3])*d}{c}], s = \text{Denom}[\text{Simplify}[\frac{(1 - \text{Sqrt}[3])*d}{c}]]], \text{Simp}[\frac{(2*d*s^3*\text{Sqrt}[a + b*x^3])}{(a*r^2*((1 + \text{Sqrt}[3])*s + r*x))}, x] - \text{Simp}[\frac{(3^{1/4}*\text{Sqrt}[2 - \text{Sqrt}[3])*d*s*(s + r*x)*\text{Sqrt}[(s^2 - r*s*x + r^2*x^2)]}{(1 + \text{Sqrt}[3])*s + r*x}^2]*\text{EllipticE}[\text{ArcSin}[\frac{(1 - \text{Sqrt}[3])*s + r*x}{(1 + \text{Sqrt}[3])*s + r*x}], -7 - 4*\text{Sqrt}[3]]/(r^2*\text{Sqrt}[a + b*x^3]*\text{Sqrt}[\frac{s*(s + r*x)}{(1 + \text{Sqrt}[3])*s + r*x}^2)], x]] /; \text{FreeQ}\{a, b, c, d, x\} \&\& \text{PosQ}[a] \&\& \text{EqQ}[b*c^3 - 2*(5 - 3*\text{Sqrt}[3])*a*d^3, 0]$

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)^{3/2} (c + dx + ex^2 + fx^3 + gx^4)}{x^2} dx &= \frac{2(a + bx^3)^{3/2} (6435cx + 5005dx^2 + 4095ex^3 + 3465fx^4 + 3003gx^5)}{45045x^2} + \frac{1}{2} \\ &= \frac{2a\sqrt{a + bx^3} (19305cx + 5005dx^2 + 2457ex^3 + 1485fx^4 + 1001gx^5)}{15015x^2} + \frac{2}{15} \\ &= -\frac{27ac\sqrt{a + bx^3}}{7x} + \frac{2a\sqrt{a + bx^3} (19305cx + 5005dx^2 + 2457ex^3 + 1485fx^4 + 1001gx^5)}{15015x^2} \\ &= -\frac{27ac\sqrt{a + bx^3}}{7x} + \frac{2a\sqrt{a + bx^3} (19305cx + 5005dx^2 + 2457ex^3 + 1485fx^4 + 1001gx^5)}{15015x^2} \\ &= -\frac{27ac\sqrt{a + bx^3}}{7x} + \frac{2a\sqrt{a + bx^3} (19305cx + 5005dx^2 + 2457ex^3 + 1485fx^4 + 1001gx^5)}{15015x^2} \\ &= \frac{2a^2g\sqrt{a + bx^3}}{15b} - \frac{27ac\sqrt{a + bx^3}}{7x} + \frac{2a\sqrt{a + bx^3} (19305cx + 5005dx^2 + 2457ex^3 + 1485fx^4 + 1001gx^5)}{15015x^2} \\ &= \frac{2a^2g\sqrt{a + bx^3}}{15b} - \frac{27ac\sqrt{a + bx^3}}{7x} + \frac{27a(13bc + 2af)\sqrt{a + bx^3}}{91b^{2/3}((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})} + \frac{2a\sqrt{a}}{15} \end{aligned}$$

Mathematica [C] time = 0.340123, size = 224, normalized size = 0.32

$$\frac{2}{9}d \left(\sqrt{a+bx^3} (4a+bx^3) - 3a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a+bx^3}}{\sqrt{a}} \right) \right) - \frac{ac\sqrt{a+bx^3} {}_2F_1 \left(-\frac{3}{2}, -\frac{1}{3}; \frac{2}{3}; -\frac{bx^3}{a} \right)}{x\sqrt{\frac{bx^3}{a}+1}} + \frac{aex\sqrt{a+bx^3} {}_2F_1 \left(-\frac{3}{2}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a} \right)}{\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2,x]

[Out] (2*g*(a + b*x^3)^(5/2))/(15*b) + (2*d*(Sqrt[a + b*x^3]*(4*a + b*x^3) - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]))/9 - (a*c*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, -1/3, 2/3, -((b*x^3)/a)])/(x*Sqrt[1 + (b*x^3)/a]) + (a*e*x*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a] + (a*f*x^2*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])

Maple [B] time = 0.009, size = 1317, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x)

[Out] 2/15*g/b*(b*x^3+a)^(5/2)+f*(2/13*b*x^5*(b*x^3+a)^(1/2)+32/91*a*x^2*(b*x^3+a)^(1/2)-18/91*I*a^2*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+e*(2/11*b*x^4*(b*x^3+a)^(1/2)+28/55*a*x*(b*x^3+a)^(1/2)-18/55*I*a^2*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)

) / b * (-b^2*a)^(1/3)) * 3^(1/2) * b / (-b^2*a)^(1/3))^(1/2) / (b*x^3+a)^(1/2) * EllipticF(1/3*3^(1/2) * (I*(x+1/2/b*(-b^2*a)^(1/3) - 1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)) * 3^(1/2) * b / (-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3) / (-3/2/b*(-b^2*a)^(1/3) + 1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))) + d * (2/9*b*x^3*(b*x^3+a)^(1/2) + 8/9*a*(b*x^3+a)^(1/2) - 2/3*a^(3/2) * arctanh((b*x^3+a)^(1/2)/a^(1/2))) + c * (-a * (b*x^3+a)^(1/2) / x + 2/7*b*x^2*(b*x^3+a)^(1/2) - 9/7*I*a*3^(1/2)*(-b^2*a)^(1/3) * (I*(x+1/2/b*(-b^2*a)^(1/3) - 1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)) * 3^(1/2) * b / (-b^2*a)^(1/3))^(1/2) * ((x-1/b*(-b^2*a)^(1/3)) / (-3/2/b*(-b^2*a)^(1/3) + 1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2) * (-I*(x+1/2/b*(-b^2*a)^(1/3) + 1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)) * 3^(1/2) * b / (-b^2*a)^(1/3))^(1/2) / (b*x^3+a)^(1/2) * ((-3/2/b*(-b^2*a)^(1/3) + 1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)) * EllipticE(1/3*3^(1/2) * (I*(x+1/2/b*(-b^2*a)^(1/3) - 1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)) * 3^(1/2) * b / (-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3) / (-3/2/b*(-b^2*a)^(1/3) + 1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)) + 1/b*(-b^2*a)^(1/3) * EllipticF(1/3*3^(1/2) * (I*(x+1/2/b*(-b^2*a)^(1/3) - 1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)) * 3^(1/2) * b / (-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3) / (-3/2/b*(-b^2*a)^(1/3) + 1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="maxima")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac)\sqrt{bx^3 + a}}{x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="fricas")

[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2 + (b*c + a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^2, x)

Sympy [A] time = 10.2973, size = 474, normalized size = 0.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)

[Out] a**(3/2)*c*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*a**(3/2)*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + a**(3/2)*e*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + a**(3/2)*f*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*c*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*e*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*b*f*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + 2*a**2*d/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*a*sqrt(b)*d*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + a*g*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + b*d*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + b*g*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="giac")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^2, x)

$$3.464 \quad \int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^3} dx$$

Optimal. Leaf size=694

$$9 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}a} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} \text{EllipticF} \left(\sin^{-1} \left(\frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}}{(1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx}} \right), -7 - 4\sqrt{3} \right) (91\sqrt[3]{b}(4af + 11bc) - 110(1$$

$$10010b^{2/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

[Out] (27*a*c*Sqrt[a + b*x^3])/(10*x^2) - (27*a*d*Sqrt[a + b*x^3])/(7*x) + (27*a*(13*b*d + 2*a*g)*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*a*Sqrt[a + b*x^3]*(27027*c*x - 19305*d*x^2 - 5005*e*x^3 - 2457*f*x^4 - 1485*g*x^5))/(15015*x^3) + (2*(a + b*x^3)^(3/2)*(9009*c*x + 6435*d*x^2 + 5005*e*x^3 + 4095*f*x^4 + 3465*g*x^5))/(45045*x^3) - (2*a^(3/2)*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13*b*d + 2*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(182*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(91*b^(1/3)*(11*b*c + 4*a*f) - 110*(1 - Sqrt[3])*a^(1/3)*(13*b*d + 2*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(10010*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 0.887517, antiderivative size = 694, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1826, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$9 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}a} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} F \left(\sin^{-1} \left(\frac{\sqrt[3]{bx} + (1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{bx} + (1+\sqrt{3})\sqrt[3]{a}} \right) \middle| -7 - 4\sqrt{3} \right) (91\sqrt[3]{b}(4af + 11bc) - 110(1 - \sqrt{3})$$

$$10010b^{2/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3,x]

[Out] (27*a*c*Sqrt[a + b*x^3])/(10*x^2) - (27*a*d*Sqrt[a + b*x^3])/(7*x) + (27*a*(13*b*d + 2*a*g)*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*a*Sqrt[a + b*x^3]*(27027*c*x - 19305*d*x^2 - 5005*e*x^3 - 2457*f*x^4 - 1485*g*x^5))/(15015*x^3) + (2*(a + b*x^3)^(3/2)*(9009*c*x + 6435*d*x^2 + 5005*e*x^3 + 4095*f*x^4 + 3465*g*x^5))/(45045*x^3) - (2*a^(3/2)*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13*b*d + 2*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(182*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(9/4)*b^(1/3)*(11*b*c + 4*a*f) - 110*(1 - Sqrt[3])*a^(1/3)*(13*b*d + 2*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(10010*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rule 1826

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[(c*x)^(m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^(m*(a + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}], x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]

Rule 1835

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x^n)^p, x], x] /; NeQ[Pq0, 0] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]

Rule 1832

Int[(Pq_)/((x_)*Sqrt[(a_) + (b_)*(x_)^(n_)]), x_Symbol] := Dist[Coeff[Pq, x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq, x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + S
qrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
```

$((1 + \text{Sqrt}[3]) * s + r * x)^2$), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + bx^3)^{3/2} (c + dx + ex^2 + fx^3 + gx^4)}{x^3} dx &= \frac{2(a + bx^3)^{3/2} (9009cx + 6435dx^2 + 5005ex^3 + 4095fx^4 + 3465gx^5)}{45045x^3} + \\
 &= -\frac{2a\sqrt{a + bx^3} (27027cx - 19305dx^2 - 5005ex^3 - 2457fx^4 - 1485gx^5)}{15015x^3} \\
 &= \frac{27ac\sqrt{a + bx^3}}{10x^2} - \frac{2a\sqrt{a + bx^3} (27027cx - 19305dx^2 - 5005ex^3 - 2457fx^4 - 1485gx^5)}{15015x^3} \\
 &= \frac{27ac\sqrt{a + bx^3}}{10x^2} - \frac{27ad\sqrt{a + bx^3}}{7x} - \frac{2a\sqrt{a + bx^3} (27027cx - 19305dx^2 - 5005ex^3 - 2457fx^4 - 1485gx^5)}{15015} \\
 &= \frac{27ac\sqrt{a + bx^3}}{10x^2} - \frac{27ad\sqrt{a + bx^3}}{7x} - \frac{2a\sqrt{a + bx^3} (27027cx - 19305dx^2 - 5005ex^3 - 2457fx^4 - 1485gx^5)}{15015} \\
 &= \frac{27ac\sqrt{a + bx^3}}{10x^2} - \frac{27ad\sqrt{a + bx^3}}{7x} - \frac{2a\sqrt{a + bx^3} (27027cx - 19305dx^2 - 5005ex^3 - 2457fx^4 - 1485gx^5)}{15015} \\
 &= \frac{27ac\sqrt{a + bx^3}}{10x^2} - \frac{27ad\sqrt{a + bx^3}}{7x} + \frac{27a(13bd + 2ag)\sqrt{a + bx^3}}{91b^{2/3} \left((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx} \right)} - \frac{2a\sqrt{a + bx^3} (27027cx - 19305dx^2 - 5005ex^3 - 2457fx^4 - 1485gx^5)}{15015} \\
 &= \frac{27ac\sqrt{a + bx^3}}{10x^2} - \frac{27ad\sqrt{a + bx^3}}{7x} + \frac{27a(13bd + 2ag)\sqrt{a + bx^3}}{91b^{2/3} \left((1 + \sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx} \right)} - \frac{2a\sqrt{a + bx^3} (27027cx - 19305dx^2 - 5005ex^3 - 2457fx^4 - 1485gx^5)}{15015}
 \end{aligned}$$

Mathematica [C] time = 0.355531, size = 232, normalized size = 0.33

$$\frac{4ex^2 \sqrt{\frac{bx^3}{a} + 1} \left(\sqrt{a + bx^3} (4a + bx^3) - 3a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + bx^3}}{\sqrt{a}} \right) \right) - 9ac\sqrt{a + bx^3} {}_2F_1 \left(-\frac{3}{2}, -\frac{2}{3}; \frac{1}{3}; -\frac{bx^3}{a} \right) - 18adx\sqrt{a + bx^3} {}_2F_1 \left(-\frac{3}{2}, -\frac{2}{3}; \frac{1}{3}; -\frac{bx^3}{a} \right) - 18a^2x^2\sqrt{a + bx^3} {}_2F_1 \left(-\frac{3}{2}, -\frac{2}{3}; \frac{1}{3}; -\frac{bx^3}{a} \right)}{18x^2 \sqrt{\frac{bx^3}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3,x]

[Out] $(4*e*x^2*\sqrt{1 + (b*x^3)/a}*(\sqrt{a + b*x^3}*(4*a + b*x^3) - 3*a^{(3/2)}*\text{ArcTanh}[\sqrt{a + b*x^3}/\sqrt{a}]) - 9*a*c*\sqrt{a + b*x^3}*\text{Hypergeometric2F1}[-3/2, -2/3, 1/3, -((b*x^3)/a)] - 18*a*d*x*\sqrt{a + b*x^3}*\text{Hypergeometric2F1}[-3/2, -1/3, 2/3, -((b*x^3)/a)] + 18*a*f*x^3*\sqrt{a + b*x^3}*\text{Hypergeometric2F1}[-3/2, 1/3, 4/3, -((b*x^3)/a)] + 9*a*g*x^4*\sqrt{a + b*x^3}*\text{Hypergeometric2F1}[-3/2, 2/3, 5/3, -((b*x^3)/a)])/(18*x^2*\sqrt{1 + (b*x^3)/a})$

Maple [B] time = 0.009, size = 1613, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x)

[Out] $g*(2/13*b*x^5*(b*x^3+a)^{(1/2)}+32/91*a*x^2*(b*x^3+a)^{(1/2)}-18/91*I*a^2*3^{(1/2)}/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3)^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3)^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}/(b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*\text{EllipticE}(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3)^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}))+1/b*(-b^2*a)^{(1/3)}*\text{EllipticF}(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3)^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)})))+f*(2/11*b*x^4*(b*x^3+a)^{(1/2)}+28/55*a*x*(b*x^3+a)^{(1/2)}-18/55*I*a^2*3^{(1/2)}/b*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3)^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3)^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}/(b*x^3+a)^{(1/2)}*\text{EllipticF}(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3)^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)})))+e*(2/9*b*x^3*(b*x^3+a)^{(1/2)}+8/9*a*(b*x^3+a)^{(1/2)}-2/3*a^{(3/2)}*\text{arctanh}((b*x^3+a)^{(1/2)}/a^{(1/2)}))+c*(-1/2*a*(b*x^3+a)^{(1/2)}/x^2+2/5*b*x*(b*x^3+a)^{(1/2)}-9/10*I*a*3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3)^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3)^{(1/2)}}$

$$\begin{aligned}
 & *b/(-b^2*a)^{(1/3))^{(1/2)}/(b*x^3+a)^{(1/2)*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b* \\
 & (-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))} *3^{(1/2)} *b/(-b^2*a)^{(1/3))^{(1/2)}, \\
 & (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))} \\
 &)^{(1/2)))+d*(-a*(b*x^3+a)^{(1/2)}/x+2/7*b*x^2*(b*x^3+a)^{(1/2)}-9/7* \\
 & I*a*3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))} *3^{(1/2)} *b/(-b^2*a)^{(1/3))^{(1/2)} * \\
 & ((x-1/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))} \\
 &)^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))} *3^{(1/2)} *b/(-b^2*a)^{(1/3))^{(1/2)}/(b*x \\
 & ^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))} *EllipticE(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))} *3^{(1/2)} *b/(-b^2*a)^{(1/3))^{(1/2)}, \\
 & (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))} \\
 &)^{(1/2)))+1/b*(-b^2*a)^{(1/3)*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))} *3^{(1/2)} *b/(-b^2*a)^{(1/3))^{(1/2)}, \\
 & (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))} \\
 &)^{(1/2))}
 \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="maxima")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac)\sqrt{bx^3 + a}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="fricas")

```
[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2 + (b*c + a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^3, x)
```

Sympy [A] time = 10.5008, size = 462, normalized size = 0.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)
```

```
[Out] a**(3/2)*c*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + a**(3/2)*d*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*a**(3/2)*e*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + a**(3/2)*f*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + a**(3/2)*g*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*c*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + sqrt(a)*b*d*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*f*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*b*g*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + 2*a**2*e/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*a*sqrt(b)*e*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + b*e*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^3, x)
```

$$3.465 \quad \int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^4} dx$$

Optimal. Leaf size=692

$$9 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} a (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (-110(1-\sqrt{3}) \sqrt[3]{a} b^{2/3} e + 28ag + 77bd) \operatorname{EllipticF} \left(\sin^{-1} \left(\frac{(1-\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{(1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}} \right) \right) \\ \hline 770 \sqrt[3]{b} \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

[Out] (a*c*Sqrt[a + b*x^3])/x^3 + (27*a*d*Sqrt[a + b*x^3])/(10*x^2) - (27*a*e*Sqrt[a + b*x^3])/(7*x) + (27*a*b^(1/3)*e*Sqrt[a + b*x^3])/(7*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*a*Sqrt[a + b*x^3]*(1155*c*x + 2079*d*x^2 - 1485*e*x^3 - 385*f*x^4 - 189*g*x^5))/(1155*x^4) + (2*(a + b*x^3)^(3/2)*(1155*c*x + 693*d*x^2 + 495*e*x^3 + 385*f*x^4 + 315*g*x^5))/(3465*x^4) - (Sqrt[a]*(3*b*c + 2*a*f)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*b^(1/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticE[ArcSin[(1 - Sqrt[3])*a^(1/3) + b^(1/3)*x]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(14*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(77*b*d - 110*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e + 28*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticF[ArcSin[(1 - Sqrt[3])*a^(1/3) + b^(1/3)*x]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(770*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 0.956585, antiderivative size = 692, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1826, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$9 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} a (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (-110(1-\sqrt{3}) \sqrt[3]{a} b^{2/3} e + 28ag + 77bd) F \left(\sin^{-1} \left(\frac{\sqrt[3]{bx} + (1-\sqrt{3}) \sqrt[3]{a}}{\sqrt[3]{bx} + (1+\sqrt{3}) \sqrt[3]{a}} \right) \right) \\ \hline 770 \sqrt[3]{b} \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4, x]

```
[Out] (a*c*Sqrt[a + b*x^3])/x^3 + (27*a*d*Sqrt[a + b*x^3])/(10*x^2) - (27*a*e*Sqrt[a + b*x^3])/(7*x) + (27*a*b^(1/3)*e*Sqrt[a + b*x^3])/(7*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*a*Sqrt[a + b*x^3]*(1155*c*x + 2079*d*x^2 - 1485*e*x^3 - 385*f*x^4 - 189*g*x^5))/(1155*x^4) + (2*(a + b*x^3)^(3/2)*(1155*c*x + 693*d*x^2 + 495*e*x^3 + 385*f*x^4 + 315*g*x^5))/(3465*x^4) - (Sqrt[a]*(3*b*c + 2*a*f)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*b^(1/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(14*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(77*b*d - 110*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e + 28*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(770*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1826

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}], x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1835

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x^n)^p, x], x] /; NeQ[Pq0, 0] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_)*(x_)^(n_)]), x_Symbol] := Dist[Coeff[Pq, x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq, x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
```

```
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3)^{3/2} (c + dx + ex^2 + fx^3 + gx^4)}{x^4} dx &= \frac{2(a + bx^3)^{3/2} (1155cx + 693dx^2 + 495ex^3 + 385fx^4 + 315gx^5)}{3465x^4} + \frac{1}{2}(9a) \\
&= -\frac{2a\sqrt{a + bx^3} (1155cx + 2079dx^2 - 1485ex^3 - 385fx^4 - 189gx^5)}{1155x^4} + \frac{2(a}{x^3} \\
&= \frac{ac\sqrt{a + bx^3}}{x^3} - \frac{2a\sqrt{a + bx^3} (1155cx + 2079dx^2 - 1485ex^3 - 385fx^4 - 189gx^5)}{1155x^4} \\
&= \frac{ac\sqrt{a + bx^3}}{x^3} + \frac{27ad\sqrt{a + bx^3}}{10x^2} - \frac{2a\sqrt{a + bx^3} (1155cx + 2079dx^2 - 1485ex^3 - 385fx^4 - 189gx^5)}{1155x^4} \\
&= \frac{ac\sqrt{a + bx^3}}{x^3} + \frac{27ad\sqrt{a + bx^3}}{10x^2} - \frac{27ae\sqrt{a + bx^3}}{7x} - \frac{2a\sqrt{a + bx^3} (1155cx + 2079dx^2 - 1485ex^3 - 385fx^4 - 189gx^5)}{1155x^4} \\
&= \frac{ac\sqrt{a + bx^3}}{x^3} + \frac{27ad\sqrt{a + bx^3}}{10x^2} - \frac{27ae\sqrt{a + bx^3}}{7x} - \frac{2a\sqrt{a + bx^3} (1155cx + 2079dx^2 - 1485ex^3 - 385fx^4 - 189gx^5)}{1155x^4} \\
&= \frac{ac\sqrt{a + bx^3}}{x^3} + \frac{27ad\sqrt{a + bx^3}}{10x^2} - \frac{27ae\sqrt{a + bx^3}}{7x} - \frac{2a\sqrt{a + bx^3} (1155cx + 2079dx^2 - 1485ex^3 - 385fx^4 - 189gx^5)}{1155x^4} \\
&= \frac{ac\sqrt{a + bx^3}}{x^3} + \frac{27ad\sqrt{a + bx^3}}{10x^2} - \frac{27ae\sqrt{a + bx^3}}{7x} + \frac{27a\sqrt[3]{be}\sqrt{a + bx^3}}{7((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx^3})} \\
&= \frac{ac\sqrt{a + bx^3}}{x^3} + \frac{27ad\sqrt{a + bx^3}}{10x^2} - \frac{27ae\sqrt{a + bx^3}}{7x} + \frac{27a\sqrt[3]{be}\sqrt{a + bx^3}}{7((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx^3})}
\end{aligned}$$

Mathematica [C] time = 0.611858, size = 243, normalized size = 0.35

$$\frac{4x^2\sqrt{\frac{bx^3}{a} + 1} \left(5a^2 f \left(\sqrt{a + bx^3} (4a + bx^3) - 3a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + bx^3}}{\sqrt{a}} \right) \right) + 3bc (a + bx^3)^{5/2} {}_2F_1 \left(2, \frac{5}{2}; \frac{7}{2}; \frac{bx^3}{a} + 1 \right) - 45a^3 d \sqrt{a + bx^3} \right)}{90a^2 x^2 \sqrt{\frac{bx^3}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4,x]

[Out] (-45*a^3*d*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, -2/3, 1/3, -((b*x^3)/a)] - 90*a^3*e*x*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, -1/3, 2/3, -((b*x^3)/a)] + 90*a^3*g*x^3*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)] + 4*x^2*Sqrt[1 + (b*x^3)/a]*(5*a^2*f*(Sqrt[a + b*x^3]*(4*a + b*x^3) - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]) + 3*b*c*(a + b*x^3)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^3)/a]))/(90*a^2*x^2*Sqrt[1 + (b*x^3)/a])

Maple [B] time = 0.01, size = 1193, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x)

[Out] g*(2/11*b*x^4*(b*x^3+a)^(1/2)+28/55*a*x*(b*x^3+a)^(1/2)-18/55*I*a^2*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3))+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))^(1/2))+f*(2/9*b*x^3*(b*x^3+a)^(1/2)+8/9*a*(b*x^3+a)^(1/2)-2/3*a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+c*(-1/3*a*(b*x^3+a)^(1/2)/x^3+2/3*b*(b*x^3+a)^(1/2)-a^(1/2)*b*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+d*(-1/2*a*(b*x^3+a)^(1/2)/x^2+2/5*b*x*(b*x^3+a)^(1/2)-9/10*I*a*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3))+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))^(1/2))+e*(-a*(b*x^3+a)^(1/2)/x+2/7*b*x^2*(b*x^3+a)^(1/2)-9/7*I*a*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3))+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(

$x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)))}^{(1/2)}+1/b*(-b^2*a)^{(1/3)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)))}^{(1/2))))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="maxima")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac)\sqrt{bx^3 + a}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="fricas")

[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2 + (b*c + a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^4, x)

Sympy [A] time = 11.9664, size = 484, normalized size = 0.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)

[Out] a**(3/2)*d*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + a**(3/2)*e*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*a**(3/2)*f*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + a**(3/2)*g*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) - sqrt(a)*b*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) + sqrt(a)*b*d*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + sqrt(a)*b*e*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*g*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + 2*a**2*f/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)*c*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*a*sqrt(b)*c/(3*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*a*sqrt(b)*f*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + 2*b**(3/2)*c*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + b*f*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="giac")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^4, x)

$$3.466 \quad \int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^5} dx$$

Optimal. Leaf size=741

$$9 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} \sqrt[3]{a} \sqrt[3]{b} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (28a^{2/3} \sqrt[3]{be} - 5(1 - \sqrt{3})(8af + 7bc)) \operatorname{EllipticF} \left(\sin^{-1} \left(\frac{(1-\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{(1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}} \right) \right)$$

$$280 \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

[Out] (27*a*c*Sqrt[a + b*x^3])/(20*x^4) + (a*d*Sqrt[a + b*x^3])/x^3 + (27*a*e*Sqrt[a + b*x^3])/(10*x^2) - (27*(7*b*c + 8*a*f)*Sqrt[a + b*x^3])/(56*x) + (27*b^(1/3)*(7*b*c + 8*a*f)*Sqrt[a + b*x^3])/(56*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*a*Sqrt[a + b*x^3]*(189*c*x + 105*d*x^2 + 189*e*x^3 - 135*f*x^4 - 35*g*x^5))/(105*x^5) + (2*(a + b*x^3)^(3/2)*(315*c*x + 105*d*x^2 + 63*e*x^3 + 45*f*x^4 + 35*g*x^5))/(315*x^5) - (Sqrt[a]*(3*b*d + 2*a*g)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(1/3)*(7*b*c + 8*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(112*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*b^(1/3)*(28*a^(2/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(7*b*c + 8*a*f))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(280*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 1.24017, antiderivative size = 741, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1826, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$9 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} \sqrt[3]{a} \sqrt[3]{b} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (28a^{2/3} \sqrt[3]{be} - 5(1 - \sqrt{3})(8af + 7bc)) F \left(\sin^{-1} \left(\frac{\sqrt[3]{bx} + (1-\sqrt{3}) \sqrt[3]{a}}{\sqrt[3]{bx} + (1+\sqrt{3}) \sqrt[3]{a}} \right) \right) -$$

$$280 \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5,x]

[Out] (27*a*c*Sqrt[a + b*x^3])/(20*x^4) + (a*d*Sqrt[a + b*x^3])/x^3 + (27*a*e*Sqrt[a + b*x^3])/(10*x^2) - (27*(7*b*c + 8*a*f)*Sqrt[a + b*x^3])/(56*x) + (27*b^(1/3)*(7*b*c + 8*a*f)*Sqrt[a + b*x^3])/(56*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*a*Sqrt[a + b*x^3]*(189*c*x + 105*d*x^2 + 189*e*x^3 - 135*f*x^4 - 35*g*x^5))/(105*x^5) + (2*(a + b*x^3)^(3/2)*(315*c*x + 105*d*x^2 + 63*e*x^3 + 45*f*x^4 + 35*g*x^5))/(315*x^5) - (Sqrt[a]*(3*b*d + 2*a*g)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(1/3)*(7*b*c + 8*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(112*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*b^(1/3)*(28*a^(2/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(7*b*c + 8*a*f))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(280*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rule 1826

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[(c*x)^(m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^(m*(a + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}], x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]

Rule 1835

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x^n)^p, x], x] /; NeQ[Pq0, 0] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]

Rule 1832

Int[(Pq_)/((x_)*Sqrt[(a_) + (b_)*(x_)^(n_)]), x_Symbol] := Dist[Coeff[Pq, x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq, x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1878

```
Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + S
qrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
```

$((1 + \text{Sqrt}[3]) * s + r * x)^2$), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + bx^3)^{3/2} (c + dx + ex^2 + fx^3 + gx^4)}{x^5} dx &= \frac{2(a + bx^3)^{3/2} (315cx + 105dx^2 + 63ex^3 + 45fx^4 + 35gx^5)}{315x^5} + \frac{1}{2}(9a) \int \\
 &= -\frac{2a\sqrt{a + bx^3} (189cx + 105dx^2 + 189ex^3 - 135fx^4 - 35gx^5)}{105x^5} + \frac{2(a + b}{ \\
 &= \frac{27ac\sqrt{a + bx^3}}{20x^4} - \frac{2a\sqrt{a + bx^3} (189cx + 105dx^2 + 189ex^3 - 135fx^4 - 35}{105x^5} \\
 &= \frac{27ac\sqrt{a + bx^3}}{20x^4} + \frac{ad\sqrt{a + bx^3}}{x^3} - \frac{2a\sqrt{a + bx^3} (189cx + 105dx^2 + 189ex^3}{105x^5} \\
 &= \frac{27ac\sqrt{a + bx^3}}{20x^4} + \frac{ad\sqrt{a + bx^3}}{x^3} + \frac{27ae\sqrt{a + bx^3}}{10x^2} - \frac{2a\sqrt{a + bx^3} (189cx +}{ \\
 &= \frac{27ac\sqrt{a + bx^3}}{20x^4} + \frac{ad\sqrt{a + bx^3}}{x^3} + \frac{27ae\sqrt{a + bx^3}}{10x^2} - \frac{27(7bc + 8af)\sqrt{a + b}}{56x} \\
 &= \frac{27ac\sqrt{a + bx^3}}{20x^4} + \frac{ad\sqrt{a + bx^3}}{x^3} + \frac{27ae\sqrt{a + bx^3}}{10x^2} - \frac{27(7bc + 8af)\sqrt{a + b}}{56x} \\
 &= \frac{27ac\sqrt{a + bx^3}}{20x^4} + \frac{ad\sqrt{a + bx^3}}{x^3} + \frac{27ae\sqrt{a + bx^3}}{10x^2} - \frac{27(7bc + 8af)\sqrt{a + b}}{56x} \\
 &= \frac{27ac\sqrt{a + bx^3}}{20x^4} + \frac{ad\sqrt{a + bx^3}}{x^3} + \frac{27ae\sqrt{a + bx^3}}{10x^2} - \frac{27(7bc + 8af)\sqrt{a + b}}{56x} \\
 &= \frac{27ac\sqrt{a + bx^3}}{20x^4} + \frac{ad\sqrt{a + bx^3}}{x^3} + \frac{27ae\sqrt{a + bx^3}}{10x^2} - \frac{27(7bc + 8af)\sqrt{a + b}}{56x}
 \end{aligned}$$

Mathematica [C] time = 0.602643, size = 246, normalized size = 0.33

$$\frac{-45a^3c\sqrt{a+bx^3} {}_2F_1\left(-\frac{3}{2}, -\frac{4}{3}; -\frac{1}{3}; -\frac{bx^3}{a}\right) + 4x^3\left(2x\sqrt{\frac{bx^3}{a}} + 1\left(5a^2g\left(\sqrt{a+bx^3}(4a+bx^3)\right) - 3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right) + 3bd\right)}{180a^2x^4\sqrt{\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5, x]

[Out] (-45*a^3*c*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, -4/3, -1/3, -((b*x^3)/a)] - 90*a^3*e*x^2*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, -2/3, 1/3, -((b*x^3)/a)] + 4*x^3*(-45*a^3*f*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, -1/3, 2/3, -((b*x^3)/a)] + 2*x*Sqrt[1 + (b*x^3)/a]*(5*a^2*g*(Sqrt[a + b*x^3]*(4*a + b*x^3) - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]) + 3*b*d*(a + b*x^3)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^3)/a]))/(180*a^2*x^4*Sqrt[1 + (b*x^3)/a])

Maple [B] time = 0.01, size = 1342, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^5, x)

[Out] g*(2/9*b*x^3*(b*x^3+a)^(1/2)+8/9*a*(b*x^3+a)^(1/2)-2/3*a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+d*(-1/3*a*(b*x^3+a)^(1/2)/x^3+2/3*b*(b*x^3+a)^(1/2)-a^(1/2)*b*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+c*(-1/4*a*(b*x^3+a)^(1/2)/x^4-11/8*b*(b*x^3+a)^(1/2)/x-9/8*I*b*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+e*(-1/2*a*(b*x^3+a)^(1/2)/x^2+2/5*b*x*(b*x^3+a)^(1/2)-9/10*I*a*3^(1/2)*(-b^2

$$\begin{aligned}
 & *a)^{(1/3)} * (I*(x+1/2/b*(-b^2*a)^{(1/3)} - 1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}) * 3^{(1/2)} \\
 &) * b / (-b^2*a)^{(1/3))^{(1/2)} * ((x-1/b*(-b^2*a)^{(1/3)}) / (-3/2/b*(-b^2*a)^{(1/3)} + 1/ \\
 & 2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)))^{(1/2)} * (-I*(x+1/2/b*(-b^2*a)^{(1/3)} + 1/2*I*3^{(1 \\
 & /2)/b*(-b^2*a)^{(1/3)}) * 3^{(1/2)} * b / (-b^2*a)^{(1/3))^{(1/2)} / (b*x^3+a)^{(1/2)} * \text{Ellip} \\
 & \text{ticF}(1/3*3^{(1/2)} * (I*(x+1/2/b*(-b^2*a)^{(1/3)} - 1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}) \\
 &) * 3^{(1/2)} * b / (-b^2*a)^{(1/3))^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)} / (-3/2/b*(-b^2*a \\
 & a)^{(1/3)} + 1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)))^{(1/2)}) + f*(-a*(b*x^3+a)^{(1/2)}/x+2 \\
 & /7*b*x^2*(b*x^3+a)^{(1/2)} - 9/7*I*a*3^{(1/2)}*(-b^2*a)^{(1/3)} * (I*(x+1/2/b*(-b^2*a \\
 &)^{(1/3)} - 1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}) * 3^{(1/2)} * b / (-b^2*a)^{(1/3))^{(1/2)} * ((x \\
 & -1/b*(-b^2*a)^{(1/3)}) / (-3/2/b*(-b^2*a)^{(1/3)} + 1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)) \\
 &)^{(1/2)} * (-I*(x+1/2/b*(-b^2*a)^{(1/3)} + 1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}) * 3^{(1/2)} \\
 &) * b / (-b^2*a)^{(1/3))^{(1/2)} / (b*x^3+a)^{(1/2)} * ((-3/2/b*(-b^2*a)^{(1/3)} + 1/2*I*3^{(1 \\
 & /2)/b*(-b^2*a)^{(1/3)} * \text{EllipticE}(1/3*3^{(1/2)} * (I*(x+1/2/b*(-b^2*a)^{(1/3)} - 1/2*I \\
 & I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}) * 3^{(1/2)} * b / (-b^2*a)^{(1/3))^{(1/2)}, (I*3^{(1/2)}/b*(- \\
 & b^2*a)^{(1/3)} / (-3/2/b*(-b^2*a)^{(1/3)} + 1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)))^{(1/2)}) \\
 & + 1/b*(-b^2*a)^{(1/3)} * \text{EllipticF}(1/3*3^{(1/2)} * (I*(x+1/2/b*(-b^2*a)^{(1/3)} - 1/2*I* \\
 & 3^{(1/2)}/b*(-b^2*a)^{(1/3)}) * 3^{(1/2)} * b / (-b^2*a)^{(1/3))^{(1/2)}, (I*3^{(1/2)}/b*(-b^ \\
 & 2*a)^{(1/3)} / (-3/2/b*(-b^2*a)^{(1/3)} + 1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)))^{(1/2)}))
 \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="maxima")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^5, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + c)\sqrt{bx^3 + a}}{x^5}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="fricas")

[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2 + (b*c + a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^5, x)

Sympy [A] time = 12.1667, size = 495, normalized size = 0.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)

[Out] a**(3/2)*c*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + a**(3/2)*e*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + a**(3/2)*f*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*a*(3/2)*g*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + sqrt(a)*b*c*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - sqrt(a)*b*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) + sqrt(a)*b*e*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + sqrt(a)*b*f*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + 2*a**2*g/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)*d*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*a*sqrt(b)*d/(3*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*a*sqrt(b)*g*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + 2*b**(3/2)*d*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + b*g*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="giac")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^5, x)

$$3.467 \quad \int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^6} dx$$

Optimal. Leaf size=689

$$9 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} \sqrt[3]{b} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \operatorname{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{(1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}\right), -7 - 4\sqrt{3}\right) (14 \sqrt[3]{b} (2af + bc) - 5(1$$

$$280 \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

[Out] $(27*b*c*\operatorname{Sqrt}[a + b*x^3])/(20*x^2) - (27*b*d*\operatorname{Sqrt}[a + b*x^3])/(8*x) + (27*b^{1/3}*(7*b*d + 8*a*g)*\operatorname{Sqrt}[a + b*x^3])/(56*((1 + \operatorname{Sqrt}[3])*a^{1/3} + b^{1/3}*x)) - (((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2 + (60*g)/x)*(a + b*x^3)^{(3/2)}/60 - (b*\operatorname{Sqrt}[a + b*x^3]*(252*c*x - 315*d*x^2 - 140*e*x^3 - 126*f*x^4 - 180*g*x^5))/(140*x^3) - \operatorname{Sqrt}[a]*b*e*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*x^3]/\operatorname{Sqrt}[a]] - (27*3^{1/4}*\operatorname{Sqrt}[2 - \operatorname{Sqrt}[3]]*a^{1/3}*b^{1/3}*(7*b*d + 8*a*g)*(a^{1/3} + b^{1/3}*x)*\operatorname{Sqrt}[(a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2)/((1 + \operatorname{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[(1 - \operatorname{Sqrt}[3])*a^{1/3} + b^{1/3}*x]/((1 + \operatorname{Sqrt}[3])*a^{1/3} + b^{1/3}*x)], -7 - 4*\operatorname{Sqrt}[3])/(112*\operatorname{Sqrt}[(a^{1/3}*(a^{1/3} + b^{1/3}*x))/((1 + \operatorname{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\operatorname{Sqrt}[a + b*x^3]) + (9*3^{3/4}*\operatorname{Sqrt}[2 + \operatorname{Sqrt}[3]]*b^{1/3}*(14*b^{1/3}*(b*c + 2*a*f) - 5*(1 - \operatorname{Sqrt}[3])*a^{1/3}*(7*b*d + 8*a*g))*(a^{1/3} + b^{1/3}*x)*\operatorname{Sqrt}[(a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2)/((1 + \operatorname{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[(1 - \operatorname{Sqrt}[3])*a^{1/3} + b^{1/3}*x]/((1 + \operatorname{Sqrt}[3])*a^{1/3} + b^{1/3}*x)], -7 - 4*\operatorname{Sqrt}[3])/(280*\operatorname{Sqrt}[(a^{1/3}*(a^{1/3} + b^{1/3}*x))/((1 + \operatorname{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\operatorname{Sqrt}[a + b*x^3])$

Rubi [A] time = 0.920922, antiderivative size = 689, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {14, 1825, 1826, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$9 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} \sqrt[3]{b} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx} + (1-\sqrt{3}) \sqrt[3]{a}}{\sqrt[3]{bx} + (1+\sqrt{3}) \sqrt[3]{a}}\right), -7 - 4\sqrt{3}\right) (14 \sqrt[3]{b} (2af + bc) - 5(1 - \sqrt{3}) \sqrt[3]{b} (2af + bc) - 5(1 - \sqrt{3}) \sqrt[3]{b} (2af + bc))$$

$$280 \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b*x^3)^{(3/2)}*(c + d*x + e*x^2 + f*x^3 + g*x^4)/x^6, x]$

```
[Out] (27*b*c*Sqrt[a + b*x^3])/(20*x^2) - (27*b*d*Sqrt[a + b*x^3])/(8*x) + (27*b^(1/3)*(7*b*d + 8*a*g)*Sqrt[a + b*x^3])/(56*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2 + (60*g)/x)*(a + b*x^3)^(3/2)/60 - (b*Sqrt[a + b*x^3]*(252*c*x - 315*d*x^2 - 140*e*x^3 - 126*f*x^4 - 180*g*x^5))/(140*x^3) - Sqrt[a]*b*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(1/3)*(7*b*d + 8*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(112*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(1/3)*(14*b^(1/3)*(b*c + 2*a*f) - 5*(1 - Sqrt[3])*a^(1/3)*(7*b*d + 8*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(280*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

Rule 1825

```
Int[(Pq_)*(x_)^ (m_)*((a_) + (b_)*(x_)^ (n_))^(p_), x_Symbol] := Module[{u = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1, 0]
```

Rule 1826

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^ (n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}], x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1835

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^ (n_))^(p_), x_Symbol] := With[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
```

$(m + 1)(Pq - Pq0)/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^{(n - 1)}, x]*(a + b*x^n)^p, x] /; NeQ[Pq0, 0] /; FreeQ[{a, b, c, p}, x] \&\& PolyQ[Pq, x] \&\& IGtQ[n, 0] \&\& LtQ[m, -1] \&\& LeQ[n - 1, Expon[Pq, x]]$

Rule 1832

$Int[(Pq_)/((x_)*Sqrt[(a_) + (b_)*(x_)^(n_)]), x_Symbol] \rightarrow Dist[Coeff[Pq, x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq, x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] \&\& PolyQ[Pq, x] \&\& IGtQ[n, 0] \&\& NeQ[Coeff[Pq, x, 0], 0]$

Rule 266

$Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] \rightarrow Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] \&\& IntegerQ[Simplify[(m + 1)/n]]$

Rule 63

$Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] \rightarrow With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] \&\& NeQ[b*c - a*d, 0] \&\& LtQ[-1, m, 0] \&\& LeQ[-1, n, 0] \&\& LeQ[Denominator[n], Denominator[m]] \&\& IntLinearQ[a, b, c, d, m, n, x]$

Rule 208

$Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] \rightarrow Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] \&\& NegQ[a/b]$

Rule 1878

$Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] \rightarrow With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x] /; FreeQ[{a, b, c, d}, x] \&\& PosQ[a] \&\& NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]$

Rule 218

$Int[1/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] \rightarrow With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)]/((1 + Sqrt[3])*s + r*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]/(3^(1/4)*r*Sqrt[a + b*x^3])*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b}, x] \&$

& PosQ[a]

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :=> With[{r = Numerator[Simplify[((1 - Sqrt[3])*d)/c]], s = Denominator[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)]], -7 - 4*Sqrt[3])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3)^{3/2} (c + dx + ex^2 + fx^3 + gx^4)}{x^6} dx &= -\frac{1}{60} \left(\frac{12c}{x^5} + \frac{15d}{x^4} + \frac{20e}{x^3} + \frac{30f}{x^2} + \frac{60g}{x} \right) (a + bx^3)^{3/2} - \frac{1}{2} (9b) \int \frac{\sqrt{a + bx^3}}{x^6} dx \\
&= -\frac{1}{60} \left(\frac{12c}{x^5} + \frac{15d}{x^4} + \frac{20e}{x^3} + \frac{30f}{x^2} + \frac{60g}{x} \right) (a + bx^3)^{3/2} - \frac{b\sqrt{a + bx^3} (252c - 27bdx^3)}{20x^2} \\
&= \frac{27bc\sqrt{a + bx^3}}{20x^2} - \frac{1}{60} \left(\frac{12c}{x^5} + \frac{15d}{x^4} + \frac{20e}{x^3} + \frac{30f}{x^2} + \frac{60g}{x} \right) (a + bx^3)^{3/2} - \frac{b\sqrt{a + bx^3} (252c - 27bdx^3)}{20x^2} \\
&= \frac{27bc\sqrt{a + bx^3}}{20x^2} - \frac{27bd\sqrt{a + bx^3}}{8x} - \frac{1}{60} \left(\frac{12c}{x^5} + \frac{15d}{x^4} + \frac{20e}{x^3} + \frac{30f}{x^2} + \frac{60g}{x} \right) (a + bx^3)^{3/2} \\
&= \frac{27bc\sqrt{a + bx^3}}{20x^2} - \frac{27bd\sqrt{a + bx^3}}{8x} - \frac{1}{60} \left(\frac{12c}{x^5} + \frac{15d}{x^4} + \frac{20e}{x^3} + \frac{30f}{x^2} + \frac{60g}{x} \right) (a + bx^3)^{3/2} \\
&= \frac{27bc\sqrt{a + bx^3}}{20x^2} - \frac{27bd\sqrt{a + bx^3}}{8x} - \frac{1}{60} \left(\frac{12c}{x^5} + \frac{15d}{x^4} + \frac{20e}{x^3} + \frac{30f}{x^2} + \frac{60g}{x} \right) (a + bx^3)^{3/2} \\
&= \frac{27bc\sqrt{a + bx^3}}{20x^2} - \frac{27bd\sqrt{a + bx^3}}{8x} + \frac{27\sqrt[3]{b}(7bd + 8ag)\sqrt{a + bx^3}}{56((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})} - \frac{1}{60} \left(\frac{12c}{x^5} + \frac{15d}{x^4} + \frac{20e}{x^3} + \frac{30f}{x^2} + \frac{60g}{x} \right) (a + bx^3)^{3/2} \\
&= \frac{27bc\sqrt{a + bx^3}}{20x^2} - \frac{27bd\sqrt{a + bx^3}}{8x} + \frac{27\sqrt[3]{b}(7bd + 8ag)\sqrt{a + bx^3}}{56((1 + \sqrt{3})\sqrt[3]{a} + \sqrt[3]{bx})} - \frac{1}{60} \left(\frac{12c}{x^5} + \frac{15d}{x^4} + \frac{20e}{x^3} + \frac{30f}{x^2} + \frac{60g}{x} \right) (a + bx^3)^{3/2}
\end{aligned}$$

Mathematica [C] time = 0.243595, size = 191, normalized size = 0.28

$$\frac{\sqrt{a + bx^3} \left(-12a^3 c {}_2F_1 \left(-\frac{5}{3}, -\frac{3}{2}; -\frac{2}{3}; -\frac{bx^3}{a} \right) - 15a^3 dx {}_2F_1 \left(-\frac{3}{2}, -\frac{4}{3}; -\frac{1}{3}; -\frac{bx^3}{a} \right) - 30a^3 fx^3 {}_2F_1 \left(-\frac{3}{2}, -\frac{2}{3}; \frac{1}{3}; -\frac{bx^3}{a} \right) - 60a^3 gx^4 {}_2F_1 \left(-\frac{3}{2}, -\frac{1}{3}; \frac{2}{3}; -\frac{bx^3}{a} \right) \right)}{60a^2 x^5 \sqrt{\frac{bx^3}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6,x]

```
[Out] (Sqrt[a + b*x^3]*(-12*a^3*c*Hypergeometric2F1[-5/3, -3/2, -2/3, -((b*x^3)/a)] - 15*a^3*d*x*Hypergeometric2F1[-3/2, -4/3, -1/3, -((b*x^3)/a)] - 30*a^3*f*x^3*Hypergeometric2F1[-3/2, -2/3, 1/3, -((b*x^3)/a)] - 60*a^3*g*x^4*Hypergeometric2F1[-3/2, -1/3, 2/3, -((b*x^3)/a)] + 8*b*e*x^5*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^3)/a]))/(60*a^2*x^5*Sqrt[1 + (b*x^3)/a])
```

Maple [B] time = 0.009, size = 1606, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^6,x)
```

```
[Out] e*(-1/3*a*(b*x^3+a)^(1/2)/x^3+2/3*b*(b*x^3+a)^(1/2)-a^(1/2)*b*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+c*(-1/5*a*(b*x^3+a)^(1/2)/x^5-13/20*b*(b*x^3+a)^(1/2)/x^2-9/20*I*b*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+d*(-1/4*a*(b*x^3+a)^(1/2)/x^4-11/8*b*(b*x^3+a)^(1/2)/x-9/8*I*b*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+f*(-1/2*a*(b*x^3+a)^(1/2)/x^2+2/5*b*x*(b*x^3+a)^(1/2)-9/10*I*a*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)
```

$$\begin{aligned} &)/b*(-b^2*a)^{(1/3)/(-3/2/b*(-b^2*a)^{(1/3)+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}} \\ & +g*(-a*(b*x^3+a)^{(1/2)/x+2/7*b*x^2*(b*x^3+a)^{(1/2)-9/7*I*a*3^{(1/2)*} \\ & -b^2*a)^{(1/3)*(I*(x+1/2/b*(-b^2*a)^{(1/3)-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))*3^{(1/2)*} \\ & b/(-b^2*a)^{(1/3))}^{(1/2)*((x-1/b*(-b^2*a)^{(1/3)))/(-3/2/b*(-b^2*a)^{(1/3) \\ &)+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)*(-I*(x+1/2/b*(-b^2*a)^{(1/3)+1/2*I* \\ & 3^{(1/2)}/b*(-b^2*a)^{(1/3))*3^{(1/2)*}b/(-b^2*a)^{(1/3))}^{(1/2)/(b*x^3+a)^{(1/2)*} \\ & (-3/2/b*(-b^2*a)^{(1/3)+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))} *EllipticE(1/3*3^{(1/2) \\ &)*(I*(x+1/2/b*(-b^2*a)^{(1/3)-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))*3^{(1/2)*}b/(-b^ \\ & 2*a)^{(1/3))}^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)/(-3/2/b*(-b^2*a)^{(1/3)+1/2*I* \\ & 3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)})+1/b*(-b^2*a)^{(1/3)*EllipticF(1/3*3^{(1/2)*} \\ & (I*(x+1/2/b*(-b^2*a)^{(1/3)-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))*3^{(1/2)*}b/(-b^2* \\ & a)^{(1/3))}^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)/(-3/2/b*(-b^2*a)^{(1/3)+1/2*I*3^{(1/2) \\ &)/b*(-b^2*a)^{(1/3))}^{(1/2))} \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^6,x, algorithm="maxima")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^6, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac)\sqrt{bx^3 + a}}{x^6}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^6,x, algorithm="fricas")

[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2 + (b*c + a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^6, x)

Sympy [A] time = 11.7593, size = 476, normalized size = 0.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**6,x)

[Out] a**(3/2)*c*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + a**(3/2)*d*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + a**(3/2)*f*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + a**(3/2)*g*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) + sqrt(a)*b*c*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + sqrt(a)*b*d*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - sqrt(a)*b*e*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) + sqrt(a)*b*f*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + sqrt(a)*b*g*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) - a*sqrt(b)*e*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*a*sqrt(b)*e/(3*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*b**(3/2)*e*x**(3/2)/(3*sqrt(a/(b*x**3) + 1))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^6,x, algorithm="giac")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^6, x)

$$3.468 \quad \int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^7} dx$$

Optimal. Leaf size=692

$$9 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} b^{2/3} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (-5(1-\sqrt{3}) \sqrt[3]{ab^{2/3}} e + 4ag + 2bd) \operatorname{EllipticF} \left(\sin^{-1} \left(\frac{(1-\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{(1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}} \right) \right) \\ \hline 40 \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

[Out] (b*c*Sqrt[a + b*x^3])/(4*x^3) + (27*b*d*Sqrt[a + b*x^3])/(20*x^2) - (27*b*e*Sqrt[a + b*x^3])/(8*x) + (27*b^(4/3)*e*Sqrt[a + b*x^3])/(8*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3 + (30*g)/x^2)*(a + b*x^3)^(3/2))/60 - (b*Sqrt[a + b*x^3]*(10*c*x + 36*d*x^2 - 45*e*x^3 - 20*f*x^4 - 18*g*x^5))/(20*x^4) - (b*(b*c + 4*a*f)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a]) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(4/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(2/3)*(2*b*d - 5*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e + 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(40*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 1.00417, antiderivative size = 692, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {14, 1825, 1826, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$9 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} b^{2/3} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (-5(1-\sqrt{3}) \sqrt[3]{ab^{2/3}} e + 4ag + 2bd) F \left(\sin^{-1} \left(\frac{\sqrt[3]{bx} + (1-\sqrt{3}) \sqrt[3]{a}}{\sqrt[3]{bx} + (1+\sqrt{3}) \sqrt[3]{a}} \right) \right) \\ \hline 40 \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7, x]

```
[Out] (b*c*Sqrt[a + b*x^3]/(4*x^3) + (27*b*d*Sqrt[a + b*x^3]/(20*x^2) - (27*b*e
*Sqrt[a + b*x^3]/(8*x) + (27*b^(4/3)*e*Sqrt[a + b*x^3]/(8*((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)) - (((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3
+ (30*g)/x^2)*(a + b*x^3)^(3/2))/60 - (b*Sqrt[a + b*x^3]*(10*c*x + 36*d*x^
2 - 45*e*x^3 - 20*f*x^4 - 18*g*x^5))/(20*x^4) - (b*(b*c + 4*a*f)*ArcTanh[Sq
rt[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a]) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)
*b^(4/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
)*x^2]/(((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticE[ArcSin[((1 - Sqrt[3]
)]*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3
]])/(16*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(2/3)*(2*b*d - 5
*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e + 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(40*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

Rule 1825

```
Int[(Pq_)*(x_)^ (m_)*((a_) + (b_)*(x_)^ (n_))^(p_), x_Symbol] := Module[{u
= IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n)
]*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x] /; FreeQ[{a, b},
x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
0]
```

Rule 1826

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^ (n_))^(p_), x_Symbol] := M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i]
*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
+ b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}],
x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1835

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^ (n_))^(p_), x_Symbol] := Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
```

$(m + 1)(Pq - Pq0)/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^{(n - 1)}, x]*(a + b*x^n)^p, x] /; NeQ[Pq0, 0] /; FreeQ[{a, b, c, p}, x] \&\& PolyQ[Pq, x] \&\& IGtQ[n, 0] \&\& LtQ[m, -1] \&\& LeQ[n - 1, Expon[Pq, x]]$

Rule 1832

$Int[(Pq_)/((x_)*Sqrt[(a_) + (b_)*(x_)^(n_)]), x_Symbol] \rightarrow Dist[Coeff[Pq, x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq, x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] \&\& PolyQ[Pq, x] \&\& IGtQ[n, 0] \&\& NeQ[Coeff[Pq, x, 0], 0]$

Rule 266

$Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] \rightarrow Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] \&\& IntegerQ[Simplify[(m + 1)/n]]$

Rule 63

$Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] \rightarrow With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] \&\& NeQ[b*c - a*d, 0] \&\& LtQ[-1, m, 0] \&\& LeQ[-1, n, 0] \&\& LeQ[Denominator[n], Denominator[m]] \&\& IntLinearQ[a, b, c, d, m, n, x]$

Rule 208

$Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] \rightarrow Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] \&\& NegQ[a/b]$

Rule 1878

$Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] \rightarrow With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x] /; FreeQ[{a, b, c, d}, x] \&\& PosQ[a] \&\& NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]$

Rule 218

$Int[1/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] \rightarrow With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)]/((1 + Sqrt[3])*s + r*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]/(3^(1/4)*r*Sqrt[a + b*x^3])*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b}, x] \&$

& PosQ[a]

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :=> With[{r = Numerator[Simplify[((1 - Sqrt[3])*d)/c]], s = Denominator[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)]], -7 - 4*Sqrt[3])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^7} dx &= -\frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} + \frac{30g}{x^2} \right) (a+bx^3)^{3/2} - \frac{1}{2} (9b) \int \frac{\sqrt{a+bx^3}}{x^7} dx \\
&= -\frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} + \frac{30g}{x^2} \right) (a+bx^3)^{3/2} - \frac{b\sqrt{a+bx^3}(10cx^4 + 12dx^3 + 15ex^2 + 20fx + 30g)}{4x^3} \\
&= \frac{bc\sqrt{a+bx^3}}{4x^3} - \frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} + \frac{30g}{x^2} \right) (a+bx^3)^{3/2} - \frac{b\sqrt{a+bx^3}(10cx^4 + 12dx^3 + 15ex^2 + 20fx + 30g)}{4x^3} \\
&= \frac{bc\sqrt{a+bx^3}}{4x^3} + \frac{27bd\sqrt{a+bx^3}}{20x^2} - \frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} + \frac{30g}{x^2} \right) (a+bx^3)^{3/2} \\
&= \frac{bc\sqrt{a+bx^3}}{4x^3} + \frac{27bd\sqrt{a+bx^3}}{20x^2} - \frac{27be\sqrt{a+bx^3}}{8x} - \frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} + \frac{30g}{x^2} \right) (a+bx^3)^{3/2} \\
&= \frac{bc\sqrt{a+bx^3}}{4x^3} + \frac{27bd\sqrt{a+bx^3}}{20x^2} - \frac{27be\sqrt{a+bx^3}}{8x} - \frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} + \frac{30g}{x^2} \right) (a+bx^3)^{3/2} \\
&= \frac{bc\sqrt{a+bx^3}}{4x^3} + \frac{27bd\sqrt{a+bx^3}}{20x^2} - \frac{27be\sqrt{a+bx^3}}{8x} - \frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} + \frac{30g}{x^2} \right) (a+bx^3)^{3/2} \\
&= \frac{bc\sqrt{a+bx^3}}{4x^3} + \frac{27bd\sqrt{a+bx^3}}{20x^2} - \frac{27be\sqrt{a+bx^3}}{8x} + \frac{27b^{4/3}e\sqrt{a+bx^3}}{8((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{b})} \\
&= \frac{bc\sqrt{a+bx^3}}{4x^3} + \frac{27bd\sqrt{a+bx^3}}{20x^2} - \frac{27be\sqrt{a+bx^3}}{8x} + \frac{27b^{4/3}e\sqrt{a+bx^3}}{8((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{b})}
\end{aligned}$$

Mathematica [C] time = 0.507349, size = 240, normalized size = 0.35

$$\frac{12a^2d\sqrt{\frac{bx^3}{a}} + {}_2F_1\left(-\frac{5}{3}, -\frac{3}{2}, -\frac{2}{3}; -\frac{bx^3}{a}\right)}{x^5} - \frac{15a^2e\sqrt{\frac{bx^3}{a}} + {}_2F_1\left(-\frac{3}{2}, -\frac{4}{3}, -\frac{1}{3}; -\frac{bx^3}{a}\right)}{x^4} + \frac{8bf(a+bx^3)^3 {}_2F_1\left(2, \frac{5}{2}, \frac{7}{2}; \frac{bx^3}{a} + 1\right)}{a^2} - \frac{30a^2g\sqrt{\frac{bx^3}{a}} + {}_2F_1\left(-\frac{3}{2}, -\frac{2}{3}, \frac{1}{3}; -\frac{bx^3}{a}\right)}{x^2}$$

$$60\sqrt{a+bx^3}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7,x]

[Out]
$$\begin{aligned} &((-15*b*c*(a + b*x^3))/x^3 - (10*c*(a + b*x^3)^2)/x^6 - 15*b^2*c*\text{Sqrt}[1 + (b*x^3)/a]*\text{ArcTanh}[\text{Sqrt}[1 + (b*x^3)/a]] - (12*a^2*d*\text{Sqrt}[1 + (b*x^3)/a]*\text{Hypergeometric2F1}[-5/3, -3/2, -2/3, -((b*x^3)/a)])/x^5 - (15*a^2*e*\text{Sqrt}[1 + (b*x^3)/a]*\text{Hypergeometric2F1}[-3/2, -4/3, -1/3, -((b*x^3)/a)])/x^4 - (30*a^2*g*\text{Sqrt}[1 + (b*x^3)/a]*\text{Hypergeometric2F1}[-3/2, -2/3, 1/3, -((b*x^3)/a)])/x^2 + \\ &(8*b*f*(a + b*x^3)^3*\text{Hypergeometric2F1}[2, 5/2, 7/2, 1 + (b*x^3)/a])/a^2)/(60*\text{Sqrt}[a + b*x^3]) \end{aligned}$$

Maple [B] time = 0.01, size = 1196, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^7,x)

[Out]
$$\begin{aligned} &f*(-1/3*a*(b*x^3+a)^{(1/2)}/x^3+2/3*b*(b*x^3+a)^{(1/2)}-a^{(1/2)}*b*\text{arctanh}((b*x^3+a)^{(1/2)}/a^{(1/2)}))+d*(-1/5*a*(b*x^3+a)^{(1/2)}/x^5-13/20*b*(b*x^3+a)^{(1/2)}/x^2-9/20*I*b^3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I^3^{(1/2)})/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)})/(b*x^3+a)^{(1/2)}*\text{EllipticF}(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)},(I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}))+e*(-1/4*a*(b*x^3+a)^{(1/2)}/x^4-11/8*b*(b*x^3+a)^{(1/2)}/x-9/8*I*b^3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)})/(b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})*\text{EllipticE}(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)},(I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}))+1/b*(-b^2*a)^{(1/3)}*\text{EllipticF}(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)},(I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}))+g*(-1/2*a*(b*x^3+a)^{(1/2)}/x^2+2/5*b*x*(b*x^3+a)^{(1/2)}-9/10*I*a^3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})^3^{(1/2)}*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I^3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}*(-I*($$

$$x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)}*b/(-b^2*a)^{(1/3))^{(1/2)}/(b*x^3+a)^{(1/2)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)}*b/(-b^2*a)^{(1/3))^{(1/2)},(I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))^{(1/2)))+c*(-1/6*a*(b*x^3+a)^{(1/2)}/x^6-5/12*b*(b*x^3+a)^{(1/2)}/x^3-1/4*b^2*arctanh((b*x^3+a)^{(1/2)}/a^{(1/2)))/a^{(1/2))}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^7,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac)\sqrt{bx^3 + a}}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^7,x, algorithm="fricas")

[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2 + (b*c + a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^7, x)

Sympy [A] time = 15.5393, size = 524, normalized size = 0.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**7,x)
```

```
[Out] a**(3/2)*d*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**5*gamma(-2/3)) + a**(3/2)*e*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + a**(3/2)*g*gamma(-2/3)*
hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) +
sqrt(a)*b*d*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)
/a)/(3*x**2*gamma(1/3)) + sqrt(a)*b*e*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - sqrt(a)*b*f*asinh(sqrt(a)/(
sqrt(b)*x**(3/2))) + sqrt(a)*b*g*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) - a**2*c/(6*sqrt(b)*x**(15/2)*sqrt(a
/(b*x**3) + 1)) - a*sqrt(b)*c/(4*x**(9/2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)
*f*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*a*sqrt(b)*f/(3*x**(3/2)*sqrt(a/(b*
x**3) + 1)) - b**(3/2)*c*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b**(3/2)*c/(12
*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*b**(3/2)*f*x**(3/2)/(3*sqrt(a/(b*x**3)
+ 1)) - b**2*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*sqrt(a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^7,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^7, x)
```


$$3.469 \quad \int \frac{(a+bx^3)^{3/2} (c+dx+ex^2+fx^3+gx^4)}{x^8} dx$$

Optimal. Leaf size=746

$$9 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} b^{4/3} \left(\sqrt[3]{a} + \sqrt[3]{bx} \right) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{\left((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx} \right)^2}} \left(28a^{2/3} \sqrt[3]{be} - 5(1 - \sqrt{3})(14af + bc) \right) \text{EllipticF} \left(\sin^{-1} \left(\frac{(1-\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{(1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}} \right) \right)$$

$$560a^{2/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{\left((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx} \right)^2}} \sqrt{a + bx^3}$$

[Out] (27*b*c*Sqrt[a + b*x^3])/(280*x^4) + (b*d*Sqrt[a + b*x^3])/(4*x^3) + (27*b*e*Sqrt[a + b*x^3])/(20*x^2) - (27*b*(b*c + 14*a*f)*Sqrt[a + b*x^3])/(112*a*x) + (27*b^(4/3)*(b*c + 14*a*f)*Sqrt[a + b*x^3])/(112*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4 + (140*g)/x^3)*(a + b*x^3)^(3/2))/420 - (b*Sqrt[a + b*x^3]*(36*c*x + 70*d*x^2 + 252*e*x^3 - 315*f*x^4 - 140*g*x^5))/(140*x^5) - (b*(b*d + 4*a*g)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a]) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(b*c + 14*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(4/3)*(28*a^(2/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(b*c + 14*a*f))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(560*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 1.28367, antiderivative size = 746, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {14, 1825, 1826, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$9 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} b^{4/3} \left(\sqrt[3]{a} + \sqrt[3]{bx} \right) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{\left((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx} \right)^2}} \left(28a^{2/3} \sqrt[3]{be} - 5(1 - \sqrt{3})(14af + bc) \right) F \left(\sin^{-1} \left(\frac{\sqrt[3]{bx} + (1-\sqrt{3}) \sqrt[3]{a}}{\sqrt[3]{bx} + (1+\sqrt{3}) \sqrt[3]{a}} \right) \right) - 7$$

$$560a^{2/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{\left((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx} \right)^2}} \sqrt{a + bx^3}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8,x]

[Out] (27*b*c*Sqrt[a + b*x^3])/(280*x^4) + (b*d*Sqrt[a + b*x^3])/(4*x^3) + (27*b*e*Sqrt[a + b*x^3])/(20*x^2) - (27*b*(b*c + 14*a*f)*Sqrt[a + b*x^3])/(112*a*x) + (27*b^(4/3)*(b*c + 14*a*f)*Sqrt[a + b*x^3])/(112*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4 + (140*g)/x^3)*(a + b*x^3)^(3/2))/420 - (b*Sqrt[a + b*x^3]*(36*c*x + 70*d*x^2 + 252*e*x^3 - 315*f*x^4 - 140*g*x^5))/(140*x^5) - (b*(b*d + 4*a*g)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a]) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(b*c + 14*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(4/3)*(28*a^(2/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(b*c + 14*a*f))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(560*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 1825

Int[(Pq_)*(x_)^ (m_.)*((a_) + (b_.)*(x_)^(n_.))^ (p_), x_Symbol] := Module[{u = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1, 0]

Rule 1826

Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^ (p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}], x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]

Rule 1835

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_)*(x_)^(n_)]), x_Symbol] := Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1878

```
Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3)^{3/2} (c + dx + ex^2 + fx^3 + gx^4)}{x^8} dx &= -\frac{1}{420} \left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} + \frac{140g}{x^3} \right) (a + bx^3)^{3/2} - \frac{1}{2} (9b) \int \frac{\sqrt{a}}{x^8} dx \\
&= -\frac{1}{420} \left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} + \frac{140g}{x^3} \right) (a + bx^3)^{3/2} - \frac{b\sqrt{a + bx^3}}{2x^2} (3) \\
&= \frac{27bc\sqrt{a + bx^3}}{280x^4} - \frac{1}{420} \left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} + \frac{140g}{x^3} \right) (a + bx^3)^{3/2} \\
&= \frac{27bc\sqrt{a + bx^3}}{280x^4} + \frac{bd\sqrt{a + bx^3}}{4x^3} - \frac{1}{420} \left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} + \frac{140g}{x^3} \right) (a + bx^3)^{3/2} \\
&= \frac{27bc\sqrt{a + bx^3}}{280x^4} + \frac{bd\sqrt{a + bx^3}}{4x^3} + \frac{27be\sqrt{a + bx^3}}{20x^2} - \frac{1}{420} \left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} + \frac{140g}{x^3} \right) (a + bx^3)^{3/2} \\
&= \frac{27bc\sqrt{a + bx^3}}{280x^4} + \frac{bd\sqrt{a + bx^3}}{4x^3} + \frac{27be\sqrt{a + bx^3}}{20x^2} - \frac{27b(bc + 14af)\sqrt{a + bx^3}}{112ax} \\
&= \frac{27bc\sqrt{a + bx^3}}{280x^4} + \frac{bd\sqrt{a + bx^3}}{4x^3} + \frac{27be\sqrt{a + bx^3}}{20x^2} - \frac{27b(bc + 14af)\sqrt{a + bx^3}}{112ax} \\
&= \frac{27bc\sqrt{a + bx^3}}{280x^4} + \frac{bd\sqrt{a + bx^3}}{4x^3} + \frac{27be\sqrt{a + bx^3}}{20x^2} - \frac{27b(bc + 14af)\sqrt{a + bx^3}}{112ax} \\
&= \frac{27bc\sqrt{a + bx^3}}{280x^4} + \frac{bd\sqrt{a + bx^3}}{4x^3} + \frac{27be\sqrt{a + bx^3}}{20x^2} - \frac{27b(bc + 14af)\sqrt{a + bx^3}}{112ax} \\
&= \frac{27bc\sqrt{a + bx^3}}{280x^4} + \frac{bd\sqrt{a + bx^3}}{4x^3} + \frac{27be\sqrt{a + bx^3}}{20x^2} - \frac{27b(bc + 14af)\sqrt{a + bx^3}}{112ax}
\end{aligned}$$

Mathematica [C] time = 0.781448, size = 240, normalized size = 0.32

$$\frac{60a^2c\sqrt{\frac{bx^3}{a}}+1_2F_1\left(-\frac{7}{3},-\frac{3}{2};-\frac{4}{3};-\frac{bx^3}{a}\right)}{x^7}-\frac{84a^2e\sqrt{\frac{bx^3}{a}}+1_2F_1\left(-\frac{5}{3},-\frac{3}{2};-\frac{2}{3};-\frac{bx^3}{a}\right)}{x^5}-\frac{105a^2f\sqrt{\frac{bx^3}{a}}+1_2F_1\left(-\frac{3}{2},-\frac{4}{3};-\frac{1}{3};-\frac{bx^3}{a}\right)}{x^4}+\frac{56bg(a+bx^3)^3_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{bx^3}{a}+1\right)}{a^2}$$

$$420\sqrt{a+bx^3}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8,x]

[Out] ((-105*b*d*(a + b*x^3))/x^3 - (70*d*(a + b*x^3)^2)/x^6 - 105*b^2*d*Sqrt[1 + (b*x^3)/a]*ArcTanh[Sqrt[1 + (b*x^3)/a]] - (60*a^2*c*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-7/3, -3/2, -4/3, -(b*x^3)/a])/x^7 - (84*a^2*e*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-5/3, -3/2, -2/3, -(b*x^3)/a])/x^5 - (105*a^2*f*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-3/2, -4/3, -1/3, -(b*x^3)/a])/x^4 + (56*b*g*(a + b*x^3)^3*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^3)/a])/a^2)/(420*Sqrt[a + b*x^3])

Maple [B] time = 0.009, size = 1375, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^8,x)

[Out] g*(-1/3*a*(b*x^3+a)^(1/2)/x^3+2/3*b*(b*x^3+a)^(1/2)-a^(1/2)*b*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+e*(-1/5*a*(b*x^3+a)^(1/2)/x^5-13/20*b*(b*x^3+a)^(1/2)/x^2-9/20*I*b^3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I^3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I^3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I^3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I^3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I^3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I^3^(1/2)/b*(-b^2*a)^(1/3))^(1/2)))+f*(-1/4*a*(b*x^3+a)^(1/2)/x^4-11/8*b*(b*x^3+a)^(1/2)/x-9/8*I*b^3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I^3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I^3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I^3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I^3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*

$(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}+1/b*(-b^2*a)^{(1/3)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}))^{(1/2)}+d*(-1/6*a*(b*x^3+a)^{(1/2)}/x^6-5/12*b*(b*x^3+a)^{(1/2)}/x^3-1/4*b^2*arctanh((b*x^3+a)^{(1/2)}/a^{(1/2)})/a^{(1/2)}+c*(-1/7*a*(b*x^3+a)^{(1/2)}/x^7-17/56*b*(b*x^3+a)^{(1/2)}/x^4-27/112/a*b^2*(b*x^3+a)^{(1/2)}/x-9/112*I/a*b^2*3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*EllipticE(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}+1/b*(-b^2*a)^{(1/3)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})^{(1/2)}))^{(1/2)}))^{(1/2)}))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^8,x, algorithm="maxima")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^8, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac)\sqrt{bx^3 + a}}{x^8}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^8,x, algorithm="fricas")
```

```
[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2 + (b*c + a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^8, x)
```

Sympy [A] time = 16.4573, size = 536, normalized size = 0.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**8,x)
```

```
[Out] a**(3/2)*c*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3)) + a**(3/2)*e*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3, ), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + a**(3/2)*f*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + sqrt(a)*b*c*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + sqrt(a)*b*e*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + sqrt(a)*b*f*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - sqrt(a)*b*g*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) - a**2*d/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)*d/(4*x**(9/2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)*g*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*a*sqrt(b)*g/(3*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**(3/2)*d*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b**(3/2)*d/(12*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*b**(3/2)*g*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) - b**2*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*sqrt(a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^8,x, algorithm="giac")
```



```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^8, x)
```

$$3.470 \quad \int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^9} dx$$

Optimal. Leaf size=705

$$9 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} b^{4/3} \left(\sqrt[3]{a} + \sqrt[3]{bx} \right) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{\left((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx} \right)^2}} \operatorname{EllipticF} \left(\sin^{-1} \left(\frac{(1-\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{(1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}} \right), -7 - 4\sqrt{3} \right) \left(7 \sqrt[3]{b} (bc - 16af) + 20 \right)$$

$$\frac{2240a \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{\left((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx} \right)^2}} \sqrt{a + bx^3}}$$

[Out] $-(b*((63*c)/x^5 + (90*d)/x^4 + (140*e)/x^3 + (252*f)/x^2 + (630*g)/x)*\operatorname{Sqrt}[a + b*x^3])/560 - (27*b^2*c*\operatorname{Sqrt}[a + b*x^3])/(320*a*x^2) - (27*b^2*d*\operatorname{Sqrt}[a + b*x^3])/(112*a*x) + (27*b^(4/3)*(b*d + 14*a*g)*\operatorname{Sqrt}[a + b*x^3])/(112*a*(1 + \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x) - (((105*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5 + (210*g)/x^4)*(a + b*x^3)^(3/2))/840 - (b^2*e*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*x^3]/\operatorname{Sqrt}[a]])/(4*\operatorname{Sqrt}[a]) - (27*3^(1/4)*\operatorname{Sqrt}[2 - \operatorname{Sqrt}[3]]*b^(4/3)*(b*d + 14*a*g)*(a^(1/3) + b^(1/3)*x)*\operatorname{Sqrt}[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x)^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[(1 - \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x]/((1 + \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*\operatorname{Sqrt}[3])/(224*a^(2/3)*\operatorname{Sqrt}[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x)^2]*\operatorname{Sqrt}[a + b*x^3]) - (9*3^(3/4)*\operatorname{Sqrt}[2 + \operatorname{Sqrt}[3]]*b^(4/3)*(7*b^(1/3)*(b*c - 16*a*f) + 20*(1 - \operatorname{Sqrt}[3])*a^(1/3)*(b*d + 14*a*g))*(a^(1/3) + b^(1/3)*x)*\operatorname{Sqrt}[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x)^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[(1 - \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x]/((1 + \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*\operatorname{Sqrt}[3])/(2240*a*\operatorname{Sqrt}[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x)^2]*\operatorname{Sqrt}[a + b*x^3])$

Rubi [A] time = 1.00792, antiderivative size = 705, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {14, 1825, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$9 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} b^{4/3} \left(\sqrt[3]{a} + \sqrt[3]{bx} \right) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{\left((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx} \right)^2}} F \left(\sin^{-1} \left(\frac{\sqrt[3]{bx} + (1-\sqrt{3}) \sqrt[3]{a}}{\sqrt[3]{bx} + (1+\sqrt{3}) \sqrt[3]{a}} \right) \middle| -7 - 4\sqrt{3} \right) \left(7 \sqrt[3]{b} (bc - 16af) + 20 (1 - \sqrt{3}) \right)$$

$$\frac{2240a \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{\left((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx} \right)^2}} \sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9,x]

[Out]
$$-\frac{b((63c)/x^5 + (90d)/x^4 + (140e)/x^3 + (252f)/x^2 + (630g)/x)\sqrt{a + b x^3}}{560} - \frac{(27b^2c\sqrt{a + b x^3})/(320a x^2) - (27b^2d\sqrt{a + b x^3})/(112a x) + (27b^{4/3}(b d + 14a g)\sqrt{a + b x^3})/(112a((1 + \sqrt{3})a^{1/3} + b^{1/3}x)) - (((105c)/x^8 + (120d)/x^7 + (140e)/x^6 + (168f)/x^5 + (210g)/x^4)(a + b x^3)^{3/2}}{840} - \frac{b^2e \operatorname{ArcTanh}[\sqrt{a + b x^3}/\sqrt{a}]}{(4\sqrt{a})} - \frac{(27 \cdot 3^{1/4} \sqrt{2 - \sqrt{3}} b^{4/3})(b d + 14a g)(a^{1/3} + b^{1/3}x)\sqrt{(a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2)}}{(1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2} \operatorname{EllipticE}[\operatorname{ArcSin}[\frac{(1 - \sqrt{3})a^{1/3} + b^{1/3}x}{(1 + \sqrt{3})a^{1/3} + b^{1/3}x}], -7 - 4\sqrt{3}]]/(224a^{2/3}\sqrt{(a^{1/3}(a^{1/3} + b^{1/3}x))})/((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2 \sqrt{a + b x^3}) - \frac{(9 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} b^{4/3})(7b^{1/3}(b c - 16a f) + 20(1 - \sqrt{3})a^{1/3}(b d + 14a g))(a^{1/3} + b^{1/3}x)\sqrt{(a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2)}}{(1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2} \operatorname{EllipticF}[\operatorname{ArcSin}[\frac{(1 - \sqrt{3})a^{1/3} + b^{1/3}x}{(1 + \sqrt{3})a^{1/3} + b^{1/3}x}], -7 - 4\sqrt{3}]]/(2240a \sqrt{(a^{1/3}(a^{1/3} + b^{1/3}x))})/((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2 \sqrt{a + b x^3})$$

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 1825

Int[(Pq_)*(x_))^(m_)*((a_) + (b_)*(x_))^(n_))^(p_), x_Symbol] := Module[{u = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1, 0]

Rule 1835

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_))^(n_))^(p_), x_Symbol] := With[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] := Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
```


Mathematica [C] time = 0.460713, size = 202, normalized size = 0.29

$$\frac{\sqrt{a+bx^3} \left(2x \left(7x \left(5 \left(3a^2 g x^2 {}_2F_1 \left(-\frac{3}{2}, -\frac{4}{3}; -\frac{1}{3}; -\frac{bx^3}{a} \right) + 3b^2 e x^6 \tanh^{-1} \left(\sqrt{\frac{bx^3}{a} + 1} \right) + a e (2a + 5bx^3) \sqrt{\frac{bx^3}{a} + 1} + 12a^2 f x \right) \right) \right)}{840ax^8 \sqrt{\frac{bx^3}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9,x]

[Out] -(Sqrt[a + b*x^3]*(105*a^2*c*Hypergeometric2F1[-8/3, -3/2, -5/3, -((b*x^3)/a)] + 2*x*(60*a^2*d*Hypergeometric2F1[-7/3, -3/2, -4/3, -((b*x^3)/a)] + 7*x*(12*a^2*f*x*Hypergeometric2F1[-5/3, -3/2, -2/3, -((b*x^3)/a)] + 5*(a*e*(2*a + 5*b*x^3)*Sqrt[1 + (b*x^3)/a] + 3*b^2*e*x^6*ArcTanh[Sqrt[1 + (b*x^3)/a]] + 3*a^2*g*x^2*Hypergeometric2F1[-3/2, -4/3, -1/3, -((b*x^3)/a)])))/840*a*x^8*Sqrt[1 + (b*x^3)/a])

Maple [B] time = 0.01, size = 1663, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^9,x)

[Out] f*(-1/5*a*(b*x^3+a)^(1/2)/x^5-13/20*b*(b*x^3+a)^(1/2)/x^2-9/20*I*b^3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I^3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I^3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3))+1/2*I^3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I^3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I^3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I^3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+g*(-1/4*a*(b*x^3+a)^(1/2)/x^4-11/8*b*(b*x^3+a)^(1/2)/x-9/8*I*b^3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I^3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I^3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3))+1/2*I^3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I^3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I^3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I^3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I^3^(1/2)/b*(-b^2*a)^(1/3))

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac)\sqrt{bx^3 + a}}{x^9}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^9,x, algorithm="fricas")

[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2 + (b*c + a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^9, x)

Sympy [A] time = 15.5129, size = 527, normalized size = 0.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**9,x)

[Out] a**(3/2)*c*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**8*gamma(-5/3)) + a**(3/2)*d*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3)) + a**(3/2)*f*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + a**(3/2)*g*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + sqrt(a)*b*c*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*b*d*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + sqrt(a)*b*f*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + sqrt(a)*b*g*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - a**2*e/(6*sqrt(b)*x*(15/2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)*e/(4*x**(9/2)*sqrt(a/(b*x**3) + 1)) - b**(3/2)*e*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b**(3/2)*e/(12*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**2*e*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*sqrt(a))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^9,x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^9, x)
```

$$3.471 \quad \int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{10}} dx$$

Optimal. Leaf size=714

$$9 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} b^{5/3} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (20(1-\sqrt{3}) \sqrt[3]{ab^{2/3}} e - 112ag + 7bd) \operatorname{EllipticF} \left(\sin^{-1} \left(\frac{(1-\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{(1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}} \right) \right)$$

$$2240a \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

[Out] $-(b*((140*c)/x^6 + (189*d)/x^5 + (270*e)/x^4 + (420*f)/x^3 + (756*g)/x^2)*\operatorname{Sqrt}[a + b*x^3])/1680 - (b^2*c*\operatorname{Sqrt}[a + b*x^3])/(24*a*x^3) - (27*b^2*d*\operatorname{Sqrt}[a + b*x^3])/(320*a*x^2) - (27*b^2*e*\operatorname{Sqrt}[a + b*x^3])/(112*a*x) + (27*b^{7/3})*e*\operatorname{Sqrt}[a + b*x^3]/(112*a*((1 + \operatorname{Sqrt}[3])*a^{1/3} + b^{1/3}*x)) - (((280*c)/x^9 + (315*d)/x^8 + (360*e)/x^7 + (420*f)/x^6 + (504*g)/x^5)*(a + b*x^3)^{(3/2)}/2520 + (b^2*(b*c - 6*a*f)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*x^3]/\operatorname{Sqrt}[a]])/(24*a^{(3/2)}) - (27*3^{1/4}*\operatorname{Sqrt}[2 - \operatorname{Sqrt}[3]]*b^{7/3}*e*(a^{1/3} + b^{1/3}*x)*\operatorname{Sqrt}[(a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2)/((1 + \operatorname{Sqrt}[3])*a^{1/3} + b^{1/3}*x)]^2)*\operatorname{EllipticE}[\operatorname{ArcSin}[(1 - \operatorname{Sqrt}[3])*a^{1/3} + b^{1/3}*x)/((1 + \operatorname{Sqrt}[3])*a^{1/3} + b^{1/3}*x)], -7 - 4*\operatorname{Sqrt}[3])/((224*a^{2/3}*\operatorname{Sqrt}[(a^{1/3}*(a^{1/3} + b^{1/3}*x))/((1 + \operatorname{Sqrt}[3])*a^{1/3} + b^{1/3}*x)]^2)*\operatorname{Sqrt}[a + b*x^3]) - (9*3^{3/4}*\operatorname{Sqrt}[2 + \operatorname{Sqrt}[3]]*b^{5/3}*(7*b*d + 20*(1 - \operatorname{Sqrt}[3])*a^{1/3}*b^{2/3})*e - 112*a*g)*(a^{1/3} + b^{1/3}*x)*\operatorname{Sqrt}[(a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2)/((1 + \operatorname{Sqrt}[3])*a^{1/3} + b^{1/3}*x)]^2)*\operatorname{EllipticF}[\operatorname{ArcSin}[(1 - \operatorname{Sqrt}[3])*a^{1/3} + b^{1/3}*x)/((1 + \operatorname{Sqrt}[3])*a^{1/3} + b^{1/3}*x)], -7 - 4*\operatorname{Sqrt}[3])/((2240*a*\operatorname{Sqrt}[(a^{1/3}*(a^{1/3} + b^{1/3}*x))/((1 + \operatorname{Sqrt}[3])*a^{1/3} + b^{1/3}*x)]^2)*\operatorname{Sqrt}[a + b*x^3])$

Rubi [A] time = 1.12299, antiderivative size = 714, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {14, 1825, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$\frac{b^2(bc - 6af) \tanh^{-1} \left(\frac{\sqrt{a+bx^3}}{\sqrt{a}} \right)}{24a^{3/2}} - \frac{9 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} b^{5/3} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (20(1-\sqrt{3}) \sqrt[3]{ab^{2/3}} e - 112ag + 7bd)}{2240a \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^10,x]

[Out] $-(b*((140*c)/x^6 + (189*d)/x^5 + (270*e)/x^4 + (420*f)/x^3 + (756*g)/x^2))*Sqrt[a + b*x^3]/1680 - (b^2*c*Sqrt[a + b*x^3])/(24*a*x^3) - (27*b^2*d*Sqrt[a + b*x^3])/(320*a*x^2) - (27*b^2*e*Sqrt[a + b*x^3])/(112*a*x) + (27*b^(7/3)*e*Sqrt[a + b*x^3])/(112*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (((280*c)/x^9 + (315*d)/x^8 + (360*e)/x^7 + (420*f)/x^6 + (504*g)/x^5)*(a + b*x^3)^(3/2)/2520 + (b^2*(b*c - 6*a*f)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(24*a^(3/2)) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(7/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(224*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(5/3)*(7*b*d + 20*(1 - Sqrt[3])*a^(1/3)*b^(2/3))*e - 112*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]/(2240*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])$

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 1825

Int[(Pq_)*(x_))^(m_)*((a_) + (b_)*(x_))^(n_))^(p_), x_Symbol] := Module[{u = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1, 0]

Rule 1835

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_))^(n_))^(p_), x_Symbol] := With[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] := Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
```


Mathematica [C] time = 0.644999, size = 226, normalized size = 0.32

$$\frac{\sqrt{a+bx^3} \left(2x \left(7x \left(5a^3 f \left(3b^2 x^6 \tanh^{-1} \left(\sqrt{\frac{bx^3}{a} + 1} \right) + a \left(2a + 5bx^3 \right) \sqrt{\frac{bx^3}{a} + 1} \right) + 12a^5 g x {}_2F_1 \left(-\frac{5}{3}, -\frac{3}{2}; -\frac{2}{3}; -\frac{bx^3}{a} \right) - 8b^3 c x^6 \right) \right)}{840a^4 x^8 \sqrt{\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^10,x]

[Out] -(Sqrt[a + b*x^3]*(105*a^5*d*Hypergeometric2F1[-8/3, -3/2, -5/3, -((b*x^3)/a)] + 2*x*(60*a^5*e*Hypergeometric2F1[-7/3, -3/2, -4/3, -((b*x^3)/a)] + 7*x*(5*a^3*f*(a*(2*a + 5*b*x^3)*Sqrt[1 + (b*x^3)/a] + 3*b^2*x^6*ArcTanh[Sqrt[1 + (b*x^3)/a]]) + 12*a^5*g*x*Hypergeometric2F1[-5/3, -3/2, -2/3, -((b*x^3)/a)] - 8*b^3*c*x^6*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[5/2, 4, 7/2, 1 + (b*x^3)/a])))/(840*a^4*x^8*Sqrt[1 + (b*x^3)/a])

Maple [B] time = 0.025, size = 1273, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^10,x)

[Out] g*(-1/5*a*(b*x^3+a)^(1/2)/x^5-13/20*b*(b*x^3+a)^(1/2)/x^2-9/20*I*b^3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I^3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I^3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3))+1/2*I^3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I^3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I^3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I^3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+d*(-1/8*a*(b*x^3+a)^(1/2)/x^8-19/80*b*(b*x^3+a)^(1/2)/x^5-27/320/a*b^2*(b*x^3+a)^(1/2)/x^2+9/320*I/a*b^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I^3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I^3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3))+1/2*I^3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I^3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I^3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I^3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+f*

$$\begin{aligned} & (-1/6*a*(b*x^3+a)^{(1/2)}/x^6-5/12*b*(b*x^3+a)^{(1/2)}/x^3-1/4*b^2*\operatorname{arctanh}((b*x^3+a)^{(1/2)}/a^{(1/2)})/a^{(1/2)})+e*(-1/7*a*(b*x^3+a)^{(1/2)}/x^7-17/56*b*(b*x^3+a)^{(1/2)}/x^4-27/112/a*b^2*(b*x^3+a)^{(1/2)}/x-9/112*I/a*b^2*3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b}/(-b^2*a)^{(1/3)})^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)})/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b}/(-b^2*a)^{(1/3)})^{(1/2)}/(b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*EllipticE(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b}/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)})+1/b*(-b^2*a)^{(1/3)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)}-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)})*3^{(1/2)*b}/(-b^2*a)^{(1/3)})^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}/(-3/2/b*(-b^2*a)^{(1/3)}+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3)}))^{(1/2)}))^{(1/2)})))+c*(-1/9*a*(b*x^3+a)^{(1/2)}/x^9-7/36*b*(b*x^3+a)^{(1/2)}/x^6-1/24/a*b^2*(b*x^3+a)^{(1/2)}/x^3+1/24/a^{(3/2)}*b^3*\operatorname{arctanh}((b*x^3+a)^{(1/2)}/a^{(1/2)})) \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^10,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac)\sqrt{bx^3 + a}}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^10,x, algorithm="fricas")

[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2 + (b*c + a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^10, x)

Sympy [A] time = 23.7195, size = 573, normalized size = 0.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**10,x)

[Out] a**(3/2)*d*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**8*gamma(-5/3)) + a**(3/2)*e*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3)) + a**(3/2)*g*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*b*d*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*b*e*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + sqrt(a)*b*g*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) - a**2*c/(9*sqrt(b)*x**(21/2)*sqrt(a/(b*x**3) + 1)) - a**2*f/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - 11*a*sqrt(b)*c/(36*x**(15/2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)*f/(4*x**(9/2)*sqrt(a/(b*x**3) + 1)) - 17*b**(3/2)*c/(72*x**(9/2)*sqrt(a/(b*x**3) + 1)) - b**(3/2)*f*sqrt(a/(b*x**3) + 1)/(3*x*(3/2)) - b**(3/2)*f/(12*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**(5/2)*c/(24*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**2*f*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*sqrt(a)) + b**3*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(24*a**(3/2))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^10,x, algorithm="giac")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^10, x)

$$3.472 \quad \int \frac{(a+bx^3)^{3/2} (c+dx+ex^2+fx^3+gx^4)}{x^{11}} dx$$

Optimal. Leaf size=764

$$9 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} b^{7/3} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (7a^{2/3} \sqrt[3]{be} - 5(1 - \sqrt{3})(bc - 4af)) \operatorname{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}{(1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx}}\right)\right)$$

$$2240a^{5/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

[Out] $-(b*((108*c)/x^7 + (140*d)/x^6 + (189*e)/x^5 + (270*f)/x^4 + (420*g)/x^3))*\operatorname{Sqrt}[a + b*x^3]/1680 - (27*b^2*c*\operatorname{Sqrt}[a + b*x^3])/((1120*a*x^4) - (b^2*d*\operatorname{Sqrt}[a + b*x^3]))/(24*a*x^3) - (27*b^2*e*\operatorname{Sqrt}[a + b*x^3])/((320*a*x^2) + (27*b^2*(b*c - 4*a*f)*\operatorname{Sqrt}[a + b*x^3]))/(448*a^2*x) - (27*b^(7/3)*(b*c - 4*a*f)*\operatorname{Sqrt}[a + b*x^3])/((448*a^2*((1 + \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x)) - ((252*c)/x^10 + (280*d)/x^9 + (315*e)/x^8 + (360*f)/x^7 + (420*g)/x^6)*(a + b*x^3)^(3/2))/2520 + (b^2*(b*d - 6*a*g)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*x^3]/\operatorname{Sqrt}[a]])/(24*a^(3/2)) + (27*3^(1/4)*\operatorname{Sqrt}[2 - \operatorname{Sqrt}[3]]*b^(7/3)*(b*c - 4*a*f)*(a^(1/3) + b^(1/3)*x)*\operatorname{Sqrt}[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x)^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[(1 - \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x)/((1 + \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*\operatorname{Sqrt}[3]])/(896*a^(5/3)*\operatorname{Sqrt}[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x)^2]*\operatorname{Sqrt}[a + b*x^3]) - (9*3^(3/4)*\operatorname{Sqrt}[2 + \operatorname{Sqrt}[3]]*b^(7/3)*(7*a^(2/3)*b^(1/3)*e - 5*(1 - \operatorname{Sqrt}[3])*(b*c - 4*a*f))*(a^(1/3) + b^(1/3)*x)*\operatorname{Sqrt}[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x)^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[(1 - \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x)/((1 + \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*\operatorname{Sqrt}[3]])/(2240*a^(5/3)*\operatorname{Sqrt}[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + \operatorname{Sqrt}[3])*a^(1/3) + b^(1/3)*x)^2]*\operatorname{Sqrt}[a + b*x^3])$

Rubi [A] time = 1.33014, antiderivative size = 764, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {14, 1825, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$9 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} b^{7/3} (\sqrt[3]{a} + \sqrt[3]{bx}) \sqrt{\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3} x^2}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} (7a^{2/3} \sqrt[3]{be} - 5(1 - \sqrt{3})(bc - 4af)) F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx} + (1-\sqrt{3}) \sqrt[3]{a}}{\sqrt[3]{bx} + (1+\sqrt{3}) \sqrt[3]{a}}\right)\right) - 7 -$$

$$2240a^{5/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} + \sqrt[3]{bx})}{((1+\sqrt{3}) \sqrt[3]{a} + \sqrt[3]{bx})^2}} \sqrt{a + bx^3}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^11,x]

[Out] $-(b*((108*c)/x^7 + (140*d)/x^6 + (189*e)/x^5 + (270*f)/x^4 + (420*g)/x^3)*\text{Sqrt}[a + b*x^3])/1680 - (27*b^2*c*\text{Sqrt}[a + b*x^3])/(1120*a*x^4) - (b^2*d*\text{Sqrt}[a + b*x^3])/(24*a*x^3) - (27*b^2*e*\text{Sqrt}[a + b*x^3])/(320*a*x^2) + (27*b^2*(b*c - 4*a*f)*\text{Sqrt}[a + b*x^3])/(448*a^2*x) - (27*b^{7/3}*(b*c - 4*a*f)*\text{Sqrt}[a + b*x^3])/(448*a^2*((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)) - (((252*c)/x^{10} + (280*d)/x^9 + (315*e)/x^8 + (360*f)/x^7 + (420*g)/x^6)*(a + b*x^3)^{3/2})/2520 + (b^2*(b*d - 6*a*g)*\text{ArcTanh}[\text{Sqrt}[a + b*x^3]/\text{Sqrt}[a]])/(24*a^{3/2}) + (27*3^{1/4}*\text{Sqrt}[2 - \text{Sqrt}[3]]*b^{7/3}*(b*c - 4*a*f)*(a^{1/3} + b^{1/3}*x)*\text{Sqrt}[(a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2)/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\text{EllipticE}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x]/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)], -7 - 4*\text{Sqrt}[3]))/(896*a^{5/3}*\text{Sqrt}[(a^{1/3}*(a^{1/3} + b^{1/3}*x))/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\text{Sqrt}[a + b*x^3]) - (9*3^{3/4}*\text{Sqrt}[2 + \text{Sqrt}[3]]*b^{7/3}*(7*a^{2/3}*b^{1/3}*e - 5*(1 - \text{Sqrt}[3])*(b*c - 4*a*f))*(a^{1/3} + b^{1/3}*x)*\text{Sqrt}[(a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2)/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\text{EllipticF}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x]/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)], -7 - 4*\text{Sqrt}[3]))/(2240*a^{5/3}*\text{Sqrt}[(a^{1/3}*(a^{1/3} + b^{1/3}*x))/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\text{Sqrt}[a + b*x^3])$

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 1825

Int[(Pq_)*(x_)^{(m_.)*((a_) + (b_.)*(x_)^{(n_.)})^{(p_)}, x_Symbol] := Module[{u = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^{(m + n)}*(a + b*x^n)^{(p - 1)}*ExpandToSum[u/x^{(m + 1)}, x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1, 0]

Rule 1835

Int[(Pq_)*((c_.)*(x_))^{(m_.)*((a_) + (b_.)*(x_)^{(n_.)})^{(p_)}, x_Symbol] := With[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^{(m + 1)}*(a + b*x^n)^{(p + 1)})/(a*c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^{(m + 1)}*ExpandToSum[(2*a*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^{(n - 1)}, x]*(a + b*x^n)^p, x], x] /; NeQ[Pq0, 0] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]

Rule 1832

Int[(Pq_)/((x_)*Sqrt[(a_) + (b_)*(x_)^(n_)]), x_Symbol] := Dist[Coeff[Pq, x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq, x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]

Rule 266

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 63

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1878

Int[((c_) + (d_)*(x_))/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

Rule 218

Int[1/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b}, x] && PosQ[a]

Rule 1877

```

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numerator[Simplify[((1 - Sqrt[3])*d)/c]], s = Denominator[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^3)^{3/2} (c+dx+ex^2+fx^3+gx^4)}{x^{11}} dx &= \frac{\left(\frac{252c}{x^{10}} + \frac{280d}{x^9} + \frac{315e}{x^8} + \frac{360f}{x^7} + \frac{420g}{x^6}\right) (a+bx^3)^{3/2}}{2520} - \frac{1}{2}(9b) \int \frac{\sqrt{a+bx^3}}{x^{11}} dx \\
&= -\frac{b\left(\frac{108c}{x^7} + \frac{140d}{x^6} + \frac{189e}{x^5} + \frac{270f}{x^4} + \frac{420g}{x^3}\right) \sqrt{a+bx^3}}{1680} - \frac{\left(\frac{252c}{x^{10}} + \frac{280d}{x^9} + \frac{315e}{x^8} + \frac{360f}{x^7} + \frac{420g}{x^6}\right) (a+bx^3)^{3/2}}{2520} \\
&= -\frac{b\left(\frac{108c}{x^7} + \frac{140d}{x^6} + \frac{189e}{x^5} + \frac{270f}{x^4} + \frac{420g}{x^3}\right) \sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a+bx^3}}{1120ax^4} - \frac{\left(\frac{252c}{x^{10}} + \frac{280d}{x^9} + \frac{315e}{x^8} + \frac{360f}{x^7} + \frac{420g}{x^6}\right) (a+bx^3)^{3/2}}{2520} \\
&= -\frac{b\left(\frac{108c}{x^7} + \frac{140d}{x^6} + \frac{189e}{x^5} + \frac{270f}{x^4} + \frac{420g}{x^3}\right) \sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a+bx^3}}{1120ax^4} - \frac{b^2\left(\frac{108c}{x^7} + \frac{140d}{x^6} + \frac{189e}{x^5} + \frac{270f}{x^4} + \frac{420g}{x^3}\right) \sqrt{a+bx^3}}{1680} \\
&= -\frac{b\left(\frac{108c}{x^7} + \frac{140d}{x^6} + \frac{189e}{x^5} + \frac{270f}{x^4} + \frac{420g}{x^3}\right) \sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a+bx^3}}{1120ax^4} - \frac{b^2\left(\frac{108c}{x^7} + \frac{140d}{x^6} + \frac{189e}{x^5} + \frac{270f}{x^4} + \frac{420g}{x^3}\right) \sqrt{a+bx^3}}{1680} \\
&= -\frac{b\left(\frac{108c}{x^7} + \frac{140d}{x^6} + \frac{189e}{x^5} + \frac{270f}{x^4} + \frac{420g}{x^3}\right) \sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a+bx^3}}{1120ax^4} - \frac{b^2\left(\frac{108c}{x^7} + \frac{140d}{x^6} + \frac{189e}{x^5} + \frac{270f}{x^4} + \frac{420g}{x^3}\right) \sqrt{a+bx^3}}{1680} \\
&= -\frac{b\left(\frac{108c}{x^7} + \frac{140d}{x^6} + \frac{189e}{x^5} + \frac{270f}{x^4} + \frac{420g}{x^3}\right) \sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a+bx^3}}{1120ax^4} - \frac{b^2\left(\frac{108c}{x^7} + \frac{140d}{x^6} + \frac{189e}{x^5} + \frac{270f}{x^4} + \frac{420g}{x^3}\right) \sqrt{a+bx^3}}{1680} \\
&= -\frac{b\left(\frac{108c}{x^7} + \frac{140d}{x^6} + \frac{189e}{x^5} + \frac{270f}{x^4} + \frac{420g}{x^3}\right) \sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a+bx^3}}{1120ax^4} - \frac{b^2\left(\frac{108c}{x^7} + \frac{140d}{x^6} + \frac{189e}{x^5} + \frac{270f}{x^4} + \frac{420g}{x^3}\right) \sqrt{a+bx^3}}{1680} \\
&= -\frac{b\left(\frac{108c}{x^7} + \frac{140d}{x^6} + \frac{189e}{x^5} + \frac{270f}{x^4} + \frac{420g}{x^3}\right) \sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a+bx^3}}{1120ax^4} - \frac{b^2\left(\frac{108c}{x^7} + \frac{140d}{x^6} + \frac{189e}{x^5} + \frac{270f}{x^4} + \frac{420g}{x^3}\right) \sqrt{a+bx^3}}{1680} \\
&= -\frac{b\left(\frac{108c}{x^7} + \frac{140d}{x^6} + \frac{189e}{x^5} + \frac{270f}{x^4} + \frac{420g}{x^3}\right) \sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a+bx^3}}{1120ax^4} - \frac{b^2\left(\frac{108c}{x^7} + \frac{140d}{x^6} + \frac{189e}{x^5} + \frac{270f}{x^4} + \frac{420g}{x^3}\right) \sqrt{a+bx^3}}{1680} \\
&= -\frac{b\left(\frac{108c}{x^7} + \frac{140d}{x^6} + \frac{189e}{x^5} + \frac{270f}{x^4} + \frac{420g}{x^3}\right) \sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a+bx^3}}{1120ax^4} - \frac{b^2\left(\frac{108c}{x^7} + \frac{140d}{x^6} + \frac{189e}{x^5} + \frac{270f}{x^4} + \frac{420g}{x^3}\right) \sqrt{a+bx^3}}{1680}
\end{aligned}$$

Mathematica [C] time = 0.506918, size = 227, normalized size = 0.3

$$\sqrt{a+bx^3} \left(2x^3 \left(35a^3gx \left(3b^2x^6 \tanh^{-1} \left(\sqrt{\frac{bx^3}{a} + 1} \right) + a \left(2a + 5bx^3 \right) \sqrt{\frac{bx^3}{a} + 1} \right) + 60a^5f {}_2F_1 \left(-\frac{7}{3}, -\frac{3}{2}; -\frac{4}{3}; -\frac{bx^3}{a} \right) - 56b^3d \right) \right)$$

$$840a^4x^{10} \sqrt{\frac{bx^3}{a} + 1}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^11,x]

[Out] $-(\sqrt{a + b x^3} (84 a^5 c \operatorname{Hypergeometric2F1}[-10/3, -3/2, -7/3, -((b x^3)/a)] + 105 a^5 e x^2 \operatorname{Hypergeometric2F1}[-8/3, -3/2, -5/3, -((b x^3)/a)] + 2 x^3 (35 a^3 g x (a (2 a + 5 b x^3) \sqrt{1 + (b x^3)/a} + 3 b^2 x^6 \operatorname{ArcTanh}[\sqrt{1 + (b x^3)/a}]) + 60 a^5 f \operatorname{Hypergeometric2F1}[-7/3, -3/2, -4/3, -((b x^3)/a)] - 56 b^3 d x^7 (a + b x^3)^2 \sqrt{1 + (b x^3)/a} \operatorname{Hypergeometric2F1}[5/2, 4, 7/2, 1 + (b x^3)/a])) / (840 a^4 x^{10} \sqrt{1 + (b x^3)/a})$

Maple [B] time = 0.008, size = 1470, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^11,x)

[Out] $e \left(-\frac{1}{8} a (b x^3 + a)^{1/2} / x^8 - \frac{19}{80} b (b x^3 + a)^{1/2} / x^5 - \frac{27}{320} a b^2 (b x^3 + a)^{1/2} / x^2 + \frac{9}{320} I / a b^2 3^{1/2} (-b^2 a)^{1/3} (I (x + 1/2 / b (-b^2 a)^{1/3}) - 1/2 I^3^{1/2} / b (-b^2 a)^{1/3})^3^{1/2} b / (-b^2 a)^{1/3} \right)^{1/2} \left((x - 1 / b (-b^2 a)^{1/3}) / (-3/2 / b (-b^2 a)^{1/3} + 1/2 I^3^{1/2} / b (-b^2 a)^{1/3}) \right)^{1/2} \left(-I (x + 1/2 / b (-b^2 a)^{1/3}) + 1/2 I^3^{1/2} / b (-b^2 a)^{1/3} \right)^3^{1/2} b / (-b^2 a)^{1/3} \right)^{1/2} / (b x^3 + a)^{1/2} \operatorname{EllipticF}(1/3 3^{1/2} (I (x + 1/2 / b (-b^2 a)^{1/3}) - 1/2 I^3^{1/2} / b (-b^2 a)^{1/3})^3^{1/2} b / (-b^2 a)^{1/3})^{1/2}, (I^3^{1/2} / b (-b^2 a)^{1/3} / (-3/2 / b (-b^2 a)^{1/3} + 1/2 I^3^{1/2} / b (-b^2 a)^{1/3}))^{1/2} \left. \right) + g \left(-\frac{1}{6} a (b x^3 + a)^{1/2} / x^6 - \frac{5}{12} b (b x^3 + a)^{1/2} / x^3 - \frac{1}{4} b^2 \operatorname{arctanh}((b x^3 + a)^{1/2} / a^{1/2}) / a^{1/2} + f \left(-\frac{1}{7} a (b x^3 + a)^{1/2} / x^7 - \frac{17}{56} b (b x^3 + a)^{1/2} / x^4 - \frac{27}{112} a b^2 (b x^3 + a)^{1/2} / x - \frac{9}{112} I / a b^2 3^{1/2} (-b^2 a)^{1/3} (I (x + 1/2 / b (-b^2 a)^{1/3}) - 1/2 I^3^{1/2} / b (-b^2 a)^{1/3})^3^{1/2} b / (-b^2 a)^{1/3} \right)^{1/2} \left((x - 1 / b (-b^2 a)^{1/3}) / (-3/2 / b (-b^2 a)^{1/3} + 1/2 I^3^{1/2} / b (-b^2 a)^{1/3}) \right)^{1/2} \left(-I (x + 1/2 / b (-b^2 a)^{1/3}) + 1/2 I^3^{1/2} / b (-b^2 a)^{1/3} \right)^3^{1/2} b / (-b^2 a)^{1/3} \right)^{1/2} / (b x^3 + a)^{1/2} \left((-3/2 / b (-b^2 a)^{1/3} + 1/2 I^3^{1/2} / b (-b^2 a)^{1/3}) \right)^{1/2} \left. \right) + 1 / b (-b^2 a)^{1/3} \operatorname{EllipticE}(1/3 3^{1/2} (I (x + 1/2 / b (-b^2 a)^{1/3}) - 1/2 I^3^{1/2} / b (-b^2 a)^{1/3})^3^{1/2} b / (-b^2 a)^{1/3})^{1/2}, (I^3^{1/2} / b (-b^2 a)^{1/3} / (-3/2 / b (-b^2 a)^{1/3} + 1/2 I^3^{1/2} / b (-b^2 a)^{1/3}))^{1/2} \left. \right) + 1 / b (-b^2 a)^{1/3} \operatorname{EllipticF}(1/3 3^{1/2} (I (x + 1/2 / b (-b^2 a)^{1/3}) - 1/2 I^3^{1/2} / b (-b^2 a)^{1/3})^3^{1/2} b / (-b^2 a)^{1/3})^{1/2}, (I^3^{1/2} / b (-b^2 a)^{1/3} / (-3/2 / b (-b^2 a)^{1/3} + 1/2 I^3^{1/2} / b (-b^2 a)^{1/3}))^{1/2} \left. \right) \left. \right) + d \left(-\frac{1}{9} a (b x^3 + a)^{1/2} / x^9 - \frac{7}{36} b (b x^3 + a)^{1/2} / x^6 - \frac{1}{24} a b^2 (b x^3 + a)^{1/2} / x^3 + \frac{1}{24} a^{3/2} b^3 \right.$

```
*arctanh((b*x^3+a)^(1/2)/a^(1/2))+c*(-1/10*a*(b*x^3+a)^(1/2)/x^10-23/140*b
*(b*x^3+a)^(1/2)/x^7-27/1120/a*b^2*(b*x^3+a)^(1/2)/x^4+27/448/a^2*b^3*(b*x^
3+a)^(1/2)/x+9/448*I/a^2*b^3*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1
/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b
*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1
/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(
-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*
a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b
*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1
/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2), (I*3^(1/2)/b*(-b^2*a)
^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^{11}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^11,x, algorithm="maxi
ma")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^11, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac)\sqrt{bx^3 + a}}{x^{11}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^11,x, algorithm="fric
as")

[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2 + (b*c +
a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^11, x)

Sympy [A] time = 24.3077, size = 576, normalized size = 0.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**11,x)

[Out] a**(3/2)*c*gamma(-10/3)*hyper((-10/3, -1/2), (-7/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**10*gamma(-7/3)) + a**(3/2)*e*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**8*gamma(-5/3)) + a**(3/2)*f*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3)) + sqrt(a)*b*c*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3)) + sqrt(a)*b*e*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*b*f*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) - a**2*d/(9*sqrt(b)*x**(21/2)*sqrt(a/(b*x**3) + 1)) - a**2*g/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - 11*a*sqrt(b)*d/(36*x**(15/2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)*g/(4*x**(9/2)*sqrt(a/(b*x**3) + 1)) - 17*b**(3/2)*d/(72*x**(9/2)*sqrt(a/(b*x**3) + 1)) - b**(3/2)*g*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b**(3/2)*g/(12*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**(5/2)*d/(24*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**2*g*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*sqrt(a)) + b**3*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(24*a**(3/2))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^{11}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^11,x, algorithm="giac")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^11, x)

$$3.473 \quad \int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{12}} dx$$

Optimal. Leaf size=796

$$\frac{e \tanh^{-1}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right) b^3}{24a^{3/2}} + \frac{27^4 \sqrt[3]{3} \sqrt{2-\sqrt{3}}(bd-4ag) (\sqrt[3]{bx} + \sqrt[3]{a}) \sqrt{\frac{b^{2/3}x^2 - \sqrt[3]{a} \sqrt[3]{bx+a^{2/3}}}{(\sqrt[3]{bx+(1+\sqrt{3})\sqrt[3]{a}})^2}} E\left(\sin^{-1}\left(\frac{\sqrt[3]{bx+(1-\sqrt{3})\sqrt[3]{a}}}{\sqrt[3]{bx+(1+\sqrt{3})\sqrt[3]{a}}}\right) \middle| -7-4\sqrt{3}\right)}{896a^{5/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{bx} + \sqrt[3]{a})}{(\sqrt[3]{bx+(1+\sqrt{3})\sqrt[3]{a}})^2} \sqrt{bx^3+a}}}$$

[Out] $-(b*((945*c)/x^8 + (1188*d)/x^7 + (1540*e)/x^6 + (2079*f)/x^5 + (2970*g)/x^4)*\text{Sqrt}[a + b*x^3])/18480 - (27*b^2*c*\text{Sqrt}[a + b*x^3])/(1760*a*x^5) - (27*b^2*d*\text{Sqrt}[a + b*x^3])/(1120*a*x^4) - (b^2*e*\text{Sqrt}[a + b*x^3])/(24*a*x^3) + (27*b^2*(7*b*c - 22*a*f)*\text{Sqrt}[a + b*x^3])/(7040*a^2*x^2) + (27*b^2*(b*d - 4*a*g)*\text{Sqrt}[a + b*x^3])/(448*a^2*x) - (27*b^(7/3)*(b*d - 4*a*g)*\text{Sqrt}[a + b*x^3])/(448*a^2*((1 + \text{Sqrt}[3])*a^(1/3) + b^(1/3)*x)) - (((2520*c)/x^11 + (2772*d)/x^10 + (3080*e)/x^9 + (3465*f)/x^8 + (3960*g)/x^7)*(a + b*x^3)^(3/2))/27720 + (b^3*e*\text{ArcTanh}[\text{Sqrt}[a + b*x^3]/\text{Sqrt}[a]])/(24*a^(3/2)) + (27*3^(1/4)*\text{Sqrt}[2 - \text{Sqrt}[3]]*b^(7/3)*(b*d - 4*a*g)*(a^(1/3) + b^(1/3)*x)*\text{Sqrt}[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + \text{Sqrt}[3])*a^(1/3) + b^(1/3)*x)^2]*\text{EllipticE}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^(1/3) + b^(1/3)*x)/((1 + \text{Sqrt}[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*\text{Sqrt}[3]])/(896*a^(5/3)*\text{Sqrt}[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + \text{Sqrt}[3])*a^(1/3) + b^(1/3)*x)^2]*\text{Sqrt}[a + b*x^3]) + (9*3^(3/4)*\text{Sqrt}[2 + \text{Sqrt}[3]]*b^(7/3)*(7*b^(1/3)*(7*b*c - 22*a*f) + 110*(1 - \text{Sqrt}[3]))*a^(1/3)*(b*d - 4*a*g)*(a^(1/3) + b^(1/3)*x)*\text{Sqrt}[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + \text{Sqrt}[3])*a^(1/3) + b^(1/3)*x)^2]*\text{EllipticF}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^(1/3) + b^(1/3)*x)/((1 + \text{Sqrt}[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*\text{Sqrt}[3]))/(49280*a^2*\text{Sqrt}[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + \text{Sqrt}[3])*a^(1/3) + b^(1/3)*x)^2]*\text{Sqrt}[a + b*x^3])$

Rubi [A] time = 1.53375, antiderivative size = 796, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {14, 1825, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$\frac{e \tanh^{-1}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right) b^3}{24a^{3/2}} + \frac{27^4 \sqrt[3]{3} \sqrt{2-\sqrt{3}}(bd-4ag) (\sqrt[3]{bx} + \sqrt[3]{a}) \sqrt{\frac{b^{2/3}x^2 - \sqrt[3]{a} \sqrt[3]{bx+a^{2/3}}}{(\sqrt[3]{bx+(1+\sqrt{3})\sqrt[3]{a}})^2}} E\left(\sin^{-1}\left(\frac{\sqrt[3]{bx+(1-\sqrt{3})\sqrt[3]{a}}}{\sqrt[3]{bx+(1+\sqrt{3})\sqrt[3]{a}}}\right) \middle| -7-4\sqrt{3}\right)}{896a^{5/3} \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{bx} + \sqrt[3]{a})}{(\sqrt[3]{bx+(1+\sqrt{3})\sqrt[3]{a}})^2} \sqrt{bx^3+a}}}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^12,x]

[Out]
$$-(b*((945*c)/x^8 + (1188*d)/x^7 + (1540*e)/x^6 + (2079*f)/x^5 + (2970*g)/x^4)*\text{Sqrt}[a + b*x^3])/18480 - (27*b^2*c*\text{Sqrt}[a + b*x^3])/(1760*a*x^5) - (27*b^2*d*\text{Sqrt}[a + b*x^3])/(1120*a*x^4) - (b^2*e*\text{Sqrt}[a + b*x^3])/(24*a*x^3) + (27*b^2*(7*b*c - 22*a*f)*\text{Sqrt}[a + b*x^3])/(7040*a^2*x^2) + (27*b^2*(b*d - 4*a*g)*\text{Sqrt}[a + b*x^3])/(448*a^2*x) - (27*b^{7/3}*(b*d - 4*a*g)*\text{Sqrt}[a + b*x^3])/(448*a^2*((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)) - (((2520*c)/x^{11} + (2772*d)/x^{10} + (3080*e)/x^9 + (3465*f)/x^8 + (3960*g)/x^7)*(a + b*x^3)^{3/2})/27720 + (b^3*e*\text{ArcTanh}[\text{Sqrt}[a + b*x^3]/\text{Sqrt}[a]])/(24*a^{3/2}) + (27*3^{1/4}*\text{Sqrt}[2 - \text{Sqrt}[3])*b^{7/3}*(b*d - 4*a*g)*(a^{1/3} + b^{1/3}*x)*\text{Sqrt}[(a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2)/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\text{EllipticE}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)], -7 - 4*\text{Sqrt}[3]])/(896*a^{5/3}*\text{Sqrt}[(a^{1/3}*(a^{1/3} + b^{1/3}*x))/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\text{Sqrt}[a + b*x^3]) + (9*3^{3/4})*\text{Sqrt}[2 + \text{Sqrt}[3])*b^{7/3}*(7*b^{1/3}*(7*b*c - 22*a*f) + 110*(1 - \text{Sqrt}[3])*a^{1/3}*(b*d - 4*a*g))*(a^{1/3} + b^{1/3}*x)*\text{Sqrt}[(a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2)/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\text{EllipticF}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)], -7 - 4*\text{Sqrt}[3]])/(49280*a^2*\text{Sqrt}[(a^{1/3}*(a^{1/3} + b^{1/3}*x))/((1 + \text{Sqrt}[3])*a^{1/3} + b^{1/3}*x)^2]*\text{Sqrt}[a + b*x^3])$$

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 1825

Int[(Pq_)*(x_)^{(m_.)*((a_) + (b_.)*(x_)^{(n_.)})^{(p_)}, x_Symbol] := Module[{u = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^{(m + n)}*(a + b*x^n)^{(p - 1)}*ExpandToSum[u/x^{(m + 1)}, x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1, 0]

Rule 1835

Int[(Pq_)*((c_.)*(x_))^{(m_.)*((a_) + (b_.)*(x_)^{(n_.)})^{(p_)}, x_Symbol] := With[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^{(m + 1)}*(a + b*x^n)^{(p + 1)})/(a*c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^{(m + 1)}*ExpandToSum[(2*a*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^{(n - 1)}, x]*(a + b*x^n)^p, x], x] /; NeQ[Pq0, 0] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&

IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]

Rule 1832

Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)])], x_Symbol] := Dist[Coeff[Pq, x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq, x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1878

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r, Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

Rule 218

Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b}, x] && PosQ[a]

Rule 1877

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Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numerator[Simplify[((1 - Sqrt[3])*d)/c]], s = Denominator[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)]], -7 - 4*Sqrt[3])]/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

```

Rubi steps

Mathematica [C] time = 0.400484, size = 194, normalized size = 0.24

$$\frac{\sqrt{a + bx^3} \left(11x^3 \left(-105a^5 f {}_2F_1 \left(-\frac{8}{3}, -\frac{3}{2}; -\frac{5}{3}; -\frac{bx^3}{a} \right) - 120a^5 g x {}_2F_1 \left(-\frac{7}{3}, -\frac{3}{2}; -\frac{4}{3}; -\frac{bx^3}{a} \right) + 112b^3 e x^8 (a + bx^3)^2 \sqrt{\frac{bx^3}{a} + 1} {}_2F_1 \left(\frac{5}{2}, 4, 7/2, 1 + \frac{bx^3}{a} \right) \right) \right)}{9240a^4 x^{11} \sqrt{\frac{bx^3}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^12,x]

[Out] (Sqrt[a + b*x^3]*(-840*a^5*c*Hypergeometric2F1[-11/3, -3/2, -8/3, -((b*x^3)/a)] - 924*a^5*d*x*Hypergeometric2F1[-10/3, -3/2, -7/3, -((b*x^3)/a)] + 11*x^3*(-105*a^5*f*Hypergeometric2F1[-8/3, -3/2, -5/3, -((b*x^3)/a)] - 120*a^5*g*x*Hypergeometric2F1[-7/3, -3/2, -4/3, -((b*x^3)/a)] + 112*b^3*e*x^8*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[5/2, 4, 7/2, 1 + (b*x^3)/a]))/(9240*a^4*x^11*Sqrt[1 + (b*x^3)/a])

Maple [B] time = 0.025, size = 1773, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^12,x)

[Out] c*(-1/11*a*(b*x^3+a)^(1/2)/x^11-25/176*b*(b*x^3+a)^(1/2)/x^8-27/1760/a*b^2*(b*x^3+a)^(1/2)/x^5+189/7040/a^2*b^3*(b*x^3+a)^(1/2)/x^2-63/7040*I/a^2*b^3*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))^3^(1/2)*b/(-b^2*a)^(1/3))^^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3))+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^^(1/2), (I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+f*(-1/8*a*(b*x^3+a)^(1/2)/x^8-19/80*b*(b*x^3+a)^(1/2)/x^5-27/320/a*b^2*(b*x^3+a)^(1/2)/x^2+9/320*I/a*b^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3))+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3))-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^^(1/2), (I*3^(1/2)/

$$\begin{aligned}
& b*(-b^2*a)^{(1/3)/(-3/2/b*(-b^2*a)^{(1/3)+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}} \\
& +g*(-1/7*a*(b*x^3+a)^{(1/2)/x^7-17/56*b*(b*x^3+a)^{(1/2)/x^4-27/112/a*b^2*(b*x^3+a)^{(1/2)/x-9/112*I/a*b^2*3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)))/(-3/2/b*(-b^2*a)^{(1/3)+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)/(b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}*EllipticE(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)/(-3/2/b*(-b^2*a)^{(1/3)+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}))^{(1/2)}+1/b*(-b^2*a)^{(1/3)*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)/(-3/2/b*(-b^2*a)^{(1/3)+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}))^{(1/2)})) \\
& +e*(-1/9*a*(b*x^3+a)^{(1/2)/x^9-7/36*b*(b*x^3+a)^{(1/2)/x^6-1/24/a*b^2*(b*x^3+a)^{(1/2)/x^3+1/24/a^{(3/2)}*b^3*arctanh((b*x^3+a)^{(1/2)/a^{(1/2))}}+d*(-1/10*a*(b*x^3+a)^{(1/2)/x^10-23/140*b*(b*x^3+a)^{(1/2)/x^7-27/1120/a*b^2*(b*x^3+a)^{(1/2)/x^4+27/448/a^2*b^3*(b*x^3+a)^{(1/2)/x+9/448*I/a^2*b^3*3^{(1/2)}*(-b^2*a)^{(1/3)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}*((x-1/b*(-b^2*a)^{(1/3)))/(-3/2/b*(-b^2*a)^{(1/3)+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}*(-I*(x+1/2/b*(-b^2*a)^{(1/3)+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)/(b*x^3+a)^{(1/2)}*((-3/2/b*(-b^2*a)^{(1/3)+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}*EllipticE(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)/(-3/2/b*(-b^2*a)^{(1/3)+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}+1/b*(-b^2*a)^{(1/3)*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/b*(-b^2*a)^{(1/3)-1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}*3^{(1/2)*b/(-b^2*a)^{(1/3))}^{(1/2)}, (I*3^{(1/2)}/b*(-b^2*a)^{(1/3)/(-3/2/b*(-b^2*a)^{(1/3)+1/2*I*3^{(1/2)}/b*(-b^2*a)^{(1/3))}^{(1/2)}+1/b*(-b^2*a)^{(1/3))}^{(1/2)}))^{(1/2)})) \\
& b*(-b^2*a)^{(1/3))^{(1/2)}}
\end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^{12}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^12,x, algorithm="maxima")

[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^12, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac)\sqrt{bx^3 + a}}{x^{12}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^12,x, algorithm="fricas")

[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2 + (b*c + a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^12, x)

Sympy [A] time = 22.8427, size = 541, normalized size = 0.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**12,x)

[Out] a**(3/2)*c*gamma(-11/3)*hyper((-11/3, -1/2), (-8/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**11*gamma(-8/3)) + a**(3/2)*d*gamma(-10/3)*hyper((-10/3, -1/2), (-7/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**10*gamma(-7/3)) + a**(3/2)*f*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**8*gamma(-5/3)) + a**(3/2)*g*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3)) + sqrt(a)*b*c*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**8*gamma(-5/3)) + sqrt(a)*b*d*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3)) + sqrt(a)*b*f*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*b*g*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) - a**2*e/(9*sqrt(b)*x**(21/2)*sqrt(a/(b*x**3) + 1)) - 11*a*sqrt(b)*e/(36*x**(15/2)*sqrt(a/(b*x**3) + 1)) - 17*b**(3/2)*e/(72*x**(9/2)*sqrt(a/(b*x**3) + 1)) - b**(5/2)*e/(24*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) + b**3*e*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(24*a**(3/2))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^{12}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^12,x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^12, x)
```

3.474 $\int (c + dx + ex^2)(a + bx^3)^p dx$

Optimal. Leaf size=102

$$\frac{cx(a + bx^3)^{p+1} {}_2F_1\left(1, p + \frac{4}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{a} + \frac{dx^2(a + bx^3)^{p+1} {}_2F_1\left(1, p + \frac{5}{3}; \frac{5}{3}; -\frac{bx^3}{a}\right)}{2a} + \frac{e(a + bx^3)^{p+1}}{3b(p+1)}$$

[Out] (e*(a + b*x^3)^(1 + p))/(3*b*(1 + p)) + (c*x*(a + b*x^3)^(1 + p)*Hypergeometric2F1[1, 4/3 + p, 4/3, -(b*x^3)/a])/a + (d*x^2*(a + b*x^3)^(1 + p)*Hypergeometric2F1[1, 5/3 + p, 5/3, -(b*x^3)/a])/(2*a)

Rubi [A] time = 0.0762832, antiderivative size = 120, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$, Rules used = {1886, 261, 1893, 246, 245, 365, 364}

$$cx(a + bx^3)^p \left(\frac{bx^3}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{3}, -p; \frac{4}{3}; -\frac{bx^3}{a}\right) + \frac{1}{2}dx^2(a + bx^3)^p \left(\frac{bx^3}{a} + 1\right)^{-p} {}_2F_1\left(\frac{2}{3}, -p; \frac{5}{3}; -\frac{bx^3}{a}\right) + \frac{e(a + bx^3)^{p+1}}{3b(p+1)}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2)*(a + b*x^3)^p,x]

[Out] (e*(a + b*x^3)^(1 + p))/(3*b*(1 + p)) + (c*x*(a + b*x^3)^p*Hypergeometric2F1[1/3, -p, 4/3, -(b*x^3)/a])/(1 + (b*x^3)/a)^p + (d*x^2*(a + b*x^3)^p*Hypergeometric2F1[2/3, -p, 5/3, -(b*x^3)/a])/(2*(1 + (b*x^3)/a)^p)

Rule 1886

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[Coeff[Pq, x, n - 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 1893

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 246

```
Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
/; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simp
lify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 365

```
Int[((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)
^m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
&& !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)]]/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int (c + dx + ex^2)(a + bx^3)^p dx &= e \int x^2 (a + bx^3)^p dx + \int (c + dx)(a + bx^3)^p dx \\
&= \frac{e(a + bx^3)^{1+p}}{3b(1+p)} + \int \left(c(a + bx^3)^p + dx(a + bx^3)^p \right) dx \\
&= \frac{e(a + bx^3)^{1+p}}{3b(1+p)} + c \int (a + bx^3)^p dx + d \int x(a + bx^3)^p dx \\
&= \frac{e(a + bx^3)^{1+p}}{3b(1+p)} + \left(c(a + bx^3)^p \left(1 + \frac{bx^3}{a} \right)^{-p} \right) \int \left(1 + \frac{bx^3}{a} \right)^p dx + \left(d(a + bx^3)^p \left(1 + \frac{bx^3}{a} \right)^{-p} \right) \int \left(1 + \frac{bx^3}{a} \right)^p dx \\
&= \frac{e(a + bx^3)^{1+p}}{3b(1+p)} + cx(a + bx^3)^p \left(1 + \frac{bx^3}{a} \right)^{-p} {}_2F_1 \left(\frac{1}{3}, -p; \frac{4}{3}; -\frac{bx^3}{a} \right) + \frac{1}{2} dx^2 (a + bx^3)^p \left(1 + \frac{bx^3}{a} \right)^{-p}
\end{aligned}$$

Mathematica [A] time = 0.0616251, size = 114, normalized size = 1.12

$$\frac{(a + bx^3)^p \left(\frac{bx^3}{a} + 1 \right)^{-p} \left(6bc(p+1)x {}_2F_1 \left(\frac{1}{3}, -p; \frac{4}{3}; -\frac{bx^3}{a} \right) + 3bd(p+1)x^2 {}_2F_1 \left(\frac{2}{3}, -p; \frac{5}{3}; -\frac{bx^3}{a} \right) + 2e(a + bx^3) \left(\frac{bx^3}{a} + 1 \right)^p \right)}{6b(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2)*(a + b*x^3)^p,x]

[Out] ((a + b*x^3)^p*(2*e*(a + b*x^3)*(1 + (b*x^3)/a)^p + 6*b*c*(1 + p)*Hypergeometric2F1[1/3, -p, 4/3, -((b*x^3)/a)] + 3*b*d*(1 + p)*x^2*Hypergeometric2F1[2/3, -p, 5/3, -((b*x^3)/a)])/(6*b*(1 + p)*(1 + (b*x^3)/a)^p)

Maple [F] time = 0.264, size = 0, normalized size = 0.

$$\int (ex^2 + dx + c)(bx^3 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d*x+c)*(b*x^3+a)^p,x)

[Out] int((e*x^2+d*x+c)*(b*x^3+a)^p,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + dx + c)(bx^3 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="maxima")

[Out] integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ex^2 + dx + c\right)\left(bx^3 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="fricas")

[Out] integral((e*x^2 + d*x + c)*(b*x^3 + a)^p, x)

Sympy [A] time = 91.6781, size = 112, normalized size = 1.1

$$\frac{a^p c x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, -p \left| \frac{bx^3 e^{i\pi}}{a} \right. \right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^p d x^2 \Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{2}{3}, -p \left| \frac{bx^3 e^{i\pi}}{a} \right. \right)}{3\Gamma\left(\frac{5}{3}\right)} + e \left(\left(\frac{a^p x^3}{3} \right. \right. \left. \left. \begin{array}{l} \text{for } b = 0 \\ \frac{(a+bx^3)^{p+1}}{p+1} \text{ for } p \neq -1 \\ \frac{\log(a+bx^3)}{3b} \text{ otherwise} \end{array} \right) \right. \left. \left. \begin{array}{l} \\ \\ \text{otherwise} \end{array} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**p,x)

[Out] a**p*c*x*gamma(1/3)*hyper((1/3, -p), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + a**p*d*x**2*gamma(2/3)*hyper((2/3, -p), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + e*Piecewise((a**p*x**3/3, Eq(b, 0)), (Piecewise(e(((a + b*x**3)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**3), True)))/(3*b

), True))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + dx + c)(bx^3 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="giac")

[Out] integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p, x)

3.475 $\int x (c + dx + ex^2) (a + bx^3)^p dx$

Optimal. Leaf size=107

$$\frac{cx^2 (a + bx^3)^{p+1} {}_2F_1\left(1, p + \frac{5}{3}; \frac{5}{3}; -\frac{bx^3}{a}\right)}{2a} + \frac{d(a + bx^3)^{p+1}}{3b(p+1)} + \frac{ex^4 (a + bx^3)^{p+1} {}_2F_1\left(1, p + \frac{7}{3}; \frac{7}{3}; -\frac{bx^3}{a}\right)}{4a}$$

[Out] (d*(a + b*x^3)^(1 + p))/(3*b*(1 + p)) + (c*x^2*(a + b*x^3)^(1 + p)*Hypergeometric2F1[1, 5/3 + p, 5/3, -((b*x^3)/a)]/(2*a) + (e*x^4*(a + b*x^3)^(1 + p)*Hypergeometric2F1[1, 7/3 + p, 7/3, -((b*x^3)/a)]/(4*a)

Rubi [A] time = 0.0911636, antiderivative size = 125, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {1893, 365, 364, 261}

$$\frac{1}{2}cx^2 (a + bx^3)^p \left(\frac{bx^3}{a} + 1\right)^{-p} {}_2F_1\left(\frac{2}{3}, -p; \frac{5}{3}; -\frac{bx^3}{a}\right) + \frac{d(a + bx^3)^{p+1}}{3b(p+1)} + \frac{1}{4}ex^4 (a + bx^3)^p \left(\frac{bx^3}{a} + 1\right)^{-p} {}_2F_1\left(\frac{4}{3}, -p; \frac{7}{3}; -\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[x*(c + d*x + e*x^2)*(a + b*x^3)^p,x]

[Out] (d*(a + b*x^3)^(1 + p))/(3*b*(1 + p)) + (c*x^2*(a + b*x^3)^p*Hypergeometric2F1[2/3, -p, 5/3, -((b*x^3)/a)]/(2*(1 + (b*x^3)/a)^p) + (e*x^4*(a + b*x^3)^p*Hypergeometric2F1[4/3, -p, 7/3, -((b*x^3)/a)]/(4*(1 + (b*x^3)/a)^p)

Rule 1893

Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n])

Rule 365

Int[((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])

Rule 364

```
Int[((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int x(c + dx + ex^2)(a + bx^3)^p dx &= \int (cx(a + bx^3)^p + dx^2(a + bx^3)^p + ex^3(a + bx^3)^p) dx \\ &= c \int x(a + bx^3)^p dx + d \int x^2(a + bx^3)^p dx + e \int x^3(a + bx^3)^p dx \\ &= \frac{d(a + bx^3)^{1+p}}{3b(1+p)} + \left(c(a + bx^3)^p \left(1 + \frac{bx^3}{a} \right)^{-p} \right) \int x \left(1 + \frac{bx^3}{a} \right)^p dx + \left(e(a + bx^3)^p \left(1 + \frac{bx^3}{a} \right)^{-p} \right) \int x^2 \left(1 + \frac{bx^3}{a} \right)^p dx \\ &= \frac{d(a + bx^3)^{1+p}}{3b(1+p)} + \frac{1}{2} cx^2 (a + bx^3)^p \left(1 + \frac{bx^3}{a} \right)^{-p} {}_2F_1 \left(\frac{2}{3}, -p; \frac{5}{3}; -\frac{bx^3}{a} \right) + \frac{1}{4} ex^4 (a + bx^3)^p \left(1 + \frac{bx^3}{a} \right)^{-p} {}_2F_1 \left(\frac{4}{3}, -p; \frac{7}{3}; -\frac{bx^3}{a} \right) \end{aligned}$$

Mathematica [A] time = 0.0620941, size = 116, normalized size = 1.08

$$\frac{(a + bx^3)^p \left(\frac{bx^3}{a} + 1 \right)^{-p} \left(6bc(p + 1)x^2 {}_2F_1 \left(\frac{2}{3}, -p; \frac{5}{3}; -\frac{bx^3}{a} \right) + 4d(a + bx^3) \left(\frac{bx^3}{a} + 1 \right)^p + 3be(p + 1)x^4 {}_2F_1 \left(\frac{4}{3}, -p; \frac{7}{3}; -\frac{bx^3}{a} \right) \right)}{12b(p + 1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + d*x + e*x^2)*(a + b*x^3)^p,x]
```

```
[Out] ((a + b*x^3)^p*(4*d*(a + b*x^3)*(1 + (b*x^3)/a)^p + 6*b*c*(1 + p)*x^2*Hyper
geometric2F1[2/3, -p, 5/3, -((b*x^3)/a)] + 3*b*e*(1 + p)*x^4*Hypergeometric
2F1[4/3, -p, 7/3, -((b*x^3)/a)])/(12*b*(1 + p)*(1 + (b*x^3)/a)^p)
```

Maple [F] time = 0.227, size = 0, normalized size = 0.

$$\int x(ex^2 + dx + c)(bx^3 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(e*x^2+d*x+c)*(b*x^3+a)^p,x)`

[Out] `int(x*(e*x^2+d*x+c)*(b*x^3+a)^p,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + dx + c)(bx^3 + a)^p x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="maxima")`

[Out] `integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p*x, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ex^3 + dx^2 + cx\right)\left(bx^3 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="fricas")`

[Out] `integral((e*x^3 + d*x^2 + c*x)*(b*x^3 + a)^p, x)`

Sympy [A] time = 145.318, size = 114, normalized size = 1.07

$$\frac{a^p c x^2 \Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{2}{3}, -p \left| \frac{b x^3 e^{i\pi}}{a} \right. \right)}{3 \Gamma\left(\frac{5}{3}\right)} + \frac{a^p e x^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{4}{3}, -p \left| \frac{b x^3 e^{i\pi}}{a} \right. \right)}{3 \Gamma\left(\frac{7}{3}\right)} + d \left(\left(\begin{array}{l} \frac{a^p x^3}{3} \\ \frac{(a + b x^3)^{p+1}}{p+1} \\ \frac{\log(a + b x^3)}{3b} \end{array} \right) \begin{array}{l} \text{for } b = 0 \\ \text{for } p \neq -1 \\ \text{otherwise} \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**p,x)
```

```
[Out] a**p*c*x**2*gamma(2/3)*hyper((2/3, -p), (5/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(5/3)) + a**p*e*x**4*gamma(4/3)*hyper((4/3, -p), (7/3,), b*x**3*exp_
polar(I*pi)/a)/(3*gamma(7/3)) + d*Piecewise((a**p*x**3/3, Eq(b, 0)), (Piece
wise(((a + b*x**3)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**3), True)))/(
3*b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + dx + c)(bx^3 + a)^p x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p*x, x)
```

3.476 $\int x^2 (c + dx + ex^2) (a + bx^3)^p dx$

Optimal. Leaf size=107

$$\frac{c(a + bx^3)^{p+1}}{3b(p+1)} + \frac{dx^4(a + bx^3)^{p+1} {}_2F_1\left(1, p + \frac{7}{3}; \frac{7}{3}; -\frac{bx^3}{a}\right)}{4a} + \frac{ex^5(a + bx^3)^{p+1} {}_2F_1\left(1, p + \frac{8}{3}; \frac{8}{3}; -\frac{bx^3}{a}\right)}{5a}$$

[Out] (c*(a + b*x^3)^(1 + p))/(3*b*(1 + p)) + (d*x^4*(a + b*x^3)^(1 + p)*Hypergeometric2F1[1, 7/3 + p, 7/3, -((b*x^3)/a)]/(4*a) + (e*x^5*(a + b*x^3)^(1 + p)*Hypergeometric2F1[1, 8/3 + p, 8/3, -((b*x^3)/a)]/(5*a))

Rubi [A] time = 0.106132, antiderivative size = 125, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1893, 261, 365, 364}

$$\frac{c(a + bx^3)^{p+1}}{3b(p+1)} + \frac{1}{4}dx^4(a + bx^3)^p \left(\frac{bx^3}{a} + 1\right)^{-p} {}_2F_1\left(\frac{4}{3}, -p; \frac{7}{3}; -\frac{bx^3}{a}\right) + \frac{1}{5}ex^5(a + bx^3)^p \left(\frac{bx^3}{a} + 1\right)^{-p} {}_2F_1\left(\frac{5}{3}, -p; \frac{8}{3}; -\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[x^2*(c + d*x + e*x^2)*(a + b*x^3)^p,x]

[Out] (c*(a + b*x^3)^(1 + p))/(3*b*(1 + p)) + (d*x^4*(a + b*x^3)^p*Hypergeometric2F1[4/3, -p, 7/3, -((b*x^3)/a)]/(4*(1 + (b*x^3)/a)^p) + (e*x^5*(a + b*x^3)^p*Hypergeometric2F1[5/3, -p, 8/3, -((b*x^3)/a)]/(5*(1 + (b*x^3)/a)^p))

Rule 1893

Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n])

Rule 261

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 365

```
Int[((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
&& !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
 \int x^2 (c + dx + ex^2) (a + bx^3)^p dx &= \int \left(cx^2 (a + bx^3)^p + dx^3 (a + bx^3)^p + ex^4 (a + bx^3)^p \right) dx \\
 &= c \int x^2 (a + bx^3)^p dx + d \int x^3 (a + bx^3)^p dx + e \int x^4 (a + bx^3)^p dx \\
 &= \frac{c (a + bx^3)^{1+p}}{3b(1+p)} + \left(d (a + bx^3)^p \left(1 + \frac{bx^3}{a} \right)^{-p} \right) \int x^3 \left(1 + \frac{bx^3}{a} \right)^p dx + \left(e (a + bx^3)^p \left(1 + \frac{bx^3}{a} \right)^{-p} \right) \int x^4 \left(1 + \frac{bx^3}{a} \right)^p dx \\
 &= \frac{c (a + bx^3)^{1+p}}{3b(1+p)} + \frac{1}{4} dx^4 (a + bx^3)^p \left(1 + \frac{bx^3}{a} \right)^{-p} {}_2F_1 \left(\frac{4}{3}, -p; \frac{7}{3}; -\frac{bx^3}{a} \right) + \frac{1}{5} ex^5 (a + bx^3)^p \left(1 + \frac{bx^3}{a} \right)^{-p} {}_2F_1 \left(\frac{5}{3}, -p; \frac{8}{3}; -\frac{bx^3}{a} \right)
 \end{aligned}$$

Mathematica [A] time = 0.0762696, size = 116, normalized size = 1.08

$$\frac{(a + bx^3)^p \left(\frac{bx^3}{a} + 1 \right)^{-p} \left(20c (a + bx^3) \left(\frac{bx^3}{a} + 1 \right)^p + 15bd(p + 1)x^4 {}_2F_1 \left(\frac{4}{3}, -p; \frac{7}{3}; -\frac{bx^3}{a} \right) + 12be(p + 1)x^5 {}_2F_1 \left(\frac{5}{3}, -p; \frac{8}{3}; -\frac{bx^3}{a} \right) \right)}{60b(p + 1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3)^p,x]
```

```
[Out] ((a + b*x^3)^p*(20*c*(a + b*x^3)*(1 + (b*x^3)/a)^p + 15*b*d*(1 + p)*x^4*Hypergeometric2F1[4/3, -p, 7/3, -((b*x^3)/a)] + 12*b*e*(1 + p)*x^5*Hypergeometric2F1[5/3, -p, 8/3, -((b*x^3)/a)])/(60*b*(1 + p)*(1 + (b*x^3)/a)^p)
```

Maple [F] time = 0.234, size = 0, normalized size = 0.

$$\int x^2 (ex^2 + dx + c) (bx^3 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^p,x)

[Out] int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^p,x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ex^4 + dx^3 + cx^2\right)\left(bx^3 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="fricas")

[Out] integral((e*x^4 + d*x^3 + c*x^2)*(b*x^3 + a)^p, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + dx + c)(bx^3 + a)^p x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p*x^2, x)
```

$$3.477 \quad \int (c + dx + ex^2 + fx^3)(a + bx^4) dx$$

Optimal. Leaf size=68

$$acx + \frac{1}{2}adx^2 + \frac{1}{3}aex^3 + \frac{1}{4}afx^4 + \frac{1}{5}bcx^5 + \frac{1}{6}bdx^6 + \frac{1}{7}bex^7 + \frac{1}{8}bfx^8$$

[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (a*f*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)/6 + (b*e*x^7)/7 + (b*f*x^8)/8

Rubi [A] time = 0.0444891, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1850}

$$acx + \frac{1}{2}adx^2 + \frac{1}{3}aex^3 + \frac{1}{4}afx^4 + \frac{1}{5}bcx^5 + \frac{1}{6}bdx^6 + \frac{1}{7}bex^7 + \frac{1}{8}bfx^8$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4), x]

[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (a*f*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)/6 + (b*e*x^7)/7 + (b*f*x^8)/8

Rule 1850

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int (c + dx + ex^2 + fx^3)(a + bx^4) dx &= \int (ac + adx + aex^2 + afx^3 + bcx^4 + bdx^5 + bex^6 + bfx^7) dx \\ &= acx + \frac{1}{2}adx^2 + \frac{1}{3}aex^3 + \frac{1}{4}afx^4 + \frac{1}{5}bcx^5 + \frac{1}{6}bdx^6 + \frac{1}{7}bex^7 + \frac{1}{8}bfx^8 \end{aligned}$$

Mathematica [A] time = 0.0053242, size = 68, normalized size = 1.

$$acx + \frac{1}{2}adx^2 + \frac{1}{3}aex^3 + \frac{1}{4}afx^4 + \frac{1}{5}bcx^5 + \frac{1}{6}bdx^6 + \frac{1}{7}bex^7 + \frac{1}{8}bfx^8$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]

[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (a*f*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)/6 + (b*e*x^7)/7 + (b*f*x^8)/8

Maple [A] time = 0.001, size = 55, normalized size = 0.8

$$acx + \frac{adx^2}{2} + \frac{aex^3}{3} + \frac{afx^4}{4} + \frac{bcx^5}{5} + \frac{bdx^6}{6} + \frac{bex^7}{7} + \frac{bfx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a),x)

[Out] a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*a*f*x^4+1/5*b*c*x^5+1/6*b*d*x^6+1/7*b*e*x^7+1/8*b*f*x^8

Maxima [A] time = 0.916526, size = 73, normalized size = 1.07

$$\frac{1}{8}bfx^8 + \frac{1}{7}bex^7 + \frac{1}{6}bdx^6 + \frac{1}{5}bcx^5 + \frac{1}{4}afx^4 + \frac{1}{3}aex^3 + \frac{1}{2}adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="maxima")

[Out] 1/8*b*f*x^8 + 1/7*b*e*x^7 + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*f*x^4 + 1/3*a*e*x^3 + 1/2*a*d*x^2 + a*c*x

Fricas [A] time = 1.38129, size = 142, normalized size = 2.09

$$\frac{1}{8}x^8fb + \frac{1}{7}x^7eb + \frac{1}{6}x^6db + \frac{1}{5}x^5cb + \frac{1}{4}x^4fa + \frac{1}{3}x^3ea + \frac{1}{2}x^2da + xca$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="fricas")

[Out] $\frac{1}{8}x^8fb + \frac{1}{7}x^7eb + \frac{1}{6}x^6db + \frac{1}{5}x^5cb + \frac{1}{4}x^4fa + \frac{1}{3}x^3ea + \frac{1}{2}x^2da + xc^a$

Sympy [A] time = 0.06287, size = 63, normalized size = 0.93

$$acx + \frac{adx^2}{2} + \frac{aex^3}{3} + \frac{afx^4}{4} + \frac{bcx^5}{5} + \frac{bdx^6}{6} + \frac{bex^7}{7} + \frac{bf^x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a),x)

[Out] $a*c*x + a*d*x**2/2 + a*e*x**3/3 + a*f*x**4/4 + b*c*x**5/5 + b*d*x**6/6 + b*e*x**7/7 + b*f*x**8/8$

Giac [A] time = 1.08806, size = 76, normalized size = 1.12

$$\frac{1}{8}bf^x^8 + \frac{1}{7}bx^7e + \frac{1}{6}bdx^6 + \frac{1}{5}bcx^5 + \frac{1}{4}afx^4 + \frac{1}{3}ax^3e + \frac{1}{2}adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="giac")

[Out] $\frac{1}{8}b*f*x^8 + \frac{1}{7}b*x^7*e + \frac{1}{6}b*d*x^6 + \frac{1}{5}b*c*x^5 + \frac{1}{4}a*f*x^4 + \frac{1}{3}a*x^3*e + \frac{1}{2}a*d*x^2 + a*c*x$

$$3.478 \quad \int x^3 (c + dx + ex^2 + fx^3) (a + bx^4) dx$$

Optimal. Leaf size=73

$$\frac{1}{4}acx^4 + \frac{1}{5}adx^5 + \frac{1}{6}aex^6 + \frac{1}{7}afx^7 + \frac{1}{8}bcx^8 + \frac{1}{9}bdx^9 + \frac{1}{10}bex^{10} + \frac{1}{11}bfx^{11}$$

[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + (a*f*x^7)/7 + (b*c*x^8)/8 + (b*d*x^9)/9 + (b*e*x^10)/10 + (b*f*x^11)/11

Rubi [A] time = 0.0641713, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {1820}

$$\frac{1}{4}acx^4 + \frac{1}{5}adx^5 + \frac{1}{6}aex^6 + \frac{1}{7}afx^7 + \frac{1}{8}bcx^8 + \frac{1}{9}bdx^9 + \frac{1}{10}bex^{10} + \frac{1}{11}bfx^{11}$$

Antiderivative was successfully verified.

[In] Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]

[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + (a*f*x^7)/7 + (b*c*x^8)/8 + (b*d*x^9)/9 + (b*e*x^10)/10 + (b*f*x^11)/11

Rule 1820

Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int x^3 (c + dx + ex^2 + fx^3) (a + bx^4) dx &= \int (acx^3 + adx^4 + aex^5 + afx^6 + bcx^7 + bdx^8 + bex^9 + bfx^{10}) dx \\ &= \frac{1}{4}acx^4 + \frac{1}{5}adx^5 + \frac{1}{6}aex^6 + \frac{1}{7}afx^7 + \frac{1}{8}bcx^8 + \frac{1}{9}bdx^9 + \frac{1}{10}bex^{10} + \frac{1}{11}bfx^{11} \end{aligned}$$

Mathematica [A] time = 0.0029445, size = 73, normalized size = 1.

$$\frac{1}{4}acx^4 + \frac{1}{5}adx^5 + \frac{1}{6}aex^6 + \frac{1}{7}afx^7 + \frac{1}{8}bcx^8 + \frac{1}{9}bdx^9 + \frac{1}{10}bex^{10} + \frac{1}{11}bfx^{11}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]

[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + (a*f*x^7)/7 + (b*c*x^8)/8 + (b*d*x^9)/9 + (b*e*x^10)/10 + (b*f*x^11)/11

Maple [A] time = 0., size = 58, normalized size = 0.8

$$\frac{acx^4}{4} + \frac{adx^5}{5} + \frac{aex^6}{6} + \frac{afx^7}{7} + \frac{bcx^8}{8} + \frac{bdx^9}{9} + \frac{bex^{10}}{10} + \frac{bfx^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a),x)

[Out] 1/4*a*c*x^4+1/5*a*d*x^5+1/6*a*e*x^6+1/7*a*f*x^7+1/8*b*c*x^8+1/9*b*d*x^9+1/10*b*e*x^10+1/11*b*f*x^11

Maxima [A] time = 0.918607, size = 77, normalized size = 1.05

$$\frac{1}{11} bfx^{11} + \frac{1}{10} bex^{10} + \frac{1}{9} bdx^9 + \frac{1}{8} bcx^8 + \frac{1}{7} afx^7 + \frac{1}{6} aex^6 + \frac{1}{5} adx^5 + \frac{1}{4} acx^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="maxima")

[Out] 1/11*b*f*x^11 + 1/10*b*e*x^10 + 1/9*b*d*x^9 + 1/8*b*c*x^8 + 1/7*a*f*x^7 + 1/6*a*e*x^6 + 1/5*a*d*x^5 + 1/4*a*c*x^4

Fricas [A] time = 1.43254, size = 155, normalized size = 2.12

$$\frac{1}{11} x^{11} fb + \frac{1}{10} x^{10} eb + \frac{1}{9} x^9 db + \frac{1}{8} x^8 cb + \frac{1}{7} x^7 fa + \frac{1}{6} x^6 ea + \frac{1}{5} x^5 da + \frac{1}{4} x^4 ca$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="fricas")

[Out] 1/11*x^11*f*b + 1/10*x^10*e*b + 1/9*x^9*d*b + 1/8*x^8*c*b + 1/7*x^7*f*a + 1/6*x^6*e*a + 1/5*x^5*d*a + 1/4*x^4*c*a

Sympy [A] time = 0.065535, size = 66, normalized size = 0.9

$$\frac{acx^4}{4} + \frac{adx^5}{5} + \frac{aex^6}{6} + \frac{afx^7}{7} + \frac{bcx^8}{8} + \frac{bdx^9}{9} + \frac{bex^{10}}{10} + \frac{bf x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a),x)

[Out] a*c*x**4/4 + a*d*x**5/5 + a*e*x**6/6 + a*f*x**7/7 + b*c*x**8/8 + b*d*x**9/9 + b*e*x**10/10 + b*f*x**11/11

Giac [A] time = 1.05782, size = 80, normalized size = 1.1

$$\frac{1}{11} b f x^{11} + \frac{1}{10} b x^{10} e + \frac{1}{9} b d x^9 + \frac{1}{8} b c x^8 + \frac{1}{7} a f x^7 + \frac{1}{6} a x^6 e + \frac{1}{5} a d x^5 + \frac{1}{4} a c x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="giac")

[Out] 1/11*b*f*x^11 + 1/10*b*x^10*e + 1/9*b*d*x^9 + 1/8*b*c*x^8 + 1/7*a*f*x^7 + 1/6*a*x^6*e + 1/5*a*d*x^5 + 1/4*a*c*x^4

$$3.479 \quad \int (c + dx + ex^2 + fx^3) (a + bx^4)^2 dx$$

Optimal. Leaf size=109

$$a^2cx + \frac{1}{2}a^2dx^2 + \frac{1}{3}a^2ex^3 + \frac{2}{5}abcx^5 + \frac{1}{3}abdx^6 + \frac{2}{7}abex^7 + \frac{f(a+bx^4)^3}{12b} + \frac{1}{9}b^2cx^9 + \frac{1}{10}b^2dx^{10} + \frac{1}{11}b^2ex^{11}$$

[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (2*a*b*c*x^5)/5 + (a*b*d*x^6)/3 + (2*a*b*e*x^7)/7 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10 + (b^2*e*x^11)/11 + (f*(a + b*x^4)^3)/(12*b)

Rubi [A] time = 0.0744559, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$, Rules used = {1582, 1657}

$$a^2cx + \frac{1}{2}a^2dx^2 + \frac{1}{3}a^2ex^3 + \frac{2}{5}abcx^5 + \frac{1}{3}abdx^6 + \frac{2}{7}abex^7 + \frac{f(a+bx^4)^3}{12b} + \frac{1}{9}b^2cx^9 + \frac{1}{10}b^2dx^{10} + \frac{1}{11}b^2ex^{11}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]

[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (2*a*b*c*x^5)/5 + (a*b*d*x^6)/3 + (2*a*b*e*x^7)/7 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10 + (b^2*e*x^11)/11 + (f*(a + b*x^4)^3)/(12*b)

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(Coeff[Px, x, n - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a + b*x^n)^p, x, m - 1], 0] && GtQ[m*x, n*p]]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[Expand[Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq,
```

, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (c + dx + ex^2 + fx^3)(a + bx^4)^2 dx &= \frac{f(a + bx^4)^3}{12b} + \int (c + dx + ex^2)(a + bx^4)^2 dx \\ &= \frac{f(a + bx^4)^3}{12b} + \int (a^2c + a^2dx + a^2ex^2 + 2abcx^4 + 2abdx^5 + 2abex^6 + b^2cx^8 + b^2fx^{12}) dx \\ &= a^2cx + \frac{1}{2}a^2dx^2 + \frac{1}{3}a^2ex^3 + \frac{2}{5}abcx^5 + \frac{1}{3}abdx^6 + \frac{2}{7}abex^7 + \frac{1}{9}b^2cx^9 + \frac{1}{10}b^2dx^{10} + \frac{1}{11}b^2fx^{11} + \frac{1}{12}b^2fx^{12} \end{aligned}$$

Mathematica [A] time = 0.0036873, size = 124, normalized size = 1.14

$$a^2cx + \frac{1}{2}a^2dx^2 + \frac{1}{3}a^2ex^3 + \frac{1}{4}a^2fx^4 + \frac{2}{5}abcx^5 + \frac{1}{3}abdx^6 + \frac{2}{7}abex^7 + \frac{1}{4}abfx^8 + \frac{1}{9}b^2cx^9 + \frac{1}{10}b^2dx^{10} + \frac{1}{11}b^2ex^{11} + \frac{1}{12}b^2fx^{12}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]

[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (a^2*f*x^4)/4 + (2*a*b*c*x^5)/5 + (a*b*d*x^6)/3 + (2*a*b*e*x^7)/7 + (a*b*f*x^8)/4 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10 + (b^2*e*x^11)/11 + (b^2*f*x^12)/12

Maple [A] time = 0., size = 103, normalized size = 0.9

$$\frac{b^2fx^{12}}{12} + \frac{b^2ex^{11}}{11} + \frac{b^2dx^{10}}{10} + \frac{b^2cx^9}{9} + \frac{fabx^8}{4} + \frac{2abex^7}{7} + \frac{abdx^6}{3} + \frac{2abcx^5}{5} + \frac{fa^2x^4}{4} + \frac{a^2ex^3}{3} + \frac{a^2dx^2}{2} + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x)

[Out] 1/12*b^2*f*x^12+1/11*b^2*e*x^11+1/10*b^2*d*x^10+1/9*b^2*c*x^9+1/4*f*a*b*x^8+2/7*a*b*e*x^7+1/3*a*b*d*x^6+2/5*a*b*c*x^5+1/4*f*a^2*x^4+1/3*a^2*e*x^3+1/2*a^2*d*x^2+a^2*c*x

Maxima [A] time = 0.914583, size = 138, normalized size = 1.27

$$\frac{1}{12} b^2 f x^{12} + \frac{1}{11} b^2 e x^{11} + \frac{1}{10} b^2 d x^{10} + \frac{1}{9} b^2 c x^9 + \frac{1}{4} a b f x^8 + \frac{2}{7} a b e x^7 + \frac{1}{3} a b d x^6 + \frac{2}{5} a b c x^5 + \frac{1}{4} a^2 f x^4 + \frac{1}{3} a^2 e x^3 + \frac{1}{2} a^2 d x^2 + \frac{1}{3} a^2 c x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="maxima")

[Out] 1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a*b*f*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4 + 1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x

Fricas [A] time = 1.48887, size = 258, normalized size = 2.37

$$\frac{1}{12} x^{12} f b^2 + \frac{1}{11} x^{11} e b^2 + \frac{1}{10} x^{10} d b^2 + \frac{1}{9} x^9 c b^2 + \frac{1}{4} x^8 f b a + \frac{2}{7} x^7 e b a + \frac{1}{3} x^6 d b a + \frac{2}{5} x^5 c b a + \frac{1}{4} x^4 f a^2 + \frac{1}{3} x^3 e a^2 + \frac{1}{2} x^2 d a^2 + x c a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="fricas")

[Out] 1/12*x^12*f*b^2 + 1/11*x^11*e*b^2 + 1/10*x^10*d*b^2 + 1/9*x^9*c*b^2 + 1/4*x^8*f*b*a + 2/7*x^7*e*b*a + 1/3*x^6*d*b*a + 2/5*x^5*c*b*a + 1/4*x^4*f*a^2 + 1/3*x^3*e*a^2 + 1/2*x^2*d*a^2 + x*c*a^2

Sympy [A] time = 0.074751, size = 121, normalized size = 1.11

$$a^2 c x + \frac{a^2 d x^2}{2} + \frac{a^2 e x^3}{3} + \frac{a^2 f x^4}{4} + \frac{2 a b c x^5}{5} + \frac{a b d x^6}{3} + \frac{2 a b e x^7}{7} + \frac{a b f x^8}{4} + \frac{b^2 c x^9}{9} + \frac{b^2 d x^{10}}{10} + \frac{b^2 e x^{11}}{11} + \frac{b^2 f x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**2,x)

[Out] a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + a**2*f*x**4/4 + 2*a*b*c*x**5/5 + a*b*d*x**6/3 + 2*a*b*e*x**7/7 + a*b*f*x**8/4 + b**2*c*x**9/9 + b**2*d*x**10/10 + b**2*e*x**11/11 + b**2*f*x**12/12

Giac [A] time = 1.08465, size = 142, normalized size = 1.3

$$\frac{1}{12} b^2 f x^{12} + \frac{1}{11} b^2 x^{11} e + \frac{1}{10} b^2 d x^{10} + \frac{1}{9} b^2 c x^9 + \frac{1}{4} a b f x^8 + \frac{2}{7} a b x^7 e + \frac{1}{3} a b d x^6 + \frac{2}{5} a b c x^5 + \frac{1}{4} a^2 f x^4 + \frac{1}{3} a^2 x^3 e + \frac{1}{2} a^2 d x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="giac")

[Out] 1/12*b^2*f*x^12 + 1/11*b^2*x^11*e + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a*b*f*x^8 + 2/7*a*b*x^7*e + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4 + 1/3*a^2*x^3*e + 1/2*a^2*d*x^2 + a^2*c*x

$$3.480 \quad \int x^3 (c + dx + ex^2 + fx^3) (a + bx^4)^2 dx$$

Optimal. Leaf size=114

$$\frac{1}{5}a^2dx^5 + \frac{1}{6}a^2ex^6 + \frac{1}{7}a^2fx^7 + \frac{c(a+bx^4)^3}{12b} + \frac{2}{9}abdx^9 + \frac{1}{5}abex^{10} + \frac{2}{11}abfx^{11} + \frac{1}{13}b^2dx^{13} + \frac{1}{14}b^2ex^{14} + \frac{1}{15}b^2fx^{15}$$

[Out] (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a^2*f*x^7)/7 + (2*a*b*d*x^9)/9 + (a*b*e*x^10)/5 + (2*a*b*f*x^11)/11 + (b^2*d*x^13)/13 + (b^2*e*x^14)/14 + (b^2*f*x^15)/15 + (c*(a + b*x^4)^3)/(12*b)

Rubi [A] time = 0.0837339, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1582, 1850}

$$\frac{1}{5}a^2dx^5 + \frac{1}{6}a^2ex^6 + \frac{1}{7}a^2fx^7 + \frac{c(a+bx^4)^3}{12b} + \frac{2}{9}abdx^9 + \frac{1}{5}abex^{10} + \frac{2}{11}abfx^{11} + \frac{1}{13}b^2dx^{13} + \frac{1}{14}b^2ex^{14} + \frac{1}{15}b^2fx^{15}$$

Antiderivative was successfully verified.

[In] Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]

[Out] (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a^2*f*x^7)/7 + (2*a*b*d*x^9)/9 + (a*b*e*x^10)/5 + (2*a*b*f*x^11)/11 + (b^2*d*x^13)/13 + (b^2*e*x^14)/14 + (b^2*f*x^15)/15 + (c*(a + b*x^4)^3)/(12*b)

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(Coeff[Px, x, n - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a + b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
```

, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned}\int x^3 (c + dx + ex^2 + fx^3) (a + bx^4)^2 dx &= \frac{c(a + bx^4)^3}{12b} + \int (a + bx^4)^2 (-cx^3 + x^3(c + dx + ex^2 + fx^3)) dx \\ &= \frac{c(a + bx^4)^3}{12b} + \int (a^2 dx^4 + a^2 ex^5 + a^2 fx^6 + 2abdx^8 + 2abex^9 + 2abfx^{10} + b^2 dx^{12} + b^2 ex^{13} + b^2 fx^{14}) dx \\ &= \frac{1}{5} a^2 dx^5 + \frac{1}{6} a^2 ex^6 + \frac{1}{7} a^2 fx^7 + \frac{2}{9} abdx^9 + \frac{1}{5} abex^{10} + \frac{2}{11} abfx^{11} + \frac{1}{13} b^2 dx^{13} + \frac{1}{14} b^2 ex^{14} + \frac{1}{15} b^2 fx^{15}\end{aligned}$$

Mathematica [A] time = 0.0041538, size = 129, normalized size = 1.13

$$\frac{1}{4} a^2 cx^4 + \frac{1}{5} a^2 dx^5 + \frac{1}{6} a^2 ex^6 + \frac{1}{7} a^2 fx^7 + \frac{1}{4} abcx^8 + \frac{2}{9} abdx^9 + \frac{1}{5} abex^{10} + \frac{2}{11} abfx^{11} + \frac{1}{12} b^2 cx^{12} + \frac{1}{13} b^2 dx^{13} + \frac{1}{14} b^2 ex^{14} + \frac{1}{15} b^2 fx^{15}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]

[Out] (a^2*c*x^4)/4 + (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a^2*f*x^7)/7 + (a*b*c*x^8)/4 + (2*a*b*d*x^9)/9 + (a*b*e*x^10)/5 + (2*a*b*f*x^11)/11 + (b^2*c*x^12)/12 + (b^2*d*x^13)/13 + (b^2*e*x^14)/14 + (b^2*f*x^15)/15

Maple [A] time = 0.001, size = 106, normalized size = 0.9

$$\frac{b^2 fx^{15}}{15} + \frac{b^2 ex^{14}}{14} + \frac{b^2 dx^{13}}{13} + \frac{b^2 cx^{12}}{12} + \frac{2 abfx^{11}}{11} + \frac{abex^{10}}{5} + \frac{2 abdx^9}{9} + \frac{abcx^8}{4} + \frac{a^2 fx^7}{7} + \frac{a^2 ex^6}{6} + \frac{a^2 dx^5}{5} + \frac{a^2 cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x)

[Out] 1/15*b^2*f*x^15+1/14*b^2*e*x^14+1/13*b^2*d*x^13+1/12*b^2*c*x^12+2/11*a*b*f*x^11+1/5*a*b*e*x^10+2/9*a*b*d*x^9+1/4*a*b*c*x^8+1/7*a^2*f*x^7+1/6*a^2*e*x^6+1/5*a^2*d*x^5+1/4*a^2*c*x^4

Maxima [A] time = 0.91656, size = 142, normalized size = 1.25

$$\frac{1}{15}b^2fx^{15} + \frac{1}{14}b^2ex^{14} + \frac{1}{13}b^2dx^{13} + \frac{1}{12}b^2cx^{12} + \frac{2}{11}abfx^{11} + \frac{1}{5}abex^{10} + \frac{2}{9}abdx^9 + \frac{1}{4}abcx^8 + \frac{1}{7}a^2fx^7 + \frac{1}{6}a^2ex^6 + \frac{1}{5}a^2dx^5 + \frac{1}{4}a^2cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="maxima")

[Out] 1/15*b^2*f*x^15 + 1/14*b^2*e*x^14 + 1/13*b^2*d*x^13 + 1/12*b^2*c*x^12 + 2/11*a*b*f*x^11 + 1/5*a*b*e*x^10 + 2/9*a*b*d*x^9 + 1/4*a*b*c*x^8 + 1/7*a^2*f*x^7 + 1/6*a^2*e*x^6 + 1/5*a^2*d*x^5 + 1/4*a^2*c*x^4

Fricas [A] time = 1.43756, size = 273, normalized size = 2.39

$$\frac{1}{15}x^{15}fb^2 + \frac{1}{14}x^{14}eb^2 + \frac{1}{13}x^{13}db^2 + \frac{1}{12}x^{12}cb^2 + \frac{2}{11}x^{11}fba + \frac{1}{5}x^{10}eba + \frac{2}{9}x^9dba + \frac{1}{4}x^8cba + \frac{1}{7}x^7fa^2 + \frac{1}{6}x^6ea^2 + \frac{1}{5}x^5da^2 + \frac{1}{4}x^4ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="fricas")

[Out] 1/15*x^15*f*b^2 + 1/14*x^14*e*b^2 + 1/13*x^13*d*b^2 + 1/12*x^12*c*b^2 + 2/11*x^11*f*b*a + 1/5*x^10*e*b*a + 2/9*x^9*d*b*a + 1/4*x^8*c*b*a + 1/7*x^7*f*a^2 + 1/6*x^6*e*a^2 + 1/5*x^5*d*a^2 + 1/4*x^4*c*a^2

Sympy [A] time = 0.079195, size = 124, normalized size = 1.09

$$\frac{a^2cx^4}{4} + \frac{a^2dx^5}{5} + \frac{a^2ex^6}{6} + \frac{a^2fx^7}{7} + \frac{abcx^8}{4} + \frac{2abdx^9}{9} + \frac{abex^{10}}{5} + \frac{2abfx^{11}}{11} + \frac{b^2cx^{12}}{12} + \frac{b^2dx^{13}}{13} + \frac{b^2ex^{14}}{14} + \frac{b^2fx^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**2,x)

[Out] a**2*c*x**4/4 + a**2*d*x**5/5 + a**2*e*x**6/6 + a**2*f*x**7/7 + a*b*c*x**8/4 + 2*a*b*d*x**9/9 + a*b*e*x**10/5 + 2*a*b*f*x**11/11 + b**2*c*x**12/12 + b**2*d*x**13/13 + b**2*e*x**14/14 + b**2*f*x**15/15

Giac [A] time = 1.08274, size = 146, normalized size = 1.28

$$\frac{1}{15} b^2 f x^{15} + \frac{1}{14} b^2 x^{14} e + \frac{1}{13} b^2 d x^{13} + \frac{1}{12} b^2 c x^{12} + \frac{2}{11} a b f x^{11} + \frac{1}{5} a b x^{10} e + \frac{2}{9} a b d x^9 + \frac{1}{4} a b c x^8 + \frac{1}{7} a^2 f x^7 + \frac{1}{6} a^2 x^6 e + \frac{1}{5} a^2 d x^5 + \frac{1}{4} a^2 c x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="giac")

[Out] 1/15*b^2*f*x^15 + 1/14*b^2*x^14*e + 1/13*b^2*d*x^13 + 1/12*b^2*c*x^12 + 2/11*a*b*f*x^11 + 1/5*a*b*x^10*e + 2/9*a*b*d*x^9 + 1/4*a*b*c*x^8 + 1/7*a^2*f*x^7 + 1/6*a^2*x^6*e + 1/5*a^2*d*x^5 + 1/4*a^2*c*x^4

$$3.481 \quad \int (c + dx + ex^2 + fx^3) (a + bx^4)^3 dx$$

Optimal. Leaf size=151

$$\frac{3}{5}a^2bcx^5 + \frac{1}{2}a^2bdx^6 + \frac{3}{7}a^2bex^7 + a^3cx + \frac{1}{2}a^3dx^2 + \frac{1}{3}a^3ex^3 + \frac{1}{3}ab^2cx^9 + \frac{3}{10}ab^2dx^{10} + \frac{3}{11}ab^2ex^{11} + \frac{f(a + bx^4)^4}{16b} + \frac{1}{13}b^3$$

[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^5)/5 + (a^2*b*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2*e*x^11)/11 + (b^3*c*x^13)/13 + (b^3*d*x^14)/14 + (b^3*e*x^15)/15 + (f*(a + b*x^4)^4)/(16*b)

Rubi [A] time = 0.108231, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$, Rules used = {1582, 1657}

$$\frac{3}{5}a^2bcx^5 + \frac{1}{2}a^2bdx^6 + \frac{3}{7}a^2bex^7 + a^3cx + \frac{1}{2}a^3dx^2 + \frac{1}{3}a^3ex^3 + \frac{1}{3}ab^2cx^9 + \frac{3}{10}ab^2dx^{10} + \frac{3}{11}ab^2ex^{11} + \frac{f(a + bx^4)^4}{16b} + \frac{1}{13}b^3$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]

[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^5)/5 + (a^2*b*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2*e*x^11)/11 + (b^3*c*x^13)/13 + (b^3*d*x^14)/14 + (b^3*e*x^15)/15 + (f*(a + b*x^4)^4)/(16*b)

Rule 1582

Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_)] /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a + b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]

Rule 1657

```
Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned} \int (c + dx + ex^2 + fx^3)(a + bx^4)^3 dx &= \frac{f(a + bx^4)^4}{16b} + \int (c + dx + ex^2)(a + bx^4)^3 dx \\ &= \frac{f(a + bx^4)^4}{16b} + \int (a^3c + a^3dx + a^3ex^2 + 3a^2bcx^4 + 3a^2bdx^5 + 3a^2bex^6 + 3ab^2cx^7 \\ &\quad + a^3cx + \frac{1}{2}a^3dx^2 + \frac{1}{3}a^3ex^3 + \frac{3}{5}a^2bcx^5 + \frac{1}{2}a^2bdx^6 + \frac{3}{7}a^2bex^7 + \frac{1}{3}ab^2cx^9 + \frac{3}{10}ab^2dx^{10} + \frac{3}{11}ab^2ex^{11} + \end{aligned}$$

Mathematica [A] time = 0.0049483, size = 180, normalized size = 1.19

$$\frac{3}{5}a^2bcx^5 + \frac{1}{2}a^2bdx^6 + \frac{3}{7}a^2bex^7 + \frac{3}{8}a^2bfx^8 + a^3cx + \frac{1}{2}a^3dx^2 + \frac{1}{3}a^3ex^3 + \frac{1}{4}a^3fx^4 + \frac{1}{3}ab^2cx^9 + \frac{3}{10}ab^2dx^{10} + \frac{3}{11}ab^2ex^{11} +$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (a^3*f*x^4)/4 + (3*a^2*b*c*x^5)/5
+ (a^2*b*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (3*a^2*b*f*x^8)/8 + (a*b^2*c*x^9)/
3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2*e*x^11)/11 + (a*b^2*f*x^12)/4 + (b^3*c*x
^13)/13 + (b^3*d*x^14)/14 + (b^3*e*x^15)/15 + (b^3*f*x^16)/16
```

Maple [A] time = 0., size = 151, normalized size = 1.

$$\frac{b^3fx^{16}}{16} + \frac{b^3ex^{15}}{15} + \frac{b^3dx^{14}}{14} + \frac{b^3cx^{13}}{13} + \frac{ab^2fx^{12}}{4} + \frac{3ab^2ex^{11}}{11} + \frac{3ab^2dx^{10}}{10} + \frac{ab^2cx^9}{3} + \frac{3fba^2x^8}{8} + \frac{3a^2bex^7}{7} + \frac{a^2bdx^6}{2} + \frac{3a^3cx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x)
```

```
[Out] 1/16*b^3*f*x^16+1/15*b^3*e*x^15+1/14*b^3*d*x^14+1/13*b^3*c*x^13+1/4*a*b^2*f
*x^12+3/11*a*b^2*e*x^11+3/10*a*b^2*d*x^10+1/3*a*b^2*c*x^9+3/8*f*b*a^2*x^8+3
```

$$/7*a^2*b*e*x^7+1/2*a^2*b*d*x^6+3/5*a^2*b*c*x^5+1/4*f*a^3*x^4+1/3*a^3*e*x^3+1/2*a^3*d*x^2+a^3*c*x$$

Maxima [A] time = 0.910649, size = 203, normalized size = 1.34

$$\frac{1}{16}b^3fx^{16} + \frac{1}{15}b^3ex^{15} + \frac{1}{14}b^3dx^{14} + \frac{1}{13}b^3cx^{13} + \frac{1}{4}ab^2fx^{12} + \frac{3}{11}ab^2ex^{11} + \frac{3}{10}ab^2dx^{10} + \frac{1}{3}ab^2cx^9 + \frac{3}{8}a^2bfx^8 + \frac{3}{7}a^2bex^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="maxima")

[Out] 1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/4*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x

Fricas [A] time = 1.38173, size = 375, normalized size = 2.48

$$\frac{1}{16}x^{16}fb^3 + \frac{1}{15}x^{15}eb^3 + \frac{1}{14}x^{14}db^3 + \frac{1}{13}x^{13}cb^3 + \frac{1}{4}x^{12}fb^2a + \frac{3}{11}x^{11}eb^2a + \frac{3}{10}x^{10}db^2a + \frac{1}{3}x^9cb^2a + \frac{3}{8}x^8fba^2 + \frac{3}{7}x^7eba^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="fricas")

[Out] 1/16*x^16*f*b^3 + 1/15*x^15*e*b^3 + 1/14*x^14*d*b^3 + 1/13*x^13*c*b^3 + 1/4*x^12*f*b^2*a + 3/11*x^11*e*b^2*a + 3/10*x^10*d*b^2*a + 1/3*x^9*c*b^2*a + 3/8*x^8*f*b*a^2 + 3/7*x^7*e*b*a^2 + 1/2*x^6*d*b*a^2 + 3/5*x^5*c*b*a^2 + 1/4*x^4*f*a^3 + 1/3*x^3*e*a^3 + 1/2*x^2*d*a^3 + x*c*a^3

Sympy [A] time = 0.087819, size = 180, normalized size = 1.19

$$a^3cx + \frac{a^3dx^2}{2} + \frac{a^3ex^3}{3} + \frac{a^3fx^4}{4} + \frac{3a^2bcx^5}{5} + \frac{a^2bdx^6}{2} + \frac{3a^2bex^7}{7} + \frac{3a^2bfx^8}{8} + \frac{ab^2cx^9}{3} + \frac{3ab^2dx^{10}}{10} + \frac{3ab^2ex^{11}}{11} + \frac{ab^2fx^{12}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**3,x)

[Out] a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + a**3*f*x**4/4 + 3*a**2*b*c*x**5/5 + a**2*b*d*x**6/2 + 3*a**2*b*e*x**7/7 + 3*a**2*b*f*x**8/8 + a*b**2*c*x**9/3 + 3*a*b**2*d*x**10/10 + 3*a*b**2*e*x**11/11 + a*b**2*f*x**12/4 + b**3*c*x**13/13 + b**3*d*x**14/14 + b**3*e*x**15/15 + b**3*f*x**16/16

Giac [A] time = 1.06258, size = 208, normalized size = 1.38

$$\frac{1}{16} b^3 f x^{16} + \frac{1}{15} b^3 x^{15} e + \frac{1}{14} b^3 d x^{14} + \frac{1}{13} b^3 c x^{13} + \frac{1}{4} a b^2 f x^{12} + \frac{3}{11} a b^2 x^{11} e + \frac{3}{10} a b^2 d x^{10} + \frac{1}{3} a b^2 c x^9 + \frac{3}{8} a^2 b f x^8 + \frac{3}{7} a^2 b x^7 e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="giac")

[Out] 1/16*b^3*f*x^16 + 1/15*b^3*x^15*e + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/4*a*b^2*f*x^12 + 3/11*a*b^2*x^11*e + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3/8*a^2*b*f*x^8 + 3/7*a^2*b*x^7*e + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*a^3*f*x^4 + 1/3*a^3*x^3*e + 1/2*a^3*d*x^2 + a^3*c*x

$$3.482 \quad \int x^3 (c + dx + ex^2 + fx^3) (a + bx^4)^3 dx$$

Optimal. Leaf size=156

$$\frac{1}{3}a^2bdx^9 + \frac{3}{10}a^2bex^{10} + \frac{3}{11}a^2bfex^{11} + \frac{1}{5}a^3dx^5 + \frac{1}{6}a^3ex^6 + \frac{1}{7}a^3fx^7 + \frac{3}{13}ab^2dx^{13} + \frac{3}{14}ab^2ex^{14} + \frac{1}{5}ab^2fx^{15} + \frac{c(a + bx^4)^3}{16b}$$

[Out] (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^3*f*x^7)/7 + (a^2*b*d*x^9)/3 + (3*a^2*b*e*x^10)/10 + (3*a^2*b*f*x^11)/11 + (3*a*b^2*d*x^13)/13 + (3*a*b^2*e*x^14)/14 + (a*b^2*f*x^15)/5 + (b^3*d*x^17)/17 + (b^3*e*x^18)/18 + (b^3*f*x^19)/19 + (c*(a + b*x^4)^4)/(16*b)

Rubi [A] time = 0.112745, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1582, 1850}

$$\frac{1}{3}a^2bdx^9 + \frac{3}{10}a^2bex^{10} + \frac{3}{11}a^2bfex^{11} + \frac{1}{5}a^3dx^5 + \frac{1}{6}a^3ex^6 + \frac{1}{7}a^3fx^7 + \frac{3}{13}ab^2dx^{13} + \frac{3}{14}ab^2ex^{14} + \frac{1}{5}ab^2fx^{15} + \frac{c(a + bx^4)^3}{16b}$$

Antiderivative was successfully verified.

[In] Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]

[Out] (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^3*f*x^7)/7 + (a^2*b*d*x^9)/3 + (3*a^2*b*e*x^10)/10 + (3*a^2*b*f*x^11)/11 + (3*a*b^2*d*x^13)/13 + (3*a*b^2*e*x^14)/14 + (a*b^2*f*x^15)/5 + (b^3*d*x^17)/17 + (b^3*e*x^18)/18 + (b^3*f*x^19)/19 + (c*(a + b*x^4)^4)/(16*b)

Rule 1582

Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_)] /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a + b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]

Rule 1850

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned} \int x^3 (c + dx + ex^2 + fx^3) (a + bx^4)^3 dx &= \frac{c(a + bx^4)^4}{16b} + \int (a + bx^4)^3 (-cx^3 + x^3(c + dx + ex^2 + fx^3)) dx \\ &= \frac{c(a + bx^4)^4}{16b} + \int (a^3 dx^4 + a^3 ex^5 + a^3 fx^6 + 3a^2 b dx^8 + 3a^2 b ex^9 + 3a^2 b fx^{10} + \\ &= \frac{1}{5} a^3 dx^5 + \frac{1}{6} a^3 ex^6 + \frac{1}{7} a^3 fx^7 + \frac{1}{3} a^2 b dx^9 + \frac{3}{10} a^2 b ex^{10} + \frac{3}{11} a^2 b fx^{11} + \frac{3}{13} ab^2 dx^{13} + \frac{3}{14} ab^2 ex^{14} + \frac{3}{15} ab^2 fx^{15} + \frac{3}{16} ab^2 dx^{17} + \frac{3}{17} ab^2 ex^{18} + \frac{3}{18} ab^2 fx^{19} \end{aligned}$$

Mathematica [A] time = 0.0052431, size = 185, normalized size = 1.19

$$\frac{3}{8} a^2 b c x^8 + \frac{1}{3} a^2 b d x^9 + \frac{3}{10} a^2 b e x^{10} + \frac{3}{11} a^2 b f x^{11} + \frac{1}{4} a^3 c x^4 + \frac{1}{5} a^3 d x^5 + \frac{1}{6} a^3 e x^6 + \frac{1}{7} a^3 f x^7 + \frac{1}{4} a b^2 c x^{12} + \frac{3}{13} a b^2 d x^{13} + \frac{3}{14} a b^2 e x^{14} + \frac{3}{15} a b^2 f x^{15} + \frac{3}{16} a b^2 d x^{17} + \frac{3}{17} a b^2 e x^{18} + \frac{3}{18} a b^2 f x^{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] (a^3*c*x^4)/4 + (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^3*f*x^7)/7 + (3*a^2*b*c*x^8)/8 + (a^2*b*d*x^9)/3 + (3*a^2*b*e*x^10)/10 + (3*a^2*b*f*x^11)/11 + (a*b^2*c*x^12)/4 + (3*a*b^2*d*x^13)/13 + (3*a*b^2*e*x^14)/14 + (a*b^2*f*x^15)/5 + (b^3*c*x^16)/16 + (b^3*d*x^17)/17 + (b^3*e*x^18)/18 + (b^3*f*x^19)/19
```

Maple [A] time = 0.001, size = 154, normalized size = 1.

$$\frac{b^3 f x^{19}}{19} + \frac{b^3 e x^{18}}{18} + \frac{b^3 d x^{17}}{17} + \frac{b^3 c x^{16}}{16} + \frac{a b^2 f x^{15}}{5} + \frac{3 a b^2 e x^{14}}{14} + \frac{3 a b^2 d x^{13}}{13} + \frac{a c b^2 x^{12}}{4} + \frac{3 a^2 b f x^{11}}{11} + \frac{3 a^2 b e x^{10}}{10} + \frac{a^2 b d x^9}{3} + \frac{a^3 f x^7}{7} + \frac{a^3 e x^6}{6} + \frac{a^3 d x^5}{5} + \frac{a^3 c x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x)
```

```
[Out] 1/19*b^3*f*x^19+1/18*b^3*e*x^18+1/17*b^3*d*x^17+1/16*b^3*c*x^16+1/5*a*b^2*f*x^15+3/14*a*b^2*e*x^14+3/13*a*b^2*d*x^13+1/4*a*c*b^2*x^12+3/11*a^2*b*f*x^11+3/10*a^2*b*e*x^10+1/3*a^2*b*d*x^9+a^3*f*x^7+a^3*e*x^6+a^3*d*x^5+a^3*c*x^4
```

$$1+3/10*a^2*b*e*x^10+1/3*a^2*b*d*x^9+3/8*b*a^2*c*x^8+1/7*a^3*f*x^7+1/6*a^3*e*x^6+1/5*a^3*d*x^5+1/4*a^3*c*x^4$$

Maxima [A] time = 0.916298, size = 207, normalized size = 1.33

$$\frac{1}{19}b^3fx^{19} + \frac{1}{18}b^3ex^{18} + \frac{1}{17}b^3dx^{17} + \frac{1}{16}b^3cx^{16} + \frac{1}{5}ab^2fx^{15} + \frac{3}{14}ab^2ex^{14} + \frac{3}{13}ab^2dx^{13} + \frac{1}{4}ab^2cx^{12} + \frac{3}{11}a^2bfx^{11} + \frac{3}{10}a^2bex^{10} + \frac{1}{3}a^2bdfx^9 + \frac{3}{8}a^2bdcx^8 + \frac{1}{7}a^3f*x^7 + \frac{1}{6}a^3e*x^6 + \frac{1}{5}a^3d*x^5 + \frac{1}{4}a^3c*x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="maxima")

[Out] 1/19*b^3*f*x^19 + 1/18*b^3*e*x^18 + 1/17*b^3*d*x^17 + 1/16*b^3*c*x^16 + 1/5*a*b^2*f*x^15 + 3/14*a*b^2*e*x^14 + 3/13*a*b^2*d*x^13 + 1/4*a*b^2*c*x^12 + 3/11*a^2*b*f*x^11 + 3/10*a^2*b*e*x^10 + 1/3*a^2*b*d*x^9 + 3/8*a^2*b*c*x^8 + 1/7*a^3*f*x^7 + 1/6*a^3*e*x^6 + 1/5*a^3*d*x^5 + 1/4*a^3*c*x^4

Fricas [A] time = 1.44069, size = 390, normalized size = 2.5

$$\frac{1}{19}x^{19}fb^3 + \frac{1}{18}x^{18}eb^3 + \frac{1}{17}x^{17}db^3 + \frac{1}{16}x^{16}cb^3 + \frac{1}{5}x^{15}fb^2a + \frac{3}{14}x^{14}eb^2a + \frac{3}{13}x^{13}db^2a + \frac{1}{4}x^{12}cb^2a + \frac{3}{11}x^{11}fba^2 + \frac{3}{10}x^{10}eba^2 + \frac{1}{3}x^9fda^2 + \frac{3}{8}x^8ebca^2 + \frac{1}{7}x^7fda^3 + \frac{1}{6}x^6eba^3 + \frac{1}{5}x^5fda^3 + \frac{1}{4}x^4fca^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="fricas")

[Out] 1/19*x^19*f*b^3 + 1/18*x^18*e*b^3 + 1/17*x^17*d*b^3 + 1/16*x^16*c*b^3 + 1/5*x^15*f*b^2*a + 3/14*x^14*e*b^2*a + 3/13*x^13*d*b^2*a + 1/4*x^12*c*b^2*a + 3/11*x^11*f*b*a^2 + 3/10*x^10*e*b*a^2 + 1/3*x^9*d*b*a^2 + 3/8*x^8*c*b*a^2 + 1/7*x^7*f*a^3 + 1/6*x^6*e*a^3 + 1/5*x^5*d*a^3 + 1/4*x^4*c*a^3

Sympy [A] time = 0.084306, size = 184, normalized size = 1.18

$$\frac{a^3cx^4}{4} + \frac{a^3dx^5}{5} + \frac{a^3ex^6}{6} + \frac{a^3fx^7}{7} + \frac{3a^2bcx^8}{8} + \frac{a^2bdx^9}{3} + \frac{3a^2bex^{10}}{10} + \frac{3a^2bfx^{11}}{11} + \frac{ab^2cx^{12}}{4} + \frac{3ab^2dx^{13}}{13} + \frac{3ab^2ex^{14}}{14} + \frac{ab^2fx^{15}}{5} + \frac{3ab^2cx^{16}}{16} + \frac{ab^2dx^{17}}{17} + \frac{ab^2ex^{18}}{18} + \frac{ab^2fx^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**3,x)

[Out] a**3*c*x**4/4 + a**3*d*x**5/5 + a**3*e*x**6/6 + a**3*f*x**7/7 + 3*a**2*b*c*x**8/8 + a**2*b*d*x**9/3 + 3*a**2*b*e*x**10/10 + 3*a**2*b*f*x**11/11 + a*b**2*c*x**12/4 + 3*a*b**2*d*x**13/13 + 3*a*b**2*e*x**14/14 + a*b**2*f*x**15/5 + b**3*c*x**16/16 + b**3*d*x**17/17 + b**3*e*x**18/18 + b**3*f*x**19/19

Giac [A] time = 1.07847, size = 212, normalized size = 1.36

$$\frac{1}{19} b^3 f x^{19} + \frac{1}{18} b^3 x^{18} e + \frac{1}{17} b^3 d x^{17} + \frac{1}{16} b^3 c x^{16} + \frac{1}{5} a b^2 f x^{15} + \frac{3}{14} a b^2 x^{14} e + \frac{3}{13} a b^2 d x^{13} + \frac{1}{4} a b^2 c x^{12} + \frac{3}{11} a^2 b f x^{11} + \frac{3}{10} a^2 b d x^{10} e + \frac{1}{3} a^2 b d x^9 + \frac{3}{8} a^2 b c x^8 + \frac{1}{7} a^3 f x^7 + \frac{1}{6} a^3 x^6 e + \frac{1}{5} a^3 d x^5 + \frac{1}{4} a^3 c x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="giac")

[Out] 1/19*b^3*f*x^19 + 1/18*b^3*x^18*e + 1/17*b^3*d*x^17 + 1/16*b^3*c*x^16 + 1/5*a*b^2*f*x^15 + 3/14*a*b^2*x^14*e + 3/13*a*b^2*d*x^13 + 1/4*a*b^2*c*x^12 + 3/11*a^2*b*f*x^11 + 3/10*a^2*b*d*x^10*e + 1/3*a^2*b*d*x^9 + 3/8*a^2*b*c*x^8 + 1/7*a^3*f*x^7 + 1/6*a^3*x^6*e + 1/5*a^3*d*x^5 + 1/4*a^3*c*x^4

3.483 $\int (c + dx + ex^2 + fx^3)(a + bx^4)^4 dx$

Optimal. Leaf size=193

$$\frac{2}{3}a^2b^2cx^9 + \frac{3}{5}a^2b^2dx^{10} + \frac{6}{11}a^2b^2ex^{11} + \frac{4}{5}a^3bcx^5 + \frac{2}{3}a^3bdx^6 + \frac{4}{7}a^3bex^7 + a^4cx + \frac{1}{2}a^4dx^2 + \frac{1}{3}a^4ex^3 + \frac{4}{13}ab^3cx^{13} + \frac{2}{7}ab^3$$

[Out] $a^4c*x + (a^4*d*x^2)/2 + (a^4*e*x^3)/3 + (4*a^3*b*c*x^5)/5 + (2*a^3*b*d*x^6)/3 + (4*a^3*b*e*x^7)/7 + (2*a^2*b^2*c*x^9)/3 + (3*a^2*b^2*d*x^{10})/5 + (6*a^2*b^2*e*x^{11})/11 + (4*a*b^3*c*x^{13})/13 + (2*a*b^3*d*x^{14})/7 + (4*a*b^3*e*x^{15})/15 + (b^4*c*x^{17})/17 + (b^4*d*x^{18})/18 + (b^4*e*x^{19})/19 + (f*(a + b*x^4)^5)/(20*b)$

Rubi [A] time = 0.15563, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$, Rules used = {1582, 1657}

$$\frac{2}{3}a^2b^2cx^9 + \frac{3}{5}a^2b^2dx^{10} + \frac{6}{11}a^2b^2ex^{11} + \frac{4}{5}a^3bcx^5 + \frac{2}{3}a^3bdx^6 + \frac{4}{7}a^3bex^7 + a^4cx + \frac{1}{2}a^4dx^2 + \frac{1}{3}a^4ex^3 + \frac{4}{13}ab^3cx^{13} + \frac{2}{7}ab^3$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]

[Out] $a^4c*x + (a^4*d*x^2)/2 + (a^4*e*x^3)/3 + (4*a^3*b*c*x^5)/5 + (2*a^3*b*d*x^6)/3 + (4*a^3*b*e*x^7)/7 + (2*a^2*b^2*c*x^9)/3 + (3*a^2*b^2*d*x^{10})/5 + (6*a^2*b^2*e*x^{11})/11 + (4*a*b^3*c*x^{13})/13 + (2*a*b^3*d*x^{14})/7 + (4*a*b^3*e*x^{15})/15 + (b^4*c*x^{17})/17 + (b^4*d*x^{18})/18 + (b^4*e*x^{19})/19 + (f*(a + b*x^4)^5)/(20*b)$

Rule 1582

Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a + b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]

Rule 1657

```
Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned} \int (c + dx + ex^2 + fx^3)(a + bx^4)^4 dx &= \frac{f(a + bx^4)^5}{20b} + \int (c + dx + ex^2)(a + bx^4)^4 dx \\ &= \frac{f(a + bx^4)^5}{20b} + \int (a^4c + a^4dx + a^4ex^2 + 4a^3bcx^4 + 4a^3bdx^5 + 4a^3bex^6 + 6a^2b^2c \\ &= a^4cx + \frac{1}{2}a^4dx^2 + \frac{1}{3}a^4ex^3 + \frac{4}{5}a^3bcx^5 + \frac{2}{3}a^3bdx^6 + \frac{4}{7}a^3bex^7 + \frac{2}{3}a^2b^2cx^9 + \frac{3}{5}a^2b^2c \end{aligned}$$

Mathematica [A] time = 0.0055653, size = 236, normalized size = 1.22

$$\frac{2}{3}a^2b^2cx^9 + \frac{3}{5}a^2b^2dx^{10} + \frac{6}{11}a^2b^2ex^{11} + \frac{1}{2}a^2b^2fx^{12} + \frac{4}{5}a^3bcx^5 + \frac{2}{3}a^3bdx^6 + \frac{4}{7}a^3bex^7 + \frac{1}{2}a^3bfx^8 + a^4cx + \frac{1}{2}a^4dx^2 + \frac{1}{3}a^4e$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + (a^4*e*x^3)/3 + (a^4*f*x^4)/4 + (4*a^3*b*c*x^5)/5
+ (2*a^3*b*d*x^6)/3 + (4*a^3*b*e*x^7)/7 + (a^3*b*f*x^8)/2 + (2*a^2*b^2*c*x
^9)/3 + (3*a^2*b^2*d*x^10)/5 + (6*a^2*b^2*e*x^11)/11 + (a^2*b^2*f*x^12)/2 +
(4*a*b^3*c*x^13)/13 + (2*a*b^3*d*x^14)/7 + (4*a*b^3*e*x^15)/15 + (a*b^3*f*
x^16)/4 + (b^4*c*x^17)/17 + (b^4*d*x^18)/18 + (b^4*e*x^19)/19 + (b^4*f*x^20
)/20
```

Maple [A] time = 0.001, size = 199, normalized size = 1.

$$\frac{fb^4x^{20}}{20} + \frac{b^4ex^{19}}{19} + \frac{b^4dx^{18}}{18} + \frac{b^4cx^{17}}{17} + \frac{fab^3x^{16}}{4} + \frac{4ab^3ex^{15}}{15} + \frac{2ab^3dx^{14}}{7} + \frac{4ab^3cx^{13}}{13} + \frac{fb^2a^2x^{12}}{2} + \frac{6a^2b^2ex^{11}}{11} + \frac{3a^2b^2c}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x)
```

[Out] $\frac{1}{20}f*b^4*x^{20} + \frac{1}{19}b^4*e*x^{19} + \frac{1}{18}b^4*d*x^{18} + \frac{1}{17}b^4*c*x^{17} + \frac{1}{4}f*a*b^3*x^{16} + \frac{4}{15}a*b^3*e*x^{15} + \frac{2}{7}a*b^3*d*x^{14} + \frac{4}{13}a*b^3*c*x^{13} + \frac{1}{2}f*b^2*a^2*x^{12} + \frac{6}{11}a^2*b^2*e*x^{11} + \frac{3}{5}a^2*b^2*d*x^{10} + \frac{2}{3}a^2*b^2*c*x^9 + \frac{1}{2}b*f*a^3*x^8 + \frac{4}{7}a^3*b*e*x^7 + \frac{2}{3}a^3*b*d*x^6 + \frac{4}{5}a^3*b*c*x^5 + \frac{1}{4}a^4*f*x^4 + \frac{1}{3}a^4*e*x^3 + \frac{1}{2}a^4*d*x^2 + a^4*c*x$

Maxima [A] time = 0.931103, size = 267, normalized size = 1.38

$$\frac{1}{20}b^4fx^{20} + \frac{1}{19}b^4ex^{19} + \frac{1}{18}b^4dx^{18} + \frac{1}{17}b^4cx^{17} + \frac{1}{4}ab^3fx^{16} + \frac{4}{15}ab^3ex^{15} + \frac{2}{7}ab^3dx^{14} + \frac{4}{13}ab^3cx^{13} + \frac{1}{2}a^2b^2fx^{12} + \frac{6}{11}a^2b^2ex^{11} + \frac{3}{5}a^2b^2dx^{10} + \frac{2}{3}a^2b^2cx^9 + \frac{1}{2}a^3bfx^8 + \frac{4}{7}a^3bex^7 + \frac{2}{3}a^3bdx^6 + \frac{4}{5}a^3bcx^5 + \frac{1}{4}a^4fx^4 + \frac{1}{3}a^4ex^3 + \frac{1}{2}a^4dx^2 + a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="maxima")`

[Out] $\frac{1}{20}b^4*f*x^{20} + \frac{1}{19}b^4*e*x^{19} + \frac{1}{18}b^4*d*x^{18} + \frac{1}{17}b^4*c*x^{17} + \frac{1}{4}a*b^3*f*x^{16} + \frac{4}{15}a*b^3*e*x^{15} + \frac{2}{7}a*b^3*d*x^{14} + \frac{4}{13}a*b^3*c*x^{13} + \frac{1}{2}a^2*b^2*f*x^{12} + \frac{6}{11}a^2*b^2*e*x^{11} + \frac{3}{5}a^2*b^2*d*x^{10} + \frac{2}{3}a^2*b^2*c*x^9 + \frac{1}{2}a^3*b*f*x^8 + \frac{4}{7}a^3*b*e*x^7 + \frac{2}{3}a^3*b*d*x^6 + \frac{4}{5}a^3*b*c*x^5 + \frac{1}{4}a^4*f*x^4 + \frac{1}{3}a^4*e*x^3 + \frac{1}{2}a^4*d*x^2 + a^4*c*x$

Fricas [A] time = 1.48056, size = 490, normalized size = 2.54

$$\frac{1}{20}x^{20}fb^4 + \frac{1}{19}x^{19}eb^4 + \frac{1}{18}x^{18}db^4 + \frac{1}{17}x^{17}cb^4 + \frac{1}{4}x^{16}fb^3a + \frac{4}{15}x^{15}eb^3a + \frac{2}{7}x^{14}db^3a + \frac{4}{13}x^{13}cb^3a + \frac{1}{2}x^{12}fb^2a^2 + \frac{6}{11}x^{11}eb^2a^2 + \frac{3}{5}x^{10}db^2a^2 + \frac{2}{3}x^9cb^2a^2 + \frac{1}{2}x^8fba^3 + \frac{4}{7}x^7eba^3 + \frac{2}{3}x^6dba^3 + \frac{4}{5}x^5cba^3 + \frac{1}{4}x^4fa^4 + \frac{1}{3}x^3ea^4 + \frac{1}{2}x^2da^4 + xca^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="fricas")`

[Out] $\frac{1}{20}x^{20}*f*b^4 + \frac{1}{19}x^{19}*e*b^4 + \frac{1}{18}x^{18}*d*b^4 + \frac{1}{17}x^{17}*c*b^4 + \frac{1}{4}x^{16}*f*b^3*a + \frac{4}{15}x^{15}*e*b^3*a + \frac{2}{7}x^{14}*d*b^3*a + \frac{4}{13}x^{13}*c*b^3*a + \frac{1}{2}x^{12}*f*b^2*a^2 + \frac{6}{11}x^{11}*e*b^2*a^2 + \frac{3}{5}x^{10}*d*b^2*a^2 + \frac{2}{3}x^9*c*b^2*a^2 + \frac{1}{2}x^8*f*b*a^3 + \frac{4}{7}x^7*e*b*a^3 + \frac{2}{3}x^6*d*b*a^3 + \frac{4}{5}x^5*c*b*a^3 + \frac{1}{4}x^4*f*a^4 + \frac{1}{3}x^3*e*a^4 + \frac{1}{2}x^2*d*a^4 + x*c*a^4$

Sympy [A] time = 0.089925, size = 241, normalized size = 1.25

$$a^4cx + \frac{a^4dx^2}{2} + \frac{a^4ex^3}{3} + \frac{a^4fx^4}{4} + \frac{4a^3bcx^5}{5} + \frac{2a^3bdx^6}{3} + \frac{4a^3bex^7}{7} + \frac{a^3bfx^8}{2} + \frac{2a^2b^2cx^9}{3} + \frac{3a^2b^2dx^{10}}{5} + \frac{6a^2b^2ex^{11}}{11} + \frac{a^2b^2fx^{12}}{2} + \frac{6a^2b^2ex^{11}}{11} + \frac{3a^2b^2dx^{10}}{5} + \frac{2a^2b^2cx^9}{3} + \frac{a^3bfx^8}{2} + \frac{4a^3bex^7}{7} + \frac{2a^3bdx^6}{3} + \frac{4a^3bcx^5}{5} + \frac{a^4fx^4}{4} + \frac{a^4ex^3}{3} + \frac{a^4dx^2}{2} + a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**4,x)

[Out] a**4*c*x + a**4*d*x**2/2 + a**4*e*x**3/3 + a**4*f*x**4/4 + 4*a**3*b*c*x**5/5 + 2*a**3*b*d*x**6/3 + 4*a**3*b*e*x**7/7 + a**3*b*f*x**8/2 + 2*a**2*b**2*c*x**9/3 + 3*a**2*b**2*d*x**10/5 + 6*a**2*b**2*e*x**11/11 + a**2*b**2*f*x**12/2 + 4*a*b**3*c*x**13/13 + 2*a*b**3*d*x**14/7 + 4*a*b**3*e*x**15/15 + a*b**3*f*x**16/4 + b**4*c*x**17/17 + b**4*d*x**18/18 + b**4*e*x**19/19 + b**4*f*x**20/20

Giac [A] time = 1.10567, size = 274, normalized size = 1.42

$$\frac{1}{20} b^4 f x^{20} + \frac{1}{19} b^4 x^{19} e + \frac{1}{18} b^4 d x^{18} + \frac{1}{17} b^4 c x^{17} + \frac{1}{4} a b^3 f x^{16} + \frac{4}{15} a b^3 x^{15} e + \frac{2}{7} a b^3 d x^{14} + \frac{4}{13} a b^3 c x^{13} + \frac{1}{2} a^2 b^2 f x^{12} + \frac{6}{11} a^2 b^2 d x^{11} e + \frac{3}{5} a^2 b^2 c x^{10} + \frac{2}{3} a^2 b^2 c x^9 + \frac{1}{2} a^3 b f x^8 + \frac{4}{7} a^3 b x^7 e + \frac{2}{3} a^3 b d x^6 + \frac{4}{5} a^3 b c x^5 + \frac{1}{4} a^4 f x^4 + \frac{1}{3} a^4 x^3 e + \frac{1}{2} a^4 d x^2 + a^4 c x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="giac")

[Out] 1/20*b^4*f*x^20 + 1/19*b^4*x^19*e + 1/18*b^4*d*x^18 + 1/17*b^4*c*x^17 + 1/4*a*b^3*f*x^16 + 4/15*a*b^3*x^15*e + 2/7*a*b^3*d*x^14 + 4/13*a*b^3*c*x^13 + 1/2*a^2*b^2*f*x^12 + 6/11*a^2*b^2*x^11*e + 3/5*a^2*b^2*d*x^10 + 2/3*a^2*b^2*c*x^9 + 1/2*a^3*b*f*x^8 + 4/7*a^3*b*x^7*e + 2/3*a^3*b*d*x^6 + 4/5*a^3*b*c*x^5 + 1/4*a^4*f*x^4 + 1/3*a^4*x^3*e + 1/2*a^4*d*x^2 + a^4*c*x

$$3.484 \quad \int x^3 (c + dx + ex^2 + fx^3) (a + bx^4)^4 dx$$

Optimal. Leaf size=198

$$\frac{6}{13}a^2b^2dx^{13} + \frac{3}{7}a^2b^2ex^{14} + \frac{2}{5}a^2b^2fx^{15} + \frac{4}{9}a^3bdx^9 + \frac{2}{5}a^3bex^{10} + \frac{4}{11}a^3bfx^{11} + \frac{1}{5}a^4dx^5 + \frac{1}{6}a^4ex^6 + \frac{1}{7}a^4fx^7 + \frac{4}{17}ab^3dx^{17}$$

[Out] (a^4*d*x^5)/5 + (a^4*e*x^6)/6 + (a^4*f*x^7)/7 + (4*a^3*b*d*x^9)/9 + (2*a^3*b*e*x^10)/5 + (4*a^3*b*f*x^11)/11 + (6*a^2*b^2*d*x^13)/13 + (3*a^2*b^2*e*x^14)/7 + (2*a^2*b^2*f*x^15)/5 + (4*a*b^3*d*x^17)/17 + (2*a*b^3*e*x^18)/9 + (4*a*b^3*f*x^19)/19 + (b^4*d*x^21)/21 + (b^4*e*x^22)/22 + (b^4*f*x^23)/23 + (c*(a + b*x^4)^5)/(20*b)

Rubi [A] time = 0.150432, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1582, 1850}

$$\frac{6}{13}a^2b^2dx^{13} + \frac{3}{7}a^2b^2ex^{14} + \frac{2}{5}a^2b^2fx^{15} + \frac{4}{9}a^3bdx^9 + \frac{2}{5}a^3bex^{10} + \frac{4}{11}a^3bfx^{11} + \frac{1}{5}a^4dx^5 + \frac{1}{6}a^4ex^6 + \frac{1}{7}a^4fx^7 + \frac{4}{17}ab^3dx^{17}$$

Antiderivative was successfully verified.

[In] Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]

[Out] (a^4*d*x^5)/5 + (a^4*e*x^6)/6 + (a^4*f*x^7)/7 + (4*a^3*b*d*x^9)/9 + (2*a^3*b*e*x^10)/5 + (4*a^3*b*f*x^11)/11 + (6*a^2*b^2*d*x^13)/13 + (3*a^2*b^2*e*x^14)/7 + (2*a^2*b^2*f*x^15)/5 + (4*a*b^3*d*x^17)/17 + (2*a*b^3*e*x^18)/9 + (4*a*b^3*f*x^19)/19 + (b^4*d*x^21)/21 + (b^4*e*x^22)/22 + (b^4*f*x^23)/23 + (c*(a + b*x^4)^5)/(20*b)

Rule 1582

Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a + b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]

Rule 1850

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned} \int x^3 (c + dx + ex^2 + fx^3) (a + bx^4)^4 dx &= \frac{c(a + bx^4)^5}{20b} + \int (a + bx^4)^4 (-cx^3 + x^3(c + dx + ex^2 + fx^3)) dx \\ &= \frac{c(a + bx^4)^5}{20b} + \int (a^4 dx^4 + a^4 ex^5 + a^4 fx^6 + 4a^3 b dx^8 + 4a^3 b ex^9 + 4a^3 b fx^{10} + \\ &= \frac{1}{5} a^4 dx^5 + \frac{1}{6} a^4 ex^6 + \frac{1}{7} a^4 fx^7 + \frac{4}{9} a^3 b dx^9 + \frac{2}{5} a^3 b ex^{10} + \frac{4}{11} a^3 b fx^{11} + \frac{6}{13} a^2 b^2 dx^{13} \end{aligned}$$

Mathematica [A] time = 0.0062771, size = 241, normalized size = 1.22

$$\frac{1}{2} a^2 b^2 c x^{12} + \frac{6}{13} a^2 b^2 d x^{13} + \frac{3}{7} a^2 b^2 e x^{14} + \frac{2}{5} a^2 b^2 f x^{15} + \frac{1}{2} a^3 b c x^8 + \frac{4}{9} a^3 b d x^9 + \frac{2}{5} a^3 b e x^{10} + \frac{4}{11} a^3 b f x^{11} + \frac{1}{4} a^4 c x^4 + \frac{1}{5} a^4 d x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

```
[Out] (a^4*c*x^4)/4 + (a^4*d*x^5)/5 + (a^4*e*x^6)/6 + (a^4*f*x^7)/7 + (a^3*b*c*x^8)/2 + (4*a^3*b*d*x^9)/9 + (2*a^3*b*e*x^10)/5 + (4*a^3*b*f*x^11)/11 + (a^2*b^2*c*x^12)/2 + (6*a^2*b^2*d*x^13)/13 + (3*a^2*b^2*e*x^14)/7 + (2*a^2*b^2*f*x^15)/5 + (a*b^3*c*x^16)/4 + (4*a*b^3*d*x^17)/17 + (2*a*b^3*e*x^18)/9 + (4*a*b^3*f*x^19)/19 + (b^4*c*x^20)/20 + (b^4*d*x^21)/21 + (b^4*e*x^22)/22 + (b^4*f*x^23)/23
```

Maple [A] time = 0.001, size = 202, normalized size = 1.

$$\frac{b^4 f x^{23}}{23} + \frac{b^4 e x^{22}}{22} + \frac{b^4 d x^{21}}{21} + \frac{c b^4 x^{20}}{20} + \frac{4 a b^3 f x^{19}}{19} + \frac{2 a b^3 e x^{18}}{9} + \frac{4 a b^3 d x^{17}}{17} + \frac{a c b^3 x^{16}}{4} + \frac{2 a^2 b^2 f x^{15}}{5} + \frac{3 a^2 b^2 e x^{14}}{7} + \frac{6 a^2 b^2 d x^{13}}{13} + \frac{2 a^2 b^2 c x^{12}}{2} + \frac{4 a^3 b f x^{11}}{11} + \frac{2 a^3 b e x^{10}}{5} + \frac{4 a^3 b d x^9}{9} + \frac{1}{2} a^3 b c x^8 + \frac{1}{4} a^4 c x^4 + \frac{1}{5} a^4 d x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x)
```

[Out] $\frac{1}{23}b^4fx^{23} + \frac{1}{22}b^4ex^{22} + \frac{1}{21}b^4dx^{21} + \frac{1}{20}c^3b^4x^{20} + \frac{4}{19}a^3b^3fx^{19} + \frac{2}{9}a^3b^3ex^{18} + \frac{4}{17}a^3b^3dx^{17} + \frac{1}{4}a^3c^3b^3x^{16} + \frac{2}{5}a^2b^2fx^{15} + \frac{3}{7}a^2b^2ex^{14} + \frac{6}{13}a^2b^2dx^{13} + \frac{1}{2}a^2b^2c^3x^{12} + \frac{4}{11}a^3b^3fx^{11} + \frac{2}{5}a^3b^3ex^{10} + \frac{4}{9}a^3b^3dx^9 + \frac{1}{2}c^3a^3b^3x^8 + \frac{1}{7}a^4fx^7 + \frac{1}{6}a^4ex^6 + \frac{1}{5}a^4dx^5 + \frac{1}{4}a^4c^3x^4$

Maxima [A] time = 0.920182, size = 271, normalized size = 1.37

$$\frac{1}{23}b^4fx^{23} + \frac{1}{22}b^4ex^{22} + \frac{1}{21}b^4dx^{21} + \frac{1}{20}b^4cx^{20} + \frac{4}{19}ab^3fx^{19} + \frac{2}{9}ab^3ex^{18} + \frac{4}{17}ab^3dx^{17} + \frac{1}{4}ab^3cx^{16} + \frac{2}{5}a^2b^2fx^{15} + \frac{3}{7}a^2b^2ex^{14} + \frac{6}{13}a^2b^2dx^{13} + \frac{1}{2}a^2b^2c^3x^{12} + \frac{4}{11}a^3b^3fx^{11} + \frac{2}{5}a^3b^3ex^{10} + \frac{4}{9}a^3b^3dx^9 + \frac{1}{2}c^3a^3b^3x^8 + \frac{1}{7}a^4fx^7 + \frac{1}{6}a^4ex^6 + \frac{1}{5}a^4dx^5 + \frac{1}{4}a^4c^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="maxima")`

[Out] $\frac{1}{23}b^4fx^{23} + \frac{1}{22}b^4ex^{22} + \frac{1}{21}b^4dx^{21} + \frac{1}{20}b^4cx^{20} + \frac{4}{19}a^3b^3fx^{19} + \frac{2}{9}a^3b^3ex^{18} + \frac{4}{17}a^3b^3dx^{17} + \frac{1}{4}a^3b^3cx^{16} + \frac{2}{5}a^2b^2fx^{15} + \frac{3}{7}a^2b^2ex^{14} + \frac{6}{13}a^2b^2dx^{13} + \frac{1}{2}a^2b^2c^3x^{12} + \frac{4}{11}a^3b^3fx^{11} + \frac{2}{5}a^3b^3ex^{10} + \frac{4}{9}a^3b^3dx^9 + \frac{1}{2}c^3a^3b^3x^8 + \frac{1}{7}a^4fx^7 + \frac{1}{6}a^4ex^6 + \frac{1}{5}a^4dx^5 + \frac{1}{4}a^4c^3x^4$

Fricas [A] time = 1.49758, size = 504, normalized size = 2.55

$$\frac{1}{23}x^{23}fb^4 + \frac{1}{22}x^{22}eb^4 + \frac{1}{21}x^{21}db^4 + \frac{1}{20}x^{20}cb^4 + \frac{4}{19}x^{19}fb^3a + \frac{2}{9}x^{18}eb^3a + \frac{4}{17}x^{17}db^3a + \frac{1}{4}x^{16}cb^3a + \frac{2}{5}x^{15}fb^2a^2 + \frac{3}{7}x^{14}eb^2a^2 + \frac{6}{13}x^{13}db^2a^2 + \frac{1}{2}x^{12}cb^2a^2 + \frac{4}{11}x^{11}fb^3a^3 + \frac{2}{5}x^{10}eb^3a^3 + \frac{4}{9}x^9db^3a^3 + \frac{1}{2}x^8cb^3a^3 + \frac{1}{7}x^7fb^4a^4 + \frac{1}{6}x^6eb^4a^4 + \frac{1}{5}x^5db^4a^4 + \frac{1}{4}x^4cb^4a^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="fricas")`

[Out] $\frac{1}{23}x^{23}fb^4 + \frac{1}{22}x^{22}eb^4 + \frac{1}{21}x^{21}db^4 + \frac{1}{20}x^{20}cb^4 + \frac{4}{19}x^{19}fb^3a + \frac{2}{9}x^{18}eb^3a + \frac{4}{17}x^{17}db^3a + \frac{1}{4}x^{16}cb^3a + \frac{2}{5}x^{15}fb^2a^2 + \frac{3}{7}x^{14}eb^2a^2 + \frac{6}{13}x^{13}db^2a^2 + \frac{1}{2}x^{12}cb^2a^2 + \frac{4}{11}x^{11}fb^3a^3 + \frac{2}{5}x^{10}eb^3a^3 + \frac{4}{9}x^9db^3a^3 + \frac{1}{2}x^8cb^3a^3 + \frac{1}{7}x^7fb^4a^4 + \frac{1}{6}x^6eb^4a^4 + \frac{1}{5}x^5db^4a^4 + \frac{1}{4}x^4cb^4a^4$

Sympy [A] time = 0.094753, size = 245, normalized size = 1.24

$$\frac{a^4cx^4}{4} + \frac{a^4dx^5}{5} + \frac{a^4ex^6}{6} + \frac{a^4fx^7}{7} + \frac{a^3bcx^8}{2} + \frac{4a^3bdx^9}{9} + \frac{2a^3bex^{10}}{5} + \frac{4a^3bfx^{11}}{11} + \frac{a^2b^2cx^{12}}{2} + \frac{6a^2b^2dx^{13}}{13} + \frac{3a^2b^2ex^{14}}{7} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**4,x)

[Out] a**4*c*x**4/4 + a**4*d*x**5/5 + a**4*e*x**6/6 + a**4*f*x**7/7 + a**3*b*c*x*
 *8/2 + 4*a**3*b*d*x**9/9 + 2*a**3*b*e*x**10/5 + 4*a**3*b*f*x**11/11 + a**2*
 b**2*c*x**12/2 + 6*a**2*b**2*d*x**13/13 + 3*a**2*b**2*e*x**14/7 + 2*a**2*b*
 *2*f*x**15/5 + a*b**3*c*x**16/4 + 4*a*b**3*d*x**17/17 + 2*a*b**3*e*x**18/9
 + 4*a*b**3*f*x**19/19 + b**4*c*x**20/20 + b**4*d*x**21/21 + b**4*e*x**22/22
 + b**4*f*x**23/23

Giac [A] time = 1.08794, size = 278, normalized size = 1.4

$$\frac{1}{23} b^4 f x^{23} + \frac{1}{22} b^4 x^{22} e + \frac{1}{21} b^4 d x^{21} + \frac{1}{20} b^4 c x^{20} + \frac{4}{19} a b^3 f x^{19} + \frac{2}{9} a b^3 x^{18} e + \frac{4}{17} a b^3 d x^{17} + \frac{1}{4} a b^3 c x^{16} + \frac{2}{5} a^2 b^2 f x^{15} + \frac{3}{7} a^2 b^2 x^{14} e + \frac{4}{11} a^2 b^2 d x^{13} + \frac{2}{5} a^2 b^2 c x^{12} + \frac{4}{11} a^3 b f x^{11} + \frac{2}{5} a^3 b x^{10} e + \frac{4}{9} a^3 b d x^9 + \frac{1}{2} a^3 b c x^8 + \frac{1}{7} a^4 f x^7 + \frac{1}{6} a^4 x^6 e + \frac{1}{5} a^4 d x^5 + \frac{1}{4} a^4 c x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="giac")

[Out] 1/23*b^4*f*x^23 + 1/22*b^4*x^22*e + 1/21*b^4*d*x^21 + 1/20*b^4*c*x^20 + 4/1
 9*a*b^3*f*x^19 + 2/9*a*b^3*x^18*e + 4/17*a*b^3*d*x^17 + 1/4*a*b^3*c*x^16 +
 2/5*a^2*b^2*f*x^15 + 3/7*a^2*b^2*x^14*e + 6/13*a^2*b^2*d*x^13 + 1/2*a^2*b^2
 *c*x^12 + 4/11*a^3*b*f*x^11 + 2/5*a^3*b*x^10*e + 4/9*a^3*b*d*x^9 + 1/2*a^3*
 b*c*x^8 + 1/7*a^4*f*x^7 + 1/6*a^4*x^6*e + 1/5*a^4*d*x^5 + 1/4*a^4*c*x^4

$$3.485 \quad \int \frac{c+dx+ex^2+fx^3}{a-bx^4} dx$$

Optimal. Leaf size=133

$$\frac{(\sqrt{bc} - \sqrt{ae}) \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{(\sqrt{ae} + \sqrt{bc}) \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{f \log(a - bx^4)}{4b}$$

[Out] ((Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)]/(2*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)]/(2*a^(3/4)*b^(3/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - (f*Log[a - b*x^4])/(4*b)

Rubi [A] time = 0.124385, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1876, 1167, 205, 208, 1248, 635, 260}

$$\frac{(\sqrt{bc} - \sqrt{ae}) \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{(\sqrt{ae} + \sqrt{bc}) \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{f \log(a - bx^4)}{4b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4), x]

[Out] ((Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)]/(2*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)]/(2*a^(3/4)*b^(3/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - (f*Log[a - b*x^4])/(4*b)

Rule 1876

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 1167

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2

$-(c*d)/(2*q), \text{Int}[1/(q + c*x^2), x], x] /; \text{FreeQ}\{a, c, d, e\}, x\} \&\& \text{NeQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[-(a*c)]$

Rule 205

$\text{Int}[(a_ + (b_.)*(x_)^2)^{-1}, x_Symbol] :> \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{PosQ}[a/b]$

Rule 208

$\text{Int}[(a_ + (b_.)*(x_)^2)^{-1}, x_Symbol] :> \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{NegQ}[a/b]$

Rule 1248

$\text{Int}[(x_)*((d_ + (e_.)*(x_)^2)^{q_})*((a_ + (c_.)*(x_)^4)^{p_}), x_Symbol] :> \text{Dist}[1/2, \text{Subst}[\text{Int}[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; \text{FreeQ}\{a, c, d, e, p, q\}, x\}$

Rule 635

$\text{Int}[(d_ + (e_.)*(x_))/((a_ + (c_.)*(x_)^2), x_Symbol] :> \text{Dist}[d, \text{Int}[1/(a + c*x^2), x], x] + \text{Dist}[e, \text{Int}[x/(a + c*x^2), x], x] /; \text{FreeQ}\{a, c, d, e\}, x\} \&\& \text{!NiceSqrtQ}[-(a*c)]$

Rule 260

$\text{Int}[(x_)^{(m_)} / ((a_ + (b_.)*(x_)^n), x_Symbol] :> \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]] / (b*n), x] /; \text{FreeQ}\{a, b, m, n\}, x\} \&\& \text{EqQ}[m, n - 1]$

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3}{a - bx^4} dx &= \int \left(\frac{c + ex^2}{a - bx^4} + \frac{x(d + fx^2)}{a - bx^4} \right) dx \\
&= \int \frac{c + ex^2}{a - bx^4} dx + \int \frac{x(d + fx^2)}{a - bx^4} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{d + fx}{a - bx^2} dx, x, x^2 \right) + \frac{1}{2} \left(-\frac{\sqrt{bc}}{\sqrt{a}} + e \right) \int \frac{1}{-\sqrt{a}\sqrt{b} - bx^2} dx + \frac{1}{2} \left(\frac{\sqrt{bc}}{\sqrt{a}} + e \right) \int \frac{1}{\sqrt{a}\sqrt{b} + bx^2} dx \\
&= \frac{(\sqrt{bc} - \sqrt{ae}) \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2a^{3/4}b^{3/4}} + \frac{(\sqrt{bc} + \sqrt{ae}) \tanh^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2a^{3/4}b^{3/4}} + \frac{1}{2} d \text{Subst} \left(\int \frac{1}{a - bx^2} dx, x, x^2 \right) \\
&= \frac{(\sqrt{bc} - \sqrt{ae}) \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2a^{3/4}b^{3/4}} + \frac{(\sqrt{bc} + \sqrt{ae}) \tanh^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2a^{3/4}b^{3/4}} + \frac{d \tanh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{b}} - \frac{f \log(a - bx^2)}{4b}
\end{aligned}$$

Mathematica [A] time = 0.0655782, size = 214, normalized size = 1.61

$$\frac{\log(\sqrt[4]{a} - \sqrt[4]{bx}) (a^{3/4}e + \sqrt[4]{a}\sqrt{bc} + \sqrt{a}\sqrt[4]{bd})}{4ab^{3/4}} - \frac{\log(\sqrt[4]{a} + \sqrt[4]{bx}) (-a^{3/4}e - \sqrt[4]{a}\sqrt{bc} + \sqrt{a}\sqrt[4]{bd})}{4ab^{3/4}} + \frac{(\sqrt[4]{a}\sqrt{bc} - a^{3/4}e) \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2ab^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4), x]

[Out] ((a^(1/4)*Sqrt[b]*c - a^(3/4)*e)*ArcTan[(b^(1/4)*x)/a^(1/4)]/(2*a*b^(3/4)) - ((a^(1/4)*Sqrt[b]*c + Sqrt[a]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) - b^(1/4)*x]/(4*a*b^(3/4)) - ((-a^(1/4)*Sqrt[b]*c) + Sqrt[a]*b^(1/4)*d - a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x]/(4*a*b^(3/4)) + (d*Log[Sqrt[a] + Sqrt[b]*x^2])/(4*Sqrt[a]*Sqrt[b]) - (f*Log[a - b*x^4])/(4*b)

Maple [A] time = 0.004, size = 177, normalized size = 1.3

$$\frac{c}{4a} \sqrt[4]{\frac{a}{b}} \ln \left(\left(x + \sqrt[4]{\frac{a}{b}} \right) \left(x - \sqrt[4]{\frac{a}{b}} \right)^{-1} \right) + \frac{c}{2a} \sqrt[4]{\frac{a}{b}} \arctan \left(x \frac{1}{\sqrt[4]{\frac{a}{b}}} \right) - \frac{d}{4} \ln \left(\left(-a + x^2 \sqrt{ab} \right) \left(-a - x^2 \sqrt{ab} \right)^{-1} \right) \frac{1}{\sqrt{ab}} - \frac{e}{2b} \arctan \left(\frac{x}{\sqrt[4]{\frac{a}{b}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x)
```

```
[Out] 1/4*c*(1/b*a)^(1/4)/a*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))+1/2*c*(1/b*a)^(1/4)/a*arctan(x/(1/b*a)^(1/4))-1/4*d/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))-1/2*e/b/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))+1/4*e/b/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))-1/4/b*f*ln(b*x^4-a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] time = 13.1386, size = 952, normalized size = 7.16

$$-\text{RootSum}\left(256t^4a^3b^4 - 256t^3a^3b^3f + t^2(96a^3b^2f^2 - 64a^2b^3ce - 32a^2b^3d^2) + t(-16a^3bf^3 + 32a^2b^2cef + 16a^2b^2d^2f - \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```



```
[Out] -RootSum(256*_t**4*a**3*b**4 - 256*_t**3*a**3*b**3*f + _t**2*(96*a**3*b**2*
f**2 - 64*a**2*b**3*c*e - 32*a**2*b**3*d**2) + _t*(-16*a**3*b*f**3 + 32*a**
2*b**2*c*e*f + 16*a**2*b**2*d**2*f - 16*a**2*b**2*d*e**2 - 16*a*b**3*c**2*d
) + a**3*f**4 - 4*a**2*b*c*e*f**2 - 2*a**2*b*d**2*f**2 + 4*a**2*b*d*e**2*f
- a**2*b*e**4 + 4*a*b**2*c**2*d*f + 2*a*b**2*c**2*e**2 - 4*a*b**2*c*d**2*e
+ a*b**2*d**4 - b**3*c**4, Lambda(_t, _t*log(x + (-64*_t**3*a**4*b**3*e**3
- 64*_t**3*a**3*b**4*c**2*e + 128*_t**3*a**3*b**4*c*d**2 + 48*_t**2*a**4*b*
*2*e**3*f + 48*_t**2*a**3*b**3*c**2*e*f - 96*_t**2*a**3*b**3*c*d**2*f + 48*
_t**2*a**3*b**3*c*d*e**2 - 32*_t**2*a**3*b**3*d**3*e - 16*_t**2*a**2*b**4*c
**3*d - 12*_t*a**4*b*e**3*f**2 - 12*_t*a**3*b**2*c**2*e*f**2 + 24*_t*a**3*b
**2*c*d**2*f**2 - 24*_t*a**3*b**2*c*d*e**2*f + 12*_t*a**3*b**2*c*e**4 + 16*
_t*a**3*b**2*d**3*e*f + 12*_t*a**3*b**2*d**2*e**3 + 8*_t*a**2*b**3*c**3*d*f
+ 16*_t*a**2*b**3*c**3*e**2 - 36*_t*a**2*b**3*c**2*d**2*e - 8*_t*a**2*b**3
*c*d**4 + 4*_t*a*b**4*c**5 + a**4*e**3*f**3 + a**3*b*c**2*e*f**3 - 2*a**3*b
*c*d**2*f**3 + 3*a**3*b*c*d*e**2*f**2 - 3*a**3*b*c*e**4*f - 2*a**3*b*d**3*e
*f**2 - 3*a**3*b*d**2*e**3*f + 3*a**3*b*d*e**5 - a**2*b**2*c**3*d*f**2 - 4*
a**2*b**2*c**3*e**2*f + 9*a**2*b**2*c**2*d**2*e*f + 2*a**2*b**2*c*d**4*f -
5*a**2*b**2*c*d**3*e**2 + 2*a**2*b**2*d**5*e - a*b**3*c**5*f + 5*a*b**3*c**
4*d*e - 5*a*b**3*c**3*d**3)/(a**3*b*e**6 + a**2*b**2*c**2*e**4 - 8*a**2*b**
2*c*d**2*e**3 + 4*a**2*b**2*d**4*e**2 - a*b**3*c**4*e**2 + 8*a*b**3*c**3*d*
*2*e - 4*a*b**3*c**2*d**4 - b**4*c**6))))
```

Giac [B] time = 1.07523, size = 419, normalized size = 3.15

$$\frac{f \log(|bx^4 - a|)}{4b} - \frac{\sqrt{2} \left(\sqrt{2} \sqrt{-abb^2d} - (-ab^3)^{\frac{1}{4}} b^2c - (-ab^3)^{\frac{3}{4}} e \right) \arctan \left(\frac{\sqrt{2} \left(2x + \sqrt{2} \left(-\frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left(-\frac{a}{b} \right)^{\frac{1}{4}}} \right)}{4ab^3} - \frac{\sqrt{2} \left(\sqrt{2} \sqrt{-abb^2d} - (-ab^3)^{\frac{1}{4}} b^2c - (-ab^3)^{\frac{3}{4}} e \right)}{4ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*f*log(abs(b*x^4 - a))/b - 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*d - (-a*
b^3)^(1/4)*b^2*c - (-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/
b)^(1/4))/(-a/b)^(1/4))/(a*b^3) - 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*d - (
-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-
a/b)^(1/4))/(-a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*c - (-
a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^3) - 1/
8*sqrt(2)*((-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(-a
/b)^(1/4) + sqrt(-a/b))/(a*b^3)
```

$$3.486 \quad \int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4} dx$$

Optimal. Leaf size=162

$$\frac{\sqrt[4]{a}(\sqrt{bd}-\sqrt{af})\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2b^{7/4}} + \frac{\sqrt[4]{a}(\sqrt{af}+\sqrt{bd})\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2b^{7/4}} + \frac{\sqrt{ae}\tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2b^{3/2}} - \frac{c\log(a-bx^4)}{4b} - \frac{dx}{b} - \frac{ex^2}{2b} - \frac{fx^3}{3b}$$

[Out] $-\left(\frac{d*x}{b}\right) - \left(\frac{e*x^2}{2*b}\right) - \left(\frac{f*x^3}{3*b}\right) + \left(a^{1/4}\right)*\left(\text{Sqrt}[b]*d - \text{Sqrt}[a]*f\right)*\text{ArcTan}\left[\left(b^{1/4}*x\right)/a^{1/4}\right]/\left(2*b^{7/4}\right) + \left(a^{1/4}\right)*\left(\text{Sqrt}[b]*d + \text{Sqrt}[a]*f\right)*\text{ArcTanh}\left[\left(b^{1/4}*x\right)/a^{1/4}\right]/\left(2*b^{7/4}\right) + \left(\text{Sqrt}[a]*e*\text{ArcTanh}\left[\left(\text{Sqrt}[b]*x^2\right)/\text{Sqrt}[a]\right]\right)/\left(2*b^{3/2}\right) - \left(c*\text{Log}[a - b*x^4]\right)/\left(4*b\right)$

Rubi [A] time = 0.202951, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.31$, Rules used = {1831, 1252, 774, 635, 208, 260, 1280, 1167, 205}

$$\frac{\sqrt[4]{a}(\sqrt{bd}-\sqrt{af})\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2b^{7/4}} + \frac{\sqrt[4]{a}(\sqrt{af}+\sqrt{bd})\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2b^{7/4}} + \frac{\sqrt{ae}\tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2b^{3/2}} - \frac{c\log(a-bx^4)}{4b} - \frac{dx}{b} - \frac{ex^2}{2b} - \frac{fx^3}{3b}$$

Antiderivative was successfully verified.

[In] $\text{Int}\left[\left(x^3*(c + d*x + e*x^2 + f*x^3)\right)/\left(a - b*x^4\right), x\right]$

[Out] $-\left(\frac{d*x}{b}\right) - \left(\frac{e*x^2}{2*b}\right) - \left(\frac{f*x^3}{3*b}\right) + \left(a^{1/4}\right)*\left(\text{Sqrt}[b]*d - \text{Sqrt}[a]*f\right)*\text{ArcTan}\left[\left(b^{1/4}*x\right)/a^{1/4}\right]/\left(2*b^{7/4}\right) + \left(a^{1/4}\right)*\left(\text{Sqrt}[b]*d + \text{Sqrt}[a]*f\right)*\text{ArcTanh}\left[\left(b^{1/4}*x\right)/a^{1/4}\right]/\left(2*b^{7/4}\right) + \left(\text{Sqrt}[a]*e*\text{ArcTanh}\left[\left(\text{Sqrt}[b]*x^2\right)/\text{Sqrt}[a]\right]\right)/\left(2*b^{3/2}\right) - \left(c*\text{Log}[a - b*x^4]\right)/\left(4*b\right)$

Rule 1831

$\text{Int}\left[\left(\text{Pq}_\right)*\left(\left(c_\right)*(x_\right)^{\left(m_\right)}\right)/\left(\left(a_\right) + \left(b_\right)*(x_\right)^{\left(n_\right)}\right), x_Symbol\right] \rightarrow \text{With}\left[\left\{v = \text{Sum}\left[\left(\left(c*x\right)^{\left(m + ii\right)}*\text{Coeff}\left[\text{Pq}, x, ii\right] + \text{Coeff}\left[\text{Pq}, x, n/2 + ii\right]*x^{\left(n/2\right)}\right)\right]/\left(c^{ii}*(a + b*x^n)\right), \{ii, 0, n/2 - 1\}\right\}, \text{Int}\left[v, x\right] /; \text{SumQ}\left[v\right] /; \text{FreeQ}\left[\{a, b, c, m\}, x\right] \&\& \text{PolyQ}\left[\text{Pq}, x\right] \&\& \text{IGtQ}\left[n/2, 0\right] \&\& \text{Expon}\left[\text{Pq}, x\right] < n$

Rule 1252

$\text{Int}\left[\left(x_\right)^{\left(m_\right)}*\left(\left(d_\right) + \left(e_\right)*(x_\right)^2\right)^{\left(q_\right)}*\left(\left(a_\right) + \left(c_\right)*(x_\right)^4\right)^{\left(p_\right)}, x_Symbol\right] \rightarrow \text{Dist}\left[1/2, \text{Subst}\left[\text{Int}\left[x^{\left((m - 1)/2\right)}*\left(d + e*x\right)^q*(a + c*x^2)^p, x\right],$

$x, x^2], x] /; \text{FreeQ}\{a, c, d, e, p, q\}, x\} \ \&\& \ \text{IntegerQ}\{(m + 1)/2\}$

Rule 774

$\text{Int}\left[\frac{((d_.) + (e_.)x_*)(f_.) + (g_.)x_*)}{(a_.) + (c_.)x_**2}, x_{\text{Symbol}}\right] \rightarrow \text{Simp}\left[\frac{e_*g_*x}{c}, x\right] + \text{Dist}\left[\frac{1}{c}, \text{Int}\left[\frac{c*d*f - a*e*g + c*(e*f + d*g)*x}{(a + c*x^2)}, x\right], x\right] /; \text{FreeQ}\{a, c, d, e, f, g\}, x\}$

Rule 635

$\text{Int}\left[\frac{(d_.) + (e_.)x_*)}{(a_.) + (c_.)x_**2}, x_{\text{Symbol}}\right] \rightarrow \text{Dist}[d, \text{Int}\left[\frac{1}{(a + c*x^2)}, x\right], x] + \text{Dist}[e, \text{Int}\left[\frac{x}{(a + c*x^2)}, x\right], x] /; \text{FreeQ}\{a, c, d, e\}, x\} \ \&\& \ \text{!NiceSqrtQ}\{-(a*c)\}$

Rule 208

$\text{Int}\left[\frac{((a_.) + (b_.)x_**2)^{-1}}{x_{\text{Symbol}}}\right] \rightarrow \text{Simp}\left[\frac{\text{Rt}\{-(a/b), 2\} * \text{ArcTanh}\left[\frac{x}{\text{Rt}\{-(a/b), 2\}}\right]}{a}, x\right] /; \text{FreeQ}\{a, b\}, x\} \ \&\& \ \text{NegQ}\{a/b\}$

Rule 260

$\text{Int}\left[\frac{x_**m}{(a_.) + (b_.)x_**n}, x_{\text{Symbol}}\right] \rightarrow \text{Simp}\left[\frac{\text{Log}\left[\text{RemoveContent}[a + b*x^n, x]\right]}{(b*n)}, x\right] /; \text{FreeQ}\{a, b, m, n\}, x\} \ \&\& \ \text{EqQ}\{m, n - 1\}$

Rule 1280

$\text{Int}\left[\frac{((f_.)x_**m)((d_.) + (e_.)x_**2)((a_.) + (c_.)x_**4)^{p_}}{x_{\text{Symbol}}}\right] \rightarrow \text{Simp}\left[\frac{e_*f_*(f_*x)^{m-1}(a + c*x^4)^{p+1}}{c*(m + 4*p + 3)}, x\right] - \text{Dist}\left[\frac{f^2}{c*(m + 4*p + 3)}, \text{Int}\left[\frac{(f_*x)^{m-2}(a + c*x^4)^p(a*e*(m-1) - c*d*(m + 4*p + 3)*x^2)}{x}, x\right], x\right] /; \text{FreeQ}\{a, c, d, e, f, p\}, x\} \ \&\& \ \text{GtQ}\{m, 1\} \ \&\& \ \text{NeQ}\{m + 4*p + 3, 0\} \ \&\& \ \text{IntegerQ}\{2*p\} \ \&\& \ (\text{IntegerQ}\{p\} \ || \ \text{IntegerQ}\{m\})$

Rule 1167

$\text{Int}\left[\frac{(d_.) + (e_.)x_**2}{(a_.) + (c_.)x_**4}, x_{\text{Symbol}}\right] \rightarrow \text{With}\left[\{q = \text{Rt}\{-(a*c), 2\}\}, \text{Dist}\left[\frac{e/2 + (c*d)/(2*q)}{2*q}, \text{Int}\left[\frac{1}{-q + c*x^2}, x\right], x\right] + \text{Dist}\left[\frac{e/2 - (c*d)/(2*q)}{2*q}, \text{Int}\left[\frac{1}{q + c*x^2}, x\right], x\right]\right] /; \text{FreeQ}\{a, c, d, e\}, x\} \ \&\& \ \text{NeQ}\{c*d^2 - a*e^2, 0\} \ \&\& \ \text{PosQ}\{-(a*c)\}$

Rule 205

$\text{Int}\left[\frac{((a_.) + (b_.)x_**2)^{-1}}{x_{\text{Symbol}}}\right] \rightarrow \text{Simp}\left[\frac{\text{Rt}\{a/b, 2\} * \text{ArcTan}\left[\frac{x}{\text{Rt}\{a/b, 2\}}\right]}{a}, x\right] /; \text{FreeQ}\{a, b\}, x\} \ \&\& \ \text{PosQ}\{a/b\}$

Rubi steps

$$\begin{aligned}
\int \frac{x^3(c + dx + ex^2 + fx^3)}{a - bx^4} dx &= \int \left(\frac{x^3(c + ex^2)}{a - bx^4} + \frac{x^4(d + fx^2)}{a - bx^4} \right) dx \\
&= \int \frac{x^3(c + ex^2)}{a - bx^4} dx + \int \frac{x^4(d + fx^2)}{a - bx^4} dx \\
&= -\frac{fx^3}{3b} + \frac{1}{2} \text{Subst} \left(\int \frac{x(c + ex)}{a - bx^2} dx, x, x^2 \right) + \frac{\int \frac{x^2(3af + 3bdx^2)}{a - bx^4} dx}{3b} \\
&= -\frac{dx}{b} - \frac{ex^2}{2b} - \frac{fx^3}{3b} + \frac{\int \frac{3abd + 3abfx^2}{a - bx^4} dx}{3b^2} - \frac{\text{Subst} \left(\int \frac{-ae - bcx}{a - bx^2} dx, x, x^2 \right)}{2b} \\
&= -\frac{dx}{b} - \frac{ex^2}{2b} - \frac{fx^3}{3b} + \frac{1}{2}c \text{Subst} \left(\int \frac{x}{a - bx^2} dx, x, x^2 \right) + \frac{(ae) \text{Subst} \left(\int \frac{1}{a - bx^2} dx, x, x^2 \right)}{2b} - \frac{\int \frac{1}{a - bx^2} dx}{2b} \\
&= -\frac{dx}{b} - \frac{ex^2}{2b} - \frac{fx^3}{3b} + \frac{\sqrt[4]{a}(\sqrt{bd} - \sqrt{af}) \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2b^{7/4}} + \frac{\sqrt[4]{a}(\sqrt{bd} + \sqrt{af}) \tanh^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2b^{7/4}} + \frac{\int \frac{1}{a - bx^2} dx}{2b}
\end{aligned}$$

Mathematica [A] time = 0.0842981, size = 221, normalized size = 1.36

$$\frac{-3 \log(\sqrt[4]{a} - \sqrt[4]{bx}) (a^{3/4}f + \sqrt[4]{a}\sqrt{bd} + \sqrt{a}\sqrt[4]{be}) + 3 \log(\sqrt[4]{a} + \sqrt[4]{bx}) (a^{3/4}f + \sqrt[4]{a}\sqrt{bd} - \sqrt{a}\sqrt[4]{be}) + 6(\sqrt[4]{a}\sqrt{bd} - a^{3/4}f) \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{12b^{7/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a - b*x^4), x]

[Out] $(-12*b^{(3/4)}*d*x - 6*b^{(3/4)}*e*x^2 - 4*b^{(3/4)}*f*x^3 + 6*(a^{(1/4)}*\text{Sqrt}[b]*d - a^{(3/4)}*f)*\text{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}] - 3*(a^{(1/4)}*\text{Sqrt}[b]*d + \text{Sqrt}[a]*b^{(1/4)}*e + a^{(3/4)}*f)*\text{Log}[a^{(1/4)} - b^{(1/4)}*x] + 3*(a^{(1/4)}*\text{Sqrt}[b]*d - \text{Sqrt}[a]*b^{(1/4)}*e + a^{(3/4)}*f)*\text{Log}[a^{(1/4)} + b^{(1/4)}*x] + 3*\text{Sqrt}[a]*b^{(1/4)}*e*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[b]*x^2] - 3*b^{(3/4)}*c*\text{Log}[a - b*x^4])/(12*b^{(7/4)})$

Maple [A] time = 0.004, size = 208, normalized size = 1.3

$$-\frac{fx^3}{3b} - \frac{ex^2}{2b} - \frac{dx}{b} + \frac{d}{2b} \sqrt[4]{\frac{a}{b}} \arctan \left(x \sqrt[4]{\frac{a}{b}} \right) + \frac{d}{4b} \sqrt[4]{\frac{a}{b}} \ln \left(\left(x + \sqrt[4]{\frac{a}{b}} \right) \left(x - \sqrt[4]{\frac{a}{b}} \right)^{-1} \right) - \frac{ae}{4b} \ln \left((-a + x^2\sqrt{ab}) (-a - x^2\sqrt{ab}) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a), x)$

[Out]
$$-1/3*f*x^3/b-1/2*e*x^2/b-d*x/b+1/2/b*d*(1/b*a)^{(1/4)}*\arctan(x/(1/b*a)^{(1/4)})+1/4/b*d*(1/b*a)^{(1/4)}*\ln((x+(1/b*a)^{(1/4)})/(x-(1/b*a)^{(1/4)}))-1/4/b*a*e/(a*b)^{(1/2)}*\ln((-a+x^2*(a*b)^{(1/2)})/(-a-x^2*(a*b)^{(1/2)}))-1/2/b^2*a*f/(1/b*a)^{(1/4)}*\arctan(x/(1/b*a)^{(1/4)})+1/4/b^2*a*f/(1/b*a)^{(1/4)}*\ln((x+(1/b*a)^{(1/4)})/(x-(1/b*a)^{(1/4)}))-1/4/b*c*\ln(b*x^4-a)$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a), x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a), x, \text{algorithm}="fricas")$

[Out] Timed out

Sympy [B] time = 12.5476, size = 887, normalized size = 5.48

$$-\text{RootSum}\left(256t^4b^7 - 256t^3b^6c + t^2(-64ab^4df - 32ab^4e^2 + 96b^5c^2) + t(-16a^2b^2ef^2 + 32ab^3cdf + 16ab^3ce^2 - 16ab^3d^2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)

[Out] -RootSum(256*_t**4*b**7 - 256*_t**3*b**6*c + _t**2*(-64*a*b**4*d*f - 32*a*b**4*e**2 + 96*b**5*c**2) + _t*(-16*a**2*b**2*e*f**2 + 32*a*b**3*c*d*f + 16*a*b**3*c*e**2 - 16*a*b**3*d**2*e - 16*b**4*c**3) - a**3*f**4 + 4*a**2*b*c*e*f**2 + 2*a**2*b*d**2*f**2 - 4*a**2*b*d*e**2*f + a**2*b*e**4 - 4*a*b**2*c**2*d*f - 2*a*b**2*c**2*e**2 + 4*a*b**2*c*d**2*e - a*b**2*d**4 + b**3*c**4, Lambda(_t, _t*log(x + (-64*_t**3*a*b**5*f**3 - 64*_t**3*b**6*d**2*f + 128*_t**3*b**6*d*e**2 + 48*_t**2*a*b**4*c*f**3 + 48*_t**2*a*b**4*d*e*f**2 - 32*_t**2*a*b**4*e**3*f + 48*_t**2*b**5*c*d**2*f - 96*_t**2*b**5*c*d*e**2 - 16*_t**2*b**5*d**3*e + 12*_t*a**2*b**2*d*f**4 + 12*_t*a**2*b**2*e**2*f**3 - 12*_t*a*b**3*c**2*f**3 - 24*_t*a*b**3*c*d*e*f**2 + 16*_t*a*b**3*c*e**3*f + 16*_t*a*b**3*d**3*f**2 - 36*_t*a*b**3*d**2*e**2*f - 8*_t*a*b**3*d*e**4 - 12*_t*b**4*c**2*d**2*f + 24*_t*b**4*c**2*d*e**2 + 8*_t*b**4*c*d**3*e + 4*_t*b**4*d**5 + 3*a**3*e*f**5 - 3*a**2*b*c*d*f**4 - 3*a**2*b*c*e**2*f**3 - 5*a**2*b*d*e**3*f**2 + 2*a**2*b*e**5*f + a*b**2*c**3*f**3 + 3*a*b**2*c**2*d*e*f**2 - 2*a*b**2*c**2*e**3*f - 4*a*b**2*c*d**3*f**2 + 9*a*b**2*c*d**2*e**2*f + 2*a*b**2*c*d*e**4 + 5*a*b**2*d**4*e*f - 5*a*b**2*d**3*e**3 + b**3*c**3*d**2*f - 2*b**3*c**3*d*e**2 - b**3*c**2*d**3*e - b**3*c*d**5)/(a**3*f**6 + a**2*b*d**2*f**4 - 8*a**2*b*d*e**2*f**3 + 4*a**2*b*e**4*f**2 - a*b**2*d**4*f**2 + 8*a*b**2*d**3*e**2*f - 4*a*b**2*d**2*e**4 - b**3*d**6))) - d*x/b - e*x**2/(2*b) - f*x**3/(3*b)

Giac [B] time = 1.07629, size = 443, normalized size = 2.73

$$\frac{c \log(|bx^4 - a|)}{4b} - \frac{\sqrt{2} \left(\sqrt{2} \sqrt{-abb^2e} - (-ab^3)^{\frac{1}{4}} b^2d - (-ab^3)^{\frac{3}{4}} f \right) \arctan \left(\frac{\sqrt{2} \left(2x + \sqrt{2} \left(-\frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left(-\frac{a}{b} \right)^{\frac{1}{4}}} \right)}{4b^4} - \frac{\sqrt{2} \left(\sqrt{2} \sqrt{-abb^2e} - (-ab^3)^{\frac{1}{4}} \right)}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")

[Out] -1/4*c*log(abs(b*x^4 - a))/b - 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*e - (-a*b^3)^(1/4)*b^2*d - (-a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/b^4 - 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*e - (-a*b^3)^(1/4)*b^2*d - (-a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/b^4 + 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*d - (-a*b^3)^(3/4)*f)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/b^4 - 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*d - (-a*b^3)^(3/4)*f)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) +

$$\sqrt{-a/b}/b^4 - 1/6*(2*b^2*f*x^3 + 3*b^2*x^2*e + 6*b^2*d*x)/b^3$$

$$3.487 \quad \int \frac{c+dx+ex^2+fx^3}{a+bx^4} dx$$

Optimal. Leaf size=293

$$\frac{(\sqrt{bc} - \sqrt{ae}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{(\sqrt{bc} - \sqrt{ae}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} - \frac{(\sqrt{ae} + \sqrt{bc}) \tan^{-1}\left(1 - \frac{(\sqrt{bc} - \sqrt{ae}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{(\sqrt{bc} - \sqrt{ae}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}$$

[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + (f*Log[a + b*x^4])/(4*b)

Rubi [A] time = 0.222421, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.44$, Rules used = {1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 205, 260}

$$\frac{(\sqrt{bc} - \sqrt{ae}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{(\sqrt{bc} - \sqrt{ae}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} - \frac{(\sqrt{ae} + \sqrt{bc}) \tan^{-1}\left(1 - \frac{(\sqrt{bc} - \sqrt{ae}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{(\sqrt{bc} - \sqrt{ae}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4), x]

[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + (f*Log[a + b*x^4])/(4*b)

Rule 1876

Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,

0] && Expon[Pq, x] < n

Rule 1168

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1248

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol]
 > Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
 [{a, c, d, e, p, q}, x]

Rule 635

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] > Dist[d, Int[1/(
 a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
 }, x] && !NiceSqrtQ[-(a*c)]

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] > Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
 /b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] > Simp[Log[RemoveConten
 t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned}
 \int \frac{c + dx + ex^2 + fx^3}{a + bx^4} dx &= \int \left(\frac{c + ex^2}{a + bx^4} + \frac{x(d + fx^2)}{a + bx^4} \right) dx \\
 &= \int \frac{c + ex^2}{a + bx^4} dx + \int \frac{x(d + fx^2)}{a + bx^4} dx \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{d + fx}{a + bx^2} dx, x, x^2 \right) + \frac{\left(\frac{\sqrt{bc}}{\sqrt{a}} - e \right) \int \frac{\sqrt{a}\sqrt{b-bx^2}}{a+bx^4} dx}{2b} + \frac{\left(\frac{\sqrt{bc}}{\sqrt{a}} + e \right) \int \frac{\sqrt{a}\sqrt{b+bx^2}}{a+bx^4} dx}{2b} \\
 &= \frac{1}{2} d \text{Subst} \left(\int \frac{1}{a + bx^2} dx, x, x^2 \right) + \frac{\left(\frac{\sqrt{bc}}{\sqrt{a}} + e \right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{b}} + x^2} dx}{4b} + \frac{\left(\frac{\sqrt{bc}}{\sqrt{a}} - e \right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{b}} + x^2} dx}{4b} \\
 &= \frac{d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{b}} - \frac{(\sqrt{bc} - \sqrt{ae}) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{(\sqrt{bc} - \sqrt{ae}) \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} \\
 &= \frac{d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{b}} - \frac{(\sqrt{bc} + \sqrt{ae}) \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{(\sqrt{bc} + \sqrt{ae}) \tan^{-1} \left(1 + \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2\sqrt{2}a^{3/4}b^{3/4}} - \frac{(\sqrt{bc} - \sqrt{ae}) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{(\sqrt{bc} - \sqrt{ae}) \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2})}{4\sqrt{2}a^{3/4}b^{3/4}}
 \end{aligned}$$

Mathematica [A] time = 0.214556, size = 296, normalized size = 1.01

$$-\sqrt{2}\sqrt[4]{b}\left(\sqrt[4]{a}\sqrt{bc}-a^{3/4}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{bx^2}\right)+\sqrt{2}\sqrt[4]{b}\left(\sqrt[4]{a}\sqrt{bc}-a^{3/4}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{bx^2}\right)-2\sqrt[4]{b}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4), x]

[Out] $(-2*a^{1/4}*b^{1/4}*(\text{Sqrt}[2]*\text{Sqrt}[b]*c + 2*a^{1/4}*b^{1/4}*d + \text{Sqrt}[2]*\text{Sqrt}[a]*e)*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{1/4}*x)/a^{1/4}] + 2*a^{1/4}*b^{1/4}*(\text{Sqrt}[2]*\text{Sqrt}[b]*c - 2*a^{1/4}*b^{1/4}*d + \text{Sqrt}[2]*\text{Sqrt}[a]*e)*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{1/4}*x)/a^{1/4}] - \text{Sqrt}[2]*b^{1/4}*(a^{1/4}*\text{Sqrt}[b]*c - a^{3/4}*e)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{1/4}*b^{1/4}*x + \text{Sqrt}[b]*x^2] + \text{Sqrt}[2]*b^{1/4}*(a^{1/4}*\text{Sqrt}[b]*c - a^{3/4}*e)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{1/4}*b^{1/4}*x + \text{Sqrt}[b]*x^2] + 2*a*f*\text{Log}[a + b*x^4])/(8*a*b)$

Maple [A] time = 0.004, size = 294, normalized size = 1.

$$\frac{c\sqrt{2}}{8a}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)+\frac{c\sqrt{2}}{4a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{c\sqrt{2}}{4a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)/(b*x^4+a), x)

[Out] $1/8*c*(1/b*a)^{1/4}/a*2^{1/2}*\ln((x^2+(1/b*a)^{1/4}*x*2^{1/2}+(1/b*a)^{1/2})/(x^2-(1/b*a)^{1/4}*x*2^{1/2}+(1/b*a)^{1/2}))+1/4*c*(1/b*a)^{1/4}/a*2^{1/2}*\arctan(2^{1/2}/(1/b*a)^{1/4}*x+1)+1/4*c*(1/b*a)^{1/4}/a*2^{1/2}*\arctan(2^{1/2}/(1/b*a)^{1/4}*x-1)+1/2*d/(a*b)^{1/2}*\arctan(x^2*(b/a)^{1/2})+1/8*e/b/(1/b*a)^{1/4}*2^{1/2}*\ln((x^2-(1/b*a)^{1/4}*x*2^{1/2}+(1/b*a)^{1/2})/(x^2+(1/b*a)^{1/4}*x*2^{1/2}+(1/b*a)^{1/2}))+1/4*e/b/(1/b*a)^{1/4}*2^{1/2}*\arctan(2^{1/2}/(1/b*a)^{1/4}*x+1)+1/4*e/b/(1/b*a)^{1/4}*2^{1/2}*\arctan(2^{1/2}/(1/b*a)^{1/4}*x-1)+1/4*f*\ln(b*x^4+a)/b$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError


```

*d**2*f**3 + 3*a**3*b*c*d*e**2*f**2 - 3*a**3*b*c*e**4*f - 2*a**3*b*d**3*e*f
**2 - 3*a**3*b*d**2*e**3*f + 3*a**3*b*d*e**5 + a**2*b**2*c**3*d*f**2 + 4*a*
**2*b**2*c**3*e**2*f - 9*a**2*b**2*c**2*d**2*e*f - 2*a**2*b**2*c*d**4*f + 5*
a**2*b**2*c*d**3*e**2 - 2*a**2*b**2*d**5*e - a*b**3*c**5*f + 5*a*b**3*c**4*
d*e - 5*a*b**3*c**3*d**3)/(a**3*b*e**6 - a**2*b**2*c**2*e**4 + 8*a**2*b**2*
c*d**2*e**3 - 4*a**2*b**2*d**4*e**2 - a*b**3*c**4*e**2 + 8*a*b**3*c**3*d**2
*e - 4*a*b**3*c**2*d**4 + b**4*c**6))))

```

Giac [A] time = 1.08104, size = 392, normalized size = 1.34

$$\frac{f \log(|bx^4 + a|)}{4b} - \frac{\sqrt{2} \left(\sqrt{2} \sqrt{abb^2d} - (ab^3)^{\frac{1}{4}} b^2c - (ab^3)^{\frac{3}{4}} e \right) \arctan \left(\frac{\sqrt{2} \left(2x + \sqrt{2} \left(\frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left(\frac{a}{b} \right)^{\frac{1}{4}}} \right)}{4ab^3} - \frac{\sqrt{2} \left(\sqrt{2} \sqrt{abb^2d} - (ab^3)^{\frac{1}{4}} b^2c - (ab^3)^{\frac{3}{4}} e \right) \arctan \left(\frac{\sqrt{2} \left(2x - \sqrt{2} \left(\frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left(\frac{a}{b} \right)^{\frac{1}{4}}} \right)}{4ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")

[Out] 1/4*f*log(abs(b*x^4 + a))/b - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d - (a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d - (a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)

$$3.488 \quad \int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4} dx$$

Optimal. Leaf size=321

$$\frac{\sqrt[4]{a}(\sqrt{bd}-\sqrt{af})\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}+\sqrt{a}+\sqrt{bx^2})}{4\sqrt{2}b^{7/4}} - \frac{\sqrt[4]{a}(\sqrt{bd}-\sqrt{af})\log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}+\sqrt{a}+\sqrt{bx^2})}{4\sqrt{2}b^{7/4}} + \frac{\sqrt[4]{a}(\sqrt{af}+\sqrt{bd})}{2\sqrt{2}b^{7/4}}$$

[Out] (d*x)/b + (e*x^2)/(2*b) + (f*x^3)/(3*b) - (Sqrt[a]*e*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*b^(3/2)) + (a^(1/4)*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(7/4)) - (a^(1/4)*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(7/4)) + (a^(1/4)*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(7/4)) - (a^(1/4)*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(7/4)) + (c*Log[a + b*x^4])/(4*b)

Rubi [A] time = 0.333813, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {1831, 1252, 774, 635, 205, 260, 1280, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{a}(\sqrt{bd}-\sqrt{af})\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}+\sqrt{a}+\sqrt{bx^2})}{4\sqrt{2}b^{7/4}} - \frac{\sqrt[4]{a}(\sqrt{bd}-\sqrt{af})\log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}+\sqrt{a}+\sqrt{bx^2})}{4\sqrt{2}b^{7/4}} + \frac{\sqrt[4]{a}(\sqrt{af}+\sqrt{bd})}{2\sqrt{2}b^{7/4}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4), x]

[Out] (d*x)/b + (e*x^2)/(2*b) + (f*x^3)/(3*b) - (Sqrt[a]*e*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*b^(3/2)) + (a^(1/4)*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(7/4)) - (a^(1/4)*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(7/4)) + (a^(1/4)*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(7/4)) - (a^(1/4)*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(7/4)) + (c*Log[a + b*x^4])/(4*b)

Rule 1831

```
Int[((Pq_)*((c_)*(x_)^(m_)))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[
{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}], Int[v, x] /; SumQ[v] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 1252

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_S
ymbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 774

```
Int[((d_) + (e_)*(x_))*((f_) + (g_)*(x_)))/((a_) + (c_)*(x_)^2), x_Sym
bol] := Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 635

```
Int[((d_) + (e_)*(x_)))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1280

```
Int[((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_
Symbol] := Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(c + dx + ex^2 + fx^3)}{a + bx^4} dx &= \int \left(\frac{x^3(c + ex^2)}{a + bx^4} + \frac{x^4(d + fx^2)}{a + bx^4} \right) dx \\
&= \int \frac{x^3(c + ex^2)}{a + bx^4} dx + \int \frac{x^4(d + fx^2)}{a + bx^4} dx \\
&= \frac{fx^3}{3b} + \frac{1}{2} \text{Subst} \left(\int \frac{x(c + ex)}{a + bx^2} dx, x, x^2 \right) - \frac{\int \frac{x^2(3af - 3bdx^2)}{a + bx^4} dx}{3b} \\
&= \frac{dx}{b} + \frac{ex^2}{2b} + \frac{fx^3}{3b} + \frac{\int \frac{-3abd - 3abfx^2}{a + bx^4} dx}{3b^2} + \frac{\text{Subst} \left(\int \frac{-ae + bcx}{a + bx^2} dx, x, x^2 \right)}{2b} \\
&= \frac{dx}{b} + \frac{ex^2}{2b} + \frac{fx^3}{3b} + \frac{1}{2}c \text{Subst} \left(\int \frac{x}{a + bx^2} dx, x, x^2 \right) - \frac{(ae) \text{Subst} \left(\int \frac{1}{a + bx^2} dx, x, x^2 \right)}{2b} - \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}}{\sqrt{b}}}{-\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}}{\sqrt{b}}} dx}{4\sqrt{2}b^{7/4}} \\
&= \frac{dx}{b} + \frac{ex^2}{2b} + \frac{fx^3}{3b} - \frac{\sqrt{ae} \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{2b^{3/2}} + \frac{c \log(a + bx^4)}{4b} + \frac{(\sqrt[4]{a}(\sqrt{bd} - \sqrt{af})) \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}}{\sqrt{b}}}{-\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}}{\sqrt{b}}} dx}{4\sqrt{2}b^{7/4}} \\
&= \frac{dx}{b} + \frac{ex^2}{2b} + \frac{fx^3}{3b} - \frac{\sqrt{ae} \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{2b^{3/2}} + \frac{\sqrt[4]{a}(\sqrt{bd} - \sqrt{af}) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2})}{4\sqrt{2}b^{7/4}} \\
&= \frac{dx}{b} + \frac{ex^2}{2b} + \frac{fx^3}{3b} - \frac{\sqrt{ae} \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{2b^{3/2}} + \frac{\sqrt[4]{a}(\sqrt{bd} + \sqrt{af}) \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} \right)}{2\sqrt{2}b^{7/4}} - \frac{\sqrt[4]{a}(\sqrt{bd} - \sqrt{af}) \log(\sqrt{a} + \sqrt{bx^2})}{4\sqrt{2}b^{7/4}} + 6b^{3/4}
\end{aligned}$$

Mathematica [A] time = 0.173454, size = 311, normalized size = 0.97

$$-3\sqrt{2}(a^{3/4}f - \sqrt[4]{a}\sqrt{bd}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}) + 3\sqrt{2}(a^{3/4}f - \sqrt[4]{a}\sqrt{bd}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}) + 6b^{3/4}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4), x]

[Out] (24*b^(3/4)*d*x + 12*b^(3/4)*e*x^2 + 8*b^(3/4)*f*x^3 + 6*a^(1/4)*(Sqrt[2]*Sqrt[b]*d + 2*a^(1/4)*b^(1/4)*e + Sqrt[2]*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 6*a^(1/4)*(Sqrt[2]*Sqrt[b]*d - 2*a^(1/4)*b^(1/4)*e + Sqrt[2]*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 3*Sqrt[2]*(-(a^(1/4)*Sqrt[b]*d) + a^(3/4)*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 3*Sqrt[2]*(-(a^(1/4)*Sqrt[b]*d) + a^(3/4)*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]

$]a^{(1/4)}b^{(1/4)}x + \text{Sqrt}[b]*x^2] + 6*b^{(3/4)}*c*\text{Log}[a + b*x^4]/(24*b^{(7/4)})$
 $)$

Maple [A] time = 0.004, size = 325, normalized size = 1.

$$\frac{fx^3}{3b} + \frac{ex^2}{2b} + \frac{dx}{b} - \frac{d\sqrt{2}}{4b} \frac{\sqrt[4]{a}}{\sqrt{b}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{a}} - 1\right) - \frac{d\sqrt{2}}{8b} \frac{\sqrt[4]{a}}{\sqrt{b}} \ln\left(\left(x^2 + \sqrt{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x^2 - \sqrt{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) - \frac{d\sqrt{2}}{8b} \frac{\sqrt[4]{a}}{\sqrt{b}} \ln\left(\left(x^2 - \sqrt{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x^2 + \sqrt{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a), x)`

[Out] $\frac{1}{3}f\frac{x^3}{b} + \frac{1}{2}e\frac{x^2}{b} + \frac{d}{b}x - \frac{1}{4}\frac{d}{b}\frac{\sqrt[4]{a}}{\sqrt{b}}\arctan\left(\frac{2\sqrt{2}}{\sqrt{a}}x - 1\right) - \frac{1}{8}\frac{d}{b}\frac{\sqrt[4]{a}}{\sqrt{b}}\arctan\left(\frac{2\sqrt{2}}{\sqrt{a}}x + 1\right) - \frac{1}{8}\frac{d}{b}\frac{\sqrt[4]{a}}{\sqrt{b}}\ln\left(\frac{x^2 + \sqrt{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}}{x^2 - \sqrt{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}}\right) - \frac{1}{8}\frac{d}{b}\frac{\sqrt[4]{a}}{\sqrt{b}}\ln\left(\frac{x^2 - \sqrt{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}}{x^2 + \sqrt{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}}\right) - \frac{1}{4}\frac{d}{b}\frac{\sqrt[4]{a}}{\sqrt{b}}\arctan\left(\frac{2\sqrt{2}}{\sqrt{a}}x - 1\right) - \frac{1}{4}\frac{d}{b}\frac{\sqrt[4]{a}}{\sqrt{b}}\arctan\left(\frac{2\sqrt{2}}{\sqrt{a}}x + 1\right) - \frac{1}{4}\frac{d}{b}\frac{\sqrt[4]{a}}{\sqrt{b}}\ln\left(\frac{x^2 + \sqrt{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}}{x^2 - \sqrt{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}}\right) - \frac{1}{4}\frac{d}{b}\frac{\sqrt[4]{a}}{\sqrt{b}}\ln\left(\frac{x^2 - \sqrt{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}}{x^2 + \sqrt{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}}\right) - \frac{1}{4}\frac{d}{b}\frac{\sqrt[4]{a}}{\sqrt{b}}\arctan\left(\frac{2\sqrt{2}}{\sqrt{a}}x - 1\right) + \frac{1}{4}c\ln\frac{b*x^4+a}{b}$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a), x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")

[Out] Timed out

Sympy [B] time = 12.1739, size = 886, normalized size = 2.76

$$\text{RootSum}\left(256t^4b^7 - 256t^3b^6c + t^2(64ab^4df + 32ab^4e^2 + 96b^5c^2) + t(-16a^2b^2ef^2 - 32ab^3cdf - 16ab^3ce^2 + 16ab^3d^2e)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)

[Out] RootSum(256*_t**4*b**7 - 256*_t**3*b**6*c + _t**2*(64*a*b**4*d*f + 32*a*b**4*e**2 + 96*b**5*c**2) + _t*(-16*a**2*b**2*e*f**2 - 32*a*b**3*c*d*f - 16*a*b**3*c*e**2 + 16*a*b**3*d**2*e - 16*b**4*c**3) + a**3*f**4 + 4*a**2*b*c*e*f**2 + 2*a**2*b*d**2*f**2 - 4*a**2*b*d*e**2*f + a**2*b*e**4 + 4*a*b**2*c**2*d*f + 2*a*b**2*c**2*e**2 - 4*a*b**2*c*d**2*e + a*b**2*d**4 + b**3*c**4, Lambda(_t, _t*log(x + (-64*_t**3*a*b**5*f**3 + 64*_t**3*b**6*d**2*f - 128*_t**3*b**6*d*e**2 + 48*_t**2*a*b**4*c*f**3 + 48*_t**2*a*b**4*d*e*f**2 - 32*_t**2*a*b**4*e**3*f - 48*_t**2*b**5*c*d**2*f + 96*_t**2*b**5*c*d*e**2 + 16*_t**2*b**5*d**3*e - 12*_t*a**2*b**2*d*f**4 - 12*_t*a**2*b**2*e**2*f**3 - 12*_t*a*b**3*c**2*f**3 - 24*_t*a*b**3*c*d*e*f**2 + 16*_t*a*b**3*c*e**3*f + 16*_t*a*b**3*d**3*f**2 - 36*_t*a*b**3*d**2*e**2*f - 8*_t*a*b**3*d*e**4 + 12*_t*b**4*c**2*d**2*f - 24*_t*b**4*c**2*d*e**2 - 8*_t*b**4*c*d**3*e - 4*_t*b**4*d**5 + 3*a**3*e*f**5 + 3*a**2*b*c*d*f**4 + 3*a**2*b*c*e**2*f**3 + 5*a**2*b*d*e**3*f**2 - 2*a**2*b*e**5*f + a*b**2*c**3*f**3 + 3*a*b**2*c**2*d*e*f**2 - 2*a*b**2*c**2*e**3*f - 4*a*b**2*c*d**3*f**2 + 9*a*b**2*c*d**2*e**2*f + 2*a*b**2*c*d*e**4 + 5*a*b**2*d**4*e*f - 5*a*b**2*d**3*e**3 - b**3*c**3*d**2*f + 2*b**3*c**3*d*e**2 + b**3*c**2*d**3*e + b**3*c*d**5)/(a**3*f**6 - a**2*b*d**2*f**4 + 8*a**2*b*d*e**2*f**3 - 4*a**2*b*e**4*f**2 - a*b**2*d**4*f**2 + 8*a*b**2*d**3*e**2*f - 4*a*b**2*d**2*e**4 + b**3*d**6))) + d*x/b + e*x**2/(2*b) + f*x**3/(3*b)

Giac [A] time = 1.10117, size = 416, normalized size = 1.3

$$\frac{c \log(|bx^4 + a|)}{4b} + \frac{\sqrt{2}\left(\sqrt{2}\sqrt{abb^2e} - (ab^3)^{\frac{1}{4}}b^2d - (ab^3)^{\frac{3}{4}}f\right) \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4b^4} + \frac{\sqrt{2}\left(\sqrt{2}\sqrt{abb^2e} - (ab^3)^{\frac{1}{4}}b^2d - (ab^3)^{\frac{3}{4}}f\right)}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")

[Out] $\frac{1}{4}c \log(\text{abs}(b x^4 + a)) / b + \frac{1}{4} \sqrt{2} (\sqrt{2} \sqrt{a b} b^2 e - (a b^3)^{1/4} b^2 d - (a b^3)^{3/4} f) \arctan(1/2 \sqrt{2} (2 x + \sqrt{2} (a/b)^{1/4})) / (a/b)^{1/4} / b^4 + \frac{1}{4} \sqrt{2} (\sqrt{2} \sqrt{a b} b^2 e - (a b^3)^{1/4} b^2 d - (a b^3)^{3/4} f) \arctan(1/2 \sqrt{2} (2 x - \sqrt{2} (a/b)^{1/4})) / (a/b)^{1/4} / b^4 - \frac{1}{8} \sqrt{2} ((a b^3)^{1/4} b^2 d - (a b^3)^{3/4} f) \log(x^2 + \sqrt{2} x (a/b)^{1/4} + \sqrt{a/b}) / b^4 + \frac{1}{8} \sqrt{2} ((a b^3)^{1/4} b^2 d - (a b^3)^{3/4} f) \log(x^2 - \sqrt{2} x (a/b)^{1/4} + \sqrt{a/b}) / b^4 + \frac{1}{6} (2 b^2 f x^3 + 3 b^2 x^2 e + 6 b^2 d x) / b^3$

$$3.489 \quad \int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^2} dx$$

Optimal. Leaf size=318

$$-\frac{(3\sqrt{bc} - \sqrt{ae}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{16\sqrt{2}a^{7/4}b^{3/4}} + \frac{(3\sqrt{bc} - \sqrt{ae}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{16\sqrt{2}a^{7/4}b^{3/4}} - \frac{(\sqrt{ae} + 3\sqrt{bc}) \tan^{-1}}{8\sqrt{2}a^{7/4}b^{3/4}}$$

[Out] $-(a*f - b*x*(c + d*x + e*x^2))/(4*a*b*(a + b*x^4)) + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^{3/2}*Sqrt[b]) - ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^{1/4}*x)/a^{1/4}])/(8*Sqrt[2]*a^{7/4}*b^{3/4}) + ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^{1/4}*x)/a^{1/4}])/(8*Sqrt[2]*a^{7/4}*b^{3/4}) - ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^{1/4}*b^{1/4}*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^{7/4}*b^{3/4}) + ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^{1/4}*b^{1/4}*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^{7/4}*b^{3/4})$

Rubi [A] time = 0.26987, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {1854, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{(3\sqrt{bc} - \sqrt{ae}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{16\sqrt{2}a^{7/4}b^{3/4}} + \frac{(3\sqrt{bc} - \sqrt{ae}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{16\sqrt{2}a^{7/4}b^{3/4}} - \frac{(\sqrt{ae} + 3\sqrt{bc}) \tan^{-1}}{8\sqrt{2}a^{7/4}b^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^2,x]

[Out] $-(a*f - b*x*(c + d*x + e*x^2))/(4*a*b*(a + b*x^4)) + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^{3/2}*Sqrt[b]) - ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^{1/4}*x)/a^{1/4}])/(8*Sqrt[2]*a^{7/4}*b^{3/4}) + ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^{1/4}*x)/a^{1/4}])/(8*Sqrt[2]*a^{7/4}*b^{3/4}) - ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^{1/4}*b^{1/4}*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^{7/4}*b^{3/4}) + ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^{1/4}*b^{1/4}*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^{7/4}*b^{3/4})$

Rule 1854

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq,
x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1})*(a + b*x^n)^(p
+ 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
```

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 204

$\text{Int}[\{(a_) + (b_.)*(x_)^2\}^{-1}, x_Symbol] \ :> \ -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2]*x]/\text{Rt}[-a, 2]]/\text{Rt}[-a, 2]*\text{Rt}[-b, 2], x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 1165

$\text{Int}[\{(d_) + (e_.)*(x_)^2\}/\{(a_) + (c_.)*(x_)^4\}, x_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[-2*d/e, 2]\}, \ \text{Dist}[e/(2*c*q), \ \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \ \text{Dist}[e/(2*c*q), \ \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

Rule 628

$\text{Int}[\{(d_) + (e_.)*(x_)\}/\{(a_.) + (b_.)*(x_) + (c_.)*(x_)^2\}, x_Symbol] \ :> \ \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3}{(a + bx^4)^2} dx &= -\frac{af - bx(c + dx + ex^2)}{4ab(a + bx^4)} - \frac{\int \frac{-3c - 2dx - ex^2}{a + bx^4} dx}{4a} \\
&= -\frac{af - bx(c + dx + ex^2)}{4ab(a + bx^4)} - \frac{\int \left(-\frac{2dx}{a + bx^4} + \frac{-3c - ex^2}{a + bx^4}\right) dx}{4a} \\
&= -\frac{af - bx(c + dx + ex^2)}{4ab(a + bx^4)} - \frac{\int \frac{-3c - ex^2}{a + bx^4} dx}{4a} + \frac{d \int \frac{x}{a + bx^4} dx}{2a} \\
&= -\frac{af - bx(c + dx + ex^2)}{4ab(a + bx^4)} + \frac{d \operatorname{Subst}\left(\int \frac{1}{a + bx^2} dx, x, x^2\right)}{4a} + \frac{\left(\frac{3\sqrt{bc}}{\sqrt{a}} - e\right) \int \frac{\sqrt{a}\sqrt{b} - bx^2}{a + bx^4} dx}{8ab} + \frac{\left(\frac{3\sqrt{bc}}{\sqrt{a}} + e\right) \int \frac{\sqrt{a}}{\sqrt{b}} dx}{16ab} \\
&= -\frac{af - bx(c + dx + ex^2)}{4ab(a + bx^4)} + \frac{d \tan^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} + \frac{\left(\frac{3\sqrt{bc}}{\sqrt{a}} + e\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{b}} + x^2} dx}{16ab} + \frac{\left(\frac{3\sqrt{bc}}{\sqrt{a}} + e\right) \int \frac{\sqrt{a}}{\sqrt{b}} dx}{16ab} \\
&= -\frac{af - bx(c + dx + ex^2)}{4ab(a + bx^4)} + \frac{d \tan^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{(3\sqrt{bc} - \sqrt{ae}) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2})}{16\sqrt{2}a^{7/4}b^{3/4}} + \frac{(3\sqrt{bc} + \sqrt{ae}) \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2})}{16\sqrt{2}a^{7/4}b^{3/4}} \\
&= -\frac{af - bx(c + dx + ex^2)}{4ab(a + bx^4)} + \frac{d \tan^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{(3\sqrt{bc} + \sqrt{ae}) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}} + \frac{(3\sqrt{bc} + \sqrt{ae}) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.340096, size = 315, normalized size = 0.99

$$\sqrt{2}\sqrt[4]{b}(a^{3/4}e - 3\sqrt[4]{a}\sqrt{bc}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}) + \sqrt{2}\sqrt[4]{b}(3\sqrt[4]{a}\sqrt{bc} - a^{3/4}e) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}) - \frac{8a^{3/4}e}{\sqrt{2}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^2, x]

[Out] ((-8*a*(a*f - b*x*(c + x*(d + e*x)))/(a + b*x^4) - 2*a^(1/4)*b^(1/4)*(3*sqrt[2]*sqrt[b]*c + 4*a^(1/4)*b^(1/4)*d + sqrt[2]*sqrt[a]*e)*ArcTan[1 - (sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*b^(1/4)*(3*sqrt[2]*sqrt[b]*c - 4*a^(1/4)*b^(1/4)*d + sqrt[2]*sqrt[a]*e)*ArcTan[1 + (sqrt[2]*b^(1/4)*x)/a^(1/4)] + sqrt[2]*b^(1/4)*(-3*a^(1/4)*sqrt[b]*c + a^(3/4)*e)*Log[sqrt[a] - sqrt[2]*a^(1/4)*b^(1/4)*x + sqrt[b]*x^2] + sqrt[2]*b^(1/4)*(3*a^(1/4)*sqrt[b]*c - a^(3/4)*e)*Log[sqrt[a] + sqrt[2]*a^(1/4)*b^(1/4)*x + sqrt[b]*x^2])

$$\frac{3}{4}e) \cdot \text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2] \cdot a^{1/4} \cdot b^{1/4} \cdot x + \text{Sqrt}[b] \cdot x^2] / (32 \cdot a^2 \cdot b)$$

Maple [A] time = 0.005, size = 362, normalized size = 1.1

$$\frac{cx}{4a(bx^4+a)} + \frac{3c\sqrt{2}}{32a^2} \sqrt[4]{\frac{a}{b}} \ln\left(\left(x^2 + \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x^2 - \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) + \frac{3c\sqrt{2}}{16a^2} \sqrt[4]{\frac{a}{b}} \arctan\left(x\sqrt{2} \frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) + \frac{3c\sqrt{2}}{16a^2} \sqrt[4]{\frac{a}{b}} \arctan\left(x\sqrt{2} \frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x)

[Out] $\frac{1}{4}c \cdot x/a / (b \cdot x^4+a) + 3/32 \cdot c/a^2 \cdot (1/b \cdot a)^{1/4} \cdot 2^{1/2} \cdot \ln((x^2+(1/b \cdot a)^{1/4}) \cdot x \cdot 2^{1/2} + (1/b \cdot a)^{1/2}) / (x^2 - (1/b \cdot a)^{1/4} \cdot x \cdot 2^{1/2} + (1/b \cdot a)^{1/2}) + 3/16 \cdot c/a^2 \cdot (1/b \cdot a)^{1/4} \cdot 2^{1/2} \cdot \arctan(2^{1/2} / (1/b \cdot a)^{1/4} \cdot x + 1) + 3/16 \cdot c/a^2 \cdot (1/b \cdot a)^{1/4} \cdot 2^{1/2} \cdot \arctan(2^{1/2} / (1/b \cdot a)^{1/4} \cdot x - 1) + 1/4 \cdot d \cdot x^2/a / (b \cdot x^4+a) + 1/4 \cdot d/a / (a \cdot b)^{1/2} \cdot \arctan(x^2 \cdot (b/a)^{1/2}) + 1/4 \cdot e \cdot x^3/a / (b \cdot x^4+a) + 1/32 \cdot e/a/b / (1/b \cdot a)^{1/4} \cdot 2^{1/2} \cdot \ln((x^2 - (1/b \cdot a)^{1/4} \cdot x \cdot 2^{1/2} + (1/b \cdot a)^{1/2}) / (x^2 + (1/b \cdot a)^{1/4} \cdot x \cdot 2^{1/2} + (1/b \cdot a)^{1/2})) + 1/16 \cdot e/a/b / (1/b \cdot a)^{1/4} \cdot 2^{1/2} \cdot \arctan(2^{1/2} / (1/b \cdot a)^{1/4} \cdot x + 1) + 1/16 \cdot e/a/b / (1/b \cdot a)^{1/4} \cdot 2^{1/2} \cdot \arctan(2^{1/2} / (1/b \cdot a)^{1/4} \cdot x - 1) + 1/4 \cdot f \cdot x^4/a / (b \cdot x^4+a)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")

[Out] Timed out

Sympy [A] time = 16.1411, size = 517, normalized size = 1.63

RootSum(65536t⁴a⁷b³ + t²(3072a⁴b²ce + 2048a⁴b²d²) + t(128a³bde² - 1152a²b²c²d) + a²e⁴ + 18abc²e² - 48abcd²e +

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)

[Out] RootSum(65536*_t**4*a**7*b**3 + _t**2*(3072*a**4*b**2*c*e + 2048*a**4*b**2*d**2) + _t*(128*a**3*b*d*e**2 - 1152*a**2*b**2*c**2*d) + a**2*e**4 + 18*a*b*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 + 81*b**2*c**4, Lambda(_t, _t*log(x + (4096*_t**3*a**7*b**2*e**3 - 36864*_t**3*a**6*b**3*c**2*e + 98304*_t**3*a**6*b**3*c*d**2 + 4608*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5*b**2*d**3*e + 13824*_t**2*a**4*b**3*c**3*d + 144*_t*a**4*b*c*e**4 + 192*_t*a**4*b*d**2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*d**2*e + 1536*_t*a**3*b**2*c*d**4 + 3888*_t*a**2*b**3*c**5 + 6*a**3*d*e**5 + 120*a**2*b*c*d**3*e**2 - 64*a**2*b*d**5*e + 810*a*b**2*c**4*d*e - 1080*a*b**2*c**3*d**3)/(a**3*e**6 - 9*a**2*b*c**2*e**4 + 96*a**2*b*c*d**2*e**3 - 64*a**2*b*d**4*e**2 - 81*a*b**2*c**4*e**2 + 864*a*b**2*c**3*d**2*e - 576*a*b**2*c**2*d**4 + 729*b**3*c**6)))) + (-a*f + b*c*x + b*d*x**2 + b*e*x**3)/(4*a**2*b + 4*a*b**2*x**4)

Giac [A] time = 1.08815, size = 427, normalized size = 1.34

$$\frac{bx^3e + bdx^2 + bcx - af}{4(bx^4 + a)ab} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{abb^2d} + 3(ab^3)^{\frac{1}{4}}b^2c + (ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2b^3} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{abb^2d} + 3\right)}{16a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")

```
[Out] 1/4*(b*x^3*e + b*d*x^2 + b*c*x - a*f)/((b*x^4 + a)*a*b) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3)
```

$$3.490 \quad \int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2} dx$$

Optimal. Leaf size=310

$$\frac{(\sqrt{bd} - 3\sqrt{af}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{16\sqrt{2}a^{3/4}b^{7/4}} + \frac{(\sqrt{bd} - 3\sqrt{af}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{16\sqrt{2}a^{3/4}b^{7/4}} - \frac{(3\sqrt{af} + \sqrt{bd}) \tan^{-1}(\dots)}{8\sqrt{2}a^{3/4}b^{7/4}}$$

[Out] $-(c + d*x + e*x^2 + f*x^3)/(4*b*(a + b*x^4)) + (e*\text{ArcTan}[(\text{Sqrt}[b]*x^2)/\text{Sqrt}[a]])/(4*\text{Sqrt}[a]*b^{(3/2)}) - ((\text{Sqrt}[b]*d + 3*\text{Sqrt}[a]*f)*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*x)/a^{(1/4)}])/(8*\text{Sqrt}[2]*a^{(3/4)}*b^{(7/4)}) + ((\text{Sqrt}[b]*d + 3*\text{Sqrt}[a]*f)*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*x)/a^{(1/4)}])/(8*\text{Sqrt}[2]*a^{(3/4)}*b^{(7/4)}) - ((\text{Sqrt}[b]*d - 3*\text{Sqrt}[a]*f)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*x + \text{Sqrt}[b]*x^2])/(16*\text{Sqrt}[2]*a^{(3/4)}*b^{(7/4)}) + ((\text{Sqrt}[b]*d - 3*\text{Sqrt}[a]*f)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*x + \text{Sqrt}[b]*x^2])/(16*\text{Sqrt}[2]*a^{(3/4)}*b^{(7/4)})$

Rubi [A] time = 0.274175, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1823, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(\sqrt{bd} - 3\sqrt{af}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{16\sqrt{2}a^{3/4}b^{7/4}} + \frac{(\sqrt{bd} - 3\sqrt{af}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{16\sqrt{2}a^{3/4}b^{7/4}} - \frac{(3\sqrt{af} + \sqrt{bd}) \tan^{-1}(\dots)}{8\sqrt{2}a^{3/4}b^{7/4}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^2, x]$

[Out] $-(c + d*x + e*x^2 + f*x^3)/(4*b*(a + b*x^4)) + (e*\text{ArcTan}[(\text{Sqrt}[b]*x^2)/\text{Sqrt}[a]])/(4*\text{Sqrt}[a]*b^{(3/2)}) - ((\text{Sqrt}[b]*d + 3*\text{Sqrt}[a]*f)*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*x)/a^{(1/4)}])/(8*\text{Sqrt}[2]*a^{(3/4)}*b^{(7/4)}) + ((\text{Sqrt}[b]*d + 3*\text{Sqrt}[a]*f)*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*x)/a^{(1/4)}])/(8*\text{Sqrt}[2]*a^{(3/4)}*b^{(7/4)}) - ((\text{Sqrt}[b]*d - 3*\text{Sqrt}[a]*f)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*x + \text{Sqrt}[b]*x^2])/(16*\text{Sqrt}[2]*a^{(3/4)}*b^{(7/4)}) + ((\text{Sqrt}[b]*d - 3*\text{Sqrt}[a]*f)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*x + \text{Sqrt}[b]*x^2])/(16*\text{Sqrt}[2]*a^{(3/4)}*b^{(7/4)})$

Rule 1823

$\text{Int}[(\text{Pq}_*)*(x_{-})^{(m_{-})}*((a_{-}) + (b_{-})*(x_{-})^{(n_{-})})^{(p_{-})}, x_Symbol] :> \text{Simp}[(\text{Pq}*(a + b*x^n)^{(p + 1)})/(b*n*(p + 1)), x] - \text{Dist}[1/(b*n*(p + 1)), \text{Int}[D[\text{Pq}, x]^*$

$(a + b*x^n)^{(p + 1), x], x] /;$ FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && EqQ[m - n + 1, 0] && LtQ[p, -1]

Rule 1876

Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 275

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1168

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2} dx &= -\frac{c+dx+ex^2+fx^3}{4b(a+bx^4)} + \frac{\int \frac{d+2ex+3fx^2}{a+bx^4} dx}{4b} \\
&= -\frac{c+dx+ex^2+fx^3}{4b(a+bx^4)} + \frac{\int \left(\frac{2ex}{a+bx^4} + \frac{d+3fx^2}{a+bx^4}\right) dx}{4b} \\
&= -\frac{c+dx+ex^2+fx^3}{4b(a+bx^4)} + \frac{\int \frac{d+3fx^2}{a+bx^4} dx}{4b} + \frac{e \int \frac{x}{a+bx^4} dx}{2b} \\
&= -\frac{c+dx+ex^2+fx^3}{4b(a+bx^4)} + \frac{e \operatorname{Subst}\left(\int \frac{1}{a+bx^2} dx, x, x^2\right)}{4b} + \frac{\left(\frac{\sqrt{bd}}{\sqrt{a}} - 3f\right) \int \frac{\sqrt{a}\sqrt{b-x^2}}{a+bx^4} dx}{8b^2} + \frac{\left(\frac{\sqrt{bd}}{\sqrt{a}}\right)}{\sqrt{a}} \\
&= -\frac{c+dx+ex^2+fx^3}{4b(a+bx^4)} + \frac{e \tan^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{4\sqrt{ab^{3/2}}} + \frac{\left(\frac{\sqrt{bd}}{\sqrt{a}} + 3f\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{b}} + x^2} dx}{16b^2} + \frac{\left(\frac{\sqrt{bd}}{\sqrt{a}}\right) \int \frac{1}{\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2}} dx}{16\sqrt{2}a^{3/4}b^{7/4}} \\
&= -\frac{c+dx+ex^2+fx^3}{4b(a+bx^4)} + \frac{e \tan^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{4\sqrt{ab^{3/2}}} - \frac{(\sqrt{bd} - 3\sqrt{a}f) \log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2}\right)}{16\sqrt{2}a^{3/4}b^{7/4}} \\
&= -\frac{c+dx+ex^2+fx^3}{4b(a+bx^4)} + \frac{e \tan^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{4\sqrt{ab^{3/2}}} - \frac{(\sqrt{bd} + 3\sqrt{a}f) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}b^{7/4}} + \frac{(\sqrt{bd} + 3f) \int \frac{1}{\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{bx^2}} dx}{16\sqrt{2}a^{3/4}b^{7/4}}
\end{aligned}$$

Mathematica [A] time = 0.269116, size = 294, normalized size = 0.95

$$\frac{2 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \left(4\sqrt[4]{a}\sqrt[4]{be} + 3\sqrt{2}\sqrt{af} + \sqrt{2}\sqrt{bd}\right)}{a^{3/4}} + \frac{2 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + 1\right) \left(-4\sqrt[4]{a}\sqrt[4]{be} + 3\sqrt{2}\sqrt{af} + \sqrt{2}\sqrt{bd}\right)}{a^{3/4}} + \frac{\sqrt{2}(3\sqrt{af} - \sqrt{bd}) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{a^{3/4}}$$

$$32b^{7/4}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^2,x]

[Out] ((-8*b^(3/4)*(c + x*(d + x*(e + f*x))))/(a + b*x^4) - (2*(Sqrt[2]*Sqrt[b]*d + 4*a^(1/4)*b^(1/4)*e + 3*Sqrt[2]*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) + (2*(Sqrt[2]*Sqrt[b]*d - 4*a^(1/4)*b^(1/4)*e + 3*Sqrt[2]*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) + (Sqrt[2]*(-Sqrt[b]*d) + 3*Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[2]*a^(1/4)*b^(1/4)*x]

$b] * x^2)) / a^{(3/4)} + (\text{Sqrt}[2] * (\text{Sqrt}[b] * d - 3 * \text{Sqrt}[a] * f) * \text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2] * a^{(1/4)} * b^{(1/4)} * x + \text{Sqrt}[b] * x^2]) / a^{(3/4)}) / (32 * b^{(7/4)})$

Maple [A] time = 0.009, size = 334, normalized size = 1.1

$$\frac{1}{bx^4 + a} \left(-\frac{fx^3}{4b} - \frac{ex^2}{4b} - \frac{dx}{4b} - \frac{c}{4b} \right) + \frac{d\sqrt{2} \sqrt[4]{a}}{32ab\sqrt{b}} \ln \left(\left(x^2 + \sqrt[4]{\frac{a}{b}} x \sqrt{2} + \sqrt{\frac{a}{b}} \right) \left(x^2 - \sqrt[4]{\frac{a}{b}} x \sqrt{2} + \sqrt{\frac{a}{b}} \right)^{-1} \right) + \frac{d\sqrt{2} \sqrt[4]{a}}{16ab\sqrt{b}} \arctan$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x)`

[Out] $(-1/4*f*x^3/b-1/4*e*x^2/b-1/4*d*x/b-1/4*c/b)/(b*x^4+a)+1/32/b*d*(1/b*a)^{(1/4)}/a*2^{(1/2)}*\ln((x^2+(1/b*a)^{(1/4)}*x*2^{(1/2)}+(1/b*a)^{(1/2)})/(x^2-(1/b*a)^{(1/4)}*x*2^{(1/2)}+(1/b*a)^{(1/2)}))+1/16/b*d*(1/b*a)^{(1/4)}/a*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)}*x+1)+1/16/b*d*(1/b*a)^{(1/4)}/a*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)}*x-1)+1/4/b*e/(a*b)^{(1/2)}*\arctan(x^2*(b/a)^{(1/2)}))+3/32/b^2*f/(1/b*a)^{(1/4)}*2^{(1/2)}*\ln((x^2-(1/b*a)^{(1/4)}*x*2^{(1/2)}+(1/b*a)^{(1/2)})/(x^2+(1/b*a)^{(1/4)}*x*2^{(1/2)}+(1/b*a)^{(1/2)}))+3/16/b^2*f/(1/b*a)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)}*x+1)+3/16/b^2*f/(1/b*a)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(1/b*a)^{(1/4)}*x-1)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")

[Out] Timed out

Sympy [A] time = 31.1828, size = 508, normalized size = 1.64

RootSum(65536t^4a^3b^7 + t^2(3072a^2b^4df + 2048a^2b^4e^2) + t(1152a^2b^2ef^2 - 128ab^3d^2e) + 81a^2f^4 + 18abd^2f^2 - 48aba

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)

[Out] RootSum(65536*_t**4*a**3*b**7 + _t**2*(3072*a**2*b**4*d*f + 2048*a**2*b**4*e**2) + _t*(1152*a**2*b**2*e*f**2 - 128*a*b**3*d**2*e) + 81*a**2*f**4 + 18*a*b*d**2*f**2 - 48*a*b*d*e**2*f + 16*a*b*e**4 + b**2*d**4, Lambda(_t, _t*log(x + (110592*_t**3*a**4*b**5*f**3 - 12288*_t**3*a**3*b**6*d**2*f + 32768*_t**3*a**3*b**6*d*e**2 + 13824*_t**2*a**3*b**4*d*e*f**2 - 12288*_t**2*a**3*b**4*e**3*f + 512*_t**2*a**2*b**5*d**3*e + 3888*_t*a**3*b**2*d*f**4 + 5184*_t*a**3*b**2*e**2*f**3 - 576*_t*a**2*b**3*d**3*f**2 + 1728*_t*a**2*b**3*d**2*e**2*f + 512*_t*a**2*b**3*d*e**4 + 16*_t*a*b**4*d**5 + 1458*a**3*e*f**5 + 360*a**2*b*d*e**3*f**2 - 192*a**2*b*e**5*f + 30*a*b**2*d**4*e*f - 40*a*b**2*d**3*e**3)/(729*a**3*f**6 - 81*a**2*b*d**2*f**4 + 864*a**2*b*d*e**2*f**3 - 576*a**2*b*e**4*f**2 - 9*a*b**2*d**4*f**2 + 96*a*b**2*d**3*e**2*f - 64*a*b**2*d**2*e**4 + b**3*d**6)))) - (c + d*x + e*x**2 + f*x**3)/(4*a*b + 4*b**2*x**4)

Giac [A] time = 1.09309, size = 409, normalized size = 1.32

$$\frac{fx^3 + x^2e + dx + c}{4(bx^4 + a)b} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{abb^2e} + (ab^3)^{\frac{1}{4}}b^2d + 3(ab^3)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16ab^4} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{abb^2e} + (ab^3)^{\frac{1}{4}}b^2d + 3(ab^3)^{\frac{3}{4}}f\right)}{16ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")

```
[Out] -1/4*(f*x^3 + x^2*e + d*x + c)/((b*x^4 + a)*b) + 1/16*sqrt(2)*(2*sqrt(2)*sq
rt(a*b)*b^2*e + (a*b^3)^(1/4)*b^2*d + 3*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)
*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + 1/16*sqrt(2)*(2*sqrt(2)
*sqrt(a*b)*b^2*e + (a*b^3)^(1/4)*b^2*d + 3*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt
(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + 1/32*sqrt(2)*((a*b^3
)^(1/4)*b^2*d - 3*(a*b^3)^(3/4)*f)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a
/b))/(a*b^4) - 1/32*sqrt(2)*((a*b^3)^(1/4)*b^2*d - 3*(a*b^3)^(3/4)*f)*log(x
^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4)
```

$$3.491 \quad \int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3} dx$$

Optimal. Leaf size=351

$$\frac{(21\sqrt{bc} - 5\sqrt{ae}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{128\sqrt{2}a^{11/4}b^{3/4}} + \frac{(21\sqrt{bc} - 5\sqrt{ae}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{128\sqrt{2}a^{11/4}b^{3/4}} - \frac{(5\sqrt{ae} + 21\sqrt{bc})}{64\sqrt{2}a^{11/4}b^{3/4}}$$

[Out] (x*(7*c + 6*d*x + 5*e*x^2))/(32*a^2*(a + b*x^4)) - (a*f - b*x*(c + d*x + e*x^2))/(8*a*b*(a + b*x^4)^2) + (3*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b]) - ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(3/4)) - ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(3/4))

Rubi [A] time = 0.31771, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.44$, Rules used = {1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(21\sqrt{bc} - 5\sqrt{ae}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{128\sqrt{2}a^{11/4}b^{3/4}} + \frac{(21\sqrt{bc} - 5\sqrt{ae}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{128\sqrt{2}a^{11/4}b^{3/4}} - \frac{(5\sqrt{ae} + 21\sqrt{bc})}{64\sqrt{2}a^{11/4}b^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^3, x]

[Out] (x*(7*c + 6*d*x + 5*e*x^2))/(32*a^2*(a + b*x^4)) - (a*f - b*x*(c + d*x + e*x^2))/(8*a*b*(a + b*x^4)^2) + (3*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b]) - ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(3/4)) - ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(3/4))

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq,
x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1})*(a + b*x^n)^(p
+ 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
+ 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))]/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
```

$(2*d)/e, 2]$, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] & EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1165

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3}{(a + bx^4)^3} dx &= -\frac{af - bx(c + dx + ex^2)}{8ab(a + bx^4)^2} - \frac{\int \frac{-7c - 6dx - 5ex^2}{(a + bx^4)^2} dx}{8a} \\
&= \frac{x(7c + 6dx + 5ex^2)}{32a^2(a + bx^4)} - \frac{af - bx(c + dx + ex^2)}{8ab(a + bx^4)^2} + \frac{\int \frac{21c + 12dx + 5ex^2}{a + bx^4} dx}{32a^2} \\
&= \frac{x(7c + 6dx + 5ex^2)}{32a^2(a + bx^4)} - \frac{af - bx(c + dx + ex^2)}{8ab(a + bx^4)^2} + \frac{\int \left(\frac{12dx}{a + bx^4} + \frac{21c + 5ex^2}{a + bx^4} \right) dx}{32a^2} \\
&= \frac{x(7c + 6dx + 5ex^2)}{32a^2(a + bx^4)} - \frac{af - bx(c + dx + ex^2)}{8ab(a + bx^4)^2} + \frac{\int \frac{21c + 5ex^2}{a + bx^4} dx}{32a^2} + \frac{(3d) \int \frac{x}{a + bx^4} dx}{8a^2} \\
&= \frac{x(7c + 6dx + 5ex^2)}{32a^2(a + bx^4)} - \frac{af - bx(c + dx + ex^2)}{8ab(a + bx^4)^2} + \frac{(3d) \text{Subst} \left(\int \frac{1}{a + bx^2} dx, x, x^2 \right)}{16a^2} + \frac{\left(\frac{21\sqrt{bc}}{\sqrt{a}} - 5e \right)}{6} \\
&= \frac{x(7c + 6dx + 5ex^2)}{32a^2(a + bx^4)} - \frac{af - bx(c + dx + ex^2)}{8ab(a + bx^4)^2} + \frac{3d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{16a^{5/2}\sqrt{b}} - \frac{\left(\frac{21\sqrt{bc}}{\sqrt{a}} - 5e \right) \int \frac{\frac{\sqrt{2}\sqrt[4]{a} + 2x}{\sqrt[4]{b}}}{-\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{b}}} dx}{128\sqrt{2}a^{9/4}b^{3/4}} \\
&= \frac{x(7c + 6dx + 5ex^2)}{32a^2(a + bx^4)} - \frac{af - bx(c + dx + ex^2)}{8ab(a + bx^4)^2} + \frac{3d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{16a^{5/2}\sqrt{b}} - \frac{\left(\frac{21\sqrt{bc}}{\sqrt{a}} - 5e \right) \log(\sqrt{a} - \sqrt{bx^2})}{128\sqrt{2}a^{9/4}} \\
&= \frac{x(7c + 6dx + 5ex^2)}{32a^2(a + bx^4)} - \frac{af - bx(c + dx + ex^2)}{8ab(a + bx^4)^2} + \frac{3d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{16a^{5/2}\sqrt{b}} - \frac{(21\sqrt{bc} + 5\sqrt{ae}) \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}} \right)}{64\sqrt{2}a^{11/4}b^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.328364, size = 347, normalized size = 0.99

$$\frac{\sqrt{2}(5a^{3/4}e - 21\sqrt[4]{a}\sqrt{bc}) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{b^{3/4}} + \frac{\sqrt{2}(21\sqrt[4]{a}\sqrt{bc} - 5a^{3/4}e) \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{b^{3/4}} - \frac{32a^2(af - bx(c + x(d + ex)))}{b(a + bx^4)^2} - \frac{2\sqrt[4]{a} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right)}{256a^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^3, x]

```
[Out] ((8*a*x*(7*c + x*(6*d + 5*e*x)))/(a + b*x^4) - (32*a^2*(a*f - b*x*(c + x*(d + e*x)))/(b*(a + b*x^4)^2) - (2*a^(1/4)*(21*Sqrt[2]*Sqrt[b]*c + 24*a^(1/4)*b^(1/4)*d + 5*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (2*a^(1/4)*(21*Sqrt[2]*Sqrt[b]*c - 24*a^(1/4)*b^(1/4)*d + 5*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (Sqrt[2]*(-21*a^(1/4)*Sqrt[b]*c + 5*a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4) + (Sqrt[2]*(21*a^(1/4)*Sqrt[b]*c - 5*a^(3/4)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(256*a^3)
```

Maple [A] time = 0.005, size = 432, normalized size = 1.2

$$\frac{cx}{8a(bx^4+a)^2} + \frac{7cx}{32a^2(bx^4+a)} + \frac{21c\sqrt{2}}{256a^3} \sqrt[4]{\frac{a}{b}} \ln\left(\left(x^2 + \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x^2 - \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) + \frac{21c\sqrt{2}}{128a^3} \sqrt[4]{\frac{a}{b}} \arctan\left(\frac{2\sqrt{2}}{1 + \sqrt{\frac{a}{b}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x)
```

```
[Out] 1/8*c*x/a/(b*x^4+a)^2+7/32*c/a^2*x/(b*x^4+a)+21/256*c/a^3*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+21/128*c/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+21/128*c/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+1/8*d*x^2/a/(b*x^4+a)^2+3/16*d/a^2*x^2/(b*x^4+a)+3/16*d/a^2/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+1/8*e*x^3/a/(b*x^4+a)^2+5/32*e/a^2*x^3/(b*x^4+a)+5/256*e/a^2/b/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+5/128*e/a^2/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+5/128*e/a^2/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+1/8*f*x^4/a/(b*x^4+a)^2+1/8*f/a^2*x^4/(b*x^4+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [A] time = 68.5143, size = 578, normalized size = 1.65

RootSum(268435456t⁴a¹¹b³ + t²(6881280a⁶b²ce + 4718592a⁶b²d²) + t(153600a⁴bde² - 2709504a³b²c²d) + 625a²e⁴)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)

[Out] RootSum(268435456*_t**4*a**11*b**3 + _t**2*(6881280*a**6*b**2*c*e + 4718592*a**6*b**2*d**2) + _t*(153600*a**4*b*d*e**2 - 2709504*a**3*b**2*c**2*d) + 625*a**2*e**4 + 22050*a*b*c**2*e**2 - 60480*a*b*c*d**2*e + 20736*a*b*d**4 + 194481*b**2*c**4, Lambda(_t, _t*log(x + (262144000*_t**3*a**10*b**2*e**3 - 4624220160*_t**3*a**9*b**3*c**2*e + 12683575296*_t**3*a**9*b**3*c*d**2 + 309657600*_t**2*a**7*b**2*c*d*e**2 - 283115520*_t**2*a**7*b**2*d**3*e + 1820786688*_t**2*a**6*b**3*c**3*d + 5040000*_t*a**5*b*c*e**4 + 6912000*_t*a**5*b*d**2*e**3 - 118540800*_t*a**4*b**2*c**3*e**2 + 365783040*_t*a**4*b**2*c**2*d**2*e + 111476736*_t*a**4*b**2*c*d**4 + 522764928*_t*a**3*b**3*c**5 + 112500*a**3*d*e**5 + 4536000*a**2*b*c*d**3*e**2 - 2488320*a**2*b*d**5*e + 58344300*a*b**2*c**4*d*e - 80015040*a*b**2*c**3*d**3)/(15625*a**3*e**6 - 275625*a**2*b*c**2*e**4 + 3024000*a**2*b*c*d**2*e**3 - 2073600*a**2*b*d**4*e**2 - 4862025*a*b**2*c**4*e**2 + 53343360*a*b**2*c**3*d**2*e - 36578304*a*b**2*c**2*d**4 + 85766121*b**3*c**6))) + (-4*a**2*f + 11*a*b*c*x + 10*a*b*d*x**2 + 9*a*b*e*x**3 + 7*b**2*c*x**5 + 6*b**2*d*x**6 + 5*b**2*e*x**7)/(32*a**4*b + 64*a**3*b**2*x**4 + 32*a**2*b**3*x**8)

Giac [A] time = 1.1009, size = 478, normalized size = 1.36

$$\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{abb^2d} + 21(ab^3)^{\frac{1}{4}}b^2c + 5(ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128a^3b^3} + \frac{\sqrt{2}\left(12\sqrt{2}\sqrt{abb^2d} + 21(ab^3)^{\frac{1}{4}}b^2c + 5(ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")

[Out] 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b^3)^(1/4)*b^2*c + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b^3)^(1/4)*b^2*c + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c - 5*(a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c - 5*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) + 1/32*(5*b^2*x^7*e + 6*b^2*d*x^6 + 7*b^2*c*x^5 + 9*a*b*x^3*e + 10*a*b*d*x^2 + 11*a*b*c*x - 4*a^2*f)/((b*x^4 + a)^2*a^2*b)

$$3.492 \quad \int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3} dx$$

Optimal. Leaf size=340

$$\frac{3(\sqrt{bd} - \sqrt{af}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{128\sqrt{2}a^{7/4}b^{7/4}} + \frac{3(\sqrt{bd} - \sqrt{af}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{128\sqrt{2}a^{7/4}b^{7/4}} - \frac{3(\sqrt{af} + \sqrt{bd}) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} - \sqrt{a} - \sqrt{bx^2}}\right)}{64\sqrt{2}a^{7/4}b^{7/4}}$$

[Out] $-(c + d*x + e*x^2 + f*x^3)/(8*b*(a + b*x^4)^2) + (x*(d + 2*e*x + 3*f*x^2))/(32*a*b*(a + b*x^4)) + (e*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^{(3/2)}*b^{(3/2)}) - (3*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^{(1/4)}*x)/a^{(1/4)}])/(64*Sqrt[2]*a^{(7/4)}*b^{(7/4)}) + (3*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^{(1/4)}*x)/a^{(1/4)}])/(64*Sqrt[2]*a^{(7/4)}*b^{(7/4)}) - (3*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^{(1/4)}*b^{(1/4)}*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^{(7/4)}*b^{(7/4)}) + (3*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^{(1/4)}*b^{(1/4)}*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^{(7/4)}*b^{(7/4)})$

Rubi [A] time = 0.328179, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {1823, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{3(\sqrt{bd} - \sqrt{af}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{128\sqrt{2}a^{7/4}b^{7/4}} + \frac{3(\sqrt{bd} - \sqrt{af}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{128\sqrt{2}a^{7/4}b^{7/4}} - \frac{3(\sqrt{af} + \sqrt{bd}) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} - \sqrt{a} - \sqrt{bx^2}}\right)}{64\sqrt{2}a^{7/4}b^{7/4}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^3,x]

[Out] $-(c + d*x + e*x^2 + f*x^3)/(8*b*(a + b*x^4)^2) + (x*(d + 2*e*x + 3*f*x^2))/(32*a*b*(a + b*x^4)) + (e*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^{(3/2)}*b^{(3/2)}) - (3*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^{(1/4)}*x)/a^{(1/4)}])/(64*Sqrt[2]*a^{(7/4)}*b^{(7/4)}) + (3*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^{(1/4)}*x)/a^{(1/4)}])/(64*Sqrt[2]*a^{(7/4)}*b^{(7/4)}) - (3*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^{(1/4)}*b^{(1/4)}*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^{(7/4)}*b^{(7/4)}) + (3*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^{(1/4)}*b^{(1/4)}*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^{(7/4)}*b^{(7/4)})$

Rule 1823

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(Pq*(
a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
+ 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])]; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3} dx &= -\frac{c+dx+ex^2+fx^3}{8b(a+bx^4)^2} + \frac{\int \frac{d+2ex+3fx^2}{(a+bx^4)^2} dx}{8b} \\
&= -\frac{c+dx+ex^2+fx^3}{8b(a+bx^4)^2} + \frac{x(d+2ex+3fx^2)}{32ab(a+bx^4)} - \frac{\int \frac{-3d-4ex-3fx^2}{a+bx^4} dx}{32ab} \\
&= -\frac{c+dx+ex^2+fx^3}{8b(a+bx^4)^2} + \frac{x(d+2ex+3fx^2)}{32ab(a+bx^4)} - \frac{\int \left(-\frac{4ex}{a+bx^4} + \frac{-3d-3fx^2}{a+bx^4}\right) dx}{32ab} \\
&= -\frac{c+dx+ex^2+fx^3}{8b(a+bx^4)^2} + \frac{x(d+2ex+3fx^2)}{32ab(a+bx^4)} - \frac{\int \frac{-3d-3fx^2}{a+bx^4} dx}{32ab} + \frac{e \int \frac{x}{a+bx^4} dx}{8ab} \\
&= -\frac{c+dx+ex^2+fx^3}{8b(a+bx^4)^2} + \frac{x(d+2ex+3fx^2)}{32ab(a+bx^4)} + \frac{e \operatorname{Subst}\left(\int \frac{1}{a+bx^2} dx, x, x^2\right)}{16ab} + \frac{3\left(\frac{\sqrt{bd}}{\sqrt{a}} - f\right)}{128ab^2} \\
&= -\frac{c+dx+ex^2+fx^3}{8b(a+bx^4)^2} + \frac{x(d+2ex+3fx^2)}{32ab(a+bx^4)} + \frac{e \tan^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{16a^{3/2}b^{3/2}} + \frac{3\left(\frac{\sqrt{bd}}{\sqrt{a}} + f\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}}{4}}}{128ab^2} \\
&= -\frac{c+dx+ex^2+fx^3}{8b(a+bx^4)^2} + \frac{x(d+2ex+3fx^2)}{32ab(a+bx^4)} + \frac{e \tan^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{16a^{3/2}b^{3/2}} - \frac{3(\sqrt{bd} - \sqrt{a}f) \log(\sqrt{a})}{128\sqrt{2}} \\
&= -\frac{c+dx+ex^2+fx^3}{8b(a+bx^4)^2} + \frac{x(d+2ex+3fx^2)}{32ab(a+bx^4)} + \frac{e \tan^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{16a^{3/2}b^{3/2}} - \frac{3(\sqrt{bd} + \sqrt{a}f) \tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{b}}\right)}{64\sqrt{2}a^{7/4}b^{7/4}}
\end{aligned}$$

Mathematica [A] time = 0.349547, size = 329, normalized size = 0.97

$$-\frac{2 \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{bx}}{\sqrt[4]{a}}\right) \left(8 \sqrt[4]{a} \sqrt[4]{be} + 3\sqrt{2} \sqrt{af} + 3\sqrt{2} \sqrt{bd}\right)}{a^{7/4}} + \frac{2 \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{bx}}{\sqrt[4]{a}} + 1\right) \left(-8 \sqrt[4]{a} \sqrt[4]{be} + 3\sqrt{2} \sqrt{af} + 3\sqrt{2} \sqrt{bd}\right)}{a^{7/4}} + \frac{3\sqrt{2}(\sqrt{af} - \sqrt{bd}) \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{bx} + \sqrt{a} + \sqrt{bx}\right)}{a^{7/4}}$$

256b^{7/4}

Antiderivative was successfully verified.

[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^3,x]

```
[Out] ((8*b^(3/4)*x*(d + x*(2*e + 3*f*x)))/(a*(a + b*x^4)) - (32*b^(3/4)*(c + x*(d + x*(e + f*x)))/(a + b*x^4)^2 - (2*(3*Sqrt[2]*Sqrt[b]*d + 8*a^(1/4)*b^(1/4)*e + 3*Sqrt[2]*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/a^(7/4) + (2*(3*Sqrt[2]*Sqrt[b]*d - 8*a^(1/4)*b^(1/4)*e + 3*Sqrt[2]*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/a^(7/4) + (3*Sqrt[2]*(-Sqrt[b]*d + Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4) + (3*Sqrt[2]*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(256*b^(7/4))
```

Maple [A] time = 0.012, size = 373, normalized size = 1.1

$$\frac{1}{(bx^4 + a)^2} \left(\frac{3fx^7}{32a} + \frac{ex^6}{16a} + \frac{dx^5}{32a} - \frac{fx^3}{32b} - \frac{ex^2}{16b} - \frac{3dx}{32b} - \frac{c}{8b} \right) + \frac{3d\sqrt{2}}{256ba^2} \sqrt[4]{\frac{a}{b}} \ln \left(\left(x^2 + \sqrt[4]{\frac{a}{b}} x \sqrt{2} + \sqrt{\frac{a}{b}} \right) \left(x^2 - \sqrt[4]{\frac{a}{b}} x \sqrt{2} + \sqrt{\frac{a}{b}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x)
```

```
[Out] (3/32*f/a*x^7+1/16/a*e*x^6+1/32*d/a*x^5-1/32*f*x^3/b-1/16*e*x^2/b-3/32*d*x/b-1/8*c/b)/(b*x^4+a)^2+3/256/b/a^2*d*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))) +3/128/b/a^2*d*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+3/128/b/a^2*d*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+1/16/b/a*e/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+3/256/b^2/a*f/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))) +3/128/b^2/a*f/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+3/128/b^2/a*f/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)

[Out] Timed out

Giac [A] time = 1.09603, size = 456, normalized size = 1.34

$$\frac{3bfx^7 + 2bx^6e + bdx^5 - afx^3 - 2ax^2e - 3adx - 4ac}{32(bx^4 + a)^2 ab} + \frac{\sqrt{2}\left(4\sqrt{2}\sqrt{abb^2e} + 3(ab^3)^{\frac{1}{4}}b^2d + 3(ab^3)^{\frac{3}{4}}f\right) \arctan\left(\frac{\sqrt{2}(2x + \sqrt{2}(a/b)^{\frac{1}{4}})}{2(a/b)^{\frac{1}{4}}}\right)}{128a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")

[Out] 1/32*(3*b*f*x^7 + 2*b*x^6*e + b*d*x^5 - a*f*x^3 - 2*a*x^2*e - 3*a*d*x - 4*a*c)/((b*x^4 + a)^2*a*b) + 1/128*sqrt(2)*(4*sqrt(2)*sqrt(a*b)*b^2*e + 3*(a*b^3)^(1/4)*b^2*d + 3*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + 1/128*sqrt(2)*(4*sqrt(2)*sqrt(a*b)*b^2*e + 3*(a*b^3)^(1/4)*b^2*d + 3*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4)

$$\begin{aligned}
& (2) * (a/b)^{(1/4)} / (a/b)^{(1/4)} / (a^2 * b^4) + 3/256 * \sqrt{2} * ((a * b^3)^{(1/4)} * b^2 * \\
& d - (a * b^3)^{(3/4)} * f) * \log(x^2 + \sqrt{2} * x * (a/b)^{(1/4)} + \sqrt{a/b}) / (a^2 * b^4) \\
& - 3/256 * \sqrt{2} * ((a * b^3)^{(1/4)} * b^2 * d - (a * b^3)^{(3/4)} * f) * \log(x^2 - \sqrt{2} * \\
& x * (a/b)^{(1/4)} + \sqrt{a/b}) / (a^2 * b^4)
\end{aligned}$$

$$3.493 \quad \int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4} dx$$

Optimal. Leaf size=382

$$\frac{(77\sqrt{bc} - 15\sqrt{ae}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{512\sqrt{2}a^{15/4}b^{3/4}} + \frac{(77\sqrt{bc} - 15\sqrt{ae}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{512\sqrt{2}a^{15/4}b^{3/4}} - \frac{(15\sqrt{ae} + 77\sqrt{bc}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{512\sqrt{2}a^{15/4}b^{3/4}}$$

[Out] (x*(11*c + 10*d*x + 9*e*x^2))/(96*a^2*(a + b*x^4)^2) + (x*(77*c + 60*d*x + 45*e*x^2))/(384*a^3*(a + b*x^4)) - (a*f - b*x*(c + d*x + e*x^2))/(12*a*b*(a + b*x^4)^3) + (5*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b]) - ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(3/4)) - ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(3/4))

Rubi [A] time = 0.405867, antiderivative size = 382, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.44$, Rules used = {1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(77\sqrt{bc} - 15\sqrt{ae}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{512\sqrt{2}a^{15/4}b^{3/4}} + \frac{(77\sqrt{bc} - 15\sqrt{ae}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{512\sqrt{2}a^{15/4}b^{3/4}} - \frac{(15\sqrt{ae} + 77\sqrt{bc}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{512\sqrt{2}a^{15/4}b^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^4, x]

[Out] (x*(11*c + 10*d*x + 9*e*x^2))/(96*a^2*(a + b*x^4)^2) + (x*(77*c + 60*d*x + 45*e*x^2))/(384*a^3*(a + b*x^4)) - (a*f - b*x*(c + d*x + e*x^2))/(12*a*b*(a + b*x^4)^3) + (5*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b]) - ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(3/4)) - ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(3/4))

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq,
x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1})*(a + b*x^n)^(p
+ 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
+ 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
```

$(2*d)/e, 2]$, $\text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x] /;$ $\text{FreeQ}\{a, c, d, e, x\} \& \& \text{EqQ}[c*d^2 - a*e^2, 0] \& \& \text{PosQ}[d*e]$

Rule 617

$\text{Int}[(a_ + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x_Symbol] := \text{With}[\{q = 1 - 4*Simplify[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /;$ $\text{RationalQ}[q] \& \& (\text{EqQ}[q^2, 1] || !\text{RationalQ}[b^2 - 4*a*c]) /;$ $\text{FreeQ}\{a, b, c\}, x] \& \& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 204

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] := -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2]*x]/\text{Rt}[-a, 2]]/\text{Rt}[-a, 2]*\text{Rt}[-b, 2], x] /;$ $\text{FreeQ}\{a, b\}, x] \& \& \text{PosQ}[a/b] \& \& (\text{LtQ}[a, 0] || \text{LtQ}[b, 0])$

Rule 1165

$\text{Int}[(d_ + (e_)*(x_)^2)/(a_ + (c_)*(x_)^4), x_Symbol] := \text{With}[\{q = \text{Rt}[-2*d]/e, 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x] /;$ $\text{FreeQ}\{a, c, d, e\}, x] \& \& \text{EqQ}[c*d^2 - a*e^2, 0] \& \& \text{NegQ}[d*e]$

Rule 628

$\text{Int}[(d_ + (e_)*(x_))/(a_ + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /;$ $\text{FreeQ}\{a, b, c, d, e\}, x] \& \& \text{EqQ}[2*c*d - b*e, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3}{(a + bx^4)^4} dx &= -\frac{af - bx(c + dx + ex^2)}{12ab(a + bx^4)^3} - \frac{\int \frac{-11c - 10dx - 9ex^2}{(a + bx^4)^3} dx}{12a} \\
&= \frac{x(11c + 10dx + 9ex^2)}{96a^2(a + bx^4)^2} - \frac{af - bx(c + dx + ex^2)}{12ab(a + bx^4)^3} + \frac{\int \frac{77c + 60dx + 45ex^2}{(a + bx^4)^2} dx}{96a^2} \\
&= \frac{x(11c + 10dx + 9ex^2)}{96a^2(a + bx^4)^2} + \frac{x(77c + 60dx + 45ex^2)}{384a^3(a + bx^4)} - \frac{af - bx(c + dx + ex^2)}{12ab(a + bx^4)^3} - \frac{\int \frac{-231c - 120dx - 45ex^2}{a + bx^4} dx}{384a^3} \\
&= \frac{x(11c + 10dx + 9ex^2)}{96a^2(a + bx^4)^2} + \frac{x(77c + 60dx + 45ex^2)}{384a^3(a + bx^4)} - \frac{af - bx(c + dx + ex^2)}{12ab(a + bx^4)^3} - \frac{\int \left(-\frac{120dx}{a + bx^4} + \frac{-231c}{a + bx^4} \right) dx}{384a^3} \\
&= \frac{x(11c + 10dx + 9ex^2)}{96a^2(a + bx^4)^2} + \frac{x(77c + 60dx + 45ex^2)}{384a^3(a + bx^4)} - \frac{af - bx(c + dx + ex^2)}{12ab(a + bx^4)^3} - \frac{\int \frac{-231c - 45ex^2}{a + bx^4} dx}{384a^3} \\
&= \frac{x(11c + 10dx + 9ex^2)}{96a^2(a + bx^4)^2} + \frac{x(77c + 60dx + 45ex^2)}{384a^3(a + bx^4)} - \frac{af - bx(c + dx + ex^2)}{12ab(a + bx^4)^3} + \frac{(5d) \text{Subst} \left(\int \frac{5d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{32a^{7/2} \sqrt{b}} \right)}{32a^{7/2} \sqrt{b}} \\
&= \frac{x(11c + 10dx + 9ex^2)}{96a^2(a + bx^4)^2} + \frac{x(77c + 60dx + 45ex^2)}{384a^3(a + bx^4)} - \frac{af - bx(c + dx + ex^2)}{12ab(a + bx^4)^3} + \frac{5d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{32a^{7/2} \sqrt{b}} \\
&= \frac{x(11c + 10dx + 9ex^2)}{96a^2(a + bx^4)^2} + \frac{x(77c + 60dx + 45ex^2)}{384a^3(a + bx^4)} - \frac{af - bx(c + dx + ex^2)}{12ab(a + bx^4)^3} + \frac{5d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{32a^{7/2} \sqrt{b}} \\
&= \frac{x(11c + 10dx + 9ex^2)}{96a^2(a + bx^4)^2} + \frac{x(77c + 60dx + 45ex^2)}{384a^3(a + bx^4)} - \frac{af - bx(c + dx + ex^2)}{12ab(a + bx^4)^3} + \frac{5d \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{32a^{7/2} \sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.408928, size = 379, normalized size = 0.99

$$\frac{3\sqrt{2}(15a^{3/4}e - 77\sqrt[4]{a}\sqrt{bc}) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx + \sqrt{a} + \sqrt{bx^2}}\right)}{b^{3/4}} + \frac{3\sqrt{2}(77\sqrt[4]{a}\sqrt{bc} - 15a^{3/4}e) \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx + \sqrt{a} + \sqrt{bx^2}}\right)}{b^{3/4}} - \frac{256a^3(af - bx(c + x(d + ex)))}{b(a + bx^4)^3} + \frac{32a^2x(11c + x(d + ex))}{(a + bx^4)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^4,x]

[Out]
$$\frac{((8ax(77c + 15x(4d + 3ex)))/(a + bx^4) + (32a^2x(11c + x(10d + 9ex)))/(a + bx^4)^2 - (256a^3(af - bxc + x(d + ex)))/(b(a + bx^4)^3) - (6a^{1/4}(77\sqrt{2}\sqrt{b}c + 80a^{1/4}b^{1/4}d + 15\sqrt{2}\sqrt{a}e)\text{ArcTan}[1 - (\sqrt{2}b^{1/4}x/a^{1/4})]/b^{3/4} + (6a^{1/4}(77\sqrt{2}\sqrt{b}c - 80a^{1/4}b^{1/4}d + 15\sqrt{2}\sqrt{a}e)\text{ArcTan}[1 + (\sqrt{2}b^{1/4}x/a^{1/4})]/b^{3/4} + (3\sqrt{2}(-77a^{1/4}\sqrt{b}c + 15a^{3/4}e)\text{Log}[\sqrt{a} - \sqrt{2}a^{1/4}b^{1/4}x + \sqrt{b}x^2])/b^{3/4} + (3\sqrt{2}(77a^{1/4}\sqrt{b}c - 15a^{3/4}e)\text{Log}[\sqrt{a} + \sqrt{2}a^{1/4}b^{1/4}x + \sqrt{b}x^2])/b^{3/4}))/3072a^4$$

Maple [A] time = 0.014, size = 400, normalized size = 1.1

$$\frac{1}{(bx^4 + a)^3} \left(\frac{15b^2ex^{11}}{128a^3} + \frac{5b^2dx^{10}}{32a^3} + \frac{77b^2cx^9}{384a^3} + \frac{21bex^7}{64a^2} + \frac{5bdx^6}{12a^2} + \frac{33bcx^5}{64a^2} + \frac{113ex^3}{384a} + \frac{11dx^2}{32a} + \frac{51cx}{128a} - \frac{f}{12b} \right) + \frac{77c}{1024a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x)

[Out]
$$\frac{15}{128} \frac{e}{a^3} b^2 x^{11} + \frac{5}{32} \frac{d}{a^3} b^2 x^{10} + \frac{77}{384} \frac{c}{a^3} b^2 x^9 + \frac{21}{64} \frac{f}{a^2} b x^7 + \frac{5}{12} \frac{bdx^6}{a^2} + \frac{33}{64} \frac{bcx^5}{a^2} + \frac{113}{384} \frac{ex^3}{a} + \frac{11}{32} \frac{dx^2}{a} + \frac{51}{128} \frac{cx}{a} - \frac{f}{12b} + \frac{77c}{1024a^4} \ln\left(\frac{x^2 + (1/ba)^{1/4}x^2 + (1/ba)^{1/2}}{x^2 - (1/ba)^{1/4}x^2 + (1/ba)^{1/2}}\right) + \frac{77}{512} \frac{e}{a^4} c (1/ba)^{1/4} 2^{1/2} \arctan\left(\frac{2^{1/2}}{(1/ba)^{1/4}x+1}\right) + \frac{77}{512} \frac{e}{a^4} c (1/ba)^{1/4} 2^{1/2} \arctan\left(\frac{2^{1/2}}{(1/ba)^{1/4}x-1}\right) + \frac{5}{32} \frac{d}{a^3} \frac{b}{(ab)^{1/2}} \arctan(x^2(b/a)^{1/2}) + \frac{15}{1024} \frac{e}{a^3} \frac{b}{(1/ba)^{1/4}} 2^{1/2} \ln\left(\frac{x^2 - (1/ba)^{1/4}x^2 + (1/ba)^{1/2}}{x^2 + (1/ba)^{1/4}x^2 + (1/ba)^{1/2}}\right) + \frac{15}{512} \frac{e}{a^3} \frac{b}{(1/ba)^{1/4}} 2^{1/2} \arctan\left(\frac{2^{1/2}}{(1/ba)^{1/4}x+1}\right) + \frac{15}{512} \frac{e}{a^3} \frac{b}{(1/ba)^{1/4}} 2^{1/2} \arctan\left(\frac{2^{1/2}}{(1/ba)^{1/4}x-1}\right)$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)

[Out] Timed out

Giac [A] time = 1.08056, size = 528, normalized size = 1.38

$$\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^2d + 77(ab^3)^{\frac{1}{4}}b^2c + 15(ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512a^4b^3} + \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^2d + 77(ab^3)^{\frac{1}{4}}b^2c + 15(ab^3)^{\frac{3}{4}}e\right)}{512a^4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")

```
[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2*c + 15*(a*
b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(
a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2
*c + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/
b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c - 15*(a*b^3)^(
3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqrt
(2)*(77*(a*b^3)^(1/4)*b^2*c - 15*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b
)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(45*b^3*x^11*e + 60*b^3*d*x^10 + 77*
b^3*c*x^9 + 126*a*b^2*x^7*e + 160*a*b^2*d*x^6 + 198*a*b^2*c*x^5 + 113*a^2*b
*x^3*e + 132*a^2*b*d*x^2 + 153*a^2*b*c*x - 32*a^3*f)/((b*x^4 + a)^3*a^3*b)
```

$$3.494 \quad \int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4} dx$$

Optimal. Leaf size=380

$$\frac{(7\sqrt{bd} - 5\sqrt{af}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{512\sqrt{2}a^{11/4}b^{7/4}} + \frac{(7\sqrt{bd} - 5\sqrt{af}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{512\sqrt{2}a^{11/4}b^{7/4}} - \frac{(5\sqrt{af} + 7\sqrt{bd}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{256\sqrt{2}a^{11/4}b^{7/4}}$$

[Out] $-(c + d*x + e*x^2 + f*x^3)/(12*b*(a + b*x^4)^3) + (x*(d + 2*e*x + 3*f*x^2))/(96*a*b*(a + b*x^4)^2) + (x*(7*d + 12*e*x + 15*f*x^2))/(384*a^2*b*(a + b*x^4)) + (e*\text{ArcTan}[(\text{Sqrt}[b]*x^2)/\text{Sqrt}[a]])/(32*a^{5/2}*b^{3/2}) - ((7*\text{Sqrt}[b]*d + 5*\text{Sqrt}[a]*f)*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{1/4}*x)/a^{1/4}])/(256*\text{Sqrt}[2]*a^{11/4}*b^{7/4}) + ((7*\text{Sqrt}[b]*d + 5*\text{Sqrt}[a]*f)*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{1/4}*x)/a^{1/4}])/(256*\text{Sqrt}[2]*a^{11/4}*b^{7/4}) - ((7*\text{Sqrt}[b]*d - 5*\text{Sqrt}[a]*f)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{1/4}*b^{1/4}*x + \text{Sqrt}[b]*x^2])/(512*\text{Sqrt}[2]*a^{11/4}*b^{7/4}) + ((7*\text{Sqrt}[b]*d - 5*\text{Sqrt}[a]*f)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{1/4}*b^{1/4}*x + \text{Sqrt}[b]*x^2])/(512*\text{Sqrt}[2]*a^{11/4}*b^{7/4})$

Rubi [A] time = 0.402178, antiderivative size = 380, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {1823, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(7\sqrt{bd} - 5\sqrt{af}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{512\sqrt{2}a^{11/4}b^{7/4}} + \frac{(7\sqrt{bd} - 5\sqrt{af}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{512\sqrt{2}a^{11/4}b^{7/4}} - \frac{(5\sqrt{af} + 7\sqrt{bd}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2})}{256\sqrt{2}a^{11/4}b^{7/4}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^4, x]$

[Out] $-(c + d*x + e*x^2 + f*x^3)/(12*b*(a + b*x^4)^3) + (x*(d + 2*e*x + 3*f*x^2))/(96*a*b*(a + b*x^4)^2) + (x*(7*d + 12*e*x + 15*f*x^2))/(384*a^2*b*(a + b*x^4)) + (e*\text{ArcTan}[(\text{Sqrt}[b]*x^2)/\text{Sqrt}[a]])/(32*a^{5/2}*b^{3/2}) - ((7*\text{Sqrt}[b]*d + 5*\text{Sqrt}[a]*f)*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{1/4}*x)/a^{1/4}])/(256*\text{Sqrt}[2]*a^{11/4}*b^{7/4}) + ((7*\text{Sqrt}[b]*d + 5*\text{Sqrt}[a]*f)*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{1/4}*x)/a^{1/4}])/(256*\text{Sqrt}[2]*a^{11/4}*b^{7/4}) - ((7*\text{Sqrt}[b]*d - 5*\text{Sqrt}[a]*f)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{1/4}*b^{1/4}*x + \text{Sqrt}[b]*x^2])/(512*\text{Sqrt}[2]*a^{11/4}*b^{7/4}) + ((7*\text{Sqrt}[b]*d - 5*\text{Sqrt}[a]*f)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{1/4}*b^{1/4}*x + \text{Sqrt}[b]*x^2])/(512*\text{Sqrt}[2]*a^{11/4}*b^{7/4})$

Rule 1823

Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(Pq*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && EqQ[m - n + 1, 0] && LtQ[p, -1]

Rule 1855

Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]

Rule 1876

Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 275

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1168

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &

& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1165

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4} dx &= \frac{c+dx+ex^2+fx^3}{12b(a+bx^4)^3} + \frac{\int \frac{d+2ex+3fx^2}{(a+bx^4)^3} dx}{12b} \\
&= \frac{c+dx+ex^2+fx^3}{12b(a+bx^4)^3} + \frac{x(d+2ex+3fx^2)}{96ab(a+bx^4)^2} - \frac{\int \frac{-7d-12ex-15fx^2}{(a+bx^4)^2} dx}{96ab} \\
&= \frac{c+dx+ex^2+fx^3}{12b(a+bx^4)^3} + \frac{x(d+2ex+3fx^2)}{96ab(a+bx^4)^2} + \frac{x(7d+12ex+15fx^2)}{384a^2b(a+bx^4)} + \frac{\int \frac{21d+24ex+15fx}{a+bx^4}}{384a^2b} \\
&= \frac{c+dx+ex^2+fx^3}{12b(a+bx^4)^3} + \frac{x(d+2ex+3fx^2)}{96ab(a+bx^4)^2} + \frac{x(7d+12ex+15fx^2)}{384a^2b(a+bx^4)} + \frac{\int \left(\frac{24ex}{a+bx^4} + \frac{21d}{a}\right)}{384a^2} \\
&= \frac{c+dx+ex^2+fx^3}{12b(a+bx^4)^3} + \frac{x(d+2ex+3fx^2)}{96ab(a+bx^4)^2} + \frac{x(7d+12ex+15fx^2)}{384a^2b(a+bx^4)} + \frac{\int \frac{21d+15fx^2}{a+bx^4} dx}{384a^2b} \\
&= \frac{c+dx+ex^2+fx^3}{12b(a+bx^4)^3} + \frac{x(d+2ex+3fx^2)}{96ab(a+bx^4)^2} + \frac{x(7d+12ex+15fx^2)}{384a^2b(a+bx^4)} + \frac{e \text{ Subst} \left(\int \frac{1}{a+b} \right)}{32a^2} \\
&= \frac{c+dx+ex^2+fx^3}{12b(a+bx^4)^3} + \frac{x(d+2ex+3fx^2)}{96ab(a+bx^4)^2} + \frac{x(7d+12ex+15fx^2)}{384a^2b(a+bx^4)} + \frac{e \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{32a^{5/2}b^{3/2}} \\
&= \frac{c+dx+ex^2+fx^3}{12b(a+bx^4)^3} + \frac{x(d+2ex+3fx^2)}{96ab(a+bx^4)^2} + \frac{x(7d+12ex+15fx^2)}{384a^2b(a+bx^4)} + \frac{e \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{32a^{5/2}b^{3/2}} \\
&= \frac{c+dx+ex^2+fx^3}{12b(a+bx^4)^3} + \frac{x(d+2ex+3fx^2)}{96ab(a+bx^4)^2} + \frac{x(7d+12ex+15fx^2)}{384a^2b(a+bx^4)} + \frac{e \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{32a^{5/2}b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.421341, size = 366, normalized size = 0.96

$$\frac{8b^{3/4}x(7d+3x(4e+5fx))}{a^2(a+bx^4)} - \frac{6 \tan^{-1} \left(1 - \frac{\sqrt{2} \sqrt[4]{bx}}{\sqrt[4]{a}} \right) \left(16 \sqrt[4]{a} \sqrt[4]{be} + 5\sqrt{2} \sqrt{af} + 7\sqrt{2} \sqrt{bd} \right)}{a^{11/4}} + \frac{6 \tan^{-1} \left(\frac{\sqrt{2} \sqrt[4]{bx}}{\sqrt[4]{a}} + 1 \right) \left(-16 \sqrt[4]{a} \sqrt[4]{be} + 5\sqrt{2} \sqrt{af} + 7\sqrt{2} \sqrt{bd} \right)}{a^{11/4}} + \frac{3\sqrt{2}(5\sqrt{af} - 7\sqrt{bd})}{3072b^{7/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^4,x]

[Out] ((32*b^(3/4)*x*(d + x*(2*e + 3*f*x)))/(a*(a + b*x^4)^2) + (8*b^(3/4)*x*(7*d + 3*x*(4*e + 5*f*x)))/(a^2*(a + b*x^4)) - (256*b^(3/4)*(c + x*(d + x*(e + f*x))))/(a + b*x^4)^3 - (6*(7*Sqrt[2]*Sqrt[b]*d + 16*a^(1/4)*b^(1/4)*e + 5*Sqrt[2]*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/a^(11/4) + (6*(7*Sqrt[2]*Sqrt[b]*d - 16*a^(1/4)*b^(1/4)*e + 5*Sqrt[2]*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/a^(11/4) + (3*Sqrt[2]*(-7*Sqrt[b]*d + 5*Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(11/4) + (3*Sqrt[2]*(7*Sqrt[b]*d - 5*Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(11/4))/(3072*b^(7/4))

Maple [A] time = 0.012, size = 403, normalized size = 1.1

$$\frac{1}{(bx^4 + a)^3} \left(\frac{5bfx^{11}}{128a^2} + \frac{bex^{10}}{32a^2} + \frac{7bdx^9}{384a^2} + \frac{7fx^7}{64a} + \frac{ex^6}{12a} + \frac{3dx^5}{64a} - \frac{5fx^3}{384b} - \frac{ex^2}{32b} - \frac{7dx}{128b} - \frac{c}{12b} \right) + \frac{7d\sqrt{2}}{1024ba^3} \sqrt[4]{\frac{a}{b}} \ln \left(\left(x^2 + \right. \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x)

[Out] (5/128*f*b/a^2*x^11+1/32/a^2*b*e*x^10+7/384/a^2*d*b*x^9+7/64*f/a*x^7+1/12/a*e*x^6+3/64*d/a*x^5-5/384*f*x^3/b-1/32*e*x^2/b-7/128*d*x/b-1/12*c/b)/(b*x^4+a)^3+7/1024/b/a^3*d*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+7/512/b/a^3*d*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+7/512/b/a^3*d*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+1/32/b/a^2*e/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+5/1024/b^2/a^2*f/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+5/512/b^2/a^2*f/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+5/512/b^2/a^2*f/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)

[Out] Timed out

Giac [A] time = 1.09185, size = 513, normalized size = 1.35

$$\frac{\sqrt{2}\left(8\sqrt{2}\sqrt{abb^2e} + 7(ab^3)^{\frac{1}{4}}b^2d + 5(ab^3)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512a^3b^4} + \frac{\sqrt{2}\left(8\sqrt{2}\sqrt{abb^2e} + 7(ab^3)^{\frac{1}{4}}b^2d + 5(ab^3)^{\frac{3}{4}}f\right)}{512a^3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")

```
[Out] 1/512*sqrt(2)*(8*sqrt(2)*sqrt(a*b)*b^2*e + 7*(a*b^3)^(1/4)*b^2*d + 5*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^4) + 1/512*sqrt(2)*(8*sqrt(2)*sqrt(a*b)*b^2*e + 7*(a*b^3)^(1/4)*b^2*d + 5*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^4) + 1/1024*sqrt(2)*(7*(a*b^3)^(1/4)*b^2*d - 5*(a*b^3)^(3/4)*f)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^4) - 1/1024*sqrt(2)*(7*(a*b^3)^(1/4)*b^2*d - 5*(a*b^3)^(3/4)*f)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^4) + 1/384*(15*b^2*f*x^11 + 12*b^2*x^10*e + 7*b^2*d*x^9 + 42*a*b*f*x^7 + 32*a*b*x^6*e + 18*a*b*d*x^5 - 5*a^2*f*x^3 - 12*a^2*x^2*e - 21*a^2*d*x - 32*a^2*c)/((b*x^4 + a)^3*a^2*b)
```

3.495 $\int x^4 (c + dx + ex^2 + fx^3) \sqrt{a + bx^4} dx$

Optimal. Leaf size=418

$$\frac{a^{7/4} (\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (7\sqrt{ae} + 5\sqrt{bc}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right) - a^2 d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right) - \frac{2a^2 ex \sqrt{a+bx^4}}{15b^{3/2} (\sqrt{a} + \sqrt{bx^2})}}{105b^{7/4} \sqrt{a+bx^4}} - \frac{a^2 d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right)}{16b^{3/2}} - \frac{2a^2 ex \sqrt{a+bx^4}}{15b^{3/2} (\sqrt{a} + \sqrt{bx^2})}$$

[Out] $(2*a*c*x*\operatorname{Sqrt}[a + b*x^4])/(21*b) - (a*d*x^2*\operatorname{Sqrt}[a + b*x^4])/(16*b) + (2*a*e*x^3*\operatorname{Sqrt}[a + b*x^4])/(45*b) - (2*a^2*e*x*\operatorname{Sqrt}[a + b*x^4])/(15*b^{3/2}*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) + (x^5*(9*c + 7*e*x^2)*\operatorname{Sqrt}[a + b*x^4])/63 + (f*x^4*(a + b*x^4)^{(3/2)})/(10*b) - ((8*a*f - 15*b*d*x^2)*(a + b*x^4)^{(3/2)})/(120*b^2) - (a^2*d*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*x^2)/\operatorname{Sqrt}[a + b*x^4]])/(16*b^{3/2}) + (2*a^{9/4}*e*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2])/(15*b^{7/4}*\operatorname{Sqrt}[a + b*x^4]) - (a^{7/4}*(5*\operatorname{Sqrt}[b]*c + 7*\operatorname{Sqrt}[a]*e)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2])/(105*b^{7/4}*\operatorname{Sqrt}[a + b*x^4])$

Rubi [A] time = 0.381155, antiderivative size = 418, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {1833, 1274, 1280, 1198, 220, 1196, 1252, 833, 780, 195, 217, 206}

$$\frac{a^{7/4} (\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (7\sqrt{ae} + 5\sqrt{bc}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right) - a^2 d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right) - \frac{2a^2 ex \sqrt{a+bx^4}}{15b^{3/2} (\sqrt{a} + \sqrt{bx^2})}}{105b^{7/4} \sqrt{a+bx^4}} - \frac{a^2 d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right)}{16b^{3/2}} - \frac{2a^2 ex \sqrt{a+bx^4}}{15b^{3/2} (\sqrt{a} + \sqrt{bx^2})} + \dots$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^4*(c + d*x + e*x^2 + f*x^3)*\operatorname{Sqrt}[a + b*x^4], x]$

[Out] $(2*a*c*x*\operatorname{Sqrt}[a + b*x^4])/(21*b) - (a*d*x^2*\operatorname{Sqrt}[a + b*x^4])/(16*b) + (2*a*e*x^3*\operatorname{Sqrt}[a + b*x^4])/(45*b) - (2*a^2*e*x*\operatorname{Sqrt}[a + b*x^4])/(15*b^{3/2}*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) + (x^5*(9*c + 7*e*x^2)*\operatorname{Sqrt}[a + b*x^4])/63 + (f*x^4*(a + b*x^4)^{(3/2)})/(10*b) - ((8*a*f - 15*b*d*x^2)*(a + b*x^4)^{(3/2)})/(120*b^2) - (a^2*d*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*x^2)/\operatorname{Sqrt}[a + b*x^4]])/(16*b^{3/2}) + (2*a^{9/4}*e*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2])/(15*b^{7/4}*\operatorname{Sqrt}[a + b*x^4]) - (a^{7/4}*(5*\operatorname{Sqrt}[b]*c + 7*\operatorname{Sqrt}[a]*e)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2])/(105*b^{7/4}*\operatorname{Sqrt}[a + b*x^4])$

)]/(105*b^(7/4)*Sqrt[a + b*x^4])

Rule 1833

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p)/c^j, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]

Rule 1274

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*p + 3)), x] + Dist[(4*a*p)/((4*p + m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p + 3) + e*(4*p + m + 1)*x^2, x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1280

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m - 1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1198

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rule 220

Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196


```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q =
  Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
  1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2]]*EllipticE[2*ArcTan[q*x],
  1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
  x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_S
ymbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
  x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 833

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p
_), x_Symbol] := Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
  c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x]
  /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
  & NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_) + (e_)*(x_))*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x
_Symbol] := Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_)*(x_)^n)^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p
+ 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
  Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
  x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x^4 (c + dx + ex^2 + fx^3) \sqrt{a + bx^4} dx &= \int \left(x^4 (c + ex^2) \sqrt{a + bx^4} + x^5 (d + fx^2) \sqrt{a + bx^4} \right) dx \\
&= \int x^4 (c + ex^2) \sqrt{a + bx^4} dx + \int x^5 (d + fx^2) \sqrt{a + bx^4} dx \\
&= \frac{1}{63} x^5 (9c + 7ex^2) \sqrt{a + bx^4} + \frac{1}{2} \text{Subst} \left(\int x^2 (d + fx) \sqrt{a + bx^2} dx, x, x^2 \right) + \frac{1}{63} \text{Subst} \left(\int x^3 (d + fx^2) \sqrt{a + bx^2} dx, x, x^2 \right) \\
&= \frac{2aex^3 \sqrt{a + bx^4}}{45b} + \frac{1}{63} x^5 (9c + 7ex^2) \sqrt{a + bx^4} + \frac{fx^4 (a + bx^4)^{3/2}}{10b} + \frac{\text{Subst} \left(\int x^3 (d + fx^2) \sqrt{a + bx^2} dx, x, x^2 \right)}{63} \\
&= \frac{2acx \sqrt{a + bx^4}}{21b} + \frac{2aex^3 \sqrt{a + bx^4}}{45b} + \frac{1}{63} x^5 (9c + 7ex^2) \sqrt{a + bx^4} + \frac{fx^4 (a + bx^4)}{10b} \\
&= \frac{2acx \sqrt{a + bx^4}}{21b} - \frac{adx^2 \sqrt{a + bx^4}}{16b} + \frac{2aex^3 \sqrt{a + bx^4}}{45b} + \frac{1}{63} x^5 (9c + 7ex^2) \sqrt{a + bx^4} \\
&= \frac{2acx \sqrt{a + bx^4}}{21b} - \frac{adx^2 \sqrt{a + bx^4}}{16b} + \frac{2aex^3 \sqrt{a + bx^4}}{45b} - \frac{2a^2 ex \sqrt{a + bx^4}}{15b^{3/2} (\sqrt{a} + \sqrt{bx^2})} + \frac{1}{63} \text{Subst} \left(\int x^3 (d + fx^2) \sqrt{a + bx^2} dx, x, x^2 \right) \\
&= \frac{2acx \sqrt{a + bx^4}}{21b} - \frac{adx^2 \sqrt{a + bx^4}}{16b} + \frac{2aex^3 \sqrt{a + bx^4}}{45b} - \frac{2a^2 ex \sqrt{a + bx^4}}{15b^{3/2} (\sqrt{a} + \sqrt{bx^2})} + \frac{1}{63} \text{Subst} \left(\int x^3 (d + fx^2) \sqrt{a + bx^2} dx, x, x^2 \right)
\end{aligned}$$

Mathematica [C] time = 0.63576, size = 202, normalized size = 0.48

$$\frac{\sqrt{a + bx^4} \left(-\frac{315a^{3/2} \sqrt{bd} \sinh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{\sqrt{\frac{bx^4}{a} + 1}} - \frac{720abcx {}_2F_1 \left(-\frac{1}{2}, \frac{5}{4}; -\frac{bx^4}{a} \right)}{\sqrt{\frac{bx^4}{a} + 1}} + 720bcx (a + bx^4) + 315bdx^2 (a + 2bx^4) - \frac{560abex^3 {}_2F_1 \left(-\frac{1}{2}, \frac{7}{4}; -\frac{bx^4}{a} \right)}{\sqrt{\frac{bx^4}{a} + 1}} \right)}{5040b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4], x]
```

```
[Out] (Sqrt[a + b*x^4]*(720*b*c*x*(a + b*x^4) + 560*b*e*x^3*(a + b*x^4) + 315*b*d*x^2*(a + 2*b*x^4) + 168*f*(a + b*x^4)*(-2*a + 3*b*x^4) - (315*a^(3/2)*Sqrt[b]*d*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[1 + (b*x^4)/a] - (720*a*b*c*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)]/Sqrt[1 + (b*x^4)/a] - (560*a*b*e*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^4)/a)]/Sqrt[1 + (b*x^4)/a]))/(5040*b^2)
```

Maple [C] time = 0.032, size = 390, normalized size = 0.9

$$-\frac{f(-3bx^4 + 2a)}{30b^2} (bx^4 + a)^{\frac{3}{2}} + \frac{ex^7}{9} \sqrt{bx^4 + a} + \frac{2aex^3}{45b} \sqrt{bx^4 + a} - \frac{2i}{15} ea^{\frac{5}{2}} \sqrt{1 - ix^2\sqrt{b}\frac{1}{\sqrt{a}}} \sqrt{1 + ix^2\sqrt{b}\frac{1}{\sqrt{a}}} \text{EllipticF}\left(x\sqrt{\frac{bx^4 + a}{a}}, \frac{1}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2), x)
```

```
[Out] -1/30*f*(b*x^4+a)^(3/2)*(-3*b*x^4+2*a)/b^2+1/9*e*x^7*(b*x^4+a)^(1/2)+2/45*a*e*x^3*(b*x^4+a)^(1/2)/b-2/15*I*e/b^(3/2)*a^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2), I)+2/15*I*e/b^(3/2)*a^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2), I)+1/8*d*x^2*(b*x^4+a)^(3/2)/b-1/16*a*d*x^2*(b*x^4+a)^(1/2)/b-1/16*d/b^(3/2)*a^2*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))+1/7*c*x^5*(b*x^4+a)^(1/2)+2/21*a*c*x*(b*x^4+a)^(1/2)/b-2/21*c/b*a^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2), I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2), x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(fx^7 + ex^6 + dx^5 + cx^4\right)\sqrt{bx^4 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="fricas")

[Out] integral((f*x^7 + e*x^6 + d*x^5 + c*x^4)*sqrt(b*x^4 + a), x)

Sympy [A] time = 7.09639, size = 252, normalized size = 0.6

$$\frac{a^{\frac{3}{2}} dx^2}{16b\sqrt{1 + \frac{bx^4}{a}}} + \frac{\sqrt{ac}x^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{5}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{3\sqrt{a}dx^6}{16\sqrt{1 + \frac{bx^4}{a}}} + \frac{\sqrt{a}ex^7\Gamma\left(\frac{7}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{7}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)} - \frac{a^2 d \operatorname{asinh}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}} + f \left\{ \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)

[Out] a**(3/2)*d*x**2/(16*b*sqrt(1 + b*x**4/a)) + sqrt(a)*c*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*d*x**6/(16*sqrt(1 + b*x**4/a)) + sqrt(a)*e*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) - a**2*d*asinh(sqrt(b)*x**2/sqrt(a))/(16*b**(3/2)) + f*Piecewise((-a**2*sqrt(a + b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x**4)/(30*b) + x**8*sqrt(a + b*x**4)/10, Ne(b, 0)), (sqrt(a)*x**8/8, True)) + b*d*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/a))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="giac")

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^4, x)
```

3.496 $\int x^3 (c + dx + ex^2 + fx^3) \sqrt{a + bx^4} dx$

Optimal. Leaf size=394

$$\frac{a^{7/4} (\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (7\sqrt{a}f + 5\sqrt{bd}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right) - a^2 e \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right) - \frac{2a^2 fx \sqrt{a+bx^4}}{15b^{3/2} (\sqrt{a} + \sqrt{bx^2})}}{105b^{7/4} \sqrt{a+bx^4}} - \frac{a^2 e \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right)}{16b^{3/2}} - \frac{2a^2 fx \sqrt{a+bx^4}}{15b^{3/2} (\sqrt{a} + \sqrt{bx^2})}$$

[Out] $(2*a*d*x*\operatorname{Sqrt}[a + b*x^4])/(21*b) - (a*e*x^2*\operatorname{Sqrt}[a + b*x^4])/(16*b) + (2*a*f*x^3*\operatorname{Sqrt}[a + b*x^4])/(45*b) - (2*a^2*f*x*\operatorname{Sqrt}[a + b*x^4])/(15*b^{(3/2)}*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) + (x^5*(9*d + 7*f*x^2)*\operatorname{Sqrt}[a + b*x^4])/63 + ((4*c + 3*e*x^2)*(a + b*x^4)^{(3/2)})/(24*b) - (a^2*e*\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*x^2)/\operatorname{Sqrt}[a + b*x^4]])/(16*b^{(3/2)}) + (2*a^{(9/4)}*f*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(15*b^{(7/4)}*\operatorname{Sqrt}[a + b*x^4]) - (a^{(7/4)}*(5*\operatorname{Sqrt}[b]*d + 7*\operatorname{Sqrt}[a]*f)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(105*b^{(7/4)}*\operatorname{Sqrt}[a + b*x^4])$

Rubi [A] time = 0.334251, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {1833, 1252, 780, 195, 217, 206, 1274, 1280, 1198, 220, 1196}

$$\frac{a^{7/4} (\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (7\sqrt{a}f + 5\sqrt{bd}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right) - a^2 e \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right) - \frac{2a^2 fx \sqrt{a+bx^4}}{15b^{3/2} (\sqrt{a} + \sqrt{bx^2})} + \dots}{105b^{7/4} \sqrt{a+bx^4}} - \frac{a^2 e \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right)}{16b^{3/2}} - \frac{2a^2 fx \sqrt{a+bx^4}}{15b^{3/2} (\sqrt{a} + \sqrt{bx^2})} + \dots$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^3*(c + d*x + e*x^2 + f*x^3)*\operatorname{Sqrt}[a + b*x^4], x]$

[Out] $(2*a*d*x*\operatorname{Sqrt}[a + b*x^4])/(21*b) - (a*e*x^2*\operatorname{Sqrt}[a + b*x^4])/(16*b) + (2*a*f*x^3*\operatorname{Sqrt}[a + b*x^4])/(45*b) - (2*a^2*f*x*\operatorname{Sqrt}[a + b*x^4])/(15*b^{(3/2)}*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) + (x^5*(9*d + 7*f*x^2)*\operatorname{Sqrt}[a + b*x^4])/63 + ((4*c + 3*e*x^2)*(a + b*x^4)^{(3/2)})/(24*b) - (a^2*e*\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*x^2)/\operatorname{Sqrt}[a + b*x^4]])/(16*b^{(3/2)}) + (2*a^{(9/4)}*f*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(15*b^{(7/4)}*\operatorname{Sqrt}[a + b*x^4]) - (a^{(7/4)}*(5*\operatorname{Sqrt}[b]*d + 7*\operatorname{Sqrt}[a]*f)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(105*b^{(7/4)}*\operatorname{Sqrt}[a + b*x^4])$

Rule 1833

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p]/c^j, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 1252

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 780

```
Int[((d_) + (e_)*(x_))*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p + 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p + 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !LeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 1274

```
Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(c*d*(m + 4*p + 3) + c*e*(4*p
```

```

+ m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*p + 3)), x] + Dist[(4*a*p)/((4*p
+ m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])

```

Rule 1280

```

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_
Symbol] := Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m]
)

```

Rule 1198

```

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]

```

Rule 220

```

Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x],
1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

```

Rule 1196

```

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]

```

Rubi steps

$$\begin{aligned}
\int x^3 (c + dx + ex^2 + fx^3) \sqrt{a + bx^4} dx &= \int \left(x^3 (c + ex^2) \sqrt{a + bx^4} + x^4 (d + fx^2) \sqrt{a + bx^4} \right) dx \\
&= \int x^3 (c + ex^2) \sqrt{a + bx^4} dx + \int x^4 (d + fx^2) \sqrt{a + bx^4} dx \\
&= \frac{1}{63} x^5 (9d + 7fx^2) \sqrt{a + bx^4} + \frac{1}{2} \text{Subst} \left(\int x(c + ex) \sqrt{a + bx^2} dx, x, x^2 \right) + \frac{1}{63} \\
&= \frac{2afx^3 \sqrt{a + bx^4}}{45b} + \frac{1}{63} x^5 (9d + 7fx^2) \sqrt{a + bx^4} + \frac{(4c + 3ex^2)(a + bx^4)^{3/2}}{24b} - \frac{1}{63} \\
&= \frac{2adx \sqrt{a + bx^4}}{21b} - \frac{aex^2 \sqrt{a + bx^4}}{16b} + \frac{2afx^3 \sqrt{a + bx^4}}{45b} + \frac{1}{63} x^5 (9d + 7fx^2) \sqrt{a + bx^4} \\
&= \frac{2adx \sqrt{a + bx^4}}{21b} - \frac{aex^2 \sqrt{a + bx^4}}{16b} + \frac{2afx^3 \sqrt{a + bx^4}}{45b} + \frac{1}{63} x^5 (9d + 7fx^2) \sqrt{a + bx^4} \\
&= \frac{2adx \sqrt{a + bx^4}}{21b} - \frac{aex^2 \sqrt{a + bx^4}}{16b} + \frac{2afx^3 \sqrt{a + bx^4}}{45b} - \frac{2a^2 fx \sqrt{a + bx^4}}{15b^{3/2} (\sqrt{a} + \sqrt{bx^2})} + \frac{1}{63}
\end{aligned}$$

Mathematica [C] time = 0.630121, size = 215, normalized size = 0.55

$$\frac{\sqrt{a + bx^4} \left(63e \left(\sqrt{bx^2} (a + 2bx^4) - \frac{a^{3/2} \sinh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{\sqrt{\frac{bx^4}{a} + 1}} \right) + 168\sqrt{bc} (a + bx^4) - \frac{144a\sqrt{bdx} {}_2F_1 \left(-\frac{1}{2}, \frac{1}{4}, \frac{5}{4}, -\frac{bx^4}{a} \right)}{\sqrt{\frac{bx^4}{a} + 1}} + 144\sqrt{bdx} (a + bx^4) \right)}{1008b^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4], x]

[Out] (Sqrt[a + b*x^4]*(168*Sqrt[b]*c*(a + b*x^4) + 144*Sqrt[b]*d*x*(a + b*x^4) + 112*Sqrt[b]*f*x^3*(a + b*x^4) + 63*e*(Sqrt[b]*x^2*(a + 2*b*x^4) - (a^(3/2)*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[1 + (b*x^4)/a]) - (144*a*Sqrt[b]*d*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)]/Sqrt[1 + (b*x^4)/a] - (112*a*Sqrt[b]*f*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^4)/a)]/Sqrt[1 + (b*x^4)/a]))/(1008*b^(3/2))

Maple [C] time = 0.008, size = 380, normalized size = 1.

$$\frac{fx^7}{9}\sqrt{bx^4+a} + \frac{2x^3af}{45b}\sqrt{bx^4+a} - \frac{2i}{15}fa^{\frac{5}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x)

[Out] $\frac{1}{9}f*x^7*(b*x^4+a)^{(1/2)} + \frac{2}{45}a*f*x^3*(b*x^4+a)^{(1/2)}/b - \frac{2}{15}I*f/b^{(3/2)}*a^{(5/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticF}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)},I) + \frac{2}{15}I*f/b^{(3/2)}*a^{(5/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticE}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)},I) + \frac{1}{8}*e*x^2*(b*x^4+a)^{(3/2)}/b - \frac{1}{16}*a*e*x^2*(b*x^4+a)^{(1/2)}/b - \frac{1}{16}*e/b^{(3/2)}*a^2*\ln(x^2*b^{(1/2)}+(b*x^4+a)^{(1/2)}) + \frac{1}{7}*d*x^5*(b*x^4+a)^{(1/2)} + \frac{2}{21}*a*d*x*(b*x^4+a)^{(1/2)}/b - \frac{2}{21}*d/b*a^2/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticF}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)},I) + \frac{1}{6}*c/b*(b*x^4+a)^{(3/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(bx^4+a)^{\frac{3}{2}}c}{6b} + \int (fx^6 + ex^5 + dx^4)\sqrt{bx^4+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="maxima")

[Out] $\frac{1}{6}*(b*x^4+a)^{(3/2)}*c/b + \text{integrate}((f*x^6+e*x^5+d*x^4)*\text{sqrt}(b*x^4+a),x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((fx^6+ex^5+dx^4+cx^3)\sqrt{bx^4+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="fricas")

[Out] integral((f*x^6 + e*x^5 + d*x^4 + c*x^3)*sqrt(b*x^4 + a), x)

Sympy [A] time = 6.29747, size = 212, normalized size = 0.54

$$\frac{a^{\frac{3}{2}}ex^2}{16b\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}dx^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{5}{4} \middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{3\sqrt{a}ex^6}{16\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}fx^7\Gamma\left(\frac{7}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{7}{4} \middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)} - \frac{a^2e \operatorname{asinh}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}} + c \left(\dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)

[Out] a**(3/2)*e*x**2/(16*b*sqrt(1 + b*x**4/a)) + sqrt(a)*d*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*e*x**6/(16*sqrt(1 + b*x**4/a)) + sqrt(a)*f*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) - a**2*e*asinh(sqrt(b)*x**2/sqrt(a))/(16*b**(3/2)) + c*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b*e*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/a))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^3, x)

3.497 $\int x^2 (c + dx + ex^2 + fx^3) \sqrt{a + bx^4} dx$

Optimal. Leaf size=369

$$\frac{a^{5/4} (\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (21\sqrt{bc} - 5\sqrt{ae}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{105b^{5/4}\sqrt{a+bx^4}} - \frac{2a^{5/4}c (\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\right)}{5b^{3/4}\sqrt{a+bx^4}}$$

[Out] $(2*a*e*x*\operatorname{Sqrt}[a + b*x^4])/(21*b) - (a*f*x^2*\operatorname{Sqrt}[a + b*x^4])/(16*b) + (2*a*c*x*\operatorname{Sqrt}[a + b*x^4])/(5*\operatorname{Sqrt}[b]*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) + (x^3*(7*c + 5*e*x^2)*\operatorname{Sqrt}[a + b*x^4])/35 + ((4*d + 3*f*x^2)*(a + b*x^4)^{(3/2)})/(24*b) - (a^2*f*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*x^2)/\operatorname{Sqrt}[a + b*x^4]])/(16*b^{(3/2)}) - (2*a^{(5/4)}*c*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(5*b^{(3/4)}*\operatorname{Sqrt}[a + b*x^4]) + (a^{(5/4)}*(21*\operatorname{Sqrt}[b]*c - 5*\operatorname{Sqrt}[a]*e)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(105*b^{(5/4)}*\operatorname{Sqrt}[a + b*x^4])$

Rubi [A] time = 0.297924, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {1833, 1274, 1280, 1198, 220, 1196, 1252, 780, 195, 217, 206}

$$\frac{a^{5/4} (\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (21\sqrt{bc} - 5\sqrt{ae}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{105b^{5/4}\sqrt{a+bx^4}} - \frac{2a^{5/4}c (\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\right)}{5b^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2*(c + d*x + e*x^2 + f*x^3)*\operatorname{Sqrt}[a + b*x^4], x]$

[Out] $(2*a*e*x*\operatorname{Sqrt}[a + b*x^4])/(21*b) - (a*f*x^2*\operatorname{Sqrt}[a + b*x^4])/(16*b) + (2*a*c*x*\operatorname{Sqrt}[a + b*x^4])/(5*\operatorname{Sqrt}[b]*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) + (x^3*(7*c + 5*e*x^2)*\operatorname{Sqrt}[a + b*x^4])/35 + ((4*d + 3*f*x^2)*(a + b*x^4)^{(3/2)})/(24*b) - (a^2*f*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*x^2)/\operatorname{Sqrt}[a + b*x^4]])/(16*b^{(3/2)}) - (2*a^{(5/4)}*c*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(5*b^{(3/4)}*\operatorname{Sqrt}[a + b*x^4]) + (a^{(5/4)}*(21*\operatorname{Sqrt}[b]*c - 5*\operatorname{Sqrt}[a]*e)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(105*b^{(5/4)}*\operatorname{Sqrt}[a + b*x^4])$

Rule 1833

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p]/c^j, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 1274

```
Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*p + 3)), x] + Dist[(4*a*p)/(4*p + m + 1)*(m + 4*p + 3), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p + 3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1280

```
Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m - 1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
```

$1/2]/(q*\text{Sqrt}[a + c*x^4]), x] /; \text{EqQ}[e + d*q^2, 0]] /; \text{FreeQ}\{a, c, d, e\}, x] \&\& \text{PosQ}[c/a]$

Rule 1252

$\text{Int}[(x_)^{(m_.)}*((d_) + (e_.)*(x_)^2)^{(q_.)}*((a_) + (c_.)*(x_)^4)^{(p_.)}, x_Symbol] := \text{Dist}[1/2, \text{Subst}[\text{Int}[x^{(m-1)/2}*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; \text{FreeQ}\{a, c, d, e, p, q\}, x] \&\& \text{IntegerQ}[(m + 1)/2]$

Rule 780

$\text{Int}[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^{(p_.)}, x_Symbol] := \text{Simp}[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^{(p + 1)})/(2*c*(p + 1)*(2*p + 3)), x] - \text{Dist}[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p + 3)), \text{Int}[(a + c*x^2)^p, x], x] /; \text{FreeQ}\{a, c, d, e, f, g, p\}, x] \&\& !\text{LeQ}[p, -1]$

Rule 195

$\text{Int}[((a_) + (b_.)*(x_)^n)^{(p_.)}, x_Symbol] := \text{Simp}[(x*(a + b*x^n)^p)/(n*p + 1), x] + \text{Dist}[(a*n*p)/(n*p + 1), \text{Int}[(a + b*x^n)^{(p-1)}, x], x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{GtQ}[p, 0] \&\& (\text{IntegerQ}[2*p] || (\text{EqQ}[n, 2] \&\& \text{IntegerQ}[4*p]) || (\text{EqQ}[n, 2] \&\& \text{IntegerQ}[3*p]) || \text{LtQ}[\text{Denominator}[p + 1/n], \text{Denominator}[p]])$

Rule 217

$\text{Int}[1/\text{Sqrt}[(a_) + (b_.)*(x_)^2], x_Symbol] := \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] /; \text{FreeQ}\{a, b\}, x] \&\& !\text{GtQ}[a, 0]$

Rule 206

$\text{Int}[((a_) + (b_.)*(x_)^2)^{-1}, x_Symbol] := \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] || \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned}
\int x^2(c + dx + ex^2 + fx^3)\sqrt{a + bx^4} dx &= \int \left(x^2(c + ex^2)\sqrt{a + bx^4} + x^3(d + fx^2)\sqrt{a + bx^4} \right) dx \\
&= \int x^2(c + ex^2)\sqrt{a + bx^4} dx + \int x^3(d + fx^2)\sqrt{a + bx^4} dx \\
&= \frac{1}{35}x^3(7c + 5ex^2)\sqrt{a + bx^4} + \frac{1}{2} \text{Subst} \left(\int x(d + fx)\sqrt{a + bx^2} dx, x, x^2 \right) + \frac{1}{35} \\
&= \frac{2aex\sqrt{a + bx^4}}{21b} + \frac{1}{35}x^3(7c + 5ex^2)\sqrt{a + bx^4} + \frac{(4d + 3fx^2)(a + bx^4)^{3/2}}{24b} - \frac{1}{35} \\
&= \frac{2aex\sqrt{a + bx^4}}{21b} - \frac{afx^2\sqrt{a + bx^4}}{16b} + \frac{1}{35}x^3(7c + 5ex^2)\sqrt{a + bx^4} + \frac{(4d + 3fx^2)}{24} \\
&= \frac{2aex\sqrt{a + bx^4}}{21b} - \frac{afx^2\sqrt{a + bx^4}}{16b} + \frac{2acx\sqrt{a + bx^4}}{5\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{1}{35}x^3(7c + 5ex^2)\sqrt{a + bx^4} \\
&= \frac{2aex\sqrt{a + bx^4}}{21b} - \frac{afx^2\sqrt{a + bx^4}}{16b} + \frac{2acx\sqrt{a + bx^4}}{5\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{1}{35}x^3(7c + 5ex^2)\sqrt{a + bx^4}
\end{aligned}$$

Mathematica [C] time = 0.72146, size = 182, normalized size = 0.49

$$\frac{1}{336}\sqrt{a + bx^4} \left(-\frac{21a^{3/2}f \sinh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{b^{3/2}\sqrt{\frac{bx^4}{a} + 1}} + \frac{112cx^3 {}_2F_1\left(-\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -\frac{bx^4}{a}\right)}{\sqrt{\frac{bx^4}{a} + 1}} + \frac{56d(a + bx^4)}{b} - \frac{48aex {}_2F_1\left(-\frac{1}{2}, \frac{1}{4}; \frac{5}{4}; -\frac{bx^4}{a}\right)}{b\sqrt{\frac{bx^4}{a} + 1}} + \dots \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4], x]

[Out] (Sqrt[a + b*x^4]*((56*d*(a + b*x^4))/b + (48*e*x*(a + b*x^4))/b + (21*f*x^2*(a + 2*b*x^4))/b - (21*a^(3/2)*f*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/(b^(3/2)*Sqrt[1 + (b*x^4)/a]) - (48*a*e*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)])/(b*Sqrt[1 + (b*x^4)/a]) + (112*c*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^4)/a)]/Sqrt[1 + (b*x^4)/a]))/336

Maple [C] time = 0.01, size = 361, normalized size = 1.

$$\frac{fx^2}{8b} (bx^4 + a)^{\frac{3}{2}} - \frac{afx^2}{16b} \sqrt{bx^4 + a} - \frac{fa^2}{16} \ln \left(x^2 \sqrt{b} + \sqrt{bx^4 + a} \right) b^{-\frac{3}{2}} + \frac{ex^5}{7} \sqrt{bx^4 + a} + \frac{2aex}{21b} \sqrt{bx^4 + a} - \frac{2a^2e}{21b} \sqrt{1 - ix^2 \sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x)

[Out] $\frac{1}{8}fx^2(bx^4+a)^{3/2}/b - \frac{1}{16}afx^2(bx^4+a)^{1/2}/b - \frac{1}{16}fa^2 \ln(x^2\sqrt{b} + \sqrt{bx^4+a})b^{-3/2} + \frac{ex^5}{7}\sqrt{bx^4+a} + \frac{2aex}{21b}\sqrt{bx^4+a} - \frac{2a^2e}{21b}\sqrt{1-ix^2\sqrt{b}}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(fx^5 + ex^4 + dx^3 + cx^2\right)\sqrt{bx^4 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="fricas")

[Out] integral((f*x^5 + e*x^4 + d*x^3 + c*x^2)*sqrt(b*x^4 + a), x)

Sympy [A] time = 6.00464, size = 212, normalized size = 0.57

$$\frac{a^{\frac{3}{2}} f x^2}{16 b \sqrt{1 + \frac{b x^4}{a}}} + \frac{\sqrt{a} c x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{3}{4} \middle| \frac{b x^4 e^{i \pi}}{a}\right)}{4 \Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{a} e x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{5}{4} \middle| \frac{b x^4 e^{i \pi}}{a}\right)}{4 \Gamma\left(\frac{9}{4}\right)} + \frac{3 \sqrt{a} f x^6}{16 \sqrt{1 + \frac{b x^4}{a}}} - \frac{a^2 f \operatorname{asinh}\left(\frac{\sqrt{b} x^2}{\sqrt{a}}\right)}{16 b^{\frac{3}{2}}} + d \left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)

[Out] a**(3/2)*f*x**2/(16*b*sqrt(1 + b*x**4/a)) + sqrt(a)*c*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + sqrt(a)*e*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*f*x**6/(16*sqrt(1 + b*x**4/a)) - a**2*f*asinh(sqrt(b)*x**2/sqrt(a))/(16*b**(3/2)) + d*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b*f*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/a))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b x^4 + a} (f x^3 + e x^2 + d x + c) x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^2, x)

3.498 $\int x (c + dx + ex^2 + fx^3) \sqrt{a + bx^4} dx$

Optimal. Leaf size=354

$$\frac{a^{5/4} (\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (21\sqrt{bd} - 5\sqrt{af}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{105b^{5/4}\sqrt{a+bx^4}} - \frac{2a^{5/4}d(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\right)}{5b^{3/4}\sqrt{a+bx^4}}$$

[Out] $(2* a * f * x * \operatorname{Sqrt}[a + b * x^4]) / (21 * b) + (c * x^2 * \operatorname{Sqrt}[a + b * x^4]) / 4 + (2 * a * d * x * \operatorname{Sqrt}[a + b * x^4]) / (5 * \operatorname{Sqrt}[b] * (\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b] * x^2)) + (x^3 * (7 * d + 5 * f * x^2) * \operatorname{Sqrt}[a + b * x^4]) / 35 + (e * (a + b * x^4)^{(3/2)}) / (6 * b) + (a * c * \operatorname{ArcTanh}[(\operatorname{Sqrt}[b] * x^2) / \operatorname{Sqrt}[a + b * x^4]]) / (4 * \operatorname{Sqrt}[b]) - (2 * a^{(5/4)} * d * (\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b] * x^2) * \operatorname{Sqrt}[(a + b * x^4) / (\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b] * x^2)^2] * \operatorname{EllipticE}[2 * \operatorname{ArcTan}[(b^{(1/4)} * x) / a^{(1/4)}], 1/2]) / (5 * b^{(3/4)} * \operatorname{Sqrt}[a + b * x^4]) + (a^{(5/4)} * (21 * \operatorname{Sqrt}[b] * d - 5 * \operatorname{Sqrt}[a] * f) * (\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b] * x^2) * \operatorname{Sqrt}[(a + b * x^4) / (\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b] * x^2)^2] * \operatorname{EllipticF}[2 * \operatorname{ArcTan}[(b^{(1/4)} * x) / a^{(1/4)}], 1/2]) / (105 * b^{(5/4)} * \operatorname{Sqrt}[a + b * x^4])$

Rubi [A] time = 0.273206, antiderivative size = 354, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {1833, 1248, 641, 195, 217, 206, 1274, 1280, 1198, 220, 1196}

$$\frac{a^{5/4} (\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (21\sqrt{bd} - 5\sqrt{af}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{105b^{5/4}\sqrt{a+bx^4}} - \frac{2a^{5/4}d(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\right)}{5b^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x * (c + d * x + e * x^2 + f * x^3) * \operatorname{Sqrt}[a + b * x^4], x]$

[Out] $(2 * a * f * x * \operatorname{Sqrt}[a + b * x^4]) / (21 * b) + (c * x^2 * \operatorname{Sqrt}[a + b * x^4]) / 4 + (2 * a * d * x * \operatorname{Sqrt}[a + b * x^4]) / (5 * \operatorname{Sqrt}[b] * (\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b] * x^2)) + (x^3 * (7 * d + 5 * f * x^2) * \operatorname{Sqrt}[a + b * x^4]) / 35 + (e * (a + b * x^4)^{(3/2)}) / (6 * b) + (a * c * \operatorname{ArcTanh}[(\operatorname{Sqrt}[b] * x^2) / \operatorname{Sqrt}[a + b * x^4]]) / (4 * \operatorname{Sqrt}[b]) - (2 * a^{(5/4)} * d * (\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b] * x^2) * \operatorname{Sqrt}[(a + b * x^4) / (\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b] * x^2)^2] * \operatorname{EllipticE}[2 * \operatorname{ArcTan}[(b^{(1/4)} * x) / a^{(1/4)}], 1/2]) / (5 * b^{(3/4)} * \operatorname{Sqrt}[a + b * x^4]) + (a^{(5/4)} * (21 * \operatorname{Sqrt}[b] * d - 5 * \operatorname{Sqrt}[a] * f) * (\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b] * x^2) * \operatorname{Sqrt}[(a + b * x^4) / (\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b] * x^2)^2] * \operatorname{EllipticF}[2 * \operatorname{ArcTan}[(b^{(1/4)} * x) / a^{(1/4)}], 1/2]) / (105 * b^{(5/4)} * \operatorname{Sqrt}[a + b * x^4])$

Rule 1833

```
Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p]/c^j, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Rule 641

```
Int[((d_) + (e_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 1274

```
Int[((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*p + 3)), x] + Dist[(4*a*p)/(4*p + m + 1)*(m + 4*p + 3), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p + 3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
```

IntegerQ[p] || IntegerQ[m])

Rule 1280

```
Int[((f_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] := Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2]]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2]]*EllipticE[2*ArcTan[q*x],
1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int x(c + dx + ex^2 + fx^3) \sqrt{a + bx^4} dx &= \int \left(x(c + ex^2) \sqrt{a + bx^4} + x^2(d + fx^2) \sqrt{a + bx^4} \right) dx \\
&= \int x(c + ex^2) \sqrt{a + bx^4} dx + \int x^2(d + fx^2) \sqrt{a + bx^4} dx \\
&= \frac{1}{35} x^3 (7d + 5fx^2) \sqrt{a + bx^4} + \frac{1}{2} \text{Subst} \left(\int (c + ex) \sqrt{a + bx^2} dx, x, x^2 \right) + \frac{1}{35} (2a) \int \frac{5af - 21b}{\sqrt{a + bx^4}} dx \\
&= \frac{2afx \sqrt{a + bx^4}}{21b} + \frac{1}{35} x^3 (7d + 5fx^2) \sqrt{a + bx^4} + \frac{e(a + bx^4)^{3/2}}{6b} - \frac{(2a) \int \frac{5af - 21b}{\sqrt{a + bx^4}} dx}{105b} \\
&= \frac{2afx \sqrt{a + bx^4}}{21b} + \frac{1}{4} cx^2 \sqrt{a + bx^4} + \frac{1}{35} x^3 (7d + 5fx^2) \sqrt{a + bx^4} + \frac{e(a + bx^4)^{3/2}}{6b} \\
&= \frac{2afx \sqrt{a + bx^4}}{21b} + \frac{1}{4} cx^2 \sqrt{a + bx^4} + \frac{2adx \sqrt{a + bx^4}}{5\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{1}{35} x^3 (7d + 5fx^2) \sqrt{a + bx^4} \\
&= \frac{2afx \sqrt{a + bx^4}}{21b} + \frac{1}{4} cx^2 \sqrt{a + bx^4} + \frac{2adx \sqrt{a + bx^4}}{5\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{1}{35} x^3 (7d + 5fx^2) \sqrt{a + bx^4}
\end{aligned}$$

Mathematica [C] time = 0.184122, size = 211, normalized size = 0.6

$$\frac{\sqrt{a + bx^4} \left(21bcx^2 \sqrt{\frac{bx^4}{a} + 1} + 21\sqrt{a}\sqrt{bc} \sinh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right) + 28bdx^3 {}_2F_1 \left(-\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -\frac{bx^4}{a} \right) + 14bex^4 \sqrt{\frac{bx^4}{a} + 1} + 14ae \sqrt{\frac{bx^4}{a} + 1} \right)}{84b \sqrt{\frac{bx^4}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[x*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4], x]

[Out] (Sqrt[a + b*x^4]*(14*a*e*Sqrt[1 + (b*x^4)/a] + 12*a*f*x*Sqrt[1 + (b*x^4)/a] + 21*b*c*x^2*Sqrt[1 + (b*x^4)/a] + 14*b*e*x^4*Sqrt[1 + (b*x^4)/a] + 12*b*f*x^5*Sqrt[1 + (b*x^4)/a] + 21*Sqrt[a]*Sqrt[b]*c*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] - 12*a*f*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)] + 28*b*d*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^4)/a)]))/(84*b*Sqrt[1 + (b*x^4)/a])

Maple [C] time = 0.013, size = 337, normalized size = 1.

$$\frac{fx^5}{7}\sqrt{bx^4+a} + \frac{2afx}{21b}\sqrt{bx^4+a} - \frac{2fa^2}{21b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right) \frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{bx^4+a}} + \frac{1}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x)`

[Out] $\frac{1}{7}f*x^5*(b*x^4+a)^{(1/2)} + \frac{2}{21}a*f*x*(b*x^4+a)^{(1/2)}/b - \frac{2}{21}f/b*a^{2/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*EllipticF(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)},I) + \frac{1}{6}*e*(b*x^4+a)^{(3/2)}/b + \frac{1}{5}*x^3*d*(b*x^4+a)^{(1/2)} + \frac{2}{5}*I*d*a^{(3/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}/b^{(1/2)}*EllipticF(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)},I) - \frac{2}{5}*I*d*a^{(3/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}/b^{(1/2)}*EllipticE(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)},I) + \frac{1}{4}*c*x^2*(b*x^4+a)^{(1/2)} + \frac{1}{4}*c*a/b^{(1/2)}*\ln(x^2*b^{(1/2)}+(b*x^4+a)^{(1/2)})$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx^4+a}(fx^4+ex^3+dx^2+cx),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="fricas")`

[Out] integral(sqrt(b*x^4 + a)*(f*x^4 + e*x^3 + d*x^2 + c*x), x)

Sympy [A] time = 4.28085, size = 158, normalized size = 0.45

$$\frac{\sqrt{ac}x^2\sqrt{1+\frac{bx^4}{a}}}{4} + \frac{\sqrt{ad}x^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{3}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{a}fx^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{5}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{ac \operatorname{asinh}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{4\sqrt{b}} + e \left\{ \left(\frac{\sqrt{ax^4}}{4} + \frac{(a+bx^4)^{\frac{3}{2}}}{6b} \right) \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)

[Out] sqrt(a)*c*x**2*sqrt(1 + b*x**4/a)/4 + sqrt(a)*d*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + sqrt(a)*f*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + a*c*asinh(sqrt(b)*x**2/sqrt(a))/(4*sqrt(b)) + e*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x, x)

3.499 $\int (c + dx + ex^2 + fx^3) \sqrt{a + bx^4} dx$

Optimal. Leaf size=331

$$\frac{a^{3/4} (\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (3\sqrt{ae} + 5\sqrt{bc}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}} - \frac{2a^{5/4}e(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\right)}{5b^{3/4}\sqrt{a+bx^4}}$$

[Out] (d*x^2*Sqrt[a + b*x^4])/4 + (2*a*e*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (x*(5*c + 3*e*x^2)*Sqrt[a + b*x^4])/15 + (f*(a + b*x^4)^(3/2))/(6*b) + (a*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (2*a^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[a + b*x^4]) + (a^(3/4)*(5*Sqrt[b]*c + 3*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4])

Rubi [A] time = 0.194189, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.37$, Rules used = {1885, 1177, 1198, 220, 1196, 1248, 641, 195, 217, 206}

$$\frac{a^{3/4} (\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (3\sqrt{ae} + 5\sqrt{bc}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}} - \frac{2a^{5/4}e(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4], x]

[Out] (d*x^2*Sqrt[a + b*x^4])/4 + (2*a*e*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (x*(5*c + 3*e*x^2)*Sqrt[a + b*x^4])/15 + (f*(a + b*x^4)^(3/2))/(6*b) + (a*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (2*a^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[a + b*x^4]) + (a^(3/4)*(5*Sqrt[b]*c + 3*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4])

Rule 1885


```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq,
x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, p},
x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 1177

```
Int[((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(x*
(d*(4*p + 3) + e*(4*p + 1)*x^2)*(a + c*x^4)^p)/((4*p + 1)*(4*p + 3)), x] +
Dist[(2*p)/((4*p + 1)*(4*p + 3)), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1198

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol]
:= Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 641

```
Int[((d_) + (e_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int (c + dx + ex^2 + fx^3) \sqrt{a + bx^4} dx &= \int \left((c + ex^2) \sqrt{a + bx^4} + x(d + fx^2) \sqrt{a + bx^4} \right) dx \\
&= \int (c + ex^2) \sqrt{a + bx^4} dx + \int x(d + fx^2) \sqrt{a + bx^4} dx \\
&= \frac{1}{15} x (5c + 3ex^2) \sqrt{a + bx^4} + \frac{1}{15} \int \frac{10ac + 6aex^2}{\sqrt{a + bx^4}} dx + \frac{1}{2} \text{Subst} \left(\int (d + fx) \sqrt{a + bx^2} dx, x, x^2 \right) \\
&= \frac{1}{15} x (5c + 3ex^2) \sqrt{a + bx^4} + \frac{f(a + bx^4)^{3/2}}{6b} + \frac{1}{2} d \text{Subst} \left(\int \sqrt{a + bx^2} dx, x, x^2 \right) \\
&= \frac{1}{4} dx^2 \sqrt{a + bx^4} + \frac{2aex \sqrt{a + bx^4}}{5\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{1}{15} x (5c + 3ex^2) \sqrt{a + bx^4} + \frac{f(a + bx^4)^{3/2}}{6b} \\
&= \frac{1}{4} dx^2 \sqrt{a + bx^4} + \frac{2aex \sqrt{a + bx^4}}{5\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{1}{15} x (5c + 3ex^2) \sqrt{a + bx^4} + \frac{f(a + bx^4)^{3/2}}{6b} \\
&= \frac{1}{4} dx^2 \sqrt{a + bx^4} + \frac{2aex \sqrt{a + bx^4}}{5\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{1}{15} x (5c + 3ex^2) \sqrt{a + bx^4} + \frac{f(a + bx^4)^{3/2}}{6b}
\end{aligned}$$

Mathematica [C] time = 0.128262, size = 171, normalized size = 0.52

$$\frac{\sqrt{a + bx^4} \left(12bcx {}_2F_1 \left(-\frac{1}{2}, \frac{1}{4}; \frac{5}{4}; -\frac{bx^4}{a} \right) + 3bdx^2 \sqrt{\frac{bx^4}{a} + 1} + 3\sqrt{a}\sqrt{bd} \sinh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right) + 4bex^3 {}_2F_1 \left(-\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -\frac{bx^4}{a} \right) + 2bfx^4 \sqrt{\frac{bx^4}{a} + 1} \right)}{12b\sqrt{\frac{bx^4}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4], x]

[Out] (Sqrt[a + b*x^4]*(2*a*f*Sqrt[1 + (b*x^4)/a] + 3*b*d*x^2*Sqrt[1 + (b*x^4)/a] + 2*b*f*x^4*Sqrt[1 + (b*x^4)/a] + 3*Sqrt[a]*Sqrt[b]*d*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + 12*b*c*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)] + 4*b*e*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^4)/a)]))/(12*b*Sqrt[1 + (b*x^4)/a])

Maple [C] time = 0.007, size = 313, normalized size = 1.

$$\frac{f}{6b} (bx^4 + a)^{\frac{3}{2}} + \frac{x^3 e}{5} \sqrt{bx^4 + a} + \frac{2i}{5} ea^{\frac{3}{2}} \sqrt{1 - ix^2 \sqrt{b} \frac{1}{\sqrt{a}}} \sqrt{1 + ix^2 \sqrt{b} \frac{1}{\sqrt{a}}} \text{EllipticF} \left(x \sqrt{i \sqrt{b} \frac{1}{\sqrt{a}}}, i \right) \frac{1}{\sqrt{i \sqrt{b} \frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{bx^4 + a}} \frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x)

[Out] $\frac{1}{6} f (bx^4 + a)^{3/2} / b + \frac{1}{5} x^3 e \sqrt{bx^4 + a} + \frac{2}{5} i e a^{3/2} \sqrt{1 - ix^2 \sqrt{b} \frac{1}{\sqrt{a}}} \sqrt{1 + ix^2 \sqrt{b} \frac{1}{\sqrt{a}}} \text{EllipticF} \left(x \sqrt{i \sqrt{b} \frac{1}{\sqrt{a}}}, i \right) \frac{1}{\sqrt{i \sqrt{b} \frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{bx^4 + a}} \frac{1}{\sqrt{b}} - \frac{2}{5} i e a^{3/2} \sqrt{1 - ix^2 \sqrt{b} \frac{1}{\sqrt{a}}} \sqrt{1 + ix^2 \sqrt{b} \frac{1}{\sqrt{a}}} \text{EllipticE} \left(x \sqrt{i \sqrt{b} \frac{1}{\sqrt{a}}}, i \right) \frac{1}{\sqrt{i \sqrt{b} \frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{bx^4 + a}} \frac{1}{\sqrt{b}} + \frac{1}{4} d x^2 \sqrt{bx^4 + a} + \frac{1}{4} d a \sqrt{bx^4 + a} \ln(x^2 \sqrt{bx^4 + a}) + \frac{1}{3} x c \sqrt{bx^4 + a} + \frac{2}{3} c a \sqrt{bx^4 + a} \text{EllipticF} \left(x \sqrt{i \sqrt{b} \frac{1}{\sqrt{a}}}, i \right) \frac{1}{\sqrt{i \sqrt{b} \frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{bx^4 + a}} \frac{1}{\sqrt{b}}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^4 + a} (fx^3 + ex^2 + dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\sqrt{bx^4 + a} (fx^3 + ex^2 + dx + c), x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c), x)

Sympy [A] time = 4.14321, size = 156, normalized size = 0.47

$$\frac{\sqrt{acx}\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{1}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{adx^2}\sqrt{1 + \frac{bx^4}{a}}}{4} + \frac{\sqrt{aex^3}\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{3}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{ad \operatorname{asinh}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{4\sqrt{b}} + f \left\{ \begin{array}{l} \frac{\sqrt{ax^4}}{4} \\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)

[Out] sqrt(a)*c*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + sqrt(a)*d*x**2*sqrt(1 + b*x**4/a)/4 + sqrt(a)*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + a*d*asinh(sqrt(b)*x**2/sqrt(a))/(4*sqrt(b)) + f*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c), x)

$$3.500 \quad \int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x} dx$$

Optimal. Leaf size=345

$$\frac{a^{3/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (3\sqrt{a}f + 5\sqrt{bd}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}} - \frac{2a^{5/4}f(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\right)}{5b^{3/4}\sqrt{a+bx^4}}$$

[Out] (2*a*f*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*c + e*x^2)*Sqrt[a + b*x^4])/4 + (x*(5*d + 3*f*x^2)*Sqrt[a + b*x^4])/15 + (a*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (Sqrt[a]*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (2*a^(5/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[a + b*x^4]) + (a^(3/4)*(5*Sqrt[b]*d + 3*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4])

Rubi [A] time = 0.255159, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {1833, 1252, 815, 844, 217, 206, 266, 63, 208, 1177, 1198, 220, 1196}

$$\frac{a^{3/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (3\sqrt{a}f + 5\sqrt{bd}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}} - \frac{2a^{5/4}f(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\right)}{5b^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x,x]

[Out] (2*a*f*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*c + e*x^2)*Sqrt[a + b*x^4])/4 + (x*(5*d + 3*f*x^2)*Sqrt[a + b*x^4])/15 + (a*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (Sqrt[a]*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (2*a^(5/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[a + b*x^4]) + (a^(3/4)*(5*Sqrt[b]*d + 3*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4])

Rule 1833

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p]/c^j, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 1252

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 815

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p]/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1177

```
Int[((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(x*
(d*(4*p + 3) + e*(4*p + 1)*x^2)*(a + c*x^4)^p)/((4*p + 1)*(4*p + 3)), x] +
Dist[(2*p)/((4*p + 1)*(4*p + 3)), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1198

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e},
```


x] && PosQ[c/a]

Rubi steps

$$\begin{aligned}
 \int \frac{(c + dx + ex^2 + fx^3) \sqrt{a + bx^4}}{x} dx &= \int \left(\frac{(c + ex^2) \sqrt{a + bx^4}}{x} + (d + fx^2) \sqrt{a + bx^4} \right) dx \\
 &= \int \frac{(c + ex^2) \sqrt{a + bx^4}}{x} dx + \int (d + fx^2) \sqrt{a + bx^4} dx \\
 &= \frac{1}{15} x (5d + 3fx^2) \sqrt{a + bx^4} + \frac{1}{15} \int \frac{10ad + 6afx^2}{\sqrt{a + bx^4}} dx + \frac{1}{2} \text{Subst} \left(\int \frac{(c + ex) \sqrt{a}}{x} dx, x, \sqrt{a + bx^4} \right) \\
 &= \frac{1}{4} (2c + ex^2) \sqrt{a + bx^4} + \frac{1}{15} x (5d + 3fx^2) \sqrt{a + bx^4} + \frac{\text{Subst} \left(\int \frac{2abc + abex}{x \sqrt{a + bx^2}} dx, x, \sqrt{a + bx^4} \right)}{4b} \\
 &= \frac{2afx \sqrt{a + bx^4}}{5\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{1}{4} (2c + ex^2) \sqrt{a + bx^4} + \frac{1}{15} x (5d + 3fx^2) \sqrt{a + bx^4} - \frac{2a^{5/2}}{5\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} \\
 &= \frac{2afx \sqrt{a + bx^4}}{5\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{1}{4} (2c + ex^2) \sqrt{a + bx^4} + \frac{1}{15} x (5d + 3fx^2) \sqrt{a + bx^4} - \frac{2a^{5/2}}{5\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} \\
 &= \frac{2afx \sqrt{a + bx^4}}{5\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{1}{4} (2c + ex^2) \sqrt{a + bx^4} + \frac{1}{15} x (5d + 3fx^2) \sqrt{a + bx^4} + \frac{ae t}{5\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} \\
 &= \frac{2afx \sqrt{a + bx^4}}{5\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{1}{4} (2c + ex^2) \sqrt{a + bx^4} + \frac{1}{15} x (5d + 3fx^2) \sqrt{a + bx^4} + \frac{ae t}{5\sqrt{b}(\sqrt{a} + \sqrt{bx^2})}
 \end{aligned}$$

Mathematica [C] time = 0.357081, size = 208, normalized size = 0.6

$$\frac{3a^{3/2} e \sqrt{\frac{bx^4}{a} + 1} \sinh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right) + 3\sqrt{b} \left((a + bx^4) (2c + ex^2) - 2\sqrt{ac} \sqrt{a + bx^4} \tanh^{-1} \left(\frac{\sqrt{a + bx^4}}{\sqrt{a}} \right) \right) + 12a\sqrt{b} dx \sqrt{\frac{bx^4}{a} + 1} {}_2F_1 \left(\dots \right)}{12\sqrt{b} \sqrt{a + bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x,x]

[Out] $(3a^{3/2}e\sqrt{1 + (bx^4)/a} \operatorname{ArcSinh}[\sqrt{b}x^2/\sqrt{a}] + 3\sqrt{b} \cdot ((2c + ex^2)(a + bx^4) - 2\sqrt{a}c\sqrt{a + bx^4}) \operatorname{ArcTanh}[\sqrt{a + bx^4}/\sqrt{a}]) + 12a\sqrt{b}dx\sqrt{1 + (bx^4)/a} \operatorname{Hypergeometric2F1}[-1/2, 1/4, 5/4, -(bx^4)/a] + 4a\sqrt{b}fx^3\sqrt{1 + (bx^4)/a} \operatorname{Hypergeometric2F1}[-1/2, 3/4, 7/4, -(bx^4)/a]) / (12\sqrt{b}\sqrt{a + bx^4})$

Maple [C] time = 0.015, size = 339, normalized size = 1.

$$\frac{fx^3}{5}\sqrt{bx^4+a} + \frac{2i}{5}fa^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\operatorname{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}} - \frac{2i}{5}fa^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}((fx^3+ex^2+dx+c)(bx^4+a)^{1/2}/x,x)$

[Out] $\frac{1}{5}fx^3(bx^4+a)^{1/2} + \frac{2}{5}Ifa^{3/2}/(I/a^{1/2}b^{1/2})^{1/2} \cdot (1-I/a^{1/2}b^{1/2})x^2)^{1/2} \cdot (1+I/a^{1/2}b^{1/2})x^2)^{1/2} / (bx^4+a)^{1/2} / b^{1/2} \cdot \operatorname{EllipticF}(x(I/a^{1/2}b^{1/2})^{1/2}, I) - \frac{2}{5}Ifa^{3/2}/(I/a^{1/2}b^{1/2})^{1/2} \cdot (1-I/a^{1/2}b^{1/2})x^2)^{1/2} \cdot (1+I/a^{1/2}b^{1/2})x^2)^{1/2} / (bx^4+a)^{1/2} / b^{1/2} \cdot \operatorname{EllipticE}(x(I/a^{1/2}b^{1/2})^{1/2}, I) + \frac{1}{4}ex^2(bx^4+a)^{1/2} + \frac{1}{4}eab^{1/2} \ln(x^2b^{1/2} + (bx^4+a)^{1/2}) + \frac{1}{3}x \cdot d(bx^4+a)^{1/2} + \frac{2}{3}da / (I/a^{1/2}b^{1/2})^{1/2} \cdot (1-I/a^{1/2}b^{1/2})x^2)^{1/2} \cdot (1+I/a^{1/2}b^{1/2})x^2)^{1/2} / (bx^4+a)^{1/2} \cdot \operatorname{EllipticF}(x(I/a^{1/2}b^{1/2})^{1/2}, I) + \frac{1}{2}c \cdot (bx^4+a)^{1/2} - \frac{1}{2}ca^{1/2} \ln((2a+2a^{1/2})(bx^4+a)^{1/2}) / x^2$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}((fx^3+ex^2+dx+c)(bx^4+a)^{1/2}/x,x, \operatorname{algorithm}="maxima")$

[Out] $\operatorname{integrate}(\sqrt{bx^4+a}(fx^3+ex^2+dx+c)/x,x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x,x, algorithm="fricas")

[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x, x)

Sympy [C] time = 8.39242, size = 204, normalized size = 0.59

$$-\frac{\sqrt{ac} \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx^2}}\right)}{2} + \frac{\sqrt{a} dx \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{1}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a} ex^2 \sqrt{1 + \frac{bx^4}{a}}}{4} + \frac{\sqrt{a} fx^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{3}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{ac}{2\sqrt{bx^2} \sqrt{\frac{a}{bx^4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x,x)

[Out] $-\sqrt{a}*c*\operatorname{asinh}(\sqrt{a}/(\sqrt{b}*x**2))/2 + \sqrt{a}*d*x*\operatorname{gamma}(1/4)*\operatorname{hyper}((-1/2, 1/4), (5/4,), b*x**4*\exp_polar(I*pi)/a)/(4*\operatorname{gamma}(5/4)) + \sqrt{a}*e*x**2*\sqrt{1 + b*x**4/a}/4 + \sqrt{a}*f*x**3*\operatorname{gamma}(3/4)*\operatorname{hyper}((-1/2, 3/4), (7/4,), b*x**4*\exp_polar(I*pi)/a)/(4*\operatorname{gamma}(7/4)) + a*c/(2*\sqrt{b}*x**2*\sqrt{a/(b*x**4) + 1}) + a*e*\operatorname{asinh}(\sqrt{b}*x**2/\sqrt{a})/(4*\sqrt{b}) + \sqrt{b}*c*x**2/(2*\sqrt{a/(b*x**4) + 1})$

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x,x, algorithm="giac")

[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x, x)

$$3.501 \quad \int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2} dx$$

Optimal. Leaf size=341

$$\frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (\sqrt{ae} + 3\sqrt{bc}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}} - \frac{\sqrt{a+bx^4}(3c-ex^2)}{3x} + \frac{2\sqrt{bcx}\sqrt{a+bx^4}}{\sqrt{a} + \sqrt{bx^2}} - \frac{2\sqrt[4]{a}\sqrt[4]{bc}}{\sqrt{a} + \sqrt{bx^2}}$$

[Out] (2*Sqrt[b]*c*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) - ((3*c - e*x^2)*Sqrt[a + b*x^4])/(3*x) + ((2*d + f*x^2)*Sqrt[a + b*x^4])/4 + (a*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (Sqrt[a]*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (2*a^(1/4)*b^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x^4] + (a^(1/4)*(3*Sqrt[b]*c + Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a + b*x^4])

Rubi [A] time = 0.263187, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {1833, 1272, 1198, 220, 1196, 1252, 815, 844, 217, 206, 266, 63, 208}

$$-\frac{\sqrt{a+bx^4}(3c-ex^2)}{3x} + \frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (\sqrt{ae} + 3\sqrt{bc}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}} + \frac{2\sqrt{bcx}\sqrt{a+bx^4}}{\sqrt{a} + \sqrt{bx^2}} - \frac{2\sqrt[4]{a}\sqrt[4]{bc}}{\sqrt{a} + \sqrt{bx^2}}$$

Antiderivative was successfully verified.

[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^2,x]

[Out] (2*Sqrt[b]*c*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) - ((3*c - e*x^2)*Sqrt[a + b*x^4])/(3*x) + ((2*d + f*x^2)*Sqrt[a + b*x^4])/4 + (a*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (Sqrt[a]*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (2*a^(1/4)*b^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x^4] + (a^(1/4)*(3*Sqrt[b]*c + Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a + b*x^4])

Rule 1833

```
Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p]/c^j, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 1272

```
Int[((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(d*(m + 4*p + 3) + e*(m + 1)*x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(4*p)/(f^2*(m + 1)*(m + 4*p + 3)), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 815

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

```

Rule 844

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]

```

Rule 217

```

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

```

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[((1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rule 266

```

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

```

Rule 63

```

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

```

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
 \int \frac{(c + dx + ex^2 + fx^3) \sqrt{a + bx^4}}{x^2} dx &= \int \left(\frac{(c + ex^2) \sqrt{a + bx^4}}{x^2} + \frac{(d + fx^2) \sqrt{a + bx^4}}{x} \right) dx \\
 &= \int \frac{(c + ex^2) \sqrt{a + bx^4}}{x^2} dx + \int \frac{(d + fx^2) \sqrt{a + bx^4}}{x} dx \\
 &= -\frac{(3c - ex^2) \sqrt{a + bx^4}}{3x} + \frac{1}{2} \text{Subst} \left(\int \frac{(d + fx) \sqrt{a + bx^2}}{x} dx, x, x^2 \right) - \frac{2}{3} \int \frac{-ae -}{\sqrt{a +}} \\
 &= -\frac{(3c - ex^2) \sqrt{a + bx^4}}{3x} + \frac{1}{4} (2d + fx^2) \sqrt{a + bx^4} + \frac{\text{Subst} \left(\int \frac{2abd + abfx}{x\sqrt{a + bx^2}} dx, x, x^2 \right)}{4b} \\
 &= \frac{2\sqrt{bcx}\sqrt{a + bx^4}}{\sqrt{a} + \sqrt{bx^2}} - \frac{(3c - ex^2) \sqrt{a + bx^4}}{3x} + \frac{1}{4} (2d + fx^2) \sqrt{a + bx^4} - \frac{2^4 \sqrt{a}^4 \sqrt{bc} (\sqrt{a + bx^4})}{4\sqrt{b}} \\
 &= \frac{2\sqrt{bcx}\sqrt{a + bx^4}}{\sqrt{a} + \sqrt{bx^2}} - \frac{(3c - ex^2) \sqrt{a + bx^4}}{3x} + \frac{1}{4} (2d + fx^2) \sqrt{a + bx^4} - \frac{2^4 \sqrt{a}^4 \sqrt{bc} (\sqrt{a + bx^4})}{4\sqrt{b}} \\
 &= \frac{2\sqrt{bcx}\sqrt{a + bx^4}}{\sqrt{a} + \sqrt{bx^2}} - \frac{(3c - ex^2) \sqrt{a + bx^4}}{3x} + \frac{1}{4} (2d + fx^2) \sqrt{a + bx^4} + \frac{af \tanh^{-1} \left(\frac{\sqrt{a + bx^4}}{\sqrt{a}} \right)}{4\sqrt{b}} \\
 &= \frac{2\sqrt{bcx}\sqrt{a + bx^4}}{\sqrt{a} + \sqrt{bx^2}} - \frac{(3c - ex^2) \sqrt{a + bx^4}}{3x} + \frac{1}{4} (2d + fx^2) \sqrt{a + bx^4} + \frac{af \tanh^{-1} \left(\frac{\sqrt{a + bx^4}}{\sqrt{a}} \right)}{4\sqrt{b}}
 \end{aligned}$$

Mathematica [C] time = 0.391284, size = 208, normalized size = 0.61

$$\frac{x \left(\sqrt{b} \sqrt{\frac{bx^4}{a}} + 1 \left(\sqrt{a + bx^4} (2d + fx^2) - 2\sqrt{ad} \tanh^{-1} \left(\frac{\sqrt{a + bx^4}}{\sqrt{a}} \right) \right) + 4\sqrt{bex} \sqrt{a + bx^4} {}_2F_1 \left(-\frac{1}{2}, \frac{1}{4}; \frac{5}{4}; -\frac{bx^4}{a} \right) + \sqrt{a} f \sqrt{a + bx^4} \right)}{4\sqrt{bx} \sqrt{\frac{bx^4}{a}} + 1}$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^2,x]

[Out] (-4*Sqrt[b]*c*Sqrt[a + b*x^4]*Hypergeometric2F1[-1/2, -1/4, 3/4, -((b*x^4)/a)] + x*(Sqrt[a]*f*Sqrt[a + b*x^4]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + Sqrt[b]*Sqrt[1 + (b*x^4)/a]*((2*d + f*x^2)*Sqrt[a + b*x^4] - 2*Sqrt[a]*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]) + 4*Sqrt[b]*e*x*Sqrt[a + b*x^4]*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)])/(4*Sqrt[b]*x*Sqrt[1 + (b*x^4)/a])

Maple [C] time = 0.012, size = 339, normalized size = 1.

$$\frac{fx^2}{4}\sqrt{bx^4+a} + \frac{af}{4}\ln\left(x^2\sqrt{b} + \sqrt{bx^4+a}\right) \frac{1}{\sqrt{b}} + \frac{ex}{3}\sqrt{bx^4+a} + \frac{2ae}{3}\sqrt{1-ix^2\sqrt{b}}\frac{1}{\sqrt{a}}\sqrt{1+ix^2\sqrt{b}}\frac{1}{\sqrt{a}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}}\frac{1}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^2,x)

[Out] 1/4*x^2*f*(b*x^4+a)^(1/2)+1/4*f*a/b^(1/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))+1/3*e*x*(b*x^4+a)^(1/2)+2/3*e*a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*d*(b*x^4+a)^(1/2)-1/2*d*a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-c/x*(b*x^4+a)^(1/2)+2*I*c*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-2*I*c*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^2,x, algorithm="maxima")

[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^2,x, algorithm="fricas")

[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)

Sympy [C] time = 5.70388, size = 206, normalized size = 0.6

$$\frac{\sqrt{ac}\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(-\frac{1}{2}, -\frac{1}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{\sqrt{ad} \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx^2}}\right)}{2} + \frac{\sqrt{aex}\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{1}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}fx^2\sqrt{1 + \frac{bx^4}{a}}}{4} + \frac{ad}{2\sqrt{bx^2}\sqrt{\frac{a}{bx^4} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**2,x)

[Out] sqrt(a)*c*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - sqrt(a)*d*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + sqrt(a)*e*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + sqrt(a)*f*x**2*sqrt(1 + b*x**4/a)/4 + a*d/(2*sqrt(b)*x**2*sqrt(a/(b*x**4) + 1)) + a*f*asinh(sqrt(b)*x**2/sqrt(a))/(4*sqrt(b)) + sqrt(b)*d*x**2/(2*sqrt(a/(b*x**4) + 1))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)
```

$$3.502 \quad \int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3} dx$$

Optimal. Leaf size=342

$$\frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (\sqrt{af} + 3\sqrt{bd}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}} - \frac{\sqrt{a+bx^4}(c-ex^2)}{2x^2} + \frac{1}{2}\sqrt{bc} \tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)$$

[Out] (2*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) - ((c - e*x^2)*Sqrt[a + b*x^4])/(2*x^2) - ((3*d - f*x^2)*Sqrt[a + b*x^4])/(3*x) + (Sqrt[b]*c*ArcTanH[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (Sqrt[a]*e*ArcTanH[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (2*a^(1/4)*b^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x^4] + (a^(1/4)*(3*Sqrt[b]*d + Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a + b*x^4])

Rubi [A] time = 0.262424, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {1833, 1252, 813, 844, 217, 206, 266, 63, 208, 1272, 1198, 220, 1196}

$$-\frac{\sqrt{a+bx^4}(c-ex^2)}{2x^2} + \frac{1}{2}\sqrt{bc} \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right) - \frac{\sqrt{a+bx^4}(3d-fx^2)}{3x} + \frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (\sqrt{af} + 3\sqrt{bd})}{3\sqrt[4]{b}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^3,x]

[Out] (2*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) - ((c - e*x^2)*Sqrt[a + b*x^4])/(2*x^2) - ((3*d - f*x^2)*Sqrt[a + b*x^4])/(3*x) + (Sqrt[b]*c*ArcTanH[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (Sqrt[a]*e*ArcTanH[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (2*a^(1/4)*b^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x^4] + (a^(1/4)*(3*Sqrt[b]*d + Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a + b*x^4])

Rule 1833

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p]/c^j, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 1252

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 813

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !RationalQ[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1272

```
Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x
_Symbol] := Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(d*(m + 4*p + 3) + e*(m + 1)*
x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(4*p)/(f^2*(m + 1)*(m + 4*p + 3)
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e},
```

x] && PosQ[c/a]

Rubi steps

$$\begin{aligned}
 \int \frac{(c + dx + ex^2 + fx^3) \sqrt{a + bx^4}}{x^3} dx &= \int \left(\frac{(c + ex^2) \sqrt{a + bx^4}}{x^3} + \frac{(d + fx^2) \sqrt{a + bx^4}}{x^2} \right) dx \\
 &= \int \frac{(c + ex^2) \sqrt{a + bx^4}}{x^3} dx + \int \frac{(d + fx^2) \sqrt{a + bx^4}}{x^2} dx \\
 &= -\frac{(3d - fx^2) \sqrt{a + bx^4}}{3x} + \frac{1}{2} \text{Subst} \left(\int \frac{(c + ex) \sqrt{a + bx^2}}{x^2} dx, x, x^2 \right) - \frac{2}{3} \int \frac{-af - 3}{\sqrt{a + bx^4}} dx \\
 &= -\frac{(c - ex^2) \sqrt{a + bx^4}}{2x^2} - \frac{(3d - fx^2) \sqrt{a + bx^4}}{3x} - \frac{1}{4} \text{Subst} \left(\int \frac{-2ae - 2bcx}{x \sqrt{a + bx^2}} dx, x, x^2 \right) \\
 &= \frac{2\sqrt{bd}x\sqrt{a + bx^4}}{\sqrt{a} + \sqrt{bx^2}} - \frac{(c - ex^2) \sqrt{a + bx^4}}{2x^2} - \frac{(3d - fx^2) \sqrt{a + bx^4}}{3x} - \frac{2\sqrt[4]{a}\sqrt[4]{bd}(\sqrt{a} + \sqrt{bx^2})}{\sqrt{a} + \sqrt{bx^2}} \\
 &= \frac{2\sqrt{bd}x\sqrt{a + bx^4}}{\sqrt{a} + \sqrt{bx^2}} - \frac{(c - ex^2) \sqrt{a + bx^4}}{2x^2} - \frac{(3d - fx^2) \sqrt{a + bx^4}}{3x} - \frac{2\sqrt[4]{a}\sqrt[4]{bd}(\sqrt{a} + \sqrt{bx^2})}{\sqrt{a} + \sqrt{bx^2}} \\
 &= \frac{2\sqrt{bd}x\sqrt{a + bx^4}}{\sqrt{a} + \sqrt{bx^2}} - \frac{(c - ex^2) \sqrt{a + bx^4}}{2x^2} - \frac{(3d - fx^2) \sqrt{a + bx^4}}{3x} + \frac{1}{2} \sqrt{bc} \tanh^{-1} \left(\frac{\sqrt{a} - \sqrt{bx^2}}{\sqrt{a} + \sqrt{bx^2}} \right) \\
 &= \frac{2\sqrt{bd}x\sqrt{a + bx^4}}{\sqrt{a} + \sqrt{bx^2}} - \frac{(c - ex^2) \sqrt{a + bx^4}}{2x^2} - \frac{(3d - fx^2) \sqrt{a + bx^4}}{3x} + \frac{1}{2} \sqrt{bc} \tanh^{-1} \left(\frac{\sqrt{a} - \sqrt{bx^2}}{\sqrt{a} + \sqrt{bx^2}} \right)
 \end{aligned}$$

Mathematica [C] time = 0.233381, size = 204, normalized size = 0.6

$$\frac{\sqrt{a}\sqrt{bc}x^2\sqrt{\frac{bx^4}{a} + 1} \sinh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right) - 2adx\sqrt{\frac{bx^4}{a} + 1} {}_2F_1 \left(-\frac{1}{2}, -\frac{1}{4}; \frac{3}{4}; -\frac{bx^4}{a} \right) - \sqrt{a}ex^2\sqrt{a + bx^4} \tanh^{-1} \left(\frac{\sqrt{a+bx^4}}{\sqrt{a}} \right) + 2afx^3\sqrt{\frac{bx^4}{a}}}{2x^2\sqrt{a + bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^3, x]

[Out] $(-(a*c) + a*e*x^2 - b*c*x^4 + b*e*x^6 + \text{Sqrt}[a]*\text{Sqrt}[b]*c*x^2*\text{Sqrt}[1 + (b*x^4)/a]*\text{ArcSinh}[(\text{Sqrt}[b]*x^2)/\text{Sqrt}[a]] - \text{Sqrt}[a]*e*x^2*\text{Sqrt}[a + b*x^4]*\text{ArcTan}[\text{Sqrt}[a + b*x^4]/\text{Sqrt}[a]] - 2*a*d*x*\text{Sqrt}[1 + (b*x^4)/a]*\text{Hypergeometric2F1}[-1/2, -1/4, 3/4, -((b*x^4)/a)] + 2*a*f*x^3*\text{Sqrt}[1 + (b*x^4)/a]*\text{Hypergeometric2F1}[-1/2, 1/4, 5/4, -((b*x^4)/a)])/(2*x^2*\text{Sqrt}[a + b*x^4])$

Maple [C] time = 0.016, size = 360, normalized size = 1.1

$$\frac{fx}{3}\sqrt{bx^4+a} + \frac{2af}{3}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}, i\right) \frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{bx^4+a}} + \frac{e}{2}\sqrt{bx^4+a} - \frac{e}{2}\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^{(1/2)}/x^3,x)$

[Out] $\frac{1}{3}x*f*(b*x^4+a)^{(1/2)} + \frac{2}{3}f*a/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)})*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)})*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticF}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I) + \frac{1}{2}*e*(b*x^4+a)^{(1/2)} - \frac{1}{2}*e*a^{(1/2)}*\ln((2*a+2*a^{(1/2)}*(b*x^4+a)^{(1/2)})/x^2) - \frac{1}{2}*c/a/x^2*(b*x^4+a)^{(3/2)} + \frac{1}{2}*c*b/a*x^2*(b*x^4+a)^{(1/2)} + \frac{1}{2}*c*b^{(1/2)}*\ln(x^2*b^{(1/2)}+(b*x^4+a)^{(1/2)}) - d/x*(b*x^4+a)^{(1/2)} + 2*I*d*b^{(1/2)}*a^{(1/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)})*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)})*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticF}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I) - 2*I*d*b^{(1/2)}*a^{(1/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)})*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)})*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticE}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^{(1/2)}/x^3,x, \text{algorithm}="maxima")$

[Out] $\text{integrate}(\text{sqrt}(b*x^4+a)*(f*x^3+e*x^2+d*x+c)/x^3,x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^3,x, algorithm="fricas")

[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^3, x)

Sympy [C] time = 5.27143, size = 230, normalized size = 0.67

$$-\frac{\sqrt{ac}}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{ad}\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(-\frac{1}{2}, -\frac{1}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{\sqrt{ae} \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx^2}}\right)}{2} + \frac{\sqrt{a}fx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{1}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{ae}{2\sqrt{bx^2}\sqrt{\frac{a}{bx^4}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**3,x)

[Out] -sqrt(a)*c/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*d*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - sqrt(a)*e*asin h(sqrt(a)/(sqrt(b)*x**2))/2 + sqrt(a)*f*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + a*e/(2*sqrt(b)*x**2*sqrt(a/(b*x**4) + 1)) + sqrt(b)*c*asinh(sqrt(b)*x**2/sqrt(a))/2 + sqrt(b)*e*x**2/(2*sqrt(a/(b*x**4) + 1)) - b*c*x**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^3,x, algorithm="giac")

[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^3, x)

$$3.503 \quad \int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4} dx$$

Optimal. Leaf size=357

$$\frac{\sqrt[4]{b}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (3\sqrt{ae} + \sqrt{bc}) \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+bx^4}} - \frac{\sqrt{a+bx^4}(c-3ex^2)}{3x^3} - \frac{\sqrt{a+bx^4}(d-fx^2)}{2x^2}$$

[Out] $(-2*e*\text{Sqrt}[a + b*x^4])/x + (2*\text{Sqrt}[b]*e*x*\text{Sqrt}[a + b*x^4])/(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2) - ((c - 3*e*x^2)*\text{Sqrt}[a + b*x^4])/(3*x^3) - ((d - f*x^2)*\text{Sqrt}[a + b*x^4])/(2*x^2) + (\text{Sqrt}[b]*d*\text{ArcTanh}[(\text{Sqrt}[b]*x^2)/\text{Sqrt}[a + b*x^4]])/2 - (\text{Sqrt}[a]*f*\text{ArcTanh}[\text{Sqrt}[a + b*x^4]/\text{Sqrt}[a]])/2 - (2*a^{(1/4)}*b^{(1/4)}*e*(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)*\text{Sqrt}[(a + b*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/ \text{Sqrt}[a + b*x^4] + (b^{(1/4)}*(\text{Sqrt}[b]*c + 3*\text{Sqrt}[a]*e)*(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)*\text{Sqrt}[(a + b*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2))/(3*a^{(1/4)}*\text{Sqrt}[a + b*x^4])$

Rubi [A] time = 0.295318, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1833, 1272, 1282, 1198, 220, 1196, 1252, 813, 844, 217, 206, 266, 63, 208}

$$-\frac{\sqrt{a+bx^4}(c-3ex^2)}{3x^3} + \frac{\sqrt[4]{b}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (3\sqrt{ae} + \sqrt{bc}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+bx^4}} - \frac{\sqrt{a+bx^4}(d-fx^2)}{2x^2} + \frac{1}{2}\sqrt{a+bx^4}$$

Antiderivative was successfully verified.

[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^4,x]

[Out] $(-2*e*\text{Sqrt}[a + b*x^4])/x + (2*\text{Sqrt}[b]*e*x*\text{Sqrt}[a + b*x^4])/(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2) - ((c - 3*e*x^2)*\text{Sqrt}[a + b*x^4])/(3*x^3) - ((d - f*x^2)*\text{Sqrt}[a + b*x^4])/(2*x^2) + (\text{Sqrt}[b]*d*\text{ArcTanh}[(\text{Sqrt}[b]*x^2)/\text{Sqrt}[a + b*x^4]])/2 - (\text{Sqrt}[a]*f*\text{ArcTanh}[\text{Sqrt}[a + b*x^4]/\text{Sqrt}[a]])/2 - (2*a^{(1/4)}*b^{(1/4)}*e*(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)*\text{Sqrt}[(a + b*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/ \text{Sqrt}[a + b*x^4] + (b^{(1/4)}*(\text{Sqrt}[b]*c + 3*\text{Sqrt}[a]*e)*(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)*\text{Sqrt}[(a + b*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2))/(3*a^{(1/4)}*\text{Sqrt}[a + b*x^4])$

])

Rule 1833

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p)/c^j, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]

Rule 1272

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(d*(m + 4*p + 3) + e*(m + 1)*x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(4*p)/(f^2*(m + 1)*(m + 4*p + 3)), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1282

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1198

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rule 220

Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],

$1/2] / (q \sqrt{a + c x^4}), x] /; \text{EqQ}[e + d q^2, 0] /; \text{FreeQ}\{a, c, d, e\}, x] \&\& \text{PosQ}[c/a]$

Rule 1252

$\text{Int}[(x_)^{(m_.)} * ((d_) + (e_.)(x_)^2)^{(q_.)} * ((a_) + (c_.)(x_)^4)^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/2, \text{Subst}[\text{Int}[x^{((m-1)/2)} * (d + e x)^q * (a + c x^2)^p, x], x, x^2], x] /; \text{FreeQ}\{a, c, d, e, p, q\}, x] \&\& \text{IntegerQ}[(m+1)/2]$

Rule 813

$\text{Int}(((d_.) + (e_.)(x_))^{(m_.)} * ((f_.) + (g_.)(x_)) * ((a_) + (c_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}(((d + e x)^{(m+1)} * (e f * (m+2 p + 2) - d g * (2 p + 1) + e g * (m+1) x) * (a + c x^2)^p) / (e^2 * (m+1) * (m+2 p + 2)), x] + \text{Dist}[p / (e^2 * (m+1) * (m+2 p + 2)), \text{Int}[(d + e x)^{(m+1)} * (a + c x^2)^{(p-1)} * \text{Simp}[g * (2 a e + 2 a e m) + (g * (2 c d + 4 c d p) - 2 c e f * (m+2 p + 2)) * x, x], x], x] /; \text{FreeQ}\{a, c, d, e, f, g, m\}, x] \&\& \text{NeQ}[c d^2 + a e^2, 0] \&\& \text{RationalQ}[p] \&\& p > 0 \&\& (\text{LtQ}[m, -1] || \text{EqQ}[p, 1] || (\text{IntegerQ}[p] \&\& !\text{RationalQ}[m])) \&\& \text{NeQ}[m, -1] \&\& !\text{ILtQ}[m+2 p + 1, 0] \&\& (\text{IntegerQ}[m] || \text{IntegerQ}[p] || \text{IntegersQ}[2 m, 2 p])$

Rule 844

$\text{Int}(((d_.) + (e_.)(x_))^{(m_.)} * ((f_.) + (g_.)(x_)) * ((a_) + (c_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[g/e, \text{Int}[(d + e x)^{(m+1)} * (a + c x^2)^p, x], x] + \text{Dist}[(e f - d g) / e, \text{Int}[(d + e x)^m * (a + c x^2)^p, x], x] /; \text{FreeQ}\{a, c, d, e, f, g, m, p\}, x] \&\& \text{NeQ}[c d^2 + a e^2, 0] \&\& !\text{IGtQ}[m, 0]$

Rule 217

$\text{Int}[1/\sqrt{(a_) + (b_.)(x_)^2}, x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b x^2), x], x, x/\sqrt{a + b x^2}] /; \text{FreeQ}\{a, b\}, x] \&\& !\text{GtQ}[a, 0]$

Rule 206

$\text{Int}(((a_) + (b_.)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 * \text{ArcTanh}[(\text{Rt}[-b, 2] * x) / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] * \text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] || \text{LtQ}[b, 0])$

Rule 266

$\text{Int}[(x_)^{(m_.)} * ((a_) + (b_.)(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1) * (a + b x)^p, x], x, x^n], x] /; \text{FreeQ}\{a, b, m, n, p\}, x] \&\& \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
 {p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
 (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
 [b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
 ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
 Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
 \int \frac{(c + dx + ex^2 + fx^3) \sqrt{a + bx^4}}{x^4} dx &= \int \left(\frac{(c + ex^2) \sqrt{a + bx^4}}{x^4} + \frac{(d + fx^2) \sqrt{a + bx^4}}{x^3} \right) dx \\
 &= \int \frac{(c + ex^2) \sqrt{a + bx^4}}{x^4} dx + \int \frac{(d + fx^2) \sqrt{a + bx^4}}{x^3} dx \\
 &= -\frac{(c - 3ex^2) \sqrt{a + bx^4}}{3x^3} + \frac{1}{2} \text{Subst} \left(\int \frac{(d + fx) \sqrt{a + bx^2}}{x^2} dx, x, x^2 \right) - \frac{2}{3} \int \frac{-3ae -}{x^2 \sqrt{a +}} \\
 &= -\frac{2e\sqrt{a + bx^4}}{x} - \frac{(c - 3ex^2) \sqrt{a + bx^4}}{3x^3} - \frac{(d - fx^2) \sqrt{a + bx^4}}{2x^2} - \frac{1}{4} \text{Subst} \left(\int \frac{-2af}{x\sqrt{a}} \right) \\
 &= -\frac{2e\sqrt{a + bx^4}}{x} - \frac{(c - 3ex^2) \sqrt{a + bx^4}}{3x^3} - \frac{(d - fx^2) \sqrt{a + bx^4}}{2x^2} + \frac{1}{2} (bd) \text{Subst} \left(\int \frac{1}{\sqrt{a + bx^2}} \right) \\
 &= -\frac{2e\sqrt{a + bx^4}}{x} + \frac{2\sqrt{bex}\sqrt{a + bx^4}}{\sqrt{a} + \sqrt{bx^2}} - \frac{(c - 3ex^2) \sqrt{a + bx^4}}{3x^3} - \frac{(d - fx^2) \sqrt{a + bx^4}}{2x^2} - \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{a + bx^2}} \right) \\
 &= -\frac{2e\sqrt{a + bx^4}}{x} + \frac{2\sqrt{bex}\sqrt{a + bx^4}}{\sqrt{a} + \sqrt{bx^2}} - \frac{(c - 3ex^2) \sqrt{a + bx^4}}{3x^3} - \frac{(d - fx^2) \sqrt{a + bx^4}}{2x^2} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{a + bx^2}} \right) \\
 &= -\frac{2e\sqrt{a + bx^4}}{x} + \frac{2\sqrt{bex}\sqrt{a + bx^4}}{\sqrt{a} + \sqrt{bx^2}} - \frac{(c - 3ex^2) \sqrt{a + bx^4}}{3x^3} - \frac{(d - fx^2) \sqrt{a + bx^4}}{2x^2} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{a + bx^2}} \right)
 \end{aligned}$$

Mathematica [C] time = 0.301453, size = 205, normalized size = 0.57

$$\frac{3x \left(\sqrt{a} \sqrt{bd} x^2 \sqrt{\frac{bx^4}{a} + 1} \sinh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right) - 2aex \sqrt{\frac{bx^4}{a} + 1} {}_2F_1 \left(-\frac{1}{2}, -\frac{1}{4}; \frac{3}{4}; -\frac{bx^4}{a} \right) - \sqrt{a} f x^2 \sqrt{a + bx^4} \tanh^{-1} \left(\frac{\sqrt{a+bx^4}}{\sqrt{a}} \right) - ad + a \right)}{6x^3 \sqrt{a + bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^4, x]

[Out] $(-2*a*c*\text{Sqrt}[1 + (b*x^4)/a]*\text{Hypergeometric2F1}[-3/4, -1/2, 1/4, -((b*x^4)/a)] + 3*x*(-(a*d) + a*f*x^2 - b*d*x^4 + b*f*x^6 + \text{Sqrt}[a]*\text{Sqrt}[b]*d*x^2*\text{Sqrt}[1 + (b*x^4)/a]*\text{ArcSinh}[(\text{Sqrt}[b]*x^2)/\text{Sqrt}[a]] - \text{Sqrt}[a]*f*x^2*\text{Sqrt}[a + b*x^4]*\text{ArcTanh}[\text{Sqrt}[a + b*x^4]/\text{Sqrt}[a]] - 2*a*e*x*\text{Sqrt}[1 + (b*x^4)/a]*\text{Hypergeometric2F1}[-1/2, -1/4, 3/4, -((b*x^4)/a)])))/(6*x^3*\text{Sqrt}[a + b*x^4])$

Maple [C] time = 0.013, size = 362, normalized size = 1.

$$\frac{f}{2} \sqrt{bx^4 + a} - \frac{f}{2} \sqrt{a} \ln \left(\frac{1}{x^2} (2a + 2\sqrt{a}\sqrt{bx^4 + a}) \right) - \frac{c}{3x^3} \sqrt{bx^4 + a} + \frac{2bc}{3} \sqrt{1 - ix^2\sqrt{b}\frac{1}{\sqrt{a}}} \sqrt{1 + ix^2\sqrt{b}\frac{1}{\sqrt{a}}} \text{EllipticF} \left(x \sqrt{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^4, x)

[Out] $1/2*f*(b*x^4+a)^{(1/2)} - 1/2*f*a^{(1/2)}*\ln((2*a+2*a^{(1/2)}*(b*x^4+a)^{(1/2)})/x^2) - 1/3*c/x^3*(b*x^4+a)^{(1/2)} + 2/3*c*b/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticF}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I) - 1/2*d/a/x^2*(b*x^4+a)^{(3/2)} + 1/2*d*b/a*x^2*(b*x^4+a)^{(1/2)} + 1/2*d*b^{(1/2)}*\ln(x^2*b^{(1/2)}+(b*x^4+a)^{(1/2)}) - e*(b*x^4+a)^{(1/2)}/x + 2*I*e*b^{(1/2)}*a^{(1/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticF}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I) - 2*I*e*b^{(1/2)}*a^{(1/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticE}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^4,x, algorithm="maxima")

[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^4,x, algorithm="fricas")

[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^4, x)

Sympy [C] time = 5.41421, size = 235, normalized size = 0.66

$$\frac{\sqrt{ac}\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, -\frac{1}{2} \\ \frac{1}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{ad}}{2x^2\sqrt{1 + \frac{bx^4}{a}}} + \frac{\sqrt{ae}\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4} \\ \frac{3}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{\sqrt{af} \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx^2}}\right)}{2} + \frac{af}{2\sqrt{bx^2}\sqrt{\frac{a}{bx^4} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**4,x)

[Out] sqrt(a)*c*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - sqrt(a)*d/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*e*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - sqrt(a)*f*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + a*f/(2*sqrt(b)*x**2*sqrt(a/(b*x**4) + 1)) + sqrt(b)*d*asinh(sqrt(b)*x**2/sqrt(a))/2 + sqrt(b)*f*x**2/(2*sqrt(a/(b*x**4) + 1)) - b*d*x**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^4,x, algorithm="giac")

[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^4, x)

$$3.504 \quad \int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5} dx$$

Optimal. Leaf size=329

$$\frac{\sqrt[4]{b}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (3\sqrt{a}f + \sqrt{bd}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+bx^4}} - \frac{1}{12} \sqrt{a+bx^4} \left(\frac{3c}{x^4} + \frac{4d}{x^3} + \frac{6e}{x^2} + \frac{12f}{x}\right) - \frac{bc \tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

[Out] -(((3*c)/x^4 + (4*d)/x^3 + (6*e)/x^2 + (12*f)/x)*Sqrt[a + b*x^4])/12 + (2*Sqrt[b]*f*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) + (Sqrt[b]*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (b*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*Sqrt[a]) - (2*a^(1/4)*b^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x^4] + (b^(1/4)*(Sqrt[b]*d + 3*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(3*a^(1/4)*Sqrt[a + b*x^4])

Rubi [A] time = 0.280873, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {14, 1825, 1832, 266, 63, 208, 1885, 275, 217, 206, 1198, 220, 1196}

$$-\frac{1}{12} \sqrt{a+bx^4} \left(\frac{3c}{x^4} + \frac{4d}{x^3} + \frac{6e}{x^2} + \frac{12f}{x}\right) - \frac{bc \tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}} + \frac{\sqrt[4]{b}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (3\sqrt{a}f + \sqrt{bd}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^5,x]

[Out] -(((3*c)/x^4 + (4*d)/x^3 + (6*e)/x^2 + (12*f)/x)*Sqrt[a + b*x^4])/12 + (2*Sqrt[b]*f*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) + (Sqrt[b]*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (b*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*Sqrt[a]) - (2*a^(1/4)*b^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x^4] + (b^(1/4)*(Sqrt[b]*d + 3*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(3*a^(1/4)*Sqrt[a + b*x^4])

Rule 14


```
Int[(u_)*((c_)*(x_)^(m_)), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{u = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x, x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1, 0]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_)*(x_)^(n_)]), x_Symbol] := Dist[Coeff[Pq, x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq, x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 275

$\text{Int}[(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{With}[\{k = \text{GCD}[m + 1, n]\}, \text{Dist}[1/k, \text{Subst}[\text{Int}[x^{(m + 1)/k - 1}*(a + b*x^{(n/k)})^p, x], x, x^k], x] \text{ /; } k \neq 1 \text{ /; } \text{FreeQ}[\{a, b, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[m]$

Rule 217

$\text{Int}[1/\text{Sqrt}[(a_) + (b_.)*(x_)^2], x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] \text{ /; } \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{!GtQ}[a, 0]$

Rule 206

$\text{Int}[(a_) + (b_.)*(x_)^2]^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] \text{ /; } \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 1198

$\text{Int}[(d_) + (e_.)*(x_)^2]/\text{Sqrt}[(a_) + (c_.)*(x_)^4], x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[c/a, 2]\}, \text{Dist}[(e + d*q)/q, \text{Int}[1/\text{Sqrt}[a + c*x^4], x], x] - \text{Dist}[e/q, \text{Int}[(1 - q*x^2)/\text{Sqrt}[a + c*x^4], x], x] \text{ /; } \text{NeQ}[e + d*q, 0] \text{ /; } \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{PosQ}[c/a]$

Rule 220

$\text{Int}[1/\text{Sqrt}[(a_) + (b_.)*(x_)^4], x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[b/a, 4]\}, \text{Simp}[(1 + q^2*x^2)*\text{Sqrt}[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*\text{EllipticF}[2*\text{ArcTan}[q*x], 1/2]]/(2*q*\text{Sqrt}[a + b*x^4]), x] \text{ /; } \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[b/a]$

Rule 1196

$\text{Int}[(d_) + (e_.)*(x_)^2]/\text{Sqrt}[(a_) + (c_.)*(x_)^4], x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[c/a, 4]\}, -\text{Simp}[(d*x*\text{Sqrt}[a + c*x^4])/(a*(1 + q^2*x^2)), x] + \text{Simp}[(d*(1 + q^2*x^2)*\text{Sqrt}[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*\text{EllipticE}[2*\text{ArcTan}[q*x], 1/2]]/(q*\text{Sqrt}[a + c*x^4]), x] \text{ /; } \text{EqQ}[e + d*q^2, 0] \text{ /; } \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{PosQ}[c/a]$

Rubi steps

$$\begin{aligned}
\int \frac{(c + dx + ex^2 + fx^3)\sqrt{a + bx^4}}{x^5} dx &= -\frac{1}{12} \left(\frac{3c}{x^4} + \frac{4d}{x^3} + \frac{6e}{x^2} + \frac{12f}{x} \right) \sqrt{a + bx^4} - (2b) \int \frac{-\frac{c}{4} - \frac{dx}{3} - \frac{ex^2}{2} - fx^3}{x\sqrt{a + bx^4}} dx \\
&= -\frac{1}{12} \left(\frac{3c}{x^4} + \frac{4d}{x^3} + \frac{6e}{x^2} + \frac{12f}{x} \right) \sqrt{a + bx^4} - (2b) \int \frac{-\frac{d}{3} - \frac{ex}{2} - fx^2}{\sqrt{a + bx^4}} dx + \frac{1}{2}(bc) \int \frac{1}{x\sqrt{a + bx^4}} dx \\
&= -\frac{1}{12} \left(\frac{3c}{x^4} + \frac{4d}{x^3} + \frac{6e}{x^2} + \frac{12f}{x} \right) \sqrt{a + bx^4} - (2b) \int \left(-\frac{ex}{2\sqrt{a + bx^4}} + \frac{-\frac{d}{3} - fx^2}{\sqrt{a + bx^4}} \right) dx \\
&= -\frac{1}{12} \left(\frac{3c}{x^4} + \frac{4d}{x^3} + \frac{6e}{x^2} + \frac{12f}{x} \right) \sqrt{a + bx^4} - (2b) \int \frac{-\frac{d}{3} - fx^2}{\sqrt{a + bx^4}} dx + \frac{1}{4}c \operatorname{Subst} \left(\int \frac{1}{x\sqrt{a + bx^4}} dx \right) \\
&= -\frac{1}{12} \left(\frac{3c}{x^4} + \frac{4d}{x^3} + \frac{6e}{x^2} + \frac{12f}{x} \right) \sqrt{a + bx^4} - \frac{bc \tanh^{-1} \left(\frac{\sqrt{a+bx^4}}{\sqrt{a}} \right)}{4\sqrt{a}} + \frac{1}{2}(bc) \operatorname{Subst} \left(\int \frac{1}{x\sqrt{a + bx^4}} dx \right) \\
&= -\frac{1}{12} \left(\frac{3c}{x^4} + \frac{4d}{x^3} + \frac{6e}{x^2} + \frac{12f}{x} \right) \sqrt{a + bx^4} + \frac{2\sqrt{b}fx\sqrt{a + bx^4}}{\sqrt{a} + \sqrt{bx^2}} - \frac{bc \tanh^{-1} \left(\frac{\sqrt{a+bx^4}}{\sqrt{a}} \right)}{4\sqrt{a}} \\
&= -\frac{1}{12} \left(\frac{3c}{x^4} + \frac{4d}{x^3} + \frac{6e}{x^2} + \frac{12f}{x} \right) \sqrt{a + bx^4} + \frac{2\sqrt{b}fx\sqrt{a + bx^4}}{\sqrt{a} + \sqrt{bx^2}} + \frac{1}{2}\sqrt{be} \tanh^{-1} \left(\frac{\sqrt{a+bx^4}}{\sqrt{a}} \right)
\end{aligned}$$

Mathematica [C] time = 0.233866, size = 175, normalized size = 0.53

$$\frac{\sqrt{\frac{bx^4}{a} + 1} \left(3ac\sqrt{\frac{bx^4}{a} + 1} + 3bcx^4 \tanh^{-1} \left(\sqrt{\frac{bx^4}{a} + 1} \right) + 4adx {}_2F_1 \left(-\frac{3}{4}, -\frac{1}{2}; \frac{1}{4}; -\frac{bx^4}{a} \right) + 6aex^2\sqrt{\frac{bx^4}{a} + 1} - 6\sqrt{a}\sqrt{b}ex^4 \sinh^{-1} \left(\frac{\sqrt{a+bx^4}}{\sqrt{a}} \right) \right)}{12x^4\sqrt{a + bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^5,x]

[Out] -(Sqrt[1 + (b*x^4)/a]*(3*a*c*Sqrt[1 + (b*x^4)/a] + 6*a*e*x^2*Sqrt[1 + (b*x^4)/a] - 6*Sqrt[a]*Sqrt[b]*e*x^4*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + 3*b*c*x^4*ArcTanh[Sqrt[1 + (b*x^4)/a]] + 4*a*d*x*Hypergeometric2F1[-3/4, -1/2, 1/4, -(b*x^4)/a] + 12*a*f*x^3*Hypergeometric2F1[-1/2, -1/4, 3/4, -(b*x^4)/a]))/(12*x^4*Sqrt[a + b*x^4])

Maple [C] time = 0.016, size = 385, normalized size = 1.2

$$-\frac{d}{3x^3}\sqrt{bx^4+a} + \frac{2bd}{3}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right) \frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{bx^4+a}} - \frac{c}{4ax^4}(bx^4+a)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^5,x)

[Out]
$$-1/3*d/x^3*(b*x^4+a)^{(1/2)}+2/3*d*b/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticF}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)},I)-1/4*c/a/x^4*(b*x^4+a)^{(3/2)}-1/4*c*b/a^{(1/2)}*\ln((2*a+2*a^{(1/2)}*(b*x^4+a)^{(1/2)})/x^2)+1/4*c*b/a*(b*x^4+a)^{(1/2)}-1/2*e/a/x^2*(b*x^4+a)^{(3/2)}+1/2*e*b/a*x^2*(b*x^4+a)^{(1/2)}+1/2*e*b^{(1/2)}*\ln(x^2*b^{(1/2)}+(b*x^4+a)^{(1/2)})-f/x*(b*x^4+a)^{(1/2)}+2*I*f*b^{(1/2)}*a^{(1/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticF}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)},I)-2*I*f*b^{(1/2)}*a^{(1/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticE}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)},I)$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^5,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^5,x, algorithm="fricas")

[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^5, x)

Sympy [C] time = 5.73538, size = 211, normalized size = 0.64

$$\frac{\sqrt{ad}\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(-\frac{3}{4}, -\frac{1}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{ae}}{2x^2\sqrt{1 + \frac{bx^4}{a}}} + \frac{\sqrt{a}f\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(-\frac{1}{2}, -\frac{1}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{\sqrt{bc}\sqrt{\frac{a}{bx^4} + 1}}{4x^2} + \frac{\sqrt{be} \operatorname{asinh}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**5,x)

[Out] sqrt(a)*d*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - sqrt(a)*e/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*f*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(4*x**2) + sqrt(b)*e*asinh(sqrt(b)*x**2/sqrt(a))/2 - b*c*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*sqrt(a)) - b*e*x**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^5,x, algorithm="giac")

[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^5, x)

$$3.505 \quad \int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6} dx$$

Optimal. Leaf size=360

$$\frac{b^{3/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (5\sqrt{ae} + 3\sqrt{bc}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}} - \frac{2b^{5/4}c(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\right)}{5a^{3/4}\sqrt{a+bx^4}}$$

[Out] -(((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2)*Sqrt[a + b*x^4])/60 - (2*b*c*Sqrt[a + b*x^4])/(5*a*x) + (2*b^(3/2)*c*x*Sqrt[a + b*x^4])/(5*a*(Sqrt[a] + Sqrt[b]*x^2)) + (Sqrt[b]*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (b*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*Sqrt[a]) - (2*b^(5/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a + b*x^4]) + (b^(3/4)*(3*Sqrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a + b*x^4])

Rubi [A] time = 0.323537, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {14, 1825, 1833, 1282, 1198, 220, 1196, 1252, 844, 217, 206, 266, 63, 208}

$$\frac{b^{3/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (5\sqrt{ae} + 3\sqrt{bc}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}} - \frac{2b^{5/4}c(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\right)}{5a^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^6,x]

[Out] -(((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2)*Sqrt[a + b*x^4])/60 - (2*b*c*Sqrt[a + b*x^4])/(5*a*x) + (2*b^(3/2)*c*x*Sqrt[a + b*x^4])/(5*a*(Sqrt[a] + Sqrt[b]*x^2)) + (Sqrt[b]*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (b*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*Sqrt[a]) - (2*b^(5/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a + b*x^4]) + (b^(3/4)*(3*Sqrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a + b*x^4])

/4)*Sqrt[a + b*x^4])

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 1825

Int[(Pq_)*(x_)^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] := Module[{u = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1, 0]

Rule 1833

Int[(Pq_)*((c_)*(x_))^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p]/c^j, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]

Rule 1282

Int[((f_)*(x_))^{(m_)}*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^{(p_)}, x_Symbol] := Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1198

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rule 220

Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q =
  Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
  1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
  1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
  x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_S
ymbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
  x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 844

```
Int[((d_) + (e_)*(x_)^(m_))*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p
_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
  e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
  x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
  Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
  Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
  Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
  , m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := With[
  {p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
  (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
  [b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```


ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
 \int \frac{(c + dx + ex^2 + fx^3)\sqrt{a + bx^4}}{x^6} dx &= -\frac{1}{60} \left(\frac{12c}{x^5} + \frac{15d}{x^4} + \frac{20e}{x^3} + \frac{30f}{x^2} \right) \sqrt{a + bx^4} - (2b) \int \frac{-\frac{c}{5} - \frac{dx}{4} - \frac{ex^2}{3} - \frac{fx^3}{2}}{x^2\sqrt{a + bx^4}} dx \\
 &= -\frac{1}{60} \left(\frac{12c}{x^5} + \frac{15d}{x^4} + \frac{20e}{x^3} + \frac{30f}{x^2} \right) \sqrt{a + bx^4} - (2b) \int \left(\frac{-\frac{c}{5} - \frac{ex^2}{3}}{x^2\sqrt{a + bx^4}} + \frac{-\frac{d}{4} - \frac{fx^2}{2}}{x\sqrt{a + bx^4}} \right) dx \\
 &= -\frac{1}{60} \left(\frac{12c}{x^5} + \frac{15d}{x^4} + \frac{20e}{x^3} + \frac{30f}{x^2} \right) \sqrt{a + bx^4} - (2b) \int \frac{-\frac{c}{5} - \frac{ex^2}{3}}{x^2\sqrt{a + bx^4}} dx - (2b) \int \frac{-\frac{d}{4} - \frac{fx^2}{2}}{x\sqrt{a + bx^4}} dx \\
 &= -\frac{1}{60} \left(\frac{12c}{x^5} + \frac{15d}{x^4} + \frac{20e}{x^3} + \frac{30f}{x^2} \right) \sqrt{a + bx^4} - \frac{2bc\sqrt{a + bx^4}}{5ax} - b \operatorname{Subst} \left(\int \frac{-\frac{d}{4} - \frac{fx^2}{2}}{x\sqrt{a + bx^4}} dx, \sqrt{bx^2} \right) \\
 &= -\frac{1}{60} \left(\frac{12c}{x^5} + \frac{15d}{x^4} + \frac{20e}{x^3} + \frac{30f}{x^2} \right) \sqrt{a + bx^4} - \frac{2bc\sqrt{a + bx^4}}{5ax} - \frac{(2b^{3/2}c) \int \frac{1 - \frac{\sqrt{bx^2}}{\sqrt{a}}}{\sqrt{a + bx^4}} dx}{5\sqrt{a}} \\
 &= -\frac{1}{60} \left(\frac{12c}{x^5} + \frac{15d}{x^4} + \frac{20e}{x^3} + \frac{30f}{x^2} \right) \sqrt{a + bx^4} - \frac{2bc\sqrt{a + bx^4}}{5ax} + \frac{2b^{3/2}cx\sqrt{a + bx^4}}{5a(\sqrt{a} + \sqrt{bx^2})} \\
 &= -\frac{1}{60} \left(\frac{12c}{x^5} + \frac{15d}{x^4} + \frac{20e}{x^3} + \frac{30f}{x^2} \right) \sqrt{a + bx^4} - \frac{2bc\sqrt{a + bx^4}}{5ax} + \frac{2b^{3/2}cx\sqrt{a + bx^4}}{5a(\sqrt{a} + \sqrt{bx^2})} + \\
 &= -\frac{1}{60} \left(\frac{12c}{x^5} + \frac{15d}{x^4} + \frac{20e}{x^3} + \frac{30f}{x^2} \right) \sqrt{a + bx^4} - \frac{2bc\sqrt{a + bx^4}}{5ax} + \frac{2b^{3/2}cx\sqrt{a + bx^4}}{5a(\sqrt{a} + \sqrt{bx^2})} +
 \end{aligned}$$

Mathematica [C] time = 0.233774, size = 179, normalized size = 0.5

$$\frac{\sqrt{a+bx^4} \left(12ac {}_2F_1 \left(-\frac{5}{4}, -\frac{1}{2}; -\frac{1}{4}; -\frac{bx^4}{a} \right) + 5x \left(3ad \sqrt{\frac{bx^4}{a} + 1} + 3bdx^4 \tanh^{-1} \left(\sqrt{\frac{bx^4}{a} + 1} \right) + 4aex {}_2F_1 \left(-\frac{3}{4}, -\frac{1}{2}; \frac{1}{4}; -\frac{bx^4}{a} \right) + 6 \right) \right)}{60ax^5 \sqrt{\frac{bx^4}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^6,x]

[Out] -(Sqrt[a + b*x^4]*(12*a*c*Hypergeometric2F1[-5/4, -1/2, -1/4, -((b*x^4)/a)] + 5*x*(3*a*d*Sqrt[1 + (b*x^4)/a] + 6*a*f*x^2*Sqrt[1 + (b*x^4)/a] - 6*Sqrt[a]*Sqrt[b]*f*x^4*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + 3*b*d*x^4*ArcTanh[Sqrt[1 + (b*x^4)/a]] + 4*a*e*x*Hypergeometric2F1[-3/4, -1/2, 1/4, -((b*x^4)/a)])))/(60*a*x^5*Sqrt[1 + (b*x^4)/a])

Maple [C] time = 0.015, size = 404, normalized size = 1.1

$$-\frac{e}{3x^3} \sqrt{bx^4 + a} + \frac{2be}{3} \sqrt{1 - ix^2 \sqrt{b} \frac{1}{\sqrt{a}}} \sqrt{1 + ix^2 \sqrt{b} \frac{1}{\sqrt{a}}} \text{EllipticF} \left(x \sqrt{i \sqrt{b} \frac{1}{\sqrt{a}}}, i \right) \frac{1}{\sqrt{i \sqrt{b} \frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{bx^4 + a}} - \frac{c}{5x^5} \sqrt{bx^4 + a} - \frac{2b}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^6,x)

[Out] -1/3*e/x^3*(b*x^4+a)^(1/2)+2/3*e*b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/5*c/x^5*(b*x^4+a)^(1/2)-2/5*b*c*(b*x^4+a)^(1/2)/a/x+2/5*I*c/a^(1/2)*b^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-2/5*I*c/a^(1/2)*b^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/4*d/a/x^4*(b*x^4+a)^(3/2)-1/4*d*b/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)+1/4*d*b/a*(b*x^4+a)^(1/2)-1/2*f/a/x^2*(b*x^4+a)^(3/2)+1/2*f*b/a*x^2*(b*x^4+a)^(1/2)+1/2*f*b^(1/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^6,x, algorithm="maxima")

[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^6,x, algorithm="fricas")

[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)

Sympy [C] time = 6.0442, size = 216, normalized size = 0.6

$$\frac{\sqrt{ac}\Gamma\left(-\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, -\frac{1}{2} \\ -\frac{1}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{\sqrt{ae}\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, -\frac{1}{2} \\ \frac{1}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{af}}{2x^2\sqrt{1 + \frac{bx^4}{a}}} - \frac{\sqrt{bd}\sqrt{\frac{a}{bx^4} + 1}}{4x^2} + \frac{\sqrt{b}f \operatorname{asinh}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**6,x)

[Out] sqrt(a)*c*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) + sqrt(a)*e*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - sqrt(a)*f/(2*x**2*sqrt(1 + b*x**4/a)) - sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(4*x**2) + sqrt(b)*f*asinh(sqrt(b)*x**2/sqrt(a))/2 - b*d*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*sqrt(a)) - b*f*x

`**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^6,x, algorithm="giac")`

[Out] `integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)`

$$3.506 \quad \int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7} dx$$

Optimal. Leaf size=352

$$\frac{b^{3/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (5\sqrt{a}f + 3\sqrt{bd}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right) - 2b^{5/4}d(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4} - 5a^{3/4}\sqrt{a+bx^4}}$$

[Out] -(((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3)*Sqrt[a + b*x^4])/60 - (b*c*Sqrt[a + b*x^4])/(6*a*x^2) - (2*b*d*Sqrt[a + b*x^4])/(5*a*x) + (2*b^(3/2)*d*x*Sqrt[a + b*x^4])/(5*a*(Sqrt[a] + Sqrt[b]*x^2)) - (b*e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*Sqrt[a]) - (2*b^(5/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a + b*x^4]) + (b^(3/4)*(3*Sqrt[b]*d + 5*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a + b*x^4])

Rubi [A] time = 0.316338, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {14, 1825, 1833, 1252, 807, 266, 63, 208, 1282, 1198, 220, 1196}

$$\frac{b^{3/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (5\sqrt{a}f + 3\sqrt{bd}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right) - 2b^{5/4}d(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4} - 5a^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^7,x]

[Out] -(((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3)*Sqrt[a + b*x^4])/60 - (b*c*Sqrt[a + b*x^4])/(6*a*x^2) - (2*b*d*Sqrt[a + b*x^4])/(5*a*x) + (2*b^(3/2)*d*x*Sqrt[a + b*x^4])/(5*a*(Sqrt[a] + Sqrt[b]*x^2)) - (b*e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*Sqrt[a]) - (2*b^(5/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a + b*x^4]) + (b^(3/4)*(3*Sqrt[b]*d + 5*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a + b*x^4])

Rule 14

```
Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_
+ (b_)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^((m_))*((a_) + (b_)*(x_)^((n_)))^(p_), x_Symbol] := Module[{u
= IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
0]
```

Rule 1833

```
Int[(Pq_)*((c_)*(x_))^(m_))*((a_) + (b_)*(x_)^((n_)))^(p_), x_Symbol] := Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p)/c^j, {j, 0,
n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] && !PolyQ[Pq, x^(n/2)]
```

Rule 1252

```
Int[(x_)^((m_))*((d_) + (e_)*(x_)^2)^(q_))*((a_) + (c_)*(x_)^4)^(p_), x_S
ymbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 807

```
Int[((d_) + (e_)*(x_))^(m_))*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p
_), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p},
x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^((m_))*((a_) + (b_)*(x_)^((n_)))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_) + (b_)*(x_))^(m_))*((c_) + (d_)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

$[b*c - a*d, 0] \ \&\& \text{LtQ}[-1, m, 0] \ \&\& \text{LeQ}[-1, n, 0] \ \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 208

$\text{Int}[(a_) + (b_)*(x_)^2]^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] \ /; \text{FreeQ}\{a, b\}, x\} \ \&\& \text{NegQ}[a/b]$

Rule 1282

$\text{Int}[(f_)*(x_)^{(m_)}*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(d*(f*x)^{(m+1)}*(a + c*x^4)^{(p+1)})/(a*f*(m+1)), x] + \text{Dist}[1/(a*f^2*(m+1)), \text{Int}[(f*x)^{(m+2)}*(a + c*x^4)^p*(a*e*(m+1) - c*d*(m+4*p+5)*x^2), x], x] \ /; \text{FreeQ}\{a, c, d, e, f, p\}, x\} \ \&\& \text{LtQ}[m, -1] \ \&\& \text{IntegerQ}[2*p] \ \&\& (\text{IntegerQ}[p] \ || \ \text{IntegerQ}[m])$

Rule 1198

$\text{Int}[(d_) + (e_)*(x_)^2]/\text{Sqrt}[(a_) + (c_)*(x_)^4], x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[c/a, 2]\}, \text{Dist}[(e + d*q)/q, \text{Int}[1/\text{Sqrt}[a + c*x^4], x], x] - \text{Dist}[e/q, \text{Int}[(1 - q*x^2)/\text{Sqrt}[a + c*x^4], x], x] \ /; \text{NeQ}[e + d*q, 0] \ /; \text{FreeQ}\{a, c, d, e\}, x\} \ \&\& \text{PosQ}[c/a]$

Rule 220

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^4], x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[b/a, 4]\}, \text{Simp}[(1 + q^2*x^2)*\text{Sqrt}[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*\text{EllipticF}[2*\text{ArcTan}[q*x], 1/2)]/(2*q*\text{Sqrt}[a + b*x^4]), x] \ /; \text{FreeQ}\{a, b\}, x\} \ \&\& \text{PosQ}[b/a]$

Rule 1196

$\text{Int}[(d_) + (e_)*(x_)^2]/\text{Sqrt}[(a_) + (c_)*(x_)^4], x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[c/a, 4]\}, -\text{Simp}[(d*x*\text{Sqrt}[a + c*x^4])/(a*(1 + q^2*x^2)), x] + \text{Simp}[(d*(1 + q^2*x^2)*\text{Sqrt}[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*\text{EllipticE}[2*\text{ArcTan}[q*x], 1/2])/(q*\text{Sqrt}[a + c*x^4]), x] \ /; \text{EqQ}[e + d*q^2, 0] \ /; \text{FreeQ}\{a, c, d, e\}, x\} \ \&\& \text{PosQ}[c/a]$

Rubi steps

$$\begin{aligned}
\int \frac{(c + dx + ex^2 + fx^3) \sqrt{a + bx^4}}{x^7} dx &= -\frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} \right) \sqrt{a + bx^4} - (2b) \int \frac{-\frac{c}{6} - \frac{dx}{5} - \frac{ex^2}{4} - \frac{fx^3}{3}}{x^3 \sqrt{a + bx^4}} dx \\
&= -\frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} \right) \sqrt{a + bx^4} - (2b) \int \left(\frac{-\frac{c}{6} - \frac{ex^2}{4}}{x^3 \sqrt{a + bx^4}} + \frac{-\frac{d}{5} - \frac{fx^2}{3}}{x^2 \sqrt{a + bx^4}} \right) dx \\
&= -\frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} \right) \sqrt{a + bx^4} - (2b) \int \frac{-\frac{c}{6} - \frac{ex^2}{4}}{x^3 \sqrt{a + bx^4}} dx - (2b) \int \frac{-\frac{d}{5} - \frac{fx^2}{3}}{x^2 \sqrt{a + bx^4}} dx \\
&= -\frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} \right) \sqrt{a + bx^4} - \frac{2bd\sqrt{a + bx^4}}{5ax} - b \operatorname{Subst} \left(\int \frac{-\frac{c}{6} - \frac{ex^2}{4}}{x^2 \sqrt{a + bx^4}} dx, x, \sqrt{a + bx^4} \right) \\
&= -\frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} \right) \sqrt{a + bx^4} - \frac{bc\sqrt{a + bx^4}}{6ax^2} - \frac{2bd\sqrt{a + bx^4}}{5ax} - \frac{(2b^{3/2}d)}{5a\sqrt{a + bx^4}} \\
&= -\frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} \right) \sqrt{a + bx^4} - \frac{bc\sqrt{a + bx^4}}{6ax^2} - \frac{2bd\sqrt{a + bx^4}}{5ax} + \frac{2b^{3/2}d}{5a\sqrt{a + bx^4}} \\
&= -\frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} \right) \sqrt{a + bx^4} - \frac{bc\sqrt{a + bx^4}}{6ax^2} - \frac{2bd\sqrt{a + bx^4}}{5ax} + \frac{2b^{3/2}d}{5a\sqrt{a + bx^4}} \\
&= -\frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} \right) \sqrt{a + bx^4} - \frac{bc\sqrt{a + bx^4}}{6ax^2} - \frac{2bd\sqrt{a + bx^4}}{5ax} + \frac{2b^{3/2}d}{5a\sqrt{a + bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.243303, size = 145, normalized size = 0.41

$$\frac{\sqrt{a + bx^4} \left(5 \left(\sqrt{\frac{bx^4}{a} + 1} (2ac + 3aex^2 + 2bcx^4) + 3bex^6 \tanh^{-1} \left(\sqrt{\frac{bx^4}{a} + 1} \right) + 4afx^3 {}_2F_1 \left(-\frac{3}{4}, -\frac{1}{2}; \frac{1}{4}; -\frac{bx^4}{a} \right) \right) + 12adx {}_2F_1 \left(-\frac{3}{4}, -\frac{1}{2}; \frac{1}{4}; -\frac{bx^4}{a} \right)}{60ax^6 \sqrt{\frac{bx^4}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^7,x]

[Out] -(Sqrt[a + b*x^4]*(12*a*d*x*Hypergeometric2F1[-5/4, -1/2, -1/4, -((b*x^4)/a)] + 5*(Sqrt[1 + (b*x^4)/a]*(2*a*c + 3*a*e*x^2 + 2*b*c*x^4) + 3*b*e*x^6*Arc

$\text{Tanh}[\text{Sqrt}[1 + (b*x^4)/a]] + 4*a*f*x^3*\text{Hypergeometric2F1}[-3/4, -1/2, 1/4, -(b*x^4)/a])]/(60*a*x^6*\text{Sqrt}[1 + (b*x^4)/a])$

Maple [C] time = 0.017, size = 361, normalized size = 1.

$$-\frac{f}{3x^3}\sqrt{bx^4+a} + \frac{2fb}{3}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}, i\right) \frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{bx^4+a}} - \frac{d}{5x^5}\sqrt{bx^4+a} - \frac{2}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^7,x)

[Out] $-1/3*f/x^3*(b*x^4+a)^{(1/2)}+2/3*f*b/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticF}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I)-1/5*d/x^5*(b*x^4+a)^{(1/2)}-2/5*b*d*(b*x^4+a)^{(1/2)}/a/x+2/5*I*d/a^{(1/2)}*b^{(3/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticF}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I)-2/5*I*d/a^{(1/2)}*b^{(3/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticE}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I)-1/4*e/a/x^4*(b*x^4+a)^{(3/2)}-1/4*e*b/a^{(1/2)}*\ln((2*a+2*a^{(1/2)}*(b*x^4+a)^{(1/2)})/x^2)+1/4*e*b/a*(b*x^4+a)^{(1/2)}-1/6*c*(b*x^4+a)^{(3/2)}/x^6/a$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{(bx^4+a)^{\frac{3}{2}}c}{6ax^6} + \int \frac{\sqrt{bx^4+a}(fx^2+ex+d)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^7,x, algorithm="maxima")

[Out] $-1/6*(b*x^4+a)^{(3/2)}*c/(a*x^6) + \text{integrate}(\text{sqrt}(b*x^4+a)*(f*x^2+e*x+d)/x^6, x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)}{x^7}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^7,x, algorithm="fricas")

[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^7, x)

Sympy [C] time = 5.70158, size = 189, normalized size = 0.54

$$\frac{\sqrt{ad}\Gamma\left(-\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, -\frac{1}{2} \\ -\frac{1}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{\sqrt{a}f\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, -\frac{1}{2} \\ \frac{1}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{bc}\sqrt{\frac{a}{bx^4} + 1}}{6x^4} - \frac{\sqrt{be}\sqrt{\frac{a}{bx^4} + 1}}{4x^2} - \frac{b^{\frac{3}{2}}c\sqrt{\frac{a}{bx^4} + 1}}{6a} - \frac{be}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**7,x)

[Out] sqrt(a)*d*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) + sqrt(a)*f*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(6*x**4) - sqrt(b)*e*sqrt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*c*sqrt(a/(b*x**4) + 1)/(6*a) - b*e*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*sqrt(a))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^7,x, algorithm="giac")

[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^7, x)

$$3.507 \quad \int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8} dx$$

Optimal. Leaf size=375

$$\frac{b^{5/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (5\sqrt{bc} - 21\sqrt{ae}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{105a^{5/4}\sqrt{a+bx^4}} - \frac{2b^{5/4}e(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}}$$

[Out] -(((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4)*Sqrt[a + b*x^4])/420 - (2*b*c*Sqrt[a + b*x^4])/(21*a*x^3) - (b*d*Sqrt[a + b*x^4])/(6*a*x^2) - (2*b*e*Sqrt[a + b*x^4])/(5*a*x) + (2*b^(3/2)*e*x*Sqrt[a + b*x^4])/(5*a*(Sqrt[a] + Sqrt[b]*x^2)) - (b*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*Sqrt[a]) - (2*b^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a + b*x^4]) - (b^(5/4)*(5*Sqrt[b]*c - 21*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*a^(5/4)*Sqrt[a + b*x^4])

Rubi [A] time = 0.361593, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {14, 1825, 1833, 1282, 1198, 220, 1196, 1252, 807, 266, 63, 208}

$$\frac{b^{5/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (5\sqrt{bc} - 21\sqrt{ae}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{105a^{5/4}\sqrt{a+bx^4}} - \frac{2b^{5/4}e(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^8,x]

[Out] -(((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4)*Sqrt[a + b*x^4])/420 - (2*b*c*Sqrt[a + b*x^4])/(21*a*x^3) - (b*d*Sqrt[a + b*x^4])/(6*a*x^2) - (2*b*e*Sqrt[a + b*x^4])/(5*a*x) + (2*b^(3/2)*e*x*Sqrt[a + b*x^4])/(5*a*(Sqrt[a] + Sqrt[b]*x^2)) - (b*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*Sqrt[a]) - (2*b^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a + b*x^4]) - (b^(5/4)*(5*Sqrt[b]*c - 21*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*a^(5/4)*Sqrt[a + b*x^4])

Rule 14

```
Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

Rule 1825

```
Int[(Pq_)*(x_)^m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{u = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1, 0]
```

Rule 1833

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p)/c^j, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 1282

```
Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q =
  Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
  1 + q^2*x^2)*Sqrt[a + c*x^4]/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
  1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e},
  x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_S
ymbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
  x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 807

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p
_), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p},
  x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
  Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
  , m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := With[
  {p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
  (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
  [b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
  ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
  Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c + dx + ex^2 + fx^3) \sqrt{a + bx^4}}{x^8} dx &= -\frac{1}{420} \left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} \right) \sqrt{a + bx^4} - (2b) \int \frac{-\frac{c}{7} - \frac{dx}{6} - \frac{ex^2}{5} - \frac{fx^3}{4}}{x^4 \sqrt{a + bx^4}} dx \\
&= -\frac{1}{420} \left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} \right) \sqrt{a + bx^4} - (2b) \int \left(\frac{-\frac{c}{7} - \frac{ex^2}{5}}{x^4 \sqrt{a + bx^4}} + \frac{-\frac{d}{6} - \frac{fx^3}{4}}{x^3 \sqrt{a + bx^4}} \right) dx \\
&= -\frac{1}{420} \left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} \right) \sqrt{a + bx^4} - (2b) \int \frac{-\frac{c}{7} - \frac{ex^2}{5}}{x^4 \sqrt{a + bx^4}} dx - (2b) \int \frac{-\frac{d}{6} - \frac{fx^3}{4}}{x^3 \sqrt{a + bx^4}} dx \\
&= -\frac{1}{420} \left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} \right) \sqrt{a + bx^4} - \frac{2bc\sqrt{a + bx^4}}{21ax^3} - b \operatorname{Subst} \left(\int \frac{-\frac{d}{6} - \frac{fx^3}{4}}{x^2 \sqrt{a + bx^4}} dx, x, \sqrt{a + bx^4} \right) \\
&= -\frac{1}{420} \left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} \right) \sqrt{a + bx^4} - \frac{2bc\sqrt{a + bx^4}}{21ax^3} - \frac{bd\sqrt{a + bx^4}}{6ax^2} - \frac{2be\sqrt{a + bx^4}}{6ax^2} \\
&= -\frac{1}{420} \left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} \right) \sqrt{a + bx^4} - \frac{2bc\sqrt{a + bx^4}}{21ax^3} - \frac{bd\sqrt{a + bx^4}}{6ax^2} - \frac{2be\sqrt{a + bx^4}}{6ax^2} \\
&= -\frac{1}{420} \left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} \right) \sqrt{a + bx^4} - \frac{2bc\sqrt{a + bx^4}}{21ax^3} - \frac{bd\sqrt{a + bx^4}}{6ax^2} - \frac{2be\sqrt{a + bx^4}}{6ax^2} \\
&= -\frac{1}{420} \left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} \right) \sqrt{a + bx^4} - \frac{2bc\sqrt{a + bx^4}}{21ax^3} - \frac{bd\sqrt{a + bx^4}}{6ax^2} - \frac{2be\sqrt{a + bx^4}}{6ax^2}
\end{aligned}$$

Mathematica [C] time = 0.240489, size = 145, normalized size = 0.39

$$\frac{\sqrt{a + bx^4} \left(60ac {}_2F_1 \left(-\frac{7}{4}, -\frac{1}{2}; -\frac{3}{4}; -\frac{bx^4}{a} \right) + 35x \left(\sqrt{\frac{bx^4}{a} + 1} (2ad + 3afx^2 + 2bdx^4) + 3bf x^6 \tanh^{-1} \left(\sqrt{\frac{bx^4}{a} + 1} \right) \right) + 84aex^2 \right)}{420ax^7 \sqrt{\frac{bx^4}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^8, x]

[Out] -(Sqrt[a + b*x^4]*(35*x*(Sqrt[1 + (b*x^4)/a]*(2*a*d + 3*a*f*x^2 + 2*b*d*x^4) + 3*b*f*x^6*ArcTanh[Sqrt[1 + (b*x^4)/a]])) + 60*a*c*Hypergeometric2F1[-7/4

, $-1/2$, $-3/4$, $-((b*x^4)/a)] + 84*a*e*x^2*Hypergeometric2F1[-5/4, -1/2, -1/4$
 $, -((b*x^4)/a)))/(420*a*x^7*sqrt[1 + (b*x^4)/a])$

Maple [C] time = 0.018, size = 385, normalized size = 1.

$$-\frac{e}{5x^5}\sqrt{bx^4+a} - \frac{2be}{5ax}\sqrt{bx^4+a} + \frac{2i}{5}eb^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}, i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^8,x)

[Out] $-1/5*e/x^5*(b*x^4+a)^{(1/2)} - 2/5*b*e*(b*x^4+a)^{(1/2)}/a/x + 2/5*I*e/a^{(1/2)}*b^{(3/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticF}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I) - 2/5*I*e/a^{(1/2)}*b^{(3/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticE}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I) - 1/4*f/a/x^4*(b*x^4+a)^{(3/2)} - 1/4*f*b/a^{(1/2)}*\ln((2*a+2*a^{(1/2)}*(b*x^4+a)^{(1/2)})/x^2) + 1/4*f*b/a*(b*x^4+a)^{(1/2)} - 1/6*d*(b*x^4+a)^{(3/2)}/x^6/a - 1/7*c/x^7*(b*x^4+a)^{(1/2)} - 2/21*b*c*(b*x^4+a)^{(1/2)}/a/x^3 - 2/21*c/a*b^{(2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticF}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^8,x, algorithm="maxima")

[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^8, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)}{x^8}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^8,x, algorithm="fricas")

[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^8, x)

Sympy [C] time = 6.31268, size = 192, normalized size = 0.51

$$\frac{\sqrt{ac}\Gamma\left(-\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{7}{4}, -\frac{1}{2} \\ -\frac{3}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{\sqrt{ae}\Gamma\left(-\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, -\frac{1}{2} \\ -\frac{1}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} - \frac{\sqrt{bd}\sqrt{\frac{a}{bx^4} + 1}}{6x^4} - \frac{\sqrt{bf}\sqrt{\frac{a}{bx^4} + 1}}{4x^2} - \frac{b^{\frac{3}{2}}d\sqrt{\frac{a}{bx^4} + 1}}{6a} - \frac{bfa}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**8,x)

[Out] sqrt(a)*c*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3/4)) + sqrt(a)*e*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) - sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(6*x**4) - sqrt(b)*f*sqrt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*d*sqrt(a/(b*x**4) + 1)/(6*a) - b*f*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*sqrt(a))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^8,x, algorithm="giac")

[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^8, x)

$$3.508 \quad \int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9} dx$$

Optimal. Leaf size=400

$$\frac{b^{5/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (5\sqrt{bd} - 21\sqrt{af}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt{a}}\right), \frac{1}{2}\right) + b^2 c \tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - 2b^{5/4} f (\sqrt{a} + \sqrt{bx^2})}{105a^{5/4}\sqrt{a+bx^4}} + \frac{b^2 c \tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - 2b^{5/4} f (\sqrt{a} + \sqrt{bx^2})}{16a^{3/2}}$$

[Out] -(((105*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5)*Sqrt[a + b*x^4])/840 - (b*c*Sqrt[a + b*x^4])/(16*a*x^4) - (2*b*d*Sqrt[a + b*x^4])/(21*a*x^3) - (b*e*Sqrt[a + b*x^4])/(6*a*x^2) - (2*b*f*Sqrt[a + b*x^4])/(5*a*x) + (2*b^(3/2)*f*x*Sqrt[a + b*x^4])/(5*a*(Sqrt[a] + Sqrt[b]*x^2)) + (b^2*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(16*a^(3/2)) - (2*b^(5/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a + b*x^4]) - (b^(5/4)*(5*Sqrt[b]*d - 21*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*a^(5/4)*Sqrt[a + b*x^4])

Rubi [A] time = 0.386205, antiderivative size = 400, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {14, 1825, 1833, 1252, 835, 807, 266, 63, 208, 1282, 1198, 220, 1196}

$$\frac{b^2 c \tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - 2b^{5/4} f (\sqrt{a} + \sqrt{bx^2})}{16a^{3/2}} - \frac{b^{5/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (5\sqrt{bd} - 21\sqrt{af}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt{a}}\right) \middle| \frac{1}{2}\right) - 2b^{5/4} f (\sqrt{a} + \sqrt{bx^2})}{105a^{5/4}\sqrt{a+bx^4}} - \frac{2b^{5/4} f (\sqrt{a} + \sqrt{bx^2})}{5a^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^9,x]

[Out] -(((105*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5)*Sqrt[a + b*x^4])/840 - (b*c*Sqrt[a + b*x^4])/(16*a*x^4) - (2*b*d*Sqrt[a + b*x^4])/(21*a*x^3) - (b*e*Sqrt[a + b*x^4])/(6*a*x^2) - (2*b*f*Sqrt[a + b*x^4])/(5*a*x) + (2*b^(3/2)*f*x*Sqrt[a + b*x^4])/(5*a*(Sqrt[a] + Sqrt[b]*x^2)) + (b^2*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(16*a^(3/2)) - (2*b^(5/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a + b*x^4]) - (b^(5/4)*(5*Sqrt[b]*d - 21*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*a^(5/4)*Sqrt[a + b*x^4])

)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2]]/(105*a^(5/4)*Sqrt[a + b*x^4])

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 1825

Int[(Pq_)*(x_)^m_)*((a_) + (b_)*(x_)^n_)^p_, x_Symbol] := Module[{u = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1, 0]

Rule 1833

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^n_)^p_, x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p]/c^j, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]

Rule 1252

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rule 835

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 807

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))

$$\frac{1}{(2(p+1)(cd^2 + ae^2))} \int (cd^2 + ae^2)^p dx + \text{Dist}\left[\frac{cd^2 + ae^2}{(cd^2 + ae^2)}, \int (d + ex)^{m+1} (a + cx^2)^p dx, x\right] /;$$

$$\text{FreeQ}\{a, c, d, e, f, g, m, p, x\} \ \&\& \ \text{NeQ}[cd^2 + ae^2, 0] \ \&\& \ \text{EqQ}[\text{Simplify}[m + 2p + 3], 0]$$

Rule 266

$$\int (x^m (a + bx)^n)^p dx \text{Symbol} \rightarrow \text{Dist}\left[\frac{1}{n}, \text{Subst}\left[\int x^{\text{Simplify}[(m+1)/n] - 1} (a + bx)^p dx, x, x^n\right], x\right] /;$$

$$\text{FreeQ}\{a, b, m, n, p, x\} \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$$

Rule 63

$$\int ((a + bx)^m (c + dx)^n) dx \text{Symbol} \rightarrow \text{With}\left[\{p = \text{Denominator}[m]\}, \text{Dist}\left[\frac{p}{b}, \text{Subst}\left[\int x^{p(m+1) - 1} (c - (ad)/b + (dx^p)/b)^n dx, x, (a + bx)^{1/p}\right], x\right] /;$$

$$\text{FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{NeQ}[bc - ad, 0] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$$

Rule 208

$$\int ((a + bx)^2)^{-1} dx \text{Symbol} \rightarrow \text{Simp}\left[\text{Rt}[-a/b, 2] \text{ArcTanh}\left[\frac{x}{\text{Rt}[-a/b, 2]}\right] / a, x\right] /;$$

$$\text{FreeQ}\{a, b, x\} \ \&\& \ \text{NegQ}[a/b]$$

Rule 1282

$$\int ((f + gx)^m (d + ex)^2 (a + cx^4)^p) dx \text{Symbol} \rightarrow \text{Simp}\left[\frac{d(f + gx)^{m+1} (a + cx^4)^{p+1}}{a f (m+1)}, x\right] + \text{Dist}\left[\frac{1}{a f^2 (m+1)}, \int (f + gx)^{m+2} (a + cx^4)^p (a e (m+1) - c d (m + 4p + 5) x^2) dx, x\right] /;$$

$$\text{FreeQ}\{a, c, d, e, f, p, x\} \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{IntegerQ}[2p] \ \&\& \ (\text{IntegerQ}[p] \ || \ \text{IntegerQ}[m])$$

Rule 1198

$$\int \frac{(d + ex)^2}{\sqrt{(a + cx^4)}} dx \text{Symbol} \rightarrow \text{With}\left[\{q = \text{Rt}[c/a, 2]\}, \text{Dist}\left[\frac{e + dq}{q}, \int \frac{1}{\sqrt{a + cx^4}} dx, x\right] - \text{Dist}\left[\frac{e}{q}, \int \frac{(1 - qx^2)}{\sqrt{a + cx^4}} dx, x\right] /;$$

$$\text{NeQ}[e + dq, 0] /;$$

$$\text{FreeQ}\{a, c, d, e, x\} \ \&\& \ \text{PosQ}[c/a]$$

Rule 220

$$\int \frac{1}{\sqrt{(a + bx^4)}} dx \text{Symbol} \rightarrow \text{With}\left[\{q = \text{Rt}[b/a, 4]\}, \text{Simp}\left[\frac{(1 + q^2 x^2) \sqrt{(a + bx^4)}}{a (1 + q^2 x^2)^2} \text{EllipticF}\left[2 \text{ArcTan}[qx], 1/2\right]\right] / (2q \sqrt{a + bx^4}), x\right] /;$$

$$\text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[b/a]$$

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
  Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
  1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
  1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
  x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c + dx + ex^2 + fx^3)\sqrt{a + bx^4}}{x^9} dx &= -\frac{1}{840} \left(\frac{105c}{x^8} + \frac{120d}{x^7} + \frac{140e}{x^6} + \frac{168f}{x^5} \right) \sqrt{a + bx^4} - (2b) \int \frac{-\frac{c}{8} - \frac{dx}{7} - \frac{ex^2}{6} - \frac{fx^3}{5}}{x^5 \sqrt{a + bx^4}} dx \\
&= -\frac{1}{840} \left(\frac{105c}{x^8} + \frac{120d}{x^7} + \frac{140e}{x^6} + \frac{168f}{x^5} \right) \sqrt{a + bx^4} - (2b) \int \left(\frac{-\frac{c}{8} - \frac{ex^2}{6}}{x^5 \sqrt{a + bx^4}} + \frac{-\frac{d}{7} - \frac{fx^3}{5}}{x^4 \sqrt{a + bx^4}} \right) dx \\
&= -\frac{1}{840} \left(\frac{105c}{x^8} + \frac{120d}{x^7} + \frac{140e}{x^6} + \frac{168f}{x^5} \right) \sqrt{a + bx^4} - (2b) \int \frac{-\frac{c}{8} - \frac{ex^2}{6}}{x^5 \sqrt{a + bx^4}} dx - (2b) \int \frac{-\frac{d}{7} - \frac{fx^3}{5}}{x^4 \sqrt{a + bx^4}} dx \\
&= -\frac{1}{840} \left(\frac{105c}{x^8} + \frac{120d}{x^7} + \frac{140e}{x^6} + \frac{168f}{x^5} \right) \sqrt{a + bx^4} - \frac{2bd\sqrt{a + bx^4}}{21ax^3} - b \operatorname{Subst} \left(\int \frac{-\frac{d}{7} - \frac{fx^3}{5}}{x^3} dx, x, \sqrt{a + bx^4} \right) \\
&= -\frac{1}{840} \left(\frac{105c}{x^8} + \frac{120d}{x^7} + \frac{140e}{x^6} + \frac{168f}{x^5} \right) \sqrt{a + bx^4} - \frac{bc\sqrt{a + bx^4}}{16ax^4} - \frac{2bd\sqrt{a + bx^4}}{21ax^3} - b \operatorname{Subst} \left(\int \frac{-\frac{d}{7} - \frac{fx^3}{5}}{x^3} dx, x, \sqrt{a + bx^4} \right) \\
&= -\frac{1}{840} \left(\frac{105c}{x^8} + \frac{120d}{x^7} + \frac{140e}{x^6} + \frac{168f}{x^5} \right) \sqrt{a + bx^4} - \frac{bc\sqrt{a + bx^4}}{16ax^4} - \frac{2bd\sqrt{a + bx^4}}{21ax^3} - b \operatorname{Subst} \left(\int \frac{-\frac{d}{7} - \frac{fx^3}{5}}{x^3} dx, x, \sqrt{a + bx^4} \right) \\
&= -\frac{1}{840} \left(\frac{105c}{x^8} + \frac{120d}{x^7} + \frac{140e}{x^6} + \frac{168f}{x^5} \right) \sqrt{a + bx^4} - \frac{bc\sqrt{a + bx^4}}{16ax^4} - \frac{2bd\sqrt{a + bx^4}}{21ax^3} - b \operatorname{Subst} \left(\int \frac{-\frac{d}{7} - \frac{fx^3}{5}}{x^3} dx, x, \sqrt{a + bx^4} \right) \\
&= -\frac{1}{840} \left(\frac{105c}{x^8} + \frac{120d}{x^7} + \frac{140e}{x^6} + \frac{168f}{x^5} \right) \sqrt{a + bx^4} - \frac{bc\sqrt{a + bx^4}}{16ax^4} - \frac{2bd\sqrt{a + bx^4}}{21ax^3} - b \operatorname{Subst} \left(\int \frac{-\frac{d}{7} - \frac{fx^3}{5}}{x^3} dx, x, \sqrt{a + bx^4} \right) \\
&= -\frac{1}{840} \left(\frac{105c}{x^8} + \frac{120d}{x^7} + \frac{140e}{x^6} + \frac{168f}{x^5} \right) \sqrt{a + bx^4} - \frac{bc\sqrt{a + bx^4}}{16ax^4} - \frac{2bd\sqrt{a + bx^4}}{21ax^3} - b \operatorname{Subst} \left(\int \frac{-\frac{d}{7} - \frac{fx^3}{5}}{x^3} dx, x, \sqrt{a + bx^4} \right)
\end{aligned}$$

Mathematica [C] time = 0.165423, size = 146, normalized size = 0.36

$$\frac{\sqrt{a+bx^4} \left(7x \left(5(a+bx^4) \sqrt{\frac{bx^4}{a}+1} \left(a^2e + b^2cx^6 {}_2F_1 \left(\frac{3}{2}, 3; \frac{5}{2}; \frac{bx^4}{a} + 1 \right) \right) + 6a^3fx {}_2F_1 \left(-\frac{5}{4}, -\frac{1}{2}; -\frac{1}{4}; -\frac{bx^4}{a} \right) \right) + 30a^3d {}_2F_1 \left(\dots \right) \right)}{210a^3x^7 \sqrt{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^9,x]

[Out] -(Sqrt[a + b*x^4]*(30*a^3*d*Hypergeometric2F1[-7/4, -1/2, -3/4, -((b*x^4)/a)]) + 7*x*(6*a^3*f*x*Hypergeometric2F1[-5/4, -1/2, -1/4, -((b*x^4)/a)] + 5*(a + b*x^4)*Sqrt[1 + (b*x^4)/a]*(a^2*e + b^2*c*x^6*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*x^4)/a]))) / (210*a^3*x^7*Sqrt[1 + (b*x^4)/a])

Maple [C] time = 0.019, size = 408, normalized size = 1.

$$-\frac{f}{5x^5} \sqrt{bx^4+a} - \frac{2fb}{5ax} \sqrt{bx^4+a} + \frac{2i}{5} fb^{\frac{3}{2}} \sqrt{1-ix^2\sqrt{b}} \frac{1}{\sqrt{a}} \sqrt{1+ix^2\sqrt{b}} \frac{1}{\sqrt{a}} \text{EllipticF} \left(x \sqrt{i\sqrt{b}} \frac{1}{\sqrt{a}}, i \right) \frac{1}{\sqrt{a}} \frac{1}{\sqrt{i\sqrt{b}} \frac{1}{\sqrt{a}}} \frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^9,x)

[Out] -1/5*f/x^5*(b*x^4+a)^(1/2)-2/5*b*f*(b*x^4+a)^(1/2)/a/x+2/5*I*f/a^(1/2)*b^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-2/5*I*f/a^(1/2)*b^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/8*c/a/x^8*(b*x^4+a)^(3/2)+1/16*c*b/a^2/x^4*(b*x^4+a)^(3/2)+1/16*c*b^2/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/16*c*b^2/a^2*(b*x^4+a)^(1/2)-1/6*e*(b*x^4+a)^(3/2)/x^6/a-1/7*d/x^7*(b*x^4+a)^(1/2)-2/21*b*d*(b*x^4+a)^(1/2)/a/x^3-2/21*d/a*b^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^9,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^9,x, algorithm="fricas")

[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^9, x)

Sympy [C] time = 7.90392, size = 246, normalized size = 0.62

$$\frac{\sqrt{ad}\Gamma\left(-\frac{7}{4}\right) {}_2F_1\left(-\frac{7}{4}, -\frac{1}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{\sqrt{a}f\Gamma\left(-\frac{5}{4}\right) {}_2F_1\left(-\frac{5}{4}, -\frac{1}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} - \frac{ac}{8\sqrt{bx^{10}}\sqrt{\frac{a}{bx^4} + 1}} - \frac{3\sqrt{bc}}{16x^6\sqrt{\frac{a}{bx^4} + 1}} - \frac{\sqrt{be}\sqrt{\frac{a}{bx^4} + 1}}{6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**9,x)

[Out] sqrt(a)*d*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3/4)) + sqrt(a)*f*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) - a*c/(8*sqrt(b)*x**10*sqrt(a/(b*x**4) + 1)) - 3*sqrt(b)*c/(16*x**6*sqrt(a/(b*x**4) + 1)) - sqrt(b)*e*sqrt(a/(b*x**4) + 1)/(6*x**4) - b**(3/2)*c/(16*a*x**2*sqrt(a/(b*x**4) + 1)) - b**(3/2)*e*sqrt(a/(b*x**4) + 1)/(6*a) + b**2*c*asinh(sqrt(a)/(sqrt(b)*x**2))/(16*a**(3/2))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^9, x)
```

$$3.509 \quad \int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^{10}} dx$$

Optimal. Leaf size=425

$$\frac{b^{7/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (5\sqrt{ae} + 7\sqrt{bc}) \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{105a^{7/4}\sqrt{a+bx^4}} - \frac{2b^{5/2}cx\sqrt{a+bx^4}}{15a^2(\sqrt{a} + \sqrt{bx^2})} + \frac{2b^2c\sqrt{a+bx^4}}{15a^2x} + \dots$$

[Out] -(((56*c)/x^9 + (63*d)/x^8 + (72*e)/x^7 + (84*f)/x^6)*Sqrt[a + b*x^4])/504 - (2*b*c*Sqrt[a + b*x^4])/(45*a*x^5) - (b*d*Sqrt[a + b*x^4])/(16*a*x^4) - (2*b*e*Sqrt[a + b*x^4])/(21*a*x^3) - (b*f*Sqrt[a + b*x^4])/(6*a*x^2) + (2*b^2*c*Sqrt[a + b*x^4])/(15*a^2*x) - (2*b^(5/2)*c*x*Sqrt[a + b*x^4])/(15*a^2*(Sqrt[a] + Sqrt[b]*x^2)) + (b^2*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(16*a^(3/2)) + (2*b^(9/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(7/4)*Sqrt[a + b*x^4]) - (b^(7/4)*(7*Sqrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*a^(7/4)*Sqrt[a + b*x^4])

Rubi [A] time = 0.433128, antiderivative size = 425, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {14, 1825, 1833, 1282, 1198, 220, 1196, 1252, 835, 807, 266, 63, 208}

$$\frac{b^{7/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (5\sqrt{ae} + 7\sqrt{bc}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{105a^{7/4}\sqrt{a+bx^4}} - \frac{2b^{5/2}cx\sqrt{a+bx^4}}{15a^2(\sqrt{a} + \sqrt{bx^2})} + \frac{2b^2c\sqrt{a+bx^4}}{15a^2x} + \frac{2b^{9/4}c(\sqrt{a} + \sqrt{bx^2})}{15a^2x}$$

Antiderivative was successfully verified.

[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^10,x]

[Out] -(((56*c)/x^9 + (63*d)/x^8 + (72*e)/x^7 + (84*f)/x^6)*Sqrt[a + b*x^4])/504 - (2*b*c*Sqrt[a + b*x^4])/(45*a*x^5) - (b*d*Sqrt[a + b*x^4])/(16*a*x^4) - (2*b*e*Sqrt[a + b*x^4])/(21*a*x^3) - (b*f*Sqrt[a + b*x^4])/(6*a*x^2) + (2*b^2*c*Sqrt[a + b*x^4])/(15*a^2*x) - (2*b^(5/2)*c*x*Sqrt[a + b*x^4])/(15*a^2*(Sqrt[a] + Sqrt[b]*x^2)) + (b^2*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(16*a^(3/2)) + (2*b^(9/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(7/4)*Sqrt[a + b*x^4]) - (b^(7/4)*(7*Sqrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*a^(7/4)*Sqrt[a + b*x^4])

) * Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2] * EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2]/(105*a^(7/4)*Sqrt[a + b*x^4])

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 1825

Int[(Pq_)*(x_)^ (m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{u = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1, 0]

Rule 1833

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p)/c^j, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]

Rule 1282

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1198

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rule 220

Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2]]*EllipticF[2*ArcTan[q*x]

, 1/2]]/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rule 1252

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rule 835

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 807

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]

Rule 266

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 63

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ

[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
 \int \frac{(c + dx + ex^2 + fx^3)\sqrt{a + bx^4}}{x^{10}} dx &= -\frac{1}{504} \left(\frac{56c}{x^9} + \frac{63d}{x^8} + \frac{72e}{x^7} + \frac{84f}{x^6} \right) \sqrt{a + bx^4} - (2b) \int \frac{-\frac{c}{9} - \frac{dx}{8} - \frac{ex^2}{7} - \frac{fx^3}{6}}{x^6 \sqrt{a + bx^4}} dx \\
 &= -\frac{1}{504} \left(\frac{56c}{x^9} + \frac{63d}{x^8} + \frac{72e}{x^7} + \frac{84f}{x^6} \right) \sqrt{a + bx^4} - (2b) \int \left(\frac{-\frac{c}{9} - \frac{ex^2}{7}}{x^6 \sqrt{a + bx^4}} + \frac{-\frac{d}{8} - \frac{fx^3}{6}}{x^5 \sqrt{a + bx^4}} \right) dx \\
 &= -\frac{1}{504} \left(\frac{56c}{x^9} + \frac{63d}{x^8} + \frac{72e}{x^7} + \frac{84f}{x^6} \right) \sqrt{a + bx^4} - (2b) \int \frac{-\frac{c}{9} - \frac{ex^2}{7}}{x^6 \sqrt{a + bx^4}} dx - (2b) \int \frac{-\frac{d}{8} - \frac{fx^3}{6}}{x^5 \sqrt{a + bx^4}} dx \\
 &= -\frac{1}{504} \left(\frac{56c}{x^9} + \frac{63d}{x^8} + \frac{72e}{x^7} + \frac{84f}{x^6} \right) \sqrt{a + bx^4} - \frac{2bc\sqrt{a + bx^4}}{45ax^5} - b \operatorname{Subst} \left(\int \frac{-\frac{d}{8} - \frac{fx^3}{6}}{x^3 \sqrt{a + bx^4}} dx, x, \frac{x^2}{b} \right) \\
 &= -\frac{1}{504} \left(\frac{56c}{x^9} + \frac{63d}{x^8} + \frac{72e}{x^7} + \frac{84f}{x^6} \right) \sqrt{a + bx^4} - \frac{2bc\sqrt{a + bx^4}}{45ax^5} - \frac{bd\sqrt{a + bx^4}}{16ax^4} - \frac{2bf\sqrt{a + bx^4}}{16ax^4} \\
 &= -\frac{1}{504} \left(\frac{56c}{x^9} + \frac{63d}{x^8} + \frac{72e}{x^7} + \frac{84f}{x^6} \right) \sqrt{a + bx^4} - \frac{2bc\sqrt{a + bx^4}}{45ax^5} - \frac{bd\sqrt{a + bx^4}}{16ax^4} - \frac{2bf\sqrt{a + bx^4}}{16ax^4} \\
 &= -\frac{1}{504} \left(\frac{56c}{x^9} + \frac{63d}{x^8} + \frac{72e}{x^7} + \frac{84f}{x^6} \right) \sqrt{a + bx^4} - \frac{2bc\sqrt{a + bx^4}}{45ax^5} - \frac{bd\sqrt{a + bx^4}}{16ax^4} - \frac{2bf\sqrt{a + bx^4}}{16ax^4} \\
 &= -\frac{1}{504} \left(\frac{56c}{x^9} + \frac{63d}{x^8} + \frac{72e}{x^7} + \frac{84f}{x^6} \right) \sqrt{a + bx^4} - \frac{2bc\sqrt{a + bx^4}}{45ax^5} - \frac{bd\sqrt{a + bx^4}}{16ax^4} - \frac{2bf\sqrt{a + bx^4}}{16ax^4} \\
 &= -\frac{1}{504} \left(\frac{56c}{x^9} + \frac{63d}{x^8} + \frac{72e}{x^7} + \frac{84f}{x^6} \right) \sqrt{a + bx^4} - \frac{2bc\sqrt{a + bx^4}}{45ax^5} - \frac{bd\sqrt{a + bx^4}}{16ax^4} - \frac{2bf\sqrt{a + bx^4}}{16ax^4}
 \end{aligned}$$

Mathematica [C] time = 0.16672, size = 148, normalized size = 0.35

$$\frac{\sqrt{a+bx^4} \left(3x^2 \left(7x(a+bx^4) \sqrt{\frac{bx^4}{a}+1} \left(a^2f + b^2dx^6 {}_2F_1 \left(\frac{3}{2}, 3; \frac{5}{2}; \frac{bx^4}{a} + 1 \right) \right) + 6a^3e {}_2F_1 \left(-\frac{7}{4}, -\frac{1}{2}; -\frac{3}{4}; -\frac{bx^4}{a} \right) \right) + 14a^3c {}_2F_1 \left(-\frac{7}{4}, -\frac{1}{2}; -\frac{3}{4}; -\frac{bx^4}{a} \right) \right)}{126a^3x^9 \sqrt{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^10,x]

[Out] -(Sqrt[a + b*x^4]*(14*a^3*c*Hypergeometric2F1[-9/4, -1/2, -5/4, -(b*x^4)/a]) + 3*x^2*(6*a^3*e*Hypergeometric2F1[-7/4, -1/2, -3/4, -(b*x^4)/a] + 7*x*(a + b*x^4)*Sqrt[1 + (b*x^4)/a]*(a^2*f + b^2*d*x^6*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*x^4)/a]))) / (126*a^3*x^9*Sqrt[1 + (b*x^4)/a])

Maple [C] time = 0.018, size = 429, normalized size = 1.

$$-\frac{d}{8ax^8} (bx^4 + a)^{\frac{3}{2}} + \frac{bd}{16a^2x^4} (bx^4 + a)^{\frac{3}{2}} + \frac{b^2d}{16} \ln \left(\frac{1}{x^2} \left(2a + 2\sqrt{a}\sqrt{bx^4 + a} \right) \right) a^{-\frac{3}{2}} - \frac{b^2d}{16a^2} \sqrt{bx^4 + a} - \frac{f}{6x^6a} (bx^4 + a)^{\frac{3}{2}} -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^10,x)

[Out] -1/8*d/a/x^8*(b*x^4+a)^(3/2)+1/16*d*b/a^2/x^4*(b*x^4+a)^(3/2)+1/16*d*b^2/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/16*d*b^2/a^2*(b*x^4+a)^(1/2)-1/6*f*(b*x^4+a)^(3/2)/x^6/a-1/7*e/x^7*(b*x^4+a)^(1/2)-2/21*b*e*(b*x^4+a)^(1/2)/a/x^3-2/21*e/a*b^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/9*c/x^9*(b*x^4+a)^(1/2)-2/45*b*c*(b*x^4+a)^(1/2)/a/x^5+2/15*b^2*c*(b*x^4+a)^(1/2)/a^2/x-2/15*I*c/a^(3/2)*b^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+2/15*I*c/a^(3/2)*b^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^10,x, algorithm="maxima")

[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^10, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^10,x, algorithm="fricas")

[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^10, x)

Sympy [C] time = 8.46443, size = 246, normalized size = 0.58

$$\frac{\sqrt{ac}\Gamma\left(-\frac{9}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{9}{4}, -\frac{1}{2} \\ -\frac{5}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^9\Gamma\left(-\frac{5}{4}\right)} + \frac{\sqrt{ae}\Gamma\left(-\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{7}{4}, -\frac{1}{2} \\ -\frac{3}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} - \frac{ad}{8\sqrt{b}x^{10}\sqrt{\frac{a}{bx^4} + 1}} - \frac{3\sqrt{bd}}{16x^6\sqrt{\frac{a}{bx^4} + 1}} - \frac{\sqrt{bf}\sqrt{\frac{a}{bx^4} + 1}}{6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**10,x)

[Out] sqrt(a)*c*gamma(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**9*gamma(-5/4)) + sqrt(a)*e*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3/4)) - a*d/(8*sqrt(b)*x**10*sqrt(a/(b*x**4) + 1)) - 3*sqrt(b)*d/(16*x**6*sqrt(a/(b*x**4) + 1)) - sqrt(b)*f*sqrt(a/(b*x**4) + 1)/(6*x**4) - b**(3/2)*d/(16*a*x**2*sqrt(a/(b*x**4) + 1))

- $b^{3/2} f \sqrt{a/(b x^4) + 1} / (6 a) + b^2 d \operatorname{asinh}(\sqrt{a} / (\sqrt{b} x^2)) / (16 a^{3/2})$

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^10,x, algorithm="giac")

[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^10, x)

$$3.510 \quad \int x^4 (c + dx + ex^2 + fx^3) (a + bx^4)^{3/2} dx$$

Optimal. Leaf size=476

$$\frac{2a^{11/4} (\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (77\sqrt{ae} + 65\sqrt{bc}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right) + a^3 d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right) - \frac{4a^3 ex \sqrt{a}}{65b^{3/2} (\sqrt{a} + \sqrt{bx^2})}}{5005b^{7/4} \sqrt{a + bx^4} - 32b^{3/2}}$$

[Out] $(4a^2cx\sqrt{a+bx^4})/(77b) - (a^2dx^2\sqrt{a+bx^4})/(32b) + (4a^2ex^3\sqrt{a+bx^4})/(195b) - (4a^3ex\sqrt{a+bx^4})/(65b^{3/2}(\sqrt{a} + \sqrt{bx^2})) + (2ax^5(117c + 77ex^2)\sqrt{a+bx^4})/3003 - (adx^2(a+bx^4)^{3/2})/(48b) + (x^5(13c + 11ex^2)(a+bx^4)^{3/2})/143 + (fx^4(a+bx^4)^{5/2})/(14b) - ((12af - 35bdx^2)(a+bx^4)^{5/2})/(420b^2) - (a^3d \operatorname{ArcTanh}[(\sqrt{b}x^2)/\sqrt{a+bx^4}])/(32b^{3/2}) + (4a^{13/4}e(\sqrt{a} + \sqrt{bx^2})\sqrt{(a+bx^4)/(\sqrt{a} + \sqrt{bx^2})^2}) \operatorname{EllipticE}[2 \operatorname{ArcTan}[(b^{1/4}x)/a^{1/4}], 1/2]/(65b^{7/4}\sqrt{a+bx^4}) - (2a^{11/4}(65\sqrt{b}c + 77\sqrt{a}e)(\sqrt{a} + \sqrt{bx^2})\sqrt{(a+bx^4)/(\sqrt{a} + \sqrt{bx^2})^2}) \operatorname{EllipticF}[2 \operatorname{ArcTan}[(b^{1/4}x)/a^{1/4}], 1/2]/(5005b^{7/4}\sqrt{a+bx^4})$

Rubi [A] time = 0.439691, antiderivative size = 476, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {1833, 1274, 1280, 1198, 220, 1196, 1252, 833, 780, 195, 217, 206}

$$\frac{2a^{11/4} (\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (77\sqrt{ae} + 65\sqrt{bc}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right) + a^3 d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right) - \frac{4a^3 ex \sqrt{a + bx^4}}{65b^{3/2} (\sqrt{a} + \sqrt{bx^2})}}{5005b^{7/4} \sqrt{a + bx^4} - 32b^{3/2}}$$

Antiderivative was successfully verified.

[In] $\int x^4(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}, x$

[Out] $(4a^2cx\sqrt{a+bx^4})/(77b) - (a^2dx^2\sqrt{a+bx^4})/(32b) + (4a^2ex^3\sqrt{a+bx^4})/(195b) - (4a^3ex\sqrt{a+bx^4})/(65b^{3/2}(\sqrt{a} + \sqrt{bx^2})) + (2ax^5(117c + 77ex^2)\sqrt{a+bx^4})/3003 - (adx^2(a+bx^4)^{3/2})/(48b) + (x^5(13c + 11ex^2)(a+bx^4)^{3/2})/143 + (fx^4(a+bx^4)^{5/2})/(14b) - ((12af - 35bdx^2)(a+bx^4)^{5/2})/(420b^2) - (a^3d \operatorname{ArcTanh}[(\sqrt{b}x^2)/\sqrt{a+bx^4}])/(32b^{3/2}) + (4a^{13/4}e(\sqrt{a} + \sqrt{bx^2})\sqrt{(a+bx^4)/(\sqrt{a} + \sqrt{bx^2})^2}) \operatorname{EllipticE}[2 \operatorname{ArcTan}[(b^{1/4}x)/a^{1/4}], 1/2]/(65b^{7/4}\sqrt{a+bx^4}) - (2a^{11/4}(65\sqrt{b}c + 77\sqrt{a}e)(\sqrt{a} + \sqrt{bx^2})\sqrt{(a+bx^4)/(\sqrt{a} + \sqrt{bx^2})^2}) \operatorname{EllipticF}[2 \operatorname{ArcTan}[(b^{1/4}x)/a^{1/4}], 1/2]/(5005b^{7/4}\sqrt{a+bx^4})$

$5b^{7/4}\sqrt{a + bx^4} - (2a^{11/4}(65\sqrt{b}c + 77\sqrt{a}e)(\sqrt{a + b}x^2)\sqrt{(a + bx^4)/(\sqrt{a} + \sqrt{b}x^2)^2})\text{EllipticF}[2 \text{ArcTan}[b^{1/4}x/a^{1/4}], 1/2]/(5005b^{7/4}\sqrt{a + bx^4})$

Rule 1833

`Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p]/c^j, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]`

Rule 1274

`Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*p + 3)), x] + Dist[(4*a*p)/(4*p + m + 1)*(m + 4*p + 3), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p + 3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])`

Rule 1280

`Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m - 1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])`

Rule 1198

`Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]`

Rule 220

`Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x, 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]`

Rule 1196

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2])*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rule 1252

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rule 833

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])

Rule 780

Int[((d_) + (e_)*(x_))*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x*(a + c*x^2)^(p + 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p + 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !LeQ[p, -1]

Rule 195

Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int x^4 (c + dx + ex^2 + fx^3) (a + bx^4)^{3/2} dx &= \int \left(x^4 (c + ex^2) (a + bx^4)^{3/2} + x^5 (d + fx^2) (a + bx^4)^{3/2} \right) dx \\
 &= \int x^4 (c + ex^2) (a + bx^4)^{3/2} dx + \int x^5 (d + fx^2) (a + bx^4)^{3/2} dx \\
 &= \frac{1}{143} x^5 (13c + 11ex^2) (a + bx^4)^{3/2} + \frac{1}{2} \text{Subst} \left(\int x^2 (d + fx) (a + bx^2)^{3/2} dx, \right. \\
 &= \frac{2ax^5 (117c + 77ex^2) \sqrt{a + bx^4}}{3003} + \frac{1}{143} x^5 (13c + 11ex^2) (a + bx^4)^{3/2} + \frac{fx^4 (a + bx^4)^{3/2}}{143} \\
 &= \frac{4a^2 ex^3 \sqrt{a + bx^4}}{195b} + \frac{2ax^5 (117c + 77ex^2) \sqrt{a + bx^4}}{3003} + \frac{1}{143} x^5 (13c + 11ex^2) (a + bx^4)^{3/2} \\
 &= \frac{4a^2 cx \sqrt{a + bx^4}}{77b} + \frac{4a^2 ex^3 \sqrt{a + bx^4}}{195b} + \frac{2ax^5 (117c + 77ex^2) \sqrt{a + bx^4}}{3003} - \frac{adx^2 \sqrt{a + bx^4}}{32b} \\
 &= \frac{4a^2 cx \sqrt{a + bx^4}}{77b} - \frac{a^2 dx^2 \sqrt{a + bx^4}}{32b} + \frac{4a^2 ex^3 \sqrt{a + bx^4}}{195b} + \frac{2ax^5 (117c + 77ex^2) \sqrt{a + bx^4}}{3003} \\
 &= \frac{4a^2 cx \sqrt{a + bx^4}}{77b} - \frac{a^2 dx^2 \sqrt{a + bx^4}}{32b} + \frac{4a^2 ex^3 \sqrt{a + bx^4}}{195b} - \frac{4a^3 ex \sqrt{a + bx^4}}{65b^{3/2} (\sqrt{a} + \sqrt{bx^2})} \\
 &= \frac{4a^2 cx \sqrt{a + bx^4}}{77b} - \frac{a^2 dx^2 \sqrt{a + bx^4}}{32b} + \frac{4a^2 ex^3 \sqrt{a + bx^4}}{195b} - \frac{4a^3 ex \sqrt{a + bx^4}}{65b^{3/2} (\sqrt{a} + \sqrt{bx^2})}
 \end{aligned}$$

Mathematica [C] time = 0.864564, size = 225, normalized size = 0.47

$$\frac{\sqrt{a + bx^4} \left(5005bdx^2 (3a^2 + 14abx^4 + 8b^2x^8) - \frac{43680a^2bcx^2 {}_2F_1\left(-\frac{3}{2}, \frac{5}{4}; -\frac{bx^4}{a}\right)}{\sqrt{\frac{bx^4}{a} + 1}} - \frac{15015a^{5/2}\sqrt{bd} \sinh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{\sqrt{\frac{bx^4}{a} + 1}} - \frac{36960a^2bcx^3 {}_2F_1\left(-\frac{3}{2}, \frac{7}{4}; -\frac{bx^4}{a}\right)}{\sqrt{\frac{bx^4}{a} + 1}} \right)}{480480b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^4*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2), x]

[Out] (Sqrt[a + b*x^4]*(43680*b*c*x*(a + b*x^4)^2 + 36960*b*e*x^3*(a + b*x^4)^2 + 6864*f*(a + b*x^4)^2*(-2*a + 5*b*x^4) + 5005*b*d*x^2*(3*a^2 + 14*a*b*x^4 + 8*b^2*x^8) - (15015*a^(5/2)*Sqrt[b]*d*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[1 + (b*x^4)/a] - (43680*a^2*b*c*x*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^4)/a)]/Sqrt[1 + (b*x^4)/a] - (36960*a^2*b*e*x^3*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^4)/a)]/Sqrt[1 + (b*x^4)/a]))/(480480*b^2)

Maple [C] time = 0.039, size = 462, normalized size = 1.

$$-\frac{f(-5bx^4 + 2a)(b^2x^8 + 2abx^4 + a^2)}{70b^2}\sqrt{bx^4 + a} + \frac{bex^{11}}{13}\sqrt{bx^4 + a} + \frac{5aex^7}{39}\sqrt{bx^4 + a} + \frac{4a^2ex^3}{195b}\sqrt{bx^4 + a} - \frac{4i}{65}ea^{\frac{7}{2}}\sqrt{1 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2), x)

[Out] -1/70*f*(b*x^4+a)^(1/2)*(-5*b*x^4+2*a)*(b^2*x^8+2*a*b*x^4+a^2)/b^2+1/13*e*b*x^11*(b*x^4+a)^(1/2)+5/39*e*a*x^7*(b*x^4+a)^(1/2)+4/195*a^2*e*x^3*(b*x^4+a)^(1/2)/b-4/65*I*e/b^(3/2)*a^(7/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2), I)+4/65*I*e/b^(3/2)*a^(7/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2), I)+1/12*d*b*x^10*(b*x^4+a)^(1/2)+7/48*d*a*x^6*(b*x^4+a)^(1/2)+1/32*a^2*d*x^2*(b*x^4+a)^(1/2)/b-1/32*d/b^(3/2)*a^3*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))+1/11*c*b*x^9*(b*x^4+a)^(1/2)+13/77*c*a*x^5*(b*x^4+a)^(1/2)+4/77*a^2*c*x*(b*x^4+a)^(1/2)/b-4/77*c/b*a^3/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2), I)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="maxima")

[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bfx^{11} + bex^{10} + bdx^9 + bcx^8 + afx^7 + aex^6 + adx^5 + acx^4\right)\sqrt{bx^4 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="fricas")

[Out] integral((b*f*x^11 + b*e*x^10 + b*d*x^9 + b*c*x^8 + a*f*x^7 + a*e*x^6 + a*d*x^5 + a*c*x^4)*sqrt(b*x^4 + a), x)

Sympy [A] time = 18.0418, size = 462, normalized size = 0.97

$$\frac{a^{\frac{5}{2}}dx^2}{32b\sqrt{1 + \frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}cx^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{5}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{17a^{\frac{3}{2}}dx^6}{96\sqrt{1 + \frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}ex^7\Gamma\left(\frac{7}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{7}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)} + \frac{\sqrt{abc}x^9\Gamma\left(\frac{9}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{9}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)

[Out] a**(5/2)*d*x**2/(32*b*sqrt(1 + b*x**4/a)) + a**(3/2)*c*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 17*a**(3/2)*d*x**6/(96*sqrt(1 + b*x**4/a)) + a**(3/2)*e*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) + sqrt(a)*b*c*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(13/4)) + 11*sqrt(a)*b*d*x**10/(48*sqrt(1 + b*x**4/a)) + sqrt(a)*b*e*x**11*gamma(11/4)*hyper((-1/2, 11/4), (15/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(15/4)) - a**3*d*asinh(sqrt(b)*x**2/sqrt(a))/(32*b**(3/2)) + a*f*Piecewise((-a**2*sqrt(a + b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x**4)/(30*b) + x**8*sqrt(a + b*x**4)/10, Ne(b, 0)), (sqrt(a)*x**8/8, True)) + b*f*Piecewise((4*a**3*sqrt(a + b*x**4)/(105*b**3) - 2*a**2*x**4*sqrt(a + b*x**4)/(105*b**2) + a*x**8*sqrt(a + b*x**4)/(70*b) + x**12*sqrt(a + b*x**4)/14, Ne(b, 0)), (sq

```
rt(a)*x**12/12, True)) + b**2*d*x**14/(12*sqrt(a)*sqrt(1 + b*x**4/a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^4, x)
```

$$3.511 \quad \int x^3 (c + dx + ex^2 + fx^3) (a + bx^4)^{3/2} dx$$

Optimal. Leaf size=452

$$\frac{2a^{11/4} (\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (77\sqrt{a}f + 65\sqrt{bd}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt{a}}\right), \frac{1}{2}\right) - a^3 e \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right) - \frac{4a^3 fx \sqrt{a}}{65b^{3/2} (\sqrt{a} + \sqrt{bx^2})}}{5005b^{7/4} \sqrt{a + bx^4} - \frac{a^3 e \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right)}{32b^{3/2}} - \frac{4a^3 fx \sqrt{a}}{65b^{3/2} (\sqrt{a} + \sqrt{bx^2})}}$$

[Out] $(4a^2 d x \sqrt{a + bx^4}) / (77b) - (a^2 e x^2 \sqrt{a + bx^4}) / (32b) + (4a^2 f x^3 \sqrt{a + bx^4}) / (195b) - (4a^3 f x \sqrt{a + bx^4}) / (65b^{3/2} (\sqrt{a} + \sqrt{bx^2})) + (2a^5 x (117d + 77f x^2) \sqrt{a + bx^4}) / 3003 - (a e x^2 (a + bx^4)^{3/2}) / (48b) + (x^5 (13d + 11f x^2) (a + bx^4)^{3/2}) / 143 + ((6c + 5e x^2) (a + bx^4)^{5/2}) / (60b) - (a^3 e \operatorname{ArcTanh}[(\sqrt{b} x^2) / \sqrt{a + bx^4}]) / (32b^{3/2}) + (4a^{13/4} f (\sqrt{a} + \sqrt{b} x^2) \sqrt{(a + bx^4) / (\sqrt{a} + \sqrt{b} x^2)^2}) \operatorname{EllipticE}[2 \operatorname{ArcTan}[(b^{1/4} x) / a^{1/4}], 1/2] / (65b^{7/4} \sqrt{a + bx^4}) - (2a^{11/4} (65 \sqrt{b} d + 77 \sqrt{a} f) (\sqrt{a} + \sqrt{b} x^2) \sqrt{(a + bx^4) / (\sqrt{a} + \sqrt{b} x^2)^2}) \operatorname{EllipticF}[2 \operatorname{ArcTan}[(b^{1/4} x) / a^{1/4}], 1/2] / (5005b^{7/4} \sqrt{a + bx^4})$

Rubi [A] time = 0.40782, antiderivative size = 452, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {1833, 1252, 780, 195, 217, 206, 1274, 1280, 1198, 220, 1196}

$$\frac{2a^{11/4} (\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (77\sqrt{a}f + 65\sqrt{bd}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt{a}}\right), \frac{1}{2}\right) - a^3 e \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right) - \frac{4a^3 fx \sqrt{a + bx^4}}{65b^{3/2} (\sqrt{a} + \sqrt{bx^2})}}{5005b^{7/4} \sqrt{a + bx^4} - \frac{a^3 e \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right)}{32b^{3/2}} - \frac{4a^3 fx \sqrt{a + bx^4}}{65b^{3/2} (\sqrt{a} + \sqrt{bx^2})}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^3 (c + dx + ex^2 + fx^3) (a + bx^4)^{3/2}, x]$

[Out] $(4a^2 d x \sqrt{a + bx^4}) / (77b) - (a^2 e x^2 \sqrt{a + bx^4}) / (32b) + (4a^2 f x^3 \sqrt{a + bx^4}) / (195b) - (4a^3 f x \sqrt{a + bx^4}) / (65b^{3/2} (\sqrt{a} + \sqrt{bx^2})) + (2a^5 x (117d + 77f x^2) \sqrt{a + bx^4}) / 3003 - (a e x^2 (a + bx^4)^{3/2}) / (48b) + (x^5 (13d + 11f x^2) (a + bx^4)^{3/2}) / 143 + ((6c + 5e x^2) (a + bx^4)^{5/2}) / (60b) - (a^3 e \operatorname{ArcTanh}[(\sqrt{b} x^2) / \sqrt{a + bx^4}]) / (32b^{3/2}) + (4a^{13/4} f (\sqrt{a} + \sqrt{b} x^2) \sqrt{(a + bx^4) / (\sqrt{a} + \sqrt{b} x^2)^2}) \operatorname{EllipticE}[2 \operatorname{ArcTan}[(b^{1/4} x) / a^{1/4}], 1/2] / (65b^{7/4} \sqrt{a + bx^4}) - (2a^{11/4} (65 \sqrt{b} d + 77 \sqrt{a} f) (\sqrt{a} + \sqrt{b} x^2) \sqrt{(a + bx^4) / (\sqrt{a} + \sqrt{b} x^2)^2}) \operatorname{EllipticF}[2 \operatorname{ArcTan}[(b^{1/4} x) / a^{1/4}], 1/2] / (5005b^{7/4} \sqrt{a + bx^4})$

*Sqrt[b]*d + 77*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2]/(5005*b^(7/4)*Sqrt[a + b*x^4])

Rule 1833

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p]/c^j, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]

Rule 1252

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rule 780

Int[((d_) + (e_)*(x_))*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p + 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p + 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !LeQ[p, -1]

Rule 195

Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1274

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x_
_Symbol] := Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(c*d*(m + 4*p + 3) + c*e*(4*p
+ m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*p + 3)), x] + Dist[(4*a*p)/((4*p
+ m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
_Symbol] := Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2]]*EllipticF[2*ArcTan[q*x],
1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2]]*EllipticE[2*ArcTan[q*x],
1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int x^3 (c + dx + ex^2 + fx^3) (a + bx^4)^{3/2} dx &= \int \left(x^3 (c + ex^2) (a + bx^4)^{3/2} + x^4 (d + fx^2) (a + bx^4)^{3/2} \right) dx \\
&= \int x^3 (c + ex^2) (a + bx^4)^{3/2} dx + \int x^4 (d + fx^2) (a + bx^4)^{3/2} dx \\
&= \frac{1}{143} x^5 (13d + 11fx^2) (a + bx^4)^{3/2} + \frac{1}{2} \text{Subst} \left(\int x(c + ex) (a + bx^2)^{3/2} dx, \right. \\
&= \frac{2ax^5 (117d + 77fx^2) \sqrt{a + bx^4}}{3003} + \frac{1}{143} x^5 (13d + 11fx^2) (a + bx^4)^{3/2} + \frac{(6c}{ \\
&= \frac{4a^2 fx^3 \sqrt{a + bx^4}}{195b} + \frac{2ax^5 (117d + 77fx^2) \sqrt{a + bx^4}}{3003} - \frac{aex^2 (a + bx^4)^{3/2}}{48b} + \frac{1}{1} \\
&= \frac{4a^2 dx \sqrt{a + bx^4}}{77b} - \frac{a^2 ex^2 \sqrt{a + bx^4}}{32b} + \frac{4a^2 fx^3 \sqrt{a + bx^4}}{195b} + \frac{2ax^5 (117d + 77fx^2) \sqrt{a + bx^4}}{3003} \\
&= \frac{4a^2 dx \sqrt{a + bx^4}}{77b} - \frac{a^2 ex^2 \sqrt{a + bx^4}}{32b} + \frac{4a^2 fx^3 \sqrt{a + bx^4}}{195b} + \frac{2ax^5 (117d + 77fx^2) \sqrt{a + bx^4}}{3003} \\
&= \frac{4a^2 dx \sqrt{a + bx^4}}{77b} - \frac{a^2 ex^2 \sqrt{a + bx^4}}{32b} + \frac{4a^2 fx^3 \sqrt{a + bx^4}}{195b} - \frac{4a^3 fx \sqrt{a + bx^4}}{65b^{3/2} (\sqrt{a} + \sqrt{bx^4})}
\end{aligned}$$

Mathematica [C] time = 0.746668, size = 238, normalized size = 0.53

$$\frac{\sqrt{a + bx^4} \left(715e \left(\sqrt{bx^2} (3a^2 + 14abx^4 + 8b^2x^8) - \frac{3a^{5/2} \sinh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{\sqrt{\frac{bx^4}{a} + 1}} \right) - \frac{6240a^2 \sqrt{b} dx {}_2F_1 \left(-\frac{3}{2}, \frac{5}{4}; \frac{5}{4}; -\frac{bx^4}{a} \right)}{\sqrt{\frac{bx^4}{a} + 1}} - \frac{5280a^2 \sqrt{b} fx^3 {}_2F_1 \left(-\frac{3}{2}, \frac{7}{4}; \frac{7}{4}; -\frac{bx^4}{a} \right)}{\sqrt{\frac{bx^4}{a} + 1}} \right)}{68640b^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2), x]

[Out] (Sqrt[a + b*x^4]*(6864*Sqrt[b]*c*(a + b*x^4)^2 + 6240*Sqrt[b]*d*x*(a + b*x^4)^2 + 5280*Sqrt[b]*f*x^3*(a + b*x^4)^2 + 715*e*(Sqrt[b]*x^2*(3*a^2 + 14*a*b*x^4 + 8*b^2*x^8) - (3*a^(5/2)*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[1 + (b*x^4)/a]) - (6240*a^2*Sqrt[b]*d*x*Hypergeometric2F1[-3/2, 1/4, 5/4, -(b*x^4)/a])/Sqrt[1 + (b*x^4)/a] - (5280*a^2*Sqrt[b]*f*x^3*Hypergeometric2F1[-3/2, 3/4, 7/4, -(b*x^4)/a])/Sqrt[1 + (b*x^4)/a))/(68640*b^(3/2))

Maple [C] time = 0.007, size = 434, normalized size = 1.

$$\frac{bfx^{11}}{13}\sqrt{bx^4+a} + \frac{5afx^7}{39}\sqrt{bx^4+a} + \frac{4a^2fx^3}{195b}\sqrt{bx^4+a} - \frac{4i}{65}fa^{\frac{7}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}, i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x)

[Out] 1/13*f*b*x^11*(b*x^4+a)^(1/2)+5/39*f*a*x^7*(b*x^4+a)^(1/2)+4/195*a^2*f*x^3*(b*x^4+a)^(1/2)/b-4/65*I*f/b^(3/2)*a^(7/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+4/65*I*f/b^(3/2)*a^(7/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/12*e*b*x^10*(b*x^4+a)^(1/2)+7/48*e*a*x^6*(b*x^4+a)^(1/2)+1/32*a^2*e*x^2*(b*x^4+a)^(1/2)/b-1/32*e/b^(3/2)*a^3*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))+1/11*d*b*x^9*(b*x^4+a)^(1/2)+13/77*d*a*x^5*(b*x^4+a)^(1/2)+4/77*a^2*d*x*(b*x^4+a)^(1/2)/b-4/77*d/b*a^3/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/10*c/b*(b*x^4+a)^(5/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(bx^4+a)^{\frac{5}{2}}c}{10b} + \int (bfx^{10} + bex^9 + bdx^8 + afx^6 + aex^5 + adx^4)\sqrt{bx^4+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="maxima")

[Out] 1/10*(b*x^4+a)^(5/2)*c/b + integrate((b*f*x^10 + b*e*x^9 + b*d*x^8 + a*f*x^6 + a*e*x^5 + a*d*x^4)*sqrt(b*x^4+a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bfx^{10} + bex^9 + bdx^8 + bcx^7 + afx^6 + aex^5 + adx^4 + acx^3\right)\sqrt{bx^4+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="fricas")

[Out] integral((b*f*x^10 + b*e*x^9 + b*d*x^8 + b*c*x^7 + a*f*x^6 + a*e*x^5 + a*d*x^4 + a*c*x^3)*sqrt(b*x^4 + a), x)

Sympy [A] time = 15.0307, size = 398, normalized size = 0.88

$$\frac{a^{\frac{5}{2}}ex^2}{32b\sqrt{1+\frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}dx^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{5}{4} \middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{17a^{\frac{3}{2}}ex^6}{96\sqrt{1+\frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}fx^7\Gamma\left(\frac{7}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{7}{4} \middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)} + \frac{\sqrt{abd}x^9\Gamma\left(\frac{9}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{9}{4} \middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)

[Out] a**(5/2)*e*x**2/(32*b*sqrt(1 + b*x**4/a)) + a**(3/2)*d*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 17*a**(3/2)*e*x**6/(96*sqrt(1 + b*x**4/a)) + a**(3/2)*f*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) + sqrt(a)*b*d*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(13/4)) + 11*sqrt(a)*b*e*x**10/(48*sqrt(1 + b*x**4/a)) + sqrt(a)*b*f*x**11*gamma(11/4)*hyper((-1/2, 11/4), (15/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(15/4)) - a**3*e*asinh(sqrt(b)*x**2/sqrt(a))/(32*b**(3/2)) + a*c*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b*c*Piecewise((-a**2*sqrt(a + b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x**4)/(30*b) + x**8*sqrt(a + b*x**4)/10, Ne(b, 0)), (sqrt(a)*x**8/8, True)) + b**2*e*x**14/(12*sqrt(a)*sqrt(1 + b*x**4/a))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="giac")

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^3, x)
```

$$3.512 \quad \int x^2 (c + dx + ex^2 + fx^3) (a + bx^4)^{3/2} dx$$

Optimal. Leaf size=427

$$\frac{2a^{9/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (77\sqrt{bc} - 15\sqrt{ae}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right) - 4a^{9/4}c(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{1155b^{5/4}\sqrt{a+bx^4} - 15b^{3/4}\sqrt{a+bx^4}}$$

[Out] $(4a^2ex\sqrt{a+bx^4})/(77b) - (a^2fx^2\sqrt{a+bx^4})/(32b) + (4a^2cx\sqrt{a+bx^4})/(15\sqrt{b}(\sqrt{a}+\sqrt{b}x^2)) + (2a^3x^3(77c+45ex^2)\sqrt{a+bx^4})/1155 - (afx^2(a+bx^4)^{3/2})/(48b) + (x^3(11c+9ex^2)(a+bx^4)^{3/2})/99 + ((6d+5fx^2)(a+bx^4)^{5/2})/(60b) - (a^3f\operatorname{ArcTan}[(\sqrt{b}x^2)/\sqrt{a+bx^4}])/(32b^{3/2}) - (4a^{9/4}c(\sqrt{a}+\sqrt{bx^2})\sqrt{(a+bx^4)/(\sqrt{a}+\sqrt{bx^2})})/(\sqrt{a}+\sqrt{bx^2})^2 * \operatorname{EllipticE}[2\operatorname{ArcTan}[(b^{1/4}x)/a^{1/4}], 1/2])/(15b^{3/4}\sqrt{a+bx^4}) + (2a^{9/4}(77\sqrt{b}c - 15\sqrt{a}e)(\sqrt{a}+\sqrt{bx^2})\sqrt{(a+bx^4)/(\sqrt{a}+\sqrt{bx^2})})/(\sqrt{a}+\sqrt{bx^2})^2 * \operatorname{EllipticF}[2\operatorname{ArcTan}[(b^{1/4}x)/a^{1/4}], 1/2])/(1155b^{5/4}\sqrt{a+bx^4})$

Rubi [A] time = 0.352719, antiderivative size = 427, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {1833, 1274, 1280, 1198, 220, 1196, 1252, 780, 195, 217, 206}

$$\frac{2a^{9/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (77\sqrt{bc} - 15\sqrt{ae}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right) - 4a^{9/4}c(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{1155b^{5/4}\sqrt{a+bx^4} - 15b^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}, x]$

[Out] $(4a^2ex\sqrt{a+bx^4})/(77b) - (a^2fx^2\sqrt{a+bx^4})/(32b) + (4a^2cx\sqrt{a+bx^4})/(15\sqrt{b}(\sqrt{a}+\sqrt{b}x^2)) + (2a^3x^3(77c+45ex^2)\sqrt{a+bx^4})/1155 - (afx^2(a+bx^4)^{3/2})/(48b) + (x^3(11c+9ex^2)(a+bx^4)^{3/2})/99 + ((6d+5fx^2)(a+bx^4)^{5/2})/(60b) - (a^3f\operatorname{ArcTan}[(\sqrt{b}x^2)/\sqrt{a+bx^4}])/(32b^{3/2}) - (4a^{9/4}c(\sqrt{a}+\sqrt{bx^2})\sqrt{(a+bx^4)/(\sqrt{a}+\sqrt{bx^2})})/(\sqrt{a}+\sqrt{bx^2})^2 * \operatorname{EllipticE}[2\operatorname{ArcTan}[(b^{1/4}x)/a^{1/4}], 1/2])/(15b^{3/4}\sqrt{a+bx^4}) + (2a^{9/4}(77\sqrt{b}c - 15\sqrt{a}e)(\sqrt{a}+\sqrt{bx^2})\sqrt{(a+bx^4)/(\sqrt{a}+\sqrt{bx^2})})/(\sqrt{a}+\sqrt{bx^2})^2 * \operatorname{EllipticF}[2\operatorname{ArcTan}[(b^{1/4}x)/a^{1/4}], 1/2])/(1155b^{5/4}\sqrt{a+bx^4})$

$4)*x)/a^{(1/4)}], 1/2)]/(1155*b^{(5/4)*Sqrt[a + b*x^4])}$

Rule 1833

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p]/c^j, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]

Rule 1274

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*p + 3)), x] + Dist[(4*a*p)/((4*p + m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p + 3) + e*(4*p + m + 1)*x^2, x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1280

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m - 1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1198

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rule 220

Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
  Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
  1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
  1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
  x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
  x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] := Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p
+ 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
  Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
  x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
  Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2} dx &= \int \left(x^2(c + ex^2)(a + bx^4)^{3/2} + x^3(d + fx^2)(a + bx^4)^{3/2} \right) dx \\
&= \int x^2(c + ex^2)(a + bx^4)^{3/2} dx + \int x^3(d + fx^2)(a + bx^4)^{3/2} dx \\
&= \frac{1}{99}x^3(11c + 9ex^2)(a + bx^4)^{3/2} + \frac{1}{2} \text{Subst} \left(\int x(d + fx)(a + bx^2)^{3/2} dx, x, x^4 \right) \\
&= \frac{2ax^3(77c + 45ex^2)\sqrt{a + bx^4}}{1155} + \frac{1}{99}x^3(11c + 9ex^2)(a + bx^4)^{3/2} + \frac{(6d + 5fx^2)\sqrt{a + bx^4}}{15\sqrt{b}} \\
&= \frac{4a^2ex\sqrt{a + bx^4}}{77b} + \frac{2ax^3(77c + 45ex^2)\sqrt{a + bx^4}}{1155} - \frac{afx^2(a + bx^4)^{3/2}}{48b} + \frac{1}{99}x^3 \\
&= \frac{4a^2ex\sqrt{a + bx^4}}{77b} - \frac{a^2fx^2\sqrt{a + bx^4}}{32b} + \frac{2ax^3(77c + 45ex^2)\sqrt{a + bx^4}}{1155} - \frac{afx^2(a + bx^4)^{3/2}}{48b} \\
&= \frac{4a^2ex\sqrt{a + bx^4}}{77b} - \frac{a^2fx^2\sqrt{a + bx^4}}{32b} + \frac{4a^2cx\sqrt{a + bx^4}}{15\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{2ax^3(77c + 45ex^2)\sqrt{a + bx^4}}{1155} \\
&= \frac{4a^2ex\sqrt{a + bx^4}}{77b} - \frac{a^2fx^2\sqrt{a + bx^4}}{32b} + \frac{4a^2cx\sqrt{a + bx^4}}{15\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{2ax^3(77c + 45ex^2)\sqrt{a + bx^4}}{1155}
\end{aligned}$$

Mathematica [C] time = 0.863065, size = 205, normalized size = 0.48

$$\sqrt{a + bx^4} \left(\frac{55f \left(\sqrt{bx^2(3a^2 + 14abx^4 + 8b^2x^8)} - \frac{3a^{5/2} \sinh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{\sqrt{\frac{bx^4}{a} + 1}} \right)}{b^{3/2}} - \frac{480a^2ex {}_2F_1 \left(-\frac{3}{2}, \frac{1}{4}, \frac{5}{4}, -\frac{bx^4}{a} \right)}{b\sqrt{\frac{bx^4}{a} + 1}} + \frac{1760acx^3 {}_2F_1 \left(-\frac{3}{2}, \frac{3}{4}, \frac{7}{4}, -\frac{bx^4}{a} \right)}{\sqrt{\frac{bx^4}{a} + 1}} + \frac{528d(a + bx^4)^2}{b} + \frac{480e(a + bx^4)^2}{b} \right)$$

5280

Antiderivative was successfully verified.

[In] Integrate[x^2*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]

[Out] (Sqrt[a + b*x^4]*((528*d*(a + b*x^4)^2)/b + (480*e*x*(a + b*x^4)^2)/b + (55*f*(Sqrt[b]*x^2*(3*a^2 + 14*a*b*x^4 + 8*b^2*x^8) - (3*a^(5/2)*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[1 + (b*x^4)/a]))/b^(3/2) - (480*a^2*e*x*Hypergeometric2F1[-3/2, 1/4, 5/4, -(b*x^4)/a])/(b*Sqrt[1 + (b*x^4)/a]) + (1760*a*c*x

$\sqrt[3]{\text{Hypergeometric2F1}[-3/2, 3/4, 7/4, -((b*x^4)/a)]}/\text{Sqrt}[1 + (b*x^4)/a])/5$
280

Maple [C] time = 0.01, size = 413, normalized size = 1.

$$\frac{bfx^{10}}{12}\sqrt{bx^4+a} + \frac{7afx^6}{48}\sqrt{bx^4+a} + \frac{a^2fx^2}{32b}\sqrt{bx^4+a} - \frac{a^3f}{32}\ln\left(x^2\sqrt{b} + \sqrt{bx^4+a}\right)b^{-\frac{3}{2}} + \frac{bex^9}{11}\sqrt{bx^4+a} + \frac{13aex^5}{77}\sqrt{bx^4+a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x)`

[Out] $1/12*f*b*x^{10}*(b*x^4+a)^{(1/2)} + 7/48*f*a*x^6*(b*x^4+a)^{(1/2)} + 1/32*a^2*f*x^2*(b*x^4+a)^{(1/2)}/b - 1/32*f/b^{(3/2)}*a^3*\ln(x^2*b^{(1/2)} + (b*x^4+a)^{(1/2)}) + 1/11*e*b*x^9*(b*x^4+a)^{(1/2)} + 13/77*e*a*x^5*(b*x^4+a)^{(1/2)} + 4/77*a^2*e*x*(b*x^4+a)^{(1/2)}/b - 4/77*e/b*a^3/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*EllipticF(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I) + 1/10*d/b*(b*x^4+a)^{(5/2)} + 1/9*c*b*x^7*(b*x^4+a)^{(1/2)} + 11/45*c*a*x^3*(b*x^4+a)^{(1/2)} + 4/15*I*c*a^{(5/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}/b^{(1/2)}*EllipticF(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I) - 4/15*I*c*a^{(5/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}/b^{(1/2)}*EllipticE(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bfx^9 + bex^8 + bdx^7 + bcx^6 + afx^5 + aex^4 + adx^3 + acx^2\right)\sqrt{bx^4 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="fricas")

[Out] integral((b*f*x^9 + b*e*x^8 + b*d*x^7 + b*c*x^6 + a*f*x^5 + a*e*x^4 + a*d*x^3 + a*c*x^2)*sqrt(b*x^4 + a), x)

Sympy [A] time = 14.209, size = 398, normalized size = 0.93

$$\frac{a^{\frac{5}{2}}fx^2}{32b\sqrt{1 + \frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}cx^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{3}{4} \middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{a^{\frac{3}{2}}ex^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{5}{4} \middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{17a^{\frac{3}{2}}fx^6}{96\sqrt{1 + \frac{bx^4}{a}}} + \frac{\sqrt{abc}x^7\Gamma\left(\frac{7}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{7}{4} \middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)

[Out] a**(5/2)*f*x**2/(32*b*sqrt(1 + b*x**4/a)) + a**(3/2)*c*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + a**(3/2)*e*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 17*a**(3/2)*f*x**6/(96*sqrt(1 + b*x**4/a)) + sqrt(a)*b*c*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) + sqrt(a)*b*e*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(13/4)) + 11*sqrt(a)*b*f*x**10/(48*sqrt(1 + b*x**4/a)) - a**3*f*asinh(sqrt(b)*x**2/sqrt(a))/(32*b**(3/2)) + a*d*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b*d*Piecewise((-a**2*sqrt(a + b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x**4)/(30*b) + x**8*sqrt(a + b*x**4)/10, Ne(b, 0)), (sqrt(a)*x**8/8, True)) + b**2*f*x**14/(12*sqrt(a)*sqrt(1 + b*x**4/a))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^2, x)
```

$$3.513 \quad \int x (c + dx + ex^2 + fx^3) (a + bx^4)^{3/2} dx$$

Optimal. Leaf size=409

$$\frac{2a^{9/4} (\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (77\sqrt{bd} - 15\sqrt{af}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{1155b^{5/4}\sqrt{a+bx^4}} - \frac{4a^{9/4}d(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}$$

[Out] (4*a^2*f*x*Sqrt[a + b*x^4])/(77*b) + (3*a*c*x^2*Sqrt[a + b*x^4])/16 + (4*a^2*d*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x^3*(77*d + 45*f*x^2)*Sqrt[a + b*x^4])/1155 + (c*x^2*(a + b*x^4)^(3/2))/8 + (x^3*(1*d + 9*f*x^2)*(a + b*x^4)^(3/2))/99 + (e*(a + b*x^4)^(5/2))/(10*b) + (3*a^2*c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b]) - (4*a^(9/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4]) + (2*a^(9/4)*(77*Sqrt[b]*d - 15*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(1155*b^(5/4)*Sqrt[a + b*x^4])

Rubi [A] time = 0.324278, antiderivative size = 409, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {1833, 1248, 641, 195, 217, 206, 1274, 1280, 1198, 220, 1196}

$$\frac{2a^{9/4} (\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (77\sqrt{bd} - 15\sqrt{af}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{1155b^{5/4}\sqrt{a+bx^4}} - \frac{4a^{9/4}d(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] Int[x*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2), x]

[Out] (4*a^2*f*x*Sqrt[a + b*x^4])/(77*b) + (3*a*c*x^2*Sqrt[a + b*x^4])/16 + (4*a^2*d*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x^3*(77*d + 45*f*x^2)*Sqrt[a + b*x^4])/1155 + (c*x^2*(a + b*x^4)^(3/2))/8 + (x^3*(1*d + 9*f*x^2)*(a + b*x^4)^(3/2))/99 + (e*(a + b*x^4)^(5/2))/(10*b) + (3*a^2*c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b]) - (4*a^(9/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4]) + (2*a^(9/4)*(77*Sqrt[b]*d - 15*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(1155*b^(5/4)*Sqrt[a + b*x^4])

$1155*b^{(5/4)}*Sqrt[a + b*x^4]$

Rule 1833

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p]/c^j, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]

Rule 1248

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]

Rule 641

Int[((d_) + (e_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

Rule 195

Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1274

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(f*x)^(m + 1)*(a + c*x^4)^p*(c*d*(m + 4*p + 3) + c*e*(4*p

```

+ m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*p + 3)), x] + Dist[(4*a*p)/((4*p
+ m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])

```

Rule 1280

```

Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] := Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m]
)

```

Rule 1198

```

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]

```

Rule 220

```

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]

```

Rule 1196

```

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]

```

Rubi steps

$$\begin{aligned}
\int x(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2} dx &= \int \left(x(c + ex^2)(a + bx^4)^{3/2} + x^2(d + fx^2)(a + bx^4)^{3/2} \right) dx \\
&= \int x(c + ex^2)(a + bx^4)^{3/2} dx + \int x^2(d + fx^2)(a + bx^4)^{3/2} dx \\
&= \frac{1}{99}x^3(11d + 9fx^2)(a + bx^4)^{3/2} + \frac{1}{2} \text{Subst} \left(\int (c + ex)(a + bx^2)^{3/2} dx, x, x^2 \right) \\
&= \frac{2ax^3(77d + 45fx^2)\sqrt{a + bx^4}}{1155} + \frac{1}{99}x^3(11d + 9fx^2)(a + bx^4)^{3/2} + \frac{e(a + bx^4)^{5/2}}{10b} \\
&= \frac{4a^2fx\sqrt{a + bx^4}}{77b} + \frac{2ax^3(77d + 45fx^2)\sqrt{a + bx^4}}{1155} + \frac{1}{8}cx^2(a + bx^4)^{3/2} + \frac{1}{99}x^3(11d + 9fx^2)(a + bx^4)^{3/2} \\
&= \frac{4a^2fx\sqrt{a + bx^4}}{77b} + \frac{3}{16}acx^2\sqrt{a + bx^4} + \frac{2ax^3(77d + 45fx^2)\sqrt{a + bx^4}}{1155} + \frac{1}{8}cx^2(a + bx^4)^{3/2} \\
&= \frac{4a^2fx\sqrt{a + bx^4}}{77b} + \frac{3}{16}acx^2\sqrt{a + bx^4} + \frac{4a^2dx\sqrt{a + bx^4}}{15\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{2ax^3(77d + 45fx^2)\sqrt{a + bx^4}}{1155} \\
&= \frac{4a^2fx\sqrt{a + bx^4}}{77b} + \frac{3}{16}acx^2\sqrt{a + bx^4} + \frac{4a^2dx\sqrt{a + bx^4}}{15\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{2ax^3(77d + 45fx^2)\sqrt{a + bx^4}}{1155}
\end{aligned}$$

Mathematica [C] time = 0.701316, size = 196, normalized size = 0.48

$$\frac{\sqrt{a + bx^4} \left(165c \left(\frac{3a^{5/2} \sqrt{\frac{bx^4}{a}} + 1 \sinh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{\sqrt{b(a + bx^4)}} + 5ax^2 + 2bx^6 \right) - \frac{240a^2fx {}_2F_1 \left(-\frac{3}{2}, \frac{5}{4}; -\frac{bx^4}{a} \right)}{b\sqrt{\frac{bx^4}{a} + 1}} + \frac{880adx^3 {}_2F_1 \left(-\frac{3}{2}, \frac{7}{4}; -\frac{bx^4}{a} \right)}{\sqrt{\frac{bx^4}{a} + 1}} + \frac{264e(a + bx^4)^2}{b} \right)}{2640}$$

Antiderivative was successfully verified.

[In] Integrate[x*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2), x]

[Out] (Sqrt[a + b*x^4]*((264*e*(a + b*x^4)^2)/b + (240*f*x*(a + b*x^4)^2)/b + 165*c*(5*a*x^2 + 2*b*x^6 + (3*a^(5/2)*Sqrt[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[b]*(a + b*x^4))) - (240*a^2*f*x*Hypergeometric2F1[-3/2, 1/4, 5/4, -(b*x^4)/a])/(b*Sqrt[1 + (b*x^4)/a]) + (880*a*d*x^3*Hypergeometric2F1[-3/2, 3/4, 7/4, -(b*x^4)/a])/Sqrt[1 + (b*x^4)/a])/2640

Maple [C] time = 0.015, size = 392, normalized size = 1.

$$\frac{bfx^9}{11}\sqrt{bx^4+a} + \frac{13afx^5}{77}\sqrt{bx^4+a} + \frac{4a^2fx}{77b}\sqrt{bx^4+a} - \frac{4a^3f}{77b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}, i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x)`

[Out] $\frac{1}{11}f*b*x^9*(b*x^4+a)^{(1/2)} + \frac{13}{77}f*a*x^5*(b*x^4+a)^{(1/2)} + \frac{4}{77}a^2*f*x*(b*x^4+a)^{(1/2)}/b - \frac{4}{77}f/b*a^3/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*EllipticF(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I) + \frac{1}{10}e*(b*x^4+a)^{(5/2)}/b + \frac{1}{9}d*b*x^7*(b*x^4+a)^{(1/2)} + \frac{11}{45}d*a*x^3*(b*x^4+a)^{(1/2)} + \frac{4}{15}I*d*a^{(5/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}/b^{(1/2)}*EllipticF(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I) - \frac{4}{15}I*d*a^{(5/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}/b^{(1/2)}*EllipticE(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I) + \frac{1}{8}c*b*x^6*(b*x^4+a)^{(1/2)} + \frac{5}{16}a*c*x^2*(b*x^4+a)^{(1/2)} + \frac{3}{16}c*a^2*\ln(x^2*b^{(1/2)}+(b*x^4+a)^{(1/2)})/b^{(1/2)}$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bfx^8 + bex^7 + bdx^6 + bcx^5 + afx^4 + aex^3 + adx^2 + acx\right)\sqrt{bx^4+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="fricas")

[Out] integral((b*f*x^8 + b*e*x^7 + b*d*x^6 + b*c*x^5 + a*f*x^4 + a*e*x^3 + a*d*x^2 + a*c*x)*sqrt(b*x^4 + a), x)

Sympy [A] time = 10.1743, size = 396, normalized size = 0.97

$$\frac{a^{\frac{3}{2}}cx^2\sqrt{1+\frac{bx^4}{a}}}{4} + \frac{a^{\frac{3}{2}}cx^2}{16\sqrt{1+\frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}dx^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{-\frac{1}{2}, \frac{3}{4}}{\frac{7}{4}} \middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{a^{\frac{3}{2}}fx^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\frac{-\frac{1}{2}, \frac{5}{4}}{\frac{9}{4}} \middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{3\sqrt{abc}x^6}{16\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{abd}}{16\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)

[Out] a**(3/2)*c*x**2*sqrt(1 + b*x**4/a)/4 + a**(3/2)*c*x**2/(16*sqrt(1 + b*x**4/a)) + a**(3/2)*d*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + a**(3/2)*f*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*b*c*x**6/(16*sqrt(1 + b*x**4/a)) + sqrt(a)*b*d*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) + sqrt(a)*b*f*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(13/4)) + 3*a**2*c*asinh(sqrt(b)*x**2/sqrt(a))/(16*sqrt(b)) + a*e*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b*e*Piecewise((-a**2*sqrt(a + b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x**4)/(30*b) + x**8*sqrt(a + b*x**4)/10, Ne(b, 0)), (sqrt(a)*x**8/8, True)) + b**2*c*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/a))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="giac")

[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x, x)

$$3.514 \quad \int (c + dx + ex^2 + fx^3) (a + bx^4)^{3/2} dx$$

Optimal. Leaf size=382

$$\frac{2a^{7/4} (\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (7\sqrt{ae} + 15\sqrt{bc}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{105b^{3/4}\sqrt{a+bx^4}} - \frac{4a^{9/4}e(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\right)}{15b^{3/4}\sqrt{a+bx^4}}$$

[Out] (3*a*d*x^2*Sqrt[a + b*x^4])/16 + (4*a^2*e*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x*(15*c + 7*e*x^2)*Sqrt[a + b*x^4])/105 + (d*x^2*(a + b*x^4)^(3/2))/8 + (x*(9*c + 7*e*x^2)*(a + b*x^4)^(3/2))/63 + (f*(a + b*x^4)^(5/2))/(10*b) + (3*a^2*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b]) - (4*a^(9/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4]) + (2*a^(7/4)*(15*Sqrt[b]*c + 7*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(3/4)*Sqrt[a + b*x^4])

Rubi [A] time = 0.254981, antiderivative size = 382, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.37$, Rules used = {1885, 1177, 1198, 220, 1196, 1248, 641, 195, 217, 206}

$$\frac{2a^{7/4} (\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (7\sqrt{ae} + 15\sqrt{bc}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{105b^{3/4}\sqrt{a+bx^4}} - \frac{4a^{9/4}e(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\right)}{15b^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2), x]

[Out] (3*a*d*x^2*Sqrt[a + b*x^4])/16 + (4*a^2*e*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x*(15*c + 7*e*x^2)*Sqrt[a + b*x^4])/105 + (d*x^2*(a + b*x^4)^(3/2))/8 + (x*(9*c + 7*e*x^2)*(a + b*x^4)^(3/2))/63 + (f*(a + b*x^4)^(5/2))/(10*b) + (3*a^2*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b]) - (4*a^(9/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4]) + (2*a^(7/4)*(15*Sqrt[b]*c + 7*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(3/4)*Sqrt[a + b*x^4])

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq,
x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*x^j*(a + b*x^n)^p, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, p},
x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*
(d*(4*p + 3) + e*(4*p + 1)*x^2)*(a + c*x^4)^p)/((4*p + 1)*(4*p + 3)), x] +
Dist[(2*p)/((4*p + 1)*(4*p + 3)), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2))] * EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2))] * EllipticE[2*ArcTan[q*x],
1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
:= Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(e*(
```

$(a + c*x^2)^{(p + 1)}/(2*c*(p + 1)), x] + \text{Dist}[d, \text{Int}[(a + c*x^2)^p, x], x] /$
 $;$ FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

Rule 195

$\text{Int}[(a_) + (b_.)*(x_)^{(n_)})^{(p_)}, x_Symbol] := \text{Simp}[(x*(a + b*x^n)^p)/(n*p$
 $+ 1), x] + \text{Dist}[(a*n*p)/(n*p + 1), \text{Int}[(a + b*x^n)^{(p - 1)}, x], x] /;$ Free
 Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
 IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])

Rule 217

$\text{Int}[1/\text{Sqrt}[(a_) + (b_.)*(x_)^2], x_Symbol] := \text{Subst}[\text{Int}[1/(1 - b*x^2), x],$
 $x, x/\text{Sqrt}[a + b*x^2]] /;$ FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

$\text{Int}[(a_) + (b_.)*(x_)^2)^{(-1)}, x_Symbol] := \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/$
 $\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
 Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int (c + dx + ex^2 + fx^3)(a + bx^4)^{3/2} dx &= \int \left((c + ex^2)(a + bx^4)^{3/2} + x(d + fx^2)(a + bx^4)^{3/2} \right) dx \\
&= \int (c + ex^2)(a + bx^4)^{3/2} dx + \int x(d + fx^2)(a + bx^4)^{3/2} dx \\
&= \frac{1}{63}x(9c + 7ex^2)(a + bx^4)^{3/2} + \frac{1}{21} \int (18ac + 14aex^2) \sqrt{a + bx^4} dx + \frac{1}{2} \text{Subst} \\
&= \frac{2}{105}ax(15c + 7ex^2) \sqrt{a + bx^4} + \frac{1}{63}x(9c + 7ex^2)(a + bx^4)^{3/2} + \frac{f(a + bx^4)^{5/2}}{10b} \\
&= \frac{2}{105}ax(15c + 7ex^2) \sqrt{a + bx^4} + \frac{1}{8}dx^2(a + bx^4)^{3/2} + \frac{1}{63}x(9c + 7ex^2)(a + bx^4)^{3/2} \\
&= \frac{3}{16}adx^2\sqrt{a + bx^4} + \frac{4a^2ex\sqrt{a + bx^4}}{15\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{2}{105}ax(15c + 7ex^2)\sqrt{a + bx^4} + \frac{1}{8}d \\
&= \frac{3}{16}adx^2\sqrt{a + bx^4} + \frac{4a^2ex\sqrt{a + bx^4}}{15\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{2}{105}ax(15c + 7ex^2)\sqrt{a + bx^4} + \frac{1}{8}d \\
&= \frac{3}{16}adx^2\sqrt{a + bx^4} + \frac{4a^2ex\sqrt{a + bx^4}}{15\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{2}{105}ax(15c + 7ex^2)\sqrt{a + bx^4} + \frac{1}{8}d
\end{aligned}$$

Mathematica [C] time = 0.552603, size = 175, normalized size = 0.46

$$\frac{1}{240}\sqrt{a + bx^4} \left(15d \left(\frac{3a^{5/2}\sqrt{\frac{bx^4}{a} + 1} \sinh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{\sqrt{b}(a + bx^4)} + 5ax^2 + 2bx^6 \right) + \frac{240acx {}_2F_1\left(-\frac{3}{2}, \frac{1}{4}; \frac{5}{4}; -\frac{bx^4}{a}\right)}{\sqrt{\frac{bx^4}{a} + 1}} + \frac{80aex^3 {}_2F_1\left(-\frac{3}{2}, \frac{3}{4}; \frac{7}{4}; -\frac{bx^4}{a}\right)}{\sqrt{\frac{bx^4}{a} + 1}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2), x]

[Out] (Sqrt[a + b*x^4]*((24*f*(a + b*x^4)^2)/b + 15*d*(5*a*x^2 + 2*b*x^6 + (3*a^(5/2)*Sqrt[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[b]*(a + b*x^4))) + (240*a*c*x*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^4)/a)]/Sqrt[1 + (b*x^4)/a] + (80*a*e*x^3*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^4)/a)]/Sqrt[1 + (b*x^4)/a]))/240

Maple [C] time = 0.006, size = 368, normalized size = 1.

$$\frac{f}{10b} (bx^4 + a)^{\frac{5}{2}} + \frac{bex^7}{9} \sqrt{bx^4 + a} + \frac{11aex^3}{45} \sqrt{bx^4 + a} + \frac{4i}{15} ea^{\frac{5}{2}} \sqrt{1 - ix^2 \sqrt{b} \frac{1}{\sqrt{a}}} \sqrt{1 + ix^2 \sqrt{b} \frac{1}{\sqrt{a}}} \text{EllipticF} \left(x \sqrt{i \sqrt{b} \frac{1}{\sqrt{a}}}, i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x)

[Out] 1/10*f*(b*x^4+a)^(5/2)/b+1/9*e*b*x^7*(b*x^4+a)^(1/2)+11/45*e*a*x^3*(b*x^4+a)^(1/2)+4/15*I*e*a^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-4/15*I*e*a^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/8*d*b*x^6*(b*x^4+a)^(1/2)+5/16*a*d*x^2*(b*x^4+a)^(1/2)+3/16*d*a^2*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2)+1/7*c*b*x^5*(b*x^4+a)^(1/2)+3/7*c*a*x*(b*x^4+a)^(1/2)+4/7*c*a^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^4 + a)^{\frac{3}{2}} (fx^3 + ex^2 + dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="maxima")

[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left((bfx^7 + bex^6 + bdx^5 + bcx^4 + afx^3 + aex^2 + adx + ac) \sqrt{bx^4 + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="fricas")

[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x + a*c)*sqrt(b*x^4 + a), x)

Sympy [A] time = 9.64242, size = 394, normalized size = 1.03

$$\frac{a^{\frac{3}{2}}cx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{1}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^{\frac{3}{2}}dx^2\sqrt{1+\frac{bx^4}{a}}}{4} + \frac{a^{\frac{3}{2}}dx^2}{16\sqrt{1+\frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}ex^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{3}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{abc}x^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{5}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)

[Out] a**(3/2)*c*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + a**(3/2)*d*x**2*sqrt(1 + b*x**4/a)/4 + a**(3/2)*d*x**2/(16*sqrt(1 + b*x**4/a)) + a**(3/2)*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + sqrt(a)*b*c*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*b*d*x**6/(16*sqrt(1 + b*x**4/a)) + sqrt(a)*b*e*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) + 3*a**2*d*a*sinh(sqrt(b)*x**2/sqrt(a))/(16*sqrt(b)) + a*f*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b*f*Piecewise((-a**2*sqrt(a + b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x**4)/(30*b) + x**8*sqrt(a + b*x**4)/10, Ne(b, 0)), (sqrt(a)*x**8/8, True)) + b**2*d*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/a))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="giac")

[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c), x)

$$3.515 \quad \int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x} dx$$

Optimal. Leaf size=403

$$\frac{2a^{7/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (7\sqrt{a}f + 15\sqrt{bd}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{105b^{3/4}\sqrt{a+bx^4}} - \frac{4a^{9/4}f(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\right)}{15b^{3/4}\sqrt{a+bx^4}}$$

[Out] $(4a^2fx\sqrt{a+bx^4})/(15\sqrt{b}(\sqrt{a} + \sqrt{b}x^2)) + (a(8c + 3ex^2)\sqrt{a+bx^4})/16 + (2ax(15d + 7fx^2)\sqrt{a+bx^4})/105 + ((4c + 3ex^2)(a+bx^4)^{3/2})/24 + (x(9d + 7fx^2)(a+bx^4)^{3/2})/63 + (3a^2e\operatorname{ArcTanh}[(\sqrt{b}x^2)/\sqrt{a+bx^4}])/(16\sqrt{b}) - (a^{3/2}c\operatorname{ArcTanh}[\sqrt{a+bx^4}/\sqrt{a}])/2 - (4a^{9/4}f(\sqrt{a} + \sqrt{b}x^2)\sqrt{(a+bx^4)/(\sqrt{a} + \sqrt{b}x^2)^2})\operatorname{EllipticE}[2\operatorname{ArcTan}[(b^{1/4}x)/a^{1/4}], 1/2]/(15b^{3/4}\sqrt{a+bx^4}) + (2a^{7/4}(15\sqrt{b}d + 7\sqrt{a}f)(\sqrt{a} + \sqrt{b}x^2)\sqrt{(a+bx^4)/(\sqrt{a} + \sqrt{b}x^2)^2})\operatorname{EllipticF}[2\operatorname{ArcTan}[(b^{1/4}x)/a^{1/4}], 1/2]/(105b^{3/4}\sqrt{a+bx^4})$

Rubi [A] time = 0.35252, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {1833, 1252, 815, 844, 217, 206, 266, 63, 208, 1177, 1198, 220, 1196}

$$\frac{2a^{7/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (7\sqrt{a}f + 15\sqrt{bd}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{105b^{3/4}\sqrt{a+bx^4}} - \frac{4a^{9/4}f(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\right)}{15b^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x,x]

[Out] $(4a^2fx\sqrt{a+bx^4})/(15\sqrt{b}(\sqrt{a} + \sqrt{b}x^2)) + (a(8c + 3ex^2)\sqrt{a+bx^4})/16 + (2ax(15d + 7fx^2)\sqrt{a+bx^4})/105 + ((4c + 3ex^2)(a+bx^4)^{3/2})/24 + (x(9d + 7fx^2)(a+bx^4)^{3/2})/63 + (3a^2e\operatorname{ArcTanh}[(\sqrt{b}x^2)/\sqrt{a+bx^4}])/(16\sqrt{b}) - (a^{3/2}c\operatorname{ArcTanh}[\sqrt{a+bx^4}/\sqrt{a}])/2 - (4a^{9/4}f(\sqrt{a} + \sqrt{b}x^2)\sqrt{(a+bx^4)/(\sqrt{a} + \sqrt{b}x^2)^2})\operatorname{EllipticE}[2\operatorname{ArcTan}[(b^{1/4}x)/a^{1/4}], 1/2]/(15b^{3/4}\sqrt{a+bx^4}) + (2a^{7/4}(15\sqrt{b}d + 7\sqrt{a}f)(\sqrt{a} + \sqrt{b}x^2)\sqrt{(a+bx^4)/(\sqrt{a} + \sqrt{b}x^2)^2})\operatorname{EllipticF}[2\operatorname{ArcTan}[(b^{1/4}x)/a^{1/4}], 1/2]/(105b^{3/4}\sqrt{a+bx^4})$

+ Sqrt[b*x^2]^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2]]/(105*b^(3/4)*Sqrt[a + b*x^4])

Rule 1833

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p]/c^j, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]

Rule 1252

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rule 815

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p]/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1))]*x, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 844

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

$Q[a, 0] \parallel LtQ[b, 0]$)

Rule 266

$\text{Int}[(x_)^{(m_.)} * ((a_) + (b_.) * (x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1) * (a + b*x)^p}, x], x, x^n], x] /; \text{FreeQ}\{a, b, m, n, p\}, x] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 63

$\text{Int}[(a_.) + (b_.) * (x_)^{(m_.)} * ((c_.) + (d_.) * (x_)^{(n_.)}), x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m + 1) - 1)} * (c - (a*d)/b + (d*x^p)/b)^n], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 208

$\text{Int}[(a_) + (b_.) * (x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2] * \text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

Rule 1177

$\text{Int}[(d_) + (e_.) * (x_)^2 * ((a_) + (c_.) * (x_)^4)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(x * (d * (4*p + 3) + e * (4*p + 1) * x^2) * (a + c * x^4)^p) / ((4*p + 1) * (4*p + 3)), x] + \text{Dist}[(2*p) / ((4*p + 1) * (4*p + 3)), \text{Int}[\text{Simp}[2*a*d * (4*p + 3) + (2*a*e * (4*p + 1)) * x^2, x] * (a + c * x^4)^{(p - 1)}, x], x] /; \text{FreeQ}\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[c * d^2 + a * e^2, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{FractionQ}[p] \ \&\& \ \text{IntegerQ}[2*p]$

Rule 1198

$\text{Int}[(d_) + (e_.) * (x_)^2 / \text{Sqrt}[(a_) + (c_.) * (x_)^4], x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[c/a, 2]\}, \text{Dist}[(e + d*q)/q, \text{Int}[1/\text{Sqrt}[a + c*x^4], x], x] - \text{Dist}[e/q, \text{Int}[(1 - q*x^2)/\text{Sqrt}[a + c*x^4], x], x] /; \text{NeQ}[e + d*q, 0]] /; \text{FreeQ}\{a, c, d, e\}, x] \ \&\& \ \text{PosQ}[c/a]$

Rule 220

$\text{Int}[1/\text{Sqrt}[(a_) + (b_.) * (x_)^4], x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[b/a, 4]\}, \text{Simp}[(1 + q^2 * x^2) * \text{Sqrt}[(a + b * x^4) / (a * (1 + q^2 * x^2)^2)] * \text{EllipticF}[2 * \text{ArcTan}[q * x, 1/2]] / (2 * q * \text{Sqrt}[a + b * x^4]), x]] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[b/a]$

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
  Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
  1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
  1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
  x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}}{x} dx &= \int \left(\frac{(c + ex^2)(a + bx^4)^{3/2}}{x} + (d + fx^2)(a + bx^4)^{3/2} \right) dx \\
&= \int \frac{(c + ex^2)(a + bx^4)^{3/2}}{x} dx + \int (d + fx^2)(a + bx^4)^{3/2} dx \\
&= \frac{1}{63} x (9d + 7fx^2) (a + bx^4)^{3/2} + \frac{1}{21} \int (18ad + 14afx^2) \sqrt{a + bx^4} dx + \frac{1}{2} \text{Sub} \\
&= \frac{2}{105} ax (15d + 7fx^2) \sqrt{a + bx^4} + \frac{1}{24} (4c + 3ex^2) (a + bx^4)^{3/2} + \frac{1}{63} x (9d + 7fx^2) \\
&= \frac{1}{16} a (8c + 3ex^2) \sqrt{a + bx^4} + \frac{2}{105} ax (15d + 7fx^2) \sqrt{a + bx^4} + \frac{1}{24} (4c + 3ex^2) \\
&= \frac{4a^2 fx \sqrt{a + bx^4}}{15\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{1}{16} a (8c + 3ex^2) \sqrt{a + bx^4} + \frac{2}{105} ax (15d + 7fx^2) \sqrt{a + } \\
&= \frac{4a^2 fx \sqrt{a + bx^4}}{15\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{1}{16} a (8c + 3ex^2) \sqrt{a + bx^4} + \frac{2}{105} ax (15d + 7fx^2) \sqrt{a + } \\
&= \frac{4a^2 fx \sqrt{a + bx^4}}{15\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{1}{16} a (8c + 3ex^2) \sqrt{a + bx^4} + \frac{2}{105} ax (15d + 7fx^2) \sqrt{a + } \\
&= \frac{4a^2 fx \sqrt{a + bx^4}}{15\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{1}{16} a (8c + 3ex^2) \sqrt{a + bx^4} + \frac{2}{105} ax (15d + 7fx^2) \sqrt{a + }
\end{aligned}$$

Mathematica [C] time = 0.512375, size = 224, normalized size = 0.56

$$\frac{1}{6}c \left(\sqrt{a+bx^4} (4a+bx^4) - 3a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a+bx^4}}{\sqrt{a}} \right) \right) + \frac{1}{16} e^{\sqrt{a+bx^4}} \left(\frac{3a^{3/2} \sinh^{-1} \left(\frac{\sqrt{bx^4}}{\sqrt{a}} \right)}{\sqrt{b} \sqrt{\frac{bx^4}{a} + 1}} + 5ax^2 + 2bx^6 \right) + \frac{adx\sqrt{a+bx^4}}{16}$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x,x]

[Out] (e*Sqrt[a + b*x^4]*(5*a*x^2 + 2*b*x^6 + (3*a^(3/2)*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[b]*Sqrt[1 + (b*x^4)/a]))/16 + (c*(Sqrt[a + b*x^4]*(4*a + b*x^4) - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]))/6 + (a*d*x*Sqrt[a + b*x^4]*Hypergeometric2F1[-3/2, 1/4, 5/4, -(b*x^4)/a])/Sqrt[1 + (b*x^4)/a] + (a*f*x^3*Sqrt[a + b*x^4]*Hypergeometric2F1[-3/2, 3/4, 7/4, -(b*x^4)/a])/(3*Sqrt[1 + (b*x^4)/a])

Maple [C] time = 0.019, size = 411, normalized size = 1.

$$\frac{bfx^7}{9} \sqrt{bx^4+a} + \frac{11x^3af}{45} \sqrt{bx^4+a} + \frac{4i}{15} fa^{\frac{5}{2}} \sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}} \sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}} \text{EllipticF} \left(x \sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}, i \right) \frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x,x)

[Out] 1/9*f*b*x^7*(b*x^4+a)^(1/2)+11/45*f*a*x^3*(b*x^4+a)^(1/2)+4/15*I*f*a^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-4/15*I*f*a^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/8*e*b*x^6*(b*x^4+a)^(1/2)+5/16*e*a*x^2*(b*x^4+a)^(1/2)+3/16*e*a^2*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2)+1/7*d*b*x^5*(b*x^4+a)^(1/2)+3/7*d*a*x*(b*x^4+a)^(1/2)+4/7*d*a^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/6*c*b*x^4*(b*x^4+a)^(1/2)+2/3*c*a*(b*x^4+a)^(1/2)-1/2*c*a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x,x, algorithm="maxima")

[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bfx^7 + bex^6 + bdx^5 + bcx^4 + afx^3 + aex^2 + adx + ac)\sqrt{bx^4 + a}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x,x, algorithm="fricas")

[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x + a*c)*sqrt(b*x^4 + a)/x, x)

Sympy [A] time = 26.8634, size = 405, normalized size = 1.

$$-\frac{a^{\frac{3}{2}}c \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx^2}}\right)}{2} + \frac{a^{\frac{3}{2}}dx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^{\frac{3}{2}}ex^2\sqrt{1 + \frac{bx^4}{a}}}{4} + \frac{a^{\frac{3}{2}}ex^2}{16\sqrt{1 + \frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}fx^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{3}{4}, \frac{3}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x,x)

[Out] -a**(3/2)*c*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + a**(3/2)*d*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + a**(3/2)*e*x**2*sqrt(1 + b*x**4/a)/4 + a**(3/2)*e*x**2/(16*sqrt(1 + b*x**4/a)) + a**(3/2)*f*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)

```
/(4*gamma(7/4)) + sqrt(a)*b*d*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*
x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*b*e*x**6/(16*sqrt(1 + b*
x**4/a)) + sqrt(a)*b*f*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x**4*e
xp_polar(I*pi)/a)/(4*gamma(11/4)) + a**2*c/(2*sqrt(b)*x**2*sqrt(a/(b*x**4)
+ 1)) + 3*a**2*e*asinh(sqrt(b)*x**2/sqrt(a))/(16*sqrt(b)) + a*sqrt(b)*c*x**
2/(2*sqrt(a/(b*x**4) + 1)) + b*c*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a
+ b*x**4)**(3/2)/(6*b), True)) + b**2*e*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/a)
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x,x, algorithm="giac")

[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x, x)

$$3.516 \quad \int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^2} dx$$

Optimal. Leaf size=404

$$\frac{2a^{5/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (5\sqrt{ae} + 21\sqrt{bc}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{35\sqrt[4]{b}\sqrt{a+bx^4}} - \frac{12a^{5/4}\sqrt[4]{bc}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E}{5\sqrt{a+bx^4}}$$

[Out] (12*a*Sqrt[b]*c*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) + (2*x*(5*a*e + 21*b*c*x^2)*Sqrt[a + b*x^4])/35 + (a*(8*d + 3*f*x^2)*Sqrt[a + b*x^4])/16 - ((7*c - e*x^2)*(a + b*x^4)^(3/2))/(7*x) + ((4*d + 3*f*x^2)*(a + b*x^4)^(3/2))/24 + (3*a^2*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b]) - (a^(3/2)*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (12*a^(5/4)*b^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(5/4)*(21*Sqrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(35*b^(1/4)*Sqrt[a + b*x^4])

Rubi [A] time = 0.348052, antiderivative size = 404, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1833, 1272, 1177, 1198, 220, 1196, 1252, 815, 844, 217, 206, 266, 63, 208}

$$\frac{2a^{5/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (5\sqrt{ae} + 21\sqrt{bc}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{35\sqrt[4]{b}\sqrt{a+bx^4}} - \frac{12a^{5/4}\sqrt[4]{bc}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{5\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^2, x]

[Out] (12*a*Sqrt[b]*c*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) + (2*x*(5*a*e + 21*b*c*x^2)*Sqrt[a + b*x^4])/35 + (a*(8*d + 3*f*x^2)*Sqrt[a + b*x^4])/16 - ((7*c - e*x^2)*(a + b*x^4)^(3/2))/(7*x) + ((4*d + 3*f*x^2)*(a + b*x^4)^(3/2))/24 + (3*a^2*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b]) - (a^(3/2)*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (12*a^(5/4)*b^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(5/4)*(21*S

$\sqrt{b} * c + 5 * \sqrt{a} * e) * (\sqrt{a} + \sqrt{b} * x^2) * \sqrt{(a + b * x^4) / (\sqrt{a} + \sqrt{b} * x^2)^2} * \text{EllipticF}[2 * \text{ArcTan}[(b^{(1/4)} * x) / a^{(1/4)}], 1/2] / (35 * b^{(1/4)} * \sqrt{a + b * x^4})$

Rule 1833

$\text{Int}[(Pq_)*((c_)*(x_))^{(m_)}*((a_)+(b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Module}[\{q = \text{Expon}[Pq, x], j, k\}, \text{Int}[\text{Sum}[(c*x)^{(m+j)}*\text{Sum}[\text{Coeff}[Pq, x, j+(k*n)/2]*x^{((k*n)/2)}, \{k, 0, (2*(q-j))/n+1\}]*\text{Sum}[(a+b*x^n)^p]/c^j, \{j, 0, n/2-1\}], x]] /; \text{FreeQ}[\{a, b, c, m, p\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[n/2, 0] \&\& !\text{PolyQ}[Pq, x^{(n/2)}]$

Rule 1272

$\text{Int}[(f_)*(x_))^{(m_)}*((d_)+(e_)*(x_)^2)*((a_)+(c_)*(x_)^4)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}*(a+c*x^4)^p*(d*(m+4*p+3)+e*(m+1)*x^2)/(f*(m+1)*(m+4*p+3)), x] + \text{Dist}[(4*p)/(f^2*(m+1)*(m+4*p+3)), \text{Int}[(f*x)^{(m+2)}*(a+c*x^4)^{(p-1)}*(a*e*(m+1)-c*d*(m+4*p+3)*x^2), x], x] /; \text{FreeQ}[\{a, c, d, e, f\}, x] \&\& \text{GtQ}[p, 0] \&\& \text{LtQ}[m, -1] \&\& m+4*p+3 \neq 0 \&\& \text{IntegerQ}[2*p] \&\& (\text{IntegerQ}[p] \parallel \text{IntegerQ}[m])$

Rule 1177

$\text{Int}[(d_)+(e_)*(x_)^2)*((a_)+(c_)*(x_)^4)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(x*(d*(4*p+3)+e*(4*p+1)*x^2)*(a+c*x^4)^p)/((4*p+1)*(4*p+3)), x] + \text{Dist}[(2*p)/((4*p+1)*(4*p+3)), \text{Int}[\text{Simp}[2*a*d*(4*p+3)+(2*a*e*(4*p+1))*x^2, x]*(a+c*x^4)^{(p-1)}, x], x] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{NeQ}[c*d^2+a*e^2, 0] \&\& \text{GtQ}[p, 0] \&\& \text{FractionQ}[p] \&\& \text{IntegerQ}[2*p]$

Rule 1198

$\text{Int}[(d_)+(e_)*(x_)^2]/\sqrt{(a_)+(c_)*(x_)^4}, x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[c/a, 2]\}, \text{Dist}[(e+d*q)/q, \text{Int}[1/\sqrt{a+c*x^4}, x], x] - \text{Dist}[e/q, \text{Int}[(1-q*x^2)/\sqrt{a+c*x^4}, x], x] /; \text{NeQ}[e+d*q, 0]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{PosQ}[c/a]$

Rule 220

$\text{Int}[1/\sqrt{(a_)+(b_)*(x_)^4}, x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[b/a, 4]\}, \text{Simp}[(1+q^2*x^2)*\sqrt{(a+b*x^4)/(a*(1+q^2*x^2)^2)}*\text{EllipticF}[2*\text{ArcTan}[q*x], 1/2]]/(2*q*\sqrt{a+b*x^4}), x]] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[b/a]$

Rule 1196


```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q =
  Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
  1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2]]*EllipticE[2*ArcTan[q*x],
  1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
  x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_S
ymbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
  x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 815

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p
_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
/; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p
_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
  e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
  x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
  Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
  Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}}{x^2} dx &= \int \left(\frac{(c + ex^2)(a + bx^4)^{3/2}}{x^2} + \frac{(d + fx^2)(a + bx^4)^{3/2}}{x} \right) dx \\
&= \int \frac{(c + ex^2)(a + bx^4)^{3/2}}{x^2} dx + \int \frac{(d + fx^2)(a + bx^4)^{3/2}}{x} dx \\
&= -\frac{(7c - ex^2)(a + bx^4)^{3/2}}{7x} + \frac{1}{2} \text{Subst} \left(\int \frac{(d + fx)(a + bx^2)^{3/2}}{x} dx, x, x^2 \right) - \frac{6}{7} \int \frac{(d + fx)(a + bx^2)^{3/2}}{x} dx \\
&= \frac{2}{35}x(5ae + 21bcx^2)\sqrt{a + bx^4} - \frac{(7c - ex^2)(a + bx^4)^{3/2}}{7x} + \frac{1}{24}(4d + 3fx^2)(a + bx^4)^{3/2} \\
&= \frac{2}{35}x(5ae + 21bcx^2)\sqrt{a + bx^4} + \frac{1}{16}a(8d + 3fx^2)\sqrt{a + bx^4} - \frac{(7c - ex^2)(a + bx^4)^{3/2}}{7x} \\
&= \frac{12a\sqrt{bcx}\sqrt{a + bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} + \frac{2}{35}x(5ae + 21bcx^2)\sqrt{a + bx^4} + \frac{1}{16}a(8d + 3fx^2)\sqrt{a + bx^4} \\
&= \frac{12a\sqrt{bcx}\sqrt{a + bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} + \frac{2}{35}x(5ae + 21bcx^2)\sqrt{a + bx^4} + \frac{1}{16}a(8d + 3fx^2)\sqrt{a + bx^4} \\
&= \frac{12a\sqrt{bcx}\sqrt{a + bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} + \frac{2}{35}x(5ae + 21bcx^2)\sqrt{a + bx^4} + \frac{1}{16}a(8d + 3fx^2)\sqrt{a + bx^4} \\
&= \frac{12a\sqrt{bcx}\sqrt{a + bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} + \frac{2}{35}x(5ae + 21bcx^2)\sqrt{a + bx^4} + \frac{1}{16}a(8d + 3fx^2)\sqrt{a + bx^4}
\end{aligned}$$

Mathematica [C] time = 0.523102, size = 222, normalized size = 0.55

$$\frac{1}{6}d \left(\sqrt{a + bx^4}(4a + bx^4) - 3a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + bx^4}}{\sqrt{a}} \right) \right) + \frac{1}{16}f\sqrt{a + bx^4} \left(\frac{3a^{3/2} \sinh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right)}{\sqrt{b}\sqrt{\frac{bx^4}{a} + 1}} + 5ax^2 + 2bx^6 \right) - \frac{ac\sqrt{a + bx^4}}{7x}$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^2,x]

```
[Out] (f*Sqrt[a + b*x^4]*(5*a*x^2 + 2*b*x^6 + (3*a^(3/2)*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[b]*Sqrt[1 + (b*x^4)/a])))/16 + (d*(Sqrt[a + b*x^4]*(4*a + b*x^4) - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]))/6 - (a*c*Sqrt[a + b*x^4]*Hypergeometric2F1[-3/2, -1/4, 3/4, -((b*x^4)/a)]/(x*Sqrt[1 + (b*x^4)/a]) + (a*e*x*Sqrt[a + b*x^4]*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a]
```

Maple [C] time = 0.013, size = 411, normalized size = 1.

$$\frac{bfx^6}{8}\sqrt{bx^4+a} + \frac{5afx^2}{16}\sqrt{bx^4+a} + \frac{3fa^2}{16}\ln\left(x^2\sqrt{b} + \sqrt{bx^4+a}\right)\frac{1}{\sqrt{b}} + \frac{bex^5}{7}\sqrt{bx^4+a} + \frac{3aex}{7}\sqrt{bx^4+a} + \frac{4a^2e}{7}\sqrt{1-ix^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^2,x)
```

```
[Out] 1/8*f*b*x^6*(b*x^4+a)^(1/2)+5/16*f*a*x^2*(b*x^4+a)^(1/2)+3/16*f*a^2*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2)+1/7*e*b*x^5*(b*x^4+a)^(1/2)+3/7*e*a*x*(b*x^4+a)^(1/2)+4/7*e*a^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/6*d*b*x^4*(b*x^4+a)^(1/2)+2/3*d*a*(b*x^4+a)^(1/2)-1/2*d*a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-c*a*(b*x^4+a)^(1/2)/x+1/5*c*b*x^3*(b*x^4+a)^(1/2)+12/5*I*c*a^(3/2)*b^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-12/5*I*c*a^(3/2)*b^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^2,x, algorithm="maxima")
```

[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bf x^7 + bex^6 + bdx^5 + bcx^4 + afx^3 + aex^2 + adx + ac)\sqrt{bx^4 + a}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^2,x, algorithm="fricas")

[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x + a*c)*sqrt(b*x^4 + a)/x^2, x)

Sympy [A] time = 11.26, size = 406, normalized size = 1.

$$\frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(-\frac{1}{2}, -\frac{1}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{a^{\frac{3}{2}}d \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx^2}}\right)}{2} + \frac{a^{\frac{3}{2}}ex\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{1}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^{\frac{3}{2}}fx^2\sqrt{1 + \frac{bx^4}{a}}}{4} + \frac{a^{\frac{3}{2}}fx^2}{16\sqrt{1 + \frac{bx^4}{a}}} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**2,x)

[Out] a**(3/2)*c*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - a**(3/2)*d*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + a**(3/2)*e*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + a**(3/2)*f*x**2*sqrt(1 + b*x**4/a)/4 + a**(3/2)*f*x**2/(16*sqrt(1 + b*x**4/a)) + sqrt(a)*b*c*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + sqrt(a)*b*e*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*b*f*x**6/(16*sqrt(1 + b*x**4/a)) + a**2*d/(2*sqrt(b)*x**2*sqrt(a/(b*x**4) + 1)) + 3*a**2*f*asinh(sqrt(b)*x**2/sqrt(a))/(16*sqrt(b)) + a*sqrt(b)*d*x**2/(2*sqrt(a/(b*x**4) + 1)) + b*d*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b**2*f*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/a))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^2,x, algorithm="giac")

[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)

$$3.517 \quad \int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^3} dx$$

Optimal. Leaf size=406

$$\frac{2a^{5/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (5\sqrt{a}f + 21\sqrt{bd}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{35\sqrt[4]{b}\sqrt{a+bx^4}} - \frac{12a^{5/4}\sqrt[4]{bd}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}}}{5\sqrt{a+bx^4}}$$

[Out] $(12*a*\operatorname{Sqrt}[b]*d*x*\operatorname{Sqrt}[a + b*x^4])/(5*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) + ((2*a*e + 3*b*c*x^2)*\operatorname{Sqrt}[a + b*x^4])/4 + (2*x*(5*a*f + 21*b*d*x^2)*\operatorname{Sqrt}[a + b*x^4])/35 - ((3*c - e*x^2)*(a + b*x^4)^{(3/2)})/(6*x^2) - ((7*d - f*x^2)*(a + b*x^4)^{(3/2)})/(7*x) + (3*a*\operatorname{Sqrt}[b]*c*\operatorname{ArcTan}h[(\operatorname{Sqrt}[b]*x^2)/\operatorname{Sqrt}[a + b*x^4]])/4 - (a^{(3/2)}*e*\operatorname{ArcTan}h[\operatorname{Sqrt}[a + b*x^4]/\operatorname{Sqrt}[a]])/2 - (12*a^{(5/4)}*b^{(1/4)}*d*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(5*\operatorname{Sqrt}[a + b*x^4]) + (2*a^{(5/4)}*(21*\operatorname{Sqrt}[b]*d + 5*\operatorname{Sqrt}[a]*f)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(35*b^{(1/4)}*\operatorname{Sqrt}[a + b*x^4])$

Rubi [A] time = 0.343212, antiderivative size = 406, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 15, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {1833, 1252, 813, 815, 844, 217, 206, 266, 63, 208, 1272, 1177, 1198, 220, 1196}

$$\frac{2a^{5/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (5\sqrt{a}f + 21\sqrt{bd}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{35\sqrt[4]{b}\sqrt{a+bx^4}} - \frac{12a^{5/4}\sqrt[4]{bd}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{5\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^{(3/2)}/x^3, x]$

[Out] $(12*a*\operatorname{Sqrt}[b]*d*x*\operatorname{Sqrt}[a + b*x^4])/(5*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) + ((2*a*e + 3*b*c*x^2)*\operatorname{Sqrt}[a + b*x^4])/4 + (2*x*(5*a*f + 21*b*d*x^2)*\operatorname{Sqrt}[a + b*x^4])/35 - ((3*c - e*x^2)*(a + b*x^4)^{(3/2)})/(6*x^2) - ((7*d - f*x^2)*(a + b*x^4)^{(3/2)})/(7*x) + (3*a*\operatorname{Sqrt}[b]*c*\operatorname{ArcTan}h[(\operatorname{Sqrt}[b]*x^2)/\operatorname{Sqrt}[a + b*x^4]])/4 - (a^{(3/2)}*e*\operatorname{ArcTan}h[\operatorname{Sqrt}[a + b*x^4]/\operatorname{Sqrt}[a]])/2 - (12*a^{(5/4)}*b^{(1/4)}*d*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(5*\operatorname{Sqrt}[a + b*x^4]) + (2*a^{(5/4)}*(21*\operatorname{Sqrt}[b]*d + 5*\operatorname{Sqrt}[a]*f)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(35*b^{(1/4)}*\operatorname{Sqrt}[a + b*x^4])$

```
rt[b]*d + 5*Sqrt[a]*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2]/(35*b^(1/4)*
Sqrt[a + b*x^4])
```

Rule 1833

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p)/c^j, {j, 0,
n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] && !PolyQ[Pq, x^(n/2)]
```

Rule 1252

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 813

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !Rational
Q[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x],
x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p
```


$_.$), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 266

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 63

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1272

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(d*(m + 4*p + 3) + e*(m + 1)*x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(4*p)/(f^2*(m + 1)*(m + 4*p + 3)), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(d*(4*p + 3) + e*(4*p + 1)*x^2)*(a + c*x^4)^p)/((4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/((4*p + 1)*(4*p + 3)), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p + 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}}{x^3} dx &= \int \left(\frac{(c + ex^2)(a + bx^4)^{3/2}}{x^3} + \frac{(d + fx^2)(a + bx^4)^{3/2}}{x^2} \right) dx \\
&= \int \frac{(c + ex^2)(a + bx^4)^{3/2}}{x^3} dx + \int \frac{(d + fx^2)(a + bx^4)^{3/2}}{x^2} dx \\
&= -\frac{(7d - fx^2)(a + bx^4)^{3/2}}{7x} + \frac{1}{2} \text{Subst} \left(\int \frac{(c + ex)(a + bx^2)^{3/2}}{x^2} dx, x, x^2 \right) - \frac{6}{7} \int \frac{(d + fx^2)(a + bx^4)^{3/2}}{x^2} dx \\
&= \frac{2}{35}x(5af + 21bdx^2)\sqrt{a + bx^4} - \frac{(3c - ex^2)(a + bx^4)^{3/2}}{6x^2} - \frac{(7d - fx^2)(a + bx^4)^{3/2}}{7x} \\
&= \frac{1}{4}(2ae + 3bcx^2)\sqrt{a + bx^4} + \frac{2}{35}x(5af + 21bdx^2)\sqrt{a + bx^4} - \frac{(3c - ex^2)(a + bx^4)^{3/2}}{6x^2} \\
&= \frac{12a\sqrt{bdx}\sqrt{a + bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} + \frac{1}{4}(2ae + 3bcx^2)\sqrt{a + bx^4} + \frac{2}{35}x(5af + 21bdx^2)\sqrt{a + bx^4} \\
&= \frac{12a\sqrt{bdx}\sqrt{a + bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} + \frac{1}{4}(2ae + 3bcx^2)\sqrt{a + bx^4} + \frac{2}{35}x(5af + 21bdx^2)\sqrt{a + bx^4} \\
&= \frac{12a\sqrt{bdx}\sqrt{a + bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} + \frac{1}{4}(2ae + 3bcx^2)\sqrt{a + bx^4} + \frac{2}{35}x(5af + 21bdx^2)\sqrt{a + bx^4} \\
&= \frac{12a\sqrt{bdx}\sqrt{a + bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} + \frac{1}{4}(2ae + 3bcx^2)\sqrt{a + bx^4} + \frac{2}{35}x(5af + 21bdx^2)\sqrt{a + bx^4}
\end{aligned}$$

Mathematica [C] time = 0.361783, size = 194, normalized size = 0.48

$$\frac{x \left(ex \sqrt{\frac{bx^4}{a}} + 1 \left(\sqrt{a + bx^4} (4a + bx^4) - 3a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + bx^4}}{\sqrt{a}} \right) \right) - 6ad \sqrt{a + bx^4} {}_2F_1 \left(-\frac{3}{2}, -\frac{1}{4}; \frac{3}{4}; -\frac{bx^4}{a} \right) + 6afx^2 \sqrt{a + bx^4} {}_2F_1 \left(-\frac{3}{2}, -\frac{1}{4}; \frac{3}{4}; -\frac{bx^4}{a} \right) \right)}{6x^2 \sqrt{\frac{bx^4}{a}} + 1}$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^3,x]

[Out] $(-3*a*c*\sqrt{a + b*x^4}*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*x^4)/a)] + x*(e*x*\sqrt{1 + (b*x^4)/a}*(\sqrt{a + b*x^4}*(4*a + b*x^4) - 3*a^{(3/2)}*ArcTanh[\sqrt{a + b*x^4}/\sqrt{a}]) - 6*a*d*\sqrt{a + b*x^4}*Hypergeometric2F1[-3/2, -1/4, 3/4, -((b*x^4)/a)] + 6*a*f*x^2*\sqrt{a + b*x^4}*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^4)/a)]))/(6*x^2*\sqrt{1 + (b*x^4)/a})$

Maple [C] time = 0.02, size = 409, normalized size = 1.

$$\frac{bfx^5}{7}\sqrt{bx^4+a} + \frac{3afx}{7}\sqrt{bx^4+a} + \frac{4fa^2}{7}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}, i\right) \frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{bx^4+a}} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^{(3/2)}/x^3, x)$

[Out] $\frac{1}{7}f*b*x^5*(b*x^4+a)^{(1/2)} + \frac{3}{7}f*a*x*(b*x^4+a)^{(1/2)} + \frac{4}{7}f*a^2/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticF}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I) + \frac{1}{6}e*b*x^4*(b*x^4+a)^{(1/2)} + \frac{2}{3}e*a*(b*x^4+a)^{(1/2)} - \frac{1}{2}e*a^{(3/2)}*\ln((2*a+2*a^{(1/2)}*(b*x^4+a)^{(1/2)})/x^2) + \frac{1}{4}c*b*x^2*(b*x^4+a)^{(1/2)} + \frac{3}{4}c*a*b^{(1/2)}*\ln(x^2*b^{(1/2)}*(b*x^4+a)^{(1/2)}) - \frac{1}{2}c*a/x^2*(b*x^4+a)^{(1/2)} - d*a*(b*x^4+a)^{(1/2)}/x + \frac{1}{5}d*b*x^3*(b*x^4+a)^{(1/2)} + \frac{12}{5}I*d*a^{(3/2)}*b^{(1/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticF}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I) - \frac{12}{5}I*d*a^{(3/2)}*b^{(1/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticE}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^{(3/2)}/x^3, x, \text{algorithm}="maxima")$

[Out] $\text{integrate}((b*x^4 + a)^{(3/2)}*(f*x^3 + e*x^2 + d*x + c)/x^3, x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bf x^7 + bex^6 + bdx^5 + bcx^4 + afx^3 + aex^2 + adx + ac)\sqrt{bx^4 + a}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^3,x, algorithm="fricas")

[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x + a*c)*sqrt(b*x^4 + a)/x^3, x)

Sympy [A] time = 9.06234, size = 377, normalized size = 0.93

$$-\frac{a^{\frac{3}{2}}c}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{1}{4}\right)_2F_1\left(-\frac{1}{2}, -\frac{1}{4}\left|\frac{bx^4e^{i\pi}}{a}\right.\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{a^{\frac{3}{2}}e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx^2}}\right)}{2} + \frac{a^{\frac{3}{2}}fx\Gamma\left(\frac{1}{4}\right)_2F_1\left(-\frac{1}{2}, \frac{1}{4}\left|\frac{bx^4e^{i\pi}}{a}\right.\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{abc}x^2\sqrt{1+\frac{bx^4}{a}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**3,x)

[Out] $-a^{3/2}c/(2x^2\sqrt{1+b*x^4/a}) + a^{3/2}d*\gamma(-1/4)*\text{hyper}((-1/2, -1/4), (3/4,), b*x^4*\exp_polar(I*\pi)/a)/(4*x*\gamma(3/4)) - a^{3/2}e*a\sinh(\sqrt{a}/(\sqrt{b}*x^2))/2 + a^{3/2}f*x*\gamma(1/4)*\text{hyper}((-1/2, 1/4), (5/4,), b*x^4*\exp_polar(I*\pi)/a)/(4*\gamma(5/4)) + \sqrt{a}*b*c*x^2*\sqrt{1+b*x^4/a}/4 - \sqrt{a}*b*c*x^2/(2*\sqrt{1+b*x^4/a}) + \sqrt{a}*b*d*x^3*\gamma(3/4)*\text{hyper}((-1/2, 3/4), (7/4,), b*x^4*\exp_polar(I*\pi)/a)/(4*\gamma(7/4)) + \sqrt{a}*b*f*x^5*\gamma(5/4)*\text{hyper}((-1/2, 5/4), (9/4,), b*x^4*\exp_polar(I*\pi)/a)/(4*\gamma(9/4)) + a^2*e/(2*\sqrt{b}*x^2*\sqrt{a/(b*x^4)+1}) + 3*a*\sqrt{b}*c*\operatorname{asinh}(\sqrt{b}*x^2/\sqrt{a})/4 + a*\sqrt{b}*e*x^2/(2*\sqrt{a/(b*x^4)+1}) + b*e*\text{Piecewise}((\sqrt{a}*x^4/4, \text{Eq}(b, 0)), ((a+b*x^4)**(3/2)/(6*b), \text{True}))$

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^3, x)
```

$$3.518 \quad \int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^4} dx$$

Optimal. Leaf size=408

$$\frac{2a^{3/4}\sqrt[4]{b}(\sqrt{a} + \sqrt{bx^2})\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}}(9\sqrt{ae} + 5\sqrt{bc})\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{15\sqrt{a+bx^4}} - \frac{12a^{5/4}\sqrt[4]{be}(\sqrt{a} + \sqrt{bx^2})\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}}}{5\sqrt{a+bx^4}}$$

[Out] $(12*a*\text{Sqrt}[b]*e*x*\text{Sqrt}[a + b*x^4])/(5*(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)) - (2*(9*a*e - 5*b*c*x^2)*\text{Sqrt}[a + b*x^4])/(15*x) + ((2*a*f + 3*b*d*x^2)*\text{Sqrt}[a + b*x^4])/4 - ((5*c - 3*e*x^2)*(a + b*x^4)^{(3/2)})/(15*x^3) - ((3*d - f*x^2)*(a + b*x^4)^{(3/2)})/(6*x^2) + (3*a*\text{Sqrt}[b]*d*\text{ArcTanh}[(\text{Sqrt}[b]*x^2)/\text{Sqrt}[a + b*x^4]])/4 - (a^{(3/2)}*f*\text{ArcTanh}[\text{Sqrt}[a + b*x^4]/\text{Sqrt}[a]])/2 - (12*a^{(5/4)}*b^{(1/4)}*e*(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)*\text{Sqrt}[(a + b*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(5*\text{Sqrt}[a + b*x^4]) + (2*a^{(3/4)}*b^{(1/4)}*(5*\text{Sqrt}[b]*c + 9*\text{Sqrt}[a]*e)*(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)*\text{Sqrt}[(a + b*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(15*\text{Sqrt}[a + b*x^4])$

Rubi [A] time = 0.335922, antiderivative size = 408, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1833, 1272, 1198, 220, 1196, 1252, 813, 815, 844, 217, 206, 266, 63, 208}

$$\frac{2a^{3/4}\sqrt[4]{b}(\sqrt{a} + \sqrt{bx^2})\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}}(9\sqrt{ae} + 5\sqrt{bc})F\left(2\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{15\sqrt{a+bx^4}} - \frac{12a^{5/4}\sqrt[4]{be}(\sqrt{a} + \sqrt{bx^2})\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^4, x]

[Out] $(12*a*\text{Sqrt}[b]*e*x*\text{Sqrt}[a + b*x^4])/(5*(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)) - (2*(9*a*e - 5*b*c*x^2)*\text{Sqrt}[a + b*x^4])/(15*x) + ((2*a*f + 3*b*d*x^2)*\text{Sqrt}[a + b*x^4])/4 - ((5*c - 3*e*x^2)*(a + b*x^4)^{(3/2)})/(15*x^3) - ((3*d - f*x^2)*(a + b*x^4)^{(3/2)})/(6*x^2) + (3*a*\text{Sqrt}[b]*d*\text{ArcTanh}[(\text{Sqrt}[b]*x^2)/\text{Sqrt}[a + b*x^4]])/4 - (a^{(3/2)}*f*\text{ArcTanh}[\text{Sqrt}[a + b*x^4]/\text{Sqrt}[a]])/2 - (12*a^{(5/4)}*b^{(1/4)}*e*(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)*\text{Sqrt}[(a + b*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(5*\text{Sqrt}[a + b*x^4]) + (2*a^{(3/4)}*b^{(1/4)}*(5*\text{Sqrt}[b]*c + 9*\text{Sqrt}[a]*e)*(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)*\text{Sqrt}[(a + b*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(15*\text{Sqrt}[a + b*x^4])$

$$\frac{b^{1/4} \cdot (5\sqrt{b}c + 9\sqrt{a}e) \cdot (\sqrt{a} + \sqrt{b}x^2) \cdot \sqrt{(a + b^2x^4)}}{(\sqrt{a} + \sqrt{b}x^2)^2 \cdot \text{EllipticF}[2\text{ArcTan}[(b^{1/4}x)/a^{1/4}], 1/2]} \\ / (15\sqrt{a + b^2x^4})$$
Rule 1833

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p)/c^j, {j, 0,
n/2 - 1}], x] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] && !PolyQ[Pq, x^(n/2)]
```

Rule 1272

```
Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x
_Symbol] := Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(d*(m + 4*p + 3) + e*(m + 1)*
x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(4*p)/(f^2*(m + 1)*(m + 4*p + 3
)), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*
x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x],
1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rule 1252


```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol]
:> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x]
/; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 813

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_)
, x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x]
+ Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x], x]
/; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !RationalQ[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_)
, x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x]
+ Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
/; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_)
, x_Symbol]
:> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol]
:> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]]
/; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol]
:> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x]
/; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

$Q[a, 0] \parallel \text{LtQ}[b, 0]$)

Rule 266

$\text{Int}[(x_)^{(m_.)} * ((a_) + (b_.) * (x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1) * (a + b*x)^p, x}], x, x^n], x] /; \text{FreeQ}[\{a, b, m, n, p\}, x] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 63

$\text{Int}[(a_.) + (b_.) * (x_)^{(m_.)} * ((c_.) + (d_.) * (x_)^{(n_.)}), x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m + 1) - 1)} * (c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 208

$\text{Int}[(a_) + (b_.) * (x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2] * \text{ArcTanh}[x / \text{Rt}[-(a/b), 2]]) / a, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

Rubi steps

$$\begin{aligned}
\int \frac{(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}}{x^4} dx &= \int \left(\frac{(c + ex^2)(a + bx^4)^{3/2}}{x^4} + \frac{(d + fx^2)(a + bx^4)^{3/2}}{x^3} \right) dx \\
&= \int \frac{(c + ex^2)(a + bx^4)^{3/2}}{x^4} dx + \int \frac{(d + fx^2)(a + bx^4)^{3/2}}{x^3} dx \\
&= -\frac{(5c - 3ex^2)(a + bx^4)^{3/2}}{15x^3} - \frac{2}{5} \int \frac{(-3ae - 5bcx^2)\sqrt{a + bx^4}}{x^2} dx + \frac{1}{2} \text{Subst} \left(\int \right) \\
&= -\frac{2(9ae - 5bcx^2)\sqrt{a + bx^4}}{15x} - \frac{(5c - 3ex^2)(a + bx^4)^{3/2}}{15x^3} - \frac{(3d - fx^2)(a + bx^4)}{6x^2} \\
&= -\frac{2(9ae - 5bcx^2)\sqrt{a + bx^4}}{15x} + \frac{1}{4}(2af + 3bdx^2)\sqrt{a + bx^4} - \frac{(5c - 3ex^2)(a + b)}{15x^3} \\
&= \frac{12a\sqrt{bex}\sqrt{a + bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} - \frac{2(9ae - 5bcx^2)\sqrt{a + bx^4}}{15x} + \frac{1}{4}(2af + 3bdx^2)\sqrt{a + bx^4} \\
&= \frac{12a\sqrt{bex}\sqrt{a + bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} - \frac{2(9ae - 5bcx^2)\sqrt{a + bx^4}}{15x} + \frac{1}{4}(2af + 3bdx^2)\sqrt{a + bx^4} \\
&= \frac{12a\sqrt{bex}\sqrt{a + bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} - \frac{2(9ae - 5bcx^2)\sqrt{a + bx^4}}{15x} + \frac{1}{4}(2af + 3bdx^2)\sqrt{a + bx^4} \\
&= \frac{12a\sqrt{bex}\sqrt{a + bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} - \frac{2(9ae - 5bcx^2)\sqrt{a + bx^4}}{15x} + \frac{1}{4}(2af + 3bdx^2)\sqrt{a + bx^4}
\end{aligned}$$

Mathematica [C] time = 0.357646, size = 194, normalized size = 0.48

$$\frac{x^2 \left(fx \sqrt{\frac{bx^4}{a}} + 1 \left(\sqrt{a + bx^4} (4a + bx^4) - 3a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a+bx^4}}{\sqrt{a}} \right) \right) - 6ae\sqrt{a + bx^4} {}_2F_1 \left(-\frac{3}{2}, -\frac{1}{4}; \frac{3}{4}; -\frac{bx^4}{a} \right) - 2ac\sqrt{a + bx^4} {}_2F_1 \left(-\frac{3}{2}, -\frac{1}{4}; \frac{3}{4}; -\frac{bx^4}{a} \right) \right)}{6x^3 \sqrt{\frac{bx^4}{a}} + 1}$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^4,x]

[Out] $(-2*a*c*\sqrt{a + b*x^4}*Hypergeometric2F1[-3/2, -3/4, 1/4, -((b*x^4)/a)] - 3*a*d*x*\sqrt{a + b*x^4}*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*x^4)/a)] + x^2*(f*x*\sqrt{1 + (b*x^4)/a}*(\sqrt{a + b*x^4}*(4*a + b*x^4) - 3*a^{(3/2)}*ArcTanh[\sqrt{a + b*x^4}/\sqrt{a}]) - 6*a*e*\sqrt{a + b*x^4}*Hypergeometric2F1[-3/2, -1/4, 3/4, -((b*x^4)/a)])) / (6*x^3*\sqrt{1 + (b*x^4)/a})$

Maple [C] time = 0.014, size = 408, normalized size = 1.

$$\frac{bfx^4}{6}\sqrt{bx^4+a} + \frac{2af}{3}\sqrt{bx^4+a} - \frac{f}{2}a^{\frac{3}{2}}\ln\left(\frac{1}{x^2}\left(2a + 2\sqrt{a}\sqrt{bx^4+a}\right)\right) - \frac{ac}{3x^3}\sqrt{bx^4+a} + \frac{bcx}{3}\sqrt{bx^4+a} + \frac{4abc}{3}\sqrt{1-ix^2\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^{(3/2)}/x^4,x)$

[Out] $1/6*f*b*x^4*(b*x^4+a)^{(1/2)}+2/3*f*a*(b*x^4+a)^{(1/2)}-1/2*f*a^{(3/2)}*\ln((2*a+2*a^{(1/2)}*(b*x^4+a)^{(1/2)})/x^2)-1/3*c*a*(b*x^4+a)^{(1/2)}/x^3+1/3*c*b*x*(b*x^4+a)^{(1/2)}+4/3*c*a*b/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*EllipticF(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)},I)+1/4*d*b*x^2*(b*x^4+a)^{(1/2)}+3/4*d*a*b^{(1/2)}*\ln(x^2*b^{(1/2)}+(b*x^4+a)^{(1/2)})-1/2*d*a/x^2*(b*x^4+a)^{(1/2)}-e*a*(b*x^4+a)^{(1/2)}/x+1/5*e*b*x^3*(b*x^4+a)^{(1/2)}+12/5*I*e*a^{(3/2)}*b^{(1/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*EllipticF(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)},I)-12/5*I*e*a^{(3/2)}*b^{(1/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*EllipticE(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)},I)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^{(3/2)}/x^4,x, \text{algorithm}="maxima")$

[Out] $\text{integrate}((b*x^4 + a)^{(3/2)}*(f*x^3 + e*x^2 + d*x + c)/x^4, x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bf x^7 + bex^6 + bdx^5 + bcx^4 + afx^3 + aex^2 + adx + ac)\sqrt{bx^4 + a}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^4,x, algorithm="fricas")

[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x + a*c)*sqrt(b*x^4 + a)/x^4, x)

Sympy [A] time = 9.12796, size = 381, normalized size = 0.93

$$\frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{3}{4}\right)_2F_1\left(-\frac{3}{4}, -\frac{1}{2}\left|\frac{bx^4e^{i\pi}}{a}\right.\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{a^{\frac{3}{2}}d}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{1}{4}\right)_2F_1\left(-\frac{1}{2}, -\frac{1}{4}\left|\frac{bx^4e^{i\pi}}{a}\right.\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{a^{\frac{3}{2}}f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx^2}}\right)}{2} + \frac{\sqrt{abc}x\Gamma\left(\frac{1}{4}\right)_2F_1\left(\frac{1}{4}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**4,x)

[Out] a**(3/2)*c*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - a**(3/2)*d/(2*x**2*sqrt(1 + b*x**4/a)) + a**(3/2)*e*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - a**(3/2)*f*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + sqrt(a)*b*c*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + sqrt(a)*b*d*x**2*sqrt(1 + b*x**4/a)/4 - sqrt(a)*b*d*x**2/(2*sqrt(1 + b*x**4/a)) + sqrt(a)*b*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + a**2*f/(2*sqrt(b)*x**2*sqrt(a/(b*x**4) + 1)) + 3*a*sqrt(b)*d*asinh(sqrt(b)*x**2/sqrt(a))/4 + a*sqrt(b)*f*x**2/(2*sqrt(a/(b*x**4) + 1)) + b*f*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^4,x, algorithm="giac")

[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^4, x)

$$3.519 \quad \int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^5} dx$$

Optimal. Leaf size=386

$$\frac{2a^{3/4}\sqrt[4]{b}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (9\sqrt{a}f + 5\sqrt{bd}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{15\sqrt{a+bx^4}} - \frac{12a^{5/4}\sqrt[4]{b}f(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}}}{5\sqrt{a+bx^4}}$$

[Out] $(12*a*\operatorname{Sqrt}[b]*f*x*\operatorname{Sqrt}[a + b*x^4])/(5*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) + (3*b*(c + e*x^2)*\operatorname{Sqrt}[a + b*x^4])/4 + (2*b*x*(5*d + 9*f*x^2)*\operatorname{Sqrt}[a + b*x^4])/15 - ((3*c)/x^4 + (4*d)/x^3 + (6*e)/x^2 + (12*f)/x)*(a + b*x^4)^{(3/2)}/12 + (3*a*\operatorname{Sqrt}[b]*e*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*x^2)/\operatorname{Sqrt}[a + b*x^4]])/4 - (3*\operatorname{Sqrt}[a]*b*c*\operatorname{ArcTan}[\operatorname{Sqrt}[a + b*x^4]/\operatorname{Sqrt}[a]])/4 - (12*a^{(5/4)}*b^{(1/4)}*f*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(5*\operatorname{Sqrt}[a + b*x^4]) + (2*a^{(3/4)}*b^{(1/4)}*(5*\operatorname{Sqrt}[b]*d + 9*\operatorname{Sqrt}[a]*f)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(15*\operatorname{Sqrt}[a + b*x^4])$

Rubi [A] time = 0.346981, antiderivative size = 386, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 15, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {14, 1825, 1833, 1252, 815, 844, 217, 206, 266, 63, 208, 1177, 1198, 220, 1196}

$$\frac{2a^{3/4}\sqrt[4]{b}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (9\sqrt{a}f + 5\sqrt{bd}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{15\sqrt{a+bx^4}} - \frac{12a^{5/4}\sqrt[4]{b}f(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{5\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^{(3/2)}/x^5, x]$

[Out] $(12*a*\operatorname{Sqrt}[b]*f*x*\operatorname{Sqrt}[a + b*x^4])/(5*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) + (3*b*(c + e*x^2)*\operatorname{Sqrt}[a + b*x^4])/4 + (2*b*x*(5*d + 9*f*x^2)*\operatorname{Sqrt}[a + b*x^4])/15 - ((3*c)/x^4 + (4*d)/x^3 + (6*e)/x^2 + (12*f)/x)*(a + b*x^4)^{(3/2)}/12 + (3*a*\operatorname{Sqrt}[b]*e*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*x^2)/\operatorname{Sqrt}[a + b*x^4]])/4 - (3*\operatorname{Sqrt}[a]*b*c*\operatorname{ArcTan}[\operatorname{Sqrt}[a + b*x^4]/\operatorname{Sqrt}[a]])/4 - (12*a^{(5/4)}*b^{(1/4)}*f*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(5*\operatorname{Sqrt}[a + b*x^4]) + (2*a^{(3/4)}*b^{(1/4)}*(5*\operatorname{Sqrt}[b]*d + 9*\operatorname{Sqrt}[a]*f)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(15*\operatorname{Sqrt}[a + b*x^4])$

$2)^2 * \text{EllipticF}[2 * \text{ArcTan}[(b^{1/4} * x) / a^{1/4}], 1/2] / (15 * \text{Sqrt}[a + b * x^4])$

Rule 14

$\text{Int}[(u_*) * ((c_*) * (x_*)^{(m_*)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c * x)^m * u, x], x] /;$ $\text{FreeQ}[\{c, m\}, x] \ \&\& \ \text{SumQ}[u] \ \&\& \ !\text{LinearQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (a_*) + (b_*) * (v_*)] /;$ $\text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{InverseFunctionQ}[v]$

Rule 1825

$\text{Int}[(Pq_*) * (x_*)^{(m_*)} * ((a_*) + (b_*) * (x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Module}[\{u = \text{IntHide}[x^m * Pq, x]\}, \text{Simp}[u * (a + b * x^n)^p, x] - \text{Dist}[b * n * p, \text{Int}[x^{(m+n)} * (a + b * x^n)^{(p-1)} * \text{ExpandToSum}[u/x^{(m+1)}, x], x], x]] /;$ $\text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{LtQ}[m + \text{Expon}[Pq, x] + 1, 0]$

Rule 1833

$\text{Int}[(Pq_*) * ((c_*) * (x_*)^{(m_*)} * ((a_*) + (b_*) * (x_*)^{(n_*)})^{(p_*)}), x_Symbol] \rightarrow \text{Module}[\{q = \text{Expon}[Pq, x], j, k\}, \text{Int}[\text{Sum}[(c * x)^{(m+j)} * \text{Sum}[\text{Coeff}[Pq, x, j + (k * n) / 2] * x^{(k * n) / 2}, \{k, 0, (2 * (q - j)) / n + 1\}] * (a + b * x^n)^p / c^j, \{j, 0, n / 2 - 1\}], x]] /;$ $\text{FreeQ}[\{a, b, c, m, p\}, x] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{IGtQ}[n / 2, 0] \ \&\& \ !\text{PolyQ}[Pq, x^{(n/2)}]$

Rule 1252

$\text{Int}[(x_*)^{(m_*)} * ((d_*) + (e_*) * (x_*)^2)^{(q_*)} * ((a_*) + (c_*) * (x_*)^4)^{(p_*)}, x_Symbol] \rightarrow \text{Dist}[1/2, \text{Subst}[\text{Int}[x^{((m-1)/2)} * (d + e * x)^q * (a + c * x^2)^p, x], x, x^2], x] /;$ $\text{FreeQ}[\{a, c, d, e, p, q\}, x] \ \&\& \ \text{IntegerQ}[(m + 1) / 2]$

Rule 815

$\text{Int}[(d_*) + (e_*) * (x_*)^{(m_*)} * ((f_*) + (g_*) * (x_*) * ((a_*) + (c_*) * (x_*)^2)^{(p_*)}), x_Symbol] \rightarrow \text{Simp}[(d + e * x)^{(m+1)} * (c * e * f * (m + 2 * p + 2) - g * c * d * (2 * p + 1) + g * c * e * (m + 2 * p + 1) * x) * (a + c * x^2)^p / (c * e^2 * (m + 2 * p + 1) * (m + 2 * p + 2)), x] + \text{Dist}[(2 * p) / (c * e^2 * (m + 2 * p + 1) * (m + 2 * p + 2)), \text{Int}[(d + e * x)^m * (a + c * x^2)^{(p-1)} * \text{Simp}[f * a * c * e^2 * (m + 2 * p + 2) + a * c * d * e * g * m - (c^2 * f * d * e * (m + 2 * p + 2) - g * (c^2 * d^2 * (2 * p + 1) + a * c * e^2 * (m + 2 * p + 1))) * x, x], x], x] /;$ $\text{FreeQ}[\{a, c, d, e, f, g, m\}, x] \ \&\& \ \text{NeQ}[c * d^2 + a * e^2, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ (\text{IntegerQ}[p] \ || \ !\text{RationalQ}[m] \ || \ (\text{GeQ}[m, -1] \ \&\& \ \text{LtQ}[m, 0])) \ \&\& \ !\text{IntegerQ}[m + 2 * p, 0] \ \&\& \ (\text{IntegerQ}[m] \ || \ \text{IntegerQ}[p] \ || \ \text{IntegersQ}[2 * m, 2 * p])$

Rule 844


```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(d*(4*p + 3) + e*(4*p + 1)*x^2)*(a + c*x^4)^p)/((4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/((4*p + 1)*(4*p + 3)), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p + 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
  Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
  Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}}{x^5} dx &= -\frac{1}{12} \left(\frac{3c}{x^4} + \frac{4d}{x^3} + \frac{6e}{x^2} + \frac{12f}{x} \right) (a + bx^4)^{3/2} - (6b) \int \frac{\left(-\frac{c}{4} - \frac{dx}{3} - \frac{ex^2}{2} - fx^3 \right) \sqrt{a + bx^4}}{x} dx \\
&= -\frac{1}{12} \left(\frac{3c}{x^4} + \frac{4d}{x^3} + \frac{6e}{x^2} + \frac{12f}{x} \right) (a + bx^4)^{3/2} - (6b) \int \left(\frac{\left(-\frac{c}{4} - \frac{ex^2}{2} \right) \sqrt{a + bx^4}}{x} + \left(-\frac{dx}{3} - fx^3 \right) \sqrt{a + bx^4} \right) dx \\
&= -\frac{1}{12} \left(\frac{3c}{x^4} + \frac{4d}{x^3} + \frac{6e}{x^2} + \frac{12f}{x} \right) (a + bx^4)^{3/2} - (6b) \int \frac{\left(-\frac{c}{4} - \frac{ex^2}{2} \right) \sqrt{a + bx^4}}{x} dx - \frac{2}{15} bx (5d + 9fx^2) \sqrt{a + bx^4} \\
&= \frac{2}{15} bx (5d + 9fx^2) \sqrt{a + bx^4} - \frac{1}{12} \left(\frac{3c}{x^4} + \frac{4d}{x^3} + \frac{6e}{x^2} + \frac{12f}{x} \right) (a + bx^4)^{3/2} - \frac{1}{5} (2b) \int \frac{\left(-\frac{c}{4} - \frac{ex^2}{2} \right) \sqrt{a + bx^4}}{x} dx \\
&= \frac{3}{4} b (c + ex^2) \sqrt{a + bx^4} + \frac{2}{15} bx (5d + 9fx^2) \sqrt{a + bx^4} - \frac{1}{12} \left(\frac{3c}{x^4} + \frac{4d}{x^3} + \frac{6e}{x^2} + \frac{12f}{x} \right) (a + bx^4)^{3/2} \\
&= \frac{12a\sqrt{b}fx\sqrt{a + bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} + \frac{3}{4} b (c + ex^2) \sqrt{a + bx^4} + \frac{2}{15} bx (5d + 9fx^2) \sqrt{a + bx^4} - \frac{1}{12} \left(\frac{3c}{x^4} + \frac{4d}{x^3} + \frac{6e}{x^2} + \frac{12f}{x} \right) (a + bx^4)^{3/2} \\
&= \frac{12a\sqrt{b}fx\sqrt{a + bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} + \frac{3}{4} b (c + ex^2) \sqrt{a + bx^4} + \frac{2}{15} bx (5d + 9fx^2) \sqrt{a + bx^4} - \frac{1}{12} \left(\frac{3c}{x^4} + \frac{4d}{x^3} + \frac{6e}{x^2} + \frac{12f}{x} \right) (a + bx^4)^{3/2} \\
&= \frac{12a\sqrt{b}fx\sqrt{a + bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} + \frac{3}{4} b (c + ex^2) \sqrt{a + bx^4} + \frac{2}{15} bx (5d + 9fx^2) \sqrt{a + bx^4} - \frac{1}{12} \left(\frac{3c}{x^4} + \frac{4d}{x^3} + \frac{6e}{x^2} + \frac{12f}{x} \right) (a + bx^4)^{3/2} \\
&= \frac{12a\sqrt{b}fx\sqrt{a + bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} + \frac{3}{4} b (c + ex^2) \sqrt{a + bx^4} + \frac{2}{15} bx (5d + 9fx^2) \sqrt{a + bx^4} - \frac{1}{12} \left(\frac{3c}{x^4} + \frac{4d}{x^3} + \frac{6e}{x^2} + \frac{12f}{x} \right) (a + bx^4)^{3/2}
\end{aligned}$$

Mathematica [C] time = 0.218816, size = 163, normalized size = 0.42

$$\frac{\sqrt{a + bx^4} \left(3x \left(-5a^3 e {}_2F_1 \left(-\frac{3}{2}, -\frac{1}{2}; \frac{1}{2}; -\frac{bx^4}{a} \right) - 10a^3 f x {}_2F_1 \left(-\frac{3}{2}, -\frac{1}{4}; \frac{3}{4}; -\frac{bx^4}{a} \right) \right) + bcx^2 (a + bx^4)^2 \sqrt{\frac{bx^4}{a} + 1} {}_2F_1 \left(2, \frac{5}{2}; \frac{7}{2}; \frac{bx^4}{a} \right) \right)}{30a^2 x^3 \sqrt{\frac{bx^4}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^5,x]

[Out] (Sqrt[a + b*x^4]*(-10*a^3*d*Hypergeometric2F1[-3/2, -3/4, 1/4, -((b*x^4)/a)] + 3*x*(-5*a^3*e*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*x^4)/a)] - 10*a^3*f*x*Hypergeometric2F1[-3/2, -1/4, 3/4, -((b*x^4)/a)] + b*c*x^2*(a + b*x^4)^2*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^4)/a]))/(30*a^2*x^3*Sqrt[1 + (b*x^4)/a])

Maple [C] time = 0.021, size = 409, normalized size = 1.1

$$-\frac{ad}{3x^3}\sqrt{bx^4+a} + \frac{bdx}{3}\sqrt{bx^4+a} + \frac{4bda}{3}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}} + \frac{b}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^5,x)

[Out] -1/3*d*a*(b*x^4+a)^(1/2)/x^3+1/3*d*b*x*(b*x^4+a)^(1/2)+4/3*d*a*b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*c*b*(b*x^4+a)^(1/2)-3/4*c*a^(1/2)*b*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/4*c*a/x^4*(b*x^4+a)^(1/2)+1/4*e*b*x^2*(b*x^4+a)^(1/2)+3/4*e*a*b^(1/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))-1/2*e*a/x^2*(b*x^4+a)^(1/2)-f*a*(b*x^4+a)^(1/2)/x+1/5*f*b*x^3*(b*x^4+a)^(1/2)+12/5*I*f*a^(3/2)*b^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-12/5*I*f*a^(3/2)*b^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^5,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bf x^7 + bex^6 + bdx^5 + bcx^4 + afx^3 + aex^2 + adx + ac)\sqrt{bx^4 + a}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^5,x, algorithm="fricas")

[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x + a*c)*sqrt(b*x^4 + a)/x^5, x)

Sympy [C] time = 10.7074, size = 379, normalized size = 0.98

$$\frac{a^{\frac{3}{2}} d \Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(-\frac{3}{4}, -\frac{1}{2} \left| \frac{bx^4 e^{i\pi}}{a} \right. \right)}{4x^3 \Gamma\left(\frac{1}{4}\right)} - \frac{a^{\frac{3}{2}} e}{2x^2 \sqrt{1 + \frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}} f \Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(-\frac{1}{2}, -\frac{1}{4} \left| \frac{bx^4 e^{i\pi}}{a} \right. \right)}{4x \Gamma\left(\frac{3}{4}\right)} - \frac{3\sqrt{abc} \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx^2}}\right)}{4} + \frac{\sqrt{abdx} \Gamma\left(\frac{1}{4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**5,x)

[Out] a**(3/2)*d*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - a**(3/2)*e/(2*x**2*sqrt(1 + b*x**4/a)) + a**(3/2)*f*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - 3*sqrt(a)*b*c*asinh(sqrt(a)/(sqrt(b)*x**2))/4 + sqrt(a)*b*d*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + sqrt(a)*b*e*x**2*sqrt(1 + b*x**4/a)/4 - sqrt(a)*b*e*x**2/(2*sqrt(1 + b*x**4/a)) + sqrt(a)*b*f*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) - a*sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(4*x**2) + a*sqrt(b)*c/(2*x**2*sqrt(a/(b*x**4) + 1)) + 3*a*sqrt(b)*e*asinh(sqrt(b)*x**2/sqrt(a))/4 + b**(3/2)*c*x**2/(2*sqrt(a/(b*x**4) + 1))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^5,x, algorithm="giac")

[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^5, x)

$$3.520 \quad \int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^6} dx$$

Optimal. Leaf size=387

$$\frac{2\sqrt[4]{ab^3/4}(\sqrt{a} + \sqrt{bx^2})\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}}(5\sqrt{ae} + 9\sqrt{bc})\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{15\sqrt{a+bx^4}} + \frac{12b^{3/2}cx\sqrt{a+bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} - \frac{12\sqrt[4]{ab^5/4}c(\sqrt{a} + \sqrt{bx^2})}{5(\sqrt{a} + \sqrt{bx^2})}$$

[Out] (12*b^(3/2)*c*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) - (2*b*(9*c - 5*e*x^2)*Sqrt[a + b*x^4])/(15*x) + (3*b*(d + f*x^2)*Sqrt[a + b*x^4])/4 - ((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2)*(a + b*x^4)^(3/2)/60 + (3*a*Sqrt[b]*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/4 - (3*Sqrt[a]*b*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/4 - (12*a^(1/4)*b^(5/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(1/4)*b^(3/4)*(9*Sqrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*Sqrt[a + b*x^4])

Rubi [A] time = 0.349002, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 15, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {14, 1825, 1833, 1272, 1198, 220, 1196, 1252, 815, 844, 217, 206, 266, 63, 208}

$$\frac{2\sqrt[4]{ab^3/4}(\sqrt{a} + \sqrt{bx^2})\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}}(5\sqrt{ae} + 9\sqrt{bc})F\left(2\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{15\sqrt{a+bx^4}} + \frac{12b^{3/2}cx\sqrt{a+bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} - \frac{12\sqrt[4]{ab^5/4}c(\sqrt{a} + \sqrt{bx^2})}{5(\sqrt{a} + \sqrt{bx^2})}$$

Antiderivative was successfully verified.

[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^6, x]

[Out] (12*b^(3/2)*c*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) - (2*b*(9*c - 5*e*x^2)*Sqrt[a + b*x^4])/(15*x) + (3*b*(d + f*x^2)*Sqrt[a + b*x^4])/4 - ((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2)*(a + b*x^4)^(3/2)/60 + (3*a*Sqrt[b]*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/4 - (3*Sqrt[a]*b*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/4 - (12*a^(1/4)*b^(5/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(1/4)*b^(3/4)*(9*Sqrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*Sqrt[a + b*x^4])

$*c + 5*\text{Sqrt}[a]*e)*(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)*\text{Sqrt}[(a + b*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2]/(15*\text{Sqrt}[a + b*x^4])$

Rule 14

$\text{Int}[(u_*)((c_.)*(x_))^{(m_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*u, x], x] /; \text{FreeQ}\{c, m\}, x] \ \&\& \ \text{SumQ}[u] \ \&\& \ !\text{LinearQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (a_ + (b_.)*(v_)] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{InverseFunctionQ}[v]$

Rule 1825

$\text{Int}[(Pq_)*(x_)^{(m_.)}*((a_ + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Module}\{u = \text{IntHide}[x^m*Pq, x]\}, \text{Simp}[u*(a + b*x^n)^p, x] - \text{Dist}[b*n*p, \text{Int}[x^{(m+n)}*(a + b*x^n)^{(p-1)}*\text{ExpandToSum}[u/x^{(m+1)}, x], x], x]] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{LtQ}[m + \text{Expon}[Pq, x] + 1, 0]$

Rule 1833

$\text{Int}[(Pq_)*((c_.)*(x_))^{(m_.)}*((a_ + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Module}\{q = \text{Expon}[Pq, x], j, k\}, \text{Int}[\text{Sum}[(c*x)^{(m+j)}*\text{Sum}[\text{Coeff}[Pq, x, j + (k*n)/2]*x^{((k*n)/2)}, \{k, 0, (2*(q-j))/n + 1\}]* (a + b*x^n)^p]/c^j, \{j, 0, n/2 - 1\}], x]] /; \text{FreeQ}\{a, b, c, m, p\}, x] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{IGtQ}[n/2, 0] \ \&\& \ !\text{PolyQ}[Pq, x^{(n/2)}]$

Rule 1272

$\text{Int}[((f_.)*(x_))^{(m_.)}*((d_ + (e_.)*(x_)^2)*((a_ + (c_.)*(x_)^4)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}*(a + c*x^4)^p*(d*(m + 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3)), x] + \text{Dist}[(4*p)/(f^2*(m + 1)*(m + 4*p + 3)), \text{Int}[(f*x)^{(m+2)}*(a + c*x^4)^{(p-1)}*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x^2), x], x] /; \text{FreeQ}\{a, c, d, e, f\}, x] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ m + 4*p + 3 \neq 0 \ \&\& \ \text{IntegerQ}[2*p] \ \&\& \ (\text{IntegerQ}[p] \ || \ \text{IntegerQ}[m])$

Rule 1198

$\text{Int}[((d_ + (e_.)*(x_)^2)/\text{Sqrt}[(a_ + (c_.)*(x_)^4], x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[c/a, 2]\}, \text{Dist}[(e + d*q)/q, \text{Int}[1/\text{Sqrt}[a + c*x^4], x], x] - \text{Dist}[e/q, \text{Int}[(1 - q*x^2)/\text{Sqrt}[a + c*x^4], x], x] /; \text{NeQ}[e + d*q, 0]] /; \text{FreeQ}\{a, c, d, e\}, x] \ \&\& \ \text{PosQ}[c/a]$

Rule 220


```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
/; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}}{x^6} dx &= -\frac{1}{60} \left(\frac{12c}{x^5} + \frac{15d}{x^4} + \frac{20e}{x^3} + \frac{30f}{x^2} \right) (a + bx^4)^{3/2} - (6b) \int \frac{\left(-\frac{c}{5} - \frac{dx}{4} - \frac{ex^2}{3} - \frac{fx^3}{2} \right)}{x^2} \\
&= -\frac{1}{60} \left(\frac{12c}{x^5} + \frac{15d}{x^4} + \frac{20e}{x^3} + \frac{30f}{x^2} \right) (a + bx^4)^{3/2} - (6b) \int \frac{\left(-\frac{c}{5} - \frac{ex^2}{3} \right) \sqrt{a + bx^4}}{x^2} \\
&= -\frac{1}{60} \left(\frac{12c}{x^5} + \frac{15d}{x^4} + \frac{20e}{x^3} + \frac{30f}{x^2} \right) (a + bx^4)^{3/2} - (6b) \int \frac{\left(-\frac{c}{5} - \frac{ex^2}{3} \right) \sqrt{a + bx^4}}{x^2} dx \\
&= -\frac{2b(9c - 5ex^2) \sqrt{a + bx^4}}{15x} - \frac{1}{60} \left(\frac{12c}{x^5} + \frac{15d}{x^4} + \frac{20e}{x^3} + \frac{30f}{x^2} \right) (a + bx^4)^{3/2} - (3b) \\
&= -\frac{2b(9c - 5ex^2) \sqrt{a + bx^4}}{15x} + \frac{3}{4} b (d + fx^2) \sqrt{a + bx^4} - \frac{1}{60} \left(\frac{12c}{x^5} + \frac{15d}{x^4} + \frac{20e}{x^3} \right) \\
&= \frac{12b^{3/2} cx \sqrt{a + bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} - \frac{2b(9c - 5ex^2) \sqrt{a + bx^4}}{15x} + \frac{3}{4} b (d + fx^2) \sqrt{a + bx^4} - \frac{1}{60} \\
&= \frac{12b^{3/2} cx \sqrt{a + bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} - \frac{2b(9c - 5ex^2) \sqrt{a + bx^4}}{15x} + \frac{3}{4} b (d + fx^2) \sqrt{a + bx^4} - \frac{1}{60} \\
&= \frac{12b^{3/2} cx \sqrt{a + bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} - \frac{2b(9c - 5ex^2) \sqrt{a + bx^4}}{15x} + \frac{3}{4} b (d + fx^2) \sqrt{a + bx^4} - \frac{1}{60} \\
&= \frac{12b^{3/2} cx \sqrt{a + bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} - \frac{2b(9c - 5ex^2) \sqrt{a + bx^4}}{15x} + \frac{3}{4} b (d + fx^2) \sqrt{a + bx^4} - \frac{1}{60}
\end{aligned}$$

Mathematica [C] time = 0.223091, size = 165, normalized size = 0.43

$$\frac{\sqrt{a + bx^4} \left(-6a^3 c {}_2F_1 \left(-\frac{3}{2}, -\frac{5}{4}; -\frac{1}{4}; -\frac{bx^4}{a} \right) - 10a^3 ex^2 {}_2F_1 \left(-\frac{3}{2}, -\frac{3}{4}; \frac{1}{4}; -\frac{bx^4}{a} \right) - 15a^3 fx^3 {}_2F_1 \left(-\frac{3}{2}, -\frac{1}{2}; \frac{1}{2}; -\frac{bx^4}{a} \right) + 3bdx^5 (a + b) \right)}{30a^2 x^5 \sqrt{\frac{bx^4}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^6,x]

[Out] (Sqrt[a + b*x^4]*(-6*a^3*c*Hypergeometric2F1[-3/2, -5/4, -1/4, -((b*x^4)/a)] - 10*a^3*e*x^2*Hypergeometric2F1[-3/2, -3/4, 1/4, -((b*x^4)/a)] - 15*a^3*f*x^3*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*x^4)/a)] + 3*b*d*x^5*(a + b*x^4)^2*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^4)/a]))/(30*a^2*x^5*Sqrt[1 + (b*x^4)/a])

Maple [C] time = 0.015, size = 409, normalized size = 1.1

$$-\frac{ae}{3x^3}\sqrt{bx^4+a} + \frac{bxe}{3}\sqrt{bx^4+a} + \frac{4aeb}{3}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}} - \frac{a}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^6,x)

[Out] -1/3*e*a*(b*x^4+a)^(1/2)/x^3+1/3*e*b*x*(b*x^4+a)^(1/2)+4/3*e*a*b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/5*c*a*(b*x^4+a)^(1/2)/x^5-7/5*c*b*(b*x^4+a)^(1/2)/x+12/5*I*c*b^(3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-12/5*I*c*b^(3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*d*b*(b*x^4+a)^(1/2)-3/4*d*a^(1/2)*b*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/4*d*a/x^4*(b*x^4+a)^(1/2)+1/4*f*b*x^2*(b*x^4+a)^(1/2)+3/4*f*a*b^(1/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))-1/2*f*a/x^2*(b*x^4+a)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^6,x, algorithm="maxima")

[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bf x^7 + bex^6 + bdx^5 + bcx^4 + afx^3 + aex^2 + adx + ac)\sqrt{bx^4 + a}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^6,x, algorithm="fricas")

[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x + a*c)*sqrt(b*x^4 + a)/x^6, x)

Sympy [C] time = 11.0208, size = 386, normalized size = 1.

$$\frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{5}{4}\right) {}_2F_1\left(-\frac{5}{4}, -\frac{1}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(-\frac{3}{4}, -\frac{1}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{a^{\frac{3}{2}}f}{2x^2\sqrt{1 + \frac{bx^4}{a}}} + \frac{\sqrt{abc}\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(-\frac{1}{4}, -\frac{1}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - 3\sqrt{a} \frac{d}{\sqrt{bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**6,x)

[Out] a**(3/2)*c*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) + a**(3/2)*e*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - a**(3/2)*f/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*b*c*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - 3*sqrt(a)*b*d*asinh(sqrt(a)/(sqrt(b)*x**2))/4 + sqrt(a)*b*e*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + sqrt(a)*b*f*x**2*sqrt(1 + b*x**4/a)/4 - sqrt(a)*b*f*x**2/(2*sqrt(1 + b*x**4/a)) - a*sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(4*x**2) + a*sqrt(b)*d/(2*x**2*sqrt(a/(b*x**4) + 1)) + 3*a*sqrt(b)*f*asinh(sqrt(b)*x**2/sqrt(a))/4 + b**(3/2)*d*x**2/(2*sqrt(a/(b*x**4) + 1))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^6,x, algorithm="giac")

[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)

$$3.521 \quad \int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^7} dx$$

Optimal. Leaf size=392

$$\frac{2\sqrt[4]{ab^3/4}(\sqrt{a} + \sqrt{bx^2})\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}}(5\sqrt{a}f + 9\sqrt{b}d)\operatorname{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{15\sqrt{a+bx^4}} + \frac{1}{2}b^{3/2}c \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right) + \frac{12b^{3/2}}{5(\sqrt{a} + \sqrt{bx^2})}$$

[Out] $(12*b^{(3/2)}*d*x*\operatorname{Sqrt}[a + b*x^4])/(5*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) - (b*(2*c - 3*e*x^2)*\operatorname{Sqrt}[a + b*x^4])/(4*x^2) - (2*b*(9*d - 5*f*x^2)*\operatorname{Sqrt}[a + b*x^4])/(15*x) - (((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3)*(a + b*x^4)^{(3/2)})/60 + (b^{(3/2)}*c*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*x^2)/\operatorname{Sqrt}[a + b*x^4]])/2 - (3*\operatorname{Sqrt}[a]*b*e*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*x^4]/\operatorname{Sqrt}[a]])/4 - (12*a^{(1/4)}*b^{(5/4)}*d*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(5*\operatorname{Sqrt}[a + b*x^4]) + (2*a^{(1/4)}*b^{(3/4)}*(9*\operatorname{Sqrt}[b]*d + 5*\operatorname{Sqrt}[a]*f)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(15*\operatorname{Sqrt}[a + b*x^4])$

Rubi [A] time = 0.342193, antiderivative size = 392, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 15, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {14, 1825, 1833, 1252, 813, 844, 217, 206, 266, 63, 208, 1272, 1198, 220, 1196}

$$\frac{1}{2}b^{3/2}c \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right) + \frac{2\sqrt[4]{ab^3/4}(\sqrt{a} + \sqrt{bx^2})\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}}(5\sqrt{a}f + 9\sqrt{b}d)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{15\sqrt{a+bx^4}} + \frac{12b^{3/2}dx\sqrt{a+bx^2}}{5(\sqrt{a} + \sqrt{bx^2})}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^{(3/2)}/x^7, x]$

[Out] $(12*b^{(3/2)}*d*x*\operatorname{Sqrt}[a + b*x^4])/(5*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) - (b*(2*c - 3*e*x^2)*\operatorname{Sqrt}[a + b*x^4])/(4*x^2) - (2*b*(9*d - 5*f*x^2)*\operatorname{Sqrt}[a + b*x^4])/(15*x) - (((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3)*(a + b*x^4)^{(3/2)})/60 + (b^{(3/2)}*c*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*x^2)/\operatorname{Sqrt}[a + b*x^4]])/2 - (3*\operatorname{Sqrt}[a]*b*e*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*x^4]/\operatorname{Sqrt}[a]])/4 - (12*a^{(1/4)}*b^{(5/4)}*d*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(5*\operatorname{Sqrt}[a + b*x^4]) + (2*a^{(1/4)}*b^{(3/4)}*(9*\operatorname{Sqrt}[b]*d + 5*\operatorname{Sqrt}[a]*f)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(15*\operatorname{Sqrt}[a + b*x^4])$

$t[b]*d + 5*\text{Sqrt}[a]*f*(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)*\text{Sqrt}[(a + b*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{1/4})*x]/a^{1/4}], 1/2)/((15*\text{Sqrt}[a + b*x^4])$

Rule 14

$\text{Int}[(u_)*((c_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*u, x], x] /; \text{FreeQ}\{c, m\}, x] \&\& \text{SumQ}[u] \&\& \text{!LinearQ}[u, x] \&\& \text{!MatchQ}[u, (a_ + (b_)*(v_)] /; \text{FreeQ}\{a, b\}, x] \&\& \text{InverseFunctionQ}[v]$

Rule 1825

$\text{Int}[(Pq_)*(x_)^{(m_)*((a_ + (b_)*(x_)^{(n_}))^{(p_)}, x_Symbol] \rightarrow \text{Module}\{u = \text{IntHide}[x^m*Pq, x]\}, \text{Simp}[u*(a + b*x^n)^p, x] - \text{Dist}[b*n*p, \text{Int}[x^{(m+n)}*(a + b*x^n)^{(p-1)}*\text{ExpandToSum}[u/x^{(m+1)}, x], x], x]] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[n, 0] \&\& \text{GtQ}[p, 0] \&\& \text{LtQ}[m + \text{Expon}[Pq, x] + 1, 0]$

Rule 1833

$\text{Int}[(Pq_)*((c_)*(x_))^{(m_)*((a_ + (b_)*(x_)^{(n_}))^{(p_)}, x_Symbol] \rightarrow \text{Module}\{q = \text{Expon}[Pq, x], j, k\}, \text{Int}[\text{Sum}[(c*x)^{(m+j)}*\text{Sum}[\text{Coeff}[Pq, x, j + (k*n)/2]*x^{((k*n)/2)}, \{k, 0, (2*(q-j))/n + 1\}]* (a + b*x^n)^p]/c^j, \{j, 0, n/2 - 1\}], x]] /; \text{FreeQ}\{a, b, c, m, p\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[n/2, 0] \&\& \text{!PolyQ}[Pq, x^{(n/2)}]$

Rule 1252

$\text{Int}[(x_)^{(m_)*((d_ + (e_)*(x_)^2)^{(q_)*((a_ + (c_)*(x_)^4)^{(p_)}, x_Symbol] \rightarrow \text{Dist}[1/2, \text{Subst}[\text{Int}[x^{((m-1)/2)}*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; \text{FreeQ}\{a, c, d, e, p, q\}, x] \&\& \text{IntegerQ}[(m+1)/2]$

Rule 813

$\text{Int}[(d_ + (e_)*(x_))^{(m_)*((f_ + (g_)*(x_))*((a_ + (c_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(d + e*x)^{(m+1)}*(e*f*(m+2*p+2) - d*g*(2*p+1) + e*g*(m+1)*x)*(a + c*x^2)^p]/(e^2*(m+1)*(m+2*p+2)), x] + \text{Dist}[p/(e^2*(m+1)*(m+2*p+2)), \text{Int}[(d + e*x)^{(m+1)}*(a + c*x^2)^{(p-1)}*\text{Simp}[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m+2*p+2))*x, x], x] /; \text{FreeQ}\{a, c, d, e, f, g, m\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{RationalQ}[p] \&\& p > 0 \&\& (\text{LtQ}[m, -1] || \text{EqQ}[p, 1] || (\text{IntegerQ}[p] \&\& \text{!RationalQ}[m])) \&\& \text{NeQ}[m, -1] \&\& \text{!ILtQ}[m + 2*p + 1, 0] \&\& (\text{IntegerQ}[m] || \text{IntegerQ}[p] || \text{IntegersQ}[2*m, 2*p])$

Rule 844

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1272

Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(d*(m + 4*p + 3) + e*(m + 1)*x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(4*p)/(f^2*(m + 1)*(m + 4*p + 3)), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +

$4*p + 3 \neq 0 \ \&\& \ \text{IntegerQ}[2*p] \ \&\& \ (\text{IntegerQ}[p] \ || \ \text{IntegerQ}[m])$

Rule 1198

$\text{Int}[\frac{(d_.) + (e_.)(x_)^2}{\text{Sqrt}[(a_) + (c_.)(x_)^4]}, x_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[c/a, 2]\}, \text{Dist}[(e + d*q)/q, \text{Int}[1/\text{Sqrt}[a + c*x^4], x], x] - \text{Dist}[e/q, \text{Int}[(1 - q*x^2)/\text{Sqrt}[a + c*x^4], x], x] \ /; \ \text{NeQ}[e + d*q, 0] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{PosQ}[c/a]$

Rule 220

$\text{Int}[1/\text{Sqrt}[(a_) + (b_.)(x_)^4], x_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[b/a, 4]\}, \text{Simp}[(1 + q^2*x^2)*\text{Sqrt}[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*\text{EllipticF}[2*\text{ArcTan}[q*x], 1/2]]/(2*q*\text{Sqrt}[a + b*x^4]), x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[b/a]$

Rule 1196

$\text{Int}[\frac{(d_.) + (e_.)(x_)^2}{\text{Sqrt}[(a_) + (c_.)(x_)^4]}, x_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[c/a, 4]\}, -\text{Simp}[(d*x*\text{Sqrt}[a + c*x^4])/(a*(1 + q^2*x^2)), x] + \text{Simp}[(d*(1 + q^2*x^2)*\text{Sqrt}[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*\text{EllipticE}[2*\text{ArcTan}[q*x], 1/2]]/(q*\text{Sqrt}[a + c*x^4]), x] \ /; \ \text{EqQ}[e + d*q^2, 0] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{PosQ}[c/a]$

Rubi steps

$$\begin{aligned}
\int \frac{(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}}{x^7} dx &= -\frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} \right) (a + bx^4)^{3/2} - (6b) \int \frac{\left(-\frac{c}{6} - \frac{dx}{5} - \frac{ex^2}{4} - \frac{fx^3}{3} \right)}{x^3} dx \\
&= -\frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} \right) (a + bx^4)^{3/2} - (6b) \int \frac{\left(-\frac{c}{6} - \frac{ex^2}{4} \right) \sqrt{a + bx^4}}{x^3} dx \\
&= -\frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} \right) (a + bx^4)^{3/2} - (6b) \int \frac{\left(-\frac{c}{6} - \frac{ex^2}{4} \right) \sqrt{a + bx^4}}{x^3} dx \\
&= -\frac{2b(9d - 5fx^2) \sqrt{a + bx^4}}{15x} - \frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} \right) (a + bx^4)^{3/2} - (3b) \int \frac{\sqrt{a + bx^4}}{x^3} dx \\
&= -\frac{b(2c - 3ex^2) \sqrt{a + bx^4}}{4x^2} - \frac{2b(9d - 5fx^2) \sqrt{a + bx^4}}{15x} - \frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} \right) (a + bx^4)^{3/2} \\
&= \frac{12b^{3/2} dx \sqrt{a + bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} - \frac{b(2c - 3ex^2) \sqrt{a + bx^4}}{4x^2} - \frac{2b(9d - 5fx^2) \sqrt{a + bx^4}}{15x} - \frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} \right) (a + bx^4)^{3/2} \\
&= \frac{12b^{3/2} dx \sqrt{a + bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} - \frac{b(2c - 3ex^2) \sqrt{a + bx^4}}{4x^2} - \frac{2b(9d - 5fx^2) \sqrt{a + bx^4}}{15x} - \frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} \right) (a + bx^4)^{3/2} \\
&= \frac{12b^{3/2} dx \sqrt{a + bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} - \frac{b(2c - 3ex^2) \sqrt{a + bx^4}}{4x^2} - \frac{2b(9d - 5fx^2) \sqrt{a + bx^4}}{15x} - \frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} \right) (a + bx^4)^{3/2} \\
&= \frac{12b^{3/2} dx \sqrt{a + bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} - \frac{b(2c - 3ex^2) \sqrt{a + bx^4}}{4x^2} - \frac{2b(9d - 5fx^2) \sqrt{a + bx^4}}{15x} - \frac{1}{60} \left(\frac{10c}{x^6} + \frac{12d}{x^5} + \frac{15e}{x^4} + \frac{20f}{x^3} \right) (a + bx^4)^{3/2}
\end{aligned}$$

Mathematica [C] time = 0.201954, size = 163, normalized size = 0.42

$$\frac{\sqrt{a + bx^4} \left(-5a^3 c {}_2F_1 \left(-\frac{3}{2}, -\frac{3}{2}; -\frac{1}{2}; -\frac{bx^4}{a} \right) - 6a^3 dx {}_2F_1 \left(-\frac{3}{2}, -\frac{5}{4}; -\frac{1}{4}; -\frac{bx^4}{a} \right) - 10a^3 fx^3 {}_2F_1 \left(-\frac{3}{2}, -\frac{3}{4}; \frac{1}{4}; -\frac{bx^4}{a} \right) + 3bex^6 (a + bx^4) \right)}{30a^2 x^6 \sqrt{\frac{bx^4}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^7,x]

[Out] (Sqrt[a + b*x^4]*(-5*a^3*c*Hypergeometric2F1[-3/2, -3/2, -1/2, -((b*x^4)/a)] - 6*a^3*d*x*Hypergeometric2F1[-3/2, -5/4, -1/4, -((b*x^4)/a)] - 10*a^3*f*x^3*Hypergeometric2F1[-3/2, -3/4, 1/4, -((b*x^4)/a)] + 3*b*e*x^6*(a + b*x^4)^2*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^4)/a]))/(30*a^2*x^6*Sqrt[1 + (b*x^4)/a])

Maple [C] time = 0.02, size = 408, normalized size = 1.

$$-\frac{af}{3x^3}\sqrt{bx^4+a} + \frac{fbx}{3}\sqrt{bx^4+a} + \frac{4abf}{3}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}} - \frac{1}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^7,x)

[Out] -1/3*f*a*(b*x^4+a)^(1/2)/x^3+1/3*f*b*x*(b*x^4+a)^(1/2)+4/3*f*a*b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/5*d*a*(b*x^4+a)^(1/2)/x^5-7/5*d*b*(b*x^4+a)^(1/2)/x+12/5*I*d*b^(3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-12/5*I*d*b^(3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*e*b*(b*x^4+a)^(1/2)-3/4*e*a^(1/2)*b*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/4*e*a/x^4*(b*x^4+a)^(1/2)+1/2*c*b^(3/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))-1/6*c*a/x^6*(b*x^4+a)^(1/2)-2/3*c*b/x^2*(b*x^4+a)^(1/2)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^7,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bf x^7 + bex^6 + bdx^5 + bcx^4 + afx^3 + aex^2 + adx + ac)\sqrt{bx^4 + a}}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^7,x, algorithm="fricas")

[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x + a*c)*sqrt(b*x^4 + a)/x^7, x)

Sympy [C] time = 9.89324, size = 406, normalized size = 1.04

$$\frac{a^{\frac{3}{2}} d \Gamma\left(-\frac{5}{4}\right) {}_2F_1\left(-\frac{5}{4}, -\frac{1}{2} \left| \frac{bx^4 e^{i\pi}}{a} \right.\right)}{4x^5 \Gamma\left(-\frac{1}{4}\right)} + \frac{a^{\frac{3}{2}} f \Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(-\frac{3}{4}, -\frac{1}{2} \left| \frac{bx^4 e^{i\pi}}{a} \right.\right)}{4x^3 \Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{abc}}{2x^2 \sqrt{1 + \frac{bx^4}{a}}} + \frac{\sqrt{abd} \Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(-\frac{1}{2}, -\frac{1}{4} \left| \frac{bx^4 e^{i\pi}}{a} \right.\right)}{4x \Gamma\left(\frac{3}{4}\right)} - 3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**7,x)

[Out] a**(3/2)*d*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) + a**(3/2)*f*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - sqrt(a)*b*c/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*b*d*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - 3*sqrt(a)*b*e*asinh(sqrt(a)/(sqrt(b)*x**2))/4 + sqrt(a)*b*f*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) - a*sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(6*x**4) - a*sqrt(b)*e*sqrt(a/(b*x**4) + 1)/(4*x**2) + a*sqrt(b)*e/(2*x**2*sqrt(a/(b*x**4) + 1)) - b**(3/2)*c*sqrt(a/(b*x**4) + 1)/6 + b**(3/2)*c*asinh(sqrt(b)*x**2/sqrt(a))/2 + b**(3/2)*e*x**2/(2*sqrt(a/(b*x**4) + 1)) - b**2*c*x**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^7,x, algorithm="giac")

[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^7, x)

$$3.522 \quad \int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^8} dx$$

Optimal. Leaf size=412

$$\frac{2b^{5/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (21\sqrt{ae} + 5\sqrt{bc}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{35\sqrt[4]{a}\sqrt{a+bx^4}} + \frac{1}{2}b^{3/2}d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right) + \frac{12b^{3/2}ex}{5(\sqrt{a} + \sqrt{bx^2})}$$

[Out] $(-12*b*e*\operatorname{Sqrt}[a + b*x^4])/(5*x) + (12*b^{(3/2)}*e*x*\operatorname{Sqrt}[a + b*x^4])/(5*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) - (2*b*(5*c - 21*e*x^2)*\operatorname{Sqrt}[a + b*x^4])/(35*x^3) - (b*(2*d - 3*f*x^2)*\operatorname{Sqrt}[a + b*x^4])/(4*x^2) - (((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4)*(a + b*x^4)^{(3/2)})/420 + (b^{(3/2)}*d*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*x^2)/\operatorname{Sqrt}[a + b*x^4]])/2 - (3*\operatorname{Sqrt}[a]*b*f*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*x^4]/\operatorname{Sqrt}[a]])/4 - (12*a^{(1/4)}*b^{(5/4)}*e*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(5*\operatorname{Sqrt}[a + b*x^4]) + (2*b^{(5/4)}*(5*\operatorname{Sqrt}[b]*c + 21*\operatorname{Sqrt}[a]*e)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(35*a^{(1/4)}*\operatorname{Sqrt}[a + b*x^4])$

Rubi [A] time = 0.393938, antiderivative size = 412, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 16, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {14, 1825, 1833, 1272, 1282, 1198, 220, 1196, 1252, 813, 844, 217, 206, 266, 63, 208}

$$\frac{2b^{5/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (21\sqrt{ae} + 5\sqrt{bc}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{35\sqrt[4]{a}\sqrt{a+bx^4}} + \frac{1}{2}b^{3/2}d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right) + \frac{12b^{3/2}ex\sqrt{a+bx^4}}{5(\sqrt{a} + \sqrt{bx^2})}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^{(3/2)}/x^8, x]$

[Out] $(-12*b*e*\operatorname{Sqrt}[a + b*x^4])/(5*x) + (12*b^{(3/2)}*e*x*\operatorname{Sqrt}[a + b*x^4])/(5*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) - (2*b*(5*c - 21*e*x^2)*\operatorname{Sqrt}[a + b*x^4])/(35*x^3) - (b*(2*d - 3*f*x^2)*\operatorname{Sqrt}[a + b*x^4])/(4*x^2) - (((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4)*(a + b*x^4)^{(3/2)})/420 + (b^{(3/2)}*d*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*x^2)/\operatorname{Sqrt}[a + b*x^4]])/2 - (3*\operatorname{Sqrt}[a]*b*f*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*x^4]/\operatorname{Sqrt}[a]])/4 - (12*a^{(1/4)}*b^{(5/4)}*e*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(5*\operatorname{Sqrt}[a + b*x^4]) + (2*b^{(5/4)}*(5*\operatorname{Sqrt}[b]*c + 21*\operatorname{Sqrt}[a]*e)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(35*a^{(1/4)}*\operatorname{Sqrt}[a + b*x^4])$

$$a + b*x^4) + (2*b^{(5/4)}*(5*sqrt[b]*c + 21*sqrt[a]*e)*(sqrt[a] + sqrt[b]*x^2)*sqrt[(a + b*x^4)/(sqrt[a] + sqrt[b]*x^2)^2]*ellipticF[2*ArcTan[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(35*a^{(1/4)}*sqrt[a + b*x^4])$$

Rule 14

```
Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^m_)*((a_) + (b_)*(x_)^n_)^p_, x_Symbol] := Module[{u = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1, 0]
```

Rule 1833

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^n_)^p_, x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p)/c^j, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 1272

```
Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^p_, x_Symbol] := Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(d*(m + 4*p + 3) + e*(m + 1)*x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(4*p)/(f^2*(m + 1)*(m + 4*p + 3)), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1282

```
Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^p_, x_Symbol] := Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198


```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q =
  Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q =
  Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_S
ymbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 813

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p
_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
) + e*g*(m + 1)*x*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !Rational
Q[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p
_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}}{x^8} dx &= -\frac{1}{420} \left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} \right) (a + bx^4)^{3/2} - (6b) \int \frac{\left(-\frac{c}{7} - \frac{dx}{6} - \frac{ex^2}{5} - \frac{fx^3}{4} \right)}{x^4} \sqrt{a + bx^4} dx \\
&= -\frac{1}{420} \left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} \right) (a + bx^4)^{3/2} - (6b) \int \frac{\left(-\frac{c}{7} - \frac{ex^2}{5} \right) \sqrt{a + bx^4}}{x^4} dx \\
&= -\frac{1}{420} \left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} \right) (a + bx^4)^{3/2} - (6b) \int \frac{\left(-\frac{c}{7} - \frac{ex^2}{5} \right) \sqrt{a + bx^4}}{x^4} dx \\
&= -\frac{2b(5c - 21ex^2) \sqrt{a + bx^4}}{35x^3} - \frac{1}{420} \left(\frac{60c}{x^7} + \frac{70d}{x^6} + \frac{84e}{x^5} + \frac{105f}{x^4} \right) (a + bx^4)^{3/2} \\
&= -\frac{12be\sqrt{a + bx^4}}{5x} - \frac{2b(5c - 21ex^2) \sqrt{a + bx^4}}{35x^3} - \frac{b(2d - 3fx^2) \sqrt{a + bx^4}}{4x^2} - \frac{1}{420} \\
&= -\frac{12be\sqrt{a + bx^4}}{5x} - \frac{2b(5c - 21ex^2) \sqrt{a + bx^4}}{35x^3} - \frac{b(2d - 3fx^2) \sqrt{a + bx^4}}{4x^2} - \frac{1}{420} \\
&= -\frac{12be\sqrt{a + bx^4}}{5x} + \frac{12b^{3/2}ex\sqrt{a + bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} - \frac{2b(5c - 21ex^2) \sqrt{a + bx^4}}{35x^3} - \frac{b(2d - 3fx^2) \sqrt{a + bx^4}}{4x^2} \\
&= -\frac{12be\sqrt{a + bx^4}}{5x} + \frac{12b^{3/2}ex\sqrt{a + bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} - \frac{2b(5c - 21ex^2) \sqrt{a + bx^4}}{35x^3} - \frac{b(2d - 3fx^2) \sqrt{a + bx^4}}{4x^2} \\
&= -\frac{12be\sqrt{a + bx^4}}{5x} + \frac{12b^{3/2}ex\sqrt{a + bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} - \frac{2b(5c - 21ex^2) \sqrt{a + bx^4}}{35x^3} - \frac{b(2d - 3fx^2) \sqrt{a + bx^4}}{4x^2}
\end{aligned}$$

Mathematica [C] time = 0.221303, size = 164, normalized size = 0.4

$$\frac{\sqrt{a + bx^4} \left(7x \left(-5a^3 d {}_2F_1 \left(-\frac{3}{2}, -\frac{3}{2}; -\frac{1}{2}; -\frac{bx^4}{a} \right) - 6a^3 ex {}_2F_1 \left(-\frac{3}{2}, -\frac{5}{4}; -\frac{1}{4}; -\frac{bx^4}{a} \right) + 3bfx^6 (a + bx^4)^2 \sqrt{\frac{bx^4}{a} + 1} {}_2F_1 \left(2, \frac{5}{2}; \frac{7}{2}; \frac{bx^4}{a} \right) \right)}{210a^2 x^7 \sqrt{\frac{bx^4}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^8,x]

[Out] (Sqrt[a + b*x^4]*(-30*a^3*c*Hypergeometric2F1[-7/4, -3/2, -3/4, -((b*x^4)/a)] + 7*x*(-5*a^3*d*Hypergeometric2F1[-3/2, -3/2, -1/2, -((b*x^4)/a)] - 6*a^3*e*x*Hypergeometric2F1[-3/2, -5/4, -1/4, -((b*x^4)/a)] + 3*b*f*x^6*(a + b*x^4)^2*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^4)/a]))/(210*a^2*x^7*Sqrt[1 + (b*x^4)/a])

Maple [C] time = 0.015, size = 411, normalized size = 1.

$$-\frac{ae}{5x^5}\sqrt{bx^4+a} - \frac{7be}{5x}\sqrt{bx^4+a} + \frac{12i}{5}eb^{\frac{3}{2}}\sqrt{a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^8,x)

[Out] -1/5*e*a*(b*x^4+a)^(1/2)/x^5-7/5*b*e*(b*x^4+a)^(1/2)/x+12/5*I*e*b^(3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-12/5*I*e*b^(3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*f*b*(b*x^4+a)^(1/2)-3/4*f*a^(1/2)*b*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/4*f*a/x^4*(b*x^4+a)^(1/2)+1/2*d*b^(3/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))-1/6*d*a/x^6*(b*x^4+a)^(1/2)-2/3*d*b/x^2*(b*x^4+a)^(1/2)-1/7*c*a*(b*x^4+a)^(1/2)/x^7-3/7*c*b*(b*x^4+a)^(1/2)/x^3+4/7*c*b^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^8,x, algorithm="maxima")

[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^8, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bfx^7 + bex^6 + bdx^5 + bcx^4 + afx^3 + aex^2 + adx + ac)\sqrt{bx^4 + a}}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^8,x, algorithm="fricas")

[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x + a*c)*sqrt(b*x^4 + a)/x^8, x)

Sympy [C] time = 11.4562, size = 415, normalized size = 1.01

$$\frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{7}{4}\right) {}_2F_1\left(-\frac{7}{4}, -\frac{1}{2}\left|\frac{bx^4e^{i\pi}}{a}\right.\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{5}{4}\right) {}_2F_1\left(-\frac{5}{4}, -\frac{1}{2}\left|\frac{bx^4e^{i\pi}}{a}\right.\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{\sqrt{abc}\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(-\frac{3}{4}, -\frac{1}{2}\left|\frac{bx^4e^{i\pi}}{a}\right.\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{abd}}{2x^2\sqrt{1 + \frac{bx^4}{a}}} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**8,x)

[Out] a**(3/2)*c*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3/4)) + a**(3/2)*e*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) + sqrt(a)*b*c*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - sqrt(a)*b*d/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*b*e*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - 3*sqrt(a)*b*f*asinh(sqrt(a)/(sqrt(b)*x**2))/4 - a*sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(6*x**4) - a*sqrt(b)*f*sqrt(a/(b*x**4) + 1)/(4*x**2) + a*sqrt(b)*f/(2*x**2*sqrt(a/(b*x**4) + 1)) - b**(3/2)*d*sqrt(a/(b*x**4) + 1)/6 + b**(3/2)*d*asinh(sqrt(b)*x**2/sqrt(a))/2 + b**(3/2)*f*x**2/(2*sqrt(a/(b*x**4) + 1)) - b**2*d*x**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^8, x)
```

$$3.523 \quad \int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^9} dx$$

Optimal. Leaf size=377

$$\frac{2b^{5/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (21\sqrt{a}f + 5\sqrt{bd}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right) - 3b^2c \tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{35\sqrt[4]{a}\sqrt{a+bx^4}} - \frac{3b^2c \tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}} + \frac{1}{2}b^{3/2}e \tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)$$

[Out] $-(b*((105*c)/x^4 + (160*d)/x^3 + (280*e)/x^2 + (672*f)/x)*\operatorname{Sqrt}[a + b*x^4])/560 + (12*b^{(3/2)}*f*x*\operatorname{Sqrt}[a + b*x^4])/(5*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) - (((105*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5)*(a + b*x^4)^{(3/2)})/840 + (b^{(3/2)}*e*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*x^2)/\operatorname{Sqrt}[a + b*x^4]])/2 - (3*b^2*c*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*x^4]/\operatorname{Sqrt}[a]])/(16*\operatorname{Sqrt}[a]) - (12*a^{(1/4)}*b^{(5/4)}*f*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(5*\operatorname{Sqrt}[a + b*x^4]) + (2*b^{(5/4)}*(5*\operatorname{Sqrt}[b]*d + 21*\operatorname{Sqrt}[a]*f)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(35*a^{(1/4)}*\operatorname{Sqrt}[a + b*x^4])$

Rubi [A] time = 0.370944, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {14, 1825, 1832, 266, 63, 208, 1885, 275, 217, 206, 1198, 220, 1196}

$$-\frac{3b^2c \tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}} + \frac{2b^{5/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (21\sqrt{a}f + 5\sqrt{bd}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{35\sqrt[4]{a}\sqrt{a+bx^4}} + \frac{1}{2}b^{3/2}e \tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^{(3/2)}/x^9, x]$

[Out] $-(b*((105*c)/x^4 + (160*d)/x^3 + (280*e)/x^2 + (672*f)/x)*\operatorname{Sqrt}[a + b*x^4])/560 + (12*b^{(3/2)}*f*x*\operatorname{Sqrt}[a + b*x^4])/(5*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) - (((105*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5)*(a + b*x^4)^{(3/2)})/840 + (b^{(3/2)}*e*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*x^2)/\operatorname{Sqrt}[a + b*x^4]])/2 - (3*b^2*c*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*x^4]/\operatorname{Sqrt}[a]])/(16*\operatorname{Sqrt}[a]) - (12*a^{(1/4)}*b^{(5/4)}*f*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(5*\operatorname{Sqrt}[a + b*x^4]) + (2*b^{(5/4)}*(5*\operatorname{Sqrt}[b]*d + 21*\operatorname{Sqrt}[a]*f)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(35*a^{(1/4)}*\operatorname{Sqrt}[a + b*x^4])$

$2)^2 * \text{EllipticF}[2 * \text{ArcTan}[(b^{1/4} * x) / a^{1/4}], 1/2] / (35 * a^{1/4} * \text{Sqrt}[a + b * x^4])$

Rule 14

$\text{Int}[(u_*) * ((c_*) * (x_*)^{(m_*)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c * x)^m * u, x], x] /;$ FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_*) * (v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

Rule 1825

$\text{Int}[(Pq_*) * (x_*)^{(m_*)} * ((a_*) + (b_*) * (x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Module}\{u = \text{IntHide}[x^m * Pq, x]\}, \text{Simp}[u * (a + b * x^n)^p, x] - \text{Dist}[b * n * p, \text{Int}[x^{(m + n)} * (a + b * x^n)^{(p - 1)} * \text{ExpandToSum}[u / x^{(m + 1)}, x], x], x]] /;$ FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1, 0]

Rule 1832

$\text{Int}[(Pq_*) / ((x_*) * \text{Sqrt}[(a_*) + (b_*) * (x_*)^{(n_*)}])), x_Symbol] \rightarrow \text{Dist}[\text{Coeff}[Pq, x, 0], \text{Int}[1 / (x * \text{Sqrt}[a + b * x^n]), x], x] + \text{Int}[\text{ExpandToSum}[(Pq - \text{Coeff}[Pq, x, 0]) / x, x] / \text{Sqrt}[a + b * x^n], x] /;$ FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]

Rule 266

$\text{Int}[(x_*)^{(m_*)} * ((a_*) + (b_*) * (x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1) * (a + b * x)^p}, x], x, x^n], x] /;$ FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 63

$\text{Int}(((a_*) + (b_*) * (x_*)^{(m_*)}) * ((c_*) + (d_*) * (x_*)^{(n_*)}), x_Symbol] \rightarrow \text{With}\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p * (m + 1) - 1) * (c - (a * d) / b + (d * x^p) / b)^n}, x], x, (a + b * x)^{(1/p)}], x]] /;$ FreeQ[{a, b, c, d}, x] && NeQ[b * c - a * d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

$\text{Int}(((a_*) + (b_*) * (x_*)^2)^{-1}), x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2] * \text{ArcTanh}[x / \text{Rt}[-(a/b), 2]]) / a, x] /;$ FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1885


```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq,
x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, p},
x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 275

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/
Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1198

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}}{x^9} dx &= -\frac{1}{840} \left(\frac{105c}{x^8} + \frac{120d}{x^7} + \frac{140e}{x^6} + \frac{168f}{x^5} \right) (a + bx^4)^{3/2} - (6b) \int \frac{\left(-\frac{c}{8} - \frac{dx}{7} - \frac{ex^2}{6} - \frac{fx^3}{5} \right)}{x^5} dx \\
&= -\frac{1}{560} b \left(\frac{105c}{x^4} + \frac{160d}{x^3} + \frac{280e}{x^2} + \frac{672f}{x} \right) \sqrt{a + bx^4} - \frac{1}{840} \left(\frac{105c}{x^8} + \frac{120d}{x^7} + \frac{140e}{x^6} \right) \\
&= -\frac{1}{560} b \left(\frac{105c}{x^4} + \frac{160d}{x^3} + \frac{280e}{x^2} + \frac{672f}{x} \right) \sqrt{a + bx^4} - \frac{1}{840} \left(\frac{105c}{x^8} + \frac{120d}{x^7} + \frac{140e}{x^6} \right) \\
&= -\frac{1}{560} b \left(\frac{105c}{x^4} + \frac{160d}{x^3} + \frac{280e}{x^2} + \frac{672f}{x} \right) \sqrt{a + bx^4} - \frac{1}{840} \left(\frac{105c}{x^8} + \frac{120d}{x^7} + \frac{140e}{x^6} \right) \\
&= -\frac{1}{560} b \left(\frac{105c}{x^4} + \frac{160d}{x^3} + \frac{280e}{x^2} + \frac{672f}{x} \right) \sqrt{a + bx^4} - \frac{1}{840} \left(\frac{105c}{x^8} + \frac{120d}{x^7} + \frac{140e}{x^6} \right) \\
&= -\frac{1}{560} b \left(\frac{105c}{x^4} + \frac{160d}{x^3} + \frac{280e}{x^2} + \frac{672f}{x} \right) \sqrt{a + bx^4} - \frac{1}{840} \left(\frac{105c}{x^8} + \frac{120d}{x^7} + \frac{140e}{x^6} \right) \\
&= -\frac{1}{560} b \left(\frac{105c}{x^4} + \frac{160d}{x^3} + \frac{280e}{x^2} + \frac{672f}{x} \right) \sqrt{a + bx^4} + \frac{12b^{3/2}fx\sqrt{a + bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} - \frac{1}{840} \left(\frac{105c}{x^8} + \frac{120d}{x^7} + \frac{140e}{x^6} \right) \\
&= -\frac{1}{560} b \left(\frac{105c}{x^4} + \frac{160d}{x^3} + \frac{280e}{x^2} + \frac{672f}{x} \right) \sqrt{a + bx^4} + \frac{12b^{3/2}fx\sqrt{a + bx^4}}{5(\sqrt{a} + \sqrt{bx^2})} - \frac{1}{840} \left(\frac{105c}{x^8} + \frac{120d}{x^7} + \frac{140e}{x^6} \right)
\end{aligned}$$

Mathematica [C] time = 0.293568, size = 174, normalized size = 0.46

$$\frac{\sqrt{a + bx^4} \left(7 \left(40a^2ex^2 {}_2F_1 \left(-\frac{3}{2}, -\frac{3}{2}; -\frac{1}{2}; -\frac{bx^4}{a} \right) + 48a^2fx^3 {}_2F_1 \left(-\frac{3}{2}, -\frac{5}{4}; -\frac{1}{4}; -\frac{bx^4}{a} \right) + 15c \left(3b^2x^8 \tanh^{-1} \left(\sqrt{\frac{bx^4}{a} + 1} \right) + a \left(2a \sqrt{\frac{bx^4}{a} + 1} \right) \right) \right)}{1680ax^8\sqrt{\frac{bx^4}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^9,x]

[Out] -(Sqrt[a + b*x^4]*(240*a^2*d*x*Hypergeometric2F1[-7/4, -3/2, -3/4, -(b*x^4)/a]) + 7*(15*c*(a*(2*a + 5*b*x^4)*Sqrt[1 + (b*x^4)/a] + 3*b^2*x^8*ArcTanh[Sqrt[1 + (b*x^4)/a]]) + 40*a^2*e*x^2*Hypergeometric2F1[-3/2, -3/2, -1/2, -(

$(b*x^4/a)] + 48*a^2*f*x^3*Hypergeometric2F1[-3/2, -5/4, -1/4, -((b*x^4)/a)$
 $])))/(1680*a*x^8*sqrt[1 + (b*x^4)/a])$

Maple [C] time = 0.023, size = 416, normalized size = 1.1

$$-\frac{af}{5x^5}\sqrt{bx^4+a} - \frac{7fb}{5x}\sqrt{bx^4+a} + \frac{12i}{5}fb^{\frac{3}{2}}\sqrt{a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^9,x)

[Out] $-1/5*f*a*(b*x^4+a)^{(1/2)}/x^5-7/5*f*b*(b*x^4+a)^{(1/2)}/x+12/5*I*f*b^{(3/2)}*a^{(1/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*EllipticF(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)},I)-12/5*I*f*b^{(3/2)}*a^{(1/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*EllipticE(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)},I)-3/16*c*b^2/a^{(1/2)}*\ln((2*a+2*a^{(1/2)}*(b*x^4+a)^{(1/2)})/x^2)-1/8*c*a/x^8*(b*x^4+a)^{(1/2)}-5/16*c*b/x^4*(b*x^4+a)^{(1/2)}+1/2*e*b^{(3/2)}*\ln(x^2*b^{(1/2)}+(b*x^4+a)^{(1/2)})-1/6*e*a/x^6*(b*x^4+a)^{(1/2)}-2/3*e*b/x^2*(b*x^4+a)^{(1/2)}-1/7*d*a*(b*x^4+a)^{(1/2)}/x^7-3/7*d*b*(b*x^4+a)^{(1/2)}/x^3+4/7*d*b^2/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*EllipticF(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)},I)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^9,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bfx^7 + bex^6 + bdx^5 + bcx^4 + afx^3 + aex^2 + adx + ac)\sqrt{bx^4 + a}}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^9,x, algorithm="fricas")

[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x + a*c)*sqrt(b*x^4 + a)/x^9, x)

Sympy [C] time = 12.9674, size = 444, normalized size = 1.18

$$\frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{7}{4}\right) {}_2F_1\left(-\frac{7}{4}, -\frac{1}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{5}{4}\right) {}_2F_1\left(-\frac{5}{4}, -\frac{1}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{\sqrt{abd}\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(-\frac{3}{4}, -\frac{1}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{abe}}{2x^2\sqrt{1 + \frac{bx^4}{a}}} + \frac{\sqrt{a}}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**9,x)

[Out] a**(3/2)*d*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3/4)) + a**(3/2)*f*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) + sqrt(a)*b*d*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - sqrt(a)*b*e/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*b*f*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - a**2*c/(8*sqrt(b)*x**10*sqrt(a/(b*x**4) + 1)) - 3*a*sqrt(b)*c/(16*x**6*sqrt(a/(b*x**4) + 1)) - a*sqrt(b)*e*sqrt(a/(b*x**4) + 1)/(6*x**4) - b**(3/2)*c*sqrt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*c/(16*x**2*sqrt(a/(b*x**4) + 1)) - b**(3/2)*e*sqrt(a/(b*x**4) + 1)/6 + b**(3/2)*e*asinh(sqrt(b)*x**2/sqrt(a))/2 - 3*b**2*c*asinh(sqrt(a)/(sqrt(b)*x**2))/(16*sqrt(a)) - b**2*e*x**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^9, x)
```

$$3.524 \quad \int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{10}} dx$$

Optimal. Leaf size=405

$$\frac{2b^{7/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (15\sqrt{ae} + 7\sqrt{bc}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{105a^{3/4}\sqrt{a+bx^4}} - \frac{4b^{9/4}c(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\right)}{15a^{3/4}\sqrt{a+bx^4}}$$

[Out] $-(b*((224*c)/x^5 + (315*d)/x^4 + (480*e)/x^3 + (840*f)/x^2)*\operatorname{Sqrt}[a + b*x^4])/1680 - (4*b^2*c*\operatorname{Sqrt}[a + b*x^4])/(15*a*x) + (4*b^(5/2)*c*x*\operatorname{Sqrt}[a + b*x^4])/(15*a*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) - (((56*c)/x^9 + (63*d)/x^8 + (72*e)/x^7 + (84*f)/x^6)*(a + b*x^4)^(3/2))/504 + (b^(3/2)*f*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*x^2)/\operatorname{Sqrt}[a + b*x^4]])/2 - (3*b^2*d*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*x^4]/\operatorname{Sqrt}[a]])/(16*\operatorname{Sqrt}[a]) - (4*b^(9/4)*c*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)*\operatorname{Sqrt}[a + b*x^4]) + (2*b^(7/4)*(7*\operatorname{Sqrt}[b]*c + 15*\operatorname{Sqrt}[a]*e)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*a^(3/4)*\operatorname{Sqrt}[a + b*x^4])$

Rubi [A] time = 0.425, antiderivative size = 405, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {14, 1825, 1833, 1282, 1198, 220, 1196, 1252, 844, 217, 206, 266, 63, 208}

$$\frac{2b^{7/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (15\sqrt{ae} + 7\sqrt{bc}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{105a^{3/4}\sqrt{a+bx^4}} - \frac{4b^{9/4}c(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\right)}{15a^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2)/x^{10}, x]$

[Out] $-(b*((224*c)/x^5 + (315*d)/x^4 + (480*e)/x^3 + (840*f)/x^2)*\operatorname{Sqrt}[a + b*x^4])/1680 - (4*b^2*c*\operatorname{Sqrt}[a + b*x^4])/(15*a*x) + (4*b^(5/2)*c*x*\operatorname{Sqrt}[a + b*x^4])/(15*a*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) - (((56*c)/x^9 + (63*d)/x^8 + (72*e)/x^7 + (84*f)/x^6)*(a + b*x^4)^(3/2))/504 + (b^(3/2)*f*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*x^2)/\operatorname{Sqrt}[a + b*x^4]])/2 - (3*b^2*d*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*x^4]/\operatorname{Sqrt}[a]])/(16*\operatorname{Sqrt}[a]) - (4*b^(9/4)*c*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)*\operatorname{Sqrt}[a + b*x^4]) + (2*b^(7/4)*(7*\operatorname{Sqrt}[b]*c + 15*\operatorname{Sqrt}[a]*e)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*$

$\text{Sqrt}[(a + b*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2]/(105*a^{(3/4)}*\text{Sqrt}[a + b*x^4])$

Rule 14

$\text{Int}[(u_)*((c_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*u, x], x] /; \text{FreeQ}\{c, m, x\} \ \&\& \ \text{SumQ}[u] \ \&\& \ !\text{LinearQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (a_ + (b_)*(v_)) /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{InverseFunctionQ}[v]]$

Rule 1825

$\text{Int}[(Pq_)*(x_)^{(m_)*((a_ + (b_)*(x_)^{(n_}))^{(p_)}, x_Symbol] \rightarrow \text{Module}\{u = \text{IntHide}[x^m*Pq, x]\}, \text{Simp}[u*(a + b*x^n)^p, x] - \text{Dist}[b*n*p, \text{Int}[x^{(m+n)}*(a + b*x^n)^{(p-1)}*\text{ExpandToSum}[u/x^{(m+1)}, x], x], x]] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{LtQ}[m + \text{Expon}[Pq, x] + 1, 0]$

Rule 1833

$\text{Int}[(Pq_)*((c_)*(x_))^{(m_)*((a_ + (b_)*(x_)^{(n_}))^{(p_)}, x_Symbol] \rightarrow \text{Module}\{q = \text{Expon}[Pq, x], j, k\}, \text{Int}[\text{Sum}[(c*x)^{(m+j)}*\text{Sum}[\text{Coeff}[Pq, x, j + (k*n)/2]*x^{((k*n)/2)}, \{k, 0, (2*(q-j))/n + 1\}]* (a + b*x^n)^p]/c^j, \{j, 0, n/2 - 1\}], x]] /; \text{FreeQ}\{a, b, c, m, p, x\} \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{IGtQ}[n/2, 0] \ \&\& \ !\text{PolyQ}[Pq, x^{(n/2)}]$

Rule 1282

$\text{Int}[(f_)*(x_))^{(m_)*((d_ + (e_)*(x_)^2)*((a_ + (c_)*(x_)^4)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(d*(f*x)^{(m+1)}*(a + c*x^4)^{(p+1)})/(a*f*(m+1)), x] + \text{Dist}[1/(a*f^2*(m+1)), \text{Int}[(f*x)^{(m+2)}*(a + c*x^4)^p*(a*e*(m+1) - c*d*(m + 4*p + 5)*x^2), x], x] /; \text{FreeQ}\{a, c, d, e, f, p, x\} \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{IntegerQ}[2*p] \ \&\& \ (\text{IntegerQ}[p] \ || \ \text{IntegerQ}[m])$

Rule 1198

$\text{Int}[(d_ + (e_)*(x_)^2)/\text{Sqrt}[(a_ + (c_)*(x_)^4], x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[c/a, 2]\}, \text{Dist}[(e + d*q)/q, \text{Int}[1/\text{Sqrt}[a + c*x^4], x], x] - \text{Dist}[e/q, \text{Int}[(1 - q*x^2)/\text{Sqrt}[a + c*x^4], x], x] /; \text{NeQ}[e + d*q, 0] /; \text{FreeQ}\{a, c, d, e, x\} \ \&\& \ \text{PosQ}[c/a]$

Rule 220

$\text{Int}[1/\text{Sqrt}[(a_ + (b_)*(x_)^4], x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[b/a, 4]\}, \text{Simp}[(1 + q^2*x^2)*\text{Sqrt}[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*\text{EllipticF}[2*\text{ArcTan}[q*x]$

, 1/2]]/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rule 1252

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rule 844

Int[((d_) + (e_)*(x_)^2)^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 266

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 63

Int[((a_) + (b_)*(x_)^m)*((c_) + (d_)*(x_)^n), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ

[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
 \int \frac{(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}}{x^{10}} dx &= -\frac{1}{504} \left(\frac{56c}{x^9} + \frac{63d}{x^8} + \frac{72e}{x^7} + \frac{84f}{x^6} \right) (a + bx^4)^{3/2} - (6b) \int \frac{\left(-\frac{c}{9} - \frac{dx}{8} - \frac{ex^2}{7} - \frac{fx^3}{6} \right)}{x^6} dx \\
 &= -\frac{b \left(\frac{224c}{x^5} + \frac{315d}{x^4} + \frac{480e}{x^3} + \frac{840f}{x^2} \right) \sqrt{a + bx^4}}{1680} - \frac{1}{504} \left(\frac{56c}{x^9} + \frac{63d}{x^8} + \frac{72e}{x^7} + \frac{84f}{x^6} \right) (a + bx^4)^{3/2} \\
 &= -\frac{b \left(\frac{224c}{x^5} + \frac{315d}{x^4} + \frac{480e}{x^3} + \frac{840f}{x^2} \right) \sqrt{a + bx^4}}{1680} - \frac{1}{504} \left(\frac{56c}{x^9} + \frac{63d}{x^8} + \frac{72e}{x^7} + \frac{84f}{x^6} \right) (a + bx^4)^{3/2} \\
 &= -\frac{b \left(\frac{224c}{x^5} + \frac{315d}{x^4} + \frac{480e}{x^3} + \frac{840f}{x^2} \right) \sqrt{a + bx^4}}{1680} - \frac{1}{504} \left(\frac{56c}{x^9} + \frac{63d}{x^8} + \frac{72e}{x^7} + \frac{84f}{x^6} \right) (a + bx^4)^{3/2} \\
 &= -\frac{b \left(\frac{224c}{x^5} + \frac{315d}{x^4} + \frac{480e}{x^3} + \frac{840f}{x^2} \right) \sqrt{a + bx^4}}{1680} - \frac{4b^2c\sqrt{a + bx^4}}{15ax} - \frac{1}{504} \left(\frac{56c}{x^9} + \frac{63d}{x^8} + \frac{72e}{x^7} + \frac{84f}{x^6} \right) (a + bx^4)^{3/2} \\
 &= -\frac{b \left(\frac{224c}{x^5} + \frac{315d}{x^4} + \frac{480e}{x^3} + \frac{840f}{x^2} \right) \sqrt{a + bx^4}}{1680} - \frac{4b^2c\sqrt{a + bx^4}}{15ax} - \frac{1}{504} \left(\frac{56c}{x^9} + \frac{63d}{x^8} + \frac{72e}{x^7} + \frac{84f}{x^6} \right) (a + bx^4)^{3/2} \\
 &= -\frac{b \left(\frac{224c}{x^5} + \frac{315d}{x^4} + \frac{480e}{x^3} + \frac{840f}{x^2} \right) \sqrt{a + bx^4}}{1680} - \frac{4b^2c\sqrt{a + bx^4}}{15ax} + \frac{4b^{5/2}cx\sqrt{a + bx^4}}{15a(\sqrt{a} + \sqrt{bx^2})} \\
 &= -\frac{b \left(\frac{224c}{x^5} + \frac{315d}{x^4} + \frac{480e}{x^3} + \frac{840f}{x^2} \right) \sqrt{a + bx^4}}{1680} - \frac{4b^2c\sqrt{a + bx^4}}{15ax} + \frac{4b^{5/2}cx\sqrt{a + bx^4}}{15a(\sqrt{a} + \sqrt{bx^2})} \\
 &= -\frac{b \left(\frac{224c}{x^5} + \frac{315d}{x^4} + \frac{480e}{x^3} + \frac{840f}{x^2} \right) \sqrt{a + bx^4}}{1680} - \frac{4b^2c\sqrt{a + bx^4}}{15ax} + \frac{4b^{5/2}cx\sqrt{a + bx^4}}{15a(\sqrt{a} + \sqrt{bx^2})}
 \end{aligned}$$

Mathematica [C] time = 0.314219, size = 174, normalized size = 0.43

$$\frac{\sqrt{a+bx^4} \left(3x \left(7 \left(8a^2 f x^2 {}_2F_1 \left(-\frac{3}{2}, -\frac{3}{2}; -\frac{1}{2}; -\frac{bx^4}{a} \right) + 9b^2 dx^8 \tanh^{-1} \left(\sqrt{\frac{bx^4}{a} + 1} \right) + 3ad(2a + 5bx^4) \sqrt{\frac{bx^4}{a} + 1} \right) + 48a^2 ex {}_2F_1 \left(-\frac{3}{2}, -\frac{3}{2}; -\frac{1}{2}; -\frac{bx^4}{a} \right) \right) \right)}{1008ax^9 \sqrt{\frac{bx^4}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^10,x]

[Out] -(Sqrt[a + b*x^4]*(112*a^2*c*Hypergeometric2F1[-9/4, -3/2, -5/4, -((b*x^4)/a)] + 3*x*(48*a^2*e*x*Hypergeometric2F1[-7/4, -3/2, -3/4, -((b*x^4)/a)] + 7*(3*a*d*(2*a + 5*b*x^4)*Sqrt[1 + (b*x^4)/a] + 9*b^2*d*x^8*ArcTanh[Sqrt[1 + (b*x^4)/a]] + 8*a^2*f*x^2*Hypergeometric2F1[-3/2, -3/2, -1/2, -((b*x^4)/a)])))/(1008*a*x^9*Sqrt[1 + (b*x^4)/a])

Maple [C] time = 0.018, size = 437, normalized size = 1.1

$$-\frac{3b^2d}{16} \ln \left(\frac{1}{x^2} \left(2a + 2\sqrt{a}\sqrt{bx^4 + a} \right) \right) \frac{1}{\sqrt{a}} - \frac{ad}{8x^8} \sqrt{bx^4 + a} - \frac{5bd}{16x^4} \sqrt{bx^4 + a} + \frac{f}{2} b^{\frac{3}{2}} \ln \left(x^2 \sqrt{b} + \sqrt{bx^4 + a} \right) - \frac{af}{6x^6} \sqrt{bx^4 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^10,x)

[Out] -3/16*d*b^2/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/8*d*a/x^8*(b*x^4+a)^(1/2)-5/16*d*b/x^4*(b*x^4+a)^(1/2)+1/2*f*b^(3/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))-1/6*f*a/x^6*(b*x^4+a)^(1/2)-2/3*f*b/x^2*(b*x^4+a)^(1/2)-1/7*e*a*(b*x^4+a)^(1/2)/x^7-3/7*e*b*(b*x^4+a)^(1/2)/x^3+4/7*e*b^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/9*c*a*(b*x^4+a)^(1/2)/x^9-11/45*c*b*(b*x^4+a)^(1/2)/x^5-4/15*b^2*c*(b*x^4+a)^(1/2)/a/x+4/15*I*c/a^(1/2)*b^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-4/15*I*c/a^(1/2)*b^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^10,x, algorithm="maxima")

[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^10, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bfx^7 + bex^6 + bdx^5 + bcx^4 + afx^3 + aex^2 + adx + ac)\sqrt{bx^4 + a}}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^10,x, algorithm="fricas")

[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x + a*c)*sqrt(b*x^4 + a)/x^10, x)

Sympy [C] time = 13.5714, size = 449, normalized size = 1.11

$$\frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{9}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{9}{4}, -\frac{1}{2} \\ -\frac{5}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^9\Gamma\left(-\frac{5}{4}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{7}{4}, -\frac{1}{2} \\ -\frac{3}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{\sqrt{abc}\Gamma\left(-\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, -\frac{1}{2} \\ -\frac{1}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{\sqrt{abe}\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4} \end{matrix} \right)}{4x^3\Gamma\left(-\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**10,x)

[Out] a**(3/2)*c*gamma(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**9*gamma(-5/4)) + a**(3/2)*e*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3/4)) + sqrt(a)*b*c*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4))

```

) + sqrt(a)*b*e*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*
pi)/a)/(4*x**3*gamma(1/4)) - sqrt(a)*b*f/(2*x**2*sqrt(1 + b*x**4/a)) - a**2
*d/(8*sqrt(b)*x**10*sqrt(a/(b*x**4) + 1)) - 3*a*sqrt(b)*d/(16*x**6*sqrt(a/(
b*x**4) + 1)) - a*sqrt(b)*f*sqrt(a/(b*x**4) + 1)/(6*x**4) - b**(3/2)*d*sqrt
(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*d/(16*x**2*sqrt(a/(b*x**4) + 1)) - b**
(3/2)*f*sqrt(a/(b*x**4) + 1)/6 + b**(3/2)*f*asinh(sqrt(b)*x**2/sqrt(a))/2 -
3*b**2*d*asinh(sqrt(a)/(sqrt(b)*x**2))/(16*sqrt(a)) - b**2*f*x**2/(2*sqrt(
a)*sqrt(1 + b*x**4/a))

```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^10,x, algorithm="giac")

[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^10, x)

$$3.525 \quad \int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{11}} dx$$

Optimal. Leaf size=399

$$\frac{2b^{7/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (15\sqrt{a}f + 7\sqrt{bd}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{105a^{3/4}\sqrt{a+bx^4}} - \frac{4b^{9/4}d(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}$$

[Out] $-(b*((168*c)/x^6 + (224*d)/x^5 + (315*e)/x^4 + (480*f)/x^3)*\operatorname{Sqrt}[a + b*x^4])/1680 - (b^2*c*\operatorname{Sqrt}[a + b*x^4])/(10*a*x^2) - (4*b^2*d*\operatorname{Sqrt}[a + b*x^4])/(15*a*x) + (4*b^{5/2}*d*x*\operatorname{Sqrt}[a + b*x^4])/(15*a*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) - ((252*c)/x^{10} + (280*d)/x^9 + (315*e)/x^8 + (360*f)/x^7)*(a + b*x^4)^{(3/2)}/2520 - (3*b^2*e*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*x^4]/\operatorname{Sqrt}[a]])/(16*\operatorname{Sqrt}[a]) - (4*b^{9/4}*d*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2])/(15*a^{3/4}*\operatorname{Sqrt}[a + b*x^4]) + (2*b^{7/4}*(7*\operatorname{Sqrt}[b]*d + 15*\operatorname{Sqrt}[a]*f)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2])/(105*a^{3/4}*\operatorname{Sqrt}[a + b*x^4])$

Rubi [A] time = 0.420172, antiderivative size = 399, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {14, 1825, 1833, 1252, 807, 266, 63, 208, 1282, 1198, 220, 1196}

$$\frac{2b^{7/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (15\sqrt{a}f + 7\sqrt{bd}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{105a^{3/4}\sqrt{a+bx^4}} - \frac{4b^{9/4}d(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^{(3/2)}/x^{11}, x]$

[Out] $-(b*((168*c)/x^6 + (224*d)/x^5 + (315*e)/x^4 + (480*f)/x^3)*\operatorname{Sqrt}[a + b*x^4])/1680 - (b^2*c*\operatorname{Sqrt}[a + b*x^4])/(10*a*x^2) - (4*b^2*d*\operatorname{Sqrt}[a + b*x^4])/(15*a*x) + (4*b^{5/2}*d*x*\operatorname{Sqrt}[a + b*x^4])/(15*a*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) - ((252*c)/x^{10} + (280*d)/x^9 + (315*e)/x^8 + (360*f)/x^7)*(a + b*x^4)^{(3/2)}/2520 - (3*b^2*e*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*x^4]/\operatorname{Sqrt}[a]])/(16*\operatorname{Sqrt}[a]) - (4*b^{9/4}*d*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2])/(15*a^{3/4}*\operatorname{Sqrt}[a + b*x^4]) + (2*b^{7/4}*(7*\operatorname{Sqrt}[b]*d + 15*\operatorname{Sqrt}[a]*f)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2])/(105*a^{3/4}*\operatorname{Sqrt}[a + b*x^4])$

$$x^4)/(\text{Sqrt}[a] + \text{Sqrt}[b*x^2]^2)*\text{EllipticF}[2*\text{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2]/(105*a^{3/4}*\text{Sqrt}[a + b*x^4])$$
Rule 14

```
Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^((m_))*((a_) + (b_)*(x_)^((n_)))^(p_), x_Symbol] := Module[{u = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1, 0]
```

Rule 1833

```
Int[(Pq_)*((c_)*(x_))^(m_))*((a_) + (b_)*(x_)^((n_)))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p)/c^j, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 1252

```
Int[(x_)^((m_))*((d_) + (e_)*(x_)^2)^(q_))*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 807

```
Int[((d_) + (e_)*(x_))^(m_))*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^((m_))*((a_) + (b_)*(x_)^((n_)))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1282

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] := Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1)/(a*f*(m + 1)), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2]]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2]]*EllipticE[2*ArcTan[q*x],
1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}}{x^{11}} dx &= -\frac{\left(\frac{252c}{x^{10}} + \frac{280d}{x^9} + \frac{315e}{x^8} + \frac{360f}{x^7}\right)(a + bx^4)^{3/2}}{2520} - (6b) \int \frac{\left(-\frac{c}{10} - \frac{dx}{9} - \frac{ex^2}{8} - \frac{fx^3}{7}\right)\sqrt{a + bx^4}}{x^7} \\
&= -\frac{b\left(\frac{168c}{x^6} + \frac{224d}{x^5} + \frac{315e}{x^4} + \frac{480f}{x^3}\right)\sqrt{a + bx^4}}{1680} - \frac{\left(\frac{252c}{x^{10}} + \frac{280d}{x^9} + \frac{315e}{x^8} + \frac{360f}{x^7}\right)(a + bx^4)^{3/2}}{2520} \\
&= -\frac{b\left(\frac{168c}{x^6} + \frac{224d}{x^5} + \frac{315e}{x^4} + \frac{480f}{x^3}\right)\sqrt{a + bx^4}}{1680} - \frac{\left(\frac{252c}{x^{10}} + \frac{280d}{x^9} + \frac{315e}{x^8} + \frac{360f}{x^7}\right)(a + bx^4)^{3/2}}{2520} \\
&= -\frac{b\left(\frac{168c}{x^6} + \frac{224d}{x^5} + \frac{315e}{x^4} + \frac{480f}{x^3}\right)\sqrt{a + bx^4}}{1680} - \frac{\left(\frac{252c}{x^{10}} + \frac{280d}{x^9} + \frac{315e}{x^8} + \frac{360f}{x^7}\right)(a + bx^4)^{3/2}}{2520} \\
&= -\frac{b\left(\frac{168c}{x^6} + \frac{224d}{x^5} + \frac{315e}{x^4} + \frac{480f}{x^3}\right)\sqrt{a + bx^4}}{1680} - \frac{4b^2d\sqrt{a + bx^4}}{15ax} - \frac{\left(\frac{252c}{x^{10}} + \frac{280d}{x^9} + \frac{315e}{x^8} + \frac{360f}{x^7}\right)(a + bx^4)^{3/2}}{2520} \\
&= -\frac{b\left(\frac{168c}{x^6} + \frac{224d}{x^5} + \frac{315e}{x^4} + \frac{480f}{x^3}\right)\sqrt{a + bx^4}}{1680} - \frac{b^2c\sqrt{a + bx^4}}{10ax^2} - \frac{4b^2d\sqrt{a + bx^4}}{15ax} - \frac{\left(\frac{252c}{x^{10}} + \frac{280d}{x^9} + \frac{315e}{x^8} + \frac{360f}{x^7}\right)(a + bx^4)^{3/2}}{2520} \\
&= -\frac{b\left(\frac{168c}{x^6} + \frac{224d}{x^5} + \frac{315e}{x^4} + \frac{480f}{x^3}\right)\sqrt{a + bx^4}}{1680} - \frac{b^2c\sqrt{a + bx^4}}{10ax^2} - \frac{4b^2d\sqrt{a + bx^4}}{15ax} + \frac{4b^2e\sqrt{a + bx^4}}{15ax} - \frac{\left(\frac{252c}{x^{10}} + \frac{280d}{x^9} + \frac{315e}{x^8} + \frac{360f}{x^7}\right)(a + bx^4)^{3/2}}{2520} \\
&= -\frac{b\left(\frac{168c}{x^6} + \frac{224d}{x^5} + \frac{315e}{x^4} + \frac{480f}{x^3}\right)\sqrt{a + bx^4}}{1680} - \frac{b^2c\sqrt{a + bx^4}}{10ax^2} - \frac{4b^2d\sqrt{a + bx^4}}{15ax} + \frac{4b^2e\sqrt{a + bx^4}}{15ax} - \frac{\left(\frac{252c}{x^{10}} + \frac{280d}{x^9} + \frac{315e}{x^8} + \frac{360f}{x^7}\right)(a + bx^4)^{3/2}}{2520} \\
&= -\frac{b\left(\frac{168c}{x^6} + \frac{224d}{x^5} + \frac{315e}{x^4} + \frac{480f}{x^3}\right)\sqrt{a + bx^4}}{1680} - \frac{b^2c\sqrt{a + bx^4}}{10ax^2} - \frac{4b^2d\sqrt{a + bx^4}}{15ax} + \frac{4b^2e\sqrt{a + bx^4}}{15ax} - \frac{\left(\frac{252c}{x^{10}} + \frac{280d}{x^9} + \frac{315e}{x^8} + \frac{360f}{x^7}\right)(a + bx^4)^{3/2}}{2520}
\end{aligned}$$

Mathematica [C] time = 0.326002, size = 171, normalized size = 0.43

$$\frac{\sqrt{a + bx^4} \left(63\sqrt{\frac{bx^4}{a}} + 1 \right) \left(2a^2(4c + 5ex^2) + abx^4(16c + 25ex^2) + 8b^2cx^8 \right) + 560a^2dx {}_2F_1\left(-\frac{9}{4}, -\frac{3}{2}; -\frac{5}{4}; -\frac{bx^4}{a}\right) + 720a^2fx^3 {}_2F_1\left(-\frac{9}{4}, -\frac{3}{2}; -\frac{5}{4}; -\frac{bx^4}{a}\right)}{5040ax^{10}\sqrt{\frac{bx^4}{a}} + 1}$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^11,x]

[Out] $-(\text{Sqrt}[a + b*x^4]*(63*\text{Sqrt}[1 + (b*x^4)/a]*(8*b^2*c*x^8 + 2*a^2*(4*c + 5*e*x^2) + a*b*x^4*(16*c + 25*e*x^2)) + 945*b^2*e*x^10*\text{ArcTanh}[\text{Sqrt}[1 + (b*x^4)/a]]) + 560*a^2*d*x*\text{Hypergeometric2F1}[-9/4, -3/2, -5/4, -((b*x^4)/a)] + 720*a^2*f*x^3*\text{Hypergeometric2F1}[-7/4, -3/2, -3/4, -((b*x^4)/a)])/(5040*a*x^10*\text{Sqrt}[1 + (b*x^4)/a])$

Maple [C] time = 0.019, size = 417, normalized size = 1.1

$$-\frac{c(b^2x^8 + 2abx^4 + a^2)}{10x^{10}a}\sqrt{bx^4 + a} - \frac{3b^2e}{16}\ln\left(\frac{1}{x^2}\left(2a + 2\sqrt{a}\sqrt{bx^4 + a}\right)\right)\frac{1}{\sqrt{a}} - \frac{ae}{8x^8}\sqrt{bx^4 + a} - \frac{5be}{16x^4}\sqrt{bx^4 + a} - \frac{af}{7x^7}\sqrt{bx^4 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^{(3/2)}/x^{11},x)$

[Out] $-1/10*c*(b*x^4+a)^{(1/2)}/x^{10}/a*(b^2*x^8+2*a*b*x^4+a^2)-3/16*e*b^2/a^{(1/2)*1}\ln((2*a+2*a^{(1/2)}*(b*x^4+a)^{(1/2)})/x^2)-1/8*e*a/x^8*(b*x^4+a)^{(1/2)}-5/16*e*b/x^4*(b*x^4+a)^{(1/2)}-1/7*f*a*(b*x^4+a)^{(1/2)}/x^7-3/7*f*b*(b*x^4+a)^{(1/2)}/x^3+4/7*f*b^2/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticF}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)},I)-1/9*d*a*(b*x^4+a)^{(1/2)}/x^9-11/45*d*b*(b*x^4+a)^{(1/2)}/x^5-4/15*b^2*d*(b*x^4+a)^{(1/2)}/a/x+4/15*I*d/a^{(1/2)}*b^{(5/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticF}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)},I)-4/15*I*d/a^{(1/2)}*b^{(5/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticE}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)},I)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{(bx^4 + a)^{\frac{5}{2}}c}{10ax^{10}} + \int \frac{(bfx^6 + bex^5 + bdx^4 + afx^2 + aex + ad)\sqrt{bx^4 + a}}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^{(3/2)}/x^{11},x, \text{algorithm}=\text{"maxima"})$

[Out] $-1/10*(b*x^4 + a)^{(5/2)}*c/(a*x^{10}) + \text{integrate}((b*f*x^6 + b*e*x^5 + b*d*x^4 + a*f*x^2 + a*e*x + a*d)*\text{sqrt}(b*x^4 + a)/x^{10}, x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bf x^7 + bex^6 + bdx^5 + bcx^4 + afx^3 + aex^2 + adx + ac)\sqrt{bx^4 + a}}{x^{11}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^11,x, algorithm="fricas")

[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x + a*c)*sqrt(b*x^4 + a)/x^11, x)

Sympy [C] time = 15.602, size = 398, normalized size = 1.

$$\frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{9}{4}\right)_2F_1\left(-\frac{9}{4}, -\frac{1}{2}\left|\frac{bx^4e^{i\pi}}{a}\right.\right)}{4x^9\Gamma\left(-\frac{5}{4}\right)} + \frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{7}{4}\right)_2F_1\left(-\frac{7}{4}, -\frac{1}{2}\left|\frac{bx^4e^{i\pi}}{a}\right.\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{\sqrt{abd}\Gamma\left(-\frac{5}{4}\right)_2F_1\left(-\frac{5}{4}, -\frac{1}{2}\left|\frac{bx^4e^{i\pi}}{a}\right.\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{\sqrt{abf}\Gamma\left(-\frac{3}{4}\right)_2F_1\left(-\frac{3}{4}, -\frac{1}{2}\left|\frac{bx^4e^{i\pi}}{a}\right.\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**11,x)

[Out] a**(3/2)*d*gamma(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**9*gamma(-5/4)) + a**(3/2)*f*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3/4)) + sqrt(a)*b*d*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) + sqrt(a)*b*f*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - a**2*e/(8*sqrt(b)*x**10*sqrt(a/(b*x**4) + 1)) - a*sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(10*x**8) - 3*a*sqrt(b)*e/(16*x**6*sqrt(a/(b*x**4) + 1)) - b**(3/2)*c*sqrt(a/(b*x**4) + 1)/(5*x**4) - b**(3/2)*e*sqrt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*e/(16*x**2*sqrt(a/(b*x**4) + 1)) - b**(5/2)*c*sqrt(a/(b*x**4) + 1)/(10*a) - 3*b**2*e*asinh(sqrt(a)/(sqrt(b)*x**2))/(16*sqrt(a))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x^{11}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^11,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^11, x)
```

$$3.526 \quad \int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{12}} dx$$

Optimal. Leaf size=424

$$\frac{2b^{9/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (15\sqrt{bc} - 77\sqrt{ae}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{1155a^{5/4}\sqrt{a+bx^4}} - \frac{4b^{9/4}e(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}$$

[Out] $-(b*((1440*c)/x^7 + (1848*d)/x^6 + (2464*e)/x^5 + (3465*f)/x^4)*\operatorname{Sqrt}[a + b*x^4])/18480 - (4*b^2*c*\operatorname{Sqrt}[a + b*x^4])/(77*a*x^3) - (b^2*d*\operatorname{Sqrt}[a + b*x^4])/(10*a*x^2) - (4*b^2*e*\operatorname{Sqrt}[a + b*x^4])/(15*a*x) + (4*b^(5/2)*e*x*\operatorname{Sqrt}[a + b*x^4])/(15*a*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) - (((360*c)/x^{11} + (396*d)/x^{10} + (440*e)/x^9 + (495*f)/x^8)*(a + b*x^4)^{(3/2)}/3960 - (3*b^2*f*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*x^4]/\operatorname{Sqrt}[a]])/(16*\operatorname{Sqrt}[a]) - (4*b^{(9/4)}*e*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(15*a^{(3/4)}*\operatorname{Sqrt}[a + b*x^4]) - (2*b^{(9/4)}*(15*\operatorname{Sqrt}[b]*c - 77*\operatorname{Sqrt}[a]*e)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(1155*a^{(5/4)}*\operatorname{Sqrt}[a + b*x^4])$

Rubi [A] time = 0.461727, antiderivative size = 424, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {14, 1825, 1833, 1282, 1198, 220, 1196, 1252, 807, 266, 63, 208}

$$\frac{2b^{9/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (15\sqrt{bc} - 77\sqrt{ae}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{1155a^{5/4}\sqrt{a+bx^4}} - \frac{4b^{9/4}e(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^{(3/2)}/x^{12}, x]$

[Out] $-(b*((1440*c)/x^7 + (1848*d)/x^6 + (2464*e)/x^5 + (3465*f)/x^4)*\operatorname{Sqrt}[a + b*x^4])/18480 - (4*b^2*c*\operatorname{Sqrt}[a + b*x^4])/(77*a*x^3) - (b^2*d*\operatorname{Sqrt}[a + b*x^4])/(10*a*x^2) - (4*b^2*e*\operatorname{Sqrt}[a + b*x^4])/(15*a*x) + (4*b^(5/2)*e*x*\operatorname{Sqrt}[a + b*x^4])/(15*a*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) - (((360*c)/x^{11} + (396*d)/x^{10} + (440*e)/x^9 + (495*f)/x^8)*(a + b*x^4)^{(3/2)}/3960 - (3*b^2*f*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*x^4]/\operatorname{Sqrt}[a]])/(16*\operatorname{Sqrt}[a]) - (4*b^{(9/4)}*e*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(15*a^{(3/4)}*\operatorname{Sqrt}[a + b*x^4]) - (2*b^{(9/4)}*(15*\operatorname{Sqrt}[b]*c - 77*\operatorname{Sqrt}[a]*e)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(1155*a^{(5/4)}*\operatorname{Sqrt}[a + b*x^4])$

$1/4]$, $1/2]$)/ $(15*a^{(3/4)*Sqrt[a + b*x^4]) - (2*b^{(9/4)*(15*Sqrt[b]*c - 77*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^{(1/4)*x)/a^{(1/4)}], 1/2])/(1155*a^{(5/4)*Sqrt[a + b*x^4])}$

Rule 14

$Int[(u_)*((c_)*(x_))^{(m_)}, x_Symbol] \rightarrow Int[ExpandIntegrand[(c*x)^m*u, x], x] /;$ FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

Rule 1825

$Int[(Pq_)*(x_)^{(m_)*((a_) + (b_)*(x_)^{(n_))^{(p_)}, x_Symbol] \rightarrow Module[{u = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^{(m + n)}*(a + b*x^n)^{(p - 1)*ExpandToSum[u/x^{(m + 1)}, x], x], x]] /;$ FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1, 0]

Rule 1833

$Int[(Pq_)*((c_)*(x_))^{(m_)*((a_) + (b_)*(x_)^{(n_))^{(p_)}, x_Symbol] \rightarrow Module[{q = Expon[Pq, x], j, k}, Int[Sum[(((c*x)^{(m + j)*Sum[Coeff[Pq, x, j + (k*n)/2]*x^{((k*n)/2)}, {k, 0, (2*(q - j))/n + 1}])*(a + b*x^n)^p)/c^j, {j, 0, n/2 - 1}], x]] /;$ FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^{(n/2)}]

Rule 1282

$Int[((f_)*(x_))^{(m_)*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^{(p_)}, x_Symbol] \rightarrow Simp[(d*(f*x)^{(m + 1)*(a + c*x^4)^{(p + 1)})/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^{(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(m + 4*p + 5)*x^2), x], x] /;$ FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1198

$Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] \rightarrow With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /;$ NeQ[e + d*q, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rule 1252

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rule 807

Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 63

Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}}{x^{12}} dx &= -\frac{\left(\frac{360c}{x^{11}} + \frac{396d}{x^{10}} + \frac{440e}{x^9} + \frac{495f}{x^8}\right)(a + bx^4)^{3/2}}{3960} - (6b) \int \frac{\left(-\frac{c}{11} - \frac{dx}{10} - \frac{ex^2}{9} - \frac{fx^3}{8}\right)\sqrt{a + bx^4}}{x^8} dx \\
&= -\frac{b\left(\frac{1440c}{x^7} + \frac{1848d}{x^6} + \frac{2464e}{x^5} + \frac{3465f}{x^4}\right)\sqrt{a + bx^4}}{18480} - \frac{\left(\frac{360c}{x^{11}} + \frac{396d}{x^{10}} + \frac{440e}{x^9} + \frac{495f}{x^8}\right)(a + bx^4)^{3/2}}{3960} \\
&= -\frac{b\left(\frac{1440c}{x^7} + \frac{1848d}{x^6} + \frac{2464e}{x^5} + \frac{3465f}{x^4}\right)\sqrt{a + bx^4}}{18480} - \frac{\left(\frac{360c}{x^{11}} + \frac{396d}{x^{10}} + \frac{440e}{x^9} + \frac{495f}{x^8}\right)(a + bx^4)^{3/2}}{3960} \\
&= -\frac{b\left(\frac{1440c}{x^7} + \frac{1848d}{x^6} + \frac{2464e}{x^5} + \frac{3465f}{x^4}\right)\sqrt{a + bx^4}}{18480} - \frac{\left(\frac{360c}{x^{11}} + \frac{396d}{x^{10}} + \frac{440e}{x^9} + \frac{495f}{x^8}\right)(a + bx^4)^{3/2}}{3960} \\
&= -\frac{b\left(\frac{1440c}{x^7} + \frac{1848d}{x^6} + \frac{2464e}{x^5} + \frac{3465f}{x^4}\right)\sqrt{a + bx^4}}{18480} - \frac{4b^2c\sqrt{a + bx^4}}{77ax^3} - \frac{\left(\frac{360c}{x^{11}} + \frac{396d}{x^{10}} + \frac{440e}{x^9} + \frac{495f}{x^8}\right)(a + bx^4)^{3/2}}{3960} \\
&= -\frac{b\left(\frac{1440c}{x^7} + \frac{1848d}{x^6} + \frac{2464e}{x^5} + \frac{3465f}{x^4}\right)\sqrt{a + bx^4}}{18480} - \frac{4b^2c\sqrt{a + bx^4}}{77ax^3} - \frac{b^2d\sqrt{a + bx^4}}{10ax^2} \\
&= -\frac{b\left(\frac{1440c}{x^7} + \frac{1848d}{x^6} + \frac{2464e}{x^5} + \frac{3465f}{x^4}\right)\sqrt{a + bx^4}}{18480} - \frac{4b^2c\sqrt{a + bx^4}}{77ax^3} - \frac{b^2d\sqrt{a + bx^4}}{10ax^2} \\
&= -\frac{b\left(\frac{1440c}{x^7} + \frac{1848d}{x^6} + \frac{2464e}{x^5} + \frac{3465f}{x^4}\right)\sqrt{a + bx^4}}{18480} - \frac{4b^2c\sqrt{a + bx^4}}{77ax^3} - \frac{b^2d\sqrt{a + bx^4}}{10ax^2} \\
&= -\frac{b\left(\frac{1440c}{x^7} + \frac{1848d}{x^6} + \frac{2464e}{x^5} + \frac{3465f}{x^4}\right)\sqrt{a + bx^4}}{18480} - \frac{4b^2c\sqrt{a + bx^4}}{77ax^3} - \frac{b^2d\sqrt{a + bx^4}}{10ax^2} \\
&= -\frac{b\left(\frac{1440c}{x^7} + \frac{1848d}{x^6} + \frac{2464e}{x^5} + \frac{3465f}{x^4}\right)\sqrt{a + bx^4}}{18480} - \frac{4b^2c\sqrt{a + bx^4}}{77ax^3} - \frac{b^2d\sqrt{a + bx^4}}{10ax^2}
\end{aligned}$$

Mathematica [C] time = 0.38134, size = 172, normalized size = 0.41

$$\frac{\sqrt{a + bx^4} \left(11x \left(9\sqrt{\frac{bx^4}{a}} + 1 \right) (2a^2(4d + 5fx^2) + abx^4(16d + 25fx^2) + 8b^2dx^8) + 80a^2ex {}_2F_1\left(-\frac{9}{4}, -\frac{3}{2}; -\frac{5}{4}; -\frac{bx^4}{a}\right) + 135b^2 \right)}{7920ax^{11} \sqrt{\frac{bx^4}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^12,x]

[Out] -(Sqrt[a + b*x^4]*(720*a^2*c*Hypergeometric2F1[-11/4, -3/2, -7/4, -((b*x^4)/a)] + 11*x*(9*Sqrt[1 + (b*x^4)/a]*(8*b^2*d*x^8 + 2*a^2*(4*d + 5*f*x^2) + a*b*x^4*(16*d + 25*f*x^2)) + 135*b^2*f*x^10*ArcTanh[Sqrt[1 + (b*x^4)/a]] + 80*a^2*e*x*Hypergeometric2F1[-9/4, -3/2, -5/4, -((b*x^4)/a)])))/(7920*a*x^11*Sqrt[1 + (b*x^4)/a])

Maple [C] time = 0.02, size = 441, normalized size = 1.

$$-\frac{ac}{11x^{11}}\sqrt{bx^4+a} - \frac{13bc}{77x^7}\sqrt{bx^4+a} - \frac{4b^2c}{77ax^3}\sqrt{bx^4+a} - \frac{4b^3c}{77a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^12,x)

[Out] -1/11*c*a*(b*x^4+a)^(1/2)/x^11-13/77*c*b*(b*x^4+a)^(1/2)/x^7-4/77*b^2*c*(b*x^4+a)^(1/2)/a/x^3-4/77*c/a*b^3/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-3/16*f*b^2/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/8*f*a/x^8*(b*x^4+a)^(1/2)-5/16*f*b/x^4*(b*x^4+a)^(1/2)-1/9*e*a*(b*x^4+a)^(1/2)/x^9-11/45*e*b*(b*x^4+a)^(1/2)/x^5-4/15*b^2*e*(b*x^4+a)^(1/2)/a/x+4/15*I*e/a^(1/2)*b^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-4/15*I*e/a^(1/2)*b^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/10*d*(b*x^4+a)^(1/2)/x^10/a*(b^2*x^8+2*a*b*x^4+a^2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4+a)^2(fx^3+ex^2+dx+c)}{x^{12}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^12,x, algorithm="maxima")

[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^12, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bf x^7 + bex^6 + bdx^5 + bcx^4 + afx^3 + aex^2 + adx + ac)\sqrt{bx^4 + a}}{x^{12}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^12,x, algorithm="fricas")

[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x + a*c)*sqrt(b*x^4 + a)/x^12, x)

Sympy [C] time = 15.4933, size = 401, normalized size = 0.95

$$\frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{11}{4}\right)_2F_1\left(-\frac{11}{4}, -\frac{1}{2}\left|\frac{bx^4e^{i\pi}}{a}\right.\right)}{4x^{11}\Gamma\left(-\frac{7}{4}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{9}{4}\right)_2F_1\left(-\frac{9}{4}, -\frac{1}{2}\left|\frac{bx^4e^{i\pi}}{a}\right.\right)}{4x^9\Gamma\left(-\frac{5}{4}\right)} + \frac{\sqrt{abc}\Gamma\left(-\frac{7}{4}\right)_2F_1\left(-\frac{7}{4}, -\frac{1}{2}\left|\frac{bx^4e^{i\pi}}{a}\right.\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{\sqrt{abe}\Gamma\left(-\frac{5}{4}\right)_2F_1\left(-\frac{5}{4}, -\frac{1}{2}\left|\frac{bx^4e^{i\pi}}{a}\right.\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**12,x)

[Out] a**(3/2)*c*gamma(-11/4)*hyper((-11/4, -1/2), (-7/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**11*gamma(-7/4)) + a**(3/2)*e*gamma(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**9*gamma(-5/4)) + sqrt(a)*b*c*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3/4)) + sqrt(a)*b*e*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) - a**2*f/(8*sqrt(b)*x**10*sqrt(a/(b*x**4) + 1)) - a*sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(10*x**8) - 3*a*sqrt(b)*f/(16*x**6*sqrt(a/(b*x**4) + 1)) - b**(3/2)*d*sqrt(a/(b*x**4) + 1)/(5*x**4) - b**(3/2)*f*sqrt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*f/(16*x**2*sqrt(a/(b*x**4) + 1)) - b**(5/2)*d*sqrt(a/(b*x**4) + 1)/(10*a) - 3*b**2*f*asinh(sqrt(a)/(sqrt(b)*x**2))/(16*sqrt(a))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x^{12}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^12,x, algorithm="giac")

[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^12, x)

$$3.527 \quad \int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{13}} dx$$

Optimal. Leaf size=449

$$\frac{2b^{9/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a} + \sqrt{bx^2})^2}} (15\sqrt{bd} - 77\sqrt{af}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right) + b^3 c \tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - 4b^{9/4} f (\sqrt{a} + \sqrt{bx^2})}{1155a^{5/4}\sqrt{a+bx^4}} + \frac{b^3 c \tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - 4b^{9/4} f (\sqrt{a} + \sqrt{bx^2})}{32a^{3/2}}$$

[Out] $-(b*((1155*c)/x^8 + (1440*d)/x^7 + (1848*e)/x^6 + (2464*f)/x^5)*\operatorname{Sqrt}[a + b*x^4])/18480 - (b^2*c*\operatorname{Sqrt}[a + b*x^4])/(32*a*x^4) - (4*b^2*d*\operatorname{Sqrt}[a + b*x^4])/(77*a*x^3) - (b^2*e*\operatorname{Sqrt}[a + b*x^4])/(10*a*x^2) - (4*b^2*f*\operatorname{Sqrt}[a + b*x^4])/(15*a*x) + (4*b^{5/2}*f*x*\operatorname{Sqrt}[a + b*x^4])/(15*a*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) - (((165*c)/x^{12} + (180*d)/x^{11} + (198*e)/x^{10} + (220*f)/x^9)*(a + b*x^4)^{3/2})/1980 + (b^3*c*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*x^4]/\operatorname{Sqrt}[a]])/(32*a^{3/2}) - (4*b^{9/4}*f*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2])* \operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2])/(15*a^{3/4}*\operatorname{Sqrt}[a + b*x^4]) - (2*b^{9/4}*(15*\operatorname{Sqrt}[b]*d - 77*\operatorname{Sqrt}[a]*f)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2])* \operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2])/(1155*a^{5/4}*\operatorname{Sqrt}[a + b*x^4])$

Rubi [A] time = 0.490132, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {14, 1825, 1833, 1252, 835, 807, 266, 63, 208, 1282, 1198, 220, 1196}

$$\frac{b^3 c \tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - 4b^{9/4} f (\sqrt{a} + \sqrt{bx^2})}{32a^{3/2}} - \frac{2b^{9/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a} + \sqrt{bx^2})^2}} (15\sqrt{bd} - 77\sqrt{af}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right) - 4b^{9/4} f (\sqrt{a} + \sqrt{bx^2})}{1155a^{5/4}\sqrt{a+bx^4}} - \frac{4b^{9/4} f (\sqrt{a} + \sqrt{bx^2})}{15}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^{3/2}]/x^{13}, x]$

[Out] $-(b*((1155*c)/x^8 + (1440*d)/x^7 + (1848*e)/x^6 + (2464*f)/x^5)*\operatorname{Sqrt}[a + b*x^4])/18480 - (b^2*c*\operatorname{Sqrt}[a + b*x^4])/(32*a*x^4) - (4*b^2*d*\operatorname{Sqrt}[a + b*x^4])/(77*a*x^3) - (b^2*e*\operatorname{Sqrt}[a + b*x^4])/(10*a*x^2) - (4*b^2*f*\operatorname{Sqrt}[a + b*x^4])/(15*a*x) + (4*b^{5/2}*f*x*\operatorname{Sqrt}[a + b*x^4])/(15*a*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) - (((165*c)/x^{12} + (180*d)/x^{11} + (198*e)/x^{10} + (220*f)/x^9)*(a + b*x^4)^{3/2})/1980 + (b^3*c*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*x^4]/\operatorname{Sqrt}[a]])/(32*a^{3/2}) - (4*b^{9/4}*f*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2])$

```
] *EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a + b*x^4]) - (2*b^(9/4)*(15*Sqrt[b]*d - 77*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(1155*a^(5/4)*Sqrt[a + b*x^4])
```

Rule 14

```
Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^m_)*((a_) + (b_)*(x_)^n_)^p_, x_Symbol] := Module[{u = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1, 0]
```

Rule 1833

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^n_)^p_, x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p)/c^j, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 1252

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 835

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1282

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] := Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

, 1/2]]/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :=> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2])*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rubi steps

$$\begin{aligned}
\int \frac{(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}}{x^{13}} dx &= -\frac{\left(\frac{165c}{x^{12}} + \frac{180d}{x^{11}} + \frac{198e}{x^{10}} + \frac{220f}{x^9}\right)(a + bx^4)^{3/2}}{1980} - (6b) \int \frac{\left(-\frac{c}{12} - \frac{dx}{11} - \frac{ex^2}{10} - \frac{fx^3}{9}\right)\sqrt{a + bx^4}}{x^9} dx \\
&= -\frac{b\left(\frac{1155c}{x^8} + \frac{1440d}{x^7} + \frac{1848e}{x^6} + \frac{2464f}{x^5}\right)\sqrt{a + bx^4}}{18480} - \frac{\left(\frac{165c}{x^{12}} + \frac{180d}{x^{11}} + \frac{198e}{x^{10}} + \frac{220f}{x^9}\right)(a + bx^4)^{3/2}}{1980} \\
&= -\frac{b\left(\frac{1155c}{x^8} + \frac{1440d}{x^7} + \frac{1848e}{x^6} + \frac{2464f}{x^5}\right)\sqrt{a + bx^4}}{18480} - \frac{\left(\frac{165c}{x^{12}} + \frac{180d}{x^{11}} + \frac{198e}{x^{10}} + \frac{220f}{x^9}\right)(a + bx^4)^{3/2}}{1980} \\
&= -\frac{b\left(\frac{1155c}{x^8} + \frac{1440d}{x^7} + \frac{1848e}{x^6} + \frac{2464f}{x^5}\right)\sqrt{a + bx^4}}{18480} - \frac{\left(\frac{165c}{x^{12}} + \frac{180d}{x^{11}} + \frac{198e}{x^{10}} + \frac{220f}{x^9}\right)(a + bx^4)^{3/2}}{1980} \\
&= -\frac{b\left(\frac{1155c}{x^8} + \frac{1440d}{x^7} + \frac{1848e}{x^6} + \frac{2464f}{x^5}\right)\sqrt{a + bx^4}}{18480} - \frac{4b^2d\sqrt{a + bx^4}}{77ax^3} - \frac{\left(\frac{165c}{x^{12}} + \frac{180d}{x^{11}} + \frac{198e}{x^{10}} + \frac{220f}{x^9}\right)(a + bx^4)^{3/2}}{1980} \\
&= -\frac{b\left(\frac{1155c}{x^8} + \frac{1440d}{x^7} + \frac{1848e}{x^6} + \frac{2464f}{x^5}\right)\sqrt{a + bx^4}}{18480} - \frac{b^2c\sqrt{a + bx^4}}{32ax^4} - \frac{4b^2d\sqrt{a + bx^4}}{77ax^3} - \frac{\left(\frac{165c}{x^{12}} + \frac{180d}{x^{11}} + \frac{198e}{x^{10}} + \frac{220f}{x^9}\right)(a + bx^4)^{3/2}}{1980} \\
&= -\frac{b\left(\frac{1155c}{x^8} + \frac{1440d}{x^7} + \frac{1848e}{x^6} + \frac{2464f}{x^5}\right)\sqrt{a + bx^4}}{18480} - \frac{b^2c\sqrt{a + bx^4}}{32ax^4} - \frac{4b^2d\sqrt{a + bx^4}}{77ax^3} - \frac{\left(\frac{165c}{x^{12}} + \frac{180d}{x^{11}} + \frac{198e}{x^{10}} + \frac{220f}{x^9}\right)(a + bx^4)^{3/2}}{1980} \\
&= -\frac{b\left(\frac{1155c}{x^8} + \frac{1440d}{x^7} + \frac{1848e}{x^6} + \frac{2464f}{x^5}\right)\sqrt{a + bx^4}}{18480} - \frac{b^2c\sqrt{a + bx^4}}{32ax^4} - \frac{4b^2d\sqrt{a + bx^4}}{77ax^3} - \frac{\left(\frac{165c}{x^{12}} + \frac{180d}{x^{11}} + \frac{198e}{x^{10}} + \frac{220f}{x^9}\right)(a + bx^4)^{3/2}}{1980} \\
&= -\frac{b\left(\frac{1155c}{x^8} + \frac{1440d}{x^7} + \frac{1848e}{x^6} + \frac{2464f}{x^5}\right)\sqrt{a + bx^4}}{18480} - \frac{b^2c\sqrt{a + bx^4}}{32ax^4} - \frac{4b^2d\sqrt{a + bx^4}}{77ax^3} - \frac{\left(\frac{165c}{x^{12}} + \frac{180d}{x^{11}} + \frac{198e}{x^{10}} + \frac{220f}{x^9}\right)(a + bx^4)^{3/2}}{1980} \\
&= -\frac{b\left(\frac{1155c}{x^8} + \frac{1440d}{x^7} + \frac{1848e}{x^6} + \frac{2464f}{x^5}\right)\sqrt{a + bx^4}}{18480} - \frac{b^2c\sqrt{a + bx^4}}{32ax^4} - \frac{4b^2d\sqrt{a + bx^4}}{77ax^3} - \frac{\left(\frac{165c}{x^{12}} + \frac{180d}{x^{11}} + \frac{198e}{x^{10}} + \frac{220f}{x^9}\right)(a + bx^4)^{3/2}}{1980}
\end{aligned}$$

Mathematica [C] time = 0.191164, size = 149, normalized size = 0.33

$$\frac{\sqrt{a + bx^4} \left(11x \left(9(a + bx^4)^2 \sqrt{\frac{bx^4}{a} + 1} \left(a^3 e - b^3 c x^{10} {}_2F_1 \left(\frac{5}{2}, 4; \frac{7}{2}; \frac{bx^4}{a} + 1 \right) \right) + 10a^5 f x {}_2F_1 \left(-\frac{9}{4}, -\frac{3}{2}; -\frac{5}{4}; -\frac{bx^4}{a} \right) \right) + 90a^5 d {}_2F_1 \left(\frac{5}{2}, 4; \frac{7}{2}; \frac{bx^4}{a} + 1 \right) \right)}{990a^4 x^{11} \sqrt{\frac{bx^4}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^13,x]

[Out] $-(\text{Sqrt}[a + b*x^4]*(90*a^5*d*\text{Hypergeometric2F1}[-11/4, -3/2, -7/4, -((b*x^4)/a)] + 11*x*(10*a^5*f*x*\text{Hypergeometric2F1}[-9/4, -3/2, -5/4, -((b*x^4)/a)] + 9*(a + b*x^4)^2*\text{Sqrt}[1 + (b*x^4)/a]*(a^3*e - b^3*c*x^{10}*\text{Hypergeometric2F1}[5/2, 4, 7/2, 1 + (b*x^4)/a]))) / (990*a^4*x^{11}*\text{Sqrt}[1 + (b*x^4)/a])$

Maple [C] time = 0.023, size = 462, normalized size = 1.

$$-\frac{ad}{11x^{11}}\sqrt{bx^4+a} - \frac{13bd}{77x^7}\sqrt{bx^4+a} - \frac{4b^2d}{77ax^3}\sqrt{bx^4+a} - \frac{4b^3d}{77a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x,\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^13,x)

[Out] $-1/11*d*a*(b*x^4+a)^{(1/2)}/x^{11}-13/77*d*b*(b*x^4+a)^{(1/2)}/x^7-4/77*b^2*d*(b*x^4+a)^{(1/2)}/a/x^3-4/77*d/a*b^3/(I/a^{(1/2)*b^{(1/2)}})^{(1/2)}*(1-I/a^{(1/2)*b^{(1/2)}})^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)*b^{(1/2)}})^{(1/2)}*(b*x^4+a)^{(1/2)}*\text{EllipticF}(x*(I/a^{(1/2)*b^{(1/2)}})^{(1/2)},I)-1/10*e*(b*x^4+a)^{(1/2)}/x^{10}/a*(b^2*x^8+2*a*b*x^4+a^2)-7/48*c*b/x^8*(b*x^4+a)^{(1/2)}-1/32*b^2*c*(b*x^4+a)^{(1/2)}/a/x^4+1/32*c/a^{(3/2)*b^3*\ln((2*a+2*a^{(1/2)}*(b*x^4+a)^{(1/2)})/x^2)-1/12*c*a/x^{12}*(b*x^4+a)^{(1/2)}-1/9*f*a*(b*x^4+a)^{(1/2)}/x^9-11/45*f*b*(b*x^4+a)^{(1/2)}/x^5-4/15*b^2*f*(b*x^4+a)^{(1/2)}/a/x+4/15*I*f/a^{(1/2)*b^{(5/2)}}/(I/a^{(1/2)*b^{(1/2)}})^{(1/2)}*(1-I/a^{(1/2)*b^{(1/2)}})^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)*b^{(1/2)}})^{(1/2)}*(b*x^4+a)^{(1/2)}*\text{EllipticF}(x*(I/a^{(1/2)*b^{(1/2)}})^{(1/2)},I)-4/15*I*f/a^{(1/2)*b^{(5/2)}}/(I/a^{(1/2)*b^{(1/2)}})^{(1/2)}*(1-I/a^{(1/2)*b^{(1/2)}})^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)*b^{(1/2)}})^{(1/2)}*(b*x^4+a)^{(1/2)}*\text{EllipticE}(x*(I/a^{(1/2)*b^{(1/2)}})^{(1/2)},I)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^13,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bf x^7 + bex^6 + bdx^5 + bcx^4 + afx^3 + aex^2 + adx + ac)\sqrt{bx^4 + a}}{x^{13}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^13,x, algorithm="fricas")

[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x + a*c)*sqrt(b*x^4 + a)/x^13, x)

Sympy [C] time = 20.1769, size = 403, normalized size = 0.9

$$\frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{11}{4}\right) {}_2F_1\left(-\frac{11}{4}, -\frac{1}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^{11}\Gamma\left(-\frac{7}{4}\right)} + \frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{9}{4}\right) {}_2F_1\left(-\frac{9}{4}, -\frac{1}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^9\Gamma\left(-\frac{5}{4}\right)} + \frac{\sqrt{abd}\Gamma\left(-\frac{7}{4}\right) {}_2F_1\left(-\frac{7}{4}, -\frac{1}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{\sqrt{abf}\Gamma\left(-\frac{5}{4}\right) {}_2F_1\left(-\frac{5}{4}, -\frac{1}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**13,x)

[Out] a**(3/2)*d*gamma(-11/4)*hyper((-11/4, -1/2), (-7/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**11*gamma(-7/4)) + a**(3/2)*f*gamma(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**9*gamma(-5/4)) + sqrt(a)*b*d*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3/4)) + sqrt(a)*b*f*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) - a**2*c/(12*sqrt(b)*x**14*sqrt(a/(b*x**4)+1)) - 11*a*sqrt(b)*c/(48*x**10*sqrt(a/(b*x**4)+1)) - a*sqrt(b)*e*sqrt(a/(b*x**4)+1)/(10*x**8) - 17*b**(3/2)*c/(96*x**6*sqrt(a/(b*x**4)+1)) - b**(3/2)*e*sqrt(a/(b*x**4)+1)/(5*x**4) - b**(5/2)*c/(32*a*x**2*sqrt(a/(b*x**4)+1)) - b**(5/2)*e*sqrt(a/(b*x**4)+1)/(10*a) + b**3*c*asinh(sqrt(a)/(sqrt(b)*x**2))/(32*a**(3/2))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x^{13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^13,x, algorithm="giac")

[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^13, x)

$$3.528 \quad \int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{14}} dx$$

Optimal. Leaf size=474

$$\frac{2b^{11/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (65\sqrt{ae} + 77\sqrt{bc}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{5005a^{7/4}\sqrt{a+bx^4}} - \frac{4b^{7/2}cx\sqrt{a+bx^4}}{65a^2(\sqrt{a} + \sqrt{bx^2})} + \frac{4b^3c\sqrt{a+bx^4}}{65a^2x}$$

[Out] $-(b*((12320*c)/x^9 + (15015*d)/x^8 + (18720*e)/x^7 + (24024*f)/x^6)*\operatorname{Sqrt}[a + b*x^4])/240240 - (4*b^2*c*\operatorname{Sqrt}[a + b*x^4])/(195*a*x^5) - (b^2*d*\operatorname{Sqrt}[a + b*x^4])/(32*a*x^4) - (4*b^2*e*\operatorname{Sqrt}[a + b*x^4])/(77*a*x^3) - (b^2*f*\operatorname{Sqrt}[a + b*x^4])/(10*a*x^2) + (4*b^3*c*\operatorname{Sqrt}[a + b*x^4])/(65*a^2*x) - (4*b^{7/2}*c*x*\operatorname{Sqrt}[a + b*x^4])/(65*a^2*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) - (((660*c)/x^{13} + (715*d)/x^{12} + (780*e)/x^{11} + (858*f)/x^{10})*(a + b*x^4)^{(3/2)})/8580 + (b^3*d*\operatorname{ArcTan}[\operatorname{Sqrt}[a + b*x^4]/\operatorname{Sqrt}[a]])/(32*a^{(3/2)}) + (4*b^{13/4}*c*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2])/(65*a^{7/4}*\operatorname{Sqrt}[a + b*x^4]) - (2*b^{11/4}*(77*\operatorname{Sqrt}[b]*c + 65*\operatorname{Sqrt}[a]*e)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2])/(5005*a^{7/4})*\operatorname{Sqrt}[a + b*x^4]$

Rubi [A] time = 0.549361, antiderivative size = 474, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {14, 1825, 1833, 1282, 1198, 220, 1196, 1252, 835, 807, 266, 63, 208}

$$\frac{2b^{11/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (65\sqrt{ae} + 77\sqrt{bc}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{5005a^{7/4}\sqrt{a+bx^4}} - \frac{4b^{7/2}cx\sqrt{a+bx^4}}{65a^2(\sqrt{a} + \sqrt{bx^2})} + \frac{4b^3c\sqrt{a+bx^4}}{65a^2x} + \frac{4b^{13}}{65a^2x}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^{(3/2)}/x^{14}, x]$

[Out] $-(b*((12320*c)/x^9 + (15015*d)/x^8 + (18720*e)/x^7 + (24024*f)/x^6)*\operatorname{Sqrt}[a + b*x^4])/240240 - (4*b^2*c*\operatorname{Sqrt}[a + b*x^4])/(195*a*x^5) - (b^2*d*\operatorname{Sqrt}[a + b*x^4])/(32*a*x^4) - (4*b^2*e*\operatorname{Sqrt}[a + b*x^4])/(77*a*x^3) - (b^2*f*\operatorname{Sqrt}[a + b*x^4])/(10*a*x^2) + (4*b^3*c*\operatorname{Sqrt}[a + b*x^4])/(65*a^2*x) - (4*b^{7/2}*c*x*\operatorname{Sqrt}[a + b*x^4])/(65*a^2*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) - (((660*c)/x^{13} + (715*d)/x^{12} + (780*e)/x^{11} + (858*f)/x^{10})*(a + b*x^4)^{(3/2)})/8580 + (b^3*d*\operatorname{ArcTan}[\operatorname{Sqrt}[a + b*x^4]/\operatorname{Sqrt}[a]])/(32*a^{(3/2)}) + (4*b^{13/4}*c*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2])/(65*a^{7/4}*\operatorname{Sqrt}[a + b*x^4]) - (2*b^{11/4}*(77*\operatorname{Sqrt}[b]*c + 65*\operatorname{Sqrt}[a]*e)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2])/(5005*a^{7/4})*\operatorname{Sqrt}[a + b*x^4]$

```
Tanh[Sqrt[a + b*x^4]/Sqrt[a]]/(32*a^(3/2)) + (4*b^(13/4)*c*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(65*a^(7/4)*Sqrt[a + b*x^4]) - (2*b^(11/4)*(77*Sqr
t[b]*c + 65*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5005*a^(7/4
)*Sqrt[a + b*x^4])
```

Rule 14

```
Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

Rule 1825

```
Int[(Pq_)*(x_)^m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{u
= IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x]] /; FreeQ[{a, b},
x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
0]
```

Rule 1833

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p)/c^j, {j, 0,
n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] && !PolyQ[Pq, x^(n/2)]
```

Rule 1282

```
Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_
Symbol] := Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rule 1252

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rule 835

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m + 2*p + 3)*x, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 807

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]

Rule 266

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}}{x^{14}} dx &= -\frac{\left(\frac{660c}{x^{13}} + \frac{715d}{x^{12}} + \frac{780e}{x^{11}} + \frac{858f}{x^{10}}\right)(a + bx^4)^{3/2}}{8580} - (6b) \int \frac{\left(-\frac{c}{13} - \frac{dx}{12} - \frac{ex^2}{11} - \frac{fx^3}{10}\right)\sqrt{a + bx^4}}{x^{10}} dx \\
&= -\frac{b\left(\frac{12320c}{x^9} + \frac{15015d}{x^8} + \frac{18720e}{x^7} + \frac{24024f}{x^6}\right)\sqrt{a + bx^4}}{240240} - \frac{\left(\frac{660c}{x^{13}} + \frac{715d}{x^{12}} + \frac{780e}{x^{11}} + \frac{858f}{x^{10}}\right)\sqrt{a + bx^4}}{8580} \\
&= -\frac{b\left(\frac{12320c}{x^9} + \frac{15015d}{x^8} + \frac{18720e}{x^7} + \frac{24024f}{x^6}\right)\sqrt{a + bx^4}}{240240} - \frac{\left(\frac{660c}{x^{13}} + \frac{715d}{x^{12}} + \frac{780e}{x^{11}} + \frac{858f}{x^{10}}\right)\sqrt{a + bx^4}}{8580} \\
&= -\frac{b\left(\frac{12320c}{x^9} + \frac{15015d}{x^8} + \frac{18720e}{x^7} + \frac{24024f}{x^6}\right)\sqrt{a + bx^4}}{240240} - \frac{\left(\frac{660c}{x^{13}} + \frac{715d}{x^{12}} + \frac{780e}{x^{11}} + \frac{858f}{x^{10}}\right)\sqrt{a + bx^4}}{8580} \\
&= -\frac{b\left(\frac{12320c}{x^9} + \frac{15015d}{x^8} + \frac{18720e}{x^7} + \frac{24024f}{x^6}\right)\sqrt{a + bx^4}}{240240} - \frac{4b^2c\sqrt{a + bx^4}}{195ax^5} - \frac{\left(\frac{660c}{x^{13}} + \frac{715d}{x^{12}} + \frac{780e}{x^{11}} + \frac{858f}{x^{10}}\right)\sqrt{a + bx^4}}{8580} \\
&= -\frac{b\left(\frac{12320c}{x^9} + \frac{15015d}{x^8} + \frac{18720e}{x^7} + \frac{24024f}{x^6}\right)\sqrt{a + bx^4}}{240240} - \frac{4b^2c\sqrt{a + bx^4}}{195ax^5} - \frac{b^2d\sqrt{a + bx^4}}{32ax^4} \\
&= -\frac{b\left(\frac{12320c}{x^9} + \frac{15015d}{x^8} + \frac{18720e}{x^7} + \frac{24024f}{x^6}\right)\sqrt{a + bx^4}}{240240} - \frac{4b^2c\sqrt{a + bx^4}}{195ax^5} - \frac{b^2d\sqrt{a + bx^4}}{32ax^4} \\
&= -\frac{b\left(\frac{12320c}{x^9} + \frac{15015d}{x^8} + \frac{18720e}{x^7} + \frac{24024f}{x^6}\right)\sqrt{a + bx^4}}{240240} - \frac{4b^2c\sqrt{a + bx^4}}{195ax^5} - \frac{b^2d\sqrt{a + bx^4}}{32ax^4} \\
&= -\frac{b\left(\frac{12320c}{x^9} + \frac{15015d}{x^8} + \frac{18720e}{x^7} + \frac{24024f}{x^6}\right)\sqrt{a + bx^4}}{240240} - \frac{4b^2c\sqrt{a + bx^4}}{195ax^5} - \frac{b^2d\sqrt{a + bx^4}}{32ax^4} \\
&= -\frac{b\left(\frac{12320c}{x^9} + \frac{15015d}{x^8} + \frac{18720e}{x^7} + \frac{24024f}{x^6}\right)\sqrt{a + bx^4}}{240240} - \frac{4b^2c\sqrt{a + bx^4}}{195ax^5} - \frac{b^2d\sqrt{a + bx^4}}{32ax^4} \\
&= -\frac{b\left(\frac{12320c}{x^9} + \frac{15015d}{x^8} + \frac{18720e}{x^7} + \frac{24024f}{x^6}\right)\sqrt{a + bx^4}}{240240} - \frac{4b^2c\sqrt{a + bx^4}}{195ax^5} - \frac{b^2d\sqrt{a + bx^4}}{32ax^4}
\end{aligned}$$

Mathematica [C] time = 0.183965, size = 151, normalized size = 0.32

$$\frac{\sqrt{a + bx^4} \left(13x^2 \left(11x(a + bx^4)^2 \sqrt{\frac{bx^4}{a} + 1} \left(a^3 f - b^3 dx^{10} {}_2F_1 \left(\frac{5}{2}, 4; \frac{7}{2}; \frac{bx^4}{a} + 1 \right) \right) + 10a^5 e {}_2F_1 \left(-\frac{11}{4}, -\frac{3}{2}; -\frac{7}{4}; -\frac{bx^4}{a} \right) \right) + 110a \right)}{1430a^4 x^{13} \sqrt{\frac{bx^4}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^14,x]

[Out] $-(\text{Sqrt}[a + b*x^4]*(110*a^5*c*\text{Hypergeometric2F1}[-13/4, -3/2, -9/4, -((b*x^4)/a)]/a) + 13*x^2*(10*a^5*e*\text{Hypergeometric2F1}[-11/4, -3/2, -7/4, -((b*x^4)/a)] + 11*x*(a + b*x^4)^2*\text{Sqrt}[1 + (b*x^4)/a]*(a^3*f - b^3*d*x^{10}*\text{Hypergeometric2F1}[5/2, 4, 7/2, 1 + (b*x^4)/a])))/(1430*a^4*x^{13}*\text{Sqrt}[1 + (b*x^4)/a])$

Maple [C] time = 0.02, size = 483, normalized size = 1.

$$-\frac{ae}{11x^{11}}\sqrt{bx^4+a} - \frac{13be}{77x^7}\sqrt{bx^4+a} - \frac{4b^2e}{77ax^3}\sqrt{bx^4+a} - \frac{4eb^3}{77a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^14,x)

[Out] $-1/11*e*a*(b*x^4+a)^{(1/2)}/x^{11}-13/77*e*b*(b*x^4+a)^{(1/2)}/x^7-4/77*b^2*e*(b*x^4+a)^{(1/2)}/a/x^3-4/77*e/a*b^3/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticF}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)},I)-1/13*c*a*(b*x^4+a)^{(1/2)}/x^{13}-5/39*c*b*(b*x^4+a)^{(1/2)}/x^9-4/195*b^2*c*(b*x^4+a)^{(1/2)}/a/x^5+4/65*b^3*c*(b*x^4+a)^{(1/2)}/a^2/x^4+4/65*I*c/a^{(3/2)}*b^{(7/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticF}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)},I)+4/65*I*c/a^{(3/2)}*b^{(7/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticE}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)},I)-7/48*d*b/x^8*(b*x^4+a)^{(1/2)}-1/32*b^2*d*(b*x^4+a)^{(1/2)}/a/x^4+1/32*d/a^{(3/2)}*b^3*\ln((2*a+2*a^{(1/2)}*(b*x^4+a)^{(1/2)})/x^2)-1/12*d*a/x^{12}*(b*x^4+a)^{(1/2)}-1/10*f*(b*x^4+a)^{(1/2)}/x^{10}/a*(b^2*x^8+2*a*b*x^4+a^2)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^2 (fx^3 + ex^2 + dx + c)}{x^{14}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^14,x, algorithm="maxima")

[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^14, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bf x^7 + bex^6 + bdx^5 + bcx^4 + afx^3 + aex^2 + adx + ac)\sqrt{bx^4 + a}}{x^{14}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^14,x, algorithm="fricas")

[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x + a*c)*sqrt(b*x^4 + a)/x^14, x)

Sympy [C] time = 21.7902, size = 403, normalized size = 0.85

$$\frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{13}{4}\right)_2F_1\left(-\frac{13}{4}, -\frac{1}{2}\left|\frac{bx^4e^{i\pi}}{a}\right.\right)}{4x^{13}\Gamma\left(-\frac{9}{4}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{11}{4}\right)_2F_1\left(-\frac{11}{4}, -\frac{1}{2}\left|\frac{bx^4e^{i\pi}}{a}\right.\right)}{4x^{11}\Gamma\left(-\frac{7}{4}\right)} + \frac{\sqrt{abc}\Gamma\left(-\frac{9}{4}\right)_2F_1\left(-\frac{9}{4}, -\frac{1}{2}\left|\frac{bx^4e^{i\pi}}{a}\right.\right)}{4x^9\Gamma\left(-\frac{5}{4}\right)} + \frac{\sqrt{abe}\Gamma\left(-\frac{7}{4}\right)_2F_1\left(-\frac{7}{4}, -\frac{1}{2}\left|\frac{bx^4e^{i\pi}}{a}\right.\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**14,x)

[Out] a**(3/2)*c*gamma(-13/4)*hyper((-13/4, -1/2), (-9/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**13*gamma(-9/4)) + a**(3/2)*e*gamma(-11/4)*hyper((-11/4, -1/2), (-7/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**11*gamma(-7/4)) + sqrt(a)*b*c*gamma(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**9*gamma(-5/4)) + sqrt(a)*b*e*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3/4)) - a**2*d/(12*sqrt(b)*x**14*sqrt(a/(b*x**4) + 1)) - 11*a*sqrt(b)*d/(48*x**10*sqrt(a/(b*x**4) + 1)) - a*sqrt(b)*f*sqrt(a/(b*x**4) + 1)/(10*x**8) - 17*b**(3/2)*d/(96*x**6*sqrt(a/(b*x**4) + 1)) - b**(3/2)*f*sqrt(a/(b*x**4) + 1)/(5*x**4) - b**(5/2)*d/(32*a*x**2*sqrt(a/(b*x**4) + 1)) - b**(5/2)*f*sqrt(a/(b*x**4) + 1)/(10*a) + b**3*d*asinh(sqrt(a)/(sqrt(b)*x**2))/(32*a**(3/2))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x^{14}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^14,x, algorithm="giac")

[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^14, x)

$$3.529 \quad \int \frac{x^4(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}} dx$$

Optimal. Leaf size=361

$$\frac{a^{3/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (9\sqrt{ae} + 5\sqrt{bc}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{30b^{7/4}\sqrt{a+bx^4}} + \frac{3a^{5/4}e(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\right)}{5b^{7/4}\sqrt{a+bx^4}}$$

[Out] (c*x*Sqrt[a + b*x^4])/(3*b) + (e*x^3*Sqrt[a + b*x^4])/(5*b) + (f*x^4*Sqrt[a + b*x^4])/(6*b) - (3*a*e*x*Sqrt[a + b*x^4])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) - ((4*a*f - 3*b*d*x^2)*Sqrt[a + b*x^4])/(12*b^2) - (a*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) + (3*a^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^4]) - (a^(3/4)*(5*Sqrt[b]*c + 9*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(30*b^(7/4)*Sqrt[a + b*x^4])

Rubi [A] time = 0.30039, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1833, 1280, 1198, 220, 1196, 1252, 833, 780, 217, 206}

$$\frac{a^{3/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (9\sqrt{ae} + 5\sqrt{bc}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{30b^{7/4}\sqrt{a+bx^4}} + \frac{3a^{5/4}e(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\right)}{5b^{7/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4], x]

[Out] (c*x*Sqrt[a + b*x^4])/(3*b) + (e*x^3*Sqrt[a + b*x^4])/(5*b) + (f*x^4*Sqrt[a + b*x^4])/(6*b) - (3*a*e*x*Sqrt[a + b*x^4])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) - ((4*a*f - 3*b*d*x^2)*Sqrt[a + b*x^4])/(12*b^2) - (a*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) + (3*a^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^4]) - (a^(3/4)*(5*Sqrt[b]*c + 9*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(30*b^(7/4)*Sqrt[a + b*x^4])

$^4)$

Rule 1833

Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p)/c^j, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]

Rule 1280

Int[((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m - 1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1198

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rule 220

Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2]]/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rule 1252

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],

$x, x^2], x] /; \text{FreeQ}\{a, c, d, e, p, q\}, x] \&\& \text{IntegerQ}\{m + 1\}/2]$

Rule 833

$\text{Int}[\{(d_.) + (e_.)(x_)\}^{(m)} \cdot \{(f_.) + (g_.)(x_)\} \cdot \{(a_) + (c_.)(x_)^2\}^{(p)}, x_Symbol] := \text{Simp}[(g*(d + e*x)^m*(a + c*x^2)^{(p + 1)})/(c*(m + 2*p + 2)), x] + \text{Dist}[1/(c*(m + 2*p + 2)), \text{Int}[(d + e*x)^{(m - 1)}*(a + c*x^2)^p * \text{Simp}[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x] /; \text{FreeQ}\{a, c, d, e, f, g, p\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{GtQ}[m, 0] \&\& \text{NeQ}[m + 2*p + 2, 0] \&\& (\text{IntegerQ}[m] || \text{IntegerQ}[p] || \text{IntegersQ}[2*m, 2*p]) \&\& !(\text{IGtQ}[m, 0] \&\& \text{EqQ}[f, 0])$

Rule 780

$\text{Int}[\{(d_.) + (e_.)(x_)\} \cdot \{(f_.) + (g_.)(x_)\} \cdot \{(a_) + (c_.)(x_)^2\}^{(p)}, x_Symbol] := \text{Simp}[\{(e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x\} \cdot (a + c*x^2)^{(p + 1)} / (2*c*(p + 1)*(2*p + 3)), x] - \text{Dist}[(a*e*g - c*d*f*(2*p + 3)) / (c*(2*p + 3)), \text{Int}[(a + c*x^2)^p, x], x] /; \text{FreeQ}\{a, c, d, e, f, g, p\}, x] \&\& !\text{LeQ}[p, -1]$

Rule 217

$\text{Int}[1/\text{Sqrt}\{(a_) + (b_.)(x_)^2\}, x_Symbol] := \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] /; \text{FreeQ}\{a, b\}, x] \&\& !\text{GtQ}[a, 0]$

Rule 206

$\text{Int}[\{(a_) + (b_.)(x_)^2\}^{-1}, x_Symbol] := \text{Simp}[(1 * \text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]]) / (\text{Rt}[a, 2] * \text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] || \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned}
\int \frac{x^4(c + dx + ex^2 + fx^3)}{\sqrt{a + bx^4}} dx &= \int \left(\frac{x^4(c + ex^2)}{\sqrt{a + bx^4}} + \frac{x^5(d + fx^2)}{\sqrt{a + bx^4}} \right) dx \\
&= \int \frac{x^4(c + ex^2)}{\sqrt{a + bx^4}} dx + \int \frac{x^5(d + fx^2)}{\sqrt{a + bx^4}} dx \\
&= \frac{ex^3\sqrt{a + bx^4}}{5b} + \frac{1}{2} \text{Subst} \left(\int \frac{x^2(d + fx)}{\sqrt{a + bx^2}} dx, x, x^2 \right) - \frac{\int \frac{x^2(3ae - 5bcx^2)}{\sqrt{a + bx^4}} dx}{5b} \\
&= \frac{cx\sqrt{a + bx^4}}{3b} + \frac{ex^3\sqrt{a + bx^4}}{5b} + \frac{fx^4\sqrt{a + bx^4}}{6b} + \frac{\int \frac{-5abc - 9abex^2}{\sqrt{a + bx^4}} dx}{15b^2} + \frac{\text{Subst} \left(\int \frac{x(-2af + 3bdx)}{\sqrt{a + bx^2}} dx \right)}{6b} \\
&= \frac{cx\sqrt{a + bx^4}}{3b} + \frac{ex^3\sqrt{a + bx^4}}{5b} + \frac{fx^4\sqrt{a + bx^4}}{6b} - \frac{(4af - 3bdx^2)\sqrt{a + bx^4}}{12b^2} - \frac{(ad) \text{Subst} \left(\int \frac{x}{\sqrt{a + bx^2}} dx \right)}{6b} \\
&= \frac{cx\sqrt{a + bx^4}}{3b} + \frac{ex^3\sqrt{a + bx^4}}{5b} + \frac{fx^4\sqrt{a + bx^4}}{6b} - \frac{3aex\sqrt{a + bx^4}}{5b^{3/2}(\sqrt{a} + \sqrt{bx^2})} - \frac{(4af - 3bdx^2)\sqrt{a + bx^4}}{12b^2} \\
&= \frac{cx\sqrt{a + bx^4}}{3b} + \frac{ex^3\sqrt{a + bx^4}}{5b} + \frac{fx^4\sqrt{a + bx^4}}{6b} - \frac{3aex\sqrt{a + bx^4}}{5b^{3/2}(\sqrt{a} + \sqrt{bx^2})} - \frac{(4af - 3bdx^2)\sqrt{a + bx^4}}{12b^2}
\end{aligned}$$

Mathematica [C] time = 0.130861, size = 212, normalized size = 0.59

$$\frac{-20a^2f - 20abcx\sqrt{\frac{bx^4}{a}} + 1 {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{bx^4}{a}\right) + 20abcx + 15abdx^2 - 15a\sqrt{bd}\sqrt{a + bx^4} \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a + bx^4}}\right) - 12abex^3\sqrt{\frac{bx^4}{a}}}{60b^2\sqrt{a + bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4], x]

[Out] (-20*a^2*f + 20*a*b*c*x + 15*a*b*d*x^2 + 12*a*b*e*x^3 - 10*a*b*f*x^4 + 20*b^2*c*x^5 + 15*b^2*d*x^6 + 12*b^2*e*x^7 + 10*b^2*f*x^8 - 15*a*Sqrt[b]*d*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 20*a*b*c*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 12*a*b*e*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])/(60*b^2*Sqrt[a + b*x^4])

Maple [C] time = 0.018, size = 335, normalized size = 0.9

$$\frac{f(-bx^4 + 2a)}{6b^2} \sqrt{bx^4 + a} + \frac{ex^3}{5b} \sqrt{bx^4 + a} - \frac{3i}{5} ea^{\frac{3}{2}} \sqrt{1 - ix^2 \sqrt{b} \frac{1}{\sqrt{a}}} \sqrt{1 + ix^2 \sqrt{b} \frac{1}{\sqrt{a}}} \text{EllipticF}\left(x \sqrt{i \sqrt{b} \frac{1}{\sqrt{a}}}, i\right) b^{-\frac{3}{2}} \frac{1}{\sqrt{i \sqrt{b} \frac{1}{\sqrt{a}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)`

[Out]
$$\begin{aligned} & -1/6*f*(b*x^4+a)^{(1/2)}*(-b*x^4+2*a)/b^2+1/5*e*x^3*(b*x^4+a)^{(1/2)}/b-3/5*I*e \\ & /b^{(3/2)}*a^{(3/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}* \\ & (1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}* \text{EllipticF}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I) \\ & +3/5*I*e/b^{(3/2)}*a^{(3/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}* \\ & (1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}* \text{EllipticE}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I) \\ & +1/4*d*x^2/b*(b*x^4+a)^{(1/2)}-1/4*d*a/b^{(3/2)}*\ln(x^2*b^{(1/2)}+(b*x^4+a)^{(1/2)})+1/3*c*x*(b*x^4+a)^{(1/2)}/b-1/3*c/b*a/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}* \text{EllipticF}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I) \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx^3 + ex^2 + dx + c)x^4}{\sqrt{bx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")`

[Out] `integrate((f*x^3 + e*x^2 + d*x + c)*x^4/sqrt(b*x^4 + a), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx^7 + ex^6 + dx^5 + cx^4}{\sqrt{bx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")`

[Out] integral((f*x^7 + e*x^6 + d*x^5 + c*x^4)/sqrt(b*x^4 + a), x)

Sympy [A] time = 6.13838, size = 177, normalized size = 0.49

$$\frac{\sqrt{a}dx^2\sqrt{1+\frac{bx^4}{a}}}{4b} - \frac{ad \operatorname{asinh}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}} + f \left\{ \begin{array}{ll} -\frac{a\sqrt{a+bx^4}}{3b^2} + \frac{x^4\sqrt{a+bx^4}}{6b} & \text{for } b \neq 0 \\ \frac{x^8}{8\sqrt{a}} & \text{otherwise} \end{array} \right\} + \frac{cx^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{5}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{ex^7\Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{7}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)

[Out] sqrt(a)*d*x**2*sqrt(1 + b*x**4/a)/(4*b) - a*d*asinh(sqrt(b)*x**2/sqrt(a))/(4*b**(3/2)) + f*Piecewise((-a*sqrt(a + b*x**4)/(3*b**2) + x**4*sqrt(a + b*x**4)/(6*b), Ne(b, 0)), (x**8/(8*sqrt(a)), True)) + c*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + e*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(11/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx^3 + ex^2 + dx + c)x^4}{\sqrt{bx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^4/sqrt(b*x^4 + a), x)

$$3.530 \quad \int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}} dx$$

Optimal. Leaf size=336

$$\frac{a^{3/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (9\sqrt{a}f + 5\sqrt{bd}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{30b^{7/4}\sqrt{a+bx^4}} + \frac{3a^{5/4}f(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\right)}{5b^{7/4}\sqrt{a+bx^4}}$$

[Out] (d*x*Sqrt[a + b*x^4])/(3*b) + (f*x^3*Sqrt[a + b*x^4])/(5*b) - (3*a*f*x*Sqrt[a + b*x^4])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*c + e*x^2)*Sqrt[a + b*x^4])/(4*b) - (a*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) + (3*a^(5/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^4]) - (a^(3/4)*(5*Sqrt[b]*d + 9*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(30*b^(7/4)*Sqrt[a + b*x^4])

Rubi [A] time = 0.265053, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {1833, 1252, 780, 217, 206, 1280, 1198, 220, 1196}

$$\frac{a^{3/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (9\sqrt{a}f + 5\sqrt{bd}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{30b^{7/4}\sqrt{a+bx^4}} + \frac{3a^{5/4}f(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\right)}{5b^{7/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4], x]

[Out] (d*x*Sqrt[a + b*x^4])/(3*b) + (f*x^3*Sqrt[a + b*x^4])/(5*b) - (3*a*f*x*Sqrt[a + b*x^4])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*c + e*x^2)*Sqrt[a + b*x^4])/(4*b) - (a*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) + (3*a^(5/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^4]) - (a^(3/4)*(5*Sqrt[b]*d + 9*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(30*b^(7/4)*Sqrt[a + b*x^4])

Rule 1833

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p]/c^j, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 1252

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 780

```
Int[((d_) + (e_)*(x_))*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p + 1)/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p + 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !LeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 1280

```
Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m - 1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
```

d, e}, x] && PosQ[c/a]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2]]/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rubi steps

$$\begin{aligned}
 \int \frac{x^3(c + dx + ex^2 + fx^3)}{\sqrt{a + bx^4}} dx &= \int \left(\frac{x^3(c + ex^2)}{\sqrt{a + bx^4}} + \frac{x^4(d + fx^2)}{\sqrt{a + bx^4}} \right) dx \\
 &= \int \frac{x^3(c + ex^2)}{\sqrt{a + bx^4}} dx + \int \frac{x^4(d + fx^2)}{\sqrt{a + bx^4}} dx \\
 &= \frac{fx^3\sqrt{a + bx^4}}{5b} + \frac{1}{2} \text{Subst} \left(\int \frac{x(c + ex)}{\sqrt{a + bx^2}} dx, x, x^2 \right) - \frac{\int \frac{x^2(3af - 5bdx^2)}{\sqrt{a + bx^4}} dx}{5b} \\
 &= \frac{dx\sqrt{a + bx^4}}{3b} + \frac{fx^3\sqrt{a + bx^4}}{5b} + \frac{(2c + ex^2)\sqrt{a + bx^4}}{4b} + \frac{\int \frac{-5abd - 9abfx^2}{\sqrt{a + bx^4}} dx}{15b^2} - \frac{(ae) \text{Subst} \left(\int \frac{1}{1 - bx^2} dx, x, \frac{x^2}{\sqrt{a + bx^4}} \right)}{4b} \\
 &= \frac{dx\sqrt{a + bx^4}}{3b} + \frac{fx^3\sqrt{a + bx^4}}{5b} + \frac{(2c + ex^2)\sqrt{a + bx^4}}{4b} - \frac{(ae) \text{Subst} \left(\int \frac{1}{1 - bx^2} dx, x, \frac{x^2}{\sqrt{a + bx^4}} \right)}{4b} \\
 &= \frac{dx\sqrt{a + bx^4}}{3b} + \frac{fx^3\sqrt{a + bx^4}}{5b} - \frac{3afx\sqrt{a + bx^4}}{5b^{3/2}(\sqrt{a} + \sqrt{bx^2})} + \frac{(2c + ex^2)\sqrt{a + bx^4}}{4b} - \frac{ae \tanh^{-1}}{4b^3}
 \end{aligned}$$

Mathematica [C] time = 0.145449, size = 212, normalized size = 0.63

$$\frac{30\sqrt{bc}(a + bx^4) - 20a\sqrt{bdx}\sqrt{\frac{bx^4}{a}} + 1 {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{bx^4}{a}\right) + 20\sqrt{bdx}(a + bx^4) + 15\sqrt{bex^2}(a + bx^4) - 15ae\sqrt{a + bx^4} \tanh^{-1}}{60b^{3/2}\sqrt{a + bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]

[Out] (30*Sqrt[b]*c*(a + b*x^4) + 20*Sqrt[b]*d*x*(a + b*x^4) + 15*Sqrt[b]*e*x^2*(a + b*x^4) + 12*Sqrt[b]*f*x^3*(a + b*x^4) - 15*a*e*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 20*a*Sqrt[b]*d*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 12*a*Sqrt[b]*f*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])/(60*b^(3/2)*Sqrt[a + b*x^4])

Maple [C] time = 0.007, size = 325, normalized size = 1.

$$\frac{fx^3}{5b}\sqrt{bx^4+a} - \frac{3i}{5}fa^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}} + \frac{3i}{5}fa^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)

[Out] 1/5*f*x^3*(b*x^4+a)^(1/2)/b-3/5*I*f/b^(3/2)*a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+3/5*I*f/b^(3/2)*a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/4*e*x^2/b*(b*x^4+a)^(1/2)-1/4*e*a/b^(3/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))+1/3*d*x*(b*x^4+a)^(1/2)/b-1/3*d/b*a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*c/b*(b*x^4+a)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{bx^4+ac}}{2b} + \int \frac{fx^6+ex^5+dx^4}{\sqrt{bx^4+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")

[Out] $1/2*\sqrt{b*x^4 + a}*c/b + \text{integrate}((f*x^6 + e*x^5 + d*x^4)/\sqrt{b*x^4 + a}, x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx^6 + ex^5 + dx^4 + cx^3}{\sqrt{bx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^{(1/2)}, x, \text{algorithm}="fricas")$

[Out] $\text{integral}((f*x^6 + e*x^5 + d*x^4 + c*x^3)/\sqrt{b*x^4 + a}, x)$

Sympy [A] time = 5.02286, size = 156, normalized size = 0.46

$$\frac{\sqrt{a}ex^2\sqrt{1 + \frac{bx^4}{a}}}{4b} - \frac{ae \operatorname{asinh}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}} + c \left(\begin{array}{l} \frac{x^4}{4\sqrt{a}} \quad \text{for } b = 0 \\ \frac{\sqrt{a+bx^4}}{2b} \quad \text{otherwise} \end{array} \right) + \frac{dx^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{5}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{fx^7\Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{7}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2), x)$

[Out] $\sqrt{a}*e*x**2*\sqrt{1 + b*x**4/a}/(4*b) - a*e*\operatorname{asinh}(\sqrt{b}*x**2/\sqrt{a})/(4*b**(3/2)) + c*\operatorname{Piecewise}((x**4/(4*\sqrt{a})), \operatorname{Eq}(b, 0)), (\sqrt{a + b*x**4}/(2*b), \operatorname{True})) + d*x**5*\operatorname{gamma}(5/4)*\operatorname{hyper}((1/2, 5/4), (9/4,), b*x**4*\exp_polar(I*\pi)/a)/(4*\sqrt{a}*\operatorname{gamma}(9/4)) + f*x**7*\operatorname{gamma}(7/4)*\operatorname{hyper}((1/2, 7/4), (11/4,), b*x**4*\exp_polar(I*\pi)/a)/(4*\sqrt{a}*\operatorname{gamma}(11/4))$

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx^3 + ex^2 + dx + c)x^3}{\sqrt{bx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^3/sqrt(b*x^4 + a), x)
```

$$3.531 \quad \int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}} dx$$

Optimal. Leaf size=308

$$\frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (3\sqrt{bc} - \sqrt{ae}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}} - \frac{\sqrt[4]{ac}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\right)}{b^{3/4}\sqrt{a+bx^4}}$$

[Out] (e*x*Sqrt[a + b*x^4])/(3*b) + (c*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*d + f*x^2)*Sqrt[a + b*x^4])/(4*b) - (a*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) - (a^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(3*Sqrt[b]*c - Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*b^(5/4)*Sqrt[a + b*x^4])

Rubi [A] time = 0.221998, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {1833, 1280, 1198, 220, 1196, 1252, 780, 217, 206}

$$\frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (3\sqrt{bc} - \sqrt{ae}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}} - \frac{\sqrt[4]{ac}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4], x]

[Out] (e*x*Sqrt[a + b*x^4])/(3*b) + (c*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*d + f*x^2)*Sqrt[a + b*x^4])/(4*b) - (a*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) - (a^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(3*Sqrt[b]*c - Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*b^(5/4)*Sqrt[a + b*x^4])

Rule 1833

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +

$(k*n)/2]*x^{((k*n)/2)}, \{k, 0, (2*(q - j))/n + 1\}*(a + b*x^n)^p/c^j, \{j, 0, n/2 - 1\}, x] /; \text{FreeQ}\{a, b, c, m, p\}, x] \&\& \text{PolyQ}\{Pq, x\} \&\& \text{IGtQ}\{n/2, 0\} \&\& \text{!PolyQ}\{Pq, x^{(n/2)}\}$

Rule 1280

$\text{Int}[(f_*)*(x_)^{(m_*)}*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(e*f*(f*x)^{(m-1)}*(a + c*x^4)^{(p+1)})/(c*(m + 4*p + 3)), x] - \text{Dist}[f^2/(c*(m + 4*p + 3)), \text{Int}[(f*x)^{(m-2)}*(a + c*x^4)^p*(a*e*(m-1) - c*d*(m + 4*p + 3)*x^2), x], x] /; \text{FreeQ}\{a, c, d, e, f, p\}, x] \&\& \text{GtQ}\{m, 1\} \&\& \text{NeQ}\{m + 4*p + 3, 0\} \&\& \text{IntegerQ}\{2*p\} \&\& (\text{IntegerQ}\{p\} || \text{IntegerQ}\{m\})$

Rule 1198

$\text{Int}[(d_) + (e_)*(x_)^2]/\text{Sqrt}[a_ + (c_)*(x_)^4], x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[c/a, 2]\}, \text{Dist}[(e + d*q)/q, \text{Int}[1/\text{Sqrt}[a + c*x^4], x], x] - \text{Dist}[e/q, \text{Int}[(1 - q*x^2)/\text{Sqrt}[a + c*x^4], x], x] /; \text{NeQ}\{e + d*q, 0\} /; \text{FreeQ}\{a, c, d, e\}, x] \&\& \text{PosQ}\{c/a\}$

Rule 220

$\text{Int}[1/\text{Sqrt}[a_ + (b_)*(x_)^4], x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[b/a, 4]\}, \text{Simp}[(1 + q^2*x^2)*\text{Sqrt}[a + b*x^4]/(a*(1 + q^2*x^2)^2)]*\text{EllipticF}[2*\text{ArcTan}[q*x], 1/2)]/(2*q*\text{Sqrt}[a + b*x^4]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}\{b/a\}$

Rule 1196

$\text{Int}[(d_) + (e_)*(x_)^2]/\text{Sqrt}[a_ + (c_)*(x_)^4], x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[c/a, 4]\}, -\text{Simp}[(d*x*\text{Sqrt}[a + c*x^4])/(a*(1 + q^2*x^2)), x] + \text{Simp}[(d*(1 + q^2*x^2)*\text{Sqrt}[a + c*x^4]/(a*(1 + q^2*x^2)^2)]*\text{EllipticE}[2*\text{ArcTan}[q*x], 1/2)]/(q*\text{Sqrt}[a + c*x^4]), x] /; \text{EqQ}\{e + d*q^2, 0\} /; \text{FreeQ}\{a, c, d, e\}, x] \&\& \text{PosQ}\{c/a\}$

Rule 1252

$\text{Int}[(x_)^{(m_*)}*((d_) + (e_)*(x_)^2)^{(q_*)}*((a_) + (c_)*(x_)^4)^{(p_)}, x_Symbol] \rightarrow \text{Dist}[1/2, \text{Subst}[\text{Int}[x^{((m-1)/2)}*(d + e*x)^q*(a + c*x^2)^p], x], x, x^2], x] /; \text{FreeQ}\{a, c, d, e, p, q\}, x] \&\& \text{IntegerQ}\{(m + 1)/2\}$

Rule 780

$\text{Int}[(d_*) + (e_)*(x_)*((f_*) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x]*(a + c*x^2)^{(p+1)}/(2*c*(p + 1)*(2*p + 3)), x] - \text{Dist}[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p$

+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !LeQ[p, -1]

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{x^2(c + dx + ex^2 + fx^3)}{\sqrt{a + bx^4}} dx &= \int \left(\frac{x^2(c + ex^2)}{\sqrt{a + bx^4}} + \frac{x^3(d + fx^2)}{\sqrt{a + bx^4}} \right) dx \\
 &= \int \frac{x^2(c + ex^2)}{\sqrt{a + bx^4}} dx + \int \frac{x^3(d + fx^2)}{\sqrt{a + bx^4}} dx \\
 &= \frac{ex\sqrt{a + bx^4}}{3b} + \frac{1}{2} \text{Subst} \left(\int \frac{x(d + fx)}{\sqrt{a + bx^2}} dx, x, x^2 \right) - \frac{\int \frac{ae - 3bcx^2}{\sqrt{a + bx^4}} dx}{3b} \\
 &= \frac{ex\sqrt{a + bx^4}}{3b} + \frac{(2d + fx^2)\sqrt{a + bx^4}}{4b} - \frac{(\sqrt{ac}) \int \frac{1 - \sqrt{bx^2}}{\sqrt{a + bx^4}} dx}{\sqrt{b}} + \frac{(\sqrt{a}(3\sqrt{bc} - \sqrt{ae})) \int \frac{1}{\sqrt{a + bx^4}} dx}{3b} \\
 &= \frac{ex\sqrt{a + bx^4}}{3b} + \frac{cx\sqrt{a + bx^4}}{\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{(2d + fx^2)\sqrt{a + bx^4}}{4b} - \frac{\sqrt[4]{ac}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a + bx^4}{(\sqrt{a} + \sqrt{bx^2})^2}}}{b^{3/4}\sqrt{a + bx^4}} \\
 &= \frac{ex\sqrt{a + bx^4}}{3b} + \frac{cx\sqrt{a + bx^4}}{\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{(2d + fx^2)\sqrt{a + bx^4}}{4b} - \frac{af \tanh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a + bx^4}} \right)}{4b^{3/2}} - \frac{\sqrt[4]{ac}}{b^{3/4}\sqrt{a + bx^4}}
 \end{aligned}$$

Mathematica [C] time = 0.168889, size = 193, normalized size = 0.63

$$\frac{4b^{3/2}cx^3 \sqrt{\frac{bx^4}{a}} + 1 {}_2F_1 \left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -\frac{bx^4}{a} \right) + 6\sqrt{bd}(a + bx^4) - 4a\sqrt{bex} \sqrt{\frac{bx^4}{a}} + 1 {}_2F_1 \left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{bx^4}{a} \right) + 4\sqrt{bex}(a + bx^4) + 3\sqrt{bf}}{12b^{3/2}\sqrt{a + bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]

[Out] (6*Sqrt[b]*d*(a + b*x^4) + 4*Sqrt[b]*e*x*(a + b*x^4) + 3*Sqrt[b]*f*x^2*(a + b*x^4) - 3*a*f*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 4*a*Sqrt[b]*e*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] + 4*b^(3/2)*c*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])/(12*b^(3/2)*Sqrt[a + b*x^4])

Maple [C] time = 0.006, size = 248, normalized size = 0.8

$$\frac{fx^2}{4b}\sqrt{bx^4+a} - \frac{af}{4}\ln\left(x^2\sqrt{b} + \sqrt{bx^4+a}\right)b^{-\frac{3}{2}} + \frac{ex}{3b}\sqrt{bx^4+a} - \frac{ae}{3b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)

[Out] 1/4*f*x^2/b*(b*x^4+a)^(1/2)-1/4*f*a/b^(3/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))+1/3*e*x*(b*x^4+a)^(1/2)/b-1/3*e/b*a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*d/b*(b*x^4+a)^(1/2)+I*c*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx^3 + ex^2 + dx + c)x^2}{\sqrt{bx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^2/sqrt(b*x^4 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx^5 + ex^4 + dx^3 + cx^2}{\sqrt{bx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")`

[Out] `integral((f*x^5 + e*x^4 + d*x^3 + c*x^2)/sqrt(b*x^4 + a), x)`

Sympy [A] time = 4.7238, size = 156, normalized size = 0.51

$$\frac{\sqrt{a}fx^2\sqrt{1+\frac{bx^4}{a}}}{4b} - \frac{af \operatorname{asinh}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}} + d \left(\begin{cases} \frac{x^4}{4\sqrt{a}} & \text{for } b = 0 \\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise} \end{cases} \right) + \frac{cx^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{3}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{ex^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{5}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)`

[Out] `sqrt(a)*f*x**2*sqrt(1 + b*x**4/a)/(4*b) - a*f*asinh(sqrt(b)*x**2/sqrt(a))/(4*b**(3/2)) + d*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)/(2*b), True)) + c*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + e*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4))`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx^3 + ex^2 + dx + c)x^2}{\sqrt{bx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")`

[Out] `integrate((f*x^3 + e*x^2 + d*x + c)*x^2/sqrt(b*x^4 + a), x)`

$$3.532 \quad \int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}} dx$$

Optimal. Leaf size=299

$$\frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (3\sqrt{bd} - \sqrt{af}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}} - \frac{\sqrt[4]{ad}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\right)}{b^{3/4}\sqrt{a+bx^4}}$$

[Out] (e*Sqrt[a + b*x^4])/(2*b) + (f*x*Sqrt[a + b*x^4])/(3*b) + (d*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (a^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(3*Sqrt[b]*d - Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*b^(5/4)*Sqrt[a + b*x^4])

Rubi [A] time = 0.198289, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1833, 1248, 641, 217, 206, 1280, 1198, 220, 1196}

$$\frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (3\sqrt{bd} - \sqrt{af}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}} - \frac{\sqrt[4]{ad}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] Int[(x*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4], x]

[Out] (e*Sqrt[a + b*x^4])/(2*b) + (f*x*Sqrt[a + b*x^4])/(3*b) + (d*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (a^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(3*Sqrt[b]*d - Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*b^(5/4)*Sqrt[a + b*x^4])

Rule 1833

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j))*Sum[Coeff[Pq, x, j +

$(k*n)/2 * x^{((k*n)/2)}$, {k, 0, (2*(q - j))/n + 1}*(a + b*x^n)^p/c^j, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]

Rule 1248

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]

Rule 641

Int[((d_) + (e_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1280

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m - 1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1198

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x]
, 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(c + dx + ex^2 + fx^3)}{\sqrt{a + bx^4}} dx &= \int \left(\frac{x(c + ex^2)}{\sqrt{a + bx^4}} + \frac{x^2(d + fx^2)}{\sqrt{a + bx^4}} \right) dx \\
&= \int \frac{x(c + ex^2)}{\sqrt{a + bx^4}} dx + \int \frac{x^2(d + fx^2)}{\sqrt{a + bx^4}} dx \\
&= \frac{fx\sqrt{a + bx^4}}{3b} + \frac{1}{2} \text{Subst} \left(\int \frac{c + ex}{\sqrt{a + bx^2}} dx, x, x^2 \right) - \frac{\int \frac{af - 3bdx^2}{\sqrt{a + bx^4}} dx}{3b} \\
&= \frac{e\sqrt{a + bx^4}}{2b} + \frac{fx\sqrt{a + bx^4}}{3b} + \frac{1}{2}c \text{Subst} \left(\int \frac{1}{\sqrt{a + bx^2}} dx, x, x^2 \right) - \frac{(\sqrt{ad}) \int \frac{1 - \sqrt{bx^2}}{\sqrt{a + bx^4}} dx}{\sqrt{b}} + \dots \\
&= \frac{e\sqrt{a + bx^4}}{2b} + \frac{fx\sqrt{a + bx^4}}{3b} + \frac{dx\sqrt{a + bx^4}}{\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} - \frac{\sqrt[4]{ad}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a + bx^4}{(\sqrt{a} + \sqrt{bx^2})^2}} E\left(2 \tan^{-1} \dots\right)}{b^{3/4}\sqrt{a + bx^4}} \\
&= \frac{e\sqrt{a + bx^4}}{2b} + \frac{fx\sqrt{a + bx^4}}{3b} + \frac{dx\sqrt{a + bx^4}}{\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{c \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a + bx^4}}\right)}{2\sqrt{b}} - \frac{\sqrt[4]{ad}(\sqrt{a} + \sqrt{bx^2})}{b^{3/4}\sqrt{a + bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.094539, size = 160, normalized size = 0.54

$$\frac{3\sqrt{bc}\sqrt{a + bx^4} \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a + bx^4}}\right) + 2bdx^3 \sqrt{\frac{bx^4}{a}} + {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -\frac{bx^4}{a}\right) - 2afx \sqrt{\frac{bx^4}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{bx^4}{a}\right) + 3ae + 2afx + 3}{6b\sqrt{a + bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]

[Out] (3*a*e + 2*a*f*x + 3*b*e*x^4 + 2*b*f*x^5 + 3*Sqrt[b]*c*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 2*a*f*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] + 2*b*d*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])/(6*b*Sqrt[a + b*x^4])

Maple [C] time = 0.005, size = 229, normalized size = 0.8

$$\frac{fx}{3b}\sqrt{bx^4+a} - \frac{af}{3b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right) \frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{bx^4+a}} + \frac{e}{2b}\sqrt{bx^4+a} + id\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)

[Out] 1/3*f*x*(b*x^4+a)^(1/2)/b-1/3*f/b*a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*e*(b*x^4+a)^(1/2)/b+I*d*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+1/2*c*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx^4 + ex^3 + dx^2 + cx}{\sqrt{bx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")

[Out] integral((f*x^4 + e*x^3 + d*x^2 + c*x)/sqrt(b*x^4 + a), x)

Sympy [A] time = 3.5603, size = 129, normalized size = 0.43

$$e^{\left(\begin{cases} \frac{x^4}{4\sqrt{a}} & \text{for } b = 0 \\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise} \end{cases}\right)} + \frac{c \operatorname{asinh}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{dx^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{3}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{fx^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{5}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)

[Out] e*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)/(2*b), True)) + c*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + d*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + f*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx^3 + ex^2 + dx + c)x}{\sqrt{bx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x/sqrt(b*x^4 + a), x)

$$3.533 \quad \int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}} dx$$

Optimal. Leaf size=276

$$\frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} \left(\frac{\sqrt{bc}}{\sqrt{a}} + e\right) \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}} - \frac{\sqrt[4]{ae}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\right)}{b^{3/4}\sqrt{a+bx^4}}$$

[Out] (f*Sqrt[a + b*x^4])/(2*b) + (e*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (a^(1/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*((Sqrt[b]*c)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])

Rubi [A] time = 0.141569, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {1885, 1198, 220, 1196, 1248, 641, 217, 206}

$$\frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} \left(\frac{\sqrt{bc}}{\sqrt{a}} + e\right) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}} - \frac{\sqrt[4]{ae}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}} + \dots$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3)/Sqrt[a + b*x^4], x]

[Out] (f*Sqrt[a + b*x^4])/(2*b) + (e*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (a^(1/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*((Sqrt[b]*c)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])

Rule 1885

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (

$2*(q - j)/n + 1}]*(a + b*x^n)^p, \{j, 0, n/2 - 1\}, x]] /; \text{FreeQ}\{a, b, p\}, x] \&\& \text{PolyQ}\{Pq, x\} \&\& \text{IGtQ}\{n/2, 0\} \&\& \text{!PolyQ}\{Pq, x^{(n/2)}\}$

Rule 1198

$\text{Int}[(d_) + (e_)*(x_)^2/\text{Sqrt}[(a_) + (c_)*(x_)^4], x_Symbol] \text{:>} \text{With}\{q = \text{Rt}[c/a, 2]\}, \text{Dist}[(e + d*q)/q, \text{Int}[1/\text{Sqrt}[a + c*x^4], x], x] - \text{Dist}[e/q, \text{Int}[(1 - q*x^2)/\text{Sqrt}[a + c*x^4], x], x] /; \text{NeQ}[e + d*q, 0] /; \text{FreeQ}\{a, c, d, e\}, x] \&\& \text{PosQ}[c/a]$

Rule 220

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^4], x_Symbol] \text{:>} \text{With}\{q = \text{Rt}[b/a, 4]\}, \text{Simp}[(1 + q^2*x^2)*\text{Sqrt}[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*\text{EllipticF}[2*\text{ArcTan}[q*x], 1/2])/(2*q*\text{Sqrt}[a + b*x^4]), x]] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[b/a]$

Rule 1196

$\text{Int}[(d_) + (e_)*(x_)^2/\text{Sqrt}[(a_) + (c_)*(x_)^4], x_Symbol] \text{:>} \text{With}\{q = \text{Rt}[c/a, 4]\}, -\text{Simp}[(d*x*\text{Sqrt}[a + c*x^4])/(a*(1 + q^2*x^2)), x] + \text{Simp}[(d*(1 + q^2*x^2)*\text{Sqrt}[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*\text{EllipticE}[2*\text{ArcTan}[q*x], 1/2])/(q*\text{Sqrt}[a + c*x^4]), x] /; \text{EqQ}[e + d*q^2, 0] /; \text{FreeQ}\{a, c, d, e\}, x] \&\& \text{PosQ}[c/a]$

Rule 1248

$\text{Int}[(x_)*((d_) + (e_)*(x_)^2)^{(q_)}*((a_) + (c_)*(x_)^4)^{(p_)}, x_Symbol] \text{:>} \text{Dist}[1/2, \text{Subst}[\text{Int}[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; \text{FreeQ}\{a, c, d, e, p, q\}, x]$

Rule 641

$\text{Int}[(d_) + (e_)*(x_))*((a_) + (c_)*(x_)^2)^{(p_)}, x_Symbol] \text{:>} \text{Simp}[(e*(a + c*x^2)^{(p + 1)})/(2*c*(p + 1)), x] + \text{Dist}[d, \text{Int}[(a + c*x^2)^p, x], x] /; \text{FreeQ}\{a, c, d, e, p\}, x] \&\& \text{NeQ}[p, -1]$

Rule 217

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \text{:>} \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] /; \text{FreeQ}\{a, b\}, x] \&\& \text{!GtQ}[a, 0]$

Rule 206

$\text{Int}[(a_) + (b_)*(x_)^2)^{-1}, x_Symbol] \text{:>} \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& \text{Gt}$

Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{c + dx + ex^2 + fx^3}{\sqrt{a + bx^4}} dx &= \int \left(\frac{c + ex^2}{\sqrt{a + bx^4}} + \frac{x(d + fx^2)}{\sqrt{a + bx^4}} \right) dx \\
 &= \int \frac{c + ex^2}{\sqrt{a + bx^4}} dx + \int \frac{x(d + fx^2)}{\sqrt{a + bx^4}} dx \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{d + fx}{\sqrt{a + bx^2}} dx, x, x^2 \right) - \frac{(\sqrt{ae}) \int \frac{1 - \frac{\sqrt{bx^2}}{\sqrt{a}}}{\sqrt{a + bx^4}} dx}{\sqrt{b}} + \left(c + \frac{\sqrt{ae}}{\sqrt{b}} \right) \int \frac{1}{\sqrt{a + bx^4}} dx \\
 &= \frac{f\sqrt{a + bx^4}}{2b} + \frac{ex\sqrt{a + bx^4}}{\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} - \frac{\sqrt[4]{ae}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a + bx^4}{(\sqrt{a} + \sqrt{bx^2})^2}} E \left(2 \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right) \middle| \frac{1}{2} \right)}{b^{3/4}\sqrt{a + bx^4}} + \dots \\
 &= \frac{f\sqrt{a + bx^4}}{2b} + \frac{ex\sqrt{a + bx^4}}{\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} - \frac{\sqrt[4]{ae}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a + bx^4}{(\sqrt{a} + \sqrt{bx^2})^2}} E \left(2 \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right) \middle| \frac{1}{2} \right)}{b^{3/4}\sqrt{a + bx^4}} + \dots \\
 &= \frac{f\sqrt{a + bx^4}}{2b} + \frac{ex\sqrt{a + bx^4}}{\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{d \tanh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a + bx^4}} \right)}{2\sqrt{b}} - \frac{\sqrt[4]{ae}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a + bx^4}{(\sqrt{a} + \sqrt{bx^2})^2}} E \left(2 \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right) \middle| \frac{1}{2} \right)}{b^{3/4}\sqrt{a + bx^4}}
 \end{aligned}$$

Mathematica [C] time = 0.0940698, size = 150, normalized size = 0.54

$$\frac{cx\sqrt{\frac{bx^4}{a} + 1} {}_2F_1 \left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{bx^4}{a} \right)}{\sqrt{a + bx^4}} + \frac{d \tanh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a + bx^4}} \right)}{2\sqrt{b}} + \frac{ex^3\sqrt{\frac{bx^4}{a} + 1} {}_2F_1 \left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -\frac{bx^4}{a} \right)}{3\sqrt{a + bx^4}} + \frac{f\sqrt{a + bx^4}}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3)/Sqrt[a + b*x^4], x]

[Out] (f*Sqrt[a + b*x^4])/(2*b) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) + (c*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -(b*x^4)/a])/Sqrt[a + b*x^4] + (e*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -(b*x^4)/a])/(3*Sqrt[a + b*x^4])

Maple [C] time = 0.004, size = 208, normalized size = 0.8

$$\frac{f}{2b} \sqrt{bx^4 + a} + ie\sqrt{a} \sqrt{1 - ix^2\sqrt{b}\frac{1}{\sqrt{a}}} \sqrt{1 + ix^2\sqrt{b}\frac{1}{\sqrt{a}}} \left(\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}, i\right) - \text{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}, i\right) \right) \frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)

[Out] 1/2*f*(b*x^4+a)^(1/2)/b+I*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+1/2*d*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2)+c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")

[Out] integral((f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)

Sympy [A] time = 2.62666, size = 128, normalized size = 0.46

$$f \left\{ \begin{array}{ll} \frac{x^4}{4\sqrt{a}} & \text{for } b = 0 \\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise} \end{array} \right\} + \frac{d \operatorname{asinh}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{cx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{ex^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{3}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)

[Out] f*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)/(2*b), True)) + d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)

$$3.534 \quad \int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}} dx$$

Optimal. Leaf size=285

$$\frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} \left(\frac{\sqrt{bd}}{\sqrt{a}} + f\right) \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}} - \frac{\sqrt[4]{a}f(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\right)}{b^{3/4}\sqrt{a+bx^4}}$$

[Out] (f*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (a^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*((Sqrt[b]*d)/Sqrt[a] + f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])

Rubi [A] time = 0.179943, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {1832, 266, 63, 208, 1885, 275, 217, 206, 1198, 220, 1196}

$$\frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} \left(\frac{\sqrt{bd}}{\sqrt{a}} + f\right) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}} - \frac{\sqrt[4]{a}f(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}} - c$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3)/(x*Sqrt[a + b*x^4]), x]

[Out] (f*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (a^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*((Sqrt[b]*d)/Sqrt[a] + f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])

Rule 1832

Int[(Pq)/((x)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq, x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,

$x, 0])/x, x]/\text{Sqrt}[a + b*x^n], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[\text{Pq}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{NeQ}[\text{Coeff}[\text{Pq}, x, 0], 0]$

Rule 266

$\text{Int}[(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \text{ :> } \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p}, x], x, x^n], x] /; \text{FreeQ}[\{a, b, m, n, p\}, x] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 63

$\text{Int}[(a_. + (b_.)*(x_))^{(m_.)}*((c_.) + (d_.)*(x_))^{(n_.)}, x_Symbol] \text{ :> } \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n}, x], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 208

$\text{Int}[(a_. + (b_.)*(x_)^2)^{-1}, x_Symbol] \text{ :> } \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{NegQ}[a/b]$

Rule 1885

$\text{Int}[(\text{Pq}_.)*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \text{ :> } \text{Module}[\{q = \text{Expon}[\text{Pq}, x], j, k\}, \text{Int}[\text{Sum}[x^j*\text{Sum}[\text{Coeff}[\text{Pq}, x, j + (k*n)/2]*x^{((k*n)/2)}, \{k, 0, (2*(q - j))/n + 1\}]* (a + b*x^n)^p, \{j, 0, n/2 - 1\}], x]] /; \text{FreeQ}[\{a, b, p\}, x] \&\& \text{PolyQ}[\text{Pq}, x] \&\& \text{IGtQ}[n/2, 0] \&\& \text{!PolyQ}[\text{Pq}, x^{(n/2)}]$

Rule 275

$\text{Int}[(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \text{ :> } \text{With}[\{k = \text{GCD}[m + 1, n]\}, \text{Dist}[1/k, \text{Subst}[\text{Int}[x^{((m + 1)/k - 1)*(a + b*x^{(n/k)})^p}, x], x, x^k], x] /; k \neq 1] /; \text{FreeQ}[\{a, b, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[m]$

Rule 217

$\text{Int}[1/\text{Sqrt}[(a_) + (b_.)*(x_)^2], x_Symbol] \text{ :> } \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{!GtQ}[a, 0]$

Rule 206

$\text{Int}[(a_. + (b_.)*(x_)^2)^{-1}, x_Symbol] \text{ :> } \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& \text{Gt}$

$Q[a, 0] \parallel LtQ[b, 0]$)

Rule 1198

$\text{Int}[\frac{(d_.) + (e_.)x^2}{\sqrt{(a_.) + (c_.)x^4}}, x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[c/a, 2]\}, \text{Dist}[\frac{e + dq}{q}, \text{Int}[\frac{1}{\sqrt{a + cx^4}}, x], x] - \text{Dist}[\frac{e}{q}, \text{Int}[\frac{1 - qx^2}{\sqrt{a + cx^4}}, x], x] \text{ ; } \text{NeQ}[e + dq, 0] \text{ ; } \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{PosQ}[c/a]$

Rule 220

$\text{Int}[\frac{1}{\sqrt{(a_.) + (b_.)x^4}}, x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[b/a, 4]\}, \text{Simp}[(1 + q^2x^2)\sqrt{(a + bx^4)/(a(1 + q^2x^2)^2)} * \text{EllipticF}[2 * \text{ArcTan}[qx], 1/2]] / (2q\sqrt{a + bx^4}), x] \text{ ; } \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[b/a]$

Rule 1196

$\text{Int}[\frac{(d_.) + (e_.)x^2}{\sqrt{(a_.) + (c_.)x^4}}, x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[c/a, 4]\}, -\text{Simp}[\frac{dx\sqrt{a + cx^4}}{a(1 + q^2x^2)}, x] + \text{Simp}[\frac{d(1 + q^2x^2)\sqrt{(a + cx^4)/(a(1 + q^2x^2)^2)} * \text{EllipticE}[2 * \text{ArcTan}[qx], 1/2]]}{q\sqrt{a + cx^4}}, x] \text{ ; } \text{EqQ}[e + dq^2, 0] \text{ ; } \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{PosQ}[c/a]$

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3}{x\sqrt{a + bx^4}} dx &= c \int \frac{1}{x\sqrt{a + bx^4}} dx + \int \frac{d + ex + fx^2}{\sqrt{a + bx^4}} dx \\
&= \frac{1}{4}c \operatorname{Subst} \left(\int \frac{1}{x\sqrt{a + bx}} dx, x, x^4 \right) + \int \left(\frac{ex}{\sqrt{a + bx^4}} + \frac{d + fx^2}{\sqrt{a + bx^4}} \right) dx \\
&= \frac{c \operatorname{Subst} \left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + bx^4} \right)}{2b} + e \int \frac{x}{\sqrt{a + bx^4}} dx + \int \frac{d + fx^2}{\sqrt{a + bx^4}} dx \\
&= -\frac{c \tanh^{-1} \left(\frac{\sqrt{a + bx^4}}{\sqrt{a}} \right)}{2\sqrt{a}} + \frac{1}{2}e \operatorname{Subst} \left(\int \frac{1}{\sqrt{a + bx^2}} dx, x, x^2 \right) - \frac{(\sqrt{a}f) \int \frac{1 - \frac{\sqrt{bx^2}}{\sqrt{a}}}{\sqrt{a + bx^4}} dx}{\sqrt{b}} + \left(d + \frac{\sqrt{a}f}{\sqrt{b}} \right) \int \frac{1}{\sqrt{a + bx^4}} dx \\
&= \frac{fx\sqrt{a + bx^4}}{\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} - \frac{c \tanh^{-1} \left(\frac{\sqrt{a + bx^4}}{\sqrt{a}} \right)}{2\sqrt{a}} - \frac{\sqrt[4]{a}f(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a + bx^4}{(\sqrt{a} + \sqrt{bx^2})^2}} E \left(2 \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt{a}} \right) \right)}{b^{3/4}\sqrt{a + bx^4}} \\
&= \frac{fx\sqrt{a + bx^4}}{\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{e \tanh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a + bx^4}} \right)}{2\sqrt{b}} - \frac{c \tanh^{-1} \left(\frac{\sqrt{a + bx^4}}{\sqrt{a}} \right)}{2\sqrt{a}} - \frac{\sqrt[4]{a}f(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a + bx^4}{(\sqrt{a} + \sqrt{bx^2})^2}}}{b^{3/4}\sqrt{a + bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.227222, size = 159, normalized size = 0.56

$$-\frac{c \tanh^{-1} \left(\frac{\sqrt{a + bx^4}}{\sqrt{a}} \right)}{2\sqrt{a}} + \frac{dx \sqrt{\frac{bx^4}{a}} + {}_2F_1 \left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{bx^4}{a} \right)}{\sqrt{a + bx^4}} + \frac{e \tanh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a + bx^4}} \right)}{2\sqrt{b}} + \frac{fx^3 \sqrt{\frac{bx^4}{a}} + {}_2F_1 \left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -\frac{bx^4}{a} \right)}{3\sqrt{a + bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x*sqrt[a + b*x^4]),x]

[Out] (e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]]/(2*Sqrt[b]) - (c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]/(2*Sqrt[a]) + (d*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -(b*x^4)/a])/Sqrt[a + b*x^4] + (f*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -(b*x^4)/a])/(3*Sqrt[a + b*x^4]))

Maple [C] time = 0.013, size = 222, normalized size = 0.8

$$i f \sqrt{a} \sqrt{1 - i x^2 \sqrt{b} \frac{1}{\sqrt{a}}} \sqrt{1 + i x^2 \sqrt{b} \frac{1}{\sqrt{a}}} \left(\text{EllipticF} \left(x \sqrt{i \sqrt{b} \frac{1}{\sqrt{a}}}, i \right) - \text{EllipticE} \left(x \sqrt{i \sqrt{b} \frac{1}{\sqrt{a}}}, i \right) \right) \frac{1}{\sqrt{i \sqrt{b} \frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{b x^4 + a}} \frac{1}{\sqrt{b}} + \frac{e}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(1/2),x)

[Out] $I * f * a^{(1/2)} / (I / a^{(1/2)} * b^{(1/2)})^{(1/2)} * (1 - I / a^{(1/2)} * b^{(1/2)} * x^2)^{(1/2)} * (1 + I / a^{(1/2)} * b^{(1/2)} * x^2)^{(1/2)} / (b * x^4 + a)^{(1/2)} / b^{(1/2)} * (\text{EllipticF}(x * (I / a^{(1/2)} * b^{(1/2)})^{(1/2)}, I) - \text{EllipticE}(x * (I / a^{(1/2)} * b^{(1/2)})^{(1/2)}, I)) + 1/2 * e * \ln(x^2 * b^{(1/2)} + (b * x^4 + a)^{(1/2)}) / b^{(1/2)} + d / (I / a^{(1/2)} * b^{(1/2)})^{(1/2)} * (1 - I / a^{(1/2)} * b^{(1/2)} * x^2)^{(1/2)} * (1 + I / a^{(1/2)} * b^{(1/2)} * x^2)^{(1/2)} / (b * x^4 + a)^{(1/2)} * \text{EllipticF}(x * (I / a^{(1/2)} * b^{(1/2)})^{(1/2)}, I) - 1/2 * c / a^{(1/2)} * \ln((2 * a + 2 * a^{(1/2)} * (b * x^4 + a)^{(1/2)}) / x^2)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{f x^3 + e x^2 + d x + c}{\sqrt{b x^4 + a x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(1/2),x, algorithm="maxima")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{b x^4 + a} (f x^3 + e x^2 + d x + c)}{b x^5 + a x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/(b*x^5 + a*x), x)

Sympy [C] time = 4.06959, size = 126, normalized size = 0.44

$$\frac{e \operatorname{asinh}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2\sqrt{b}} - \frac{c \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx^2}}\right)}{2\sqrt{a}} + \frac{dx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{fx^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{3}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)/x/(b*x**4+a)**(1/2),x)

[Out] e*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) - c*asinh(sqrt(a)/(sqrt(b)*x**2))/(2*sqrt(a)) + d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + f*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x), x)

$$3.535 \quad \int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}} dx$$

Optimal. Leaf size=309

$$\frac{(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (\sqrt{ae} + \sqrt{bc}) \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{2a^{3/4} \sqrt[4]{b} \sqrt{a+bx^4}} - \frac{\sqrt[4]{bc} (\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\right)}{a^{3/4} \sqrt{a+bx^4}}$$

[Out] -((c*Sqrt[a + b*x^4])/(a*x)) + (Sqrt[b]*c*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) + (f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (b^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*c + Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(1/4)*Sqrt[a + b*x^4])

Rubi [A] time = 0.224348, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {1833, 1282, 1198, 220, 1196, 1252, 844, 217, 206, 266, 63, 208}

$$\frac{(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (\sqrt{ae} + \sqrt{bc}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{2a^{3/4} \sqrt[4]{b} \sqrt{a+bx^4}} - \frac{\sqrt[4]{bc} (\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{a^{3/4} \sqrt{a+bx^4}} - \frac{c \sqrt{a+bx^4}}{a^{3/4} \sqrt[4]{b} \sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3)/(x^2*Sqrt[a + b*x^4]), x]

[Out] -((c*Sqrt[a + b*x^4])/(a*x)) + (Sqrt[b]*c*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) + (f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (b^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*c + Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(1/4)*Sqrt[a + b*x^4])

Rule 1833

```
Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p]/c^j, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 1282

```
Int[((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 844

```
Int[((d_)*(x_)^(m_))*((f_)*(x_) + (g_)*(x_)^2)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

Int[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 266

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 63

Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3}{x^2\sqrt{a + bx^4}} dx &= \int \left(\frac{c + ex^2}{x^2\sqrt{a + bx^4}} + \frac{d + fx^2}{x\sqrt{a + bx^4}} \right) dx \\
&= \int \frac{c + ex^2}{x^2\sqrt{a + bx^4}} dx + \int \frac{d + fx^2}{x\sqrt{a + bx^4}} dx \\
&= -\frac{c\sqrt{a + bx^4}}{ax} + \frac{1}{2} \text{Subst} \left(\int \frac{d + fx}{x\sqrt{a + bx^2}} dx, x, x^2 \right) - \frac{\int \frac{-ae - bcx^2}{\sqrt{a + bx^4}} dx}{a} \\
&= -\frac{c\sqrt{a + bx^4}}{ax} - \frac{(\sqrt{bc}) \int \frac{1 - \sqrt{bx^2}}{\sqrt{a + bx^4}} dx}{\sqrt{a}} + \frac{1}{2} d \text{Subst} \left(\int \frac{1}{x\sqrt{a + bx^2}} dx, x, x^2 \right) + \left(\frac{\sqrt{bc}}{\sqrt{a}} + e \right) \int \frac{1}{\sqrt{a + bx^4}} dx \\
&= -\frac{c\sqrt{a + bx^4}}{ax} + \frac{\sqrt{bcx}\sqrt{a + bx^4}}{a(\sqrt{a} + \sqrt{bx^2})} - \frac{\sqrt[4]{bc}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a + bx^4}{(\sqrt{a} + \sqrt{bx^2})^2}} E \left(2 \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right) \middle| \frac{1}{2} \right)}{a^{3/4}\sqrt{a + bx^4}} + \dots \\
&= -\frac{c\sqrt{a + bx^4}}{ax} + \frac{\sqrt{bcx}\sqrt{a + bx^4}}{a(\sqrt{a} + \sqrt{bx^2})} + \frac{f \tanh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a + bx^4}} \right)}{2\sqrt{b}} - \frac{\sqrt[4]{bc}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a + bx^4}{(\sqrt{a} + \sqrt{bx^2})^2}} E \left(2 \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right) \middle| \frac{1}{2} \right)}{a^{3/4}\sqrt{a + bx^4}} \\
&= -\frac{c\sqrt{a + bx^4}}{ax} + \frac{\sqrt{bcx}\sqrt{a + bx^4}}{a(\sqrt{a} + \sqrt{bx^2})} + \frac{f \tanh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a + bx^4}} \right)}{2\sqrt{b}} - \frac{d \tanh^{-1} \left(\frac{\sqrt{a + bx^4}}{\sqrt{a}} \right)}{2\sqrt{a}} - \frac{\sqrt[4]{bc}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a + bx^4}{(\sqrt{a} + \sqrt{bx^2})^2}} E \left(2 \tan^{-1} \left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right) \middle| \frac{1}{2} \right)}{a^{3/4}\sqrt{a + bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.227375, size = 157, normalized size = 0.51

$$\frac{c\sqrt{\frac{bx^4}{a}} + 1 {}_2F_1 \left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; -\frac{bx^4}{a} \right)}{x\sqrt{a + bx^4}} - \frac{d \tanh^{-1} \left(\frac{\sqrt{a + bx^4}}{\sqrt{a}} \right)}{2\sqrt{a}} + \frac{ex\sqrt{\frac{bx^4}{a}} + 1 {}_2F_1 \left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{bx^4}{a} \right)}{\sqrt{a + bx^4}} + \frac{f \tanh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a + bx^4}} \right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^2*Sqrt[a + b*x^4]),x]

[Out] (f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]]/(2*Sqrt[b]) - (d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]/(2*Sqrt[a]) - (c*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-1/4, 1/2, 3/4, -(b*x^4)/a])/(x*Sqrt[a + b*x^4]) + (e*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -(b*x^4)/a])/Sqrt[a + b*x^4])

Maple [C] time = 0.009, size = 299, normalized size = 1.

$$\frac{f}{2} \ln\left(x^2\sqrt{b} + \sqrt{bx^4 + a}\right) \frac{1}{\sqrt{b}} + e \sqrt{1 - ix^2\sqrt{b} \frac{1}{\sqrt{a}}} \sqrt{1 + ix^2\sqrt{b} \frac{1}{\sqrt{a}}} \text{EllipticF}\left(x \sqrt{i\sqrt{b} \frac{1}{\sqrt{a}}}, i\right) \frac{1}{\sqrt{i\sqrt{b} \frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{bx^4 + a}} - \frac{d}{2} \ln\left(\frac{1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(1/2),x)

[Out] 1/2*f*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2)+e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/2*d/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-c*(b*x^4+a)^(1/2)/a/x+I*c*b^(1/2)/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-I*c*b^(1/2)/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(1/2),x, algorithm="maxima")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)}{bx^6 + ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/(b*x^6 + a*x^2), x)

Sympy [C] time = 3.3097, size = 128, normalized size = 0.41

$$\frac{f \operatorname{asinh}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{c\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{ax}\Gamma\left(\frac{3}{4}\right)} - \frac{d \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx^2}}\right)}{2\sqrt{a}} + \frac{ex\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)/x**2/(b*x**4+a)**(1/2),x)

[Out] f*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x*gamma(3/4)) - d*asinh(sqrt(a)/(sqrt(b)*x**2))/(2*sqrt(a)) + e*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^2), x)

$$3.536 \quad \int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}} dx$$

Optimal. Leaf size=300

$$\frac{(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (\sqrt{a}f + \sqrt{bd}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt{a}}\right), \frac{1}{2}\right) + \sqrt[4]{bd} (\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt{a}}\right)\right)}{2a^{3/4} \sqrt[4]{b} \sqrt{a+bx^4} - a^{3/4} \sqrt{a+bx^4}}$$

[Out] $-(c\sqrt{a+bx^4})/(2ax^2) - (d\sqrt{a+bx^4})/(ax) + (\sqrt{b}dx\sqrt{a+bx^4})/(a(\sqrt{a} + \sqrt{b}x^2)) - (e\operatorname{ArcTanh}[\sqrt{a+bx^4}/\sqrt{a}])/(2\sqrt{a}) - (b^{1/4}d(\sqrt{a} + \sqrt{b}x^2)\sqrt{a+bx^4})/(\sqrt{a} + \sqrt{b}x^2)^2 \operatorname{EllipticE}[2\operatorname{ArcTan}[(b^{1/4}x)/a^{1/4}], 1/2]/(a^{3/4}\sqrt{a+bx^4}) + ((\sqrt{b}d + \sqrt{a}f)(\sqrt{a} + \sqrt{b}x^2)\sqrt{a+bx^4})/(\sqrt{a} + \sqrt{b}x^2)^2 \operatorname{EllipticF}[2\operatorname{ArcTan}[(b^{1/4}x)/a^{1/4}], 1/2]/(2a^{3/4}b^{1/4}\sqrt{a+bx^4})$

Rubi [A] time = 0.215868, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1833, 1252, 807, 266, 63, 208, 1282, 1198, 220, 1196}

$$\frac{(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (\sqrt{a}f + \sqrt{bd}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt{a}}\right) \middle| \frac{1}{2}\right) + \sqrt[4]{bd} (\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt{a}}\right) \middle| \frac{1}{2}\right)}{2a^{3/4} \sqrt[4]{b} \sqrt{a+bx^4} - a^{3/4} \sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c + dx + ex^2 + fx^3)/(x^3\sqrt{a+bx^4}), x]$

[Out] $-(c\sqrt{a+bx^4})/(2ax^2) - (d\sqrt{a+bx^4})/(ax) + (\sqrt{b}dx\sqrt{a+bx^4})/(a(\sqrt{a} + \sqrt{b}x^2)) - (e\operatorname{ArcTanh}[\sqrt{a+bx^4}/\sqrt{a}])/(2\sqrt{a}) - (b^{1/4}d(\sqrt{a} + \sqrt{b}x^2)\sqrt{a+bx^4})/(\sqrt{a} + \sqrt{b}x^2)^2 \operatorname{EllipticE}[2\operatorname{ArcTan}[(b^{1/4}x)/a^{1/4}], 1/2]/(a^{3/4}\sqrt{a+bx^4}) + ((\sqrt{b}d + \sqrt{a}f)(\sqrt{a} + \sqrt{b}x^2)\sqrt{a+bx^4})/(\sqrt{a} + \sqrt{b}x^2)^2 \operatorname{EllipticF}[2\operatorname{ArcTan}[(b^{1/4}x)/a^{1/4}], 1/2]/(2a^{3/4}b^{1/4}\sqrt{a+bx^4})$

Rule 1833

$\operatorname{Int}[(Pq) * ((c_.) * (x_.)^{(m_.)} * ((a_.) + (b_.) * (x_.)^{(n_.)})^{(p_.)}), x_Symbol] := \operatorname{Module}\{\{q = \operatorname{Expon}[Pq, x], j, k\}, \operatorname{Int}[\operatorname{Sum}[(c*x)^{(m+j)} * \operatorname{Sum}[\operatorname{Coeff}[Pq, x, j +$

```
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}*(a + b*x^n)^p/c^j, {j, 0,
n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0]
] && !PolyQ[Pq, x^(n/2)]
```

Rule 1252

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol]
:> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 807

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol]
:> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1)) / (2*(p + 1)*(c*d^2 + a*e^2)), x]
+ Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /;
FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x]
&& IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x],
x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0]
&& LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x]
/; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1282

```
Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)),
Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(m + 4*p + 5)*x^2), x], x] /;
FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
  Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
  Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3}{x^3\sqrt{a + bx^4}} dx &= \int \left(\frac{c + ex^2}{x^3\sqrt{a + bx^4}} + \frac{d + fx^2}{x^2\sqrt{a + bx^4}} \right) dx \\
&= \int \frac{c + ex^2}{x^3\sqrt{a + bx^4}} dx + \int \frac{d + fx^2}{x^2\sqrt{a + bx^4}} dx \\
&= -\frac{d\sqrt{a + bx^4}}{ax} + \frac{1}{2} \text{Subst} \left(\int \frac{c + ex}{x^2\sqrt{a + bx^2}} dx, x, x^2 \right) - \frac{\int \frac{-af - bdx^2}{\sqrt{a + bx^4}} dx}{a} \\
&= -\frac{c\sqrt{a + bx^4}}{2ax^2} - \frac{d\sqrt{a + bx^4}}{ax} - \frac{(\sqrt{bd}) \int \frac{1 - \frac{\sqrt{bx^2}}{\sqrt{a}}}{\sqrt{a + bx^4}} dx}{\sqrt{a}} + \frac{1}{2} e \text{Subst} \left(\int \frac{1}{x\sqrt{a + bx^2}} dx, x, x^2 \right) + \left(\frac{\sqrt{bd}}{\sqrt{a}} \right) \int \frac{1}{\sqrt{a + bx^4}} dx \\
&= -\frac{c\sqrt{a + bx^4}}{2ax^2} - \frac{d\sqrt{a + bx^4}}{ax} + \frac{\sqrt{bd}x\sqrt{a + bx^4}}{a(\sqrt{a} + \sqrt{bx^2})} - \frac{\sqrt[4]{bd}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a + bx^4}{(\sqrt{a} + \sqrt{bx^2})^2}} E \left(2 \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right) \right)}{a^{3/4}\sqrt{a + bx^4}} \\
&= -\frac{c\sqrt{a + bx^4}}{2ax^2} - \frac{d\sqrt{a + bx^4}}{ax} + \frac{\sqrt{bd}x\sqrt{a + bx^4}}{a(\sqrt{a} + \sqrt{bx^2})} - \frac{\sqrt[4]{bd}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a + bx^4}{(\sqrt{a} + \sqrt{bx^2})^2}} E \left(2 \tan^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right) \right)}{a^{3/4}\sqrt{a + bx^4}} \\
&= -\frac{c\sqrt{a + bx^4}}{2ax^2} - \frac{d\sqrt{a + bx^4}}{ax} + \frac{\sqrt{bd}x\sqrt{a + bx^4}}{a(\sqrt{a} + \sqrt{bx^2})} - \frac{e \tanh^{-1} \left(\frac{\sqrt{a + bx^4}}{\sqrt{a}} \right)}{2\sqrt{a}} - \frac{\sqrt[4]{bd}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a + bx^4}{(\sqrt{a} + \sqrt{bx^2})^2}}}{a^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.139661, size = 148, normalized size = 0.49

$$-\frac{c\sqrt{a + bx^4}}{2ax^2} - \frac{d\sqrt{\frac{bx^4}{a} + 1} {}_2F_1 \left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; -\frac{bx^4}{a} \right)}{x\sqrt{a + bx^4}} - \frac{e \tanh^{-1} \left(\frac{\sqrt{a + bx^4}}{\sqrt{a}} \right)}{2\sqrt{a}} + \frac{fx\sqrt{\frac{bx^4}{a} + 1} {}_2F_1 \left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{bx^4}{a} \right)}{\sqrt{a + bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^3*Sqrt[a + b*x^4]),x]

[Out] -(c*Sqrt[a + b*x^4])/(2*a*x^2) - (e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (d*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-1/4, 1/2, 3/4, -(b*x^4)/a])/(x*Sqrt[a + b*x^4]) + (f*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -(b*x^4)/a])/Sqrt[a + b*x^4]

Maple [C] time = 0.015, size = 293, normalized size = 1.

$$f \sqrt{1 - ix^2 \sqrt{b} \frac{1}{\sqrt{a}}} \sqrt{1 + ix^2 \sqrt{b} \frac{1}{\sqrt{a}}} \text{EllipticF} \left(x \sqrt{i \sqrt{b} \frac{1}{\sqrt{a}}}, i \right) \frac{1}{\sqrt{i \sqrt{b} \frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{bx^4 + a}} - \frac{e}{2} \ln \left(\frac{1}{x^2} \left(2a + 2 \sqrt{a} \sqrt{bx^4 + a} \right) \right) \frac{1}{\sqrt{a}} - \frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(1/2),x)

[Out] $f/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticF}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)},I)-1/2*e/a^{(1/2)}*\ln((2*a+2*a^{(1/2)}*(b*x^4+a)^{(1/2)})/x^2)-1/2*c*(b*x^4+a)^{(1/2)}/a/x^2-d*(b*x^4+a)^{(1/2)}/a/x+I*d*b^{(1/2)}/a^{(1/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticF}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)},I)-I*d*b^{(1/2)}/a^{(1/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticE}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)},I)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + ax^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(1/2),x, algorithm="maxima")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)}{bx^7 + ax^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(1/2),x, algorithm="fricas")

[Out] `integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/(b*x^7 + a*x^3), x)`

Sympy [C] time = 3.09551, size = 126, normalized size = 0.42

$$-\frac{\sqrt{bc}\sqrt{\frac{a}{bx^4}+1}}{2a} + \frac{d\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{ax}\Gamma\left(\frac{3}{4}\right)} - \frac{e \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx^2}}\right)}{2\sqrt{a}} + \frac{fx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x**3+e*x**2+d*x+c)/x**3/(b*x**4+a)**(1/2), x)`

[Out] `-sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(2*a) + d*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x*gamma(3/4)) - e*asinh(sqrt(a)/(sqrt(b)*x**2))/(2*sqrt(a)) + f*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + ax^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(1/2), x, algorithm="giac")`

[Out] `integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^3), x)`

$$3.537 \quad \int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}} dx$$

Optimal. Leaf size=323

$$\frac{\sqrt[4]{b}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (\sqrt{bc} - 3\sqrt{ae}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}} - \frac{\sqrt[4]{be}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\right)}{a^{3/4}\sqrt{a+bx^4}}$$

[Out] $-(c*\operatorname{Sqrt}[a + b*x^4])/(3*a*x^3) - (d*\operatorname{Sqrt}[a + b*x^4])/(2*a*x^2) - (e*\operatorname{Sqrt}[a + b*x^4])/(a*x) + (\operatorname{Sqrt}[b]*e*x*\operatorname{Sqrt}[a + b*x^4])/(a*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) - (f*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*x^4]/\operatorname{Sqrt}[a]])/(2*\operatorname{Sqrt}[a]) - (b^{(1/4)}*e*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(a^{(3/4)}*\operatorname{Sqrt}[a + b*x^4]) - (b^{(1/4)}*(\operatorname{Sqrt}[b]*c - 3*\operatorname{Sqrt}[a]*e)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(6*a^{(5/4)}*\operatorname{Sqrt}[a + b*x^4])$

Rubi [A] time = 0.258399, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1833, 1282, 1198, 220, 1196, 1252, 807, 266, 63, 208}

$$\frac{\sqrt[4]{b}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (\sqrt{bc} - 3\sqrt{ae}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}} - \frac{\sqrt[4]{be}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c + d*x + e*x^2 + f*x^3)/(x^4*\operatorname{Sqrt}[a + b*x^4]), x]$

[Out] $-(c*\operatorname{Sqrt}[a + b*x^4])/(3*a*x^3) - (d*\operatorname{Sqrt}[a + b*x^4])/(2*a*x^2) - (e*\operatorname{Sqrt}[a + b*x^4])/(a*x) + (\operatorname{Sqrt}[b]*e*x*\operatorname{Sqrt}[a + b*x^4])/(a*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) - (f*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*x^4]/\operatorname{Sqrt}[a]])/(2*\operatorname{Sqrt}[a]) - (b^{(1/4)}*e*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(a^{(3/4)}*\operatorname{Sqrt}[a + b*x^4]) - (b^{(1/4)}*(\operatorname{Sqrt}[b]*c - 3*\operatorname{Sqrt}[a]*e)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(6*a^{(5/4)}*\operatorname{Sqrt}[a + b*x^4])$

Rule 1833


```
Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p]/c^j, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 1282

```
Int[((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 807

```
Int[((d_)*(x_)^(m_))*((f_)*(x_) + (g_)*(x_)^2)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
```

$$\frac{1}{2(p+1)(cd^2 + ae^2)} + \text{Dist}[(c*d*f + a*e*g)/(c*d^2 + a*e^2), \text{Int}[(d + e*x)^{(m+1)}(a + c*x^2)^p, x], x] /;$$

$$\text{FreeQ}\{a, c, d, e, f, g, m, p\}, x \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{EqQ}[\text{Simplify}[m + 2*p + 3], 0]$$

Rule 266

$$\text{Int}[(x_)^{(m_)}((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a + b*x)^p}, x], x, x^n], x] /;$$

$$\text{FreeQ}\{a, b, m, n, p\}, x \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$$

Rule 63

$$\text{Int}[(a_.) + (b_)*(x_)^{(m_)}((c_.) + (d_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1) - 1)}(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /;$$

$$\text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$$

Rule 208

$$\text{Int}[(a_) + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /;$$

$$\text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b]$$

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3}{x^4 \sqrt{a + bx^4}} dx &= \int \left(\frac{c + ex^2}{x^4 \sqrt{a + bx^4}} + \frac{d + fx^2}{x^3 \sqrt{a + bx^4}} \right) dx \\
&= \int \frac{c + ex^2}{x^4 \sqrt{a + bx^4}} dx + \int \frac{d + fx^2}{x^3 \sqrt{a + bx^4}} dx \\
&= -\frac{c\sqrt{a + bx^4}}{3ax^3} + \frac{1}{2} \text{Subst} \left(\int \frac{d + fx}{x^2 \sqrt{a + bx^2}} dx, x, x^2 \right) - \frac{\int \frac{-3ae + bcx^2}{x^2 \sqrt{a + bx^4}} dx}{3a} \\
&= -\frac{c\sqrt{a + bx^4}}{3ax^3} - \frac{d\sqrt{a + bx^4}}{2ax^2} - \frac{e\sqrt{a + bx^4}}{ax} + \frac{\int \frac{-abc + 3abex^2}{\sqrt{a + bx^4}} dx}{3a^2} + \frac{1}{2} f \text{Subst} \left(\int \frac{1}{x \sqrt{a + bx^2}} dx, x, x^2 \right) \\
&= -\frac{c\sqrt{a + bx^4}}{3ax^3} - \frac{d\sqrt{a + bx^4}}{2ax^2} - \frac{e\sqrt{a + bx^4}}{ax} - \frac{(\sqrt{be}) \int \frac{1 - \sqrt{bx^2}}{\sqrt{a + bx^4}} dx}{\sqrt{a}} - \frac{(\sqrt{b}(\sqrt{bc} - 3\sqrt{ae})) \int \frac{1}{\sqrt{a + bx^2}} dx}{3a} \\
&= -\frac{c\sqrt{a + bx^4}}{3ax^3} - \frac{d\sqrt{a + bx^4}}{2ax^2} - \frac{e\sqrt{a + bx^4}}{ax} + \frac{\sqrt{bex}\sqrt{a + bx^4}}{a(\sqrt{a} + \sqrt{bx^2})} - \frac{\sqrt[4]{be}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a + bx^4}{(\sqrt{a} + \sqrt{bx^2})^2}}}{a^{3/4}\sqrt{a + bx^4}} \\
&= -\frac{c\sqrt{a + bx^4}}{3ax^3} - \frac{d\sqrt{a + bx^4}}{2ax^2} - \frac{e\sqrt{a + bx^4}}{ax} + \frac{\sqrt{bex}\sqrt{a + bx^4}}{a(\sqrt{a} + \sqrt{bx^2})} - \frac{f \tanh^{-1} \left(\frac{\sqrt{a + bx^4}}{\sqrt{a}} \right)}{2\sqrt{a}} - \frac{\sqrt[4]{be}(\sqrt{a} + \sqrt{bx^2})}{a^{3/4}\sqrt{a + bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.157824, size = 149, normalized size = 0.46

$$\frac{-2ac\sqrt{\frac{bx^4}{a}} + {}_2F_1\left(-\frac{3}{4}, \frac{1}{2}; \frac{1}{4}; -\frac{bx^4}{a}\right) - 3x\left(2aex\sqrt{\frac{bx^4}{a}} + {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; -\frac{bx^4}{a}\right) + \sqrt{a}fx^2\sqrt{a + bx^4} \tanh^{-1}\left(\frac{\sqrt{a + bx^4}}{\sqrt{a}}\right) + ad + b\right)}{6ax^3\sqrt{a + bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^4*Sqrt[a + b*x^4]), x]

[Out] (-2*a*c*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-3/4, 1/2, 1/4, -((b*x^4)/a)] - 3*x*(a*d + b*d*x^4 + Sqrt[a]*f*x^2*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]] + 2*a*e*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((b*x^4)/a)])/(6*a*x^3*Sqrt[a + b*x^4])

Maple [C] time = 0.01, size = 316, normalized size = 1.

$$-\frac{f}{2} \ln\left(\frac{1}{x^2} \left(2a + 2\sqrt{a}\sqrt{bx^4 + a}\right)\right) \frac{1}{\sqrt{a}} - \frac{c}{3ax^3} \sqrt{bx^4 + a} - \frac{bc}{3a} \sqrt{1 - ix^2\sqrt{b}\frac{1}{\sqrt{a}}} \sqrt{1 + ix^2\sqrt{b}\frac{1}{\sqrt{a}}} \text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}, i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(1/2),x)

[Out] $-\frac{1}{2} \frac{f}{a^{1/2}} \ln\left(\frac{(2a + 2a^{1/2}(bx^4 + a)^{1/2})}{x^2}\right) - \frac{1}{3} \frac{c}{a^{1/2}} (bx^4 + a)^{1/2} / a/x^3 - \frac{1}{3} \frac{c}{a^{1/2}} \frac{b/a}{(I/a^{1/2} b^{1/2})^{1/2}} (1 - I/a^{1/2} b^{1/2} x^2)^{1/2} * (1 + I/a^{1/2} b^{1/2} x^2)^{1/2} / (bx^4 + a)^{1/2} * \text{EllipticF}(x * (I/a^{1/2} b^{1/2})^{1/2}, I) - \frac{1}{2} \frac{d}{a^{1/2}} (bx^4 + a)^{1/2} / a/x^2 - \frac{e}{a^{1/2}} (bx^4 + a)^{1/2} / a/x + \frac{1}{2} \frac{e}{a^{1/2}} \frac{b/a}{(I/a^{1/2} b^{1/2})^{1/2}} (1 - I/a^{1/2} b^{1/2} x^2)^{1/2} * (1 + I/a^{1/2} b^{1/2} x^2)^{1/2} / (bx^4 + a)^{1/2} * \text{EllipticF}(x * (I/a^{1/2} b^{1/2})^{1/2}, I) - \frac{1}{2} \frac{e}{a^{1/2}} \frac{b/a}{(I/a^{1/2} b^{1/2})^{1/2}} (1 - I/a^{1/2} b^{1/2} x^2)^{1/2} * (1 + I/a^{1/2} b^{1/2} x^2)^{1/2} / (bx^4 + a)^{1/2} * \text{EllipticE}(x * (I/a^{1/2} b^{1/2})^{1/2}, I)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + ax^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(1/2),x, algorithm="maxima")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^4), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)}{bx^8 + ax^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/(b*x^8 + a*x^4), x)

Sympy [C] time = 3.44096, size = 131, normalized size = 0.41

$$-\frac{\sqrt{bd}\sqrt{\frac{a}{bx^4}+1}}{2a} + \frac{c\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(-\frac{3}{4}, \frac{1}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{ax^3}\Gamma\left(\frac{1}{4}\right)} + \frac{e\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{ax}\Gamma\left(\frac{3}{4}\right)} - \frac{f \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx^2}}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)/x**4/(b*x**4+a)**(1/2),x)

[Out] -sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(2*a) + c*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x**3*gamma(1/4)) + e*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x*gamma(3/4)) - f*asinh(sqrt(a)/(sqrt(b)*x**2))/(2*sqrt(a))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + ax^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^4), x)

$$3.538 \quad \int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}} dx$$

Optimal. Leaf size=346

$$\frac{\sqrt[4]{b}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (\sqrt{bd} - 3\sqrt{af}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right) + bc \tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - \sqrt[4]{b}f(\sqrt{a} + \sqrt{bx^2})}{6a^{5/4}\sqrt{a+bx^4}} + \frac{bc \tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{3/2}}$$

[Out] $-(c\sqrt{a+bx^4})/(4ax^4) - (d\sqrt{a+bx^4})/(3ax^3) - (e\sqrt{a+bx^4})/(2ax^2) - (f\sqrt{a+bx^4})/(ax) + (\sqrt{b}f x \sqrt{a+bx^4})/(a(\sqrt{a} + \sqrt{bx^2})) + (bc \operatorname{ArcTanh}[\sqrt{a+bx^4}/\sqrt{a}])/(4a^{3/2}) - (b^{1/4}f(\sqrt{a} + \sqrt{bx^2})\sqrt{(a+bx^4)/(\sqrt{a} + \sqrt{bx^2})^2}) \operatorname{EllipticE}[2 \operatorname{ArcTan}[(b^{1/4}x)/a^{1/4}], 1/2]/(a^{3/4}\sqrt{a+bx^4}) - (b^{1/4}(\sqrt{b}d - 3\sqrt{af})(\sqrt{a} + \sqrt{bx^2})\sqrt{(a+bx^4)/(\sqrt{a} + \sqrt{bx^2})^2}) \operatorname{EllipticF}[2 \operatorname{ArcTan}[(b^{1/4}x)/a^{1/4}], 1/2]/(6a^{5/4}\sqrt{a+bx^4})$

Rubi [A] time = 0.2813, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {1833, 1252, 835, 807, 266, 63, 208, 1282, 1198, 220, 1196}

$$\frac{bc \tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{3/2}} - \frac{\sqrt[4]{b}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (\sqrt{bd} - 3\sqrt{af}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}} - \frac{\sqrt[4]{b}f(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}}}{a^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c + dx + ex^2 + fx^3)/(x^5\sqrt{a+bx^4}), x]$

[Out] $-(c\sqrt{a+bx^4})/(4ax^4) - (d\sqrt{a+bx^4})/(3ax^3) - (e\sqrt{a+bx^4})/(2ax^2) - (f\sqrt{a+bx^4})/(ax) + (\sqrt{b}f x \sqrt{a+bx^4})/(a(\sqrt{a} + \sqrt{bx^2})) + (bc \operatorname{ArcTanh}[\sqrt{a+bx^4}/\sqrt{a}])/(4a^{3/2}) - (b^{1/4}f(\sqrt{a} + \sqrt{bx^2})\sqrt{(a+bx^4)/(\sqrt{a} + \sqrt{bx^2})^2}) \operatorname{EllipticE}[2 \operatorname{ArcTan}[(b^{1/4}x)/a^{1/4}], 1/2]/(a^{3/4}\sqrt{a+bx^4}) - (b^{1/4}(\sqrt{b}d - 3\sqrt{af})(\sqrt{a} + \sqrt{bx^2})\sqrt{(a+bx^4)/(\sqrt{a} + \sqrt{bx^2})^2}) \operatorname{EllipticF}[2 \operatorname{ArcTan}[(b^{1/4}x)/a^{1/4}], 1/2]/(6a^{5/4}\sqrt{a+bx^4})$

Rule 1833

```
Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j))*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p]/c^j, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 1252

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 835

```
Int[((d_) + (e_)*(x_)^(m_))*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 807

```
Int[((d_) + (e_)*(x_)^(m_))*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1282

```
Int[((f_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] := Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x
, 1/2])]/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x,
1/2])]/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3}{x^5 \sqrt{a + bx^4}} dx &= \int \left(\frac{c + ex^2}{x^5 \sqrt{a + bx^4}} + \frac{d + fx^2}{x^4 \sqrt{a + bx^4}} \right) dx \\
&= \int \frac{c + ex^2}{x^5 \sqrt{a + bx^4}} dx + \int \frac{d + fx^2}{x^4 \sqrt{a + bx^4}} dx \\
&= -\frac{d\sqrt{a + bx^4}}{3ax^3} + \frac{1}{2} \text{Subst} \left(\int \frac{c + ex}{x^3 \sqrt{a + bx^2}} dx, x, x^2 \right) - \frac{\int \frac{-3af + bdx^2}{x^2 \sqrt{a + bx^4}} dx}{3a} \\
&= -\frac{c\sqrt{a + bx^4}}{4ax^4} - \frac{d\sqrt{a + bx^4}}{3ax^3} - \frac{f\sqrt{a + bx^4}}{ax} + \frac{\int \frac{-abd + 3abfx^2}{\sqrt{a + bx^4}} dx}{3a^2} - \frac{\text{Subst} \left(\int \frac{-2ae + bcx}{x^2 \sqrt{a + bx^2}} dx, x, x^2 \right)}{4a} \\
&= -\frac{c\sqrt{a + bx^4}}{4ax^4} - \frac{d\sqrt{a + bx^4}}{3ax^3} - \frac{e\sqrt{a + bx^4}}{2ax^2} - \frac{f\sqrt{a + bx^4}}{ax} - \frac{(bc) \text{Subst} \left(\int \frac{1}{x \sqrt{a + bx^2}} dx, x, x^2 \right)}{4a} \\
&= -\frac{c\sqrt{a + bx^4}}{4ax^4} - \frac{d\sqrt{a + bx^4}}{3ax^3} - \frac{e\sqrt{a + bx^4}}{2ax^2} - \frac{f\sqrt{a + bx^4}}{ax} + \frac{\sqrt{b}fx\sqrt{a + bx^4}}{a(\sqrt{a} + \sqrt{bx^2})} - \frac{\sqrt[4]{b}f(\sqrt{a} + \sqrt{bx^2})}{\sqrt{a + bx^4}} \\
&= -\frac{c\sqrt{a + bx^4}}{4ax^4} - \frac{d\sqrt{a + bx^4}}{3ax^3} - \frac{e\sqrt{a + bx^4}}{2ax^2} - \frac{f\sqrt{a + bx^4}}{ax} + \frac{\sqrt{b}fx\sqrt{a + bx^4}}{a(\sqrt{a} + \sqrt{bx^2})} - \frac{\sqrt[4]{b}f(\sqrt{a} + \sqrt{bx^2})}{\sqrt{a + bx^4}} \\
&= -\frac{c\sqrt{a + bx^4}}{4ax^4} - \frac{d\sqrt{a + bx^4}}{3ax^3} - \frac{e\sqrt{a + bx^4}}{2ax^2} - \frac{f\sqrt{a + bx^4}}{ax} + \frac{\sqrt{b}fx\sqrt{a + bx^4}}{a(\sqrt{a} + \sqrt{bx^2})} + \frac{bc \tanh^{-1} \left(\frac{\sqrt{a + bx^4}}{\sqrt{a}} \right)}{4a^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.147496, size = 147, normalized size = 0.42

$$\frac{\sqrt{a + bx^4} \left(3ac \sqrt{\frac{bx^4}{a} + 1} - 3bcx^4 \tanh^{-1} \left(\sqrt{\frac{bx^4}{a} + 1} \right) + 4adx {}_2F_1 \left(-\frac{3}{4}, \frac{1}{2}; \frac{1}{4}; -\frac{bx^4}{a} \right) + 6aex^2 \sqrt{\frac{bx^4}{a} + 1} + 12afx^3 {}_2F_1 \left(-\frac{1}{4}, \frac{1}{2} \right) \right)}{12a^2x^4 \sqrt{\frac{bx^4}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^5*Sqrt[a + b*x^4]),x]

[Out] -(Sqrt[a + b*x^4]*(3*a*c*Sqrt[1 + (b*x^4)/a] + 6*a*e*x^2*Sqrt[1 + (b*x^4)/a] - 3*b*c*x^4*ArcTanh[Sqrt[1 + (b*x^4)/a]] + 4*a*d*x*Hypergeometric2F1[-3/4, 1/2, 1/4, -(b*x^4)/a]) + 12*a*f*x^3*Hypergeometric2F1[-1/4, 1/2, 3/4, -(

$(b*x^4/a)])))/(12*a^2*x^4*sqrt[1 + (b*x^4)/a])$

Maple [C] time = 0.017, size = 335, normalized size = 1.

$$-\frac{d}{3ax^3}\sqrt{bx^4+a} - \frac{bd}{3a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right) - \frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}} - \frac{c}{4ax^4}\sqrt{bx^4+a} + \frac{b}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)/x^5/(b*x^4+a)^(1/2),x)

[Out] $-1/3*d*(b*x^4+a)^{(1/2)}/a/x^3 - 1/3*d*b/a/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*EllipticF(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)},I) - 1/4*c*(b*x^4+a)^{(1/2)}/a/x^4 + 1/4*c*b/a^{(3/2)}*\ln((2*a+2*a^{(1/2)}*(b*x^4+a)^{(1/2)})/x^2) - 1/2*e*(b*x^4+a)^{(1/2)}/a/x^2 - f*(b*x^4+a)^{(1/2)}/a/x + I*f*b^{(1/2)}/a^{(1/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*EllipticF(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)},I) - I*f*b^{(1/2)}/a^{(1/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*EllipticE(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)},I)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/x^5/(b*x^4+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{bx^9+ax^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/x^5/(b*x^4+a)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/(b*x^9 + a*x^5), x)

Sympy [C] time = 4.53686, size = 158, normalized size = 0.46

$$-\frac{\sqrt{bc}\sqrt{\frac{a}{bx^4}+1}}{4ax^2} - \frac{\sqrt{be}\sqrt{\frac{a}{bx^4}+1}}{2a} + \frac{d\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(-\frac{3}{4}, \frac{1}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{ax}^3\Gamma\left(\frac{1}{4}\right)} + \frac{f\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{ax}\Gamma\left(\frac{3}{4}\right)} + \frac{bc \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx^2}}\right)}{4a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)/x**5/(b*x**4+a)**(1/2),x)

[Out] -sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(4*a*x**2) - sqrt(b)*e*sqrt(a/(b*x**4) + 1)/(2*a) + d*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x**3*gamma(1/4)) + f*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x*gamma(3/4)) + b*c*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*a**(3/2))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + ax^5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/x^5/(b*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^5), x)

$$3.539 \quad \int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}} dx$$

Optimal. Leaf size=377

$$\frac{b^{3/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (5\sqrt{ae} + 9\sqrt{bc}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{30a^{7/4}\sqrt{a+bx^4}} - \frac{3b^{3/2}cx\sqrt{a+bx^4}}{5a^2(\sqrt{a} + \sqrt{bx^2})} + \frac{3b^{5/4}c(\sqrt{a} + \sqrt{bx^2})}{5a^{7/4}\sqrt{a+bx^4}}$$

[Out] $-(c\sqrt{a+bx^4})/(5ax^5) - (d\sqrt{a+bx^4})/(4ax^4) - (e\sqrt{a+bx^4})/(3ax^3) - (f\sqrt{a+bx^4})/(2ax^2) + (3bc\sqrt{a+bx^4})/(5a^2x) - (3b^{3/2}cx\sqrt{a+bx^4})/(5a^2(\sqrt{a} + \sqrt{bx^2})) + (b^2 \operatorname{ArcTanh}[\sqrt{a+bx^4}/\sqrt{a}])/(4a^{3/2}) + (3b^{5/4}c(\sqrt{a} + \sqrt{bx^2})\sqrt{a+bx^4})/(5a^{7/4}(\sqrt{a} + \sqrt{bx^2})) - (b^{3/4}(9\sqrt{b}c + 5\sqrt{a}e)(\sqrt{a} + \sqrt{bx^2})\sqrt{a+bx^4})/(30a^{7/4}(\sqrt{a} + \sqrt{bx^2}))$

Rubi [A] time = 0.326546, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {1833, 1282, 1198, 220, 1196, 1252, 835, 807, 266, 63, 208}

$$\frac{b^{3/4}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (5\sqrt{ae} + 9\sqrt{bc}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{30a^{7/4}\sqrt{a+bx^4}} - \frac{3b^{3/2}cx\sqrt{a+bx^4}}{5a^2(\sqrt{a} + \sqrt{bx^2})} + \frac{3b^{5/4}c(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}}}{5a^{7/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c + dx + ex^2 + fx^3)/(x^6\sqrt{a+bx^4}), x]$

[Out] $-(c\sqrt{a+bx^4})/(5ax^5) - (d\sqrt{a+bx^4})/(4ax^4) - (e\sqrt{a+bx^4})/(3ax^3) - (f\sqrt{a+bx^4})/(2ax^2) + (3bc\sqrt{a+bx^4})/(5a^2x) - (3b^{3/2}cx\sqrt{a+bx^4})/(5a^2(\sqrt{a} + \sqrt{bx^2})) + (b^2 \operatorname{ArcTanh}[\sqrt{a+bx^4}/\sqrt{a}])/(4a^{3/2}) + (3b^{5/4}c(\sqrt{a} + \sqrt{bx^2})\sqrt{a+bx^4})/(5a^{7/4}(\sqrt{a} + \sqrt{bx^2})) - (b^{3/4}(9\sqrt{b}c + 5\sqrt{a}e)(\sqrt{a} + \sqrt{bx^2})\sqrt{a+bx^4})/(30a^{7/4}(\sqrt{a} + \sqrt{bx^2}))$

Rule 1833

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p]/c^j, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 1282

```
Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3}{x^6 \sqrt{a + bx^4}} dx &= \int \left(\frac{c + ex^2}{x^6 \sqrt{a + bx^4}} + \frac{d + fx^2}{x^5 \sqrt{a + bx^4}} \right) dx \\
&= \int \frac{c + ex^2}{x^6 \sqrt{a + bx^4}} dx + \int \frac{d + fx^2}{x^5 \sqrt{a + bx^4}} dx \\
&= -\frac{c\sqrt{a + bx^4}}{5ax^5} + \frac{1}{2} \text{Subst} \left(\int \frac{d + fx}{x^3 \sqrt{a + bx^2}} dx, x, x^2 \right) - \frac{\int \frac{-5ae + 3bcx^2}{x^4 \sqrt{a + bx^4}} dx}{5a} \\
&= -\frac{c\sqrt{a + bx^4}}{5ax^5} - \frac{d\sqrt{a + bx^4}}{4ax^4} - \frac{e\sqrt{a + bx^4}}{3ax^3} + \frac{\int \frac{-9abc - 5abex^2}{x^2 \sqrt{a + bx^4}} dx}{15a^2} - \frac{\text{Subst} \left(\int \frac{-2af + bdx}{x^2 \sqrt{a + bx^2}} dx, x, x^2 \right)}{4a} \\
&= -\frac{c\sqrt{a + bx^4}}{5ax^5} - \frac{d\sqrt{a + bx^4}}{4ax^4} - \frac{e\sqrt{a + bx^4}}{3ax^3} - \frac{f\sqrt{a + bx^4}}{2ax^2} + \frac{3bc\sqrt{a + bx^4}}{5a^2x} - \frac{\int \frac{5a^2be + 9ab^2cx^2}{\sqrt{a + bx^4}} dx}{15a^3} \\
&= -\frac{c\sqrt{a + bx^4}}{5ax^5} - \frac{d\sqrt{a + bx^4}}{4ax^4} - \frac{e\sqrt{a + bx^4}}{3ax^3} - \frac{f\sqrt{a + bx^4}}{2ax^2} + \frac{3bc\sqrt{a + bx^4}}{5a^2x} + \frac{(3b^{3/2}c) \int \frac{1 - \sqrt{bx^2}}{\sqrt{a + bx^4}} dx}{5a^{3/2}} \\
&= -\frac{c\sqrt{a + bx^4}}{5ax^5} - \frac{d\sqrt{a + bx^4}}{4ax^4} - \frac{e\sqrt{a + bx^4}}{3ax^3} - \frac{f\sqrt{a + bx^4}}{2ax^2} + \frac{3bc\sqrt{a + bx^4}}{5a^2x} - \frac{3b^{3/2}cx\sqrt{a + bx^4}}{5a^2(\sqrt{a} + \sqrt{bx^2})} + \\
&= -\frac{c\sqrt{a + bx^4}}{5ax^5} - \frac{d\sqrt{a + bx^4}}{4ax^4} - \frac{e\sqrt{a + bx^4}}{3ax^3} - \frac{f\sqrt{a + bx^4}}{2ax^2} + \frac{3bc\sqrt{a + bx^4}}{5a^2x} - \frac{3b^{3/2}cx\sqrt{a + bx^4}}{5a^2(\sqrt{a} + \sqrt{bx^2})} +
\end{aligned}$$

Mathematica [C] time = 0.208193, size = 134, normalized size = 0.36

$$\frac{\sqrt{a + bx^4} \left(12ac {}_2F_1 \left(-\frac{5}{4}, \frac{1}{2}; -\frac{1}{4}; -\frac{bx^4}{a} \right) + 5x \left(3a \sqrt{\frac{bx^4}{a}} + 1 (d + 2fx^2) - 3bdx^4 \tanh^{-1} \left(\sqrt{\frac{bx^4}{a}} + 1 \right) + 4aex {}_2F_1 \left(-\frac{3}{4}, \frac{1}{2}; \frac{1}{4}; -\frac{bx^4}{a} \right) \right)}{60a^2x^5 \sqrt{\frac{bx^4}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^6*Sqrt[a + b*x^4]),x]

[Out] -(Sqrt[a + b*x^4]*(12*a*c*Hypergeometric2F1[-5/4, 1/2, -1/4, -((b*x^4)/a)] + 5*x*(3*a*(d + 2*f*x^2)*Sqrt[1 + (b*x^4)/a] - 3*b*d*x^4*ArcTanh[Sqrt[1 + (b*x^4)/a]] + 4*a*e*x*Hypergeometric2F1[-3/4, 1/2, 1/4, -((b*x^4)/a)])))/(60*a^2*x^5*Sqrt[1 + (b*x^4)/a])

Maple [C] time = 0.01, size = 354, normalized size = 0.9

$$-\frac{e}{3ax^3}\sqrt{bx^4+a} - \frac{be}{3a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}} - \frac{c}{5ax^5}\sqrt{bx^4+a} + \frac{3}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)/x^6/(b*x^4+a)^(1/2),x)

[Out] $-\frac{1}{3}e*(b*x^4+a)^{(1/2)}/a/x^3 - \frac{1}{3}e*b/a/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticF}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)},I) - \frac{1}{5}c*(b*x^4+a)^{(1/2)}/a/x^5 + \frac{3}{5}b*c*(b*x^4+a)^{(1/2)}/a^2/x - \frac{3}{5}I*c/a^{(3/2)}*b^{(3/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticF}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)},I) + \frac{3}{5}I*c/a^{(3/2)}*b^{(3/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticE}(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)},I) - \frac{1}{4}d*(b*x^4+a)^{(1/2)}/a/x^4 + \frac{1}{4}d*b/a^{(3/2)}*\ln((2*a+2*a^{(1/2)}*(b*x^4+a)^{(1/2)})/x^2) - \frac{1}{2}f*(b*x^4+a)^{(1/2)}/a/x^2$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + ax^6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/x^6/(b*x^4+a)^(1/2),x, algorithm="maxima")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^6), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{bx^{10}+ax^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/x^6/(b*x^4+a)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/(b*x^10 + a*x^6), x)

Sympy [C] time = 4.88255, size = 163, normalized size = 0.43

$$-\frac{\sqrt{bd}\sqrt{\frac{a}{bx^4}+1}}{4ax^2} - \frac{\sqrt{bf}\sqrt{\frac{a}{bx^4}+1}}{2a} + \frac{c\Gamma\left(-\frac{5}{4}\right) {}_2F_1\left(-\frac{5}{4}, \frac{1}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{ax^5}\Gamma\left(-\frac{1}{4}\right)} + \frac{e\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(-\frac{3}{4}, \frac{1}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{ax^3}\Gamma\left(\frac{1}{4}\right)} + \frac{bd \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx^2}}\right)}{4a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)/x**6/(b*x**4+a)**(1/2),x)

[Out] -sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(4*a*x**2) - sqrt(b)*f*sqrt(a/(b*x**4) + 1)/(2*a) + c*gamma(-5/4)*hyper((-5/4, 1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x**5*gamma(-1/4)) + e*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x**3*gamma(1/4)) + b*d*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*a**(3/2))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + ax^6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/x^6/(b*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^6), x)

$$3.540 \quad \int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}} dx$$

Optimal. Leaf size=365

$$\frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (9\sqrt{bc} - 5\sqrt{ae}) \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{12b^{9/4}\sqrt{a+bx^4}} + \frac{x(ae + afx - bcx^2 - bdx^3)}{2b^2\sqrt{a+bx^4}} + \frac{3cx\sqrt{a+bx^4}}{2b^{3/2}(\sqrt{a} + \sqrt{bx^2})}$$

[Out] (x*(a*e + a*f*x - b*c*x^2 - b*d*x^3))/(2*b^2*Sqrt[a + b*x^4]) + (d*Sqrt[a + b*x^4])/b^2 + (e*x*Sqrt[a + b*x^4])/(3*b^2) + (f*x^2*Sqrt[a + b*x^4])/(4*b^2) + (3*c*x*Sqrt[a + b*x^4])/(2*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) - (3*a*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(5/2)) - (3*a^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(7/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(9*Sqrt[b]*c - 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(12*b^(9/4)*Sqrt[a + b*x^4])

Rubi [A] time = 0.522866, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {1828, 1885, 1888, 1198, 220, 1196, 1819, 1815, 641, 217, 206}

$$\frac{x(ae + afx - bcx^2 - bdx^3)}{2b^2\sqrt{a+bx^4}} + \frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (9\sqrt{bc} - 5\sqrt{ae}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{12b^{9/4}\sqrt{a+bx^4}} + \frac{3cx\sqrt{a+bx^4}}{2b^{3/2}(\sqrt{a} + \sqrt{bx^2})}$$

Antiderivative was successfully verified.

[In] Int[(x^6*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x]

[Out] (x*(a*e + a*f*x - b*c*x^2 - b*d*x^3))/(2*b^2*Sqrt[a + b*x^4]) + (d*Sqrt[a + b*x^4])/b^2 + (e*x*Sqrt[a + b*x^4])/(3*b^2) + (f*x^2*Sqrt[a + b*x^4])/(4*b^2) + (3*c*x*Sqrt[a + b*x^4])/(2*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) - (3*a*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(5/2)) - (3*a^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(7/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(9*Sqrt[b]*c - 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(12*b^(9/4)*Sqrt[a + b*x^4])

*Sqrt[a + b*x^4])

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^ (p_), x_Symbol] := With[{q =
m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
+ b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^ (p_), x_Symbol] := Module[{q = Expon[Pq,
x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^ (p_), x_Symbol] := With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)]) /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2])*EllipticF[2*ArcTan[q*x],
1/2]]/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
  Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
  1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
  1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
  x] && PosQ[c/a]
```

Rule 1819

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/(m
  + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
  , x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
  Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Expon[Pq, x],
  e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
  (q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
  m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
  ], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(e*(
  a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
  ; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
  x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
  Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
  Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6(c + dx + ex^2 + fx^3)}{(a + bx^4)^{3/2}} dx &= \frac{x(ae + afx - bcx^2 - bdx^3)}{2b^2\sqrt{a + bx^4}} - \frac{\int \frac{a^2be + 2a^2bfx - 3ab^2cx^2 - 4ab^2dx^3 - 2ab^2ex^4 - 2ab^2fx^5}{\sqrt{a + bx^4}} dx}{2ab^3} \\
&= \frac{x(ae + afx - bcx^2 - bdx^3)}{2b^2\sqrt{a + bx^4}} - \frac{\int \left(\frac{a^2be - 3ab^2cx^2 - 2ab^2ex^4}{\sqrt{a + bx^4}} + \frac{x(2a^2bf - 4ab^2dx^2 - 2ab^2fx^4)}{\sqrt{a + bx^4}} \right) dx}{2ab^3} \\
&= \frac{x(ae + afx - bcx^2 - bdx^3)}{2b^2\sqrt{a + bx^4}} - \frac{\int \frac{a^2be - 3ab^2cx^2 - 2ab^2ex^4}{\sqrt{a + bx^4}} dx}{2ab^3} - \frac{\int \frac{x(2a^2bf - 4ab^2dx^2 - 2ab^2fx^4)}{\sqrt{a + bx^4}} dx}{2ab^3} \\
&= \frac{x(ae + afx - bcx^2 - bdx^3)}{2b^2\sqrt{a + bx^4}} + \frac{ex\sqrt{a + bx^4}}{3b^2} - \frac{\int \frac{5a^2b^2e - 9ab^3cx^2}{\sqrt{a + bx^4}} dx}{6ab^4} - \frac{\text{Subst} \left(\int \frac{2a^2bf - 4ab^2dx^2}{\sqrt{a + bx^4}} dx \right)}{4ab^3} \\
&= \frac{x(ae + afx - bcx^2 - bdx^3)}{2b^2\sqrt{a + bx^4}} + \frac{ex\sqrt{a + bx^4}}{3b^2} + \frac{fx^2\sqrt{a + bx^4}}{4b^2} - \frac{\text{Subst} \left(\int \frac{6a^2b^2f - 8ab^3dx}{\sqrt{a + bx^4}} dx \right)}{8ab^4} \\
&= \frac{x(ae + afx - bcx^2 - bdx^3)}{2b^2\sqrt{a + bx^4}} + \frac{d\sqrt{a + bx^4}}{b^2} + \frac{ex\sqrt{a + bx^4}}{3b^2} + \frac{fx^2\sqrt{a + bx^4}}{4b^2} + \frac{3cx\sqrt{a + bx^4}}{2b^{3/2}(\sqrt{a + bx^4})} \\
&= \frac{x(ae + afx - bcx^2 - bdx^3)}{2b^2\sqrt{a + bx^4}} + \frac{d\sqrt{a + bx^4}}{b^2} + \frac{ex\sqrt{a + bx^4}}{3b^2} + \frac{fx^2\sqrt{a + bx^4}}{4b^2} + \frac{3cx\sqrt{a + bx^4}}{2b^{3/2}(\sqrt{a + bx^4})} \\
&= \frac{x(ae + afx - bcx^2 - bdx^3)}{2b^2\sqrt{a + bx^4}} + \frac{d\sqrt{a + bx^4}}{b^2} + \frac{ex\sqrt{a + bx^4}}{3b^2} + \frac{fx^2\sqrt{a + bx^4}}{4b^2} + \frac{3cx\sqrt{a + bx^4}}{2b^{3/2}(\sqrt{a + bx^4})}
\end{aligned}$$

Mathematica [C] time = 0.173399, size = 220, normalized size = 0.6

$$\frac{-9a^{3/2}f\sqrt{\frac{bx^4}{a}} + 1 \sinh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right) - 12b^{3/2}cx^3\sqrt{\frac{bx^4}{a}} + 1 {}_2F_1\left(\frac{3}{4}, \frac{3}{2}; \frac{7}{4}; -\frac{bx^4}{a}\right) + 12a\sqrt{bd} - 10a\sqrt{bex}\sqrt{\frac{bx^4}{a}} + 1 {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{bx^4}{a}\right)}{12b^{5/2}\sqrt{a + bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^6*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x]

[Out] (12*a*Sqrt[b]*d + 10*a*Sqrt[b]*e*x + 9*a*Sqrt[b]*f*x^2 + 12*b^(3/2)*c*x^3 + 6*b^(3/2)*d*x^4 + 4*b^(3/2)*e*x^5 + 3*b^(3/2)*f*x^6 - 9*a^(3/2)*f*Sqrt[1 +

$$\frac{(b*x^4)/a * \text{ArcSinh}[(\text{Sqrt}[b]*x^2)/\text{Sqrt}[a]] - 10*a*\text{Sqrt}[b]*e*x*\text{Sqrt}[1 + (b*x^4)/a] * \text{Hypergeometric2F1}[1/4, 1/2, 5/4, -((b*x^4)/a)] - 12*b^{(3/2)}*c*x^3*\text{Sqrt}[1 + (b*x^4)/a] * \text{Hypergeometric2F1}[3/4, 3/2, 7/4, -((b*x^4)/a)]}{(12*b^{(5/2)}*\text{Sqrt}[a + b*x^4])}$$

Maple [C] time = 0.027, size = 378, normalized size = 1.

$$\frac{fx^6}{4b} \frac{1}{\sqrt{bx^4+a}} + \frac{3afx^2}{4b^2} \frac{1}{\sqrt{bx^4+a}} - \frac{3af}{4} \ln\left(x^2\sqrt{b} + \sqrt{bx^4+a}\right) b^{-\frac{5}{2}} + \frac{aex}{2b^2} \frac{1}{\sqrt{\left(x^4 + \frac{a}{b}\right)b}} + \frac{ex}{3b^2} \sqrt{bx^4+a} - \frac{5ae}{6b^2} \sqrt{1-ix^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x)

[Out] $\frac{1}{4} * f * x^6 / b / (b * x^4 + a)^{(1/2)} + \frac{3}{4} * f / b^2 * a * x^2 / (b * x^4 + a)^{(1/2)} - \frac{3}{4} * f / b^{(5/2)} * a * \ln(x^2 * b^{(1/2)} + (b * x^4 + a)^{(1/2)}) + \frac{1}{2} * e / b^2 * x * a / ((x^4 + 1/b * a) * b)^{(1/2)} + \frac{1}{3} * e * x * (b * x^4 + a)^{(1/2)} / b^2 - \frac{5}{6} * e / b^2 * a / (I/a^{(1/2)} * b^{(1/2)})^{(1/2)} * (1 - I/a^{(1/2)} * b^{(1/2)} * x^2)^{(1/2)} * (1 + I/a^{(1/2)} * b^{(1/2)} * x^2)^{(1/2)} / (b * x^4 + a)^{(1/2)} * \text{EllipticF}(x * (I/a^{(1/2)} * b^{(1/2)})^{(1/2)}, I) + \frac{1}{2} * d * (b * x^4 + 2 * a) / (b * x^4 + a)^{(1/2)} / b^2 - \frac{1}{2} * c / b * x^3 / ((x^4 + 1/b * a) * b)^{(1/2)} + \frac{3}{2} * I * c / b^{(3/2)} * a^{(1/2)} / (I/a^{(1/2)} * b^{(1/2)})^{(1/2)} * (1 - I/a^{(1/2)} * b^{(1/2)} * x^2)^{(1/2)} * (1 + I/a^{(1/2)} * b^{(1/2)} * x^2)^{(1/2)} / (b * x^4 + a)^{(1/2)} * \text{EllipticF}(x * (I/a^{(1/2)} * b^{(1/2)})^{(1/2)}, I) - \frac{3}{2} * I * c / b^{(3/2)} * a^{(1/2)} / (I/a^{(1/2)} * b^{(1/2)})^{(1/2)} * (1 - I/a^{(1/2)} * b^{(1/2)} * x^2)^{(1/2)} * (1 + I/a^{(1/2)} * b^{(1/2)} * x^2)^{(1/2)} / (b * x^4 + a)^{(1/2)} * \text{EllipticE}(x * (I/a^{(1/2)} * b^{(1/2)})^{(1/2)}, I)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx^3 + ex^2 + dx + c)x^6}{(bx^4 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^6/(b*x^4 + a)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(fx^9 + ex^8 + dx^7 + cx^6)\sqrt{bx^4 + a}}{b^2x^8 + 2abx^4 + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")

[Out] integral((f*x^9 + e*x^8 + d*x^7 + c*x^6)*sqrt(b*x^4 + a)/(b^2*x^8 + 2*a*b*x^4 + a^2), x)

Sympy [A] time = 34.6687, size = 202, normalized size = 0.55

$$d \left(\begin{cases} \frac{a}{b^2\sqrt{a+bx^4}} + \frac{x^4}{2b\sqrt{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^8}{8a^{\frac{3}{2}}} & \text{otherwise} \end{cases} \right) + f \left(\frac{3\sqrt{a}x^2}{4b^2\sqrt{1+\frac{bx^4}{a}}} - \frac{3a \operatorname{asinh}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}} + \frac{x^6}{4\sqrt{ab}\sqrt{1+\frac{bx^4}{a}}} \right) + \frac{cx^7\Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\frac{3}{2}, \frac{7}{4} \middle| \frac{11}{4}\right) \sqrt{bx^4+a}}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)

[Out] d*Piecewise((a/(b**2*sqrt(a + b*x**4)) + x**4/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**8/(8*a**(3/2)), True)) + f*(3*sqrt(a)*x**2/(4*b**2*sqrt(1 + b*x**4/a)) - 3*a*asinh(sqrt(b)*x**2/sqrt(a))/(4*b**(5/2)) + x**6/(4*sqrt(a)*b*sqrt(1 + b*x**4/a))) + c*x**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(11/4)) + e*x**9*gamma(9/4)*hyper((3/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(13/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx^3 + ex^2 + dx + c)x^6}{(bx^4 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^6/(b*x^4 + a)^(3/2), x)
```


$$3.541 \quad \int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}} dx$$

Optimal. Leaf size=343

$$\frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (9\sqrt{bd} - 5\sqrt{af}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{12b^{9/4}\sqrt{a+bx^4}} + \frac{x(af - bcx - bdx^2 - bex^3)}{2b^2\sqrt{a+bx^4}} + \frac{c \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right)}{2b^3}$$

[Out] (x*(a*f - b*c*x - b*d*x^2 - b*e*x^3))/(2*b^2*Sqrt[a + b*x^4]) + (e*Sqrt[a + b*x^4])/b^2 + (f*x*Sqrt[a + b*x^4])/(3*b^2) + (3*d*x*Sqrt[a + b*x^4])/(2*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*b^(3/2)) - (3*a^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(7/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(9*Sqrt[b]*d - 5*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(12*b^(9/4)*Sqrt[a + b*x^4])

Rubi [A] time = 0.367229, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1828, 1885, 1248, 641, 217, 206, 1888, 1198, 220, 1196}

$$\frac{x(af - bcx - bdx^2 - bex^3)}{2b^2\sqrt{a+bx^4}} + \frac{c \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right)}{2b^{3/2}} + \frac{\sqrt[4]{a}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (9\sqrt{bd} - 5\sqrt{af}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{12b^{9/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x]

[Out] (x*(a*f - b*c*x - b*d*x^2 - b*e*x^3))/(2*b^2*Sqrt[a + b*x^4]) + (e*Sqrt[a + b*x^4])/b^2 + (f*x*Sqrt[a + b*x^4])/(3*b^2) + (3*d*x*Sqrt[a + b*x^4])/(2*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*b^(3/2)) - (3*a^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(7/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(9*Sqrt[b]*d - 5*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(12*b^(9/4)*Sqrt[a + b*x^4])

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q =
m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
+ b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Module[{q = Expon[Pq,
x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
:= Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Expon[Pq, x
]}, With[{Pq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
```

```
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1)), x] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*n)]) /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1198

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5(c + dx + ex^2 + fx^3)}{(a + bx^4)^{3/2}} dx &= \frac{x(af - bcx - bdx^2 - bex^3)}{2b^2\sqrt{a + bx^4}} - \frac{\int \frac{a^2f - 2abcx - 3abdx^2 - 4abex^3 - 2abfx^4}{\sqrt{a + bx^4}} dx}{2ab^2} \\
&= \frac{x(af - bcx - bdx^2 - bex^3)}{2b^2\sqrt{a + bx^4}} - \frac{\int \left(\frac{x(-2abc - 4abex^2)}{\sqrt{a + bx^4}} + \frac{a^2f - 3abdx^2 - 2abfx^4}{\sqrt{a + bx^4}} \right) dx}{2ab^2} \\
&= \frac{x(af - bcx - bdx^2 - bex^3)}{2b^2\sqrt{a + bx^4}} - \frac{\int \frac{x(-2abc - 4abex^2)}{\sqrt{a + bx^4}} dx}{2ab^2} - \frac{\int \frac{a^2f - 3abdx^2 - 2abfx^4}{\sqrt{a + bx^4}} dx}{2ab^2} \\
&= \frac{x(af - bcx - bdx^2 - bex^3)}{2b^2\sqrt{a + bx^4}} + \frac{fx\sqrt{a + bx^4}}{3b^2} - \frac{\int \frac{5a^2bf - 9ab^2dx^2}{\sqrt{a + bx^4}} dx}{6ab^3} - \frac{\text{Subst}\left(\int \frac{-2abc - 4abex}{\sqrt{a + bx^2}} dx, x, x^2\right)}{4ab^2} \\
&= \frac{x(af - bcx - bdx^2 - bex^3)}{2b^2\sqrt{a + bx^4}} + \frac{e\sqrt{a + bx^4}}{b^2} + \frac{fx\sqrt{a + bx^4}}{3b^2} + \frac{c \text{Subst}\left(\int \frac{1}{\sqrt{a + bx^2}} dx, x, x^2\right)}{2b} \\
&= \frac{x(af - bcx - bdx^2 - bex^3)}{2b^2\sqrt{a + bx^4}} + \frac{e\sqrt{a + bx^4}}{b^2} + \frac{fx\sqrt{a + bx^4}}{3b^2} + \frac{3dx\sqrt{a + bx^4}}{2b^{3/2}(\sqrt{a} + \sqrt{bx^2})} - \frac{3^4\sqrt{ad}}{\dots} \\
&= \frac{x(af - bcx - bdx^2 - bex^3)}{2b^2\sqrt{a + bx^4}} + \frac{e\sqrt{a + bx^4}}{b^2} + \frac{fx\sqrt{a + bx^4}}{3b^2} + \frac{3dx\sqrt{a + bx^4}}{2b^{3/2}(\sqrt{a} + \sqrt{bx^2})} + \frac{c \tanh^{-1}}{2}
\end{aligned}$$

Mathematica [C] time = 0.151856, size = 176, normalized size = 0.51

$$\frac{3\sqrt{a}\sqrt{bc}\sqrt{\frac{bx^4}{a} + 1} \sinh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right) - 6bdx^3\sqrt{\frac{bx^4}{a} + 1} {}_2F_1\left(\frac{3}{4}, \frac{3}{2}; \frac{7}{4}; -\frac{bx^4}{a}\right) - 5afx\sqrt{\frac{bx^4}{a} + 1} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{bx^4}{a}\right) + 6ae + 5afx - 3}{6b^2\sqrt{a + bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x]

[Out] (6*a*e + 5*a*f*x - 3*b*c*x^2 + 6*b*d*x^3 + 3*b*e*x^4 + 2*b*f*x^5 + 3*sqrt[a]*sqrt[b]*c*sqrt[1 + (b*x^4)/a]*ArcSinh[(sqrt[b]*x^2)/sqrt[a]] - 5*a*f*x*sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 6*b*d*x^3*sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(6*b^2*sqrt[a + b*x^4])

Maple [C] time = 0.016, size = 358, normalized size = 1.

$$\frac{afx}{2b^2} \frac{1}{\sqrt{\left(x^4 + \frac{a}{b}\right)b}} + \frac{fx}{3b^2} \sqrt{bx^4 + a} - \frac{5af}{6b^2} \sqrt{1 - ix^2\sqrt{b}} \frac{1}{\sqrt{a}} \sqrt{1 + ix^2\sqrt{b}} \frac{1}{\sqrt{a}} \text{EllipticF}\left(x\sqrt{i\sqrt{b}} \frac{1}{\sqrt{a}}, i\right) \frac{1}{\sqrt{i\sqrt{b}} \frac{1}{\sqrt{a}}} \frac{1}{\sqrt{bx^4 + a}} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x)`

[Out] $\frac{1}{2} \frac{f}{b^2} \frac{x^4 + \frac{a}{b}}{b} \sqrt{\left(x^4 + \frac{a}{b}\right)b} + \frac{1}{3} \frac{f}{b^2} \frac{x^3}{b} \sqrt{bx^4 + a} - \frac{5}{6} \frac{af}{b^2} \sqrt{1 - ix^2\sqrt{b}} \frac{1}{\sqrt{a}} \sqrt{1 + ix^2\sqrt{b}} \frac{1}{\sqrt{a}} \text{EllipticF}\left(x\sqrt{i\sqrt{b}} \frac{1}{\sqrt{a}}, i\right) \frac{1}{\sqrt{i\sqrt{b}} \frac{1}{\sqrt{a}}} \frac{1}{\sqrt{bx^4 + a}} + \frac{1}{2} \frac{e}{b^2} \frac{x^2}{b} \sqrt{bx^4 + a} + \frac{1}{2} \frac{d}{b^2} \frac{x}{b} \sqrt{bx^4 + a} + \frac{1}{2} \frac{c}{b^2} \ln\left(x^2 \sqrt{bx^4 + a} + (bx^4 + a)^{3/2}\right)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(fx^8 + ex^7 + dx^6 + cx^5)\sqrt{bx^4 + a}}{b^2x^8 + 2abx^4 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")

[Out] integral((f*x^8 + e*x^7 + d*x^6 + c*x^5)*sqrt(b*x^4 + a)/(b^2*x^8 + 2*a*b*x^4 + a^2), x)

Sympy [A] time = 25.7538, size = 172, normalized size = 0.5

$$c \left(\frac{\operatorname{asinh}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}} - \frac{x^2}{2\sqrt{ab}\sqrt{1 + \frac{bx^4}{a}}} \right) + e \left(\begin{cases} \frac{a}{b^2\sqrt{a+bx^4}} + \frac{x^4}{2b\sqrt{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^8}{8a^{\frac{3}{2}}} & \text{otherwise} \end{cases} \right) + \frac{dx^7\Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\frac{3}{2}, \frac{7}{4} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)} + \frac{fx^9\Gamma\left(\frac{9}{4}\right) {}_2F_1}{4a^{\frac{3}{2}}\Gamma}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)

[Out] c*(asinh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - x**2/(2*sqrt(a)*b*sqrt(1 + b*x**4/a))) + e*Piecewise((a/(b**2*sqrt(a + b*x**4)) + x**4/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**8/(8*a**(3/2)), True)) + d*x**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(11/4)) + f*x**9*gamma(9/4)*hyper((3/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(13/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx^3 + ex^2 + dx + c)x^5}{(bx^4 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^5/(b*x^4 + a)^(3/2), x)

$$3.542 \quad \int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}} dx$$

Optimal. Leaf size=314

$$\frac{(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (3\sqrt{ae} + \sqrt{bc}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{4\sqrt[4]{ab}^{7/4}\sqrt{a+bx^4}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right)}{2b^{3/2}} + \frac{3ex\sqrt{a+bx^4}}{2b^{3/2}(\sqrt{a} + \sqrt{bx^2})} - \frac{3\sqrt[4]{ae}(\sqrt{a} + \sqrt{bx^2})}{2b^{3/2}(\sqrt{a} + \sqrt{bx^2})}$$

[Out] $-(x*(c + d*x + e*x^2 + f*x^3))/(2*b*\operatorname{Sqrt}[a + b*x^4]) + (f*\operatorname{Sqrt}[a + b*x^4])/b^2 + (3*e*x*\operatorname{Sqrt}[a + b*x^4])/(2*b^{(3/2)}*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) + (d*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*x^2)/\operatorname{Sqrt}[a + b*x^4]])/(2*b^{(3/2)}) - (3*a^{(1/4)}*e*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(2*b^{(7/4)}*\operatorname{Sqrt}[a + b*x^4]) + ((\operatorname{Sqrt}[b]*c + 3*\operatorname{Sqrt}[a]*e)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(4*a^{(1/4)}*b^{(7/4)}*\operatorname{Sqrt}[a + b*x^4])$

Rubi [A] time = 0.26789, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {1828, 1885, 1198, 220, 1196, 1248, 641, 217, 206}

$$\frac{(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (3\sqrt{ae} + \sqrt{bc}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{4\sqrt[4]{ab}^{7/4}\sqrt{a+bx^4}} + \frac{d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right)}{2b^{3/2}} + \frac{3ex\sqrt{a+bx^4}}{2b^{3/2}(\sqrt{a} + \sqrt{bx^2})} - \frac{3\sqrt[4]{ae}(\sqrt{a} + \sqrt{bx^2})}{2b^{3/2}(\sqrt{a} + \sqrt{bx^2})}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x^4*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^{(3/2)}, x]$

[Out] $-(x*(c + d*x + e*x^2 + f*x^3))/(2*b*\operatorname{Sqrt}[a + b*x^4]) + (f*\operatorname{Sqrt}[a + b*x^4])/b^2 + (3*e*x*\operatorname{Sqrt}[a + b*x^4])/(2*b^{(3/2)}*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) + (d*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*x^2)/\operatorname{Sqrt}[a + b*x^4]])/(2*b^{(3/2)}) - (3*a^{(1/4)}*e*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(2*b^{(7/4)}*\operatorname{Sqrt}[a + b*x^4]) + ((\operatorname{Sqrt}[b]*c + 3*\operatorname{Sqrt}[a]*e)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(4*a^{(1/4)}*b^{(7/4)}*\operatorname{Sqrt}[a + b*x^4])$

Rule 1828

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q =
m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
+ b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq,
x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 1198

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x]] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x],
1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
1/2])/(q*Sqrt[a + c*x^4]), x]] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol]
:= Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x]] /; FreeQ
[{a, c, d, e, p, q}, x]
```


Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}} dx &= -\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}} - \frac{\int \frac{-abc-2abdx-3abex^2-4abfx^3}{\sqrt{a+bx^4}} dx}{2ab^2} \\
&= -\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}} - \frac{\int \left(\frac{-abc-3abex^2}{\sqrt{a+bx^4}} + \frac{x(-2abd-4abfx^2)}{\sqrt{a+bx^4}} \right) dx}{2ab^2} \\
&= -\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}} - \frac{\int \frac{-abc-3abex^2}{\sqrt{a+bx^4}} dx}{2ab^2} - \frac{\int \frac{x(-2abd-4abfx^2)}{\sqrt{a+bx^4}} dx}{2ab^2} \\
&= -\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}} - \frac{\text{Subst} \left(\int \frac{-2abd-4abfx}{\sqrt{a+bx^2}} dx, x, x^2 \right)}{4ab^2} - \frac{(3\sqrt{ae}) \int \frac{1-\frac{\sqrt{bx^2}}{\sqrt{a}}}{\sqrt{a+bx^4}} dx}{2b^{3/2}} + \frac{(\sqrt{ae}) \int \frac{\sqrt{bx^2}}{\sqrt{a+bx^4}} dx}{2b^{3/2}} \\
&= -\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}} + \frac{f\sqrt{a+bx^4}}{b^2} + \frac{3ex\sqrt{a+bx^4}}{2b^{3/2}(\sqrt{a}+\sqrt{bx^2})} - \frac{3^4\sqrt{ae}(\sqrt{a}+\sqrt{bx^2})\sqrt{\frac{a}{a+bx^4}}}{2b^{7/4}\sqrt{a+bx^4}} \\
&= -\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}} + \frac{f\sqrt{a+bx^4}}{b^2} + \frac{3ex\sqrt{a+bx^4}}{2b^{3/2}(\sqrt{a}+\sqrt{bx^2})} - \frac{3^4\sqrt{ae}(\sqrt{a}+\sqrt{bx^2})\sqrt{\frac{a}{a+bx^4}}}{2b^{7/4}\sqrt{a+bx^4}} \\
&= -\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}} + \frac{f\sqrt{a+bx^4}}{b^2} + \frac{3ex\sqrt{a+bx^4}}{2b^{3/2}(\sqrt{a}+\sqrt{bx^2})} + \frac{d \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right)}{2b^{3/2}} - \frac{3^4\sqrt{ae}(\sqrt{a}+\sqrt{bx^2})\sqrt{\frac{a}{a+bx^4}}}{2b^{7/4}\sqrt{a+bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.145698, size = 166, normalized size = 0.53

$$\frac{bcx\sqrt{\frac{bx^4}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{bx^4}{a}\right) + \sqrt{a}\sqrt{bd}\sqrt{\frac{bx^4}{a}} + 1 \sinh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right) - 2bex^3\sqrt{\frac{bx^4}{a}} + {}_2F_1\left(\frac{3}{4}, \frac{3}{2}; \frac{7}{4}; -\frac{bx^4}{a}\right) + 2af - bcx - bdx^2}{2b^2\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x]

[Out] (2*a*f - b*c*x - b*d*x^2 + 2*b*e*x^3 + b*f*x^4 + Sqrt[a]*Sqrt[b]*d*Sqrt[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + b*c*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -(b*x^4)/a] - 2*b*e*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -(b*x^4)/a])/(2*b^2*Sqrt[a + b*x^4])

Maple [C] time = 0.008, size = 340, normalized size = 1.1

$$\frac{f(bx^4 + 2a)}{2b^2} \frac{1}{\sqrt{bx^4 + a}} - \frac{ex^3}{2b} \frac{1}{\sqrt{(x^4 + \frac{a}{b})b}} + \frac{3i}{2} e\sqrt{a} \sqrt{1 - ix^2\sqrt{b}} \frac{1}{\sqrt{a}} \sqrt{1 + ix^2\sqrt{b}} \frac{1}{\sqrt{a}} \text{EllipticF}\left(x\sqrt{i\sqrt{b}} \frac{1}{\sqrt{a}}, i\right) b^{-\frac{3}{2}} \frac{1}{\sqrt{i\sqrt{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2), x)`

[Out] $\frac{1}{2} * f * (b * x^4 + 2 * a) / (b * x^4 + a)^{(1/2)} / b^2 - 1/2 * e / b * x^3 / ((x^4 + 1/b * a) * b)^{(1/2)} + 3/2 * I * e / b^{(3/2)} * a^{(1/2)} / (I/a^{(1/2)} * b^{(1/2)})^{(1/2)} * (1 - I/a^{(1/2)} * b^{(1/2)} * x^2)^{(1/2)} * (1 + I/a^{(1/2)} * b^{(1/2)} * x^2)^{(1/2)} / (b * x^4 + a)^{(1/2)} * \text{EllipticF}(x * (I/a^{(1/2)} * b^{(1/2)})^{(1/2)}, I) - 3/2 * I * e / b^{(3/2)} * a^{(1/2)} / (I/a^{(1/2)} * b^{(1/2)})^{(1/2)} * (1 - I/a^{(1/2)} * b^{(1/2)} * x^2)^{(1/2)} * (1 + I/a^{(1/2)} * b^{(1/2)} * x^2)^{(1/2)} / (b * x^4 + a)^{(1/2)} * \text{EllipticE}(x * (I/a^{(1/2)} * b^{(1/2)})^{(1/2)}, I) - 1/2 * d * x^2 / b / (b * x^4 + a)^{(1/2)} + 1/2 * d / b^{(3/2)} * \ln(x^2 * b^{(1/2)} + (b * x^4 + a)^{(1/2)}) - 1/2 * c / b * x / ((x^4 + 1/b * a) * b)^{(1/2)} + 1/2 * c / b / (I/a^{(1/2)} * b^{(1/2)})^{(1/2)} * (1 - I/a^{(1/2)} * b^{(1/2)} * x^2)^{(1/2)} * (1 + I/a^{(1/2)} * b^{(1/2)} * x^2)^{(1/2)} / (b * x^4 + a)^{(1/2)} * \text{EllipticF}(x * (I/a^{(1/2)} * b^{(1/2)})^{(1/2)}, I)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx^3 + ex^2 + dx + c)x^4}{(bx^4 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2), x, algorithm="maxima")`

[Out] `integrate((f*x^3 + e*x^2 + d*x + c)*x^4/(b*x^4 + a)^(3/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(fx^7 + ex^6 + dx^5 + cx^4)\sqrt{bx^4 + a}}{b^2x^8 + 2abx^4 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")

[Out] integral((f*x^7 + e*x^6 + d*x^5 + c*x^4)*sqrt(b*x^4 + a)/(b^2*x^8 + 2*a*b*x^4 + a^2), x)

Sympy [A] time = 19.9864, size = 172, normalized size = 0.55

$$d \left(\frac{\operatorname{asinh}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}} - \frac{x^2}{2\sqrt{ab}\sqrt{1 + \frac{bx^4}{a}}} \right) + f \left(\left\{ \begin{array}{ll} \frac{a}{b^2\sqrt{a+bx^4}} + \frac{x^4}{2b\sqrt{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^8}{8a^{\frac{3}{2}}} & \text{otherwise} \end{array} \right. \right) + \frac{cx^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\frac{5}{4}, \frac{3}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{ex^7\Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\frac{7}{4}, \frac{3}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)

[Out] d*(asinh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - x**2/(2*sqrt(a)*b*sqrt(1 + b*x**4/a))) + f*Piecewise((a/(b**2*sqrt(a + b*x**4)) + x**4/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**8/(8*a**(3/2)), True)) + c*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4)) + e*x**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(11/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx^3 + ex^2 + dx + c)x^4}{(bx^4 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^4/(b*x^4 + a)^(3/2), x)

$$3.543 \quad \int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}} dx$$

Optimal. Leaf size=297

$$\frac{(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (3\sqrt{a}f + \sqrt{bd}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right) + \frac{e \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right)}{2b^{3/2}} + \frac{3fx\sqrt{a+bx^4}}{2b^{3/2}(\sqrt{a} + \sqrt{bx^2})} - \frac{3\sqrt{a}f(\sqrt{a} + \sqrt{bx^2})}{4\sqrt[4]{ab^{7/4}}\sqrt{a+bx^4}}}{4\sqrt[4]{ab^{7/4}}\sqrt{a+bx^4}} + \frac{e \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right)}{2b^{3/2}} + \frac{3fx\sqrt{a+bx^4}}{2b^{3/2}(\sqrt{a} + \sqrt{bx^2})} - \frac{3\sqrt{a}f(\sqrt{a} + \sqrt{bx^2})}{4\sqrt[4]{ab^{7/4}}\sqrt{a+bx^4}}$$

[Out] $-(c + d*x + e*x^2 + f*x^3)/(2*b*\operatorname{Sqrt}[a + b*x^4]) + (3*f*x*\operatorname{Sqrt}[a + b*x^4])/(2*b^{(3/2)}*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) + (e*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*x^2)/\operatorname{Sqrt}[a + b*x^4]])/(2*b^{(3/2)}) - (3*a^{(1/4)}*f*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(2*b^{(7/4)}*\operatorname{Sqrt}[a + b*x^4]) + ((\operatorname{Sqrt}[b]*d + 3*\operatorname{Sqrt}[a]*f)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(4*a^{(1/4)}*b^{(7/4)}*\operatorname{Sqrt}[a + b*x^4])$

Rubi [A] time = 0.199901, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1823, 1885, 275, 217, 206, 1198, 220, 1196}

$$\frac{(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (3\sqrt{a}f + \sqrt{bd}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right) + \frac{e \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right)}{2b^{3/2}} + \frac{3fx\sqrt{a+bx^4}}{2b^{3/2}(\sqrt{a} + \sqrt{bx^2})} - \frac{3\sqrt{a}f(\sqrt{a} + \sqrt{bx^2})}{4\sqrt[4]{ab^{7/4}}\sqrt{a+bx^4}}}{4\sqrt[4]{ab^{7/4}}\sqrt{a+bx^4}} + \frac{e \tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right)}{2b^{3/2}} + \frac{3fx\sqrt{a+bx^4}}{2b^{3/2}(\sqrt{a} + \sqrt{bx^2})} - \frac{3\sqrt{a}f(\sqrt{a} + \sqrt{bx^2})}{4\sqrt[4]{ab^{7/4}}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^{(3/2)}, x]$

[Out] $-(c + d*x + e*x^2 + f*x^3)/(2*b*\operatorname{Sqrt}[a + b*x^4]) + (3*f*x*\operatorname{Sqrt}[a + b*x^4])/(2*b^{(3/2)}*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) + (e*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*x^2)/\operatorname{Sqrt}[a + b*x^4]])/(2*b^{(3/2)}) - (3*a^{(1/4)}*f*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(2*b^{(7/4)}*\operatorname{Sqrt}[a + b*x^4]) + ((\operatorname{Sqrt}[b]*d + 3*\operatorname{Sqrt}[a]*f)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(4*a^{(1/4)}*b^{(7/4)}*\operatorname{Sqrt}[a + b*x^4])$

Rule 1823

Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(Pq*(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && EqQ[m - n + 1, 0] && LtQ[p, -1]

Rule 1885

Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]* (a + b*x^n)^p, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]

Rule 275

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1198

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rule 220

Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q =
  Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
  1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
  1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
  x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(c + dx + ex^2 + fx^3)}{(a + bx^4)^{3/2}} dx &= -\frac{c + dx + ex^2 + fx^3}{2b\sqrt{a + bx^4}} + \frac{\int \frac{d+2ex+3fx^2}{\sqrt{a+bx^4}} dx}{2b} \\
&= -\frac{c + dx + ex^2 + fx^3}{2b\sqrt{a + bx^4}} + \frac{\int \left(\frac{2ex}{\sqrt{a+bx^4}} + \frac{d+3fx^2}{\sqrt{a+bx^4}} \right) dx}{2b} \\
&= -\frac{c + dx + ex^2 + fx^3}{2b\sqrt{a + bx^4}} + \frac{\int \frac{d+3fx^2}{\sqrt{a+bx^4}} dx}{2b} + \frac{e \int \frac{x}{\sqrt{a+bx^4}} dx}{b} \\
&= -\frac{c + dx + ex^2 + fx^3}{2b\sqrt{a + bx^4}} + \frac{e \operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}} dx, x, x^2\right)}{2b} - \frac{(3\sqrt{a}f) \int \frac{1-\sqrt{bx^2}}{\sqrt{a+bx^4}} dx}{2b^{3/2}} + \frac{\left(d + \frac{3\sqrt{a}}{\sqrt{b}}\right)}{2b^{3/2}} \\
&= -\frac{c + dx + ex^2 + fx^3}{2b\sqrt{a + bx^4}} + \frac{3fx\sqrt{a + bx^4}}{2b^{3/2}(\sqrt{a} + \sqrt{bx^2})} - \frac{3\sqrt[4]{a}f(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a} + \sqrt{bx^2})^2}} E\left(2 \operatorname{atanh}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right)\right)}{2b^{7/4}\sqrt{a + bx^4}} \\
&= -\frac{c + dx + ex^2 + fx^3}{2b\sqrt{a + bx^4}} + \frac{3fx\sqrt{a + bx^4}}{2b^{3/2}(\sqrt{a} + \sqrt{bx^2})} + \frac{e \operatorname{tanh}^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a+bx^4}}\right)}{2b^{3/2}} - \frac{3\sqrt[4]{a}f(\sqrt{a} + \sqrt{bx^2})}{2b^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.128945, size = 181, normalized size = 0.61

$$\frac{\sqrt{b}dx\sqrt{\frac{bx^4}{a} + 1} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{bx^4}{a}\right) + \sqrt{ae}\sqrt{\frac{bx^4}{a} + 1} \sinh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right) - 2\sqrt{b}fx^3\sqrt{\frac{bx^4}{a} + 1} {}_2F_1\left(\frac{3}{4}, \frac{3}{2}; \frac{7}{4}; -\frac{bx^4}{a}\right) - \sqrt{bc} - \sqrt{b}dx - \sqrt{a}}{2b^{3/2}\sqrt{a + bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x]

[Out] $(-\sqrt{b}c - \sqrt{b}dx - \sqrt{a} \operatorname{ArcSinh}[\sqrt{b}x^2/\sqrt{a}] + \sqrt{b}d \operatorname{ArcSinh}[\sqrt{b}x^2/\sqrt{a}] + \sqrt{b}fx^3 \operatorname{ArcSinh}[\sqrt{b}x^2/\sqrt{a}] + \sqrt{b}e \operatorname{ArcSinh}[\sqrt{b}x^2/\sqrt{a}]) / (2b^{3/2}\sqrt{a + bx^4})$

$*x^4/a]*\text{Hypergeometric2F1}[1/4, 1/2, 5/4, -((b*x^4)/a)] - 2*\text{Sqrt}[b]*f*x^3*\text{Sqrt}[1 + (b*x^4)/a]*\text{Hypergeometric2F1}[3/4, 3/2, 7/4, -((b*x^4)/a)]/(2*b^(3/2)*\text{Sqrt}[a + b*x^4])$

Maple [C] time = 0.007, size = 331, normalized size = 1.1

$$-\frac{fx^3}{2b} \frac{1}{\sqrt{(x^4 + \frac{a}{b})b}} + \frac{3i}{2} f\sqrt{a} \sqrt{1 - ix^2\sqrt{b}\frac{1}{\sqrt{a}}} \sqrt{1 + ix^2\sqrt{b}\frac{1}{\sqrt{a}}} \text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}, i\right) b^{-\frac{3}{2}} \frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{bx^4 + a}} - \frac{3i}{2} f\sqrt{a} \sqrt{1 - ix^2\sqrt{b}\frac{1}{\sqrt{a}}} \sqrt{1 + ix^2\sqrt{b}\frac{1}{\sqrt{a}}} \text{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}, i\right) b^{-\frac{3}{2}} \frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2), x)

[Out] $-\frac{1}{2} \frac{f}{b} \frac{x^3}{((x^4+1/b*a)*b)^{1/2}} + \frac{3}{2} \frac{I*f}{b^{3/2}} \frac{a^{1/2}}{(I/a^{1/2}*b^{1/2})^{1/2}} \frac{1}{(1-I/a^{1/2}*b^{1/2}*x^2)^{1/2}} \frac{1}{(1+I/a^{1/2}*b^{1/2}*x^2)^{1/2}} \frac{1}{(b*x^4+a)^{1/2}} \text{EllipticF}(x*(I/a^{1/2}*b^{1/2})^{1/2}, I) - \frac{3}{2} \frac{I*f}{b^{3/2}} \frac{a^{1/2}}{(I/a^{1/2}*b^{1/2})^{1/2}} \frac{1}{(1-I/a^{1/2}*b^{1/2}*x^2)^{1/2}} \frac{1}{(1+I/a^{1/2}*b^{1/2}*x^2)^{1/2}} \frac{1}{(b*x^4+a)^{1/2}} \text{EllipticE}(x*(I/a^{1/2}*b^{1/2})^{1/2}, I) - \frac{1}{2} \frac{e*x^2}{b} \frac{1}{(b*x^4+a)^{1/2}} + \frac{1}{2} \frac{e}{b^{3/2}} \ln(x^2*b^{1/2} + (b*x^4+a)^{1/2}) - \frac{1}{2} \frac{d}{b*x} \frac{1}{((x^4+1/b*a)*b)^{1/2}} + \frac{1}{2} \frac{d}{b} \frac{1}{(I/a^{1/2}*b^{1/2})^{1/2}} \frac{1}{(1-I/a^{1/2}*b^{1/2}*x^2)^{1/2}} \frac{1}{(1+I/a^{1/2}*b^{1/2}*x^2)^{1/2}} \frac{1}{(b*x^4+a)^{1/2}} \text{EllipticF}(x*(I/a^{1/2}*b^{1/2})^{1/2}, I) - \frac{1}{2} \frac{c}{b} \frac{1}{(b*x^4+a)^{1/2}}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c}{2\sqrt{bx^4 + ab}} + \int \frac{fx^6 + ex^5 + dx^4}{(bx^4 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2), x, algorithm="maxima")

[Out] $-\frac{1}{2} \frac{c}{\sqrt{bx^4 + a}*b} + \int (fx^6 + ex^5 + dx^4)/(bx^4 + a)^{3/2}, x$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(fx^6 + ex^5 + dx^4 + cx^3)\sqrt{bx^4 + a}}{b^2x^8 + 2abx^4 + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")

[Out] integral((f*x^6 + e*x^5 + d*x^4 + c*x^3)*sqrt(b*x^4 + a)/(b^2*x^8 + 2*a*b*x^4 + a^2), x)

Sympy [A] time = 17.2315, size = 156, normalized size = 0.53

$$c \left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases} \right) + e \left(\frac{\operatorname{asinh}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}} - \frac{x^2}{2\sqrt{ab}\sqrt{1+\frac{bx^4}{a}}} \right) + \frac{dx^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\frac{5}{4}, \frac{3}{2} \middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{fx^7\Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\frac{7}{4}, \frac{7}{4} \middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)

[Out] c*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), True)) + e*(asinh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - x**2/(2*sqrt(a)*b*sqrt(1 + b*x**4/a))) + d*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4)) + f*x**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(11/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx^3 + ex^2 + dx + c)x^3}{(bx^4 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^4 + a)^(3/2), x)
```

$$3.544 \quad \int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}} dx$$

Optimal. Leaf size=333

$$\frac{(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (\sqrt{bc} - \sqrt{ae}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{4a^{3/4}b^{5/4}\sqrt{a+bx^4}} + \frac{c(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}$$

[Out] $-(x*(a*e + a*f*x - b*c*x^2 - b*d*x^3))/(2*a*b*\operatorname{Sqrt}[a + b*x^4]) - (d*\operatorname{Sqrt}[a + b*x^4])/(2*a*b) - (c*x*\operatorname{Sqrt}[a + b*x^4])/(2*a*\operatorname{Sqrt}[b]*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) + (f*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*x^2)/\operatorname{Sqrt}[a + b*x^4]])/(2*b^{(3/2)}) + (c*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(2*a^{(3/4)}*b^{(3/4)}*\operatorname{Sqrt}[a + b*x^4]) - ((\operatorname{Sqrt}[b]*c - \operatorname{Sqrt}[a]*e)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(4*a^{(3/4)}*b^{(5/4)}*\operatorname{Sqrt}[a + b*x^4])$

Rubi [A] time = 0.250941, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {1828, 1885, 1198, 220, 1196, 1248, 641, 217, 206}

$$\frac{(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (\sqrt{bc} - \sqrt{ae}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{4a^{3/4}b^{5/4}\sqrt{a+bx^4}} + \frac{c(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}} + \dots$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x^2*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^{(3/2)}, x]$

[Out] $-(x*(a*e + a*f*x - b*c*x^2 - b*d*x^3))/(2*a*b*\operatorname{Sqrt}[a + b*x^4]) - (d*\operatorname{Sqrt}[a + b*x^4])/(2*a*b) - (c*x*\operatorname{Sqrt}[a + b*x^4])/(2*a*\operatorname{Sqrt}[b]*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) + (f*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*x^2)/\operatorname{Sqrt}[a + b*x^4]])/(2*b^{(3/2)}) + (c*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(2*a^{(3/4)}*b^{(3/4)}*\operatorname{Sqrt}[a + b*x^4]) - ((\operatorname{Sqrt}[b]*c - \operatorname{Sqrt}[a]*e)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/(4*a^{(3/4)}*b^{(5/4)}*\operatorname{Sqrt}[a + b*x^4])$

Rule 1828

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q =
m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
+ b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq,
x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 1198

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x]] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x],
1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
1/2])/(q*Sqrt[a + c*x^4]), x]] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol]
:= Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x]] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(c + dx + ex^2 + fx^3)}{(a + bx^4)^{3/2}} dx &= \frac{x(ae + afx - bcx^2 - bdx^3)}{2ab\sqrt{a + bx^4}} - \frac{\int \frac{-abe - 2abfx + b^2cx^2 + 2b^2dx^3}{\sqrt{a + bx^4}} dx}{2ab^2} \\
&= \frac{x(ae + afx - bcx^2 - bdx^3)}{2ab\sqrt{a + bx^4}} - \frac{\int \left(\frac{-abe + b^2cx^2}{\sqrt{a + bx^4}} + \frac{x(-2abf + 2b^2dx^2)}{\sqrt{a + bx^4}} \right) dx}{2ab^2} \\
&= \frac{x(ae + afx - bcx^2 - bdx^3)}{2ab\sqrt{a + bx^4}} - \frac{\int \frac{-abe + b^2cx^2}{\sqrt{a + bx^4}} dx}{2ab^2} - \frac{\int \frac{x(-2abf + 2b^2dx^2)}{\sqrt{a + bx^4}} dx}{2ab^2} \\
&= \frac{x(ae + afx - bcx^2 - bdx^3)}{2ab\sqrt{a + bx^4}} - \frac{\text{Subst} \left(\int \frac{-2abf + 2b^2dx}{\sqrt{a + bx^2}} dx, x, x^2 \right)}{4ab^2} + \frac{c \int \frac{1 - \sqrt{bx^2}}{\sqrt{a + bx^4}} dx}{2\sqrt{a}\sqrt{b}} - \frac{\left(\frac{\sqrt{bc}}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{b}} \\
&= \frac{x(ae + afx - bcx^2 - bdx^3)}{2ab\sqrt{a + bx^4}} - \frac{d\sqrt{a + bx^4}}{2ab} - \frac{cx\sqrt{a + bx^4}}{2a\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{c(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{1}{\sqrt{a}}}}{2a^{3/4}} \\
&= \frac{x(ae + afx - bcx^2 - bdx^3)}{2ab\sqrt{a + bx^4}} - \frac{d\sqrt{a + bx^4}}{2ab} - \frac{cx\sqrt{a + bx^4}}{2a\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{c(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{1}{\sqrt{a}}}}{2a^{3/4}} \\
&= \frac{x(ae + afx - bcx^2 - bdx^3)}{2ab\sqrt{a + bx^4}} - \frac{d\sqrt{a + bx^4}}{2ab} - \frac{cx\sqrt{a + bx^4}}{2a\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{f \tanh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a + bx^4}} \right)}{2b^{3/2}} + \dots
\end{aligned}$$

Mathematica [C] time = 0.196435, size = 165, normalized size = 0.5

$$\frac{3a^{3/2}f\sqrt{\frac{bx^4}{a} + 1} \sinh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right) + 2b^{3/2}cx^3\sqrt{\frac{bx^4}{a} + 1} {}_2F_1 \left(\frac{3}{4}, \frac{3}{2}; \frac{7}{4}; -\frac{bx^4}{a} \right) - 3a\sqrt{b}(d + x(e + fx)) + 3a\sqrt{b}ex\sqrt{\frac{bx^4}{a} + 1} {}_2F_1 \left(\frac{1}{4}, \frac{1}{2}; -\frac{bx^4}{a} \right)}{6ab^{3/2}\sqrt{a + bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x]

[Out] (-3*a*Sqrt[b]*(d + x*(e + f*x)) + 3*a^(3/2)*f*Sqrt[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + 3*a*Sqrt[b]*e*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] + 2*b^(3/2)*c*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(6*a*b^(3/2)*Sqrt[a + b*x^4])

Maple [C] time = 0.009, size = 331, normalized size = 1.

$$-\frac{fx^2}{2b} \frac{1}{\sqrt{bx^4+a}} + \frac{f}{2} \ln\left(x^2\sqrt{b} + \sqrt{bx^4+a}\right) b^{-\frac{3}{2}} - \frac{ex}{2b} \frac{1}{\sqrt{\left(x^4 + \frac{a}{b}\right)b}} + \frac{e}{2b} \sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}} \sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}} \text{EllipticF}\left(x\sqrt{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x)`

[Out]
$$-1/2*f*x^2/b/(b*x^4+a)^{(1/2)}+1/2*f/b^{(3/2)}*\ln(x^2*b^{(1/2)}+(b*x^4+a)^{(1/2)})-1/2*e/b*x/((x^4+1/b*a)*b)^{(1/2)}+1/2*e/b/(I/a^{(1/2)*b^{(1/2)}})^{(1/2)}*(1-I/a^{(1/2)*b^{(1/2)*x^2}})^{(1/2)}*(1+I/a^{(1/2)*b^{(1/2)*x^2}})^{(1/2)}/(b*x^4+a)^{(1/2)}*\text{EllipticF}(x*(I/a^{(1/2)*b^{(1/2)}})^{(1/2)},I)-1/2*d/b/(b*x^4+a)^{(1/2)}+1/2*c*x^3/a/((x^4+1/b*a)*b)^{(1/2)}-1/2*I*c/a^{(1/2)}/(I/a^{(1/2)*b^{(1/2)}})^{(1/2)}*(1-I/a^{(1/2)*b^{(1/2)*x^2}})^{(1/2)}*(1+I/a^{(1/2)*b^{(1/2)*x^2}})^{(1/2)}/(b*x^4+a)^{(1/2)}/b^{(1/2)}*\text{EllipticF}(x*(I/a^{(1/2)*b^{(1/2)}})^{(1/2)},I)+1/2*I*c/a^{(1/2)}/(I/a^{(1/2)*b^{(1/2)}})^{(1/2)}*(1-I/a^{(1/2)*b^{(1/2)*x^2}})^{(1/2)}*(1+I/a^{(1/2)*b^{(1/2)*x^2}})^{(1/2)}/(b*x^4+a)^{(1/2)}/b^{(1/2)}*\text{EllipticE}(x*(I/a^{(1/2)*b^{(1/2)}})^{(1/2)},I)$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx^3 + ex^2 + dx + c)x^2}{(bx^4 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")`

[Out] `integrate((f*x^3 + e*x^2 + d*x + c)*x^2/(b*x^4 + a)^(3/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(fx^5 + ex^4 + dx^3 + cx^2)\sqrt{bx^4 + a}}{b^2x^8 + 2abx^4 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")

[Out] integral((f*x^5 + e*x^4 + d*x^3 + c*x^2)*sqrt(b*x^4 + a)/(b^2*x^8 + 2*a*b*x^4 + a^2), x)

Sympy [A] time = 15.565, size = 156, normalized size = 0.47

$$d \left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases} \right) + f \left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}} - \frac{x^2}{2\sqrt{ab}\sqrt{1+\frac{bx^4}{a}}} \right) + \frac{cx^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{3}{4}, \frac{3}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{ex^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\frac{5}{4}, \frac{3}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)

[Out] d*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), True)) + f*(asinh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - x**2/(2*sqrt(a)*b*sqrt(1 + b*x**4/a))) + c*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(7/4)) + e*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx^3 + ex^2 + dx + c)x^2}{(bx^4 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^2/(b*x^4 + a)^(3/2), x)

$$3.545 \quad \int \frac{x(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}} dx$$

Optimal. Leaf size=303

$$\frac{(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (\sqrt{bd} - \sqrt{af}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{4a^{3/4}b^{5/4}\sqrt{a+bx^4}} + \frac{d(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}$$

[Out] $-(x*(a*f - b*c*x - b*d*x^2 - b*e*x^3))/(2*a*b*\operatorname{Sqrt}[a + b*x^4]) - (e*\operatorname{Sqrt}[a + b*x^4])/(2*a*b) - (d*x*\operatorname{Sqrt}[a + b*x^4])/(2*a*\operatorname{Sqrt}[b]*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) + (d*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2])/(2*a^{3/4}*b^{3/4}* \operatorname{Sqrt}[a + b*x^4]) - ((\operatorname{Sqrt}[b]*d - \operatorname{Sqrt}[a]*f)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2])/(4*a^{3/4}*b^{5/4}* \operatorname{Sqrt}[a + b*x^4])$

Rubi [A] time = 0.192557, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1828, 1885, 261, 1198, 220, 1196}

$$\frac{(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (\sqrt{bd} - \sqrt{af}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{4a^{3/4}b^{5/4}\sqrt{a+bx^4}} + \frac{d(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}} - x$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^{3/2}, x]$

[Out] $-(x*(a*f - b*c*x - b*d*x^2 - b*e*x^3))/(2*a*b*\operatorname{Sqrt}[a + b*x^4]) - (e*\operatorname{Sqrt}[a + b*x^4])/(2*a*b) - (d*x*\operatorname{Sqrt}[a + b*x^4])/(2*a*\operatorname{Sqrt}[b]*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) + (d*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2])/(2*a^{3/4}*b^{3/4}* \operatorname{Sqrt}[a + b*x^4]) - ((\operatorname{Sqrt}[b]*d - \operatorname{Sqrt}[a]*f)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2])/(4*a^{3/4}*b^{5/4}* \operatorname{Sqrt}[a + b*x^4])$

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p_, x_Symbol] := With[{q =
m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
+ b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^p_, x_Symbol] := Module[{q = Expon[Pq,
x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p_, x_Symbol] := Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
NeQ[p, -1]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x],
1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(c + dx + ex^2 + fx^3)}{(a + bx^4)^{3/2}} dx &= -\frac{x(af - bcx - bdx^2 - bex^3)}{2ab\sqrt{a + bx^4}} - \frac{\int \frac{-af + bdx^2 + 2bex^3}{\sqrt{a + bx^4}} dx}{2ab} \\
&= -\frac{x(af - bcx - bdx^2 - bex^3)}{2ab\sqrt{a + bx^4}} - \frac{\int \left(\frac{2bex^3}{\sqrt{a + bx^4}} + \frac{-af + bdx^2}{\sqrt{a + bx^4}} \right) dx}{2ab} \\
&= -\frac{x(af - bcx - bdx^2 - bex^3)}{2ab\sqrt{a + bx^4}} - \frac{\int \frac{-af + bdx^2}{\sqrt{a + bx^4}} dx}{2ab} - \frac{e \int \frac{x^3}{\sqrt{a + bx^4}} dx}{a} \\
&= -\frac{x(af - bcx - bdx^2 - bex^3)}{2ab\sqrt{a + bx^4}} - \frac{e\sqrt{a + bx^4}}{2ab} + \frac{d \int \frac{1 - \frac{\sqrt{bx^2}}{\sqrt{a}}}{\sqrt{a + bx^4}} dx}{2\sqrt{a}\sqrt{b}} - \frac{\left(\frac{\sqrt{bd}}{\sqrt{a}} - f\right) \int \frac{1}{\sqrt{a + bx^4}} dx}{2b} \\
&= -\frac{x(af - bcx - bdx^2 - bex^3)}{2ab\sqrt{a + bx^4}} - \frac{e\sqrt{a + bx^4}}{2ab} - \frac{dx\sqrt{a + bx^4}}{2a\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} + \frac{d(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{1}{a + bx^4}}}{2a^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.0769455, size = 116, normalized size = 0.38

$$\frac{2bdx^3 \sqrt{\frac{bx^4}{a}} + {}_2F_1\left(\frac{3}{4}, \frac{3}{2}; \frac{7}{4}; -\frac{bx^4}{a}\right) + 3afx \sqrt{\frac{bx^4}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{bx^4}{a}\right) - 3ae - 3afx + 3bcx^2}{6ab\sqrt{a + bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]

[Out] (-3*a*e - 3*a*f*x + 3*b*c*x^2 + 3*a*f*x*sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -(b*x^4)/a] + 2*b*d*x^3*sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -(b*x^4)/a])/(6*a*b*sqrt[a + b*x^4])

Maple [C] time = 0.011, size = 250, normalized size = 0.8

$$f \left(-\frac{x}{2b} \frac{1}{\sqrt{\left(x^4 + \frac{a}{b}\right)b}} + \frac{1}{2b} \sqrt{1 - ix^2\sqrt{b}\frac{1}{\sqrt{a}}} \sqrt{1 + ix^2\sqrt{b}\frac{1}{\sqrt{a}}} \text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}, i\right) \frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{bx^4 + a}} \right) - \frac{e}{2b} \frac{1}{\sqrt{bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x)`

[Out] $f*(-1/2/b*x/((x^4+1/b*a)*b)^{(1/2)}+1/2/b/(I/a^{(1/2)*b^{(1/2)}})^{(1/2)}*(1-I/a^{(1/2)*b^{(1/2)*x^2}})^{(1/2)}*(1+I/a^{(1/2)*b^{(1/2)*x^2}})^{(1/2)}/(b*x^4+a)^{(1/2)}*EllipticF(x*(I/a^{(1/2)*b^{(1/2)}})^{(1/2)},I))-1/2*e/b/(b*x^4+a)^{(1/2)}+d*(1/2*x^3/a/((x^4+1/b*a)*b)^{(1/2)}-1/2*I/a^{(1/2)}/(I/a^{(1/2)*b^{(1/2)}})^{(1/2)}*(1-I/a^{(1/2)*b^{(1/2)*x^2}})^{(1/2)}*(1+I/a^{(1/2)*b^{(1/2)*x^2}})^{(1/2)}/(b*x^4+a)^{(1/2)}/b^{(1/2)}*(EllipticF(x*(I/a^{(1/2)*b^{(1/2)}})^{(1/2)},I)-EllipticE(x*(I/a^{(1/2)*b^{(1/2)}})^{(1/2)},I)))+1/2*c/(b*x^4+a)^{(1/2)}/a*x^2$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{cx^2}{2\sqrt{bx^4+aa}} + \int \frac{fx^4+ex^3+dx^2}{(bx^4+a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")`

[Out] `1/2*c*x^2/(sqrt(b*x^4+a)*a) + integrate((f*x^4+e*x^3+d*x^2)/(b*x^4+a)^(3/2),x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4+a}(fx^4+ex^3+dx^2+cx)}{b^2x^8+2abx^4+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*x^4+a)*(f*x^4+e*x^3+d*x^2+c*x)/(b^2*x^8+2*a*b*x^4+a^2),x)`

Sympy [A] time = 14.6385, size = 133, normalized size = 0.44

$$e^{\left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases} \right) + \frac{cx^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}} + \frac{dx^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{3}{4}, \frac{3}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{fx^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\frac{5}{4}, \frac{3}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)

[Out] e*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), True)) + c*x**2/(2*a**(3/2)*sqrt(1 + b*x**4/a)) + d*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(7/4)) + f*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx^3 + ex^2 + dx + c)x}{(bx^4 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x/(b*x^4 + a)^(3/2), x)

$$3.546 \quad \int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}} dx$$

Optimal. Leaf size=275

$$\frac{(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (\sqrt{bc} - \sqrt{ae}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{4a^{5/4}b^{3/4}\sqrt{a+bx^4}} + \frac{e(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}$$

[Out] $-(e*x*\operatorname{Sqrt}[a + b*x^4])/(2*a*\operatorname{Sqrt}[b]*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) - (a*f - b*x*(c + d*x + e*x^2))/(2*a*b*\operatorname{Sqrt}[a + b*x^4]) + (e*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2])/(2*a^{3/4}*b^{3/4}*\operatorname{Sqrt}[a + b*x^4]) + ((\operatorname{Sqrt}[b]*c - \operatorname{Sqrt}[a]*e)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2])/(4*a^{5/4}*b^{3/4}*\operatorname{Sqrt}[a + b*x^4])$

Rubi [A] time = 0.117198, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1854, 1198, 220, 1196}

$$\frac{(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (\sqrt{bc} - \sqrt{ae}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{4a^{5/4}b^{3/4}\sqrt{a+bx^4}} + \frac{e(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}} - \frac{af - b}{2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^{(3/2)}, x]$

[Out] $-(e*x*\operatorname{Sqrt}[a + b*x^4])/(2*a*\operatorname{Sqrt}[b]*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) - (a*f - b*x*(c + d*x + e*x^2))/(2*a*b*\operatorname{Sqrt}[a + b*x^4]) + (e*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2])/(2*a^{3/4}*b^{3/4}*\operatorname{Sqrt}[a + b*x^4]) + ((\operatorname{Sqrt}[b]*c - \operatorname{Sqrt}[a]*e)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2])/(4*a^{5/4}*b^{3/4}*\operatorname{Sqrt}[a + b*x^4])$

Rule 1854

$\operatorname{Int}[(Pq_*)*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \operatorname{Module}\{q = \operatorname{Expon}[Pq, x], i\}, \operatorname{Simp}[(a*\operatorname{Coeff}[Pq, x, q] - b*x*\operatorname{ExpandToSum}[Pq - \operatorname{Coeff}[Pq, x, q]*x^q, x])*(a + b*x^n)^{(p+1)}/(a*b*n*(p+1)), x] + \operatorname{Dist}[1/(a*n*(p+1)), \operatorname{Int}$

```
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p + 1), x], x] /; q == n - 1 /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2]]/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned} \int \frac{c + dx + ex^2 + fx^3}{(a + bx^4)^{3/2}} dx &= -\frac{af - bx(c + dx + ex^2)}{2ab\sqrt{a + bx^4}} - \frac{\int \frac{-c+ex^2}{\sqrt{a+bx^4}} dx}{2a} \\ &= -\frac{af - bx(c + dx + ex^2)}{2ab\sqrt{a + bx^4}} + \frac{e \int \frac{1-\frac{\sqrt{bx^2}}{\sqrt{a}}}{\sqrt{a+bx^4}} dx}{2\sqrt{a}\sqrt{b}} + \frac{\left(c - \frac{\sqrt{ae}}{\sqrt{b}}\right) \int \frac{1}{\sqrt{a+bx^4}} dx}{2a} \\ &= -\frac{ex\sqrt{a + bx^4}}{2a\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} - \frac{af - bx(c + dx + ex^2)}{2ab\sqrt{a + bx^4}} + \frac{e(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a} + \sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)\right)}{2a^{3/4}b^{3/4}\sqrt{a + bx^4}} \end{aligned}$$

Mathematica [C] time = 0.0540039, size = 116, normalized size = 0.42

$$\frac{3bcx\sqrt{\frac{bx^4}{a}} + 1 {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{bx^4}{a}\right) + 2bex^3\sqrt{\frac{bx^4}{a}} + 1 {}_2F_1\left(\frac{3}{4}, \frac{3}{2}; \frac{7}{4}; -\frac{bx^4}{a}\right) - 3af + 3bcx + 3bdx^2}{6ab\sqrt{a + bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^(3/2), x]

[Out] (-3*a*f + 3*b*c*x + 3*b*d*x^2 + 3*b*c*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] + 2*b*e*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(6*a*b*Sqrt[a + b*x^4])

Maple [C] time = 0.006, size = 250, normalized size = 0.9

$$-\frac{f}{2b} \frac{1}{\sqrt{bx^4 + a}} + e \left(\frac{x^3}{2a} \frac{1}{\sqrt{(x^4 + \frac{a}{b})b}} - \frac{i}{2} \sqrt{1 - ix^2\sqrt{b}} \frac{1}{\sqrt{a}} \sqrt{1 + ix^2\sqrt{b}} \frac{1}{\sqrt{a}} \left(\text{EllipticF}\left(x\sqrt{i\sqrt{b}} \frac{1}{\sqrt{a}}, i\right) - \text{EllipticE}\left(x\sqrt{i\sqrt{b}} \frac{1}{\sqrt{a}}, i\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2), x)

[Out] -1/2*f/b/(b*x^4+a)^(1/2)+e*(1/2*x^3/a/((x^4+1/b*a)*b)^(1/2)-1/2*I/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2), I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2), I))+1/2*d/(b*x^4+a)^(1/2)/a*x^2+c*(1/2*x/a/((x^4+1/b*a)*b)^(1/2)+1/2/a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2), I))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{(bx^4 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{b^2x^8+2abx^4+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/(b^2*x^8 + 2*a*b*x^4 + a^2), x)

Sympy [A] time = 13.9834, size = 131, normalized size = 0.48

$$f\left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{cx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{3}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)} + \frac{dx^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}} + \frac{ex^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{3}{4}, \frac{3}{2} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)

[Out] f*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), True)) + c*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(5/4)) + d*x**2/(2*a**(3/2)*sqrt(1 + b*x**4/a)) + e*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(7/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{(bx^4 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^(3/2), x)
```

$$3.547 \quad \int \frac{c+dx+ex^2+fx^3}{x(a+bx^4)^{3/2}} dx$$

Optimal. Leaf size=323

$$\frac{(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (\sqrt{bd} - \sqrt{a}f) \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{4a^{5/4}b^{3/4}\sqrt{a+bx^4}} + \frac{f(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}$$

[Out] (x*(a*d + a*e*x + a*f*x^2 - b*c*x^3))/(2*a^2*Sqrt[a + b*x^4]) + (c*Sqrt[a + b*x^4])/(2*a^2) - (f*x*Sqrt[a + b*x^4])/(2*a*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) - (c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*a^(3/2)) + (f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*d - Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*b^(3/4)*Sqrt[a + b*x^4])

Rubi [A] time = 0.291992, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1829, 1832, 266, 63, 208, 1885, 261, 1198, 220, 1196}

$$\frac{(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (\sqrt{bd} - \sqrt{a}f) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{4a^{5/4}b^{3/4}\sqrt{a+bx^4}} + \frac{f(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}} + x$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3)/(x*(a + b*x^4)^(3/2)), x]

[Out] (x*(a*d + a*e*x + a*f*x^2 - b*c*x^3))/(2*a^2*Sqrt[a + b*x^4]) + (c*Sqrt[a + b*x^4])/(2*a^2) - (f*x*Sqrt[a + b*x^4])/(2*a*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) - (c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*a^(3/2)) + (f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*d - Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*b^(3/4)*Sqrt[a + b*x^4])

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_.)]), x_Symbol] := Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Module[{q = Expon[Pq,
x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^(((k*n)/2), {k, 0, (
2*(q - j))/n + 1})*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 261

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rule 1198

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3}{x(a + bx^4)^{3/2}} dx &= \frac{x(ad + aex + afx^2 - bcx^3)}{2a^2\sqrt{a + bx^4}} - \frac{\int \frac{-2bc - bdx + bfx^3 - \frac{2b^2cx^4}{a}}{x\sqrt{a+bx^4}} dx}{2ab} \\
&= \frac{x(ad + aex + afx^2 - bcx^3)}{2a^2\sqrt{a + bx^4}} - \frac{\int \frac{-bd + bfx^2 - \frac{2b^2cx^3}{a}}{\sqrt{a+bx^4}} dx}{2ab} + \frac{c \int \frac{1}{x\sqrt{a+bx^4}} dx}{a} \\
&= \frac{x(ad + aex + afx^2 - bcx^3)}{2a^2\sqrt{a + bx^4}} - \frac{\int \left(-\frac{2b^2cx^3}{a\sqrt{a+bx^4}} + \frac{-bd + bfx^2}{\sqrt{a+bx^4}} \right) dx}{2ab} + \frac{c \operatorname{Subst} \left(\int \frac{1}{x\sqrt{a+bx}} dx, x, x^4 \right)}{4a} \\
&= \frac{x(ad + aex + afx^2 - bcx^3)}{2a^2\sqrt{a + bx^4}} - \frac{\int \frac{-bd + bfx^2}{\sqrt{a+bx^4}} dx}{2ab} + \frac{c \operatorname{Subst} \left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + bx^4} \right)}{2ab} + \frac{(bc) \int \frac{1}{\sqrt{a+bx^4}} dx}{a} \\
&= \frac{x(ad + aex + afx^2 - bcx^3)}{2a^2\sqrt{a + bx^4}} + \frac{c\sqrt{a + bx^4}}{2a^2} - \frac{c \tanh^{-1} \left(\frac{\sqrt{a+bx^4}}{\sqrt{a}} \right)}{2a^{3/2}} + \frac{f \int \frac{1 - \frac{\sqrt{bx^2}}{\sqrt{a}}}{\sqrt{a+bx^4}} dx}{2\sqrt{a}\sqrt{b}} + \frac{\left(d - \frac{\sqrt{a}f}{\sqrt{b}} \right)}{2} \\
&= \frac{x(ad + aex + afx^2 - bcx^3)}{2a^2\sqrt{a + bx^4}} + \frac{c\sqrt{a + bx^4}}{2a^2} - \frac{fx\sqrt{a + bx^4}}{2a\sqrt{b}(\sqrt{a} + \sqrt{bx^2})} - \frac{c \tanh^{-1} \left(\frac{\sqrt{a+bx^4}}{\sqrt{a}} \right)}{2a^{3/2}} + \frac{f(\sqrt{a} - \sqrt{bx^2})}{2\sqrt{b}}
\end{aligned}$$

Mathematica [C] time = 0.126972, size = 125, normalized size = 0.39

$$\frac{3c {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{bx^4}{a} + 1\right) + x \left(3d\sqrt{\frac{bx^4}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{bx^4}{a}\right) + 2fx^2\sqrt{\frac{bx^4}{a}} + {}_2F_1\left(\frac{3}{4}, \frac{3}{2}; \frac{7}{4}; -\frac{bx^4}{a}\right) + 3d + 3ex \right)}{6a\sqrt{a + bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x*(a + b*x^4)^(3/2)),x]

[Out] (3*c*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^4)/a] + x*(3*d + 3*e*x + 3*d*
Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] + 2*f*x^2*
Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)]))/(6*a*
Sqrt[a + b*x^4])

Maple [C] time = 0.017, size = 336, normalized size = 1.

$$\frac{fx^3}{2a} \frac{1}{\sqrt{\left(x^4 + \frac{a}{b}\right)b}} - \frac{i}{2} f \sqrt{1 - ix^2 \sqrt{b}} \frac{1}{\sqrt{a}} \sqrt{1 + ix^2 \sqrt{b}} \frac{1}{\sqrt{a}} \text{EllipticF}\left(x \sqrt{i \sqrt{b}} \frac{1}{\sqrt{a}}, i\right) \frac{1}{\sqrt{a}} \frac{1}{\sqrt{i \sqrt{b}} \frac{1}{\sqrt{a}}} \frac{1}{\sqrt{bx^4 + a}} \frac{1}{\sqrt{b}} + \frac{i}{2} f \sqrt{1 - i}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(3/2),x)

[Out] $\frac{1}{2} f x^3 / a / ((x^4 + 1/b * a) * b)^{(1/2)} - \frac{1}{2} * I * f / a^{(1/2)} / (I/a^{(1/2)} * b^{(1/2)})^{(1/2)}$
 $* (1 - I/a^{(1/2)} * b^{(1/2)} * x^2)^{(1/2)} * (1 + I/a^{(1/2)} * b^{(1/2)} * x^2)^{(1/2)} / (b * x^4 + a)^{(1/2)}$
 $/ b^{(1/2)} * \text{EllipticF}(x * (I/a^{(1/2)} * b^{(1/2)})^{(1/2)}, I) + \frac{1}{2} * I * f / a^{(1/2)} / (I/a^{(1/2)}$
 $* b^{(1/2)})^{(1/2)} * (1 - I/a^{(1/2)} * b^{(1/2)} * x^2)^{(1/2)} * (1 + I/a^{(1/2)} * b^{(1/2)} * x^2)^{(1/2)}$
 $/ (b * x^4 + a)^{(1/2)} / b^{(1/2)} * \text{EllipticE}(x * (I/a^{(1/2)} * b^{(1/2)})^{(1/2)}, I)$
 $+ \frac{1}{2} * e / (b * x^4 + a)^{(1/2)} / a * x^2 + \frac{1}{2} * d * x / a / ((x^4 + 1/b * a) * b)^{(1/2)} + \frac{1}{2} * d / a / (I/a^{(1/2)}$
 $* b^{(1/2)})^{(1/2)} * (1 - I/a^{(1/2)} * b^{(1/2)} * x^2)^{(1/2)} * (1 + I/a^{(1/2)} * b^{(1/2)} * x^2)^{(1/2)}$
 $/ (b * x^4 + a)^{(1/2)} * \text{EllipticF}(x * (I/a^{(1/2)} * b^{(1/2)})^{(1/2)}, I) + \frac{1}{2} * c / a / (b * x^4 + a)^{(1/2)}$
 $- \frac{1}{2} * c / a^{(3/2)} * \ln((2 * a + 2 * a^{(1/2)} * (b * x^4 + a)^{(1/2)}) / x^2)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{(bx^4 + a)^{\frac{3}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(3/2),x, algorithm="maxima")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)}{b^2x^9 + 2abx^5 + a^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/(b^2*x^9 + 2*a*b*x^5 + a^2*x), x)

Sympy [C] time = 24.4472, size = 289, normalized size = 0.89

$$c \left(\frac{2a^3 \sqrt{1 + \frac{bx^4}{a}}}{4a^{\frac{9}{2}} + 4a^{\frac{7}{2}}bx^4} + \frac{a^3 \log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}} + 4a^{\frac{7}{2}}bx^4} - \frac{2a^3 \log\left(\sqrt{1 + \frac{bx^4}{a}} + 1\right)}{4a^{\frac{9}{2}} + 4a^{\frac{7}{2}}bx^4} + \frac{a^2 bx^4 \log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}} + 4a^{\frac{7}{2}}bx^4} - \frac{2a^2 bx^4 \log\left(\sqrt{1 + \frac{bx^4}{a}} + 1\right)}{4a^{\frac{9}{2}} + 4a^{\frac{7}{2}}bx^4} \right) + \frac{dx \Gamma\left(\frac{1}{4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)/x/(b*x**4+a)**(3/2),x)

[Out] c*(2*a**3*sqrt(1 + b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**3*log(b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**3*log(sqrt(1 + b*x**4/a) + 1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**2*b*x**4*log(b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**2*b*x**4*log(sqrt(1 + b*x**4/a) + 1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + d*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(5/4)) + e*x**2/(2*a**(3/2)*sqrt(1 + b*x**4/a)) + f*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(7/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{(bx^4 + a)^{\frac{3}{2}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(3/2),x, algorithm="giac")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x), x)

$$3.548 \quad \int \frac{c+dx+ex^2+fx^3}{x^2(a+bx^4)^{3/2}} dx$$

Optimal. Leaf size=344

$$\frac{(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (\sqrt{ae} + 3\sqrt{bc}) \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt{a}}\right), \frac{1}{2}\right)}{4a^{7/4} \sqrt[4]{b} \sqrt{a+bx^4}} + \frac{x(ae + afx - bcx^2 - bdx^3)}{2a^2 \sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{a^2 x} + \frac{3\sqrt{bcx}\sqrt{a}}{2a^2}$$

[Out] (x*(a*e + a*f*x - b*c*x^2 - b*d*x^3))/(2*a^2*Sqrt[a + b*x^4]) + (d*Sqrt[a + b*x^4])/(2*a^2) - (c*Sqrt[a + b*x^4])/(a^2*x) + (3*Sqrt[b]*c*x*Sqrt[a + b*x^4])/(2*a^2*(Sqrt[a] + Sqrt[b]*x^2)) - (d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*a^(3/2)) - (3*b^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(7/4)*Sqrt[a + b*x^4]) + ((3*Sqrt[b]*c + Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(7/4)*b^(1/4)*Sqrt[a + b*x^4])

Rubi [A] time = 0.382575, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {1829, 1833, 1835, 1584, 1198, 220, 1196, 21, 266, 50, 63, 208}

$$\frac{x(ae + afx - bcx^2 - bdx^3)}{2a^2 \sqrt{a+bx^4}} + \frac{(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (\sqrt{ae} + 3\sqrt{bc}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt{a}}\right) \middle| \frac{1}{2}\right)}{4a^{7/4} \sqrt[4]{b} \sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{a^2 x} + \frac{3\sqrt{bcx}\sqrt{a}}{2a^2 (\sqrt{a} + \sqrt{bx^2})}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3)/(x^2*(a + b*x^4)^(3/2)), x]

[Out] (x*(a*e + a*f*x - b*c*x^2 - b*d*x^3))/(2*a^2*Sqrt[a + b*x^4]) + (d*Sqrt[a + b*x^4])/(2*a^2) - (c*Sqrt[a + b*x^4])/(a^2*x) + (3*Sqrt[b]*c*x*Sqrt[a + b*x^4])/(2*a^2*(Sqrt[a] + Sqrt[b]*x^2)) - (d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*a^(3/2)) - (3*b^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(7/4)*Sqrt[a + b*x^4]) + ((3*Sqrt[b]*c + Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(7/4)*b^(1/4)*Sqrt[a + b*x^4])

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p)/c^j, {j, 0,
n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] && !PolyQ[Pq, x^(n/2)]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

, 1/2]]/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2]]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rule 21

Int[(u_)*((a_) + (b_)*(v_))^(m_)*((c_) + (d_)*(v_))^(n_), x_Symbol] := Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x, a + b*x])

Rule 266

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 50

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3}{x^2(a + bx^4)^{3/2}} dx &= \frac{x(ae + afx - bcx^2 - bdx^3)}{2a^2\sqrt{a + bx^4}} - \frac{\int \frac{-2bc - 2bdx - bex^2 - \frac{b^2cx^4}{a} - \frac{2b^2dx^5}{a}}{x^2\sqrt{a + bx^4}} dx}{2ab} \\
&= \frac{x(ae + afx - bcx^2 - bdx^3)}{2a^2\sqrt{a + bx^4}} - \frac{\int \left(\frac{-2bc - bex^2 - \frac{b^2cx^4}{a}}{x^2\sqrt{a + bx^4}} + \frac{-2bd - \frac{2b^2dx^4}{a}}{x\sqrt{a + bx^4}} \right) dx}{2ab} \\
&= \frac{x(ae + afx - bcx^2 - bdx^3)}{2a^2\sqrt{a + bx^4}} - \frac{\int \frac{-2bc - bex^2 - \frac{b^2cx^4}{a}}{x^2\sqrt{a + bx^4}} dx}{2ab} - \frac{\int \frac{-2bd - \frac{2b^2dx^4}{a}}{x\sqrt{a + bx^4}} dx}{2ab} \\
&= \frac{x(ae + afx - bcx^2 - bdx^3)}{2a^2\sqrt{a + bx^4}} - \frac{c\sqrt{a + bx^4}}{a^2x} + \frac{\int \frac{2abex + 6b^2cx^3}{x\sqrt{a + bx^4}} dx}{4a^2b} + \frac{d \int \frac{\sqrt{a + bx^4}}{x} dx}{a^2} \\
&= \frac{x(ae + afx - bcx^2 - bdx^3)}{2a^2\sqrt{a + bx^4}} - \frac{c\sqrt{a + bx^4}}{a^2x} + \frac{\int \frac{2abe + 6b^2cx^2}{\sqrt{a + bx^4}} dx}{4a^2b} + \frac{d \operatorname{Subst}\left(\int \frac{\sqrt{a + bx}}{x} dx, x, x^4\right)}{4a^2} \\
&= \frac{x(ae + afx - bcx^2 - bdx^3)}{2a^2\sqrt{a + bx^4}} + \frac{d\sqrt{a + bx^4}}{2a^2} - \frac{c\sqrt{a + bx^4}}{a^2x} - \frac{(3\sqrt{bc}) \int \frac{1 - \frac{\sqrt{bx^2}}{\sqrt{a}}}{\sqrt{a + bx^4}} dx}{2a^{3/2}} + \frac{d \operatorname{Subst}\left(\int \frac{\sqrt{a + bx}}{x} dx, x, x^4\right)}{4a^2} \\
&= \frac{x(ae + afx - bcx^2 - bdx^3)}{2a^2\sqrt{a + bx^4}} + \frac{d\sqrt{a + bx^4}}{2a^2} - \frac{c\sqrt{a + bx^4}}{a^2x} + \frac{3\sqrt{bc}x\sqrt{a + bx^4}}{2a^2(\sqrt{a} + \sqrt{bx^2})} - \frac{3^4\sqrt{bc}(\sqrt{a} + \sqrt{bx^2})}{2a^2(\sqrt{a} + \sqrt{bx^2})} \\
&= \frac{x(ae + afx - bcx^2 - bdx^3)}{2a^2\sqrt{a + bx^4}} + \frac{d\sqrt{a + bx^4}}{2a^2} - \frac{c\sqrt{a + bx^4}}{a^2x} + \frac{3\sqrt{bc}x\sqrt{a + bx^4}}{2a^2(\sqrt{a} + \sqrt{bx^2})} - \frac{d \tanh^{-1}\left(\frac{\sqrt{a + bx}}{\sqrt{a}}\right)}{2a^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.114255, size = 123, normalized size = 0.36

$$\frac{-2c\sqrt{\frac{bx^4}{a}} + 1 {}_2F_1\left(-\frac{1}{4}, \frac{3}{2}; \frac{3}{4}; -\frac{bx^4}{a}\right) + dx {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{bx^4}{a} + 1\right) + x^2 \left(e\sqrt{\frac{bx^4}{a}} + 1 {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{bx^4}{a}\right) + e + fx \right)}{2ax\sqrt{a + bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^2*(a + b*x^4)^(3/2)), x]

[Out] $(d*x*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^4)/a] - 2*c*sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-1/4, 3/2, 3/4, -((b*x^4)/a)] + x^2*(e + f*x + e*sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)]))/(2*a*x*sqrt[a + b*x^4])$

Maple [C] time = 0.01, size = 355, normalized size = 1.

$$\frac{fx^2}{2a} \frac{1}{\sqrt{bx^4 + a}} + \frac{ex}{2a} \frac{1}{\sqrt{(x^4 + \frac{a}{b})b}} + \frac{e}{2a} \sqrt{1 - ix^2\sqrt{b}\frac{1}{\sqrt{a}}} \sqrt{1 + ix^2\sqrt{b}\frac{1}{\sqrt{a}}} \text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}, i\right) \frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{bx^4 + a}} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^{(3/2)}, x)$

[Out] $1/2*f/(b*x^4+a)^{(1/2)}/a*x^2+1/2*e*x/a/((x^4+1/b*a)*b)^{(1/2)}+1/2*e/a/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*EllipticF(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I)+1/2*d/a/(b*x^4+a)^{(1/2)}-1/2*d/a^{(3/2)}*\ln((2*a+2*a^{(1/2)}*(b*x^4+a)^{(1/2)})/x^2)-1/2*c*b*x^3/a^2/((x^4+1/b*a)*b)^{(1/2)}-c*(b*x^4+a)^{(1/2)}/a^2/x+3/2*I*c*b^{(1/2)}/a^{(3/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*EllipticF(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I)-3/2*I*c*b^{(1/2)}/a^{(3/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*EllipticE(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{(bx^4 + a)^{\frac{3}{2}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^{(3/2)}, x, \text{algorithm}="maxima")$

[Out] $\text{integrate}((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^{(3/2)}*x^2), x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)}{b^2x^{10} + 2abx^6 + a^2x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/(b^2*x^10 + 2*a*b*x^6 + a^2*x^2), x)

Sympy [C] time = 34.1444, size = 291, normalized size = 0.85

$$d \left(\frac{2a^3 \sqrt{1 + \frac{bx^4}{a}}}{4a^{\frac{9}{2}} + 4a^{\frac{7}{2}}bx^4} + \frac{a^3 \log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}} + 4a^{\frac{7}{2}}bx^4} - \frac{2a^3 \log\left(\sqrt{1 + \frac{bx^4}{a}} + 1\right)}{4a^{\frac{9}{2}} + 4a^{\frac{7}{2}}bx^4} + \frac{a^2bx^4 \log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}} + 4a^{\frac{7}{2}}bx^4} - \frac{2a^2bx^4 \log\left(\sqrt{1 + \frac{bx^4}{a}} + 1\right)}{4a^{\frac{9}{2}} + 4a^{\frac{7}{2}}bx^4} \right) + \frac{c\Gamma\left(-\frac{1}{4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)/x**2/(b*x**4+a)**(3/2),x)

[Out] d*(2*a**3*sqrt(1 + b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**3*log(b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**3*log(sqrt(1 + b*x**4/a) + 1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**2*b*x**4*log(b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**2*b*x**4*log(sqrt(1 + b*x**4/a) + 1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4)) + c*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*x*gamma(3/4)) + e*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(5/4)) + f*x**2/(2*a**(3/2)*sqrt(1 + b*x**4/a))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{(bx^4 + a)^{\frac{3}{2}}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^2), x)
```

$$3.549 \quad \int \frac{c+dx+ex^2+fx^3}{x^3(a+bx^4)^{3/2}} dx$$

Optimal. Leaf size=367

$$\frac{(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (\sqrt{a}f + 3\sqrt{bd}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{4a^{7/4} \sqrt[4]{b} \sqrt{a+bx^4}} + \frac{x(af - bcx - bdx^2 - bex^3)}{2a^2 \sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{2a^2 x^2} - \frac{d\sqrt{a+bx^4}}{a^2 x}$$

[Out] (x*(a*f - b*c*x - b*d*x^2 - b*e*x^3))/(2*a^2*Sqrt[a + b*x^4]) + (e*Sqrt[a + b*x^4])/(2*a^2) - (c*Sqrt[a + b*x^4])/(2*a^2*x^2) - (d*Sqrt[a + b*x^4])/(a^2*x) + (3*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(2*a^2*(Sqrt[a] + Sqrt[b]*x^2)) - (e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*a^(3/2)) - (3*b^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(7/4)*Sqrt[a + b*x^4]) + ((3*Sqrt[b]*d + Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(7/4)*b^(1/4)*Sqrt[a + b*x^4])

Rubi [A] time = 0.477778, antiderivative size = 367, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {1829, 1833, 1835, 1584, 1198, 220, 1196, 21, 266, 50, 63, 208}

$$\frac{x(af - bcx - bdx^2 - bex^3)}{2a^2 \sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{2a^2 x^2} + \frac{(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (\sqrt{a}f + 3\sqrt{bd}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{4a^{7/4} \sqrt[4]{b} \sqrt{a+bx^4}} - \frac{d\sqrt{a+bx^4}}{a^2 x}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3)/(x^3*(a + b*x^4)^(3/2)), x]

[Out] (x*(a*f - b*c*x - b*d*x^2 - b*e*x^3))/(2*a^2*Sqrt[a + b*x^4]) + (e*Sqrt[a + b*x^4])/(2*a^2) - (c*Sqrt[a + b*x^4])/(2*a^2*x^2) - (d*Sqrt[a + b*x^4])/(a^2*x) + (3*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(2*a^2*(Sqrt[a] + Sqrt[b]*x^2)) - (e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*a^(3/2)) - (3*b^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(7/4)*Sqrt[a + b*x^4]) + ((3*Sqrt[b]*d + Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(7/4)*b^(1/4)*Sqrt[a + b*x^4])

+ b*x^4])

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1833

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p)/c^j, {j, 0,
n/2 - 1}], x] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0]
] && !PolyQ[Pq, x^(n/2)]
```

Rule 1835

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1198

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e}, x] && PosQ[c/a]

Rule 21

Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] := Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x, a + b*x])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
 \int \frac{c + dx + ex^2 + fx^3}{x^3(a + bx^4)^{3/2}} dx &= \frac{x(af - bcx - bdx^2 - bex^3)}{2a^2\sqrt{a + bx^4}} - \frac{\int \frac{-2bc - 2bdx - 2bex^2 - bfx^3 - \frac{b^2dx^5}{a} - \frac{2b^2ex^6}{a}}{x^3\sqrt{a + bx^4}} dx}{2ab} \\
 &= \frac{x(af - bcx - bdx^2 - bex^3)}{2a^2\sqrt{a + bx^4}} - \frac{\int \left(\frac{-2bd - bfx^2 - \frac{b^2dx^4}{a}}{x^2\sqrt{a + bx^4}} + \frac{-2bc - 2bex^2 - \frac{2b^2ex^6}{a}}{x^3\sqrt{a + bx^4}} \right) dx}{2ab} \\
 &= \frac{x(af - bcx - bdx^2 - bex^3)}{2a^2\sqrt{a + bx^4}} - \frac{\int \frac{-2bd - bfx^2 - \frac{b^2dx^4}{a}}{x^2\sqrt{a + bx^4}} dx}{2ab} - \frac{\int \frac{-2bc - 2bex^2 - \frac{2b^2ex^6}{a}}{x^3\sqrt{a + bx^4}} dx}{2ab} \\
 &= \frac{x(af - bcx - bdx^2 - bex^3)}{2a^2\sqrt{a + bx^4}} - \frac{c\sqrt{a + bx^4}}{2a^2x^2} - \frac{d\sqrt{a + bx^4}}{a^2x} + \frac{\int \frac{8abex + 8b^2ex^5}{x^2\sqrt{a + bx^4}} dx}{8a^2b} + \frac{\int \frac{2abfx + 6b^2dx^3}{x\sqrt{a + bx^4}} dx}{4a^2b} \\
 &= \frac{x(af - bcx - bdx^2 - bex^3)}{2a^2\sqrt{a + bx^4}} - \frac{c\sqrt{a + bx^4}}{2a^2x^2} - \frac{d\sqrt{a + bx^4}}{a^2x} + \frac{\int \frac{8abe + 8b^2ex^4}{x\sqrt{a + bx^4}} dx}{8a^2b} + \frac{\int \frac{2abf + 6b^2dx^2}{\sqrt{a + bx^4}} dx}{4a^2b} \\
 &= \frac{x(af - bcx - bdx^2 - bex^3)}{2a^2\sqrt{a + bx^4}} - \frac{c\sqrt{a + bx^4}}{2a^2x^2} - \frac{d\sqrt{a + bx^4}}{a^2x} - \frac{(3\sqrt{bd}) \int \frac{1 - \frac{\sqrt{bx^2}}{\sqrt{a}}}{\sqrt{a + bx^4}} dx}{2a^{3/2}} + \frac{e \int \frac{\sqrt{a + bx^4}}{x} dx}{a^2} \\
 &= \frac{x(af - bcx - bdx^2 - bex^3)}{2a^2\sqrt{a + bx^4}} - \frac{c\sqrt{a + bx^4}}{2a^2x^2} - \frac{d\sqrt{a + bx^4}}{a^2x} + \frac{3\sqrt{bd}x\sqrt{a + bx^4}}{2a^2(\sqrt{a} + \sqrt{bx^2})} - \frac{3^4\sqrt{bd}(\sqrt{a} + \sqrt{bx^2})}{2a^2} \\
 &= \frac{x(af - bcx - bdx^2 - bex^3)}{2a^2\sqrt{a + bx^4}} + \frac{e\sqrt{a + bx^4}}{2a^2} - \frac{c\sqrt{a + bx^4}}{2a^2x^2} - \frac{d\sqrt{a + bx^4}}{a^2x} + \frac{3\sqrt{bd}x\sqrt{a + bx^4}}{2a^2(\sqrt{a} + \sqrt{bx^2})} \\
 &= \frac{x(af - bcx - bdx^2 - bex^3)}{2a^2\sqrt{a + bx^4}} + \frac{e\sqrt{a + bx^4}}{2a^2} - \frac{c\sqrt{a + bx^4}}{2a^2x^2} - \frac{d\sqrt{a + bx^4}}{a^2x} + \frac{3\sqrt{bd}x\sqrt{a + bx^4}}{2a^2(\sqrt{a} + \sqrt{bx^2})} \\
 &= \frac{x(af - bcx - bdx^2 - bex^3)}{2a^2\sqrt{a + bx^4}} + \frac{e\sqrt{a + bx^4}}{2a^2} - \frac{c\sqrt{a + bx^4}}{2a^2x^2} - \frac{d\sqrt{a + bx^4}}{a^2x} + \frac{3\sqrt{bd}x\sqrt{a + bx^4}}{2a^2(\sqrt{a} + \sqrt{bx^2})}
 \end{aligned}$$

Mathematica [C] time = 0.106415, size = 140, normalized size = 0.38

$$\frac{-2adx\sqrt{\frac{bx^4}{a}} + 1 {}_2F_1\left(-\frac{1}{4}, \frac{3}{2}; \frac{3}{4}; -\frac{bx^4}{a}\right) + aex^2 {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{bx^4}{a} + 1\right) + afx^3\sqrt{\frac{bx^4}{a}} + 1 {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{bx^4}{a}\right) - ac + afx^3 - 2bcx^4}{2a^2x^2\sqrt{a + bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^3*(a + b*x^4)^(3/2)), x]

[Out] $(-(a*c) + a*f*x^3 - 2*b*c*x^4 + a*e*x^2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^4)/a] - 2*a*d*x*sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-1/4, 3/2, 3/4, -(b*x^4)/a] + a*f*x^3*sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -(b*x^4)/a])/(2*a^2*x^2*sqrt[a + b*x^4])$

Maple [C] time = 0.016, size = 363, normalized size = 1.

$$\frac{fx}{2a} \frac{1}{\sqrt{(x^4 + \frac{a}{b})b}} + \frac{f}{2a} \sqrt{1 - ix^2\sqrt{b}} \frac{1}{\sqrt{a}} \sqrt{1 + ix^2\sqrt{b}} \frac{1}{\sqrt{a}} \text{EllipticF}\left(x\sqrt{i\sqrt{b}} \frac{1}{\sqrt{a}}, i\right) \frac{1}{\sqrt{i\sqrt{b}} \frac{1}{\sqrt{a}}} \frac{1}{\sqrt{bx^4 + a}} + \frac{e}{2a} \frac{1}{\sqrt{bx^4 + a}} - \frac{e}{2} \ln$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(3/2), x)

[Out] $\frac{1}{2}f*x/a/((x^4+1/b*a)*b)^{(1/2)} + \frac{1}{2}f/a/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*EllipticF(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I) + \frac{1}{2}e/a/(b*x^4+a)^{(1/2)} - \frac{1}{2}e/a^{(3/2)}*\ln((2*a+2*a^{(1/2)}*(b*x^4+a)^{(1/2)})/x^2) - \frac{1}{2}c/x^2*(2*b*x^4+a)/(b*x^4+a)^{(1/2)}/a^2 - \frac{1}{2}d*b*x^3/a^2/((x^4+1/b*a)*b)^{(1/2)} - d*(b*x^4+a)^{(1/2)}/a^2/x + \frac{3}{2}I*d*b^{(1/2)}/a^{(3/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*EllipticF(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I) - \frac{3}{2}I*d*b^{(1/2)}/a^{(3/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+I/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(b*x^4+a)^{(1/2)}*EllipticE(x*(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{(bx^4 + a)^{\frac{3}{2}}x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(3/2),x, algorithm="maxima")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{b^2x^{11}+2abx^7+a^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/(b^2*x^11 + 2*a*b*x^7 + a^2*x^3), x)

Sympy [C] time = 42.4373, size = 316, normalized size = 0.86

$$c\left(-\frac{1}{2a\sqrt{bx^4}\sqrt{\frac{a}{bx^4}+1}}-\frac{\sqrt{b}}{a^2\sqrt{\frac{a}{bx^4}+1}}\right)+e\left(\frac{2a^3\sqrt{1+\frac{bx^4}{a}}}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^3\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^2bx^4\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)/x**3/(b*x**4+a)**(3/2),x)

[Out] c*(-1/(2*a*sqrt(b)*x**4*sqrt(a/(b*x**4) + 1)) - sqrt(b)/(a**2*sqrt(a/(b*x**4) + 1))) + e*(2*a**3*sqrt(1 + b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**3*log(b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**3*log(sqrt(1 + b*x**4/a) + 1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**2*b*x**4*log(b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**2*b*x**4*log(sqrt(1 + b*x**4/a) + 1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4)) + d*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*x*gamma(3/4)) + f*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(5/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{(bx^4 + a)^{\frac{3}{2}}x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(3/2),x, algorithm="giac")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^3), x)

$$3.550 \quad \int \frac{c+dx+ex^2+fx^3}{x^4(a+bx^4)^{3/2}} dx$$

Optimal. Leaf size=387

$$\frac{\sqrt[4]{b}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (5\sqrt{bc} - 9\sqrt{ae}) \operatorname{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{12a^{9/4}\sqrt{a+bx^4}} - \frac{x(bc + bdx + bex^2 + bfx^3)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{3a^2x^3}$$

[Out] $-(x*(b*c + b*d*x + b*e*x^2 + b*f*x^3))/(2*a^2*\operatorname{Sqrt}[a + b*x^4]) + (f*\operatorname{Sqrt}[a + b*x^4])/(2*a^2) - (c*\operatorname{Sqrt}[a + b*x^4])/(3*a^2*x^3) - (d*\operatorname{Sqrt}[a + b*x^4])/(2*a^2*x^2) - (e*\operatorname{Sqrt}[a + b*x^4])/(a^2*x) + (3*\operatorname{Sqrt}[b]*e*x*\operatorname{Sqrt}[a + b*x^4])/(2*a^2*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) - (f*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*x^4]/\operatorname{Sqrt}[a]])/(2*a^{3/2}) - (3*b^{1/4}*e*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2])/(2*a^{7/4}* \operatorname{Sqrt}[a + b*x^4]) - (b^{1/4}*(5*\operatorname{Sqrt}[b]*c - 9*\operatorname{Sqrt}[a]*e)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2])/(12*a^{9/4}* \operatorname{Sqrt}[a + b*x^4])$

Rubi [A] time = 0.606259, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {1829, 1833, 1835, 1585, 1584, 1198, 220, 1196, 21, 266, 50, 63, 208}

$$\frac{x(bc + bdx + bex^2 + bfx^3)}{2a^2\sqrt{a+bx^4}} - \frac{\sqrt[4]{b}(\sqrt{a} + \sqrt{bx^2}) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} (5\sqrt{bc} - 9\sqrt{ae}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{12a^{9/4}\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{3a^2x^3} - \frac{d\sqrt{a+bx^4}}{2a^2x^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c + d*x + e*x^2 + f*x^3)/(x^4*(a + b*x^4)^{(3/2)}), x]$

[Out] $-(x*(b*c + b*d*x + b*e*x^2 + b*f*x^3))/(2*a^2*\operatorname{Sqrt}[a + b*x^4]) + (f*\operatorname{Sqrt}[a + b*x^4])/(2*a^2) - (c*\operatorname{Sqrt}[a + b*x^4])/(3*a^2*x^3) - (d*\operatorname{Sqrt}[a + b*x^4])/(2*a^2*x^2) - (e*\operatorname{Sqrt}[a + b*x^4])/(a^2*x) + (3*\operatorname{Sqrt}[b]*e*x*\operatorname{Sqrt}[a + b*x^4])/(2*a^2*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)) - (f*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*x^4]/\operatorname{Sqrt}[a]])/(2*a^{3/2}) - (3*b^{1/4}*e*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticE}[2*\operatorname{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2])/(2*a^{7/4}* \operatorname{Sqrt}[a + b*x^4]) - (b^{1/4}*(5*\operatorname{Sqrt}[b]*c - 9*\operatorname{Sqrt}[a]*e)*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)*\operatorname{Sqrt}[(a + b*x^4)/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]*x^2)^2]*\operatorname{EllipticF}[2*\operatorname{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2])/(12*a^{9/4}* \operatorname{Sqrt}[a + b*x^4])$

$*x)/a^{(1/4)}], 1/2)]/(12*a^{(9/4)}*Sqrt[a + b*x^4])$

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1), x]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p)/c^j, {j, 0,
n/2 - 1}], x] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] && !PolyQ[Pq, x^(n/2)]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1198


```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q =
  Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (c_)*(x_)^4], x_Symbol] := With[{q =
  Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rule 21

```
Int[(u_)*((a_) + (b_)*(v_))^(m_)*((c_) + (d_)*(v_))^(n_), x_Symbol] :=
Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx + ex^2 + fx^3}{x^4(a + bx^4)^{3/2}} dx &= -\frac{x(bc + bdx + bex^2 + bfx^3)}{2a^2\sqrt{a + bx^4}} - \frac{\int \frac{-2bc - 2bdx - 2bex^2 - 2bfx^3 + \frac{b^2cx^4}{a} - \frac{b^2ex^6}{a} - \frac{2b^2fx^7}{a}}{x^4\sqrt{a + bx^4}} dx}{2ab} \\
&= -\frac{x(bc + bdx + bex^2 + bfx^3)}{2a^2\sqrt{a + bx^4}} - \frac{\int \left(\frac{-2bc - 2bex^2 + \frac{b^2cx^4}{a} - \frac{b^2ex^6}{a}}{x^4\sqrt{a + bx^4}} + \frac{-2bd - 2bfx^2 - \frac{2b^2fx^6}{a}}{x^3\sqrt{a + bx^4}} \right) dx}{2ab} \\
&= -\frac{x(bc + bdx + bex^2 + bfx^3)}{2a^2\sqrt{a + bx^4}} - \frac{\int \frac{-2bc - 2bex^2 + \frac{b^2cx^4}{a} - \frac{b^2ex^6}{a}}{x^4\sqrt{a + bx^4}} dx}{2ab} - \frac{\int \frac{-2bd - 2bfx^2 - \frac{2b^2fx^6}{a}}{x^3\sqrt{a + bx^4}} dx}{2ab} \\
&= -\frac{x(bc + bdx + bex^2 + bfx^3)}{2a^2\sqrt{a + bx^4}} - \frac{c\sqrt{a + bx^4}}{3a^2x^3} - \frac{d\sqrt{a + bx^4}}{2a^2x^2} + \frac{\int \frac{12abex - 10b^2cx^3 + 6b^2ex^5}{x^3\sqrt{a + bx^4}} dx}{12a^2b} + \frac{\int \frac{8ab}{x}}{12a^2b} \\
&= -\frac{x(bc + bdx + bex^2 + bfx^3)}{2a^2\sqrt{a + bx^4}} - \frac{c\sqrt{a + bx^4}}{3a^2x^3} - \frac{d\sqrt{a + bx^4}}{2a^2x^2} + \frac{\int \frac{12abe - 10b^2cx^2 + 6b^2ex^4}{x^2\sqrt{a + bx^4}} dx}{12a^2b} + \frac{\int \frac{8ab}{x}}{12a^2b} \\
&= -\frac{x(bc + bdx + bex^2 + bfx^3)}{2a^2\sqrt{a + bx^4}} - \frac{c\sqrt{a + bx^4}}{3a^2x^3} - \frac{d\sqrt{a + bx^4}}{2a^2x^2} - \frac{e\sqrt{a + bx^4}}{a^2x} - \frac{\int \frac{20ab^2cx - 36ab^2ex^3}{x\sqrt{a + bx^4}} dx}{24a^3b} \\
&= -\frac{x(bc + bdx + bex^2 + bfx^3)}{2a^2\sqrt{a + bx^4}} - \frac{c\sqrt{a + bx^4}}{3a^2x^3} - \frac{d\sqrt{a + bx^4}}{2a^2x^2} - \frac{e\sqrt{a + bx^4}}{a^2x} - \frac{\int \frac{20ab^2c - 36ab^2ex^2}{\sqrt{a + bx^4}} dx}{24a^3b} \\
&= -\frac{x(bc + bdx + bex^2 + bfx^3)}{2a^2\sqrt{a + bx^4}} + \frac{f\sqrt{a + bx^4}}{2a^2} - \frac{c\sqrt{a + bx^4}}{3a^2x^3} - \frac{d\sqrt{a + bx^4}}{2a^2x^2} - \frac{e\sqrt{a + bx^4}}{a^2x} - \frac{(3\sqrt{b})}{2a^2} \\
&= -\frac{x(bc + bdx + bex^2 + bfx^3)}{2a^2\sqrt{a + bx^4}} + \frac{f\sqrt{a + bx^4}}{2a^2} - \frac{c\sqrt{a + bx^4}}{3a^2x^3} - \frac{d\sqrt{a + bx^4}}{2a^2x^2} - \frac{e\sqrt{a + bx^4}}{a^2x} + \frac{3\sqrt{b}}{2a^2} \\
&= -\frac{x(bc + bdx + bex^2 + bfx^3)}{2a^2\sqrt{a + bx^4}} + \frac{f\sqrt{a + bx^4}}{2a^2} - \frac{c\sqrt{a + bx^4}}{3a^2x^3} - \frac{d\sqrt{a + bx^4}}{2a^2x^2} - \frac{e\sqrt{a + bx^4}}{a^2x} + \frac{3\sqrt{b}}{2a^2}
\end{aligned}$$

Mathematica [C] time = 0.119275, size = 136, normalized size = 0.35

$$\frac{-2ac\sqrt{\frac{bx^4}{a}} + 1 {}_2F_1\left(-\frac{3}{4}, \frac{3}{2}; \frac{1}{4}; -\frac{bx^4}{a}\right) - 3x\left(2aex\sqrt{\frac{bx^4}{a}} + 1 {}_2F_1\left(-\frac{1}{4}, \frac{3}{2}; \frac{3}{4}; -\frac{bx^4}{a}\right) - afx^2 {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{bx^4}{a} + 1\right) + ad + 2bdx^4\right)}{6a^2x^3\sqrt{a + bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^4*(a + b*x^4)^(3/2)),x]

[Out] (-2*a*c*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-3/4, 3/2, 1/4, -((b*x^4)/a)] - 3*x*(a*d + 2*b*d*x^4 - a*f*x^2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^4)/a] + 2*a*e*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-1/4, 3/2, 3/4, -((b*x^4)/a)]))/(6*a^2*x^3*Sqrt[a + b*x^4])

Maple [C] time = 0.011, size = 383, normalized size = 1.

$$\frac{f}{2a} \frac{1}{\sqrt{bx^4 + a}} - \frac{f}{2} \ln \left(\frac{1}{x^2} \left(2a + 2\sqrt{a}\sqrt{bx^4 + a} \right) \right) a^{-\frac{3}{2}} - \frac{c}{3x^3a^2} \sqrt{bx^4 + a} - \frac{bcx}{2a^2} \frac{1}{\sqrt{\left(x^4 + \frac{a}{b}\right)b}} - \frac{5bc}{6a^2} \sqrt{1 - ix^2\sqrt{b}\frac{1}{\sqrt{a}}} \sqrt{1 + ix^2\sqrt{b}\frac{1}{\sqrt{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(3/2),x)

[Out] 1/2*f/a/(b*x^4+a)^(1/2)-1/2*f/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/3*c*(b*x^4+a)^(1/2)/x^3/a^2-1/2*c*b*x/a^2/((x^4+1/b*a)*b)^(1/2)-5/6*c*b/a^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2), I)-1/2*d/x^2*(2*b*x^4+a)/(b*x^4+a)^(1/2)/a^2-1/2*e*b*x^3/a^2/((x^4+1/b*a)*b)^(1/2)-e*(b*x^4+a)^(1/2)/a^2/x+3/2*I*e*b^(1/2)/a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2), I)-3/2*I*e*b^(1/2)/a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2), I)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{(bx^4 + a)^{\frac{3}{2}}x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(3/2),x, algorithm="maxima")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^4), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{b^2x^{12}+2abx^8+a^2x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/(b^2*x^12 + 2*a*b*x^8 + a^2*x^4), x)

Sympy [C] time = 41.3974, size = 321, normalized size = 0.83

$$d\left(-\frac{1}{2a\sqrt{bx^4}\sqrt{\frac{a}{bx^4}+1}}-\frac{\sqrt{b}}{a^2\sqrt{\frac{a}{bx^4}+1}}\right)+f\left(\frac{2a^3\sqrt{1+\frac{bx^4}{a}}}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^3\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^2bx^4\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)/x**4/(b*x**4+a)**(3/2),x)

[Out] d*(-1/(2*a*sqrt(b)*x**4*sqrt(a/(b*x**4)+1))-sqrt(b)/(a**2*sqrt(a/(b*x**4)+1))) + f*(2*a**3*sqrt(1+b*x**4/a)/(4*a**(9/2)+4*a**(7/2)*b*x**4) + a**3*log(b*x**4/a)/(4*a**(9/2)+4*a**(7/2)*b*x**4) - 2*a**3*log(sqrt(1+b*x**4/a)+1)/(4*a**(9/2)+4*a**(7/2)*b*x**4) + a**2*b*x**4*log(b*x**4/a)/(4*a**(9/2)+4*a**(7/2)*b*x**4) - 2*a**2*b*x**4*log(sqrt(1+b*x**4/a)+1)/(4*a**(9/2)+4*a**(7/2)*b*x**4)) + c*gamma(-3/4)*hyper((-3/4, 3/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*x**3*gamma(1/4)) + e*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*x*gamma(3/4))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3+ex^2+dx+c}{(bx^4+a)^{\frac{3}{2}}x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^4), x)
```

$$3.551 \quad \int (gx)^m (c + dx + ex^2 + fx^3) (a + bx^4)^p dx$$

Optimal. Leaf size=269

$$\frac{c(gx)^{m+1} (a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+1}{4}, -p; \frac{m+5}{4}; -\frac{bx^4}{a}\right)}{g(m+1)} + \frac{d(gx)^{m+2} (a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+2}{4}, -p; \frac{m+6}{4}; -\frac{bx^4}{a}\right)}{g^2(m+2)} + \dots$$

[Out] $(c*(g*x)^{(1+m)*(a+b*x^4)^p*Hypergeometric2F1[(1+m)/4, -p, (5+m)/4, -(b*x^4)/a]})/(g*(1+m)*(1+(b*x^4)/a)^p) + (d*(g*x)^{(2+m)*(a+b*x^4)^p*Hypergeometric2F1[(2+m)/4, -p, (6+m)/4, -(b*x^4)/a]})/(g^2*(2+m)*(1+(b*x^4)/a)^p) + (e*(g*x)^{(3+m)*(a+b*x^4)^p*Hypergeometric2F1[(3+m)/4, -p, (7+m)/4, -(b*x^4)/a]})/(g^3*(3+m)*(1+(b*x^4)/a)^p) + (f*(g*x)^{(4+m)*(a+b*x^4)^p*Hypergeometric2F1[(4+m)/4, -p, (8+m)/4, -(b*x^4)/a]})/(g^4*(4+m)*(1+(b*x^4)/a)^p)$

Rubi [A] time = 0.256017, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1833, 1336, 365, 364}

$$\frac{c(gx)^{m+1} (a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+1}{4}, -p; \frac{m+5}{4}; -\frac{bx^4}{a}\right)}{g(m+1)} + \frac{d(gx)^{m+2} (a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+2}{4}, -p; \frac{m+6}{4}; -\frac{bx^4}{a}\right)}{g^2(m+2)} + \dots$$

Antiderivative was successfully verified.

[In] $\text{Int}[(g*x)^m*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p, x]$

[Out] $(c*(g*x)^{(1+m)*(a+b*x^4)^p*Hypergeometric2F1[(1+m)/4, -p, (5+m)/4, -(b*x^4)/a]})/(g*(1+m)*(1+(b*x^4)/a)^p) + (d*(g*x)^{(2+m)*(a+b*x^4)^p*Hypergeometric2F1[(2+m)/4, -p, (6+m)/4, -(b*x^4)/a]})/(g^2*(2+m)*(1+(b*x^4)/a)^p) + (e*(g*x)^{(3+m)*(a+b*x^4)^p*Hypergeometric2F1[(3+m)/4, -p, (7+m)/4, -(b*x^4)/a]})/(g^3*(3+m)*(1+(b*x^4)/a)^p) + (f*(g*x)^{(4+m)*(a+b*x^4)^p*Hypergeometric2F1[(4+m)/4, -p, (8+m)/4, -(b*x^4)/a]})/(g^4*(4+m)*(1+(b*x^4)/a)^p)$

Rule 1833

$\text{Int}[(Pq_*)*((c_*)*(x_*)^{(m_*)*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x_Symbol] :> \text{Module}[\{q = \text{Expon}[Pq, x], j, k\}, \text{Int}[\text{Sum}[(c*x)^{(m+j)}*\text{Sum}[\text{Coeff}[Pq, x, j + (k*n)/2]*x^{((k*n)/2)}, \{k, 0, (2*(q-j))/n + 1\}]* (a + b*x^n)^p]/c^j, \{j, 0, n/2 - 1\}], x]] /; \text{FreeQ}[\{a, b, c, m, p\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[n/2, 0]$

] && !PolyQ[Pq, x^(n/2)]

Rule 1336

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] | IntegersQ[m, q])

Rule 365

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])

Rule 364

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a)])/ (c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned}
\int (gx)^m (c + dx + ex^2 + fx^3) (a + bx^4)^p dx &= \int \left((gx)^m (c + ex^2) (a + bx^4)^p + \frac{(gx)^{1+m} (d + fx^2) (a + bx^4)^p}{g} \right) dx \\
&= \frac{\int (gx)^{1+m} (d + fx^2) (a + bx^4)^p dx}{g} + \int (gx)^m (c + ex^2) (a + bx^4)^p dx \\
&= \frac{\int \left(d(gx)^{1+m} (a + bx^4)^p + \frac{f(gx)^{3+m} (a + bx^4)^p}{g^2} \right) dx}{g} + \int \left(c(gx)^m (a + bx^4)^p + \frac{e(gx)^{2+m} (a + bx^4)^p}{g^2} \right) dx \\
&= c \int (gx)^m (a + bx^4)^p dx + \frac{f \int (gx)^{3+m} (a + bx^4)^p dx}{g^3} + \frac{e \int (gx)^{2+m} (a + bx^4)^p dx}{g^2} \\
&= \left(c (a + bx^4)^p \left(1 + \frac{bx^4}{a} \right)^{-p} \right) \int (gx)^m \left(1 + \frac{bx^4}{a} \right)^p dx + \frac{\left(f (a + bx^4)^p \left(1 + \frac{bx^4}{a} \right)^{-p} \right) \int (gx)^{3+m} (a + bx^4)^p dx}{g^3} + \frac{\left(e (a + bx^4)^p \left(1 + \frac{bx^4}{a} \right)^{-p} \right) \int (gx)^{2+m} (a + bx^4)^p dx}{g^2} \\
&= \frac{c (gx)^{1+m} (a + bx^4)^p \left(1 + \frac{bx^4}{a} \right)^{-p} {}_2F_1 \left(\frac{1+m}{4}, -p; \frac{5+m}{4}; -\frac{bx^4}{a} \right)}{g(1+m)} + \frac{d (gx)^{2+m} (a + bx^4)^p \left(1 + \frac{bx^4}{a} \right)^{-p} {}_2F_1 \left(\frac{2+m}{4}, -p; \frac{6+m}{4}; -\frac{bx^4}{a} \right)}{g^2} + \frac{e (gx)^{3+m} (a + bx^4)^p \left(1 + \frac{bx^4}{a} \right)^{-p} {}_2F_1 \left(\frac{3+m}{4}, -p; \frac{7+m}{4}; -\frac{bx^4}{a} \right)}{g^3}
\end{aligned}$$

Mathematica [A] time = 0.214322, size = 174, normalized size = 0.65

$$x(gx)^m (a + bx^4)^p \left(\frac{bx^4}{a} + 1 \right)^{-p} \left(\frac{c {}_2F_1 \left(\frac{m+1}{4}, -p; \frac{m+5}{4}; -\frac{bx^4}{a} \right)}{m+1} + x \left(\frac{d {}_2F_1 \left(\frac{m+2}{4}, -p; \frac{m+6}{4}; -\frac{bx^4}{a} \right)}{m+2} + x \left(\frac{e {}_2F_1 \left(\frac{m+3}{4}, -p; \frac{m+7}{4}; -\frac{bx^4}{a} \right)}{m+3} + x \left(\frac{f {}_2F_1 \left(\frac{m+4}{4}, -p; \frac{m+8}{4}; -\frac{bx^4}{a} \right)}{m+4} + \dots \right) \right) \right) \right) / (1 + (bx^4)/a)^p$$

Antiderivative was successfully verified.

[In] Integrate[(g*x)^m*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]

[Out] (x*(g*x)^m*(a + b*x^4)^p*((c*Hypergeometric2F1[(1 + m)/4, -p, (5 + m)/4, -(b*x^4)/a])/(1 + m) + x*(d*Hypergeometric2F1[(2 + m)/4, -p, (6 + m)/4, -(b*x^4)/a])/(2 + m) + x*(e*Hypergeometric2F1[(3 + m)/4, -p, (7 + m)/4, -(b*x^4)/a])/(3 + m) + (f*x*Hypergeometric2F1[(4 + m)/4, -p, (8 + m)/4, -(b*x^4)/a])/(4 + m)))/(1 + (b*x^4)/a)^p

Maple [F] time = 0.314, size = 0, normalized size = 0.

$$\int (gx)^m (fx^3 + ex^2 + dx + c) (bx^4 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)`

[Out] `int((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (fx^3 + ex^2 + dx + c)(bx^4 + a)^p (gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="maxima")`

[Out] `integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*(g*x)^m, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(fx^3 + ex^2 + dx + c\right)\left(bx^4 + a\right)^p (gx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="fricas")`

[Out] `integral((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*(g*x)^m, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x)**m*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**p,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx^3 + ex^2 + dx + c)(bx^4 + a)^p (gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="giac")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*(g*x)^m, x)

3.552 $\int (c + dx + ex^2 + fx^3) (a + bx^4)^p dx$

Optimal. Leaf size=143

$$\frac{cx(a+bx^4)^{p+1} {}_2F_1\left(1, p + \frac{5}{4}; \frac{5}{4}; -\frac{bx^4}{a}\right)}{a} + \frac{dx^2(a+bx^4)^{p+1} {}_2F_1\left(1, p + \frac{3}{2}; \frac{3}{2}; -\frac{bx^4}{a}\right)}{2a} + \frac{ex^3(a+bx^4)^{p+1} {}_2F_1\left(1, p + \frac{7}{4}; \frac{7}{4}; -\frac{bx^4}{a}\right)}{3a}$$

[Out] (f*(a + b*x^4)^(1 + p))/(4*b*(1 + p)) + (c*x*(a + b*x^4)^(1 + p)*Hypergeometric2F1[1, 5/4 + p, 5/4, -((b*x^4)/a)])/a + (d*x^2*(a + b*x^4)^(1 + p)*Hypergeometric2F1[1, 3/2 + p, 3/2, -((b*x^4)/a)]/(2*a) + (e*x^3*(a + b*x^4)^(1 + p)*Hypergeometric2F1[1, 7/4 + p, 7/4, -((b*x^4)/a)]/(3*a)

Rubi [A] time = 0.131025, antiderivative size = 170, normalized size of antiderivative = 1.19, number of steps used = 12, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.32$, Rules used = {1885, 1204, 246, 245, 365, 364, 1248, 641}

$$cx(a+bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -\frac{bx^4}{a}\right) + \frac{1}{2}dx^2(a+bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^4}{a}\right) + \frac{1}{3}ex^3(a+bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} {}_2F_1\left(\frac{3}{4}, -p; \frac{7}{4}; -\frac{bx^4}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]

[Out] (f*(a + b*x^4)^(1 + p))/(4*b*(1 + p)) + (c*x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)]/(1 + (b*x^4)/a)^p + (d*x^2*(a + b*x^4)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^4)/a)]/(2*(1 + (b*x^4)/a)^p) + (e*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)]/(3*(1 + (b*x^4)/a)^p)

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*((a_ + b*x^n)^p), {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Rule 1204

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
```

NeQ[c*d^2 + a*e^2, 0]

Rule 246

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

Rule 245

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rule 365

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])

Rule 364

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rule 1248

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]

Rule 641

Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(e*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
\int (c + dx + ex^2 + fx^3)(a + bx^4)^p dx &= \int \left((c + ex^2)(a + bx^4)^p + x(d + fx^2)(a + bx^4)^p \right) dx \\
&= \int (c + ex^2)(a + bx^4)^p dx + \int x(d + fx^2)(a + bx^4)^p dx \\
&= \frac{1}{2} \text{Subst} \left(\int (d + fx)(a + bx^2)^p dx, x, x^2 \right) + \int \left(c(a + bx^4)^p + ex^2(a + bx^4)^p \right) dx \\
&= \frac{f(a + bx^4)^{1+p}}{4b(1+p)} + c \int (a + bx^4)^p dx + \frac{1}{2} d \text{Subst} \left(\int (a + bx^2)^p dx, x, x^2 \right) + e \int (a + bx^4)^p dx \\
&= \frac{f(a + bx^4)^{1+p}}{4b(1+p)} + \left(c(a + bx^4)^p \left(1 + \frac{bx^4}{a} \right)^{-p} \right) \int \left(1 + \frac{bx^4}{a} \right)^p dx + \frac{1}{2} \left(d(a + bx^4)^p \right) \int (a + bx^4)^p dx \\
&= \frac{f(a + bx^4)^{1+p}}{4b(1+p)} + cx(a + bx^4)^p \left(1 + \frac{bx^4}{a} \right)^{-p} {}_2F_1 \left(\frac{1}{4}, -p; \frac{5}{4}; -\frac{bx^4}{a} \right) + \frac{1}{2} dx^2 (a + bx^4)^p
\end{aligned}$$

Mathematica [A] time = 0.111298, size = 147, normalized size = 1.03

$$\frac{1}{12} (a + bx^4)^p \left(12cx \left(\frac{bx^4}{a} + 1 \right)^{-p} {}_2F_1 \left(\frac{1}{4}, -p; \frac{5}{4}; -\frac{bx^4}{a} \right) + 6dx^2 \left(\frac{bx^4}{a} + 1 \right)^{-p} {}_2F_1 \left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^4}{a} \right) + 4ex^3 \left(\frac{bx^4}{a} + 1 \right)^{-p} {}_2F_1 \left(\frac{3}{4}, -p; \frac{7}{4}; -\frac{bx^4}{a} \right) \right) / (1 + (bx^4)/a)^p$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]

[Out] ((a + b*x^4)^p*((3*f*(a + b*x^4))/(b*(1 + p)) + (12*c*x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)/a])/(1 + (b*x^4)/a)^p + (6*d*x^2*Hypergeometric2F1[1/2, -p, 3/2, -(b*x^4)/a])/(1 + (b*x^4)/a)^p + (4*e*x^3*Hypergeometric2F1[3/4, -p, 7/4, -(b*x^4)/a])/(1 + (b*x^4)/a)^p)/12

Maple [F] time = 0.25, size = 0, normalized size = 0.

$$\int (fx^3 + ex^2 + dx + c)(bx^4 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)

[Out] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (fx^3 + ex^2 + dx + c)(bx^4 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="maxima")

[Out] integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(fx^3 + ex^2 + dx + c\right)\left(bx^4 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="fricas")

[Out] integral((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p, x)

Sympy [A] time = 70.5717, size = 141, normalized size = 0.99

$$\frac{a^p c x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, -p \left| \frac{bx^4 e^{i\pi}}{a} \right. \right)}{4 \Gamma\left(\frac{5}{4}\right)} + \frac{a^p d x^2 {}_2F_1\left(\frac{1}{2}, -p \left| \frac{bx^4 e^{i\pi}}{a} \right. \right)}{2} + \frac{a^p e x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{3}{4}, -p \left| \frac{bx^4 e^{i\pi}}{a} \right. \right)}{4 \Gamma\left(\frac{7}{4}\right)} + f \begin{cases} \frac{a^p x^4}{4} & \text{for } p = - \\ \frac{(a+bx^4)^{p+1}}{p+1} & \text{otherwise} \\ \log(a+bx^4) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**p,x)

[Out] a**p*c*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + a**p*d*x**2*hyper((1/2, -p), (3/2,), b*x**4*exp_polar(I*pi)/a)/2 + a**p*e*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + f*Piecewise((a**p*x**4/4, Eq(b, 0)), (Piecewise(((a +

```
b*x**4)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**4), True))/(4*b), True)
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx^3 + ex^2 + dx + c)(bx^4 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p, x)
```


$$3.553 \quad \int x^3 (c + dx + ex^2 + fx^3) (a + bx^4)^p dx$$

Optimal. Leaf size=175

$$\frac{c(a + bx^4)^{p+1}}{4b(p+1)} + \frac{1}{5} dx^5 (a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} {}_2F_1\left(\frac{5}{4}, -p; \frac{9}{4}; -\frac{bx^4}{a}\right) + \frac{1}{6} ex^6 (a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; -\frac{bx^4}{a}\right)$$

[Out] (c*(a + b*x^4)^(1 + p))/(4*b*(1 + p)) + (d*x^5*(a + b*x^4)^p*Hypergeometric2F1[5/4, -p, 9/4, -((b*x^4)/a)]/(5*(1 + (b*x^4)/a)^p) + (e*x^6*(a + b*x^4)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^4)/a)]/(6*(1 + (b*x^4)/a)^p) + (f*x^7*(a + b*x^4)^p*Hypergeometric2F1[7/4, -p, 11/4, -((b*x^4)/a)]/(7*(1 + (b*x^4)/a)^p)

Rubi [A] time = 0.18225, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1833, 1252, 764, 261, 365, 364, 1336}

$$\frac{c(a + bx^4)^{p+1}}{4b(p+1)} + \frac{1}{5} dx^5 (a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} {}_2F_1\left(\frac{5}{4}, -p; \frac{9}{4}; -\frac{bx^4}{a}\right) + \frac{1}{6} ex^6 (a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; -\frac{bx^4}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]

[Out] (c*(a + b*x^4)^(1 + p))/(4*b*(1 + p)) + (d*x^5*(a + b*x^4)^p*Hypergeometric2F1[5/4, -p, 9/4, -((b*x^4)/a)]/(5*(1 + (b*x^4)/a)^p) + (e*x^6*(a + b*x^4)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^4)/a)]/(6*(1 + (b*x^4)/a)^p) + (f*x^7*(a + b*x^4)^p*Hypergeometric2F1[7/4, -p, 11/4, -((b*x^4)/a)]/(7*(1 + (b*x^4)/a)^p)

Rule 1833

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p]/c^j, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
:> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
NeQ[p, -1]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^m*(1 + (b*x^n)/a)^p, x], x]
/; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a])/
(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1336

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

Rubi steps

$$\begin{aligned}
\int x^3 (c + dx + ex^2 + fx^3) (a + bx^4)^p dx &= \int \left(x^3 (c + ex^2) (a + bx^4)^p + x^4 (d + fx^2) (a + bx^4)^p \right) dx \\
&= \int x^3 (c + ex^2) (a + bx^4)^p dx + \int x^4 (d + fx^2) (a + bx^4)^p dx \\
&= \frac{1}{2} \text{Subst} \left(\int x(c + ex) (a + bx^2)^p dx, x, x^2 \right) + \int \left(dx^4 (a + bx^4)^p + fx^6 (a + bx^4)^p \right) dx \\
&= \frac{1}{2} c \text{Subst} \left(\int x (a + bx^2)^p dx, x, x^2 \right) + d \int x^4 (a + bx^4)^p dx + \frac{1}{2} e \text{Subst} \left(\int x^3 (a + bx^2)^p dx, x, x^2 \right) \\
&= \frac{c (a + bx^4)^{1+p}}{4b(1+p)} + \left(d (a + bx^4)^p \left(1 + \frac{bx^4}{a} \right)^{-p} \right) \int x^4 \left(1 + \frac{bx^4}{a} \right)^p dx + \frac{1}{2} \left(e (a + bx^4)^p \left(1 + \frac{bx^4}{a} \right)^{-p} \right) \int x^6 \left(1 + \frac{bx^4}{a} \right)^p dx \\
&= \frac{c (a + bx^4)^{1+p}}{4b(1+p)} + \frac{1}{5} dx^5 (a + bx^4)^p \left(1 + \frac{bx^4}{a} \right)^{-p} {}_2F_1 \left(\frac{5}{4}, -p; \frac{9}{4}; -\frac{bx^4}{a} \right) + \frac{1}{6} ex^6 (a + bx^4)^p \left(1 + \frac{bx^4}{a} \right)^{-p} {}_2F_1 \left(\frac{3}{2}, -p; \frac{5}{2}; -\frac{bx^4}{a} \right)
\end{aligned}$$

Mathematica [A] time = 0.114865, size = 145, normalized size = 0.83

$$\frac{(a + bx^4)^p \left(\frac{bx^4}{a} + 1 \right)^{-p} \left(105c (a + bx^4) \left(\frac{bx^4}{a} + 1 \right)^p + 84bd(p + 1)x^5 {}_2F_1 \left(\frac{5}{4}, -p; \frac{9}{4}; -\frac{bx^4}{a} \right) + 70be(p + 1)x^6 {}_2F_1 \left(\frac{3}{2}, -p; \frac{5}{2}; -\frac{bx^4}{a} \right) \right)}{420b(p + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]

[Out] ((a + b*x^4)^p*(105*c*(a + b*x^4)*(1 + (b*x^4)/a)^p + 84*b*d*(1 + p)*x^5*Hypergeometric2F1[5/4, -p, 9/4, -((b*x^4)/a)] + 70*b*e*(1 + p)*x^6*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^4)/a)] + 60*b*f*(1 + p)*x^7*Hypergeometric2F1[7/4, -p, 11/4, -((b*x^4)/a)])/(420*b*(1 + p)*(1 + (b*x^4)/a)^p)

Maple [F] time = 0.222, size = 0, normalized size = 0.

$$\int x^3 (fx^3 + ex^2 + dx + c) (bx^4 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)

[Out] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(fx^6 + ex^5 + dx^4 + cx^3\right)\left(bx^4 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="fricas")`

[Out] `integral((f*x^6 + e*x^5 + d*x^4 + c*x^3)*(b*x^4 + a)^p, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**p,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx^3 + ex^2 + dx + c)(bx^4 + a)^p x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*x^3, x)
```

$$3.554 \quad \int \frac{1+x+x^2+x^3+x^4}{1-x^5} dx$$

Optimal. Leaf size=8

$$-\log(1-x)$$

[Out] -Log[1 - x]

Rubi [A] time = 0.0098654, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1586, 31}

$$-\log(1-x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^2 + x^3 + x^4)/(1 - x^5), x]

[Out] -Log[1 - x]

Rule 1586

Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p+q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\int \frac{1+x+x^2+x^3+x^4}{1-x^5} dx = \int \frac{1}{1-x} dx = -\log(1-x)$$

Mathematica [A] time = 0.0010466, size = 8, normalized size = 1.

$$-\log(1-x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + x^2 + x^3 + x^4)/(1 - x^5), x]

[Out] -Log[1 - x]

Maple [A] time = 0., size = 7, normalized size = 0.9

$$-\ln(-1 + x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+x^3+x^2+x+1)/(-x^5+1), x)

[Out] -ln(-1+x)

Maxima [A] time = 0.91599, size = 8, normalized size = 1.

$$-\log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+x^3+x^2+x+1)/(-x^5+1), x, algorithm="maxima")

[Out] -log(x - 1)

Fricas [A] time = 1.34621, size = 18, normalized size = 2.25

$$-\log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+x^3+x^2+x+1)/(-x^5+1), x, algorithm="fricas")

[Out] -log(x - 1)

Sympy [A] time = 0.059698, size = 5, normalized size = 0.62

$$-\log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+x**3+x**2+x+1)/(-x**5+1),x)

[Out] -log(x - 1)

Giac [A] time = 1.0536, size = 9, normalized size = 1.12

$$-\log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+x^3+x^2+x+1)/(-x^5+1),x, algorithm="giac")

[Out] -log(abs(x - 1))

$$3.555 \quad \int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6} dx$$

Optimal. Leaf size=10

$$\frac{1}{2} \log(2x + 3)$$

[Out] Log[3 + 2*x]/2

Rubi [A] time = 0.0146238, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1586, 31}

$$\frac{1}{2} \log(2x + 3)$$

Antiderivative was successfully verified.

[In] Int[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6), x]

[Out] Log[3 + 2*x]/2

Rule 1586

Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\begin{aligned} \int \frac{243 - 162x + 108x^2 - 72x^3 + 48x^4 - 32x^5}{729 - 64x^6} dx &= \int \frac{1}{3 + 2x} dx \\ &= \frac{1}{2} \log(3 + 2x) \end{aligned}$$

Mathematica [A] time = 0.0010186, size = 10, normalized size = 1.

$$\frac{1}{2} \log(2x + 3)$$

Antiderivative was successfully verified.

[In] Integrate[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6), x]

[Out] Log[3 + 2*x]/2

Maple [A] time = 0.001, size = 9, normalized size = 0.9

$$\frac{\ln(3 + 2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729), x)

[Out] 1/2*ln(3+2*x)

Maxima [A] time = 0.928414, size = 11, normalized size = 1.1

$$\frac{1}{2} \log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729), x, algorithm="maxima")

[Out] 1/2*log(2*x + 3)

Fricas [A] time = 1.34647, size = 24, normalized size = 2.4

$$\frac{1}{2} \log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729),x, algorithm="fricas")
```

```
[Out] 1/2*log(2*x + 3)
```

Sympy [A] time = 0.070157, size = 7, normalized size = 0.7

$$\frac{\log(2x + 3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-32*x**5+48*x**4-72*x**3+108*x**2-162*x+243)/(-64*x**6+729),x)
```

```
[Out] log(2*x + 3)/2
```

Giac [A] time = 1.05907, size = 12, normalized size = 1.2

$$\frac{1}{2} \log(|2x + 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(2*x + 3))
```

$$3.556 \quad \int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6} dx$$

Optimal. Leaf size=10

$$-\frac{1}{2} \log(3-2x)$$

[Out] -Log[3 - 2*x]/2

Rubi [A] time = 0.0133504, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1586, 31}

$$-\frac{1}{2} \log(3-2x)$$

Antiderivative was successfully verified.

[In] Int[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6), x]

[Out] -Log[3 - 2*x]/2

Rule 1586

Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p+q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\begin{aligned} \int \frac{243 + 162x + 108x^2 + 72x^3 + 48x^4 + 32x^5}{729 - 64x^6} dx &= \int \frac{1}{3 - 2x} dx \\ &= -\frac{1}{2} \log(3 - 2x) \end{aligned}$$

Mathematica [A] time = 0.0010264, size = 10, normalized size = 1.

$$-\frac{1}{2} \log(3 - 2x)$$

Antiderivative was successfully verified.

[In] Integrate[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6), x]

[Out] -Log[3 - 2*x]/2

Maple [A] time = 0.001, size = 9, normalized size = 0.9

$$-\frac{\ln(-3 + 2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729), x)

[Out] -1/2*ln(-3+2*x)

Maxima [A] time = 0.920725, size = 11, normalized size = 1.1

$$-\frac{1}{2} \log(2x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729), x, algorithm="maxima")

[Out] -1/2*log(2*x - 3)

Fricas [A] time = 1.35855, size = 26, normalized size = 2.6

$$-\frac{1}{2} \log(2x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729),x, algorithm="fricas")
```

```
[Out] -1/2*log(2*x - 3)
```

Sympy [A] time = 0.068114, size = 8, normalized size = 0.8

$$-\frac{\log(2x - 3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x**5+48*x**4+72*x**3+108*x**2+162*x+243)/(-64*x**6+729),x)
```

```
[Out] -log(2*x - 3)/2
```

Giac [A] time = 1.05218, size = 12, normalized size = 1.2

$$-\frac{1}{2} \log(|2x - 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(2*x - 3))
```

$$3.557 \quad \int \frac{81+36x^2+16x^4}{729-64x^6} dx$$

Optimal. Leaf size=10

$$\frac{1}{6} \tanh^{-1}\left(\frac{2x}{3}\right)$$

[Out] ArcTanh[(2*x)/3]/6

Rubi [A] time = 0.009933, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1586, 206}

$$\frac{1}{6} \tanh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

[In] Int[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6), x]

[Out] ArcTanh[(2*x)/3]/6

Rule 1586

Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p+q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{81 + 36x^2 + 16x^4}{729 - 64x^6} dx &= \int \frac{1}{9 - 4x^2} dx \\ &= \frac{1}{6} \tanh^{-1}\left(\frac{2x}{3}\right) \end{aligned}$$

Mathematica [B] time = 0.0026819, size = 21, normalized size = 2.1

$$\frac{1}{12} \log(2x + 3) - \frac{1}{12} \log(3 - 2x)$$

Antiderivative was successfully verified.

[In] Integrate[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6), x]

[Out] -Log[3 - 2*x]/12 + Log[3 + 2*x]/12

Maple [B] time = 0.006, size = 18, normalized size = 1.8

$$\frac{\ln(3 + 2x)}{12} - \frac{\ln(-3 + 2x)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((16*x^4+36*x^2+81)/(-64*x^6+729), x)

[Out] 1/12*ln(3+2*x)-1/12*ln(-3+2*x)

Maxima [B] time = 0.895089, size = 23, normalized size = 2.3

$$\frac{1}{12} \log(2x + 3) - \frac{1}{12} \log(2x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((16*x^4+36*x^2+81)/(-64*x^6+729), x, algorithm="maxima")

[Out] 1/12*log(2*x + 3) - 1/12*log(2*x - 3)

Fricas [B] time = 1.35319, size = 53, normalized size = 5.3

$$\frac{1}{12} \log(2x + 3) - \frac{1}{12} \log(2x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((16*x^4+36*x^2+81)/(-64*x^6+729),x, algorithm="fricas")
```

```
[Out] 1/12*log(2*x + 3) - 1/12*log(2*x - 3)
```

Sympy [B] time = 0.0933, size = 15, normalized size = 1.5

$$-\frac{\log\left(x - \frac{3}{2}\right)}{12} + \frac{\log\left(x + \frac{3}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x**4+36*x**2+81)/(-64*x**6+729),x)
```

```
[Out] -log(x - 3/2)/12 + log(x + 3/2)/12
```

Giac [B] time = 1.04777, size = 20, normalized size = 2.

$$\frac{1}{12} \log\left(\left|x + \frac{3}{2}\right|\right) - \frac{1}{12} \log\left(\left|x - \frac{3}{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x^4+36*x^2+81)/(-64*x^6+729),x, algorithm="giac")
```

```
[Out] 1/12*log(abs(x + 3/2)) - 1/12*log(abs(x - 3/2))
```

$$3.558 \quad \int \frac{81+54x-24x^3-16x^4}{729-64x^6} dx$$

Optimal. Leaf size=24

$$-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{3\sqrt{3}}$$

[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(3*Sqrt[3])

Rubi [A] time = 0.0237435, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$, Rules used = {1586, 618, 204}

$$-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6), x]

[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(3*Sqrt[3])

Rule 1586

Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p+q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{81 + 54x - 24x^3 - 16x^4}{729 - 64x^6} dx &= \int \frac{1}{9 - 6x + 4x^2} dx \\ &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{-108 - x^2} dx, x, -6 + 8x\right)\right) \\ &= -\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{3\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.0063597, size = 24, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6), x]

[Out] ArcTan[(-3 + 4*x)/(3*Sqrt[3])]/(3*Sqrt[3])

Maple [A] time = 0.003, size = 17, normalized size = 0.7

$$\frac{\sqrt{3}}{9} \arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729), x)

[Out] 1/9*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))

Maxima [A] time = 1.37219, size = 22, normalized size = 0.92

$$\frac{1}{9} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x-3)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729),x, algorithm="maxima")

[Out] 1/9*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3))

Fricas [A] time = 1.35482, size = 58, normalized size = 2.42

$$\frac{1}{9} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x - 3)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729),x, algorithm="fricas")

[Out] 1/9*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3))

Sympy [A] time = 0.114066, size = 24, normalized size = 1.

$$\frac{\sqrt{3} \operatorname{atan}\left(\frac{4\sqrt{3}x}{9} - \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x**4-24*x**3+54*x+81)/(-64*x**6+729),x)

[Out] sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/9

Giac [A] time = 1.05202, size = 22, normalized size = 0.92

$$\frac{1}{9} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x - 3)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729),x, algorithm="giac")

```
[Out] 1/9*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3))
```

$$3.559 \quad \int \frac{3-2x}{729-64x^6} dx$$

Optimal. Leaf size=50

$$-\frac{1}{972} \log(4x^2 - 6x + 9) + \frac{1}{486} \log(2x + 3) + \frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{162\sqrt{3}}$$

[Out] ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(162*Sqrt[3]) + Log[3 + 2*x]/486 - Log[9 - 6*x + 4*x^2]/972

Rubi [A] time = 0.049007, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1586, 2058, 628, 618, 204}

$$-\frac{1}{972} \log(4x^2 - 6x + 9) + \frac{1}{486} \log(2x + 3) + \frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{162\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(3 - 2*x)/(729 - 64*x^6), x]

[Out] ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(162*Sqrt[3]) + Log[3 + 2*x]/486 - Log[9 - 6*x + 4*x^2]/972

Rule 1586

Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 2058

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{3-2x}{729-64x^6} dx &= \int \frac{1}{243+162x+108x^2+72x^3+48x^4+32x^5} dx \\
 &= \int \left(\frac{1}{243(3+2x)} + \frac{3-4x}{486(9-6x+4x^2)} + \frac{1}{54(9+6x+4x^2)} \right) dx \\
 &= \frac{1}{486} \log(3+2x) + \frac{1}{486} \int \frac{3-4x}{9-6x+4x^2} dx + \frac{1}{54} \int \frac{1}{9+6x+4x^2} dx \\
 &= \frac{1}{486} \log(3+2x) - \frac{1}{972} \log(9-6x+4x^2) - \frac{1}{27} \text{Subst} \left(\int \frac{1}{-108-x^2} dx, x, 6+8x \right) \\
 &= \frac{\tan^{-1} \left(\frac{3+4x}{3\sqrt{3}} \right)}{162\sqrt{3}} + \frac{1}{486} \log(3+2x) - \frac{1}{972} \log(9-6x+4x^2)
 \end{aligned}$$

Mathematica [A] time = 0.0169689, size = 50, normalized size = 1.

$$-\frac{1}{972} \log(4x^2 - 6x + 9) + \frac{1}{486} \log(2x + 3) + \frac{\tan^{-1} \left(\frac{4x+3}{3\sqrt{3}} \right)}{162\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - 2*x)/(729 - 64*x^6), x]

[Out] ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(162*Sqrt[3]) + Log[3 + 2*x]/486 - Log[9 - 6*x + 4*x^2]/972

Maple [A] time = 0.007, size = 39, normalized size = 0.8

$$\frac{\ln(3+2x)}{486} + \frac{\sqrt{3}}{486} \arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right) - \frac{\ln(4x^2-6x+9)}{972}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3-2*x)/(-64*x^6+729),x)

[Out] 1/486*ln(3+2*x)+1/486*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))-1/972*ln(4*x^2-6*x+9)

Maxima [A] time = 1.38798, size = 51, normalized size = 1.02

$$\frac{1}{486} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x+3)\right) - \frac{1}{972} \log(4x^2-6x+9) + \frac{1}{486} \log(2x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3-2*x)/(-64*x^6+729),x, algorithm="maxima")

[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/972*log(4*x^2 - 6*x + 9) + 1/486*log(2*x + 3)

Fricas [A] time = 1.43269, size = 128, normalized size = 2.56

$$\frac{1}{486} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x+3)\right) - \frac{1}{972} \log(4x^2-6x+9) + \frac{1}{486} \log(2x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3-2*x)/(-64*x^6+729),x, algorithm="fricas")

[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/972*log(4*x^2 - 6*x + 9) + 1/486*log(2*x + 3)

Sympy [A] time = 0.162269, size = 46, normalized size = 0.92

$$\frac{\log\left(x + \frac{3}{2}\right)}{486} - \frac{\log(4x^2 - 6x + 9)}{972} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{4\sqrt{3}x}{9} + \frac{\sqrt{3}}{3}\right)}{486}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3-2*x)/(-64*x**6+729),x)

[Out] log(x + 3/2)/486 - log(4*x**2 - 6*x + 9)/972 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/486

Giac [A] time = 1.08276, size = 53, normalized size = 1.06

$$\frac{1}{486} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x + 3)\right) - \frac{1}{972} \log(4x^2 - 6x + 9) + \frac{1}{486} \log(|2x + 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3-2*x)/(-64*x^6+729),x, algorithm="giac")

[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/972*log(4*x^2 - 6*x + 9) + 1/486*log(abs(2*x + 3))

$$3.560 \quad \int \frac{3+2x}{729-64x^6} dx$$

Optimal. Leaf size=50

$$\frac{1}{972} \log(4x^2 + 6x + 9) - \frac{1}{486} \log(3 - 2x) - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{162\sqrt{3}}$$

[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(162*Sqrt[3]) - Log[3 - 2*x]/486 + Log[9 + 6*x + 4*x^2]/972

Rubi [A] time = 0.0481794, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1586, 2058, 618, 204, 628}

$$\frac{1}{972} \log(4x^2 + 6x + 9) - \frac{1}{486} \log(3 - 2x) - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{162\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(3 + 2*x)/(729 - 64*x^6), x]

[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(162*Sqrt[3]) - Log[3 - 2*x]/486 + Log[9 + 6*x + 4*x^2]/972

Rule 1586

Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 2058

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

$x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 204

$\text{Int}[\frac{(a_.) + (b_.)(x_)^2}{(a_. + (b_.)(x_) + (c_.)(x_)^2)}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[\frac{\text{Rt}[-b, 2]*x}{\text{Rt}[-a, 2]}] / (\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] / ; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{LtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rule 628

$\text{Int}[\frac{(d_.) + (e_.)(x_)}{(a_.) + (b_.)(x_) + (c_.)(x_)^2}, x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] / ; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

Rubi steps

$$\begin{aligned} \int \frac{3+2x}{729-64x^6} dx &= \int \frac{1}{243-162x+108x^2-72x^3+48x^4-32x^5} dx \\ &= \int \left(-\frac{1}{243(-3+2x)} + \frac{1}{54(9-6x+4x^2)} + \frac{3+4x}{486(9+6x+4x^2)} \right) dx \\ &= -\frac{1}{486} \log(3-2x) + \frac{1}{486} \int \frac{3+4x}{9+6x+4x^2} dx + \frac{1}{54} \int \frac{1}{9-6x+4x^2} dx \\ &= -\frac{1}{486} \log(3-2x) + \frac{1}{972} \log(9+6x+4x^2) - \frac{1}{27} \text{Subst} \left(\int \frac{1}{-108-x^2} dx, x, -6+8x \right) \\ &= -\frac{\tan^{-1} \left(\frac{3-4x}{3\sqrt{3}} \right)}{162\sqrt{3}} - \frac{1}{486} \log(3-2x) + \frac{1}{972} \log(9+6x+4x^2) \end{aligned}$$

Mathematica [A] time = 0.0134169, size = 46, normalized size = 0.92

$$\frac{1}{972} \left(\log(4x^2 + 6x + 9) - 2 \log(3 - 2x) + 2\sqrt{3} \tan^{-1} \left(\frac{4x - 3}{3\sqrt{3}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 2*x)/(729 - 64*x^6), x]

[Out] (2*sqrt[3]*ArcTan[(-3 + 4*x)/(3*sqrt[3])] - 2*Log[3 - 2*x] + Log[9 + 6*x + 4*x^2])/972

Maple [A] time = 0.007, size = 39, normalized size = 0.8

$$-\frac{\ln(-3+2x)}{486} + \frac{\ln(4x^2+6x+9)}{972} + \frac{\sqrt{3}}{486} \arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3+2*x)/(-64*x^6+729),x)

[Out] -1/486*ln(-3+2*x)+1/972*ln(4*x^2+6*x+9)+1/486*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))

Maxima [A] time = 1.3875, size = 51, normalized size = 1.02

$$\frac{1}{486} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x-3)\right) + \frac{1}{972} \log(4x^2+6x+9) - \frac{1}{486} \log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+2*x)/(-64*x^6+729),x, algorithm="maxima")

[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/972*log(4*x^2 + 6*x + 9) - 1/486*log(2*x - 3)

Fricas [A] time = 1.40525, size = 128, normalized size = 2.56

$$\frac{1}{486} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x-3)\right) + \frac{1}{972} \log(4x^2+6x+9) - \frac{1}{486} \log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+2*x)/(-64*x^6+729),x, algorithm="fricas")

[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/972*log(4*x^2 + 6*x + 9) - 1/486*log(2*x - 3)

Sympy [A] time = 0.16509, size = 46, normalized size = 0.92

$$-\frac{\log\left(x - \frac{3}{2}\right)}{486} + \frac{\log(4x^2 + 6x + 9)}{972} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{4\sqrt{3}x}{9} - \frac{\sqrt{3}}{3}\right)}{486}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+2*x)/(-64*x**6+729),x)

[Out] -log(x - 3/2)/486 + log(4*x**2 + 6*x + 9)/972 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/486

Giac [A] time = 1.04905, size = 53, normalized size = 1.06

$$\frac{1}{486} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x - 3)\right) + \frac{1}{972} \log(4x^2 + 6x + 9) - \frac{1}{486} \log(|2x - 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+2*x)/(-64*x^6+729),x, algorithm="giac")

[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/972*log(4*x^2 + 6*x + 9) - 1/486*log(abs(2*x - 3))

$$3.561 \quad \int \frac{9-6x+4x^2}{729-64x^6} dx$$

Optimal. Leaf size=60

$$-\frac{1}{324} \log(4x^2 + 6x + 9) - \frac{1}{324} \log(3 - 2x) + \frac{1}{108} \log(2x + 3) + \frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{54\sqrt{3}}$$

[Out] ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(54*Sqrt[3]) - Log[3 - 2*x]/324 + Log[3 + 2*x]/108 - Log[9 + 6*x + 4*x^2]/324

Rubi [A] time = 0.0512375, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {1586, 2058, 634, 618, 204, 628}

$$-\frac{1}{324} \log(4x^2 + 6x + 9) - \frac{1}{324} \log(3 - 2x) + \frac{1}{108} \log(2x + 3) + \frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{54\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(9 - 6*x + 4*x^2)/(729 - 64*x^6), x]

[Out] ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(54*Sqrt[3]) - Log[3 - 2*x]/324 + Log[3 + 2*x]/108 - Log[9 + 6*x + 4*x^2]/324

Rule 1586

Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 2058

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ

`[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rule 618

`Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

Rule 204

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

Rule 628

`Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`

Rubi steps

$$\begin{aligned}
 \int \frac{9 - 6x + 4x^2}{729 - 64x^6} dx &= \int \frac{1}{81 + 54x - 24x^3 - 16x^4} dx \\
 &= \int \left(-\frac{1}{162(-3 + 2x)} + \frac{1}{54(3 + 2x)} + \frac{3 - 2x}{81(9 + 6x + 4x^2)} \right) dx \\
 &= -\frac{1}{324} \log(3 - 2x) + \frac{1}{108} \log(3 + 2x) + \frac{1}{81} \int \frac{3 - 2x}{9 + 6x + 4x^2} dx \\
 &= -\frac{1}{324} \log(3 - 2x) + \frac{1}{108} \log(3 + 2x) - \frac{1}{324} \int \frac{6 + 8x}{9 + 6x + 4x^2} dx + \frac{1}{18} \int \frac{1}{9 + 6x + 4x^2} dx \\
 &= -\frac{1}{324} \log(3 - 2x) + \frac{1}{108} \log(3 + 2x) - \frac{1}{324} \log(9 + 6x + 4x^2) - \frac{1}{9} \text{Subst} \left(\int \frac{1}{-108 - x^2} dx, x, 6x + 9 \right) \\
 &= \frac{\tan^{-1} \left(\frac{3+4x}{3\sqrt{3}} \right)}{54\sqrt{3}} - \frac{1}{324} \log(3 - 2x) + \frac{1}{108} \log(3 + 2x) - \frac{1}{324} \log(9 + 6x + 4x^2)
 \end{aligned}$$

Mathematica [A] time = 0.0126314, size = 56, normalized size = 0.93

$$\frac{1}{324} \left(-\log(4x^2 + 6x + 9) - \log(3 - 2x) + 3 \log(2x + 3) + 2\sqrt{3} \tan^{-1} \left(\frac{4x + 3}{3\sqrt{3}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(9 - 6*x + 4*x^2)/(729 - 64*x^6),x]

[Out] (2*sqrt(3)*ArcTan[(3 + 4*x)/(3*sqrt(3))] - Log[3 - 2*x] + 3*Log[3 + 2*x] - Log[9 + 6*x + 4*x^2])/324

Maple [A] time = 0.007, size = 47, normalized size = 0.8

$$\frac{\ln(3+2x)}{108} - \frac{\ln(-3+2x)}{324} - \frac{\ln(4x^2+6x+9)}{324} + \frac{\sqrt{3}}{162} \arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^2-6*x+9)/(-64*x^6+729),x)

[Out] 1/108*ln(3+2*x)-1/324*ln(-3+2*x)-1/324*ln(4*x^2+6*x+9)+1/162*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))

Maxima [A] time = 1.37365, size = 62, normalized size = 1.03

$$\frac{1}{162} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x+3)\right) - \frac{1}{324} \log(4x^2+6x+9) + \frac{1}{108} \log(2x+3) - \frac{1}{324} \log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2-6*x+9)/(-64*x^6+729),x, algorithm="maxima")

[Out] 1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/324*log(4*x^2 + 6*x + 9) + 1/108*log(2*x + 3) - 1/324*log(2*x - 3)

Fricas [A] time = 1.43479, size = 157, normalized size = 2.62

$$\frac{1}{162} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x+3)\right) - \frac{1}{324} \log(4x^2+6x+9) + \frac{1}{108} \log(2x+3) - \frac{1}{324} \log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2-6*x+9)/(-64*x^6+729),x, algorithm="fricas")

[Out] $\frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) - \frac{1}{324}\log(4x^2+6x+9) + \frac{1}{108}\log(2x+3) - \frac{1}{324}\log(2x-3)$

Sympy [A] time = 0.181276, size = 56, normalized size = 0.93

$$-\frac{\log\left(x-\frac{3}{2}\right)}{324} + \frac{\log\left(x+\frac{3}{2}\right)}{108} - \frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{324} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{162}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x**2-6*x+9)/(-64*x**6+729),x)`

[Out] $-\log(x-3/2)/324 + \log(x+3/2)/108 - \log(x^2+3x/2+9/4)/324 + \sqrt{3}\operatorname{atan}(4\sqrt{3}x/9 + \sqrt{3}/3)/162$

Giac [A] time = 1.04567, size = 65, normalized size = 1.08

$$\frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) - \frac{1}{324}\log(4x^2+6x+9) + \frac{1}{108}\log(|2x+3|) - \frac{1}{324}\log(|2x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x^2-6*x+9)/(-64*x^6+729),x, algorithm="giac")`

[Out] $\frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) - \frac{1}{324}\log(4x^2+6x+9) + \frac{1}{108}\log(\operatorname{abs}(2x+3)) - \frac{1}{324}\log(\operatorname{abs}(2x-3))$

$$3.562 \quad \int \frac{9+6x+4x^2}{729-64x^6} dx$$

Optimal. Leaf size=60

$$\frac{1}{324} \log(4x^2 - 6x + 9) - \frac{1}{108} \log(3 - 2x) + \frac{1}{324} \log(2x + 3) - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{54\sqrt{3}}$$

[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(54*Sqrt[3]) - Log[3 - 2*x]/108 + Log[3 + 2*x]/324 + Log[9 - 6*x + 4*x^2]/324

Rubi [A] time = 0.0485636, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {1586, 2058, 634, 618, 204, 628}

$$\frac{1}{324} \log(4x^2 - 6x + 9) - \frac{1}{108} \log(3 - 2x) + \frac{1}{324} \log(2x + 3) - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{54\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(9 + 6*x + 4*x^2)/(729 - 64*x^6),x]

[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(54*Sqrt[3]) - Log[3 - 2*x]/108 + Log[3 + 2*x]/324 + Log[9 - 6*x + 4*x^2]/324

Rule 1586

Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 2058

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ

`[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rule 618

`Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

Rule 204

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

Rule 628

`Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`

Rubi steps

$$\begin{aligned}
 \int \frac{9 + 6x + 4x^2}{729 - 64x^6} dx &= \int \frac{1}{81 - 54x + 24x^3 - 16x^4} dx \\
 &= \int \left(-\frac{1}{54(-3 + 2x)} + \frac{1}{162(3 + 2x)} + \frac{3 + 2x}{81(9 - 6x + 4x^2)} \right) dx \\
 &= -\frac{1}{108} \log(3 - 2x) + \frac{1}{324} \log(3 + 2x) + \frac{1}{81} \int \frac{3 + 2x}{9 - 6x + 4x^2} dx \\
 &= -\frac{1}{108} \log(3 - 2x) + \frac{1}{324} \log(3 + 2x) + \frac{1}{324} \int \frac{-6 + 8x}{9 - 6x + 4x^2} dx + \frac{1}{18} \int \frac{1}{9 - 6x + 4x^2} dx \\
 &= -\frac{1}{108} \log(3 - 2x) + \frac{1}{324} \log(3 + 2x) + \frac{1}{324} \log(9 - 6x + 4x^2) - \frac{1}{9} \text{Subst} \left(\int \frac{1}{-108 - x^2} dx, x, -6x + 9 \right) \\
 &= -\frac{\tan^{-1} \left(\frac{3-4x}{3\sqrt{3}} \right)}{54\sqrt{3}} - \frac{1}{108} \log(3 - 2x) + \frac{1}{324} \log(3 + 2x) + \frac{1}{324} \log(9 - 6x + 4x^2)
 \end{aligned}$$

Mathematica [A] time = 0.0108217, size = 52, normalized size = 0.87

$$\frac{1}{324} \left(\log(4x^2 - 6x + 9) - 3 \log(3 - 2x) + \log(2x + 3) + 2\sqrt{3} \tan^{-1} \left(\frac{4x - 3}{3\sqrt{3}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(9 + 6*x + 4*x^2)/(729 - 64*x^6),x]

[Out] (2*sqrt(3)*ArcTan[(-3 + 4*x)/(3*sqrt(3))] - 3*Log[3 - 2*x] + Log[3 + 2*x] + Log[9 - 6*x + 4*x^2])/324

Maple [A] time = 0.006, size = 47, normalized size = 0.8

$$\frac{\ln(3+2x)}{324} - \frac{\ln(-3+2x)}{108} + \frac{\ln(4x^2-6x+9)}{324} + \frac{\sqrt{3}}{162} \arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^2+6*x+9)/(-64*x^6+729),x)

[Out] 1/324*ln(3+2*x)-1/108*ln(-3+2*x)+1/324*ln(4*x^2-6*x+9)+1/162*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))

Maxima [A] time = 1.38407, size = 62, normalized size = 1.03

$$\frac{1}{162} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x-3)\right) + \frac{1}{324} \log(4x^2-6x+9) + \frac{1}{324} \log(2x+3) - \frac{1}{108} \log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2+6*x+9)/(-64*x^6+729),x, algorithm="maxima")

[Out] 1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/324*log(4*x^2 - 6*x + 9) + 1/324*log(2*x + 3) - 1/108*log(2*x - 3)

Fricas [A] time = 1.43353, size = 157, normalized size = 2.62

$$\frac{1}{162} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x-3)\right) + \frac{1}{324} \log(4x^2-6x+9) + \frac{1}{324} \log(2x+3) - \frac{1}{108} \log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2+6*x+9)/(-64*x^6+729),x, algorithm="fricas")

[Out] $\frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) + \frac{1}{324}\log(4x^2-6x+9) + \frac{1}{324}\log(2x+3) - \frac{1}{108}\log(2x-3)$

Sympy [A] time = 0.179838, size = 56, normalized size = 0.93

$$-\frac{\log\left(x-\frac{3}{2}\right)}{108} + \frac{\log\left(x+\frac{3}{2}\right)}{324} + \frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{324} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{162}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x**2+6*x+9)/(-64*x**6+729),x)`

[Out] $-\log(x-3/2)/108 + \log(x+3/2)/324 + \log(x^2-3x/2+9/4)/324 + \sqrt{3}\operatorname{atan}(4\sqrt{3}x/9-\sqrt{3}/3)/162$

Giac [A] time = 1.072, size = 65, normalized size = 1.08

$$\frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) + \frac{1}{324}\log(4x^2-6x+9) + \frac{1}{324}\log(|2x+3|) - \frac{1}{108}\log(|2x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x^2+6*x+9)/(-64*x^6+729),x, algorithm="giac")`

[Out] $\frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) + \frac{1}{324}\log(4x^2-6x+9) + \frac{1}{324}\log(\operatorname{abs}(2x+3)) - \frac{1}{108}\log(\operatorname{abs}(2x-3))$

$$3.563 \quad \int \frac{27-8x^3}{729-64x^6} dx$$

Optimal. Leaf size=50

$$-\frac{1}{108} \log(4x^2 - 6x + 9) + \frac{1}{54} \log(2x + 3) - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}}$$

[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(18*Sqrt[3]) + Log[3 + 2*x]/54 - Log[9 - 6*x + 4*x^2]/108

Rubi [A] time = 0.0262205, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {26, 200, 31, 634, 618, 204, 628}

$$-\frac{1}{108} \log(4x^2 - 6x + 9) + \frac{1}{54} \log(2x + 3) - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(27 - 8*x^3)/(729 - 64*x^6), x]

[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(18*Sqrt[3]) + Log[3 + 2*x]/54 - Log[9 - 6*x + 4*x^2]/108

Rule 26

Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_.))^(p_.), x_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && GtQ[a, 0] && LtQ[d, 0]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 31

Int[((a_) + (b_)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^-1, x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{27 - 8x^3}{729 - 64x^6} dx &= \int \frac{1}{27 + 8x^3} dx \\
 &= \frac{1}{27} \int \frac{1}{3 + 2x} dx + \frac{1}{27} \int \frac{6 - 2x}{9 - 6x + 4x^2} dx \\
 &= \frac{1}{54} \log(3 + 2x) - \frac{1}{108} \int \frac{-6 + 8x}{9 - 6x + 4x^2} dx + \frac{1}{6} \int \frac{1}{9 - 6x + 4x^2} dx \\
 &= \frac{1}{54} \log(3 + 2x) - \frac{1}{108} \log(9 - 6x + 4x^2) - \frac{1}{3} \text{Subst} \left(\int \frac{1}{-108 - x^2} dx, x, -6 + 8x \right) \\
 &= -\frac{\tan^{-1} \left(\frac{3-4x}{3\sqrt{3}} \right)}{18\sqrt{3}} + \frac{1}{54} \log(3 + 2x) - \frac{1}{108} \log(9 - 6x + 4x^2)
 \end{aligned}$$

Mathematica [A] time = 0.0056041, size = 50, normalized size = 1.

$$-\frac{1}{108} \log(4x^2 - 6x + 9) + \frac{1}{54} \log(2x + 3) + \frac{\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)}{18\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(27 - 8*x^3)/(729 - 64*x^6), x]

[Out] ArcTan[(-3 + 4*x)/(3*Sqrt[3])]/(18*Sqrt[3]) + Log[3 + 2*x]/54 - Log[9 - 6*x + 4*x^2]/108

Maple [A] time = 0.005, size = 39, normalized size = 0.8

$$\frac{\ln(3 + 2x)}{54} - \frac{\ln(4x^2 - 6x + 9)}{108} + \frac{\sqrt{3}}{54} \arctan\left(\frac{(8x - 6)\sqrt{3}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-8*x^3+27)/(-64*x^6+729), x)

[Out] 1/54*ln(3+2*x)-1/108*ln(4*x^2-6*x+9)+1/54*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))

Maxima [A] time = 1.37319, size = 51, normalized size = 1.02

$$\frac{1}{54} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x - 3)\right) - \frac{1}{108} \log(4x^2 - 6x + 9) + \frac{1}{54} \log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-8*x^3+27)/(-64*x^6+729), x, algorithm="maxima")

[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/108*log(4*x^2 - 6*x + 9) + 1/54*log(2*x + 3)

Fricas [A] time = 1.3748, size = 126, normalized size = 2.52

$$\frac{1}{54} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x-3)\right) - \frac{1}{108} \log(4x^2 - 6x + 9) + \frac{1}{54} \log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-8*x^3+27)/(-64*x^6+729),x, algorithm="fricas")

[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/108*log(4*x^2 - 6*x + 9) + 1/54*log(2*x + 3)

Sympy [A] time = 0.138369, size = 48, normalized size = 0.96

$$\frac{\log\left(x + \frac{3}{2}\right)}{54} - \frac{\log\left(x^2 - \frac{3x}{2} + \frac{9}{4}\right)}{108} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{4\sqrt{3}x}{9} - \frac{\sqrt{3}}{3}\right)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-8*x**3+27)/(-64*x**6+729),x)

[Out] log(x + 3/2)/54 - log(x**2 - 3*x/2 + 9/4)/108 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/54

Giac [A] time = 1.04442, size = 47, normalized size = 0.94

$$\frac{1}{54} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x-3)\right) - \frac{1}{108} \log\left(x^2 - \frac{3}{2}x + \frac{9}{4}\right) + \frac{1}{54} \log\left(\left|x + \frac{3}{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-8*x^3+27)/(-64*x^6+729),x, algorithm="giac")

[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/108*log(x^2 - 3/2*x + 9/4) + 1/54*log(abs(x + 3/2))

$$3.564 \quad \int \frac{27+36x+24x^2+8x^3}{729-64x^6} dx$$

Optimal. Leaf size=50

$$\frac{1}{36} \log(4x^2 - 6x + 9) - \frac{1}{18} \log(3 - 2x) - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}}$$

[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(18*Sqrt[3]) - Log[3 - 2*x]/18 + Log[9 - 6*x + 4*x^2]/36

Rubi [A] time = 0.0428354, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {1586, 2058, 634, 618, 204, 628}

$$\frac{1}{36} \log(4x^2 - 6x + 9) - \frac{1}{18} \log(3 - 2x) - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6), x]

[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(18*Sqrt[3]) - Log[3 - 2*x]/18 + Log[9 - 6*x + 4*x^2]/36

Rule 1586

Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 2058

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ

`[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rule 618

`Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

Rule 204

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

Rule 628

`Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`

Rubi steps

$$\begin{aligned}
 \int \frac{27 + 36x + 24x^2 + 8x^3}{729 - 64x^6} dx &= \int \frac{1}{27 - 36x + 24x^2 - 8x^3} dx \\
 &= \int \left(-\frac{1}{9(-3 + 2x)} + \frac{2x}{9(9 - 6x + 4x^2)} \right) dx \\
 &= -\frac{1}{18} \log(3 - 2x) + \frac{2}{9} \int \frac{x}{9 - 6x + 4x^2} dx \\
 &= -\frac{1}{18} \log(3 - 2x) + \frac{1}{36} \int \frac{-6 + 8x}{9 - 6x + 4x^2} dx + \frac{1}{6} \int \frac{1}{9 - 6x + 4x^2} dx \\
 &= -\frac{1}{18} \log(3 - 2x) + \frac{1}{36} \log(9 - 6x + 4x^2) - \frac{1}{3} \text{Subst} \left(\int \frac{1}{-108 - x^2} dx, x, -6 + 8x \right) \\
 &= -\frac{\tan^{-1} \left(\frac{3-4x}{3\sqrt{3}} \right)}{18\sqrt{3}} - \frac{1}{18} \log(3 - 2x) + \frac{1}{36} \log(9 - 6x + 4x^2)
 \end{aligned}$$

Mathematica [A] time = 0.0097146, size = 50, normalized size = 1.

$$\frac{1}{36} \log(4x^2 - 6x + 9) - \frac{1}{18} \log(3 - 2x) + \frac{\tan^{-1} \left(\frac{4x-3}{3\sqrt{3}} \right)}{18\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6), x]

[Out] ArcTan[(-3 + 4*x)/(3*Sqrt[3])]/(18*Sqrt[3]) - Log[3 - 2*x]/18 + Log[9 - 6*x + 4*x^2]/36

Maple [A] time = 0.004, size = 39, normalized size = 0.8

$$-\frac{\ln(-3 + 2x)}{18} + \frac{\ln(4x^2 - 6x + 9)}{36} + \frac{\sqrt{3}}{54} \arctan\left(\frac{(8x - 6)\sqrt{3}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*x^3+24*x^2+36*x+27)/(-64*x^6+729), x)

[Out] -1/18*ln(-3+2*x)+1/36*ln(4*x^2-6*x+9)+1/54*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))

Maxima [A] time = 1.36732, size = 51, normalized size = 1.02

$$\frac{1}{54} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x - 3)\right) + \frac{1}{36} \log(4x^2 - 6x + 9) - \frac{1}{18} \log(2x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729), x, algorithm="maxima")

[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/36*log(4*x^2 - 6*x + 9) - 1/18*log(2*x - 3)

Fricas [A] time = 1.41497, size = 124, normalized size = 2.48

$$\frac{1}{54} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x - 3)\right) + \frac{1}{36} \log(4x^2 - 6x + 9) - \frac{1}{18} \log(2x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729),x, algorithm="fricas")

[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/36*log(4*x^2 - 6*x + 9) - 1/18*log(2*x - 3)

Sympy [A] time = 0.145883, size = 48, normalized size = 0.96

$$-\frac{\log\left(x - \frac{3}{2}\right)}{18} + \frac{\log\left(x^2 - \frac{3x}{2} + \frac{9}{4}\right)}{36} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{4\sqrt{3}x}{9} - \frac{\sqrt{3}}{3}\right)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x**3+24*x**2+36*x+27)/(-64*x**6+729),x)

[Out] -log(x - 3/2)/18 + log(x**2 - 3*x/2 + 9/4)/36 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/54

Giac [A] time = 1.06169, size = 53, normalized size = 1.06

$$\frac{1}{54} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x - 3)\right) + \frac{1}{36} \log(4x^2 - 6x + 9) - \frac{1}{18} \log(|2x - 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729),x, algorithm="giac")

[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/36*log(4*x^2 - 6*x + 9) - 1/18*log(abs(2*x - 3))

$$3.565 \quad \int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{(729-64x^6)^2} dx$$

Optimal. Leaf size=110

$$-\frac{\log(4x^2-6x+9)}{17496} - \frac{\log(4x^2+6x+9)}{17496} - \frac{1}{2916(2x+3)} - \frac{\log(3-2x)}{17496} + \frac{5\log(2x+3)}{17496} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{8748\sqrt{3}} + \frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{2916\sqrt{3}}$$

[Out] -1/(2916*(3 + 2*x)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(8748*Sqrt[3]) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(2916*Sqrt[3]) - Log[3 - 2*x]/17496 + (5*Log[3 + 2*x])/17496 - Log[9 - 6*x + 4*x^2]/17496 - Log[9 + 6*x + 4*x^2]/17496

Rubi [A] time = 0.118137, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {1586, 2074, 634, 618, 204, 628}

$$-\frac{\log(4x^2-6x+9)}{17496} - \frac{\log(4x^2+6x+9)}{17496} - \frac{1}{2916(2x+3)} - \frac{\log(3-2x)}{17496} + \frac{5\log(2x+3)}{17496} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{8748\sqrt{3}} + \frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{2916\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6)^2,x]

[Out] -1/(2916*(3 + 2*x)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(8748*Sqrt[3]) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(2916*Sqrt[3]) - Log[3 - 2*x]/17496 + (5*Log[3 + 2*x])/17496 - Log[9 - 6*x + 4*x^2]/17496 - Log[9 + 6*x + 4*x^2]/17496

Rule 1586

Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p+q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 2074

Int[(P_)^(p_)*(Q_)^(q_), x_Symbol] := With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] && PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{243 - 162x + 108x^2 - 72x^3 + 48x^4 - 32x^5}{(729 - 64x^6)^2} dx &= \int \frac{1}{(3 + 2x)^2 (243 - 162x + 108x^2 - 72x^3 + 48x^4 - 32x^5)} dx \\
 &= \int \left(-\frac{1}{8748(-3 + 2x)} + \frac{1}{1458(3 + 2x)^2} + \frac{5}{8748(3 + 2x)} + \frac{3}{4374(9 - 6x + 4x^2)} \right) dx \\
 &= -\frac{1}{2916(3 + 2x)} - \frac{\log(3 - 2x)}{17496} + \frac{5 \log(3 + 2x)}{17496} + \frac{\int \frac{3-2x}{9-6x+4x^2} dx}{4374} + \frac{\int \frac{3-2x}{9-6x+4x^2} dx}{4374} \\
 &= -\frac{1}{2916(3 + 2x)} - \frac{\log(3 - 2x)}{17496} + \frac{5 \log(3 + 2x)}{17496} - \frac{\int \frac{-6+8x}{9-6x+4x^2} dx}{17496} - \frac{\int \frac{-6+8x}{9-6x+4x^2} dx}{17496} \\
 &= -\frac{1}{2916(3 + 2x)} - \frac{\log(3 - 2x)}{17496} + \frac{5 \log(3 + 2x)}{17496} - \frac{\log(9 - 6x + 4x^2)}{17496} \\
 &= -\frac{1}{2916(3 + 2x)} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{8748\sqrt{3}} + \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{2916\sqrt{3}} - \frac{\log(3 - 2x)}{17496} + \frac{5 \log(3 + 2x)}{17496}
 \end{aligned}$$

Mathematica [A] time = 0.0872429, size = 100, normalized size = 0.91

$$\frac{-3 \log(4x^2 - 6x + 9) - 3 \log(4x^2 + 6x + 9) - \frac{18}{2x+3} - 3 \log(3 - 2x) + 15 \log(2x + 3) + 2\sqrt{3} \tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right) + 6\sqrt{3} \tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{52488}$$

Antiderivative was successfully verified.

[In] Integrate[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6)^2, x]

[Out] (-18/(3 + 2*x) + 2*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] + 6*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 3*Log[3 - 2*x] + 15*Log[3 + 2*x] - 3*Log[9 - 6*x + 4*x^2] - 3*Log[9 + 6*x + 4*x^2])/52488

Maple [A] time = 0.01, size = 85, normalized size = 0.8

$$-\frac{1}{8748 + 5832x} + \frac{5 \ln(3 + 2x)}{17496} - \frac{\ln(-3 + 2x)}{17496} - \frac{\ln(4x^2 + 6x + 9)}{17496} + \frac{\sqrt{3}}{8748} \arctan\left(\frac{(8x + 6)\sqrt{3}}{18}\right) - \frac{\ln(4x^2 - 6x + 9)}{17496}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729)^2, x)

[Out] -1/2916/(3+2*x)+5/17496*ln(3+2*x)-1/17496*ln(-3+2*x)-1/17496*ln(4*x^2+6*x+9)+1/8748*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))-1/17496*ln(4*x^2-6*x+9)+1/26244*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))

Maxima [A] time = 1.40531, size = 113, normalized size = 1.03

$$\frac{1}{8748} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x + 3)\right) + \frac{1}{26244} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x - 3)\right) - \frac{1}{2916(2x + 3)} - \frac{1}{17496} \log(4x^2 + 6x + 9) - \frac{1}{17496}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729)^2, x, algorithm="maxima")

[Out] 1/8748*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/26244*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/2916/(2*x + 3) - 1/17496*log(4*x^2 + 6*x + 9) - 1/17496

$$96 \log(4x^2 - 6x + 9) + 5/17496 \log(2x + 3) - 1/17496 \log(2x - 3)$$

Fricas [A] time = 1.49653, size = 342, normalized size = 3.11

$$\frac{6\sqrt{3}(2x+3)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) + 2\sqrt{3}(2x+3)\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) - 3(2x+3)\log(4x^2+6x+9) - 3(2x-3)\log(4x^2-6x+9) + 15(2x+3)\log(2x+3) - 3(2x+3)\log(2x-3) - 18}{52488(2x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729)^2,x, algorith="fricas")

[Out] 1/52488*(6*sqrt(3)*(2*x + 3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 2*sqrt(3)*(2*x + 3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 3*(2*x + 3)*log(4*x^2 + 6*x + 9) - 3*(2*x + 3)*log(4*x^2 - 6*x + 9) + 15*(2*x + 3)*log(2*x + 3) - 3*(2*x + 3)*log(2*x - 3) - 18)/(2*x + 3)

Sympy [A] time = 0.358786, size = 105, normalized size = 0.95

$$-\frac{\log\left(x - \frac{3}{2}\right)}{17496} + \frac{5\log\left(x + \frac{3}{2}\right)}{17496} - \frac{\log\left(x^2 - \frac{3x}{2} + \frac{9}{4}\right)}{17496} - \frac{\log\left(x^2 + \frac{3x}{2} + \frac{9}{4}\right)}{17496} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9} - \frac{\sqrt{3}}{3}\right)}{26244} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9} + \frac{\sqrt{3}}{3}\right)}{8748}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-32*x**5+48*x**4-72*x**3+108*x**2-162*x+243)/(-64*x**6+729)**2,x)

[Out] -log(x - 3/2)/17496 + 5*log(x + 3/2)/17496 - log(x**2 - 3*x/2 + 9/4)/17496 - log(x**2 + 3*x/2 + 9/4)/17496 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/26244 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/8748 - 1/(5832*x + 8748)

Giac [A] time = 1.07194, size = 116, normalized size = 1.05

$$\frac{1}{8748}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) + \frac{1}{26244}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) - \frac{1}{2916(2x+3)} - \frac{1}{17496}\log(4x^2+6x+9) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729)^2,x, algo  
rithm="giac")
```

```
[Out] 1/8748*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/26244*sqrt(3)*arctan(1/9*s  
qrt(3)*(4*x - 3)) - 1/2916/(2*x + 3) - 1/17496*log(4*x^2 + 6*x + 9) - 1/174  
96*log(4*x^2 - 6*x + 9) + 5/17496*log(abs(2*x + 3)) - 1/17496*log(abs(2*x -  
3))
```

$$3.566 \quad \int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{(729-64x^6)^2} dx$$

Optimal. Leaf size=110

$$\frac{\log(4x^2 - 6x + 9)}{17496} + \frac{\log(4x^2 + 6x + 9)}{17496} + \frac{1}{2916(3 - 2x)} - \frac{5 \log(3 - 2x)}{17496} + \frac{\log(2x + 3)}{17496} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{2916\sqrt{3}} + \frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{8748\sqrt{3}}$$

[Out] 1/(2916*(3 - 2*x)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(2916*Sqrt[3]) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(8748*Sqrt[3]) - (5*Log[3 - 2*x])/17496 + Log[3 + 2*x]/17496 + Log[9 - 6*x + 4*x^2]/17496 + Log[9 + 6*x + 4*x^2]/17496

Rubi [A] time = 0.117263, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {1586, 2074, 634, 618, 204, 628}

$$\frac{\log(4x^2 - 6x + 9)}{17496} + \frac{\log(4x^2 + 6x + 9)}{17496} + \frac{1}{2916(3 - 2x)} - \frac{5 \log(3 - 2x)}{17496} + \frac{\log(2x + 3)}{17496} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{2916\sqrt{3}} + \frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{8748\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6)^2,x]

[Out] 1/(2916*(3 - 2*x)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(2916*Sqrt[3]) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(8748*Sqrt[3]) - (5*Log[3 - 2*x])/17496 + Log[3 + 2*x]/17496 + Log[9 - 6*x + 4*x^2]/17496 + Log[9 + 6*x + 4*x^2]/17496

Rule 1586

Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] :> Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 2074

Int[(P_)^(p_)*(Q_)^(q_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{243 + 162x + 108x^2 + 72x^3 + 48x^4 + 32x^5}{(729 - 64x^6)^2} dx &= \int \frac{1}{(3 - 2x)^2 (243 + 162x + 108x^2 + 72x^3 + 48x^4 + 32x^5)} dx \\ &= \int \left(\frac{1}{1458(-3 + 2x)^2} - \frac{5}{8748(-3 + 2x)} + \frac{1}{8748(3 + 2x)} + \frac{3 + 2x}{4374(9 - 6x + 4x^2)} \right) dx \\ &= \frac{1}{2916(3 - 2x)} - \frac{5 \log(3 - 2x)}{17496} + \frac{\log(3 + 2x)}{17496} + \frac{\int \frac{3+2x}{9-6x+4x^2} dx}{4374} + \frac{\int \frac{3}{9+6x} dx}{4374} \\ &= \frac{1}{2916(3 - 2x)} - \frac{5 \log(3 - 2x)}{17496} + \frac{\log(3 + 2x)}{17496} + \frac{\int \frac{-6+8x}{9-6x+4x^2} dx}{17496} + \frac{\int \frac{6}{9+6x} dx}{4374} \\ &= \frac{1}{2916(3 - 2x)} - \frac{5 \log(3 - 2x)}{17496} + \frac{\log(3 + 2x)}{17496} + \frac{\log(9 - 6x + 4x^2)}{17496} + \frac{\log(9 + 6x)}{4374} \\ &= \frac{1}{2916(3 - 2x)} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{2916\sqrt{3}} + \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{8748\sqrt{3}} - \frac{5 \log(3 - 2x)}{17496} + \frac{\log(9 + 6x)}{4374} \end{aligned}$$

Mathematica [A] time = 0.076873, size = 97, normalized size = 0.88

$$\frac{3 \left(\log(4x^2 - 6x + 9) + \log(4x^2 + 6x + 9) + \frac{6}{3-2x} - 5 \log(3-2x) + \log(2x+3) \right) + 6\sqrt{3} \tan^{-1} \left(\frac{4x-3}{3\sqrt{3}} \right) + 2\sqrt{3} \tan^{-1} \left(\frac{4x+3}{3\sqrt{3}} \right)}{52488}$$

Antiderivative was successfully verified.

[In] Integrate[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6)^2,x]

[Out] (6*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] + 2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] + 3*(6/(3 - 2*x) - 5*Log[3 - 2*x] + Log[3 + 2*x] + Log[9 - 6*x + 4*x^2] + Log[9 + 6*x + 4*x^2]))/52488

Maple [A] time = 0.009, size = 85, normalized size = 0.8

$$\frac{\ln(3+2x)}{17496} - \frac{1}{-8748+5832x} - \frac{5 \ln(-3+2x)}{17496} + \frac{\ln(4x^2+6x+9)}{17496} + \frac{\sqrt{3}}{26244} \arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right) + \frac{\ln(4x^2-6x+9)}{17496}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729)^2,x)

[Out] 1/17496*ln(3+2*x)-1/2916/(-3+2*x)-5/17496*ln(-3+2*x)+1/17496*ln(4*x^2+6*x+9)+1/26244*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))+1/17496*ln(4*x^2-6*x+9)+1/8748*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))

Maxima [A] time = 1.38071, size = 113, normalized size = 1.03

$$\frac{1}{26244} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x+3)\right) + \frac{1}{8748} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x-3)\right) - \frac{1}{2916(2x-3)} + \frac{1}{17496} \log(4x^2+6x+9) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729)^2,x, algorith="maxima")

[Out] 1/26244*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/8748*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/2916/(2*x - 3) + 1/17496*log(4*x^2 + 6*x + 9) + 1/174

$$96 \log(4x^2 - 6x + 9) + 1/17496 \log(2x + 3) - 5/17496 \log(2x - 3)$$

Fricas [A] time = 1.43763, size = 342, normalized size = 3.11

$$\frac{2\sqrt{3}(2x-3)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) + 6\sqrt{3}(2x-3)\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) + 3(2x-3)\log(4x^2+6x+9) + 3(2x-3)\log(2x+3) - 5(2x-3)\log(2x-3) - 18}{52488(2x-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729)^2,x, algorith="fricas")

[Out] 1/52488*(2*sqrt(3)*(2*x - 3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 6*sqrt(3)*(2*x - 3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 3*(2*x - 3)*log(4*x^2 + 6*x + 9) + 3*(2*x - 3)*log(4*x^2 - 6*x + 9) + 3*(2*x - 3)*log(2*x + 3) - 15*(2*x - 3)*log(2*x - 3) - 18)/(2*x - 3)

Sympy [A] time = 0.351426, size = 105, normalized size = 0.95

$$-\frac{5 \log\left(x - \frac{3}{2}\right)}{17496} + \frac{\log\left(x + \frac{3}{2}\right)}{17496} + \frac{\log\left(x^2 - \frac{3x}{2} + \frac{9}{4}\right)}{17496} + \frac{\log\left(x^2 + \frac{3x}{2} + \frac{9}{4}\right)}{17496} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{4\sqrt{3}x}{9} - \frac{\sqrt{3}}{3}\right)}{8748} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{4\sqrt{3}x}{9} + \frac{\sqrt{3}}{3}\right)}{26244}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((32*x**5+48*x**4+72*x**3+108*x**2+162*x+243)/(-64*x**6+729)**2,x)

[Out] -5*log(x - 3/2)/17496 + log(x + 3/2)/17496 + log(x**2 - 3*x/2 + 9/4)/17496 + log(x**2 + 3*x/2 + 9/4)/17496 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/8748 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/26244 - 1/(5832*x - 8748)

Giac [A] time = 1.07353, size = 116, normalized size = 1.05

$$\frac{1}{26244} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x+3)\right) + \frac{1}{8748} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x-3)\right) - \frac{1}{2916(2x-3)} + \frac{1}{17496} \log(4x^2+6x+9) + \frac{1}{17496} \log(2x+3) - \frac{5}{17496} \log(2x-3) - \frac{18}{17496}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] 1/26244*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/8748*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/2916/(2*x - 3) + 1/17496*log(4*x^2 + 6*x + 9) + 1/17496*log(4*x^2 - 6*x + 9) + 1/17496*log(abs(2*x + 3)) - 5/17496*log(abs(2*x - 3))
```

$$3.567 \quad \int \frac{81+36x^2+16x^4}{(729-64x^6)^2} dx$$

Optimal. Leaf size=81

$$\frac{1}{17496(3-2x)} - \frac{1}{17496(2x+3)} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{13122\sqrt{3}} + \frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{13122\sqrt{3}} + \frac{\tanh^{-1}\left(\frac{2x}{3}\right)}{8748}$$

[Out] 1/(17496*(3 - 2*x)) - 1/(17496*(3 + 2*x)) - ArcTan[(3 - 4*x)/(3*sqrt[3])]/(13122*sqrt[3]) + ArcTan[(3 + 4*x)/(3*sqrt[3])]/(13122*sqrt[3]) + ArcTanh[(2*x)/3]/8748

Rubi [A] time = 0.0692554, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1586, 1170, 207, 618, 204}

$$\frac{1}{17496(3-2x)} - \frac{1}{17496(2x+3)} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{13122\sqrt{3}} + \frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{13122\sqrt{3}} + \frac{\tanh^{-1}\left(\frac{2x}{3}\right)}{8748}$$

Antiderivative was successfully verified.

[In] Int[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6)^2,x]

[Out] 1/(17496*(3 - 2*x)) - 1/(17496*(3 + 2*x)) - ArcTan[(3 - 4*x)/(3*sqrt[3])]/(13122*sqrt[3]) + ArcTan[(3 + 4*x)/(3*sqrt[3])]/(13122*sqrt[3]) + ArcTanh[(2*x)/3]/8748

Rule 1586

Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p+q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 1170

Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{81 + 36x^2 + 16x^4}{(729 - 64x^6)^2} dx &= \int \frac{1}{(9 - 4x^2)^2 (81 + 36x^2 + 16x^4)} dx \\
 &= \int \left(\frac{1}{8748(-3 + 2x)^2} + \frac{1}{8748(3 + 2x)^2} - \frac{1}{1458(-9 + 4x^2)} + \frac{1}{4374(9 - 6x + 4x^2)} + \frac{1}{4374(9 + 6x + 4x^2)} \right) dx \\
 &= \frac{1}{17496(3 - 2x)} - \frac{1}{17496(3 + 2x)} + \frac{\int \frac{1}{9 - 6x + 4x^2} dx}{4374} + \frac{\int \frac{1}{9 + 6x + 4x^2} dx}{4374} - \frac{\int \frac{1}{-9 + 4x^2} dx}{1458} \\
 &= \frac{1}{17496(3 - 2x)} - \frac{1}{17496(3 + 2x)} + \frac{\tanh^{-1}\left(\frac{2x}{3}\right)}{8748} - \frac{\text{Subst}\left(\int \frac{1}{-108 - x^2} dx, x, -6 + 8x\right)}{2187} - \frac{\text{Subst}\left(\int \frac{1}{108 - x^2} dx, x, -6 + 8x\right)}{2187} \\
 &= \frac{1}{17496(3 - 2x)} - \frac{1}{17496(3 + 2x)} - \frac{\tan^{-1}\left(\frac{3 - 4x}{3\sqrt{3}}\right)}{13122\sqrt{3}} + \frac{\tan^{-1}\left(\frac{3 + 4x}{3\sqrt{3}}\right)}{13122\sqrt{3}} + \frac{\tanh^{-1}\left(\frac{2x}{3}\right)}{8748}
 \end{aligned}$$

Mathematica [C] time = 0.430905, size = 122, normalized size = 1.51

$$\frac{36x}{9 - 4x^2} - 9 \log(3 - 2x) + 9 \log(2x + 3) + 3\sqrt{3} \tan^{-1}\left(\frac{1}{3}(\sqrt{3} - i)x\right) + 4i\sqrt{3} \tanh^{-1}\left(\frac{1}{3}(1 - i\sqrt{3})x\right) + \left(-3 + \frac{2}{\sqrt{\frac{1}{6}(1 + i\sqrt{3})}}\right) \tan^{-1}\left(\frac{1}{3}(1 + i\sqrt{3})x\right)$$

157464

Warning: Unable to verify antiderivative.

[In] Integrate[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6)^2,x]

[Out] ((36*x)/(9 - 4*x^2) + 3*Sqrt[3]*ArcTan[(-I + Sqrt[3])*x]/3] + (4*I)*Sqrt[3]*ArcTanh[((1 - I*Sqrt[3])*x)/3] + (-3 + 2/Sqrt[(1 + I*Sqrt[3])/6])*ArcTanh[(x + I*Sqrt[3])*x]/3] - 9*Log[3 - 2*x] + 9*Log[3 + 2*x])/157464

Maple [A] time = 0.01, size = 68, normalized size = 0.8

$$-\frac{1}{52488 + 34992x} + \frac{\ln(3 + 2x)}{17496} - \frac{1}{-52488 + 34992x} - \frac{\ln(-3 + 2x)}{17496} + \frac{\sqrt{3}}{39366} \arctan\left(\frac{(8x + 6)\sqrt{3}}{18}\right) + \frac{\sqrt{3}}{39366} \arctan\left(\frac{(8x - 6)\sqrt{3}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((16*x^4+36*x^2+81)/(-64*x^6+729)^2,x)

[Out] -1/17496/(3+2*x)+1/17496*ln(3+2*x)-1/17496/(-3+2*x)-1/17496*ln(-3+2*x)+1/39366*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))+1/39366*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))

Maxima [A] time = 1.37462, size = 82, normalized size = 1.01

$$\frac{1}{39366} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x + 3)\right) + \frac{1}{39366} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x - 3)\right) - \frac{x}{4374(4x^2 - 9)} + \frac{1}{17496} \log(2x + 3) - \frac{1}{17496} \log(2x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((16*x^4+36*x^2+81)/(-64*x^6+729)^2,x, algorithm="maxima")

[Out] 1/39366*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/39366*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(4*x^2 - 9) + 1/17496*log(2*x + 3) - 1/17496*log(2*x - 3)

Fricas [A] time = 1.52705, size = 259, normalized size = 3.2

$$\frac{4\sqrt{3}(4x^2 - 9) \arctan\left(\frac{4}{81} \sqrt{3}(2x^3 + 9x)\right) + 4\sqrt{3}(4x^2 - 9) \arctan\left(\frac{2}{9} \sqrt{3}x\right) + 9(4x^2 - 9) \log(2x + 3) - 9(4x^2 - 9) \log(2x - 3)}{157464(4x^2 - 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((16*x^4+36*x^2+81)/(-64*x^6+729)^2,x, algorithm="fricas")

[Out] 1/157464*(4*sqrt(3)*(4*x^2 - 9)*arctan(4/81*sqrt(3)*(2*x^3 + 9*x)) + 4*sqrt(3)*(4*x^2 - 9)*arctan(2/9*sqrt(3)*x) + 9*(4*x^2 - 9)*log(2*x + 3) - 9*(4*x^2 - 9)*log(2*x - 3) - 36*x)/(4*x^2 - 9)

Sympy [A] time = 0.201229, size = 70, normalized size = 0.86

$$-\frac{x}{17496x^2 - 39366} + \frac{\sqrt{3}\left(2\operatorname{atan}\left(\frac{2\sqrt{3}x}{9}\right) + 2\operatorname{atan}\left(\frac{8\sqrt{3}x^3}{81} + \frac{4\sqrt{3}x}{9}\right)\right)}{78732} - \frac{\log\left(x - \frac{3}{2}\right)}{17496} + \frac{\log\left(x + \frac{3}{2}\right)}{17496}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((16*x**4+36*x**2+81)/(-64*x**6+729)**2,x)

[Out] -x/(17496*x**2 - 39366) + sqrt(3)*(2*atan(2*sqrt(3)*x/9) + 2*atan(8*sqrt(3)*x**3/81 + 4*sqrt(3)*x/9))/78732 - log(x - 3/2)/17496 + log(x + 3/2)/17496

Giac [A] time = 1.06105, size = 85, normalized size = 1.05

$$\frac{1}{39366} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x + 3)\right) + \frac{1}{39366} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x - 3)\right) - \frac{x}{4374(4x^2 - 9)} + \frac{1}{17496} \log(|2x + 3|) - \frac{1}{17496} \log(|2x - 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((16*x^4+36*x^2+81)/(-64*x^6+729)^2,x, algorithm="giac")

[Out] 1/39366*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/39366*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(4*x^2 - 9) + 1/17496*log(abs(2*x + 3)) - 1/17496*log(abs(2*x - 3))

$$3.568 \quad \int \frac{81+54x-24x^3-16x^4}{(729-64x^6)^2} dx$$

Optimal. Leaf size=92

$$\frac{x}{4374(4x^2-6x+9)} - \frac{\log(4x^2-6x+9)}{157464} + \frac{\log(4x^2+6x+9)}{52488} - \frac{\log(3-2x)}{26244} + \frac{\log(2x+3)}{78732} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{4374\sqrt{3}}$$

[Out] x/(4374*(9 - 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(4374*Sqrt[3]) - Log[3 - 2*x]/26244 + Log[3 + 2*x]/78732 - Log[9 - 6*x + 4*x^2]/157464 + Log[9 + 6*x + 4*x^2]/52488

Rubi [A] time = 0.116453, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {1586, 2074, 638, 618, 204, 634, 628}

$$\frac{x}{4374(4x^2-6x+9)} - \frac{\log(4x^2-6x+9)}{157464} + \frac{\log(4x^2+6x+9)}{52488} - \frac{\log(3-2x)}{26244} + \frac{\log(2x+3)}{78732} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{4374\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6)^2,x]

[Out] x/(4374*(9 - 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(4374*Sqrt[3]) - Log[3 - 2*x]/26244 + Log[3 + 2*x]/78732 - Log[9 - 6*x + 4*x^2]/157464 + Log[9 + 6*x + 4*x^2]/52488

Rule 1586

Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p+q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 2074

Int[(P_)^(p_)*(Q_)^(q_), x_Symbol] := With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] && PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{81 + 54x - 24x^3 - 16x^4}{(729 - 64x^6)^2} dx &= \int \frac{1}{(9 - 6x + 4x^2)^2 (81 + 54x - 24x^3 - 16x^4)} dx \\
&= \int \left(-\frac{1}{13122(-3 + 2x)} + \frac{1}{39366(3 + 2x)} + \frac{3 - x}{729(9 - 6x + 4x^2)^2} + \frac{39 - 4x}{78732(9 - 6x + 4x^2)} \right) dx \\
&= -\frac{\log(3 - 2x)}{26244} + \frac{\log(3 + 2x)}{78732} + \frac{\int \frac{39-4x}{9-6x+4x^2} dx}{78732} + \frac{\int \frac{3+4x}{9+6x+4x^2} dx}{26244} + \frac{1}{729} \int \frac{3-x}{(9-6x+4x^2)^2} dx \\
&= \frac{x}{4374(9-6x+4x^2)} - \frac{\log(3-2x)}{26244} + \frac{\log(3+2x)}{78732} + \frac{\log(9+6x+4x^2)}{52488} - \frac{\int \frac{-6+8x}{9-6x+4x^2} dx}{157464} \\
&= \frac{x}{4374(9-6x+4x^2)} - \frac{\log(3-2x)}{26244} + \frac{\log(3+2x)}{78732} - \frac{\log(9-6x+4x^2)}{157464} + \frac{\log(9+6x+4x^2)}{52488} \\
&= \frac{x}{4374(9-6x+4x^2)} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{4374\sqrt{3}} - \frac{\log(3-2x)}{26244} + \frac{\log(3+2x)}{78732} - \frac{\log(9-6x+4x^2)}{157464}
\end{aligned}$$

Mathematica [A] time = 0.029823, size = 84, normalized size = 0.91

$$\frac{\frac{36x}{4x^2-6x+9} - \log(4x^2 - 6x + 9) + 3 \log(4x^2 + 6x + 9) - 6 \log(3 - 2x) + 2 \log(2x + 3) + 12\sqrt{3} \tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)}{157464}$$

Antiderivative was successfully verified.

[In] Integrate[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6)^2,x]

[Out] ((36*x)/(9 - 6*x + 4*x^2) + 12*sqrt(3)*ArcTan[(-3 + 4*x)/(3*sqrt(3))] - 6*Log[3 - 2*x] + 2*Log[3 + 2*x] - Log[9 - 6*x + 4*x^2] + 3*Log[9 + 6*x + 4*x^2])/157464

Maple [A] time = 0.01, size = 73, normalized size = 0.8

$$\frac{\ln(3 + 2x)}{78732} - \frac{\ln(-3 + 2x)}{26244} + \frac{\ln(4x^2 + 6x + 9)}{52488} + \frac{x}{17496} \left(x^2 - \frac{3x}{2} + \frac{9}{4}\right)^{-1} - \frac{\ln(4x^2 - 6x + 9)}{157464} + \frac{\sqrt{3}}{13122} \arctan\left(\frac{8x - 3}{3\sqrt{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729)^2,x)`

[Out] $\frac{1}{78732} \ln(3+2x) - \frac{1}{26244} \ln(-3+2x) + \frac{1}{52488} \ln(4x^2+6x+9) + \frac{1}{17496} \frac{x}{(x^2-3/2x+9/4)} - \frac{1}{157464} \ln(4x^2-6x+9) + \frac{1}{13122} 3^{(1/2)} \arctan(1/18(8x-6)3^{(1/2)})$

Maxima [A] time = 1.38606, size = 100, normalized size = 1.09

$$\frac{1}{13122} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x-3)\right) + \frac{x}{4374(4x^2-6x+9)} + \frac{1}{52488} \log(4x^2+6x+9) - \frac{1}{157464} \log(4x^2-6x+9)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729)^2,x, algorithm="maxima")`

[Out] $\frac{1}{13122} \sqrt{3} \arctan(1/9 \sqrt{3}(4x-3)) + \frac{1}{4374} \frac{x}{(4x^2-6x+9)} + \frac{1}{52488} \log(4x^2+6x+9) - \frac{1}{157464} \log(4x^2-6x+9) + \frac{1}{78732} \log(2x+3) - \frac{1}{26244} \log(2x-3)$

Fricas [A] time = 1.65431, size = 338, normalized size = 3.67

$$\frac{12 \sqrt{3}(4x^2-6x+9) \arctan\left(\frac{1}{9} \sqrt{3}(4x-3)\right) + 3(4x^2-6x+9) \log(4x^2+6x+9) - (4x^2-6x+9) \log(4x^2-6x+9)}{157464(4x^2-6x+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729)^2,x, algorithm="fricas")`

[Out] $\frac{1}{157464} (12 \sqrt{3}(4x^2-6x+9) \arctan(1/9 \sqrt{3}(4x-3)) + 3(4x^2-6x+9) \log(4x^2+6x+9) - (4x^2-6x+9) \log(4x^2-6x+9) + 2(4x^2-6x+9) \log(2x+3) - 6(4x^2-6x+9) \log(2x-3) + 36x) / (4x^2-6x+9)$

Sympy [A] time = 0.284686, size = 82, normalized size = 0.89

$$\frac{x}{17496x^2-26244x+39366} - \frac{\log\left(x-\frac{3}{2}\right)}{26244} + \frac{\log\left(x+\frac{3}{2}\right)}{78732} - \frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{157464} + \frac{\log(4x^2+6x+9)}{52488} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{4\sqrt{3}x}{9}\right)}{13122}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x**4-24*x**3+54*x+81)/(-64*x**6+729)**2,x)

[Out] x/(17496*x**2 - 26244*x + 39366) - log(x - 3/2)/26244 + log(x + 3/2)/78732 - log(x**2 - 3*x/2 + 9/4)/157464 + log(4*x**2 + 6*x + 9)/52488 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/13122

Giac [A] time = 1.05603, size = 103, normalized size = 1.12

$$\frac{1}{13122} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x-3)\right) + \frac{x}{4374(4x^2-6x+9)} + \frac{1}{52488} \log(4x^2+6x+9) - \frac{1}{157464} \log(4x^2-6x+9) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729)^2,x, algorithm="giac")

[Out] 1/13122*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(4*x^2 - 6*x + 9) + 1/52488*log(4*x^2 + 6*x + 9) - 1/157464*log(4*x^2 - 6*x + 9) + 1/78732*log(abs(2*x + 3)) - 1/26244*log(abs(2*x - 3))

$$3.569 \quad \int \frac{3-2x}{(729-64x^6)^2} dx$$

Optimal. Leaf size=148

$$\frac{3-x}{708588(4x^2-6x+9)} + \frac{x}{236196(4x^2+6x+9)} - \frac{\log(4x^2-6x+9)}{944784} + \frac{\log(4x^2+6x+9)}{8503056} - \frac{1}{708588(2x+3)} - \frac{\log(3-2x)}{4251528} + \frac{\log(3+2x)}{472392} - \frac{\log(9-6x+4x^2)}{944784} + \frac{\log(9+6x+4x^2)}{8503056}$$

[Out] -1/(708588*(3 + 2*x)) + (3 - x)/(708588*(9 - 6*x + 4*x^2)) + x/(236196*(9 + 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(1417176*Sqrt[3]) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(157464*Sqrt[3]) - Log[3 - 2*x]/4251528 + Log[3 + 2*x]/472392 - Log[9 - 6*x + 4*x^2]/944784 + Log[9 + 6*x + 4*x^2]/8503056

Rubi [A] time = 0.171598, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1586, 2074, 638, 618, 204, 634, 628}

$$\frac{3-x}{708588(4x^2-6x+9)} + \frac{x}{236196(4x^2+6x+9)} - \frac{\log(4x^2-6x+9)}{944784} + \frac{\log(4x^2+6x+9)}{8503056} - \frac{1}{708588(2x+3)} - \frac{\log(3-2x)}{4251528} + \frac{\log(3+2x)}{472392} - \frac{\log(9-6x+4x^2)}{944784} + \frac{\log(9+6x+4x^2)}{8503056}$$

Antiderivative was successfully verified.

[In] Int[(3 - 2*x)/(729 - 64*x^6)^2,x]

[Out] -1/(708588*(3 + 2*x)) + (3 - x)/(708588*(9 - 6*x + 4*x^2)) + x/(236196*(9 + 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(1417176*Sqrt[3]) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(157464*Sqrt[3]) - Log[3 - 2*x]/4251528 + Log[3 + 2*x]/472392 - Log[9 - 6*x + 4*x^2]/944784 + Log[9 + 6*x + 4*x^2]/8503056

Rule 1586

Int[(u_)*(P_x_)^(p_)*(Q_x_)^(q_), x_Symbol] := Int[u*PolynomialQuotient[P_x, Q_x, x]^p*Q_x^(p+q), x] /; FreeQ[q, x] && PolyQ[P_x, x] && PolyQ[Q_x, x] && EqQ[PolynomialRemainder[P_x, Q_x, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 2074

Int[(P_)^(p_)*(Q_)^(q_), x_Symbol] := With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] &&

PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rule 638

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
\int \frac{3-2x}{(729-64x^6)^2} dx &= \int \frac{1}{(3-2x)(243+162x+108x^2+72x^3+48x^4+32x^5)^2} dx \\
&= \int \left(-\frac{1}{2125764(-3+2x)} + \frac{1}{354294(3+2x)^2} + \frac{1}{236196(3+2x)} - \frac{x}{39366(9-6x+4x^2)^2} + \frac{x}{708588(9-6x+4x^2)} \right) dx \\
&= -\frac{1}{708588(3+2x)} - \frac{\log(3-2x)}{4251528} + \frac{\log(3+2x)}{472392} + \frac{\int \frac{33+2x}{9+6x+4x^2} dx}{2125764} + \frac{\int \frac{7-6x}{9-6x+4x^2} dx}{708588} - \frac{\int \frac{x}{(9-6x+4x^2)^2} dx}{39366} \\
&= -\frac{1}{708588(3+2x)} + \frac{3-x}{708588(9-6x+4x^2)} + \frac{x}{236196(9+6x+4x^2)} - \frac{\log(3-2x)}{4251528} + \frac{\log(3+2x)}{472392} \\
&= -\frac{1}{708588(3+2x)} + \frac{3-x}{708588(9-6x+4x^2)} + \frac{x}{236196(9+6x+4x^2)} - \frac{\log(3-2x)}{4251528} + \frac{\log(3+2x)}{472392} \\
&= -\frac{1}{708588(3+2x)} + \frac{3-x}{708588(9-6x+4x^2)} + \frac{x}{236196(9+6x+4x^2)} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{1417176\sqrt{3}} + \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{15746}
\end{aligned}$$

Mathematica [A] time = 0.0569492, size = 119, normalized size = 0.8

$$\frac{1944x}{32x^5+48x^4+72x^3+108x^2+162x+243} - 9 \log(4x^2 - 6x + 9) + \log(4x^2 + 6x + 9) - 2 \log(3 - 2x) + 18 \log(2x + 3) + 2\sqrt{3} \tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right) - \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{15746}$$

8503056

Antiderivative was successfully verified.

[In] Integrate[(3 - 2*x)/(729 - 64*x^6)^2,x]

[Out] ((1944*x)/(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5) + 2*sqrt(3)*ArcTan[(-3 + 4*x)/(3*sqrt(3))] + 18*sqrt(3)*ArcTan[(3 + 4*x)/(3*sqrt(3))] - 2*Log[3 - 2*x] + 18*Log[3 + 2*x] - 9*Log[9 - 6*x + 4*x^2] + Log[9 + 6*x + 4*x^2])/8503056

Maple [A] time = 0.016, size = 115, normalized size = 0.8

$$-\frac{1}{2125764 + 1417176x} + \frac{\ln(3+2x)}{472392} - \frac{\ln(-3+2x)}{4251528} + \frac{x}{944784} \left(x^2 + \frac{3x}{2} + \frac{9}{4} \right)^{-1} + \frac{\ln(4x^2 + 6x + 9)}{8503056} + \frac{\sqrt{3}}{472392} \arctan\left(\frac{3-4x}{3\sqrt{3}}\right) - \frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{15746}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3-2*x)/(-64*x^6+729)^2,x)

[Out] -1/708588/(3+2*x)+1/472392*ln(3+2*x)-1/4251528*ln(-3+2*x)+1/944784*x/(x^2+3/2*x+9/4)+1/8503056*ln(4*x^2+6*x+9)+1/472392*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))-1/708588*(1/4*x-3/4)/(x^2-3/2*x+9/4)-1/944784*ln(4*x^2-6*x+9)+1/4251528*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))

Maxima [A] time = 1.37397, size = 142, normalized size = 0.96

$$\frac{1}{472392} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x+3)\right) + \frac{1}{4251528} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x-3)\right) + \frac{x}{4374(32x^5 + 48x^4 + 72x^3 + 108x^2 + 162x + 243)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3-2*x)/(-64*x^6+729)^2,x, algorithm="maxima")

[Out] 1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/4251528*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 + 162*x + 243) + 1/8503056*log(4*x^2 + 6*x + 9) - 1/944784*log(4*x^2 - 6*x + 9) + 1/472392*log(2*x + 3) - 1/4251528*log(2*x - 3)

Fricas [B] time = 1.68333, size = 736, normalized size = 4.97

$$18\sqrt{3}(32x^5 + 48x^4 + 72x^3 + 108x^2 + 162x + 243) \arctan\left(\frac{1}{9} \sqrt{3}(4x+3)\right) + 2\sqrt{3}(32x^5 + 48x^4 + 72x^3 + 108x^2 + 162x + 243)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3-2*x)/(-64*x^6+729)^2,x, algorithm="fricas")

[Out] 1/8503056*(18*sqrt(3)*(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 + 162*x + 243)*arctan(1/9*sqrt(3)*(4*x + 3)) + 2*sqrt(3)*(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 + 162*x + 243)*arctan(1/9*sqrt(3)*(4*x - 3)) + (32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 + 162*x + 243)*log(4*x^2 + 6*x + 9) - 9*(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 + 162*x + 243)*log(4*x^2 - 6*x + 9) + 18*(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 + 162*x + 243)*log(2*x + 3) - 2*(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 + 162*x + 243)*log(2*x - 3) + 1944*x)/(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 + 162*x + 243)

Sympy [A] time = 0.502706, size = 124, normalized size = 0.84

$$\frac{x}{139968x^5 + 209952x^4 + 314928x^3 + 472392x^2 + 708588x + 1062882} - \frac{\log\left(x - \frac{3}{2}\right)}{4251528} + \frac{\log\left(x + \frac{3}{2}\right)}{472392} - \frac{\log\left(x^2 - \frac{3x}{2} + \frac{9}{4}\right)}{944784}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3-2*x)/(-64*x**6+729)**2,x)

[Out] x/(139968*x**5 + 209952*x**4 + 314928*x**3 + 472392*x**2 + 708588*x + 1062882) - log(x - 3/2)/4251528 + log(x + 3/2)/472392 - log(x**2 - 3*x/2 + 9/4)/944784 + log(x**2 + 3*x/2 + 9/4)/8503056 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/4251528 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/472392

Giac [A] time = 1.07046, size = 150, normalized size = 1.01

$$\frac{1}{472392} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x + 3)\right) + \frac{1}{4251528} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x - 3)\right) + \frac{x}{4374(4x^2 + 6x + 9)(4x^2 - 6x + 9)(2x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3-2*x)/(-64*x^6+729)^2,x, algorithm="giac")

[Out] 1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/4251528*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/4374*x/((4*x^2 + 6*x + 9)*(4*x^2 - 6*x + 9)*(2*x + 3)) + 1/8503056*log(4*x^2 + 6*x + 9) - 1/944784*log(4*x^2 - 6*x + 9) + 1/472392*log(abs(2*x + 3)) - 1/4251528*log(abs(2*x - 3))

$$3.570 \quad \int \frac{3+2x}{(729-64x^6)^2} dx$$

Optimal. Leaf size=146

$$\frac{x}{236196(4x^2-6x+9)} - \frac{x+3}{708588(4x^2+6x+9)} - \frac{\log(4x^2-6x+9)}{8503056} + \frac{\log(4x^2+6x+9)}{944784} + \frac{1}{708588(3-2x)} - \frac{\log(3-2x)}{472392}$$

[Out] 1/(708588*(3 - 2*x)) + x/(236196*(9 - 6*x + 4*x^2)) - (3 + x)/(708588*(9 + 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(157464*Sqrt[3]) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(1417176*Sqrt[3]) - Log[3 - 2*x]/472392 + Log[3 + 2*x]/4251528 - Log[9 - 6*x + 4*x^2]/8503056 + Log[9 + 6*x + 4*x^2]/944784

Rubi [A] time = 0.172511, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1586, 2074, 638, 618, 204, 634, 628}

$$\frac{x}{236196(4x^2-6x+9)} - \frac{x+3}{708588(4x^2+6x+9)} - \frac{\log(4x^2-6x+9)}{8503056} + \frac{\log(4x^2+6x+9)}{944784} + \frac{1}{708588(3-2x)} - \frac{\log(3-2x)}{472392}$$

Antiderivative was successfully verified.

[In] Int[(3 + 2*x)/(729 - 64*x^6)^2,x]

[Out] 1/(708588*(3 - 2*x)) + x/(236196*(9 - 6*x + 4*x^2)) - (3 + x)/(708588*(9 + 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(157464*Sqrt[3]) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(1417176*Sqrt[3]) - Log[3 - 2*x]/472392 + Log[3 + 2*x]/4251528 - Log[9 - 6*x + 4*x^2]/8503056 + Log[9 + 6*x + 4*x^2]/944784

Rule 1586

Int[(u_)*(P_x_)^(p_)*(Q_x_)^(q_), x_Symbol] := Int[u*PolynomialQuotient[P_x, Q_x, x]^p*Q_x^(p+q), x] /; FreeQ[q, x] && PolyQ[P_x, x] && PolyQ[Q_x, x] && EqQ[PolynomialRemainder[P_x, Q_x, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 2074

Int[(P_)^(p_)*(Q_)^(q_), x_Symbol] := With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] &&

PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rule 638

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 628

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
\int \frac{3+2x}{(729-64x^6)^2} dx &= \int \frac{1}{(3+2x)(243-162x+108x^2-72x^3+48x^4-32x^5)^2} dx \\
&= \int \left(\frac{1}{354294(-3+2x)^2} - \frac{1}{236196(-3+2x)} + \frac{1}{2125764(3+2x)} + \frac{3-x}{39366(9-6x+4x^2)^2} + \frac{1}{2125764(3+2x)} \right) dx \\
&= \frac{1}{708588(3-2x)} - \frac{\log(3-2x)}{472392} + \frac{\log(3+2x)}{4251528} + \frac{\int \frac{33-2x}{9-6x+4x^2} dx}{2125764} + \frac{\int \frac{7+6x}{9+6x+4x^2} dx}{708588} + \frac{\int \frac{3-x}{(9-6x+4x^2)^2} dx}{39366} \\
&= \frac{1}{708588(3-2x)} + \frac{x}{236196(9-6x+4x^2)} - \frac{3+x}{708588(9+6x+4x^2)} - \frac{\log(3-2x)}{472392} + \frac{\log(3+2x)}{4251528} \\
&= \frac{1}{708588(3-2x)} + \frac{x}{236196(9-6x+4x^2)} - \frac{3+x}{708588(9+6x+4x^2)} - \frac{\log(3-2x)}{472392} + \frac{\log(3+2x)}{4251528} \\
&= \frac{1}{708588(3-2x)} + \frac{x}{236196(9-6x+4x^2)} - \frac{3+x}{708588(9+6x+4x^2)} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}} + \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{1417176\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.0817094, size = 121, normalized size = 0.83

$$\frac{1944x}{-32x^5+48x^4-72x^3+108x^2-162x+243} - \log(4x^2-6x+9) + 9\log(4x^2+6x+9) - 18\log(3-2x) + 2\log(2x+3) + 18\sqrt{3}\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right) + \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{1417176\sqrt{3}}$$

8503056

Antiderivative was successfully verified.

[In] Integrate[(3 + 2*x)/(729 - 64*x^6)^2, x]

[Out] ((1944*x)/(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5) + 18*sqrt(3)*ArcTan[(-3 + 4*x)/(3*sqrt(3))] + 2*sqrt(3)*ArcTan[(3 + 4*x)/(3*sqrt(3))] - 18*Log[3 - 2*x] + 2*Log[3 + 2*x] - Log[9 - 6*x + 4*x^2] + 9*Log[9 + 6*x + 4*x^2])/8503056

Maple [A] time = 0.013, size = 115, normalized size = 0.8

$$\frac{\ln(3+2x)}{4251528} - \frac{1}{-2125764+1417176x} - \frac{\ln(-3+2x)}{472392} + \frac{1}{708588} \left(-\frac{x}{4} - \frac{3}{4} \right) \left(x^2 + \frac{3x}{2} + \frac{9}{4} \right)^{-1} + \frac{\ln(4x^2+6x+9)}{944784} + \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}} + \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{1417176\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3+2*x)/(-64*x^6+729)^2,x)

[Out] 1/4251528*ln(3+2*x)-1/708588/(-3+2*x)-1/472392*ln(-3+2*x)+1/708588*(-1/4*x-3/4)/(x^2+3/2*x+9/4)+1/944784*ln(4*x^2+6*x+9)+1/4251528*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))+1/944784*x/(x^2-3/2*x+9/4)-1/8503056*ln(4*x^2-6*x+9)+1/472392*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))

Maxima [A] time = 1.38129, size = 142, normalized size = 0.97

$$\frac{1}{4251528} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x+3)\right) + \frac{1}{472392} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x-3)\right) - \frac{x}{4374(32x^5 - 48x^4 + 72x^3 - 108x^2 + 162x - 243)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+2*x)/(-64*x^6+729)^2,x, algorithm="maxima")

[Out] 1/4251528*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2 + 162*x - 243) + 1/944784*log(4*x^2 + 6*x + 9) - 1/8503056*log(4*x^2 - 6*x + 9) + 1/4251528*log(2*x + 3) - 1/472392*log(2*x - 3)

Fricas [B] time = 1.50641, size = 736, normalized size = 5.04

$$2\sqrt{3}(32x^5 - 48x^4 + 72x^3 - 108x^2 + 162x - 243) \arctan\left(\frac{1}{9} \sqrt{3}(4x+3)\right) + 18\sqrt{3}(32x^5 - 48x^4 + 72x^3 - 108x^2 + 162x - 243)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+2*x)/(-64*x^6+729)^2,x, algorithm="fricas")

[Out] 1/8503056*(2*sqrt(3)*(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2 + 162*x - 243)*arctan(1/9*sqrt(3)*(4*x + 3)) + 18*sqrt(3)*(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2 + 162*x - 243)*arctan(1/9*sqrt(3)*(4*x - 3)) + 9*(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2 + 162*x - 243)*log(4*x^2 + 6*x + 9) - (32*x^5 - 48*x^4 + 72*x^3 - 108*x^2 + 162*x - 243)*log(4*x^2 - 6*x + 9) + 2*(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2 + 162*x - 243)*log(2*x + 3) - 18*(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2 + 162*x - 243)*log(2*x - 3) - 1944*x)/(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2 + 162*x - 243)

Sympy [A] time = 0.501011, size = 124, normalized size = 0.85

$$\frac{x}{139968x^5 - 209952x^4 + 314928x^3 - 472392x^2 + 708588x - 1062882} - \frac{\log\left(x - \frac{3}{2}\right)}{472392} + \frac{\log\left(x + \frac{3}{2}\right)}{4251528} - \frac{\log\left(x^2 - \frac{3x}{2} + \frac{9}{4}\right)}{8503056}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+2*x)/(-64*x**6+729)**2,x)

[Out] -x/(139968*x**5 - 209952*x**4 + 314928*x**3 - 472392*x**2 + 708588*x - 1062882) - log(x - 3/2)/472392 + log(x + 3/2)/4251528 - log(x**2 - 3*x/2 + 9/4)/8503056 + log(x**2 + 3*x/2 + 9/4)/944784 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/472392 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/4251528

Giac [A] time = 1.06199, size = 150, normalized size = 1.03

$$\frac{1}{4251528} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x+3)\right) + \frac{1}{472392} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x-3)\right) - \frac{x}{4374(4x^2+6x+9)(4x^2-6x+9)(2x-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+2*x)/(-64*x^6+729)^2,x, algorithm="giac")

[Out] 1/4251528*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 + 6*x + 9)*(4*x^2 - 6*x + 9)*(2*x - 3)) + 1/944784*log(4*x^2 + 6*x + 9) - 1/8503056*log(4*x^2 - 6*x + 9) + 1/4251528*log(abs(2*x + 3)) - 1/472392*log(abs(2*x - 3))

$$3.571 \quad \int \frac{9-6x+4x^2}{(729-64x^6)^2} dx$$

Optimal. Leaf size=142

$$\frac{4x+3}{236196(4x^2+6x+9)} - \frac{\log(4x^2-6x+9)}{944784} - \frac{5\log(4x^2+6x+9)}{2834352} + \frac{1}{472392(3-2x)} - \frac{1}{157464(2x+3)} - \frac{\log(3-2x)}{354294}$$

[Out] 1/(472392*(3 - 2*x)) - 1/(157464*(3 + 2*x)) + (3 + 4*x)/(236196*(9 + 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(472392*Sqrt[3]) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(52488*Sqrt[3]) - Log[3 - 2*x]/354294 + Log[3 + 2*x]/118098 - Log[9 - 6*x + 4*x^2]/944784 - (5*Log[9 + 6*x + 4*x^2])/2834352

Rubi [A] time = 0.148439, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$, Rules used = {1586, 2074, 634, 618, 204, 628, 614}

$$\frac{4x+3}{236196(4x^2+6x+9)} - \frac{\log(4x^2-6x+9)}{944784} - \frac{5\log(4x^2+6x+9)}{2834352} + \frac{1}{472392(3-2x)} - \frac{1}{157464(2x+3)} - \frac{\log(3-2x)}{354294}$$

Antiderivative was successfully verified.

[In] Int[(9 - 6*x + 4*x^2)/(729 - 64*x^6)^2,x]

[Out] 1/(472392*(3 - 2*x)) - 1/(157464*(3 + 2*x)) + (3 + 4*x)/(236196*(9 + 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(472392*Sqrt[3]) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(52488*Sqrt[3]) - Log[3 - 2*x]/354294 + Log[3 + 2*x]/118098 - Log[9 - 6*x + 4*x^2]/944784 - (5*Log[9 + 6*x + 4*x^2])/2834352

Rule 1586

Int[(u_.)*(P_x_)^(p_.)*(Q_x_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[P_x, Q_x, x]^p*Q_x^(p+q), x] /; FreeQ[q, x] && PolyQ[P_x, x] && PolyQ[Q_x, x] && EqQ[PolynomialRemainder[P_x, Q_x, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 2074

Int[(P_)^(p_.)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&

PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rubi steps

$$\begin{aligned}
\int \frac{9 - 6x + 4x^2}{(729 - 64x^6)^2} dx &= \int \frac{1}{(9 - 6x + 4x^2)(81 + 54x - 24x^3 - 16x^4)^2} dx \\
&= \int \left(\frac{1}{236196(-3 + 2x)^2} - \frac{1}{177147(-3 + 2x)} + \frac{1}{78732(3 + 2x)^2} + \frac{1}{59049(3 + 2x)} + \frac{3 - 2x}{236196(9 - 6x + 4x^2)} \right) dx \\
&= \frac{1}{472392(3 - 2x)} - \frac{1}{157464(3 + 2x)} - \frac{\log(3 - 2x)}{354294} + \frac{\log(3 + 2x)}{118098} + \frac{\int \frac{21-10x}{9+6x+4x^2} dx}{708588} + \frac{\int \frac{3-2x}{9-6x+4x^2} dx}{236196} \\
&= \frac{1}{472392(3 - 2x)} - \frac{1}{157464(3 + 2x)} + \frac{3 + 4x}{236196(9 + 6x + 4x^2)} - \frac{\log(3 - 2x)}{354294} + \frac{\log(3 + 2x)}{118098} - \frac{\int \frac{21-10x}{9+6x+4x^2} dx}{708588} \\
&= \frac{1}{472392(3 - 2x)} - \frac{1}{157464(3 + 2x)} + \frac{3 + 4x}{236196(9 + 6x + 4x^2)} - \frac{\log(3 - 2x)}{354294} + \frac{\log(3 + 2x)}{118098} - \frac{\log\left(\frac{9 + 6x + 4x^2}{9}\right)}{708588} \\
&= \frac{1}{472392(3 - 2x)} - \frac{1}{157464(3 + 2x)} + \frac{3 + 4x}{236196(9 + 6x + 4x^2)} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{472392\sqrt{3}} + \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{52488\sqrt{3}} - \frac{\log\left(\frac{9 + 6x + 4x^2}{9}\right)}{708588}
\end{aligned}$$

Mathematica [A] time = 0.0630649, size = 111, normalized size = 0.78

$$\frac{\frac{648x}{-16x^4 - 24x^3 + 54x + 81} - 3 \log(4x^2 - 6x + 9) - 5 \log(4x^2 + 6x + 9) - 8 \log(3 - 2x) + 24 \log(2x + 3) + 2\sqrt{3} \tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right) + 1}{2834352}$$

Antiderivative was successfully verified.

[In] Integrate[(9 - 6*x + 4*x^2)/(729 - 64*x^6)^2, x]

[Out] ((648*x)/(81 + 54*x - 24*x^3 - 16*x^4) + 2*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] + 18*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 8*Log[3 - 2*x] + 24*Log[3 + 2*x] - 3*Log[9 - 6*x + 4*x^2] - 5*Log[9 + 6*x + 4*x^2])/2834352

Maple [A] time = 0.013, size = 111, normalized size = 0.8

$$-\frac{1}{472392 + 314928x} + \frac{\ln(3 + 2x)}{118098} - \frac{1}{-1417176 + 944784x} - \frac{\ln(-3 + 2x)}{354294} - \frac{1}{708588} \left(-3x - \frac{9}{4}\right) \left(x^2 + \frac{3x}{2} + \frac{9}{4}\right)^{-1} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{472392\sqrt{3}} + \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{52488\sqrt{3}} - \frac{\log\left(\frac{9 + 6x + 4x^2}{9}\right)}{708588}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^2-6*x+9)/(-64*x^6+729)^2,x)

[Out] -1/157464/(3+2*x)+1/118098*ln(3+2*x)-1/472392/(-3+2*x)-1/354294*ln(-3+2*x)-
1/708588*(-3*x-9/4)/(x^2+3/2*x+9/4)-5/2834352*ln(4*x^2+6*x+9)+1/157464*3^(1
/2)*arctan(1/18*(8*x+6)*3^(1/2))-1/944784*ln(4*x^2-6*x+9)+1/1417176*3^(1/2)
arctan(1/18(8*x-6)*3^(1/2))

Maxima [A] time = 1.37607, size = 128, normalized size = 0.9

$$\frac{1}{157464} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x+3)\right) + \frac{1}{1417176} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x-3)\right) - \frac{x}{4374(16x^4+24x^3-54x-81)} - \frac{5}{2834352}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2-6*x+9)/(-64*x^6+729)^2,x, algorithm="maxima")

[Out] 1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/1417176*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(16*x^4 + 24*x^3 - 54*x - 81) - 5/2834352*
log(4*x^2 + 6*x + 9) - 1/944784*log(4*x^2 - 6*x + 9) + 1/118098*log(2*x + 3
) - 1/354294*log(2*x - 3)

Fricas [A] time = 1.46151, size = 539, normalized size = 3.8

$$18\sqrt{3}(16x^4+24x^3-54x-81)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) + 2\sqrt{3}(16x^4+24x^3-54x-81)\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) - 5(1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2-6*x+9)/(-64*x^6+729)^2,x, algorithm="fricas")

[Out] 1/2834352*(18*sqrt(3)*(16*x^4 + 24*x^3 - 54*x - 81)*arctan(1/9*sqrt(3)*(4*x
+ 3)) + 2*sqrt(3)*(16*x^4 + 24*x^3 - 54*x - 81)*arctan(1/9*sqrt(3)*(4*x -
3)) - 5*(16*x^4 + 24*x^3 - 54*x - 81)*log(4*x^2 + 6*x + 9) - 3*(16*x^4 + 24
*x^3 - 54*x - 81)*log(4*x^2 - 6*x + 9) + 24*(16*x^4 + 24*x^3 - 54*x - 81)*1
og(2*x + 3) - 8*(16*x^4 + 24*x^3 - 54*x - 81)*log(2*x - 3) - 648*x)/(16*x^4
+ 24*x^3 - 54*x - 81)

Sympy [A] time = 0.474707, size = 116, normalized size = 0.82

$$\frac{x}{69984x^4 + 104976x^3 - 236196x - 354294} - \frac{\log\left(x - \frac{3}{2}\right)}{354294} + \frac{\log\left(x + \frac{3}{2}\right)}{118098} - \frac{\log\left(x^2 - \frac{3x}{2} + \frac{9}{4}\right)}{944784} - \frac{5 \log\left(x^2 + \frac{3x}{2} + \frac{9}{4}\right)}{2834352} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x**2-6*x+9)/(-64*x**6+729)**2,x)

[Out] -x/(69984*x**4 + 104976*x**3 - 236196*x - 354294) - log(x - 3/2)/354294 + log(x + 3/2)/118098 - log(x**2 - 3*x/2 + 9/4)/944784 - 5*log(x**2 + 3*x/2 + 9/4)/2834352 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/1417176 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/157464

Giac [A] time = 1.06484, size = 143, normalized size = 1.01

$$\frac{1}{157464} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x+3)\right) + \frac{1}{1417176} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x-3)\right) - \frac{x}{4374(4x^2+6x+9)(2x+3)(2x-3)} -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2-6*x+9)/(-64*x^6+729)^2,x, algorithm="giac")

[Out] 1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/1417176*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 + 6*x + 9)*(2*x + 3)*(2*x - 3)) - 5/2834352*log(4*x^2 + 6*x + 9) - 1/944784*log(4*x^2 - 6*x + 9) + 1/118098*log(abs(2*x + 3)) - 1/354294*log(abs(2*x - 3))

$$3.572 \quad \int \frac{9+6x+4x^2}{(729-64x^6)^2} dx$$

Optimal. Leaf size=142

$$-\frac{3-4x}{236196(4x^2-6x+9)} + \frac{5 \log(4x^2-6x+9)}{2834352} + \frac{\log(4x^2+6x+9)}{944784} + \frac{1}{157464(3-2x)} - \frac{1}{472392(2x+3)} - \frac{\log(3-2x)}{118098}$$

[Out] 1/(157464*(3 - 2*x)) - 1/(472392*(3 + 2*x)) - (3 - 4*x)/(236196*(9 - 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(52488*Sqrt[3]) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(472392*Sqrt[3]) - Log[3 - 2*x]/118098 + Log[3 + 2*x]/354294 + (5*Log[9 - 6*x + 4*x^2])/2834352 + Log[9 + 6*x + 4*x^2]/944784

Rubi [A] time = 0.144815, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$, Rules used = {1586, 2074, 614, 618, 204, 634, 628}

$$-\frac{3-4x}{236196(4x^2-6x+9)} + \frac{5 \log(4x^2-6x+9)}{2834352} + \frac{\log(4x^2+6x+9)}{944784} + \frac{1}{157464(3-2x)} - \frac{1}{472392(2x+3)} - \frac{\log(3-2x)}{118098}$$

Antiderivative was successfully verified.

[In] Int[(9 + 6*x + 4*x^2)/(729 - 64*x^6)^2,x]

[Out] 1/(157464*(3 - 2*x)) - 1/(472392*(3 + 2*x)) - (3 - 4*x)/(236196*(9 - 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(52488*Sqrt[3]) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(472392*Sqrt[3]) - Log[3 - 2*x]/118098 + Log[3 + 2*x]/354294 + (5*Log[9 - 6*x + 4*x^2])/2834352 + Log[9 + 6*x + 4*x^2]/944784

Rule 1586

Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p+q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 2074

Int[(P_)^(p_.)*(Q_)^(q_.), x_Symbol] := With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] &&

PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
\int \frac{9 + 6x + 4x^2}{(729 - 64x^6)^2} dx &= \int \frac{1}{(9 + 6x + 4x^2)(81 - 54x + 24x^3 - 16x^4)^2} dx \\
&= \int \left(\frac{1}{78732(-3 + 2x)^2} - \frac{1}{59049(-3 + 2x)} + \frac{1}{236196(3 + 2x)^2} + \frac{1}{177147(3 + 2x)} + \frac{1}{4374(9 - 6x + 4x^2)} \right) dx \\
&= \frac{1}{157464(3 - 2x)} - \frac{1}{472392(3 + 2x)} - \frac{\log(3 - 2x)}{118098} + \frac{\log(3 + 2x)}{354294} + \frac{\int \frac{21+10x}{9-6x+4x^2} dx}{708588} + \frac{\int \frac{3+2x}{9+6x+4x^2} dx}{236196} \\
&= \frac{1}{157464(3 - 2x)} - \frac{1}{472392(3 + 2x)} - \frac{3 - 4x}{236196(9 - 6x + 4x^2)} - \frac{\log(3 - 2x)}{118098} + \frac{\log(3 + 2x)}{354294} + \frac{\int \frac{6}{9+6x+4x^2} dx}{9} \\
&= \frac{1}{157464(3 - 2x)} - \frac{1}{472392(3 + 2x)} - \frac{3 - 4x}{236196(9 - 6x + 4x^2)} - \frac{\log(3 - 2x)}{118098} + \frac{\log(3 + 2x)}{354294} + \frac{5 \log}{\dots} \\
&= \frac{1}{157464(3 - 2x)} - \frac{1}{472392(3 + 2x)} - \frac{3 - 4x}{236196(9 - 6x + 4x^2)} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{52488\sqrt{3}} + \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{472392\sqrt{3}} - \frac{\log}{\dots}
\end{aligned}$$

Mathematica [A] time = 0.0619316, size = 111, normalized size = 0.78

$$\frac{\frac{648x}{-16x^4+24x^3-54x+81} + 5 \log(4x^2 - 6x + 9) + 3 \log(4x^2 + 6x + 9) - 24 \log(3 - 2x) + 8 \log(2x + 3) + 18\sqrt{3} \tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right) + 2}{2834352}$$

Antiderivative was successfully verified.

[In] Integrate[(9 + 6*x + 4*x^2)/(729 - 64*x^6)^2,x]

[Out] ((648*x)/(81 - 54*x + 24*x^3 - 16*x^4) + 18*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] + 2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 24*Log[3 - 2*x] + 8*Log[3 + 2*x] + 5*Log[9 - 6*x + 4*x^2] + 3*Log[9 + 6*x + 4*x^2])/2834352

Maple [A] time = 0.013, size = 111, normalized size = 0.8

$$-\frac{1}{1417176 + 944784x} + \frac{\ln(3 + 2x)}{354294} - \frac{1}{-472392 + 314928x} - \frac{\ln(-3 + 2x)}{118098} + \frac{\ln(4x^2 + 6x + 9)}{944784} + \frac{\sqrt{3}}{1417176} \arctan\left(\frac{8}{\dots}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^2+6*x+9)/(-64*x^6+729)^2,x)

[Out] -1/472392/(3+2*x)+1/354294*ln(3+2*x)-1/157464/(-3+2*x)-1/118098*ln(-3+2*x)+
1/944784*ln(4*x^2+6*x+9)+1/1417176*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))+1/7
08588*(3*x-9/4)/(x^2-3/2*x+9/4)+5/2834352*ln(4*x^2-6*x+9)+1/157464*3^(1/2)*
arctan(1/18*(8*x-6)*3^(1/2))

Maxima [A] time = 1.37685, size = 128, normalized size = 0.9

$$\frac{1}{1417176} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x+3)\right) + \frac{1}{157464} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x-3)\right) - \frac{x}{4374(16x^4 - 24x^3 + 54x - 81)} + \frac{1}{944784}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2+6*x+9)/(-64*x^6+729)^2,x, algorithm="maxima")

[Out] 1/1417176*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/157464*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(16*x^4 - 24*x^3 + 54*x - 81) + 1/944784*1
og(4*x^2 + 6*x + 9) + 5/2834352*log(4*x^2 - 6*x + 9) + 1/354294*log(2*x + 3
) - 1/118098*log(2*x - 3)

Fricas [A] time = 1.4611, size = 539, normalized size = 3.8

$$2\sqrt{3}(16x^4 - 24x^3 + 54x - 81) \arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) + 18\sqrt{3}(16x^4 - 24x^3 + 54x - 81) \arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) + 3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2+6*x+9)/(-64*x^6+729)^2,x, algorithm="fricas")

[Out] 1/2834352*(2*sqrt(3)*(16*x^4 - 24*x^3 + 54*x - 81)*arctan(1/9*sqrt(3)*(4*x
+ 3)) + 18*sqrt(3)*(16*x^4 - 24*x^3 + 54*x - 81)*arctan(1/9*sqrt(3)*(4*x -
3)) + 3*(16*x^4 - 24*x^3 + 54*x - 81)*log(4*x^2 + 6*x + 9) + 5*(16*x^4 - 24
*x^3 + 54*x - 81)*log(4*x^2 - 6*x + 9) + 8*(16*x^4 - 24*x^3 + 54*x - 81)*lo
g(2*x + 3) - 24*(16*x^4 - 24*x^3 + 54*x - 81)*log(2*x - 3) - 648*x)/(16*x^4
- 24*x^3 + 54*x - 81)

Sympy [A] time = 0.46865, size = 116, normalized size = 0.82

$$\frac{x}{69984x^4 - 104976x^3 + 236196x - 354294} - \frac{\log\left(x - \frac{3}{2}\right)}{118098} + \frac{\log\left(x + \frac{3}{2}\right)}{354294} + \frac{5\log\left(x^2 - \frac{3x}{2} + \frac{9}{4}\right)}{2834352} + \frac{\log\left(x^2 + \frac{3x}{2} + \frac{9}{4}\right)}{944784} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x**2+6*x+9)/(-64*x**6+729)**2,x)

[Out] -x/(69984*x**4 - 104976*x**3 + 236196*x - 354294) - log(x - 3/2)/118098 + log(x + 3/2)/354294 + 5*log(x**2 - 3*x/2 + 9/4)/2834352 + log(x**2 + 3*x/2 + 9/4)/944784 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/157464 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/1417176

Giac [A] time = 1.06027, size = 143, normalized size = 1.01

$$\frac{1}{1417176} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x + 3)\right) + \frac{1}{157464} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x - 3)\right) - \frac{x}{4374(4x^2 - 6x + 9)(2x + 3)(2x - 3)} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2+6*x+9)/(-64*x^6+729)^2,x, algorithm="giac")

[Out] 1/1417176*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 - 6*x + 9)*(2*x + 3)*(2*x - 3)) + 1/944784*log(4*x^2 + 6*x + 9) + 5/2834352*log(4*x^2 - 6*x + 9) + 1/354294*log(abs(2*x + 3)) - 1/118098*log(abs(2*x - 3))

$$3.573 \quad \int \frac{27-8x^3}{(729-64x^6)^2} dx$$

Optimal. Leaf size=113

$$\frac{x}{4374(8x^3 + 27)} - \frac{7 \log(4x^2 - 6x + 9)}{944784} + \frac{\log(4x^2 + 6x + 9)}{314928} - \frac{\log(3 - 2x)}{157464} + \frac{7 \log(2x + 3)}{472392} - \frac{7 \tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}} + \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{52488\sqrt{3}}$$

[Out] x/(4374*(27 + 8*x^3)) - (7*ArcTan[(3 - 4*x)/(3*Sqrt[3])])/(157464*Sqrt[3]) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(52488*Sqrt[3]) - Log[3 - 2*x]/157464 + (7*Log[3 + 2*x])/472392 - (7*Log[9 - 6*x + 4*x^2])/944784 + Log[9 + 6*x + 4*x^2]/314928

Rubi [A] time = 0.0818849, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {1404, 414, 522, 200, 31, 634, 618, 204, 628}

$$\frac{x}{4374(8x^3 + 27)} - \frac{7 \log(4x^2 - 6x + 9)}{944784} + \frac{\log(4x^2 + 6x + 9)}{314928} - \frac{\log(3 - 2x)}{157464} + \frac{7 \log(2x + 3)}{472392} - \frac{7 \tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}} + \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{52488\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(27 - 8*x^3)/(729 - 64*x^6)^2,x]

[Out] x/(4374*(27 + 8*x^3)) - (7*ArcTan[(3 - 4*x)/(3*Sqrt[3])])/(157464*Sqrt[3]) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(52488*Sqrt[3]) - Log[3 - 2*x]/157464 + (7*Log[3 + 2*x])/472392 - (7*Log[9 - 6*x + 4*x^2])/944784 + Log[9 + 6*x + 4*x^2]/314928

Rule 1404

Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbol] := Int[(d + e*x^n)^(p + q)*(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, c, d, e, n, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
- Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
 imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{27 - 8x^3}{(729 - 64x^6)^2} dx &= \int \frac{1}{(27 - 8x^3)(27 + 8x^3)^2} dx \\
 &= \frac{x}{4374(27 + 8x^3)} - \frac{\int \frac{-1080 + 128x^3}{(27 - 8x^3)(27 + 8x^3)} dx}{34992} \\
 &= \frac{x}{4374(27 + 8x^3)} + \frac{\int \frac{1}{27 - 8x^3} dx}{2916} + \frac{7 \int \frac{1}{27 + 8x^3} dx}{8748} \\
 &= \frac{x}{4374(27 + 8x^3)} + \frac{\int \frac{1}{3 - 2x} dx}{78732} + \frac{\int \frac{6 + 2x}{9 + 6x + 4x^2} dx}{78732} + \frac{7 \int \frac{1}{3 + 2x} dx}{236196} + \frac{7 \int \frac{6 - 2x}{9 - 6x + 4x^2} dx}{236196} \\
 &= \frac{x}{4374(27 + 8x^3)} - \frac{\log(3 - 2x)}{157464} + \frac{7 \log(3 + 2x)}{472392} + \frac{\int \frac{6 + 8x}{9 + 6x + 4x^2} dx}{314928} - \frac{7 \int \frac{-6 + 8x}{9 - 6x + 4x^2} dx}{944784} + \frac{\int \frac{1}{9 + 6x + 4x^2} dx}{17496} \\
 &= \frac{x}{4374(27 + 8x^3)} - \frac{\log(3 - 2x)}{157464} + \frac{7 \log(3 + 2x)}{472392} - \frac{7 \log(9 - 6x + 4x^2)}{944784} + \frac{\log(9 + 6x + 4x^2)}{314928} \\
 &= \frac{x}{4374(27 + 8x^3)} - \frac{7 \tan^{-1}\left(\frac{3 - 4x}{3\sqrt{3}}\right)}{157464\sqrt{3}} + \frac{\tan^{-1}\left(\frac{3 + 4x}{3\sqrt{3}}\right)}{52488\sqrt{3}} - \frac{\log(3 - 2x)}{157464} + \frac{7 \log(3 + 2x)}{472392} - \frac{7 \log(9 - 6x + 4x^2)}{944784}
 \end{aligned}$$

Mathematica [A] time = 0.0501018, size = 103, normalized size = 0.91

$$\frac{\frac{216x}{8x^3 + 27} - 7 \log(4x^2 - 6x + 9) + 3 \log(4x^2 + 6x + 9) - 6 \log(3 - 2x) + 14 \log(2x + 3) + 14\sqrt{3} \tan^{-1}\left(\frac{4x - 3}{3\sqrt{3}}\right) + 6\sqrt{3} \tan^{-1}\left(\frac{4x + 3}{3\sqrt{3}}\right)}{944784}$$

Antiderivative was successfully verified.

[In] Integrate[(27 - 8*x^3)/(729 - 64*x^6)^2, x]

[Out] ((216*x)/(27 + 8*x^3) + 14*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])]) + 6*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 6*Log[3 - 2*x] + 14*Log[3 + 2*x] - 7*Log

$[9 - 6*x + 4*x^2] + 3*\text{Log}[9 + 6*x + 4*x^2])/944784$

Maple [A] time = 0.013, size = 102, normalized size = 0.9

$$-\frac{1}{236196 + 157464x} + \frac{7 \ln(3 + 2x)}{472392} - \frac{\ln(-3 + 2x)}{157464} + \frac{\ln(4x^2 + 6x + 9)}{314928} + \frac{\sqrt{3}}{157464} \arctan\left(\frac{(8x + 6)\sqrt{3}}{18}\right) - \frac{1}{118098} \left($$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-8*x^3+27)/(-64*x^6+729)^2,x)

[Out] -1/78732/(3+2*x)+7/472392*ln(3+2*x)-1/157464*ln(-3+2*x)+1/314928*ln(4*x^2+6*x+9)+1/157464*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))-1/118098*(-3/4*x-9/8)/(x^2-3/2*x+9/4)-7/944784*ln(4*x^2-6*x+9)+7/472392*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))

Maxima [A] time = 1.38086, size = 117, normalized size = 1.04

$$\frac{1}{157464} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x + 3)\right) + \frac{7}{472392} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x - 3)\right) + \frac{x}{4374(8x^3 + 27)} + \frac{1}{314928} \log(4x^2 + 6x + 9)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-8*x^3+27)/(-64*x^6+729)^2,x, algorithm="maxima")

[Out] 1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 7/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(8*x^3 + 27) + 1/314928*log(4*x^2 + 6*x + 9) - 7/944784*log(4*x^2 - 6*x + 9) + 7/472392*log(2*x + 3) - 1/157464*log(2*x - 3)

Fricas [A] time = 1.42664, size = 377, normalized size = 3.34

$$\frac{6\sqrt{3}(8x^3 + 27) \arctan\left(\frac{1}{9}\sqrt{3}(4x + 3)\right) + 14\sqrt{3}(8x^3 + 27) \arctan\left(\frac{1}{9}\sqrt{3}(4x - 3)\right) + 3(8x^3 + 27) \log(4x^2 + 6x + 9) - 944784(8x^3 + 27)}{944784(8x^3 + 27)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-8*x^3+27)/(-64*x^6+729)^2,x, algorithm="fricas")

[Out] 1/944784*(6*sqrt(3)*(8*x^3 + 27)*arctan(1/9*sqrt(3)*(4*x + 3)) + 14*sqrt(3)*(8*x^3 + 27)*arctan(1/9*sqrt(3)*(4*x - 3)) + 3*(8*x^3 + 27)*log(4*x^2 + 6*x + 9) - 7*(8*x^3 + 27)*log(4*x^2 - 6*x + 9) + 14*(8*x^3 + 27)*log(2*x + 3) - 6*(8*x^3 + 27)*log(2*x - 3) + 216*x)/(8*x^3 + 27)

Sympy [A] time = 0.3835, size = 110, normalized size = 0.97

$$\frac{x}{34992x^3 + 118098} - \frac{\log\left(x - \frac{3}{2}\right)}{157464} + \frac{7\log\left(x + \frac{3}{2}\right)}{472392} - \frac{7\log\left(x^2 - \frac{3x}{2} + \frac{9}{4}\right)}{944784} + \frac{\log\left(x^2 + \frac{3x}{2} + \frac{9}{4}\right)}{314928} + \frac{7\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9} - \frac{\sqrt{3}}{3}\right)}{472392}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-8*x**3+27)/(-64*x**6+729)**2,x)

[Out] x/(34992*x**3 + 118098) - log(x - 3/2)/157464 + 7*log(x + 3/2)/472392 - 7*log(x**2 - 3*x/2 + 9/4)/944784 + log(x**2 + 3*x/2 + 9/4)/314928 + 7*sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/472392 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/157464

Giac [A] time = 1.05941, size = 120, normalized size = 1.06

$$\frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) + \frac{7}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) + \frac{x}{4374(8x^3+27)} + \frac{1}{314928}\log(4x^2+6x+9) - \frac{7}{944784}\log(4x^2-6x+9) + \frac{7}{472392}\log(\operatorname{abs}(2x+3)) - \frac{1}{157464}\log(\operatorname{abs}(2x-3))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-8*x^3+27)/(-64*x^6+729)^2,x, algorithm="giac")

[Out] 1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 7/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(8*x^3 + 27) + 1/314928*log(4*x^2 + 6*x + 9) - 7/944784*log(4*x^2 - 6*x + 9) + 7/472392*log(abs(2*x + 3)) - 1/157464*log(abs(2*x - 3))

$$3.574 \quad \int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2} dx$$

Optimal. Leaf size=131

$$-\frac{3-2x}{26244(4x^2-6x+9)} + \frac{17 \log(4x^2-6x+9)}{944784} + \frac{\log(4x^2+6x+9)}{314928} + \frac{1}{26244(3-2x)} - \frac{7 \log(3-2x)}{157464} + \frac{\log(2x+3)}{472392} -$$

[Out] 1/(26244*(3 - 2*x)) - (3 - 2*x)/(26244*(9 - 6*x + 4*x^2)) - (11*ArcTan[(3 - 4*x)/(3*Sqrt[3])])/(157464*Sqrt[3]) - ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(157464*Sqrt[3]) - (7*Log[3 - 2*x])/157464 + Log[3 + 2*x]/472392 + (17*Log[9 - 6*x + 4*x^2])/944784 + Log[9 + 6*x + 4*x^2]/314928

Rubi [A] time = 0.147723, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {1586, 2074, 638, 618, 204, 634, 628}

$$-\frac{3-2x}{26244(4x^2-6x+9)} + \frac{17 \log(4x^2-6x+9)}{944784} + \frac{\log(4x^2+6x+9)}{314928} + \frac{1}{26244(3-2x)} - \frac{7 \log(3-2x)}{157464} + \frac{\log(2x+3)}{472392} -$$

Antiderivative was successfully verified.

[In] Int[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6)^2,x]

[Out] 1/(26244*(3 - 2*x)) - (3 - 2*x)/(26244*(9 - 6*x + 4*x^2)) - (11*ArcTan[(3 - 4*x)/(3*Sqrt[3])])/(157464*Sqrt[3]) - ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(157464*Sqrt[3]) - (7*Log[3 - 2*x])/157464 + Log[3 + 2*x]/472392 + (17*Log[9 - 6*x + 4*x^2])/944784 + Log[9 + 6*x + 4*x^2]/314928

Rule 1586

Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p+q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 2074

Int[(P_)^(p_)*(Q_)^(q_), x_Symbol] := With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] &&

PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rule 638

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 628

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
\int \frac{27 + 36x + 24x^2 + 8x^3}{(729 - 64x^6)^2} dx &= \int \frac{1}{(27 - 36x + 24x^2 - 8x^3)^2 (27 + 36x + 24x^2 + 8x^3)} dx \\
&= \int \left(\frac{1}{13122(-3 + 2x)^2} - \frac{7}{78732(-3 + 2x)} + \frac{1}{236196(3 + 2x)} + \frac{3 + 2x}{4374(9 - 6x + 4x^2)^2} + \frac{11}{157464\sqrt{3}} \tan^{-1} \left(\frac{3-4x}{3\sqrt{3}} \right) \right) dx \\
&= \frac{1}{26244(3 - 2x)} - \frac{7 \log(3 - 2x)}{157464} + \frac{\log(3 + 2x)}{472392} + \frac{\int \frac{3+17x}{9-6x+4x^2} dx}{118098} + \frac{\int \frac{x}{9+6x+4x^2} dx}{39366} + \frac{\int \frac{3}{(9-6x+4x^2)^2} dx}{4} \\
&= \frac{1}{26244(3 - 2x)} - \frac{3 - 2x}{26244(9 - 6x + 4x^2)} - \frac{7 \log(3 - 2x)}{157464} + \frac{\log(3 + 2x)}{472392} + \frac{\int \frac{6+8x}{9+6x+4x^2} dx}{314928} + \frac{11 \tan^{-1} \left(\frac{3-4x}{3\sqrt{3}} \right)}{157464\sqrt{3}} \\
&= \frac{1}{26244(3 - 2x)} - \frac{3 - 2x}{26244(9 - 6x + 4x^2)} - \frac{7 \log(3 - 2x)}{157464} + \frac{\log(3 + 2x)}{472392} + \frac{17 \log(9 - 6x + 4x^2)}{944784} \\
&= \frac{1}{26244(3 - 2x)} - \frac{3 - 2x}{26244(9 - 6x + 4x^2)} - \frac{11 \tan^{-1} \left(\frac{3-4x}{3\sqrt{3}} \right)}{157464\sqrt{3}} - \frac{\tan^{-1} \left(\frac{3+4x}{3\sqrt{3}} \right)}{157464\sqrt{3}} - \frac{7 \log(3 - 2x)}{157464}
\end{aligned}$$

Mathematica [A] time = 0.0637757, size = 111, normalized size = 0.85

$$\frac{\frac{216x}{-8x^3+24x^2-36x+27} + 17 \log(4x^2 - 6x + 9) + 3 \log(4x^2 + 6x + 9) - 42 \log(3 - 2x) + 2 \log(2x + 3) + 22\sqrt{3} \tan^{-1} \left(\frac{4x-3}{3\sqrt{3}} \right) - 2 \tan^{-1} \left(\frac{3+4x}{3\sqrt{3}} \right)}{944784}$$

Antiderivative was successfully verified.

[In] Integrate[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6)^2, x]

[Out] ((216*x)/(27 - 36*x + 24*x^2 - 8*x^3) + 22*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] - 2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 42*Log[3 - 2*x] + 2*Log[3 + 2*x] + 17*Log[9 - 6*x + 4*x^2] + 3*Log[9 + 6*x + 4*x^2])/944784

Maple [A] time = 0.013, size = 102, normalized size = 0.8

$$\frac{\ln(3 + 2x)}{472392} - \frac{1}{-78732 + 52488x} - \frac{7 \ln(-3 + 2x)}{157464} + \frac{\ln(4x^2 + 6x + 9)}{314928} - \frac{\sqrt{3}}{472392} \arctan\left(\frac{(8x + 6)\sqrt{3}}{18}\right) + \frac{1}{118098} \left(\frac{9x}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*x^3+24*x^2+36*x+27)/(-64*x^6+729)^2,x)

[Out] 1/472392*ln(3+2*x)-1/26244/(-3+2*x)-7/157464*ln(-3+2*x)+1/314928*ln(4*x^2+6*x+9)-1/472392*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))+1/118098*(9/4*x-27/8)/(x^2-3/2*x+9/4)+17/944784*ln(4*x^2-6*x+9)+11/472392*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))

Maxima [A] time = 1.3948, size = 128, normalized size = 0.98

$$-\frac{1}{472392} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x+3)\right) + \frac{11}{472392} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x-3)\right) - \frac{x}{4374(8x^3-24x^2+36x-27)} + \frac{1}{314928}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729)^2,x, algorithm="maxima")

[Out] -1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 11/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(8*x^3 - 24*x^2 + 36*x - 27) + 1/314928*log(4*x^2 + 6*x + 9) + 17/944784*log(4*x^2 - 6*x + 9) + 1/472392*log(2*x + 3) - 7/157464*log(2*x - 3)

Fricas [A] time = 1.44256, size = 531, normalized size = 4.05

$$2\sqrt{3}(8x^3-24x^2+36x-27)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) - 22\sqrt{3}(8x^3-24x^2+36x-27)\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) - 3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729)^2,x, algorithm="fricas")

[Out] -1/944784*(2*sqrt(3)*(8*x^3 - 24*x^2 + 36*x - 27)*arctan(1/9*sqrt(3)*(4*x + 3)) - 22*sqrt(3)*(8*x^3 - 24*x^2 + 36*x - 27)*arctan(1/9*sqrt(3)*(4*x - 3)) - 3*(8*x^3 - 24*x^2 + 36*x - 27)*log(4*x^2 + 6*x + 9) - 17*(8*x^3 - 24*x^2 + 36*x - 27)*log(4*x^2 - 6*x + 9) - 2*(8*x^3 - 24*x^2 + 36*x - 27)*log(2*x + 3) + 42*(8*x^3 - 24*x^2 + 36*x - 27)*log(2*x - 3) + 216*x)/(8*x^3 - 24*x^2 + 36*x - 27)

Sympy [A] time = 0.46846, size = 119, normalized size = 0.91

$$\frac{x}{34992x^3 - 104976x^2 + 157464x - 118098} - \frac{7 \log\left(x - \frac{3}{2}\right)}{157464} + \frac{\log\left(x + \frac{3}{2}\right)}{472392} + \frac{17 \log\left(x^2 - \frac{3x}{2} + \frac{9}{4}\right)}{944784} + \frac{\log\left(x^2 + \frac{3x}{2} + \frac{9}{4}\right)}{314928} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x**3+24*x**2+36*x+27)/(-64*x**6+729)**2,x)

[Out] -x/(34992*x**3 - 104976*x**2 + 157464*x - 118098) - 7*log(x - 3/2)/157464 + log(x + 3/2)/472392 + 17*log(x**2 - 3*x/2 + 9/4)/944784 + log(x**2 + 3*x/2 + 9/4)/314928 + 11*sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/472392 - sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/472392

Giac [A] time = 1.04995, size = 134, normalized size = 1.02

$$-\frac{1}{472392} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x+3)\right) + \frac{11}{472392} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x-3)\right) - \frac{x}{4374(4x^2-6x+9)(2x-3)} + \frac{1}{314928} \log(4x^2+6x+9) + 17/944784 \log(4x^2-6x+9) + 1/472392 \log(\text{abs}(2x+3)) - 7/157464 \log(\text{abs}(2x-3))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729)^2,x, algorithm="giac")

[Out] -1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 11/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 - 6*x + 9)*(2*x - 3)) + 1/314928*log(4*x^2 + 6*x + 9) + 17/944784*log(4*x^2 - 6*x + 9) + 1/472392*log(abs(2*x + 3)) - 7/157464*log(abs(2*x - 3))

$$3.575 \quad \int \frac{x(27-2x^3)}{729-64x^6} dx$$

Optimal. Leaf size=99

$$\frac{5}{576} \log(4x^2 - 6x + 9) + \frac{1}{192} \log(4x^2 + 6x + 9) - \frac{1}{96} \log(3 - 2x) - \frac{5}{288} \log(2x + 3) - \frac{5 \tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{96\sqrt{3}} - \frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{32\sqrt{3}}$$

[Out] $(-5*\text{ArcTan}[(3 - 4*x)/(3*\text{Sqrt}[3]))]/(96*\text{Sqrt}[3]) - \text{ArcTan}[(3 + 4*x)/(3*\text{Sqrt}[3])]/(32*\text{Sqrt}[3]) - \text{Log}[3 - 2*x]/96 - (5*\text{Log}[3 + 2*x])/288 + (5*\text{Log}[9 - 6*x + 4*x^2])/576 + \text{Log}[9 + 6*x + 4*x^2]/192$

Rubi [A] time = 0.0627053, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1511, 292, 31, 634, 618, 204, 628}

$$\frac{5}{576} \log(4x^2 - 6x + 9) + \frac{1}{192} \log(4x^2 + 6x + 9) - \frac{1}{96} \log(3 - 2x) - \frac{5}{288} \log(2x + 3) - \frac{5 \tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{96\sqrt{3}} - \frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{32\sqrt{3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x*(27 - 2*x^3))/(729 - 64*x^6), x]$

[Out] $(-5*\text{ArcTan}[(3 - 4*x)/(3*\text{Sqrt}[3]))]/(96*\text{Sqrt}[3]) - \text{ArcTan}[(3 + 4*x)/(3*\text{Sqrt}[3])]/(32*\text{Sqrt}[3]) - \text{Log}[3 - 2*x]/96 - (5*\text{Log}[3 + 2*x])/288 + (5*\text{Log}[9 - 6*x + 4*x^2])/576 + \text{Log}[9 + 6*x + 4*x^2]/192$

Rule 1511

$\text{Int}[(((f_.)*(x_.))^{(m_.)}*((d_.) + (e_.)*(x_.)^{(n_.)}))/((a_.) + (c_.)*(x_.)^{(n2_.)})$, x_Symbol] \rightarrow $\text{With}\{q = \text{Rt}[-(a*c), 2]\}$, $-\text{Dist}[e/2 + (c*d)/(2*q), \text{Int}[(f*x)^m/(q - c*x^n), x], x] + \text{Dist}[e/2 - (c*d)/(2*q), \text{Int}[(f*x)^m/(q + c*x^n), x], x]$ /; $\text{FreeQ}\{a, c, d, e, f, m\}, x$ && $\text{EqQ}[n2, 2*n]$ && $\text{IGtQ}[n, 0]$

Rule 292

$\text{Int}[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol]$ \rightarrow $-\text{Dist}[(3*\text{Rt}[a, 3]*\text{Rt}[b, 3])^{(-1)}, \text{Int}[1/(\text{Rt}[a, 3] + \text{Rt}[b, 3]*x), x], x] + \text{Dist}[1/(3*\text{Rt}[a, 3]*\text{Rt}[b, 3]), \text{Int}[(\text{Rt}[a, 3] + \text{Rt}[b, 3]*x)/(\text{Rt}[a, 3]^2 - \text{Rt}[a, 3]*\text{Rt}[b, 3]*x + \text{Rt}[b, 3]^2*x^2), x], x]$ /; $\text{FreeQ}\{a, b\}, x]$

Rule 31

```
Int[((a_) + (b_.)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^( -1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^( -1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(27-2x^3)}{729-64x^6} dx &= 3 \int \frac{x}{216-64x^3} dx + 5 \int \frac{x}{216+64x^3} dx \\
&= \frac{1}{24} \int \frac{1}{6-4x} dx - \frac{1}{24} \int \frac{6-4x}{36+24x+16x^2} dx - \frac{5}{72} \int \frac{1}{6+4x} dx + \frac{5}{72} \int \frac{6+4x}{36-24x+16x^2} dx \\
&= -\frac{1}{96} \log(3-2x) - \frac{5}{288} \log(3+2x) + \frac{1}{192} \int \frac{24+32x}{36+24x+16x^2} dx + \frac{5}{576} \int \frac{-24+32x}{36-24x+16x^2} dx - \\
&= -\frac{1}{96} \log(3-2x) - \frac{5}{288} \log(3+2x) + \frac{5}{576} \log(9-6x+4x^2) + \frac{1}{192} \log(9+6x+4x^2) + \frac{3}{4} \text{Subst} \\
&= -\frac{5 \tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{96\sqrt{3}} - \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{32\sqrt{3}} - \frac{1}{96} \log(3-2x) - \frac{5}{288} \log(3+2x) + \frac{5}{576} \log(9-6x+4x^2) +
\end{aligned}$$

Mathematica [A] time = 0.0139777, size = 91, normalized size = 0.92

$$\frac{1}{576} \left(5 \log(4x^2 - 6x + 9) + 3 \log(4x^2 + 6x + 9) - 6 \log(3 - 2x) - 10 \log(2x + 3) + 10\sqrt{3} \tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right) - 6\sqrt{3} \tan^{-1}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(x*(27 - 2*x^3))/(729 - 64*x^6),x]

[Out] (10*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] - 6*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 6*Log[3 - 2*x] - 10*Log[3 + 2*x] + 5*Log[9 - 6*x + 4*x^2] + 3*Log[9 + 6*x + 4*x^2])/576

Maple [A] time = 0.007, size = 76, normalized size = 0.8

$$-\frac{5 \ln(3+2x)}{288} - \frac{\ln(-3+2x)}{96} + \frac{\ln(4x^2+6x+9)}{192} - \frac{\sqrt{3}}{96} \arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right) + \frac{5 \ln(4x^2-6x+9)}{576} + \frac{5\sqrt{3}}{288} \arctan\left(\frac{4x-3}{3\sqrt{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(-2*x^3+27)/(-64*x^6+729),x)

[Out] -5/288*ln(3+2*x)-1/96*ln(-3+2*x)+1/192*ln(4*x^2+6*x+9)-1/96*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))+5/576*ln(4*x^2-6*x+9)+5/288*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))

Maxima [A] time = 1.37515, size = 101, normalized size = 1.02

$$-\frac{1}{96} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x+3)\right) + \frac{5}{288} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x-3)\right) + \frac{1}{192} \log(4x^2+6x+9) + \frac{5}{576} \log(4x^2-6x+9)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-2*x^3+27)/(-64*x^6+729),x, algorithm="maxima")

[Out] -1/96*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 5/288*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/192*log(4*x^2 + 6*x + 9) + 5/576*log(4*x^2 - 6*x + 9) - 5/288*log(2*x + 3) - 1/96*log(2*x - 3)

Fricas [A] time = 1.44422, size = 257, normalized size = 2.6

$$-\frac{1}{96} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x+3)\right) + \frac{5}{288} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x-3)\right) + \frac{1}{192} \log(4x^2+6x+9) + \frac{5}{576} \log(4x^2-6x+9)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-2*x^3+27)/(-64*x^6+729),x, algorithm="fricas")

[Out] -1/96*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 5/288*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/192*log(4*x^2 + 6*x + 9) + 5/576*log(4*x^2 - 6*x + 9) - 5/288*log(2*x + 3) - 1/96*log(2*x - 3)

Sympy [A] time = 0.327847, size = 102, normalized size = 1.03

$$-\frac{\log\left(x - \frac{3}{2}\right)}{96} - \frac{5 \log\left(x + \frac{3}{2}\right)}{288} + \frac{5 \log\left(x^2 - \frac{3x}{2} + \frac{9}{4}\right)}{576} + \frac{\log\left(x^2 + \frac{3x}{2} + \frac{9}{4}\right)}{192} + \frac{5\sqrt{3} \operatorname{atan}\left(\frac{4\sqrt{3}x}{9} - \frac{\sqrt{3}}{3}\right)}{288} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{4\sqrt{3}x}{9} + \frac{\sqrt{3}}{3}\right)}{96}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-2*x**3+27)/(-64*x**6+729),x)

[Out] -log(x - 3/2)/96 - 5*log(x + 3/2)/288 + 5*log(x**2 - 3*x/2 + 9/4)/576 + log(x**2 + 3*x/2 + 9/4)/192 + 5*sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/288 -

$\sqrt{3} \operatorname{atan}(4\sqrt{3}x/9 + \sqrt{3}/3)/96$

Giac [A] time = 1.06298, size = 93, normalized size = 0.94

$$-\frac{1}{96} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x+3)\right) + \frac{5}{288} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3}(4x-3)\right) + \frac{1}{192} \log\left(x^2 + \frac{3}{2}x + \frac{9}{4}\right) + \frac{5}{576} \log\left(x^2 - \frac{3}{2}x + \frac{9}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-2*x^3+27)/(-64*x^6+729),x, algorithm="giac")

[Out] -1/96*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 5/288*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/192*log(x^2 + 3/2*x + 9/4) + 5/576*log(x^2 - 3/2*x + 9/4) - 5/288*log(abs(x + 3/2)) - 1/96*log(abs(x - 3/2))

$$3.576 \quad \int \frac{(cx)^m (d + ex^n + fx^{2n} + gx^{3n})}{a + bx^n} dx$$

Optimal. Leaf size=162

$$\frac{(cx)^{m+1} (a^2bf + a^3(-g) - ab^2e + b^3d) {}_2F_1\left(1, \frac{m+1}{n}; \frac{m+n+1}{n}; -\frac{bx^n}{a}\right)}{ab^3c(m+1)} + \frac{(cx)^{m+1} (a^2g - abf + b^2e)}{b^3c(m+1)} + \frac{x^{n+1}(cx)^m (bf - ag)}{b^2(m+n+1)} + \frac{g}{b}$$

[Out] ((b*f - a*g)*x^(1 + n)*(c*x)^m)/(b^2*(1 + m + n)) + (g*x^(1 + 2*n)*(c*x)^m)/(b*(1 + m + 2*n)) + ((b^2*e - a*b*f + a^2*g)*(c*x)^(1 + m))/(b^3*c*(1 + m)) + ((b^3*d - a*b^2*e + a^2*b*f - a^3*g)*(c*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(a*b^3*c*(1 + m))

Rubi [A] time = 0.167104, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1844, 20, 30, 364}

$$\frac{(cx)^{m+1} (a^2bf + a^3(-g) - ab^2e + b^3d) {}_2F_1\left(1, \frac{m+1}{n}; \frac{m+n+1}{n}; -\frac{bx^n}{a}\right)}{ab^3c(m+1)} + \frac{(cx)^{m+1} (a^2g - abf + b^2e)}{b^3c(m+1)} + \frac{x^{n+1}(cx)^m (bf - ag)}{b^2(m+n+1)} + \frac{g}{b}$$

Antiderivative was successfully verified.

[In] Int[((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/(a + b*x^n), x]

[Out] ((b*f - a*g)*x^(1 + n)*(c*x)^m)/(b^2*(1 + m + n)) + (g*x^(1 + 2*n)*(c*x)^m)/(b*(1 + m + 2*n)) + ((b^2*e - a*b*f + a^2*g)*(c*x)^(1 + m))/(b^3*c*(1 + m)) + ((b^3*d - a*b^2*e + a^2*b*f - a^3*g)*(c*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(a*b^3*c*(1 + m))

Rule 1844

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) && !IGtQ[m, 0]

Rule 20

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !

IntegerQ[m + n]

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N eQ[m, -1]

Rule 364

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^ p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a)])]/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt Q[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned} \int \frac{(cx)^m (d + ex^n + fx^{2n} + gx^{3n})}{a + bx^n} dx &= \int \left(\frac{(b^2e - abf + a^2g)(cx)^m}{b^3} + \frac{(bf - ag)x^n(cx)^m}{b^2} + \frac{gx^{2n}(cx)^m}{b} + \frac{(b^3d - ab^2e + a^3g)(cx)^m}{b^3(a + bx^n)} \right) dx \\ &= \frac{(b^2e - abf + a^2g)(cx)^{1+m}}{b^3c(1+m)} + \frac{g \int x^{2n}(cx)^m dx}{b} + \frac{(bf - ag) \int x^n(cx)^m dx}{b^2} + \frac{(b^3d - ab^2e + a^3g)(cx)^{1+m}}{b^3(a + bx^n)} \\ &= \frac{(b^2e - abf + a^2g)(cx)^{1+m}}{b^3c(1+m)} + \frac{(b^3d - ab^2e + a^2bf - a^3g)(cx)^{1+m} {}_2F_1\left(1, \frac{1+m}{n}; \frac{1+m+n}{n}, -\frac{bx^n}{a}\right)}{ab^3c(1+m)} \\ &= \frac{(bf - ag)x^{1+n}(cx)^m}{b^2(1+m+n)} + \frac{gx^{1+2n}(cx)^m}{b(1+m+2n)} + \frac{(b^2e - abf + a^2g)(cx)^{1+m}}{b^3c(1+m)} + \frac{(b^3d - ab^2e + a^3g)(cx)^{1+m}}{b^3(a + bx^n)} \end{aligned}$$

Mathematica [A] time = 0.341273, size = 130, normalized size = 0.8

$$\frac{x(cx)^m \left(\frac{(a^2bf + a^3(-g) - ab^2e + b^3d) {}_2F_1\left(1, \frac{m+1}{n}; \frac{m+n+1}{n}; -\frac{bx^n}{a}\right)}{a(m+1)} + \frac{a^2g - abf + b^2e}{m+1} + \frac{bx^n(bf - ag)}{m+n+1} + \frac{b^2gx^{2n}}{m+2n+1} \right)}{b^3}$$

Antiderivative was successfully verified.

[In] Integrate[((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/(a + b*x^n), x]

[Out] (x*(c*x)^m*((b^2*e - a*b*f + a^2*g)/(1 + m) + (b*(b*f - a*g)*x^n)/(1 + m + n) + (b^2*g*x^(2*n))/(1 + m + 2*n) + ((b^3*d - a*b^2*e + a^2*b*f - a^3*g)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(a*(1 + m)))/

b^3

Maple [F] time = 0.384, size = 0, normalized size = 0.

$$\int \frac{(cx)^m (d + ex^n + fx^{2n} + gx^{3n})}{a + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n), x)

[Out] int((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$(b^3c^m d - ab^2c^m e + a^2bc^m f - a^3c^m g) \int \frac{x^m}{b^4x^n + ab^3} dx + \frac{(m^2 + m(n+2) + n + 1)b^2c^m g x e^{(m \log(x) + 2n \log(x))} + ((m^2 + m(3n+2) + 2n^2 + 3n + 1)b^2c^m e - (m^2 + m(3n+2) + 2n^2 + 3n + 1)a^2bc^m f + (m^2 + m(3n+2) + 2n^2 + 3n + 1)a^2c^m g) x x^m + ((m^2 + 2m(n+1) + 2n + 1)b^2c^m f - (m^2 + 2m(n+1) + 2n + 1)a^2bc^m g) x e^{(m \log(x) + n \log(x))}}{(m^3 + 3m^2(n+1) + (2n^2 + 6n + 3)m + 2n^2 + 3n + 1)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n), x, algorithm="maxima")

[Out] (b^3*c^m*d - a*b^2*c^m*e + a^2*b*c^m*f - a^3*c^m*g)*integrate(x^m/(b^4*x^n + a*b^3), x) + ((m^2 + m*(n + 2) + n + 1)*b^2*c^m*g*x*e^(m*log(x) + 2*n*log(x)) + ((m^2 + m*(3*n + 2) + 2*n^2 + 3*n + 1)*b^2*c^m*e - (m^2 + m*(3*n + 2) + 2*n^2 + 3*n + 1)*a^2*b*c^m*f + (m^2 + m*(3*n + 2) + 2*n^2 + 3*n + 1)*a^2*c^m*g)*x*x^m + ((m^2 + 2*m*(n + 1) + 2*n + 1)*b^2*c^m*f - (m^2 + 2*m*(n + 1) + 2*n + 1)*a^2*b*c^m*g)*x*e^(m*log(x) + n*log(x)))/((m^3 + 3*m^2*(n + 1) + (2*n^2 + 6*n + 3)*m + 2*n^2 + 3*n + 1)*b^3)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(gx^{3n} + fx^{2n} + ex^n + d)(cx)^m}{bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(c*x)^m/(b*x^n + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(d+e*x**n+f*x**(2*n)+g*x**(3*n))/(a+b*x**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^{3n} + fx^{2n} + ex^n + d)(cx)^m}{bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(c*x)^m/(b*x^n + a), x)
```

$$3.577 \quad \int (c + dx^{-1+n}) (a + bx^n)^3 dx$$

Optimal. Leaf size=84

$$\frac{3a^2bcx^{n+1}}{n+1} + a^3cx + \frac{3ab^2cx^{2n+1}}{2n+1} + \frac{d(a+bx^n)^4}{4bn} + \frac{b^3cx^{3n+1}}{3n+1}$$

[Out] $a^3cx + (3a^2bcx^{n+1})/(n+1) + (3ab^2cx^{2n+1})/(2n+1) + (d(a+bx^n)^4)/(4bn) + (b^3cx^{3n+1})/(3n+1)$

Rubi [A] time = 0.0557268, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1891, 244, 261}

$$\frac{3a^2bcx^{n+1}}{n+1} + a^3cx + \frac{3ab^2cx^{2n+1}}{2n+1} + \frac{d(a+bx^n)^4}{4bn} + \frac{b^3cx^{3n+1}}{3n+1}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^(-1 + n))*(a + b*x^n)^3, x]

[Out] $a^3cx + (3a^2bcx^{n+1})/(n+1) + (3ab^2cx^{2n+1})/(2n+1) + (d(a+bx^n)^4)/(4bn) + (b^3cx^{3n+1})/(3n+1)$

Rule 1891

Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :=
Dist[A, Int[(a + b*x^n)^p, x], x] + Dist[B, Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]

Rule 244

Int[((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
\int (c + dx^{-1+n})(a + bx^n)^3 dx &= c \int (a + bx^n)^3 dx + d \int x^{-1+n} (a + bx^n)^3 dx \\
&= \frac{d(a + bx^n)^4}{4bn} + c \int (a^3 + 3a^2bx^n + 3ab^2x^{2n} + b^3x^{3n}) dx \\
&= a^3cx + \frac{3a^2bcx^{1+n}}{1+n} + \frac{3ab^2cx^{1+2n}}{1+2n} + \frac{b^3cx^{1+3n}}{1+3n} + \frac{d(a + bx^n)^4}{4bn}
\end{aligned}$$

Mathematica [A] time = 0.163315, size = 108, normalized size = 1.29

$$\frac{x(c + dx^{n-1}) \left(\frac{12a^2bcx^{n+1}}{n+1} + 4a^3cx + \frac{12ab^2cx^{2n+1}}{2n+1} + \frac{d(a+bx^n)^4}{bn} + \frac{4b^3cx^{3n+1}}{3n+1} \right)}{4(cx + dx^n)}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^(-1 + n))*(a + b*x^n)^3, x]

[Out] (x*(c + d*x^(-1 + n))*(4*a^3*c*x + (12*a^2*b*c*x^(1 + n))/(1 + n) + (12*a*b^2*c*x^(1 + 2*n))/(1 + 2*n) + (4*b^3*c*x^(1 + 3*n))/(1 + 3*n) + (d*(a + b*x^n)^4)/(b*n)))/(4*(c*x + d*x^n))

Maple [A] time = 0.015, size = 130, normalized size = 1.6

$$a^3cx + \frac{a^3de^{n\ln(x)}}{n} + \frac{ab^2d(e^{n\ln(x)})^3}{n} + \frac{b^3cx(e^{n\ln(x)})^3}{1+3n} + \frac{b^3d(e^{n\ln(x)})^4}{4n} + \frac{3a^2bd(e^{n\ln(x)})^2}{2n} + 3\frac{acb^2x(e^{n\ln(x)})^2}{1+2n} + 3\frac{a^2bcx}{1+3n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d*x^(-1+n))*(a+b*x^n)^3, x)

[Out] a^3*c*x+a^3*d/n*exp(n*ln(x))+a*b^2*d/n*exp(n*ln(x))^3+b^3*c/(1+3*n)*x*exp(n*ln(x))^3+1/4*b^3*d/n*exp(n*ln(x))^4+3/2*a^2*b*d/n*exp(n*ln(x))^2+3*a*c*b^2/(1+2*n)*x*exp(n*ln(x))^2+3*b*a^2*c/(1+n)*x*exp(n*ln(x))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*x^(-1+n))*(a+b*x^n)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 1.54533, size = 660, normalized size = 7.86

$$4(6a^3cn^4 + 11a^3cn^3 + 6a^3cn^2 + a^3cn)x + (6b^3dn^3 + 11b^3dn^2 + 6b^3dn + b^3d)x^{4n} + 4(6ab^2dn^3 + 11ab^2dn^2 + 6ab^2dn$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*x^(-1+n))*(a+b*x^n)^3,x, algorithm="fricas")

[Out]
$$\frac{1}{4} \cdot (4 \cdot (6a^3cn^4 + 11a^3cn^3 + 6a^3cn^2 + a^3cn)x + (6b^3dn^3 + 11b^3dn^2 + 6b^3dn + b^3d)x^{4n} + 4 \cdot (6ab^2dn^3 + 11ab^2dn^2 + 6ab^2dn^2 + 6ab^2dn^2 + 6ab^2dn^2 + a^2b^2d + (2b^3cn^3 + 3b^3cn^2 + b^3cn)x)x^{3n} + 6 \cdot (6a^2b^2dn^3 + 11a^2b^2dn^2 + 6a^2b^2dn + a^2b^2d + 2 \cdot (3ab^2cn^3 + 4ab^2cn^2 + a^2b^2cn)x)x^{2n} + 4 \cdot (6a^3dn^3 + 11a^3dn^2 + 6a^3dn + a^3d + 3 \cdot (6a^2b^2cn^3 + 5a^2b^2cn^2 + a^2b^2cn)x)x^n) / (6n^4 + 11n^3 + 6n^2 + n)$$

Sympy [A] time = 6.76931, size = 1251, normalized size = 14.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*x**(-1+n))*(a+b*x**n)**3,x)

[Out] Piecewise((a**3*c*x - a**3*d/x + 3*a**2*b*c*log(x) - 3*a**2*b*d/(2*x**2) - 3*a*b**2*c/x - a*b**2*d/x**3 - b**3*c/(2*x**2) - b**3*d/(4*x**4), Eq(n, -1)), (a**3*c*x - 2*a**3*d/sqrt(x) + 6*a**2*b*c*sqrt(x) - 3*a**2*b*d/x + 3*a*b

```

**2*c*log(x) - 2*a*b**2*d/x**(3/2) - 2*b**3*c/sqrt(x) - b**3*d/(2*x**2), Eq
(n, -1/2)), (a**3*c*x - 3*a**3*d/x**(1/3) + 9*a**2*b*c*x**(2/3)/2 - 9*a**2*
b*d/(2*x**(2/3)) + 9*a*b**2*c*x**(1/3) - 3*a*b**2*d/x + b**3*c*log(x) - 3*b
**3*d/(4*x**(4/3)), Eq(n, -1/3)), ((a + b)**3*(c*x + d*log(x)), Eq(n, 0)),
(24*a**3*c*n**4*x/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 44*a**3*c*n**3*x/(2
4*n**4 + 44*n**3 + 24*n**2 + 4*n) + 24*a**3*c*n**2*x/(24*n**4 + 44*n**3 + 2
4*n**2 + 4*n) + 4*a**3*c*n*x/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 24*a**3*
d*n**3*x**n/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 44*a**3*d*n**2*x**n/(24*n
**4 + 44*n**3 + 24*n**2 + 4*n) + 24*a**3*d*n*x**n/(24*n**4 + 44*n**3 + 24*n
**2 + 4*n) + 4*a**3*d*x**n/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 72*a**2*b*
c*n**3*x*x**n/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 60*a**2*b*c*n**2*x*x**n
/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 12*a**2*b*c*n*x*x**n/(24*n**4 + 44*n
**3 + 24*n**2 + 4*n) + 36*a**2*b*d*n**3*x**(2*n)/(24*n**4 + 44*n**3 + 24*n*
**2 + 4*n) + 66*a**2*b*d*n**2*x**(2*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) +
36*a**2*b*d*n*x**(2*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 6*a**2*b*d*x*
*(2*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 36*a*b**2*c*n**3*x*x**(2*n)/(2
4*n**4 + 44*n**3 + 24*n**2 + 4*n) + 48*a*b**2*c*n**2*x*x**(2*n)/(24*n**4 +
44*n**3 + 24*n**2 + 4*n) + 12*a*b**2*c*n*x*x**(2*n)/(24*n**4 + 44*n**3 + 24
*n**2 + 4*n) + 24*a*b**2*d*n**3*x**(3*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n
) + 44*a*b**2*d*n**2*x**(3*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 24*a*b*
**2*d*n*x**(3*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 4*a*b**2*d*x**(3*n)/(
24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 8*b**3*c*n**3*x*x**(3*n)/(24*n**4 + 44
*n**3 + 24*n**2 + 4*n) + 12*b**3*c*n**2*x*x**(3*n)/(24*n**4 + 44*n**3 + 24*
n**2 + 4*n) + 4*b**3*c*n*x*x**(3*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 6
*b**3*d*n**3*x**(4*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 11*b**3*d*n**2*
x**(4*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 6*b**3*d*n*x**(4*n)/(24*n**4
+ 44*n**3 + 24*n**2 + 4*n) + b**3*d*x**(4*n)/(24*n**4 + 44*n**3 + 24*n**2
+ 4*n), True))

```

Giac [B] time = 1.06869, size = 529, normalized size = 6.3

$$24 a^3 c n^4 x + 8 b^3 c n^3 x x^{3n} + 36 a b^2 c n^3 x x^{2n} + 72 a^2 b c n^3 x x^n + 44 a^3 c n^3 x + 6 b^3 d n^3 x^{4n} + 24 a b^2 d n^3 x^{3n} + 12 b^3 c n^2 x x^{3n} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*x^(-1+n))*(a+b*x^n)^3,x, algorithm="giac")

[Out] 1/4*(24*a^3*c*n^4*x + 8*b^3*c*n^3*x*x^(3*n) + 36*a*b^2*c*n^3*x*x^(2*n) + 72*a^2*b*c*n^3*x*x^n + 44*a^3*c*n^3*x + 6*b^3*d*n^3*x^(4*n) + 24*a*b^2*d*n^3*x^(3*n) + 12*b^3*c*n^2*x*x^(3*n) + 36*a^2*b*d*n^3*x^(2*n) + 48*a*b^2*c*n^2*x*x^(2*n) + 24*a^3*d*n^3*x^n + 60*a^2*b*c*n^2*x*x^n + 24*a^3*c*n^2*x + 11*b

$$\begin{aligned} &^3d^n^2x^{(4n)} + 44ab^2d^n^2x^{(3n)} + 4b^3c^nxx^{(3n)} + 66a^2b* \\ &d^n^2x^{(2n)} + 12ab^2c^nxx^{(2n)} + 44a^3d^n^2x^n + 12a^2b*c^n*x* \\ &x^n + 4a^3c^n*x + 6b^3d^n*x^{(4n)} + 24ab^2*d^n*x^{(3n)} + 36a^2*b*d*n \\ &*x^{(2n)} + 24a^3*d*n*x^n + b^3*d*x^{(4n)} + 4ab^2*d*x^{(3n)} + 6a^2*b*d*x \\ &^{(2n)} + 4a^3*d*x^n)/(6n^4 + 11n^3 + 6n^2 + n) \end{aligned}$$

$$3.578 \quad \int (c + dx^{-1+n}) (a + bx^n)^2 dx$$

Optimal. Leaf size=61

$$a^2cx + \frac{2abcx^{n+1}}{n+1} + \frac{d(a+bx^n)^3}{3bn} + \frac{b^2cx^{2n+1}}{2n+1}$$

[Out] $a^2c*x + (2*a*b*c*x^{(1+n)})/(1+n) + (b^2*c*x^{(1+2*n)})/(1+2*n) + (d*(a + b*x^n)^3)/(3*b*n)$

Rubi [A] time = 0.0402319, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1891, 244, 261}

$$a^2cx + \frac{2abcx^{n+1}}{n+1} + \frac{d(a+bx^n)^3}{3bn} + \frac{b^2cx^{2n+1}}{2n+1}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^(-1 + n))*(a + b*x^n)^2,x]

[Out] $a^2c*x + (2*a*b*c*x^{(1+n)})/(1+n) + (b^2*c*x^{(1+2*n)})/(1+2*n) + (d*(a + b*x^n)^3)/(3*b*n)$

Rule 1891

Int[((A_) + (B_)*(x_)^(m_.))*((a_) + (b_)*(x_)^(n_.))^(p_.), x_Symbol] :>
Dist[A, Int[(a + b*x^n)^p, x], x] + Dist[B, Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]

Rule 244

Int[((a_) + (b_)*(x_)^(n_.))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]

Rule 261

Int[(x_)^(m_.)*((a_) + (b_)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
\int (c + dx^{-1+n})(a + bx^n)^2 dx &= c \int (a + bx^n)^2 dx + d \int x^{-1+n} (a + bx^n)^2 dx \\
&= \frac{d(a + bx^n)^3}{3bn} + c \int (a^2 + 2abx^n + b^2x^{2n}) dx \\
&= a^2cx + \frac{2abcx^{1+n}}{1+n} + \frac{b^2cx^{1+2n}}{1+2n} + \frac{d(a + bx^n)^3}{3bn}
\end{aligned}$$

Mathematica [A] time = 0.112459, size = 120, normalized size = 1.97

$$\frac{3a^2b(2n^2 + 3n + 1)(cnx + dx^n) + a^3d(2n^2 + 3n + 1) + 3ab^2(2n + 1)x^n(2cnx + d(n + 1)x^n) + b^3(n + 1)x^{2n}(3cnx + d(2n + 1))}{3bn(n + 1)(2n + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^(-1 + n))*(a + b*x^n)^2,x]

[Out] (a^3*d*(1 + 3*n + 2*n^2) + 3*a^2*b*(1 + 3*n + 2*n^2)*(c*n*x + d*x^n) + 3*a*b^2*(1 + 2*n)*x^n*(2*c*n*x + d*(1 + n)*x^n) + b^3*(1 + n)*x^(2*n)*(3*c*n*x + d*(1 + 2*n)*x^n))/(3*b*n*(1 + n)*(1 + 2*n))

Maple [A] time = 0.012, size = 87, normalized size = 1.4

$$a^2cx + \frac{a^2de^{n \ln(x)}}{n} + \frac{bda(e^{n \ln(x)})^2}{n} + \frac{b^2cx(e^{n \ln(x)})^2}{1 + 2n} + \frac{b^2d(e^{n \ln(x)})^3}{3n} + 2 \frac{abcxe^{n \ln(x)}}{1 + n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d*x^(-1+n))*(a+b*x^n)^2,x)

[Out] a^2*c*x+a^2*d/n*exp(n*ln(x))+b*d*a/n*exp(n*ln(x))^2+b^2*c/(1+2*n)*x*exp(n*ln(x))^2+1/3*b^2*d/n*exp(n*ln(x))^3+2*a*b*c/(1+n)*x*exp(n*ln(x))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*x^(-1+n))*(a+b*x^n)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 1.53706, size = 347, normalized size = 5.69

$$\frac{3(2a^2cn^3 + 3a^2cn^2 + a^2cn)x + (2b^2dn^2 + 3b^2dn + b^2d)x^{3n} + 3(2abdn^2 + 3abdn + abd + (b^2cn^2 + b^2cn)x)x^{2n} + 3(2n^3 + 3n^2 + n)}{3(2n^3 + 3n^2 + n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*x^(-1+n))*(a+b*x^n)^2,x, algorithm="fricas")

[Out] $\frac{1}{3} \cdot (3 \cdot (2 \cdot a^2 \cdot c \cdot n^3 + 3 \cdot a^2 \cdot c \cdot n^2 + a^2 \cdot c \cdot n) \cdot x + (2 \cdot b^2 \cdot d \cdot n^2 + 3 \cdot b^2 \cdot d \cdot n + b^2 \cdot d) \cdot x^{3n} + 3 \cdot (2 \cdot a \cdot b \cdot d \cdot n^2 + 3 \cdot a \cdot b \cdot d \cdot n + a \cdot b \cdot d + (b^2 \cdot c \cdot n^2 + b^2 \cdot c \cdot n) \cdot x) \cdot x^{2n} + 3 \cdot (2 \cdot a^2 \cdot d \cdot n^2 + 3 \cdot a^2 \cdot d \cdot n + a^2 \cdot d + 2 \cdot (2 \cdot a \cdot b \cdot c \cdot n^2 + a \cdot b \cdot c \cdot n) \cdot x) \cdot x^n) / (2 \cdot n^3 + 3 \cdot n^2 + n)$

Sympy [A] time = 3.25191, size = 552, normalized size = 9.05

$$\left\{ \begin{array}{l} a^2cx - \frac{a^2d}{x} + 2abc \log(x) - \frac{abd}{x^2} - \frac{b^2c}{x} - \frac{b^2d}{3x^3} \\ a^2cx - \frac{2a^2d}{\sqrt{x}} + 4abc\sqrt{x} - \frac{2abd}{x} + b^2c \log(x) - \frac{2b^2d}{3x^2} \\ (a+b)^2(cx + d \log(x)) \\ \frac{6a^2cn^3x}{6n^3+9n^2+3n} + \frac{9a^2cn^2x}{6n^3+9n^2+3n} + \frac{3a^2cnx}{6n^3+9n^2+3n} + \frac{6a^2dn^2x^n}{6n^3+9n^2+3n} + \frac{9a^2dnx^n}{6n^3+9n^2+3n} + \frac{3a^2dx^n}{6n^3+9n^2+3n} + \frac{12abcn^2xx^n}{6n^3+9n^2+3n} + \frac{6abcnxx^n}{6n^3+9n^2+3n} + \frac{6abdn^2x^{2n}}{6n^3+9n^2+3n} + \dots \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*x**(-1+n))*(a+b*x**n)**2,x)

[Out] Piecewise((a**2*c*x - a**2*d/x + 2*a*b*c*log(x) - a*b*d/x**2 - b**2*c/x - b**2*d/(3*x**3), Eq(n, -1)), (a**2*c*x - 2*a**2*d/sqrt(x) + 4*a*b*c*sqrt(x) - 2*a*b*d/x + b**2*c*log(x) - 2*b**2*d/(3*x**(3/2)), Eq(n, -1/2)), ((a + b)**2*(c*x + d*log(x)), Eq(n, 0)), (6*a**2*c*n**3*x/(6*n**3 + 9*n**2 + 3*n) + 9*a**2*c*n**2*x/(6*n**3 + 9*n**2 + 3*n) + 3*a**2*c*n*x/(6*n**3 + 9*n**2 + 3*n) + 6*a**2*d*n**2*x**n/(6*n**3 + 9*n**2 + 3*n) + 9*a**2*d*n*x**n/(6*n**3 + 9*n**2 + 3*n) + 3*a**2*d*x**n/(6*n**3 + 9*n**2 + 3*n) + 12*a*b*c*n**2*x**n/(6*n**3 + 9*n**2 + 3*n) + 6*a*b*c*n*x**n/(6*n**3 + 9*n**2 + 3*n) + 6*a*b*d*n**2*x**n/(6*n**3 + 9*n**2 + 3*n) + 6*a*b*d*x**n/(6*n**3 + 9*n**2 + 3*n) + 3*a*b*d/(6*n**3 + 9*n**2 + 3*n), Eq(n, 1)), (a**2*c*x - a**2*d/x + 2*a*b*c*log(x) - a*b*d/x**2 - b**2*c/x - b**2*d/(3*x**3), Eq(n, -1)), (a**2*c*x - 2*a**2*d/sqrt(x) + 4*a*b*c*sqrt(x) - 2*a*b*d/x + b**2*c*log(x) - 2*b**2*d/(3*x**(3/2)), Eq(n, -1/2)), ((a + b)**2*(c*x + d*log(x)), Eq(n, 0)), (6*a**2*c*n**3*x/(6*n**3 + 9*n**2 + 3*n) + 9*a**2*c*n**2*x/(6*n**3 + 9*n**2 + 3*n) + 3*a**2*c*n*x/(6*n**3 + 9*n**2 + 3*n) + 6*a**2*d*n**2*x**n/(6*n**3 + 9*n**2 + 3*n) + 9*a**2*d*n*x**n/(6*n**3 + 9*n**2 + 3*n) + 3*a**2*d*x**n/(6*n**3 + 9*n**2 + 3*n) + 12*a*b*c*n**2*x**n/(6*n**3 + 9*n**2 + 3*n) + 6*a*b*c*n*x**n/(6*n**3 + 9*n**2 + 3*n) + 6*a*b*d*n**2*x**n/(6*n**3 + 9*n**2 + 3*n) + 6*a*b*d*x**n/(6*n**3 + 9*n**2 + 3*n) + 3*a*b*d/(6*n**3 + 9*n**2 + 3*n), Eq(n, 1))

```

+ 9*n**2 + 3*n) + 3*a**2*d*x**n/(6*n**3 + 9*n**2 + 3*n) + 12*a*b*c*n**2*x*
x**n/(6*n**3 + 9*n**2 + 3*n) + 6*a*b*c*n*x*x**n/(6*n**3 + 9*n**2 + 3*n) + 6
*a*b*d*n**2*x**(2*n)/(6*n**3 + 9*n**2 + 3*n) + 9*a*b*d*n*x**(2*n)/(6*n**3 +
9*n**2 + 3*n) + 3*a*b*d*x**(2*n)/(6*n**3 + 9*n**2 + 3*n) + 3*b**2*c*n**2*x
*x**(2*n)/(6*n**3 + 9*n**2 + 3*n) + 3*b**2*c*n*x*x**(2*n)/(6*n**3 + 9*n**2
+ 3*n) + 2*b**2*d*n**2*x**(3*n)/(6*n**3 + 9*n**2 + 3*n) + 3*b**2*d*n*x**(3*
n)/(6*n**3 + 9*n**2 + 3*n) + b**2*d*x**(3*n)/(6*n**3 + 9*n**2 + 3*n), True)
)

```

Giac [B] time = 1.07394, size = 265, normalized size = 4.34

$$\frac{6a^2cn^3x + 3b^2cn^2xx^{2n} + 12abcn^2xx^n + 9a^2cn^2x + 2b^2dn^2x^{3n} + 6abdn^2x^{2n} + 3b^2cnxx^{2n} + 6a^2dn^2x^n + 6abcnxx^n + 3a^2dncn^2x^{2n} + 3b^2dncn^2x^{3n} + 3a^2dncn^2x^{2n} + 3b^2dncn^2x^{3n} + 3a^2dncn^2x^{2n} + 3b^2dncn^2x^{3n}}{3(2n^3 + 3n^2 + n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] 1/3*(6*a^2*c*n^3*x + 3*b^2*c*n^2*x*x^(2*n) + 12*a*b*c*n^2*x*x^n + 9*a^2*c*n
^2*x + 2*b^2*d*n^2*x^(3*n) + 6*a*b*d*n^2*x^(2*n) + 3*b^2*c*n*x*x^(2*n) + 6
a^2*d*n^2*x^n + 6*a*b*c*n*x*x^n + 3*a^2*c*n*x + 3*b^2*d*n*x^(3*n) + 9*a*b*d
*n*x^(2*n) + 9*a^2*d*n*x^n + b^2*d*x^(3*n) + 3*a*b*d*x^(2*n) + 3*a^2*d*x^n)
/(2*n^3 + 3*n^2 + n)
```


$$3.579 \quad \int (c + dx^{-1+n}) (a + bx^n) dx$$

Optimal. Leaf size=41

$$acx + \frac{adx^n}{n} + \frac{bcx^{n+1}}{n+1} + \frac{bdx^{2n}}{2n}$$

[Out] a*c*x + (a*d*x^n)/n + (b*d*x^(2*n))/(2*n) + (b*c*x^(1 + n))/(1 + n)

Rubi [A] time = 0.0227337, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1891, 14}

$$acx + \frac{adx^n}{n} + \frac{bcx^{n+1}}{n+1} + \frac{bdx^{2n}}{2n}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^(-1 + n))*(a + b*x^n), x]

[Out] a*c*x + (a*d*x^n)/n + (b*d*x^(2*n))/(2*n) + (b*c*x^(1 + n))/(1 + n)

Rule 1891

```
Int[((A_) + (B_)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Dist[A, Int[(a + b*x^n)^p, x], x] + Dist[B, Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int (c + dx^{-1+n})(a + bx^n) dx &= c \int (a + bx^n) dx + d \int x^{-1+n} (a + bx^n) dx \\
&= acx + \frac{bcx^{1+n}}{1+n} + d \int (ax^{-1+n} + bx^{-1+2n}) dx \\
&= acx + \frac{adx^n}{n} + \frac{bdx^{2n}}{2n} + \frac{bcx^{1+n}}{1+n}
\end{aligned}$$

Mathematica [A] time = 0.0962162, size = 42, normalized size = 1.02

$$\frac{2a(cnx + dx^n) + bx^n \left(\frac{2cnx}{n+1} + dx^n \right)}{2n}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^(-1 + n))*(a + b*x^n), x]

[Out] (2*a*(c*n*x + d*x^n) + b*x^n*((2*c*n*x)/(1 + n) + d*x^n))/(2*n)

Maple [A] time = 0.011, size = 45, normalized size = 1.1

$$acx + \frac{ade^{n \ln(x)}}{n} + \frac{bcxe^{n \ln(x)}}{1+n} + \frac{bd(e^{n \ln(x)})^2}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d*x^(-1+n))*(a+b*x^n), x)

[Out] a*c*x+a*d/n*exp(n*ln(x))+b*c/(1+n)*x*exp(n*ln(x))+1/2*b*d/n*exp(n*ln(x))^2

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*x^(-1+n))*(a+b*x^n), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.48533, size = 128, normalized size = 3.12

$$\frac{2(acn^2 + acn)x + (bdn + bd)x^{2n} + 2(bcnx + adn + ad)x^n}{2(n^2 + n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*x^(-1+n))*(a+b*x^n),x, algorithm="fricas")

[Out] 1/2*(2*(a*c*n^2 + a*c*n)*x + (b*d*n + b*d)*x^(2*n) + 2*(b*c*n*x + a*d*n + a*d)*x^n)/(n^2 + n)

Sympy [A] time = 1.50908, size = 163, normalized size = 3.98

$$\begin{cases} acx - \frac{ad}{x} + bc \log(x) - \frac{bd}{2x^2} & \text{for } n = -1 \\ (a + b)(cx + d \log(x)) & \text{for } n = 0 \\ \frac{2acn^2x}{2n^2+2n} + \frac{2acnx}{2n^2+2n} + \frac{2adnx^n}{2n^2+2n} + \frac{2adx^n}{2n^2+2n} + \frac{2bcnxx^n}{2n^2+2n} + \frac{bdnx^{2n}}{2n^2+2n} + \frac{bdx^{2n}}{2n^2+2n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*x**(-1+n))*(a+b*x**n),x)

[Out] Piecewise((a*c*x - a*d/x + b*c*log(x) - b*d/(2*x**2), Eq(n, -1)), ((a + b)*(c*x + d*log(x)), Eq(n, 0)), (2*a*c*n**2*x/(2*n**2 + 2*n) + 2*a*c*n*x/(2*n**2 + 2*n) + 2*a*d*n*x**n/(2*n**2 + 2*n) + 2*a*d*x**n/(2*n**2 + 2*n) + 2*b*c*n*x*x**n/(2*n**2 + 2*n) + b*d*n*x**(2*n)/(2*n**2 + 2*n) + b*d*x**(2*n)/(2*n**2 + 2*n), True))

Giac [A] time = 1.05632, size = 88, normalized size = 2.15

$$\frac{2acn^2x + 2bcnxx^n + 2acnx + bdnx^{2n} + 2adnx^n + bdx^{2n} + 2adx^n}{2(n^2 + n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*c*n^2*x + 2*b*c*n*x*x^n + 2*a*c*n*x + b*d*n*x^(2*n) + 2*a*d*n*x^n  
+ b*d*x^(2*n) + 2*a*d*x^n)/(n^2 + n)
```

$$3.580 \quad \int (c + dx^{-1+n}) dx$$

Optimal. Leaf size=12

$$cx + \frac{dx^n}{n}$$

[Out] c*x + (d*x^n)/n

Rubi [A] time = 0.0028014, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$cx + \frac{dx^n}{n}$$

Antiderivative was successfully verified.

[In] Int[c + d*x^(-1 + n), x]

[Out] c*x + (d*x^n)/n

Rubi steps

$$\int (c + dx^{-1+n}) dx = cx + \frac{dx^n}{n}$$

Mathematica [A] time = 0.0013776, size = 12, normalized size = 1.

$$cx + \frac{dx^n}{n}$$

Antiderivative was successfully verified.

[In] Integrate[c + d*x^(-1 + n), x]

[Out] c*x + (d*x^n)/n

Maple [A] time = 0.041, size = 13, normalized size = 1.1

$$cx + \frac{dx^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(c+d*x^(-1+n),x)

[Out] c*x+d*x^n/n

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(c+d*x^(-1+n),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.50071, size = 36, normalized size = 3.

$$\frac{cnx + dxx^{n-1}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(c+d*x^(-1+n),x, algorithm="fricas")

[Out] (c*n*x + d*x*x^(n - 1))/n

Sympy [A] time = 0.057868, size = 15, normalized size = 1.25

$$cx + d \begin{cases} \frac{x^n}{n} & \text{for } n - 1 \neq -1 \\ \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c+d*x**(-1+n),x)
```

```
[Out] c*x + d*Piecewise((x**n/n, Ne(n - 1, -1)), (log(x), True))
```

Giac [A] time = 1.04143, size = 16, normalized size = 1.33

$$cx + \frac{dx^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c+d*x^(-1+n),x, algorithm="giac")
```

```
[Out] c*x + d*x^n/n
```

$$3.581 \quad \int \frac{c+dx^{-1+n}}{a+bx^n} dx$$

Optimal. Leaf size=42

$$\frac{cx {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a} + \frac{d \log(a + bx^n)}{bn}$$

[Out] (c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a + (d*Log[a + b*x^n])/(b*n)

Rubi [A] time = 0.0304338, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1891, 245, 260}

$$\frac{cx {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a} + \frac{d \log(a + bx^n)}{bn}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^(-1 + n))/(a + b*x^n), x]

[Out] (c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a + (d*Log[a + b*x^n])/(b*n)

Rule 1891

```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Dist[A, Int[(a + b*x^n)^p, x], x] + Dist[B, Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 260


```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned} \int \frac{c + dx^{-1+n}}{a + bx^n} dx &= c \int \frac{1}{a + bx^n} dx + d \int \frac{x^{-1+n}}{a + bx^n} dx \\ &= \frac{cx {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a} + \frac{d \log(a + bx^n)}{bn} \end{aligned}$$

Mathematica [A] time = 0.0561254, size = 42, normalized size = 1.

$$\frac{cx {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a} + \frac{d \log(a + bx^n)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^(-1 + n))/(a + b*x^n), x]
```

```
[Out] (c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a + (d*Log[a + b*x^n])/(b*n)
```

Maple [F] time = 0.339, size = 0, normalized size = 0.

$$\int \frac{c + dx^{-1+n}}{a + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^(-1+n))/(a+b*x^n), x)
```

```
[Out] int((c+d*x^(-1+n))/(a+b*x^n), x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{d \log(x)}{b} + \int \frac{bcx - ad}{b^2xx^n + abx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*x^(-1+n))/(a+b*x^n),x, algorithm="maxima")

[Out] d*log(x)/b + integrate((b*c*x - a*d)/(b^2*x*x^n + a*b*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx^{n-1} + c}{bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*x^(-1+n))/(a+b*x^n),x, algorithm="fricas")

[Out] integral((d*x^(n - 1) + c)/(b*x^n + a), x)

Sympy [A] time = 9.52058, size = 65, normalized size = 1.55

$$d \left(\begin{array}{ll} \left(\frac{\log(x)}{x^n} \right) & \text{for } b = 0 \wedge n = 0 \\ \left(\frac{a^n}{\log(x)} \right) & \text{for } b = 0 \\ \left(\frac{a+b}{\log\left(\frac{a}{b} + x^n\right)} \right) & \text{for } n = 0 \\ \left(\frac{a}{bn} \right) & \text{otherwise} \end{array} \right) + \frac{cx\Phi\left(\frac{bx^n e^{i\pi}}{a}, 1, \frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*x**(-1+n))/(a+b*x**n),x)

[Out] d*Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0)), (x**n/(a*n), Eq(b, 0)), (log(x)/(a + b), Eq(n, 0)), (log(a/b + x**n)/(b*n), True)) + c*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(a*n**2*gamma(1 + 1/n))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^{n-1} + c}{bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((d*x^(n - 1) + c)/(b*x^n + a), x)
```

$$3.582 \quad \int \frac{c+dx^{-1+n}}{(a+bx^n)^2} dx$$

Optimal. Leaf size=44

$$\frac{cx {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2} - \frac{d}{bn(a+bx^n)}$$

[Out] $-(d/(b*n*(a + b*x^n))) + (c*x*Hypergeometric2F1[2, n^{(-1)}, 1 + n^{(-1)}, -(b*x^n)/a])/a^2$

Rubi [A] time = 0.0288111, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1891, 245, 261}

$$\frac{cx {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2} - \frac{d}{bn(a+bx^n)}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^(-1 + n))/(a + b*x^n)^2, x]

[Out] $-(d/(b*n*(a + b*x^n))) + (c*x*Hypergeometric2F1[2, n^{(-1)}, 1 + n^{(-1)}, -(b*x^n)/a])/a^2$

Rule 1891

Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :=
Dist[A, Int[(a + b*x^n)^p, x], x] + Dist[B, Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]

Rule 245

Int[((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rule 261

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{c + dx^{-1+n}}{(a + bx^n)^2} dx = c \int \frac{1}{(a + bx^n)^2} dx + d \int \frac{x^{-1+n}}{(a + bx^n)^2} dx$$

$$= -\frac{d}{bn(a + bx^n)} + \frac{cx {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2}$$

Mathematica [A] time = 0.0754271, size = 44, normalized size = 1.

$$\frac{cx {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2} - \frac{d}{abn + b^2nx^n}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^(-1 + n))/(a + b*x^n)^2, x]

[Out] -(d/(a*b*n + b^2*n*x^n)) + (c*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -(b*x^n)/a])/a^2

Maple [F] time = 0.36, size = 0, normalized size = 0.

$$\int \frac{c + dx^{-1+n}}{(a + bx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d*x^(-1+n))/(a+b*x^n)^2,x)

[Out] int((c+d*x^(-1+n))/(a+b*x^n)^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$c(n-1) \int \frac{1}{abnx^n + a^2n} dx + \frac{bcx - ad}{ab^2nx^n + a^2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*x^(-1+n))/(a+b*x^n)^2,x, algorithm="maxima")

[Out] c*(n - 1)*integrate(1/(a*b*n*x^n + a^2*n), x) + (b*c*x - a*d)/(a*b^2*n*x^n + a^2*b*n)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx^{n-1} + c}{b^2x^{2n} + 2abx^n + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*x^(-1+n))/(a+b*x^n)^2,x, algorithm="fricas")

[Out] integral((d*x^(n - 1) + c)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)

Sympy [C] time = 22.7527, size = 313, normalized size = 7.11

$$c \left(\frac{nx\Phi\left(\frac{bx^n e^{i\pi}}{a}, 1, \frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{a\left(an^3\Gamma\left(1 + \frac{1}{n}\right) + bn^3x^n\Gamma\left(1 + \frac{1}{n}\right)\right)} + \frac{nx\Gamma\left(\frac{1}{n}\right)}{a\left(an^3\Gamma\left(1 + \frac{1}{n}\right) + bn^3x^n\Gamma\left(1 + \frac{1}{n}\right)\right)} - \frac{x\Phi\left(\frac{bx^n e^{i\pi}}{a}, 1, \frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{a\left(an^3\Gamma\left(1 + \frac{1}{n}\right) + bn^3x^n\Gamma\left(1 + \frac{1}{n}\right)\right)} + \frac{1}{a^2} \left(\frac{1}{a\left(an^3\Gamma\left(1 + \frac{1}{n}\right) + bn^3x^n\Gamma\left(1 + \frac{1}{n}\right)\right)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*x**(-1+n))/(a+b*x**n)**2,x)

[Out] c*(n*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(a*(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) + n*x*gamma(1/n)/(a*(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) - x*lerchphi(b*x**n*exp_polar(I*pi)

```

i)/a, 1, 1/n)*gamma(1/n)/(a*(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 +
1/n))) + b*n*x*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(
a**2*(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) - b*x*x**n*lerch
phi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(a**2*(a*n**3*gamma(1 + 1/
n) + b*n**3*x**n*gamma(1 + 1/n))) + d*Piecewise((zoo*log(x), Eq(a, 0) & Eq
(b, 0) & Eq(n, 0)), (-x**(-n)/(b**2*n), Eq(a, 0)), (zoo*x**n/n, Eq(b, -a*x*
*(-n))), (log(x)/(a + b)**2, Eq(n, 0)), (x**n/(a**2*n + a*b*n*x**n), True))

```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^{n-1} + c}{(bx^n + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^(n - 1) + c)/(b*x^n + a)^2, x)
```

$$3.583 \quad \int \frac{c+dx^{-1+n}}{(a+bx^n)^3} dx$$

Optimal. Leaf size=46

$$\frac{cx {}_2F_1\left(3, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^3} - \frac{d}{2bn(a+bx^n)^2}$$

[Out] -d/(2*b*n*(a + b*x^n)^2) + (c*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -(b*x^n)/a])/a^3

Rubi [A] time = 0.0295255, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1891, 245, 261}

$$\frac{cx {}_2F_1\left(3, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^3} - \frac{d}{2bn(a+bx^n)^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^(-1 + n))/(a + b*x^n)^3, x]

[Out] -d/(2*b*n*(a + b*x^n)^2) + (c*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -(b*x^n)/a])/a^3

Rule 1891

```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :=
Dist[A, Int[(a + b*x^n)^p, x], x] + Dist[B, Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -(b*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 261


```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{c + dx^{-1+n}}{(a + bx^n)^3} dx = c \int \frac{1}{(a + bx^n)^3} dx + d \int \frac{x^{-1+n}}{(a + bx^n)^3} dx$$

$$= -\frac{d}{2bn(a + bx^n)^2} + \frac{cx {}_2F_1\left(3, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^3}$$

Mathematica [A] time = 0.0744282, size = 63, normalized size = 1.37

$$\frac{2bcnx(a + bx^n)^2 {}_2F_1\left(3, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right) - a^3d}{2a^3bn(a + bx^n)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^(-1 + n))/(a + b*x^n)^3, x]
```

```
[Out] (-a^3*d) + 2*b*c*n*x*(a + b*x^n)^2*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((b*x^n)/a)]/(2*a^3*b*n*(a + b*x^n)^2)
```

Maple [F] time = 0.355, size = 0, normalized size = 0.

$$\int \frac{c + dx^{-1+n}}{(a + bx^n)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^(-1+n))/(a+b*x^n)^3,x)
```

```
[Out] int((c+d*x^(-1+n))/(a+b*x^n)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$(2n^2 - 3n + 1)c \int \frac{1}{2(a^2bn^2x^n + a^3n^2)} dx + \frac{b^2c(2n-1)xx^n + abc(3n-1)x - a^2dn}{2(a^2b^3n^2x^{2n} + 2a^3b^2n^2x^n + a^4bn^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*x^(-1+n))/(a+b*x^n)^3,x, algorithm="maxima")

[Out] (2*n^2 - 3*n + 1)*c*integrate(1/2/(a^2*b*n^2*x^n + a^3*n^2), x) + 1/2*(b^2*c*(2*n - 1)*x*x^n + a*b*c*(3*n - 1)*x - a^2*d*n)/(a^2*b^3*n^2*x^(2*n) + 2*a^3*b^2*n^2*x^n + a^4*b*n^2)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx^{n-1} + c}{b^3x^{3n} + 3ab^2x^{2n} + 3a^2bx^n + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*x^(-1+n))/(a+b*x^n)^3,x, algorithm="fricas")

[Out] integral((d*x^(n - 1) + c)/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n + a^3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*x**(-1+n))/(a+b*x**n)**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^{n-1} + c}{(bx^n + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))/(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x^(n - 1) + c)/(b*x^n + a)^3, x)
```

$$3.584 \quad \int \frac{(cx)^m (d + ex^n + fx^{2n} + gx^{3n})}{\sqrt{a + bx^n}} dx$$

Optimal. Leaf size=305

$$\frac{d(cx)^{m+1} \sqrt{\frac{bx^n}{a} + 1} {}_2F_1\left(\frac{1}{2}, \frac{m+1}{n}; \frac{m+n+1}{n}; -\frac{bx^n}{a}\right)}{c(m+1)\sqrt{a+bx^n}} + \frac{ex^{n+1}(cx)^m \sqrt{\frac{bx^n}{a} + 1} {}_2F_1\left(\frac{1}{2}, \frac{m+n+1}{n}; \frac{m+2n+1}{n}; -\frac{bx^n}{a}\right)}{(m+n+1)\sqrt{a+bx^n}} + \frac{fx^{2n+1}(cx)^m \sqrt{\frac{bx^n}{a} + 1} {}_2F_1\left(\frac{1}{2}, \frac{m+2n+1}{n}; \frac{m+3n+1}{n}; -\frac{bx^n}{a}\right)}{(m+2n+1)\sqrt{a+bx^n}}$$

[Out] (d*(c*x)^(1+m)*Sqrt[1+(b*x^n)/a]*Hypergeometric2F1[1/2, (1+m)/n, (1+m+n)/n, -(b*x^n)/a])/(c*(1+m)*Sqrt[a+b*x^n]) + (e*x^(1+n)*(c*x)^m*Sqrt[1+(b*x^n)/a]*Hypergeometric2F1[1/2, (1+m+n)/n, (1+m+2*n)/n, -(b*x^n)/a])/((1+m+n)*Sqrt[a+b*x^n]) + (f*x^(1+2*n)*(c*x)^m*Sqrt[1+(b*x^n)/a]*Hypergeometric2F1[1/2, (1+m+2*n)/n, (1+m+3*n)/n, -(b*x^n)/a])/((1+m+2*n)*Sqrt[a+b*x^n]) + (g*x^(1+3*n)*(c*x)^m*Sqrt[1+(b*x^n)/a]*Hypergeometric2F1[1/2, (1+m+3*n)/n, (1+m+4*n)/n, -(b*x^n)/a])/((1+m+3*n)*Sqrt[a+b*x^n])

Rubi [A] time = 0.231849, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1844, 365, 364, 20}

$$\frac{d(cx)^{m+1} \sqrt{\frac{bx^n}{a} + 1} {}_2F_1\left(\frac{1}{2}, \frac{m+1}{n}; \frac{m+n+1}{n}; -\frac{bx^n}{a}\right)}{c(m+1)\sqrt{a+bx^n}} + \frac{ex^{n+1}(cx)^m \sqrt{\frac{bx^n}{a} + 1} {}_2F_1\left(\frac{1}{2}, \frac{m+n+1}{n}; \frac{m+2n+1}{n}; -\frac{bx^n}{a}\right)}{(m+n+1)\sqrt{a+bx^n}} + \frac{fx^{2n+1}(cx)^m \sqrt{\frac{bx^n}{a} + 1} {}_2F_1\left(\frac{1}{2}, \frac{m+2n+1}{n}; \frac{m+3n+1}{n}; -\frac{bx^n}{a}\right)}{(m+2n+1)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

[In] Int[((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/Sqrt[a + b*x^n], x]

[Out] (d*(c*x)^(1+m)*Sqrt[1+(b*x^n)/a]*Hypergeometric2F1[1/2, (1+m)/n, (1+m+n)/n, -(b*x^n)/a])/(c*(1+m)*Sqrt[a+b*x^n]) + (e*x^(1+n)*(c*x)^m*Sqrt[1+(b*x^n)/a]*Hypergeometric2F1[1/2, (1+m+n)/n, (1+m+2*n)/n, -(b*x^n)/a])/((1+m+n)*Sqrt[a+b*x^n]) + (f*x^(1+2*n)*(c*x)^m*Sqrt[1+(b*x^n)/a]*Hypergeometric2F1[1/2, (1+m+2*n)/n, (1+m+3*n)/n, -(b*x^n)/a])/((1+m+2*n)*Sqrt[a+b*x^n]) + (g*x^(1+3*n)*(c*x)^m*Sqrt[1+(b*x^n)/a]*Hypergeometric2F1[1/2, (1+m+3*n)/n, (1+m+4*n)/n, -(b*x^n)/a])/((1+m+3*n)*Sqrt[a+b*x^n])

Rule 1844

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n

, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) && !IGtQ[m, 0]

Rule 365

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])

Rule 364

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a])]/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rule 20

Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

Rubi steps

$$\begin{aligned}
 \int \frac{(cx)^m (d + ex^n + fx^{2n} + gx^{3n})}{\sqrt{a + bx^n}} dx &= \int \left(\frac{d(cx)^m}{\sqrt{a + bx^n}} + \frac{ex^n(cx)^m}{\sqrt{a + bx^n}} + \frac{fx^{2n}(cx)^m}{\sqrt{a + bx^n}} + \frac{gx^{3n}(cx)^m}{\sqrt{a + bx^n}} \right) dx \\
 &= d \int \frac{(cx)^m}{\sqrt{a + bx^n}} dx + e \int \frac{x^n(cx)^m}{\sqrt{a + bx^n}} dx + f \int \frac{x^{2n}(cx)^m}{\sqrt{a + bx^n}} dx + g \int \frac{x^{3n}(cx)^m}{\sqrt{a + bx^n}} dx \\
 &= (ex^{-m}(cx)^m) \int \frac{x^{m+n}}{\sqrt{a + bx^n}} dx + (fx^{-m}(cx)^m) \int \frac{x^{m+2n}}{\sqrt{a + bx^n}} dx + (gx^{-m}(cx)^m) \int \frac{x^{m+3n}}{\sqrt{a + bx^n}} dx \\
 &= \frac{d(cx)^{1+m} \sqrt{1 + \frac{bx^n}{a}} {}_2F_1\left(\frac{1}{2}, \frac{1+m}{n}; \frac{1+m+n}{n}; -\frac{bx^n}{a}\right)}{c(1+m)\sqrt{a + bx^n}} + \frac{(ex^{-m}(cx)^m \sqrt{1 + \frac{bx^n}{a}}) \int \frac{x^{m+n}}{\sqrt{1 + \frac{bx^n}{a}}} dx}{\sqrt{a + bx^n}} \\
 &= \frac{d(cx)^{1+m} \sqrt{1 + \frac{bx^n}{a}} {}_2F_1\left(\frac{1}{2}, \frac{1+m}{n}; \frac{1+m+n}{n}; -\frac{bx^n}{a}\right)}{c(1+m)\sqrt{a + bx^n}} + \frac{ex^{1+n}(cx)^m \sqrt{1 + \frac{bx^n}{a}} {}_2F_1\left(\frac{1}{2}, \frac{1+m}{n}; \frac{1+m+n}{n}; -\frac{bx^n}{a}\right)}{(1+m+n)\sqrt{a + bx^n}}
 \end{aligned}$$

Mathematica [A] time = 0.373558, size = 206, normalized size = 0.68

$$\frac{x(cx)^m \sqrt{\frac{bx^n}{a} + 1} \left(\frac{{}_2F_1\left(\frac{1}{2}, \frac{m+1}{n}; \frac{m+n+1}{n}; -\frac{bx^n}{a}\right)}{m+1} + x^n \left(\frac{{}_2F_1\left(\frac{1}{2}, \frac{m+n+1}{n}; \frac{m+2n+1}{n}; -\frac{bx^n}{a}\right)}{m+n+1} + x^n \left(\frac{{}_2F_1\left(\frac{1}{2}, \frac{m+2n+1}{n}; \frac{m+3n+1}{n}; -\frac{bx^n}{a}\right)}{m+2n+1} + \frac{gx^n {}_2F_1\left(\frac{1}{2}, \frac{m+3n+1}{n}; \frac{m+4n+1}{n}; -\frac{bx^n}{a}\right)}{m+3n+1} \right) \right) \right)}{\sqrt{a + bx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/Sqrt[a + b*x^n],x]

[Out] (x*(c*x)^m*Sqrt[1 + (b*x^n)/a]*((d*Hypergeometric2F1[1/2, (1 + m)/n, (1 + m + n)/n, -(b*x^n)/a])/(1 + m) + x^n*((e*Hypergeometric2F1[1/2, (1 + m + n)/n, (1 + m + 2*n)/n, -(b*x^n)/a])/(1 + m + n) + x^n*((f*Hypergeometric2F1[1/2, (1 + m + 2*n)/n, (1 + m + 3*n)/n, -(b*x^n)/a])/(1 + m + 2*n) + (g*x^n*Hypergeometric2F1[1/2, (1 + m + 3*n)/n, (1 + m + 4*n)/n, -(b*x^n)/a])/(1 + m + 3*n))))/Sqrt[a + b*x^n]

Maple [F] time = 0.464, size = 0, normalized size = 0.

$$\int (cx)^m (d + ex^n + fx^{2n} + gx^{3n}) \frac{1}{\sqrt{a + bx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n)^(1/2),x)

[Out] int((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n)^(1/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^{3n} + fx^{2n} + ex^n + d)(cx)^m}{\sqrt{bx^n + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n)^(1/2),x, algorithm="maxima")

[Out] integrate((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(c*x)^m/sqrt(b*x^n + a), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n)^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)**m*(d+e*x**n+f*x**(2*n)+g*x**(3*n))/(a+b*x**n)**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^{3n} + fx^{2n} + ex^n + d)(cx)^m}{\sqrt{bx^n + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n)^(1/2),x, algorithm="giac")

[Out] integrate((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(c*x)^m/sqrt(b*x^n + a), x)

$$3.585 \quad \int \frac{-ahx^{-1+\frac{n}{4}} + bfx^{-1+\frac{n}{2}} + bgx^{-1+n} + bhx^{-1+\frac{5n}{4}}}{(a+bx^n)^{3/2}} dx$$

Optimal. Leaf size=45

$$\frac{2(ag + 2ahx^{n/4} - bfx^{n/2})}{an\sqrt{a + bx^n}}$$

[Out] $(-2*(a*g + 2*a*h*x^{(n/4)} - b*f*x^{(n/2)}))/(a*n*\text{Sqrt}[a + b*x^n])$

Rubi [A] time = 0.394729, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {6741, 1816}

$$\frac{2(ag + 2ahx^{n/4} - bfx^{n/2})}{an\sqrt{a + bx^n}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-a*h*x^{(-1 + n/4)} + b*f*x^{(-1 + n/2)} + b*g*x^{(-1 + n)} + b*h*x^{(-1 + (5*n)/4)})/(a + b*x^n)^{(3/2)}, x]$

[Out] $(-2*(a*g + 2*a*h*x^{(n/4)} - b*f*x^{(n/2)}))/(a*n*\text{Sqrt}[a + b*x^n])$

Rule 6741

$\text{Int}[u_, x_Symbol] \rightarrow \text{With}[\{v = \text{NormalizeIntegrand}[u, x]\}, \text{Int}[v, x] /; v \neq u]$

Rule 1816

$\text{Int}[\frac{(x_)^{(m_.)}*((e_) + (h_.)*(x_)^{(n_.)} + (f_.)*(x_)^{(q_.)} + (g_.)*(x_)^{(r_.)})}{((a_) + (c_.)*(x_)^{(n_.)})^{(3/2)}}, x_Symbol] \rightarrow -\text{Simp}[\frac{2*a*g + 4*a*h*x^{(n/4)} - 2*c*f*x^{(n/2)}}{(a*c*n*\text{Sqrt}[a + c*x^n])}, x] /; \text{FreeQ}\{a, c, e, f, g, h, m, n\}, x] \&\& \text{EqQ}[q, n/4] \&\& \text{EqQ}[r, (3*n)/4] \&\& \text{EqQ}[4*m - n + 4, 0] \&\& \text{EqQ}[c*e + a*h, 0]$

Rubi steps

$$\int \frac{-ahx^{-1+\frac{n}{4}} + bfx^{-1+\frac{n}{2}} + bgx^{-1+n} + bhx^{-1+\frac{5n}{4}}}{(a+bx^n)^{3/2}} dx = \int \frac{x^{-1+\frac{n}{4}}(-ah + bfx^{n/4} + bgx^{3n/4} + bhx^n)}{(a+bx^n)^{3/2}} dx$$

$$= -\frac{2(ag + 2ahx^{n/4} - bfx^{n/2})}{an\sqrt{a+bx^n}}$$

Mathematica [A] time = 0.21546, size = 45, normalized size = 1.

$$\frac{2bfx^{n/2} - 2a(g + 2hx^{n/4})}{an\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[(-a*h*x^(-1 + n/4)) + b*f*x^(-1 + n/2) + b*g*x^(-1 + n) + b*h*x^(-1 + (5*n)/4)]/(a + b*x^n)^(3/2), x]

[Out] (2*b*f*x^(n/2) - 2*a*(g + 2*h*x^(n/4)))/(a*n*Sqrt[a + b*x^n])

Maple [F] time = 0.487, size = 0, normalized size = 0.

$$\int \left(-ahx^{-1+\frac{n}{4}} + bfx^{-1+\frac{n}{2}} + bgx^{-1+n} + bhx^{-1+\frac{5n}{4}} \right) (a+bx^n)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a*h*x^(-1+1/4*n)+b*f*x^(-1+1/2*n)+b*g*x^(-1+n)+b*h*x^(-1+5/4*n))/(a+b*x^n)^(3/2), x)

[Out] int((-a*h*x^(-1+1/4*n)+b*f*x^(-1+1/2*n)+b*g*x^(-1+n)+b*h*x^(-1+5/4*n))/(a+b*x^n)^(3/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b hx^{\frac{5}{4}n-1} + b g x^{n-1} + b f x^{\frac{1}{2}n-1} - a h x^{\frac{1}{4}n-1}}{(b x^n + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a*h*x^(-1+1/4*n)+b*f*x^(-1+1/2*n)+b*g*x^(-1+n)+b*h*x^(-1+5/4*n))/(a+b*x^n)^(3/2),x, algorithm="maxima")

[Out] integrate((b*h*x^(5/4*n - 1) + b*g*x^(n - 1) + b*f*x^(1/2*n - 1) - a*h*x^(1/4*n - 1))/(b*x^n + a)^(3/2), x)

Fricas [A] time = 1.895, size = 153, normalized size = 3.4

$$\frac{2\sqrt{bx^4x^{n-4} + a}\left(bfx^2x^{\frac{1}{2}n-2} - 2ahxx^{\frac{1}{4}n-1} - ag\right)}{abnx^4x^{n-4} + a^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a*h*x^(-1+1/4*n)+b*f*x^(-1+1/2*n)+b*g*x^(-1+n)+b*h*x^(-1+5/4*n))/(a+b*x^n)^(3/2),x, algorithm="fricas")

[Out] 2*sqrt(b*x^4*x^(n - 4) + a)*(b*f*x^2*x^(1/2*n - 2) - 2*a*h*x*x^(1/4*n - 1) - a*g)/(a*b*n*x^4*x^(n - 4) + a^2*n)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a*h*x**(-1+1/4*n)+b*f*x**(-1+1/2*n)+b*g*x**(-1+n)+b*h*x**(-1+5/4*n))/(a+b*x**n)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bhx^{\frac{5}{4}n-1} + bgx^{n-1} + bfx^{\frac{1}{2}n-1} - ahx^{\frac{1}{4}n-1}}{(bx^n + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*h*x^(-1+1/4*n)+b*f*x^(-1+1/2*n)+b*g*x^(-1+n)+b*h*x^(-1+5/4*n)
)/(a+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*h*x^(5/4*n - 1) + b*g*x^(n - 1) + b*f*x^(1/2*n - 1) - a*h*x^(1
/4*n - 1))/(b*x^n + a)^(3/2), x)
```

3.586 $\int (cx)^m (d + ex + fx^2 + gx^3) (a + bx^n)^p dx$

Optimal. Leaf size=273

$$\frac{e(cx)^{m+2} (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+2}{n}, -p; \frac{m+n+2}{n}; -\frac{bx^n}{a}\right)}{c^2(m+2)} + \frac{f(cx)^{m+3} (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+3}{n}, -p; \frac{m+n+3}{n}; -\frac{bx^n}{a}\right)}{c^3(m+3)}$$

[Out] (d*(c*x)^(1+m)*(a+b*x^n)^p*Hypergeometric2F1[(1+m)/n, -p, (1+m+n)/n, -((b*x^n)/a)]/(c*(1+m)*(1+(b*x^n)/a)^p) + (e*(c*x)^(2+m)*(a+b*x^n)^p*Hypergeometric2F1[(2+m)/n, -p, (2+m+n)/n, -((b*x^n)/a)]/(c^2*(2+m)*(1+(b*x^n)/a)^p) + (f*(c*x)^(3+m)*(a+b*x^n)^p*Hypergeometric2F1[(3+m)/n, -p, (3+m+n)/n, -((b*x^n)/a)]/(c^3*(3+m)*(1+(b*x^n)/a)^p) + (g*(c*x)^(4+m)*(a+b*x^n)^p*Hypergeometric2F1[(4+m)/n, -p, (4+m+n)/n, -((b*x^n)/a)]/(c^4*(4+m)*(1+(b*x^n)/a)^p)

Rubi [A] time = 0.18508, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {1844, 365, 364}

$$\frac{e(cx)^{m+2} (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+2}{n}, -p; \frac{m+n+2}{n}; -\frac{bx^n}{a}\right)}{c^2(m+2)} + \frac{f(cx)^{m+3} (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+3}{n}, -p; \frac{m+n+3}{n}; -\frac{bx^n}{a}\right)}{c^3(m+3)}$$

Antiderivative was successfully verified.

[In] Int[(c*x)^m*(d + e*x + f*x^2 + g*x^3)*(a + b*x^n)^p,x]

[Out] (d*(c*x)^(1+m)*(a+b*x^n)^p*Hypergeometric2F1[(1+m)/n, -p, (1+m+n)/n, -((b*x^n)/a)]/(c*(1+m)*(1+(b*x^n)/a)^p) + (e*(c*x)^(2+m)*(a+b*x^n)^p*Hypergeometric2F1[(2+m)/n, -p, (2+m+n)/n, -((b*x^n)/a)]/(c^2*(2+m)*(1+(b*x^n)/a)^p) + (f*(c*x)^(3+m)*(a+b*x^n)^p*Hypergeometric2F1[(3+m)/n, -p, (3+m+n)/n, -((b*x^n)/a)]/(c^3*(3+m)*(1+(b*x^n)/a)^p) + (g*(c*x)^(4+m)*(a+b*x^n)^p*Hypergeometric2F1[(4+m)/n, -p, (4+m+n)/n, -((b*x^n)/a)]/(c^4*(4+m)*(1+(b*x^n)/a)^p)

Rule 1844

Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) && !IGtQ[m, 0]

Rule 365

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
&& !(ILtQ[p, 0] || GtQ[a, 0])

Rule 364

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned} \int (cx)^m (d + ex + fx^2 + gx^3) (a + bx^n)^p dx &= \int \left(d(cx)^m (a + bx^n)^p + \frac{e(cx)^{1+m} (a + bx^n)^p}{c} + \frac{f(cx)^{2+m} (a + bx^n)^p}{c^2} + \frac{g(cx)^{3+m} (a + bx^n)^p}{c^3} \right) dx \\ &= d \int (cx)^m (a + bx^n)^p dx + \frac{e \int (cx)^{1+m} (a + bx^n)^p dx}{c} + \frac{f \int (cx)^{2+m} (a + bx^n)^p dx}{c^2} + \frac{g \int (cx)^{3+m} (a + bx^n)^p dx}{c^3} \\ &= \left(d(a + bx^n)^p \left(1 + \frac{bx^n}{a} \right)^{-p} \right) \int (cx)^m \left(1 + \frac{bx^n}{a} \right)^p dx + \frac{\left(e(a + bx^n)^p \left(1 + \frac{bx^n}{a} \right)^{-p} \right) \int (cx)^{1+m} \left(1 + \frac{bx^n}{a} \right)^p dx}{c} \\ &+ \frac{\left(f(a + bx^n)^p \left(1 + \frac{bx^n}{a} \right)^{-p} \right) \int (cx)^{2+m} \left(1 + \frac{bx^n}{a} \right)^p dx}{c^2} + \frac{\left(g(a + bx^n)^p \left(1 + \frac{bx^n}{a} \right)^{-p} \right) \int (cx)^{3+m} \left(1 + \frac{bx^n}{a} \right)^p dx}{c^3} \\ &= \frac{d(cx)^{1+m} (a + bx^n)^p \left(1 + \frac{bx^n}{a} \right)^{-p} {}_2F_1\left(\frac{1+m}{n}, -p; \frac{1+m+n}{n}; -\frac{bx^n}{a}\right)}{c(1+m)} + \frac{e(cx)^{2+m} (a + bx^n)^p \left(1 + \frac{bx^n}{a} \right)^{-p} {}_2F_1\left(\frac{2+m}{n}, -p; \frac{2+m+n}{n}; -\frac{bx^n}{a}\right)}{c^2} \\ &+ \frac{f(cx)^{3+m} (a + bx^n)^p \left(1 + \frac{bx^n}{a} \right)^{-p} {}_2F_1\left(\frac{3+m}{n}, -p; \frac{3+m+n}{n}; -\frac{bx^n}{a}\right)}{c^3} + \frac{g(cx)^{4+m} (a + bx^n)^p \left(1 + \frac{bx^n}{a} \right)^{-p} {}_2F_1\left(\frac{4+m}{n}, -p; \frac{4+m+n}{n}; -\frac{bx^n}{a}\right)}{c^4} \end{aligned}$$

Mathematica [A] time = 0.24311, size = 178, normalized size = 0.65

$$x(cx)^m (a + bx^n)^p \left(\frac{bx^n}{a} + 1 \right)^{-p} \left(\frac{d {}_2F_1\left(\frac{m+1}{n}, -p; \frac{m+n+1}{n}; -\frac{bx^n}{a}\right)}{m+1} + x \left(\frac{e {}_2F_1\left(\frac{m+2}{n}, -p; \frac{m+n+2}{n}; -\frac{bx^n}{a}\right)}{m+2} + x \left(\frac{f {}_2F_1\left(\frac{m+3}{n}, -p; \frac{m+n+3}{n}; -\frac{bx^n}{a}\right)}{m+3} + x \left(\frac{g {}_2F_1\left(\frac{m+4}{n}, -p; \frac{m+n+4}{n}; -\frac{bx^n}{a}\right)}{m+4} \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c*x)^m*(d + e*x + f*x^2 + g*x^3)*(a + b*x^n)^p,x]

[Out] (x*(c*x)^m*(a + b*x^n)^p*((d*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)/n, -((b*x^n)/a)])/(1 + m) + x*((e*Hypergeometric2F1[(2 + m)/n, -p, (2 + m + n)/n, -((b*x^n)/a)])/(2 + m) + x*((f*Hypergeometric2F1[(3 + m)/n, -p, (3 + m + n)/n, -((b*x^n)/a)])/(3 + m) + (g*x*Hypergeometric2F1[(4 + m)/n, -p, (4 + m + n)/n, -((b*x^n)/a)])/(4 + m))

$+ m + n)/n, -((b*x^n)/a)]/(4 + m))))/(1 + (b*x^n)/a)^p$

Maple [F] time = 0.437, size = 0, normalized size = 0.

$$\int (cx)^m (gx^3 + fx^2 + ex + d) (a + bx^n)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^m*(g*x^3+f*x^2+e*x+d)*(a+b*x^n)^p,x)`

[Out] `int((c*x)^m*(g*x^3+f*x^2+e*x+d)*(a+b*x^n)^p,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (gx^3 + fx^2 + ex + d)(bx^n + a)^p (cx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^m*(g*x^3+f*x^2+e*x+d)*(a+b*x^n)^p,x, algorithm="maxima")`

[Out] `integrate((g*x^3 + f*x^2 + e*x + d)*(b*x^n + a)^p*(c*x)^m, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((gx^3 + fx^2 + ex + d)(bx^n + a)^p (cx)^m, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^m*(g*x^3+f*x^2+e*x+d)*(a+b*x^n)^p,x, algorithm="fricas")`

[Out] `integral((g*x^3 + f*x^2 + e*x + d)*(b*x^n + a)^p*(c*x)^m, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**m*(g*x**3+f*x**2+e*x+d)*(a+b*x**n)**p,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (gx^3 + fx^2 + ex + d)(bx^n + a)^p (cx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^m*(g*x^3+f*x^2+e*x+d)*(a+b*x^n)^p,x, algorithm="giac")`

[Out] `integrate((g*x^3 + f*x^2 + e*x + d)*(b*x^n + a)^p*(c*x)^m, x)`

$$3.587 \quad \int (cx)^m (a + bx^n)^p (d + ex^n + fx^{2n} + gx^{3n}) dx$$

Optimal. Leaf size=297

$$\frac{d(cx)^{m+1} (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+1}{n}, -p; \frac{m+n+1}{n}; -\frac{bx^n}{a}\right)}{c(m+1)} + \frac{ex^{n+1}(cx)^m (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+n+1}{n}, -p; \frac{m+2n+1}{n}; -\frac{bx^n}{a}\right)}{m+n+1}$$

[Out] (d*(c*x)^(1+m)*(a+b*x^n)^p*Hypergeometric2F1[(1+m)/n, -p, (1+m+n)/n, -((b*x^n)/a)]/(c*(1+m)*(1+(b*x^n)/a)^p) + (e*x^(1+n)*(c*x)^m*(a+b*x^n)^p*Hypergeometric2F1[(1+m+n)/n, -p, (1+m+2*n)/n, -((b*x^n)/a)])/((1+m+n)*(1+(b*x^n)/a)^p) + (f*x^(1+2*n)*(c*x)^m*(a+b*x^n)^p*Hypergeometric2F1[(1+m+2*n)/n, -p, (1+m+3*n)/n, -((b*x^n)/a)])/((1+m+2*n)*(1+(b*x^n)/a)^p) + (g*x^(1+3*n)*(c*x)^m*(a+b*x^n)^p*Hypergeometric2F1[(1+m+3*n)/n, -p, (1+m+4*n)/n, -((b*x^n)/a)])/((1+m+3*n)*(1+(b*x^n)/a)^p)

Rubi [A] time = 0.208185, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1844, 365, 364, 20}

$$\frac{d(cx)^{m+1} (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+1}{n}, -p; \frac{m+n+1}{n}; -\frac{bx^n}{a}\right)}{c(m+1)} + \frac{ex^{n+1}(cx)^m (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+n+1}{n}, -p; \frac{m+2n+1}{n}; -\frac{bx^n}{a}\right)}{m+n+1}$$

Antiderivative was successfully verified.

[In] Int[(c*x)^m*(a + b*x^n)^p*(d + e*x^n + f*x^(2*n) + g*x^(3*n)),x]

[Out] (d*(c*x)^(1+m)*(a+b*x^n)^p*Hypergeometric2F1[(1+m)/n, -p, (1+m+n)/n, -((b*x^n)/a)]/(c*(1+m)*(1+(b*x^n)/a)^p) + (e*x^(1+n)*(c*x)^m*(a+b*x^n)^p*Hypergeometric2F1[(1+m+n)/n, -p, (1+m+2*n)/n, -((b*x^n)/a)])/((1+m+n)*(1+(b*x^n)/a)^p) + (f*x^(1+2*n)*(c*x)^m*(a+b*x^n)^p*Hypergeometric2F1[(1+m+2*n)/n, -p, (1+m+3*n)/n, -((b*x^n)/a)])/((1+m+2*n)*(1+(b*x^n)/a)^p) + (g*x^(1+3*n)*(c*x)^m*(a+b*x^n)^p*Hypergeometric2F1[(1+m+3*n)/n, -p, (1+m+4*n)/n, -((b*x^n)/a)])/((1+m+3*n)*(1+(b*x^n)/a)^p)

Rule 1844

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n

, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) && !IGtQ[m, 0]

Rule 365

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])

Rule 364

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rule 20

Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

Rubi steps

$$\begin{aligned}
 \int (cx)^m (a + bx^n)^p (d + ex^n + fx^{2n} + gx^{3n}) dx &= \int (d(cx)^m (a + bx^n)^p + ex^n (cx)^m (a + bx^n)^p + fx^{2n} (cx)^m (a + bx^n)^p + gx^{3n} (cx)^m (a + bx^n)^p) dx \\
 &= d \int (cx)^m (a + bx^n)^p dx + e \int x^n (cx)^m (a + bx^n)^p dx + f \int x^{2n} (cx)^m (a + bx^n)^p dx + g \int x^{3n} (cx)^m (a + bx^n)^p dx \\
 &= (ex^{-m} (cx)^m) \int x^{m+n} (a + bx^n)^p dx + (fx^{-m} (cx)^m) \int x^{m+2n} (a + bx^n)^p dx + (gx^{-m} (cx)^m) \int x^{m+3n} (a + bx^n)^p dx \\
 &= \frac{d(cx)^{1+m} (a + bx^n)^p \left(1 + \frac{bx^n}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{n}, -p; \frac{1+m+n}{n}; -\frac{bx^n}{a}\right)}{c(1+m)} + \left(ex^{-m} (cx)^m\right) \int x^{m+n} (a + bx^n)^p dx \\
 &= \frac{d(cx)^{1+m} (a + bx^n)^p \left(1 + \frac{bx^n}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{n}, -p; \frac{1+m+n}{n}; -\frac{bx^n}{a}\right)}{c(1+m)} + \frac{ex^{1+n} (cx)^m (a + bx^n)^p \left(1 + \frac{bx^n}{a}\right)^{-p} {}_2F_1\left(\frac{1+m+n}{n}, -p; \frac{1+m+n+n}{n}; -\frac{bx^n}{a}\right)}{c(1+m)} + \dots
 \end{aligned}$$

Mathematica [A] time = 0.273287, size = 204, normalized size = 0.69

$$x(cx)^m (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} \left(\frac{d {}_2F_1\left(\frac{m+1}{n}, -p; \frac{m+n+1}{n}; -\frac{bx^n}{a}\right)}{m+1} + x^n \left(\frac{e {}_2F_1\left(\frac{m+n+1}{n}, -p; \frac{m+2n+1}{n}; -\frac{bx^n}{a}\right)}{m+n+1} + x^n \left(\frac{f {}_2F_1\left(\frac{m+2n+1}{n}, -p; \frac{m+2n+1+n}{n}; -\frac{bx^n}{a}\right)}{m} + \dots \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c*x)^m*(a + b*x^n)^p*(d + e*x^n + f*x^(2*n) + g*x^(3*n)),x]

[Out] (x*(c*x)^m*(a + b*x^n)^p*((d*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)/n, -(b*x^n)/a]))/(1 + m) + x^n*((e*Hypergeometric2F1[(1 + m + n)/n, -p, (1 + m + 2*n)/n, -(b*x^n)/a]))/(1 + m + n) + x^n*((f*Hypergeometric2F1[(1 + m + 2*n)/n, -p, (1 + m + 3*n)/n, -(b*x^n)/a]))/(1 + m + 2*n) + (g*x^n*Hypergeometric2F1[(1 + m + 3*n)/n, -p, (1 + m + 4*n)/n, -(b*x^n)/a]))/(1 + m + 3*n))))/(1 + (b*x^n)/a)^p

Maple [F] time = 0.385, size = 0, normalized size = 0.

$$\int (cx)^m (a + bx^n)^p (d + ex^n + fx^{2n} + gx^{3n}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^m*(a+b*x^n)^p*(d+e*x^n+f*x^(2*n)+g*x^(3*n)),x)

[Out] int((c*x)^m*(a+b*x^n)^p*(d+e*x^n+f*x^(2*n)+g*x^(3*n)),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (gx^{3n} + fx^{2n} + ex^n + d)(bx^n + a)^p (cx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^m*(a+b*x^n)^p*(d+e*x^n+f*x^(2*n)+g*x^(3*n)),x, algorithm="maxima")

[Out] integrate((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(b*x^n + a)^p*(c*x)^m, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((gx^{3n} + fx^{2n} + ex^n + d)(bx^n + a)^p (cx)^m, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a+b*x^n)^p*(d+e*x^n+f*x^(2*n)+g*x^(3*n)),x, algorithm="fricas")
```

```
[Out] integral((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(b*x^n + a)^p*(c*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(a+b*x**n)**p*(d+e*x**n+f*x**(2*n)+g*x**(3*n)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a+b*x^n)^p*(d+e*x^n+f*x^(2*n)+g*x^(3*n)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.588 \quad \int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2} dx$$

Optimal. Leaf size=162

$$\frac{x(ae - bc(1 - n)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2bn} - \frac{x^{\frac{n+2}{2}}(bd(2 - n) - af(n + 2)) {}_2F_1\left(1, \frac{1}{2}\left(1 + \frac{2}{n}\right); \frac{1}{2}\left(3 + \frac{2}{n}\right); -\frac{bx^n}{a}\right)}{a^2bn(n + 2)} + \frac{x(x^{n/2}(bd - a))}{abn(a + bx^n)}$$

[Out] (x*(b*c - a*e + (b*d - a*f)*x^(n/2)))/(a*b*n*(a + b*x^n)) - ((b*d*(2 - n) - a*f*(2 + n))*x^((2 + n)/2)*Hypergeometric2F1[1, (1 + 2/n)/2, (3 + 2/n)/2, -((b*x^n)/a)]/(a^2*b*n*(2 + n)) + ((a*e - b*c*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)]/(a^2*b*n))

Rubi [A] time = 0.124185, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1892, 1418, 245, 364}

$$\frac{x(ae - bc(1 - n)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2bn} - \frac{x^{\frac{n+2}{2}}(bd(2 - n) - af(n + 2)) {}_2F_1\left(1, \frac{1}{2}\left(1 + \frac{2}{n}\right); \frac{1}{2}\left(3 + \frac{2}{n}\right); -\frac{bx^n}{a}\right)}{a^2bn(n + 2)} + \frac{x(x^{n/2}(bd - a))}{abn(a + bx^n)}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^(n/2) + e*x^n + f*x^((3*n)/2))/(a + b*x^n)^2,x]

[Out] (x*(b*c - a*e + (b*d - a*f)*x^(n/2)))/(a*b*n*(a + b*x^n)) - ((b*d*(2 - n) - a*f*(2 + n))*x^((2 + n)/2)*Hypergeometric2F1[1, (1 + 2/n)/2, (3 + 2/n)/2, -((b*x^n)/a)]/(a^2*b*n*(2 + n)) + ((a*e - b*c*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)]/(a^2*b*n))

Rule 1892

Int[(P3_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{A = Coeff[P3, x^(n/2), 0], B = Coeff[P3, x^(n/2), 1], C = Coeff[P3, x^(n/2), 2], D = Coeff[P3, x^(n/2), 3]}, -Simp[(x*(b*A - a*C + (b*B - a*D)*x^(n/2))*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[1/(2*a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Simp[2*a*C - 2*b*A*(n*(p + 1) + 1) + (a*D*(n + 2) - b*B*(n*(2*p + 3) + 2))*x^(n/2), x], x] /; FreeQ[{a, b, n}, x] && PolyQ[P3, x^(n/2), 3] && ILtQ[p, -1]

Rule 1418

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] := Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] || !IntegerQ[n])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{c + dx^{n/2} + ex^n + fx^{3n/2}}{(a + bx^n)^2} dx &= \frac{x(bc - ae + (bd - af)x^{n/2})}{abn(a + bx^n)} + \frac{\int \frac{2(ae - bc(1-n) - (bd(2-n) - af(2+n))x^{n/2}}{a + bx^n} dx}{2abn} \\ &= \frac{x(bc - ae + (bd - af)x^{n/2})}{abn(a + bx^n)} + \frac{(ae - bc(1-n)) \int \frac{1}{a + bx^n} dx}{abn} - \frac{(bd(2-n) - af(2+n)) \int \frac{1}{a + bx^n} dx}{2abn} \\ &= \frac{x(bc - ae + (bd - af)x^{n/2})}{abn(a + bx^n)} - \frac{(bd(2-n) - af(2+n))x^{\frac{2+n}{2}} {}_2F_1\left(1, \frac{1}{2}\left(1 + \frac{2}{n}\right); \frac{1}{2}\left(3 + \frac{2}{n}\right)\right)}{a^2bn(2+n)} \end{aligned}$$

Mathematica [A] time = 0.338448, size = 147, normalized size = 0.91

$$\frac{x \left((bc - ae) {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right) + \frac{2x^{n/2}(bd - af) {}_2F_1\left(2, \frac{1}{2} + \frac{1}{n}, \frac{3}{2} + \frac{1}{n}; -\frac{bx^n}{a}\right)}{n+2} + ae {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right) + \frac{2afx^{n/2} {}_2F_1\left(1, \frac{1}{2} + \frac{1}{n}, \frac{3}{2} + \frac{1}{n}; -\frac{bx^n}{a}\right)}{n+2} \right)}{a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^(n/2) + e*x^n + f*x^((3*n)/2))/(a + b*x^n)^2, x]
```

[Out] $(x*((2*a*f*x^{(n/2)}*Hypergeometric2F1[1, 1/2 + n^{(-1)}, 3/2 + n^{(-1)}, -((b*x^n)/a)])/(2 + n) + a*e*Hypergeometric2F1[1, n^{(-1)}, 1 + n^{(-1)}, -((b*x^n)/a)] + (2*(b*d - a*f)*x^{(n/2)}*Hypergeometric2F1[2, 1/2 + n^{(-1)}, 3/2 + n^{(-1)}, -((b*x^n)/a)])/(2 + n) + (b*c - a*e)*Hypergeometric2F1[2, n^{(-1)}, 1 + n^{(-1)}, -((b*x^n)/a)]))/(a^2*b)$

Maple [F] time = 0.432, size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^n)^2} \left(c + dx^{\frac{n}{2}} + ex^n + fx^{\frac{3n}{2}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x)`

[Out] `int((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(bd - af)xx^{\frac{1}{2}n} + (bc - ae)x}{ab^2nx^n + a^2bn} + \int \frac{2bc(n - 1) + 2ae + (af(n + 2) + bd(n - 2))x^{\frac{1}{2}n}}{2(ab^2nx^n + a^2bn)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x, algorithm="maxima")`

[Out] `((b*d - a*f)*x*x^(1/2*n) + (b*c - a*e)*x)/(a*b^2*n*x^n + a^2*b*n) + integrate(1/2*(2*b*c*(n - 1) + 2*a*e + (a*f*(n + 2) + b*d*(n - 2))*x^(1/2*n))/(a*b^2*n*x^n + a^2*b*n), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx^{\frac{3}{2}n} + dx^{\frac{1}{2}n} + ex^n + c}{b^2x^{2n} + 2abx^n + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((f*x^(3/2*n) + d*x^(1/2*n) + e*x^n + c)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**(1/2*n)+e*x**n+f*x**(3/2*n))/(a+b*x**n)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^{\frac{3}{2}n} + dx^{\frac{1}{2}n} + ex^n + c}{(bx^n + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((f*x^(3/2*n) + d*x^(1/2*n) + e*x^n + c)/(b*x^n + a)^2, x)
```

$$3.589 \quad \int \frac{ac+2(bc+ad)x^2+3bdx^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}} dx$$

Optimal. Leaf size=24

$$x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

[Out] x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]

Rubi [A] time = 0.0525376, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.022$, Rules used = {1590}

$$x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

Antiderivative was successfully verified.

[In] Int[(a*c + 2*(b*c + a*d)*x^2 + 3*b*d*x^4)/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x]

[Out] x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] := With[{p = Expon[Pp, x], q =
  Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{ac + 2(bc + ad)x^2 + 3bdx^4}{\sqrt{a + bx^2}\sqrt{c + dx^2}} dx = x\sqrt{a + bx^2}\sqrt{c + dx^2}$$

Mathematica [A] time = 0.148485, size = 24, normalized size = 1.

$$x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a*c + 2*(b*c + a*d)*x^2 + 3*b*d*x^4)/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]

[Out] x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]

Maple [A] time = 0.009, size = 21, normalized size = 0.9

$$x\sqrt{bx^2 + a}\sqrt{dx^2 + c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*c+2*(a*d+b*c)*x^2+3*b*d*x^4)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)

[Out] x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)

Maxima [A] time = 1.10455, size = 27, normalized size = 1.12

$$\sqrt{bx^2 + a}\sqrt{dx^2 + cx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*c+2*(a*d+b*c)*x^2+3*b*d*x^4)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")

[Out] sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x

Fricas [A] time = 1.61799, size = 47, normalized size = 1.96

$$\sqrt{bx^2 + a}\sqrt{dx^2 + cx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*c+2*(a*d+b*c)*x^2+3*b*d*x^4)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")

[Out] $\sqrt{bx^2 + a} \sqrt{dx^2 + c} x$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ac + 2adx^2 + 2bcx^2 + 3bdx^4}{\sqrt{a + bx^2} \sqrt{c + dx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*c+2*(a*d+b*c)*x**2+3*b*d*x**4)/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)`

[Out] `Integral((a*c + 2*a*d*x**2 + 2*b*c*x**2 + 3*b*d*x**4)/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3bdx^4 + 2(bc + ad)x^2 + ac}{\sqrt{bx^2 + a} \sqrt{dx^2 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*c+2*(a*d+b*c)*x^2+3*b*d*x^4)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")`

[Out] `integrate((3*b*d*x^4 + 2*(b*c + a*d)*x^2 + a*c)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)`

$$3.590 \quad \int \frac{1+x^3}{(1-x^4)\sqrt[4]{1+x^4}} dx$$

Optimal. Leaf size=103

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{x^4+1}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{x^4+1}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}$$

[Out] ArcTan[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(2*2^(1/4)) - ArcTan[(1 + x^4)^(1/4)/2^(1/4)]/(2*2^(1/4)) + ArcTanh[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(2*2^(1/4)) + ArcTanh[(1 + x^4)^(1/4)/2^(1/4)]/(2*2^(1/4))

Rubi [A] time = 0.10257, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1899, 377, 212, 206, 203, 444, 63, 298}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{x^4+1}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{x^4+1}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^3)/((1 - x^4)*(1 + x^4)^(1/4)), x]

[Out] ArcTan[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(2*2^(1/4)) - ArcTan[(1 + x^4)^(1/4)/2^(1/4)]/(2*2^(1/4)) + ArcTanh[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(2*2^(1/4)) + ArcTanh[(1 + x^4)^(1/4)/2^(1/4)]/(2*2^(1/4))

Rule 1899

Int[((A_) + (B_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[A, Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x] + Dist[B, Int[x^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, A, B, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 377

Int[((a_) + (b_)*(x_)^(n_))^(p_)/((c_) + (d_)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b),
  2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
  x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
  Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
  [a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
  , 0] || GtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_
  ), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
  ] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
  1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[
  {p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
  (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
  [b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
  ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b)
  ], 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
  ], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
  tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^3}{(1-x^4)\sqrt[4]{1+x^4}} dx &= \int \frac{1}{(1-x^4)\sqrt[4]{1+x^4}} dx + \int \frac{x^3}{(1-x^4)\sqrt[4]{1+x^4}} dx \\
&= \frac{1}{4} \text{Subst} \left(\int \frac{1}{(1-x)\sqrt[4]{1+x}} dx, x, x^4 \right) + \text{Subst} \left(\int \frac{1}{1-2x^4} dx, x, \frac{x}{\sqrt[4]{1+x^4}} \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{1-\sqrt{2}x^2} dx, x, \frac{x}{\sqrt[4]{1+x^4}} \right) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{1+\sqrt{2}x^2} dx, x, \frac{x}{\sqrt[4]{1+x^4}} \right) + \text{Subst} \left(\int \frac{1}{\sqrt{2}-x^2} dx, x, \frac{x}{\sqrt[4]{1+x^4}} \right) \\
&= \frac{\tan^{-1} \left(\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}} \right)}{2\sqrt[4]{2}} + \frac{\tanh^{-1} \left(\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}} \right)}{2\sqrt[4]{2}} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{2}-x^2} dx, x, \sqrt[4]{1+x^4} \right) - \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{2}+x^2} dx, x, \sqrt[4]{1+x^4} \right) \\
&= \frac{\tan^{-1} \left(\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}} \right)}{2\sqrt[4]{2}} - \frac{\tan^{-1} \left(\frac{\sqrt[4]{1+x^4}}{\sqrt[4]{2}} \right)}{2\sqrt[4]{2}} + \frac{\tanh^{-1} \left(\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}} \right)}{2\sqrt[4]{2}} + \frac{\tanh^{-1} \left(\frac{\sqrt[4]{1+x^4}}{\sqrt[4]{2}} \right)}{2\sqrt[4]{2}}
\end{aligned}$$

Mathematica [C] time = 0.168213, size = 93, normalized size = 0.9

$$\frac{1}{4} x^4 F_1 \left(1; \frac{1}{4}, 1, 2; -x^4, x^4 \right) + \frac{-\log \left(1 - \frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}} \right) + \log \left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}} + 1 \right) + 2 \tan^{-1} \left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}} \right)}{4\sqrt[4]{2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 + x^3)/((1 - x^4)*(1 + x^4)^(1/4)), x]

[Out] (x^4*AppellF1[1, 1/4, 1, 2, -x^4, x^4])/4 + (2*ArcTan[(2^(1/4)*x)/(1 + x^4)^(1/4)] - Log[1 - (2^(1/4)*x)/(1 + x^4)^(1/4)] + Log[1 + (2^(1/4)*x)/(1 + x^4)^(1/4)])/(4*2^(1/4))

Maple [F] time = 0.387, size = 0, normalized size = 0.

$$\int \frac{x^3+1}{-x^4+1} \frac{1}{\sqrt[4]{x^4+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+1)/(-x^4+1)/(x^4+1)^(1/4), x)

[Out] $\text{int}((x^3+1)/(-x^4+1)/(x^4+1)^{(1/4)}, x)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^3 + 1}{(x^4 + 1)^{\frac{1}{4}}(x^4 - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((x^3+1)/(-x^4+1)/(x^4+1)^{(1/4)}, x, \text{algorithm}="maxima")$

[Out] $-\text{integrate}((x^3 + 1)/((x^4 + 1)^{(1/4)}*(x^4 - 1)), x)$

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((x^3+1)/(-x^4+1)/(x^4+1)^{(1/4)}, x, \text{algorithm}="fricas")$

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{x}{x^3\sqrt[4]{x^4+1} - x^2\sqrt[4]{x^4+1} + x\sqrt[4]{x^4+1} - \sqrt[4]{x^4+1}} dx - \int \frac{x^2}{x^3\sqrt[4]{x^4+1} - x^2\sqrt[4]{x^4+1} + x\sqrt[4]{x^4+1} - \sqrt[4]{x^4+1}} dx - \int \frac{1}{x^3\sqrt[4]{x^4+1} - x^2\sqrt[4]{x^4+1} + x\sqrt[4]{x^4+1} - \sqrt[4]{x^4+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((x**3+1)/(-x**4+1)/(x**4+1)**(1/4), x)$

[Out] $-\text{Integral}(-x/(x**3*(x**4 + 1)**(1/4) - x**2*(x**4 + 1)**(1/4) + x*(x**4 + 1)**(1/4) - (x**4 + 1)**(1/4)), x) - \text{Integral}(x**2/(x**3*(x**4 + 1)**(1/4) - x**2*(x**4 + 1)**(1/4) + x*(x**4 + 1)**(1/4) - (x**4 + 1)**(1/4)), x) - \text{Integral}(1/(x**3*(x**4 + 1)**(1/4) - x**2*(x**4 + 1)**(1/4) + x*(x**4 + 1)**(1/4) - (x**4 + 1)**(1/4)), x)$

$1/4) - (x^{**4} + 1)**(1/4)), x)$

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^3 + 1}{(x^4 + 1)^{\frac{1}{4}}(x^4 - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+1)/(-x^4+1)/(x^4+1)^(1/4),x, algorithm="giac")`

[Out] `integrate(-(x^3 + 1)/((x^4 + 1)^(1/4)*(x^4 - 1)), x)`

$$3.591 \quad \int (a + bx^n)^{\frac{-1-n}{n}} (c + dx^n)^{\frac{-1-n}{n}} (ac - bdx^{2n}) dx$$

Optimal. Leaf size=28

$$x(a + bx^n)^{-1/n} (c + dx^n)^{-1/n}$$

[Out] x/((a + b*x^n)^n^(-1)*(c + d*x^n)^n^(-1))

Rubi [A] time = 0.101101, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = {1898}

$$x(a + bx^n)^{-1/n} (c + dx^n)^{-1/n}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^n)^((-1 - n)/n)*(c + d*x^n)^((-1 - n)/n)*(a*c - b*d*x^(2*n)), x]

[Out] x/((a + b*x^n)^n^(-1)*(c + d*x^n)^n^(-1))

Rule 1898

Int[((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(p_.)*((e_) + (g_.)*(x_)^(n2_.)), x_Symbol] := Simp[(e*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(p + 1))/(a*c), x] /; FreeQ[{a, b, c, d, e, g, n, p}, x] && EqQ[n2, 2*n] && EqQ[n*(p + 1) + 1, 0] && EqQ[a*c*g - b*d*e*(2*n*(p + 1) + 1), 0]

Rubi steps

$$\int (a + bx^n)^{\frac{-1-n}{n}} (c + dx^n)^{\frac{-1-n}{n}} (ac - bdx^{2n}) dx = x(a + bx^n)^{-1/n} (c + dx^n)^{-1/n}$$

Mathematica [A] time = 0.306663, size = 28, normalized size = 1.

$$x(a + bx^n)^{-1/n} (c + dx^n)^{-1/n}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^n)^((-1 - n)/n)*(c + d*x^n)^((-1 - n)/n)*(a*c - b*d*x^(2*n)),x]

[Out] x/((a + b*x^n)^n^(-1)*(c + d*x^n)^n^(-1))

Maple [F] time = 0.817, size = 0, normalized size = 0.

$$\int (a + bx^n)^{\frac{-1-n}{n}} (c + dx^n)^{\frac{-1-n}{n}} (ac - bdx^{2n}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*x^n)^((-1-n)/n)*(c+d*x^n)^((-1-n)/n)*(a*c-b*d*x^(2*n)),x)

[Out] int((a+b*x^n)^((-1-n)/n)*(c+d*x^n)^((-1-n)/n)*(a*c-b*d*x^(2*n)),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$- \int \frac{bdx^{2n} - ac}{(bx^n + a)^{\frac{n+1}{n}} (dx^n + c)^{\frac{n+1}{n}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^n)^((-1-n)/n)*(c+d*x^n)^((-1-n)/n)*(a*c-b*d*x^(2*n)),x, algorithm="maxima")

[Out] -integrate((b*d*x^(2*n) - a*c)/((b*x^n + a)^((n + 1)/n)*(d*x^n + c)^((n + 1)/n)), x)

Fricas [B] time = 2.00106, size = 128, normalized size = 4.57

$$\frac{bdxx^{2n} + acx + (bc + ad)xx^n}{(bx^n + a)^{\frac{n+1}{n}} (dx^n + c)^{\frac{n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^n)^((-1-n)/n)*(c+d*x^n)^((-1-n)/n)*(a*c-b*d*x^(2*n)),x, algorithm="fricas")

[Out] (b*d*x*x^(2*n) + a*c*x + (b*c + a*d)*x*x^n)/((b*x^n + a)^((n + 1)/n)*(d*x^n + c)^((n + 1)/n))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x**n)**((-1-n)/n)*(c+d*x**n)**((-1-n)/n)*(a*c-b*d*x**(2*n)), x)

[Out] Timed out

Giac [B] time = 1.11932, size = 308, normalized size = 11.

$$bdxx^{2n}e^{\left(-\frac{n\log(bx^n+a)+\log(bx^n+a)}{n}-\frac{n\log(dx^n+c)+\log(dx^n+c)}{n}\right)} + bcxx^n e^{\left(-\frac{n\log(bx^n+a)+\log(bx^n+a)}{n}-\frac{n\log(dx^n+c)+\log(dx^n+c)}{n}\right)} + adxx^n e^{\left(-\frac{n\log(bx^n+a)+\log(bx^n+a)}{n}-\frac{n\log(dx^n+c)+\log(dx^n+c)}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^n)^((-1-n)/n)*(c+d*x^n)^((-1-n)/n)*(a*c-b*d*x^(2*n)),x, algorithm="giac")

[Out] b*d*x*x^(2*n)*e^(-(n*log(b*x^n + a) + log(b*x^n + a))/n - (n*log(d*x^n + c) + log(d*x^n + c))/n) + b*c*x*x^n*e^(-(n*log(b*x^n + a) + log(b*x^n + a))/n - (n*log(d*x^n + c) + log(d*x^n + c))/n) + a*d*x*x^n*e^(-(n*log(b*x^n + a) + log(b*x^n + a))/n - (n*log(d*x^n + c) + log(d*x^n + c))/n) + a*c*x*e^(-(n*log(b*x^n + a) + log(b*x^n + a))/n - (n*log(d*x^n + c) + log(d*x^n + c))/n)

$$3.592 \quad \int (hx)^{-1-n-np} (a + bx^n)^p (c + dx^n)^p (ac - bdx^{2n}) dx$$

Optimal. Leaf size=45

$$\frac{(hx)^{-n(p+1)} (a + bx^n)^{p+1} (c + dx^n)^{p+1}}{hn(p+1)}$$

[Out] -(((a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(h*n*(1 + p)*(h*x)^(n*(1 + p))))

Rubi [A] time = 0.156862, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.022$, Rules used = {1849}

$$\frac{(hx)^{-n(p+1)} (a + bx^n)^{p+1} (c + dx^n)^{p+1}}{hn(p+1)}$$

Antiderivative was successfully verified.

[In] Int[(h*x)^(-1 - n - n*p)*(a + b*x^n)^p*(c + d*x^n)^p*(a*c - b*d*x^(2*n)),x]

[Out] -(((a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(h*n*(1 + p)*(h*x)^(n*(1 + p))))

Rule 1849

Int[((h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(p_.)*((e_) + (g_.)*(x_)^(n2_.)), x_Symbol] :> Simp[(e*(h*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(p + 1))/(a*c*h*(m + 1)), x] /; FreeQ[{a, b, c, d, e, g, h, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[m + n*(p + 1) + 1, 0] && EqQ[a*c*g*(m + 1) - b*d*e*(m + 2*n*(p + 1) + 1), 0] && NeQ[m, -1]

Rubi steps

$$\int (hx)^{-1-n-np} (a + bx^n)^p (c + dx^n)^p (ac - bdx^{2n}) dx = -\frac{(hx)^{-n(1+p)} (a + bx^n)^{1+p} (c + dx^n)^{1+p}}{hn(1+p)}$$

Mathematica [A] time = 0.354939, size = 46, normalized size = 1.02

$$\frac{(hx)^{n(-p)-n} (a + bx^n)^{p+1} (c + dx^n)^{p+1}}{hnp + hn}$$

Antiderivative was successfully verified.

[In] Integrate[(h*x)^(-1 - n - n*p)*(a + b*x^n)^p*(c + d*x^n)^p*(a*c - b*d*x^(2*n)), x]

[Out] -(((h*x)^(-n - n*p)*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(h*n + h*n*p))

Maple [C] time = 0.51, size = 138, normalized size = 3.1

$$\frac{x (bd (x^n)^2 + adx^n + bcx^n + ac) (a + bx^n)^p (c + dx^n)^p}{n(1+p)} e^{-\frac{(np+n+1)(-i\pi (\operatorname{csgn}(ihx))^3 + i\pi (\operatorname{csgn}(ihx))^2 \operatorname{csgn}(ih) + i\pi (\operatorname{csgn}(ihx))^2 \operatorname{csgn}(ix) - i\pi \operatorname{csgn}(ihx) \operatorname{csgn}(ix))}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x)^(-n*p-n-1)*(a+b*x^n)^p*(c+d*x^n)^p*(a*c-b*d*x^(2*n)), x)

[Out] -(a+b*x^n)^p*exp(-1/2*(n*p+n+1)*(-I*Pi*csgn(I*h*x)^3+I*Pi*csgn(I*h*x)^2*csgn(I*h)+I*Pi*csgn(I*h*x)^2*csgn(I*x)-I*Pi*csgn(I*h*x)*csgn(I*h)*csgn(I*x)+2*ln(h)+2*ln(x)))*(b*d*(x^n)^2+a*d*x^n+b*c*x^n+a*c)*x/n/(1+p)*(c+d*x^n)^p

Maxima [A] time = 1.36532, size = 104, normalized size = 2.31

$$\frac{(bdx^{2n} + ac + (bc + ad)x^n)h^{-np-n-1}e^{(-np \log(x) + p \log(bx^n+a) + p \log(dx^n+c) - n \log(x))}}{n(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x)^(-n*p-n-1)*(a+b*x^n)^p*(c+d*x^n)^p*(a*c-b*d*x^(2*n)), x, algorithm="maxima")

[Out] -(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n)*h^(-n*p - n - 1)*e^(-n*p*log(x) + p*log(b*x^n + a) + p*log(d*x^n + c) - n*log(x))/(n*(p + 1))

Fricas [B] time = 2.0071, size = 312, normalized size = 6.93

$$\frac{\left(bdx x^{2n} e^{-(np+n+1)\log(h)-(np+n+1)\log(x)} + acx e^{-(np+n+1)\log(h)-(np+n+1)\log(x)} + (bc + ad)xx^n e^{-(np+n+1)\log(h)-(np+n+1)\log(x)}\right)}{np + n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x)^(-n*p-n-1)*(a+b*x^n)^p*(c+d*x^n)^p*(a*c-b*d*x^(2*n)),x, algorithm="fricas")

[Out] $-(b*d*x*x^{(2*n)}*e^{-(n*p + n + 1)*\log(h) - (n*p + n + 1)*\log(x)} + a*c*x*e^{-(n*p + n + 1)*\log(h) - (n*p + n + 1)*\log(x)} + (b*c + a*d)*x*x^n*e^{-(n*p + n + 1)*\log(h) - (n*p + n + 1)*\log(x)})*(b*x^n + a)^p*(d*x^n + c)^p/(n*p + n)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x)**(-n*p-n-1)*(a+b*x**n)**p*(c+d*x**n)**p*(a*c-b*d*x**(2*n)),x)

[Out] Timed out

Giac [B] time = 1.1456, size = 320, normalized size = 7.11

$$\frac{(bx^n + a)^p(dx^n + c)^p bdx x^{2n} e^{(-np \log(h) - np \log(x) - n \log(h) - n \log(x) - \log(h) - \log(x))} + (bx^n + a)^p(dx^n + c)^p bcx x^n e^{(-np \log(h) - np \log(x) - n \log(h) - n \log(x) - \log(h) - \log(x))}}{(bx^n + a)^p(dx^n + c)^p bdx x^{2n} e^{(-np \log(h) - np \log(x) - n \log(h) - n \log(x) - \log(h) - \log(x))} + (bx^n + a)^p(dx^n + c)^p bcx x^n e^{(-np \log(h) - np \log(x) - n \log(h) - n \log(x) - \log(h) - \log(x))}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x)^(-n*p-n-1)*(a+b*x^n)^p*(c+d*x^n)^p*(a*c-b*d*x^(2*n)),x, algorithm="giac")

```
[Out] -((b*x^n + a)^p*(d*x^n + c)^p*b*d*x*x^(2*n)*e^(-n*p*log(h) - n*p*log(x) - n
*log(h) - n*log(x) - log(h) - log(x)) + (b*x^n + a)^p*(d*x^n + c)^p*b*c*x*x
^n*e^(-n*p*log(h) - n*p*log(x) - n*log(h) - n*log(x) - log(h) - log(x)) + (
b*x^n + a)^p*(d*x^n + c)^p*a*d*x*x^n*e^(-n*p*log(h) - n*p*log(x) - n*log(h)
- n*log(x) - log(h) - log(x)) + (b*x^n + a)^p*(d*x^n + c)^p*a*c*x*e^(-n*p*
log(h) - n*p*log(x) - n*log(h) - n*log(x) - log(h) - log(x)))/(n*p + n)
```

$$3.593 \quad \int (a + bx^n)^p (c + dx^n)^p \left(e + \frac{(bc+ad)e(1+n+np)x^n}{ac} + \frac{bde(1+2n+2np)}{ac} \right) dx$$

Optimal. Leaf size=31

$$\frac{ex(a + bx^n)^{p+1}(c + dx^n)^{p+1}}{ac}$$

[Out] (e*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c)

Rubi [A] time = 0.205318, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 69, $\frac{\text{number of rules}}{\text{integrand size}} = 0.014$, Rules used = {1897}

$$\frac{ex(a + bx^n)^{p+1}(c + dx^n)^{p+1}}{ac}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c + a*d)*e*(1 + n + n*p)*x^n)/(a*c) + (b*d*e*(1 + 2*n + 2*n*p)*x^(2*n))/(a*c)), x]

[Out] (e*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c)

Rule 1897

Int[((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(p_.)*((e_) + (f_.)*(x_)^(n_.) + (g_.)*(x_)^(n2_.)), x_Symbol] :> Simp[(e*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(p + 1))/(a*c), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[n2, 2*n] && EqQ[a*c*f - e*(b*c + a*d)*(n*(p + 1) + 1), 0] && EqQ[a*c*g - b*d*e*(2*n*(p + 1) + 1), 0]

Rubi steps

$$\int (a + bx^n)^p (c + dx^n)^p \left(e + \frac{(bc + ad)e(1 + n + np)x^n}{ac} + \frac{bde(1 + 2n + 2np)x^{2n}}{ac} \right) dx = \frac{ex(a + bx^n)^{1+p}(c + dx^n)^{1+p}}{ac}$$

Mathematica [A] time = 0.523959, size = 31, normalized size = 1.

$$\frac{ex(a + bx^n)^{p+1}(c + dx^n)^{p+1}}{ac}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c + a*d)*e*(1 + n + n*p)*x^n)/(a*c) + (b*d*e*(1 + 2*n + 2*n*p)*x^(2*n))/(a*c)),x]

[Out] (e*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c)

Maple [A] time = 0.153, size = 52, normalized size = 1.7

$$\frac{xe\left(bd(x^n)^2 + adx^n + bcx^n + ac\right)(a + bx^n)^p(c + dx^n)^p}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+n+1)*x^n/a/c+b*d*e*(2*n*p+2*n+1)*x^(2*n)/a/c),x)

[Out] (a+b*x^n)^p*(b*d*(x^n)^2+a*d*x^n+b*c*x^n+a*c)*e*x/a/c*(c+d*x^n)^p

Maxima [A] time = 1.25654, size = 80, normalized size = 2.58

$$\frac{(bdexx^{2n} + acex + (bce + ade)xx^n)e^{(p\log(bx^n+a)+p\log(dx^n+c))}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+n+1)*x^n/a/c+b*d*e*(2*n*p+2*n+1)*x^(2*n)/a/c),x, algorithm="maxima")

[Out] (b*d*e*x*x^(2*n) + a*c*e*x + (b*c*e + a*d*e)*x*x^n)*e^(p*log(b*x^n + a) + p*log(d*x^n + c))/(a*c)

Fricas [A] time = 1.79826, size = 115, normalized size = 3.71

$$\frac{(bdexx^{2n} + acex + (bc + ad)exx^n)(bx^n + a)^p(dx^n + c)^p}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c))*e*(n*p+n+1)*x^n/a/c+b*d*e*(2
*n*p+2*n+1)*x^(2*n)/a/c),x, algorithm="fricas")
```

```
[Out] (b*d*e*x*x^(2*n) + a*c*e*x + (b*c + a*d)*e*x*x^n)*(b*x^n + a)^p*(d*x^n + c)
^p/(a*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p*(c+d*x**n)**p*(e+(a*d+b*c))*e*(n*p+n+1)*x**n/a/c+b*d
*e*(2*n*p+2*n+1)*x**(2*n)/a/c),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17098, size = 155, normalized size = 5.

$$\frac{(bx^n + a)^p(dx^n + c)^p b dx x^{2n} e + (bx^n + a)^p(dx^n + c)^p bc x x^n e + (bx^n + a)^p(dx^n + c)^p ad x x^n e + (bx^n + a)^p(dx^n + c)^p ac x e}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c))*e*(n*p+n+1)*x^n/a/c+b*d*e*(2
*n*p+2*n+1)*x^(2*n)/a/c),x, algorithm="giac")
```

```
[Out] ((b*x^n + a)^p*(d*x^n + c)^p*b*d*x*x^(2*n)*e + (b*x^n + a)^p*(d*x^n + c)^p*
b*c*x*x^n*e + (b*x^n + a)^p*(d*x^n + c)^p*a*d*x*x^n*e + (b*x^n + a)^p*(d*x^n
+ c)^p*a*c*x*e)/(a*c)
```

$$3.594 \quad \int (hx)^m (a + bx^n)^p (c + dx^n)^p \left(e + \frac{(bc+ad)e(1+m+n+np)x^n}{ac(1+m)} + \frac{bde(1+m+2n+2np)x^{2n}}{ac(1+m)} \right) dx = \frac{e(hx)^{1+m} (a + bx^n)^{p+1} (c + dx^n)^{p+1}}{ach(1+m)}$$

Optimal. Leaf size=45

$$\frac{e(hx)^{m+1} (a + bx^n)^{p+1} (c + dx^n)^{p+1}}{ach(m+1)}$$

[Out] (e*(h*x)^(1+m)*(a+b*x^n)^(1+p)*(c+d*x^n)^(1+p))/(a*c*h*(1+m))

Rubi [A] time = 0.553113, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 86, $\frac{\text{number of rules}}{\text{integrand size}} = 0.012$, Rules used = {1848}

$$\frac{e(hx)^{m+1} (a + bx^n)^{p+1} (c + dx^n)^{p+1}}{ach(m+1)}$$

Antiderivative was successfully verified.

[In] Int[(h*x)^m*(a+b*x^n)^p*(c+d*x^n)^p*(e+((b*c+a*d)*e*(1+m+n+n*p)*x^n)/(a*c*(1+m))+ (b*d*e*(1+m+2*n+2*n*p)*x^(2*n))/(a*c*(1+m))),x]

[Out] (e*(h*x)^(1+m)*(a+b*x^n)^(1+p)*(c+d*x^n)^(1+p))/(a*c*h*(1+m))

Rule 1848

Int[((h_.)*(x_))^(m_.)*((a_.)+(b_.)*(x_)^(n_.))^(p_.)*((c_.)+(d_.)*(x_)^(n_.))^(p_.)*((e_.)+(f_.)*(x_)^(n_.)+(g_.)*(x_)^(n2_.)), x_Symbol] :> Simp[(e*(h*x)^(m+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(p+1))/(a*c*h*(m+1)), x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[a*c*f*(m+1)-e*(b*c+a*d)*(m+n*(p+1)+1), 0] && EqQ[a*c*g*(m+1)-b*d*e*(m+2*n*(p+1)+1), 0] && NeQ[m, -1]

Rubi steps

$$\int (hx)^m (a + bx^n)^p (c + dx^n)^p \left(e + \frac{(bc+ad)e(1+m+n+np)x^n}{ac(1+m)} + \frac{bde(1+m+2n+2np)x^{2n}}{ac(1+m)} \right) dx = \frac{e(hx)^{1+m} (a + bx^n)^{p+1} (c + dx^n)^{p+1}}{ach(1+m)}$$

Mathematica [A] time = 0.767357, size = 41, normalized size = 0.91

$$\frac{ex(hx)^m (a + bx^n)^{p+1} (c + dx^n)^{p+1}}{ac(m + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(h*x)^m*(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c + a*d)*e*(1 + m + n + n*p)*x^n)/(a*c*(1 + m)) + (b*d*e*(1 + m + 2*n + 2*n*p)*x^(2*n))/(a*c*(1 + m))),x]

[Out] (e*x*(h*x)^m*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c*(1 + m))

Maple [C] time = 0.429, size = 136, normalized size = 3.

$$\frac{xe \left(bd(x^n)^2 + adx^n + bcx^n + ac \right) (a + bx^n)^p (c + dx^n)^p e^{-\frac{m \left(i \operatorname{csgn}(ihx)^3 \pi - i \pi \operatorname{csgn}(ihx)^2 \operatorname{csgn}(ih) - i \pi \operatorname{csgn}(ihx)^2 \operatorname{csgn}(ix) + i \operatorname{csgn}(ihx) \operatorname{csgn}(ih) \operatorname{csgn}(ix) \right)}{2}}}{ac(1 + m)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x)^m*(a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+m+n+1)*x^n/a/c/(1+m)+b*d*e*(2*n*p+m+2*n+1)*x^(2*n)/a/c/(1+m)),x)

[Out] (a+b*x^n)^p*exp(-1/2*m*(I*csgn(I*h*x)^3*Pi-I*Pi*csgn(I*h*x)^2*csgn(I*h)-I*Pi*csgn(I*h*x)^2*csgn(I*x)+I*csgn(I*h*x)*csgn(I*h)*csgn(I*x)*Pi-2*ln(h)-2*ln(x)))*(b*d*(x^n)^2+a*d*x^n+b*c*x^n+a*c)*e*x/a/c/(1+m)*(c+d*x^n)^p

Maxima [B] time = 1.36721, size = 124, normalized size = 2.76

$$\frac{\left(aceh^m x x^m + bdeh^m x e^{(m \log(x) + 2n \log(x))} + (bceh^m + adeh^m) x e^{(m \log(x) + n \log(x))} \right) e^{(p \log(bx^n + a) + p \log(dx^n + c))}}{ac(m + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x)^m*(a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+m+n+1)*x^n/a/c/(1+m)+b*d*e*(2*n*p+m+2*n+1)*x^(2*n)/a/c/(1+m)),x, algorithm="maxima")

[Out] (a*c*e*h^m*x*x^m + b*d*e*h^m*x*e^(m*log(x) + 2*n*log(x)) + (b*c*e*h^m + a*d*e*h^m)*x*e^(m*log(x) + n*log(x)))*e^(p*log(b*x^n + a) + p*log(d*x^n + c))/

$(a*c*(m + 1))$

Fricas [A] time = 1.7337, size = 223, normalized size = 4.96

$$\frac{(bdexx^{2n}e^{(m \log(h)+m \log(x))} + acexe^{(m \log(h)+m \log(x))} + (bc + ad)exx^n e^{(m \log(h)+m \log(x))})(bx^n + a)^p(dx^n + c)^p}{acm + ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x)^m*(a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e^(n*p+m+n+1)*x^n/a/c/(1+m)+b*d*e*(2*n*p+m+2*n+1)*x^(2*n)/a/c/(1+m)),x, algorithm="fricas")

[Out] (b*d*e*x*x^(2*n)*e^(m*log(h) + m*log(x)) + a*c*e*x*e^(m*log(h) + m*log(x)) + (b*c + a*d)*e*x*x^n*e^(m*log(h) + m*log(x)))*(b*x^n + a)^p*(d*x^n + c)^p/(a*c*m + a*c)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x)**m*(a+b*x**n)**p*(c+d*x**n)**p*(e+(a*d+b*c)*e^(n*p+m+n+1)*x**n/a/c/(1+m)+b*d*e*(2*n*p+m+2*n+1)*x**(2*n)/a/c/(1+m)),x)

[Out] Timed out

Giac [B] time = 1.18857, size = 209, normalized size = 4.64

$$\frac{(bx^n + a)^p(dx^n + c)^p bdx^{2n}e^{(m \log(h)+m \log(x)+1)} + (bx^n + a)^p(dx^n + c)^p bcx^n e^{(m \log(h)+m \log(x)+1)} + (bx^n + a)^p(dx^n + c)^p adxx}{acm + ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x)^m*(a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e^(n*p+m+n+1)*x^n/a/c/(1+m)+b*d*e*(2*n*p+m+2*n+1)*x^(2*n)/a/c/(1+m)),x, algorithm="giac")

```
[Out] ((b*x^n + a)^p*(d*x^n + c)^p*b*d*x*x^(2*n)*e^(m*log(h) + m*log(x) + 1) + (b
*x^n + a)^p*(d*x^n + c)^p*b*c*x*x^n*e^(m*log(h) + m*log(x) + 1) + (b*x^n +
a)^p*(d*x^n + c)^p*a*d*x*x^n*e^(m*log(h) + m*log(x) + 1) + (b*x^n + a)^p*(d
*x^n + c)^p*a*c*x*e^(m*log(h) + m*log(x) + 1))/(a*c*m + a*c)
```


Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*      is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*      antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```

22     If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25     "C"],
26 If[FreeQ[result,Integrate] && FreeQ[result,Int],
27     "C",
28     "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
38 (*5 = hyperpergeometric function*)
39 (*6 = appell function*)
40 (*7 = rootsum function*)
41 (*8 = integrate function*)
42 (*9 = unknown function*)
43
44
45 ExpnType[expn_] :=
46     If[AtomQ[expn],
47         1,
48     If[ListQ[expn],
49         Max[Map[ExpnType,expn]],
50     If[Head[expn]===Power,
51         If[IntegerQ[expn[[2]]],
52             ExpnType[expn[[1]]],
53         If[Head[expn[[2]]]===Rational,
54             If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
55                 1,
56                 Max[ExpnType[expn[[1]],2]],
57             Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
58     If[Head[expn]===Plus || Head[expn]===Times,
59         Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
60     If[ElementaryFunctionQ[Head[expn]],
61         Max[3,ExpnType[expn[[1]]],
62     If[SpecialFunctionQ[Head[expn]],
63         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
64     If[HypergeometricFunctionQ[Head[expn]],
65         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
66     If[AppellFunctionQ[Head[expn]],
67         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
68     If[Head[expn]===RootSum,

```



```

69   Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
70   If[Head[expn]===Integrate || Head[expn]===Int,
71     Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
72   9]]]]]]]]]]
73
74
75 ElementaryFunctionQ[func_] :=
76   MemberQ[{
77     Exp, Log,
78     Sin, Cos, Tan, Cot, Sec, Csc,
79     ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
80     Sinh, Cosh, Tanh, Coth, Sech, Csch,
81     ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
82   }, func]
83
84
85 SpecialFunctionQ[func_] :=
86   MemberQ[{
87     Erf, Erfc, Erfi,
88     FresnelS, FresnelC,
89     ExpIntegralE, ExpIntegralEi, LogIntegral,
90     SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
91     Gamma, LogGamma, PolyGamma,
92     Zeta, PolyLog, ProductLog,
93     EllipticF, EllipticE, EllipticPi
94   }, func]
95
96
97 HypergeometricFunctionQ[func_] :=
98   MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]
99
100
101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1}, func]

```

4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 #
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 #
11 see problem 156, file Apostol_Problems

```

```

11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;
14
15     leaf_count_result:=leafcount(result);
16     #do NOT call ExpnType() if leaf size is too large. Recursion problem
17     if leaf_count_result > 500000 then
18         return "B";
19     fi;
20
21     leaf_count_optimal:=leafcount(optimal);
22
23     ExpnType_result:=ExpnType(result);
24     ExpnType_optimal:=ExpnType(optimal);
25
26     if debug then
27         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
28     fi;
29
30 # If result and optimal are mathematical expressions,
31 # GradeAntiderivative[result,optimal] returns
32 #   "F" if the result fails to integrate an expression that
33 #     is integrable
34 #   "C" if result involves higher level functions than necessary
35 #   "B" if result is more than twice the size of the optimal
36 #     antiderivative
37 #   "A" if result can be considered optimal
38
39 #This check below actually is not needed, since I only
40 #call this grading only for passed integrals. i.e. I check
41 #for "F" before calling this. But no harm of keeping it here.
42 #just in case.
43
44
45 if not type(result,freeof('int')) then
46     return "F";
47 end if;
48
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then

```

```

56     if debug then
57         print("both result and optimal complex");
58     fi;
59     #both result and optimal complex
60     if leaf_count_result<=2*leaf_count_optimal then
61         return "A";
62     else
63         return "B";
64     end if
65     else #result contains complex but optimal is not
66         if debug then
67             print("result contains complex but optimal is not");
68         fi;
69         return "C";
70     end if
71     else # result do not contain complex
72         # this assumes optimal do not as well
73         if debug then
74             print("result do not contain complex, this assumes optimal do not
as well");
75         fi;
76         if leaf_count_result<=2*leaf_count_optimal then
77             if debug then
78                 print("leaf_count_result<=2*leaf_count_optimal");
79             fi;
80             return "A";
81         else
82             if debug then
83                 print("leaf_count_result>2*leaf_count_optimal");
84             fi;
85             return "B";
86         end if
87     end if
88     else #ExpnType(result) > ExpnType(optimal)
89         if debug then
90             print("ExpnType(result) > ExpnType(optimal)");
91         fi;
92         return "C";
93     end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417

```

```

102 is_contains_complex:= proc(expression)
103   return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)
119   if type(expn,'atomic') then
120     1
121   elif type(expn,'list') then
122     apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124     if type(op(1,expn),'rational') then
125       1
126     else
127       max(2,ExpnType(op(1,expn)))
128     end if
129   elif type(expn,'^^') then
130     if type(op(2,expn),'integer') then
131       ExpnType(op(1,expn))
132     elif type(op(2,expn),'rational') then
133       if type(op(1,expn),'rational') then
134         1
135       else
136         max(2,ExpnType(op(1,expn)))
137       end if
138     else
139       max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140     end if
141   elif type(expn,'^+^') or type(expn,'`*`') then
142     max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144     max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146     max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148     max(5,apply(max,map(ExpnType,[op(expn)])))

```

```

149   elif AppellFunctionQ(op(0,expn)) then
150       max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152       max(8,apply(max,map(ExpnType,[op(expn)]))) else
153       9
154   end if
155 end proc:
156
157
158 ElementaryFunctionQ := proc(func)
159     member(func,[
160         exp,log,ln,
161         sin,cos,tan,cot,sec,csc,
162         arcsin,arccos,arctan,arccot,arcsec,arccsc,
163         sinh,cosh,tanh,coth,sech,csch,
164         arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165 end proc:
166
167 SpecialFunctionQ := proc(func)
168     member(func,[
169         erf,erfc,erfi,
170         FresnelS,FresnelC,
171         Ei,Ei,Li,Si,Ci,Shi,Chi,
172         GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173         EllipticF,EllipticE,EllipticPi])
174 end proc:
175
176 HypergeometricFunctionQ := proc(func)
177     member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178 end proc:
179
180 AppellFunctionQ := proc(func)
181     member(func,[AppellF1])
182 end proc:
183
184 # u is a sum or product. rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple

```

```

196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #           Port of original Maple grading function by
3 #           Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #           added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                    asinh,acosh,atanh,acoth,asech,acsch
25                    ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                    gamma,loggamma,digamma,zeta,polylog,LambertW,
31                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                    ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:

```

```

42     if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43         return True
44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,'^^')
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
72 ))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+' or type
77 (expn,'*')
78         m1 = expnType(expn.args[0])
79         m2 = expnType(list(expn.args[1:]))
80         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82         return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84         m1 = max(map(expnType, list(expn.args)))
85         return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88         m1 = max(map(expnType, list(expn.args)))

```

```

85     return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
86 elif is_appell_function(expn.func):
87     m1 = max(map(expnType, list(expn.args)))
88     return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
89 elif isinstance(expn,RootSum):
90     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
91     return max(7,m1)
92 elif str(expn).find("Integral") != -1:
93     m1 = max(map(expnType, list(expn.args)))
94     return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
95 else:
96     return 9
97
98 #main function
99 def grade_antiderivative(result,optimal):
100
101     leaf_count_result  = leaf_count(result)
102     leaf_count_optimal = leaf_count(optimal)
103
104     expnType_result  = expnType(result)
105     expnType_optimal = expnType(optimal)
106
107     if str(result).find("Integral") != -1:
108         return "F"
109
110     if expnType_result <= expnType_optimal:
111         if result.has(I):
112             if optimal.has(I): #both result and optimal complex
113                 if leaf_count_result <= 2*leaf_count_optimal:
114                     return "A"
115                 else:
116                     return "B"
117             else: #result contains complex but optimal is not
118                 return "C"
119         else: # result do not contain complex, this assumes optimal do not as
well
120             if leaf_count_result <= 2*leaf_count_optimal:
121                 return "A"
122             else:
123                 return "B"
124     else:
125         return "C"

```

4.0.4 SageMath grading function

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by


```

2 #           Albert Rich to use with Sagemath. This is used to
3 #           grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #           'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
33         flatten(tree(anti))))
34         return round(1.35*len(flatten(tree(anti)))) #fudge factor
35         #since this estimate of leaf count is bit lower than
36         #what it should be compared to Mathematica's
37
38 def is_sqrt(expr):
39     debug=False;
40     if expr.operator() == operator.pow: #isinstance(expr,Pow):
41         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
42             if debug: print ("expr is sqrt")
43             return True
44         else:
45             return False
46     else:
47         return False

```

```

48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func ," is special_function")
83         else:
84             print ("func ", func ," is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
91     ']
92
93 def is_appell_function(func):

```

```

93     return func.name() in ['hypergeometric']    #[appellf1] can't find this in
          sagemath
94
95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
104             return expn in expn.parent().base_ring() or expn in expn.parent().
          gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list:    #isinstance(expn,list):
121         return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
          Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0]))    #max(2,expnType(expn.
          args[0]))
127     elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer:    #isinstance(expn.args[1],Integer)
129             return expnType(expn.operands()[0])    #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational:    #isinstance(expn.args[1],
          Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
          Rational)
132                 return 1

```

```

133         else:
134             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137         elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138             m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139             m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140             return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
141         elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
142             return max(3,expnType(expn.operands()[0]))
143         elif is_special_function(expn.operator()): #is_special_function(expn.func)
144             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
145             return max(4,m1) #max(4,m1)
146         elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
148             return max(5,m1) #max(5,m1)
149         elif is_appell_function(expn.operator()):
150             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
151             return max(6,m1) #max(6,m1)
152         elif str(expn).find("Integral") != -1: #this will never happen, since it
153             #is checked before calling the grading function that is passed.
154             #but kept it here.
155             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
156             return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
157         else:
158             return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

```

```
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
176                     expnType_optimal)
177
178     if expnType_result <= expnType_optimal:
179         if result.has(I):
180             if optimal.has(I): #both result and optimal complex
181                 if leaf_count_result <= 2*leaf_count_optimal:
182                     return "A"
183                 else:
184                     return "B"
185             else: #result contains complex but optimal is not
186                 return "C"
187         else: # result do not contain complex, this assumes optimal do not as
188             well
189                 if leaf_count_result <= 2*leaf_count_optimal:
190                     return "A"
191                 else:
192                     return "B"
193         else:
194             return "C"
```